# The Early Mathematics of Welfare: The Contribution of Bruno de Finetti

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Bruno de Finetti (1906–1985) is well known as the founder of the subjective theory of probability (Feduzi, Runde, and Zappia 2013). Less known, with a few exceptions (Rossignoli 1999; Lunghini 2007; Scazzieri 2009), is his contribution to economic theory during the early stage of his scientific career. In the second half of the 1930s, the young de Finetti was passionately involved in the field of economics, particularly in welfare economics. To provide a theoretical framework for evaluating social welfare and to help in designing public policies, he advanced a new mathematical tool: the theory of simultaneous maxima. Using this analytical approach, he criticized the laissez-faire interpretation of Vilfredo Pareto's theory and advanced the idea of a social welfare function, albeit one quite different from that introduced in 1938 by Abram Bergson, reflecting the debate on economic planning among Italian corporatist economists.

It is still interesting to analyze de Finetti's contribution for many reasons, not only for historical purposes. His criticism was not driven merely by theoretical interests. Through his critical analysis of the Paretian system, he wanted to identify the theoretical causes of the failure of the economic systems in the 1930s. Moreover, his contribution touched on some fundamental questions that are still relevant today, such as the relationships between the descriptive and normative aspects of economic theory. The fact that de Finetti's criticism has rarely been taken into consideration may

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be due to accidental circumstances, such as the modest standing of the journals in which his work was published. However, it may also be due to much deeper reasons, which highlights that the evolution of economic theory is deeply influenced by the historical and cultural situation of the time.

## 1. De Finetti's and the Corporatist's Economic Theory

What was the economic background of the young de Finetti in the 1930s? We can find a rare suggestion in a passage from the introduction to the 1969 book, *Un matematico e l'economia* (A mathematician and economics):

> I started with very poor preparation. However, it was very effective, at least to arouse interest and to begin to understand the use of mathematics for economic problems. The only contact with economic theory was in my university studies: it was the course taught by Ulisse Gobbi at the Polytechnic of Milan. The doubts on some points touched twelve years later in the 1936 conference [date back to my university period]. . . . The doubts resurfaced and became more troubling when, in the 1930s, the topic of the crisis (in the system or of the system), of the X-Crise, as defined by a French economist underlining the paradox of overproduction coupled with misery, the Keynesian ideas, news on the Soviet economy, and the perspectives of a planned economy (albeit in a corporate sense) in Italy too, were the subject of heated discussions and lively interest. I found in Pareto, then, the most satisfactory basis for trying to reflect (especially in the article of *Enzyklopaedie der Mathematischen Wissenschaften*, and after in the Appendix of the *Manuel*). (de Finetti 1969: 26)[1]

Attending Ulisse Gobbi's industrial economics course as a third-year student at the Polytechnic in Milan was a fortunate coincidence that proved to be one of the elements that aroused de Finetti's interest in economics. It had a long-lasting influence on his thinking. Gobbi (1859–1936) was a lawyer. After switching his intellectual career to economics, he became an exponent of the economic circle that grew around the personality of Luigi Cossa in Pavia. He was not a scholar interested in pure theory (Barucci 2007). He devoted his research entirely to the study of the

1. This conference was held at the Seminario attuariale in Rome. The X-Crise was a French technocratic group created in 1931 by former students of the École Polytechnique opposing classical liberalism in favor of the economic planning. Translations from Italian are mine if not otherwise indicated.

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social problems of his time, in particular to the theories of cooperation and social insurance. His main work was the 1919 *Trattato di Economia* (Treatise on economic policy), a broad treatise integrating the economic analysis with long legal discussions. In this book he demonstrated a lively interest in applied economics and a strong suspicion of the pure theoretical advancements of the discipline, together with a general dissatisfaction with the individualism implicit in the new marginalist approach. For Gobbi, the main aim of economic theory was to study the principles of economic welfare, not in reference to the individual agent but to the entire society. And this anti-individualistic vision of economic science also characterized de Finetti's approach to welfare theory. Gobbi's influence on the young de Finetti also had a more specific character. In 1899, Gobbi wrote a book titled *L'assicurazione in generale* (Insurance theory) which can be considered the first systematic writing on the topic of insurance in Italy. Particularly interesting was chapter 10, "La probabilità nei fatti economico-sociali" (Probability in economic and social phenomena), in which he discussed the limits of the frequentist approach in the case of economic and social phenomena. As a tribute to his teacher, de Finetti dedicated his book on insurance, *Economia delle assicurazioni* (The economic theory of insurance [1967]), to Gobbi and inserted in the text many quotations from Gobbi's book written almost a century earlier. In the full spirit of his teacher, the last part was dedicated to social security, which for De Finetti, as for Gobbi, represented a central aspect of the economic welfare of the entire society.

The young de Finetti entered the economic debate during a period of crisis and rethinking of economic theory. The 1929 crisis raised questions about the capacity of the capitalistic system to self-regulate through the system of competitive prices. On the contrary, laissez-faire policies seemed to aggravate the already difficult economic conditions. Toward the middle of the 1930s, Italian economists, both those of the Marshallian and Paretian School, shifted toward corporatism. The theoretical ground was prepared by Luigi Amoroso, an economist of the highest international reputation, in an article written in 1934 with Alberto De Stefani, "La logica del sistema corporativo." This paved the way for the scientific legitimation of the corporatist view of economic theory. In the same year, with the creation of the Corporazioni, which had been envisaged as early as 1926, the corporative system moved out of purely doctrinal debate. The Corporazioni were intended to be the regime's principal institutional innovations in economic policy, and they required scientific interpretation on the part of economists. This attempt to introduce corporatist topics into

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traditional economic theory also had immediate repercussions at the academic level with the change, from the mid-1930s onwards, in the official titles of economics courses, which by law became courses in corporatist economics and corporatist economic policy.

