An h–φ Cell–Method Formulation for Solving Eddy–Current Problems in Multiply–Connected Domains Without Cuts

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***Abstract*—A novel h–φ formulation for solving time–harmonic eddy current problems is presented. It makes it possible to limit the number of degrees of freedom required for the discretization likewise T–Ω formulation, while overcoming topological issues related to it when multiply connected domains are considered. Global basis functions, needed for representing magnetic field in the air region, are obtained by a fast iterative solver. The computation of both source fields and thick cuts by high–complexity computational topology tools is thus avoided.**

***Index Terms*—Eddy currents, Finite element method, Cell Method, Multiply connected, Cut.**

I. Introduction

The $\mathbf{T}$–Ω method is known to be one of the most efficient methods for eddy–current problems [1]. This formulation requires however high–complexity algorithms for generating source fields and thick cuts. A novel $\mathbf{H}$–Φ formulation for multiply connected problems has been proposed in [2]. By generating topological bases with an iterative LSMR solver, cuts for Φ are avoided in [3]. Starting from this approach, an $\mathbf{h}$–φ formulation for eddy current problems is here presented.

II. h–φ Formulation

The computational domain $\Omega \subset \mathbb{R}^3$ is made of multiply connected conductive Ω_C and insulating Ω_I subdomains, with $\Gamma = \overline{\Omega}_C \cap \overline{\Omega}_I$. In order to avoid cuts, the key idea is that the reduced magnetic field in Ω_I splits as $\mathbf{H} = \nabla\varphi + \sum_{k=1}^{\beta} I_k\,\mathbf{H}_k$, where φ, I_k are not necessarily a magnetic scalar potential and currents, and $\mathbf{H}_k$ are curl–free fields, i.e., a cohomology basis in Ω_I, with possibly unconstrained circulations around Ω_C.

The time–harmonic diffusion equation of the magnetic field in Ω_C, discretized by the Cell Method (CM), becomes:

$$\left(\mathbf{C}^{\mathrm{T}}_{\Omega_C}\mathbf{M}_{\rho,\Omega_C}\mathbf{C}_{\Omega_C} + \jmath\,\omega\,\mathbf{M}_{\mu,\Omega_C}\right)\mathbf{h}_{\Omega_C} + \widetilde{\mathbf{C}}_{\Omega_C\Gamma}\,\widetilde{\mathbf{e}}_\Gamma = \mathbf{0}, \quad (1)$$

where $\mathbf{M}_{\rho,\Omega_C}$, $\mathbf{M}_{\mu,\Omega_C}$ are electric and magnetic constitutive matrices, $\mathbf{C}_{\Omega_C}$, $\widetilde{\mathbf{C}}_{\Omega_C\Gamma}$ are the (primal) edge–to–face and the boundary (dual) face–to–edge incidence matrices, and $\mathbf{h}_{\Omega_C}$, $\widetilde{\mathbf{e}}_\Gamma$ are the arrays of mmfs and emfs on primal and dual edges of Ω_C and Γ. The magnetostatic equation in Ω_I becomes:

$$\mathbf{G}^{\mathrm{T}}_{\Omega_I}\mathbf{M}_{\mu,\Omega_I}\mathbf{G}_{\Omega_I}\,\varphi_{\Omega_I} + \widetilde{\mathbf{D}}_{\Omega_I\Gamma}\widetilde{\mathbf{b}}_\Gamma + I\,\mathbf{G}^{\mathrm{T}}_{\Omega_I}\mathbf{M}_{\mu,\Omega_I}\mathbf{h}_{\Omega_I} = = -\mathbf{G}^{\mathrm{T}}_{\Omega_I}\mathbf{M}_{\mu,\Omega_I}\mathbf{h}_s, \quad (2)$$

where $\mathbf{M}_{\mu,\Omega_I}$ is the magnetic constitutive matrix, $\widetilde{\mathbf{D}}_{\Omega\Gamma_I}$ are the (primal) node–to–edge and the boundary (dual) face–to–volume incidence matrices, and $\widetilde{\mathbf{b}}_\Gamma$ is the array of magnetic fluxes through Γ. A key feature is that arrays $\mathbf{h}_{\Omega_I}$, i.e., mmfs of field $\mathbf{H}_1$ in Ω_I, and $\mathbf{h}_s$, i.e. mmfs of the source magnetic field, are computed by a fast iterative LSMR solver. $\mathbf{h}_{\Omega_I}$ is obtained by solving system $\mathbf{C}_{\Omega_I}\mathbf{h}_{\Omega_I} = \mathbf{0}$, where $\mathbf{C}_{\Omega_I}$ is the (primal) edge–to–face incidence matrix in Ω_I and a coefficient of $\mathbf{h}_{\Omega_I}$, corresponding to an arbitrary edge of Γ, is set to one. Complementing (1) and (2) with the magnetostatic energy balance in Ω_I and matching conditions on Γ yields a saddle–point matrix system to be solved by direct LU solver.

The method is validated with 3$^{\mathrm{rd}}$ ord. axisymmetric FEM. A torus shell Ω_C (5 mm inner radius, 5 mm thick, 4 cm long, $\mu_r = 2$ relative permeability, $\sigma = 25$ MS/m conductivity) is excited by a current–driven coil Ω_S (4 mm × 4 mm square cross–section, 15 mm radius, 10^6 A m^{-2} current density, 100 Hz frequency) centered on the shell axis. A very coarse mesh is used for 3–D CM (i.e. 5,867 tets for the shell, 213 tets for the coil, 11,811 tets for the air), whereas 2–D FEM is refined up to convergence. Topological field is solved in 7 ms by the LSMR solver. Fig. 1 shows that 3–D CM results are very accurate even with a coarse discretization (12,539 DoFs).

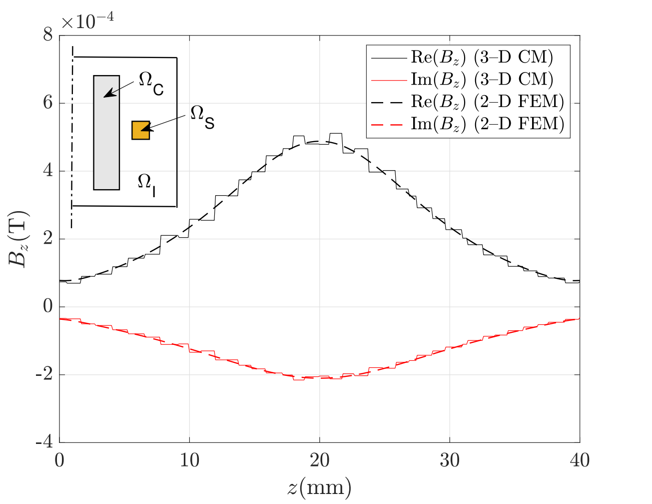

Fig. 1. *Real and imaginary parts of the magnetic flux density z–axis component along the vertical line $x = 4$ mm, $y = 0$, $z = [0, 40]$ mm.*

References

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