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Thermal load-induced notch stress intensity factors derived from averaged strain energy density

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Abstract

Under the hypothesis of steady-state heat transfer and plane-strain conditions, the intensity of the stress distributions ahead of sharp V-notch tips can be expressed in terms of thermal notch stress intensity factors (thermal NSIFs) which can be used for fatigue strength assessments of notched components. The calculation of thermal NSIFs requires both an uncoupled thermal-mechanical numerical analysis and a very refined mesh. For these reasons, the numerical simulation becomes considerably expensive and time-consuming above all if large 2D or 3D models have to be solved. Refined meshes are not necessary when the aim of the finite element analysis is to determine the mean value of the local strain energy density on a control volume surrounding the points of stress singularity. On the other hand, the NSIFs value can be directly determined by the strain energy density. In this work, the method for rapid calculations of NSIFs based on averaged strain energy density, recently published in literature, is extended to thermal problems.

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Keywords: Thermal Stress; Strain Energy Density; Thermal Notch Stress Intensity Factors; Finite Element Method; Thermal Fatigue

1. Introduction

In presence of components with geometric discontinuities like sharp V-notches, the stress distribution near such regions is singular. Under linear-elastic hypothesis and plain strain or plain stress conditions, the solution of this problem was first obtained by Williams (1952) and later extended to the elastic-plastic regime by Hutchinson (1968), Rice and Rosengreen (1968) (HRR). It was found that the singularity grade of the stress asymptotic distribution

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depends only on the V-notch opening angle, while its intensity is given by the notch stress intensity factor (NSIF). The definition of this last parameter allowed to formulate a local approach useful to predict the static and fatigue resistance of V-notched components. For example, when mode II contribution is not singular, the mode I NSIF was used to summarize the high-cycle fatigue strength of welded joints having very different geometries (Lazzarin and Tovo (1998), Atzori et al. (1999), Lazzarin and Livieri (2001), Atzori and Meneghetti (2001)). In those works, the local parameter K_I was correlated only with the geometry and the external loads without including the influence of residual stresses.

In order to take into account the effect of thermal loads on fatigue strength of notched components, the local approach based on NSIFs was extended to thermal problems. First works about this topic date back to 1979 and extend until 1998 (Stern (1979), Babuska and Miller (1984), Szabó and Yosibash (1996), Yosibash (1997), Yosibash (1998)). Many years later, this problem was analytically solved by P. Ferro at al. (2006) by using the 'stress function approach'. The thermal load-induced NSIFs were found to be natural extension of the conventional force-induced NSIFs. In that work, the authors introduced also the concept of Residual-NSIF which is the parameter used to quantify the intensity of residual singular stress field arising near the weld toe and induced by the solidification of the fusion zone (FZ). The residual stress field near the weld toe was then extensively studied in further works that highlighted how the intensity and the sign of the induced residual stress depend on boundaries conditions and phase transformations (Ferro and Petrone (2009), Ferro (2012)). Finally, a local model was developed to predict the fatigue strength of welded joints which takes into account the effects of the residual stress (Ferro (2014)). Unfortunately, the local approach based on NSIFs suffers from two drawbacks. Fatigue strength of notched components with different V-opening angles can not be compared each other because of the different singularities degree which appears in the NSIF units; furthermore, a very fine mesh is required near the notch tip to capture the real NSIF value. This last drawback is particularly heavy in 3D numerical modelling and even more in the 3D simulation of welding process since a non-linear-transient thermo-metallurgical and mechanical analysis is required. In order to overcome these problems, a local energy based approach was developed by Lazzarin and Zambardi (2001). In this formulation it is hypothesized that the static and fatigue behaviour of components with sharp V-shaped notches depends on the strain energy density (SED) averaged over a circular sector (structural volume) of radius R_C near the notch tip. The local strain energy density is directly linked to the relevant NSIF for modes I and II while R_c is treated as a material property. This criterion was verified over a great number of experimental tests collected in different papers by Lazzarin and co-workers (Livieri and Lazzarin (2005), Berto and Lazzarin (2009)). The great advantage of the SED approach is that the units of the strain energy density do not change with the opening angle and its value is related to the nodal displacements. This last characteristic makes the strain energy density almost mesh independent. In other words, a coarse mesh is sufficient to capture the SED value averaged over the control volume, which makes this approach easy to use also when 3D and large components have to be modelled (Lazzarin et al. (2010)). Another interesting consequence of the above mentioned SED characteristic is that the NSIF value can be indirectly derived from the numerically calculated value of the strain energy density averaged over a control volume of radius R by means of a coarse mesh (Lazzarin et al. (2010)). Starting from previous results, the possibility to derive the thermal load-induced NSIFs through the SED approach and a coarse mesh is in this work demonstrated.

