A Rapid Estimation of the Rotor Losses in High Speed Synchronous PM Machines

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***Abstract*—The rotor losses evaluation is a fundamental task in the electromagnetic design of synchronous machines, especially when the reached speeds are so high to exacerbate eddy-current phenomenon. Commonly Time-Stepping Finite-Element Analyses are applied for predicting synchronous permanent magnets machine performance, in terms of iron losses due to eddy-currents. This procedure offers benefits and drawbacks regarding precision and computing time. Moreover in a Time-Stepping simulation there is the need to reach a steady state condition to obtain meaningful results. In this paper the iron losses occurring in iron rotor and permanent magnets are calculated by means of both Time-Harmonic and Time-Stepping Finite Element Analysis. Such analyses are carried out on various models. It is shown that the results obtained by a Time-Harmonic analysis are comparable to those extracted by a Time-Stepping one, with particular care of the model. Furthermore, the proposed alternative takes shorter simulation times.**

I. Introduction

The evaluation of rotor losses is a fundamental task in the electromagnetic design of synchronous machines, especially when the reached speeds are so high to exacerbate eddy-current phenomenon.

Eddy-current rotor losses are caused by air-gap flux variation due to (a) non sinusoidal current waveform [1], [2], (b) the Magneto Motive Force (MMF) space harmonics generated by the discrete position of coils within slots [3] and (c) slot opening [4]. In addition to a decrease of efficiency they result in an increase of the rotor temperature. This leads to a reduction of both magnet residual flux density and coercive force risking irreversible demagnetization. As reported in [5], in this specific case, the main source of eddy-current rotor losses are Time Harmonics (TH) due to non-sinusoidal current behavior.

In the past years, particular attention has been paid on rotor eddy-current losses in permanent magnet (PM) machines with fractional-slot windings [6]–[8]. In fact, such a stator structure gives large air-gap field Space Harmonics (SH), which rotate asynchronously with respect to the rotor. They produce flux pulsations and eddy-current losses in rotor parts, resulting in possible overheating and demagnetization issues. The computation of rotor eddy-current losses becomes even more important when stator currents have not a sinusoidal behavior (i.e. affected by Time Harmonic (TH) contents) [9]; this causes a worsening in air-gap field harmonic contents due to the presence of such SH.

Analytical methods have been developed to solve the problem in a faster way [10]. However, the causes involved in rotor eddy-current generation are difficult to take into account of in such a way. Thus, these methods may lead to an imprecise prediction, that is, over or under-estimation of rotor losses. An accurate Time-Stepping Finite Element Analysis (TS-FEA) is preferable for loss computation. Stator slotting and current distortion effects are taken into account. However, TS-FEA is affected by some disadvantages. It requires long computational time. Moreover, the power loss evaluation needs to reach a steady state condition. The full transient phenomenon is analysed. For the aforementioned reasons, TS-FEA method is not eligible to be the cleverest way to compute iron losses.

This paper proposes an alternative for rotor eddy-current loss evaluation: it is a set of Time-Harmonic Finite-Element Analyses (TH-FEA), one for each TH component. In every TH-FEA simulation, no mechanical rotation is included. The unknown state variables and the derived physical quantities (magnetic field strength and magnetic flux density) are assumed to be harmonic (sinusoidal) time dependent. The complex representation is used, and the solution can be obtained in a single solving process. TS-FEAs are used as a term of comparison to verify the accuracy of TH-FEA method.

To describe the proposed method a surface permanent magnet (SPM) machine is considered. Its main data are reported in Table I. SPM machine is connected to the grid by a three-phase full bridge rectifier; thus, it carries distorted stator currents. Eddy-current losses resulting from TH-FEAs are compared to those obtained by TS-FEA. TH-FEA approach leads to a significant decrease of computational times (about 1:200) while deriving reasonable results and offers a smarter way for rotor iron losses computation.