In the Italian context developed in the 1930s, de Finetti can be considered a corporatist economist. Many years later, he defined himself as a "left-wing fascist" (de Finetti 1969: 16). To understand de Finetti's position as a corporatist economist, we have to briefly consider the development of the economic theory of corporatism in Italy (Guidi 2000), in which we can distinguish two phases. The early phase, from 1925 to 1934, was the period of maximum affirmation and expansion of corporatist economics. The main theoreticians of corporatism (Ugo Spirito, Nino Massimo Fovel, Gino Arias, and Filippo Carli) proclaimed that economics should be rebuilt on a new theoretical basis, by replacing the homo economicus with homo corporativus. Orthodox economists made a tactical retreat. This was also the period in which the corporative legislation was promulgated and the fascist economic policy, after Alberto De Stefani's resignation,[2] caused a marked interventionist turnaround. The fascist propaganda repeated that this turn was the result of an attempt to institutionalize the corporatist economy.

The second phase, ranging from 1934 to 1943, was more constructive. In this period the orthodox economists returned to the stage. Most of them put into brackets their laissez-faire beliefs and went in search of an honorable compromise with the fascist regime. Some of them insisted that if the corporative system was to be efficient and competitive, the negotiation for wages and prices could not reach solutions that differed from those that a market in conditions of free competition would have spontaneously individuated. Other economists found an occasion in this discussion on the role of the state to debate the problems related to the causes of the crisis of 1929 and to the transformation of the economy in an oligopolistic system.

De Finetti's approach to corporatist economic theory was deeply influenced by Ugo Spirito, a follower of the Italian philosopher Giovanni Gentile. In the second half of the 1920s Spirito began to deal extensively with economic theory and developed a strong criticism of economic science, attempting to outline a system of corporatist political economy (Spirito 1930, 1932). Spirito's purpose was to demonstrate that, although the laws

2. Alberto De Stefani (1879–1969) was an Italian economist. He was finance minister from 1922 to 1925 in the first period of the fascist regime. His action was characterized by laissez-faire principles.

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of pure economics seemed to be logical deductions from neutral and general axioms and hypotheses, they in fact were, on the contrary, based on a liberal and individualistic ideology. In his research Spirito turned his attention to a number of subjects that were foremost in the contemporary debate, from the role of homo economicus to the theory of marginal utility, from the problem of society's maximum welfare to the comparison between market and socialist economies, and to the incompatibility between decreasing costs and free competition. In an essay written in 1934, according to his ideology, he concluded that the economic problem consisted in defining the maximum welfare of society, a problem that could be solved only by ethics, since this permitted the combining of individual and general interests. The main idea developed by Spirito in the field of economic theory was the *economia programmata* (planned economy), emphasizing the role of the state in all of economic life.

In the debate on corporatism, de Finetti supported Spirito's view.[3] He confronted Spirito's economic perspective in the seminal 1935 article, "Vilfredo Pareto di fronte ai suoi critici odierni." In the beginning, de Finetti admitted that the *economia programmata* advanced by Spirito was "the idea of economic theory closer to me" (513). From Spirito he derived his criticism of economic hedonism and the impossibility of separating economic theory from the moral dimension. In contrast to Spirito, he defended the wide use of mathematical tools in economic theory, following the Paretian methodology. Spirito's influence contributed to introducing the young de Finetti to the realm of welfare economics, wherein, like Spirito, he believed the true scope of economics was the study of society, not the choices made by individuals.

## 2. The Relevance of Pareto

In the discussion on welfare economics de Finetti focused his attention on Paretian theory and specifically on the appendix of the French edition of *Manuale* (1909). There were a number of reasons that explain why de Finetti considered Pareto as the starting point. The first was that de Finetti had always declared himself to be a mathematician and not an economist. He was never interested in questions of economic theory outside of welfare economics and did not follow the debates among economists on other issues. Second, in the years up to 1946, de Finetti worked as a statistician

3. De Finetti also corresponded with Spirito during 1935–36.

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at the company Assicurazioni Generali in Trieste. Only in 1954 did he become a professor at the University of Rome, which enabled him to devote himself fully to conducting scientific research.

There is a third and fundamental reason why de Finetti focused solely on Pareto's writings. In the 1930s, welfare economics inside the Paretian tradition had still not taken any steps forward. Indeed, the entire theory of static general equilibrium was still dependent on the results formulated by Pareto. In the 1930s, Pareto's followers were not interested in the analytical developments of the theory of static general equilibrium. An early, important exception was the 1928 article by Amoroso, "Discussione del sistema di equazioni che definiscono l'equilibrio del consumatore" (An analysis of the system of equations defining consumer behavior), in which he discussed the mathematical problem of the existence and uniqueness of the equilibrium position in the case of a single consumer. In this strict mathematical contribution, Amoroso anticipated some aspects of the axiomatic turning point of the theory of general economic equilibrium (Guccione and Minelli 1999). In the subsequent years, Amoroso did not continue along this line of research but changed direction toward dynamic analysis (Amoroso 1929). The real issue for Italian economists in the 1930s was building a business cycle theory, and the static theory was abandoned.

