Analytical Movement Optimization of Dual-Arm Planar Robots with Rotating Platform

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Abstract. In the last years some industrial robots have been realized with two robot arms connected to a single rotating platform. Such a redundant structure allows moving the platform to adjust each robot base for optimizing each arm movement. In this paper, a planar model of two 7-axis robots connected to a rotating platform is proposed, and a novel analytical optimization procedure, retrieved from a geometrical analysis, is presented.

Keywords: Dual arm robot · Kinematics · Robotics

1 Introduction

Redundant robots have become more and more popular in the last years thanks to new commercial solutions, especially in the field of collaborative robotics [1]. These solutions, however, mostly rely on the addition of a supplementary joint to enhance the performance of traditional robots [2], by increasing their degrees of freedom.

In the field of industrial robots there is also a small branch in which it can be found a particular type of robots: dual arm robots. These are generally made up of two manipulators attached to a common platform [3]. In this way, the movements of the two robots are decoupled [4] (except for the possible mutual interference [5,6]), and the work-cell itself improves in both flexibility and capabilities [7,8]. Its flexibility can be further improved by adding a rotating platform, so that the two robot shoulders move around a common axis. In this way the two arms are capable of a wider workspace, but this solution has the downside of influencing the movement of both the arms. A proper trajectory planning of the rotating platform allows the movement of the two arms to improve the work-cell throughput. This is important because commercial solutions are already available.

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Research have already addressed the problem of the optimization of dual arm robots, but most of them are based on optimization algorithms that may not be reliable and depend on the starting conditions [9–11].

The aim of this paper is to develop an analytical method to rapidly provide a solution of the movement of the platform so that the two arms' elbow joints perform the same displacement in moving between consecutive positions. This method can be used to choose specific arm configuration depending on the task, e.g. in case of vibrations. The proposed method is specific to 7-axis robots with spherical shoulder and wrist joints, since with such configuration it is possible to change the redundant angle value without affecting the elbow joint, adapting the configuration of the distal joints to the movement time of the elbow joint.

The method, although approximated, does not rely on any optimization algorithm that may decrease performances while obtaining only local minima. Due to its fast calculation, the method can be used to optimize the work-cell throughput by using specific task sequencing algorithms [12].

2 Analytical Model

Let's consider the kinematic scheme shown in Fig. 1 in which a manipulator is made of two arms on the edge of a rotating platform of radius R_0. The manipulator is considered to be the planar projection of two 7-axis robots mounted on a rotating platform working on a plane. An example of such robot is the Yaskawa SDA series robot, in which two 7-axis robot are attached on a common rotating base[1]. Each arm is built in such a way that the first three joints and the last three joints can be considered as two different spherical joints (commonly named "shoulder" and "wrist" joints respectively), leaving the fourth joint ("elbow") to rotate lonely.

In this scheme S_i is the shoulder of the arm i and P_i is the corresponding target position. The rotation of the platform is identified by the angle φ. For simplicity, the first arm's shoulder (S_1) is located on the radius defined by the angle φ, while the second arm's shoulder (S_2) is located around the same circumference of S_1 (centered in platform center O) at an angular distance of α from S_1.

From this scheme, the shoulder positions S_1 and S_2 can be easily calculated:

$$S_1 = \begin{Bmatrix} R_0 \cos(\varphi) \\ R_0 \sin(\varphi) \end{Bmatrix} \quad , \quad S_2 = \begin{Bmatrix} R_0 \cos(\varphi + \alpha) \\ R_0 \sin(\varphi + \alpha) \end{Bmatrix} \tag{1}$$

For simplicity the two arms are considered to be equal with link lengths a_1 (first link) and a_2 (second link), and the robot arms are considered to be lying on the xy-plane. In this way, joints 1-2-3 can be reduced to a single joint on the shoulder, and joints 5-6-7 are reduced on the wrist. This simplification can be justified by the fact that joints 1-2-3 act as a single spherical joint on

[1] https://www.motoman.com/en-us/products/robots/industrial/assembly/sda.

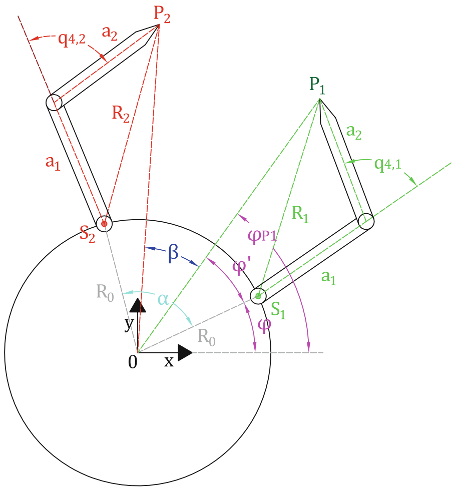

Fig. 1. Scheme of the dual arm robot with rotating platform.

the shoulder, allowing the arm to increase its height. In this case, on the xy-plane all the points would be projected, but the shoulder-wrist distance will only depend on the link lengths a_1 and a_2. In fact, for the proposed method only shoulder-wrist distance and platform radius are needed.