II. Magneto motive force space harmonic contents

Losses are induced in the rotor by SH that move asynchronously with respect to the rotor. It can be shown that, for an integral-slot windings, the space MMF distribution can be split in its SH components whose order ν is equal to $(6k+1)$, $k \in \mathbb{Z}$. Referring to a sine-wave current three phase system, the MMF SH distribution is expressed as a function of time t and stator electrical angle coordinate θ_s^e as:

$$f^s(t, \theta_s^e) = \sum_{\nu=1}^{\infty} F_\nu \sin(\omega t - \nu\theta_s^e) \qquad (1)$$

where F_ν is the amplitude of the ν-th MMF SH and ω is the fundamental electrical speed.

TABLE I
MAIN DATA OF THE SPM MACHINE UNDER ANALYSIS.

MOTOR OUTPUT			
Rated power	P_N	550	kW
Frequency	f	300	Hz
Mechanical speed	n	9000	rpm
GEOMETRICAL DATA			
Outer diameter	D_e	375	mm
Inner diameter	D_i	240	mm
Stack length	L_{stk}	470	mm
Number of poles	$2p$	4	-
Number of slots	Q	48	-
Slot height	h_s	25	mm
Tooth width	w_t	5	mm
Air-gap	g	4	mm
WINDING			
Conductors in slot	n_c	2	-
Number of parallel paths	n_{pp}	4	-
Number of conductors per phase	N_s	8	-
Coil pitch	y_q	10	-
MATERIALS			
Iron Lamination (stator)		N020	
Solid Steel (rotor)		C45	
Permanent Magnet		N38UH	

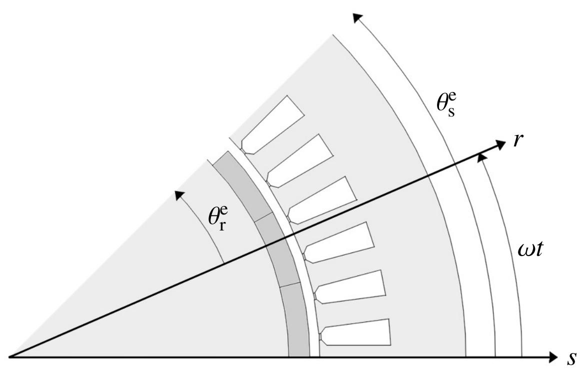

Fig. 1. Stator and rotor reference frame transformation.

In presence of non-sinusoidal stator currents, characterized by the TH order n, the MMF SH distribution becomes:

$$f^s(t,\theta_s^e) = \sum_{n=1}^{\infty}\left(\sum_{\nu=1}^{\infty} F_{n\nu}\sin\left(n\omega t - \nu\theta_s^e\right)\right) \tag{2}$$

where n is the time harmonic order. It can be expressed as $(6h+1)$, $h \in \mathbb{Z}$.

By acting a change of reference frame, from stator to rotor as shown in Fig. 1, θ_s^e is re-written as $\theta_r^e + \omega t$, where θ_r^e is the rotor electrical angle coordinate. (2) becomes:

$$f^r(t,\theta_r^e) = \sum_{n=1}^{\infty}\left(\sum_{\nu=1}^{\infty} F_{n\nu}\sin\left[n\omega t - \nu\left(\theta_r^e + \omega t\right)\right]\right) \tag{3}$$

In a rotor reference an equivalent MMF SH distribution is expressed as:

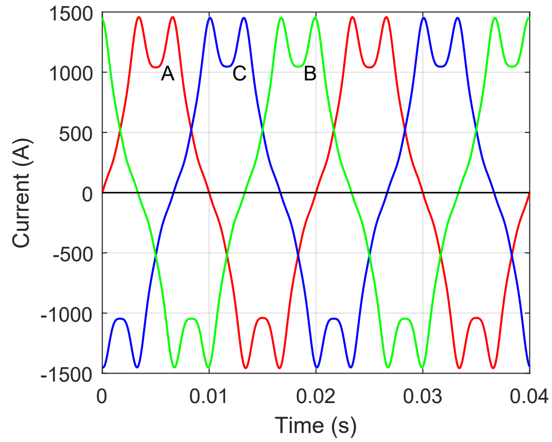

Fig. 2. Test Current.