The only economist within the Paretian tradition who was able to offer some analytical contributions to static general equilibrium theory was Arrigo Bordin, a Paretian of the second generation. He addressed the problem of further developing the Paretian theory in a dense article titled "Schemi di varianti nella teoria dell'equilibrio economico generale" (Mathematical developments of the general economic theory), which was published in *Annali di Economia* in 1930. A more detailed exposition appeared in his *Lezioni di Economia Politica* (Lectures on political economy [1934]). The main problem considered by Bordin was the lack of uniqueness of the general equilibrium solution. As was well known, considering by example the case of consumption, in the final position the allocation of resources among economic agents remains undetermined since any position on the contract curve is acceptable. To try to solve this problem, Bordin introduced the concept of the hedonic force of economic agents, defined as the capacity that each agent has to turn the exchange in his favor. He sought a formal definition of this sociological aspect of the real dynamics of exchange. In the case of simple exchange, the hedonistic force can be represented as a vector orthogonal to the indifference curve at the initial position. Following a mechanical analogy, Bordin showed

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that the vector sum of the two hedonistic forces gave the missing equation indicating the direction in which the exchange will move (Bordin 1934: 120–24). The final result was a path of prices from the initial to the final position depending on initial endowment, the indifference curves, and a new parameter indicating the ratio between hedonistic forces. Bordin demonstrated that, given the value of this ratio, the problem was perfectly determined both in the case of two consumers and in the case of $n$ consumers. He called his model, in which prices are the result of the intensity of hedonistic forces, the generalized equilibrium model, to distinguish it from Pareto's model in which prices are exogenous parameters.

Bordin's approach can be considered the most important attempt to develop, in a realistic sense, the theory of economic equilibrium within the Paretian tradition. In the lectures of the early 1930s, he took another step forward. He was probably the first to characterize, in a modern sense, the first-order conditions of general economic equilibrium, both in consumption and in production, following the approach of Francis Ysidro Edgeworth. Considering the case of consumption, he observed that a situation of general equilibrium required that the ratio between the marginal utilities of two goods (in modern words, the marginal rate of substitution between goods) were equal to the ratio of their prices for all agents (Bordin 1934: 178–81). He obtained the relevant analytical results in his lectures, but did not grasp their theoretical value because his aim was to analyze the process of price formation without discussing the properties of final equilibrium in the common case of parametric prices. As this analytic achievement remained confined to minor texts, we can say that when the mathematician, de Finetti, began to consider the state of welfare economics, the starting point was still the Paretian texts.

## 3. De Finetti and the Tragic Sophism of Liberalism

In three articles published in 1935 and 1936, de Finetti introduced the general perspective of his criticism of the Paretian system with respect to welfare theory, which he subsequently developed in mathematical terms in the two articles published in 1937. These articles, which we will analyze in detail, exemplify how mathematical research in economics is driven by the need to carry out an in-depth analysis of an economic problem. The main target of de Finetti's criticism was what is now known as the first theorem of welfare economics, the idea that a perfectly competitive

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equilibrium was Pareto optimum. Pareto, in his *Manuale* (1906), introduced the idea of a social evaluation of the economic system. Pareto's purpose was to rid economic theory of utilitarian philosophy, and thus transform economics into a positive science, like rational mechanics (McLure 2001). This criterion allowed for stating something about the welfare of society while ruling out interpersonal comparisons of utility. The first welfare theorem indicated the way to obtain this result. In the case of perfect competition, a situation in which economic agents were unable to directly influence prices, the competitive equilibrium was Pareto optimal. Forgotten for many decades (Blaug 1997: 59–78), this proposition was modernized in mathematical terms first by Arrow (1951) and subsequently by Arrow and Debreu (1954).

In his writings from this period, de Finetti tried to demonstrate that this proposition, albeit mathematically correct, was a sophism, that is, a reasoning that was misleading in terms of its political claim that the free market produces maximum welfare. It is worth noting that de Finetti used the same term used thirty years earlier by the young mathematician Gaetano Scorza (1902) in his debate with Pareto on the same issue (Gattei and Guerraggio 1991). Also, Scorza considered the identification of the result of free market competition and the optimal allocation of the resources for the society to be a *sofisma* (sophism), which de Finetti considered tragic because it could potentially produce very harmful consequences for society. The term *sofisma* was also used by Spirito to criticize the classical economists (Sipirto 1930).

De Finetti observed:

> A more serious and hateful sophism is added to the error at the basis of research into the optimum, through the adoption of certain means of approach: the optimistic sophism of liberalism, the superstition of self-regulating anarchy, according to which the most simple and secure way to reach the maximum welfare for all consists in allowing each individual to try to realize the maximum egoistic profit. (de Finetti 1935b: 440)

To better understand de Finetti's position, it is necessary to consider that his criticism was not aimed at questioning the validity of Pareto's theory as such but, instead, to free it from what he considered to be a merely contingent and ideological interpretation. In de Finetti's view, it was not correct to consider Pareto as the main exponent of economic liberalism. To reach this conclusion, he made an epistemological distinction

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between a general plane in economic reasoning, which he referred to as the geometry of utility (de Finetti 1936, 1943), and on the institutional and historical plane, the mechanics of utility. The geometry of utility represented the invariant core of economic theory—and of Pareto's theory—and therefore was not amendable. This point of view required the full acceptance of the marginalist principle, according to which every economic action has to be assessed for small variations until reaching the point of equilibrium. From this epistemological perspective, de Finetti considered himself a defender of Pareto against those who questioned his excessive abstractness of thought.