2. Preliminary considerations

In a previous work, Lazzarin et al. (2010) observed how the total elastic stain energy (E_t) stored in a finite element depends only on the nodal displacements and finite element stiffness matrix according to the following relation:

$$\mathbf{E_t = \int_V WdV = \frac{1}{2}\{d\}^t[K]\{d\}} \tag{1}$$

with V and [K] being the volume and the stiffness matrix of the finite element, respectively, while {d} being the vector of nodal displacements. Thus, the elastic strain energy determination does not require any strain and stress calculation; it can be derived directly from nodal displacements. On the other hand, the calculation of stresses involves the displacements derivative. For this reason, the mesh refinement necessary to capture the stress values must be much higher than that required for the accurate determination of the strain energy. Now, in a thermal-mechanical problem, it can be observed that the displacements are related to temperature through the thermal expansion coefficient, α. It is thus evident that the elastic strain energy induced by thermal loads depends only on nodal temperatures and shape functions of the finite element. No derivation or integration process are involved in the calculation. In such conditions, the degree of mesh refinement required for the determination of the strain energy induced by thermal loads is expected to be low as in the previous case.

3. Steady-state thermal problem

Under steady-state conditions, the temperature distribution near a sharp V-notch (Fig. 1) of an isotropic and homogeneous material is given by the following equation (Ferro et al. (2006)):

$$\mathbf{T}(\mathbf{r},\boldsymbol{\theta}) = \sum_{1}^{\infty} \mathbf{C_i r^{s_i} f_i}(\boldsymbol{\theta}) \tag{2}$$

where T is the temperature, r and θ are the polar coordinates in the cylindrical reference system of Fig. 1, C_i are the generalized flux intensity factors (GFIFs) and s_i and f_i are the eigenvalues and the eigenfunctions of the problem, respectively. These last parameters depend only on the V-notch angle and boundary conditions on the free edges of the notches (Γ_1 and Γ_2 in Fig. 1).
It was found (Ferro et al. (2006)) that Eq. (2) can be simplified as follows:

$$\mathbf{T}(\mathbf{r},\boldsymbol{\theta}) = \mathbf{C_s r^{s_s} f_s}(\boldsymbol{\theta}) + \mathbf{C_a r^{s_a} f_a}(\boldsymbol{\theta}) \tag{3}$$

where subscripts s and a stand for symmetric and anti-symmetric component, respectively. Furthermore, the following relations hold true:

$$\mathbf{s_s = 2 s_a} \tag{4}$$

$$\mathbf{f_s}(\boldsymbol{\theta}) = \mathbf{cos}(\boldsymbol{\theta}\mathbf{s_s}), \quad \mathbf{f_a}(\boldsymbol{\theta}) = \mathbf{sin}(\boldsymbol{\theta}\mathbf{s_a}), \tag{5}$$

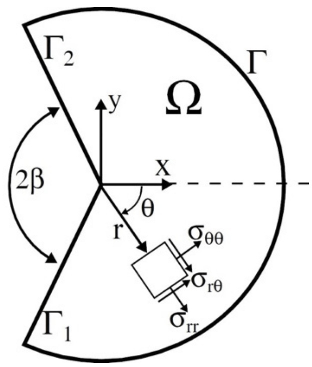


Fig. 1. Domain Ω for the sharp V-notch problem

The thermal load was applied in terms of symmetric (Eq. 6) and anti-symmetric (Eq. 7) heat fluxes on the inner line (Γ) shown in Fig. 1. The V-notch surfaces (Γ_1 and Γ_2) were considered adiabatic while a reference value for the temperature was imposed in the notch apex (T(0,0) = 0).