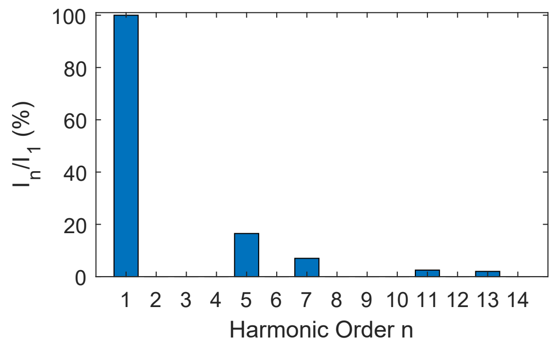

Fig. 3. Test current time harmonic content.

$$f^r(t,\theta_r^e) = \sum_{n=1}^{\infty}\left(\sum_{\nu=1}^{\infty} F_{n\nu}\sin\left[(n-\nu)\,\omega t - \nu\theta_r^e\right]\right) \tag{4}$$

Let's consider the machine operating as a generator connected to the rectifier. The TH of the Test Current (TC), which is shown in Fig. 2, are almost forced by the rectifier itself. The TH amplitudes are given in Fig. 3. the test Current TH electrical speeds are calculated by applying (4). Results are reported in Table II. It shows how MMF SH distribution can be reproduced by simply keeping the rotor part as fixed and changing properly the TH electrical speeds.

III. ROTOR LOSSES COMPUTATION

This section describes the analysis of the SPM generator whose main data are reported in Table I. An accurate bi-dimensional FE model has been carried out. TS-FEA and TH-FEA allow to evaluate the rotor iron and magnet losses due to both TH and SH.

Due to the geometry symmetry, anti-periodic boundaries are imposed to reduce the FEM model to one pole pitch, as shown in Fig. 5. All materials are assumed to be isotropic. Stator core is made of iron sheet N020 and rotor core of solid steel iron C45. Materials non-linearity are taken into account by the characteristics shown in Fig. 4. NdFeB magnets are used,

TABLE II
MMF ELECTRICAL SPEEDS IN THE ROTOR REFERENCE FRAME

n/ν	1	-5	7	-11	13
1	0	$-\frac{6\omega}{5}$	$-\frac{6\omega}{7}$	$-\frac{12\omega}{11}$	$-\frac{12\omega}{13}$
-5	6ω	0	$\frac{12\omega}{7}$	$-\frac{6\omega}{11}$	$\frac{18\omega}{13}$
7	6ω	$-\frac{12\omega}{5}$	0	$-\frac{18\omega}{11}$	$-\frac{6\omega}{13}$
-11	12ω	$\frac{6\omega}{5}$	$\frac{6\omega}{7}$	0	$-\frac{24\omega}{13}$
13	12ω	$-\frac{18\omega}{5}$	$\frac{6\omega}{7}$	$-\frac{24\omega}{11}$	0

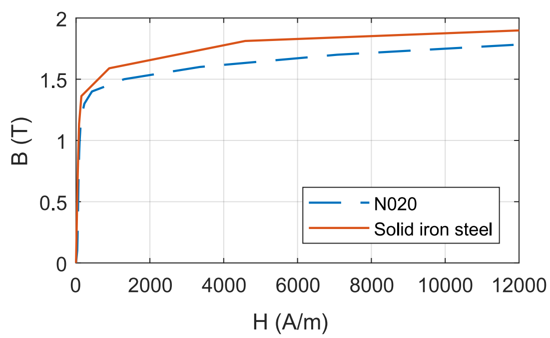

Fig. 4. The $B-H$ curves of used magnetic materials.

characterized by residual flux density $B_r = 1.09\,\text{T}$ at machine working temperature of 120 °C.