Concrete restrictions shape this general geometry of utility and affect the outcome of the economic behavior. De Finetti defined this aspect as the mechanics of utility, which is conditioned by institutional and historical circumstances. As long as there is only one geometry of utility, there are many mechanics; in de Finetti's view, the free market approach represented a specific case in which prices are taken as parametric. According to him, this situation corresponded to the initial phase of capitalism, but it was not useful to describe the current state as dominated by the role of large oligopolistic firms (de Finetti 1943: 55). He deemed it fully legitimate to imagine many other versions of economic mechanics, starting from that of a planned economy strongly conditioned by the state. De Finetti directed his most penetrating criticism toward the mechanics implicit in neoclassical theory, both for theoretical reasons and for the difficulty of coping with the economic problems of his time.

De Finetti's criticism of general economic equilibrium theory was based mainly on three critical issues. The first, which we will analyze in detail, concerns the problem of the nonuniqueness of Pareto's optimum position. The second refers to the classic issue of the impossibility of establishing a Pareto optimum when production is characterized by increasing returns; in fact, in such a situation, in equilibrium, marginal costs diverge from average costs, and it is always possible to have an exogenous, redistributive intervention that increases social welfare. De Finetti derived this argument from a long chapter in Spirito's 1930 *Critica del liberismo economico*, titled "I sofismi dell'economia pura," which adoped this point from Enrico Barone's 1925 *Principii di economia politica*. The third concerns the existence of relevant effects of externality in the economic system. In this case, the system of competitive prices is unable to capture all the advantages of exchange, and it is necessary for the state to intervene.

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It is easy to see how these three critical aspects differ in their epistemological status. While the first is related to the invariant core of economic theory, the second and third deal with the issue of realism. It is not a coincidence that the latter two, in general, can be eliminated simply by hypothesis. Textbooks that deal with general economic equilibrium theory assume decreasing returns to scale and no effects of externality (Starr 1997). These assumptions set aside the two problematic issues and put off their empirical relevance to a later verification. However, the problem of the nonuniqueness of the position of optimum cannot be simply ignored because it belongs to the essential part of the general economic equilibrium theory. De Finetti concentrated on this aspect, which is also the field where he made his most interesting contribution to economics with the application of the theory of simultaneous maxima.

## 4. De Finetti's Theory of Simultaneous Maxima

De Finetti's criticism of Pareto's general equilibrium theory is based on the mathematical theory of simultaneous maxima, which is his contribution to the field of pure theory. Much later this topic of simultaneous maximization would be considered by Stephen Smale (1974, 1976) and applied to the study of general equilibrium. This approach is very general and de Finetti considers also some examples obtained from the physical sciences, as the case of maximization the illumination of a plane by a spotlight. The criticism of identification between free competition and the social optimum was one of the possible applications to economics. The starting point was a problem that remained open in Pareto's system. In fact, from a given initial allocation, the Pareto optimum position could not be determined uniquely, there could be more than one optimum position, as in the well-known case of the Edgeworth box. Pareto considered this problem, especially in the discussion of the barter system, but it did not have a central position in his welfare analysis of the appendix of *Manuale* ([1909] 2014). Pareto introduced his criterion mainly to analyze institutional settings, as in the case of free competition. This aspect did not attract the attention of Paretian economists. In the *Lezioni di economia matematica* by Amoroso (1921: paragraph 39) or in the *Lezioni di economia razionale e sperimentale* by Alfonso de Pietri Tonelli (1921: 260–75), the main textbooks of Pareto's followers, the issue of nonuniqueness is only incidentally mentioned. This problem had no solution within the Pareto approach because the utilities of each individual are not comparable, as they are in

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traditional utilitarianism. The simultaneous maxima theory advanced by de Finetti offered a radical and definitive solution to the problem of general equilibrium indeterminacy in the sense that he demonstrated the impossibility of a unique solution in the very general case.

In his first 1937 article "Problemi di 'Optimum,'" de Finetti dealt with a very general context, which he defined as simultaneous maximization. This type of optimization differs from traditional constraint optimization, which is the common case considered by economists, because the problem is to obtain the maximum values of many functions at the same time in a particular way. These maximum results are simultaneous in the sense that it is not possible to increase the value of a function without decreasing that of another. The analogy with the case of the Pareto optimum—whose simultaneous maximization constitutes a generalization—is evident. De Finetti's theory is one of the first applications of the vector calculus in mathematical economics. For the sake of simplicity we consider the simplified case of two functions in two variables, $f(x,y)$, $g(x,y)$.

De Finetti starts from the assumption that to obtain a solution of simultaneous optima, it is necessary for the total differentials of the two functions to cancel out; otherwise, it would be possible to increase the value of one of the two without decreasing that of the other. In our simplified case, the following expressions must not be greater than zero:

$$df = f_x dx + f_y dy \tag{1}$$
$$dg = g_x dx + g_y dy$$

because if all equations in the system 1 are positive, no point of coordinates *x, y* can be the optimal one. For both differentials of 1 to be null, it is necessary that the determinant of the coefficients $dx,dy$ cancels out. Therefore, the following must occur:

$$\begin{vmatrix} f_x & f_y \\ g_x & g_y \end{vmatrix} = 0 \tag{2}$$

This equation represents the curve that contains the optimum points. De Finetti concluded this first step by oserving how, in general, the points of the simultaneous maxima belong to a manifold of $n - 1$ dimensions, on which the determinant of the matrix of partial derivatives of the functions to maximize is zero.

