Control of the vibrations of a cartesian automatic machine

Matteo Bottin[0000−0001−6611−173X], Giulio Cipriani[0000−0002−6980−7377], Domenico Tommasino[0000−0002−5466−4270], and Alberto Doria[0000−0003−0896−7796]

Department of Industrial Engineering, University of Padova, Padova 35131 , Italy
alberto.doria@unipd.it
https://research.dii.unipd.it/mmsa/

Abstract. This paper deals with the control of the vibrations of an automatic cutting machine with a cartesian kinematic chain. A particular application of the method of structural modifications is considered, since a small number of frequency response function is available. The Sherman-Morrison formula is used to predict the effect of a tuned vibration absorber (TVA) on the vibrations of the machine generated by the cutting tool; the advantages and the limits of this method are discussed. In order to improve the cutting speed, the cutting head moves along the linear guides with relevant accelerations that can excite the TVA, motion induced vibrations are analyzed with a lumped element model.

Keywords: cutting · tuned vibration absorber · automatic machine · dynamic modeling.

1 Introduction

The increase in the working speed of robots and automatic machines is a typical trend of manufacturing industry. Sometimes high levels of noise and vibrations are detected in machines modified in order to increase the working speed, this happens because the increased speed leads to the excitation of modes of vibration that were not considered in the first design of the machine. Vibrations of the modified machine can be controlled carrying out structural modifications, e.g. increasing stiffness or adding lumped masses or tuned vibration absorbers (TVA). In recent years some effective methods for predicting the effects of structural modifications starting from the measured frequency response functions (FRFs) have been developed [14, 12, 4, 10]. These methods are based on the modal analysis approach and typically require a large set of experimentally measured FRFs. In actual industrial applications, usually small slots of time are available for carrying out experimental tests on existing (and working) machines and robots. For this reason, the engineer in charge of vibration control has to predict the effect of structural modifications using a very limited set of measured FRFs.

This paper focuses on this particular aspect of the method of structural modifications. Only 2 measured FRFs are used for predicting the effect of a TVA on an automatic cutting machine. The next section briefly describes the cutting machine and the experimental tests. In section 3 the fundamentals of the method of structural modifications are summarized. In section 4 the Sherman-Morrison formula is used to predict the effect of a TVA on the vibrations of the machine generated by the cutting tool; the advantages and the limits of this method are discussed. The cutting head moves along the linear guides with relevant accelerations that can excite the TVA; hence motion induced vibrations are analyzed in section 5 adopting a lumped element approach. Finally, conclusions are drawn.

2 Experimental tests on the cutting machine

The cutting machine considered in the framework of this research has a cartesian kinematic chain. The cutting head moves at high speed in the Y and Z directions, whereas the cutting tool operates in the X-direction, as shown in Fig. 1. The cutting tool is operated as a pneumatic actuator by inflating and deflating the air inside the cylinder. The high-speed switching needed for cutting is the cause of noisy vibrations. Moreover, the tool speed was increased from 8000 rpm to 13000 rpm to make the machine able to cut hard materials.

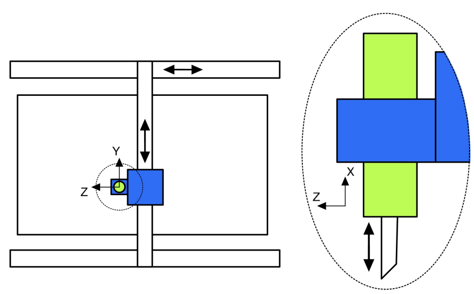


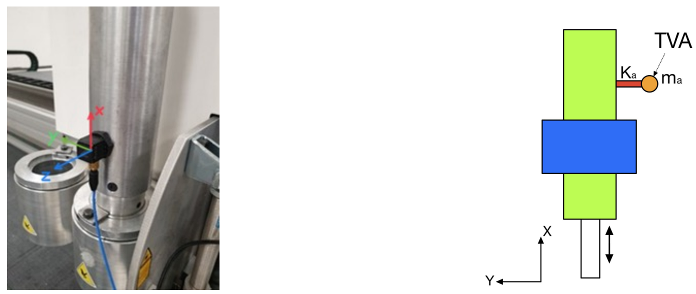
Fig. 1: Scheme of the cutting machine.

