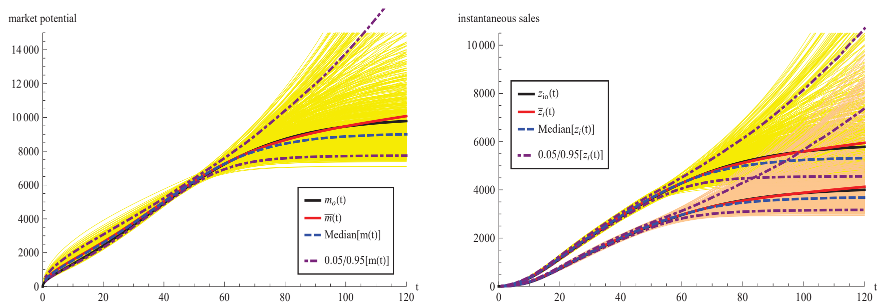

Fig A5.19. *Case BE(1,0.5), ${}_{\omega}\sigma_i = 0.15$. Left side: $\hat{m}_k(t)$ (yellow trajectories), true $m_0(t)$, $\overline{m}(t) = \frac{1}{1000}\sum_{k=1}^{1000}\hat{m}_k(t)$, median trajectory, and quantile trajectories (0.05 and 0.95). Right side: $\hat{z}_{ik}(t)$ (yellow and orange trajectories), true $z_{i0}(t)$, $\overline{z}_i(t) = \frac{1}{1000}\sum_{k=1}^{1000}\hat{z}_{ik}(t)$, median trajectory, and quantile trajectories (0.05 and 0.95).*

Table A5.7

MSE of the parameter estimates for $(p_1, q_1, p_2, q_2, \delta)$ with alternative $m(t)$ specifications.

	parameter (β_{j0})	p_1 (0.02)	q_1 (0.03)	p_2 (0.005)	q_2 (0.1)	δ (0.05)
BE(0.5,0.5)	${}_{\omega}\sigma_i = 0.05$	$1.1897*10^{-5}$	$2.7558*10^{-3}$	$1.4823*10^{-6}$	$3.5251*10^{-3}$	$8.9532*10^{-3}$
	${}_{\omega}\sigma_i = 0.10$	$2.5986*10^{-5}$	$8.3951*10^{-3}$	$6.5940*10^{-6}$	$9.1042*10^{-3}$	$2.4722*10^{-2}$
	${}_{\omega}\sigma_i = 0.15$	$5.5103*10^{-5}$	$3.1755*10^{-2}$	$2.1710*10^{-5}$	$3.2648*10^{-2}$	$9.0742*10^{-2}$
BE(0.25,0.5)	${}_{\omega}\sigma_i = 0.05$	$1.1852*10^{-5}$	$1.8281*10^{-3}$	$1.1023*10^{-6}$	$2.5001*10^{-3}$	$6.1818*10^{-3}$
	${}_{\omega}\sigma_i = 0.10$	$2.7446*10^{-5}$	$6.6698*10^{-3}$	$5.0748*10^{-6}$	$7.4257*10^{-3}$	$1.9925*10^{-2}$
	${}_{\omega}\sigma_i = 0.15$	$4.9248*10^{-5}$	$2.1805*10^{-2}$	$1.3392*10^{-5}$	$2.3126*10^{-2}$	$6.1728*10^{-2}$
BE(1,0.5)	${}_{\omega}\sigma_i = 0.05$	$3.7755*10^{-6}$	$4.8813*10^{-3}$	$3.1662*10^{-6}$	$5.4007*10^{-3}$	$1.4122*10^{-2}$
	${}_{\omega}\sigma_i = 0.10$	$1.4818*10^{-5}$	$1.9343*10^{-2}$	$1.1618*10^{-5}$	$2.0172*10^{-2}$	$5.5211*10^{-2}$
	${}_{\omega}\sigma_i = 0.15$	$3.3855*10^{-5}$	$6.4107*10^{-2}$	$2.9979*10^{-5}$	$6.4740*10^{-2}$	$1.8183*10^{-1}$
TW(20)	${}_{\omega}\sigma_i = 0.05$	$1.1022*10^{-4}$	2.5203	$1.9994*10^{-6}$	3.5076	6.1608
	${}_{\omega}\sigma_i = 0.10$	$1.1130*10^{-4}$	2.5217	$2.6444*10^{-6}$	3.5249	6.3360
	${}_{\omega}\sigma_i = 0.15$	$1.1872*10^{-4}$	2.5613	$3.0510*10^{-6}$	3.5735	6.4498

