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An Approximate Analytic Solution to a Non-Linear ODE for Air Jet Velocity Decay through Tree Crops Using Piecewise Linear Emulations and Rectangle Functions

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Featured Application: to protect orchards and vineyards from pests, the correct speed of air that carries the agrochemical drops in the canopy is very important. Too slow an air jet compromises the uniformity of spray deposition, while an excessive air speed leads to significant loss of agrochemicals in the environment ($\geq 50\%$). The present work proposes a mathematical tool to develop a self-regulation system for the air-fans of orchard sprayers.

Abstract: The velocity of air that crosses the canopy of tree crops when using orchard sprayers is a variable that affects pesticide dispersion in the environment. Therefore, having an equation to describe air velocity decay through the canopy is of interest. It was necessary to start from a more general non-linear ordinary differential equation (ODE) obtained from the momentum theorem. After approximating the non-linearity with some piecewise linear terms, analytic solutions were found. Subsequently, to obtain a single equation for velocity decay, a combination of these solutions was proposed by using rectangle functions formed through the hyperbolic tangent function. This single equation was assessed in comparison to the experimental value obtained on a vineyard row by measuring the air velocity at exit of canopy. The results have shown good correspondence, with a mean relative error of 6.6%; moreover, there was no significant difference. To simplify, a combination of only two linearized solutions was also proposed. Again, there was no significant difference between the experimental value and the predicted one, but the mean relative error between the two equations was 3.6%.

Keywords: non-linear ODE; mathematical solution; air jet velocity; air-assisted sprayer; environmental pollution

1. Introduction

In agrochemical applications on orchards and vineyards, the idea of producing droplets by using nozzles and transporting them to vegetation, by using an air jet generated by a fan, is now the generally accepted method. Some studies [1–6] have demonstrated that stream speed strongly influences the effectiveness of spray treatment. If the speed is too low, the distribution of chemicals on vegetation tends to be non-uniform. On the contrary, if the speed is too high, the leaves align with the air stream, and much of the spray is not captured by the vegetation. Spray drift increases with velocity until approximately 50% of the spray does not land on the vegetation [3,4] and, consequently, causes both economic loss and environmental pollution.

Therefore, to help understand the experimental results, it is useful to find an equation that relates the air jet velocity vs. the distance from the output of the sprayer fan when air passes through the

canopy. This equation could also be helpful to equip sprayers with an automatic controller that sets the fan speed in order to maintain the optimum air jet velocity at exit of the canopy, both for the row width [1] and for the growth stage [7].

In recent work [8], an equation was developed for the air velocity decay when the air jet crosses the canopy of tree rows. The equation was obtained by applying the momentum theorem under three assumptions:

- (a) The foliage is uniformly distributed in space, and subsequently, the LAD (leaf area density) is constant.
- (b) The forward motion of the sprayer is ignored since its velocity is one order of magnitude lower than the air velocity from fan exit, and no effect of travel speed on spray deposition in the canopy was experimentally observed [9].
- (c) The inverse relationship between the drag coefficient and air velocity is considered.

Assumption (c), also proposed by other authors [10,11], was verified by wind tunnel tests [8]. All three works confirmed this hypothesis for air jet velocity v_x , which is higher than a fixed value (inferior limit value).

Due to the third assumption, (c), the result of applying the momentum theorem was a linear ordinary differential equation (ODE). By integrating the ODE, an equation to describe air speed decay through the vegetation was found.

If the air speed is under an inferior limit value, the decay equation is not valid because the third assumption, (c), is no longer valid.

However, air jet velocity was observed during experiments [8]; in some situations, when the vegetation is highly developed, the air jet velocity decreases under the limit set by the model. Consequently, the aim of the present work was to find a new equation for the decay speed that would also be valid under the limit when v_x approaches zero. In order to obtain the equation, we started from the non-linear ODE and found analytic solutions by approximating the non-linearity with piecewise linear terms. The method to approximate non-linearity by linear pieces has previously been conducted in other situations [12–16]. We used the approach here to obtain a series of analytical solutions. Finally, to obtain a unique equation for the v_x decay, a combination of piecewise solutions was proposed here, which used multipliers consisting of rectangle functions.

2. Mathematical Modeling of the Air Jet Velocity in the Canopy

2.1. Previous Results

In a previous paper [8], an approach to obtain the speed decay equation was presented. It was based applying the momentum theorem on the control volume within the canopy. Here, we now applied the momentum theorem, but only under the first two assumptions (assumption (a) and (b)) listed above:

- (a) Foliage is uniformly distributed in space,
- (b) Forward motion of the sprayer is ignored.