De Finetti then took an additional original step with the aim of offering an operational version of the condition of simultaneous maxima. If the

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determinant of 2 is zero, it follows that equations 1 will be represented by a linear relationship. In this case, we can find two coefficients $\lambda_1$, $\lambda_2$ connected in the following relationship:

$$\lambda_1 df + \lambda_2 dg = 0 \tag{3}$$

Substituting equation 1 in equation 3, and rearranging in term of *dx* and *dy*, the system obtained is as follows:

$$\begin{aligned} \lambda_1 f_x + \lambda_2 g_x &= 0 \\ \lambda_1 f_y + \lambda_2 g_y &= 0 \end{aligned} \tag{4}$$

De Finetti showed that in order to verify that a specific arbitrary vector represents a position of simultaneous maxima for *n* functions, two conditions must be met: the determinant of the Jacobian matrix must be zero and the parameters, $\lambda_1, \lambda_2$, must all be of the same sign. These are necessary conditions that become sufficient as soon as some restrictions are added, such as concavity. As de Finetti (1937a: 56) stated: "The optimal point belongs to a manifold of $n - 1$ dimensions, for which the determinant of the partial derivatives cancels out. Knowing the value of the n cofactors, $\lambda_1, \lambda_2, \ldots, \lambda_n$ we can exclude that it is a point of optimum if two of them have opposite signs."

De Finetti analyzed also some topological aspects of the solutions obtained. He identified the entire set of optimal points, starting from the evident property that each point of the maximum of one of the *n* functions, given the value of the others, resulted in an optimal point. Since this property was true for all optimal points related to *n* functions, the conclusion was the topological hypothesis that the set of optimal points formed a simplex of the $n - 1$ dimensions, whose face was composed of the optimal points of a function with $n - 1$ components, the $(n / 2)$ corners for those of $n - 2$ components, and so on, up to the *n* vertexes, each one representing the maximum of one of the *n* functions. This topological analysis of the positions of simultaneous optima occupied a significant part of de Finetti's 1937 essay, demonstrating the high level of mathematics involved.

## 5. From Mathematics to Economics

De Finetti (1937b) considered the application of the theory of simultaneous maxima to economics in the companion paper, "*Problemi di optimum vincolato*" (Problems of constrained optima). For the analysis of constrained optima in the general case of economic theory, he observed: "A

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particularly noteworthy case of constrained 'optimum' is what one might call the allocation problem, which constitutes the simplest 'optimum' problem of economics" (18). De Finetti dealt only with the case of consumption and showed that the mathematical structure of the problem remained the same. The only difference was that, in this case, the matrix of the first derivatives 2 had to be augmented in order to consider the existence of the resource constraint. The final result did not change: the maximum obtained was merely one of an infinite number of Pareto optima, among which the economic system had to choose.

In the simple case of consumption of two goods and two agents the solution required the following conditions:

$$\frac{f_x}{f_y} = \frac{g_x}{g_y} \tag{5}$$

Equation 5 expressed the familiar condition of equality of the marginal rates of substitution between two goods. De Finetti noted that equation 5, which characterized the optimal allocation could easily be extended to the case of *m* goods and *n* consumers, a condition in which all the rows of the matrix 2 were proportional to one another. In de Finetti's (1937b: 20) words: "The conclusion constitutes, in the case of the economic allocation problem, the classical result of Jevons-Walras, basis of their masterly treatment of Vilfredo Pareto." From a historical point of view, we can say that the theory of simultaneous maxima led de Finetti to express the marginal conditions of optimal allocation of the resources, albeit in an implicit form and considering only the side of consumption, one year before the classical contribution by Bergson in 1938.

De Finetti was not interested in deepening his analysis of the other marginal conditions that characterized the general equilibrium, considering, for example, the side of production. He emphasized the nonuniqueness in the consumption of the optimal position, concluding that the identification between the Pareto optimum and free competition would be purely arbitrary, in the sense that anarchic market forces reached only one of the many positions that had this property. He believed that he had revealed the logical weakness of economic liberalism:

> This fundamental theorem, under the hypotheses that are necessary to establish it rigorously, therefore has an indisputable validity, but it is not legitimate to interpret it in a more concrete sense with the consequence that liberalism leads to an *optimum*; and even if that were true, it would be necessary to observe that there is not a single point of *optimum*, but an infinity of optima.

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> We demonstrate that normally in the case of n individuals, the points of optimum are $\infty^{n-1}$. Suppose set the ophelimities $\Theta_1 = a_1$, $\Theta_2 = a_2$, . . . ., $\Theta_{n-1} = a_{n-1}$ of $n - 1$ individuals; on the variety so defined, the $\Theta_n = a_n$ will admit a maximum value, and therefore at least a point of optimum. Of such points there are at least $\infty^{n-1}$; they actually constitute a manifold of $n - 1$ dimensions. (De Finetti 1937b: 552)

Therefore the theory of simultaneous maxima as applied to welfare economics led to a negative result, that is, the impossibility of determining the social optimum through the ordinary instruments of economic analysis.

