Efficient representation of supply and demand curves on day-ahead electricity markets

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Abstract

Our paper aims to model supply and demand curves of electricity day-ahead auctions in a parsimonious way. Our main task is to build an appropriate algorithm to present the information about electricity prices and demands with far less parameters than the original one. We represent each curve using mesh-free interpolation techniques based on radial basis function approximation. We describe results of this method for the day-ahead IPEX spot price of Italy. Then we use these representations with the aim of forecasting supply and demand curves and finding the intersection of the predicted curves in order to obtain the market clearing price.

1 Introduction

Accurate modeling and forecasting electricity demand and prices are very important issues for decision making in deregulated electricity markets. Different techniques were developed to describe and forecast the dynamics of electricity load. Short term forecast proved to be a very challenging task due to various unstable factors. For example, Figures 1 and 2 demonstrate changing of electricity equilibrium price and quantity during one week. Functional data analysis is extensively used in other fields of science, but it has been little explored in the electricity market setting.

We consider the Italian electricity market (IPEX). IPEX consists of different markets, including a day-ahead market. The day-ahead market is managed by Gestore del Mercato Elettrico, where prices and demand are determined by crossing supply and demand the day before the delivery. Supply and demand curves on day-ahead electricity markets are the results of thousands of bid and ask entries in the day-ahead auction, this for all the 24 hours. In principle, it would be possible to represent, and forecast, these curves by taking into account each production and each consumption unit as a separate time series, and then joining these together to construct the final curves, and thus the resulting price. However, the huge number of these units makes this naive strategy infeasible, unless one has extremely high computing capacity with complex machine learning algorithms available.

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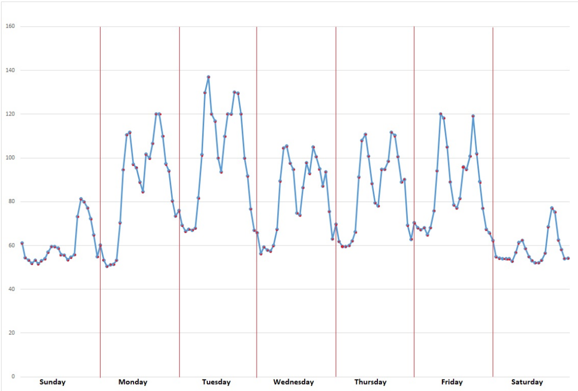


Figure 1: Electricity equilibrium prices during a week.

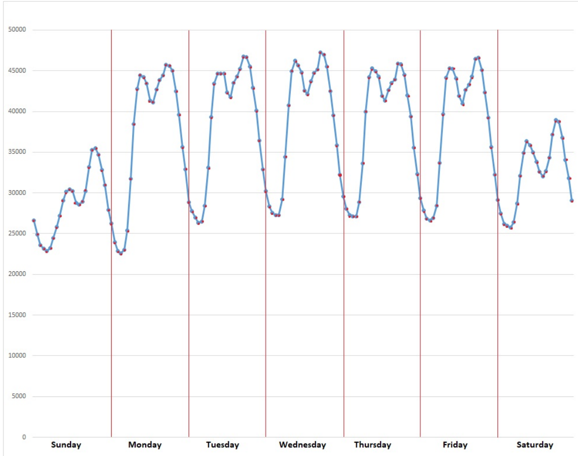


Figure 2: Electricity equilibrium quantities during a week.

In this paper, we are going to present a more parsimonious approach. In fact, the idea is to represent each curve using non-parametric mesh-free interpolation techniques, so that we can obtain an approximation of the original curve with far less parameters than the original one. The original curve, in fact, in principle depends on about hundreds of parameters and is obtained as follow.

The producers submit offers where they specify the quantities and the minimum price at which they are willing to sell. The demanders submit bids where they specify the quantities and the maximum price at which they are willing to buy. They are then aggregated by an independent system operator (ISO) in order to construct the supply and demand curves. Once the offers and bids are received by the ISO, supply and demand curves are established by summing up individual supply and demand schedules. In the case of demand, the first step is to replace ”zero prices“ bids by the market maximum price (for Italian electricity market, the market maximum price is 3000 Euro) without changing the corresponding quantities. After this replacement, the bids are sorted from the highest to the lowest with respect to prices. The corresponding value of the quantities is obtained by cumulating each single demand bid. For supply curve, in contrast, the offers are sorted from the lowest to the highest with respect to prices and the corresponding value of the quantities is obtained by cumulating each single supply offer. The market equilibrium is the point where both curves intersect each other and the price balances supply and demand schedules (see e.g. Figure 3). This point determines the market clearing price and the traded quantity. Accepted offers and bids are those that fall to the left of the intersection of the two curves, and all of them are exchanged at the resulting price.

In the beginning of the 2000s, the amount of papers focused on electricity price forecasting started to increase dramatically. A great variety of methods and models occurred during the last twenty years. Weron [18] (2014) made an overview of the existing literature on electricity

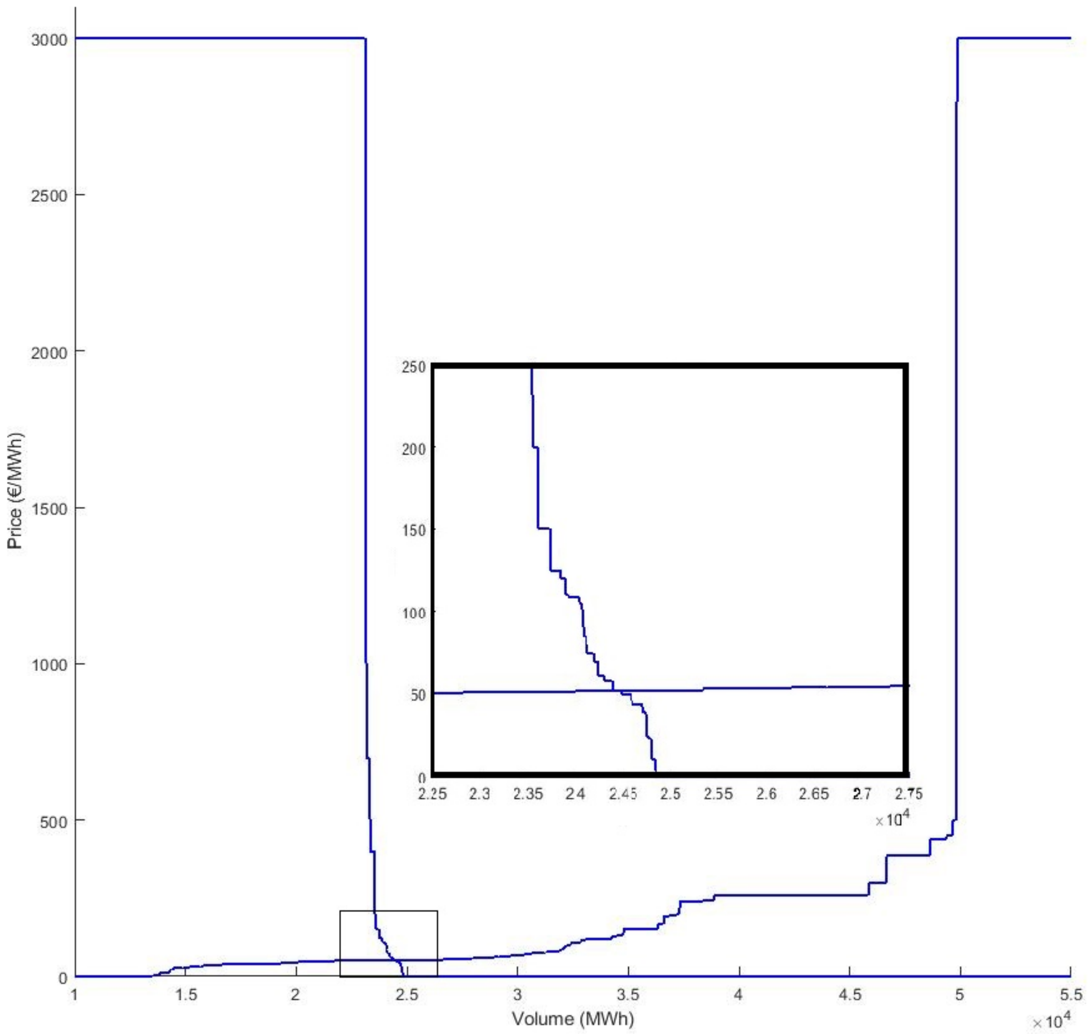


Figure 3: The market equilibrium point between demand and supply.

price forecasting and divided electricity price models into five different groups: multi-agent, fundamental, reduced-form, statistical and computational intelligence models. A review of probabilistic forecasting was done in [10] (2018) by Weron and Nowotarski. Most models have in common that they focus on the price itself or related time series. In such a way these models do not take into account the underlying mechanic which determines the price process – the intersection between the electricity supply and demand.

