



# Unresolved Binaries and Galactic Clusters' Mass Estimates

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## Abstract

Binary stars are present in all stellar systems, yet their role is far from being fully understood. We investigate the effect of unresolved binaries in the derivation of open clusters' mass by star counts. We start from the luminosity functions of five open clusters: IC 2714, NGC 1912, NGC 2099, NGC 6834, and NGC 7142. Luminosity functions are obtained via star counts extracted from the 2MASS database. The fraction of binaries is considered to be independent of stellar magnitude. We take into account different assumptions for the binary mass ratio distribution and assign binary masses using the so-called *luminosity-limited pairing* method and Monte-Carlo simulations. We show that cluster masses increase when binary stars are appropriately taken into account.

*Key words:* binaries: general – open clusters and associations: general – stars: luminosity function, mass function

## 1. Introduction

Haffner & Heckmann (1937) provided one of the first indications that star clusters harbor a large number of unresolved binary stars. Maeder (1974) showed what position binary stars have in the color–magnitude diagram (CMD) as a function of their mass ratio  $q = M_2/M_1$  (where  $M_2$  is the mass of the secondary while  $M_1$  is the mass of the primary component). Hurley & Tout (1998) demonstrated that the secondary sequences routinely seen above the main sequence (MS) in clusters' CMDs are actually made of binaries with wide mass ratio ranges (and not merely by equal mass binaries). A summary of the results on the binary stars content of star clusters is presented by Duchêne & Kraus (2013).

The binary fraction  $\alpha$  in Galactic globular clusters is relatively small and usually does not exceed  $\sim 10\%$  (Milone et al. 2012), with only rare exceptions. For instance, Li et al. (2017) found a much larger binary fraction for just three globular clusters ( $\alpha = 0.6\text{--}0.8$ ). Open clusters (OCI), on the other hand, host a more significant fraction of binaries with  $\alpha \geq 30\%$  (Bonifazi et al. 1990; Khalaj & Baumgardt 2013; Sarro et al. 2014; Sheikhi et al. 2016; Li et al. 2017). This percentage is, however, smaller than the one among field stars in the solar vicinity (Duquennoy et al. 1991). It has also been noted that the binary percentage increases at increasing a primary mass. This fact is often linked to the dynamical evolution of clusters (Kaczmarek et al. 2011; Dorval et al. 2017). Nevertheless, it does not seem to be universal since, for instance, Patience et al. (2002) found an increase of the companion-star fraction toward smaller masses in  $\alpha$  Persei and Praesepe.

A fundamental quantity is the mass ratio  $q$  distribution. Unfortunately, a consensus is still lacking. According to Duquennoy & Mayor (1991), the distribution of masses of the secondary in the field does not show a maximum close to unity. Instead, this distribution is continuously increased toward the low-mass end. Fisher et al. (2005), however, found a  $q$  distribution peaking near  $q = 1$  for field stars. The same peak was found by Maxted et al. (2008) for the low-mass spectroscopic binaries in the young clusters around  $\sigma$  Ori and  $\lambda$  Ori. Raghavan et al. (2010) support this point of view, showing that the mass ratio distribution shows a preference for

like-mass pairs, which occur more frequently in relatively close pairs. Reggiani & Meyer (2013) argue for a universal form of the  $q$  distribution both for solar-type and for M-dwarfs in the general Galactic field:

$$dN/dq \sim q^\beta \quad (1)$$

with the  $\beta = 0.25 \pm 0.29$  (flat within the errors). Also, Milone et al. (2012) claim that in the interval  $q \in [0.5, 1.0]$  the distribution of  $q$  is nearly flat, with few possible deviations among Galactic globular clusters. Kouwenhoven et al. (2009) introduces two different  $q$  distributions: a power law (1) for  $q \in [q_0, 1]$  and different  $\beta$  values, and a Gaussian one

$$dN/dq \sim \exp[-(q - \mu_q)^2/2\sigma_q^2] \quad (2)$$

for  $q \in (0, 1]$  with  $\mu_q = 0.23$  and  $\sigma_q^2 = 0.42$ . According to Patience et al. (2002), the  $q$  distribution depends on the stellar mass interval: the higher-mass systems reveal a decreasing mass ratio distribution, and the lower-mass systems reveal a deficit of low mass ratio companions (see Figure 8 in Patience et al. 2002). As a result, the combined samples show the deficiency of  $q > 0.85$ . However, a flat distribution is not ruled out (see Figure 6 in Patience et al. 2002).

