UNIVERSITÀ DEGLI STUDI DI PADOVA

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Optimal Offering and Operating Strategies in Electricity Markets

SCUOLA DI DOTTORATO DI RICERCA IN INGEGNERIA INDUSTRIALE

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Abstract

In recent years the power sector has experienced a significant growth of the deployment of renewable energy sources, such as wind and solar power. This brings new challenges to the optimal operation and management of power systems. The output of these stochastic sources can only be predicted with limited accuracy, leading to real-time deviations from their contracted schedule. Therefore, such stochastic sources need to be operated differently than conventional generation units, such as gas- or coal-fired plants. The operation of conventional production units may also be strongly affected by the massive deployment of renewable energy sources as their growing penetration is leading to a decrease in the market prices and an increase in the balancing energy need.

The thesis focuses on the optimal participation of power producers in competitive electricity markets with high penetration of renewable energy sources. We propose to derive the optimal offering strategy of a power producer with a modular approach. Indeed, we provide a general formulation of the optimal offering strategy, and each "block" (i.e., a set of constraints) of the optimization model can be replaced depending on the electricity market structure considered or the specific power production unit.

The thesis analyses three case studies, i.e., the optimal market participation of a stochastic power producer, a conventional power producer and a group of stochastic and conventional production units. We start from a stochastic power producer trading in an electricity market. The real-time deviations are settled under two alternative imbalance settlement schemes available in the Italian electricity market. These schemes add a tolerance band around the quantity of energy contracted in the day-ahead market. The portion of the imbalance within the band is priced differently than the part exceeding the band. We conclude that those imbalance pricing schemes may lead to a market distortion and therefore they may not be a preferable alternative to conventional schemes (e.g., the dual-price scheme).

Then, the thesis takes the perspective of a conventional power producer. Even though European day-ahead electricity markets are mostly settled under a uniform pricing scheme, several balancing markets (e.g., in Germany and Italy) are settled

under a pay-as-bid pricing scheme. We propose an innovative approach that allows modeling the trading problem under a pay-as-bid pricing scheme as a linear programming model. Thanks to this novel formulation, we derive the optimization problem that a conventional power producer would solve to evaluate its optimal day-ahead market offers while considering the future expected revenues from the balancing market. The proposed model is tested on a realistic case study against a sequential offering approach, showing the capability of increasing profits in expectation.

Lastly, we consider the case of a virtual power plant, defined as a cluster of conventional generating units, stochastic power units, and storage systems, which together act as a single participant in the electricity market. We introduce a novel structure of the balancing market, which allows an active/passive participation of the virtual power plant. Indeed, the virtual power plant can decide to be an active actor (i.e., offering regulating energy) in some trading intervals, and a passive one (i.e., deviating from the contracted schedule) in the remaining hours. The model is tested in a realistic case study against alternative benchmark strategies (e.g., active-only and passive-only participation).

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Chapter 1

Introduction

1.1 Context and Motivation

In recent years, the power sector has experienced major structural changes. On one side, the liberalization of the electric energy industry has brought to the unbundling of vertically integrated utilities and has enhanced competitivity through the creation of electricity markets trading platforms. On the other side, the need of replacing fossil fuels with environmental-friendly energy sources has led to an increasing deployment of renewable energy production.

Traditionally, a single vertically integrated utility was responsible for the whole chain, from the generation to the delivery of electricity. The inflexible consumers' demand was covered by few centralized conventional units, generally fueled by fossil energy sources, which required to be dispatched well in advance as several technical limitations constrain their operation. As an example, ramping constraints limit the capability of varying the power output of a conventional production unit in a short time span. Accordingly, the typical pattern was to use "slow" (i.e., with important ramping limitations) and cheap technologies, such as coal-fired units or nuclear power plants, to cover the base load. Differently, pick loads were covered by "fast" but expensive technologies, such as gas-fired turbines or diesel units. In this context, the few uncertainties in the operation of power systems were related to relatively small forecasting errors in the consumers' energy demand and unpredictable units' failures.

The appeal for competition and innovation has motivated a progressive establishment of electricity markets across the worldwide. The critical feature of the process is the separation between activities of generation, transmission, distribution, and retail while banning the vertical integration among different sectors. Competition is promoted mainly in generation and retail, while the transmission and distribution sectors are still natural monopolies, due to the prohibitive investment cost of transmission and distribution lines. The idea is to develop a marketplace managed by a non-commercial entity that matches the needs of numerous participants in both the generation and demand side, providing transparent and non-discriminatory price signals. However, this competitive environment must consider several physical aspects that differentiate electricity from other commodities. As an example, the storage of large quantities of electricity is still not convenient in an economic perspective. Consequently, injections and withdrawals of electric energy need to be matched continuously to ensure a safe and stable operation of the power system. The structure of electricity markets was initially tailored for the technical and economic requirements of conventional production units with non-negligible fuel costs. The main short-term trading floor, i.e., the day-ahead market stage, is usually cleared from 12 to 36 hours before the delivery of energy. In this way, conventional units can schedule their production pattern well in advance of the real-time operation.

The increasing deployment of renewable energy sources, such as wind and solar power, has brought significant changes in the electricity market context. Indeed, these sources are non-dispatchable, i.e., their power production is only partially controllable, and stochastic, i.e., their power output can only be predicted with limited accuracy, which decreases as the time horizon of the forecast increases. This naturally brings a higher level of uncertainty in the optimal management of power systems as these stochastic units cannot follow the production schedule contracted in the day-ahead market and they create imbalances in the real-time. From an economic perspective, these energy sources are usually traded at zero marginal cost, thus lowering the average prices while increasing their variability. All the participants in electricity markets are affected by this revolution. On the one hand, stochastic sources need to be traded in a market platform not yet restructured to accommodate this increasing level of uncertainty. In fact, stochastic producers may incur in important financial penalties for the creation of imbalances, which are inherent in the nature of their energy source. On the other hand, conventional units may have to adapt their offering and operating strategy as their profits from the day-ahead market are affected by the decrease of the market prices. Moreover, they are generally required to be more flexible in the real-time to compensate the imbalances created by the stochastic producers.

In this context, it is important to develop offering and operating strategies for the different power producers involved. Indeed, the importance of an efficient market participation of power producers is twofold. From the producer's point of view, it results in reaching higher market profits. From the system's perspective, this may result in prices that can reflect the operational cost of the system and in having a sufficient level of flexibility to safely operate it. As an example, a conventional producer that does not schedule its power unit in the day-ahead market due to the low market prices is limited from possible huge profits deriving from offering regulating energy in the real-time. Hence, this penalizes both the producer, that gains lower profits, and the system, which has less available flexibility in the real-time.

1.2 Thesis Description and Contributions

The thesis proposes a comprehensive approach to derive the optimal offering strategy of a power producer in a competitive electricity market. The offering strategies are formulated as mathematical models, i.e., optimization models, that can be used as decision-making tools for the different power producers to tackle the increasing level of uncertainty in the electricity market context.

We focus on a price-taker and risk-neutral power producer trading in a short-term electricity market, composed of a day-ahead and a balancing market. Thanks to the price-taker assumption, the uncertain market prices can be represented through marginal distributions or discrete sets of possible scenarios, as we neglect the influence of the producer's decisions on the market clearing process. The idea is to develop these optimal offering strategies with a modular approach. This translates in deriving a compact formulation of a general offering strategy, where each set of variables and constraints within the general formulation can be replaced depending on the electricity market structure considered or the specific production units. As we assume a risk-neutral producer, the objective function of the general strategy maximizes the expectation of the market profit, incorporating the participation in both the day-ahead and the balancing market, and including the production cost of the producer. An energy balance imposes that the amount of energy exchanged with the electricity market matches the total power production of the producer. Then, three set of constraints model the trading problem in the different market stages, i.e., the day-ahead market, the balancing market as an active participant (i.e., a producer that offers its regulating energy), and the balancing market as a passive participant (i.e., a producer that deviates from the contracted day-ahead schedule). For the trading problem, we intend a set of constraints and variables that simulate the market clearing mechanism, the market pricing scheme, and other additional constraints associated with the market offers. The general formulation is concluded by two extra set of constraints related to the specific production unit or cluster of units. The first yields the expected production cost while the second imposes the feasible operating region of the units.

Together with the general model of an offering strategy, the thesis provides an extensive series of formulations aimed at representing the trading problem in the different market stages. For the day-ahead market, we start considering single price-quantity offers and a continuous distribution of the uncertain market prices. Then, we extend the formulation including non-decreasing step-wise offer curves that allow the producer to schedule an increasing amount of energy production as the day-ahead market price realization increases. In both cases, the result is a non-linear model, where the non-linearities arise from the product of prices and quantities to compute the producer's market income. A linear alternative is proposed by mean of a stochastic-programming approach. The idea is to use the market price scenarios as potential offer prices for the price-taker producer building its optimal offer curves. Then, we develop the trading problem for an active participant offering in the balancing market. Similarly to the day-ahead market, we start from single price-quantity offers, and then we extend it to offer curves. Note that for the balancing market trading problem we consider both a uniform pricing and a pay-as-bid pricing scheme. The thesis proposes an innovative formulation that allows casting the trading problem with offer curves under a pay-as-bid pricing scheme as a linear programming (LP) program. Conversely, the alternative formulations available within the literature are non-linear and may limit the possibility of deriving offering strategies including multiple market stages and complex cost functions or operating regions of the units. Finally, we formulate the trading problem in the balancing market for a passive participant, considering both a single-price and dual-price imbalance settlement scheme.

Subsequently, we take the perspective of different power producers offering in the electricity market. We start from a stochastic power producer that includes the future imbalance revenue from the balancing market while deriving its optimal day-ahead quantity offer. We adapt the general formulation of the offering strategy to the characteristics of a stochastic energy source (e.g., we assume that the producer offers its energy at zero marginal cost in the day-ahead market), deriving its optimal offering strategy under a single-price and a dual-price imbalance settlement scheme (in the balancing market). Then, we formulate its optimal trading strategy under two alternative imbalance settlement schemes used in the Italian electricity market. Such schemes consider tolerance margins, i.e., they introduce a tolerance band around the quantity of energy contracted in the day-ahead market. For both the pricing schemes we prove that the market offer that maximizes the producer's expected profit is unique. We also investigate how these alternative schemes may influence the real-time imbalance of a rational stochastic power producer, i.e., a producer seeking at maximizing its expected profit. Note that a formulation of the optimal trading strategy under those pricing schemes is novel, together with the analysis of their influence on the real-time imbalance of a rational producer.

Then, we move to a conventional power producer that offers in an electricity market where the balancing stage is cleared under a pay-as-bid pricing scheme. We formulate the operating region and the production cost function of the unit as a mixed-integer and linear (MILP) problem. We derive two alternative offering strategies in the day-ahead market. The first considers the day-ahead market only. Differently, the second models the future decisions in the balancing market as recourse decisions in a stochastic programming setup. The trading problems (for the day-ahead and the balancing market) are merged with the MILP operating region of the unit to derive the optimal offering strategy of the conventional power producer. As mentioned before, the models available in the literature for the trading problem under a pay-as-bid pricing scheme are non-linear. Introducing the feasibility region of the production unit would result in a mixed-integer non-linear problem, which may have high computational cost and, generally, does not guarantee the optimality of the solution. Thanks to our novel linear approach, the result is instead a MILP two-stage stochastic programming problem with recourse. We show how an accurate modeling of the expected revenues from the balancing market is of increasing importance as the penetration of the renewable energy production in the market increases. Indeed, an increasing share of renewable energy generation translates into lower day-ahead market prices and a rising need for regulating energy. Therefore, it may happen that a producer that uses a sequential offering strategy (i.e., considering one market stage at the time) is not going to schedule its unit at the day-ahead stage due to the low market prices. Then, at the balancing stage, it has few possibilities of offering regulating energy. Differently, by co-optimizing its offering strategy in the day-ahead and the balancing market, the power producer may be willing to operate in the day-ahead even when the prices are not convenient (i.e., it is in a negative position after the day-ahead market clearing). However, by being scheduled in the day-ahead market, it can offer both upward and downward regulation to the balancing market and increase its total market profit.

Finally, we take the perspective of a Virtual Power Plant (VPP). A VPP is defined as a cluster of conventional generating units, stochastic generating units, storage systems, and flexible loads, which together act as a single participant in the electricity market. We assume that the balancing market allows an *Active-Passive* participation of the VPP. With *Active-Passive* participation, we mean that the VPP can decide to be an active actor in some trading periods, and a passive one in the remaining. Indeed, given the presence of some dispatchable units and storage units, in some hours it may be able to provide regulating power to the system. Differently, the available models in the literature only consider a passive-only participation. The innovative offering model is tested in a realistic case study against alternative benchmark strategies (e.g., passive-only participation), showing the capability of increasing profits in expectations. We also analyze in which condition the VPP is willing to be active (less flexibility of operation but more favorable prices) and when passive (more flexibility but penalized prices). This market setup may be also interesting from the system's perspective. Indeed, by allowing this *Active-Passive* participation the system may have more regulating energy available for some trading intervals.

1.3 Thesis Structure

The thesis is structured as follows. Chapter 2 presents the main principles and structure of European Electricity markets, focusing on the short-term trading floors. It also introduces a simplified electricity market model, while describing the submission process and its clearing mechanism. Chapter 3 provides the general formulation of the producer's offering strategy together with the trading problems in the different market stages. Chapter 4 proposes a methodology to obtain a discrete set of trajectories of the stochastic processes, e.g., electricity market prices or wind and solar power production, to be used within a stochastic programming framework. Then, Chapters 5, 6, and 7 take the perspective of a stochastic power producer, of a conventional power producer, and of a VPP, deriving their optimal day-ahead offering strategy, respectively. The conclusions are drawn in Chapter 8.

Chapter 2

Electricity Markets

2.1 Introduction

Over the last decades, power systems moved to new frameworks aiming at enhancing the competition in the sector. Traditionally, a single vertically integrated entity was entitled to manage and control the whole power system. This state-owned company was in charge of the four main activities, i.e., generation, transmission, distribution and retail of electric energy. A progressive movement toward the liberalization of the power sector pushed several countries to ban the vertical integration among different areas while promoting competition in the activities of generation and retail. Differently, a natural monopoly still holds in the transmission sector due to the prohibitive investment cost of transmission lines. The essential role of operating and managing the transmission grid is carried out by non-commercial entities, called Transmission System Operator in Europe and Independent System Operator in the US.

The deregulation of the electricity supply sector helped to attract new investors but brought challenges due to the nature of the commodity traded in the competitive market framework. Different from similar commodities, such as natural gas, the electric energy is not suitable to be stored in large quantities and the long term, at least from an economic point of view. Hence, the injections and withdrawals of electricity in the grid need to be continuously matched to ensure a safe operation of the system. Historically, a vertically integrated company was scheduling the power production units (fully controllable conventional units) to follow an inflexible and quite well predictable electric energy demand. This process is nowadays delegated to a market structure that needs to ensure the reliability of the system while keeping a competitive and transparent trading environment. Additionally, it is usually impossible to distinguish between electric energy generated by different energy sources. Similarly, it is not possible to track the path of electric energy within the transmission grid. Consequently, electricity markets are driven by aggregated curves obtained as the sum of single production units or single loads.

This chapter introduces the reader to the structure of electricity markets and their timeline. It is organized as follows. Section 2.2 gives a general overview of the structure of European electricity markets, distinguishing between long-term and short-term electricity markets. Then, Section 2.3 presents a simplified but realistic electricity market model. It is used to explain the submission process of the different participants and the clearing mechanism used to evaluate the accepted offers and to compute the market price.

2.2 Overview of various markets and their time-line

In electricity markets, two main trading floors can be typically distinguished, depending on the proximity of the trading. Medium/long-term markets (e.g., futures markets) allow trading on long-term horizons. The participants in the market can trade both physical and financial products. Examples of financial trading are forward contracts and options. A forward contract is signed between a seller that undertakes to produce a certain amount of energy, and a buyer that is willing consumes that energy. These usually deal with standard products, e.g., base-load contracts include all the hours of the contracted time span, while peak-load include only the hours with high energy demand, e.g., from 8 a.m. to 7 p.m. of working days. A forward contract can also be coupled with options, which allow the buyer to decide after the agreement whether to benefit or not of the forward contract.

Closer to the real-time operation, short-term markets (i.e., *electricity pools* or *power exchanges*) allow trading electricity on a daily and hourly horizon. They include several trading floors, i.e., day-ahead, intra-day adjustment and balancing markets. The *electricity pool* model is, originally, a centralized market, where the producers operate under a cost recovery mechanism. Indeed, they recover their operating costs through some fees, which have been approved by the market regulator and are paid by the participants in the pool. Differently, *power exchanges* are more open and decentralized markets where the producers' cost recovery is not guaranteed. These markets are accessible to every participant that satisfies specific admission requirements. The main goal of *power exchanges* is to ensure a transparent and reliable market price formation, generally obtained by matching the aggregated supply and demand curves of the market actors.

Power producers can participate in both futures and short-term markets. Usually, part of the thermal plants capacity is contracted in medium/long-term contracts, since these ensure fixed revenues for the producers, avoiding the uncertainties of the short-term trading. The remaining capacity is contracted in the short-term markets. Contrariwise, renewable energy plants, e.g., wind farms and PV solar plants, are stochastic and can only be predicted with a limited accuracy. Therefore, they are not suitable for long-term contracts, as it is hard to guarantee a certain level of production long time ahead of the real-time operation. This chapter (and the thesis as well) focuses on short-term electricity markets. Section 2.2.1 presents the day-ahead market trading floor. Sections 2.2.2 and 2.2.3 do the same for the intra-day and the balancing markets.

2.2.1 Day-ahead market

The day-ahead market hosts transactions for selling and buying electric energy one day before the delivery day. It is the prominent and more liquid among the different electricity market floors. Buyers and sellers submit their offers to a Market Operator, which acts as the central counterpart. A market offer includes a quantity of energy and the price at which the market participant wants to produce or consume it. In case of sell/buy offers/bids the price denotes the minimum/maximum price at which the seller/buyer is willing to provide/consume electricity.

The day-ahead market gate closure occurs the day before the actual delivery day, usually at noon. It includes 24 separate auctions, one per each hour of the day, cleared simultaneously but independently (in the European Electricity markets). After the gate closure, the market operator clears the market and informs each seller/buyer of their production/consumption schedule. Typically, the US approach is to include the full representation of the transmission grid within the clearing algorithm, thus leading to a different market price for each node of the network (i.e., nodal pricing). Differently, the European approach is to give a poor representation of the grid by including only the more limiting transmission constraints. Accordingly, it translates in market zones were each dispatch within the zone is assumed to be feasible (at least from a transmission point of view) and leads to zonal market prices, i.e., the same market price for all the participants within the same market zone.

2.2.2 Intra-day market and continuous trading

The intra-day market is the market for sale/purchase energy during the day of delivery. It opens after the day-ahead market gate closure and closes from hours to minutes before the real-time operation. The intra-day market can be a useful tool for the market participants which can use this additional floor to adjust their market position. Indeed, conventional producers may access the intra-day market for fixing an infeasible production schedule, as inter-temporal constraints (e.g., ramping limits) cannot, usually, be directly included in the day-ahead market offers. On the other hand, stochastic producers can use this additional trading floor to modify their market position as their power production forecasts are more accurate closer to the real-time operation. The trading process in the intra-day market is, usually, continuous. The negotiation mechanism is based on the automatic matching of demand bids and supply offers, which allows a continuous submission of new offers/bids during the whole session. Similarly to the day-ahead market, the intra-day market is managed by the Market Operator.

2.2.3 Balancing market

The balancing market is the last stage for trading electric energy. It plays an essential role, as production and consumption levels must match during the entire operation of power systems. Balancing markets are usually single-period markets, i.e., each trading period has a separate associated session. They allow the possibility to trade, in addition to electric energy, *ancillary services* (e.g., voltage control) needed to maintain the stability of the power system.

Conventional producers, usually, participate in the balancing market as active participants. They offer regulating energy, both in upward (i.e., increasing their power production) and downward (i.e., decreasing their power output) directions. Differently, stochastic producers access the balancing stage as passive participants. They use this last trading floor to settle the deviations from their contracted schedule. The System Operator manages the balancing market. It uses the regulating offers submitted by the active participants to compensate the system imbalance generated by the passive participants (both in the supply and demand side).

2.3 Electricity Market Model

This section presents a simplified market model used as a reference for the remaining of the thesis. We consider a two-settlement electricity market framework, which includes a day-ahead and a balancing stage. The day-ahead market is cleared once a day, at noon, simultaneously for the 24 hourly trading periods of the following day. Subsequently, a separate balancing market is cleared for each hourly interval, one hour before the real-time operation. The intra-day adjustment trading floor is neglected for the sake of simplicity. The idea is to use a simple market structure, provided that it maintains the key stages and properties of a real electricity market. Indeed, the day-ahead stage is the main trading floor, and it is a reference for the following stages (i.e., the intra-day adjustment and the balancing ones). Then, the balancing market is of crucial importance as it allows the System Operator to schedule the regulation energy needed to compensate the real-time imbalance of the system and ensure a safe operation. Figure 2.1 shows a schematic representation of the simplified electricity market structure. Note that the day-ahead market for the day $d1$ closes at noon of the day $d0$ and hosts transaction for the 24 trading intervals (blue shaded area) of the day $d1$. Differently, a separate balancing market is cleared every hour, one hour before the operation. E.g., the balancing market for the interval $k1$ (i.e., from midnight to 1 a.m. of the day $d1$) closes at 11 p.m. of the day $d0$. Note that for the balancing market the trading intervals are illustrated with a red and a green area. It highlights the fact that different producers may access the balancing stage for various purposes. The green shaded area indicates an active participation in the balancing stage, i.e., the submission of regulating energy offers to the System Operator. Differently, other participants may use the balancing market to settle their deviations from the production schedule contracted in the day-ahead market. A red shaded are illustrates this passive participation.

Thanks to this simple structure, we can derive producers' offering strategies that are general and not customized for a specific electricity market. Indeed, a part of the optimization problem aimed at obtaining the optimal market offers models the electricity market rules and mechanism. This piece, called the trading problem, is

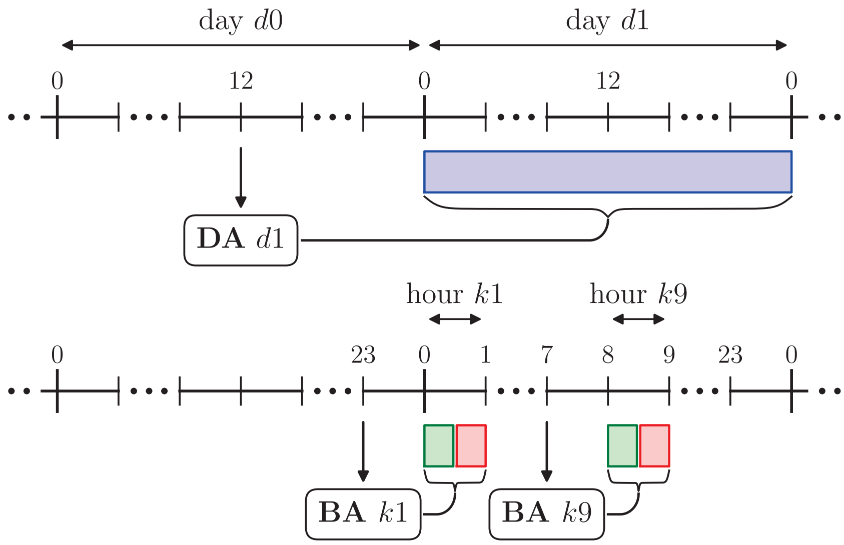


FIGURE 2.1: Schematic representation of the electricity market structure. The day-ahead (DA) market is cleared at noon, simultaneously for the 24 hourly interval of the following day. A balancing (BA) market is cleared separately per each hourly interval, one hour before the real-time.

strongly influenced by the electricity market structure. Besides, a trading problem based on a simplified market can easily be adapted to electricity markets with additional stages (e.g., the intra-day adjustment stage) or with different size of the trading intervals (e.g., 15 minutes trading intervals in the balancing market).

Power producers, either conventional (e.g., coal- or gas-fired power unit) or stochastic (e.g., wind and solar power producers) submit their market offers to the Market Operator. The ensemble of the sell offers builds the so-called aggregated supply curve. Stochastic producers are assumed to provide their energy production at zero marginal cost. As for the sell offers, aggregating the buy bids (i.e., bids for consuming electricity) the Market Operator obtains the demand curve. The intersection between those two curves gives the market clearing price and quantity. In this simplified model, we consider an inflexible aggregated demand curve, i.e., the amount of energy consumption does not depend on the market price.

At the balancing stage, we assume that only conventional producers can provide regulating energy, either for upward or downward regulation (active participation). Differently, stochastic producers use the balancing stage to settle their deviations from their day-ahead contracted schedule (passive participation).

2.3.1 Day-ahead Market Clearing

At noon of the day before the delivery of energy, the Market Operator collects the producers' offers for the day-ahead market. Let K be the set of the 24 hourly trading intervals of the following day and k be the index of the trading intervals, so that $k \in K$. The Market Operator receives the market offers for the 24 hourly trading intervals of the following day, simultaneously. Let N_k^{O} be the number of sell offers submitted by conventional generators for the hourly trading interval k, while letting o be the index for this set of offers. An offer o includes a quantity of energy $\overline{Q}_{ok}^{\mathrm{DA}}$ (MWh) and a price $\alpha_{ok}^{\mathrm{DA}}$ (€/MWh). The price-quantity offer $\left(\alpha_{ok}^{\mathrm{DA}}, \overline{Q}_{ok}^{\mathrm{DA}}\right)$ indicates that the power producer is willing to generate the quantity of energy $\overline{Q}_{ok}^{\mathrm{DA}}$ at k, provided that the day-ahead market price λ_k^{DA} (€/MWh) is greater or equal to its offered price $\alpha_{ok}^{\mathrm{DA}}$. The Market Operator also receives N_k^{S} sell offers submitted by the stochastic producers. Let s be the index for this set of sell offers. The offer s includes a quantity of energy $\overline{E}_{sk}^{\mathrm{DA}}$ (MWh) offered at 0 €/MWh. Figure 2.2 shows a schematic representation of the day-ahead market submission process.

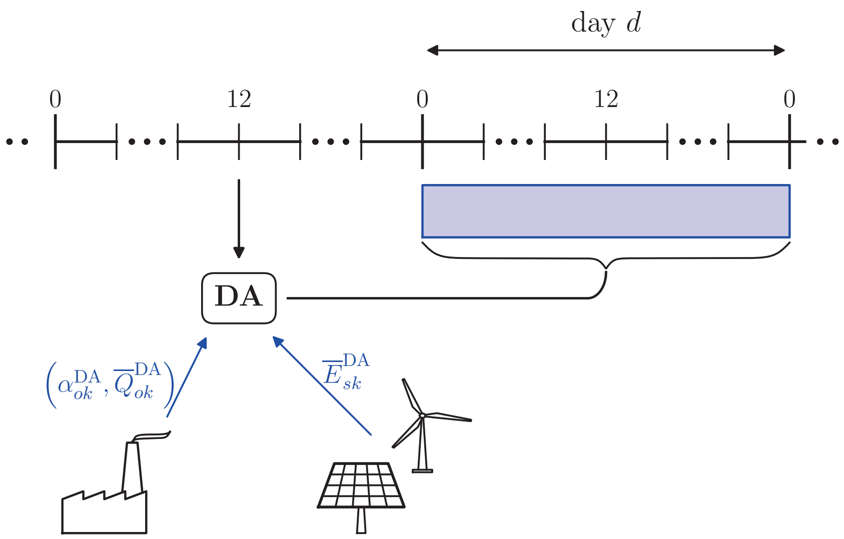


FIGURE 2.2: Schematic representation of the day-ahead (DA) electricity market submission process. Conventional producers submit price quantity offers $\left(\alpha_{ok}^{\mathrm{DA}}, \overline{Q}_{ok}^{\mathrm{DA}}\right)$, while stochastic producers quantity offers $\overline{E}_{sk}^{\mathrm{DA}}$.

The Market Operator wants to evaluate the optimal dispatch of energy to satisfy the inflexible energy demand D_k^{DA} (MWh) at each interval k. It solves a separate economic dispatch per each hourly interval k, computing the amount of energy q_{ok}^{DA} (MWh) scheduled of each offer o and the energy e_{sk}^{DA} (MWh) contracted of each offer s. Then, it computes the market clearing price λ_k^{DA} at each interval k. The economic dispatch problem at k is formulated as

$$\underset{\{q_{ok}^{\mathrm{DA}}, e_{sk}^{\mathrm{DA}}\}}{\text{Min}} \quad \sum_{o=1}^{N_k^{\mathrm{O}}} \alpha_{ok}^{\mathrm{DA}} q_{ok}^{\mathrm{DA}} \tag{2.1a}$$

$$\text{s.t.} \quad \sum_{o=1}^{N_k^{\mathrm{O}}} q_{ok}^{\mathrm{DA}} + \sum_{s=1}^{N_k^{\mathrm{S}}} e_{sk}^{\mathrm{DA}} = D_k^{\mathrm{DA}} \ : \ (\lambda_k^{\mathrm{DA}}) \tag{2.1b}$$

$$0 \leq q_{ok}^{\mathrm{DA}} \leq \overline{Q}_{ok}^{\mathrm{DA}}, \ \forall o \in \{1, ..., N_k^{\mathrm{O}}\} \tag{2.1c}$$

$$0 \leq e_{sk}^{\mathrm{DA}} \leq \overline{E}_{sk}^{\mathrm{DA}}, \ \forall s \in \{1, ..., N_k^{\mathrm{S}}\} \tag{2.1d}$$

The objective function (2.1a) minimizes the cost for satisfying the energy demand D_k^{DA}. Constraint (2.1b) imposes the balance between production (i.e., $\sum_{o=1}^{N_k^{\mathrm{O}}} q_{ok}^{\mathrm{DA}} + \sum_{s=1}^{N_k^{\mathrm{S}}} e_{sk}^{\mathrm{DA}}$) and consumption (i.e., D_k^{DA}). Constraint (2.1c) forces q_{ok}^{DA} to lie in its feasibility region, i.e., between 0 and $\overline{Q}_{ok}^{\mathrm{DA}}$. Similarly, constraint (2.1d) imposes that e_{sk}^{DA} is bounded between 0 and $\overline{E}_{sk}^{\mathrm{DA}}$. Finally, the market price λ_k^{DA} is obtained by computing the sensitivity, i.e., the dual variable, of constraint (2.1b).

Figure 2.3 illustrates the day-ahead market clearing mechanism. The aggregated supply curve is shown in blue, while the demand curve in black. Note that the supply curve is built with both the stochastic producers' offers and the conventional producers' ones. As the energy $\overline{E}_{sk}^{\mathrm{DA}}$ is offered at 0 €/MWh, it appears on the left side of the curve. Differently, the conventional producers' offers $\left(\alpha_{ok}^{\mathrm{DA}}, \overline{Q}_{ok}^{\mathrm{DA}}\right)$ are ranked based on a merit order resulting in a step-wise curve. The intersection between the two curves gives the day-ahead market clearing price λ_k^{DA}. Note that an increase of the stochastic producers' quantity offers would shift the supply curve towards right thus resulting in lower values of the market price λ_k^{DA}.

2.3.2 Balancing Market Clearing

Close to the real-time operation, the System Operator manages the balancing market. The balancing market is used to ensure the continuous matching between injections and withdrawals of electricity in the grid, as it is the last available trading floor. Stochastic producers are usually not able to fulfill the production schedule

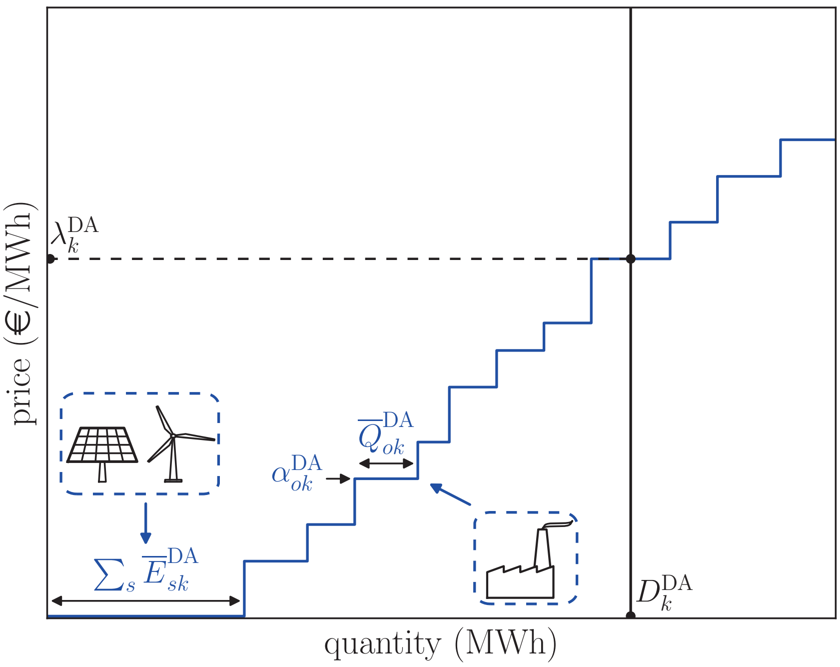


FIGURE 2.3: Schematic representation of the day-ahead electricity market clearing mechanism. The intersection between the supply curve (blue) and the demand curve (black) identifies the market clearing price.

contracted in the day-ahead market and access the balancing stage to settle their deviations. This, together with potential forecasting errors in the energy demand, results in a discrepancy between the scheduled energy generation and consumption. Conventional generators offer their availability to upward or downward adjust their power production to compensate the system imbalance.

In this context, we can distinguish between two kinds of balancing market participation. The first, called active participation, refers to power producers that offer their available regulating energy to the System Operator, thus ensuring a more flexible operation of the power system. The second, called passive participation, refers to power producers that deviate from their production schedule and contribute to the whole system imbalance. Conventional units are usually active actors in the balancing markets, while stochastic producers are passive participants and are prevented from offering regulating energy.

A conventional producer can submit in the balancing market both upward and downward regulation offers. An up-regulation offer $\left(\alpha_{ok}^{\mathrm{UP}}, \overline{Q}_{ok}^{\mathrm{UP}}\right)$ indicates that the power producer can produce additional $\overline{Q}_{ok}^{\mathrm{UP}}$ (MWh) provided that it is remunerated at a price higher or equal to $\alpha_{ok}^{\mathrm{UP}}$ (€/MWh). Note that usually $\alpha_{ok}^{\mathrm{UP}} \geq \lambda_k^{\mathrm{DA}}$ as the marginal cost for producing the energy $\overline{Q}_{ok}^{\mathrm{UP}}$ is likely to be higher than the day-ahead market clearing price λ_k^{DA}. Differently, a down-regulation offer $\left(\alpha_{ok}^{\mathrm{DW}}, \overline{Q}_{ok}^{\mathrm{DW}}\right)$ expresses that the producer can decrease its power production of $\overline{Q}_{ok}^{\mathrm{DW}}$ (MWh) when this energy is priced equally or lower than $\alpha_{ok}^{\mathrm{DW}}$ (€/MWh). As the quantity $\overline{Q}_{ok}^{\mathrm{DW}}$ was previously contracted in the day-ahead market at price λ_k^{DA}, by "buying" back this amount at $\alpha_{ok}^{\mathrm{DW}}$ the producer would receive a payment of $\left(\lambda_k^{\mathrm{DA}} - \alpha_{ok}^{\mathrm{DW}}\right) \overline{Q}_{ok}^{\mathrm{DW}}$. Note that this payment increases as $\alpha_{ok}^{\mathrm{DW}}$ decreases, which explains the inverted merit order compared to the up-regulation offers. Moreover, as a rule, $\alpha_{ok}^{\mathrm{DW}} \leq \lambda_k^{\mathrm{DA}}$ since differently the producer may incur in a negative profit from the sell of downward regulation.

Let us denote with E_{sk}^{DA} (MWh) the amount of energy contracted by the stochastic producer s in the day-ahead market, while E_{sk} (MWh) is the real-time production of its power unit during the hourly interval k. In the balancing market the stochastic producer settles a deviation $\left(E_{sk} - E_{sk}^{\mathrm{DA}}\right)$ in order to balance its position. Consequently, the real-time system imbalance δ_k^{BA} is obtained as

$$\delta_k^{\mathrm{BA}} = \left(D_k - D_k^{\mathrm{DA}}\right) - \sum_{s=1}^{N_k^{\mathrm{S}}} \left(E_{sk} - E_{sk}^{\mathrm{DA}}\right), \quad \forall k \in K. \tag{2.2}$$

where D_k (MWh) is the real-time aggregate consumption demand.

The System Operator receives N_k^{O} upward and downward regulation offers, i.e., $\left(\alpha_{ok}^{\mathrm{UP}}, \overline{Q}_{ok}^{\mathrm{UP}}\right)$ and $\left(\alpha_{ok}^{\mathrm{DW}}, \overline{Q}_{ok}^{\mathrm{DW}}\right)$, respectively. In the real-time, it receives the deviations $\left(E_{sk} - E_{sk}^{\mathrm{DA}}\right)$ settled by the stochastic producers and computes the system imbalance δ_k^{BA}. A schematic representation of balancing market submission process is shown in Figure 2.4. Note that the offers from the conventional units are shown in green (active participation), while the deviations settled by the stochastic producers are illustrated in red (passive participation). The System Operator computes the more convenient (from an economic perspective) re-dispatch of the conventional units aimed at compensating the system imbalance δ_k^{BA}. Let q_{ok}^{UP} and q_{ok}^{DW} be the amount of upward and downward regulating energy scheduled of producer o at

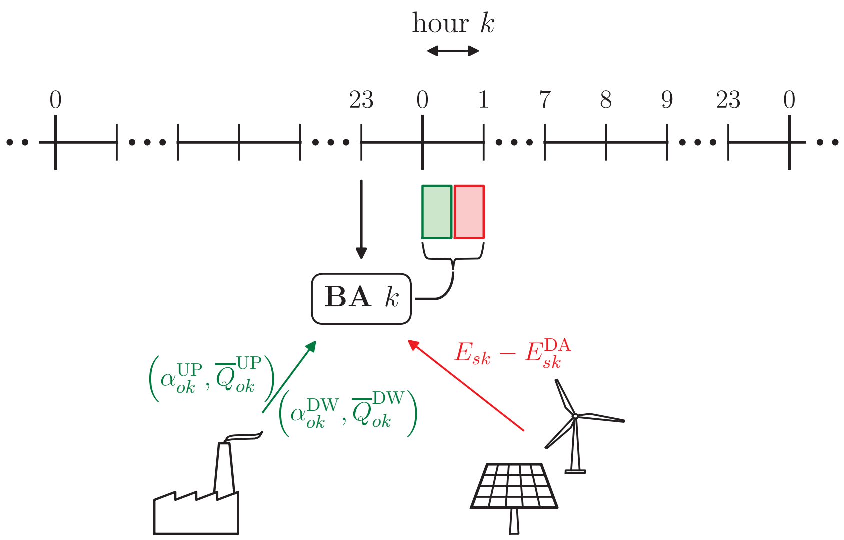


FIGURE 2.4: Schematic representation of the balancing (BA) market submission process. Conventional producers submit price-quantity offers for upward $\left(\alpha_{ok}^{\mathrm{UP}}, \overline{Q}_{ok}^{\mathrm{UP}}\right)$ and downward $\left(\alpha_{ok}^{\mathrm{DW}}, \overline{Q}_{ok}^{\mathrm{DW}}\right)$ regulation, while stochastic producers settle the deviations $E_{sk} - E_{sk}^{\mathrm{DA}}$.

time k. The optimization problem solved to evaluate the optimal re-dispatch of conventional generators is

$$\underset{\{q_{ok}^{\mathrm{UP}}, q_{ok}^{\mathrm{DW}}\}}{\mathrm{Min}} \quad \sum_{o=1}^{N_k^{\mathrm{O}}} \alpha_{ok}^{\mathrm{UP}} q_{ok}^{\mathrm{UP}} - \sum_{o=1}^{N_k^{\mathrm{O}}} \alpha_{ok}^{\mathrm{DW}} q_{ok}^{\mathrm{DW}} \tag{2.3a}$$

$$\text{s.t.} \quad \sum_{o=1}^{N_k^{\mathrm{O}}} \left(q_{ok}^{\mathrm{UP}} - q_{ok}^{\mathrm{DW}}\right) = \left(D_k - D_k^{\mathrm{DA}}\right) - \sum_{s=1}^{N_k^{\mathrm{S}}} \left(E_{sk} - E_{sk}^{\mathrm{DA}}\right) \ : \ \left(\lambda_k^{\mathrm{BA}}\right) \tag{2.3b}$$

$$0 \leq q_{ok}^{\mathrm{UP}} \leq \overline{Q}_{ok}^{\mathrm{UP}}, \ \forall o \in \{1, ..., N_k^{\mathrm{O}}\} \tag{2.3c}$$

$$0 \leq q_{ok}^{\mathrm{DW}} \leq \overline{Q}_{ok}^{\mathrm{DW}}, \ \forall o \in \{1, ..., N_k^{\mathrm{O}}\} \tag{2.3d}$$

The objective function (2.3a) minimizes the balancing cost, evaluated as the sum of the up-regulation cost (positive) and the down-regulation cost (negative). Constraint (2.3b) imposes that the system imbalance δ_k^{BA} is compensated by the total regulation energy scheduled (upward or downward). Note that δ_k^{BA} is replaced with its formulation provided in Equation 2.2. Constraints (2.3c) and (2.3d) force q_{ok}^{UP} and q_{ok}^{DW} to lie between 0 and $\overline{Q}_{ok}^{\mathrm{UP}}$, and between 0 and $\overline{Q}_{ok}^{\mathrm{DW}}$, respectively. The dual variable of constraint (2.3b) gives the balancing market clearing price λ_k^{BA}.

A graphical interpretation of the balancing market clearing mechanism is shown in Figures 2.5 and 2.6. The regulating offers of the conventional generators are ranked based on their offered price, thus obtain a step-wise supply curve (shown in green). Differently, the new demand curve (shown in red), is obtained as the sum of D_k^{DA} and the aggregate deviation of the stochastic producers. In this example, we assume that $D_k = D_k^{\mathrm{DA}}$, so the system imbalance δ_k^{BA} is only given by the stochastic producers' deviation, i.e., $\sum_{s=1}^{N_k^{\mathrm{S}}} \left(E_{sk} - E_{sk}^{\mathrm{DA}}\right)$. Figure 2.5 illustrates an example when $\delta_k^{\mathrm{BA}} > 0$ and the System Operator schedules upward adjustments of the power production of conventional units. This case results in $\lambda_k^{\mathrm{BA}} \geq \lambda_k^{\mathrm{DA}}$. Indeed, as the total demand to fulfill is greater than the day-ahead one, generators with higher marginal cost are required to operate. Differently, Figure 2.6 considers an example when $\delta_k^{\mathrm{BA}} < 0$ and the System Operator schedules downward adjustments of the conventional production units. This leads to $\lambda_k^{\mathrm{BA}} \leq \lambda_k^{\mathrm{DA}}$, as the total energy demand is lower than the day-ahead one and the more expensive generators (among the scheduled ones) are shut down.

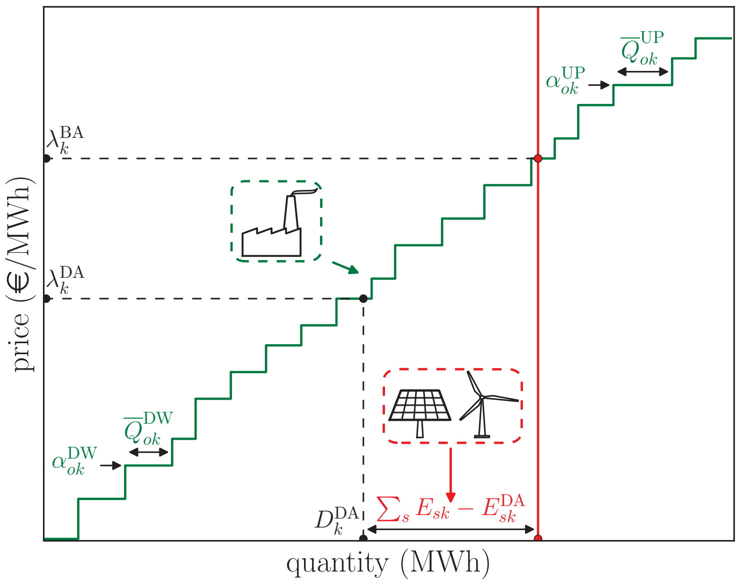


FIGURE 2.5: Schematic representation of the balancing market clearing mechanism when upward regulation is required.

