Energy optimal design of jerk-continuous trajectories for industrial robots

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Abstract In this paper a method for planning smooth and energy-efficient trajectories for industrial robots is presented. The motion design, which is based on the use of piecewise polynomial functions, is optimized for achieving minimum energy consumption when executing a trajectory that passes through a sequence of via-points with C^3 continuity. For robots with simple kinematics, such as the Cartesian robot presented in this work, the energy consumption estimation can be performed using equations based on inverse dynamic models that allow fast and reliable numerical computing.

Key words: energy saving, trajectory planning, robot, energy

1 Introduction

Modern robotics is now facing the challenge of energy efficiency, as the reduction of the energy consumption is fostered by the growing environment awareness and and by a general increase of the energy cost. This attention is testified by a recent flourishing literature on energy saving for automatic machines [1], which list several methods that can bring an effective energy improvement. One method relies on the use of lightweight robots, or on the use of energy sharing [2] or energy storage devices [3]. If the robotic architecture cannot be altered, one possibility is to tackle the problem by designing energy-efficient trajectories, i.e. by moving the robot using motion profiles that minimize the energy consumption, but mixed approaches can be used as well [4, 5]. Also kinematic redundancy can be exploited to enhance energy efficiency, as shown in [6]. The influence of motion planning on the consumption of electric driven mechatronic devices has been largely investigated, and

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several numerical, experimental [7] and analytic models [8, 9] have been developed to establish a precise relation between motion and energy consumption.

This work is aimed at developing a method for the design of energy efficient trajectories for industrial robots. Such a method is quite general, since it is based on the solution of the inverse dynamic problem and the application of a simple electric model of the robot actuators. It is also shown that in some simple cases, such as the test-case used in this work, the numerical solution of the inverse dynamic problem is not needed, since it can be replaced by the analytic expressions that are developed in the following. The motion design is based on splines, which allow to generate a motion that interpolates a sequence of via-points, ensuring continuity up to jerk for minimal motion-induced vibrations.

2 Spline-based trajectory planning

The trajectory planning method of choice for this work uses via-points as the mean to specify basic information on the path of the end-effector. A trajectory planning algorithm will then take care of interpolating such point with piecewise polynomial segments, often referred to as splines [10]. In particular, a trajectory defined by a set of via-points can be described by a set of interpolating splines of different degrees, arranged so that continuity up to the third derivative, i.e. jerk, can be achieved. One possible method is to use the '445' method [11], which uses a fourth-degree polynomials to describe all the segments with the exception of the last one, for which a fifth-degree polynomial is used. Other methods that achieve C^3 continuity are the '545' and the '5455' methods, which are capable of enforcing null initial jerk, and null initial and final jerk, respectively. A detailed overview of such methods, together with their analytic formulation, is available in [12]. If the desired end-effector motion is described by a set of N via-points, the trajectory must be defined so that it comprises $N-1$ segments, each one described by a m-th degree polynomial form such as:

$$F_k(t) = b_{0,k} + b_{1,k}t + b_{2,k}t^2 + \ldots + b_{m,k}t^m = \sum_{i=0}^{m} b_{i,k}t^i \tag{1}$$

Equation (1) describes the time evolution of the position of the end-effector, for $t \in [0, T_k]$, being T_k the duration of the k-th segment of the trajectory. Formula (1) refers to a single axis of motion, since the extension to multiple axes is straightforward. The corresponding speed and acceleration profiles are:

$$\dot{F}_k(t) = \sum_{i=1}^{m} i\, b_{i,k}t^{i-1} \qquad \ddot{F}_k(t) = \sum_{i=2}^{m} i(i-1)\, b_{i,k}t^{i-2} \tag{2}$$

The vector $\mathbf{T} = [T_1, \ldots, T_{N-1}]$ collects the time distances between consecutive via-points, the polynomial coefficients $b_{i,n}$ are determined for all $N-1$ segments at once to ensure the enforcement of boundary and continuity conditions [12]. Each trajectory is conveniently parametrized by $\mathbf{T}$: given a set of via-points, each choice of **T** uniquely defines the values the polynomial coefficients $b_{i,n}$. This parametrization is used to enforce bounded speed, acceleration and jerk profiles, and, as it will be shown in the following section, to minimize the energy consumption associated with the execution of a trajectory.