In order to solve the problem of defining a single optimum for the society it was necessary to develop a different research strategy. De Finetti tackled this problem in his 1943 article, "La crisi dei principi e l'economia matematica" (The crisis of principles and mathematical economics), his last relevant contribution to the field of economic theory. The article opened, as was usual for de Finetti, with a sharp criticism of the individualistic and laissez-faire approach of classical (and neoclassical) economists. After a brief clarification of the previous mathematical results, de Finetti advanced a possible solution to the problem of the determination of the social optimum, the maximum maximorum. He proposed introducing a collective preference function (*funzione di preferibilità collettiva*). This was not a new idea in the Italian context. Bordin, in an article about Cournot (1939), had already suggested introducing a function of collective welfare, meant as an expression of the government's preferences. De Finetti formulated the same approach in a more rigorous way:

> The criteria of preference $\Phi_h$, that can be set in a certain ethical social system, will consist in making the situation of each single individual as preferable as possible and the circumstances directly involving the society as preferable as possible.
>
> These collective needs can give place to more functions of preference, $\Phi_1, \Phi_2, \ldots, \Phi_n$, each one concerning, for example, the preference towards the interests in the army, the navy, the air forces, etc., but can be summarized—as we will assume—in a single function of collective preference $\Phi_2 = F(\Phi_1, \Phi_2, \ldots, \Phi_n)$ that summarizes the preference of a body (government) that coordinates and manages the various collective needs. (de Finnetti 1943: 43)

In this article, de Finetti did not develop the issue further. He simply observed that this social preference function could be based on individual

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preferences or could be expressed by the choices made by the political power. De Finetti openly opted for the second possibility. In his view, only the strong intervention of the state in the economy could correct some of the economic evils of his time. He made no mention at all in the article of the social welfare function advanced by Bergson a few years before.

After the Second World War, de Finetti returned several times, although briefly, to this topic. On many occasions he tried to specify the main elements of his welfare approach. His conclusive evaluation can be found in his 1969 essay, "L'apporto della matematica allo sviluppo del pensiero economico" (The contribution of mathematics to the development of economic theory). In the central part, de Finetti clarified that, in introducing the function of collective preference, his aim was to overcome the false dichotomy of the descriptive and normative aspects of economic analysis implicit in the notion of Pareto's optimality. He argued that it was necessary to "renew Paretian theory in the normative dressing" (de Finetti 1969: 218).[4] To do so, the starting point was still to acknowledge that the Paretian principle of optimality was only a necessary condition for evaluating an economic system. He insisted on this point:

> It is important to clarify again this aspect. Paretian optimum is just a necessary condition for a point to be maximum, but this fact does not mean that it is "good," while the use of it shows that economists often undergo this tendency. It would be like thinking that the points where the first derivative is zero should be preferable to others, for those who want a great value, even if they are point of minimum or inflection points. (220)

Moreover, he clarified some features of his function of collective preference with respect to other formulations, for example that proposed by Bergson and Samuelson. His collective function was not based on individual preferences or utilities. On the contrary, for de Finetti, the social welfare function indicated the aims pursued by society, expressed through the evaluation of political institutions. He observed: "the main characteristic of the introduction of a welfare function consists in establishing, according to criteria of general interest or of another nature beyond individual egoisms, a preference among the various distributions which, according to prevailing concepts, should remain excluded from the ambit of economic investigations" (de Finetti 1969: 221).

4. A general discussion of the normative turn after the Second World War is offered in Erickson et al. 2013.

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De Finetti's approach presents two main features. First, it did not, by definition, suffer the problem of needing to aggregate individual preferences. De Finetti was not interested in this aspect, and he entirely accepted the conclusions of the Arrow impossibility theorem (de Finetti 1969: 220). In his view, the domain of his social welfare function was represented by the main values shared by society in particular historical phases. Second, de Finetti insisted on the necessity of determining an operational form of this collective function. From this point of view, de Finetti's position was very close to that of Ragnar Frisch and Jan Tinbergen (de Finetti 1965), developed in the postwar period. Frisch introduced the concept of "preference political function" to indicate that quantitative economic models need some welfare criterion to evaluate the virtue of different economic policy choices (Long 1979). Tinbergen's (1952) social welfare function was constructed from an ethical principle whose legitimacy was a matter of assumption rather than debate. For de Finetti, as for Frish and Tinbergen, the social welfare function was instrumental to economic planning.

From the point of view of the evolution of economic ideas, it is interesting to note that this approach is not as far from the Paretian perspective as de Finetti assumed it to be. Yet it is not the Pareto economist but the Pareto sociologist who is worth considering. In the last part of his *Treatise of Sociology* (1916), Pareto discussed the problem of a comparative evaluation of the various states in which a society can find itself. Pareto introduced the fundamental distinction between a maximum utility *for* the society and a maximum utility *of* the society. In the former case, the society is considered from an atomistic viewpoint—as a set of molecules using a physical metaphor—and the optimal position is reached when an increase in the utility of an individual cannot be obtained without causing a detriment to others. The case of the maximum utility of the society is different. In this hypothesis, the society is treated as a single entity, and Pareto's solution was to entrust governments with the task of establishing the characteristics of a hypothetical function of social utility (Tarascio 1969). Pareto stated the following:

> Suppose a community whose conditions are such that the only choice is between having a very rich community with great inequality of revenues among its components, or a very poor one with an income basically the same for all. The search for the maximum utility of the community can lead to the first state, while the research of the maximum for the community to the second. And it is so because the effect depends on the coefficients used to make homogenous the heterogeneous utilities of the various social classes. The admirer of the "superman" will

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> assign a coefficient of approximately zero to the utility of the lower classes, and get a point of equilibrium very close to a state where larger inequalities prevail. The lover of equality will assign a high coefficient to the utility of the lower classes, and get a point of equilibrium very close to the equalitarian conditions. (Pareto 1963: 208–9)

More specifically, what Pareto had in mind was a linear social welfare function in which the weights assigned to each welfare element represent assessments of the government (Bergson 1983). De Finetti could agree with this normative way of approaching the general problem of evaluating what is the best position for the society.