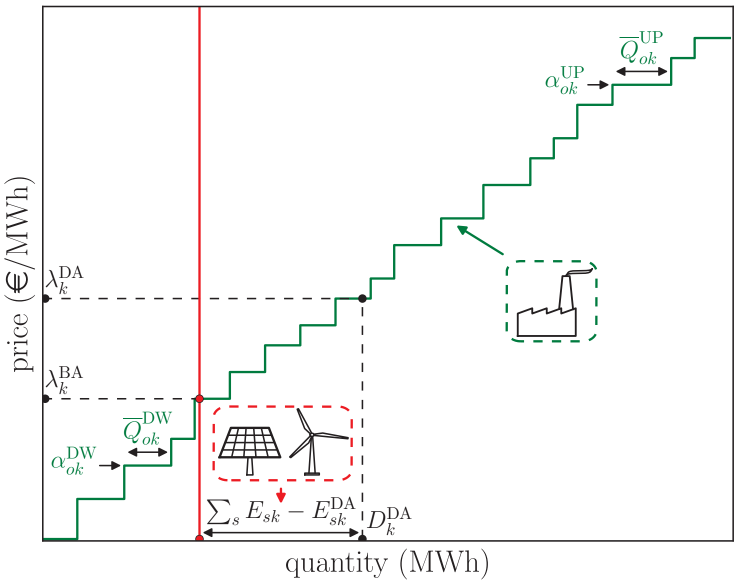


FIGURE 2.6: Schematic representation of the balancing market clearing mechanism, when downward regulation is needed.

Chapter 3

A General Model for the Offering Strategy

3.1 Introduction

A power producer offering in electricity markets bases its strategy on some decision-making tool, usually in the form of a mathematical optimization problem. The literature proposes several models aimed at driving the optimal trading strategy of a specific power producer. As examples, references (Bremnes, 2004; Pinson, Chevallier, and Kariniotakis, 2007; Morales, Conejo, and Pérez-Ruiz, 2010) develop offering models for a price-taker, i.e., that does not influence the market price, stochastic power producer. Similarly, references (Conejo, Nogales, and Arroyo, 2002; Ni, Luh, and Rourke, 2004; Maenhoudt and Deconinck, 2014) propose offering models for a price-taker conventional power producer, while references (Mashhour and Moghaddas-Tafreshi, 2011a; Mashhour and Moghaddas-Tafreshi, 2011b; Pandžić, Kuzle, and Capuder, 2013; Pandžić et al., 2013) consider an aggregate of different technologies (e.g., conventional units, stochastic power units and storages) offering in the electricity market as a single price-taker participant. Those trading models can also be extended to include the producer's price-maker (i.e., that influences the market price outcome) effect on the market clearing mechanism, for a stochastic power producer (Zugno et al., 2013; Baringo and Conejo, 2013), a conventional producer (Gountis and Bakirtzis, 2004; Bakirtzis et al., 2007) or an aggregate of units (Kardakos, Simoglou, and Bakirtzis, 2016).

This chapter wants to provide a more general approach to build the offering strategy of a generic power producer. By analyzing the optimization models mentioned above, we derive a general structure of the offering strategy for optimal electricity market participation. It is developed with a modular approach. Accordingly, the general model is composed of blocks (in the form of sets of constraints) that can be replaced depending on the specific electricity market structure or regulation and on the particular production units. Some of these blocks simulate the trading problem in different market stages. For the trading problem, we intend a set of constraints and variables that can model the market clearing mechanism (endogenously or exogenously), the market pricing scheme (uniform or pay-as-bid pricing), and other additional constraints associated with the market offers (e.g., the non-decreasing condition of offer curves). Together with the general offering strategy, this chapter provides several formulations for modeling the trading problem in the different market stages.

The more general approach is to consider the power producer as a price-maker in the market. The price-maker trading problem can be implemented through a residual demand model (Baillo et al., 2004) or using a bilevel optimization setup to include the market clearing mechanism within the optimal offering strategy (Gountis and Bakirtzis, 2004; Bakirtzis et al., 2007). The result is a Mathematical Problem with Equilibrium Constraints (MPEC), where the optimization problem that simulates the market clearing mechanism is formulated as a set of constraints, obtaining a single-level optimization problem. In this context, it is essential to mention the work of Ruiz and Conejo (2009), that shows how to cast an MPEC as a mixed-integer and linear optimization problem. They reformulate the non-linear producer's market income (where the non-linearity arises from the product between the market price and the quantity offer) as a linear one by exploiting the strong duality of the lower level problem (i.e., the market clearing optimization problem).

Although more general, MPECs may have high computational cost and rely on strong assumptions on opponents' behavior. Hence, when the power producer has a small impact on the market, a price-maker setup may not be the preferable choice. In this case, it is possible to assume that the power producer is price-taker as it strongly simplifies the formulation of the trading problem. Indeed, the producer now sees the market price as a parameter and no more as a variable. Its uncertainty can accordingly be represented by a continuous marginal distribution or through a discrete set of possible outcomes. When the market is settled under a uniform pricing scheme, i.e., all the accepted offers are remunerated at the market price (disregarding the offered price), the price-taker trading problem can be formulated as linear programming (LP) problem. E.g., Conejo, Carrión, and Morales (2010) show how a price-taker power producer can derive its optimal offer curves by exploiting a stochastic programming approach. The idea is to use the discrete set of possible market price outcomes as the possible offering prices for the price-taker producer. Differently, if the market is settled under a pay-as-bid pricing scheme, i.e., the accepted offers are remunerated at the offered price (disregarding the market clearing price), fewer trading models are available. Indeed, the topic of trading under a pay-as-bid scheme and price uncertainty has not been extensively addressed in the literature. Ren and Galiana (2004a) and Ren and Galiana (2004b) present an analysis on optimal offering under pay-as-bid and uniform pricing schemes. The authors obtain the profit expectation and variance for both pricing schemes while assuming that the market price follows a uniform distribution. Swider and Weber (2007) propose a methodology that aims to maximize the profit expectation in a pay-as-bid auction. Other formulations of the trading problem under a pay-as-bid pricing scheme are also proposed by Swider (2007), Khorasani and Mashhadi (2012) and Sadeh, Mashhadi, and Latifi (2009). These references show how to model the price-taker trading problem under a pay-as-bid pricing scheme using a non-linear programming (NLP) approach. Compared to the available literature, this chapter provides a novel approach that allows casting the price-taker trading problem in pay-as-bid markets under price uncertainty as an LP problem. For that purpose, we represent continuous random variables (i.e., market-clearing prices) as discrete variables. Then, following the idea of Conejo, Carrión, and Morales (2010), we use the market price scenarios as potential offering prices of the price-taker power producer. It is worth mentioning that Khorasani and Mashhadi (2012) propose to solve the trading problem in pay-as-bid markets under price uncertainty with a two-step approach, obtaining the expected profit as a linear function of the quantity offer. However, this approach is not applicable in case of problems with inter-temporal constraints or with more complex cost functions.

The remaining of the chapter is organized as follows. Section 3.2 presents the general formulation of an offering strategy for a producer trading in a two-settlement electricity market. Section 3.3 yields a formulation of the trading problem, both for the day-ahead and the balancing market, for a price-maker power producer. Then, Section 3.4 introduces the assumptions of a price-taker and risk-neutral power producer.

3.2 General Formulation of an Offering Strategy

We consider a two-settlement electricity market, composed of a day-ahead and a balancing market. The power producer submits to the day-ahead market the quantity $\overline{q}_k^{\mathrm{DA}}$ (MWh) that it wants to produce during the hourly interval k. In the remaining of the chapter, we develop the trading problem for a single hourly interval k. Consequently, we skip the subscript k for the clarity of the notation, e.g., $\overline{q}_k^{\mathrm{DA}} \rightarrow \overline{q}^{\mathrm{DA}}$. Then, at the balancing stage, it can offer to adjust upward its power production of $\overline{q}^{\mathrm{UP}}$ (MWh) or to decrease it of $\overline{q}^{\mathrm{DW}}$ (MWh). These market offers are related to an active participation in the balancing stage, i.e., the power producer is offering to the System Operator its available regulating energy. Differently, a passive participation allows the producer to create a deviation q^{BA} (MWh) from its day-ahead contracted schedule $\overline{q}^{\mathrm{DA}}$. The total amount of energy exchanged with the market platform has to match the energy production q^{A} (MWh) of the power unit, i.e.,

$$\overline{q}^{\mathrm{DA}} + \overline{q}^{\mathrm{UP}} - \overline{q}^{\mathrm{DW}} + q^{\mathrm{BA}} = q^{\mathrm{A}}. \tag{3.1}$$

Note that Equation (3.1) considers both an active and passive participation in the balancing market, which is usually not allowed in a real-world electricity market. Indeed, a power producer cannot simultaneously offer regulating energy while deviating from its contracted schedule. However, as we want to provide a general formulation, we include both of them. A schematic representation of the interface between the producer and the electricity market is shown in Figure 3.1, where the question mark represents a generic power producer.

The day-ahead quantity offer $\overline{q}^{\mathrm{DA}}$ yields an income ρ^{DA} (€) to the power producer, and it is associated with a feasibility region. Consequently, we impose

$$\overline{q}^{\mathrm{DA}}, \rho^{\mathrm{DA}} \in \Pi^{\mathrm{DA}}, \tag{3.2}$$

where Π^{DA} is a set of constraints associated with the day-ahead market offer. Similarly, the balancing market incomes ρ^{UP} (€) and ρ^{DW} (€) are linked to the upward

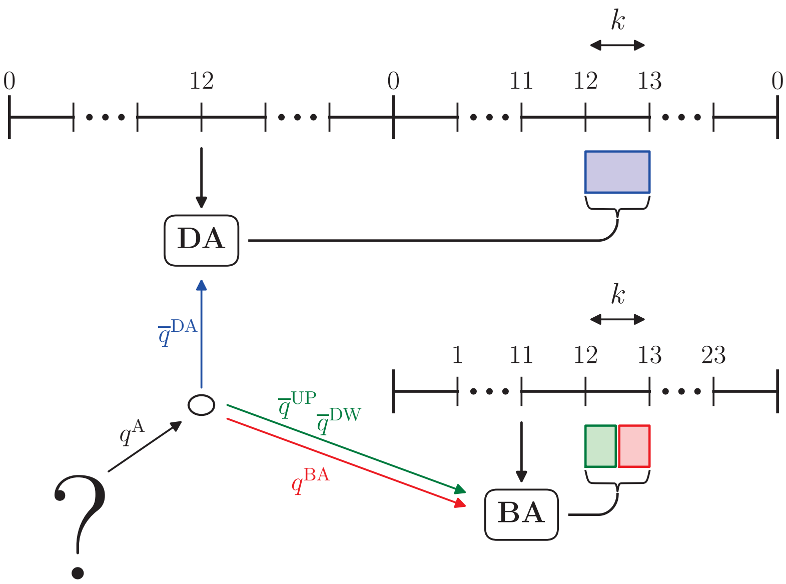


FIGURE 3.1: Illustration of the energy balance between the power producer and the electricity market, considering a single hourly interval k. The question mark indicates a generic power production unit.

and the downward regulation offers, i.e., $\overline{q}^{\mathrm{UP}}$ and $\overline{q}^{\mathrm{DW}}$. Accordingly, we impose

$$\overline{q}^{\mathrm{UP}}, \overline{q}^{\mathrm{DW}}, \rho^{\mathrm{UP}}, \rho^{\mathrm{DW}} \in \Pi^{\mathrm{BA}}_{\mathrm{Act}}, \tag{3.3}$$

where $\Pi^{\mathrm{BA}}_{\mathrm{Act}}$ is a set of constraints associated with the balancing market offers. Likewise the active participation, also a passive deviation q^{BA} is related to a market income ρ^{BA} (€), given by

$$q^{\mathrm{BA}}, \rho^{\mathrm{BA}} \in \Pi^{\mathrm{BA}}_{\mathrm{Pas}}. \tag{3.4}$$

These three set of constraints, i.e., Π^{DA}, $\Pi^{\mathrm{BA}}_{\mathrm{Act}}$, and $\Pi^{\mathrm{BA}}_{\mathrm{Pas}}$, are a compact representation of the trading problem in different market stages. Together with these constraints, the energy production q^{A} has a feasible operating region, generically described by the set of constraints Ω, i.e.,

$$q^{\mathrm{A}} \in \Omega, \tag{3.5}$$

where the set Ω depends on the production unit considered. Then, the cost c (€) for producing the quantity q^{A} is evaluated by mean of the generic function $h(\cdot)$. This writes

$$c = h(q^{\mathrm{A}}) \,. \tag{3.6}$$

The total profit μ (€) of the power producer is computed as the sum of incomes from the electricity markets minus the production cost c, thus leading to

$$\mu = \rho^{\mathrm{DA}} + \rho^{\mathrm{UP}} + \rho^{\mathrm{DW}} + \rho^{\mathrm{BA}} - c. \tag{3.7}$$

The producer is usually solving a stochastic model as several market parameters are still uncertain at the moment of submitting its market offers. Consequently, the producer's profit μ is a random value. We introduce the generic function $g(\cdot)$ transforming a random variable into a deterministic one. As an example, the function $g(\cdot)$ can be the expectation or the worst case realization of μ. The generic structure of the offering strategy is formulated as

$$\begin{aligned}
\underset{\Gamma}{\mathrm{Max}} \quad & g\big(\rho^{\mathrm{DA}} + \rho^{\mathrm{UP}} + \rho^{\mathrm{DW}} + \rho^{\mathrm{BA}} - c\big) && \text{(3.8a)}\\
\text{s.t.} \quad & \overline{q}^{\mathrm{DA}} + \overline{q}^{\mathrm{UP}} - \overline{q}^{\mathrm{DW}} + q^{\mathrm{BA}} = q^{\mathrm{A}}, && \text{(3.8b)}\\
& \overline{q}^{\mathrm{DA}}, \rho^{\mathrm{DA}} \in \Pi^{\mathrm{DA}}, && \text{(3.8c)}\\
& \overline{q}^{\mathrm{UP}}, \overline{q}^{\mathrm{DW}}, \rho^{\mathrm{UP}}, \rho^{\mathrm{DW}} \in \Pi^{\mathrm{BA}}_{\mathrm{Act}}, && \text{(3.8d)}\\
& q^{\mathrm{BA}}, \rho^{\mathrm{BA}} \in \Pi^{\mathrm{BA}}_{\mathrm{Pas}}, && \text{(3.8e)}\\
& c = h\big(q^{\mathrm{A}}\big)\,, && \text{(3.8f)}\\
& q^{\mathrm{A}} \in \Omega, && \text{(3.8g)}
\end{aligned}$$

where

$$\Gamma = \{q^{\mathrm{A}}, \overline{q}^{\mathrm{DA}}, \overline{q}^{\mathrm{UP}}, \overline{q}^{\mathrm{DW}}, q^{\mathrm{BA}}, \rho^{\mathrm{DA}}, \rho^{\mathrm{UP}}, \rho^{\mathrm{DW}}, \rho^{\mathrm{BA}}, c\}. \tag{3.9}$$

The objective function (3.8a) maximizes a certain function $g(\cdot)$ of the producer's profit μ. The energy balance between the power production q^{A} and the total amount of energy exchanged with the market platform is enforced by constraint (3.8b). Then, constraints (3.8c) and (3.8d) impose a set of constraints related to the day-ahead and the balancing market offers. Constraint (3.8e) enforces a set of constraints related to a passive participation in the balancing market. Then, constraint (3.8f) yields the cost c for producing the energy q^{A}. Finally, constraint (3.8g) imposes the feasible operating region of the power production units.

3.3 General Formulation of the Trading Problem

This section derives the formulation of the trading for a price-maker power producer, i.e., a producer that can influence the market price outcome with its decisions. Section 3.3.1 considers the day-ahead market trading problem, while Section 3.3.2 the balancing market one.

3.3.1 Day-Ahead Market Trading Problem

Let us assume that the power producer submits a single price-quantity offer $(p^{\mathrm{DA}}, \overline{q}^{\mathrm{DA}})$ in the day-ahead market. With the market offer $(p^{\mathrm{DA}}, \overline{q}^{\mathrm{DA}})$ the power producer indicates its willingness to produce $\overline{q}^{\mathrm{DA}}$, provided that it is remunerated at a price higher or equal to p^{DA} (€/MWh). Together with the producer's offer $(p^{\mathrm{DA}}, \overline{q}^{\mathrm{DA}})$, the Market Operator receives $N^{\mathrm{O}} + N^{\mathrm{S}}$ opponents' offers. N^{O} price-quantity offers are submitted by conventional producers, i.e., $\{(\alpha_o^{\mathrm{DA}}, \overline{Q}_o^{\mathrm{DA}}),\ o = 1, ..., N^{\mathrm{O}}\}$, and N^{S} quantity offers by the stochastic producers, i.e., $\{(\overline{E}_s^{\mathrm{DA}}),\ s = 1, ..., N^{\mathrm{S}}\}$. The Market Operator clears the market and computes the day-ahead market price λ^{DA} by solving the economic dispatch presented in Section 2.3.1, i.e.,

$$\underset{\{q^{\mathrm{DA}}, q_o^{\mathrm{DA}}, e_s^{\mathrm{DA}}\}}{\mathrm{Min}} \quad \sum_{o=1}^{N^{\mathrm{O}}} \alpha_o^{\mathrm{DA}} q_o^{\mathrm{DA}} + p^{\mathrm{DA}} q^{\mathrm{DA}} \tag{3.10a}$$

$$\text{s.t.} \quad \sum_{o=1}^{N^{\mathrm{O}}} q_o^{\mathrm{DA}} + q^{\mathrm{DA}} + \sum_{s=1}^{N^{\mathrm{S}}} e_s^{\mathrm{DA}} = D^{\mathrm{DA}} \ : \ \left(\lambda^{\mathrm{DA}}\right) \tag{3.10b}$$

$$0 \le q_o^{\mathrm{DA}} \le \overline{Q}_o^{\mathrm{DA}},\ \forall o \in \{1, ..., N^{\mathrm{O}}\} \tag{3.10c}$$

$$0 \le q^{\mathrm{DA}} \le \overline{q}^{\mathrm{DA}} \tag{3.10d}$$

$$0 \le e_s^{\mathrm{DA}} \le \overline{E}_s^{\mathrm{DA}},\ \forall s \in \{1, ..., N^{\mathrm{S}}\} \tag{3.10e}$$

where q^{DA} is the amount of energy that the power producer is contracted to produce in the day-ahead market. Note that we add the producer's offer to the economic dispatch formulation (2.1) as we assume the the offer $(p^{\mathrm{DA}}, \overline{q}^{\mathrm{DA}})$ influences the market outcome.

The market revenue ρ^{DA} of the producer is computed differently, depending on the pricing scheme considered. We can distinguish among two main pricing schemes, i.e., uniform pricing and pay-as-bid pricing. In a uniform pricing scheme, all the accepted offers are remunerated at the day-ahead market price λ^{DA}, disregarding the offered price. Differently, under a pay-as-bid pricing scheme, the accepted offers are paid at the proposed price p^{DA}. Since the structure of the day-ahead electricity markets across the European countries has been harmonized and considers a uniform pricing scheme, we assume that the day-ahead market is settled under a uniform pricing scheme. Accordingly, the market income ρ^{DA} is computed as

$$\rho^{\mathrm{DA}} = \lambda^{\mathrm{DA}} q^{\mathrm{DA}}. \tag{3.11}$$

A price-maker power producer that submits a single price-quantity offer in the day-ahead market formulates the set Π^{DA} of the general model (3.8) as

$$\rho^{\mathrm{DA}} = \lambda^{\mathrm{DA}} \, q^{\mathrm{DA}} \tag{3.12a}$$

$$q^{\mathrm{DA}}, \lambda^{\mathrm{DA}} = \arg\{(3.10)\} \tag{3.12b}$$

$$\underline{Q} \leq \overline{q}^{\mathrm{DA}} \leq \overline{Q} \tag{3.12c}$$

Constraint (3.12a) gives the day-ahead market income ρ^{DA} under a uniform pricing scheme. Constraint (3.12b) solves the economic dispatch (3.10) and computes the accepted quantity q^{DA} and the market price λ^{DA}. Finally, constraint (3.12c) limits the quantity offer $\overline{q}^{\mathrm{DA}}$ between $\underline{Q}$ and $\overline{Q}$. Usually, $\overline{Q}$ is the unit's capacity while $\underline{Q}$ is 0 MWh for a production unit. However, $\underline{Q}$ can also be a negative value in case the power unit can buy (i.e., consume) electricity. An example is an electric storage system, which is producing energy when discharging and consuming it while charging. Note that the general formulation (3.12) would lead to a bi-level optimization problem (3.8), as constraint (3.12b) is an optimization problem itself. We do not investigate how to formulate an MPEC as a single-level optimization problem, as the thesis focuses on a price-taker power producer. We refer the interested reader to (Gountis and Bakirtzis, 2004; Bakirtzis et al., 2007; Ruiz and Conejo, 2009) for an extensive coverage of the topic.

3.3.2 Balancing Market Trading Problem

At the moment of offering in the balancing market, the power producer knows the amount of energy $\overline{q}^{\mathrm{DA}}$ contracted in the day-ahead market. We assume that the producer submits single price-quantity offers in the balancing market, as an active participant. It can offer to increase its power production of a quantity $\overline{q}^{\mathrm{UP}}$ at price p^{UP} or to decrease it of a quantity $\overline{q}^{\mathrm{DW}}$ at price p^{DW}. Let consider that the producer submits both an up-regulation and a down-regulation offer in the balancing market. Moreover, as a passive participant, it can deviate of a quantity q^{BA} from its day-ahead contracted schedule $\overline{q}^{\mathrm{DA}}$. As mentioned in Section 3.2, a simultaneous active and passive participation at the balancing stage is not allowed by the System Operator. However, to provide a general formulation of the balancing market trading problem we include both of them.

The System Operator also receives N^{O} opponents' offers for upward regulation $\{(\alpha_o^{\mathrm{UP}}, \overline{Q}_o^{\mathrm{UP}}), i = 1, ..., N^{\mathrm{O}}\}$. Similarly, it receives N^{O} downward regulation offers $\{(\alpha_o^{\mathrm{DW}}, \overline{Q}_o^{\mathrm{DW}}), i = 1, ..., N^{\mathrm{O}}\}$. We indicate with δ^{BA} the system imbalance (without the producer's deviation) that needs to be restored through the balancing market. The System Operator clears the balancing market by solving the economic dispatch presented in Section 2.3.2, adapted with the producer's participation, i.e.,

$$\underset{\{q_o^{\mathrm{UP}}, q^{\mathrm{UP}}, q_o^{\mathrm{DW}}, q^{\mathrm{DW}}\}}{\mathrm{Min}} \quad \sum_{o=1}^{N^{\mathrm{O}}} \alpha_o^{\mathrm{UP}} q_o^{\mathrm{UP}} + p^{\mathrm{UP}} q^{\mathrm{UP}} - \sum_{o=1}^{N^{\mathrm{O}}} \alpha_o^{\mathrm{DW}} q_o^{\mathrm{DW}} - p^{\mathrm{DW}} q^{\mathrm{DW}} \tag{3.13a}$$

$$\text{s.t.} \quad \sum_{o=1}^{N^{\mathrm{O}}} \left(q_o^{\mathrm{UP}} - q_o^{\mathrm{DW}}\right) + q^{\mathrm{UP}} - q^{\mathrm{DW}} = \delta^{\mathrm{BA}} - q^{\mathrm{BA}} \; : \; \left(\lambda^{\mathrm{BA}}\right) \tag{3.13b}$$

$$0 \le q_o^{\mathrm{UP}} \le \overline{Q}_o^{\mathrm{UP}}, \; o = 1, ..., N^{\mathrm{O}} \tag{3.13c}$$

$$0 \le q^{\mathrm{UP}} \le \overline{q}^{\mathrm{UP}}, \tag{3.13d}$$

$$0 \le q_o^{\mathrm{DW}} \le Q_o^{\mathrm{DW}}, \; o = 1, ..., N^{\mathrm{O}} \tag{3.13e}$$

$$0 \le q^{\mathrm{DW}} \le \overline{q}^{\mathrm{DW}}, \tag{3.13f}$$

where q^{UP} and q^{DW} are the producer's quantity of up-regulation and down-regulation energy scheduled by the System Operator, while λ^{BA} is the balancing market clearing price. Note that the producer's decision variables, i.e., $\overline{q}^{\mathrm{UP}}$, $\overline{q}^{\mathrm{DW}}$, and q^{BA}, are not variables but parameters for the economic dispatch in (3.13), as the System Operator solves it after having received the producer's offers and deviation.

The market revenues ρ^{UP} and ρ^{DW} are computed differently, depending on the pricing scheme considered. Differently than the day-ahead market, which is mainly settled under a uniform pricing scheme, several European balancing markets, e.g., Italy and Germany (Wang et al., 2015), are settled under a pay-as-bid pricing scheme.

Accordingly, we provide a formulation of the up- and down-regulation market incomes under both the pricing schemes, i.e.,

$$\rho^{\mathrm{UP}} = \begin{cases} \lambda^{\mathrm{BA}},\ q^{\mathrm{UP}} & \text{if uniform pricing scheme} \\ p^{\mathrm{UP}},\ q^{\mathrm{UP}} & \text{if pay-as-bid pricing scheme} \end{cases} \tag{3.14a}$$

$$\rho^{\mathrm{DW}} = \begin{cases} -\lambda^{\mathrm{BA}},\ q^{\mathrm{DW}} & \text{if uniform pricing scheme} \\ -p^{\mathrm{DW}},\ q^{\mathrm{DW}} & \text{if pay-as-bid pricing scheme} \end{cases} \tag{3.14b}$$

The income ρ^{BA}, linked to a passive participation, is evaluated based on the imbalance settlement scheme considered. Two main imbalance pricing schemes can be distinguished in the European electricity markets, i.e., the single- and the dual-price imbalance settlement scheme. Under a single-price imbalance settlement scheme, imbalances are priced at the balancing market price λ^{BA}, disregarding the sign of the deviation q^{BA}. Differently, under a dual-price imbalance settlement scheme, the imbalance q^{BA} is priced at the least convenient (for the producer) price between the balancing market price λ^{BA} and the day-ahead one λ^{DA}. This leads to

$$\rho^{\mathrm{BA}} = \begin{cases} \lambda^{\mathrm{BA}} q^{\mathrm{BA}}, & \text{if single-price scheme} \\ \min\left(\lambda^{\mathrm{BA}} q^{\mathrm{BA}}, \lambda^{\mathrm{DA}} q^{\mathrm{BA}}\right), & \text{if dual-price scheme} \end{cases} \tag{3.15}$$

Consequently, a price-maker producer that submits single price-quantity regulation offers and deviates from its contracted schedule, can formulate the sets $\Pi^{\mathrm{BA}}_{\mathrm{Act}}$ and $\Pi^{\mathrm{BA}}_{\mathrm{Pas}}$ of the general model (3.8) as

$$\rho^{\mathrm{UP}} = \begin{cases} \lambda^{\mathrm{BA}},\ q^{\mathrm{UP}} & \text{if uniform pricing scheme} \\ p^{\mathrm{UP}},\ q^{\mathrm{UP}} & \text{if pay-as-bid pricing scheme} \end{cases} \tag{3.16a}$$

$$\rho^{\mathrm{DW}} = \begin{cases} -\lambda^{\mathrm{BA}},\ q^{\mathrm{DW}} & \text{if uniform pricing scheme} \\ -p^{\mathrm{DW}},\ q^{\mathrm{DW}} & \text{if pay-as-bid pricing scheme} \end{cases} \tag{3.16b}$$

$$\rho^{\mathrm{BA}} = \begin{cases} \lambda^{\mathrm{BA}} q^{\mathrm{BA}}, & \text{if single-price scheme} \\ \min\left(\lambda^{\mathrm{BA}} q^{\mathrm{BA}}, \lambda^{\mathrm{DA}} q^{\mathrm{BA}}\right), & \text{if dual-price scheme} \end{cases} \tag{3.16c}$$

$$q^{\mathrm{UP}}, q^{\mathrm{DW}}, \lambda^{\mathrm{BA}} = \arg\{(3.13)\} \tag{3.16d}$$

Constraints (3.16a) and (3.16b) yield the producer's upward and downward regulation incomes, considering both a uniform and a pay-as-bid pricing schemes. Constraint (3.16c) gives the income associated with the uncontracted deviation q^{BA}. Finally, constraint (3.16d), given the producer's decisions ($\overline{q}^{\mathrm{UP}}$, $\overline{q}^{\mathrm{DW}}$, and q^{BA}), simulates the balancing market clearing process and computes the quantity of up- and down-regulation energy scheduled, i.e., q^{UP} and q^{DW}, and the market price λ^{BA}. As for the day-ahead market trading problem (3.12), we do not present the methodology to reformulate the bi-level structure of (3.16) into a single-level one, as the thesis focuses on a price-taker producer. We refer the interested reader to (Gountis and Bakirtzis, 2004; Bakirtzis et al., 2007; Ruiz and Conejo, 2009) for an extensive coverage of the topic.

3.4 Trading Problem for a Price-Taker and Risk-Neutral Producer

This section introduces two fundamental assumptions on the power producer. First, we consider a risk-neutral power producer. This translates in maximizing the expectation of its future marker profit, disregarding possible huge losses. Consequently, the objective function (3.8a) of the general offering strategy, can be written as

$$g\big(\rho^{\mathrm{DA}} + \rho^{\mathrm{UP}} + \rho^{\mathrm{DW}} + \rho^{\mathrm{BA}} - c\big) = \hat{\rho}^{\mathrm{DA}} + \hat{\rho}^{\mathrm{UP}} + \hat{\rho}^{\mathrm{DW}} + \hat{\rho}^{\mathrm{BA}} - \hat{c}, \tag{3.17}$$

where the $\hat{\cdot}$ symbol indicates the mean expected value, e.g., $\hat{\rho}^{\mathrm{DA}} = \mathbb{E}\big[\rho^{\mathrm{DA}}\big]$. Accordingly, we reformulate the general offering model (3.8) as

$$\begin{aligned}
\underset{\Gamma}{\text{Max}} \quad & \hat{\rho}^{\mathrm{DA}} + \hat{\rho}^{\mathrm{UP}} + \hat{\rho}^{\mathrm{DW}} + \hat{\rho}^{\mathrm{BA}} - \hat{c} && \text{(3.18a)}\\
\text{s.t.} \quad & \overline{q}^{\mathrm{DA}} + \overline{q}^{\mathrm{UP}} - \overline{q}^{\mathrm{DW}} + q^{\mathrm{BA}} = q^{\mathrm{A}}, && \text{(3.18b)}\\
& \overline{q}^{\mathrm{DA}}, \hat{\rho}^{\mathrm{DA}} \in \Pi^{\mathrm{DA}}, && \text{(3.18c)}\\
& \overline{q}^{\mathrm{UP}}, \overline{q}^{\mathrm{DW}}, \hat{\rho}^{\mathrm{UP}}, \hat{\rho}^{\mathrm{DW}} \in \Pi^{\mathrm{BA}}_{\mathrm{Act}}, && \text{(3.18d)}\\
& q^{\mathrm{BA}}, \hat{\rho}^{\mathrm{BA}} \in \Pi^{\mathrm{BA}}_{\mathrm{Pas}}, && \text{(3.18e)}\\
& \hat{c} = h\big(q^{\mathrm{A}}\big)\,, && \text{(3.18f)}\\
& q^{\mathrm{A}} \in \Omega, && \text{(3.18g)}
\end{aligned}$$

where

$$\Gamma = \{q^{\mathrm{A}}, \overline{q}^{\mathrm{DA}}, \overline{q}^{\mathrm{UP}}, \overline{q}^{\mathrm{DW}}, q^{\mathrm{BA}}, \hat{\rho}^{\mathrm{DA}}, \hat{\rho}^{\mathrm{UP}}, \hat{\rho}^{\mathrm{DW}}, \hat{\rho}^{\mathrm{BA}}, \hat{c}\}. \tag{3.19}$$

Then, we consider that the power producer is price-taker in the electricity market. It means that the influence of its decisions on the market clearing process of both the day-ahead and the balancing market is negligible. This assumption is, generally, acceptable for producers with a small market power. As a consequence, the day-ahead market clearing problem in (3.10), under the price-taker assumption, can be simplified as

$$\begin{aligned}
\underset{\{q_o^{\mathrm{DA}}, e_s^{\mathrm{DA}}\}}{\text{Min}} \quad & \sum_{o=1}^{N^{\mathrm{O}}} \alpha_o^{\mathrm{DA}} q_o^{\mathrm{DA}} && (3.20\text{a})\\
\text{s.t.} \quad & \sum_{o=1}^{N^{\mathrm{O}}} q_o^{\mathrm{DA}} + \sum_{s=1}^{N^{\mathrm{S}}} e_s^{\mathrm{DA}} = D^{\mathrm{DA}} \; : \; \left(\lambda^{\mathrm{DA}}\right) && (3.20\text{b})\\
& 0 \le q_o^{\mathrm{DA}} \le \overline{Q}_o^{\mathrm{DA}}, \; \forall o \in \{1, ..., N^{\mathrm{O}}\} && (3.20\text{c})\\
& 0 \le e_s^{\mathrm{DA}} \le \overline{E}_s^{\mathrm{DA}}, \; \forall s \in \{1, ..., N^{\mathrm{S}}\} && (3.20\text{d})
\end{aligned}$$

The day-ahead market clearing model (3.20) can be solved exogenously to the trading problem, as λ^{DA} is no more influenced by the producer's offer $(p^{\mathrm{DA}}, \overline{q}^{\mathrm{DA}})$. Hence, the uncertain market price λ^{DA} can be modeled as a random variable following the density function $f_\lambda^{\mathrm{DA}} : \mathbb{R}^+ \mapsto \mathbb{R}^+$.

Similarly, the price-taker assumption allows to reformulate the balancing market clearing problem in (3.13) as

$$\begin{aligned}
\underset{\{q_o^{\mathrm{UP}}, q_o^{\mathrm{DW}}\}}{\text{Min}} \quad & \sum_{o=1}^{N^{\mathrm{O}}} \alpha_o^{\mathrm{UP}} q_o^{\mathrm{UP}} - \sum_{o=1}^{N^{\mathrm{O}}} \alpha_o^{\mathrm{DW}} q_o^{\mathrm{DW}} && (3.21\text{a})\\
\text{s.t.} \quad & \sum_{o=1}^{N^{\mathrm{O}}} \left(q_o^{\mathrm{UP}} - q_o^{\mathrm{DW}}\right) = \delta^{\mathrm{BA}} \; : \; \left(\lambda^{\mathrm{BA}}\right) && (3.21\text{b})\\
& 0 \le q_o^{\mathrm{UP}} \le \overline{Q}_o^{\mathrm{UP}}, \; o = 1, ..., N^{\mathrm{O}} && (3.21\text{c})\\
& 0 \le q_o^{\mathrm{DW}} \le Q_o^{\mathrm{DW}}, \; o = 1, ..., N^{\mathrm{O}} && (3.21\text{d})
\end{aligned}$$

The balancing market economic dispatch in (3.21) can be solved as well exogenously to the trading problem. Consequently, for a given realization of the day-ahead market price λ^{DA}, the balancing market one, i.e., λ^{BA}, can be modeled as a random variable following the density function $f_\lambda^{\mathrm{BA}} : \mathbb{R} \mapsto \mathbb{R}^+$.

Instead of representing the uncertain market prices λ^{DA} and λ^{BA} with continuous distributions, the power producer can use a discrete representation. This is generally required in a stochastic programming framework. Under such approach, the continuous distributions of random variables are replaced by discrete distributions. In our problem, we can indeed represent the uncertain market price λ^{DA} using a set I of possible scenarios $\{\lambda_i^{\mathrm{DA}},\ i \in I\}$, where each price scenario λ_i^{DA} is associated with a probability π_i^{DA} of occurrence, such that $\sum_i \pi_i^{\mathrm{DA}} = 1$. Similarly, we can do for the balancing market price λ^{BA}. For each day-ahead market scenario $i \in I$, we represent the uncertain λ^{BA} using a set J of scenarios, i.e., $\{\lambda_{ij}^{\mathrm{BA}},\ i \in I, j \in J\}$. Each scenario (ij) is associated with a discrete probability π_{ij}^{BA}, such that $\sum_j \pi_{ij}^{\mathrm{BA}} = 1,\ \forall i$. Figure 3.2 illustrates the difference between a continuous distribution (in blue) and a discrete one (in green).

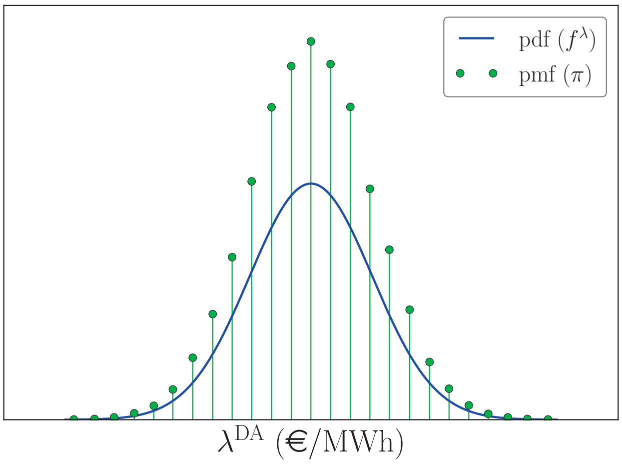


FIGURE 3.2: Example of a probability density function (pdf) in blue *vs* a probability mass function (pmf) in green

The remaining of the section is organized as follows. Section 3.4.1 develops the trading problem in the day-ahead market when the producer submits a single price-quantity offer. Section 3.4.2 extends the formulation by considering non-decreasing step-wise offer curves. Then, Section 3.4.3 uses a stochastic programming approach to formulate the day-ahead market trading problem with offer curves as an LP problem. Section 3.4.4 formulates the trading problem in the balancing market for an active participant submitting single price-quantities offers. Subsequently, Sections 3.4.5 and 3.4.6 extend the problem to offer curves and to a stochastic programming framework, respectively. Finally, Section 3.4.7 models the trading problem for a passive participant in the balancing market while Section 3.4.8 does the same based on a stochastic programming approach.

3.4.1 Day-Ahead Market Trading Problem with Single Offers

We consider that the power producer submits a single price-quantity offer $(p^{\mathrm{DA}}, \overline{q}^{\mathrm{DA}})$ in the day-ahead market and we denote with q^{DA} the quantity accepted by the Market Operator. With the price-taker assumption, q^{DA} is no more computed endogenously by the market clearing model as in Equation (3.12b). However, it can be evaluated as

$$q^{\mathrm{DA}} = \begin{cases} 0, & \text{if} \quad \lambda^{\mathrm{DA}} < p^{\mathrm{DA}}, \\ \overline{q}^{\mathrm{DA}}, & \text{if} \quad \lambda^{\mathrm{DA}} \geq p^{\mathrm{DA}}. \end{cases} \tag{3.22}$$

Indeed, the accepted quantity q^{DA} is equal to $\overline{q}^{\mathrm{DA}}$ when the offer price p^{DA} is lower or equal to the day-ahead market price λ^{DA} and 0 otherwise. Figure 3.3 shows an illustrative example, where the continuous blue line represents the accepted quantity q^{DA} as a function of the future realization of the uncertain day-ahead market price λ^{DA}. Therefore, the probability that the Market Operator accepts the market offer

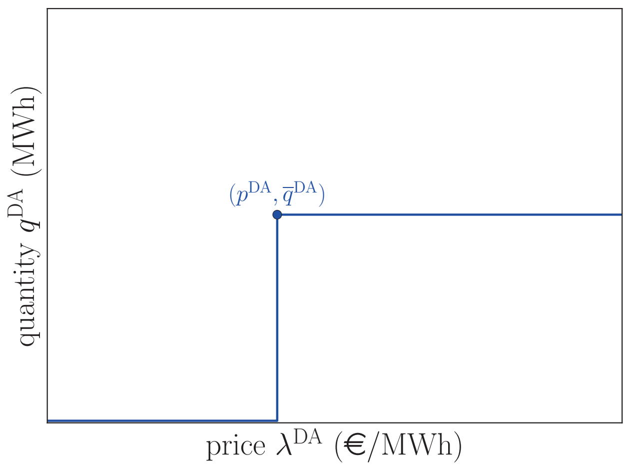


FIGURE 3.3: Example of a single offer $(p^{\mathrm{DA}}, \overline{q}^{\mathrm{DA}})$ in the day-ahead market.

$(p^{\mathrm{DA}}, \overline{q}^{\mathrm{DA}})$ is given by

$$\mathbb{P}\left[p^{\mathrm{DA}} \leq \lambda^{\mathrm{DA}}\right] = \int_{p^{\mathrm{DA}}}^{\infty} f_{\lambda}^{\mathrm{DA}}(\ell)\, \mathrm{d}\ell, \tag{3.23}$$

as it is only accepted when $p^{\mathrm{DA}} \leq \lambda^{\mathrm{DA}}$, in accordance with Equation (3.22). The variable ℓ in Equation (3.23) is an auxiliary integration variable. A graphical interpretation of the probability of acceptance of the single price-quantity market offer is shown in Figure 3.4. Let $p^{\mathrm{DA}*}$ be the remuneration price provided that the of-

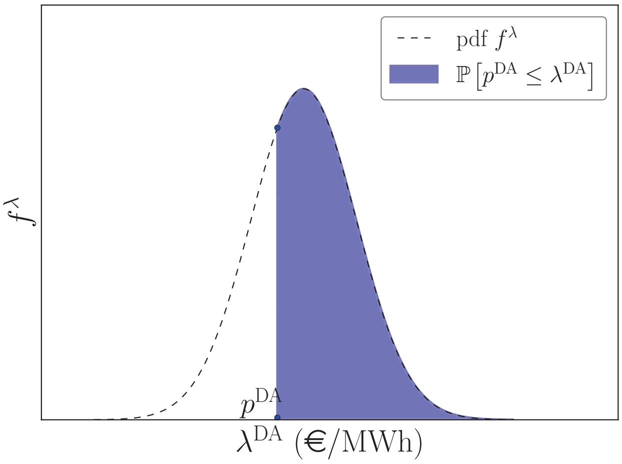


FIGURE 3.4: Graphic representation of the probability of acceptance of the price-quantity offer.

fer $(p^{\mathrm{DA}}, \overline{q}^{\mathrm{DA}})$ is being accepted (i.e., that $p^{\mathrm{DA}} \leq \lambda^{\mathrm{DA}}$) by the Market Operator. Its expected value can be evaluated as

$$\mathbb{E}\left[p^{\mathrm{DA}*} \mid p^{\mathrm{DA}} \leq \lambda^{\mathrm{DA}}\right] = \frac{\int_{p^{\mathrm{DA}}}^{\infty} \ell \, f_{\lambda}^{\mathrm{DA}}(\ell) \, \mathrm{d}\ell}{\int_{p^{\mathrm{DA}}}^{\infty} f_{\lambda}^{\mathrm{DA}}(\ell) \, \mathrm{d}\ell}. \tag{3.24}$$

Then, the expected market revenue $\hat{\rho}^{\mathrm{DA}}$ is given by the product of the offered quantity $\overline{q}^{\mathrm{DA}}$, the probability of acceptance of the offer, and the expected remuneration price provided that the offer is being accepted, i.e.,

$$\hat{\rho}^{\mathrm{DA}} = \overline{q}^{\mathrm{DA}} \, \mathbb{P}\left[\lambda^{\mathrm{DA}} \geq p^{\mathrm{DA}}\right] \, \mathbb{E}\left[p^{*} \mid \lambda^{\mathrm{DA}} \geq p^{\mathrm{DA}}\right]. \tag{3.25}$$

Let us rewrite Equation (3.25) by replacing the term $\mathbb{P}\left[p^{\mathrm{DA}} \leq \lambda^{\mathrm{DA}}\right]$ with Equation (3.23), and $\mathbb{E}\left[p^{\mathrm{DA}*} \mid p^{\mathrm{DA}} \leq \lambda^{\mathrm{DA}}\right]$ with Equation (3.24). This leads to

$$\hat{\rho}^{\mathrm{DA}} = \overline{q}^{\mathrm{DA}} \int_{p^{\mathrm{DA}}}^{\infty} \ell \, f_{\lambda}^{\mathrm{DA}}(\ell) \, \mathrm{d}\ell, \tag{3.26}$$

The set Π^{DA} in the risk-neutral offering strategy (3.18), can be replaced by the following set of constraints:

$$\hat{\rho}^{\mathrm{DA}} = \overline{q}^{\mathrm{DA}} \int_{p^{\mathrm{DA}}}^{\infty} \ell \, f_{\lambda}^{\mathrm{DA}}(\ell) \, \mathrm{d}\ell, \tag{3.27a}$$

$$\underline{Q} \leq \overline{q}^{\mathrm{DA}} \leq \overline{Q}. \tag{3.27b}$$

Constraint (3.27a) yields the expected day-ahead market income $\hat{\rho}^{\mathrm{DA}}$, given the single price-quantity offer $(p^{\mathrm{DA}}, \overline{q}^{\mathrm{DA}})$. Then, constraint (3.27b) limits the quantity offer $\overline{q}^{\mathrm{DA}}$ between its minimum and maximum values, i.e., $\underline{Q}$ and $\overline{Q}$.