The  $q$  distribution keeps a memory of the primordial binaries' properties. Some numerical experiments were carried out along this line (Kroupa 2011; Geller et al. 2013; Parker & Reggiani 2013). Geller et al. (2013) performed  $N$ -body simulations of the old open cluster NGC 188 and showed that the distribution of orbital parameters for short-period ( $P < 1000^d$ ) solar-type binaries would not be changed significantly for several Gyr of evolution. This fact means that observations of the present-day binaries even in the oldest open clusters can bring essential information on the primordial binary population. On the other hand, Parker & Reggiani (2013) showed that while the overall binary fraction decreases, the shape of the  $q$  distribution remains unaltered during the evolution. The presence of unresolved binaries in star clusters affects any estimate of their mass, both photometric (via star counts) and dynamical (via velocity dispersion and the virial theorem). In the latter case, if the sample of stars selected for velocity dispersion calculation (through radial velocities)

**Table 1**  
Star Cluster Characteristics

Cluster	$\log t$ $t$ in years	$(m - M)_0^a$ (mag)	$E(B - V)^a$ (mag)	$d_{\text{PHOT}}$ (pc)	$d_{\text{GAIA}}$ (pc)	$M_{\text{min}}$ ( $M_{\odot}$ )	$M_{\text{max}}$ ( $M_{\odot}$ )
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
IC 2714	8.6	10.48	0.34	1250	1390	0.73	2.82
NGC 1912	8.3	10.29	0.25	1140	1150	0.68	3.60
NGC 2099	8.7	10.74	0.30	1410	1510	0.76	2.77
NGC 6834	7.9	11.59	0.71	2080	3570	1.07	5.12
NGC 7142	9.2	11.25	0.39	1780	2600	0.87	1.80

**Note.**

<sup>a</sup> Loktin & Popova (2017).

contains spectroscopic binaries, one can indeed artificially inflate the velocity dispersion, and hence increase the mass. This point has been recently underlined by Kouwenhoven & de Grijs (2009), Bianchini et al. (2016), and by Seleznev et al. (2017).

When the cluster mass is evaluated through the luminosity function (LF) obtained via star counts, the mass estimate derived neglecting unresolved binaries would be smaller than the actual mass. This is straightforward to show, since the mass of a binary system is larger than the mass of a single star at the same magnitude due to the strong mass dependence of the stars' luminosity (approximately  $(L/L_{\odot}) \sim (M/M_{\odot})^4$  for the MS stars, see Figure 7 on page 209 in Carroll & Ostlie 2014).

If a single star and a binary system have the same magnitude, their luminosities are also equal  $L_s = L_1 + L_2$ , where suffix  $s$  marks the single star, while 1 and 2 are the primary and secondary, respectively. Therefore,  $M_s^4 = M_1^4 + M_2^4$ . Instead,  $(M_1 + M_2)^4 = M_1^4 + M_2^4 + 4M_1M_2^3 + 6M_1^2M_2^2 + 4M_1^3M_2 = M_s^4 + 4M_1M_2^3 + 6M_1^2M_2^2 + 4M_1^3M_2$ . Since all terms are positive,  $(M_1 + M_2)^4 > M_s^4$ , and  $M_1 + M_2 > M_s$ .

For example, the presence of unresolved binaries was taken into account by Khalaj & Baumgardt (2013) to estimate the Praesepe cluster mass. Khalaj & Baumgardt (2013) found a binary fraction of  $35\% \pm 5\%$  in Praesepe and used a correction (multiplicative) factor of 1.35 for the cluster mass estimate. Following them, the same correction was applied by Seleznev (2016a) to estimate of NGC 1502 stellar mass. Unfortunately, Khalaj & Baumgardt (2013) provided little information on how they obtained the multiplicative correction factor 1.35, which leaves room for further investigation.