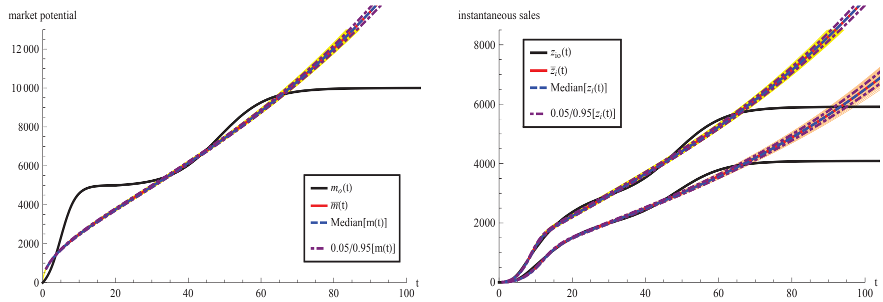

Fig A5.20. *Case TW(20),* $_\omega\sigma_i = 0.05$. *Left side:* $\hat{m}_k(t)$ *(yellow trajectories), true* $m_0(t)$, $\overline{m}(t) = \frac{1}{1000}\sum_{k=1}^{1000}\hat{m}_k(t)$, *median trajectory, and quantile trajectories (0.05 and 0.95). Right side:* $\hat{z}_{ik}(t)$ *(yellow and orange trajectories), true* $z_{i0}(t)$, $\overline{z}_i(t) = \frac{1}{1000}\sum_{k=1}^{1000}\hat{z}_{ik}(t)$, *median trajectory, and quantile trajectories (0.05 and 0.95).*

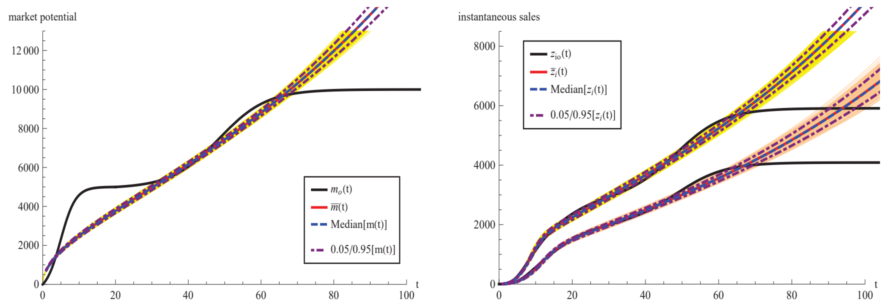

Fig A5.21. *Case TW(20),* $_\omega\sigma_i = 0.10$. *Left side:* $\hat{m}_k(t)$ *(yellow trajectories), true* $m_0(t)$, $\overline{m}(t) = \frac{1}{1000}\sum_{k=1}^{1000}\hat{m}_k(t)$, *median trajectory, and quantile trajectories (0.05 and 0.95). Right side:* $\hat{z}_{ik}(t)$ *(yellow and orange trajectories), true* $z_{i0}(t)$, $\overline{z}_i(t) = \frac{1}{1000}\sum_{k=1}^{1000}\hat{z}_{ik}(t)$, *median trajectory, and quantile trajectories (0.05 and 0.95).*

alternative $m(t)$ specifications. We can see that when $m(t)$ belongs to the Bemmaor family, the misspecification has a very small impact on the precision of the evolutionary parameters. For two-wave $m(t)$, the impact is larger.

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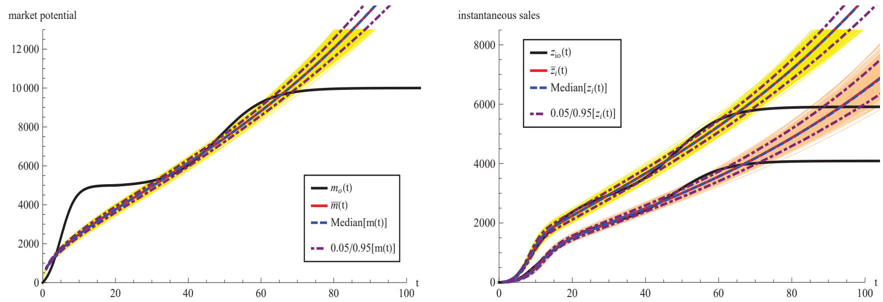

Fig A5.22. *Case TW(20), ${}_{\omega}\sigma_i = 0.15$. Left side: $\hat{m}_k(t)$ (yellow trajectories), true $m_0(t)$, $\overline{m}(t) = \frac{1}{1000}\sum_{k=1}^{1000}\hat{m}_k(t)$, median trajectory, and quantile trajectories (0.05 and 0.95). Right side: $\hat{z}_{ik}(t)$ (yellow and orange trajectories), true $z_{i0}(t)$, $\overline{z}_i(t) = \frac{1}{1000}\sum_{k=1}^{1000}\hat{z}_{ik}(t)$, median trajectory, and quantile trajectories (0.05 and 0.95).*

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