2.1 Energy consumption estimation

The estimation of the energy consumption of a robot requires to use both its dynamic and the electric model. Assuming that the robot is actuated by brushless motors, DC motors or induction motors, an equivalent DC motor model can be effectively used to describe its electric behavior. According to it, the current $I(t)$ drawn by a motor is proportional to the exerted torque τ_m thorough the motor constant k_t:

$$\tau_m(t) = k_t I(t) \tag{3}$$

The voltage drop across the motor is due to the voltage drop across the motor equivalent resistance R, plus the voltage generated by the equivalent speed-controller back-emf generator, according to:

$$V(t) = RI(t) + k_b \dot{q}(t) \tag{4}$$

in which k_b is the back-emf constant of the motor, and $\dot{q}(t)$ is its revolution speed. The voltage-current product can then be used to compute the instantaneous motor electric power absorption:

$$W_e(t) = V(t)I(t) \tag{5}$$

which can be used to measure the energy consumption over a time frame $[t_a, t_b]$ through the time integral:

$$E = \int_{t_a}^{t_b} W_e(t)dt = \int_{t_a}^{t_b} V(t)I(t)dt \tag{6}$$

Equation (6) shows that the computation of the energy consumption of a robot requires to compute the speed and the torque generated by each motor: the first one is known once the trajectory is defined, the latter need to be computed using an inverse dynamic model of the robot. The computation of the integral in Eq. (6) is usually performed numerically, but in some cases closed-form formulas can be used. Here a method to outline these formulas is presented for the simple case of a three axis Cartesian robot. In order to avoid confusing formulations, all formulas are reported for a single axis of the robot, being the extension to multiple axis straightforward for kinematically independent axes.

The starting point is the computation of the electric power absorption during the execution of the $k-th$ segment of the trajectory, for which a closed-formula expression is sought. According to Eq. (5), the electric power can be written as:

$$W_{e,k}(t) = V(t)\,I(t) = RI^2(t) + k_b\,I(t)\,\dot{q}(t) = W_{Joule,k}(t) + W_{m,k}(t) \tag{7}$$

Equation (7) highlights the contributions due to the Joule losses $W_{Joule,k}$ and to the mechanical power $W_{m,k}$. The first one is proportional to the square value of the current, i.e. to the squared value of the motor torque, according to Eq. (3), while the second one is simply the motor torque-speed product. The robot inverse dynamic model is required to compute them: for the simple case under consideration, it can be written, for any of the three axes of the robot, as:

$$\tau_m(t) = J\ddot{q}(t) + f_v\,\dot{q}(t) + T_{ext} \tag{8}$$

The simple model of Eq. (8) accounts for the equivalent moment of inertia at the motor shaft, J, the equivalent viscous damping coefficient f_v, and the equivalent constant torque T_{ext} at the motor shaft. The latter is used to account for the constant gravity load. Eq. (8) can be used in Eq. (7) leading to:

$$\begin{aligned} W_{Joule,k}(t) &= \frac{R}{k_t^2}\left(J\ddot{q}(t) + f_v\,\dot{q}(t) + T_{ext}\right)^2 \\ W_{m,k}(t) &= \frac{k_b}{k_t}\left(J\ddot{q}(t) + f_v\,\dot{q}(t) + T_{ext}\right)\dot{q}(t) \end{aligned} \tag{9}$$

Equation (9) clearly highlights that the instantaneous energy consumption is a function of some constant parameters, as well as of the joint speed $\dot{q}(t)$ and the acceleration $\ddot{q}(t)$. Since both the joint speed and acceleration take a polynomial form, according to Eq. (2), then also the electric power takes a polynomial form, that can be condensed by the simple formula:

$$W_{e,k} = \sum_{i=0}^{n} w_{i,k} t^i = \mathbf{w}_k \begin{bmatrix} t^0 \\ t^1 \\ \vdots \\ t^n \end{bmatrix} \tag{10}$$

in which $\mathbf{w}_k$ is a $1 \times n$ vector of coefficients. Such a vector needs to be computed using Eq.(1-9), which require some lengthy calculations that are omitted here. As a matter of example, the entries $w_{i,k}$ for a segment described by a fourth-degree polynomial, such as in all but the last segment of the 445 motion profile, take the following form:

$$w_{0,k} = \frac{\left(b_2 k_b + \frac{R(T_{ext}+2Jb_3+b_2 f_v)}{k_t}\right)(T_{ext} + 2Jb_3 + b_2 f_v)}{k_t} \tag{11}$$

$$w_{1,k} = \frac{2k_b\left(T_{ext} b_3 + 2J{b_3}^2 + 3Jb_2 b_4 + 2b_2 b_3 f_v\right)}{k_t} + \frac{4R(3Jb_4 + b_3 f_v)(T_{ext} + 2Jb_3 + b_2 f_v)}{{k_t}^2} \tag{12}$$

$$w_{2,k} = \frac{(6Jb_4 + 2b_3 f_v)\left(2b_3 k_b + \frac{R(6Jb_4+2b_3 f_v)}{k_t}\right)}{k_t} + \frac{(12Jb_5 + 3b_4 f_v)\left(b_2 k_b + \frac{R(T_{ext}+2Jb_3+b_2 f_v)}{k_t}\right)}{k_t} + \frac{\left(3b_4 k_b + \frac{R(12Jb_5+3b_4 f_v)}{k_t}\right)(T_{ext} + 2Jb_3 + b_2 f_v)}{k_t} \tag{13}$$

$$w_{3,k} = \frac{4R\left(36b_5 J^2 b_4 + 9Jb_4{}^2 f_v + 16b_3 b_5 J f_v + 3b_3 b_4 f_v{}^2 + 2b_2 b_5 f_v{}^2 + 2T_{ext} b_5 f_v\right)}{k_t{}^2} + \frac{2k_b\left(9Jb_4{}^2 + 6b_3 f_v b_4 + 2T_{ext} b_5 + 16Jb_3 b_5 + 4b_2 b_5 f_v\right)}{k_t} \tag{14}$$

$$w_{4,k} = \frac{R\left(144J^2 b_5{}^2 + 120Jb_4 b_5 f_v + 9b_4{}^2 f_v{}^2 + 16b_3 b_5 f_v{}^2\right)}{k_t{}^2} + \frac{k_b\left(9f_v b_4{}^2 + 60Jb_5 b_4 + 16b_3 b_5 f_v\right)}{k_t} \tag{15}$$

$$w_{5,k} = \frac{24b_5 k_b (2Jb_5 + b_4 f_v)}{k_t} + \frac{24Rb_5 f_v (4Jb_5 + b_4 f_v)}{k_t{}^2} \tag{16}$$

$$w_{6,k} = \frac{16b_5{}^2 f_v (Rf_v + k_b k_t)}{k_t{}^2} \tag{17}$$

The equivalent formulation for trajectory segments described by fifth-degree polynomial is omitted due to the space constraints of this paper. The polynomial expression of the electric power of Eq. (10) is particularly convenient for the the computation of the time integral that defined the energy consumption associated with the execution of the $k-th$ segment of the trajectory as:

$$E_k = \int_0^{T_k} W_{e,k} dt = \sum_{i=0}^{n} \frac{w_{i,k}}{i+1} T_k^{i+1} \tag{18}$$

Eq. (18) is very straightforward to compute, since it relates the time durations of the segments, T_k, to the energy consumption once the via-points positions are known, with the aid of the polynomial coefficients b_i. This formulation can be used for manipulators with coupled axis dynamics too, provided that a sufficiently simple robotic architecture is taken into account.

3 Energy optimization

The formulas reported above, and in particular Eq. (18), can be used to formulate an energy optimization problem. The total energy consumption E is parametrized by the vector of time intervals $\mathbf{T} = [T_1, \ldots, T_{N-1}]$ and therefore Eq. (18) can be directly used as the cost function. The following optimization problem is proposed:

$$\begin{cases} \min\limits_{[T_1,..,T_{N-1}]} E_{tot} = \min\limits_{[T_1,..,T_{N-1}]} \sum\limits_{j=1}^{3} \sum\limits_{k=1}^{N-1} E_{k,j} \\ \\ \text{subject to: bounded speed, acceleration and jerk} \end{cases} \quad (19)$$