To amend this, in this work, we present a novel approach and estimate the mass of five open clusters of different ages and metallicities, starting from their LF. In this case, one can provide an independent estimate of this correction factor and assess its dependence both on binary fraction  $\alpha$  and on  $q$  distribution.

The layout of the paper is as follows. Section 2 is devoted to the description of our approach and the associated algorithms. Section 3 contains our results for NGC 1912, NGC 2099, NGC 6834, NGC 7142, and IC 2714. Section 4 is dedicated to a summary of our results, provides the paper conclusions, and discusses some future perspectives.

## 2. Model and Algorithm

Two ingredients are needed to derive the correction factor for applying to the photometric mass because of the presence of unresolved binaries. The first one is the binary fraction. In this study, we adopt a binary fraction independent on magnitude. A

larger binary fraction for brighter stars would not increase cluster mass significantly since bright, massive stars typically only number a few. We consider a binary fraction in the range of 10%–90%. The second one is the mass ratio  $q$  distribution. We explore four different distribution functions for  $q$ :

1. A  $\delta$  function with  $q = 1$ .
2. A flat distribution function.
3. A Gaussian distribution (2) as in Kouwenhoven et al. (2009).
4. A Gaussian distribution (2) with mode shifted to  $q = 1$  to reproduce Fisher et al. (2005), Maxted et al. (2008), and Raghavan et al. (2010) function.

The last distribution was taken with  $\mu_q = 0.60$  and  $\sigma_q^2 = 0.42$ , the latter is the same as in Kouwenhoven et al. (2009).

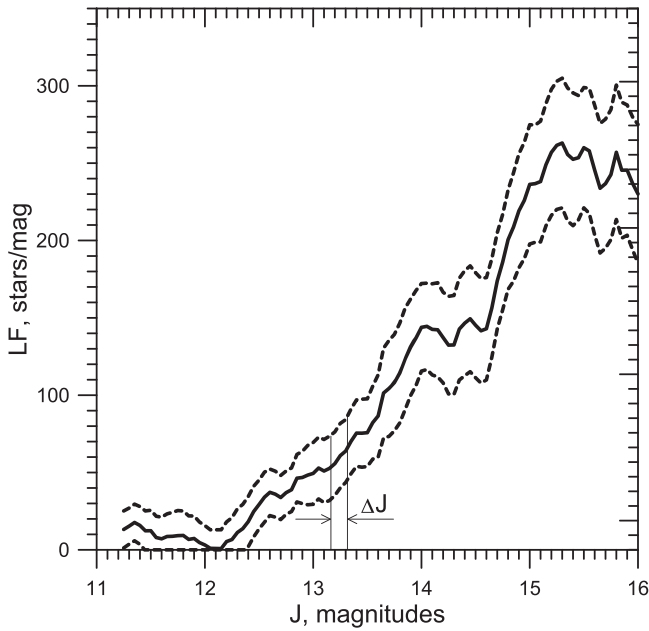
Kouwenhoven et al. (2009) summarized different methods of assignment of the mass values to the binary components, “pairing” methods, as they called them. Our task is different because at the beginning we have the stellar magnitude of the binary and require the mass of each component. The procedure described below could then be called “luminosity-limited pairing”, following the terminology of Kouwenhoven et al. (2009).

We use a quadratic mass–luminosity relation following Eker et al. (2015):

$$\log L = -(0.705 \pm 0.041)(\log M)^2 + (4.655 \pm 0.042)(\log M) - (0.025 \pm 0.010) \quad (3)$$

where  $L$  is the luminosity, and  $M$  is the stellar mass (in solar units). Relation (3) refers to single MS stars; consequently, we assume that all unresolved binaries have their components at the MS and have not experienced mass transfer. It is reasonable for stars below the MS turn-off. Then, only NGC 7142 with an age logarithm of 9.2 (see Table 1 below) could contain a detectable number of binary stars after this stage of evolution. Nevertheless, even for stars above the turn-off in NGC 7142, we could find only a few binary stars after the mass transfer. There could probably be five blue stragglers (see Figures 3 and 4 in Straižys et al. 2014) and one to two evolved (yellow) stragglers among the upper part of CMD.