The eddy-currents in magnet and iron rotor parts are included in this model by assuming both magnets and rotor as a solid conductors. They are characterized by a conductivity σ equal to:

- C45: $\sigma = 10.44\,\frac{\text{MS}}{\text{m}}$;

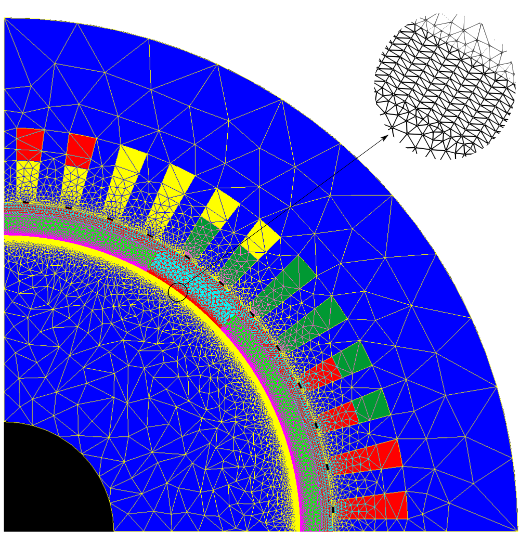

Fig. 5. Mesh adopted in finite element modeling.

- NdFeB: $\sigma = 0.667\,\frac{\text{MS}}{\text{m}}$.

In computing the eddy currents, in each magnet the total current is constrained to be zero. This is because PMs are individually segmented (see Fig. 6) and insulated so as to obstacle eddy currents inside them. This is done by imposing:

$$\int_{S_{\text{PM}}} \mathbf{J}_z(t)\,\mathrm{d}S = 0. \tag{5}$$

Thus, in the TS-FEA, losses can be expressed as:

$$P = \frac{1}{T}\int_T \frac{1}{\sigma}\int_V \mathbf{J}_z(t)^2\,\mathrm{d}V\mathrm{d}t \tag{6}$$

where $\mathbf{J}_z(t)$ is the eddy-current density along the axial direction.

In a TH-FEA losses can be expressed as:

$$P = \frac{1}{\sigma}\int_V \mathbf{J}_z \cdot \tilde{\mathbf{J}}_z\,\mathrm{d}V \tag{7}$$

where $\tilde{\mathbf{J}}_z(t)$ is the complex conjugate of $\mathbf{J}_z(t)$.

The motor is simulated with coarse mesh, characterized by second order element, in the solid stator. Mesh is finer in the iron rotor part closest to the air-gap. In this case, as shown in Fig. 5, it is refined 0.5 mm using ten triangular elements in radial direction. This is because the rotor solid iron configuration adopted constrains the eddy currents in the outer part of itself. The finite element mesh consists of approximately 48 000 triangular elements.

IV. TIME-STEPPING SIMULATIONS

A. Stator reference frame

In this first case the simulations are carried out in the stator reference frame. Rotor is moving at each simulation step. This analysis takes into account the losses due to MMF space harmonics, stator current time harmonics and slotting effect. Losses caused by any air-gap flux variation are considered. The mechanical speed is set to be constant and equal to 9000 rpm.

Two kinds of TS-FEA simulations are carried out. In the first one, each n-th TH is imposed through the three stator phases. Separately a further simulation is carried out by imposing all the Test Current. As a result the amount of losses computed with this TS-FEA is the most accurate and can be considered a reference value for further investigations.

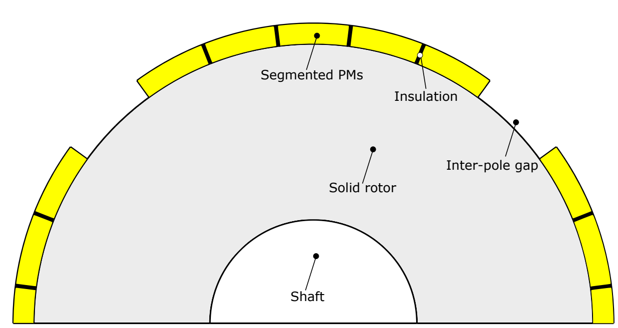

Fig. 6. Rotor sketch.