$$\begin{cases} q(r = r_{max}, \theta) = -\lambda \left(\frac{\partial T}{\partial r}\right)_{r=r_{max}} = X\left(1 - \frac{2}{112.5}\theta\right) \quad for \quad 0° < \theta < 112.5° \\ q(r = r_{max}, \theta) = -\lambda \left(\frac{\partial T}{\partial r}\right)_{r=r_{max}} = X\left(1 + \frac{2}{112.5}\theta\right) \quad for \quad -112.5° < \theta < 0° \end{cases} \tag{6}$$

$$q(r = r_{max}, \theta) = -\lambda \left(\frac{\partial T}{\partial r}\right)_{r=r_{max}} = X\left(\frac{\theta}{112.5}\right) \quad for \quad -112.5° < \theta < 112.5° \tag{7}$$

The parameters of the thermal problem are summarized in Table 1. The material properties refer to AISI 1008 steel.

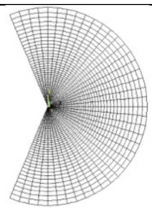
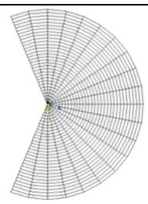
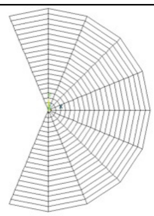
Table 1. Parameters of the analysed problem

2β (°)	r_{max} (mm)	X (W/m^2)	λ (thermal conductivity) (W/(mK))	ρ (mass density) (kg/m^3)	c (specific heat) (J/(kg°C))	α linear expansion coefficient) (m/(m°C))	E (Young modulus) (GPa)
135	10	5	65.2	7872	481	13.1×10^{-6}	200

The thermal problem was solved by using three different mesh densities in order to evaluate the mesh sensitivity to the temperature distribution (Tab. 2). In particular, the analyses were carried out with ANSYS numerical code by using the finite element PLANE 77.

Fig. 2 shows the temperature distribution along the notch bisector induced by the symmetric heat flux and the temperature distribution along one adiabatic notch surface induced by the anti-symmetric heat flux. Values obtained with the highest mesh density are plotted with continuous lines, while those obtained with the lowest mesh density are marked with circles. Even if the results of such thermal problem were just published in a previous work (Ferro et al. (2006)), it is shown here that they are almost insensitive to the mesh density like nodal displacement values derived from mechanical analysis.