Using these two assumptions, we obtained the subsequent non-linear ODE:

$$\frac{dv_x}{dx} + \frac{(2x - x_0 + r_0)}{2(x - x_0)(x + r_0)} v_x + \frac{1}{4} \rho_l \cdot c_r(v_x) \cdot v_x = 0, \quad (1)$$

where (see Figure 1 and [8]) v_x is the maximum air jet velocity at the centerline (m/s); x is the distance travelled by the air jet from the output (m); x_0 is the distance, which is normally negative, between the vertex of the diffusion triangle and the output section (m); r_0 is the radius of the edge of the air outlet from the sprayer (vertical view of Figure 1); $c_r(v_x)$ is an adimensional drag coefficient of the canopy that is dependent on v_x ; and ρ_l (m^{-1}) is coincident with the LAD.

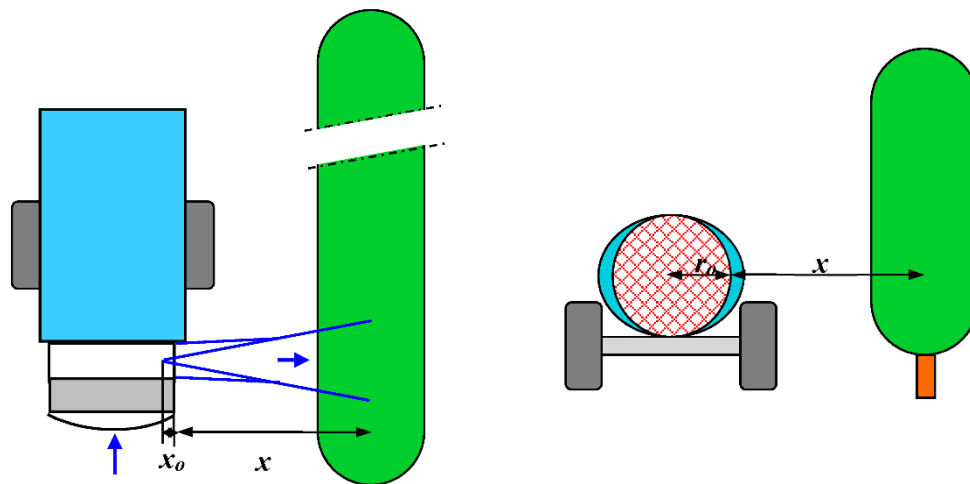


Figure 1. Sprayer during agrochemical applications to the vineyard: vertical view (**right**) and plane view (**left**).

In the previous paper [8], the result of experimental activity conducted in a wind tunnel was presented in order to find the function $c_r = c_r(v_x)$.

This function was formally transformed into $c_r = \frac{k(v_x)}{v_x}$, and the result is presented as in Figure 2, where parameter $k(v_x)$ vs. v_x appears. For a higher air velocity in the tunnel ($v_x = v_{xm} \geq 3$ m/s), parameter $k(v_x)$ is nearly constant and equal to a mean value of 1.1 m/s, with an error <5%, thus substantially confirming the results found by [17] and the validity of the third assumption (c) “inverse relationship between the drag coefficient and air velocity, $c_r = \frac{k}{v_x} = \frac{1.1}{v_x}$, for $v_x \geq 3$ m/s”. The fact that a velocity limit exists under which the inverse relationship between the drag coefficient and air velocity is not valid is shown in Figure 2, where k decreases approximately linearly until the null value when air velocity v_x tends to zero, which is also confirmed by [11,12].

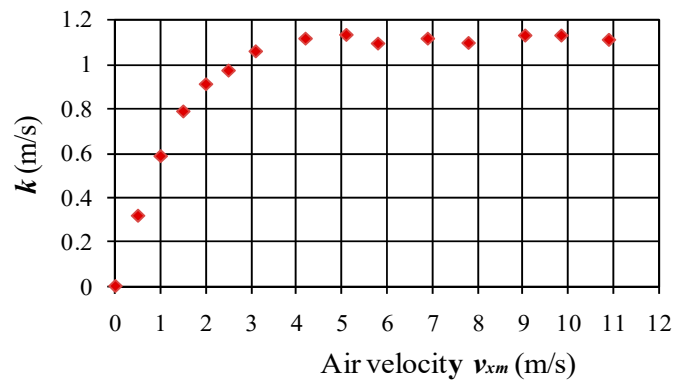


Figure 2. Relationship between the k parameter and mean air velocity within the tunnel, v_{xm} .