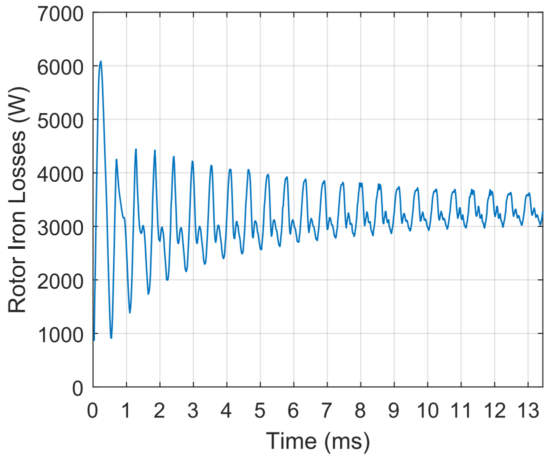

Fig. 7. Rotor iron losses due to a 5th time harmonic.

The simulation period T_{sim} is fixed for every harmonic order and it is equal to two mechanical periods. In this way the machine reaches a steady state operation and average losses can be computed. Due to the fact that there are two machine pole pairs, the mechanical frequency is 150 Hz. Thus, the simulation period is $T_{sim} = 2/150 = 13.13$ ms. The time step is set to $T_{n\text{-th}}/45$, where $T_{n\text{-th}}$ is the electrical period of every n-th TH. For instance, the time step regarding the fifth TH is set to be $1/(1500{\cdot}45) = 14.81$ µs. Since the iron losses are a function of time, once the machine has reached a steady state operation as shown in Fig. 7, losses are calculated using the mean value of the n-th harmonic simulation period. Results are reported in Table III. Its last row shows the comparison between the losses computed imposing each n-th TH and those obtained with the whole TC. The main difference is due to the fact that each n-th TH analysis takes into account the interaction between stator tooth and magnets.

Considering 7 s for the computation of each time step, the total time of simulation is at most 4.6 hours.

TABLE III
TS-FEA$_s$ ROTOR LOSSES COMPUTATION IMPOSING, ONE BY ONE, EACH HARMONIC COMPONENTS.

TS-FEA$_s$							
n	f	T_s	N_{steps}	I_{pk}	P_{tot}	P_{fe}	P_{PM}
	(Hz)	(µs)		(A)	(W)	(W)	(W)
1	300	18.51	360	1300	380	5	375
5	1500	14.81	887	214.5	3719	3270	449
7	2100	10.58	1241	97.5	1064	851	213
11	3300	6.73	1951	32.5	286	162	124
13	3900	5.69	2337	26.0	224	114	110
TH TOTAL SUM					**5673**	4402	1271
TEST CURRENT **(I)**					**4688**	3884	804

B. Rotor reference frame

In the second case simulations are carried out in the rotor reference frame that is at zero mechanical speed (i.e. seen from an observer moving synchronously with the rotor). Thus, losses caused by stator slotting and MMF SH due to the fundamental current component are not considered. In this case only TH causes rotor losses. The first four TC TH are considered. As explained in Table II, the harmonic speeds are:

- f = 1800 Hz, for both fifth (16.5% amplitude) and seventh harmonics (7% amplitude),
- f = 3600 Hz for both eleventh (2.5% amplitude) and thirteenth harmonics (2% amplitude).

The machine reaches a steady state operation after ten TH periods. Thus, the simulation period is not fixed as the previous case. It depends upon the harmonic order. As before, the time step is set to $T_{n\text{-th}}/45$. Results are reported in Table IV.

TABLE IV
TS-FEA$_r$ ROTOR LOSSES COMPUTATION IMPOSING, ONE BY ONE, EACH HARMONIC COMPONENTS.

TS-FEA$_r$							
n	f	T_s	N_{steps}	I_{pk}	P_{tot}	P_{fe}	P_{PM}
	(Hz)	(µs)		(A)	(W)	(W)	(W)
5	1800	12.34	450	214.5	3548	3163	385
7	1800	12.34	450	97.5	964	831	133
11	3600	6.17	450	32.5	200	151	49
13	3600	6.17	450	26.0	136	101	35
TOTAL SUM **(II)**					**4848**	4246	602

Considering 7 s for the computation of each time step, the total time of simulation is at most 52 minutes.