We use the cluster LF  $\varphi(J)$  to count the number of stars in different magnitude intervals. The LFs are evaluated statistically with the use of 2MASS database (Skrutskie et al. 2006); that is, we obtain the LFs for the cluster region (“cluster plus field”) and an equal area nearby reference field (“field”) and get the cluster LF as the difference between “cluster plus field” and “field”. With this approach, we do not take into account a



**Figure 1.** Luminosity function of NGC 7142 (solid line) and its  $2\sigma$  confidence interval (dotted lines). A  $J$ -band bin is showed for illustration purposes.

possible difference in the mass function between the cluster center and outskirts. This procedure has been described in detail in Seleznev (1998), Seleznev et al. (2000), Prisinzano et al. (2001), Seleznev (2016b), and Seleznev et al. (2017). The magnitude distribution is binned in intervals  $\Delta J$ , and in each of them, we count number of stars and then derive the number of binaries, using a binary fraction  $\alpha$ :

$$N = \int_J^{J+\Delta J} \varphi(J) dJ, \quad N_b = \alpha \int_J^{J+\Delta J} \varphi(J) dJ. \quad (4)$$

We round star numbers to integers and tune the number of intervals to obtain each bin occupied by at least one star. Figure 1 illustrates the process and shows the LF of NGC 7142 obtained as in Seleznev (2016b). For each magnitude bin, the mean magnitude is considered for further calculations.

Stellar magnitudes are converted into luminosities with the use of the isochrone tables (Bressan et al. 2012) as follows. An isochrone corresponding to the cluster age is first selected. We took the cluster ages from Loktin & Popova (2017), but then refined them by comparing with isochrones (Bressan et al. 2012) the cluster CMD for the probable cluster members selected from *Gaia* DR2 (Gaia Collaboration et al. 2016, 2018), filtering by parallaxes and proper motions. Then, absolute magnitude is obtained from cluster distance modulus and color excess. The set of adopted cluster data is listed in Table 1. The photometric distances are then compared with *Gaia* DR2 distances, derived from parallaxes. We find that for distances closer than about 1.5 kpc, photometric and *Gaia* distances agree exceptionally well. Beyond this distance, the figures provided by *Gaia* tend to be significantly larger than the photometric ones. We tentatively impute such differences to the actual *Gaia* release, which is probably not very precise for large distances. Future releases will surely alleviate these differences.

Then, the star mass and the luminosity value are extracted from the isochrone table corresponding to each cluster age. Table 1 contains the limits in the star masses covered by our

LFs: column 7 contains the minimum mass (it corresponds to  $J = 16$  mag with exception to NGC 6834, where the minimum mass corresponds to  $J = 15.9$  mag; these magnitudes, in turn, correspond to the completeness limit of the 2MASS data) and column 8 contains the maximum mass. Stars with masses close to the upper mass limit have been evolved from the MS. Due to this reason, we use another isochrone table with an age of  $4 \cdot 10^7$  yr to determine the luminosity of the evolved stars at the MS stage with the same mass as evolved star mass. For each binary, the following system of equations holds:

$$\begin{cases} L = L_1 + L_2 \\ \log L_1 = -0.705(\log M_1)^2 + 4.655(\log M_1) - 0.025 \\ \log L_2 = -0.705(\log M_2)^2 + 4.655(\log M_2) - 0.025 \\ q = M_2/M_1 \end{cases} \quad (5)$$

where  $L$  is luminosity of binary star,  $L_1$  and  $L_2$  are luminosities of the binary components, and  $M_1$  and  $M_2$  are masses of the primary and secondary components of the binary star, respectively. For each binary star, we extract mass ratio  $q$  from the component mass ratio distribution from Monte-Carlo simulations.