Therefore, under assumption (c), the previous non-linear ODE (1) becomes linear and non-homogeneous:

$$\frac{dv_x}{dx} + \frac{(2x - x_0 + r_0)}{2(x - x_0)(x + r_0)}v_x + \frac{1}{4}\rho_l \cdot k = 0. \tag{2}$$

Whereas, as previously stated, the non-linear ODE (1), when $v_x < 3$ m/s, remains to be solved:

$$\frac{dv_x}{dx} + \frac{(2x - x_0 + r_0)}{2(x - x_0)(x + r_0)}v_x + \frac{1}{4}\rho_l \cdot k(v_x) = 0. \tag{3}$$

2.2. Linearization of the Non-Linear ODE

The form (Figure 2) of function $k(v_x)$ can be approximated with a polygonal curve made up of four linear segments.

As a result, the emulator curve $k(v_x)$ will be:

$$k = a \cdot v_x + b, \tag{4}$$

where constants a and b are shown in Table 1 for each linear piecewise section.

Table 1. Values of the constants a and b of the emulator curve and v_{xin} , x_{in} , and v_{xen} for each piecewise linear n .

n° Linear Piecewise n	Δv_x (m/s)	a	b (m/s)	v_{xin} (m/s)	x_{in} (m)	v_{xen} (m/s)
1	≥ 3	0	1.10	7.70	0.71	3.00
2	$3 \div 2$	0.18	0.56	3.00	1.35	2.00
3	$2 \div 1$	0.33	0.26	2.00	1.63	1.00
4	$1 \div 0$	0.59	0	1.00	1.95	0

Therefore, the ODE (3) becomes a linear-ODE:

$$\frac{dv_x}{dx} + \left[\frac{(2x - x_0 + r_0)}{2(x - x_0)(x + r_0)} + \frac{\rho_l}{4} \cdot a \right] v_x + \frac{\rho_l}{4} \cdot b = 0. \tag{5}$$

2.3. Solution of the Linear ODE Series and Their Combination

For the first linear piecewise n ($n = 1$ of Table 1), the solution of (5) is the same as that obtained in previous work [8] and refers to the ODE (2) when it is posed as $k = b$:

$$v_x = v_{xin} \frac{G(x_{in})}{G(x)} - \frac{\rho_l}{16} b \left[(2x - x_0 + r_0) - (2x_{in} - x_0 + r_0) \frac{G(x_{in})}{G(x)} \right], \tag{6}$$

where: $n = 1$; $G(x) = \sqrt{(x - x_0) \cdot (x + r_0)}$; and $G(x_{in}) = G(x_{i1}) = \sqrt{(x_{i1} - x_0) \cdot (x_{i1} + r_0)}$. The integration constant was obtained by imposing the conditions $v_x = v_{xi1}$ for $x = x_{i1}$ (Table 1, $n = 1$ row); x_i and v_{xi} are the experimental values of the coordinate and the velocity of air entering the foliage, respectively; x_0 is the distance (normally negative) between the vertex of the diffusion triangle and the outlet section (m; Figure 1); and r_0 is the radius of the border of the air outlet from the sprayer (vertical view of Figure 1). Equation (6), as indicated in [8], is an approximation of the analytical solution that is useful for making the same (6) more manageable. The error introduced with the approximation is limited to less than 2% because of the low values of x_0 and r_0 with respect to x in the tree crops. It would be zero if x_0 and r_0 were null, and this fact corresponds to having $dG(x)/dx = 1$.

For the second and third piecewise portions ($n^\circ 2$ and 3 of Table 1), the solution of the L-ODE (5) becomes:

$$v_x = \frac{16b}{\rho_l^2 a^3 G(x_{in})^2} \left\{ \left[-2 + \frac{\rho_l}{4} a (2x + r_0 - x_0) - \frac{\rho_l^2}{16} a^2 G(x)^2 \right] - \left[-2 + \frac{\rho_l}{4} a (2x_{in} + r_0 - x_0) - \frac{\rho_l^2}{16} a^2 G(x_{in})^2 \right] e^{-\frac{\rho_l}{4} a (x - x_{in})} \right\} + v_{xin} \frac{G(x_{in})^2}{G(x)^2} e^{-\frac{\rho_l}{4} a (x - x_{in})} \tag{7}$$