V. TIME-HARMONIC SIMULATIONS

An alternative method for the iron losses computation is the TH-FEA. This is carried out in the rotor reference frame. In this case flux and current densities are supposed to be sinusoidal, over time, in any point. By applying the superposition principle, all the TH effects are added together. The sum leads to results comparable to those obtained in Subsection IV-B. Moreover, the computation takes less time then a TS-FEA. The rotor is fixed so that all calculations are conducted at zero mechanical speed. The results are reported in Table V.

Considering 4 s for each iteration and 89 steps to achieve the convergence of each TH simulation, the required time is at most 6 minutes.

In order to verify if it is possible a further reduction of computational time, different analyses are carried out.

TABLE V
TH-FEA$_r$ ROTOR LOSSES COMPUTATION IMPOSING, ONE BY ONE, EACH HARMONIC COMPONENTS.

TH-FEA$_r$					
n	f_r	I_{pk}	P_{tot}	P_{fe}	P_{PM}
	(Hz)	(A)	(W)	(W)	(W)
5	1800	214.5	3580	3220	373
7	1800	97.5	962	841	128
11	3600	32.5	231	178	53
13	3600	26.0	152	114	38
TOTAL SUM **(III)**			**4925**	4353	593

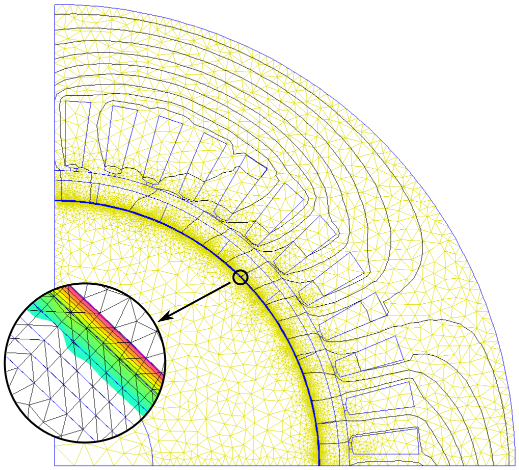

Fig. 8. Refined mesh.

Firstly the TH-FEAs are performed using the same number of mesh elements than previous cases, with one difference: the element order is relaxed to the first one. This relaxation leads to less time spending (only 1 minute is required instead of 6), though losses are slightly over-estimated. The computation results are reported in the upper part of Table VI.

Then, with a view to improving the simulation accuracy, TH-FEAs are carried out as follows. Mesh is further refined in the 0.075 mm of rotor iron part closest to magnets. This is because the skin depth of solid rotor iron at 3600 Hz is about 0.02 mm. The number of triangular elements placed in radial direction is five as shown in Fig. 8, therefore the total number of first order mesh elements results in 286 000. Since the eddy-current phenomenon is confined in the refined domain region, in order to reduce computational time is possible to use non linear material only in there, while in the rotor inner part linear iron is considered. The results are reported in Table VI. It should be noted that further mesh refining results in higher computational time while the arising amount losses are about the same.

Finally, TH-FEAs are carried out for both first order mesh above described, imposing the rotor iron completely linear. Results are reported in Table VII. It should be noted that linear iron leads to rotor losses over estimation, this because the non linearity in solid iron near PMs can not be neglected. These results are not acceptable at all.

TABLE VI
TH-FEA$_r$ ROTOR LOSSES COMPUTATION USING TWO DIFFERENT NUMBERS OF FIRST ORDER ELEMENTS AND NON-LINEAR ROTOR IRON.