3.4.2 Day-Ahead Market Trading Problem with Offer Curves

The market offers in a real-world electricity market for the generation-side are, generally, non-decreasing step-wise functions called offer curves. Such offer curves allow the producer to schedule more production as the market price increases. Figure 3.5 illustrates an example of multiple offer curves with three blocks. The continuous red line represents the quantity q^{DA} that would be contracted, depending on the future realization of the uncertain market price λ^{DA}. Such accepted quantity q^{DA} is

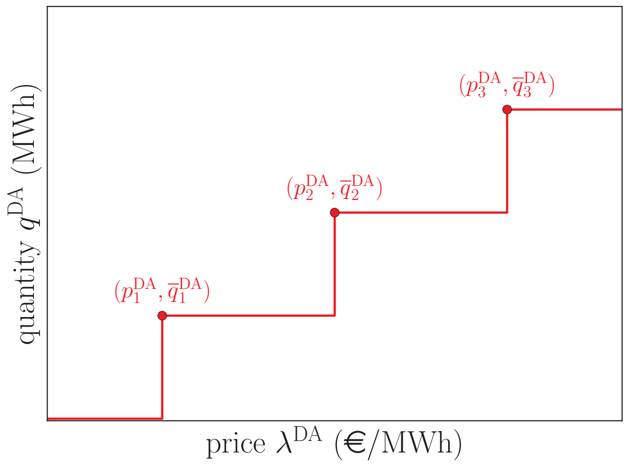


FIGURE 3.5: Example of a 3 blocks offer curve in the day-ahead market.

evaluated as

$$q^{\mathrm{DA}} = \begin{cases} 0, & \text{if} \quad \lambda^{\mathrm{DA}} < p_1^{\mathrm{DA}}, \\ \overline{q}_1^{\mathrm{DA}}, & \text{if} \quad p_1^{\mathrm{DA}} \leq \lambda^{\mathrm{DA}} < p_2^{\mathrm{DA}}, \\ \overline{q}_2^{\mathrm{DA}}, & \text{if} \quad p_2^{\mathrm{DA}} \leq \lambda^{\mathrm{DA}} < p_3^{\mathrm{DA}}, \\ \overline{q}_3^{\mathrm{DA}}, & \text{if} \quad p_3^{\mathrm{DA}} \leq \lambda^{\mathrm{DA}}. \end{cases} \tag{3.28}$$

The first bock offer $(p_1^{\mathrm{DA}}, \overline{q}_1^{\mathrm{DA}})$ is accepted if the market price λ^{DA} is greater or equal to p_1^{DA} but lower than p_2^{DA}. Indeed, if λ^{DA} is greater or equal to p_2^{DA} the second (or third) block would be contracted. The probability of acceptance of the first block is computed as

$$\mathbb{P}\left[p_1^{\mathrm{DA}} \leq \lambda^{\mathrm{DA}} < p_2^{\mathrm{DA}}\right] = \int_{p_1^{\mathrm{DA}}}^{p_2^{\mathrm{DA}}} f_\lambda^{\mathrm{DA}}(\ell) \, \mathrm{d}\ell, \tag{3.29}$$

and similarly for the second block, i.e.,

$$\mathbb{P}\left[p_2^{\mathrm{DA}} \leq \lambda^{\mathrm{DA}} < p_3^{\mathrm{DA}}\right] = \int_{p_2^{\mathrm{DA}}}^{p_3^{\mathrm{DA}}} f_\lambda^{\mathrm{DA}}(\ell)\, \mathrm{d}\ell. \tag{3.30}$$

The third block is, in this example, the last one and it is accepted when λ^{DA} is greater or equal to p_3^{DA}, without an upper bound as for the first two blocks. In this case, we replace the upper bound of the integral with ∞, thus obtaining

$$\mathbb{P}\left[p_3^{\mathrm{DA}} \leq \lambda^{\mathrm{DA}}\right] = \int_{p_3^{\mathrm{DA}}}^{\infty} f_\lambda^{\mathrm{DA}}(\ell)\, \mathrm{d}\ell. \tag{3.31}$$

Figure 3.6 shows a graphical interpretation of the acceptance probability of the three blocks as in Equations (3.29), (3.30), and (3.31).

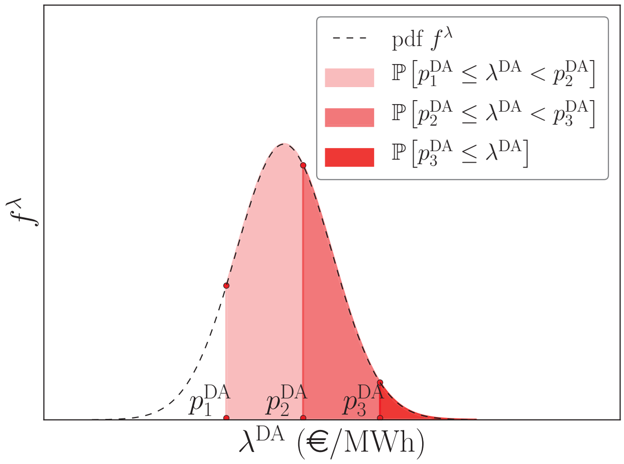


FIGURE 3.6: Graphic representation of the probability of acceptance of the offer curves composed of three blocks.

Let $\hat{\rho}_1^{\mathrm{DA}}$, $\hat{\rho}_2^{\mathrm{DA}}$, and $\hat{\rho}_3^{\mathrm{DA}}$ be the expected revenues associated with the blocks 1, 2, and 3 composing the offer curve, respectively. They can be computed similarly to the case of a single market offer in Equation (3.25), i.e.,

$$\hat{\rho}_1^{\mathrm{DA}} = \overline{q}_1^{\mathrm{DA}} \int_{p_1^{\mathrm{DA}}}^{p_2^{\mathrm{DA}}} \ell\, f_\lambda^{\mathrm{DA}}(\ell)\, \mathrm{d}\ell, \tag{3.32a}$$

$$\hat{\rho}_2^{\mathrm{DA}} = \overline{q}_2^{\mathrm{DA}} \int_{p_2^{\mathrm{DA}}}^{p_3^{\mathrm{DA}}} \ell\, f_\lambda^{\mathrm{DA}}(\ell)\, \mathrm{d}\ell, \tag{3.32b}$$

$$\hat{\rho}_3^{\mathrm{DA}} = \overline{q}_3^{\mathrm{DA}} \int_{p_3^{\mathrm{DA}}}^{\infty} \ell\, f_\lambda^{\mathrm{DA}}(\ell)\, \mathrm{d}\ell. \tag{3.32c}$$

Note that for blocks 1 and 2 we replace the upper bound of the integral (i.e., ∞ for the single market offer) with their associated upper bound p_2^{DA} and p_3^{DA}, respectively.

Then, the day-ahead market expected income $\hat{\rho}^{\mathrm{DA}}$ can be computed as

$$\hat{\rho}^{\mathrm{DA}} = \hat{\rho}_1^{\mathrm{DA}} + \hat{\rho}_2^{\mathrm{DA}} + \hat{\rho}_3^{\mathrm{DA}}. \tag{3.33}$$

The formulation for the offer curve made of three blocks can be generalized to an offer curve with B blocks. Let $(p_b^{\mathrm{DA}}, \overline{q}_b^{\mathrm{DA}})$ be the price-quantity offer block b of the offer curve. The offer curve needs to be non-decreasing, as it is usually a requirement of electricity markets. This condition is imposed by the following constraints:

$$\overline{q}_{b+1}^{\mathrm{DA}} \geq \overline{q}_b^{\mathrm{DA}}, \quad b = 1, ..., B-1, \tag{3.34a}$$

$$p_{b+1}^{\mathrm{DA}} \geq p_b^{\mathrm{DA}}, \quad b = 1, ..., B-1. \tag{3.34b}$$

Then, the expected market revenue of the multiple block offer curve $\{(p_b^{\mathrm{DA}}, \overline{q}_b^{\mathrm{DA}}), b = 1, ..., B\}$ is given by

$$\hat{\rho}^{\mathrm{DA}} = \sum_{b=1}^{B} \overline{q}_b^{\mathrm{DA}} \int_{p_b^{\mathrm{DA}}}^{p_{b+1}^{\mathrm{DA}}} \ell \, f_\lambda^{\mathrm{DA}}(\ell) \, \mathrm{d}\ell, \tag{3.35}$$

where p_{B+1}^{DA}, i.e., the upper integral limit of the last offer block of the curve, is equal to ∞.

When the price-taker producer submits offer curves, the set Π^{DA} in the risk-neutral offering strategy (3.18), is replaced by

$$\hat{\rho}^{\mathrm{DA}} = \sum_{b=1}^{B} \overline{q}_b^{\mathrm{DA}} \int_{p_b^{\mathrm{DA}}}^{p_{b+1}^{\mathrm{DA}}} \ell \, f_\lambda^{\mathrm{DA}}(\ell) \, \mathrm{d}\ell, \tag{3.36a}$$

$$\overline{q}_{b+1}^{\mathrm{DA}} \geq \overline{q}_b^{\mathrm{DA}}, \quad b = 1, ..., B-1 \tag{3.36b}$$

$$p_{b+1}^{\mathrm{DA}} \geq p_b^{\mathrm{DA}}, \quad b = 1, ..., B-1 \tag{3.36c}$$

$$\underline{Q} \leq \overline{q}_b^{\mathrm{DA}} \leq \overline{Q}, \quad b = 1, ..., B \tag{3.36d}$$

Constraint (3.36a) gives the expected day-ahead market income associated with the offer curve $\{(p_b^{\mathrm{DA}}, \overline{q}_b^{\mathrm{DA}}), b = 1, ..., B\}$. Constraints (3.36b) and (3.36c) impose the non-decreasing condition of the offer curve. Finally, constraint (3.36d) limits the day-ahead market offers $\overline{q}_b^{\mathrm{DA}}$ between $\underline{Q}$ and $\overline{Q}$.

3.4.3 Day-Ahead Market Trading Problem *via* Stochastic Programming

The trading problem (3.36) derived in Section 3.4.2 is formulated as a non-linear problem. Since the uncertain market clearing price λ^{DA} is modeled as a random variable with a continuous distribution function (i.e., $f_{\lambda}^{\mathrm{DA}}$), the probability of acceptance of each block of the curve needs to integrate the density function $f_{\lambda}^{\mathrm{DA}}$ over the price domain. Moreover, to evaluate the expected market income, we need to compute the product of quantities and prices, thus resulting in a non-linear model. A stochastic programming approach can provide an alternative linear formulation. As mentioned in Section 3.4, under such approach, the continuous distribution of random variables is replaced by a discrete distribution. Indeed, the uncertain market price λ^{DA} is represented by mean of a discrete set I of possible realizations $\{\lambda_i^{\mathrm{DA}},\ i \in I\}$, where each scenario i is associated with a probability π_i^{DA} of occurrence, such that $\sum_i \pi_i^{\mathrm{DA}} = 1$.

As the possible outcomes of the random variable λ^{DA} belongs to the discrete set I, we consider each price scenario λ_i^{DA} as the potential offer price of the price-taker producer. This simplifies the trading problem, as the offer price of each block of the curve is now a parameter instead of a variable. Moreover, it seems a natural choice as any offer price different than λ_i^{DA} would have a null probability of occurrence. For each potential offer price, we evaluate the optimal quantity $\overline{q}_i^{\mathrm{DA}}$ to be offered in the market, provided that the market price outcome is λ_i^{DA}. The result is a collection of N price-quantity offers, i.e., $(\lambda_i^{\mathrm{DA}}, \overline{q}_i^{\mathrm{DA}})$ that allows building the offer curve of the producer. Similarly to the continuous case, we need to enforce that the offer curve is non-decreasing. The following constraints impose such condition:

$$\overline{q}_i^{\mathrm{DA}} \geq \overline{q}_{i'}^{\mathrm{DA}} \quad \text{if} \quad \lambda_i^{\mathrm{DA}} \geq \lambda_{i'}^{\mathrm{DA}}, \qquad \forall i, i', \tag{3.37a}$$

$$\overline{q}_i^{\mathrm{DA}} = \overline{q}_{i'}^{\mathrm{DA}} \quad \text{if} \quad \lambda_i^{\mathrm{DA}} = \lambda_{i'}^{\mathrm{DA}}, \qquad \forall i, i', \tag{3.37b}$$

where i and i' are both indices of the market price scenarios. Constraint (3.37a) imposes the non-decreasing requirement, while constraint (3.37b) the non-anticipativity one. Indeed, constraint (3.37b) prevents the power producer from offering different quantities at the same market price. Note that this offer curve $\{(\lambda_i^{\mathrm{DA}}, \overline{q}_i^{\mathrm{DA}}), i \in I\}$ is still scenario-independent, i.e., it is adapted to the all set I of scenarios, though it is build based on scenario-dependent price-quantity offers.

Let us initially consider an offer curve composed of three blocks, similar to the one shown in Figure 3.5, though adapted to the new discrete formulation. Let λ_1^{DA}, λ_2^{DA}, and λ_3^{DA} be the three scenarios of λ^{DA}, with π_1^{DA}, π_2^{DA}, and π_3^{DA} their associated probability, respectively. We assume that this set is in growing order, i.e., $\lambda_1^{\mathrm{DA}} \leq \lambda_2^{\mathrm{DA}} \leq \lambda_3^{\mathrm{DA}}$. An example of such offer curve is shown in Figure 3.7. The probability

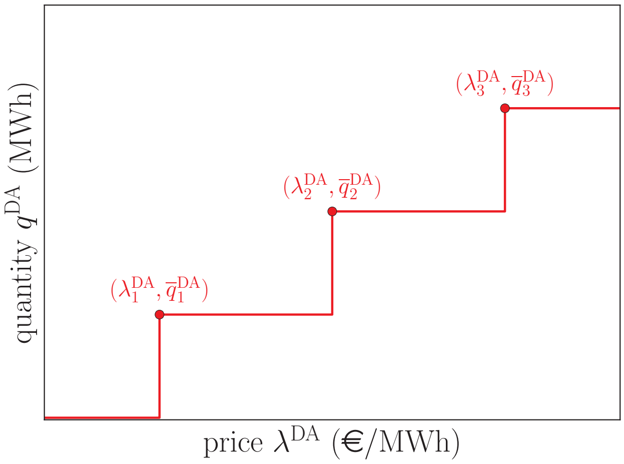


FIGURE 3.7: Example of a 3 blocks offer curve under a stochastic programming approach.

of acceptance of the first block can be computed as

$$\mathbb{P}\left[\lambda_1^{\mathrm{DA}} \leq \lambda^{\mathrm{DA}} < \lambda_2^{\mathrm{DA}}\right] = \mathbb{P}\left[\lambda^{\mathrm{DA}} = \lambda_1^{\mathrm{DA}}\right] = \pi_1^{\mathrm{DA}}. \tag{3.38}$$

Similarly, the probability of acceptance of the second block is

$$\mathbb{P}\left[\lambda_2^{\mathrm{DA}} \leq \lambda^{\mathrm{DA}} < \lambda_3^{\mathrm{DA}}\right] = \mathbb{P}\left[\lambda^{\mathrm{DA}} = \lambda_2^{\mathrm{DA}}\right] = \pi_2^{\mathrm{DA}}, \tag{3.39}$$

and the one of the third block is

$$\mathbb{P}\left[\lambda_3^{\mathrm{DA}} \leq \lambda^{\mathrm{DA}}\right] = \mathbb{P}\left[\lambda^{\mathrm{DA}} = \lambda_3^{\mathrm{DA}}\right] = \pi_3^{\mathrm{DA}}. \tag{3.40}$$

Then, we evaluate the expected revenue of the 3 blocks composing the offer curve (i.e., $\hat{\rho}_1^{\mathrm{DA}}$, $\hat{\rho}_2^{\mathrm{DA}}$, and $\hat{\rho}_3^{\mathrm{DA}}$) as

$$\hat{\rho}_1^{\mathrm{DA}} = \pi_1^{\mathrm{DA}} \, \lambda_1^{\mathrm{DA}} \, \overline{q}_1^{\mathrm{DA}}, \tag{3.41a}$$

$$\hat{\rho}_2^{\mathrm{DA}} = \pi_2^{\mathrm{DA}} \, \lambda_2^{\mathrm{DA}} \, \overline{q}_2^{\mathrm{DA}}, \tag{3.41b}$$

$$\hat{\rho}_3^{\mathrm{DA}} = \pi_3^{\mathrm{DA}} \, \lambda_3^{\mathrm{DA}} \, \overline{q}_3^{\mathrm{DA}}. \tag{3.41c}$$

and the total day-ahead income as

$$\hat{\rho}^{\text{DA}} = \hat{\rho}_1^{\text{DA}} + \hat{\rho}_2^{\text{DA}} + \hat{\rho}_3^{\text{DA}}. \tag{3.42}$$

Let us now generalize to the case with a set I of market price scenarios. In this case, the expected revenue $\hat{\rho}^{\text{DA}}$ is given by

$$\hat{\rho}^{\text{DA}} = \sum_i \pi_i^{\text{DA}} \, \lambda_i^{\text{DA}} \, \overline{q}_i^{\text{DA}}, \tag{3.43}$$

Accordingly, under a stochastic programming approach, the set Π^{DA} in (3.18) is replaced by

$$\hat{\rho}^{\text{DA}} = \sum_i \pi_i^{\text{DA}} \, \lambda_i^{\text{DA}} \, \overline{q}_i^{\text{DA}} \tag{3.44a}$$

$$\overline{q}_i^{\text{DA}} \geq \overline{q}_{i'}^{\text{DA}} \quad \text{if} \quad \lambda_i^{\text{DA}} \geq \lambda_{i'}^{\text{DA}}, \quad \forall i, i', \tag{3.44b}$$

$$\overline{q}_i^{\text{DA}} = \overline{q}_{i'}^{\text{DA}} \quad \text{if} \quad \lambda_i^{\text{DA}} = \lambda_{i'}^{\text{DA}}, \quad \forall i, i', \tag{3.44c}$$

$$\underline{Q} \leq \overline{q}_i^{\text{DA}} \leq \overline{Q}, \quad \forall i \tag{3.44d}$$

Constraint (3.44a) yields the expected day-ahead market income of the power producer. The non-decreasing and non-anticipativity conditions are imposed by constraints (3.44b) and (3.44c), respectively. Lastly, constraint (3.44d) forces the day-ahead market offers $\overline{q}_i^{\text{DA}}$ between $\underline{Q}$ and $\overline{Q}$.

3.4.4 Balancing Market (Active) Trading Problem with Single Offers

We assume that the power producer submits a single price-quantity offer for both upward and downward regulation, i.e., $(p^{\text{UP}}, \overline{q}^{\text{UP}})$ and $(p^{\text{DW}}, \overline{q}^{\text{DW}})$, respectively. This section restricts the balancing market participation to the active one, i.e., the power producer can offer regulating energy but it can not passively deviate from its contracted production schedule. In the general formulation of the balancing market trading problem, i.e., (3.16), the accepted regulation adjustments q^{UP} and q^{DW} are endogenously computed within the offering strategy by constraint (3.16d), as well as the balancing market price λ^{BA}. Differently, the price-taker producer, given the already revealed day-ahead market price λ^{DA}, considers the balancing market price λ^{BA} as a random variable with marginal distribution $f_\lambda^{\text{BA}} : \mathbb{R} \mapsto \mathbb{R}^+$. Then, it evaluates the accepted regulation adjustments q^{UP} and q^{DW} as

$$q^{\mathrm{UP}} = \begin{cases} 0, & \text{if} \quad \lambda^{\mathrm{BA}} < p^{\mathrm{UP}}, \\ \overline{q}^{\mathrm{UP}}, & \text{if} \quad \lambda^{\mathrm{BA}} \geq p^{\mathrm{UP}}. \end{cases} \tag{3.45}$$

$$q^{\mathrm{DW}} = \begin{cases} 0, & \text{if} \quad \lambda^{\mathrm{BA}} > p^{\mathrm{DW}}, \\ \overline{q}^{\mathrm{DW}}, & \text{if} \quad \lambda^{\mathrm{BA}} \leq p^{\mathrm{DW}}. \end{cases} \tag{3.46}$$

In Equation (3.46) the merit order of q^{DW} is inverse compared to q^{UP}. Indeed, it is more convenient for the System Operator to schedule down-regulation energy offered at higher price p^{DW}, as it is going to receive a payment of $p^{\mathrm{DW}} q^{\mathrm{DW}}$ from the producer. Figure 3.8 shows the accepted quantities q^{UP} and q^{DW}, given the single offers $(p^{\mathrm{UP}}, \overline{q}^{\mathrm{UP}})$ and $(p^{\mathrm{DW}}, \overline{q}^{\mathrm{DW}})$. In order to show that the down-regulation is actually a decrease of production, Figure 3.8 illustrates $-q^{\mathrm{DW}}$ instead of q^{DW}. Then, in

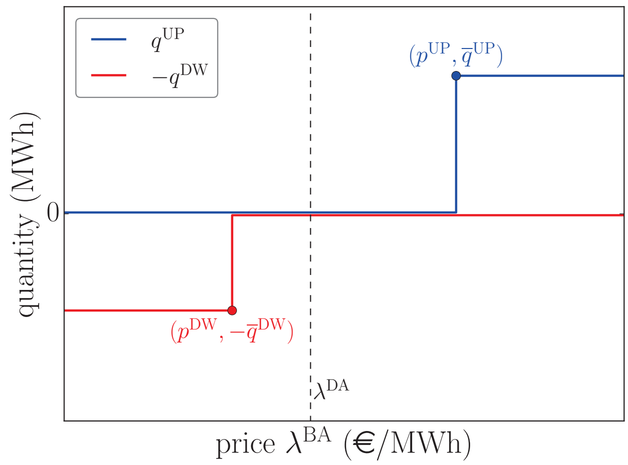


FIGURE 3.8: Example of single offers $(p^{\mathrm{UP}}, \overline{q}^{\mathrm{UP}})$ and $(p^{\mathrm{DW}}, \overline{q}^{\mathrm{DW}})$ in the balancing market.

accordance with Equation (3.45), the probability of acceptance of the up-regulation offer is evaluated as

$$\mathbb{P}\left[p^{\mathrm{UP}} \leq \lambda^{\mathrm{BA}}\right] = \int_{p^{\mathrm{UP}}}^{\infty} f_{\lambda}^{\mathrm{BA}}(\ell)\, \mathrm{d}\ell, \tag{3.47}$$

since $(p^{\mathrm{UP}}, \overline{q}^{\mathrm{UP}})$ is going to be accepted if λ^{BA} is greater or equal the up-regulation offer price p^{UP}. Following Equation (3.46), the acceptance probability of the down-regulation offer is given by

$$\mathbb{P}\left[p^{\mathrm{DW}} \geq \lambda^{\mathrm{BA}}\right] = \int_{-\infty}^{p^{\mathrm{DW}}} f_{\lambda}^{\mathrm{BA}}(\ell)\, \mathrm{d}\ell. \tag{3.48}$$

Indeed, the down-regulation offer $(p^{\mathrm{DW}}, \overline{q}^{\mathrm{DW}})$ is accepted if λ^{BA} is lower or equal to p^{DW}. A graphical interpretation of the probability of acceptance of the price-quantity offers in the balancing market is shown in Figure (3.9). The acceptance probability of the up-regulation offer is the blue area, while the one of the down-regulation offer is the red area. Let $p^{\mathrm{UP}*}$ be the remuneration price provided that the up-regulation

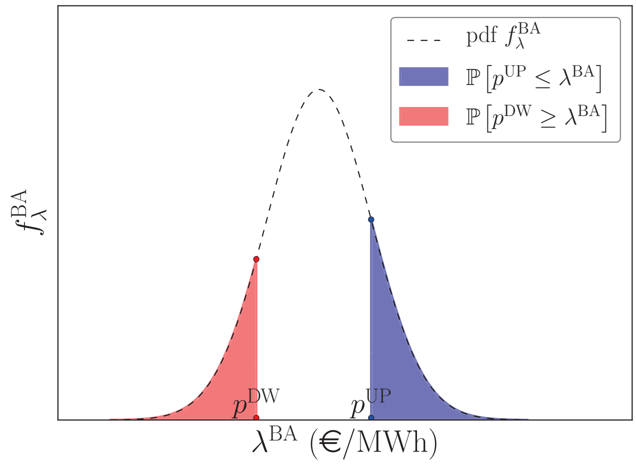


FIGURE 3.9: Graphic representation of the offer probability acceptance of the up-regulation offer (blue) and the down-regulation one (red).

offer is being accepted by the System Operator. Its expected value is computed as

$$\mathbb{E}\left[p^{\mathrm{UP}*} \mid \lambda^{\mathrm{BA}} \geq p^{\mathrm{UP}}\right] = \begin{cases} \dfrac{\int_{p^{\mathrm{UP}}}^{\infty} \ell \, f_\lambda^{\mathrm{BA}}(\ell) \, \mathrm{d}\ell}{\int_{p^{\mathrm{UP}}}^{\infty} f_\lambda^{\mathrm{BA}}(\ell) \, \mathrm{d}\ell}, & \text{if uniform pricing,} \\ p^{\mathrm{UP}}, & \text{if pay-as-bid pricing,} \end{cases} \tag{3.49}$$

Similarly, being $p^{\mathrm{DW}*}$ the remuneration price given that the down-regulation offer is accepted, its expected value is evaluated as

$$\mathbb{E}\left[p^{\mathrm{DW}*} \mid \lambda^{\mathrm{BA}} \leq p^{\mathrm{DW}}\right] = \begin{cases} \dfrac{\int_{-\infty}^{p^{\mathrm{DW}}} \ell \, f_\lambda^{\mathrm{BA}}(\ell) \, \mathrm{d}\ell}{\int_{-\infty}^{p^{\mathrm{DW}}} f_\lambda^{\mathrm{BA}}(\ell) \, \mathrm{d}\ell}, & \text{if uniform pricing,} \\ p^{\mathrm{DW}}, & \text{if pay-as-bid pricing.} \end{cases} \tag{3.50}$$

Then, the expected revenues $\hat{\rho}^{\mathrm{UP}}$ and $\hat{\rho}^{\mathrm{DW}}$ are computed as

$$\hat{\rho}^{\mathrm{UP}} = \begin{cases} \overline{q}^{\mathrm{UP}} \int_{p^{\mathrm{UP}}}^{\infty} \ell \, f_{\lambda}^{\mathrm{BA}}(\ell) \, \mathrm{d}\ell, & \text{if uniform pricing,} \\ \overline{q}^{\mathrm{UP}} \, p^{\mathrm{UP}} \int_{p^{\mathrm{UP}}}^{\infty} f_{\lambda}^{\mathrm{BA}}(\ell) \, \mathrm{d}\ell, & \text{if pay-as-bid pricing,} \end{cases} \tag{3.51a}$$

$$\hat{\rho}^{\mathrm{DW}} = \begin{cases} -\overline{q}^{\mathrm{DW}} \int_{-\infty}^{p^{\mathrm{DW}}} \ell \, f_{\lambda}^{\mathrm{BA}}(\ell) \, \mathrm{d}\ell, & \text{if uniform pricing,} \\ -\overline{q}^{\mathrm{DW}} \, p^{\mathrm{DW}} \int_{-\infty}^{p^{\mathrm{DW}}} f_{\lambda}^{\mathrm{BA}}(\ell) \, \mathrm{d}\ell, & \text{if pay-as-bid pricing,} \end{cases} \tag{3.51b}$$

The set $\Pi_{\mathrm{Act}}^{\mathrm{BA}}$ in the risk-neutral offer strategy (3.18), can be replaced by the following set of constraints:

$$\hat{\rho}^{\mathrm{UP}} = \begin{cases} \overline{q}^{\mathrm{UP}} \int_{p^{\mathrm{UP}}}^{\infty} \ell \, f_{\lambda}^{\mathrm{BA}}(\ell) \, \mathrm{d}\ell, & \text{if uniform pricing,} \\ \overline{q}^{\mathrm{UP}} \, p^{\mathrm{UP}} \int_{p^{\mathrm{UP}}}^{\infty} f_{\lambda}^{\mathrm{BA}}(\ell) \, \mathrm{d}\ell, & \text{if pay-as-bid pricing,} \end{cases} \tag{3.52a}$$

$$\hat{\rho}^{\mathrm{DW}} = \begin{cases} -\overline{q}^{\mathrm{DW}} \int_{-\infty}^{p^{\mathrm{DW}}} \ell \, f_{\lambda}^{\mathrm{BA}}(\ell) \, \mathrm{d}\ell, & \text{if uniform pricing,} \\ -\overline{q}^{\mathrm{DW}} \, p^{\mathrm{DW}} \int_{-\infty}^{p^{\mathrm{DW}}} f_{\lambda}^{\mathrm{BA}}(\ell) \, \mathrm{d}\ell, & \text{if pay-as-bid pricing,} \end{cases} \tag{3.52b}$$

$$\overline{q}^{\mathrm{UP}}, \overline{q}^{\mathrm{DW}} \geq 0. \tag{3.52c}$$

Constraint (3.52a) yields the expected market income from offering up-regulation energy. Constraint (3.52b) does the same for the down-regulation market income. Note that (3.52a) and (3.52b) give the formulation for both an uniform and a pay-as-bid pricing scheme. Finally, constraint (3.52c) imposes that the quantities $\overline{q}^{\mathrm{UP}}$ and $\overline{q}^{\mathrm{DW}}$ are non-negative.

3.4.5 Balancing Market (Active) Trading Problem with Offer Curves

The possibility of submitting a market offer through non-decreasing step-wise curves is usually available even in the balancing market. We first develop the balancing market trading problem with offer curves for a balancing market settled under a uniform pricing scheme. Subsequently, we assume a pay-as-bid balancing market.

Uniform Pricing Scheme

Figure 3.10 shows an example of offer curves for upward (blue) and downward (red) regulation in the balancing market, which is considered settled under a uniform pricing scheme. The probability of acceptance of the first block $(p_1^{\mathrm{UP}}, \overline{q}_1^{\mathrm{UP}})$ of the

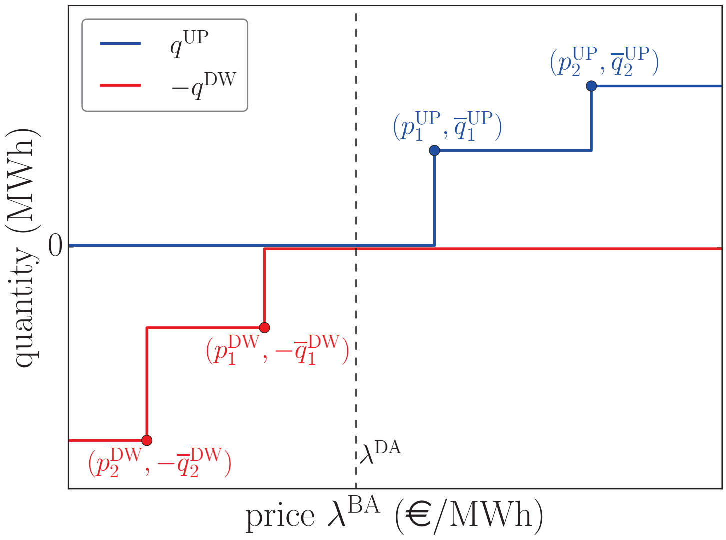


FIGURE 3.10: Example of a 2 blocks offer curve for both upward (blue) and downward (red) regulation in the balancing market.

up-regulation offer curve is computed as

$$\mathbb{P}\left[p_1^{\mathrm{UP}} \leq \lambda^{\mathrm{BA}} < p_2^{\mathrm{UP}}\right] = \int_{p_1^{\mathrm{UP}}}^{p_2^{\mathrm{UP}}} f_\lambda^{\mathrm{BA}}(\ell)\, \mathrm{d}\ell. \tag{3.53}$$

Similarly, the one of the second (and last) block $(p_2^{\mathrm{UP}}, \overline{q}_2^{\mathrm{UP}})$ is given by

$$\mathbb{P}\left[p_2^{\mathrm{UP}} \leq \lambda^{\mathrm{BA}}\right] = \int_{p_2^{\mathrm{UP}}}^{\infty} f_\lambda^{\mathrm{BA}}(\ell)\, \mathrm{d}\ell, \tag{3.54}$$

Conversely, the down-regulation offer curves are non-increasing if the quantity q^{DW} is modeled as a positive variable. Note that the downward regulation offer curve in Figure 3.10 is non-decreasing as it shows $-q^{\mathrm{DW}}$ instead of q^{DW}. In this case, the probability of acceptance of the first block $(p_1^{\mathrm{DW}}, \overline{q}_1^{\mathrm{DW}})$ of the down-regulation offer curve is computed as

$$\mathbb{P}\left[p_1^{\mathrm{DW}} \geq \lambda^{\mathrm{BA}} > p_2^{\mathrm{DW}}\right] = \int_{p_2^{\mathrm{DW}}}^{p_1^{\mathrm{DW}}} f_\lambda^{\mathrm{BA}}(\ell)\, \mathrm{d}\ell, \tag{3.55}$$

while the one of the second (and last) block $(p_2^{\mathrm{UP}}, \overline{q}_2^{\mathrm{UP}})$ as

$$\mathbb{P}\left[p_2^{\mathrm{DW}} \geq \lambda^{\mathrm{BA}}\right] = \int_{-\infty}^{p_2^{\mathrm{DW}}} f_\lambda^{\mathrm{BA}}(\ell)\, \mathrm{d}\ell. \tag{3.56}$$

Figure 3.11 shows a graphical interpretation of the acceptance probability of the upward (blue) and downward (red) regulation offer curves illustrated in Figure 3.10.

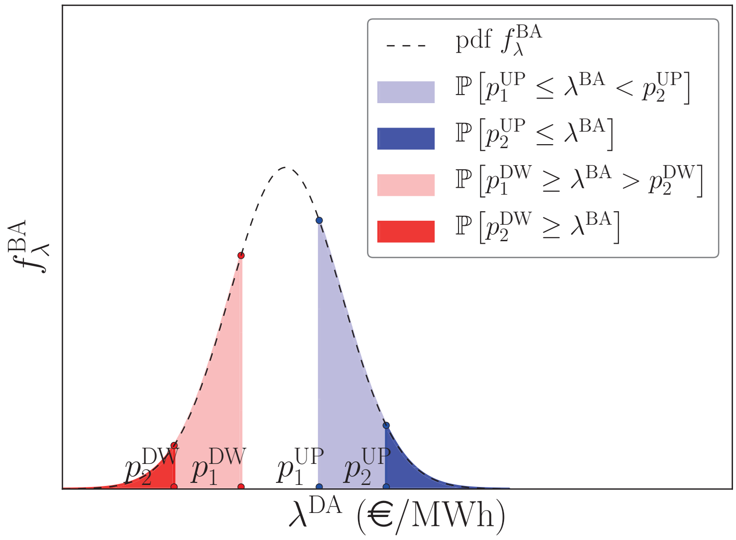


FIGURE 3.11: Graphic representation of the probability acceptance of the two blocks composing the offer curve for upward (blue) and downward (red) regulation.

Let us now consider an offer curve composed of B blocks, for both up- and down-regulation. For the up-regulation offer curve, the non-decreasing condition is enforced by the following constraints:

$$\overline{q}_{b+1}^{UP} \geq \overline{q}_b^{UP}, \quad b = 1, ..., B-1, \tag{3.57a}$$

$$p_{b+1}^{UP} \geq p_b^{UP}, \quad b = 1, ..., B-1, \tag{3.57b}$$

while the non-increasing condition of the down-regulation offer curve as

$$\overline{q}_{b+1}^{DW} \geq \overline{q}_b^{DW}, \quad b = 1, ..., B-1, \tag{3.58a}$$

$$p_{b+1}^{DW} \leq p_b^{DW}, \quad b = 1, ..., B-1. \tag{3.58b}$$

Then, the expected market income $\hat{\rho}^{UP}$ associated with the up-regulation offer curve $\{(p_b^{UP}, \overline{q}_b^{UP}), b = 1, ..., B\}$ is given by

$$\hat{\rho}^{UP} = \sum_{b=1}^{B} \overline{q}_b^{UP} \int_{p_b^{UP}}^{p_{b+1}^{UP}} \ell \, f_\lambda^{BA}(\ell) \, d\ell, \tag{3.59}$$

where p_{B+1}^{UP}, i.e., the upper integral limit of the last block of the curve, is equal to ∞. Similarly, the expected market revenue $\hat{\rho}^{DW}$ of the down-regulation offer curve

$\{(p_b^{\text{DW}}, \overline{q}_b^{\text{DW}}), b = 1, ..., B\}$ is evaluated as

$$\hat{\rho}^{\text{DW}} = \sum_{b=1}^{B} \overline{q}_b^{\text{DW}} \int_{p_{b+1}^{\text{DW}}}^{p_b^{\text{DW}}} \ell \, f_\lambda^{\text{BA}}(\ell) \, \mathrm{d}\ell, \tag{3.60}$$

where p_{B+1}^{DW}, i.e., the lower integral limit of the last block of the curve, is equal to $-\infty$.

When the price-taker producer submits offer curves in the balancing market (under a uniform pricing scheme), the set $\Pi_{\text{Act}}^{\text{BA}}$ in the risk-neutral offer strategy (3.18) is replaced by the following set of constraints:

$$\hat{\rho}^{\text{UP}} = \sum_{b=1}^{B} \overline{q}_b^{\text{UP}} \int_{p_b^{\text{UP}}}^{p_{b+1}^{\text{UP}}} \ell \, f_\lambda^{\text{BA}}(\ell) \, \mathrm{d}\ell, \tag{3.61a}$$

$$\hat{\rho}^{\text{DW}} = \sum_{b=1}^{B} \overline{q}_b^{\text{DW}} \int_{p_{b+1}^{\text{DW}}}^{p_b^{\text{DW}}} \ell \, f_\lambda^{\text{BA}}(\ell) \, \mathrm{d}\ell, \tag{3.61b}$$

$$\overline{q}_{b+1}^{\text{UP}} \geq \overline{q}_b^{\text{UP}}, \quad b = 1, ..., B-1, \tag{3.61c}$$

$$p_{b+1}^{\text{UP}} \geq p_b^{\text{UP}}, \quad b = 1, ..., B-1, \tag{3.61d}$$

$$\overline{q}_{b+1}^{\text{DW}} \geq \overline{q}_b^{\text{DW}}, \quad b = 1, ..., B-1, \tag{3.61e}$$

$$p_{b+1}^{\text{DW}} \leq p_b^{\text{DW}}, \quad b = 1, ..., B-1, \tag{3.61f}$$

$$\overline{q}_b^{\text{UP}}, \overline{q}_b^{\text{DW}} \geq 0 \quad b = 1, ..., B. \tag{3.61g}$$

Constraints (3.61a) and (3.61b) compute the expected market incomes $\hat{\rho}^{\text{UP}}$ and $\hat{\rho}^{\text{DW}}$ under a uniform pricing scheme. Constraints (3.61c) and (3.61d) enforce the non-decreasing condition of the up-regulation offer curve. Similarly, constraints (3.61e) and (3.61f) impose the non-increasing requirement of the down-regulation offer curve. Finally, constraint (3.61g) imposes that the offer quantities $\overline{q}_b^{\text{UP}}$ and $\overline{q}_b^{\text{DW}}$ are non-negative.