Since target positions P_1 and P_2 are known, the shoulder-wrist distance is:

$$\begin{gathered}\overrightarrow{R_i} = \overrightarrow{P_i} - \overrightarrow{S_i}(\varphi) \\ R_i^2 = (x_{P,i}^2 + y_{P,i}^2) + R_0^2 - 2R_0\sqrt{x_{P,i}^2 + y_{P,i}^2}\cos(\varphi - \varphi_{P,i})\end{gathered} \tag{2}$$

where $x_{P,i}$ and $y_{P,i}$ are the coordinates of the target position P_i on the plane and $\varphi_{P,i}$ is the angle about x-axis defined by the vector $\overrightarrow{P_i}$.

The objective of the optimization is to find an angle φ_f so that the two arms, in moving from a position $P_{i,0}$ to $P_{i,f}$, move equally the fourth joint $q_{4,i}$. This is possible since in a 7-axis robot the value of q_4 only depends on the shoulder-wrist distance. In fact, since a 7-axis robot is kinematically redundant, it is possible to change the redundant angle value to change the values of the other joints (except for q_4) still maintaining functionality (e.g. with the method proposed in [13]), decoupling the problem [14].

In this scenario, it is important to impose (if possible) that the movement of the elbow joints ($\Delta q_{4,i}$) are equal. The equation holds:

$$\Delta q_{4,i}(\varphi_f) = \arccos\left(\frac{a_1^2 + a_2^2 - R_{1,f}^2}{2a_1a_2}\right) - \arccos\left(\frac{a_1^2 + a_2^2 - R_{1,0}^2}{2a_1a_2}\right) \tag{3}$$

Equation 3 is highly non linear and it is difficult to solve with analytical tools. It is possible to approximate this equation by using the appropriate Taylor series, that, in the case of the arccosine, is $\arccos(x) = \frac{\pi}{2} - x + o(x^3)$. By stopping at the first order of the series, Eq. 3 becomes:

$$\Delta q_{4,i}(\varphi_f) = \frac{R_{1,0}^2 - R_{1,f}^2}{2a_1 a_2} \tag{4}$$

In such a way, since the two arms are equal and the objective of our optimization is to equal the two elbow joint displacements $\Delta q_{4,1}$ and $\Delta q_{4,2}$:

$$|R_{1,f}^2 - R_{1,0}^2| = |R_{2,f}^2 - R_{2,0}^2| \tag{5}$$

where the absolute values of the movements are taken since the electric motors equipped on the robots provide equal performance in both directions.

The initial system configuration is considered to be known, so $R_{i,0}$ and $R_{2,0}$ are fully defined (due to the initial platform rotation φ_0). To calculate φ_f, from Eq. 2 and Fig. 1 it is possible to retrieve the values of $R_{1,f}^2$ and $R_{2,f}^2$:

$$\begin{aligned} R_{1,f}^2 &= (x_{P1,f}^2 + y_{P1,f}^2) + R_0^2 - 2R_0\sqrt{x_{P1,f}^2 + y_{P1,f}^2}\cos(\varphi_f - \varphi_{P1,f}) \\ &= (x_{P1,f}^2 + y_{P1,f}^2) + R_0^2 - 2R_0\sqrt{x_{P1,f}^2 + y_{P1,f}^2}\cos(\varphi'_f) \end{aligned} \tag{6}$$

$$\begin{aligned} R_{2,f}^2 &= (x_{P2,f}^2 + y_{P2,f}^2) + R_0^2 - 2R_0\sqrt{x_{P2,f}^2 + y_{P2,f}^2}\cos(\varphi_f - \varphi_{P1,f} + \alpha - \beta) \\ &= (x_{P2,f}^2 + y_{P2,f}^2) + R_0^2 - 2R_0\sqrt{x_{P2,f}^2 + y_{P2,f}^2}\cos(\varphi'_f + \gamma) \end{aligned} \tag{7}$$

with $\gamma = \alpha - \beta$ and $\varphi'_f = \varphi_f - \varphi_{P1,f}$.

