

# Three-body Description of $2n$ -Halo and Unbound $2n$ -Systems: $^{22}\text{C}$ and $^{26}\text{O}$

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We study the two-neutron correlations in the ground state of the weakly-bound two-neutron halo nucleus  $^{22}\text{C}$  sitting at the edge of the neutron-drip line and also in the unbound nucleus  $^{26}\text{O}$  sitting beyond the neutron dripline. For the present study, we employ a three-body (core +  $n$  +  $n$ ) structure model designed for describing the two-neutron halo system by explicit inclusion of unbound continuum states of the subsystem (core +  $n$ ). We use either a density-independent or a density-dependent contact-delta interaction to describe the neutron-neutron interaction and its strength is varied to fix the binding energy. We report the configuration mixing in the ground state of these systems for different choices of pairing interactions.

**KEYWORDS:** Two-neutron halo, Two-neutron unbound system, Two-neutron correlations.

## 1. Introduction

The new generation of radioactive ion beam facilities around the various parts of the globe has provided the access to the neutron-rich side of the nuclear chart. Due to this, there has been a rapidly increasing interest in the physics of the two-neutron ( $2n$ ) halo nuclei sitting right on the top of neutron driplines and decays of  $2n$ -unbound systems beyond the neutron dripline. These systems demand a three-body ( $3b$ ) description with proper treatment of continuum, the conventional shell-model assumptions being insufficient. The well established  $2n$ -halo  $^6\text{He}$  has long history of studies in  $3b$ -framework and thus it can be safely remarked that the understanding of  $3b$ -dynamics is fairly established for  $p$ -shell nuclei, whereas for the  $s$ - $d$  shell nuclei still the situation is in progress stage [1]. In the present study we consider two different  $s$ - $d$  shell nuclei, the  $2n$ -halo  $^{22}\text{C}$  and the  $2n$ -unbound system  $^{26}\text{O}$ . Very recently a high precision measurement of the interaction cross-section for  $^{22}\text{C}$  was made on a carbon target at 235 MeV/nucleon [2] and also the unbound nucleus  $^{26}\text{O}$  has been investigated, using invariant-mass spectroscopy [3] at RIKEN. The structural spectroscopy of the two-body subsystem plays a vital role in the understanding the  $3b$ -system. These high precision measurements and the sensitivity of the structural spectroscopy of subsystem with the structure of  $3b$ -system (core+ $n$  +  $n$ ), are the motivation for selecting these nuclei for the present study. We have studied the pairing collectivity in the ground state of the  $2n$ -halo  $^{22}\text{C}$  and in the the  $2n$ -unbound system  $^{26}\text{O}$ . For this study we have used our recently implemented  $3b$ -structure model (core+ $n$  +  $n$ ) for the ground and continuum states of the  $2n$ -halo nuclei [4–6]. We have explored the role of different pairing interactions such as density independent (DI) contact-delta pairing interaction and density dependent (DD) contact-delta pairing interaction with the configuration mixing in the ground-state of these systems.

## 2. Model Formulation

In our approach we consider a 3b-system consisting of an inert core nucleus and two valence neutrons, which is specified by Hamiltonian

$$H = -\frac{\hbar^2}{2\mu} \sum_{i=1}^2 \nabla_i^2 + \sum_{i=1}^2 V_{\text{core}+n}(\vec{r}_i) + V_{12}(\vec{r}_1, \vec{r}_2) \quad (1)$$

where  $\mu = A_c m_N / (A_c + 1)$  is the reduced mass, and  $m_N$  and  $A_c$  are the nucleon mass and mass number of the core nucleus, respectively. The recoil term is neglected in the present study, as  $A_c = 20$  and 24 are large enough to ignore it.  $V_{\text{core}+n}$  is the core- $n$  potential and  $V_{12}$  is  $n$ - $n$  potential. The neutron single-particle unbound  $s$ -,  $p$ -,  $d$ - and  $f$ -wave continuum states of the subsystem  $^{21}\text{C}$  and  $^{25}\text{O}$  are calculated in a simple shell model picture for the converged model parameter, bin width ( $\Delta E = 0.1$ ), by using the Dirac delta normalization and are checked with a more refined phase-shift analysis. These core +  $n$  continuum wave functions are used to construct the two-particle states of the core +  $n$  +  $n$  system by proper angular momentum couplings. We use a density-independent (DI) contact-delta pairing interaction for simplicity, and its strength is the parameter which will be fixed to reproduce the ground-state energy. We have also used a density-dependent (DD) contact-delta pairing interaction. For a detailed formulation one can refer to [4–6].