Unfortunately, the decision came up after the design and construction of the machine. It turned out that at 13000 rpm, the system is excited near a natural frequency and vibrations increase reaching annoying levels.

The experimental tests involved a modal analysis on the existing system and the solution is proposed trying to minimize modifications. The experimental analysis was subjected to two major restrictions:

1. To reduce day-offs, it had to be carried out in just one working day.
2. The actual system's geometry prevents excitation parallel to surfaces and the location of sensors in many points, hence the measures were limited to two points, one on the tool and one on its seat.

The analysis is carried out using an instrumental hammer and a triaxial accelerometer, shown in Fig. 2. The excitation is performed near the accelerometer and along its Z-axis.



(a) (b)

Fig. 2: (a) Testing equipment and (b) scheme of the proposed solution (in orange the TVA added to the tool).

The FRFs, shown in Figs. 3 and 4, are calculated from the measured accelerations and hammer force, processing the measured signals by means of LabView.

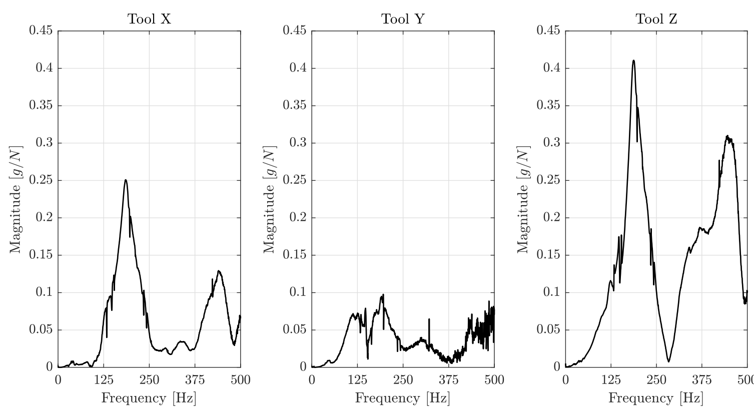


Fig. 3: FRFs measured on the tool.

Figs. 3 and 4 highlight the presence of a resonance peak at 187.7 Hz, which can be excited by the tool operating at 13000 rpm (216.7 Hz). Vibrations are critical radially (along accelerometer Z-direction) and, to a lesser degree, axially (along accelerometer X-direction). The FRFs measured on the seat highlight that the mode at 187.7 Hz involves only the tool.

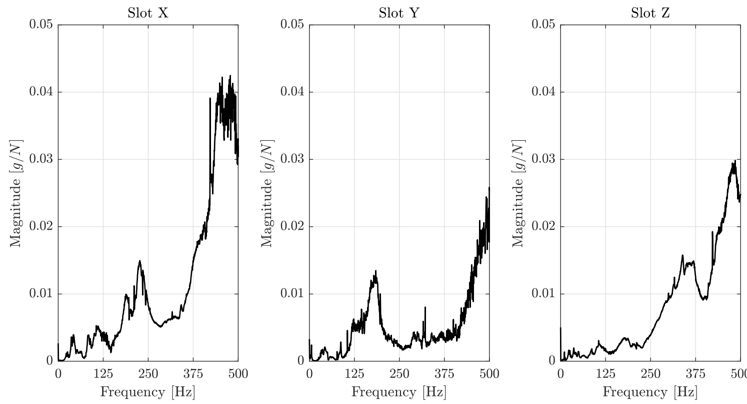


Fig. 4: FRFs measured on the tool seat.