## 6. Simultaneous Maxima and the Theory Of Oligopoly

The conclusion of the Second World War marked a watershed in Italian economic thought. With the fall of the fascist regime, corporate economic theory was quickly set aside and young Italian economists turned to the most recent development of economic theory in the international debate. This also happened in the field of welfare economics. Basically, two research approaches emerged, defined as the British approach and the American approach (Baujard 2016). The British approach, essentially coming from the London School of Economics, proposed a new criterion that considered the possibility of hypothetical compensation. The American approach formalized the concept of social welfare as a function of the individual utilities that each individual derives from the social state. The Italian economists took an eclectic position, as witnessed by the long reviews (Lombardini 1954; Caffè 1956), and did not produce an original position. Among the Paretians, there was an attempt to propose the general scheme of an economy regulated according to public purposes in a long essay of 1948, "La convenienza economica collettiva." In it La Volpe tried to formalize the widespread idea among the Italian economists that the collective welfare could not be evaluated on an individual basis but, instead, needed to be judged in relation to the public aims that the community intended to pursue. In the Italian debate on welfare economics, we no longer find any reference to de Finetti's mathematical contribution.

The approach of simultaneous maxima would have passed unnoticed if it had not been adopted by another Paretian economist, Emilio Zaccagnini, who applied it also to a different field of economic theory, oligopoly theory. With a degree in mathematics, Zaccagnini belonged to the third Paretian generation. He followed the Paretian approach of his teacher,

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Arrigo Bordin, in Torino. His first contributions were wholly orthodox, dealing with barter theory and the problem of obtaining the demand function from the preference relations. Subsequently, the main focus of his research was on the application of de Finetti's simultaneous maxima theory to some topics of economic theory, in particular oligopoly theory. In several articles, Zaccagnini (1947, 1953, 1958) attempted to offer the theory of simultaneous maxima as a general methodology for studying many economic phenomena. His main contribution was the 1947 article, "Massimi simultanei in economia pura," which was included in the first edition of *International Economic Papers* (1951) under the title "Simultaneous Maxima in Pure Economics." It is interesting to consider this contribution in order to evaluate the application of de Finetti's approach to economics outside the field of welfare theory. This article contained also a long introductive part in which Zaccagnini illustrated in detail the mathematical theory of simultaneous maxima in the case of three functions and three variables. For our purposes, we limit our attention to the theories of duopoly and oligopoly. Zaccagnini's project was to return to Pareto via the mathematical lens offered by de Finetti.

His starting point was the traditional criticism of the lack of realism of Cournot's approach to duopoly. This criticism was widespread and had reached its peak in the 1930s (von Stackelberg 1934). The problematic point was the fundamental assumption that in the process of maximizing choice, for each of the two firms considered, the production of one would consider constant that of the other. Zaccagnini (1947: 262) observed:

> From this point of view, Cournot's hypothesis clashes with common sense and daily observations. It is evident that without experience it is not possible to conceive of any rational economic operation; how it is possible to admit that this experience teaches nothing to the two single firms in the market and it allows them to assume the Cournot hypothesis about the assumption of each operator regarding the quantity negotiated by the rival?

In addition, in Zaccagnini's view, Bowley's solution of conjectural variations in the case of duopoly could not be considered as an adequate answer, as it introduced a psychological element that was extraneous to economic reasoning. Zaccagnini recognized the need to return to Pareto's position and revise it in light of de Finetti's new mathematical contribution:

> To Cournot's solution and to these criticisms, Pareto opposed an important observation which in our opinion is decisive for the general approach to the problem. In fact, writes the ingenious scientist, the two quantities

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> $q_1$ and $q_2$ are both variables and have to be considered in the two total profit functions to be maximized. The solution requires considering four partial derivatives. In this case, the number of variables is greater with respect to the number of equations and the problem is indeterminate. (Zaccagnini 1947: 264)

Moreover,

> The second solution originates from Paretian criticism and it is really general, as it excludes any subjective hypothesis of an operator on the behavior of the other, and considers only the hedonistic postulate in its more general expression and the simultaneity of the solutions. But it was the technical means adopted by Pareto that were unsuitable and that led the Author to an erroneous conclusion: it is not possible to resolve the two maxima separately when they behave simultaneously. By applying a proper mathematical technique, the problem does not seem determined, as was the case with Cournot, nor was it impossible, as Pareto stated. (266)

In Zaccagnini's view, the tool that could allow for overcoming the analytic impasse in Pareto's theory was the application of de Finetti's simultaneous maxima methodology. With this approach it was possible, on the one hand, to maintain the Paretian assumptions and, on the other hand, to overcome the problem of the modest interpretative capacity of Cournot's perspective, which had never been quite convincing to economists in the interwar period. Zaccagnini treated the classic case of duopoly without cost functions and with a linear demand curve, comparing the solution of the simultaneous maxima with that of Cournot. Directly applying de Finetti's methodology (and hence equation 3), Zaccagnini obtained the following expression for price:

$$p = -p'(q_1 + q_2) \tag{6}$$

Equation 6, as Zaccagnini observed, was not new; it corresponded to the case in which firms maximize their joint profits. The price depends on the derivative of the demand function and total production, while the level of production of each firm remains indeterminate. As Zaccagnini (1947: 268) concluded: "To determine the equilibrium between $q_1$ and $q_2$ it is necessary to: 1) abandon the static hypothesis of simultaneity and thus admit a particular sequence of actions and reactions based on specific assumptions of each operator on the behavior of the rival; or 2) impose additional conditions that cancel the indeterminacy of the problem (arbitration, hedonistic strength, etc.)."

