











and calculated intensity patterns obtained at different distances  $d$  from the lens in the left panel of Fig. 3. The theoretical intensity patterns were evaluated by inserting in the CiBs of (3) the above values of  $q_0(d)$  and  $\xi(d)$ . Four of the eight measured patterns and the corresponding similarities  $S$  are shown (the lowest of the eight measured similarity is 0.885): they demonstrate that CiB correctly models the propagation of such beam. We notice that the chosen position of the lens is such that no real imaging of the  $q$ -plate takes place in zone C.

We finally observed the beam in the “zone D”, that is after a second lens leading to the formation of a real image of the  $q$ -plate vortex source. In this case we used  $f_1 = 300 \text{ mm}$  and  $f_2 = 200 \text{ mm}$ , with the lens  $f_1$  placed at distance  $d_1 = f_1$  from the  $q$ -plate and the lens  $f_2$  placed at distance  $f_1 + f_2$  from the first lens. We now indicate by  $d$  the distance after the lens  $f_2$ . The ABCD matrix in this case is given by  $M = \begin{pmatrix} -f_2/f_1 & f_1 - d f_1/f_2 \\ 0 & -f_1/f_2 \end{pmatrix}$  corresponding to  $q_0(d) = d - f_1 + i z_0 f_2^2 / f_1^2$  and  $\xi_D = 1$ . In Fig. 3 (right) we show three of the seven measured patterns and the corresponding similarities  $S$  (now the lowest measured similarity was 0.928). It is worth noticing the almost complete closure of the vortex: at the plane  $d = 200 \text{ mm}$  the vortex at the center almost completely disappears, even if the OAM content is still non-vanishing. The disappearance is actually not complete: a more accurate analysis (to be reported elsewhere) shows that the vortex radius reduces to a minimum value that depends on the numerical aperture of the optical imaging system, according to the standard resolution limits imposed by wave theory. However, this minimum vortex size can easily be orders of magnitude smaller than the beam size in the same plane. We further notice that an approximate theory based on LG vortex beams does not describe properly this vortex imaging phenomenon.

#### 4. Conclusions

We have studied the generation and the propagation of an optical vortex created by superimposing an azimuthal phase pattern imprinted by a  $q$ -plate on a Gaussian beam. The application of such a phase mask is the principle on which all current approaches to generate and measure OAM eigenstates (spiral phase plates, fork holograms,  $q$ -plates) are based. By using an optical system with two lenses, we have experimentally verified for the first time the recently introduced ABCD law for Circular beams [17]. Our results demonstrate that the CiBs are very useful to analytically model the propagation through a generic optical system of OAM beams. We stress that many well known beams carrying OAM are included in the CiB family with particular values of the beam parameters. The accuracy of the  $q$ -plate in the generation of the singular phase profile  $\exp(i\ell\phi)$  to the beam was essential to generate a highly stigmatic beam that perfectly matches the theoretical predictions.

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