3 Structural modifications

The measurement of a set of FRFs between some response coordinates (p_n) and some excitation coordinates (q_n) is the basis of modal analysis methods. The measured FRFs not only allow the identification of the modal parameters of the system [9, 5, 2], but also allow the prediction of the effect of modifications of the structure without building an actual prototype. The great advantages in terms of reduction in development times and costs have fostered the research in this field and the results of many relevant researches have been published in recent years [12, 9, 3].

The Sherman-Morrison method is a linear algebra technique [6] that have been successfully used to solve many engineering problems including structural modifications [12]. In particular, the effects of structural modifications in milling machines [7] and trusses [11] have been predicted with this method. In [10] the effect of a TVA on a multi-DOF mechanical system has been predicted by means of the Sherman-Morrison method, and in [1] some experimental results have been shown. The Sherman-Morrison approach can be used to predict the effect of lumped masses, stiffnesses and TVAs in dynamic systems described by many coordinates, but can it be used even if there are only a response, an excitation and a modification coordinate. For this reason, the Sherman-Morrison method is suited to industrial applications in which few FRFs are available.

The measured FRFs of a vibrating system can be collected in a receptance matrix $[A(\omega)]$. It is the inverse of the dynamic stiffness matrix $[Z(\omega)]$, which is related to the mass $[M]$, stiffness $[K]$ and damping matrices $[C]$ of the dynamic system:

$$[A(\omega)] = [[K] + i\omega[C] - \omega^2[M]]^{-1} = [Z(\omega)]^{-1} \tag{1}$$

Modifications in $[Z(\omega)]$ lead to modifications in $[A(\omega)]$ that can be calculated with the Sherman-Morrison formula, if $[A(\omega)]$ is a square non-singular matrix.

In particular, if the modification in $[Z(\omega)]$ can be expressed as the product of two vectors $\{u\}$ and $\{v\}$:

$$[\Delta Z(\omega)] = \{u\}\{v\}^T \tag{2}$$

The modified receptance matrix $[A_m(\omega)]$ is given by:

$$[A_m(\omega)] = [A(\omega)] - \frac{([A(\omega)]\{u\})(\{v\}^T[A(\omega)])}{1 + \{v\}^T[A(\omega)]\{u\}} \tag{3}$$

In particular, if only one modification at coordinate r is carried out adding a lumped element, all the elements of vectors $\{u\}$ and $\{v\}$ are zero except the r^{th} elements. The r^{th} element of $\{v\}$ is the dynamic stiffness of the lumped element $(z_l(\omega))$, whereas the r^{th} element of $\{u\}$ is 1.

When experimental results relative to only a response, an excitation and a modification coordinate are available the modified transfer function $(a_{pqm}(\omega))$ between the response and the excitation coordinate can be calculated according to this equation [4]:

$$a_{pqm}(\omega) = \frac{a_{pq}(\omega) + z_l(\omega)(a_{rr}(\omega)a_{pq}(\omega) - a_{pr}(\omega)a_{rq}(\omega))}{1 + z_l(\omega)a_{rr}(\omega)} \tag{4}$$

In the specific case considered in this research, the structural modification is a TVA located at coordinate r, hence this dynamic stiffness [12] has to be considered in Eq. 4

$$z_l(\omega) = \frac{(-\omega^2 m_a k_a - i\omega^3 m_a c_a)}{(k_a - \omega^2 m_a + i\omega c_a)} \tag{5}$$

In which k_a, c_a and m_a are the stiffness, damping and mass of the TVA.