Some of the recent approaches try to to analyse the real offered volumes for selling and purchasing electricity. This commonly leads to a problem of a large amount of data and, therefore, high complexity. In particular, Eichler, Sollie, Tuerk in 2012 [5] investigated a new approach that exploits information available in the supply and demand curves for the German day-ahead market. They proposed the idea that the form of the supply and demand curves or, more precisely, the spread between supply and demand, reflects the risk of extreme price fluctuations. They utilize the curves to model a scaled supply and demand spread using an autoregressive time series model in order to construct a flexible model adapted to changing market conditions. Furthermore, Aneiros, Vilar, Cao, San Roque in 2013 [2] dealt with the prediction of residual demand curve in electricity spot markets using two functional models. They tested this method as a tool for optimizing bidding strategies for the Spanish day-ahead market. Then Ziel and Steinert in 2016 [19] proposed a model for the German European Power Exchange (EPEX) market, which considers all the supply and demand information of the system and discussed the effects of the changes in supply and demand. Their idea was to fill the gap between research done in time-series analysis, where the structure of the market is usually left out, and research done in structural analysis, where empirical data is used very rarely and even less thoroughly. They provided deep insight on the bidding behavior of market participants. They also showed that incorporating the sale and purchase data yields promising results for forecasting the likelihood of extreme price events. In 2016 Shah [15] also considered the idea of modeling the daily supply and demand curves, predicting them and finding the intersection of the predicted curves in order to find the predicted market clearing price and volume. He used the functional approach, namely, B-spline approximation, to convert the resulted piecewise constant curves into smooth functions. However, as far as we know, non-parametric mesh-free interpolation techniques were never considered for the problem of modeling the daily supply and demand curves.

We are going to use a relatively new modeling technique based on functional data analysis for demand and price forecasting. The first task for this purpose is to make an appropriate algorithm to present the information about electricity prices and demands, in particular to approximate a monotone piecewise constant function.

We want to make an appropriate algorithm to present this information, in particular, to approximate a monotone piecewise constant function. Accuracy of the approximation and running time are very important for us. As we already said, the basic novelty of our problem is that we are going to present the information about electricity prices and demands using functional data analysis approach. The main idea behind functional data analysis is, instead of considering a collection of data points, to consider the data as a single structured object. This allows to use additional information contained in the functional structure of the data. Once the data are converted to functional form, it can be evaluated at all values over some interval.

The most promising technique to do so is the use of (integrals of) Radial Basis Functions, which are been used in several other applications (image reconstruction, medical imaging, geology, etc.) and allow a very flexible adaptation of the interpolating curves to real data. The use of radial basis functions have attracted increasing attention in recent years as an elegant scheme for high-dimensional scattered data approximation, an accepted method for machine learning, one of the foundations of meshfree methods and so on. The initial motivation for RBF methods came from geodesy, mapping, and meteorology. RBF methods were first studied by Roland Hardy, an Iowa State geodesist, in 1968, when he developed one of the first effective methods for the interpolation of scattered data. Later in 1986 Charles Micchelli, an IBM mathematician, developed the theory behind the multiquadric method. Micchelli made the connection between scattered data interpolation and positive definite functions [9]. RBF methods are now considered an effective way to solve partial differential equations, to represent topographical surfaces as well as other intricate three-dimensional shapes, having been successfully applied in such diverse areas as climate modeling, facial recognition, topographical map production, auto and aircraft design, ocean floor mapping, and medical imaging (see, for example, [4], [7], [11]). Now RBF methods are an active area of mathematical research, as many open questions still remain. We will present different techniques for this interpolation, with their advantages and drawbacks, and with an application to the Italian day-ahead market.

The paper is organized as follow. Section 2 describes the theoretical background, namely, mesh-free interpolation techniques based on radial basis function approximation. Section 3 presents the database from the Italian electricity market. Section 4 is devoted to a short description of the numerical schemes and to the analysis of the results. In Section 5 we forecast next-day electricity demand and prices using approximated supply and demand curves and we compare the results obtained with our functional approach with corresponding univariate price predictions. Section 6 concludes the paper.

2 Meshless approximation

Let us briefly notice some features of supply and demand curves that are relevant for our modeling.

- By construction, the curves are monotone.
- The values attained by the supply curve are roughly clustered around **layers**, corresponding to different production technologies. For example, in Italy they are non-dispatchable renewables, gas, coal, hydro, oil.
- The fact that renewables are the first ones make the supply curve intrinsically "meshless".
- Demand is much more inelastic than supply.

Thus, we are dealing with a scattered data interpolation problem. We have a large amount of points (each point representing price and amount of electricity) that we want to approximate. We can formalize this problem as follows.

Given a set of N distinct *data points* $X_N = \{x_i : i = 1, 2, \dots, N\}$ arbitrarily distributed on a domain $\Omega \subset \mathbb{R}$ and a set of *data values* (or function values) $Y_N = \{y_i : i = 1, 2, \dots, N\} \subset \mathbb{R}$, the data interpolation problem consists in finding a function $s_f : \Omega \to \mathbb{R}$ such that

$$s_f(x_i) = y_i,\ i = 1, \dots, N. \tag{2.1}$$

Let us recall briefly the most popular methods for the interpolation problem. Polynomial interpolation is the interpolation of a given data set by the polynomial of lowest possible degree that passes through the points of the dataset. For given data sites X_N and function values Y_N there exists exactly one polynomial $p \in \pi_{N-1}(\mathbb{R})$ that interpolates the data at the data sites. Therefore the space $\pi_{N-1}(\mathbb{R})$ depends neither on the data sites nor on the function values but only on the number of points. However, Runge's phenomenon [12] shows that, for high values of N, the interpolation polynomial may oscillate wildly between the data points. Besides, the polynomial interpolation does not guarantee of monotonicity of the curves (see Figure 4).

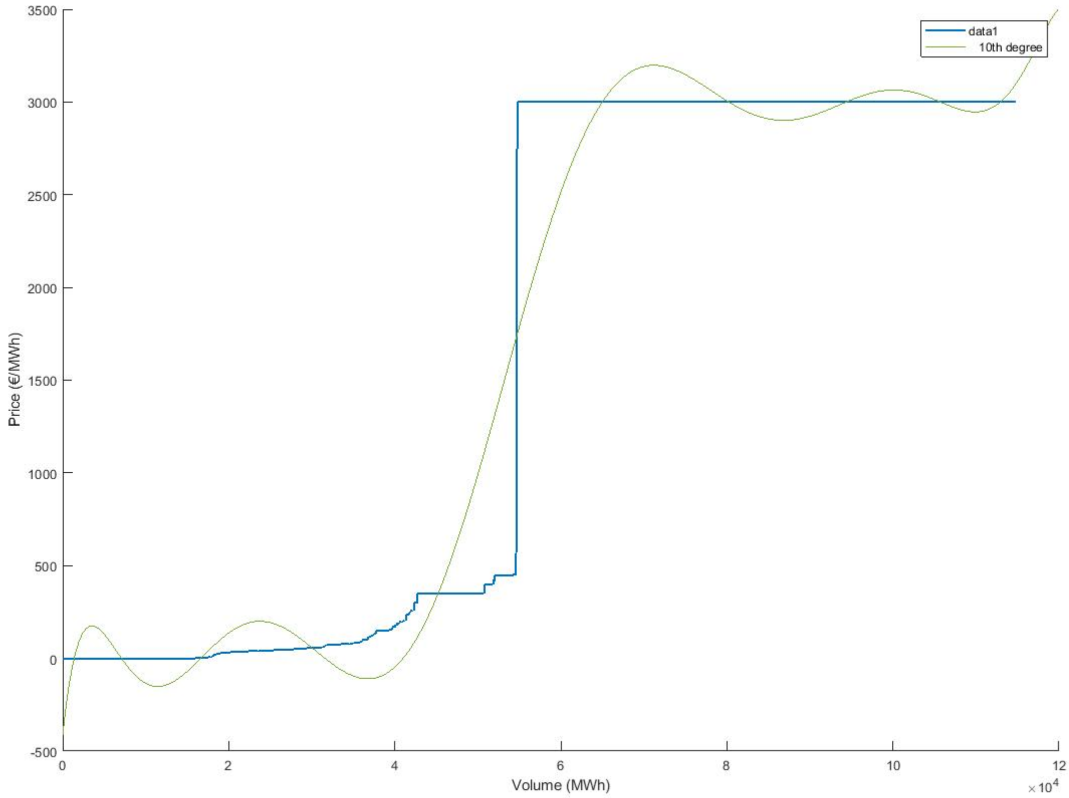


Figure 4: Approximation of supply curve with polynomials.