## 3. Two-body unbound subsystems (core + $n$ )

The investigation of the two-body (core +  $n$ ) subsystem is crucial in understanding the three-body system (core +  $n$  +  $n$ ). The interaction of the core with the valence neutron ( $n$ ) plays a vital role in the binding mechanism of the core +  $n$  +  $n$  system. The elementary concern over the choice of a core +  $n$  potential is the scarce experimental information about the core-neutron systems. We employ the following core +  $n$  potential

$$V_{\text{core}+n} = \left( V_0^l + V_{ls} \vec{l} \cdot \vec{s} \frac{1}{r} \frac{d}{dr} \right) \frac{1}{1 + \exp\left(\frac{r-R_c}{a}\right)}, \quad (2)$$

where  $R_c = r_0 A_c^{\frac{1}{3}}$  with  $r_0$  and  $a$  are the radius and diffuseness parameter of the Woods-Saxon potential.

### 3.1 $^{21}\text{C}$

For  $^{21}\text{C}$ , not much is known beyond that it is unbound. The only available experimental study using the single-proton removal reaction reported the limit to the scattering length  $|a_0| < 2.8$  fm and due to the low statistics of this experimental data at low energies, the possibility of low-lying resonance states can not be ruled out [7]. In the view of exploring the sensitivity of the core- $n$  potential to the possible resonances and configuration mixing in the ground state of  $^{22}\text{C}$ , very recently we examined in detail the four different potential sets (for details see text and Table 1 of Ref. [6]). Here we will discuss the results corresponding to potential Set-1, which is adopted from the literature [9], because of its acceptance by different 3-body models leading to good explanation of the observed properties for  $^{22}\text{C}$ . The subshell closure of the neutron number 14 is assumed for the core configuration given by  $(0s_{1/2})^2(0p_{3/2})^4(0p_{1/2})^2(0d_{5/2})^6$ . The seven valence neutron continuum orbits, i.e.,  $s_{1/2}$ ,  $d_{3/2}$ ,  $f_{7/2}$ ,  $p_{3/2}$ ,  $f_{5/2}$ ,  $p_{1/2}$  and  $d_{5/2}$  are considered in the present calculations for  $^{21}\text{C}$ .

### 3.2 $^{25}\text{O}$

In the recent measurement conducted at RIKEN [3], along with high accuracy measurement of ground state of  $^{26}\text{O}$ , they have also reported the  $d_{3/2}$  resonance state at 749(10) keV with width of 88(6) keV for  $^{25}\text{O}$ . This information will serve as input for fixing the core+ $n$  potential parameters. For  $^{25}\text{O}$ , we adopt the same value for the diffuseness parameter ( $a$ ) and the radius parameter ( $r_0$ ), as in Ref. [10]. For the Wood-Saxon depth parameter ( $V_0$ ) and the strength of spin-orbit potential ( $V_{ls}$ ) parameter tabulated in Table I, we use the information for the energy of unbound  $d_{3/2}$  state. Our parameters for  $V_0$  and  $V_{ls}$  are consistent with the one reported in Ref. [10]. The neutron number 16 is assumed for the core configuration given by  $(0s_{1/2})^2(0p_{3/2})^4(0p_{1/2})^2(0d_{5/2})^6(1s_{1/2})^2$ . The three

**Table I.** Parameter sets of the core- $n$  potential for  $l = 1, 2, 3$  states of a  $^{24}\text{O}+n$  system. The possible resonances with resonance energy  $E_R$  and decay width  $\Gamma$  in MeV are also tabulated.