Pay-as-Bid Pricing Scheme

Figure 3.12 provides an example of offer curves for upward (blue) and downward (red) regulation in the balancing market, which is considered settled under a pay-as-bid pricing scheme. To model the pay-as-bid pricing scheme, we introduce the additional variables o_1^{UP} and o_2^{UP} for the up-regulation offer curve, and o_1^{DW} and o_2^{DW} for the down-regulation curve. They represent the additional amount of energy offered provided that their block offer is being accepted. E.g., the first block of the

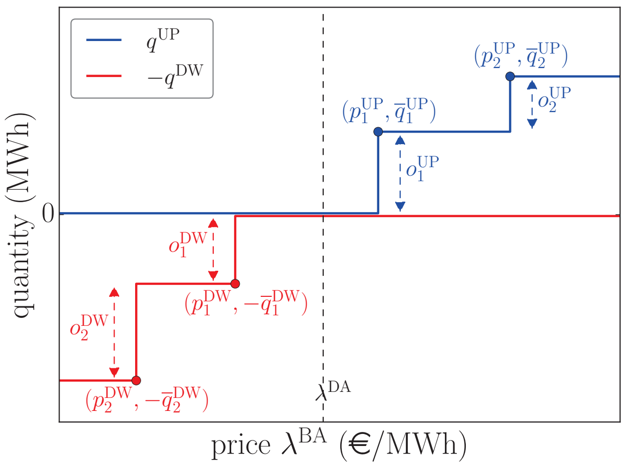


FIGURE 3.12: Example of a 2 blocks offer curve for both upward (blue) and downward (red) regulation in the balancing market.

up-regulation offer is $(p_1^{\mathrm{UP}}, \bar{q}_1^{\mathrm{UP}})$ and

$$\bar{q}_1^{\mathrm{UP}} = o_1^{\mathrm{UP}}. \tag{3.62}$$

The second block of the offer is $(p_2^{\mathrm{UP}}, \bar{q}_2^{\mathrm{UP}})$ and, if accepted, the producer is contracted to produce $\bar{q}_2^{\mathrm{UP}}$. We split this amount between o_1^{UP}, i.e., the size of the first block offered at p_1^{UP}, and o_2^{UP}, i.e., the additional energy sold at p_2^{UP}, as those quantities would be priced differently by the System Operator. Consequently, $\bar{q}_2^{\mathrm{UP}}$ is obtained as

$$\bar{q}_2^{\mathrm{UP}} = o_1^{\mathrm{UP}} + o_2^{\mathrm{UP}}, \tag{3.63}$$

i.e., it is computed as the sum of the sizes of the two blocks. Similarly, for the down-regulation offer curve $\{(p_1^{\mathrm{DW}}, \bar{q}_1^{\mathrm{DW}}), (p_2^{\mathrm{DW}}, \bar{q}_2^{\mathrm{DW}})\}$, the following equalities hold:

$$\bar{q}_1^{\mathrm{DW}} = o_1^{\mathrm{DW}}, \tag{3.64}$$

$$\bar{q}_2^{\mathrm{DW}} = o_1^{\mathrm{DW}} + o_2^{\mathrm{DW}}. \tag{3.65}$$

Aiming at computing the expected revenues associated with the offer curves, it is convenient to work with the incremental sizes of the offer blocks, i.e., $\{(p_1^{\mathrm{UP}}, o_1^{\mathrm{UP}}), (p_2^{\mathrm{UP}}, o_2^{\mathrm{UP}})\}$ and $\{(p_1^{\mathrm{DW}}, o_1^{\mathrm{DW}}), (p_2^{\mathrm{DW}}, o_2^{\mathrm{DW}})\}$. The probability of acceptance of the first step $(p_1^{\mathrm{UP}}, o_1^{\mathrm{UP}})$ can be computed as

$$\mathbb{P}\left[p_1^{\mathrm{UP}} \leq \lambda^{\mathrm{BA}}\right] = \int_{p_1^{\mathrm{UP}}}^{\infty} f_\lambda^{\mathrm{BA}}(\ell)\, \mathrm{d}\ell, \tag{3.66}$$

as the quantity o_1^{UP} is scheduled also when the second block offer $(p_2^{\mathrm{UP}}, \overline{q}_2^{\mathrm{UP}})$ is accepted. Then, the probability of acceptance of the second step $(p_2^{\mathrm{UP}}, o_2^{\mathrm{UP}})$ is given by

$$\mathbb{P}\big[p_2^{\mathrm{UP}} \leq \lambda^{\mathrm{BA}}\big] = \int_{p_2^{\mathrm{UP}}}^{\infty} f_\lambda^{\mathrm{BA}}(\ell)\, \mathrm{d}\ell. \tag{3.67}$$

Similarly, for the down-regulation offer curve, the probability of acceptance of the first step $(p_1^{\mathrm{DW}}, o_1^{\mathrm{DW}})$ is evaluated as

$$\mathbb{P}\big[p_1^{\mathrm{DW}} \geq \lambda^{\mathrm{BA}}\big] = \int_{-\infty}^{p_1^{\mathrm{DW}}} f_\lambda^{\mathrm{BA}}(\ell)\, \mathrm{d}\ell, \tag{3.68}$$

while the one of the second step $(p_2^{\mathrm{UP}}, o_2^{\mathrm{UP}})$ as

$$\mathbb{P}\big[p_2^{\mathrm{DW}} \geq \lambda^{\mathrm{BA}}\big] = \int_{-\infty}^{p_2^{\mathrm{DW}}} f_\lambda^{\mathrm{BA}}(\ell)\, \mathrm{d}\ell, \tag{3.69}$$

The expected up-regulation income $\hat{\rho}^{\mathrm{UP}}$ is accordingly given by

$$\hat{\rho}^{\mathrm{UP}} = o_1^{\mathrm{UP}}\, p_1^{\mathrm{UP}} \int_{p_1^{\mathrm{UP}}}^{\infty} f_\lambda^{\mathrm{BA}}(\ell)\, \mathrm{d}\ell \;+\; o_2^{\mathrm{UP}}\, p_2^{\mathrm{UP}} \int_{p_2^{\mathrm{UP}}}^{\infty} f_\lambda^{\mathrm{BA}}(\ell)\, \mathrm{d}\ell, \tag{3.70}$$

while the down-regulation one, i.e., $\hat{\rho}^{\mathrm{DW}}$, by

$$\hat{\rho}^{\mathrm{DW}} = -o_1^{\mathrm{DW}}\, p_1^{\mathrm{DW}} \int_{-\infty}^{p_1^{\mathrm{DW}}} f_\lambda^{\mathrm{BA}}(\ell)\, \mathrm{d}\ell \;-\; o_2^{\mathrm{DW}}\, p_2^{\mathrm{DW}} \int_{-\infty}^{p_2^{\mathrm{DW}}} f_\lambda^{\mathrm{BA}}(\ell)\, \mathrm{d}\ell. \tag{3.71}$$

Then, extending the formulation to offer curves with B blocks, where $(p_b^{\mathrm{UP}}, o_b^{\mathrm{UP}})$ and $(p_b^{\mathrm{DW}}, o_b^{\mathrm{DW}})$ are the steps b of the two offer curves. The quantities $\overline{q}_b^{\mathrm{UP}}$ and $\overline{q}_b^{\mathrm{DW}}$ are evaluates as

$$\overline{q}_b^{\mathrm{UP}} = \sum_{b'=1}^{b} o_{b'}^{\mathrm{UP}}, \tag{3.72a}$$

$$\overline{q}_b^{\mathrm{DW}} = \sum_{b'=1}^{b} o_{b'}^{\mathrm{DW}}, \tag{3.72b}$$

where both b and b' are indices of the block offers. The non-decreasing and non-increasing requirements of the upward and downward regulation offer curves, are enforced through the following constraints:

$$p_{b+1}^{\mathrm{UP}} \geq p_b^{\mathrm{UP}}, \quad b = 1, ..., B-1, \tag{3.73a}$$

$$p_{b+1}^{\mathrm{DW}} \leq p_b^{\mathrm{DW}}, \quad b = 1, ..., B-1. \tag{3.73b}$$

In this case, it is necessary to enforce the non-decreasing (or non-increasing) condition only on the prices, as for the quantities it is inherently enforced in the way $\overline{q}_b^{\mathrm{UP}}$ and $\overline{q}_b^{\mathrm{DW}}$ are computed, i.e., Equations (3.72a) and (3.72b). The expected revenues $\hat{\rho}^{\mathrm{UP}}$ and $\hat{\rho}^{\mathrm{DW}}$ are computed as

$$\hat{\rho}^{\mathrm{UP}} = \sum_{b=1}^{B} o_b^{\mathrm{UP}} \; p_b^{\mathrm{UP}} \int_{p_b^{\mathrm{UP}}}^{\infty} f_\lambda^{\mathrm{BA}}(\ell) \; \mathrm{d}\ell, \tag{3.74a}$$

$$\hat{\rho}^{\mathrm{DW}} = -\sum_{b=1}^{B} o_b^{\mathrm{DW}} \; p_b^{\mathrm{DW}} \int_{-\infty}^{p_b^{\mathrm{DW}}} f_\lambda^{\mathrm{BA}}(\ell) \; \mathrm{d}\ell, \tag{3.74b}$$

When the price-taker producer submits offer curves in the balancing market (under a pay-a-bid pricing scheme), the set $\Pi_{\mathrm{Act}}^{\mathrm{BA}}$ in the risk-neutral offer strategy (3.18) is formulated

$$\hat{\rho}^{\mathrm{UP}} = \sum_{b=1}^{B} o_b^{\mathrm{UP}} \; p_b^{\mathrm{UP}} \int_{p_b^{\mathrm{UP}}}^{\infty} f_\lambda^{\mathrm{BA}}(\ell) \; \mathrm{d}\ell, \tag{3.75a}$$

$$\hat{\rho}^{\mathrm{DW}} = -\sum_{b=1}^{B} o_b^{\mathrm{DW}} \; p_b^{\mathrm{DW}} \int_{-\infty}^{p_b^{\mathrm{DW}}} f_\lambda^{\mathrm{BA}}(\ell) \; \mathrm{d}\ell, \tag{3.75b}$$

$$\overline{q}_b^{\mathrm{UP}} = \sum_{b'=1}^{b} o_{b'}^{\mathrm{UP}}, \quad b = 1, ..., B \tag{3.75c}$$

$$\overline{q}_b^{\mathrm{DW}} = \sum_{b'=1}^{b} o_{b'}^{\mathrm{DW}}, \quad b = 1, ..., B \tag{3.75d}$$

$$p_{b+1}^{\mathrm{UP}} \geq p_b^{\mathrm{UP}}, \quad b = 1, ..., B-1, \tag{3.75e}$$

$$p_{b+1}^{\mathrm{DW}} \leq p_b^{\mathrm{DW}}, \quad b = 1, ..., B-1, \tag{3.75f}$$

$$o_b^{\mathrm{UP}}, o_b^{\mathrm{DW}} \geq 0, \quad b = 1, ..., B \tag{3.75g}$$

Constraints (3.75a) and (3.75b) yield the expected market incomes $\hat{\rho}^{\mathrm{UP}}$ and $\hat{\rho}^{\mathrm{DW}}$ given the up- and down-regulation offer curve in a balancing market settled under a pay-as-bid pricing scheme. Constraint (3.75c) computes the total up-regulation quantity $\overline{q}_b^{\mathrm{UP}}$ scheduled when the block offer b is accepted. Similarly, constraint (3.75d) computes the down-regulation quantity $\overline{q}_b^{\mathrm{DW}}$ contracted when the block offer

b is accepted. Constraints (3.75e) and (3.75f) enforce the non-decreasing and non-increasing requirement of the up- and down-regulation offer curves, respectively. Finally, constraint (3.75g) forces o_b^{UP} and o_b^{DW} to be positive.

3.4.6 Balancing Market (Active) Trading Problem *via* Stochastic Programming

The trading problems (3.61) and (3.75) derived in Section 3.4.5 are formulated as a non-linear problem. An alternative linear formulation can be obtained through a stochastic programming approach. For a given realization i of the day the day-ahead market price, i.e., λ_i^{DA}, the uncertain balancing market price λ_i^{BA} is represented using a discrete set J possible scenarios $\{\lambda_{ij}^{\text{BA}},\ j \in J\}$, where each scenario j is associated with a probability π_{ij}^{BA}, such that $\sum_j \pi_{ij}^{\text{BA}} = 1$. We first develop the balancing market trading problem using a stochastic programming approach for a balancing market settled under a uniform pricing scheme. Subsequently, we consider a pay-as-bid balancing market.

Uniform Pricing Scheme

Figure 3.13 shows an example of offer curves for upward (blue) and downward (red) regulation in a balancing market settled under a uniform pricing scheme. We con-

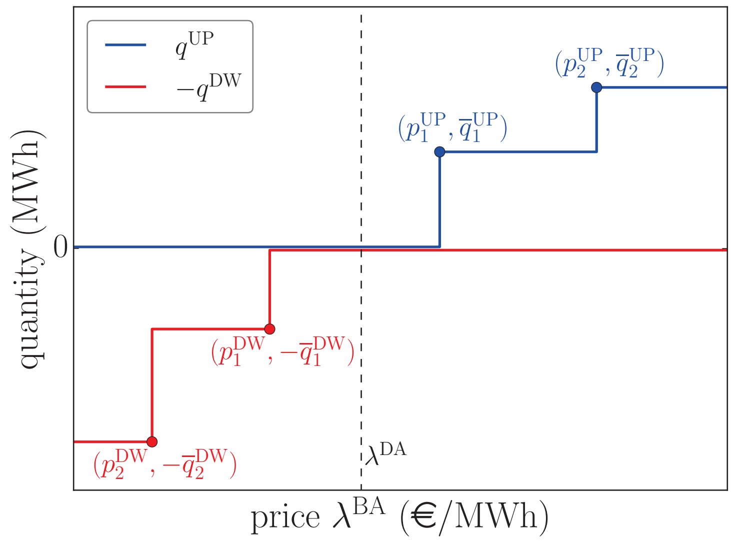


FIGURE 3.13: Example of a 2 blocks offer curve for both upward (blue) and downward (red) regulation in the balancing market (uniform pricing scheme).

sider each price scenario λ_{ij}^{BA} as a potential offer price of the price-taker producer, for both upward and downward regulation. The result is a collection of price-quantity offers, i.e., $\{(\lambda_{ij}^{\mathrm{BA}}, \overline{q}_{ij}^{\mathrm{UP}}),\ j \in J\}$ and $\{(\lambda_{ij}^{\mathrm{BA}}, \overline{q}_{ij}^{\mathrm{DW}}),\ j \in J\}$, that allows to build the up-regulation and down-regulation offer curves to be submitted in the balancing market. We impose the non-decreasing requirement of the up-regulation offer curve as

$$\overline{q}_{ij}^{\mathrm{UP}} \geq \overline{q}_{ij'}^{\mathrm{UP}} \quad \text{if} \quad \lambda_{ij}^{\mathrm{BA}} \geq \lambda_{ij'}^{\mathrm{BA}}, \quad \forall j, j', \tag{3.76a}$$

$$\overline{q}_{ij}^{\mathrm{UP}} = \overline{q}_{ij'}^{\mathrm{UP}} \quad \text{if} \quad \lambda_{ij}^{\mathrm{BA}} = \lambda_{ij'}^{\mathrm{BA}}, \quad \forall j, j', \tag{3.76b}$$

where j and j' are both indices of the balancing market price scenarios. Similarly, the non-increasing condition of the down-regulation curve is enforced by

$$\overline{q}_{ij}^{\mathrm{DW}} \leq \overline{q}_{ij'}^{\mathrm{DW}} \quad \text{if} \quad \lambda_{ij}^{\mathrm{BA}} \geq \lambda_{ij'}^{\mathrm{BA}}, \quad \forall j, j', \tag{3.77a}$$

$$\overline{q}_{ij}^{\mathrm{DW}} = \overline{q}_{ij'}^{\mathrm{DW}} \quad \text{if} \quad \lambda_{ij}^{\mathrm{BA}} = \lambda_{ij'}^{\mathrm{BA}}, \quad \forall j, j'. \tag{3.77b}$$

Then, we impose that the producer contracts its up-regulation offers when the system needs upward regulation. An indicator of the status of the system imbalance is the market price scenario $\lambda_{ij}^{\mathrm{BA}}$ compared to the day-ahead market price λ_i^{DA} (already revealed at the balancing stage), i.e.,

if $\lambda_{ij}^{\mathrm{BA}} > \lambda_i^{\mathrm{DA}}$ up-regulation required when scenario j realizes

if $\lambda_{ij}^{\mathrm{BA}} = \lambda_i^{\mathrm{DA}}$ no regulation required when scenario j realizes

if $\lambda_{ij}^{\mathrm{BA}} < \lambda_i^{\mathrm{DA}}$ down-regulation required when scenario j realizes

As a consequence, we impose that the variables $\overline{q}_{ij}^{\mathrm{UP}}$ and $\overline{q}_{ij}^{\mathrm{DW}}$ are null when the system does not require up-regulation and down-regulation, respectively. This writes

$$\overline{q}_{ij}^{\mathrm{UP}} = 0 \quad \text{if} \quad \lambda_{ij}^{\mathrm{BA}} \leq \lambda_i^{\mathrm{DA}},\ \forall j, \tag{3.79a}$$

$$\overline{q}_{ij}^{\mathrm{DW}} = 0 \quad \text{if} \quad \lambda_{ij}^{\mathrm{BA}} \geq \lambda_i^{\mathrm{DA}},\ \forall j. \tag{3.79b}$$

The expected revenues $\hat{\rho}^{\text{UP}}$ and $\hat{\rho}^{\text{DW}}$, based on the stochastic programming approach, are evaluated as

$$\hat{\rho}_i^{\text{UP}} = \sum_j \pi_{ji}^{\text{BA}} \lambda_{ij}^{\text{BA}} \overline{q}_{ij}^{\text{UP}}, \tag{3.80a}$$

$$\hat{\rho}_i^{\text{DW}} = -\sum_j \pi_{ij}^{\text{BA}} \lambda_{ij}^{\text{BA}} \overline{q}_{ij}^{\text{DW}}, \tag{3.80b}$$

Accordingly, under a stochastic programming approach, the set $\Pi_{\text{Act}}^{\text{BA}}$ in (3.18), for a balancing market settled under a uniform pricing scheme, is given by the following set of constraints:

$$\hat{\rho}_i^{\text{UP}} = \sum_j \pi_{ji}^{\text{BA}} \lambda_{ij}^{\text{BA}} \overline{q}_{ij}^{\text{UP}}, \tag{3.81a}$$

$$\hat{\rho}_i^{\text{DW}} = -\sum_j \pi_{ij}^{\text{BA}} \lambda_{ij}^{\text{BA}} \overline{q}_{ij}^{\text{DW}}, \tag{3.81b}$$

$$\overline{q}_{ij}^{\text{UP}} \geq \overline{q}_{ij'}^{\text{UP}} \quad \text{if} \quad \lambda_{ij}^{\text{BA}} \geq \lambda_{ij'}^{\text{BA}}, \quad \forall j, j', \tag{3.81c}$$

$$\overline{q}_{ij}^{\text{UP}} = \overline{q}_{ij'}^{\text{UP}} \quad \text{if} \quad \lambda_{ij}^{\text{BA}} = \lambda_{ij'}^{\text{BA}}, \quad \forall j, j', \tag{3.81d}$$

$$\overline{q}_{ij}^{\text{DW}} \leq \overline{q}_{ij'}^{\text{DW}} \quad \text{if} \quad \lambda_{ij}^{\text{BA}} \geq \lambda_{ij'}^{\text{BA}}, \quad \forall j, j', \tag{3.81e}$$

$$\overline{q}_{ij}^{\text{DW}} = \overline{q}_{ij'}^{\text{DW}} \quad \text{if} \quad \lambda_{ij}^{\text{BA}} = \lambda_{ij'}^{\text{BA}}, \quad \forall j, j', \tag{3.81f}$$

$$\overline{q}_{ij}^{\text{UP}} = 0 \quad \text{if} \quad \lambda_{ij}^{\text{BA}} \leq \lambda_i^{\text{DA}}, \quad \forall j, \tag{3.81g}$$

$$\overline{q}_{ij}^{\text{DW}} = 0 \quad \text{if} \quad \lambda_{ij}^{\text{BA}} \geq \lambda_i^{\text{DA}}, \quad \forall j, \tag{3.81h}$$

$$\overline{q}_{ij}^{\text{UP}}, \overline{q}_{ij}^{\text{DW}} \geq 0, \quad \forall j. \tag{3.81i}$$

Constraints (3.81a) and (3.81b) compute the expected market revenues from offering up-regulation and down-regulation, respectively. The non-decreasing and non-anticipativity requirements of the up-regulation offer curve are imposed by constraints (3.81c) and (3.81d), while the down-regulation offer curve is forced to be non-increasing through constraints (3.81e) and (3.81f). Constraint (3.81g) restricts the offering of up-regulation energy to the scenarios in which it is required. Similarly, constraint (3.81h) does for the down-regulation energy. Finally, constraint (3.81i) forces the market offers $\overline{q}_{ij}^{\text{UP}}$ and $\overline{q}_{ij}^{\text{DW}}$ to be non-negative.

Pay-as-Bid Pricing Scheme

Figure 3.14 provides an example of offer curves for upward (blue) and downward (red) regulation in a balancing market settled under a pay-as-bid pricing scheme.

We introduce the variables o_{ij}^{UP} and o_{ij}^{DW}, representing the additional quantity to be

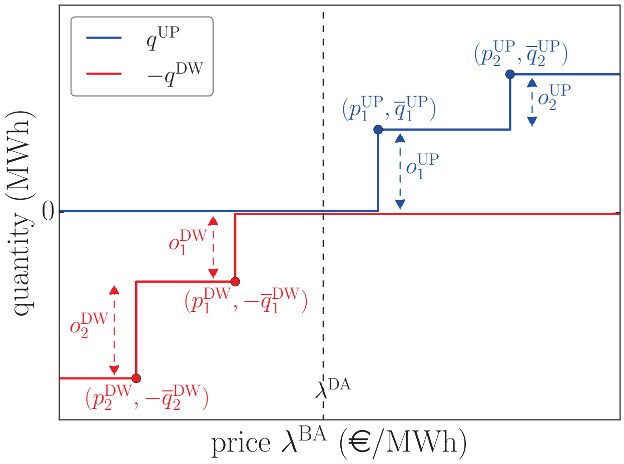


FIGURE 3.14: Example of a 2 blocks offer curve for both upward (blue) and downward (red) regulation in the balancing market (pay-as-bid pricing scheme).

offered at price λ_{ij}^{BA}, for upward and downward regulation, respectively. Given that the step j of the up-regulation curve is $(\lambda_{ij}^{\text{BA}}, o_{ij}^{\text{UP}})$, the total up-regulation energy $\overline{q}_{ij}^{\text{UP}}$ scheduled when scenario (ij) realizes can be computed as

$$\overline{q}_{ij}^{\text{UP}} = \sum_{j'} M_{jj'}^{\text{UP}} o_{ij'}^{\text{UP}}, \quad \forall j, \tag{3.82}$$

where j and j' are both indices of the balancing market price scenarios. The element (jj') of the matrix M^{UP} is defined as

$$M_{jj'}^{\text{UP}} = \begin{cases} 1, & \lambda_{ij}^{\text{BA}} \geq \lambda_{ij'}^{\text{BA}}, \\ 0, & \text{otherwise.} \end{cases} \tag{3.83}$$

Similarly, the total down-regulation energy $\overline{q}_{ij}^{\text{DW}}$ scheduled under scenario j is

$$\overline{q}_{ij}^{\text{DW}} = \sum_{j'} M_{jj'}^{\text{DW}} o_{ij'}^{\text{DW}}, \quad \forall j, \tag{3.84}$$

where M^{DW} is defined as

$$M_{jj'}^{\text{DW}} = \begin{cases} 1, & \lambda_{ij}^{\text{BA}} \leq \lambda_{ij'}^{\text{BA}}, \\ 0, & \text{otherwise.} \end{cases} \tag{3.85}$$

Note that the way M^{UP} and M^{DW} are built inherently imposes the non-decreasing and non-increasing requirement of the upward and downward regulation curves, respectively. However, we need to force the variables o_j^{UP} and o_j^{DW} to be null when the system does not require up-regulation and down-regulation, respectively, i.e.,

$$o_{ij'}^{\mathrm{UP}} = 0 \quad \text{if} \quad \lambda_{ij'}^{\mathrm{BA}} \leq \lambda_i^{\mathrm{DA}}, \quad \forall j', \tag{3.86a}$$

$$o_{ij'}^{\mathrm{DW}} = 0 \quad \text{if} \quad \lambda_{ij'}^{\mathrm{BA}} \geq \lambda_i^{\mathrm{DA}}, \quad \forall j'. \tag{3.86b}$$

Let $\hat{\rho}_{ij'}^{\mathrm{UP}}$ be the expected income associated to the step $(\lambda_{ij'}^{\mathrm{BA}}, o_{ij'}^{\mathrm{UP}})$. Its probability of acceptance can be computed as

$$\mathbb{P}\big[\lambda_i^{\mathrm{BA}} \geq \lambda_{ij'}^{\mathrm{BA}}\big] = \sum_j M_{jj'}^{\mathrm{UP}} \, \pi_{ij}^{\mathrm{BA}}, \tag{3.87}$$

while the expectation of the remuneration price $p_{j'}^{\mathrm{UP}*}$, provided that the offer is being accepted, as

$$\mathbb{E}\big[p_{j'}^{\mathrm{UP}*} \mid \lambda_i^{\mathrm{BA}} \geq \lambda_{ij'}^{\mathrm{BA}}\big] = \lambda_{ij'}^{\mathrm{BA}}, \tag{3.88}$$

thus resulting in an expected revenue $\hat{\rho}_{ij'}^{\mathrm{UP}}$ given by

$$\hat{\rho}_{ij'}^{\mathrm{UP}} = o_{ij'}^{\mathrm{UP}} \, \lambda_{ij'}^{\mathrm{BA}} \sum_j M_{jj'}^{\mathrm{UP}} \, \pi_{ij}^{\mathrm{BA}}. \tag{3.89}$$

The expected revenue $\hat{\rho}_i^{\mathrm{UP}}$ of the whole offer curve is computed by summing up the revenues $\hat{\rho}_{ij'}^{\mathrm{UP}}$ over the steps of the curve, i.e.,

$$\hat{\rho}_i^{\mathrm{UP}} = \sum_{j'} o_{ij'}^{\mathrm{UP}} \, \lambda_{ij'}^{\mathrm{BA}} \sum_j M_{jj'}^{\mathrm{UP}} \, \pi_{ij}^{\mathrm{BA}}. \tag{3.90}$$

The expected revenue $\hat{\rho}_i^{\mathrm{UP}}$ can alternatively be seen as $\sum_j \hat{\rho}_{ij}^{\mathrm{UP}}$, being $\hat{\rho}_{ij}^{\mathrm{UP}}$ the revenue when scenario j realizes. This leads to

$$\hat{\rho}_i^{\mathrm{UP}} = \sum_j \pi_{ij}^{\mathrm{BA}} \sum_{j'} M_{jj'}^{\mathrm{UP}} \, o_{ij'}^{\mathrm{UP}} \, \lambda_{ij'}^{\mathrm{BA}}. \tag{3.91}$$

Note that the formulation in Equation (3.91) is equivalent to (3.90), while the terms are rearranged in a more practical fashion. Similarly, the expected revenue $\hat{\rho}_i^{\mathrm{DW}}$ of the whole down-regulation offer curve can be computed as

$$\hat{\rho}_i^{\mathrm{DW}} = -\sum_j \pi_{ij}^{\mathrm{BA}} \sum_{j'} M_{jj'}^{\mathrm{DW}} \, o_{ij'}^{\mathrm{DW}} \, \lambda_{ij'}^{\mathrm{BA}}. \tag{3.92}$$

Consequently, under a stochastic programming approach, the set $\Pi^{\text{BA}}_{\text{Act}}$ in (3.18), for a pay-as-bid balancing market, is formulated as

$$\hat{\rho}_i^{\text{UP}} = \sum_j \pi_{ij}^{\text{BA}} \sum_{j'} M_{jj'}^{\text{UP}} \, o_{ij'}^{\text{UP}} \, \lambda_{ij'}^{\text{BA}}, \tag{3.93a}$$

$$\hat{\rho}_i^{\text{DW}} = -\sum_j \pi_{ij}^{\text{BA}} \sum_{j'} M_{jj'}^{\text{DW}} \, o_{ij'}^{\text{DW}} \, \lambda_{ij'}^{\text{BA}}, \tag{3.93b}$$

$$\overline{q}_{ij}^{\text{UP}} = \sum_{j'} M_{jj'}^{\text{UP}} o_{ij'}^{\text{UP}}, \quad \forall j, \tag{3.93c}$$

$$\overline{q}_{ij}^{\text{DW}} = \sum_{j'} M_{jj'}^{\text{DW}} o_{ij'}^{\text{DW}}, \quad \forall j, \tag{3.93d}$$

$$o_{ij'}^{\text{UP}} = 0 \quad \text{if} \quad \lambda_{ij'}^{\text{BA}} \leq \lambda_i^{\text{DA}}, \quad \forall j', \tag{3.93e}$$

$$o_{ij'}^{\text{DW}} = 0 \quad \text{if} \quad \lambda_{ij'}^{\text{BA}} \geq \lambda_i^{\text{DA}}, \quad \forall j', \tag{3.93f}$$

$$o_{ij'}^{\text{UP}}, o_{ij'}^{\text{DW}} \geq 0, \quad \forall j'. \tag{3.93g}$$

Constraints (3.93a) and (3.93b) yield expected market revenue from offering up-regulation and down-regulation, respectively. Constraints (3.93c) and (3.93d) evaluate the total amount of energy $\overline{q}_{ij}^{\text{UP}}$ and $\overline{q}_{ij}^{\text{DW}}$ scheduled provided that the block offer j is being accepted. Constraint (3.93e) restricts the offering of up-regulation energy to the scenarios in which it is required. Similarly, constraint (3.93f) does for the down-regulation energy. Finally, constraint (3.93g) forces $o_{ij'}^{\text{UP}}$ and $o_{ij'}^{\text{DW}}$ to be non-negative.

3.4.7 Balancing Market (Passive) Trading Problem

A passive participant in the balancing market deviates of a quantity q^{BA} from its day-ahead contracted schedule $\overline{q}^{\text{DA}}$. Being a price-taker in the market, it models the balancing market price λ^{BA} as a random variable with marginal distribution $f_\lambda^{\text{BA}} : \mathbb{R} \mapsto \mathbb{R}^+$, given the (already revealed) day-ahead market price λ^{DA}. As mentioned in Section 3.3.2 two main imbalance settlement schemes can be distinguished, i.e., the single-price and the dual-price imbalance settlement scheme.

Single-Price Imbalance Settlement Scheme

Under a single-price imbalance settlement scheme, the deviation q^{BA} is priced at the balancing market price λ^{BA}, disregarding the mutual sign of the producer's and system's imbalance. It follows that the expected revenue $\hat{\rho}^{\text{BA}}$ of the passive producer is given by

$$\hat{\rho}^{\mathrm{BA}} = \hat{\lambda}^{\mathrm{BA}} q^{\mathrm{BA}}, \tag{3.94}$$

where the term $\hat{\lambda}^{\mathrm{BA}}$ is computed as

$$\hat{\lambda}^{\mathrm{BA}} = \int_{-\infty}^{\infty} \ell \, f_{\lambda}^{\mathrm{BA}}(\ell) \, \mathrm{d}\ell. \tag{3.95}$$

Consequently, the set $\Pi_{\mathrm{Pas}}^{\mathrm{BA}}$ for a price-taker and risk-neutral passive participant in a single-price balancing market can be formulated as

$$\hat{\rho}^{\mathrm{BA}} = \hat{\lambda}^{\mathrm{BA}} q^{\mathrm{BA}} \tag{3.96}$$

Constraint (3.96) gives the the expected balancing market income $\hat{\rho}^{\mathrm{BA}}$ linked to the deviation q^{BA}.

Dual-Price Imbalance Settlement Scheme

Differently, when a dual-price imbalance settlement scheme is used to price the deviations of the passive producers, q^{BA} is priced differently, depending on the mutual sign of the producer's and system's imbalance. Let us introduce the artificial prices $\lambda^{(+)}$ and $\lambda^{(-)}$. The first, i.e., $\lambda^{(+)}$, represents the remuneration price of positive deviations, i.e., when $q^{\mathrm{BA}} > 0$. The second, i.e., $\lambda^{(-)}$, is used to price negative deviations, i.e., when $q^{\mathrm{BA}} < 0$. They are computed, starting from λ^{DA} and λ^{BA}, as

$$\lambda^{(+)} = \begin{cases} \lambda^{\mathrm{DA}}, & \text{if } \lambda^{\mathrm{BA}} > \lambda^{\mathrm{DA}} \\ \lambda^{\mathrm{BA}}, & \text{otherwise} \end{cases} \tag{3.97a}$$

$$\lambda^{(-)} = \begin{cases} \lambda^{\mathrm{DA}}, & \text{if } \lambda^{\mathrm{BA}} < \lambda^{\mathrm{DA}} \\ \lambda^{\mathrm{BA}}, & \text{otherwise} \end{cases} \tag{3.97b}$$

Note that a positive q^{BA} is priced at $\lambda^{(+)}$, i.e., at the lower between λ^{DA} and λ^{BA}. Indeed, as it is an additional quantity of energy that the passive producer is generating, it is remunerated at the less convenient price. Contrariwise, a negative q^{BA} is priced at $\lambda^{(-)}$, i.e., at the higher between λ^{DA} and λ^{BA}. Indeed, a negative q^{BA} is a quantity that the producer is "buying" back from the market, and $\lambda^{(-)}$ is the less convenient price for "buying" back this quantity. Accordingly, the producer's expected profit $\hat{\rho}^{\mathrm{BA}}$ is evaluated as

$$\hat{\rho}^{\mathrm{BA}} = \begin{cases} \hat{\lambda}^{(+)} q^{\mathrm{BA}}, & \text{if } q^{\mathrm{BA}} > 0 \\ \hat{\lambda}^{(-)} q^{\mathrm{BA}}, & \text{otherwise} \end{cases} \tag{3.98}$$

where the expected market prices $\hat{\lambda}^{(+)}$ and $\hat{\lambda}^{(-)}$, according to Equations (3.97a) and (3.97b), are given by

$$\hat{\lambda}^{(+)} = \int_{-\infty}^{\lambda^{\mathrm{DA}}} \ell\, f_{\lambda}^{\mathrm{BA}}(\ell)\, \mathrm{d}\ell + \lambda^{\mathrm{DA}} \int_{\lambda^{\mathrm{DA}}}^{\infty} f_{\lambda}^{\mathrm{BA}}(\ell)\, \mathrm{d}\ell \tag{3.99a}$$

$$\hat{\lambda}^{(-)} = \lambda^{\mathrm{DA}} \int_{-\infty}^{\lambda^{\mathrm{DA}}} f_{\lambda}^{\mathrm{BA}}(\ell)\, \mathrm{d}\ell + \int_{\lambda^{\mathrm{DA}}}^{\infty} \ell\, f_{\lambda}^{\mathrm{BA}}(\ell)\, \mathrm{d}\ell \tag{3.99b}$$

The set $\Pi_{\mathrm{Pas}}^{\mathrm{BA}}$ for a price-taker and risk-neutral passive participant in a dual-price balancing market is formulated as

$$\hat{\rho}^{\mathrm{BA}} = \begin{cases} \hat{\lambda}^{(+)} q^{\mathrm{BA}}, & \text{if } q^{\mathrm{BA}} > 0 \\ \hat{\lambda}^{(-)} q^{\mathrm{BA}}, & \text{otherwise} \end{cases} \tag{3.100}$$

Constraint (3.100) yields the expected balancing market income $\hat{\rho}^{\mathrm{BA}}$ associated with the deviation q^{BA}.

3.4.8 Balancing Market (Passive) Trading Problem *via* Stochastic Programming

Under a stochastic programming approach we represent the uncertain market price λ_i^{BA}, for a given day-ahead market price realization λ_i^{DA}, by mean of a set J of possible scenarios $\{\lambda_{ij}^{\mathrm{BA}},\ j \in J\}$, where each price scenario j is associated with a probability π_{ij}^{BA} ($\sum_j \pi_{ij}^{\mathrm{BA}} = 1$).

Single-Price Imbalance Settlement Scheme

Under a single-price imbalance settlement scheme, the expected producer's market income $\hat{\rho}_i^{\mathrm{BA}}$ linked with the deviation q_i^{BA} is given by

$$\hat{\rho}_i^{\mathrm{BA}} = \sum_j \pi_{ij}^{\mathrm{BA}} \lambda_{ij}^{\mathrm{BA}} q_i^{\mathrm{BA}}, \tag{3.101}$$

as $\hat{\lambda}_i^{\mathrm{BA}} = \sum_j \pi_{ij}^{\mathrm{BA}} \lambda_{ij}^{\mathrm{BA}}$. This leads to the following formulation of the set $\Pi_{\mathrm{Pas}}^{\mathrm{BA}}$:

$$\hat{\rho}_i^{\mathrm{BA}} = \sum_j \pi_{ij}^{\mathrm{BA}} \lambda_{ij}^{\mathrm{BA}} q_i^{\mathrm{BA}}. \tag{3.102}$$

Constraint (3.102) yields the the expected balancing market income $\hat{\rho}_i^{\mathrm{BA}}$ related to a deviation q_i^{BA} from the day-ahead schedule.

Dual-Price Imbalance Settlement Scheme

Differently, for the dual-price imbalance settlement scheme, it is convenient to split the deviation q_i^{BA} into its positive and negative part, i.e., $q_i^{(+)}$ and $q_i^{(-)}$, respectively. Accordingly, we impose

$$q_i^{\mathrm{BA}} = q_i^{(+)} - q_i^{(-)}, \tag{3.103}$$

while enforcing $q_i^{(+)} \geq 0$ and $q_i^{(-)} \geq 0$. Consequently, we compute separately the expected revenue for the creation of a positive and negative deviation, i.e., $\hat{\rho}_i^{(+)}$ and $\hat{\rho}_i^{(-)}$, given by

$$\hat{\rho}_i^{(+)} = \sum_j \pi_{ij}^{\mathrm{BA}} \lambda_{ij}^{(+)} q_i^{(+)} \tag{3.104a}$$

$$\hat{\rho}_i^{(-)} = -\sum_j \pi_{ij}^{\mathrm{BA}} \lambda_{ij}^{(-)} q_i^{(-)} \tag{3.104b}$$

where $\lambda_{ij}^{(+)} = \min\left(\lambda_{ij}^{\mathrm{BA}}, \lambda_i^{\mathrm{DA}}\right)$ and $\lambda_{ij}^{(-)} = \max\left(\lambda_{ij}^{\mathrm{BA}}, \lambda_i^{\mathrm{DA}}\right)$, according to their definition in Equations (3.97a) and (3.97b). Then, the expected profit $\hat{\rho}_i^{\mathrm{BA}}$ is obtained as

$$\hat{\rho}_i^{\mathrm{BA}} = \hat{\rho}_i^{(+)} + \hat{\rho}_i^{(-)} \tag{3.105}$$

This leads a formulation of the set $\Pi_{\mathrm{Pas}}^{\mathrm{BA}}$ which is

$$\hat{\rho}_i^{\mathrm{BA}} = \sum_j \pi_{ij}^{\mathrm{BA}} \left(\lambda_{ij}^{(+)} q_i^{(+)} - \lambda_{ij}^{(-)} q_i^{(-)}\right), \tag{3.106a}$$

$$q_i^{\mathrm{BA}} = q_i^{(+)} - q_i^{(-)}, \tag{3.106b}$$

$$q_i^{(+)}, q_i^{(-)} \geq 0. \tag{3.106c}$$

Constraint (3.106a) evaluates the the expected balancing market income $\hat{\rho}_i^{\mathrm{BA}}$, given by the sum of the the incomes associated to a positive and a negative deviation. Constraint (3.106b) imposes the balance between q_i^{BA} and its positive and negative parts $q_i^{(+)}$ and $q_i^{(-)}$. Finally, constraint (3.106c) forces $q_i^{(+)}$ and $q_i^{(-)}$ to be positive.

Chapter 4

Uncertainty Characterization

4.1 Introduction

This chapter presents a methodology to generate a discrete set of scenarios to be used by a power producer building its offering strategy under a stochastic programming approach. Section 4.2 introduces to the concept of probabilistic forecasts of wind and solar power generation. Differently than point forecasts, the probabilistic ones give a full representation of the marginal distribution of the random variable of interest (e.g., wind power production) for each look-ahead time interval. Then, Section 4.3 presents a fundamental market model aimed at producing realistic day-ahead and balancing market price forecasts. Section 4.4 shows a methodology to generate trajectories of a random process (e.g., market prices or renewable energy power production) while respecting the correlation among different lead times. Finally, Section 4.5 shows a technique to reduce the set of generated scenarios to a limited amount of representative ones while keeping most of the information embedded in the generated set.

4.2 Wind & Solar Power Forecasts

Stochastic energy sources, such as wind and solar power, cannot be controlled but only predicted with limited accuracy. Such forecasts are crucial for a stochastic producer making informed decisions when offering in an electricity market. As presented in Chapter 2, the main trading floor, i.e., the day-ahead market, is cleared from 12 to 36 hours before the real-time operation. Having accurate information on the future power production may have a significant impact on the profit of a power producer, which may be penalized for deviations from its day-ahead position.

Let t be the time when the prediction is made, and k be the lead time between t and the real-time production of the stochastic source. Let E_{t+k} be the measured power production of the stochastic source during the hourly interval $t+k$. This power production is considered as the realization of the random variable $\widetilde{E}_{t+k}$. A simple forecast of E_{t+k} is a point (or deterministic) forecast. We denote it as $\hat{E}_{t+k|t}$. The point forecast $\hat{E}_{t+k|t}$ can be considered as the conditional expectation (i.e., the mean expected value) of the random variable $\widetilde{E}_{t+k}$. A point forecast is relatively easy to handle as it summarizes the randomness into a single statistic, i.e., the mean expected value. However, also additional information other than the center of the distribution of $\widetilde{E}_{t+k}$ (e.g., its variance) may be required for some analysis.

A complete way of providing forecasts of E_{t+k} are the so-called probabilistic forecasts. They can be obtained from meteorological ensembles (Nielsen et al., 2006), based of physical considerations (Lange and Focken, 2006) or from one of the several statistical models (Bremnes, 2006; Gneiting et al., 2006; Juban, Siebert, and Kariniotakis, 2007; Møller, Nielsen, and Madsen, 2008) available within the literature. Compared to point forecasts, probabilistic aim at providing a complete information of the random variable $\widetilde{E}_{t+k}$, instead of the simple mean value. Probabilistic forecasts may take different forms, e.g., quantile, interval or density forecasts. However, the basic format is the quantile forecasts, as the interval or the density forecasts can be expressed as a combination of at least two quantile forecasts. Let us denote with f_{t+k} the probability density function of the random variable $\widetilde{E}_{t+k}$, defined as

$$\mathbb{P}[a \leq \widetilde{E}_{t+k} \leq b] = \int_a^b f_{t+k}(\ell)\, \mathrm{d}\ell, \tag{4.1}$$

where ℓ in an auxiliary integration variable. With $\mathbb{P}[a \leq \widetilde{E}_{t+k} \leq b]$ we intend the probability that the realization of $\widetilde{E}_{t+k}$ will lie between a and b. Then, we introduce the cumulative distribution function F_{t+k}, relative to f_{t+k}. It is defined as

$$F_{t+k}(b) = \int_{-\infty}^b f_{t+k}(\ell)\, \mathrm{d}\ell = \mathbb{P}[\widetilde{E}_{t+k} \leq b] \tag{4.2}$$

Given that F_{t+k} is strictly increasing by definition, the quantile $\xi_{t+k}^{(\alpha)}$ of proportion $\alpha \in [0,1]$ is given by

$$\mathbb{P}\Big[\widetilde{E}_{t+k} \leq \xi_{t+k}^{(\alpha)}\Big] = \alpha, \tag{4.3}$$

or alternatively

$$\xi_{t+k}^{(\alpha)} = F_{t+k}^{-1}(\alpha). \tag{4.4}$$

Let $\hat{f}_{t+k|t}$ and $\hat{\xi}_{t+k|t}^{(\alpha)}$ be the estimate of f_{t+k} and $\xi_{t+k}^{(\alpha)}$ at time t, respectively. A generic nonparametric forecast $\hat{f}_{t+k|t}$ of the probability density function of $\widetilde{E}_{t+k}$ can be represented through a set of n quantile forecasts, i.e.,

$$\hat{f}_{t+k|t} = \{\xi_{t+k}^{(\alpha_i)} \mid 0 \leq \alpha_1 \leq ... \leq \alpha_i \leq ... \leq \alpha_n \leq 1\}. \tag{4.5}$$

These types of probabilistic forecasts are referred as predictive distributions. Figure 4.1 shows an example of a set of probabilistic forecasts that provide the prediction of the wind power production for the following 24 hourly intervals. The blue areas indicate the prediction intervals of increasing nominal coverage rate. E.g., the 10% prediction interval is given by the two quantiles $\xi_{t+k}^{(0.45)}$ and $\xi_{t+k}^{(0.55)}$, as

$$\mathbb{P}\Big[\xi_{t+k}^{(0.45)} \leq \widetilde{E}_{t+k} \leq \xi_{t+k}^{(0.55)}\Big] = 0.10. \tag{4.6}$$

Similarly, Figure 4.2 illustrates an example of a set of probabilistic forecasts of the solar power production for the following 24 hourly intervals.

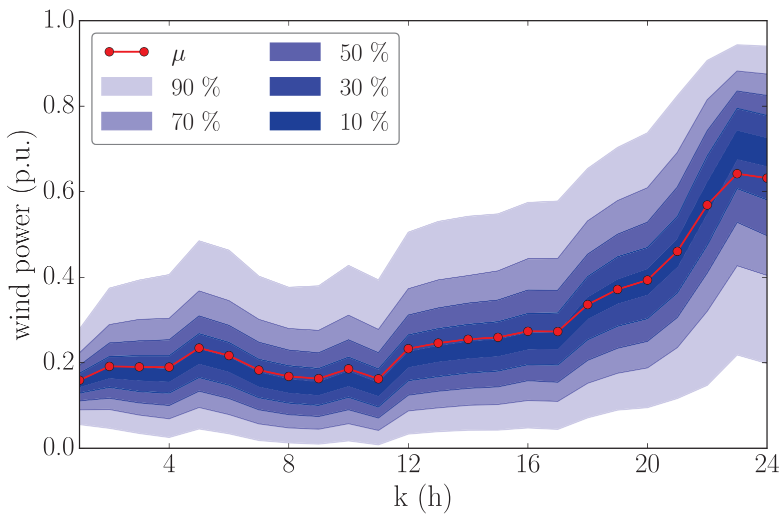


FIGURE 4.1: Example of probabilistic forecast of wind power production with a look ahead of 24 hours. The nominal coverage rates of the prediction intervals (blue areas) are 10, 30, 50, 70, and 90%. The mean expected value is shown in red.