Table 2. Mesh densities used for the thermal numerical solution

Number FE	r_{max} = 10 mm
4158	
748	
206	

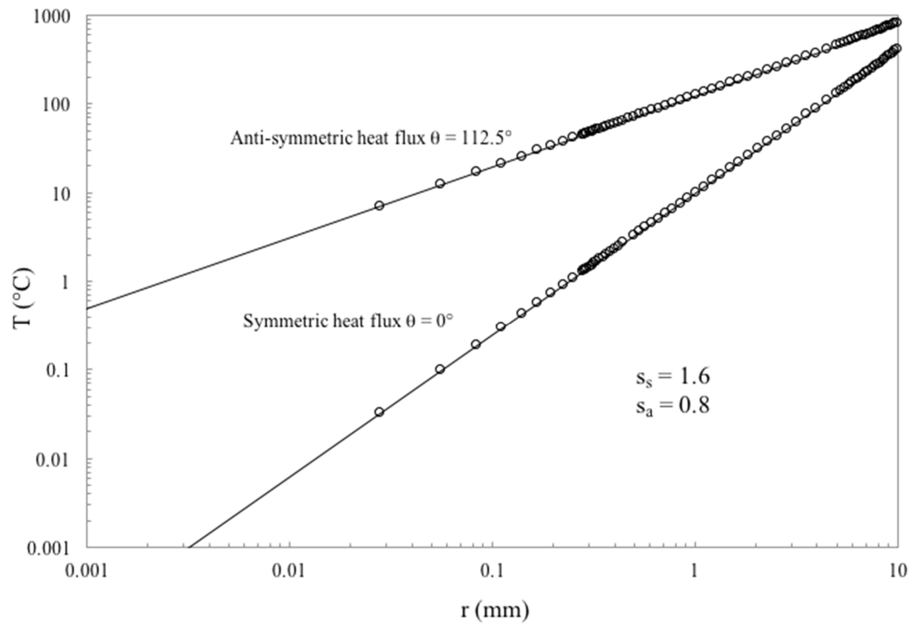


Fig. 2. Comparison between thermal solutions obtained with the highest (continuous lines) and lowest (circles) mesh density

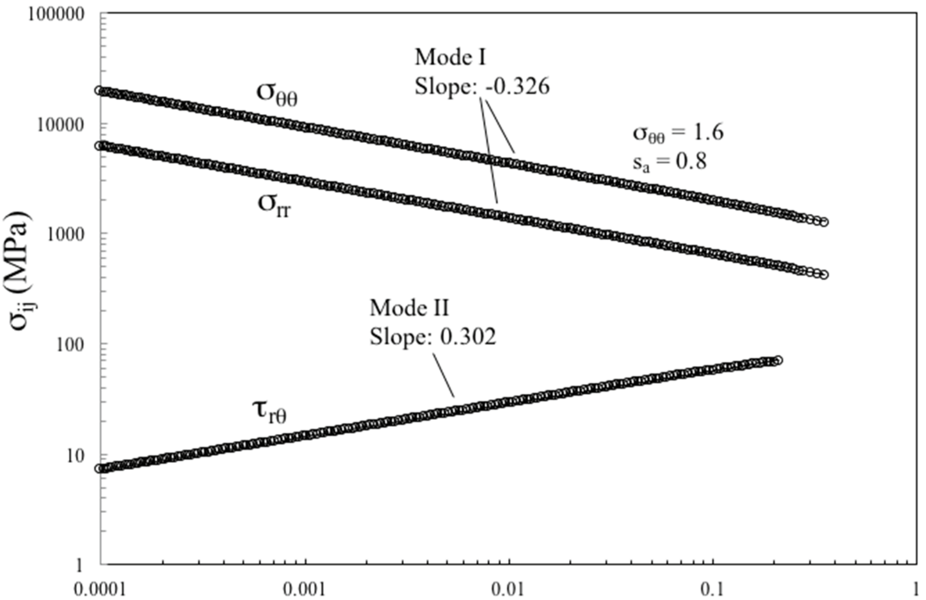


Fig. 3. Plots of the stress components along the notch bisector (θ = 0)

4. Strain energy density induced by thermal loads

Under linear thermo-elastic conditions, the stress distribution induced by a thermal load near the V-notch was shown to follow the William's solution (Ferro et al. (2006)):

$$\boldsymbol{\sigma_{ij}(r,\theta) = \frac{K_1^{th}}{r^{1-\lambda_1}} g_{ij}^{(1)}(\theta) + \frac{K_2^{th}}{r^{1-\lambda_2}} g_{ij}^{(2)}(\theta) \quad (i,j = r,\theta)} \tag{8}$$

where λ_1 and λ_2 are the first eigenvalues for mode I and II, respectively, while K^{th}_1 and K^{th}_2 are the thermo-elastic stress intensity factors (TSIFs) which can be determined according to the definition proposed by Gross and Mendelson (1972):

$$\boldsymbol{K_1^{th} = \sqrt{2\pi} \lim_{r \to 0^+} r^{1-\lambda_1} \sigma_{\theta\theta}(r, \theta = 0)} \tag{9}$$

$$\boldsymbol{K_2^{th} = \sqrt{2\pi} \lim_{r \to 0^+} r^{1-\lambda_2} \tau_{r\theta}(r, \theta = 0)} \tag{10}$$