It is a well-established fact that a large data set is better dealt with splines than with polynomials. An aspect to notice in contrast to polynomials is that the accuracy of the interpolation process using splines is not based on the polynomial degree but on the spacing of the data sites. In particular, cubic splines are widely used to fit a smooth continuous function through discrete data. However, spline interpolation requires a fixed mesh.

Notice that for all methods, the interpolant s_f is expressed as a linear combination of some basis functions B_i , i.e.

$$s_f(x) = \sum_{k=1}^{d} c_k B_k(x).$$

The basis functions in e.g. polynomial interpolation does not depend on the data points. Another approach is to use a basis which depends on the data points.

One simple way to solve problem (2.1) is to use the technique of radial basis functions. This consists in choosing a fixed function $\phi : \mathbb{R} \to \mathbb{R}$ and forming the interpolant as

$$s_f(x) = \sum_{i=1}^{N} \alpha_i \phi(\|x - x_i\|), \tag{2.2}$$

where the coefficients α_i are determined by the interpolation conditions $s_f(x_i) = y_i$. Therefore, the scattered data interpolation problem leads to the solution of a linear system

$$A\alpha = y, \text{ where } A_{i,j} = \phi(|x_i - x_j|).$$

The solution of the system requires that the matrix A is non-singular. It is enough to know in advance that the matrix is positive definite (see [17] for more details). Let us recall the definition of strictly positive definite function.

Definition 2.1. A real-valued function $\Phi : \mathbb{R} \longrightarrow \mathbb{R}$ is called *positive semi-definite* if, for all $m \in \mathbb{N}$ and for any set of pairwise distinct points $x_1, x_2, \ldots, x_m$, the $m \times m$ matrix

$$A = (\Phi(x_i - x_j))_{i,j=1}^{m}$$

is positive semi-definite, i.e. for every column vector z of m real numbers the scalar $z^T Az \geqslant 0$. The function $\Phi : \mathbb{R} \longrightarrow \mathbb{R}$ is called (strictly) *positive definite* if the matrix A is positive definite, i.e. for every non-zero column vector z of m real numbers the scalar $z^T Az > 0$.

The most important property of positive semi-definite matrices is that their eigenvalues are positive and so is its determinant.

A radial function is a real-valued positive semi-definite function whose value depends only on the distance from the center $\mathbf{c}$. One useful characterization for positive semi-definite univariate functions was given by Schoenberg in 1938 in the terms of completely monotone functions: a continuous function $\phi : [0, \infty) \to \mathbb{R}$ is positive semi-definite if and only if $\phi \in C^\infty(0, \infty)$ and $(-1)^k \phi^{(k)}(r) \geqslant 0$ for all $r \geqslant 0$, for $k = 0, 1, \ldots$.

Some standard radial basis functions are

- $\phi(r) = e^{-(\varepsilon r)^2}$ (Gaussian),
- $\phi(r) = e^{-\varepsilon r}(\varepsilon r + 1)$ (Matérn),
- $\phi(r) = (1 - \varepsilon r)_+^4 (4\varepsilon r + 1)$ (Wendland),

where $\varepsilon > 0$ denote a shape parameter, $r = \|x\|_2$.

The idea of meshless approximation with radial basis functions is to find an approximant of f in the following form of Equation (2.2), where:

- the coefficients α_i and the **centers** x_i are to be chosen so that the interpolant is as near as possible as the original function f;
- $\phi : \mathbb{R} \to \mathbb{R}$ is a **radial basis function** (RBF).

Notice that the radial basis function $\phi \geqslant 0$, with $\alpha_i \geqslant 0$, so

$$\sum_{i=1}^{M} \alpha_i \phi(\|x - x_i\|) \geqslant 0.$$

As we need to approximate piecewise constant monotone function from $[0, M]$ to $\mathbb{R}^+$, we decide to use the integrals of RBF. Namely, we want to find an approximant of the form

$$s_f(t) = \int_0^t \sum_{i=1}^{M} \alpha_i \phi(\lambda_i \|x - x_i\|)\, dx = \sum_{i=1}^{M} \alpha_i \int_0^t \phi(\lambda_i \|x - x_i\|)\, dx$$

where λ_i is a shape parameter for every center x_i. As radial basis functions, we choose Gaussian functions for analytical tractability.

Evidently, any supply curve and any demand curve can be approximated by a combination of error functions, which is the integral of a normalized Gaussian function. The standard error function is defined as:

$$\mathrm{erf}(x) = \frac{1}{\sqrt{\pi}} \int_{-x}^{x} e^{-t^2}\, dt = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt.$$

In order to find unknown coefficients α_i, λ_i, x_i we need to solve the global minimization problem:

$$\min_p \|s_f(x_i, p) - y_i\|_2^2, \tag{2.3}$$

where $p = (\alpha_i, \lambda_i, x_i)_{i=1,\ldots,N}$ and

$$s_f(t, p) := \sum_{i=1}^{M} \alpha_i \int_0^t \phi(\lambda_i \|x - x_i\|)\, dx$$

and $\phi(t) = (\mathrm{erf}(t) + 1)/2$ is the primitive of a Gaussian kernel. However, optimization problem (2.3) is very heavy, as it is a nonlinear and nonconvex minimization over $p \in \mathbb{R}^{3M}$.

For this reason, we divide our global problem in simpler subproblems, with lower dimensionality, so that the final result is faster. We describe two realization of this approach in Section 4.

3 Data set

We now use the data about supply and demand bids from the Italian day-ahead electricity market from the GME website www.mercatoelettrico.org. We consider the time period from 01.01 to 31.12.2017. These data are in aggregated form, i.e. bids coming from different agents, but with the same price, are aggregated in the same price layer. Even in this form, we are dealing with a massive amount of data. For instance, **2 800 687** offer and **558 926** bid layers were observed during this period.

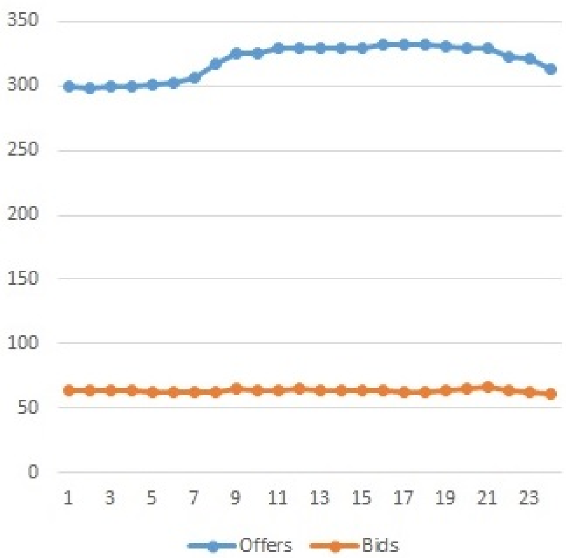
Table 1: Data.

Date	Hour	Volume (MW)	Price (Euro)
01-01-2017	1	13392.7	0
01-01-2017	1	25	0.1
01-01-2017	1	113.8	1
01-01-2017	1	11	3.5
01-01-2017	1	270.3	5
01-01-2017	1	0.5	6
....	......	....	....
31-12-2017	24	370	554.2
31-12-2017	24	352	554.3
31-12-2017	24	365	554.5
31-12-2017	24	97	700
31-12-2017	24	60000	3000

This means, that on average there are 324 offer and 65 bid layers for each hour of the year, which corresponds to one supply curve and one demand curve respectively.