Non linear rotor iron					
n	f	I_{pk}	P_{tot}	P_{fe}	P_{PM}
	(Hz)	(A)	(W)	(W)	(W)
5	1800	214.5	4251	3944	307
7	1800	97.5	1125	1016	109
11	3600	32.5	346	287	58
13	3600	26.0	223	185	38
Total sum (**IV**)			**5944**	5432	512
Non linear refined region					
n	f	I_{pk}	P_{tot}	P_{fe}	P_{PM}
	(Hz)	(A)	(W)	(W)	(W)
5	1800	214.5	4252	3868	383
7	1800	97.5	1110	974	136
11	3600	32.5	260	200	60
13	3600	26.0	176	131	44
Total sum (**V**)			**5796**	5173	623

TABLE VII
TH-FEA$_r$ ROTOR LOSSES COMPUTATION USING TWO DIFFERENT NUMBERS OF FIRST ORDER ELEMENTS AND LINEAR ROTOR IRON. THE RESULTS WITH THIS SIMPLIFICATION ARE NOT ACCEPTABLE AT ALL.

Linear rotor iron					
n	f	I_{pk}	P_{tot}	P_{fe}	P_{PM}
	(Hz)	(A)	(W)	(W)	(W)
5	1800	214.5	10598	8218	2379
7	1800	97.5	2189	1697	491
11	3600	32.5	359	282	76
13	3600	26.0	230	181	49
Total sum			**13373**	10378	2995
Linear rotor iron and refined region near PMs					
n	f	I_{pk}	P_{tot}	P_{fe}	P_{PM}
	(Hz)	(A)	(W)	(W)	(W)
5	1800	214.5	8174	5305	2868
7	1800	97.5	1688	1096	592
11	3600	32.5	339	205	134
13	3600	26.0	217	131	85
Total sum			**10416**	6737	3679

VI. Conclusions

This paper deals with a comparison among different procedures to compute eddy-current rotor losses. TS-FEA in stator reference frame is the most accurate to compute these losses, since spatial, time harmonics and slotting effect are taken into account. On the other hand, the required computational time is the highest among all the analyzed methods as shown in Fig. 9(b). This is because there is the need, from the power of losses point of view, to reach a steady state condition to obtain meaningful results.

It has been shown that moving on the rotor reference frame TS-FEA leads to a time savings (about 75% less) and rotor losses amount computed are almost the same.

In order to obtain a further computational time reduction TH-FEA is proposed. In particular a first order mesh element and a linear model, except for the rotor part closer to the magnets, it is a good trade off between accuracy and time savings. In this last case (**IV**), the computational time is drastically cut down. This model may be implemented in situations where short simulation time is a requirement, such as in machine design optimization problem. It has been also shown that a linear model leads to not acceptable rotor losses over estimation, since the linearity in rotor periphery avoid the iron saturation hence it exacerbate the eddy current phenomena.

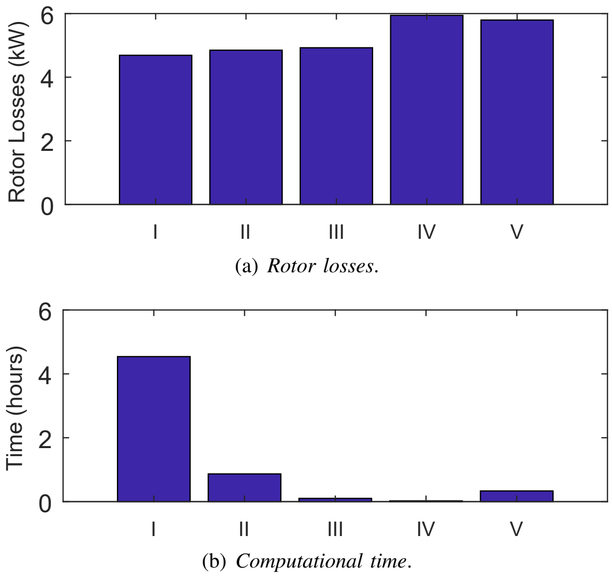

Fig. 9. Rotor losses and computational time required for all the procedure described. I) TS-FEA$_s$ II) TS-FEA$_r$ III) TH-FEA$_r$ IV) TH-FEA$_r$ V) TH-FEA$_r$

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