Equations 6 and 7 can be simplified by grouping the constant terms:

$$R_{1,f}^2 = A_1 + B_1\cos(\varphi'_f) \tag{8}$$
$$R_{2,f}^2 = A_2 + B_2\cos(\varphi'_f) + C_2\sin(\varphi'_f) \tag{9}$$

where:

$$A_1 = x_{P1,f}^2 + y_{P1,f}^2 + R_0^2 \qquad B_1 = -2R_0\sqrt{x_{P1,f}^2 + y_{P1,f}^2} \tag{10}$$
$$A_2 = x_{P2,f}^2 + y_{P2,f}^2 + R_0^2 \qquad B_2 = -2R_0\sqrt{x_{P2,f}^2 + y_{P2,f}^2}\cos(\gamma) \qquad C_2 = 2R_0\sqrt{x_{P2,f}^2 + y_{P2,f}^2}\sin(\gamma) \tag{11}$$

By imposing the equality of Eq. 5 it is possible to calculate the value of φ_f. The modulus defines two different equations, each one with opposite signs. It results in:

$$A + B\cos(\varphi'_f) + C\sin(\varphi'_f) = 0 \tag{12}$$

$$A = (A_1 \mp A_2) - (R_{1,0}^2 \mp R_{2,0}^2) \quad , \quad B = B_1 \mp B_2 \quad , \quad C = \mp C_2 \tag{13}$$

where the sign depends on which one of the two solutions provided by the modulus is being calculated.

It is possible to apply the double-angle formulae to Eq. 12, and by solving the equation it is possible to obtain the platform angle that equals the movement of the two arms:

$$\varphi_f = 2 \cdot \arctan\left(\frac{-C \pm \sqrt{-A^2 + B^2 + C^2}}{A - B}\right) + \varphi_{P1,f} \tag{14}$$

Equation 14 produces 2 solutions, which are doubled thanks to the absolute value equality expressed in Eq. 5.

It is worth to notice how the proposed method does not consider movement times nor mechanical limits. In fact, only reachability is considered, which may remove some solutions: if Eq. 5 provides no solution (i.e. the function never reaches null value), the optimal φ'_f can be obtained by deriving Eq. 5 (with Eqs. 8 and 9) by φ'_f. In this way it is possible to fin the base rotation angle that minimize the difference of the two elbow displacements. The solutions, in this case, are:

$$\varphi_f = 2 \cdot \arctan\left(\frac{-B \mp \sqrt{B^2 + C^2}}{C}\right) + \varphi_{P1,f} \tag{15}$$

Future work will include movement times and mechanical limits for a more comprehensive method. Nonetheless, from the proposed method a first good solution can be found.

3 Simulation Results and Discussion

To test the algorithm, some simulations have been carried out. In this paragraph an example will be provided. Let's consider an application as defined via the parameters of Table 1. The two arms are made of identical links and are placed around the platform at the same distance R_0.

By applying the method described above in moving the first arm from $P_{1,0}$ to $P_{1,f}$ and the second arm from $P_{2,0}$ to $P_{2,f}$, four possible φ_f values are obtained. Among these, in our application the closest to φ_0 is chosen (#3 of Table 2).

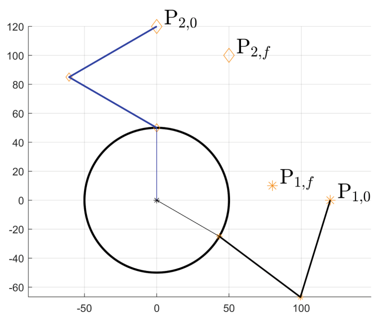

Fig. 2. Initial configuration and points to be reached

Table 1. Parameters used in the simulation

Parameter	Value
R_0	50 [mm]
α	120 [°]
a_1 , a_2	70 [mm]
φ_0	-30 [°]
$P_{1,0}$	[120,0] [mm]
$P_{2,0}$	[0,120] [mm]
$P_{1,f}$	[80,10] [mm]
$P_{2,f}$	[50,100] [mm]

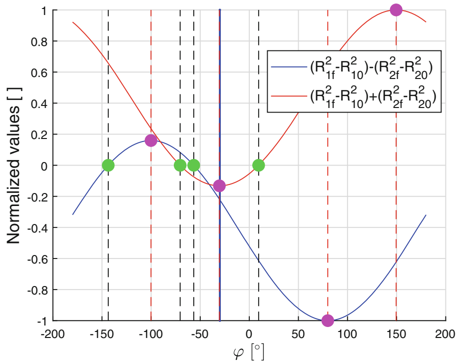

Fig. 3. Shape of the Eq. 5. The two lines are normalized to the respective maximum absolute values. Dashed lines provide the solutions of Eq. 14 (black) and 15 (red) with the parameters of Table 1. (Color figure online)

Table 2. Values and difference of the displacements of the arms for each solution of Eq. 14 (#1–#4) and Eq. 15 (#5–#8) with the parameters of Table 1. Some solutions are not available due to the unreachability of the final point (two red lines to the left of Fig. 4).