The relevant eigenfunctions g_{ij} have closed form expressions (Lazzarin and Tovo (1998)). Now, under plane stress or plane strain conditions, the strain energy density averaged over a control volume of radius R is given by:

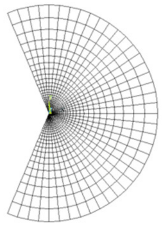
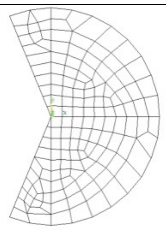
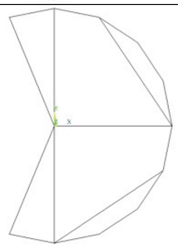
$$\boldsymbol{\bar{W} = \frac{e_1}{E}\left[\frac{K_1^{th}}{R^{1-\lambda_1}}\right]^2 + \frac{e_2}{E}\left[\frac{K_2^{th}}{R^{1-\lambda_2}}\right]^2} \tag{11}$$

where E is the Young's modulus, e_1 and e_2 are parameters which vary under plane stress or plane strain conditions and depend also on the notch opening angle 2β and the Poisson's ratio ν (Lazzarin and Zambardi (2001)). R is a material parameter which in this work was set equal to 0.28 mm according to the value obtained for steel welded joints (Berto and Lazzarin (2009)). By rearranging Eq. (11) and by considering separately pure mode I and pure mode II loading, the TSIFs can by derived through the strain energy density as follows:

$$\boldsymbol{K_1^{th} = R^{1-\lambda_1}\sqrt{\frac{E \times \bar{W}}{e_1}}\ ;\ K_2^{th} = R^{1-\lambda_2}\sqrt{\frac{E \times \bar{W}}{e_2}}} \tag{12}$$

The mechanical analyses were carried out by using the same mashes of the previous thermal simulations (Tab 2) and by switching the thermal to mechanical element (PLANE 183). The temperature distribution obtained in the previous step was used as load for the stress and strain calculation. Material properties are summarized in Table 1. Node displacements belonging to the Γ surface were set equal to 0. In particular, Table 3 shows the mesh densities used to model the circular sector of radius R = 0.28 mm and the most relevant results obtained. Fig. 3 shows the thermal stress distribution along the notch bisector obtained with the finest mesh. It is noted the very good correlation between the numerical and theoretical solution by Williams (1952). In this case, by using Eqs. (9) and (10), the theoretical values of K^{th}_1 and K^{th}_2 were found equal to 2388.99 MPa $mm^{0.326}$ and 289.48 MPa $mm^{-0.302}$, respectively. It is evident from Table 3 that the strain energy density averaged over a control volume of radius R is almost mesh insensitive. This phenomenon was expected in force of the direct correlation between nodal temperatures and displacements and it is now demonstrated. It is interesting to note that the K^{th}_1 and K^{th}_2 values obtained with the coarsest mesh and Eq. (12) differ from the theoretical values of 1.71% and 3.49%, respectively.

Table 3. Mesh densities used for the thermo-mechanical problem numerical solution

Number FE	R = 0.28 mm	$\overline{W_1}$ (MJ/m^3)	K_1^{th} (MPa mm$^{0.326}$)	$\bar{W}_2$ (MJ/m^3)	K_2^{th} (MPa mm$^{-0.302}$)
2158		7.459692	2341	0.030105	301
148		7.466512	2342	0.030053	301
6		7.500357	2348	0.029744	299

5. Conclusions

The possibility to calculate *a posteriori* the thermal notch intensity factor through the strain energy density averaged over a control volume is demonstrated. The advantage of this approach is due to the possibility to use a very coarse mesh. As a matter of fact, it was shown that the strain energy value is almost mesh insensitive since it can be determined via the nodal displacements, without involving their derivatives. In thermo-mechanical analysis nodal temperatures are correlated to displacements. Thus, the strain energy density can be directly derived from nodal temperatures. This result is important in view of the possibility to calculate the thermal notch intensity factor values of 3D large structures without the necessity of high density meshes.