Let be  $x = \log M_1$ ,  $a = -0.705$ ,  $b = 4.655$ , and  $c = -0.025$ . After some algebra, the luminosity reads:

$$\ln L = \ln 10 \cdot (ax^2 + bx + c) + \ln(1 + e^{\ln 10 \cdot (a(\log q)^2 + \log q(b+2ax))}). \quad (6)$$

The goal is to define  $x$ , so that we build up a function  $f(x)$ , which is equal to zero when a solution to the system (5) is found:

$$f(x) = \ln 10 \cdot (ax^2 + bx + c) + \ln(1 + e^{\ln 10 \cdot (a(\log q)^2 + \log q(b+2ax))}) - \ln L. \quad (7)$$

To solve this equation, we use the Newton–Raphson method as

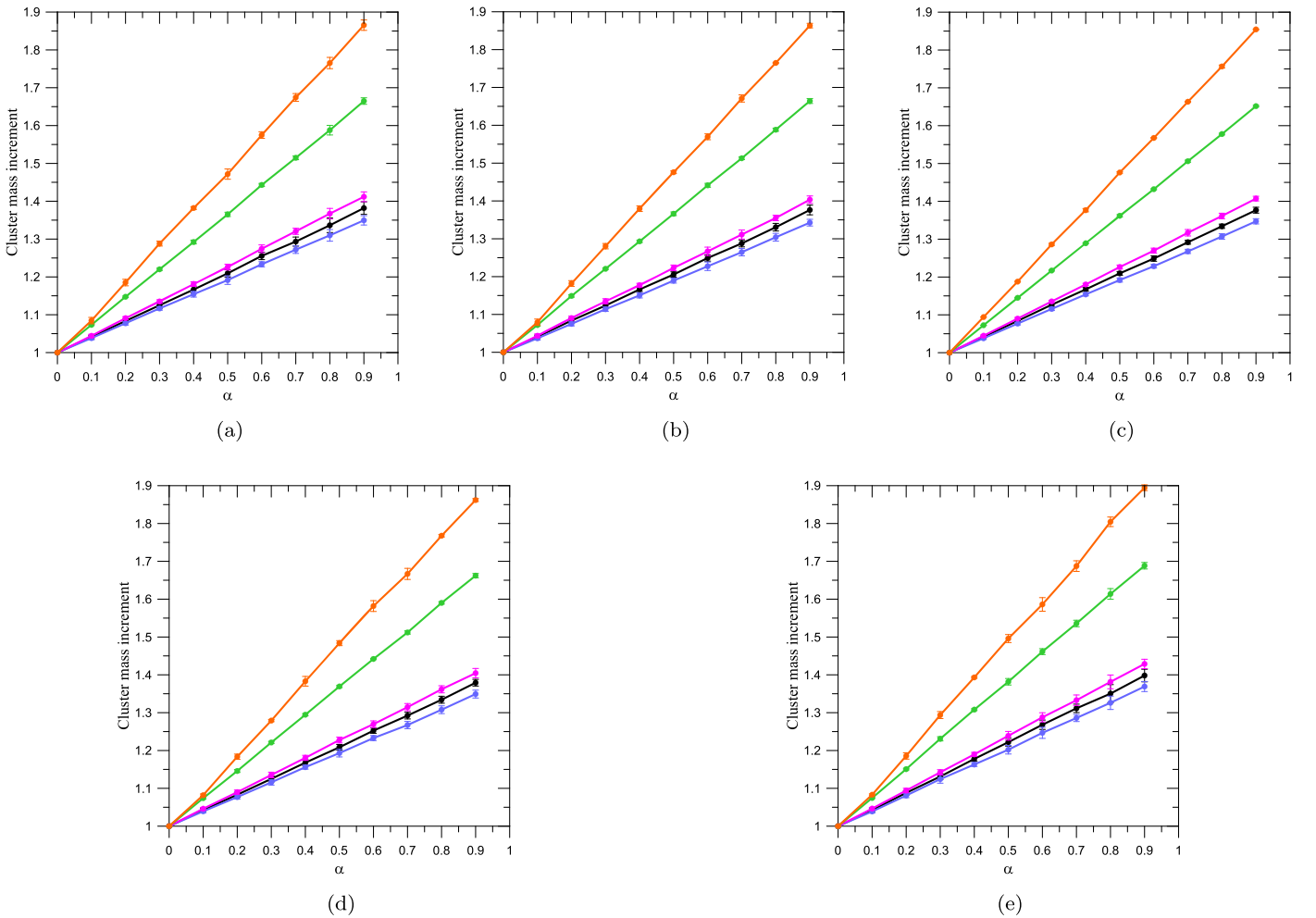
$$x_{k+1} = x_k - f(x_k)/f'(x_k), \quad \text{where } f'(x) = df(x)/dx \quad (8)$$