The optimization problem in Eq. (18) is solved for a set of 9 via-points, resulting in the path shown in Fig. 1, which refers to the optimal solution for the 445 motion profile. The starting point of the trajectory is located on Cartesian coordinates $(0, -.45, 0)$ m, the last on is located on $(.6, .45, .15)$ m. The energy-optimal motion profile is shown in Fig. 2, in which the acceleration and the jerk profile for the Y axis of the robot are shown, comparing the results of the 445 interpolation and of the 5455 one. In particular, Fig. 2 shows that the jerk profile shown in black color, i.e. the 445 one, is continuous, however it takes large values at initial and final point. In particular, during the last segment the jerk of the Y axis can reach values as high as 7 m/s^3, so it is expected that some residual vibration could be detected when executing this motion task. This occurrence is avoided by the 5455 motion profile, since it can enforce null jerk at both boundary points. The corresponding energy-optimal trajectory is shown in Fig. 2 as well, using a gray line. The improved smoothness however results in an increased total execution time. A comparison of the execution time and of the energy consumption is provided in Table 1, which reports the results for the same via-points, using several trajectory primitives. The comparison highlights that smoother trajectory require more energy and a longer execution time, however such difference is at most 5%. A minor energy improvement can be obtained by using a less smooth trajectory, as the 434 trajectory [13] requires 3% less time and 2.58% less energy than the 445 trajectory.

Table 1 Energy optimal trajectories: comparison of total execution time and energy

Motion profile	Total time $[s]$	ΔT	Energy $[J]$	ΔE
445	8.379	-	157.1	-
545	8.578	$+2.37\%$	160.2	$+2\%$
5455	8.805	$+5.07\%$	164.0	$+4.4\%$
434	8.130	-2.98%	153.0	-2.58%

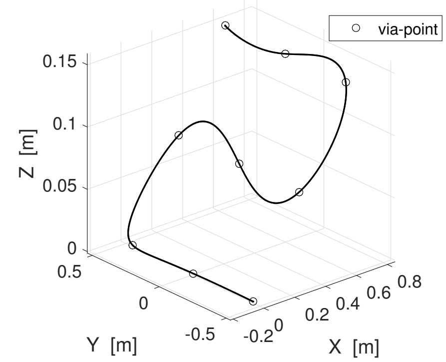

Fig. 1 Energy-optimal motion profile for the 445 algorithm: path in the operational space

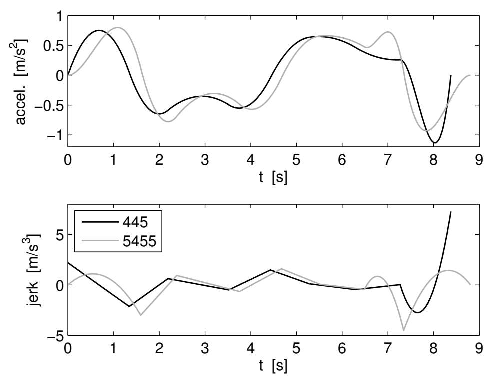

Fig. 2 Energy-optimal motion profiles: acceleration and jerk profiles, Y axis, 445 and 5455 trajectories

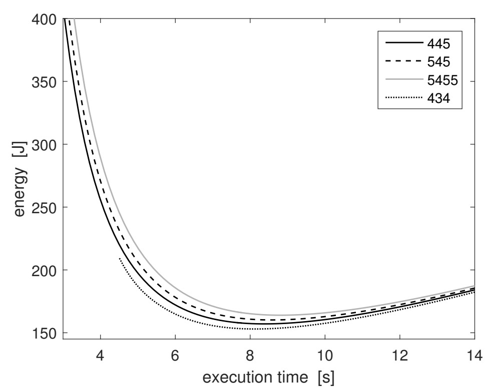

Fig. 3 Energy vs total execution time: comparison between interpolation methods

4 Conclusion

In this work a method to reduce the energy consumption of industrial robots for motion tasks defined by via-points has been presented. The method allows to achieve an exact interpolation of via-points, with continuity up to the jerk for improved smoothness and reduced motion-induced vibrations. The trajectory is parametrized by the time distance between consecutive via-points, providing a simple parametrization that is exploited to reduce the energy consumption of the robot. The estimation of the robot energy consumption is provided by analytic expressions, which are developed in this work. The availability of simple formulas allows to avoid the time-consuming numerical integration of the robot dynamics that is usually needed to perform this kind of optimization. The work provides also a comparison between the effect on the overall energy consumption as the result of the choice of the trajectory primitive. The data highlight that smoother trajectories are slightly less energy efficient and require a slightly longer execution time.

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