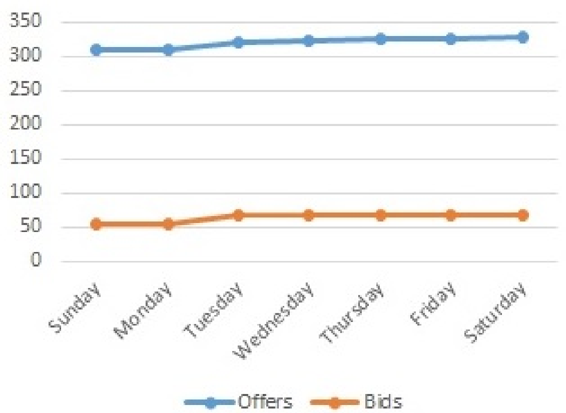
It is a known fact that the dynamics of electricity trade displays a set of features: dependence of the consumption on external weather conditions, the hour of the day, the day of the week, and time of the year. Variation in prices are all dependent on the principles of demand and supply. First of all, on the day-ahead market the energy is typically traded on an hourly basis and this means that the prices can and will vary across hours. For example, at 9:00 a.m. there could be a price peak, while at 4:00 a.m. prices could be only half of the peak price. Second, the weekly seasonal behaviour matters. Usually, it is necessary to differentiate between the two weekend days (Saturday and Sunday), the first business day of the week (Monday), the last business day of the week (Friday) and the remaining business days (see e.g. [1]). Thirdly, electricity spot prices display a strong yearly seasonal pattern: for instance, demand increases in summer, as consumers turn their air conditioners on, and also in winter because of electric heating in housing.

As far as the number of offers (or bids) affects directly the complexity of approximation, we decided to explore the relationship between the number of bids and offers and such a characteristics as the hour of the day, the day of the week, and the month of the year. Based on the dependence between this three factors and electricity prices we could expect that some hours, days have much less offers and bids than another one. This analysis is presented on Figures 5 – 7. The main conclusion that we have made is that there is no direct relationship between the number of offer and bid layers and the hour of the day, the day of the week, and the time of the year. In particular, during 24 hour of the day the number of offer layers varies between 299 and 332, and the number of bid layers varies between 61 and 66. With regard to dependence of the day of the week the number of offer layers varies between 310 and 320, and the number of bid layers varies between 55 and 68. Based on this observation we decided to choose the same number of basis functions independently of the hour of the day, the day of the week, and the time of the year.



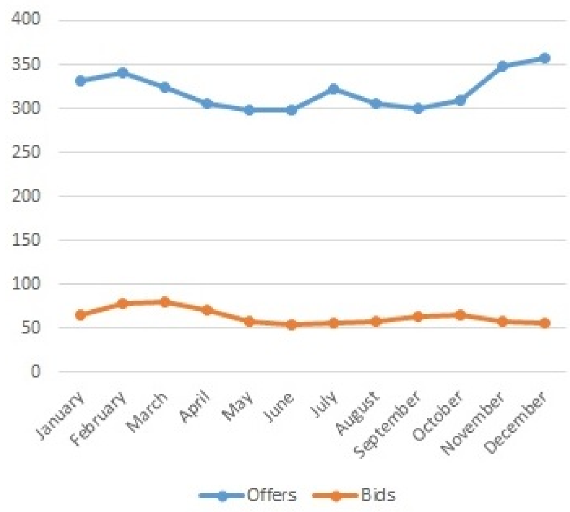
Hour	Number		Hour	Number	
	of offers	of bids		of offers	of bids
1	300	64	**13**	329	64
2	299	64	**14**	329	64
3	300	64	**15**	330	64
4	300	64	**16**	332	64
5	301	63	**17**	332	63
6	303	63	**18**	332	63
7	307	62	**19**	331	64
8	318	63	**20**	329	65
9	325	65	**21**	329	66
10	326	64	**22**	323	64
11	329	64	**23**	321	63
12	329	65	**24**	314	61

Figure 5: Hour dependence of the number of offer and bid layers.



Month	Number	
	of offers	of bids
Sunday	310	55
Monday	310	56
Tuesday	322	68
Wednesday	324	67
Thursday	326	68
Friday	327	68
Saturday	329	68

Figure 6: Weekly dependence of the number of offer and bid layers.



Month	Number of offers	Number of bids
January	331	65
February	341	79
March	324	81
April	305	72
May	298	57
June	298	54
July	322	55
August	305	58
September	300	64
October	309	66
November	348	58
December	357	57

Figure 7: Monthly dependence of the number of offer and bid layers.

4 Numerical experiments

Since the maximum market clearing price for the period under review (i.e. from 01.01.2017 to 31.12.2017) is 350 €, in all the experiments we restricted ourselves to a maximum price of 400 €. For the implementation of our algorithm we are using the function `lsqcurvefit` from MATLAB Optimization Toolbox.

First, we choose the number of basis function M. After that, as we mentioned that the global optimization problem (2.3) is nonconvex, we choose to divide our problem into M sub-problems. Then each part of the supply curve must be approximated by one error function.

Our first attempt (Method 1) was just to divide y-axis uniformly into M equal intervals (see Figure 8). However this approach proved to be ineffective, as a huge jump concentrates on itself, keeping uselessly many components.

To resolve this problem we created a simple algorithm - Method 2 - that finds the points $p_1, \ldots, p_M$ on the y-axis such that our supply curve takes the value exactly p_i on some non-trivial interval (see Figure 9). Then we solve the same optimization problem for the values of the supply curve between p_i and p_{i+1} using function `lsqcurvefit` (see Figure 10). On each part we need to find only the coefficients a_i, b_i, c_i of the function

$$G(x) = \sum_{i=1}^{k} a_i(\operatorname{erf}(c_i \cdot (x - b_i)) + 1). \tag{4.1}$$

Here, for convenience of representation we are using $\{\operatorname{erf}(c_i \cdot (x - b_i)) + 1\}$ instead of $\{\operatorname{erf}(c_i \cdot (x - b_i))\}$, as our data values are never negative.

The `lsqcurvefit` function solves nonlinear data-fitting problems in least-squares sense. Suppose that we have data points $X_N = \{x_i : i = 1, 2, \ldots, N\}$ and data values $Y_N = \{y_i :$

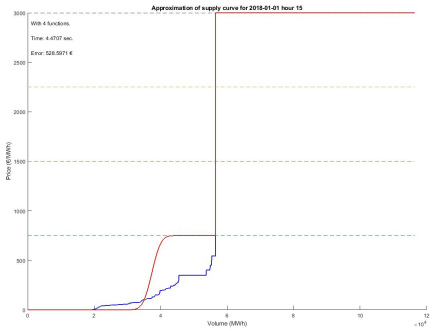


Figure 8: Method 1.

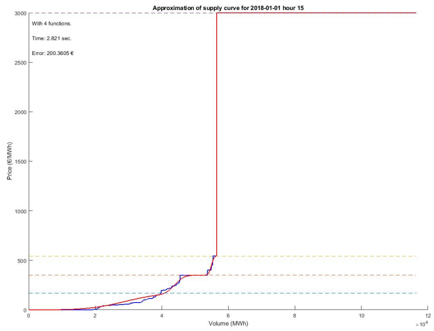


Figure 9: Method 2.

$i = 1, 2, \ldots, N\} \subset \mathbb{R}$ and we want to find a function f such that $f(x_i) \approx y_i$, $i = 1, \ldots, N$. We can consider the family of functions $\{f(x, p) : p \in \mathbb{R}^k\}$, depending of some parameter $p \in \mathbb{R}^k$. Let $p_0 \in \mathbb{R}^k$ be an "initial guess" such that $f(x_i, p)$ is reasonably close to y_i. The function `lsqcurvefit` starts at p_0 and finds coefficients p from some neighborhood of p_0 to best fit the data set Y_N:

$$\min_p \|f(x_i, p) - y_i\|_2^2.$$

For optimizing the numerical procedure we solved some parts of the optimization problem by ourselves: in fact, when the interval $[p_i, p_{i+1}]$ contains only one jump, then

$$a_i := f(p_{i+1}) - f(p_i)$$

for any kernel function ϕ with unit integral.

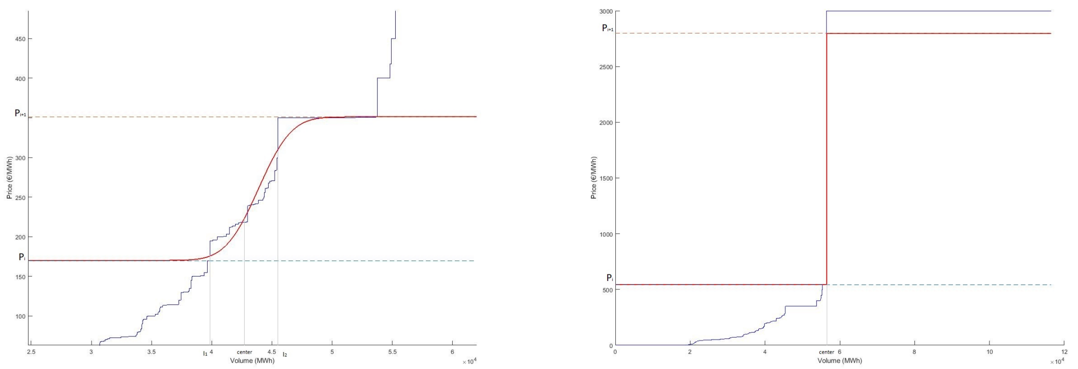


Figure 10: Local interpolation by one error function with `lsqcurvefit` function.