until the difference  $|x_{k+1} - x_k|$  reaches the requested accuracy.

The Newton–Raphson method converges only if certain conditions are met. First, one needs to choose initial trial values, which are not too far from the root. Therefore, we build a *for-loop* with intervals of the mass  $[0.08; 10] M_\odot$ , (or  $x \in [-1.097, 1]$ ) with a small increase. The loop ends when we find those  $x_i$  that give  $f(x_i) \cdot f(x_{i+1}) < 0$ ; this implies that the root is in the interval  $x \in [x_i, x_{i+1}]$ . We then consider  $x_i$  as a starting point for iteration. Second, the function  $f(x)$  should be smooth in its domain; this is easy to prove, as  $f(x)$  is a combination of smooth functions.

The final  $x_k$  will be the solution of the equation and, in turn,  $M_1 = 10^x$  the value for the primary component mass from the system (5). Hence, we can define the secondary component mass  $M_2$  from the fourth equation of the system (5), and, finally, the total mass of the binary star  $M_1 + M_2$ . The described procedure is repeated for all  $N_b$  stars to eventually derive the total mass of binaries in the interval  $J \in [J; J + \Delta J]$ . When extended to all magnitude bins, the procedure yields the total mass of the cluster binaries  $M_b$  in these bins.

Finally, to define the mass of the cluster, we need to find the mass stored in single stars  $M_s$  (whose number is  $N_s = N - N_b$  in each magnitude interval). For these stars, we use an



**Figure 2.** Dependence of the cluster mass increment on the binary fraction  $\alpha$  with different assumptions on the binary component mass ratio  $q$  distribution. Green line: equal components; orange line: equal components taking into account the multiple (triple and quadruple) systems; black line: flat distribution; electric blue line: Gaussian distribution with a maximum close to zero; magenta line: Gaussian distribution with a maximum close to unity. (a) IC 2714; (b) NGC 1912; (c) NGC 2099; (d) NGC 6834; (e) NGC 7142.

isochrone table, where we determine the mass according to the magnitude and the cluster parameters from Table 1 (see a description of the procedure above). As a result, we obtain the cluster mass  $M = M_b + M_s$  in the considered magnitude interval.

Let us now define  $M_{\text{wob}}$  as the cluster mass obtained assuming that all stars are single. Then the ratio  $M/M_{\text{wob}}$  would naturally give the cluster mass increment due to unresolved binaries.

### 3. Results for the Program Clusters

In this work, we start from the LF of five open clusters: IC 2714, NGC 1912, NGC 2099, NGC 6834, and NGC 7142 obtained by star counts with 2MASS as described above.

For each cluster, we repeated the procedure described in the previous section up to 30 times both for cluster LF and for boundaries of the LF confidence interval. This procedure allowed us to evaluate the scatter of the mass increment factors. We explored the whole parameter space made of binary fraction  $\alpha$  and mass ratio  $q$  distribution to quantify the spread in the estimates of the cluster mass when unresolved binaries are taken into account.

We considered two cases of equal mass components. The first case is when we take into account binary systems only. In the second case, we also take into account the multiple (triple and quadruple) systems following Tokovinin (2014), who found for systems with multiplicity of 1:2:3:4:5 (“1” means single star) the relative abundance ratio of 54:33:8:4:1. It is worthwhile because at distances of  $\sim 1$  kpc a hierarchical triple of separation  $\sim 100$  au has an angular separation of about 0.1 arcsec, then a triple system or a “binary of binaries” could be missed, just like tight unresolved binaries.