where: the constant of integration was obtained by imposing the initial conditions $v_x = v_{xin}$ for $x = x_{in}$; and x_{in} is the coordinate corresponding to air velocity, v_{xin} . For the second linear piecewise n , $x = x_{in} = x_{i2}$ was obtained (Table 1) by the solution of the implicit equation (6) by imposing $v_x = 3$ m/s (Table 1). Whereas, for the third linear piecewise n , the new $x = x_{in} = x_{i3}$ was obtained (Table 1) by the solution of the implicit Equation (7), relative to the second linear piecewise n , by imposing $v_x = 2$ m/s. Solution (7)

can be applied to the second linear piecewise n by inserting $n = 2$ and taking the number (7'). Similarly, the same (7) can be applied to the third linear piecewise n by inserting $n = 3$ and taking the number (7'').

$$v_x = \frac{16b}{\rho_l^2 a^3 G(x_{i2})^2} \left\{ \left[-2 + \frac{\rho_l}{4} a(2x + r_o - x_o) - \frac{\rho_l^2}{16} a^2 G(x)^2 \right] - \left[-2 + \frac{\rho_l}{4} a(2x_{i2} + r_o - x_o) - \frac{\rho_l^2}{16} a^2 G(x_{i2})^2 \right] e^{-\frac{\rho_l}{4} a(x-x_{i2})} \right\} + v_{xi2} \frac{G(x_{i2})^2}{G(x)^2} e^{-\frac{\rho_l}{4} a(x-x_{i2})} \tag{7'}$$

$$v_x = \frac{16b}{\rho_l^2 a^3 G(x_{i3})^2} \left\{ \left[-2 + \frac{\rho_l}{4} a(2x + r_o - x_o) - \frac{\rho_l^2}{16} a^2 G(x)^2 \right] - \left[-2 + \frac{\rho_l}{4} a(2x_{i3} + r_o - x_o) - \frac{\rho_l^2}{16} a^2 G(x_{i3})^2 \right] e^{-\frac{\rho_l}{4} a(x-x_{i3})} \right\} + v_{xi3} \frac{G(x_{i3})^2}{G(x)^2} e^{-\frac{\rho_l}{4} a(x-x_{i3})} \tag{7''}$$

Finally, for the fourth linear piecewise n , (n° 4 of Table 1), the solution of the L-ODE (5) is:

$$v_x = \frac{G(x_{in})}{G(x)} v_{xin} \cdot \exp\left[-\frac{\rho_l}{4} a(x - x_{in})\right] = \frac{G(x_{i4})}{G(x)} v_{xi4} \cdot \exp\left[-\frac{\rho_l}{4} a(x - x_{i4})\right], \tag{8}$$

where: $x = x_{in} = x_{i4}$ was obtained by the solution of implicit equation (7''), relative to the third piecewise n , by imposing $v_x = 1$ m/s (Table 1).

Now, there needs to be a unique equation to be able to completely represent the function $v_x = f(x)$ along the entire domain of variable x . To solve this problem, a combination of the solutions of (6), (7'), (7''), and (8), found for each L-ODE obtained in correspondence with each linear piecewise presented in Table 1, was studied and proposed here. In this combination, the coefficients were obtained by using a rectangle function built through the hyperbolic tangent function. For each (Table 1) linear stepwise n ,

$$\Pi_n = \frac{1}{2} \tanh[10^3(v_{xn} - v_{xen})] - \frac{1}{2} \tanh[(10^3(v_{xn} - v_{xin}))], \tag{9}$$

where: v_{xn} is obtained by introducing Equations (6), (7'), (7'') and (8), respectively, for $n = 1, 2, 3$, and 4; and v_{xin} and v_{xen} are reported in Table 1 for each n value.

For example, the rectangle function, Π_2 vs. $v_x = v_{x2}$, is shown in Figure 3.

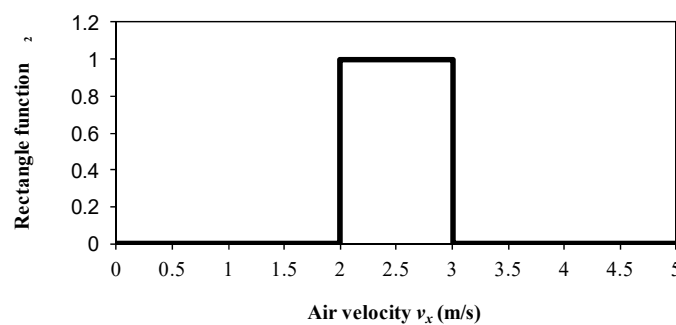


Figure 3. Rectangle Function Π_2 vs. v_x .