A summary of the results is shown in Table 2. For all experiments we proceed with the data for period from 01.01.2017 to 31.12.2017. We used different numbers of basis function to approximate supply and demand curves, and then compared the equilibrium price, which was received as intersection of approximants (P_{appr}), with the correct equilibrium price (P). We did this for each hour of each day, and then computed the average value of $|P - P_{appr}|$ (Error) for all 8 664 hours of the year and the maximum value of $|P - P_{appr}|$ (Max error).

This empirical results show that the accuracy of our approximation is good enough, if we use 5 basis function for the demand curve and 15 basis function for the supply curve. Above these values, the increase in the number of functions leads to more time consumption, but the increase of the accuracy is less significant.

Table 2: Results of numerical experiment.

Number of functions		Results		
For demand	For supply	Error	Max error	Running time
5	5	3.9 €	28.6 €	69 min.
5	10	2.2 €	14.9 €	82 min.
5	15	1.5 €	11.1 €	103 min.
5	20	1.3 €	9.1 €	110 min.
5	25	1.2 €	9.3 €	135 min.
5	30	1.2 €	9.4 €	159 min.
5	35	1.2 €	9.8 €	177 min.
5	40	1.2 €	9.6 €	190 min.
5	45	1.2 €	9.6 €	199 min.
5	50	1.2 €	9.6 €	207 min.
10	5	3.9 €	39.5 €	100 min.
10	10	2.1 €	14.9 €	128 min.
10	15	1.4 €	8.9 €	146 min.
10	20	1.2 €	9.1 €	162 min.
10	25	1.1 €	9.5 €	183 min.
10	30	1.1 €	9.3 €	199 min.
10	35	1.0 €	9.4 €	223 min.
10	40	0.98 €	9.8 €	241 min.
10	45	0.98 €	9.6 €	255 min.
10	50	0.98 €	9.6 €	273 min.

As a last step, we analyzed the stability of the coefficients for the case when we approximate the supply curve with 10 basis functions and the demand curve with 5 basis functions for the same period of time, as

$$S(x) = \sum_{i=1}^{10} A_i(\mathrm{erf}(C_i \cdot (x - B_i)) + 1) \quad \text{and}$$

$$D(x) = \sum_{i=1}^{5} E_i(\mathrm{erf}(K_i \cdot (x - L_i)) + 1).$$

From Table 3 we can see that these coefficients do not have a stable behavior (namely, maximum values, minimum values and mean values are presented). Although the values attained by the supply curve are clustered around layers, which correspond to different production technologies, we came to the conclusion that we have no chance to choose these coefficients uniformly for all curves, but we need to calculate them for all supply and demand curves.

Figure 11: Supply curve approximated with 10 basis functions.

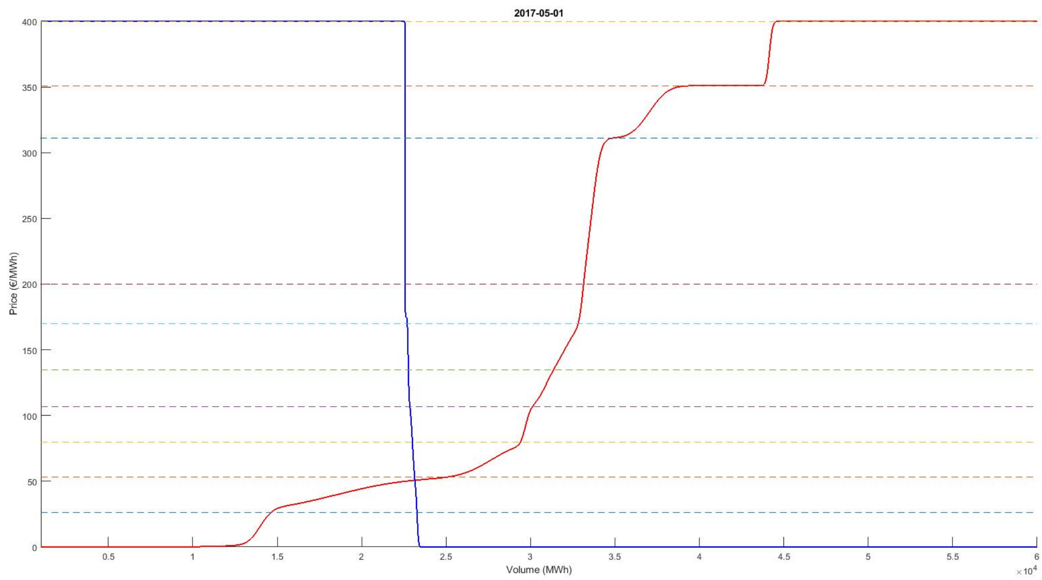


Table 3: Stability of the coefficients.

	Min	Mean	Max
Coefficients for supply curve			
A_1	10	14.76981	18
A_2	10.5	15.15519	21
A_3	10.5	15.21438	19.5
A_4	11	15.53944	22
A_5	11	16.8968	27.5
A_6	12.5	20.44287	27
A_7	14.5	22.15457	33
A_8	19	29.69132	57.5
A_9	17	24.48784	48
A_{10}	21	25.64777	50
Coefficients for demand curve			
E_1	12	30.95154	37.5
E_2	25	34.31039	58.5
E_3	25	36.24469	50
E_4	33	40.19715	50
E_5	50	58.29623	75

5 Price and demand forecasting based on supply and demand curves

Our main goal in this section is to forecast next-day electricity demand and prices using approximated supply and demand curves and to compare different modeling techniques. The classical models do not explain the relationships between market clearing price and different influential factors that can be essential in the problem of price prediction. To this purpose, we want to compare commonly used autoregressive models, based just on the clearing price, with ours, based on supply and demand curves. For this test, we are using again the data about supply bids from the Italian electricity market considering the time period from 01.01.2017 to 31.12.2017. In particular, our training set includes data from 01.01.2017 to 31.10.2017, while the test set which is used for forecasting to test the performance of the model on out-of-sample data is from 01.11.2017 to 31.12.2017. We will consider a linear parametric autoregressive (AR) model for univariate price prediction and functional autoregressive (FAR) models for the prediction of supply and demand curves.

We performed electricity price forecasting using six different methods: autoregressive model of order 1 with (SAR(1)) and without seasonality (AR(1)) for the closing price; functional autoregressive model of order 1 applied to the modeled supply and demand curves, where for the representation of demand curve we used one basis function and for the representation of supply curve we used 5 or 10 functions (FAR(1) (5 functions) and FAR(1) (10 functions), respectively) together with the corresponding seasonal models (SFAR(1) (5 functions) and SFAR(1)

(10 functions), respectively). In all the seasonal versions, dummy variables corresponding to weekdays were introduced. These models were applied to each market hour separately.

While formulations of AR(1) and SAR(1) models for the closing prices are quite standard (thus we do not give details on them here), we feel that a description of our implementation of FAR(1) and SFAR(1) models for supply and demand curves are needed. We considered the simplified representation of the supply curve $S_{d,h}(x)$ with M basis functions, and the demand curve $D_{d,h}(x)$ with one basis function, at day d and hour h, keeping the shape parameter constantly equal to 1

$$\begin{aligned} S_{d,h}(x) &= \sum_{i=1}^{M} A_{d,h,i} \cdot (\mathrm{erf}((x - B_{d,h,i})) + 1), \quad M = 5 \text{ or } M = 10, \\ D_{d,h}(x) &= 200 \cdot \mathrm{erf}((x - L_{d,h})) + 1. \end{aligned}$$

Then we provide a model for the process $X_{d,h} = (X^1_{d,h}, X^2_{d,h}, \ldots, X^{2M}_{d,h})$, where

$$\begin{aligned} X^i_{d,h} &= A_{d,h,i}, \qquad i = 1, \ldots, M-1, \\ X^{i+M-1}_{d,h} &= B_{d,h,i},\ i = 1, \ldots, M, \\ X^{2M}_{d,h} &= L_{d,h}. \end{aligned}$$

Notice that, as we restricted ourselves to a maximum price (and so the maximum of supply and demand curves) of 400 €, we need to exclude the parameter $A_{d,h,M}$ from the model, as it is linearly dependent on others. The considered time series model FAR(1) for $X_{d,h}$ for each hour h is given by

$$X_{d,h} = \nu_d + \Phi_d X_{d,h-1} + \varepsilon_{d,h}$$

with the $2M \times 2M$ matrix Φ_d, and the $2M$-dimentional vector ν_d as parameters, and $\varepsilon_{d,h}$ as error term. We assume that the error process $\varepsilon_{d,h}$ is a $2M$-dimensional white noise process.