Figure 2 shows the dependence of the cluster mass increment on the binary fraction for the five clusters. Each panel corresponds to a cluster, and different colors are used to indicate the various  $q$  distributions. At first glance, one can easily see that the equal mass component model significantly deviates from the other models, which do not appear much different.

Khalaj & Baumgardt (2013) found the cluster mass increment value of 1.35 for a binary fraction of 0.35. According to our study, the increment value should be between 1.10 and 1.15 for *realistic*  $q$  distribution (see Figure 2). However, taking into account the possible presence of the multiple (triple and quadruple) systems in the cluster would increase the value of the increment on the average 1.32 times for the case of equal



**Table 2**  
Linear Approximation  $y = A + B\alpha$  for the Cluster Mass Increment Dependence on the Binary Fraction

$q$ Distribution Model	Cluster	$A$	$B$	$\chi^2$	$Q$
(1)	(2)	(3)	(4)	(5)	(6)
Equal component masses	IC 2714	$1.000 \pm 0.002$	$0.736 \pm 0.005$	0.673	1.000
	NGC 1912	$1.000 \pm 0.002$	$0.735 \pm 0.005$	1.162	0.997
	NGC 2099	$1.000 \pm 0.000$	$0.722 \pm 0.001$	1.639	0.990
	NGC 6834	$1.000 \pm 0.002$	$0.736 \pm 0.004$	1.025	0.998
	NGC 7142	$0.997 \pm 0.001$	$0.773 \pm 0.006$	1.263	0.996
Equal component masses with triple and quadruple systems	IC 2714	$0.994 \pm 0.005$	$0.968 \pm 0.012$	1.690	0.989
	NGC 1912	$0.991 \pm 0.004$	$0.968 \pm 0.006$	2.316	0.970
	NGC 2099	$0.999 \pm 0.000$	$0.949 \pm 0.001$	5.274	0.728
	NGC 6834	$0.987 \pm 0.003$	$0.975 \pm 0.005$	5.582	0.694
	NGC 7142	$0.983 \pm 0.003$	$1.020 \pm 0.007$	6.496	0.592
Flat distribution	IC 2714	$0.999 \pm 0.004$	$0.423 \pm 0.011$	0.181	1.000
	NGC 1912	$1.000 \pm 0.004$	$0.414 \pm 0.009$	0.227	1.000
	NGC 2099	$1.000 \pm 0.002$	$0.417 \pm 0.005$	0.194	1.000
	NGC 6834	$1.000 \pm 0.003$	$0.419 \pm 0.008$	0.185	1.000
	NGC 7142	$0.998 \pm 0.004$	$0.447 \pm 0.011$	0.190	1.000
Gaussian distribution $\mu_q = 0.23$	IC 2714	$1.000 \pm 0.004$	$0.389 \pm 0.010$	0.086	1.000
	NGC 1912	$0.999 \pm 0.004$	$0.381 \pm 0.009$	0.061	1.000
	NGC 2099	$1.000 \pm 0.003$	$0.384 \pm 0.006$	0.273	1.000
	NGC 6834	$1.000 \pm 0.003$	$0.386 \pm 0.008$	0.256	1.000
	NGC 7142	$0.998 \pm 0.004$	$0.411 \pm 0.010$	0.167	1.000
Gaussian distribution $\mu_q = 0.60$	IC 2714	$0.998 \pm 0.003$	$0.459 \pm 0.009$	0.218	1.000
	NGC 1912	$1.000 \pm 0.004$	$0.446 \pm 0.009$	0.265	1.000
	NGC 2099	$1.000 \pm 0.002$	$0.452 \pm 0.006$	0.073	1.000
	NGC 6834	$1.001 \pm 0.003$	$0.450 \pm 0.009$	0.161	1.000
	NGC 7142	$0.999 \pm 0.003$	$0.478 \pm 0.010$	0.058	1.000