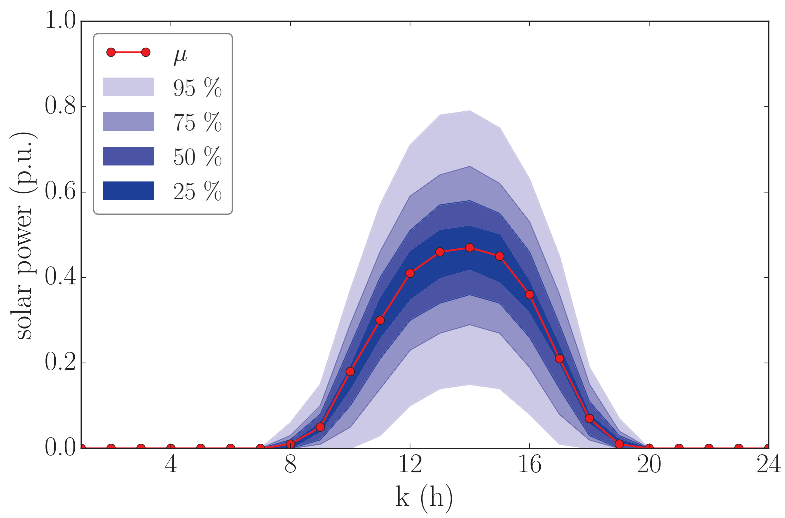


FIGURE 4.2: Example of probabilistic forecast of solar power production with a look ahead of 24 hours. The nominal coverage rates of the prediction intervals (blue areas) are 25, 50, 75, and 95%. The mean expected value is shown in red.

4.3 Electricity Market Price Forecasts

A price-taker power producer building its offering strategy is interested in predicting the outcome of the future and uncertain market price. In this case, it is essential to have probabilistic forecasts instead of deterministic ones, mainly when the producer is willing to submit an offer curve, as seen in Chapter 3. However, instead of using real market price forecasts, this section presents a fundamental market model used to generate realistic forecasts for the market prices in both the day-ahead and balancing market stages. For fundamental model, we intend a simplified market model which respects the essential characteristics and mechanisms of a real electricity market. The proposed model is designed to be used as a tool to generate realistic market price forecasts to test the effectiveness of an offering strategy, but not to predict real market prices. Section 4.3.1 presents the electricity market framework and the assumptions of the fundamental model, which is subsequently described in Section 4.3.2.

4.3.1 Market Framework and Assumptions

We consider a two-settlement electricity market framework as the one presented in Section 2.3. The day-ahead market is cleared once a day, at noon, simultaneously for the whole 24 hourly trading periods of the following day. Then, a balancing market is cleared separately per each hourly interval, one hour before the real-time operation. We neglect the intra-day trading floor for the sake of simplicity.

We use a fundamental market model to generate realistic market price scenarios. In the fundamental market model, we assume that the only stochastic generation is wind power generation. We use a dataset of wind power forecasts for a wind farm located in Denmark. The wind power forecasts are re-scaled and assumed representative of the aggregated wind power production in the market area. At the day-ahead stage, we assume that the demand curve is linear, known, and different per each hourly interval. Conversely, the supply curve of conventional producers is quadratic and uncertain. To model this uncertainty, we consider the coefficient of the second-degree term (i.e., γ_k) as a random variable with known marginal distribution. The methodology for fitting such distribution is beyond the scope of the thesis. The coefficient of the first-degree term is also considered known to simplify the process of scenario generation. Then, we assume that the stochastic generation is offered in the day-ahead market at its mean forecast and zero marginal cost. At the balancing stage, the supply curve is considered known but different from the day-ahead one. Indeed, the participants in the balancing market may not offer their marginal cost, since they may have to internalize the expected revenues into their market offers (e.g., under pay-as-bid pricing scheme). Therefore, we fix a negative price floor λ_0 and impose $\gamma_k^{\text{BA}} = \eta\gamma_k$ ($\eta > 1$), where γ_k^{BA} is referred to the supply curve in the balancing market. Several factors may cause the real-time power imbalance in the system, e.g., errors in load and wind forecasts. For the sake of simplicity, we consider the wind stochasticity as the only source of uncertainty at the balancing stage. This simplifies the scenario generation process for the balancing market prices.

4.3.2 Market Model

The demand curve of the day-ahead market at hourly interval k is modeled as

$$p_k^{\text{DA,d}} = \alpha_k + \delta\, e_k^{\text{DA,d}}, \tag{4.7}$$

where $e_k^{\text{DA,d}}$ is the amount of energy demand at price $p_k^{\text{DA,d}}$. The parameters α_k and δ control the shape of the demand curve. For the same interval k, the supply curve is

$$p_k^{\mathrm{DA},s} = \begin{cases} 0, & \text{if } e_k^{\mathrm{DA,s}} \leq W_k^{\mathrm{DA}} \\ \beta \left(e_k^{\mathrm{DA},s} - W_k^{\mathrm{DA}} \right) + \gamma_k \left(e_k^{\mathrm{DA},s} - W_k^{\mathrm{DA}} \right)^2, & \text{otherwise} \end{cases} \tag{4.8}$$

where $p_k^{\mathrm{DA},s}$ is the price for scheduling the quantity $e_k^{\mathrm{DA},s}$, and W_k^{DA} is the amount of wind power production offered in the day-ahead market. The first segment of the supply curve, i.e., $e_k^{\mathrm{DA,s}} \in \left[0, W_k^{\mathrm{DA}}\right]$, is generated by the market offers of the wind power producers, which are assumed to offer their energy at zero marginal cost. Then, the second segment of the curve, i.e., $e_k^{\mathrm{DA,s}} > W_k^{\mathrm{DA}}$ represents the supply curve of conventional generators. The shape of this supply curve is controlled by β and γ_k. The quantity of wind energy W_k^{DA} offered in the day-ahead market is computed as

$$W_k^{\mathrm{DA}} = \mathbb{E}\left[w_k\right] \overline{W}, \tag{4.9}$$

where w_k is the normalized value ($w_k \in [0,1]$) of wind power production, and $\overline{W}$ is the total installed wind capacity. The uncertain parameter γ_k follows a Normal distribution, i.e.,

$$\gamma_k \sim \mathcal{N}\left(\mu_\gamma, \sigma_\gamma^2\right), \tag{4.10}$$

where μ_γ and σ_γ^2 are the mean value and variance of γ_k, respectively. Given the supply and demand curves, the market clearing price λ_k^{DA} and quantity e_k^{DA} are obtained as the intersection of the two curves. Figure 4.3 shows an example of the clearing mechanism of the day-ahead market model.

Then, in the balancing market, the supply curve at time interval k is defined as

$$p_k^{\mathrm{BA},s} = \begin{cases} \lambda_0, & \text{if } e_k^{\mathrm{BA},s} \leq e_k^0 \\ \beta_k^{\mathrm{BA}} \left(e_k^{\mathrm{BA},s} - e_k^0 \right) + \gamma_k^{\mathrm{BA}} \left(e_k^{\mathrm{BA},s} - e_k^0 \right)^2 + \lambda_0, & \text{otherwise} \end{cases} \tag{4.11}$$

where the variables $p_k^{\mathrm{BA},s}$ and $e_k^{\mathrm{BA},s}$ are the price and quantity of the balancing market supply curve, respectively. The quantity e_k^0 is the amount of energy offered at price λ_0. The term $e_k^{\mathrm{BA},s}$ is the difference between e_k^{DA} and the imbalance generated by the stochastic generation, i.e.,

$$e_k^{\mathrm{BA},s} = e_k^{\mathrm{DA}} - \left(w_k \overline{W} - W_k^{\mathrm{DA}} \right). \tag{4.12}$$

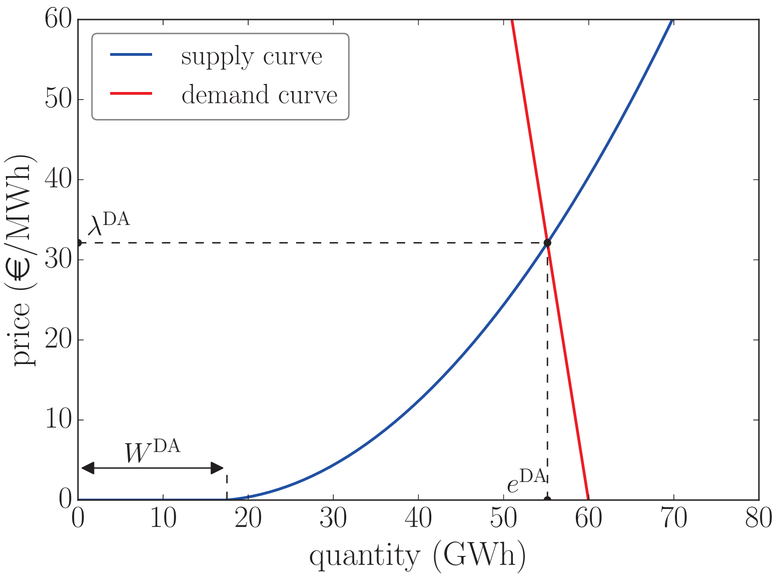


FIGURE 4.3: Example of the day-ahead market clearing model. The supply curve is shown in blue, while the demand curve in red. The intersection of the two curves identifies the market clearing price λ^{DA} and the market clearing quantity e^{DA}.

The term γ_k^{BA} is equal to $\eta\gamma_k$ and it is considered known. The parameters β_k^{BA} and e_k^0 are evaluated by imposing that $p_k^{\mathrm{BA},s} = \lambda_k^{\mathrm{DA}}$ when $e_k^{\mathrm{BA},s} = e_k^{\mathrm{DA}}$. This ensures that the day-ahead and the balancing market prices coincide when no balancing power is required. Figure 4.4 illustrates how the balancing market supply curve (continuous blue line) is derived from the day-ahead market one (dashed blue line). Then, Figures 4.5 and 4.6 show the balancing market clearing mechanism in case of a negative and positive real-time imbalance, respectively. Note that the demand curve is inflexible as the system operator has to contract the exact amount of energy to compensate the deviation of the stochastic production. When the real-time production of the wind generators is lower than the amount contracted in the day-ahead market (see Figure 4.5), the System Operator schedules more production from the conventional generators. The result is $\lambda^{\mathrm{BA}} \geq \lambda^{\mathrm{DA}}$. Differently, when the wind power production is higher than the contracted one (see Figure 4.6), it generates a positive real-time imbalance in the system. This is compensated by scheduling a downward adjustment of conventional generators, resulting in $\lambda^{\mathrm{BA}} \leq \lambda^{\mathrm{DA}}$.

Given this fundamental market model, it is possible to generate probabilistic forecasts of the market prices. E.g., the day-ahead market price λ_k^{DA} uncertainty is given by the uncertainty of the the supply curve ($\gamma_k \sim \mathcal{N}\left(\mu_\gamma, \sigma_\gamma^2\right)$). Figure 4.7 illustrates an example of probabilistic forecasts of λ_k^{DA} generated with the proposed

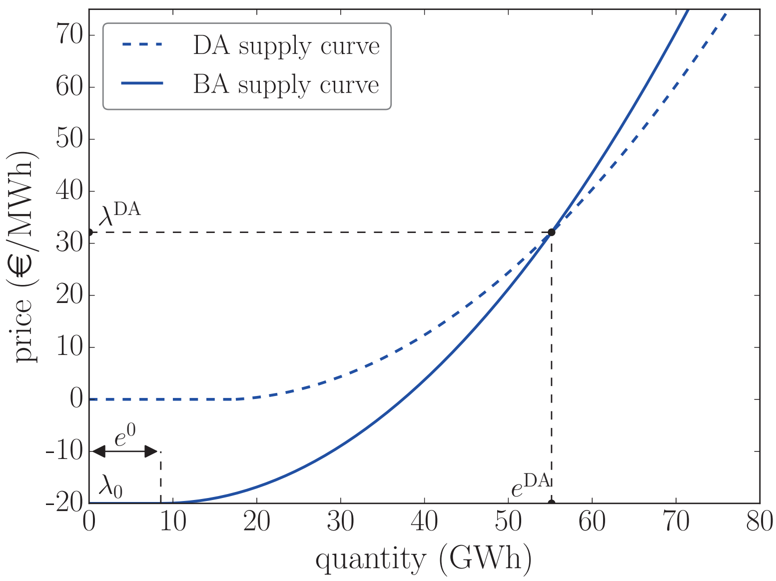


FIGURE 4.4: Example of the balancing (BA) market supply (continuous blue line), derived from the day-ahead (DA) one (dashed blue line). Note that the balancing market supply curve still passes in $(e^{\mathrm{DA}}, \lambda^{\mathrm{DA}})$.

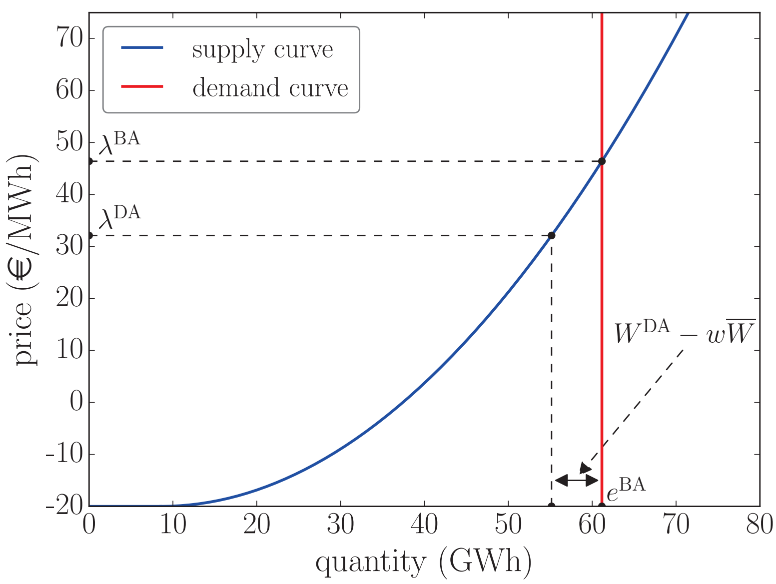


FIGURE 4.5: Example of the balancing clearing model, when the wind power production in the real-time ($(w\overline{W})$ is lower than W^{DA}. In this case the system operator schedules more energy from the conventional generators. Here, $\lambda^{\mathrm{BA}} \geq \lambda^{\mathrm{DA}}$.

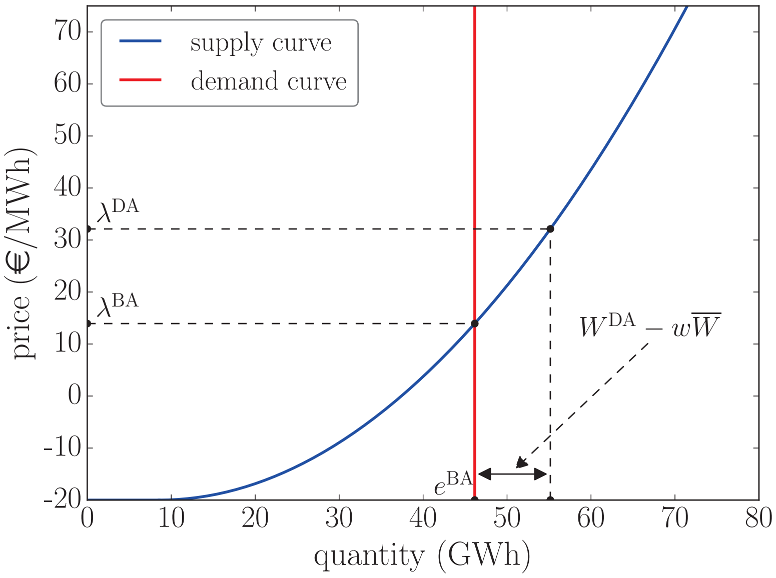


FIGURE 4.6: Example of the balancing market clearing model, when the wind power production in the real-time $((w\overline{W})$ is higher than W^{DA}. In this case the system operator reduces the energy scheduled by conventional generators. Here, $\lambda^{\mathrm{BA}} \leq \lambda^{\mathrm{DA}}$.

fundamental model.

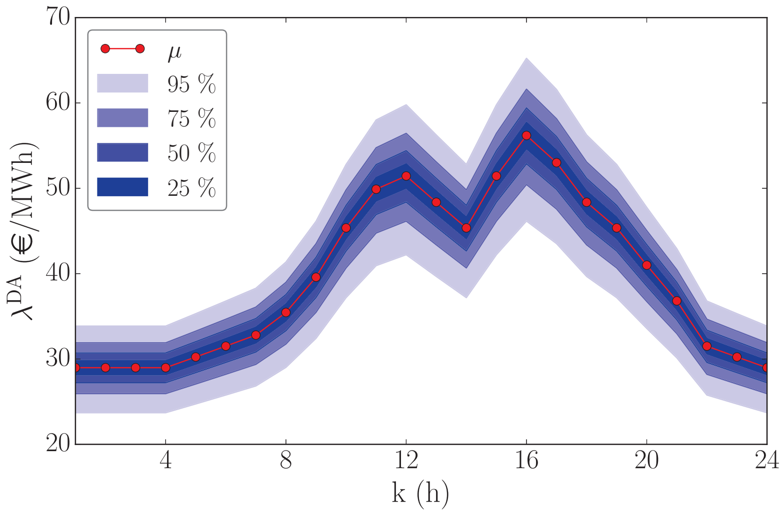


FIGURE 4.7: Example of probabilistic forecast of the day-ahead market price with a look ahead of 24 hours. The nominal coverage rates of the prediction intervals (blue areas) are 25, 50, 75, and 95%. The mean expected value is shown in red.

4.4 Scenario Generation

Given the probabilistic forecasts described in Sections 4.2 and 4.3, the power producer is interested in sampling possible trajectories of the random processes (e.g., wind or solar power production, and market prices). When offering in the day-ahead market, the power producer has to submit the market offers for the 24 hourly trading intervals of the following day, simultaneously. Therefore, we want to include in our scenarios the interdependence structure of the forecast errors among different time intervals. However, probabilistic forecasts are provided as a set of estimated density functions $\{\hat{f}_{t+k|t} \mid k = 1, ..., K\}$, without any inter-temporal correlations among them. Several scenario generation algorithms can be found in the literature, while we consider the methodology presented by Pinson et al. (2009) and Pinson and Girard (2012). The key idea is to exploit a fundamental property of probabilistic forecasts, i.e., that prediction errors can be transformed into Gaussian errors through a suitable transformation. Then, an appropriate interdependence structure can be represented by a single covariance matrix.

The goal is to sample a set of N_I possible trajectories of the random process $\{\widetilde{E}_{t+k},\ k = 1, ..., K\}$, where

$$\widetilde{E}_{t+k} \sim \hat{f}_{t+k|t},\ k = 1, ..., K. \tag{4.13}$$

However, in this format, the random process is hard to handle as the estimated density function $\hat{f}_{t+k|t}$ is different for each lead time k. First, we operate a change of variable from $\widetilde{E}_{t+k}$ to $\widetilde{Y}_k$, defined as

$$\widetilde{Y}_k = \hat{F}_{t+k|t}\Big(\widetilde{E}_{t+k}\Big)\ ,\ k = 1, ..., K. \tag{4.14}$$

A nice property of $\widetilde{Y}_k$ is that it is uniformly distributed between 0 and 1, i.e., $\widetilde{Y}_k \sim \mathcal{U}[0,1],\ k = 1, ..., K$. Then, we apply a second transformation and define the random variable $\widetilde{X}_k$ as

$$\widetilde{X}_k = \Phi^{-1}\Big(\widetilde{Y}_k\Big)\ ,\ k = 1, ..., K, \tag{4.15}$$

where Φ^{-1} is the *probit* function. The new random variable $\widetilde{X}_k$ follows a Gaussian distribution with zero mean and unitary standard deviation, i.e., $\widetilde{X}_k \sim \mathcal{N}(0,1),\ k = 1, ..., K$. Now, we want to sample possible trajectories of the random process $\widetilde{\mathbf{X}} = \{\widetilde{X}_k,\ k = 1, ..., K\}$, which is assumed to follow a multivariate Gaussian distribution, i.e.,

$$\widetilde{\mathbf{X}} \sim \mathcal{N}(\mu_0, \Sigma), \tag{4.16}$$

where μ_0 is a vector of zeros and Σ is the covariance matrix. The matrix Σ summarizes the information about the variance-covariance among the random variables $\widetilde{X}_{t+k},\ k = 1, ..., K$. Such covariance matrix Σ could be recursively estimated by past forecasts and observations (Pinson et al., 2009), if available. For our purpose we use a simplified version of Σ, assuming that we do not have historical data to estimate it. A proposal is given by Pinson and Girard (2012), which suggest to use an exponential covariance function to model Σ, i.e.,

$$\Sigma_{k_1,k_2} = \text{cov}\left(\widetilde{X}_{k_1}, \widetilde{X}_{k_2}\right) = \exp\left(-\frac{|k_1 - k_2|}{v}\right),\ 1 \leq k_1, k_2 \leq K, \tag{4.17}$$

where the parameter v controls the correlation among different lead times. As an example, Pinson and Girard (2012) indicate to use $v = 7$ for on-shore wind power forecasts.

Now, we can describe the scenario generation algorithm. Starting from $\{\hat{f}_{t+k|t} \mid k = 1, ..., K\}$ we want to generate a set of N_I possible trajectories of the random process $\{\widetilde{E}_{t+k},\ k = 1, ..., K\}$.

- We randomly sample N_I realizations of the random variable $\widetilde{\mathbf{X}}$ $\left(\widetilde{\mathbf{X}} \sim \mathcal{N}(\mu_0, \Sigma)\right)$. Let $\widetilde{\mathbf{X}}^{(i)}$ be the i^{th} of the N_I realizations.

- We apply the inverse *probit* function Φ to perform a change of variable from $\widetilde{\mathbf{X}}$ to $\widetilde{\mathbf{Y}}$, i.e.,

$$\widetilde{Y}_k^{(i)} = \Phi\left(\widetilde{X}_k^{(i)}\right),\ \forall i, k. \tag{4.18}$$

- We apply an additional change of variable, from $\widetilde{\mathbf{Y}}$ to $\widetilde{\mathbf{E}}$, i.e., the random variable of interest. To do so, we use the set of cumulative distribution functions $\{\hat{F}_{t+k|t} \mid k = 1, ..., K\}$ that can be fit starting from the quantile probabilistic forecasts available. This leads to

$$\widetilde{E}_{t+k|t}^{(i)} = \hat{F}_{t+k|t}^{-1}\left(\widetilde{Y}_k^{(i)}\right),\ \forall i, k. \tag{4.19}$$

This algorithm allows generating a set of N_I trajectories of the random process of interest when a set of predictive distributions (one per lead time) is available. Such set of scenarios is now suitable to be used as input to decision-making tools based on stochastic programming, where continuous random variables or processes are approximated (and linearized) by making them into a discrete set of possible realizations, each of one is associated with a probability of occurrence.

Figure 4.8 shows an example of 100 trajectories of wind power production, given the probability forecasts shown in Figure 4.1.

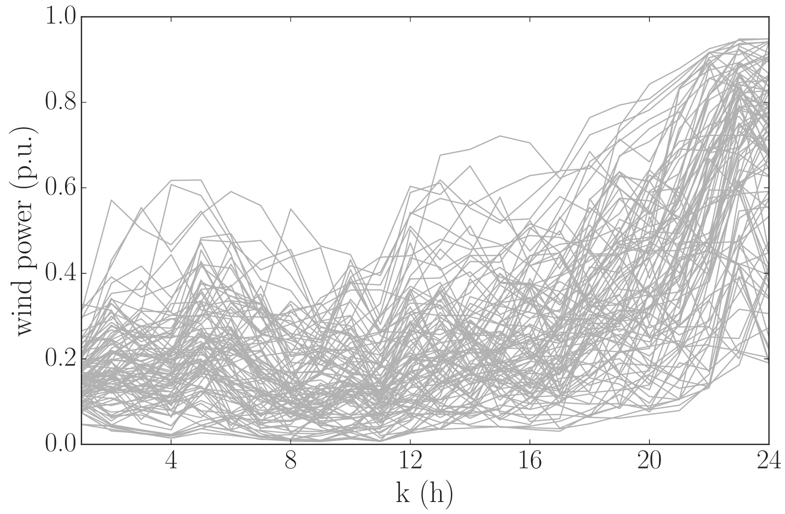


FIGURE 4.8: Example of 100 time trajectories of wind power production, with a look ahead of 24 hours.

4.5 Scenario Reduction

Section 4.4 presented a methodology to generate statistically correlated trajectories of stochastic processes, such as wind power production and electricity market prices. The size of the discrete set of scenarios necessary to accurately represent the continuous random variables or stochastic processes is usually large, which may render the stochastic programming problem intractable. Therefore, this section presents a technique to reduce the number of scenarios while maintaining the stochastic information enclosed in such scenarios.

Several scenario reduction techniques are available within the literature. This section presents a *forward selection* algorithm (Growe-Kuska, Heitsch, and Romisch, 2003). Let $\{\widetilde{\mathbf{E}}^{(i)},\ i \in I\}$ be a set of N_I scenarios of the stochastic process $\{\widetilde{E}_{t+k},\ k = 1, ..., K\}$, where $\widetilde{\mathbf{E}}^{(i)} = \{\widetilde{E}^{(i)}_{t+k},\ k = 1, ..., K\}$. Each scenario i of the set I is associated with a discrete probability π_i of occurrence, such that $\sum_{i \in I} \pi_i = 1$. Note that if the N_I scenarios are generated with the technique presented in Section 4.4, then

$\pi_i = {}^1\!/_{N_I},\ \forall i \in I$. We define the cost function $c(\cdot)$ as

$$c(i,i') = \|\widetilde{\mathbf{E}}^{(i)} - \widetilde{\mathbf{E}}^{(i')}\|, \quad \forall i, i' \in I. \tag{4.20}$$

The goal is to perform an iterative selection of the N_S ($N_S \leq N_I$) more representative scenarios. The algorithm can be described as follows:

- Let I_S be the set of the selected scenarios, while I_O is the set of the non selected ones. At the beginning of the process (i.e., $n = 0$) they are defined as $I_S^{[0]} = \varnothing$ and $I_O^{[0]} = I$. Then, compute the cost function $c(i,i')$ per each pair of $i, i' \in I$.

- Compute the starting scenario i_1 to be included in I_S. It is given by

$$i_1 = \arg\left\{\min_{i' \in I} \sum_{i \in I} \pi_i\, c(i,i')\right\}. \tag{4.21}$$

  The scenario i_1 can be seen as the average scenario of the set I. Then, we impose $I_S^{[1]} = \{i_1\}$ and $I_O^{[1]} = I \setminus \{i_1\}$.

- For $n = 2, ..., N_S$, compute

$$i_n = \arg\left\{\min_{i' \in I_O^{[n-1]}} \sum_{i \in I_O^{[n-1]} \setminus \{i'\}} \pi_i \min_{i'' \in I_S^{[n-1]} \cup \{i\}} c(i,i'')\right\}. \tag{4.22}$$

  Impose $I_S^{[n]} = I_S^{[n-1]} \cup \{i_n\}$ and $I_O^{[n]} = I_O^{[n-1]} \setminus \{i_n\}$.

The result of the scenario reduction algorithm are the sets I_S^* and I_O^* of the selected and non selected scenarios, respectively. Note that $I_S^* \cup I_O^* = I$. Now, we want to properly distribute the probabilities of the non-selected scenario to the selected ones. For each selected scenario $i \in I_S^*$, let Γ_i be the set of the non-selected scenarios $i' \in I_O^*$ for which i is the closest selected scenario, according to the cost function c. It is evaluated as

$$\Gamma_i = \left\{i' \in I_O^* \mid i = \arg\min_{i'' \in I_S^*} c(i',i'')\right\}, \quad \forall i \in I_S^*. \tag{4.23}$$

Then, the updated probabilities π_i^* of the selected scenarios can be computed as

$$\pi_i^* = \pi_i + \sum_{i' \in \Gamma_i} \pi_{i'}, \quad \forall i \in I_S^*. \tag{4.24}$$

A more extensive description of the *forward selection* algorithm is presented by Growe-Kuska, Heitsch, and Romisch (2003) and Morales et al. (2009).

Figure 4.8 shows an example of 100 scenarios of wind power production (gray) reduced to the 15 more representative ones (blue). The 100 trajectories are the ones shown previously in Figure 4.8.

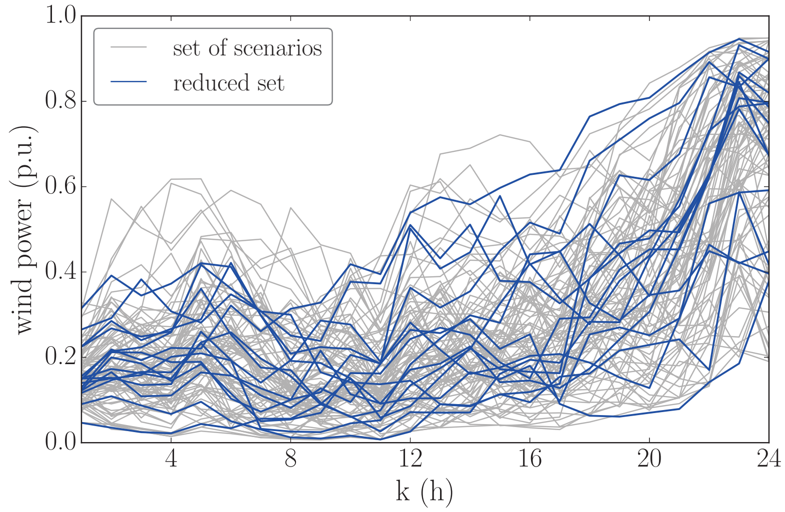


FIGURE 4.9: Example of 100 time trajectories (gray) of wind power production reduced to the 15 significant ones (blue).

Chapter 5

Trading of Renewable Energy Sources

5.1 Introduction

The deployment of renewable energy sources has constantly increased over the last decade. Many of these renewable energy sources, such as wind and solar power, have a stochastic nature. Their power production can only be predicted with a limited accuracy, which degrades as the time horizon of the forecast increases. Initially, stochastic energy sources were supported through feed-in tariffs to facilitate their deployment. By receiving a fixed price for the energy generated, stochastic power producers were exempt from their balancing responsibility. As their penetration has continuously increased, these producers are asked to participate in the electricity market under similar rules as conventional generators. It translates to being financially responsible for the creation of real-time imbalances. This, together with uncertain market prices, yield to uncertain market profits. Renewable energy producers can tackle this high level of uncertainty by exploiting the information enclosed in their forecasts.

The problem of participation of renewable energy producers in electricity markets has received an increasing interest over the last years. One of the first work on this topic is that of Bathurst, Weatherill, and Strbac (2002). The authors show how, through risk-analysis, expected profits can be increased by exploiting the information of a point forecast model of wind power production. Subsequently, several alternative participation models were proposed, highlighting the value of probabilistic forecasts in increasing the expected wind power producer's profit. Assuming a simplified market setup, i.e., no intra-day market, reference (Bremnes, 2004) shows that the optimal market quantity to be submitted in the day-ahead market is a quantile of the power production forecasts. Then, Pinson, Chevallier, and Kariniotakis (2007) give analytical expressions for the optimal day-ahead quantity to contract in the market, based on probabilistic forecasts. The authors focus on the utility function of the wind power producer, considering both profit maximization and risk management. An extensive analysis on the topic is the one of Bitar et al. (2012). The authors highlight the link between the accuracy of the probabilistic wind power forecasts and the expected producer's profit. The intra-day trading floor is introduced by Morales, Conejo, and Pérez-Ruiz, 2010 within a stochastic programming framework, as well as a risk-aversion trough the conditional value at risk. Finally, some authors (Zugno et al., 2013; Baringo and Conejo, 2013) relax the price-taker assumption by computing market prices endogenously in the trading problem.

The optimal offering strategy of a stochastic producer is strongly influenced by the imbalance settlement scheme used to price the real-time deviations from the day-ahead contracted schedule. Naturally, a rational stochastic producer is only seeking at maximizing its profit while deriving its optimal market offers. However, the aggregate of the stochastic producers' imbalance needs to be compensated by scheduling upward or downward adjustments from conventional generators at the balancing stage. Therefore, different imbalance settlement schemes may lead to a different balancing cost for the System Operator. Some works (Vandezande et al., 2010; Batlle, Pérez-Arriaga, and Zambrano-Barragán, 2012; Chaves-Ávila, Hakvoort, and Ramos, 2014) compare the effectiveness of different pricing and support schemes for renewable energy generation. They highlight the importance of an accurate market signal in encouraging stochastic producers to reduce their real-time imbalance. In this context, Bueno-Lorenzo, Moreno, and Usaola (2013) propose a new imbalance settlement scheme that may promote the participation in the intra-day market stage. Other works (Winkler et al., 2016; Batalla-Bejerano and Trujillo-Baute, 2016) motivate further research in this field, highlighting the impact of renewable energy sources on the balancing cost of the system. The definition of an optimal imbalance settlement structure is a complicated issue. On one side, stochastic producers should not be excessively penalized for their real-time imbalances, as they are inherent in their nature and high penalties may discourage new investments in renewable energy generation. On the other side, stochastic producers should be pushed in improving their forecasting skills and reduce their real-time imbalances.

With the purpose of mitigating the imbalance responsibility of stochastic producers, some electricity markets, e.g., Belgium (De Vos and Driesen, 2009) and Italy (Giannitrapani et al., 2013), introduced imbalance settlement schemes with tolerance margins. Under such pricing schemes, the stochastic producer is penalized differently if its imbalance exceeds a specified tolerance band, which is proportional to the market quantity offer. Some works (De Vos, Driesen, and Belmans, 2011; Chaves-Ávila, Hakvoort, and Ramos, 2014) suggest that the tolerance margins may lead to possible market distortions, and therefore are not beneficial from a system perspective. However, these conclusions are drawn after numerical simulations, without analytically proving why those tolerance margins may distort the market offers. On this topic, it is interesting the work of Giannitrapani et al. (2013), which investigates how a wind power producer can maximize its expected profits when bidding in a soft penalized market (i.e., with tolerance margins). They prove the uniqueness of the optimal solution, under the price-taker producer assumption and no intra-day trading. Nevertheless, the work focuses on maximizing the expected producer's profit of the producer, without analyzing eventual market distortions.

The Italian Electricity Market presents two alternative imbalance settlement schemes within the market structure. Different than the single- and the dual-price imbalance settlement scheme seen in Section 3.3.2, they consider tolerance margins. The resolution of Italian Regulatory Authority for Electricity Gas and Water, 2014 presents the imbalance settlement schemes in the Italian electricity market. One, denoted in the chapter as *band-dual*, includes a tolerance band proportional to the energy quantity contracted in the day-ahead market. Imbalances within the tolerance band are priced at the day-ahead market price, while the amount of the imbalance that exceeds the band is priced at the less convenient between the day-ahead market price and balancing market price (i.e., as the dual-price settlement scheme). The relative width of the band is different per each renewable energy source, e.g., 45% and 25% for wind and solar power, respectively. Subsequently, the resolution of Italian Regulatory Authority for Electricity Gas and Water, 2016 introduces an additional imbalance settlement scheme, called *single-dual* pricing scheme. Similarly to the *band-dual* pricing scheme, a dual-pricing is used for the amount of the imbalance that exceeds the tolerance band. Differently, the imbalances within the band are priced at the balancing market price (i.e., as the single-price settlement scheme). This scheme is initially tested on units different than renewable energy ones (e.g., consumption units).

However, it may be extended to renewable energy producers in the next years.

This chapter shows how a stochastic producer, starting from the general model of Chapter 3, can derive its optimal offering strategy. Moreover, we extend the analysis by considering the optimal offering strategies under the *band-dual* and the *single-dual* imbalance pricing scheme of the Italian electricity market. We show that the market quantity that maximizes the producer's expected profit is unique and we provide a formulation to evaluate such optimal quantity. To our best knowledge, the formulation of the optimal offering strategy under the *single-dual* imbalance settlement scheme is not available in the literature. Then, we investigate how this optimal market quantity influences the expected real-time imbalance of the stochastic power producer, distinguishing between imbalances that are expected to "help" the system (i.e., they reduce the imbalance of the system) from the ones that are expected to "hurt" the system (i.e., they increase the imbalance of the system). A detailed analysis on the expected real-time imbalances of a stochastic producer under the *band-dual* and the *single-dual* imbalance pricing scheme in novel in the field. This brings information about possible market distortions introduced by the tolerance band of the *band-dual* and the *single-dual* imbalance pricing scheme.

The remaining of the chapter is organized as follows. Section 5.2 introduces the electricity market framework and the assumptions needed to derive the analytical formulation of the optimal market offers. Section 5.3 presents the optimal offering models under conventional imbalance settlement schemes, i.e., the single- and the dual-price settlement scheme. Section 5.4 derives optimal offering strategies for the *band-dual* and *single-dual* imbalance pricing schemes. Moreover, it shows how to link the optimal offering strategy to the expected real-time imbalance of the stochastic producer. A summary of the chapter is given in Section 5.5.

5.2 Electricity Market Framework and Assumptions

We consider an electricity market similar to the one presented in Section 2.3. Such market includes a day-ahead and a balancing stage, while the intra-day adjustment market is neglected for the sake of simplicity. The day-ahead market closes at noon of the day before energy delivery and hosts transaction the whole 24 hourly intervals of the following day. Then, closer to the real-time operation, a separate balancing market is cleared for each hourly interval. Stochastic producers access the balancing market to contract their deviations from the day-ahead market schedule. These imbalances are priced differently depending on the imbalance settlement scheme adopted. The most common imbalance settlement schemes are the single-price and the dual-price scheme, as discussed in Section 3.3.2. A schematic representation of electricity market submission process is shown in Figure 5.1.

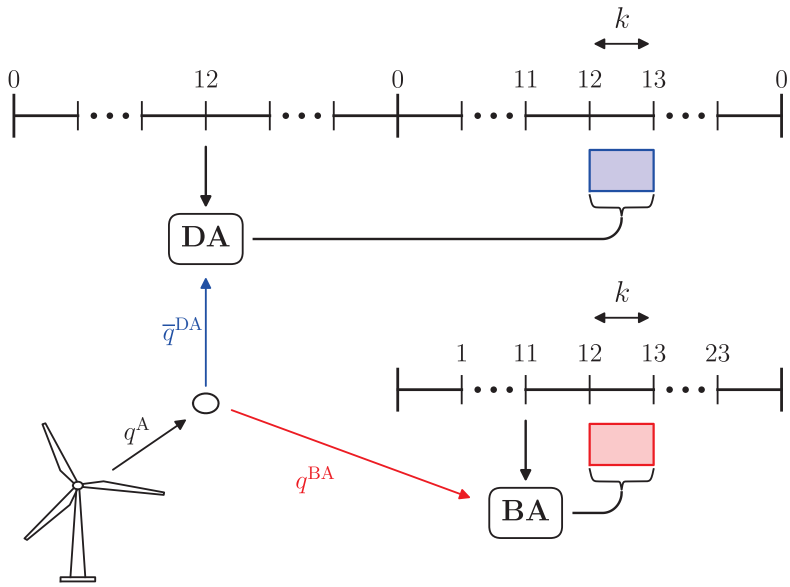


FIGURE 5.1: Schematic representation of the electricity market framework. The stochastic producer submits the quantity offer $\overline{q}^{\mathrm{DA}}$ in the day-ahead (DA) market, while settling its deviation q^{BA} in the balancing (BA) market.

This chapter takes the perspective of a renewable energy producer that has to contract its position in the day-ahead market. The producer is assumed to be price-taker in both the day-ahead and the balancing market. Let λ_k^{DA} and λ_k^{BA} be the day-ahead and the balancing market price of the hourly interval k. Under the price-taker assumption, the day-ahead market price λ_k^{DA} is considered as a random variable following the density function $f_{\lambda_k}^{\mathrm{DA}} : \mathbb{R}^+ \mapsto \mathbb{R}^+$. Then, let ϕ_k^{BA} be the differential price between the balancing and the day-ahead market price, i.e.,

$$\phi_k^{\mathrm{BA}} = \lambda_k^{\mathrm{BA}} - \lambda_k^{\mathrm{DA}}. \tag{5.1}$$

Given that the producer is considered a price-taker also in the balancing market, ϕ_k^{BA} is modeled as a random variable of marginal distribution $f_{\phi_k}^{\mathrm{BA}} : \mathbb{R} \mapsto \mathbb{R}^+$. We also assume that the producer is risk-neutral and that it offers its energy production at zero marginal cost. It is provided with probabilistic forecasts of the future power production E_k of the stochastic source (e.g., wind power), i.e., the producer knows the marginal distribution $f_{E_k} : \mathbb{R}^+ \mapsto \mathbb{R}^+$ of E_k at interval k.

This chapter uses a wind power producer as an example. However, the proposed models are all valid for the trading of stochastic energy sources different than wind, e.g., solar power. We assume that the random variable E_k follows a Beta distribution (Morales et al., 2014; Liu, 2011) of mean expected value μ_{E_k} and variance $\sigma^2_{E_k}$. We consider that $\sigma^2_{E_k}$ and μ_{E_k} are linked together by

$$\sigma^2_{E_k} = 4\, \mu^{\nu}_{E_k} \left(1 - \mu^{\nu}_{E_k}\right)\, \overline{\sigma}^2, \tag{5.2}$$

where ν and $\overline{\sigma}^2$ are parameters that control the correlation between μ_{E_k} and $\sigma^2_{E_k}$. Thanks to that, the marginal distribution $f_{E_k}(\cdot)$ is uniquely defined by knowing its mean expected value μ_{E_k}. Consequently, with $f(\ell, \mu_{E_k})$ we intend the probability that the level of production ℓ realizes, given that the mean expected value of E_k is μ_{E_k}. Lastly, we assume that μ_{E_k} follows as well a Beta distribution E_k, i.e.,

$$\mu_{E_k} \sim \text{Beta}(\alpha_\mu, \beta_\mu), \tag{5.3}$$

and we denote with $f_\mu(\cdot)$ the probability density function of μ_{E_k}.

5.3 Optimal Day-ahead Offering Strategy

This section derives the optimal offering strategy that a stochastic power producer may use to obtain its best day-ahead market offer. Starting from the general price-taker and risk-neutral offering strategy shown in Section 3.4, Section 5.3.1 adapts it to the characteristics of the wind power producer. Then, Section 5.3.2 and 5.3.3 focus on the single- and the dual-price settlement scheme, respectively.