For modeling the day of the week impact in SFAR(1) models we define additionally function $W(d)$ that gives a number that corresponds to the weekday of day d ($W(d) = 1$ for a Sunday, for a Monday $W(d) = 2$ up to $W(d) = 7$ for a Saturday), and the weekday indicators

$$W_k(d) = \begin{cases} 1, \text{if } W(d) = k \\ 0, \text{if } W(d) \neq k \end{cases} .$$

We introduced parameters $D_{d,h,k}$ for the weekday effect. Thus, the corresponding SFAR(1) model for $X_{d,h}$ for each hour h and is written, in terms of coefficients, as

$$X_{d,h} = \nu_d + \Phi_d X_{d,h-1} + \sum_{k=1}^{7} W_k(d) D_{d,h,k} + \varepsilon_{d,h}.$$

We compared the results obtained with our functional approach with corresponding univariate price prediction. Three different summary measures, namely, mean absolute error (MAE), root mean square error (RMSE) and mean absolute percentage error (MAPE) were used to evaluate the out-of-sample forecasting performance. Let us denote E_{dh} and $\hat{E}_{dh}$ the observed and the predicted values for day d, $d = 1, \ldots, T = 61$ and hour h, $h = 1, \ldots, 24$. We computed

$$MAE = \frac{\sum_{i=1}^{T} |E_{ih} - \hat{E}_{dh}|}{T}, \quad h = 1, \ldots, 24;$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{T} (E_{ih} - \hat{E}_{dh})^2}{T}}, \quad h = 1, \ldots, 24;$$

$$MAPE = \frac{\sum_{i=1}^{T} |E_{ih} - \hat{E}_{dh}|/E_{ih}}{T}, \quad h = 1, \ldots, 24;$$

Table 4 provide summary statistics of errors for the forecasting of next-day electricity price. In order to facilitate the comparison between different methods we plot the errors for each of the six methods on Figures 12, 13 and 14.

As expected, SAR(1) performs better than AR(1). Surprisingly, instead, functional autoregressive models without seasonality gives better results than corresponding seasonal models. By comparing functional autoregressive models with 5 and 10 functions we can see similar results, so increasing the number of parameters does not lead to the improvement of the prediction accuracy. These two outcomes could be possibly due to overfitting effects. These results shows that we should use FAR(1) (5 functions) as this method is less time-consuming than the one with 10 functions. Finally, our method FAR(1) (5 functions) gives considerably more accurate results compared to the SAR(1) model for all hours. In particular, not only SAR(1) gives an average of the MAPE equal to 16.51% while FAR(1) (5 functions) gives 14.98%, but we can see that FAR(1) (5 functions) performs significantly better than SAR(1) on every single hour. Also comparing MAE and RMSE we obtain similar results.

Due to the superior performance of FAR(1) (5 functions) method, we also conducted prediction of electricity demand with just three methods: AR(1), SAR(1), and FAR(1) (5 functions). Table 5 provide summary statistics of errors for the forecasting of next-day electricity demand also represented in Figures 15, 16, 17. In this case AR(1) gives an average of the mean absolute percentage error 12.82%, SAR(1) gives 11.33% and FAR(1) (5 functions) gives 10.04%. Moreover, FAR(1) (5 functions) for the demand forecasting again gives more accurate results compared to the AR(1) model for all hours and also compared to the SAR(1) model. The same is true for MAE and RMSE.

Figure 12: Mean absolute error for price forecasting.

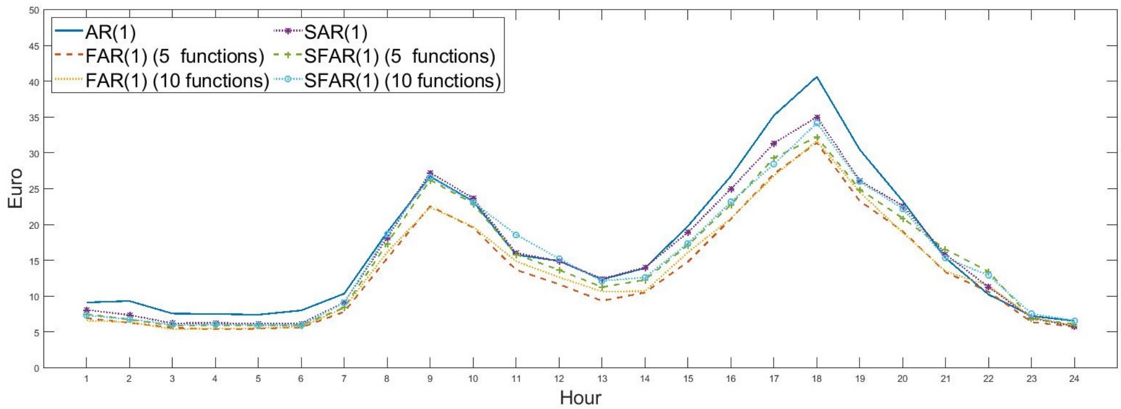


Figure 13: Root mean square error for price forecasting.

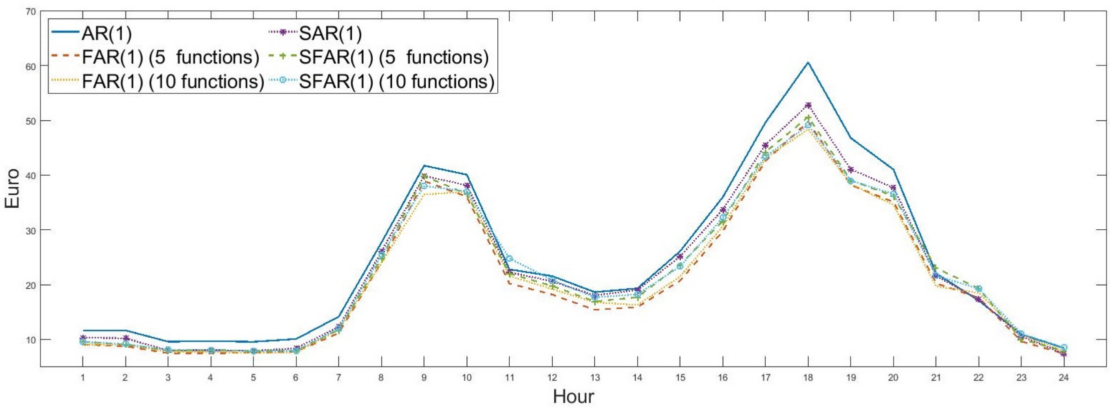


Figure 14: Mean absolute percentage error for price forecasting.

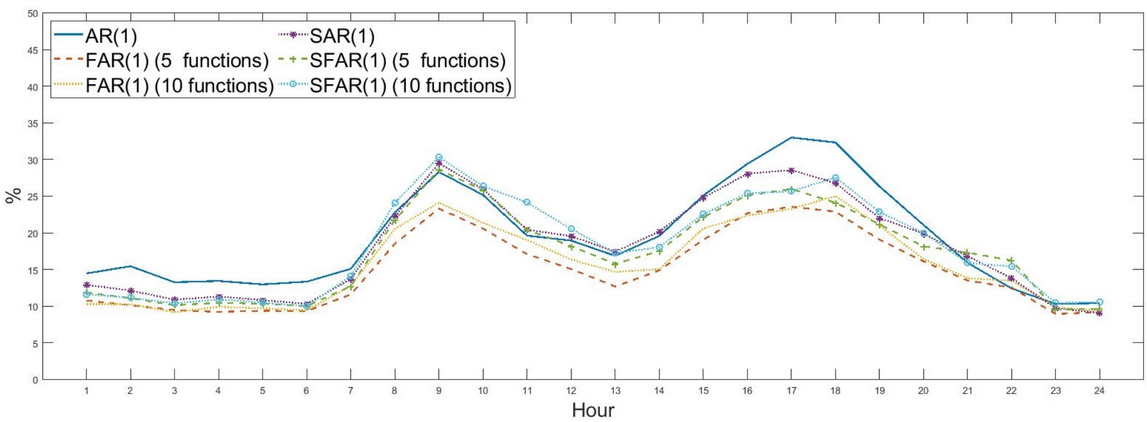


Table 4: Price prediction accuracy statistics.