components. Then the value of 1.35 for the cluster mass increment found by Khalaj & Baumgardt (2013) for the Praesepe cluster is reasonable. We fitted the dependencies of the increment on the binary fraction via linear regression and provide fitting formulae in Table 2. The columns of Table 2 are: the binary components mass ratio model, the cluster, the coefficients  $A$  and  $B$  of the linear regression  $y = A + B\alpha$  (where  $y$  is the cluster mass increment, and  $\alpha$  is the binary fraction), the  $\chi^2$  of the fit, and the goodness-of-fit probability  $Q$  (Press et al. 1992). Coefficient  $A$  does not differ significantly from the unity in virtually all cases. The coefficients  $B$  for the clusters lie within the limits of the  $q$  distribution model (except for NGC 7142, the oldest one). This fact demonstrates that the shape of the LF does not affect the dependence of the cluster mass increment on the binary fraction  $\alpha$  significantly.

The LFs used in the present work are limited in magnitude because of the completeness limit of 2MASS. Therefore, we miss stars with masses lower than the limit listed in the seventh column of Table 1. How can the missing low-mass stars affect our results? We consider the binary fraction  $\alpha$  independent of the stellar magnitude. In such a case, the cluster mass increment should be independent of the magnitude (and the mass) limit. In order to make this suggestion more solid, we performed the following experiment. For NGC 2099, we calculated the mass increment for a set of limiting magnitudes  $J = 14, 15, 16$  mag in the case of flat  $q$  distribution. It turned out that the mass increment slightly increases with the limiting magnitude. For instance, for  $\alpha = 0.8$   $y = 1.322 \pm 0.013$  for  $J_{\text{lim}} = 14$  mag,  $y = 1.328 \pm 0.009$  for  $J_{\text{lim}} = 15$  mag, and  $y = 1.334 \pm 0.006$  for  $J_{\text{lim}} = 16$  mag. If the binary fraction increases with the stellar

magnitude, the cluster mass increment would most probably increase with the stellar magnitude. If the binary fraction decreases with the stellar magnitude, we would expect the cluster mass increment to be independent of the stellar magnitude or even to decrease with the stellar magnitude.

In any case, we underline that even applying the mass increment, one would not obtain the total mass of the cluster but only slightly improve a lower limit estimate of it.

#### 4. Conclusions

In this work, we attempt to quantify the increase of the cluster mass estimate—obtained by star counts—produced by the presence of unresolved binaries. The results are illustrated in Figure 2 and summarized in Table 2.

The most relevant results of this study are

1. The dependence of the cluster mass increment on binary fraction is linear in most cases.
2. The dependence of the cluster mass increment on the binary fraction  $\alpha$  does not vary significantly for the *realistic*  $q$  distributions considered here. We checked three *realistic* distributions: a Gaussian distribution (2) with  $\mu_q = 0.23$ , a flat distribution, and a Gaussian distribution (2) with  $\mu_q = 0.60$ . An inspection of Figure 2 and Table 2 shows that the closer the distribution mode to unity, the higher the expected cluster mass increment.
3. The dependence of the cluster mass increment on the binary fraction  $\alpha$  within the limits of a specific  $q$  distribution model does not differ substantially among the

selected clusters (except for NGC 7142, the oldest one). Then we can safely conclude that the form of the LF does not affect this dependence considerably.

- For the particular case of a binary fraction  $\alpha = 0.35$ , the cluster mass increment is confined between 1.10 and 1.15 (for *realistic*  $q$  distributions, see Figure 2). However, taking into account the possible presence of the multiple (triple and quadruple) systems in the cluster would increase the value of the increment (in the mean 1.32 times for the case of equal components). Then the value of 1.35 for the cluster mass increment for the Praesepe cluster obtained by Khalaj & Baumgardt (2013) is reasonable.

Our results will help to improve the estimate of the mass of clusters containing unresolved binary stars in the broad range of the binary ratios  $\alpha$  and with different assumptions on the distribution of the binary component mass ratio  $q$ .

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