5.3.1 Problem Formulation

At the day-ahead stage, the stochastic producer submits its market offers for the whole 24 trading hours of the following day. To derive its optimal offering strategy, we start from the general formulation (3.18) for the price-taker and risk-neutral producer, extended to account the 24 trading intervals, i.e.,

$$\underset{\Gamma}{\text{Max}} \quad \sum_k \hat{\rho}_k^{\text{DA}} + \hat{\rho}_k^{\text{UP}} + \hat{\rho}_k^{\text{DW}} + \hat{\rho}_k^{\text{BA}} - \hat{c}_k \tag{5.4a}$$

$$\text{s.t.} \quad \overline{q}_k^{\text{DA}} + \overline{q}_k^{\text{UP}} - \overline{q}_k^{\text{DW}} + q_k^{\text{BA}} = q_k^{\text{A}}, \quad \forall k, \tag{5.4b}$$

$$\overline{q}_k^{\text{DA}}, \hat{\rho}_k^{\text{DA}} \in \Pi^{\text{DA}}, \quad \forall k, \tag{5.4c}$$

$$\overline{q}_k^{\text{UP}}, \overline{q}_k^{\text{DW}}, \hat{\rho}_k^{\text{UP}}, \hat{\rho}_k^{\text{DW}} \in \Pi_{\text{Act}}^{\text{BA}}, \quad \forall k, \tag{5.4d}$$

$$q_k^{\text{BA}}, \hat{\rho}_k^{\text{BA}} \in \Pi_{\text{Pas}}^{\text{BA}}, \quad \forall k, \tag{5.4e}$$

$$\hat{c}_k = h\big(q_k^{\text{A}}\big), \quad \forall k, \tag{5.4f}$$

$$q_k^{\text{A}} \in \Omega, \quad \forall k, \tag{5.4g}$$

where

$$\Gamma = \{q_k^{\text{A}}, \overline{q}_k^{\text{DA}}, \overline{q}_k^{\text{UP}}, \overline{q}_k^{\text{DW}}, q_k^{\text{BA}}, \hat{\rho}_k^{\text{DA}}, \hat{\rho}_k^{\text{UP}}, \hat{\rho}_k^{\text{DW}}, \hat{\rho}_k^{\text{BA}}, \hat{c}_k\}. \tag{5.5}$$

First, we remove all the variables and constraints associated with an active participation in the balancing market. To that purpose, we eliminate the variables $\hat{\rho}_k^{\text{UP}}$ and $\hat{\rho}_k^{\text{DW}}$ from the objective function (5.4a) and $\overline{q}_k^{\text{UP}}$ and $\overline{q}_k^{\text{DW}}$ from constraint (5.4b), and we remove constraint (5.4d). Similarly, as we assume that the wind power producer has a production cost of 0 €/MWh, we eliminate constraint (5.4f) and we remove the variable $\hat{c}_k$ from the objective function (5.4a). Then, the set Ω of constraint (5.4g), describing the feasible region of the production unit, can be replaced by

$$q_k^{\text{A}} = E_k, \quad \forall k, \tag{5.6}$$

and we reformulate constraint (5.4b) by replacing q_k^{A} with its formulation in (5.6), i.e.,

$$q_k^{\text{BA}} = E_k - \overline{q}_k^{\text{DA}} \tag{5.7}$$

Finally, we replace the set Π^{DA} of constraint (5.4c) with the formulation provided in Equations (3.27), which considers the submission of single price-quantity offers $(p_k^{\text{DA}}, \overline{q}_k^{\text{DA}})$ in the day-ahead market. Note that we assume to offer the quantity $\overline{q}_k^{\text{DA}}$ at $p_k^{\text{DA}} = 0$ €/MWh, thus leading to the following formulation of constraint (5.4c):

$$\hat{\rho}_k^{\text{DA}} = \overline{q}_k^{\text{DA}} \int_0^{\infty} \ell \, f_{\lambda_k}^{\text{DA}}(\ell) \, \mathrm{d}\ell = \hat{\lambda}_k^{\text{DA}} \overline{q}_k^{\text{DA}}, \tag{5.8a}$$

$$\underline{Q} \leq \overline{q}^{\text{DA}} \leq \overline{Q}. \tag{5.8b}$$

where $\underline{Q} = 0$ and $\overline{Q} = \overline{E}$ (i.e., the capacity of the wind farm). Accordingly, the general offering strategy tailored to the wind power producer is

$$\underset{\Gamma}{\text{Max}} \quad \sum_{k} \hat{\lambda}_k^{\text{DA}} \overline{q}_k^{\text{DA}} + \hat{\rho}_k^{\text{BA}} \tag{5.9a}$$

$$\text{s.t.} \quad q_k^{\text{BA}} = E_k - \overline{q}_k^{\text{DA}}, \quad \forall k, \tag{5.9b}$$

$$q_k^{\text{BA}}, \hat{\rho}_k^{\text{BA}} \in \Pi_{\text{Pas}}^{\text{BA}}, \quad \forall k, \tag{5.9c}$$

$$0 \leq \overline{q}_k^{\text{DA}} \leq \overline{E}, \quad \forall k, \tag{5.9d}$$

where

$$\Gamma = \{\overline{q}_k^{\text{DA}}, q_k^{\text{BA}}, \hat{\rho}_k^{\text{BA}}\}. \tag{5.10}$$

The formulation in (5.9) can be decomposed in 24 optimization problems, as there are no inter-temporal constraints linking the decision variables of different trading intervals. Consequently, we solve the offering strategy for a single hourly interval k, without loss of generality. For the clarity of the notation, in the remaining of the chapter we skip the subscript k. Finally, we need to reformulate the set $\Pi_{\text{Pas}}^{\text{BA}}$ in constraint (5.9c), which is conditional to the imbalance settlement scheme considered. Therefore, we develop a different offering model for the single- and the dual-price imbalance settlement scheme.

5.3.2 Optimal Offering under a Single-Price Settlement Scheme

Under a single-price imbalance settlement scheme, the deviation q^{BA} is priced at the balancing market price λ^{BA}, disregarding the sign of the imbalance. We replace the set $\Pi_{\text{Pas}}^{\text{BA}}$ in constraint (5.9c) with its formulation in Equation (3.96), i.e.,

$$\hat{\rho}^{\text{BA}} = \hat{\lambda}^{\text{BA}} q^{\text{BA}}, \tag{5.11}$$

and we reformulate the objective function (5.9a) as follows:

$$\hat{\mu} = \mathbb{E}\Big[\hat{\lambda}^{\text{DA}} \overline{q}^{\text{DA}} + \hat{\lambda}^{\text{BA}} \left(E - \overline{q}^{\text{DA}}\right)\Big], \tag{5.12}$$

where q^{BA} is replaced by $E - \overline{q}^{\text{DA}}$, according to the equality constraint (5.9b). By rearranging the terms in (5.12) we obtain

$$\hat{\mu} = \mathbb{E}\Big[\hat{\lambda}^{\text{DA}} E + \hat{\phi}^{\text{BA}} \left(E - \overline{q}^{\text{DA}}\right)\Big]. \tag{5.13}$$

The first term in Equation (5.13) is the product between the day-ahead market price and the future value of wind power production. It represents the market profit in case the producer would submit the quantity $\overline{q}^{\text{DA}} = E$ in the day-ahead market. Besides, as it is the product of two parameters, it can be removed from the objective function. The second term, i.e., $\hat{\phi}^{\text{BA}}\left(E - \overline{q}^{\text{DA}}\right)$, is the income due to the creation of the real-time imbalance. We denote it with $\mathcal{L}$. Note that also $\hat{\phi}^{\text{BA}}E$ is the product of two parameters, and can be removed from the objective function.

Therefore, the optimal offering strategy under a single-price imbalance settlement scheme is

$$\underset{\overline{q}^{\text{DA}}}{\text{Max}} \quad -\hat{\phi}^{\text{BA}}\overline{q}^{\text{DA}} \tag{5.14a}$$

$$\text{s.t.} \quad 0 \leq \overline{q}^{\text{DA}} \leq \overline{E} \tag{5.14b}$$

The optimization problem (5.14) has a trivial solution, i.e.,

- if $\hat{\phi}^{\text{BA}} > 0$, the optimal day-ahead market offer $\overline{q}^{\text{DA}*}$ is equal to 0,
- if $\hat{\phi}^{\text{BA}} < 0$, the optimal day-ahead market offer $\overline{q}^{\text{DA}*}$ is equal to the unit's capacity $\overline{E}$,
- if $\hat{\phi}^{\text{BA}} = 0$, each market offer $\overline{q}^{\text{DA}} \in [0, \overline{E}]$ brings the same expected revenue.

Under a single-price imbalance settlement scheme, the producer is willing to offer 0 or the full capacity $\overline{E}$, depending on the expectation of ϕ^{BA}, i.e., if it is expecting a balancing market price higher or lower than the day-ahead one. Note that the forecasts of the future power production E do not influence the optimal solution of (5.14). Indeed, the power producer bases its decision on the expectation of the difference between the balancing market price and the day-ahead market one.

5.3.3 Optimal Offering under a Dual-price Settlement Scheme

Under a dual-price imbalance settlement scheme, the deviation q^{BA} is priced differently, depending on the mutual sign of the producer's and system's imbalance. For this pricing scheme, we replace the set $\Pi^{\text{BA}}_{\text{Pas}}$ in constraint (5.9c) with its formulation in Equation (3.100), i.e.,

$$\hat{\rho}^{\mathrm{BA}} = \begin{cases} \hat{\lambda}^{(+)} q^{\mathrm{BA}}, & \text{if } q^{\mathrm{BA}} > 0, \\ \hat{\lambda}^{(-)} q^{\mathrm{BA}}, & \text{otherwise.} \end{cases} \tag{5.15}$$

Similarly to Section 5.3.2, we replace q^{BA} with $E - \overline{q}^{\mathrm{DA}}$ and we reformulate the objective function (5.9a) as

$$\hat{\mu} = \mathbb{E}\Big[\hat{\lambda}^{\mathrm{DA}} E + \mathcal{L}\Big], \tag{5.16}$$

where

$$\mathcal{L} = \begin{cases} \left(\hat{\lambda}^{(+)} - \hat{\lambda}^{\mathrm{DA}}\right)\left(E - \overline{q}^{\mathrm{DA}}\right) & \text{if } E \geq \overline{q}^{\mathrm{DA}}, \\ \left(\hat{\lambda}^{(-)} - \hat{\lambda}^{\mathrm{DA}}\right)\left(E - \overline{q}^{\mathrm{DA}}\right) & \text{otherwise.} \end{cases} \tag{5.17}$$

Let us now introduce the artificial penalties $\phi^{(+)}$ and $\phi^{(+)}$, defined as $\hat{\lambda}^{(+)} - \hat{\lambda}^{\mathrm{DA}}$ and $\hat{\lambda}^{(-)} - \hat{\lambda}^{\mathrm{DA}}$, respectively. They are computed as

$$\phi^{(+)} = \begin{cases} \phi^{\mathrm{BA}}, & \text{if } \phi^{\mathrm{BA}} \leq 0 \\ 0, & \text{otherwise} \end{cases} \tag{5.18}$$

$$\phi^{(-)} = \begin{cases} \phi^{\mathrm{BA}}, & \text{if } \phi^{\mathrm{BA}} \geq 0 \\ 0, & \text{otherwise} \end{cases} \tag{5.19}$$

From the producer's perspective, $\phi^{(+)}$ is the penalty for the creation of a positive imbalance, and $\phi^{(-)}$ is the penalty for the creation of a negative imbalance. The offering strategy of the wind power producer under a dual-price imbalance settlement scheme can consequently be formulated as

$$\underset{\overline{q}^{\mathrm{DA}}}{\text{Max}} \quad \mathbb{E}[\mathcal{L}] \tag{5.20a}$$

$$\text{s.t.} \quad \mathcal{L} = \begin{cases} \hat{\phi}^{(+)}\left(E - \overline{q}^{\mathrm{DA}}\right) & \text{if } E \geq \overline{q}^{\mathrm{DA}}, \\ \hat{\phi}^{(-)}\left(E - \overline{q}^{\mathrm{DA}}\right) & \text{otherwise.} \end{cases} \tag{5.20b}$$

$$0 \leq \overline{q}^{\mathrm{DA}} \leq \overline{E}. \tag{5.20c}$$

Hence, the aim of the wind power producer is to maximize the expectation of $\mathcal{L}$, while imposing that $0 \leq \overline{q}^{\mathrm{DA}} \leq \overline{E}$. The expected values of the imbalance penalties $\phi^{(+)}$ and $\phi^{(-)}$, according to their definition in Equations (5.18) and (5.19), are computed as

$$\hat{\phi}^{(+)} = \int_{-\infty}^{0} \ell\, f_{\phi}^{\mathrm{BA}}(\ell)\, \mathrm{d}\ell, \tag{5.21}$$

$$\hat{\phi}^{(-)} = \int_{0}^{\infty} \ell\, f_{\phi}^{\mathrm{BA}}(\ell)\, \mathrm{d}\ell. \tag{5.22}$$

In the remaining of the chapter, we assume that $\hat{\phi}^{(+)} < 0$ and $\hat{\phi}^{(-)} > 0$, which is a condition usually respected in real electricity markets. It translates to not being sure of the sign of future differential price ϕ^{BA}. Moreover, differently, the optimization problem in (5.20) becomes trivial. Additionally, from here to the remaining of the chapter, the formulation is developed in per unit (p.u.), e.g., $\overline{E} \rightarrow 1$ p.u. The expected imbalance revenue $\hat{\mathcal{L}}$ is evaluated as

$$\begin{aligned} \hat{\mathcal{L}} &= \hat{\phi}^{(-)} \int_{0}^{\overline{q}^{\mathrm{DA}}} \left(\ell - \overline{q}^{\mathrm{DA}}\right)\ f_E(\ell)\ \mathrm{d}\ell + \hat{\phi}^{(+)} \int_{\overline{q}^{\mathrm{DA}}}^{1} \left(\ell - \overline{q}^{\mathrm{DA}}\right)\ f_E(\ell)\ \mathrm{d}\ell \\ &= \hat{\phi}^{(-)} \left(\int_{0}^{\overline{q}^{\mathrm{DA}}} \ell\ f_E(\ell)\ \mathrm{d}\ell - \overline{q}^{\mathrm{DA}} F_E(\overline{q}^{\mathrm{DA}}) \right) + \\ &\quad \hat{\phi}^{(+)} \left(\int_{\overline{q}^{\mathrm{DA}}}^{1} \ell\ f_E(\ell)\ \mathrm{d}\ell - \overline{q}^{\mathrm{DA}} \left(1 - F_E(\overline{q}^{\mathrm{DA}})\right) \right), \end{aligned} \tag{5.23}$$

In order to prove that the optimization problem (5.20) has a unique solution for $\overline{q}^{\mathrm{DA}} \in [0,1]$, we demonstrate that its second derivative with respect to $\overline{q}^{\mathrm{DA}}$ is always negative. The first derivative $\hat{\mathcal{L}}'$ is given by

$$\begin{aligned} \hat{\mathcal{L}}' = \hat{\phi}^{(-)} &\left(\overline{q}^{\mathrm{DA}} f_E(\overline{q}^{\mathrm{DA}}) - F_E(\overline{q}^{\mathrm{DA}}) - \overline{q}^{\mathrm{DA}} f_E(\overline{q}^{\mathrm{DA}})\right) + \\ &\hat{\phi}^{(+)} \left(-\overline{q}^{\mathrm{DA}} f_E(\overline{q}^{\mathrm{DA}}) - \left(1 - F_E(\overline{q}^{\mathrm{DA}})\right) + \overline{q}^{\mathrm{DA}} f_E(\overline{q}^{\mathrm{DA}})\right) \\ = &- \hat{\phi}^{(-)} F_E(\overline{q}^{\mathrm{DA}}) - \hat{\phi}^{(+)} \left(1 - F_E(\overline{q}^{\mathrm{DA}})\right). \end{aligned} \tag{5.24}$$

Then, its second derivative $\hat{\mathcal{L}}''$ is

$$\hat{\mathcal{L}}'' = -\hat{\phi}^{(-)} f_E(\overline{q}^{\mathrm{DA}}) + \hat{\phi}^{(+)} f_E(\overline{q}^{\mathrm{DA}}). \tag{5.25}$$

Since $\hat{\phi}^{(-)} > 0$ and $\hat{\phi}^{(+)} < 0$, it can be inferred that $\hat{\mathcal{L}}'' < 0$ for $\overline{q}^{\mathrm{DA}} \in [0,1]$. Hence, we prove that the value of $\overline{q}^{\mathrm{DA}}$ that maximizes $\mathcal{L}$ is unique. Then, the values of the first derivative $\hat{\mathcal{L}}'$ at the limits, i.e., in 0 and in 1 p.u., are the followings:

$$\hat{\mathcal{L}}' = \begin{cases} -\hat{\phi}^{(+)} \geq 0 & \text{if } \overline{q}^{\mathrm{DA}} = 0, \\ -\hat{\phi}^{(-)} \leq 0 & \text{if } \overline{q}^{\mathrm{DA}} = 1. \end{cases} \tag{5.26}$$

It implies that the optimal solution $\overline{q}^{\mathrm{DA}*}$ lies between 0 and 1 p.u. Such optimal solution can be obtain by imposing $\hat{\mathcal{L}}' = 0$, thus leading to

$$F_E(\overline{q}^{\mathrm{DA}*}) = \frac{-\hat{\phi}^{(+)}}{-\hat{\phi}^{(+)} + \hat{\phi}^{(-)}} = \frac{|\hat{\phi}^{(+)}|}{|\hat{\phi}^{(+)}| + \hat{\phi}^{(-)}}, \tag{5.27}$$

and consequently,

$$\overline{q}^{\mathrm{DA}*} = F_E^{-1}\left(\frac{|\hat{\phi}^{(+)}|}{|\hat{\phi}^{(+)}| + \hat{\phi}^{(-)}}\right). \tag{5.28}$$

The optimal solution $\overline{q}^{\mathrm{DA}*}$ is a quantile of the cumulative distribution function of the wind power production E. The nominal level of this optimal quantile only depends on the expectation of the imbalance penalties. In particular, if $\hat{\phi}^{(-)} > |\hat{\phi}^{(+)}|$ the producer wants to underestimate its future power production, as it is expected to be more penalized for the creation of negative imbalances. Conversely, if $\hat{\phi}^{(-)} < |\hat{\phi}^{(+)}|$ it overestimates the future production as positive imbalances are expected to be more penalized than the negative ones. Finally, if $\hat{\phi}^{(-)} = |\hat{\phi}^{(+)}|$ it offers the median of the distribution of E, as it is expected to be equally penalized for positive and negative imbalances. For a broader analysis on this topic, we refer the interested reader to the work of Bitar et al. (2012).

5.4 Alternative Imbalance Settlement Schemes with Tolerance Band

This section considers two alternative imbalance settlement schemes introduced in the Italian electricity market structure, i.e., the *band-dual* and the *single-dual* imbalance settlement scheme. These schemes introduce a tolerance band anchored to the day-ahead market quantity offer $\overline{q}^{\mathrm{DA}}$. The part of the imbalance that lies within the band is priced differently than the part outside the band. The *band-dual* and the *single-dual* imbalance settlement scheme are presented in Section 5.4.1. Subsequently, Sections 5.4.2 and 5.4.3 derive the optimal offering strategy under the *band-dual* and the *single-dual* scheme, respectively. Finally, Section 5.4.4 links the optimal offering strategy with the expected real-time imbalances of the wind power producer.

5.4.1 Band-dual & Single-dual Imbalance Settlement Schemes

Two different imbalance settlement schemes are considered in the Italian regulation, and both of them introduce a tolerance band of width $2\tau\overline{q}^{\mathrm{DA}}$ around the level of energy scheduled in the day-ahead market. The part of the imbalance that exceeds such band is priced under a dual-price settlement scheme, for both the settlement schemes. However, in the *band-dual* scheme, the portion of the imbalance within the band is priced at the day-ahead market price. Differently, in the *single-dual* scheme, the balancing market price is used to price the part of the imbalance within the band. Let $\overline{\tau} = 1 + \tau$ and $\underline{\tau} = 1 - \tau$ be the upper and lower relative margins of the band, respectively. Consequently, the balancing market revenue associated with the real-time deviation q^{BA} under the *band-dual* settlement scheme, i.e., ρ^{BD}, is evaluated as

$$\rho^{\mathrm{BD}} = \begin{cases} \lambda^{(+)}\left(E - \overline{\tau}\overline{q}^{\mathrm{DA}}\right) + \lambda^{\mathrm{DA}}\left(\overline{\tau}\overline{q}^{\mathrm{DA}} - \overline{q}^{\mathrm{DA}}\right), & \text{if } E \geq \overline{\tau}\overline{q}^{\mathrm{DA}}, \\ \lambda^{(-)}\left(E - \underline{\tau}\overline{q}^{\mathrm{DA}}\right) + \lambda^{\mathrm{DA}}\left(\underline{\tau}\overline{q}^{\mathrm{DA}} - \overline{q}^{\mathrm{DA}}\right), & \text{if } E \leq \underline{\tau}\overline{q}^{\mathrm{DA}}, \\ \lambda^{\mathrm{DA}}\left(E - \overline{q}^{\mathrm{DA}}\right), & \text{otherwise.} \end{cases} \tag{5.29}$$

Indeed, when the imbalance $E - \overline{q}^{\mathrm{DA}}$ lies within the tolerance band $\left[\underline{\tau}\overline{q}^{\mathrm{DA}}, \overline{\tau}\overline{q}^{\mathrm{DA}}\right]$, it is priced at λ^{DA}. Differently, when it exceeds the band (e.g., when $E > \overline{\tau}\overline{q}^{\mathrm{DA}}$), the portion of the imbalance within the band ($\overline{\tau}\overline{q}^{\mathrm{DA}} - \overline{q}^{\mathrm{DA}}$) is priced at λ^{DA}, while the exceeding part ($E - \overline{\tau}\overline{q}^{\mathrm{DA}}$) under a dual-price settlement scheme (i.e., $\lambda^{(+)}$ in this example). Conversely, under the *single-dual* scheme, the balancing market revenue, denoted with ρ^{SD}, is given by

$$\rho^{\mathrm{SD}} = \begin{cases} \lambda^{(+)}\left(E - \overline{\tau}\overline{q}^{\mathrm{DA}}\right) + \lambda^{\mathrm{BA}}\left(\overline{\tau}\overline{q}^{\mathrm{DA}} - \overline{q}^{\mathrm{DA}}\right), & \text{if } E \geq \overline{\tau}\overline{q}^{\mathrm{DA}}, \\ \lambda^{(-)}\left(E - \underline{\tau}\overline{q}^{\mathrm{DA}}\right) + \lambda^{\mathrm{BA}}\left(\underline{\tau}\overline{q}^{\mathrm{DA}} - \overline{q}^{\mathrm{DA}}\right), & \text{if } E \leq \underline{\tau}\overline{q}^{\mathrm{DA}}, \\ \lambda^{\mathrm{BA}}\left(E - \overline{q}^{\mathrm{DA}}\right), & \text{otherwise,} \end{cases} \tag{5.30}$$

as in this case the portion of the imbalance $E - \overline{q}^{\mathrm{DA}}$ that falls outside the tolerance band $\left[\underline{\tau}\overline{q}^{\mathrm{DA}}, \overline{\tau}\overline{q}^{\mathrm{DA}}\right]$ is priced as λ^{BA}.

Similar to Section 5.3.3, we want to formulate the optimal offering strategy by rearranging the profit formulation and removing the profit in case of perfect information, i.e., $\lambda^{\mathrm{DA}} E$. This leads to

$$\underset{\overline{q}^{\mathrm{DA}}}{\mathrm{Max}} \quad \mathbb{E}[\mathcal{L}] \tag{5.31a}$$

$$\text{s.t.} \quad 0 \leq \overline{q}^{\mathrm{DA}} \leq 1, \tag{5.31b}$$

where the imbalance revenue under a *band-dual* settlement scheme, denoted as $\mathcal{L}^{\mathrm{BD}}$, is evaluated as

$$\mathcal{L}^{\mathrm{BD}} = \begin{cases} \phi^{(+)} \left(E - \overline{\tau}\overline{q}^{\mathrm{DA}}\right), & \text{if } E \geq \overline{\tau}\overline{q}^{\mathrm{DA}}, \\ \phi^{(-)} \left(E - \underline{\tau}\overline{q}^{\mathrm{DA}}\right), & \text{if } E \leq \underline{\tau}\overline{q}^{\mathrm{DA}}, \\ 0, & \text{otherwise.} \end{cases} \tag{5.32}$$

Differently, under the *single-dual* pricing scheme, the imbalance revenue $\mathcal{L}^{\mathrm{SD}}$ is computed as

$$\mathcal{L}^{\mathrm{SD}} = \begin{cases} \phi^{(+)} \left(E - \overline{\tau}\overline{q}^{\mathrm{DA}}\right) + \phi^{\mathrm{BA}} \left(\overline{\tau}\overline{q}^{\mathrm{DA}} - \overline{q}^{\mathrm{DA}}\right), & \text{if } E \geq \overline{\tau}\overline{q}^{\mathrm{DA}}, \\ \phi^{(-)} \left(E - \underline{\tau}\overline{q}^{\mathrm{DA}}\right) + \phi^{\mathrm{BA}} \left(\underline{\tau}\overline{q}^{\mathrm{DA}} - \overline{q}^{\mathrm{DA}}\right), & \text{if } E \leq \underline{\tau}\overline{q}^{\mathrm{DA}}, \\ \phi^{\mathrm{BA}} \left(E - \overline{q}^{\mathrm{DA}}\right), & \text{otherwise.} \end{cases} \tag{5.33}$$

Given the expected values of the imbalance penalties, i.e., $\hat{\phi}^{(+)}$, $\hat{\phi}^{(-)}$, and $\hat{\phi}^{\mathrm{BA}}$, and for a fixed value of $\overline{q}^{\mathrm{DA}}$, the power producer can compute the expected imbalance revenue $\hat{\mathcal{L}}$ as function of the future wind power production E. Figure 5.2 shows the expected imbalance revenue as function of E for three different imbalance settlement schemes, i.e., the dual-price (red), the *band-dual* (blue), and the *single-dual* (green) scheme. It is worth mentioning that the value of $\mathcal{L}$ in both the dual-price and *band-dual* scheme is always lower or equal to 0. Therefore, it decreases the total profit of the producer. Conversely, under the *single-dual* scheme the term $\mathcal{L}$ can be positive and increase the profit of the producer.

5.4.2 Optimal Offering under a Band-dual Settlement Scheme

This section derives the optimal offering strategy under the *band-dual* imbalance settlement scheme. We will proceed similarly to what we did for the dual-price imbalance settlement scheme in Section 5.3.3. First, we prove the uniqueness of the solution by showing that $\left(\hat{\mathcal{L}}^{\mathrm{BD}}\right)'' < 0$ for $\overline{q}^{\mathrm{DA}} \in [0, 1]$. The expected imbalance

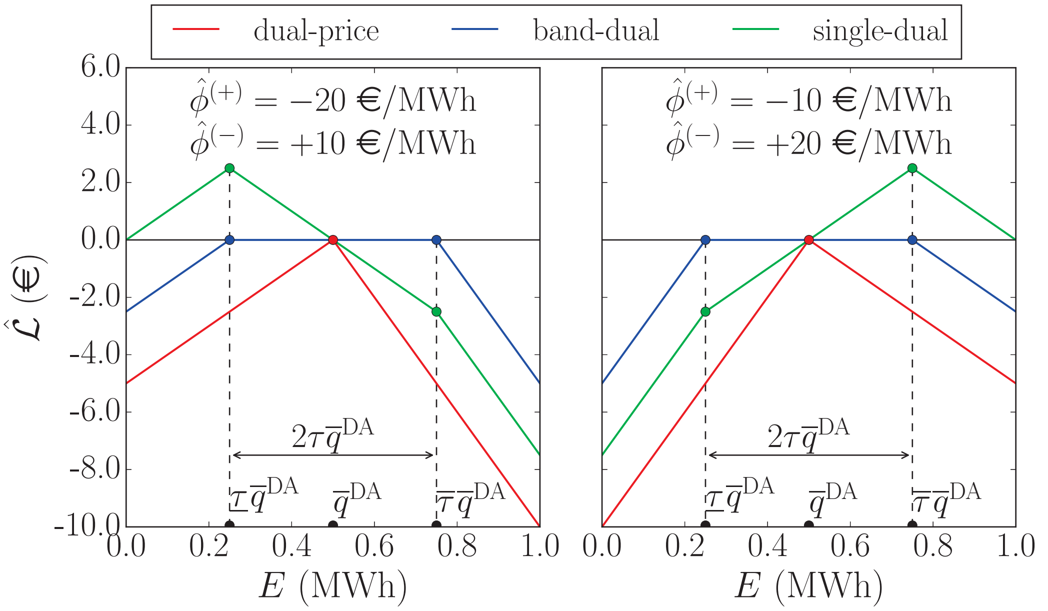


FIGURE 5.2: Example of the expected imbalance cost as function of the real-time wind power production E for different imbalance settlement schemes.

revenue $\hat{\mathcal{L}}^{\mathrm{BD}}$ can be computed, following Equation (5.32), as

$$\begin{aligned}
\hat{\mathcal{L}}^{\mathrm{BD}} &= \hat{\phi}^{(-)} \int_0^{\underline{\tau}\overline{q}^{\mathrm{DA}}} \left(\ell - \underline{\tau}\overline{q}^{\mathrm{DA}}\right) \, f_E(\ell)\, \mathrm{d}\ell + \hat{\phi}^{(+)} \int_{\overline{\tau}\overline{q}^{\mathrm{DA}}}^{1} \left(\ell - \overline{\tau}\overline{q}^{\mathrm{DA}}\right) \, f_E(\ell)\, \mathrm{d}\ell \\
&= \hat{\phi}^{(-)} \left(\int_0^{\underline{\tau}\overline{q}^{\mathrm{DA}}} \ell \, f_E(\ell)\, \mathrm{d}\ell - \underline{\tau}\overline{q}^{\mathrm{DA}} F_E(\underline{\tau}\overline{q}^{\mathrm{DA}}) \right) + \\
&\quad \hat{\phi}^{(+)} \left(\int_{\overline{\tau}\overline{q}^{\mathrm{DA}}}^{1} \ell \, f_E(\ell)\, \mathrm{d}\ell - \overline{\tau}\overline{q}^{\mathrm{DA}} \left(1 - F_E(\overline{\tau}\overline{q}^{\mathrm{DA}})\right) \right).
\end{aligned} \tag{5.34}$$

Then, the first derivative $\left(\hat{\mathcal{L}}^{\mathrm{BD}}\right)'$ with respect to $\overline{q}^{\mathrm{DA}}$ is given by

$$\begin{aligned}
\left(\hat{\mathcal{L}}^{\mathrm{BD}}\right)' &= \hat{\phi}^{(-)} \left(\underline{\tau}^2 \overline{q}^{\mathrm{DA}} f_E(\underline{\tau}\overline{q}^{\mathrm{DA}}) - \underline{\tau} F_E(\underline{\tau}\overline{q}^{\mathrm{DA}}) - \underline{\tau}^2 \overline{q}^{\mathrm{DA}} f_E(\underline{\tau}\overline{q}^{\mathrm{DA}})\right) + \\
&\quad \hat{\phi}^{(+)} \left(-\overline{\tau}^2 \overline{q}^{\mathrm{DA}} f_E(\overline{\tau}\overline{q}^{\mathrm{DA}}) - \overline{\tau}\left(1 - F_E(\overline{\tau}\overline{q}^{\mathrm{DA}})\right) + \overline{\tau}^2 \overline{q}^{\mathrm{DA}} f_E(\overline{\tau}\overline{q}^{\mathrm{DA}})\right) \\
&= -\hat{\phi}^{(-)} \underline{\tau} F_E(\underline{\tau}\overline{q}^{\mathrm{DA}}) - \hat{\phi}^{(+)} \overline{\tau}\left(1 - F_E(\overline{\tau}\overline{q}^{\mathrm{DA}})\right),
\end{aligned} \tag{5.35}$$

and its second derivative $\left(\hat{\mathcal{L}}^{\mathrm{BD}}\right)''$ by

$$\left(\hat{\mathcal{L}}^{\mathrm{BD}}\right)'' = -\hat{\phi}^{(-)} \underline{\tau}^2 f_E(\underline{\tau}\overline{q}^{\mathrm{DA}}) + \hat{\phi}^{(+)} \overline{\tau}^2 f_E(\overline{\tau}\overline{q}^{\mathrm{DA}}). \tag{5.36}$$

Since $\hat{\phi}^{(-)} > 0$ and $\hat{\phi}^{(+)} < 0$, it follows that $\left(\hat{\mathcal{L}}^{\mathrm{BD}}\right)'' < 0$, i.e., the objective function $\hat{\mathcal{L}}^{\mathrm{BD}}$ to be maximized is concave. Then, we prove that the optimal solution lies in $[0, 1]$ as the following border conditions hold:

$$\left(\hat{\mathcal{L}}^{\mathrm{BD}}\right)' = \begin{cases} -\overline{\tau}\hat{\phi}^{(+)} \geq 0 & \text{if } \overline{q}^{\mathrm{DA}} = 0 \\ -\underline{\tau}\hat{\phi}^{(-)} \leq 0 & \text{if } \overline{q}^{\mathrm{DA}} = 1 \end{cases} \tag{5.37}$$

In this case, we can even restrict this region to $[0, 1/\overline{\tau}]$. Indeed, when $\overline{q}^{\mathrm{DA}} = 1/\overline{\tau}$, the upper limit of the unpenalized band of width $2\tau\overline{q}^{\mathrm{DA}}$ is the unit's capacity, i.e., 1 p.u. Offering at $\overline{q}^{\mathrm{DA}} > 1/\overline{\tau}$ would result in "losing" the part of the unpenalized band that exceeds the unit's capacity. This can be proved by showing that

$$\left(\hat{\mathcal{L}}^{\mathrm{BD}}\right)' = -\underline{\tau}\hat{\phi}^{(-)}F_E\Big(\frac{\underline{\tau}}{\overline{\tau}}\Big) \leq 0 \qquad \text{if } \overline{q}^{\mathrm{DA}} = \frac{1}{\overline{\tau}}. \tag{5.38}$$

The optimal solution $\overline{q}^{\mathrm{DA}*} \in [0, 1/\overline{\tau}]$ is obtained by imposing $\left(\hat{\mathcal{L}}^{\mathrm{BD}}\right)' = 0$. This leads to

$$-\hat{\phi}^{(-)}\underline{\tau}F_E(\underline{\tau}\,\overline{q}^{\mathrm{DA}*}) - \hat{\phi}^{(+)}\overline{\tau}\left(1 - F_E(\overline{\tau}\,\overline{q}^{\mathrm{DA}*})\right) = 0. \tag{5.39}$$

By solving the non-linear Equation (5.39) the wind power producer evaluates its optimal quantity offer $\overline{q}^{\mathrm{DA}*}$ to be submitted in the day-ahead market.

5.4.3 Optimal Offering under a Single-Dual Settlement Scheme

This section derives the optimal offering strategy under the *single-dual* imbalance settlement scheme. First, we prove that the market quantity that maximizes the producer's imbalance revenue is unique. To do so, we show that $\left(\hat{\mathcal{L}}^{\mathrm{SD}}\right)'' < 0$ for $\overline{q}^{\mathrm{DA}} \in [0, 1]$. The expected imbalance revenue $\hat{\mathcal{L}}^{\mathrm{SD}}$ can be evaluated, in accordance with Equation (5.33), as

$$
\begin{aligned}
\hat{\mathcal{L}}^{\mathrm{SD}} = \hat{\phi}^{(-)} &\int_0^{\underline{\tau}\overline{q}^{\mathrm{DA}}} \left(\ell - \underline{\tau}\overline{q}^{\mathrm{DA}}\right) \, f_E(\ell) \, \mathrm{d}\ell + \hat{\phi}^{(+)} \int_{\overline{\tau}\overline{q}^{\mathrm{DA}}}^{1} \left(\ell - \overline{\tau}\overline{q}^{\mathrm{DA}}\right) \, f_E(\ell) \, \mathrm{d}\ell + \\
&\hat{\phi}^{\mathrm{BA}} \Bigg(\int_0^{\underline{\tau}\overline{q}^{\mathrm{DA}}} \left(\underline{\tau}\overline{q}^{\mathrm{DA}} - \overline{q}^{\mathrm{DA}}\right) \, f_E(\ell) \, \mathrm{d}\ell + \int_{\underline{\tau}\overline{q}^{\mathrm{DA}}}^{\overline{\tau}\overline{q}^{\mathrm{DA}}} \left(\ell - \overline{q}^{\mathrm{DA}}\right) \, f_E(\ell) \, \mathrm{d}\ell + \\
&\int_{\overline{\tau}\overline{q}^{\mathrm{DA}}}^{1} \left(\overline{\tau}\overline{q}^{\mathrm{DA}} - \overline{q}^{\mathrm{DA}}\right) \, f_E(\ell) \, \mathrm{d}\ell \Bigg) \\
= \hat{\mathcal{L}}^{\mathrm{BD}} &+ \hat{\phi}^{\mathrm{BA}} \left(\underline{\tau}\overline{q}^{\mathrm{DA}} - \overline{q}^{\mathrm{DA}}\right) F_E(\underline{\tau}\overline{q}^{\mathrm{DA}}) + \hat{\phi}^{\mathrm{BA}} \left(\overline{\tau}\overline{q}^{\mathrm{DA}} - \overline{q}^{\mathrm{DA}}\right) \left(1 - F_E(\overline{\tau}\overline{q}^{\mathrm{DA}})\right) + \\
&\hat{\phi}^{\mathrm{BA}} \left(\int_{\underline{\tau}\overline{q}^{\mathrm{DA}}}^{\overline{\tau}\overline{q}^{\mathrm{DA}}} \ell \, f_E(\ell) \, \mathrm{d}\ell - \overline{q}^{\mathrm{DA}} \left(F_E(\overline{\tau}\overline{q}^{\mathrm{DA}}) - F_E(\underline{\tau}\overline{q}^{\mathrm{DA}})\right) \right) \\
= \hat{\mathcal{L}}^{\mathrm{BD}} &+ \hat{\phi}^{\mathrm{BA}} \Bigg(\underline{\tau}\overline{q}^{\mathrm{DA}} F_E(\underline{\tau}\overline{q}^{\mathrm{DA}}) - \overline{\tau}\overline{q}^{\mathrm{DA}} F_E(\overline{\tau}\overline{q}^{\mathrm{DA}}) + \left(\overline{\tau}\overline{q}^{\mathrm{DA}} - \overline{q}^{\mathrm{DA}}\right) + \\
&\int_{\underline{\tau}\overline{q}^{\mathrm{DA}}}^{\overline{\tau}\overline{q}^{\mathrm{DA}}} \ell \, f_E(\ell) \, \mathrm{d}\ell \Bigg)
\end{aligned}
\tag{5.40}
$$

Its first derivative $\left(\hat{\mathcal{L}}^{\mathrm{SD}}\right)'$ with respect to $\overline{q}^{\mathrm{DA}}$ is computed as

$$
\begin{aligned}
\left(\hat{\mathcal{L}}^{\mathrm{SD}}\right)' = &\left(\hat{\mathcal{L}}^{\mathrm{BD}}\right)' + \hat{\phi}^{\mathrm{BA}} \big(\underline{\tau} F_E(\underline{\tau}\overline{q}^{\mathrm{DA}}) + \underline{\tau}^2 \overline{q}^{\mathrm{DA}} f_E(\underline{\tau}\overline{q}^{\mathrm{DA}}) - \overline{\tau} F_E(\overline{\tau}\overline{q}^{\mathrm{DA}}) - \\
&\overline{\tau}^2 \overline{q}^{\mathrm{DA}} f_E(\overline{\tau}\overline{q}^{\mathrm{DA}}) + \overline{\tau} - 1 + \overline{\tau}^2 \overline{q}^{\mathrm{DA}} f_E(\overline{\tau}\overline{q}^{\mathrm{DA}}) - \underline{\tau}^2 \overline{q}^{\mathrm{DA}} f_E(\underline{\tau}\overline{q}^{\mathrm{DA}}) \big) \\
= &\left(\hat{\mathcal{L}}^{\mathrm{BD}}\right)' + \hat{\phi}^{\mathrm{BA}} \big(-1 + \overline{\tau} - \overline{\tau} F_E(\overline{\tau}\overline{q}^{\mathrm{DA}}) + \underline{\tau} F_E(\underline{\tau}\overline{q}^{\mathrm{DA}}) \big)
\end{aligned}
\tag{5.41}
$$

Then, we replace the term $\left(\hat{\mathcal{L}}^{\mathrm{BD}}\right)'$ in Equation (5.41) with the formulation provided in Equation (5.35). Moreover, according to the definition of $\hat{\phi}^{(+)}$ and $\hat{\phi}^{(-)}$ in Equations (5.21) and (5.22), the following equality holds:

$$
\hat{\phi}^{\mathrm{BA}} = \hat{\phi}^{(+)} + \hat{\phi}^{(-)}. \tag{5.42}
$$

This leads to

$$
\begin{aligned}
\left(\hat{\mathcal{L}}^{\mathrm{SD}}\right)' = &-\hat{\phi}^{(-)} \underline{\tau} F_E(\underline{\tau}\overline{q}^{\mathrm{DA}}) - \hat{\phi}^{(+)} \overline{\tau} \left(1 - F_E(\overline{\tau}\overline{q}^{\mathrm{DA}})\right) - \hat{\phi}^{\mathrm{BA}} + \\
&\left(\hat{\phi}^{(-)} + \hat{\phi}^{(+)}\right) \left(\overline{\tau} - \overline{\tau} F_E(\overline{\tau}\overline{q}^{\mathrm{DA}}) + \underline{\tau} F_E(\underline{\tau}\overline{q}^{\mathrm{DA}})\right) \\
= &-\hat{\phi}^{\mathrm{BA}} + \hat{\phi}^{(-)} \overline{\tau} \left(1 - F_E(\overline{\tau}\overline{q}^{\mathrm{DA}})\right) + \hat{\phi}^{(+)} \underline{\tau} F_E(\underline{\tau}\overline{q}^{\mathrm{DA}})
\end{aligned}
\tag{5.43}
$$

The second derivative $\left(\hat{\mathcal{L}}^{\mathrm{SD}}\right)''$ can be evaluated as

$$\left(\hat{\mathcal{L}}^{\mathrm{SD}}\right)'' = -\hat{\phi}^{(-)}\overline{\tau}^2 f_E(\overline{\tau}\,\overline{q}^{\mathrm{DA}}) + \hat{\phi}^{(+)}\underline{\tau}^2 f_E(\underline{\tau}\,\overline{q}^{\mathrm{DA}}). \tag{5.44}$$

Also under the *single-dual* settlement scheme, the objective function (i.e., the imbalance revenue) is concave, thus proving the uniqueness of the optimal solution $\overline{q}^{\mathrm{DA}*}$. Then, we investigate if such optimal solution lies in the feasibility region $[0,1]$. To do so, we check the sign of the first derivative at the borders. For the lower bound, i.e., $\overline{q}^{\mathrm{DA}} = 0$, we obtain

$$\left(\hat{\mathcal{L}}^{\mathrm{SD}}\right)' = -\hat{\phi}^{(+)} + \hat{\phi}^{(-)}\left(\overline{\tau} - 1\right) \geq 0 \qquad \text{if } \overline{q}^{\mathrm{DA}} = 0. \tag{5.45}$$

Consequently, the optimal solution $\overline{q}^{\mathrm{DA}*}$ is greater or equal to 0. Conversely, for the upper bound, i.e., $\overline{q}^{\mathrm{DA}} = 1$, the value of the first derivative $\left(\hat{\mathcal{L}}^{\mathrm{SD}}\right)'$ is

$$\left(\hat{\mathcal{L}}^{\mathrm{SD}}\right)' = -\hat{\phi}^{(-)} - \hat{\phi}^{(+)}\left(\underline{\tau} + 1\right) \qquad \text{if } \overline{q}^{\mathrm{DA}} = 1. \tag{5.46}$$

In this case, $\left(\hat{\mathcal{L}}^{\mathrm{SD}}\right)' \leq 0$ if and only if $\hat{\phi}^{(-)} \geq -\hat{\phi}^{(+)}\left(\underline{\tau}+1\right)$. So, if $\hat{\phi}^{(-)} \geq -\hat{\phi}^{(+)}\left(\underline{\tau}+1\right)$ the optimal solution $\overline{q}^{\mathrm{DA}*}$ lies in $[0,1]$. Differently, if $\hat{\phi}^{(-)} < -\hat{\phi}^{(+)}\left(\underline{\tau}+1\right)$, the optimal solution $\overline{q}^{\mathrm{DA}*}$ is greater than 1 p.u. However, the feasibility region $[0,1]$ can be artificially imposed, thus obtaining the optimal market offer $\overline{q}^{\mathrm{DA}*}$ as

$$\overline{q}^{\mathrm{DA}*} = \begin{cases} \overline{q}^{\mathrm{DA}} \mid \left(\hat{\mathcal{L}}^{\mathrm{SD}}\right)' = 0, & \text{if} \quad \hat{\phi}^{(-)} \geq -\hat{\phi}^{(+)}\left(\underline{\tau}+1\right) \\ 1, & \text{otherwise} \end{cases} \tag{5.47}$$

The idea is to check if the solution lies or not in $[0,1]$. If this is true, we solve the following equation to compute the optimal market offer $\overline{q}^{\mathrm{DA}*}$:

$$-\hat{\phi}^{\mathrm{BA}} + \hat{\phi}^{(-)}\overline{\tau}\left(1 - F_E(\overline{\tau}\,\overline{q}^{\mathrm{DA}*})\right) + \hat{\phi}^{(+)}\underline{\tau} F_E(\underline{\tau}\,\overline{q}^{\mathrm{DA}*}) = 0. \tag{5.48}$$

Differently, we impose $\overline{q}^{\mathrm{DA}*} = 1$ p.u.

