Distributed characterization of coupling in multimode and multicore fibers

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***Abstract*—Distributed characterization of mode coupling is key to understanding the behavior of multimode and multicore fibers. This paper presents a theoretical framework that precisely assess limits and potentialities of mode-selective distributed measurements based on Rayleigh backscattering.**

***Index Terms*—mode coupling, SDM, distributed, Rayleigh scattering**

I. INTRODUCTION

Spatial division multiplexing (SDM) is the new paradigm in optical fiber communications proposed as a solution to the foreseen *capacity crunch* of backbone networks; at the base of this new technology are few-mode and multi-core optical fibers (FMF and MCF, respectively) [1]. An intense research activity is focused on the propagation properties of these waveguides, whose characteristic traits are mode coupling and the related modal dispersion [2], [3]. Owing to these phenomena, FMFs and MCFs behave like channels affected by multi-path interference [4], and while in general mode coupling can be seen as a detrimental effect [5], it has an important beneficial role when optical nonlinearities come in to play [6]. In this perspective, an accurate knowledge of mode coupling is key to an effective description and clear understanding of propagation in FMFs and MCFs.

Being a local property, mode coupling has to be investigated with distributed techniques similarly to what has been done for PMD in single-mode fibers [7]. To this aim, preliminary experimental and theoretical works have already shown how the analysis of Rayleigh-backscattered light can provide information at least on some parameters, such as power coupling, modal birefringence and differential modal delay [8]–[10]. In this perspective, we present here a theoretical analysis showing how, under proper assumptions, backscattering measurements can actually provide an almost complete distributed characterization of the transmission Jones matrix.

II. THEORETICAL ANALYSIS

We describe light propagation along the optical fiber by the N-dimensional vector $\bar{a}(z)$, whose elements are the complex amplitudes of the N propagating modes (counting both polarization and spatial ones); note that this model can be used both for FMFs and MCFs. The first assumption we make is that there are no mode-dependent losses, so $\bar{a}(z) = \mathbf{F}(z)\bar{a}(0)$ where $\mathbf{F}$ is the unitary $N \times N$ Jones matrix describing forward propagation in the fiber. We can also write $\partial_z \bar{a} = -j\mathbf{K}\bar{a}$, where ∂_z represents derivative with respect to z and $\mathbf{K}$ is the Hermitian *coupling matrix*, which describes all the coupling properties of the fiber. Ideally, we would like to be able to determine $\mathbf{F}(z)$ or, equivalently, $\mathbf{K}(z)$, which would be possible if we were able to measure $\bar{a}(z)$.

The work is partly supported by the Italian Ministry for Education, University and Research (MIUR, "Departments of Excellence"—law 232/2016, and PRIN 2017—project 2017HP5K) and by the University of Padova (BIRD 2019—project MACFIBER).

A. Round-trip propagation

While with cannot measure the light forward propagating inside the fiber, we can measure the light backscattered at the fiber input by Rayleigh scattering using techniques analogous to those used for the transmitted light [11]. Let $\bar{b}(z)$ be the N-dimensional complex vector describing the light backscattered by the point z along the fiber. The relationship between $\bar{a}(z)$ and $\bar{b}(z)$ is in general involved and depends on the fiber dispersion. However, assuming that this can be neglected, we can write $\bar{b}(z) = \mathbf{B}(z)\mathbf{F}(z)\bar{a}(z)$, where $\mathbf{B}$ is the unitary matrix representing backward propagation from z to the fiber input and we used also the fact that Rayleigh backscattering can be described, to a quite good approximation, by the identity matrix (neglecting its intrinsic losses, immaterial to the present analysis). The last hypothesis we make is that the fiber is reciprocal, so that $\mathbf{B} = \mathbf{F}^T$ [7].

It can now be shown that $\partial_z \bar{b} = -j\mathbf{K}_B\bar{b}$ with

$$\mathbf{K}_B(z) = 2\mathbf{B}(z)\mathsf{Re}[\mathbf{K}(z)]\mathbf{B}^\dagger(z), \tag{1}$$

where $\mathsf{Re}[\cdot]$ is the real part and † the transpose conjugate. Therefore, since we can measure $\bar{b}(z)$, we can calculate $\mathbf{K}_B$, but the relationship between this matrix and $\mathbf{B}$ and $\mathbf{K}$, the quantities we are interested in, seems quite involved. Nevertheless, note that

$$\partial_z \mathbf{B} = -j\left[\mathbf{B}(z)\mathbf{K}^T(z)\mathbf{B}^\dagger(z)\right]\mathbf{B}(z)\,, \tag{2}$$

so that if $\mathbf{K}$ were real, we could calculate $\mathbf{B}$ by solving

$$\partial_z \mathbf{B} = -j\tfrac{1}{2}\mathbf{K}_B(z)\mathbf{B}(z) \qquad (\text{if } \mathsf{Im}[\mathbf{K}] = 0)\,. \tag{3}$$

B. The role of the imaginary part of $\mathbf{K}(z)$

The elements of $\mathbf{K}(z)$ can be calculated with the coupled-mode theory, which shows that only two physical phenomena contribute to the imaginary part of $\mathbf{K}$; these are Faraday rotation and twisting the fiber [2]. The first effect has been already ruled out when we assumed reciprocity. Regarding the second, we can straightforwardly conclude that when the fiber is not twisted, (3) holds and so $\mathbf{B}(z)$ can be calculated from the measured values of $\mathbf{K}_B(z)$, solving our problem. We can reach, however, a more general conclusion.