Finally, the obtained combination is:

$$v_x = \sum_{n=1}^4 \Pi_n \cdot v_{xn} = \Pi_1 \cdot v_{x1} + \Pi_2 \cdot v_{x2} + \Pi_3 \cdot v_{x3} + \Pi_4 \cdot v_{x4}. \tag{10}$$

3. Experimental Evaluation

Theoretical Equations (6), (7'), (7''), and (8) and their combination (10) were verified by experimental measurements of the velocity of air when exiting (v_{xe}) a vineyard row. A vane probe (10-mm diameter) was connected to a data logger and calibrated to record each test as a mean of 64 values. Each test was

replicated five times by moving the sprayer forward 1 meter each time along the inter-rows. Each repetition considered the measurement at a constant height from the ground at 1 meter.

The velocity of air, v_{xi} , entering the foliage was 7.70 m/s, and the values (as an average of five measurements at a constant 1 meter from the ground) of the ingoing and outgoing horizontal coordinates of the row, x_i and x_e respectively, are reported in Table 2. In the same table, the values of the quantities r_o and b_o (measured on the sprayer) and x_o and ρ_l (calculated in previous work [8]) are also shown.

Table 2. Geometrical data measured on the crops and on the sprayer as well as measured and predicted air velocities.

Geometrical Data	Value
r_o (m)	0.47
b_o (m)	0.075
x_o (m)	-0.085
x_t (m)	0.373
x_i (m)	0.71
x_e (m)	1.94
v_{xi} (m/s)	7.70
v_{xe} (m/s) experim.	0.98
S.D. of v_{xe}	0.25
ρ_l (s ⁻¹)	11.1
v_{xe} (m/s) Equation (10)	1.05
v_{xe} (m/s) Equation (11)	0.96

4. Results

Table 2 reports the results of the field test reflecting the mean values of maximum air velocity v_{xe} out of the vineyard row and the relative standard deviation (S.D.).

To compare the maximum velocities (centerline) of the air jet within the canopy that was predicted by combination (10) (- -□- - -) with the one that was predicted by Equation (6) (- - - - -), Figure 4 shows the respective curves of the air velocity decay, v_x , vs. the horizontal distance, x , from the outlet of the sprayer. Equation (6) gives incorrect negative values for high x values. In addition, the experimental v_{xe} value and the relative standard deviation bars are shown in correspondence with the foliage exit x_e . There was no significant difference between the experimental value and the predicted value by combination (10).

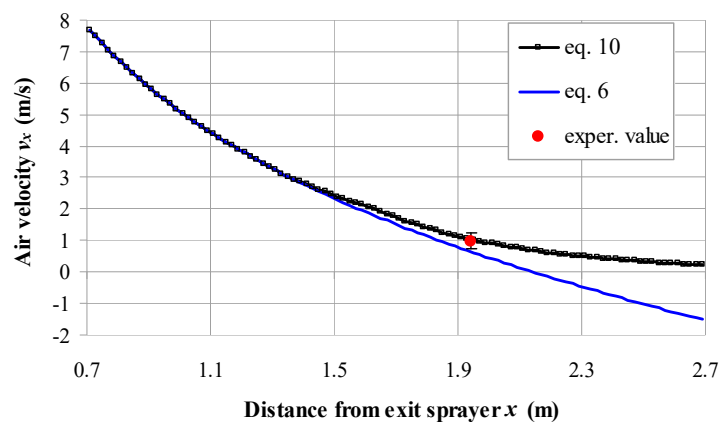


Figure 4. The maximum velocity (centerline) of the air jet in the foliage, predicted by combination (10) (- -□- - -), is shown in comparison to the predicted maximum velocity by Equation (6) (- - - - -), which can result in incorrect negative values and in the experimental value (•).

Due to the complexity of Equation (7), which was carried over to the second and third linear piecewise portions, (7') and (7''), respectively, a simpler combination was made based only on Equations (6) and (8). Equation (6) was extended from 7.70 to 2 m/s, and (8) was extended from 0 to 2 m/s, as shown in Table 3.