$lj$	$r_0(\text{fm})$	$a(\text{fm})$	$V_0(\text{MeV})$	$V_{ls}(\text{MeV})$	$E_R(\text{MeV})$	$\Gamma(\text{MeV})$
$d_{3/2}$			-44.10	22.84	0.740	0.086
$p_{3/2}$	1.25	0.72	-48.67	22.84	0.570	1.382
$f_{7/2}$			-44.10	22.84	2.440	0.206

valence neutron continuum orbits, i.e.,  $d_{3/2}$ ,  $p_{3/2}$  and  $f_{7/2}$  are considered in the present calculations for  $^{25}\text{O}$ .

#### 4. Results and Discussions

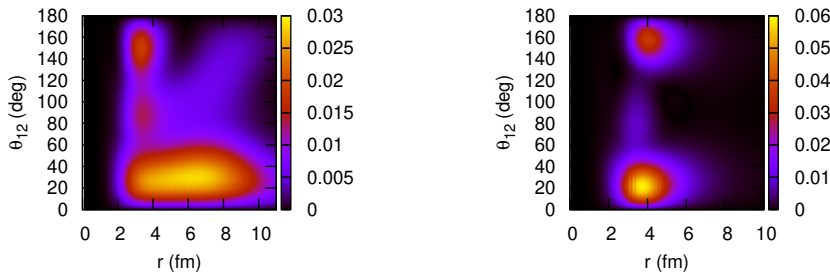
The 3b-model with two non-interacting particles in the above single-particle levels of  $^{21}\text{C}$  and  $^{25}\text{O}$  produces different parity states, when two neutrons are placed in different unbound orbits. The seven configurations,  $(s_{1/2})^2$ ,  $(p_{1/2})^2$ ,  $(p_{3/2})^2$ ,  $(d_{3/2})^2$ ,  $(d_{5/2})^2$ ,  $(f_{5/2})^2$  and  $(f_{7/2})^2$  couple to  $J^\pi = 0^+$  for  $^{22}\text{C}$  and three configurations  $(d_{3/2})^2$ ,  $(p_{3/2})^2$  and  $(f_{7/2})^2$  couple to  $J^\pi = 0^+$  for  $^{26}\text{O}$ . In the 3b-calculations, along with the core- $n$  potential the other important ingredient is the  $n$ - $n$  interaction. An attractive contact-delta pairing interaction is used,  $g\delta(\vec{r}_1 - \vec{r}_2)$  for simplicity, with the only adjustable parameter being  $g$ . We also use the DD contact-delta pairing interaction to explore the role of different pairing interactions. In the DD contact-delta pairing interaction (defined by Eq. (8) of Ref. [6]), the strength of the DI part is given as  $v_0 = 2\pi^2 \frac{\hbar^2}{m_N} \frac{2a_{nn}}{\pi - 2k_c a_{nn}}$ , where  $a_{nn}$  is the scattering length for the free neutron-neutron scattering and  $k_c$  is related to the cutoff energy,  $e_c$ , as  $k_c = \sqrt{\frac{m_N e_c}{\hbar^2}}$ . We use  $a_{nn} = 15$  fm and  $e_c = 30$  MeV [10], which leads to  $v_0 = 857.2$  MeV fm<sup>3</sup>. For the parameters of the DD part, we determine them so as to fix the ground-state energy of  $^{22}\text{C}$  and  $^{26}\text{O}$ ,  $E = -0.140$  MeV [8] and  $0.018$  MeV [3] respectively. The values of the parameters that we employ are  $R_\rho = 1.25 \times A_c^{\frac{1}{3}}$  ( $A_c = 20, 24$ ),  $a = 0.65$  fm. and  $v_\rho = 591.55$  and  $1058.70$  MeV fm<sup>3</sup> for  $^{22}\text{C}$  and  $^{26}\text{O}$  respectively. We found that configuration mixing in the ground state of  $^{22}\text{C}$  does not change much with the choice of  $n - n$  interaction. We present the numbers for our potential Set 1 in Table II which are consistent with results of Ref. [9], and the same behavior is observed for the other sets [6]. Whereas for the  $^{26}\text{O}$  case our results report different magnitude of configuration mixing for the different interactions and the results corresponding to the DD part that are consistent with the results of Ref. [10], where they have also used the DD interaction. One possible reason for this difference is that  $^{26}\text{O}$  is unbound by 18 keV whereas  $^{22}\text{C}$  is bound by 0.140 MeV. In order to reach a final conclusion we need more analysis, which will be reported elsewhere. The two particle density of  $^{22}\text{C}$  and  $^{26}\text{O}$  as a function of two radial coordinates,  $r_1$  and  $r_2$ , for valence neutrons, and the angle between them,  $\theta_{12}$  in the LS-coupling scheme is calculated by following Refs. [5, 10]. The distribution at smaller and larger  $\theta_{12}$  are referred to as ‘‘di-neutron’’ and ‘‘cigar-like’’ configurations, respectively. One can see in Fig. 1 that the two-particle density is well concentrated around  $\theta_{12} \leq 90^\circ$ , which is the clear indication of the di-neutron correlation and the di-neutron component has a relatively higher density in comparison to the small cigar-like component for both  $^{22}\text{C}$  and  $^{26}\text{O}$ . The reflection of dominance of  $s$ -component in ground state of  $^{22}\text{C}$  can be seen in left panel of Fig. 1 showing extended di-neutron component in comparison to  $^{26}\text{O}$  (in right panel of Fig. 1), which has sharper dineutron component due to the mixing of  $l > 0$  components in its ground-state.