5.4.4 Effect on the Expected Imbalances

Decisions on the quantities offered in the day-ahead market do not only affect the imbalance revenues of the wind producers. Indeed, as the deviation q^{BA} is obtained from the difference of the real-time production E and the day-ahead market offer $\overline{q}^{\mathrm{DA}}$, the producers' real-time imbalances are influenced by their day-ahead offering strategy. Decisions that are optimal for the producers, who are only seeking in maximizing their profit, may not be optimal from a system perspective. Given the optimal quantity $\overline{q}^{\mathrm{DA}*}$ contracted in the day-ahead market, we can indeed evaluate the deviation that the stochastic power producer is expecting to create in the real-time. Let us denote with $\hat{\Delta}$ the expected size of the real-time imbalance and with $\hat{\psi}$ the ratio between the expected imbalance penalties, i.e.,

$$\hat{\psi} = \frac{\hat{\phi}^{(-)}}{|\hat{\phi}^{(+)}|}. \tag{5.49}$$

Moreover, we introduce the function $\theta(\tau, \hat{\psi}, \mu_E)$ which gives the optimal day-ahead market offer $\overline{q}^{\mathrm{DA}*}$, depending on the imbalance settlement scheme considered. For given τ, $\hat{\psi}$ and μ_E, the expected size of the real-time imbalance $\hat{\Delta}$ can be computed as

$$\hat{\Delta} \,\Big|_{\tau,\hat{\psi},\mu_E} = \int_0^1 \big|\, \ell - \theta(\tau, \hat{\psi}, \mu_E) \,\big|\, f(\ell, \mu_E) \, \mathrm{d}\ell. \tag{5.50}$$

where $f(\ell, \mu_E)$ has been defined in Section 5.2, together with the probability density function of μ_E, i.e., f_μ. Given the probability distribution of μ_E, we evaluate $\hat{\Delta}$ for given values of τ and $\hat{\psi}$ as

$$\hat{\Delta} \,\Big|_{\tau,\hat{\psi}} = \int_0^1 \int_0^1 \big|\, \ell - \theta(\tau, \hat{\psi}, y) \,\big|\, f(\ell, y) \; f_\mu(y) \, \mathrm{d}\ell \, \mathrm{d}y. \tag{5.51}$$

where y is an additional integration variable. The value of $\hat{\Delta}$ for different values of the tolerance margin τ and different imbalance settlement schemes can be a useful indicator to evaluate the effect of the tolerance band on the real-time imbalances of the producers. However, Equation (5.51) computes the expected size of the producer's imbalance, disregarding its sign. Besides, it does not distinguish imbalances that may "help" in restoring the system's imbalance from imbalances that may contribute to increasing it. Indeed, a market structure that pushes stochastic producers in having higher expected sizes of their imbalances but more likely to be in opposite sign with the system's one, may be acceptable from a system perspective. Therefore, it is interesting to distinguish between imbalances that are expected to reduce or to increase the imbalance of the system. To do so, we initially evaluate the expected size of the positive and the negative imbalance, i.e., $\hat{\Delta}^{(+)}$ and $\hat{\Delta}^{(-)}$, respectively. For given τ and $\hat{\psi}$, they are computed as

$$\hat{\Delta}^{(+)} \mid_{\tau,\hat{\psi}} = \int_0^1 \int_{\theta(\tau,\hat{\psi},y)}^1 \Big(\ell - \theta(\tau,\hat{\psi},y)\Big)\, f(\ell,y)\; f_\mu(y)\; \mathrm{d}\ell\; \mathrm{d}y, \tag{5.52}$$

$$\hat{\Delta}^{(-)} \mid_{\tau,\hat{\psi}} = \int_0^1 \int_0^{\theta(\tau,\hat{\psi},y)} \Big(\theta(\tau,\hat{\psi},y) - \ell\Big)\, f(\ell,y)\; f_\mu(y)\; \mathrm{d}\ell\; \mathrm{d}y. \tag{5.53}$$

Then, let $\hat{\Delta}^{(\uparrow)}$ and $\hat{\Delta}^{(\downarrow)}$ be the sizes of imbalances that are expected to "help" and "hurt" the system, respectively. Helping the system means that the producer's deviation is of opposite sign to the system's one, thus reducing it. Differently, a deviation that is expected to hurt the system has the same sign of the system's imbalance and consequently increases it. To evaluate such expected imbalances we need additional information regarding the probability of the system to require upward or downward regulation. This information is partly enclosed in the expected imbalance penalties $\hat{\phi}^{(+)}$ and $\hat{\phi}^{(-)}$. Indeed, the penalty $\hat{\phi}^{(+)}$ can be evaluated as the product of the probability of the system to be in up-regulation (i.e., $\hat{\pi}^{(+)}$), and the expected penalty given that the system is in up-regulation (i.e., $\hat{\phi}^{(+|+)}$). The imbalance penalty $\hat{\phi}^{(-)}$ is estimated similarly. This writes

$$\hat{\phi}^{(+)} = \hat{\pi}^{(+)}\; \hat{\phi}^{(+|+)} \tag{5.54}$$

$$\hat{\phi}^{(-)} = \hat{\pi}^{(-)}\; \hat{\phi}^{(-|-)} \tag{5.55}$$

Therefore, if we assume that $\hat{\phi}^{(+|+)} = -\hat{\phi}^{(-|-)}$, asymmetries in the imbalance penalties can be directly linked to asymmetries in the system's probability to require upward or downward regulation. For instance, if $\hat{\phi}^{(-)} = 10$ €/MWh and $\hat{\phi}^{(+)} = -20$ €/MWh (i.e., $\hat{\psi} = 0.5$), the probability $\hat{\pi}^{(-)}$ and $\hat{\pi}^{(+)}$ would be 33.3% and 66.7%, respectively. The expected values $\hat{\Delta}^{(\uparrow)}$ and $\hat{\Delta}^{(\downarrow)}$ are computed as

$$\hat{\Delta}^{(\uparrow)} = \hat{\pi}^{(-)}\hat{\Delta}^{(+)} + \hat{\pi}^{(+)}\hat{\Delta}^{(-)}, \tag{5.56}$$

$$\hat{\Delta}^{(\downarrow)} = \hat{\pi}^{(+)}\hat{\Delta}^{(+)} + \hat{\pi}^{(-)}\hat{\Delta}^{(-)}. \tag{5.57}$$

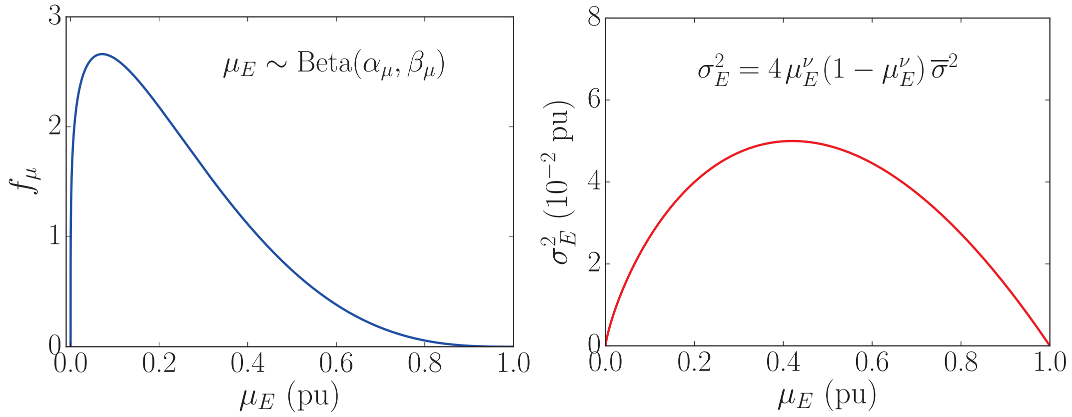
5.5 Analysis

This section presents the results of the optimal offering strategy for different tolerance margin τ and the associated expected real-time imbalances. All the results are provided in p.u. The parameters defining the wind forecast model, i.e., Equations

(5.2) and (5.3), are shown in Table 5.1. Figure 5.3a illustrates the probability density function of μ_E and Figure 5.3b the value of σ_E^2 as function of μ_E.

TABLE 5.1: Parameters of wind power forecasts

α_μ	β_μ	$\overline{\sigma^2}$	ν
1.22	3.86	0.05	0.8



(A) Illustration of the probability distribution of μ_E.

(B) Illustration of the correlation between σ_E^2 and μ_E, controlled by the parameter ν.

FIGURE 5.3: Images related to the wind forecast modeling.

The remaining of the section is organized as follows. Section 5.5.1 provides an analysis of the optimal offering strategies under the *band-dual* and the *single-dual* imbalance pricing scheme. These optimal market quantities are linked to the expected real-time imbalance of the price-taker producer in Section 5.5.2.

5.5.1 Analysis of the Optimal Offering Strategies

Section 5.3.3 presented the optimization problem that the wind power producer would solve for deriving its best day-ahead market offer when the balancing stage is settled under a dual-pricing scheme. We showed that the optimal quantity $\overline{q}^{\text{DA}*}$ is a quantile of the probability distribution of the future power production E. Moreover, we showed that the nominal level of such optimal quantile only depends on the expected imbalance penalties $\hat{\phi}^{(+)}$ and $\hat{\phi}^{(-)}$. Figure 5.4 illustrates the nominal level of the optimal quantile, i.e., $F_E\big(\overline{q}^{\text{DA}*}\big)$, as function of μ_E, for values of the imbalance penalty ratio $\hat{\psi}$ of 0.5, 1, and 2, respectively. The case without tolerance band (i.e., $\tau = 0$) is represented with red squares. In accordance with Equation (5.27), $F_E\big(\overline{q}^{\text{DA}*}\big)$ is constant for $\mu_E \in [0, 1]$, since its optimal value is uniquely determined by the penalty ratio $\hat{\psi}$. Differently, when a tolerance band of width $2\tau\overline{q}^{\mathrm{DA}}$ is introduced within the penalty scheme, $F_E(\overline{q}^{\mathrm{DA}*})$ is influenced by the shape of the probability density function f_E. The optimal values of $F_E(\overline{q}^{\mathrm{DA}*})$ under the *band-dual* settlement scheme are shown in Figure 5.4 with blue dots, from light blue for $\tau = 0.1$ to dark blue for $\tau = 0.5$. Notice that the tolerance band incentives the power producer to overestimate its future power production, for low values of μ_E, compared to the case with $\tau = 0$. Indeed, since the width of the tolerance band is proportional to $\overline{q}^{\mathrm{DA}}$, it is convenient to overestimate the power production in order to gain a wider tolerance band. Conversely, as μ_E gets closer to 1 p.u., the optimal trading strategy suggests to underestimate the power production, compared to the dual-price settlement scheme. Indeed, we showed that, under the *band-dual* settlement scheme, the optimal solution $\overline{q}^{\mathrm{DA}*}$ lies in the interval $[0, 1/\overline{\tau}]$, since when $\overline{q}^{\mathrm{DA}*} = 1/\overline{\tau}$ the upper limit of the band $\overline{\tau}\overline{q}^{\mathrm{DA}*}$ is 1 p.u. (i.e., the capacity of the unit) and there is no advantage in offering higher values of $\overline{q}^{\mathrm{DA}*}$.

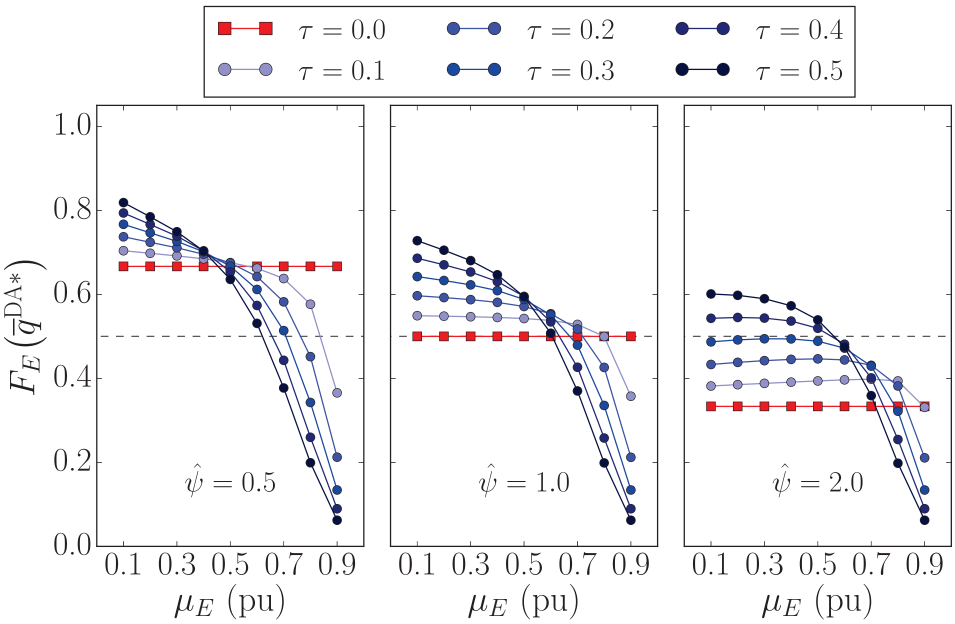


FIGURE 5.4: Nominal level of the optimal quantile, i.e., $F_E(\overline{q}^{\mathrm{DA}*})$, as function of μ_E. We consider a *band-dual* scheme for different values of τ and $\hat{\psi}$.

Figure 5.5 shows the difference between the optimal market quantity $\overline{q}^{\mathrm{DA}*}$ and the expected value μ_E as function of μ_E, for values of the imbalance penalty ratio $\hat{\psi}$ of 0.5, 1, and 2, respectively. The case of $\tau = 0$ is illustrated with red squares, while the cases with tolerance band (i.e., $\tau > 0$) are shown with blue dots, darker as the value of τ increases. The figure shows how the producer is over/underestimating its future power production in terms of quantity. Positive values in the ordinate indicate that the producer is willing to offer more than the expected value μ_E in the day-ahead market. Conversely, negative values show that the optimal day-ahead market offer $\overline{q}^{\text{DA}*}$ is lower than μ_E. Also in Figure 5.5 we can notice that the tolerance band pushes the stochastic producer to increase its market offer $\overline{q}^{\text{DA}*}$ when $\mu_E \to 0$ p.u. and to decrease it when $\mu_E \to 1$ p.u.

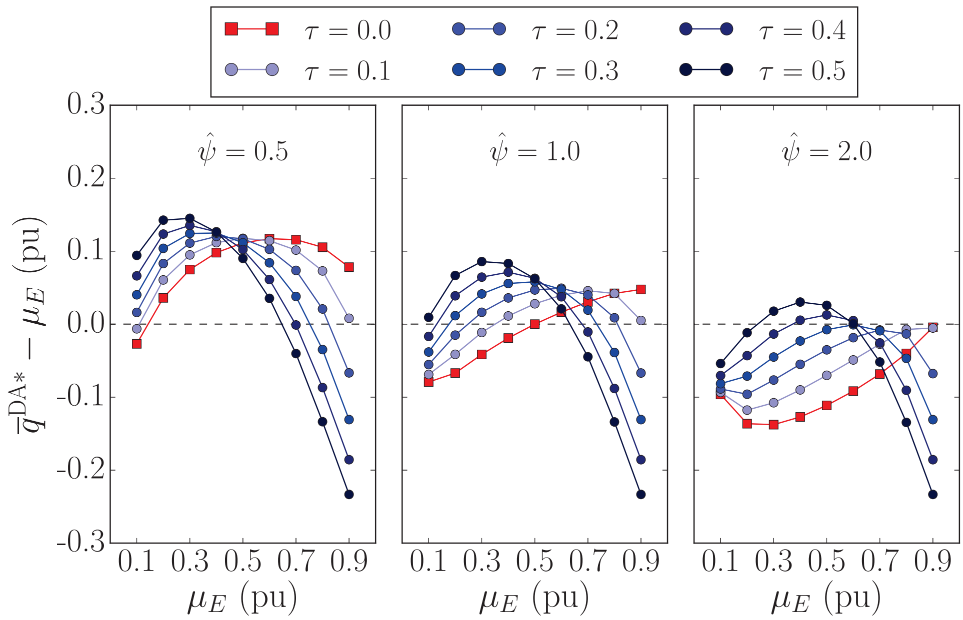


FIGURE 5.5: Difference between the optimal market quantity $\overline{q}^{\text{DA}*}$ and the expected value μ_E, as function of μ_E. We consider a *band-dual* scheme for different values of τ and of $\hat{\psi}$.

Now, we consider the *single-dual* imbalance settlement scheme. Figure 5.6 shows the difference between the optimal market quantity $\overline{q}^{\text{DA}*}$ and the expected value μ_E, as function of μ_E, for values of the imbalance penalty ratio $\hat{\psi}$ of 0.5, 1, and 2, respectively. The red squares represent the case with $\tau = 0$, while the blue dots the cases with the tolerance band, from light blue for $\tau = 0.1$ to dark blue for $\tau = 0.5$. When $\hat{\psi} = 1$, i.e., $\hat{\phi}^{(-)} = |\hat{\phi}^{(+)}|$, the expected value of ϕ^{BA} is 0. In such condition, the *single-dual* and the *dual-band* schemes are equivalent and bring the same optimal solution $\overline{q}^{\text{DA}*}$. Conversely, when $\hat{\psi} = 0.5$, i.e., $|\hat{\phi}^{(+)}| = 2\hat{\phi}^{(-)}$, the expected value of ϕ^{BA} is negative. In a single-price settlement scheme, the power producer would offer the full capacity in the market (see Section 5.3.2), since it is expected to receive an extra-profit for the creation of a negative imbalance (i.e., when $E \leq \overline{q}^{\text{DA}}$). Under the *single-dual* scheme its offering strategy is more conservative as outside the band the deviations are priced under a dual-price settlement scheme. However, the power producer is encouraged to overestimate its future power production for increasing the width of the tolerance band and for being more likely to create a negative imbalance. The optimal quantity offer is indeed often the full capacity, i.e., $\overline{q}^{\text{DA}*} = 1$ p.u., even if it means to "lose" half of the width of the tolerance band. It is still more convenient to be sure to create a negative imbalance, even at the price of "losing" a portion of the tolerance band. When $\hat{\psi} = 2$, i.e., when $\hat{\phi}^{(-)} = 2|\hat{\phi}^{(+)}|$, the expected value of ϕ^{BA} is positive. The power producer would offer 0 p.u. in a single-price settlement scheme since it is expected to receive an extra-profit for the creation of a positive imbalance. Nevertheless, under the *single-dual* scheme, offering at $\overline{q}^{\text{DA}} = 0$ p.u. would result in shrinking the tolerance band to 0 and losing the extra-profit of the single-price scheme. On one side, the producer is encouraged to underestimate its production for being more likely to create a positive imbalance. On the other side, it wants to increase its market offer to gain a wider tolerance band. These two conflicting interests prevent the power producer from offering at $\overline{q}^{\text{DA}*} = 0$ p.u. as suggested in a single-price settlement scheme.

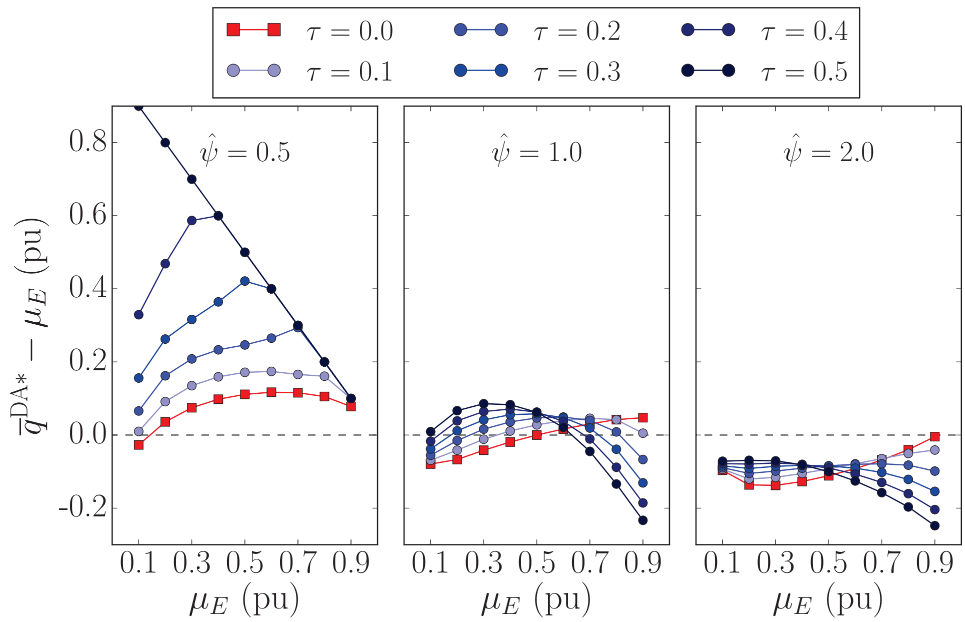


FIGURE 5.6: Difference between the optimal market quantity $\overline{q}^{\text{DA}*}$ and the expected value μ_E, as function of μ_E. We consider a *single-dual* scheme for different values of τ and of $\hat{\psi}$.

Similar conclusions can be drawn by analyzing Figure 5.7, which shows the nominal level of the optimal quantile, i.e., $F_E(\overline{q}^{\mathrm{DA}*})$, as function of μ_{E_k}, for values of the imbalance penalty ratio $\hat{\psi}$ of 0.5, 1, and 2, respectively.

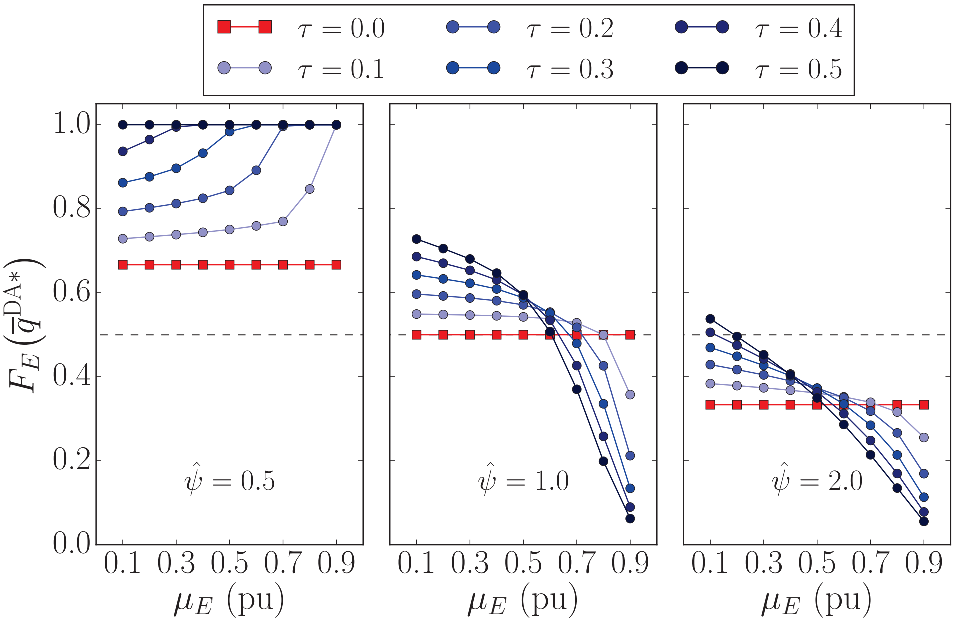


FIGURE 5.7: Nominal level of the optimal quantile, i.e., $F_E(\overline{q}^{\mathrm{DA}*})$ as function of μ_E. We consider a *single-dual* scheme for different values of τ and of $\hat{\psi}$.

5.5.2 Analysis on the Expected Imbalances

This section analyses how the results of the optimal trading strategies shown in Section 5.5.1 affect the expected real-time imbalances of the stochastic producers. Figure 5.8 shows the value of $\hat{\Delta}$, defined in Equation (5.51), under the *band-dual* settlement scheme as function of τ, for values of the imbalance penalty ratio $\hat{\psi}$ of 0.5, 1, and 2, respectively. Similarly, Figure 5.9 does for the *single-dual* settlement scheme. When $\hat{\psi} = 0.5$, the expected imbalance $\hat{\Delta}$ increases as the relative width of the tolerance band increases, for both the imbalance settlement schemes. For instance, under the *band-dual* settlement scheme $\hat{\Delta}$ is 5.8% higher for $\tau = 0.2$ and 15.8% higher for $\tau = 0.5$ with respect to a dual-price imbalance settlement scheme (i.e., $\tau = 0$). This increment is strongly emphasized when considering a *single-dual* settlement scheme. Indeed, for such settlement scheme $\hat{\Delta}$ is 317.3% higher for $\tau = 0.5$ with respect to $\tau = 0$. This huge growth is mainly due to strong a overestimation of the future power production when $\hat{\psi} < 0$. When $\hat{\psi} = 1$, $\hat{\Delta}$ increases as τ grows, in the same manner for both the settlement schemes. As mentioned in Section 5.5.1, when $\hat{\psi} = 1$, the two settlement schemes can be considered equivalent. The expected value of Δ grows of 3.9% and of 9.95% compared to $\tau = 0$, for τ equal to 0.3 and 0.5, respectively. Conversely, when $\hat{\psi} = 2$, $\hat{\Delta}$ is lower when $\tau > 0$, for both *band-dual* and *single-dual* settlement schemes. Under the *band-dual* settlement scheme, $\hat{\Delta}$ decreases of 7.6% for $\tau = 0.3$ and 6.3% for $\tau = 0.5$, compared to $\tau = 0$.

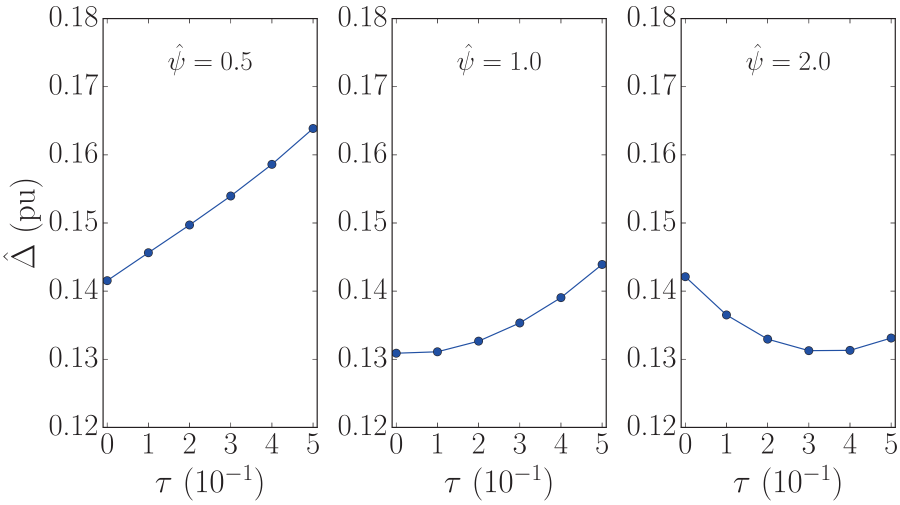


FIGURE 5.8: Expected value of the imbalance $\hat{\Delta}$ as function of τ, for different values of the $\hat{\psi}$ and considering a *band-dual* settlement scheme.

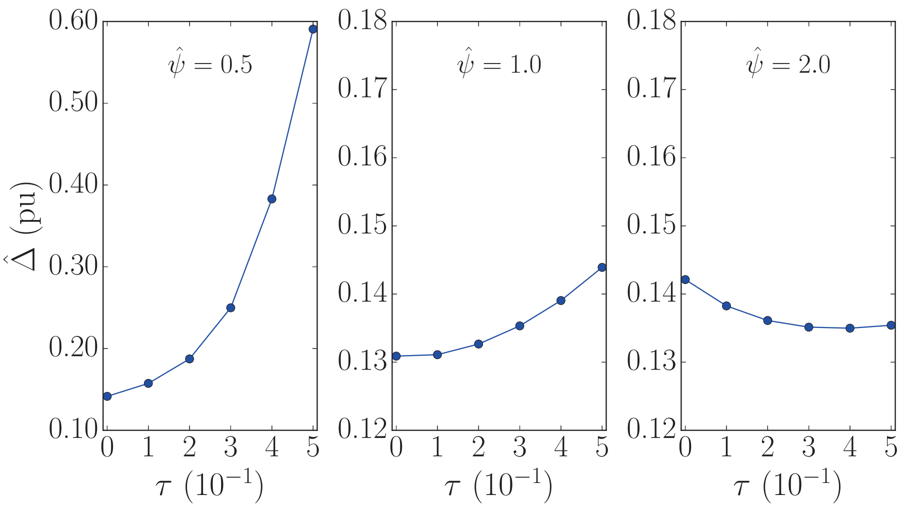


FIGURE 5.9: Expected value of the imbalance $\hat{\Delta}$ as function of τ, for different values of the $\hat{\psi}$ and considering a *single-dual* settlement scheme.

As mentioned in Section 5.4.4, we are also interested in understanding if the imbalance $\hat{\Delta}$ is expected to help in restoring the whole system imbalance or to contribute to it. First, we distinguish between imbalances of different sign, i.e., $\hat{\Delta}^{(+)}$ and $\hat{\Delta}^{(-)}$, evaluated with Equations (5.52) and (5.53), respectively. Figure 5.10 shows the expected values $\hat{\Delta}^{(+)}$ and $\hat{\Delta}^{(-)}$, for the *dual-band* settlement scheme, as function of τ. Similarly, Figure 5.11 does for the *single-dual* settlement scheme.

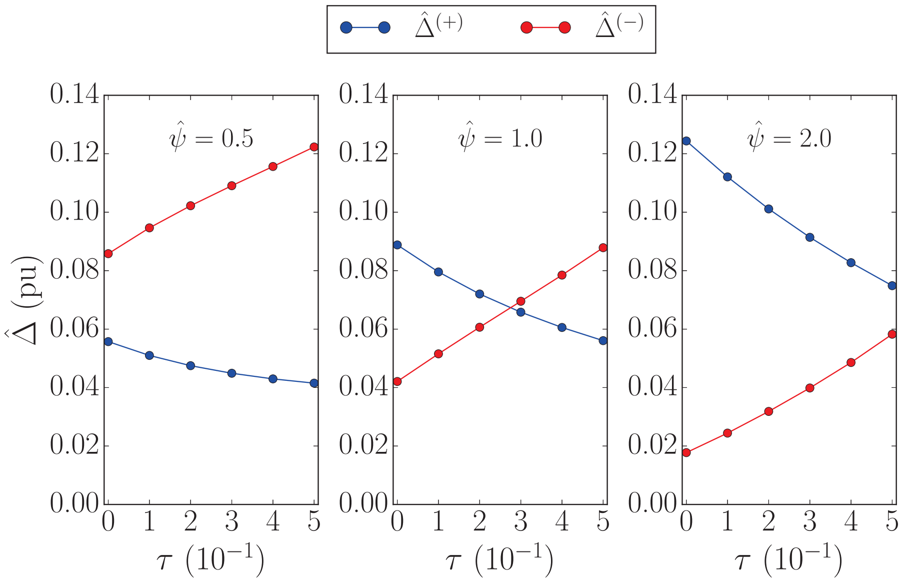


FIGURE 5.10: Expected value of the positive imbalance $\hat{\Delta}^{(+)}$ and the negative imbalance $\hat{\Delta}^{(-)}$ as function of τ, for different values of the $\hat{\psi}$ and considering a *band-dual* settlement scheme.

Then, the size of the imbalances that are expected to "help" the system, i.e., $\hat{\Delta}^{(\uparrow)}$, and to "hurt" the system, i.e., $\hat{\Delta}^{(\downarrow)}$, are computed with Equations (5.56) and (5.57), respectively. Figure 5.12 shows the values of $\hat{\Delta}^{(\uparrow)}$ and $\hat{\Delta}^{(\downarrow)}$ as function of τ, for different values of the $\hat{\psi}$, for the *dual-band* settlement scheme. Figure 5.13 does the same for the *single-dual* settlement scheme. The expected imbalance $\hat{\Delta}^{(\uparrow)}$ for $\hat{\psi} = 0.5$ increases of 26% for the *band-dual* scheme and of 420% for the *single-dual* when $\tau = 0.5$ compared to the case with $\tau = 0$. Conversely, for $\hat{\psi} = 0.5$ and for $\tau = 0.5$, $\hat{\Delta}^{(\downarrow)}$ grows of 4% for the *band-dual* scheme and of 200% for the *single-dual*, with respect to $\tau = 0$. It is interesting to compare Figures 5.8 and 5.9 with Figures 5.12 and 5.13. As a matter of fact, we mentioned that when $\hat{\psi} = 0.5$, the expected imbalance $\hat{\Delta}$ increases as τ grows, for both the imbalance schemes. From Figures 5.12 and 5.13 it can be noticed that both $\hat{\Delta}^{(\uparrow)}$ and $\hat{\Delta}^{(\downarrow)}$ increase with τ, even if $\hat{\Delta}^{(\uparrow)}$ shows a higher grow rate. Moving to the case of $\hat{\psi} = 2$, we highlighted before that $\hat{\Delta}$ first decreases up to $\tau = 0.3$, while start growing for τ greater than 0.3. However, analyzing Figures

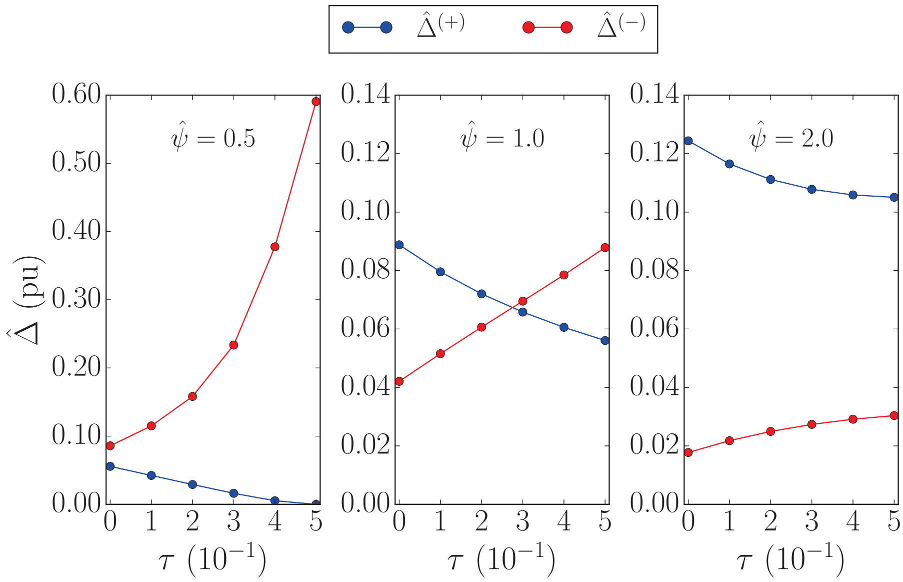


FIGURE 5.11: Expected value of the positive imbalance $\hat{\Delta}^{(+)}$ and the negative imbalance $\hat{\Delta}^{(-)}$ as function of τ, for different values of the $\hat{\psi}$ and considering a *single-dual* settlement scheme.

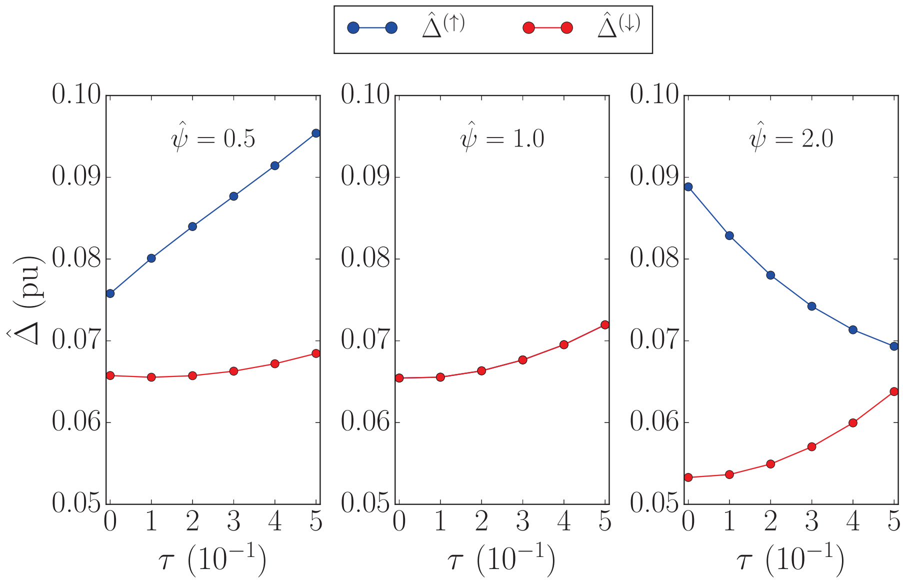


FIGURE 5.12: Expected value of the helping imbalance $\hat{\Delta}^{(\uparrow)}$ and the hurting imbalance $\hat{\Delta}^{(\downarrow)}$ as function of τ, for different values of the $\hat{\psi}$ and considering a *band-dual* settlement scheme.

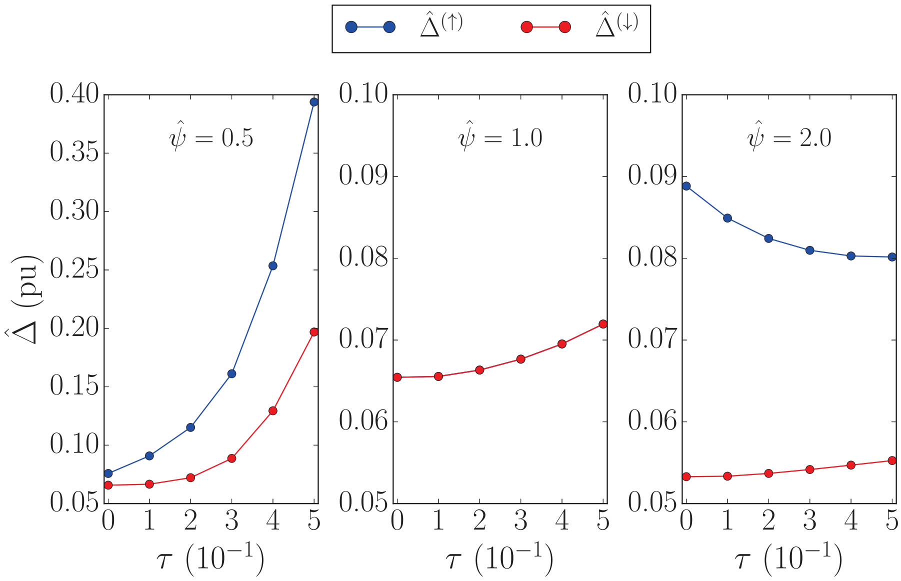


FIGURE 5.13: Expected value of the helping imbalance $\hat{\Delta}^{(\uparrow)}$ and the hurting imbalance $\hat{\Delta}^{(\downarrow)}$ as function of τ, for different values of the $\hat{\psi}$ and considering a *single-dual* settlement scheme.

5.12 and 5.13 it can be noticed that $\hat{\Delta}^{(\uparrow)}$ decreases as τ increases, while $\hat{\Delta}^{(\downarrow)}$ increases with τ. Thus, the slight decrease of the expected imbalance up to $\tau = 0.3$ is due to a decrease of the expected beneficial imbalance and a lower increase of the harmful imbalance, for both the imbalance settlement schemes.

5.6 Summary

This chapter takes the perspective of a price-taker and risk-neutral stochastic power producer offering in a two-settlement electricity market. Starting from the general offering strategy of Section 3.4 we tailor it to the characteristics of the stochastic producer. We show how, under a single-price imbalance settlement scheme, the quantity offered by the power producer in the day-ahead market is only influenced by the expectation on the balancing market penalties, while the power production forecasts do not influence its optimal decision. Indeed, the producer decides to offer 0 or the full capacity depending on the market price forecasts. Differently, we prove that under a dual-price imbalance settlement scheme the optimal market offer is a quantile of the cumulative distribution function of the future renewable energy production. The nominal level of such optimal quantile is influenced by the expected balancing market penalties. This translates in overestimating the future power production if it is expected to be more penalized for the creation of a positive imbalance, while to underestimate it if the penalty for the creation of a negative imbalance is higher than the positive one, in expectation.

The chapter also investigates the effects of imbalance settlement schemes with tolerance margins on the optimal offering strategy of the stochastic producers. We prove that also under the *band-dual* and the *single-dual* imbalance settlement scheme the market quantity offer that maximizes the producer's expected profit is unique. Then, we analyze the effects of the tolerance margins from a system perspective. We formulate the expected real-time imbalance of a stochastic producer seeking at maximizing its expected market profit, for both the *band-dual* and the *single-dual* scheme. We also differentiate between imbalances that are expecting to "hurt" or to "help" in restoring the system's imbalance.

Through parametrized curves, we show the effect of the tolerance margins on the optimal amount of energy to be contracted in the day-ahead market for the two schemes. We demonstrate that under the *single-dual* scheme the power producer is encouraged to strongly overestimate its power production when it is expecting a balancing market price lower than the day-ahead one. Then, we also show that the tolerance margins within the penalty scheme may generate larger real-time imbalances for the stochastic producers, mainly for the *single-dual* scheme.

As a result of this analysis, the imbalance pricing scheme introduced in the Italian electricity market to mitigate the balancing responsibility of the stochastic producers may not be optimal from a system perspective. Our analysis suggests that they may bring more significant real-time imbalances of the stochastic producer and, supposedly, a higher balancing cost for the system. It is worth mentioning that this analysis is based on a price-taker producer who is not supposed to influence the whole system imbalance with its deviation. However, a producer may offer as a price-taker producer while affecting the system imbalance.

Chapter 6

Trading of Conventional Generators

6.1 Introduction

This chapter takes the perspective of a conventional power producer. Differently than stochastic producers, conventional ones have control of the power output of their production units. However, the operation of a conventional production unit (e.g., coal- or gas-fired power plant) is constrained by technical limitations, e.g., ramping constraints. Those ramping restrictions, which constraint the capability of the unit to vary its power production over time, are of particular interest in the context of optimal participation in the electricity market. Indeed, in Chapter 2 we show that market participants submit a separate offer curve per each trading interval. Therefore, such intertemporal constraints cannot be explicitly included in the market offers. The power producer has to internalize them in its offering strategy, together with non-convex costs, such as start-up and shut-down costs. Traditionally, this was not a prominent issue, as "slow" technologies (e.g., coal-fired or nuclear power plant), which are the more constrained ones in their operation, had marginal costs below the day-ahead market price and were operated to cover the base loads. Differently, the pick-loads were covered by "fast" technologies (e.g., gas-fired or diesel units), which flexibility almost allows to neglect intertemporal constraints when considering hourly trading intervals. However, in recent years the power sector has experienced a significant increase in the deployment of renewable energy sources, such as wind and solar power. These sources are usually traded at zero marginal cost and their growing penetration is leading to a decrease of the prices in the day-ahead market (Sensfuss, Ragwitz, and Genoese, 2008; Clò, Cataldi, and Zoppoli, 2015). Moreover, they can only be predicted with a limited accuracy, thus leading to real-time imbalances and increasing the need for balancing energy. These changes may affect the strategy of the conventional producers in both the day-ahead and the balancing market. E.g., a "slow" production unit may not be able to solely rely on the revenues of the day-ahead market, due to the decrease of its prices, but may have to be more strategic and considering possible additional profits from selling regulating power in the balancing market.