Table 3. Values of the constants *a* and *b* of the emulator curve and v_{xin} , x_{in} , and v_{xen} for each linear piecewise *n*.

n° Linear Piecewise <i>n</i>	Δv_x (m/s)	<i>a</i>	<i>b</i> (m/s)	v_{xin} (m/s)	x_{in} (m)	v_{xen} (m/s)
1	≥ 2	0	1.10	7.70	0.71	2.00
2	$2 \div 0$	0.55	0	2.00	1.59	0

The new simplified combination is:

$$v_x = \sum_{n=1}^2 \Pi_n \cdot v_{xn} = \Pi_1 \cdot v_{x1} + \Pi_2 \cdot v_{x2}, \tag{11}$$

where: v_{x1} is provided by Equation (6) and v_{x2} is provided by Equation (8).

The new curve (11) of air velocity decay v_x vs. the horizontal distance x is shown in Figure 5 with curve (6) and the experimental v_{xe} value. There was no significant difference between the experimental value and the one that was predicted by combination (11). The mean relative error (MRE) of Equation (11) vs. Equation (10) was 3.6%, and the maximum error was 10.7% at $x = 1.63$ m.

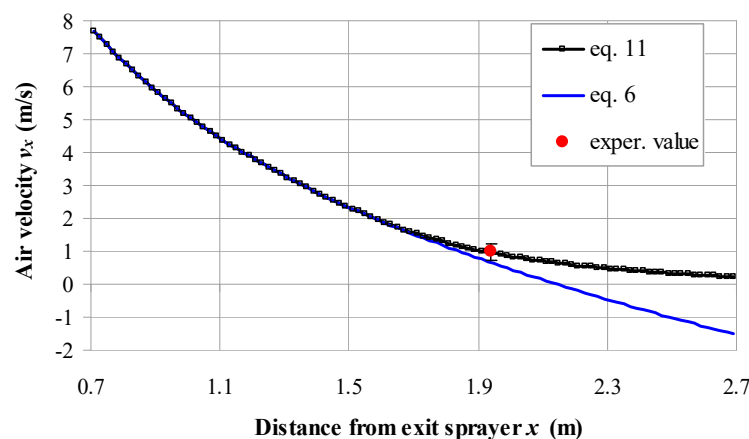


Figure 5. The maximum velocity (centerline) of the air jet in the foliage, predicted by simplified combination (11) (- - □ - -), is shown in comparison to the predicted maximum velocity by Equation (6) (- - - -) and to the experimental value (●).

5. Conclusions

The results of several experimental works have widely demonstrated the importance of the correct velocity of air carrying agrochemicals drops to the canopy of an orchard or vineyard, which must be protected. It has been known that a slow air jet compromises the uniformity of the spray deposition, while an excessive air speed produces significant losses of pesticide to the environment ($\geq 50\%$).

An equation was obtained in previous work [8] to predict the decay of air velocity during tree row crossings in an attempt to furnish a mathematical instrument to improve the interpretation of previous experimental data and to foresee the development of a self-adjusting system for orchard and vineyard fan sprayers. The velocity decay equation that was proposed was not valid under a limit velocity of $v_x = 3$ m/s because the equation was obtained assuming an inverse relationship between the drag coefficient c_r and velocity v_x .

However, a previous experiment [8] has shown that, in some situations when the vegetation is highly developed, the air speed decreases under this limit of 3 m/s.

Therefore, finding a solution to the more general non-linear ODE is important in order to have a velocity decay equation for the entire domain of v_x , namely, until its null value.

Therefore, starting from the non-linear ODE, four analytic solutions were found by approximating the non-linearity with four piecewise linear terms. Subsequently, to obtain a single equation for decay v_x (10), a combination of these solutions was proposed by using multipliers consisting of rectangle functions built by the hyperbolic tangent function, tanh. Finally, the obtained equation (10) was verified in-field on a vineyard row by measuring air velocity v_{xe} at exit of the canopy. The results showed good correspondence between the experimental mean value and the predicted one, with a relative error of 6.6%. In addition, there was no significant difference between the experimental and predicted curves.

As the procedure to build a unique equation by combining the four solutions of the four differential linearized equations tends to be challenging, we simplified the procedure by reducing the number of linearized equations from four to two. Therefore, the combination of only two solutions produced a velocity decay equation (11) that was much more manageable. Again, there was no significant difference between the experimental value and the predicted one by combination (11), but the mean relative error (MRE) of Equation (11) vs. Equation (10) was 3.6%, and the maximum error was 10.7% at $x = 1.63$ m. This is an acceptable error, compared to the variation of experimental measurements of air velocity v_{xe} at the exit of the foliage, which was confirmed by the wide standard deviation.

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