Start by considering the transformation $\mathbf{T}(z)$ of the N-dimensional Jones space given by

$$\partial_z \mathbf{T} = \mathsf{Im}[\mathbf{K}^T(z)]\mathbf{T}(z)\,, \qquad \mathbf{T}(0) = \mathbf{I}\,; \tag{4}$$

it can be easily proved that since $\mathbf{K}$ is Hermitian, $\mathbf{T}$ is orthogonal (i.e. $\mathbf{T}\mathbf{T}^T = \mathbf{I}$). With respect to this new reference frame, the forward propagating field is described by the vector $\hat{a}(z) = \mathbf{T}^T(z)\bar{a}(z)$ that obeys the equation

$$\partial_z \hat{a} = -j\mathbf{T}^T\mathsf{Re}[\mathbf{K}]\mathbf{T}\hat{a}(z) = -j\mathbf{K}_A(z)\hat{a}(z)\,; \tag{5}$$

notice that $\mathbf{K}_A(z)$ is real by construction.

Another important fact is that $\bar{b}(z)$ is invariant under the transformation $\mathbf{T}$. Indeed, $\hat{a}(z) = \mathbf{T}^T(z)\mathbf{F}(z)\bar{a}(0)$ and hence the forward-propagation matrix in the transformed space is $\hat{\mathbf{F}}(z) = \mathbf{T}^T(z)\mathbf{F}(z)$; therefore, $\hat{b}(z) = \hat{\mathbf{F}}^T(z)\hat{\mathbf{F}}(z)\bar{a}(0) = \bar{b}(z)$, where the last equality is a trivial consequence of the orthogonality of $\mathbf{T}$.

C. Physical interpretation

The above mathematical results have a clear physical interpretation. To begin with, it can be shown that the transformation $\mathbf{T}(z)$ corresponds to rotating the physical reference frame around the fiber axis by the same angle of rotation induced by the twist on the modes [7]. Thus, the mathematical invariance of $\bar{b}(z)$ is just representing the physical fact that $\bar{b}(z)$ is measured at the fiber input, where the reference frame is not changed since the twist-induced rotation is still zero ($\mathbf{T}(0) = \mathbf{I}$). Moreover, owing to (5), we can conclude that, from the point of view of the backscattering measurement, the real fiber with coupling matrix $\mathbf{K}(z)$ is equivalent to a fiber with the apparent coupling matrix

$$\mathbf{K}_A(z) = \mathbf{T}^T(z)\mathsf{Re}[\mathbf{K}(z)]\mathbf{T}(z). \tag{6}$$

Since $\mathbf{K}_A$ is real, we can use (3) to calculate the backward-propagation matrix $\hat{\mathbf{B}}(z)$ with respect the rotate reference frame and, finally, use (1) to calculate

$$\mathbf{K}_A(z) = \tfrac{1}{2}\hat{\mathbf{B}}^\dagger(z)\mathbf{K}_B(z)\hat{\mathbf{B}}(z)\,. \tag{7}$$

The matrix $\mathsf{Re}[\mathbf{K}]$ represents the effects of linear birefringence; in particular, its eigenvectors are the local modes of propagation, whereas the eigenvalues are the corresponding propagation constants. Given that $\mathbf{T}$ is orthogonal, $\mathsf{Re}[\mathbf{K}]$ and $\mathbf{K}_A$ share the same eigenvalues, while their respective eigenvectors are related by $\bar{v} = \mathbf{T}\bar{v}_A$.

We can therefore conclude that, by measuring $\bar{b}(z)$ we can calculate $\mathbf{K}_A(z)$ and hence the propagation constants of the local modes along the optical fiber; regarding the eigenvectors describing these local modes, they can be measured up to the unknown rotation represented by $\mathbf{T}(z)$, which is due to the possibly present twist. This is an intrinsic limit of distributed measurements and is the N-dimensional generalisation of what happens for polarization birefringence in single-mode fibers, where the birefringence orientation and the apparent rotation due to twist cannot be distinguished [12].

III. Concluding Remarks

The theoretical analysis presented in this work shows that distributed measurements can provide an almost complete characterization of the complex transfer matrix of the fiber as a function of distance, with a residual uncertainty related to twist. This can be however removed by carefully preparing the experiment to avoid twist or by comparing measurements taken in different and known twist conditions.

The theory is based on three assumptions. The first two are absence of mode dependent loss and reciprocity. These are not critical, since FMFs and MCFs typically have negligible mode dependent loss and only strong magnetic fields can break reciprocity; in any case, the theory can be generalized also to nonreciprocal fibers. Differently, the third assumption–i.e. negligible modal dispersion, is more critical and limits the applicability of the proposed method to short samples of low-dispersion fibers. For this reason, graded-index FMFs and coulped-core MCFs seem the best candidates for successful experimental tests.

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