References

Atzori, B., Meneghetti, G., 2001. Fatigue strength of fillet welded structural steels: finite elements, strain gauges and reality. Int. J. Fatigue 23, 713–721.

Atzori, B., Lazzarin, P., Tovo, R., 1999. From the local stress approach to fracture mechanics: a comprehensive evaluation of the fatigue strength of welded joints. Fatigue Fract. Engng. Mater. Struct. 22, 369–381.

Babuska, I., Miller, A., 1984. The post-processing approach in the finite element method – Part 2: the calculation of stress intensity factors. Int. J. Numer. Methods Eng. 20, 1111–1129.

Berto, F., Lazzarin, P., 2009. A review of the volume-based strain energy density approach applied to V-notches and welded structures. Theoretical and Applied Fracture Mechanics 52(3) 184-194

Ferro P., Berto F., Lazzarin P., 2006. Generalized stress intensity factors due to steady and transient thermal loads with applications to welded joints. Fatigue Fract Engng Mater Struct 29, 440–453.

Ferro, P, Petrone N, 2009. Asymptotic thermal and residual stress distribution due to transient thermal loads. Fatigue Fract. Eng. Mater. Struct., 32, 936–948

Ferro, P, 2012. Influence of phase transformations on the asymptotic residual stress distribution arising near a sharp V- notch tip. Modell. Simul.

Mater. Sci. Eng., 20, DOI: 10.1088/ 0965-0393/20/8/085003
Ferro, P, 2014. The local strain energy density approach applied to pre-stressed components subjected to cyclic load. Fatigue Fract. Eng. Mater. Struct., 37, 1268–1280
Gross, R., Mendelson, A., 1972. Plane elastoplastic analysis of V-notched plates Int. J. Fract. Mech, 8, 267-76
Hutchinson, J. W., 1968. Singular behaviour at the end of a tensile crack in a hardening material. J. Mech. Phys. Solids 16, 13–31.
Lazzarin, P., Livieri, P., 2001. Notch Stress Intensity Factors and fatigue strength of aluminium and steel welded joints. Int. J. Fatigue 23, 225–232.
Lazzarin, P., Tovo, R., 1998. A notch intensity approach to the stress analysis of welds. Fatigue Fract. Engng. Mater. Struct. 21, 1089–1104.
Lazzarin, P., Zambardi R, 2001. A finite-volume-energy based approach to predict the static and fatigue behavior of components with sharp V-shaped notches. Int. J. Fract., 112, 275–298
Lazzarin, P., Berto F., Zappalorto M., 2010. Rapid calculations of notch stress intensity factors based on averaged strain energy density from coarse meshes: Theoretical bases and applications. Int. J. Fatigue 32, 1559–1567.
Livieri, P, Lazzarin P, 2005. Fatigue strength of steel and aluminium welded joints based on generalised stress intensity factors and local strain energy values. Int. J. Fract., 133, 247–76
Rice, J. R., Rosengren, G. F.,1968. Plane strain deformation near a crack tip in a power-law hardening material. J. Mech. Phys. Solids 16, 1–12.
Stern, M., 1979. The numerical study calculation of thermally induced stress intensity factors. J. Elasticity 9, 91–95
Szabo´, B.A., Yosibash, Z., 1996. Numerical analysis of singularities in two-dimensions. Part 2: computation of the generalized flux/stress intensity factors. Int. J. Numer. Methods Eng. 39, 409–434.
Williams, M. L., 1952. Stress singularities resulting from various boundary conditions in angular corners of plates in extension. J. Appl. Mech. 19, 526–528.
Yosibash, Z., 1997. On solutions of two-dimensional linear elastostatic and heat-transfer problems in the vicinity of singular points. Int. J. Solids Struct. 34, 243–274.
Yosibash, Z., 1998. Thermal generalized stress intensity factors in 2D domains. Comput. Methods Appl. Mech. Engng. 157, 365–385.