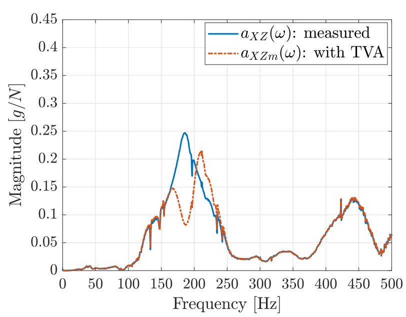
4 Prediction of the effect of a TVA by means of the Sherman-Morrison formula

The TVA consists of a steel cantilever beam with a tip mass m_a. The stiffness, damping and mass properties of the TVA determine the natural frequency and the damping ratio of the device:

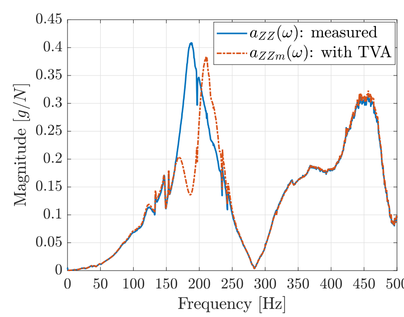
$$\omega_n = \sqrt{\frac{k_a}{m_a}} \tag{6}$$

$$\zeta_a = \frac{c_a}{2\sqrt{k_a\, m_a}} \tag{7}$$

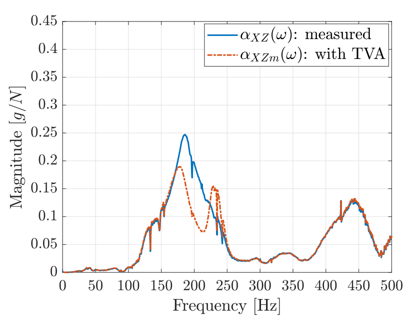
Mass m_a is related to the mass (M) of the original machine, and typically $m_a = (0.05 \div 0.1)M$ [14]. The natural frequency of the TVA is tuned to the frequency of excitation or to the natural frequency of the machine, hence Eq. 6 makes it possible to calculate the stiffness of the cantilever beam. Damping essentially depends on the material of the beam. Typical values of ζ_a for a steel beam are in the range $0.01 \div 0.05$. Stiffness k_a refers to barely deformation of the cantilever in the plane perpendicular to the cantilever's surface, hence the TVA is mounted on the tool as shown in Fig. 2b. The modification due to the TVA is applied to the Z-coordinate. Figs. 5a and 5b compare the measured FRF of the tool with the FRF predicted by means of the Sherman-Morrison formula, considering the effect of the vibration absorber tuned to the natural frequency of the tool ($f_n =$ 187.7 Hz). Fig. 5a compares $a_{XZ}(\omega)$ with $a_{XZm}(\omega)$, whereas Fig.5b compares $a_{ZZ}(\omega)$ with $a_{ZZm}(\omega)$. The TVA parameters are $m_a = 0.05$ kg, $k_a = 69544$ Nm^{-1} and $c_a = 5.90$ Nsm^{-1}. Figs. 5a and 5b show that the TVA reduces by about 66% the magnitude of the FRFs at the natural frequency and splits the first resonance peak in two peaks. Nevertheless, the second peak is still high (0.22 g/N at 211.7 Hz in X; 0.38 g/N at 210.1 Hz in Z). Typically, when the dynamic system is dominated by only one resonance peak, the TVA generates two new peaks having similar heights [8]. The different heights shown in Figs. 5a and 5b are due to the presence of higher modes of vibration.



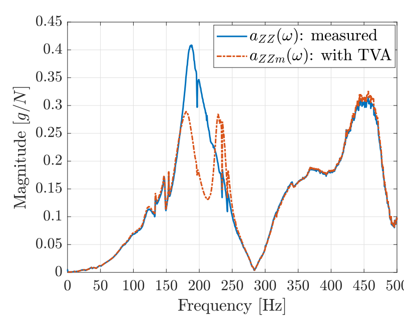
(a) $a_{XZ}(\omega)$, $a_{XZm}(\omega)$, $f_n = 187.7$ Hz



(b) $a_{ZZ}(\omega)$, $a_{ZZm}(\omega)$, $f_n = 187.7$ Hz



(c) $a_{XZ}(\omega)$, $a_{XZm}(\omega)$, $f_n = 216.7$ Hz



(d) $a_{ZZ}(\omega)$, $a_{ZZm}(\omega)$, $f_n = 216.7$ Hz

Fig. 5: Comparison between the measured FRF and the predicted FRF, considering a vibration absorber tuned to f_n.