Model	Hour	MAE euro	RMSE euro	MAPE %	Hour	MAE euro	RMSE euro	MAPE %
AR(1)	**1**	9.12	11.59	14.47	**13**	12.33	18.62	16.86
FAR(1) (5 functions)		6.92	9.06	10.77		9.35	15.37	12.68
FAR(1) (10 functions)		6.57	9.07	10.28		10.66	16.76	14.68
SAR(1)		8.1	10.31	12.92		12.43	18.04	17.43
SFAR(1) (5 functions)		7.5	9.60	11.86		11.27	16.91	15.77
SFAR(1) (10 functions)		7.32	9.55	11.62		12.14	17.67	17.22
AR(1)	**2**	9.32	11.64	15.45	**14**	13.92	19.31	19.58
FAR(1) (5 functions)		6.29	8.75	10.14		10.51	15.91	14.88
FAR(1) (10 functions)		6.39	9.03	10.27		10.68	16.28	15.05
SAR(1)		7.37	10.20	12.13		14.02	19.01	20.17
SFAR(1) (5 functions)		6.74	9.06	11.09		12.28	17.70	17.51
SFAR(1) (10 functions)		6.80	9.13	11.16		12.60	18.23	18.07
AR(1)	**3**	7.58	9.58	13.26	**15**	19.78	26.10	25.10
FAR(1) (5 functions)		5.59	7.47	9.47		14.80	20.73	19.05
FAR(1) (10 functions)		5.39	7.76	9.18		16.12	21.49	20.60
SAR(1)		6.22	7.97	10.89		18.91	25.16	24.76
SFAR(1) (5 functions)		5.88	7.98	10.13		17.14	23.63	22.16
SFAR(1) (10 functions)		6.00	8.13	10.39		17.37	23.29	22.55
AR(1)	**4**	7.51	9.67	13.44	**16**	26.77	35.98	29.41
FAR(1) (5 functions)		5.36	7.48	9.22		20.78	29.76	22.68
FAR(1) (10 functions)		5.48	7.74	9.91		20.86	30.67	22.36
SAR(1)		6.27	8.02	11.31		24.95	33.72	28.07
SFAR(1) (5 functions)		5.96	8.07	10.46		22.76	31.65	25.11
SFAR(1) (10 functions)		6.05	8.04	10.93		23.15	32.27	25.41
AR(1)	**5**	7.41	9.55	12.97	**17**	35.21	49.61	33.00
FAR(1) (5 functions)		5.47	7.55	9.38		27.07	42.61	23.56
FAR(1) (10 functions)		5.54	7.50	9.71		26.78	43.08	23.27
SAR(1)		6.17	7.92	10.83		31.34	45.55	28.56
SFAR(1) (5 functions)		5.94	7.86	10.34		29.29	44.22	25.99
SFAR(1) (10 functions)		5.95	7.85	10.49		28.40	43.37	25.66
AR(1)	**6**	8.01	10.06	13.34	**18**	40.62	60.62	32.32
FAR(1) (5 functions)		5.65	7.76	9.32		31.41	49.74	22.90
FAR(1) (10 functions)		5.70	7.66	9.44		31.65	48.41	25.03
SAR(1)		6.19	8.36	10.31		35.01	52.87	26.79
SFAR(1) (5 functions)		5.95	7.95	9.99		32.21	50.63	24.12
SFAR(1) (10 functions)		5.95	7.89	10.02		34.17	49.10	27.56

Table 4: Price prediction accuracy statistics.

Model	Hour	MAE euro	RMSE euro	MAPE %	Hour	MAE euro	RMSE euro	MAPE %
AR(1)	**7**	10.35	14.15	15.09	**19**	30.43	46.78	26.33
FAR(1) (5 functions)	**7**	7.80	11.15	11.60	**19**	23.27	38.21	19.06
FAR(1) (10 functions)	**7**	8.36	11.77	12.77	**19**	24.53	38.40	20.99
SAR(1)	**7**	9.13	12.29	13.70	**19**	26.13	41.06	22.01
SFAR(1) (5 functions)	**7**	8.42	11.75	12.74	**19**	24.91	39.04	21.16
SFAR(1) (10 functions)	**7**	9.07	11.99	14.11	**19**	25.98	38.99	22.84
AR(1)	**8**	18.91	27.67	22.79	**20**	23.26	41.01	21.08
FAR(1) (5 functions)	**8**	15.27	24.08	18.51	**20**	19.08	35.09	16.11
FAR(1) (10 functions)	**8**	16.13	23.97	20.55	**20**	18.85	34.54	16.43
SAR(1)	**8**	18.14	26.08	22.25	**20**	22.62	37.70	19.90
SFAR(1) (5 functions)	**8**	17.37	24.80	21.74	**20**	20.87	36.16	18.16
SFAR(1) (10 functions)	**8**	18.60	25.46	24.09	**20**	22.22	36.56	20.00
AR(1)	**9**	26.71	41.73	28.29	**21**	15.29	22.04	15.91
FAR(1) (5 functions)	**9**	22.56	38.91	23.33	**21**	13.34	20.24	13.49
FAR(1) (10 functions)	**9**	22.40	36.46	24.12	**21**	13.47	19.69	13.83
SAR(1)	**9**	27.24	39.85	29.50	**21**	15.85	21.60	16.80
SFAR(1) (5 functions)	**9**	26.21	39.85	28.64	**21**	16.51	23.04	17.30
SFAR(1) (10 functions)	**9**	26.62	38.01	30.35	**21**	15.28	21.66	15.88
AR(1)	**10**	23.25	40.09	25.17	**22**	10.21	17.07	12.41
FAR(1) (5 functions)	**10**	19.58	36.07	20.59	**22**	10.61	17.56	12.54
FAR(1) (10 functions)	**10**	19.70	36.96	21.37	**22**	11.45	18.48	13.46
SAR(1)	**10**	23.68	38.14	25.96	**22**	11.25	17.26	13.79
SFAR(1) (5 functions)	**10**	22.92	36.55	25.82	**22**	13.43	19.27	16.23
SFAR(1) (10 functions)	**10**	23.12	37.02	26.37	**22**	12.91	19.23	15.43
AR(1)	**11**	15.77	22.79	19.62	**23**	7.23	10.92	10.31
FAR(1) (5 functions)	**11**	13.66	20.20	17.10	**23**	6.37	9.62	8.92
FAR(1) (10 functions)	**11**	14.88	21.57	19.02	**23**	7.09	10.76	9.87
SAR(1)	**11**	16.04	22.24	20.42	**23**	6.97	10.54	9.72
SFAR(1) (5 functions)	**11**	15.82	22.02	20.42	**23**	6.84	10.00	9.54
SFAR(1) (10 functions)	**11**	18.59	24.78	24.15	**23**	7.55	11.02	10.45
AR(1)	**12**	14.92	21.56	18.95	**24**	6.55	8.37	10.43
FAR(1) (5 functions)	**12**	11.67	18.19	15.07	**24**	5.74	7.36	9.23
FAR(1) (10 functions)	**12**	12.63	19.20	16.37	**24**	5.83	7.79	9.35
SAR(1)	**12**	14.90	20.61	19.56	**24**	5.74	7.41	9.08
SFAR(1) (5 functions)	**12**	13.66	19.76	18.19	**24**	6.07	7.88	9.67
SFAR(1) (10 functions)	**12**	15.23	20.83	20.54	**24**	6.58	8.50	10.55

Figure 15: Mean absolute error for demand forecasting.