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In equation 6, the old non-uniqueness problem of the equilibrium solution reappeared, but this fact did not seem to be a problem for Zaccagnini. The final result was not a single equilibrium position but an entire path depending on the exogenous choice of firms, as in Bertrand's case. From a geometric point of view, the equilibrium points identified by equation 6 could be found on the tangency of the isoprofit curves of duopolists. This equilibrium differed from that of Cournot, with higher prices and lower quantities produced. However, the problem of the distribution of production between the two firms remained unresolved, and, as in Pareto, it could not been solved using pure economic theory. Therefore, for Zaccagnini as before for Pareto, the theory of oligopolistic markets remained an open theory depending on institutional, sociological, and historical elements. In the following years, Zaccagnini attempted to extend his interpretation of simultaneous maxima in various directions, especially in the case of a socialist economy and the labor market (Zaccagnini 1958).

The application of the simultaneous maxima approach to economic problems had a mixed result in the Italian context. The oligopolistic theory was rejected for its lack of rationality. This conclusion was put forward by Claudio Napoleoni in a survey of the oligopoly theory published in the *Dizionario di Economia* (1956). In his view, the idea implicit in the simultaneous maximization approach, that is, the assumption that the oligopolistic firms could behave as monopolists, was inconsistent. Instead, he fully accepted the idea that the free market solution of the general equilibrium problem was only one of the possible results based on a specific intepretation of the economic process. In Napoleoni's words: "One can therefore say that perfect competition determines a configuration of equilibrium which is one of the optimal configurations (in the Paretian sense)" (Napoleoni 1956: 429). De Finetti's theory of simultaneous maxima was a way to prove this result in a rigorous form (Zaccagnini 1958).

## 7. A Final Assessment

De Finetti was a critical and original scholar. At the beginning of his research, he tried to investigate the theoretical and practical problems in the Paretian theory of general equilibrium from the point of view of society. De Finetti's theory of simultaneous maxima can be considered as one of the most relevant mathematical contributions to the static general equilibrium theory in the Italian tradition between the two world wars. The two articles that de Finetti published in 1937 clarified the range and limits

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of the idea of the Paretian optimum, one year before of the publication of Bergson's fundamental article. De Finetti aimed to demonstrate that free competition was one of the many ways to take the economy toward an optimal condition for the society in Pareto's sense. Given the dramatic economic situation of the 1930s, it is easy to understand why the first theorem of welfare economics appeared not only as an empty abstraction for many economists, but also as a theory that offered wrong recipes for economic policy. From this point of view, de Finetti was very critical of neoclassical economics and argued that Pareto's optimal points could be very bad for society. Following his intellectual path he converged toward a social welfare function. The arguments of this function were the social values expressed by the society throughout some form of political power. The theory of simultaneous maxima opened the door to corporatist economics. In de Finetti's view, as in his teacher Sipirito's, economics was not a positive and neutral science.

His theory did not arouse a great deal of interest in the period after the Second World War, even in the Italian context. There are several reasons that can help to explain the marginalization of the de Finetti's contribution. First, his articles were published in an actuarial journal not well known among Italian economists. Second, in the Italian context the troubles with the static general equilibrium were not analytical difficulties but the lack of its interpretative power (for example, Dei Nardi 1941; Bordin 1950). With the exception of the mathematician Zaccagnini, the internal difficulties of static general equilibrium as a mathematical object were neglected. On the international level, the axiomatic approach produced a new theoretical lift for propositions that were already widely demonstrated within the limits of the hypotheses assumed (Lange 1942). Economic equilibrium metaphysics, as de Finetti sometimes defined the general equilibrium theory (de Finetti 1969: 219), regained its place and became stronger than before because it was supported by new and refined analytical tools. Moreover, de Finetti himself changed his research direction and did not contribute to the developments of the debate on planned economies from the point of view of economic welfare. Also, his interpretation of the theory of simultaneous maxima in terms of corporatist theory contributed to overshadowing de Finetti's mathematical contribution. Lastly, from a strict mathematical point of view the theory of simultaneous maxima could be considerated as a special case of the more general method of constrained maximization.

The application of the oligopoly theory by Zaccagnini illuminated other and more fundamental aspects of the simultaneous optima theory

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that can explain why this approach, albeit mathematically rigorous, had been abandoned during the Second world War. As we saw when considering the case of oligopolistic theory, the simultaneous maxima approach did not resolve the problem of the non-uniqueness of the solution found in the process of maximization. In general, this approach is characterized by the existence of many solutions. In this perspective, economic reasoning is not a closed system but an open system influenced by sociological, historical, or ethical elements. This view was common among the Italian economists who considered it necessary to supplement economic reasoning with other evaluations to make the economic theory more realistic and useful for interpreting economic phenomena. However, during the postwar period, a completely different approach emerged at the international level, according to which the lack of uniqueness was a fatal flaw in the mathematical modeling of economic phenomena.

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