Parametric simulations were carried out in order to evaluate the tuning frequency of the absorber, which has the largest effect on the first resonance peak. The largest reduction in the first resonance peak was obtained with a tuning frequency of 216.7 Hz, which corresponds to the cutting speed of the tool at 13000 rpm. Figs. 5c and 5d consider the vibration absorber tuned to the frequency $f =$ 216.7 Hz. The parameters of the TVA are $m_a = 0.05$ kg, $k_a = 92693\ Nm^{-1}$ and $c_a = 6.81\ Nsm^{-1}$. Figs. 5c and 5d shows that the TVA splits the first resonance peak in two smaller peaks and reduces the magnitude of the FRFs by about 42% at 216.7 Hz. This reduction is assumed enough to embedded a prototype of the TVA in the real machine to validate the models.

5 Effect of machine acceleration

Whilst the structure of the cartesian robot holding the tool is considered to be infinitely rigid for our dissertation (due to its bulky structure), it is true that the cartesian robot must move within the workspace to reach the working positions. Therefore, fast movement are performed by the robot, and high accelerations are exerted to the tool both in Y and Z axis, which can excite tool vibrations. To analyze this phenomenon, a lumped element two-DOFs model of the cutting head equipped with the TVA has been developed.

To test if the absorber does influence the excitation of the tool during the movement, a velocity profile along Z-axis is applied. This velocity profile is symmetric and trapezoidal with a maximum speed of 2 m/s, an acceleration time of 0.2 s and a total movement time of 4 s.

Results of this analysis are shown in Figs. 6a and 6b. The absorber slightly influences the response of the tool (Fig. 6a), but the difference is so limited to be considered negligible. In fact, the acceleration of the tool is much more constrained than the one of the absorber (Fig. 6b). Despite the high fluctuations of both tool and absorber, the damping allows to reduce the vibrations in less than 80 ms. This behaviour could be further improved if smoother trajectory profiles would be used, in which no discontinuities in accelerations would be exerted, e.g. third/five degree polynomials.

6 Conclusion

The results presented in this paper show that the Sherman-Morrison method is suited to develop a TVA for an actual machine, when few measurements are available. A series of parametric simulations made it possible to improve the tuning of the TVA.

Finally, since the TVA is installed on a cartesian robot, a dynamic analysis of its behavior during robot movement was carried out. Future work will implement the TVA on the real machine for validation.

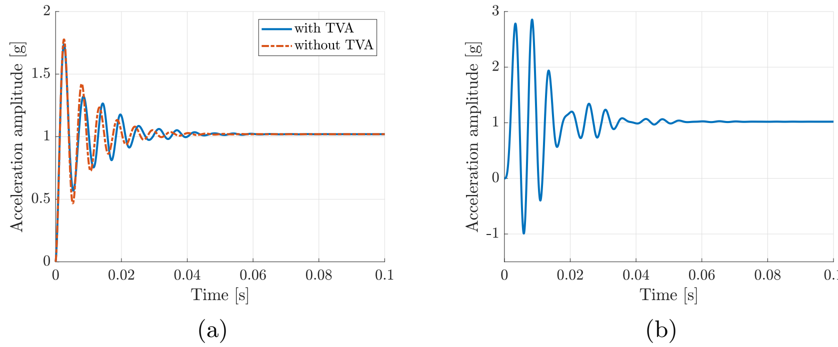


Fig. 6: (a) Comparison of the amplitude of acceleration of the tool with and without absorber; (b) Acceleration amplitude of the response of the absorber.

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