N	$\varphi_f - \varphi_0$ [°]	$\Delta q_{4,1}$ [°]	$\Delta q_{4,2}$ [°]	$\lvert\Delta q_{4,1}\rvert - \lvert\Delta q_{4,2}\rvert$ [°]
1	−113.63	59.14	−58.17	−0.97
2	−40.43	4.62	5.24	0.62
3	−26.57	−6.87	7.61	0.74
4	39.49	−45.01	−33.88	−11.13
5	−70.10	28.95	−12.75	−16.20
6	109.90	0.65	–	–
7	179.53	54.4	–	–
8	−0.47	−27.55	−0.40	−27.15

The application of the method is straightforward: no parameter optimization is required.

In Figs. 2 to 5 the results are shown. Figure 3 shows the two functions provided by Eq. 5, where the values have been normalized to the respective maximum absolute values. The values of φ_f are obtained from the null values of these two functions. As can be noted, the two functions are not symmetric with respect to the x-axis. This means that in some cases it could be possible that the functions do not result in null values, thus not providing any solutions (Eq. 14). As stated in previous Section, in this scenario the best solution to φ_f to be used can be obtained by finding the stationary points of the function (Eq. 15). In this case the two arms will not have the same displacement (leftmost column of Table 2, #5–#8), but it will be minimized by the approximation of the proposed method.

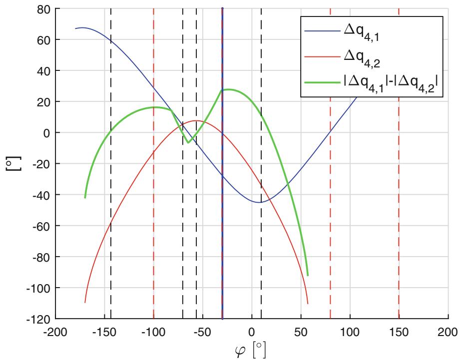

Fig. 4. Displacement of elbow joints for the right (blue) and left (red) arms. The green line shows the difference $|\Delta q_{1,4}| - |\Delta q_{4,2}|$. Dashed lines provide the solutions of Eq. 5 (black) and 15 (red) with the parameters of Table 1. (Color figure online)

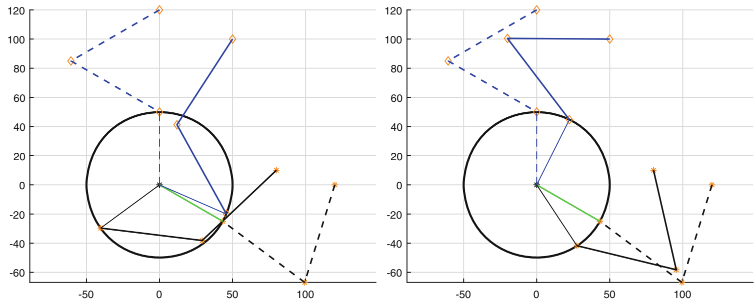

Fig. 5. Solution #1 (to the left) and #3 (to the right) as of Table 2. Green line shows the initial position of the first shoulder; dashed lines show the starting configuration. (Color figure online)

In fact, the proposed method is approximating the displacement of the two arms. As can be noted from the #4 solution of Table 2 (leftmost black line of Fig. 4), the solutions provided by Eq. 14 do not reflect on a completely equal displacement of the two arms. This approximation, however, is very small in most of the solutions. In fact, as can be noted by Table 2, the error is usually lower than 1°, a very small error if it is considered that is provided by an analytical solution and is then computationally efficient (in our tests the computational time is around 4 ms for each run). Since Eq. 14 provides some solution, Eq. 15 is not used. In fact, the solution provided by this equation (#5–#8 of Table 2) are not optimal for our application. Difference between calculated and actual joint displacements may be due to the expansion of the Taylor series, which is calculated around $x = 0$ even if q_4 can be very far from 0. However it should be noted that our objective is to find the difference of the two Taylor series, so the error should be limited (as shown by the results of Table 2).

4 Conclusions

In this paper a novel analytical method for the definition of the movement of the platform of a dual arm robot is presented. This approach aims at finding the proper rotation of the platform so that the two arms, in moving between positions, perform the same elbow joint displacement. In this way none of the elbow joints will result to be the bottleneck of the movement. The other joints will be optimized exploiting redundant configurations. In fact, the elbow joint value only depends from the distance between shoulder and wrist, and hence it is decoupled from the rest of the arm. Future work will consider the overall movement time in the model.

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