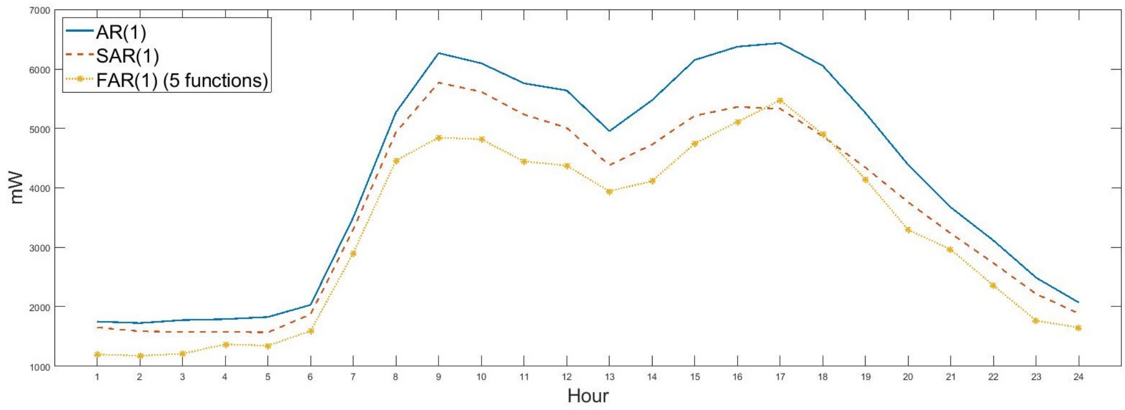


Figure 16: Root mean square error for demand forecasting.

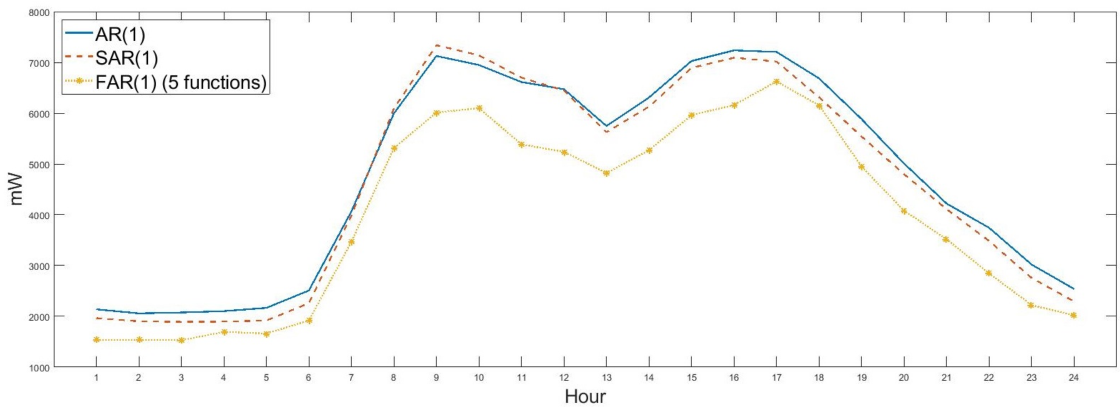


Figure 17: Mean absolute percentage error for demand forecasting.

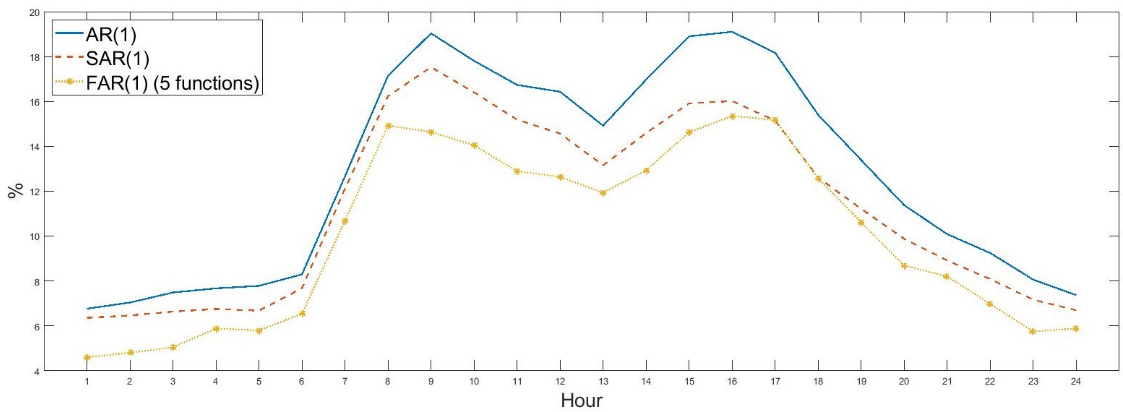


Table 5: Demand prediction accuracy statistics.

Model	Hour	MAE mW	RMSE mW	MAPE %	Hour	MAE mW	RMSE mW	MAPE %
AR(1)	**1**	1749	2134	6.7575	**13**	4955	5750	14.9272
SAR(1)		1650	1960	6.349		4381	5629	13.1578
FAR(1) (5 functions)		1197	1534	4.5906		3941	4824	11.9352
AR(p)	**2**	1723	2054	7.0308	**14**	5477	6313	16.9839
SAR(1)		1584	1897	6.458		4731	6132	14.596
FAR(1) (5 functions)		1173	1531	4.7919		4117	5272	12.9417
AR(p)	**3**	1773	2071	7.4811	**15**	6154	7033	18.9165
SAR(1)		1573	1887	6.6261		5214	6897	15.9249
FAR(1) (5 functions)		1206	1523	5.0349		4747	5964	14.6323
AR(p)	**4**	1789	2098	7.6647	**16**	6378	7243	19.1183
SAR(1)		1576	1892	6.7444		5364	7098	16.0223
FAR(1) (5 functions)		1365	1690	5.8724		5113	6164	15.3625
AR(p)	**5**	1825	2162	7.7768	**17**	6439	7211	18.1779
SAR(1)		1566	1912	6.6696		5334	7022	15.1515
FAR(1) (5 functions)		1343	1656	5.7846		5476	6630	15.1782
AR(p)	**6**	2029	2505	8.2893	**18**	6055	6690	15.406
SAR(1)		1870	2254	7.6712		4869	6316	12.6027
FAR(1) (5 functions)		1589	1913	6.5373		4905	6158	12.5699
AR(p)	**7**	3502	4065	12.6681	**19**	5259	5887	13.3915
SAR(1)		3299	3987	12.125		4341	5543	11.2166
FAR(1) (5 functions)		2903	3464	10.6912		4142	4948	10.6123
AR(p)	**8**	5272	6000	17.1605	**20**	4387	5009	11.3723
SAR(1)		4938	6097	16.2434		3758	4798	9.8719
FAR(1) (5 functions)		4461	5316	14.9315		3294	4079	8.6847
AR(p)	**9**	6270	7132	19.0464	**21**	3670	4220	10.0831
SAR(1)		5772	7345	17.5386		3231	4109	8.909
FAR(1) (5 functions)		4847	6016	14.6453		2960	3516	8.1968
AR(p)	**10**	6098	6954	17.8199	**22**	3111	3741	9.2407
SAR(1)		5618	7145	16.4139		2732	3483	8.0774
FAR(1) (5 functions)		4818	6103	14.0522		2353	2842	6.9697
AR(p)	**11**	5761	6618	16.7452	**23**	2485	3016	8.0533
SAR(1)		5236	6704	15.1949		2211	2752	7.1536
FAR(1) (5 functions)		4445	5389	12.8906		1764	2216	5.7381
AR(p)	**12**	5641	6476	16.4409	**24**	2067	2534	7.3591
SAR(1)		5011	6449	14.573		1883	2289	6.6889
FAR(1) (5 functions)		4377	5240	12.6463		1646	2015	5.8758

6 Conclusions

We presented a parsimonious way for representing supply and demand curves, using a mesh-free method based on Radial Basis Functions. Using the tools of functional data analysis, we are able to approximate the original curves with far less parameters than the original ones. Namely, in order to approximate piecewise constant monotone functions, we are using linear combinations of integrals of Gaussian functions.

The real data about supply and demand bids from the Italian day-ahead electricity market showed that there is no direct relationship between the number of offer and bid layers and the hour of the day, the day of the week, and the time of the year. Based on this observation, we decided to choose the same number of basis functions independently of these three seasonality modes. The numerical results showed that the accuracy of our approximation is good enough, if we use 5 basis function for the demand curve and 10 basis function for the supply curve, and then the increase in the number of functions leads to more time-consumption, but the increase of the accuracy is less significant.

We also tested this new approach with the aim of forecasting supply and demand curves and finding the intersection of the predicted curves in order to obtain the market clearing price. In assess the goodness of our method, we compared it with models with similar complexity in terms of dependence of the past, but only based on the clearing price. Our forecasting errors are smaller compared with these univariate models. In particular, our analyses show that our multivariate approach leads to better results than the univariate one in terms of error measures like MAE, MAPE and RMSE.

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