#### 5. Conclusions

In the present study we present the emergence of bound  $2n$ -halo ground state of  $^{22}\text{C}$  from the coupling of seven unbound  $spdf$ -waves in the continuum of  $^{21}\text{C}$  and  $2n$ -unbound ground state of  $^{26}\text{O}$  from the coupling of three unbound  $pdf$ -waves in the continuum of  $^{25}\text{O}$  due to presence of

**Table II.** Components of the ground state ( $0^+$ ) of  $^{22}\text{C}$  and  $^{26}\text{O}$ , with model parameter energy cut  $E_{cut}$ . For  $^{22}\text{C}$  we have used the potential Set 1 of Table 1 of [6] for the shallow case with the ground-state energy,  $-0.140$  MeV and for  $^{26}\text{O}$  we have used the potential tabulated in Table I. In last column of the table, the comparison has been made with the Ref. [9] for  $^{22}\text{C}$  and Ref. [10] for  $^{26}\text{O}$ .

System	$E_{cut}$ (MeV)	$lj$	$DI$	$DD$	Reference
<b>Present work</b>					
$^{22}\text{C}$	5	$(s_{1/2})^2$	0.923	0.899	0.915
		$(p_{1/2})^2$	0.004	0.008	0.009
		$(p_{3/2})^2$	0.022	0.029	0.024
		$(d_{3/2})^2$	0.045	0.047	0.033
		$(d_{5/2})^2$	0.001	0.005	0.003
		$(f_{5/2})^2$	0.0003	0.001	0.033
		$(f_{7/2})^2$	0.003	0.007	0.007
$^{26}\text{O}$	10	$(d_{3/2})^2$	0.798	0.643	0.661
		$(p_{3/2})^2$	0.024	0.088	0.105
		$(f_{7/2})^2$	0.178	0.268	0.183



**Fig. 1.** Two-particle density for the ground state of  $^{22}\text{C}$  (left-panel) and  $^{26}\text{O}$  (right-panel) as a function  $r_1 = r_2 = r$  and the opening angle between the valence neutrons  $\theta_{12}$  for settings mentioned in caption of Table II .

pairing interaction. Contribution of different configurations has been presented along with the  $2n$ -correlations. More investigations are needed to comment on the role of different pairing interactions and to explain in detail the role of each wavefunction component.

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