The optimal offering strategy and self-scheduling of conventional thermal units have already been widely studied in the literature. Arroyo and Conejo (2000) address the optimal response of a thermal generator to a given set of electricity market prices in terms of both energy and reserve. A MILP problem is developed considering a non-convex cost function, as well as its start-up costs, ramp rates and minimum-up and -down constraints. The same authors (Arroyo and Conejo, 2004) propose a detailed formulation to model start-up and shut-down characteristics of a thermal generator. Other works, such as (Conejo et al., 2004; Jabr, 2005), include risk measures while optimizing the self-scheduling problem of thermal units. References (Arroyo and Conejo, 2000; Arroyo and Conejo, 2004; Conejo et al., 2004; Jabr, 2005) demonstrate that a detailed modeling of the generator feasibility region and its production cost function may be essential for deriving its optimal self-scheduling. Indeed, the inter-temporal constraints (e.g., ramping constraints) and non-convex costs (e.g., start-up and shut-down costs) may affect the optimal solution. In this context, the pioneering paper (Conejo, Nogales, and Arroyo, 2002) presents an offering strategy for a price-taker producer under price uncertainty. It develops a set of rules that aim to translate the results of a self-scheduling problem into market offers. Ni, Luh, and Rourke (2004) present an algorithm for offering and self-scheduling of a unit including risk management. Maenhoudt and Deconinck (2014) derive an offering strategy for a price-taker power producer that aims to maximize profit expectation while hedging against possible infeasible schedules. Other works relax the price-taker assumption and develop tools for strategic offering considering the impact of power producer's decisions on market prices. This can be done through a residual demand model (Baillo et al., 2004) or a bilevel optimization setup (Gountis and Bakirtzis, 2004; Bakirtzis et al., 2007).

By analyzing the optimization models mentioned above, we can identify two different sets of variables and constraints. The first set defines the feasibility region and the cost function of the production unit. For instance, references (Arroyo and Conejo, 2000; Arroyo and Conejo, 2004) show how to successfully model it as a MILP problem. The second set simulates the trading problem, i.e., how the power producer participates in the market (e.g., through non-decreasing step-wise offering curves), while considering the market clearing mechanism (endogenously or exogenously) and the pricing scheme (e.g., uniform or pay-as-bid). The trading problem can be modeled using a LP approach (Conejo, Carrión, and Morales, 2010), under price-taker assumptions and uniform pricing scheme. However, even though European day-ahead electricity markets are mostly settled under a uniform pricing scheme, several balancing markets, e.g., in Germany and Italy (Wang et al., 2015), are settled under a pay-as-bid pricing scheme.

The topic of trading under a pay-as-bid scheme and price uncertainty has not been extensively addressed in the literature. References (Ren and Galiana, 2004a; Ren and Galiana, 2004b; Swider and Weber, 2007; Swider, 2007; Khorasani and Mashhadi, 2012; Sadeh, Mashhadi, and Latifi, 2009) show how to model the trading problem under a pay-as-bid pricing scheme using a non-linear programming (NLP) approach. However, they do not consider an accurate modeling of production unit's operational constraints. Introducing the feasibility region would result in a MINLP model, which may have high computational cost and, generally, do not guarantee the optimality of the solution. Differently, Chapter 3 provides a novel approach that allows casting the optimal price-taker trading problem in pay-as-bid markets under price uncertainty as an LP problem. For that purpose, continuous random variables (i.e., the market-clearing prices) are represented as discrete variables. We refer the interested reader to Chapter 3 for a more extensive literature review on the topic of trading in pay-as-bid electricity markets.

We use the proposed LP approach to build a multi-stage stochastic programming problem with recourse. This efficient decision-making tool could be used by a price-taker conventional producer to derive its best day-ahead market offer curves. In line with current practice in several European electricity markets, we consider a two-settlement market framework, in which the day-ahead market is cleared based on a uniform pricing scheme, while a pay-as-bid pricing scheme is used in the balancing stage. The market prices in both stages are given but uncertain. This uncertainty is properly characterized by generating a set of foreseen scenarios. The resulting model is a stochastic MILP problem, where non-convexities (i.e., binary variables) arise from the unit commitment constraints. To the best of our knowledge, this kind of stochastic MILP optimization model for obtaining the offering strategy of a price-taker thermal producer in a two-settlement electricity market with a pay-as-bid pricing scheme in the balancing stage is not available in the literature. It is worth mentioning that Bakirtzis et al. (2007) provide a formulation for obtaining optimal offering curves in markets settled under a pay-as-bid pricing scheme for a price-maker producer. However, market problems with equilibrium constraints may have high computational cost and rely on strong assumptions on opponents' behavior. Hence, when the production unit has a negligible impact on the market, a price-maker setup may not be the preferable choice.

The remaining of the chapter is organized as follows. Section 6.2 introduces the electricity market framework and a set of assumptions used to formulate the offering strategies. Section 6.3 shows a general MILP formulation to represent the feasible operating region and the non-convex cost function of the thermal unit. Section 6.4 presents two offering strategies to be used by a conventional power producer to derive its optimal day-ahead offer curves. A case study is presented in Section 6.5, while Section 6.6 summarizes the contents of the chapter.

6.2 Electricity Market Framework and Assumptions

We consider a single conventional producer that trades in a two-settlement electricity market framework, similar to the one introduced in Section 2.3. The day-ahead market is cleared once a day, at noon, simultaneously for the whole 24 hourly trading intervals of the following day. Generators are remunerated under a uniform pricing scheme in the day-ahead market. Then, a balancing market is cleared separately per each hourly interval, one hour before the real-time operation. The provision of balancing energy is remunerated under a pay-as-bid pricing scheme. Both the day-ahead and balancing market give the possibility to submit non-decreasing offer curves. The intra-day trading floor is neglected for the sake of simplicity. A schematic representation of the electricity market submission process is illustrated in Figure 6.1.

The power producer is assumed to be price-taker in both the day-ahead and the balancing market. Hence, the market prices within the offering strategy problem of

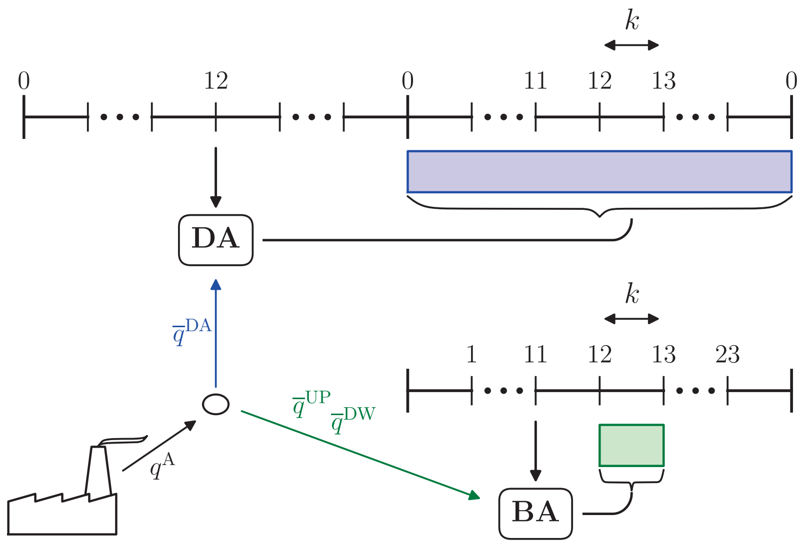


FIGURE 6.1: Schematic representation of the electricity market framework. The conventional producer submits the quantity offer $\overline{q}^{\mathrm{DA}}$ in the day-ahead (DA) market, while submitting upward $\overline{q}^{\mathrm{UP}}$ and downward $\overline{q}^{\mathrm{DW}}$ regulation offers in the balancing (BA) market.

the producer are exogenous, but still uncertain. We model those uncertainties using a set of scenarios. Uncertainty characterization is a critical input to stochastic optimization. The quality of the solution of a stochastic optimization model is indeed strongly influenced by the quality of the scenarios provided as input. Given that the purpose is to analyze and test an optimization model, we exploit the fundamental model presented in Section 4.3 for generating market prices, instead of using real market data. This fundamental model generates a set of electricity market price forecasts, which is required as an input to our proposed offering strategy.

The scenario generation and reduction algorithm in Chapter 4 provides a set of day-ahead market price trajectories $\{\lambda^{\mathrm{DA}}_{ik} : \ \forall i \in I, \ \forall k \in K\}$, being i the index of the day-ahead market price scenarios and k the index of the time intervals. Each scenario i is associated with a probability π^{DA}_{i} of occurrence. Then, for each possible day-ahead realization i, it provides a set of balancing market price trajectories $\{\lambda^{\mathrm{BA}}_{ijk} : \forall i \in I, \ \forall j \in J, \ \forall k \in K\}$, being j the index of the balancing market price scenarios. The scenario (ij) has a probability $\pi^{\mathrm{DA}}_{i}\pi^{\mathrm{BA}}_{ij}$ of occurrence.

6.3 Thermal Unit Model

This section derives an approximate model of the feasibility region and the cost function of a conventional production unit (e.g., coal- or gas-fired power plant) through a MILP formulation. The operation of conventional production units is constrained by several technical limitations. It is important to properly model them through a set of constraints to replace constraint (3.18e) (i.e., the set Ω) and constraint (3.18f) (i.e., the function $h(\cdot)$) of the general offering strategy formulation (3.18) in Section 3.4. It is also important to evaluate an appropriate trade-off between the complexity and accuracy of such model. On the one hand, a too accurate model may be too complex, thus leading to an intractable optimization problem when merged with the offering strategy. On the other hand, a too simple model may not be able to represent the fundamental characteristics of the production unit. This problem has already been widely explored within the literature (Arroyo and Conejo, 2000; Arroyo and Conejo, 2004). They show how a MILP formulation can approximate such feasible region with an acceptable level of accuracy.

6.3.1 Feasible Operating Region of the Unit

This section provides a formulation to represent the feasible region of a thermal unit, used to replace constrain (3.18e) in the general offering strategy (3.18), i.e.,

$$q_k^{\mathrm{A}} \in \Omega, \ \forall k, \tag{6.1}$$

where q_k^{A} is the total amount of energy generated by the power producer. Let d_k (MWh) be the quantity of energy produced in the real-time by the thermal unit. Since we consider a single production unit, we impose

$$q_k^{\mathrm{A}} = d_k, \ \forall k, \tag{6.2}$$

In case the power producer manages a set H of units, where d_{hk} is the power output of unit h at time k, it would impose $q_k^{\mathrm{A}} = \sum_h d_{hk}$. Consequently, even if this chapter considers a single production unit, it is easily adaptable to multiple production units. Let us define with $\underline{D}$ and $\overline{D}$ the minimum power output and the capacity of the unit, respectively. This means that when the unit is on, its production d_k has to lie between $\underline{D}$ (MWh) and $\overline{D}$ (MWh), i.e., $d_k \in [\underline{D}, \overline{D}]$. Differently, when the unit is not operating, its power output is 0, i.e., $d_k = 0$. The result is a non convex operating region, as the power production d_k can either be 0 or in $[\underline{D}, \overline{D}]$. Let u_k be the binary variable, i.e., $u_k \in \{0, 1\}$, representing the commitment status of the production unit. A value of u_k equal to 0 means that the unit is not operating. Differently, u_k is equal to 1 when the unit is on. Accordingly, the operating region of d_k can be formulated as

$$u_k \underline{D} \leq d_k \leq u_k \overline{D}, \quad \forall k. \tag{6.3}$$

Notice that when u_k is 0, i.e., the unit is off, constraint (6.3) becomes $0 \leq d_k \leq 0$, which is equivalent to impose $d_k = 0$. Then, if $u_k = 1$, i.e., the unit is on, constraint (6.3) is equivalent to $\underline{D} \leq d_k \leq \overline{D}$. Figure 6.2 shows an illustrative representation of the feasibility region imposed by constraint (6.3), which is shown in red.

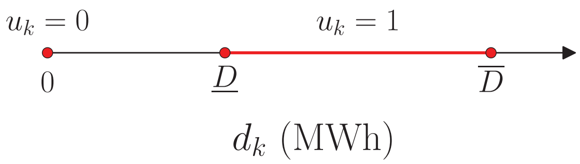


FIGURE 6.2: Graphic representation of the non-convex feasibility constraint (6.3).

Then, thermal units also have inter-temporal limitations. Indeed, they have ramping constraints, which limit the capability of the unit to change its power production over short time intervals. Let R^{UP} (MW/h) be the upward ramping limit of the unit. As an example, if $R^{\mathrm{UP}} = 10$ MW/h and the unit is producing 20 MW, in one hour it can increase its power production up to 30 MW and 40 MW in two hours, and so on. However, as we only consider hourly intervals, we define R^{UP} as the maximum upward variation of energy production in one hour, i.e., in MWh. The upward ramping limitation of d_k is expressed as

$$d_k - d_{(k-1)} \leq R^{\mathrm{UP}}, \quad \forall k \in K - \{k_1\}, \tag{6.4a}$$

$$d_{k_1} - d_0 \leq R^{\mathrm{UP}}, \tag{6.4b}$$

where d_0 is the initial production level, while k_1 is the first hourly interval of K. Similarly, we define R^{DW} (MWh) as the maximum downward variation in one hour. E.g., if $R^{\mathrm{DW}} = 5$ MWh and $d_{(k-1)} = 30$ MWh, then d_k cannot be lower than 25 MWh.

The downward ramping limitation of d_k can be imposed as

$$d_{(k-1)} - d_k \leq R^{\mathrm{DW}}, \quad \forall k \in K - \{k_1\}, \tag{6.5a}$$

$$d_0 - d_{k_1} \leq R^{\mathrm{DW}}, \tag{6.5b}$$

A graphic representation of the ramping limits of a conventional unit is given in Figure 6.3. Indeed, if the amount of production $d_{(k-1)}$ at $k-1$ is known, then d_k is limited between $d_{(k-1)} + R^{\mathrm{UP}}$ and $d_{(k-1)} - R^{\mathrm{DW}}$.

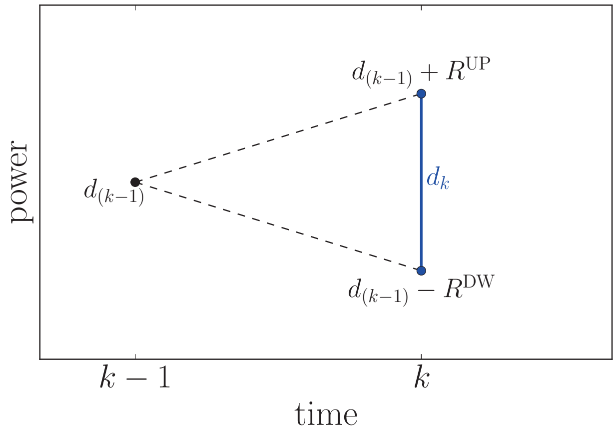


FIGURE 6.3: Illustration of upward and downward limitations of the conventional unit.

Consequently, the set Ω representing the feasible region of the unit can be replaced by

$$q_k^{\mathrm{A}} = d_k, \ \forall k, \tag{6.6a}$$

$$u_k \underline{D} \leq d_k \leq u_k \overline{D}, \ \forall k, \tag{6.6b}$$

$$d_k - d_{(k-1)} \leq R^{\mathrm{UP}}, \ \forall k, \tag{6.6c}$$

$$d_{(k-1)} - d_k \leq R^{\mathrm{DW}}, \ \forall k. \tag{6.6d}$$

$$u_k \in \{0, 1\}, \ \forall k. \tag{6.6e}$$

Constraint (6.6a) imposes the balance between q_k^{A} and d_k. Constraint (6.6b) forces d_k to operate in its feasible region, i.e., between $\underline{D}$ and $\overline{D}$ when on, and to be equal to 0 when off. Constraints (6.6c) and (6.6d) impose the ramp up and down limitations of the production unit. Finally, constraint (6.6e) imposes that u_k is binary. Notice thatconstraints (6.6c) and (6.6d) are a compact formulation of the ramping limits. Indeed, they require the initial production level d_0 of the unit.

It is worth mentioning that (6.6) is simplified feasibility region, which usually includes additional constraints, e.g., the minimum up- and down-time of the unit. We refer the interested reader to references (Arroyo and Conejo, 2000; Arroyo and Conejo, 2004) for a more detailed representation of such feasible operating region.

6.3.2 Cost Function of the Thermal Unit

This section deals with the cost function of a thermal unit. We derive a formulation to replace constrain (3.18f) in the general offering strategy formulation (3.18), i.e.,

$$\hat{c}_k = h\big(q_k^{\mathrm{A}}\big)\,,\ \forall k \tag{6.7}$$

where $q_k^{\mathrm{A}} = d_k$. We initially consider that such cost only depends on the actual amount of energy production d_k. Such curve can usually be approximated with a quadratic cost curve, i.e.,

$$h(d_k) = E + F\,d_k + G\,(d_k)^2\,, \quad \forall k \in K, \tag{6.8}$$

where the parameters E , F, and G, control the shape of the curve. The formulation of $h(d_k)$ in Equation (6.8) is quadratic, meanwhile we would like a MILP model of the unit. To obtain an alternative linear formulation of Equation (6.8) we use a piecewise linear interpolation. This approximation can be applied only if the marginal cost $h'(d_k)$ is non decreasing. The marginal cost is the cost for selling an additional unit of energy, while from a mathematical point of view, it is evaluated as the derivative of $h(d_k)$ with respect to d_k. Therefore, $h'(d_k)$ can be computed as

$$h'(d_k) = F + 2G\,d_k, \quad \forall k \in K. \tag{6.9}$$

It is straightforward to prove that $h'(d_k)$ is non decreasing, for $d_k \in \big[\underline{D}, \overline{D}\big]$, if $G \geq 0$. Such condition is usually verified for real cost curves. The idea of the piecewise linear approximation is subdividing the interval $\big[\underline{D}, \overline{D}\big]$ into N^S intervals and considering a constant marginal cost within each block. We define with s the index identifying a production block, while S is the set of those blocks. We introduce x_{sk} (MWh) as the amount of energy produced with the block s. Then, being C_s (€/MWh) the marginal cost of the block s and C_0 (€) the production cost at the minimum output level of the unit, the cost function $h(d_k)$ can be approximated as

$$h(d_k) = C_0 u_k + \sum_s C_s x_{sk}, \quad \forall k \tag{6.10a}$$

$$d_k = \underline{D} u_k + \sum_s x_{sk}, \quad \forall k \tag{6.10b}$$

$$0 \leq x_{sk} \leq \overline{X}_s, \quad \forall s, \forall k \tag{6.10c}$$

where $\overline{X}_s$ (MWh) is the size of the production block s. When the objective function is to minimize the production cost (or to maximize the profit) the formulation (6.10) provides a successful approximation to the quadratic cost function in Equation (6.8).

Figure 6.4 illustrates an example of piecewise linear approximation of a quadratic cost curve. The quadratic curve is shown in blue, while its piecewise linear approximation with four production blocks in red. Then, the marginal cost function of the two curves is illustrated in Figure 6.5. Note that, as mentioned before, the marginal cost is constant over each of the four production blocks. Finally, we also show the average production cost of the real cost curve (blue) and the piecewise linear approximation (in red) in Figure 6.6.

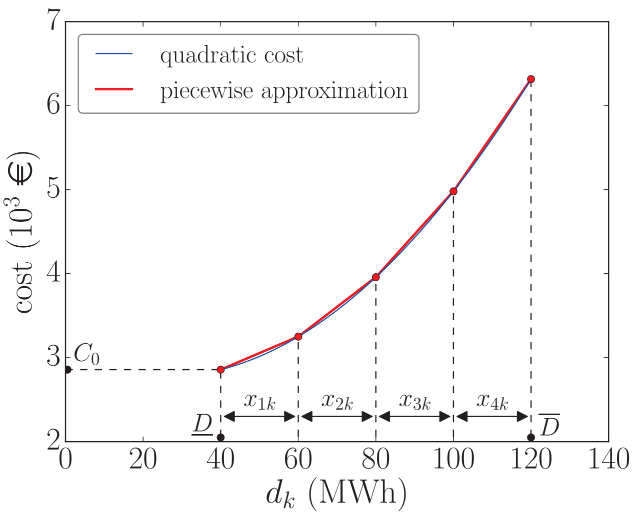


FIGURE 6.4: Example of a quadratic cost function (blue) and its piecewise linear approximation (red) with 4 production blocks.

Now, we introduce other costs, non-proportional to d_k, that contribute to the total production cost of the unit. An example is the start-up cost. To model it, we introduce the binary variable y_k, representing the start-up status of the unit during the interval k. $y_k = 1$ if the unit is turned on during the interval k, and 0 otherwise.

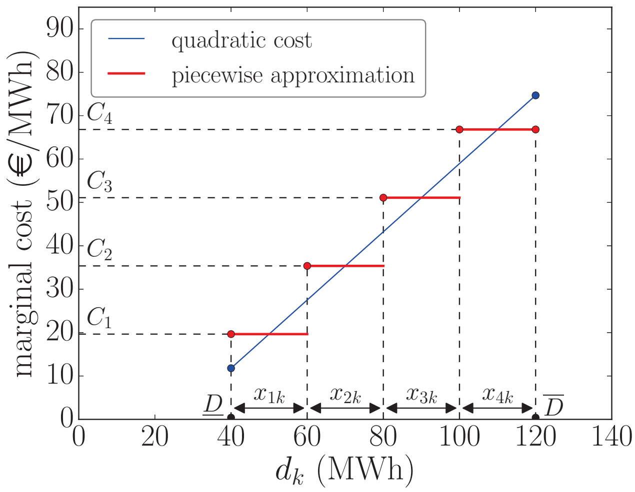


FIGURE 6.5: Marginal cost function of the cost curves in Figure 6.4.

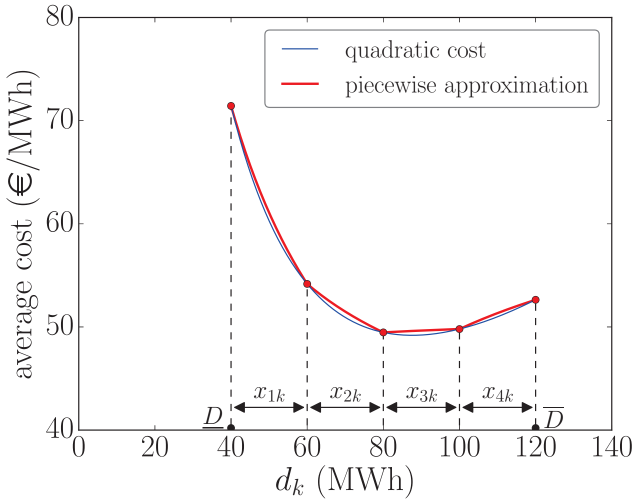


FIGURE 6.6: Average cost function of the cost curves in Figure 6.4.

The value of y_k can be evaluated by imposing

$$u_k - u_{(k-1)} \leq y_k, \quad \forall k \in K - \{k_1\}, \tag{6.11a}$$

$$u_{k_1} - u_0 \leq y_{(k_1)}, \tag{6.11b}$$

where u_0 is the initial commitment status of the unit. Note that the term $u_k - u_{(k-1)}$ is equal to 1 if and only if $u_k = 1$ and $u_{(k-1)} = 0$, which represents a start-up situation associated with a cost C^{UP} (€). In this case, y_k is forced to be equal to 1. For the other 3 combinations, $u_k - u_{(k-1)}$ is equal to either 0 or -1, thus allowing y_k to be 0.

Similarly, we can include the shut-down cost. The binary variable z_k represents the shut-down status of the unit during the interval k, while C^{DW} (€) is the cost for turning off the unit. The shut-down cost can be evaluated as $C^{\text{DW}} z_k$, while the value of z_k can be computed by imposing

$$u_{(k-1)} - u_k \leq z_k, \quad \forall k \in K - \{k_1\}, \tag{6.12a}$$

$$u_0 - u_{k_1} \leq z_{(k_1)}, \tag{6.12b}$$

The term $u_{(k-1)} - u_k$ is equal to 1 if and only if $u_k = 0$ and $u_{(k-1)} = 1$, which represents a shut-down situation at k. Combining all those costs in a unique formulation results in the following set of constraints, which can be used to replace $h(q_k^{\text{A}})$ in the offering strategy:

$$\hat{c}_k = \mathbb{E}[c_k], \quad \forall k, \tag{6.13a}$$

$$c_k = C_0 u_k + \sum_s C_s x_{sk} + C^{\text{UP}} y_k + C^{\text{DW}} z_k, \quad \forall k, \tag{6.13b}$$

$$d_k = \underline{D} u_k + \sum_s x_{sk}, \quad \forall k, \tag{6.13c}$$

$$0 \leq x_{sk} \leq \overline{X}_s, \quad \forall s, \forall k, \tag{6.13d}$$

$$u_k - u_{(k-1)} \leq y_k, \quad \forall k, \tag{6.13e}$$

$$u_{(k-1)} - u_k \leq z_k, \quad \forall k, \tag{6.13f}$$

$$y_k, z_k \in \{0, 1\}, \quad \forall k. \tag{6.13g}$$

Constraint (6.13b) yields the total production cost c_k of the unit. Constraints (6.13c) and (6.13d) are auxiliary constraints to the piecewise linear formulation of the quadratic cost function. Constraints (6.13e) and (6.13f) compute the start-up (y_k) and shut-down (z_k) status of the unit. Finally, constraint (6.13g) imposes y_k and z_k to be binary variables.

6.4 Optimal Offering Strategy

This section formulates the offering strategy that the power producer may use to derive its best day-ahead market offers. We start from the generic formulation for the price-taker and risk-neutral producer in Equations (3.18), extended to the 24 trading intervals. First, we remove the variables and constraints associated with a passive participation in the balancing market. Indeed, we eliminate constraint (3.18e) and we remove the variables $\hat{\rho}_k^{\mathrm{BA}}$ and q_k^{BA} from the objective function (3.18a) and from constraint (3.18b), respectively. Finally, we replace q_k^{A} with d_k, in accordance with Equation (6.2). This leads to

$$\underset{\Gamma}{\text{Max}} \quad \sum_k \hat{\rho}_k^{\mathrm{DA}} + \hat{\rho}_k^{\mathrm{UP}} + \hat{\rho}_k^{\mathrm{DW}} - \hat{c}_k \tag{6.14a}$$

$$\text{s.t.} \quad \overline{q}_k^{\mathrm{DA}} + \overline{q}_k^{\mathrm{UP}} - \overline{q}_k^{\mathrm{DW}} = d_k, \quad \forall k, \tag{6.14b}$$

$$\overline{q}_k^{\mathrm{DA}}, \hat{\rho}_k^{\mathrm{DA}} \in \Pi^{\mathrm{DA}}, \quad \forall k, \tag{6.14c}$$

$$\overline{q}_k^{\mathrm{UP}}, \overline{q}_k^{\mathrm{DW}}, \hat{\rho}_k^{\mathrm{UP}}, \hat{\rho}_k^{\mathrm{DW}} \in \Pi_{\mathrm{Act}}^{\mathrm{BA}}, \quad \forall k, \tag{6.14d}$$

$$\hat{c}_k = h(d_k)\,, \quad \forall k, \tag{6.14e}$$

$$d_k \in \Omega, \quad \forall k, \tag{6.14f}$$

where

$$\Gamma = \{d_k, \overline{q}_k^{\mathrm{DA}}, \overline{q}_k^{\mathrm{UP}}, \overline{q}_k^{\mathrm{DW}}, \hat{\rho}_k^{\mathrm{DA}}, \hat{\rho}_k^{\mathrm{UP}}, \hat{\rho}_k^{\mathrm{DW}}, \hat{c}_k\}. \tag{6.15}$$

Section 6.4.1 presents a sequential offering strategy. It describes the case in which the producer is only considering the day-ahead stage while offering in the day-ahead market, without considering eventual profits from the balancing stage. Differently, Section 6.4.2 presents a co-optimized offering strategy, where the conventional producer co-optimizes the offering strategy in the day-ahead and the balancing market.

6.4.1 Sequential Offering Strategy

Under the sequential offering strategy, the power producer only considers the day-ahead stage while deriving its optimal offer curves. To that purpose, it is provided with a set I of day-ahead market price scenarios $\{\lambda_{ik}^{\text{DA}} : \forall i \in I, \forall k \in K\}$. As the producer does not consider the future revenues from the balancing stage, we remove from the offering model (6.14) the variables and constraints associated with the balancing market. Indeed, we eliminate constraint (6.14d) and we remove variables $\overline{q}_k^{\text{UP}}$ and $\overline{q}_k^{\text{DW}}$ from constraint (6.14b). Moreover, we eliminate $\hat{\rho}_k^{\text{UP}}$ and $\hat{\rho}_k^{\text{DW}}$ from the objective function (6.14a). Figure 6.7 illustrates the stochastic programming framework of the sequential offering strategy, where $\overline{q}_k^{\text{DA}}$ is modeled as a fist-stage decision. Fi-

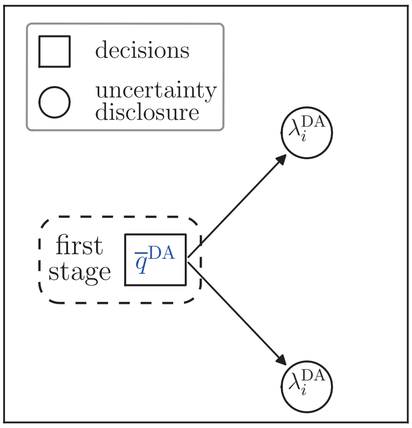


FIGURE 6.7: Schematic illustration of the stochastic programming framework of the sequential offering strategy.

nally, we introduce the subscript i in accordance with the stochastic programming approach, i.e, $d_k \rightarrow d_{ik}$ and $\overline{q}_k^{\text{DA}} \rightarrow \overline{q}_{ik}^{\text{DA}}$. Note that $\overline{q}_{ik}^{\text{DA}}$ is made scenario dependent for building the offer curve (see Section 3.4.3). This leads to

$$\underset{\Gamma^{\text{seq}}}{\text{Max}} \quad \sum_k \hat{\rho}_k^{\text{DA}} - \hat{c}_k \tag{6.16a}$$

$$\text{s.t.} \quad \overline{q}_{ik}^{\text{DA}} = d_{ik}, \quad \forall k, \forall i \tag{6.16b}$$

$$\overline{q}_{ik}^{\text{DA}}, \hat{\rho}_k^{\text{DA}} \in \Pi^{\text{DA}}, \quad \forall k, \forall i \tag{6.16c}$$

$$\hat{c}_k = h(d_{ik}), \quad \forall k, \tag{6.16d}$$

$$d_{ik} \in \Omega, \quad \forall k, \tag{6.16e}$$

where

$$\Gamma^{\text{seq}} = \{d_{ik}, \overline{q}_{ik}^{\text{DA}}, \hat{\rho}_k^{\text{DA}}, \hat{c}_k\}. \tag{6.17}$$

The objective function (6.16a) maximizes the expected profit of the producer for selling energy in the day-ahead market. Constraint (6.16b) balances the total energy exchanged with the market, i.e., $\overline{q}_{ik}^{\text{DA}}$, with the production d_{ik}. Constraint (6.16c) is a set of constraints associated with the day-ahead market offer curves. Constraint (6.16d) computes the production cost of the power unit. Finally, constraint (6.16e) forces the unit to operate in its feasible region.

Linear Formulation of Π^{DA}

The set Π^{DA} in (6.16) is a set of constraints related to the day-ahead market offer curves. The formulation of Π^{DA} is presented in Equations (3.44) in Section 3.4.3. Its formulation, adapted to the conventional producer (i.e., $\underline{Q} = 0$ and $\overline{Q} = \overline{D}$) and extended to the set K of the trading intervals is

$$\hat{\rho}_k^{\mathrm{DA}} = \sum_i \pi_i^{\mathrm{DA}} \, \lambda_{ik}^{\mathrm{DA}} \, \overline{q}_{ik}^{\mathrm{DA}}, \quad \forall k, \tag{6.18a}$$

$$\overline{q}_{ik}^{\mathrm{DA}} \geq \overline{q}_{i'k}^{\mathrm{DA}} \quad \text{if} \quad \lambda_{ik}^{\mathrm{DA}} \geq \lambda_{i'k}^{\mathrm{DA}}, \quad \forall i, i', \forall k \tag{6.18b}$$

$$\overline{q}_{ik}^{\mathrm{DA}} = \overline{q}_{i'k}^{\mathrm{DA}} \quad \text{if} \quad \lambda_{ik}^{\mathrm{DA}} = \lambda_{i'k}^{\mathrm{DA}}, \quad \forall i, i', \forall k \tag{6.18c}$$

$$0 \leq \overline{q}_{ik}^{\mathrm{DA}} \leq \overline{D}, \quad \forall i, \forall k \tag{6.18d}$$

Constraint (6.18a) yields the expected day-ahead market income of the power producer. Constraints (6.18b) and (6.18c) impose the non-decreasing and non-anticipativity conditions, respectively. Lastly, constraint (6.18d) forces the day-ahead market offers $\overline{q}_{ik}^{\mathrm{DA}}$ to lie between 0 and the unit's capacity $\overline{D}$.

Linear Formulation of the Cost Function

Constraint (6.16d) yielding the production cost, can be replaced with the formulation provided in Section 6.3.2, i.e., Equations (6.13). Its formulation, adapted to the stochastic programming framework, is the following:

$$\hat{c}_k = \sum_i \pi_i^{\mathrm{DA}} c_{ik}, \quad \forall k, \tag{6.19a}$$

$$c_{ik} = C_0 u_{ik} + \sum_s C_s x_{isk} + C^{\mathrm{UP}} y_{ik} + C^{\mathrm{DW}} z_{ik}, \quad \forall i, \forall k, \tag{6.19b}$$

$$d_{ik} = \underline{D} u_{ik} + \sum_s x_{isk}, \quad \forall i, \forall k, \tag{6.19c}$$

$$0 \leq x_{isk} \leq \overline{X}_s, \quad \forall i, \forall s, \forall k, \tag{6.19d}$$

$$u_{ik} - u_{i(k-1)} \leq y_{ik}, \quad \forall i, \forall k, \tag{6.19e}$$

$$u_{i(k-1)} - u_{ik} \leq z_{ik}, \quad \forall i, \forall k, \tag{6.19f}$$

$$y_{ik}, z_{ik} \in \{0, 1\}, \quad \forall i, \forall k. \tag{6.19g}$$

Constraint (6.19a) computes the expected total cost $\hat{c}_k$ per each interval k. Constraint (6.19b) yields the total production cost c_{ik} of the unit when scenario i realizes at k. Constraints (6.19c) and (6.19d) are auxiliary constraints to the piecewise linear formulation of the quadratic cost function. Constraints (6.19e) and (6.19f) compute the start-up (y_{ik}) and shut-down (z_{ik}) status of the unit. Finally, constraint (6.19g) imposes y_{ik} and z_{ik} to be binary variables.

Linear Formulation of Ω

The set Ω in constraint (6.16e) imposes the feasible operation of the unit. We replace it with the set of constraints (6.6) presented in Section 6.3.1, though adapted to the stochastic programming formulation. This writes:

$$u_{ik} \underline{D} \leq d_{ik} \leq u_{ik} \overline{D}, \quad \forall i, \forall k, \tag{6.20a}$$

$$d_{ik} - d_{i(k-1)} \leq R^{\mathrm{UP}}, \quad \forall i, \forall k, \tag{6.20b}$$

$$d_{i(k-1)} - d_{ik} \leq R^{\mathrm{DW}}, \quad \forall i, \forall k, \tag{6.20c}$$

$$u_{ik} \in \{0, 1\}, \quad \forall i, \forall k. \tag{6.20d}$$

Constraint (6.20a) forces d_{ik} to operate between $\underline{D}$ and $\overline{D}$ when on-line, and to be equal to 0 when off-line. Constraints (6.20b) and (6.20c) enforce the ramp up and down limitations of the production unit. Finally, constraint (6.20d) imposes that u_{ik} is binary.

6.4.2 Co-optimized Offering Strategy

This section considers a co-optimized offering strategy for building the day-ahead and the balancing marker offer curves. Consequently, the power producer is provided, together with the set I of day-ahead market price scenarios $\{\lambda_{ik}^{\mathrm{DA}} : \forall i \in I, \forall k \in K\}$, with a set J of balancing market price scenarios per each day-ahead possible realization i, i.e., $\{\lambda_{ijk}^{\mathrm{BA}} : \forall i \in I, \forall j \in J, \forall k \in K\}$. Of course, at the moment of offering in the day-ahead market, it does not have to submit the balancing market offers, but only the day-ahead ones. However, it includes the balancing market decisions within its optimization problem as recourse variables. The result is a two-stage stochastic optimization problem, where the producer simultaneously maximizes the profit from the two market stages, in the sense that it endogenously determines its future balancing actions while deriving its optimal day-ahead offers. Accordingly, we model the day-ahead production level $\overline{q}_k^{\mathrm{DA}}$ as a first-stage decision, and the upward and the downward production adjustment, i.e., $\overline{q}_{ik}^{\mathrm{UP}}$ and $\overline{q}_{ik}^{\mathrm{DW}}$, as second-stage decisions. Figure 6.7 illustrates the stochastic programming framework of the co-optimized offering strategy.

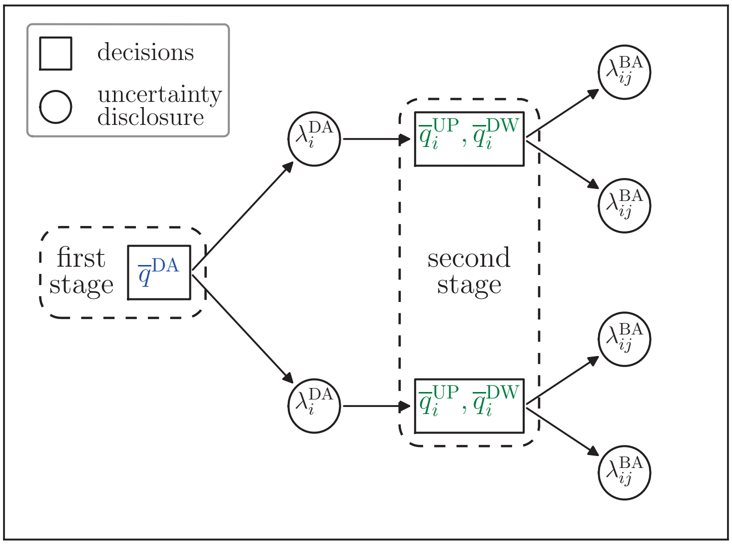


FIGURE 6.8: Schematic illustration of the stochastic programming framework of the co-optimized offering strategy.

Then, we make the day-ahead production variable $\overline{q}_k^{\mathrm{DA}}$ scenario-dependent (i.e., $\overline{q}_k^{\mathrm{DA}} \rightarrow \overline{q}_{ik}^{\mathrm{DA}}$) to model the offer curve, as seen in Section 3.4.3. Similarly, the real-time adjustments $\overline{q}_{ik}^{\mathrm{UP}}$ and $\overline{q}_{ik}^{\mathrm{DW}}$ are made scenario-dependent (i.e., $\overline{q}_{ik}^{\mathrm{UP}} \rightarrow \overline{q}_{ijk}^{\mathrm{UP}}$ and $\overline{q}_{ik}^{\mathrm{DW}} \rightarrow \overline{q}_{ijk}^{\mathrm{DW}}$) for obtaining the offer curves in the balancing stage, as seen in Section 3.4.4.

Finally, we also adapt the operational variables to the stochastic programming setup, e.g., $d_k \rightarrow d_{ijk}$. The offering strategy in (6.14) is accordingly adapted as follows:

$$\underset{\Gamma^{\text{coop}}}{\text{Max}} \quad \sum_k \hat{\rho}_k^{\text{DA}} + \hat{\rho}_k^{\text{UP}} + \hat{\rho}_k^{\text{DW}} - \hat{c}_k \tag{6.21a}$$

$$\text{s.t.} \quad \overline{q}_{ik}^{\text{DA}} + \overline{q}_{ijk}^{\text{UP}} - \overline{q}_{ijk}^{\text{DW}} = d_{ijk}, \quad \forall i, \forall j, \forall k, \tag{6.21b}$$

$$\overline{q}_{ik}^{\text{DA}}, \hat{\rho}_k^{\text{DA}} \in \Pi^{\text{DA}}, \quad \forall i, \forall k, \tag{6.21c}$$

$$\overline{q}_{ijk}^{\text{UP}}, \overline{q}_{ijk}^{\text{DW}}, \hat{\rho}_k^{\text{UP}}, \hat{\rho}_k^{\text{DW}} \in \Pi_{\text{Act}}^{\text{BA}}, \quad \forall i, \forall j, \forall k, \tag{6.21d}$$

$$\hat{c}_k = h(d_{ijk}), \quad \forall k, \tag{6.21e}$$

$$d_{ijk} \in \Omega, \quad \forall i, \forall j, \forall k, \tag{6.21f}$$

where

$$\Gamma^{\text{coop}} = \{d_{ijk}, \overline{q}_{ik}^{\text{DA}}, \overline{q}_{ijk}^{\text{UP}}, \overline{q}_{ijk}^{\text{DW}}, \hat{\rho}_k^{\text{DA}}, \hat{\rho}_k^{\text{UP}}, \hat{\rho}_k^{\text{DW}}, \hat{c}_k\}. \tag{6.22}$$

The objective function (6.21a) maximizes the expected producer's profit, modeling the participation in both the day-ahead and the balancing market. Constraint (6.21b) imposes the balance between the total energy exchanged with the market, i.e., $\overline{q}_{ik}^{\text{DA}} + \overline{q}_{ijk}^{\text{UP}} - \overline{q}_{ijk}^{\text{DW}}$, and the energy production of the power unit d_{ijk}. Constraint (6.21c) is a set of constraints associated with the day-ahead market offer curves, while constraint (6.21d) a set related to the balancing market offer curves. Constraint (6.21e) yields the expected production cost of the power unit, while constraint (6.21f) forces the power unit to operate in its feasible region Ω.

Linear Formulation of Π^{DA}

The set Π^{DA} is a set of constraints associated with the day-ahead market offer curves, which is presented in Equations (3.44), i.e.,

$$\hat{\rho}_k^{\text{DA}} = \sum_i \pi_i^{\text{DA}} \, \lambda_{ik}^{\text{DA}} \, \overline{q}_{ik}^{\text{DA}}, \quad \forall k, \tag{6.23a}$$

$$\overline{q}_{ik}^{\text{DA}} \geq \overline{q}_{i'k}^{\text{DA}} \quad \text{if} \quad \lambda_{ik}^{\text{DA}} \geq \lambda_{i'k}^{\text{DA}}, \quad \forall i, i', \forall k \tag{6.23b}$$

$$\overline{q}_{ik}^{\text{DA}} = \overline{q}_{i'k}^{\text{DA}} \quad \text{if} \quad \lambda_{ik}^{\text{DA}} = \lambda_{i'k}^{\text{DA}}, \quad \forall i, i', \forall k \tag{6.23c}$$

$$0 \leq \overline{q}_{ik}^{\text{DA}} \leq \overline{D}, \quad \forall i, \forall k \tag{6.23d}$$

Constraint (6.23a) computes the expectation of the day-ahead market income. Constraints (6.23b) and (6.23c) impose the non-decreasing and non-anticipativity conditions of the offer curves, respectively. Finally, constraint (6.23d) bounds the day-ahead market offer $\overline{q}_{ik}^{\mathrm{DA}}$ between 0 and $\overline{D}$.

Linear Formulation of $\Pi_{\mathrm{Act}}^{\mathrm{BA}}$

The set $\Pi_{\mathrm{Act}}^{\mathrm{BA}}$ in (6.21) is a set of constraints associated with the balancing market offer curves. The formulation for a price-taker and risk-neutral producer submitting offer curves in a pay-as-bid balancing market via stochastic programming is shown in Equations (3.93), Section 3.4.6. Adapting it to the present stochastic programming framework leads to:

$$\hat{\rho}_k^{\mathrm{UP}} = \sum_{ij} \pi_i^{\mathrm{DA}} \pi_{ij}^{\mathrm{BA}} \sum_{j'} M_{ijj'}^{\mathrm{UP}} \, o_{ij'k}^{\mathrm{UP}} \, \lambda_{ij'k}^{\mathrm{BA}}, \quad \forall k, \tag{6.24a}$$

$$\hat{\rho}_k^{\mathrm{DW}} = -\sum_{ij} \pi_i^{\mathrm{DA}} \pi_{ij}^{\mathrm{BA}} \sum_{j'} M_{ijj'}^{\mathrm{DW}} \, o_{ij'k}^{\mathrm{DW}} \, \lambda_{ij'k}^{\mathrm{BA}}, \quad \forall k, \tag{6.24b}$$

$$\overline{q}_{ijk}^{\mathrm{UP}} = \sum_{j'} M_{ijj'}^{\mathrm{UP}} o_{ij'k}^{\mathrm{UP}}, \quad \forall i, \forall j, \forall k, \tag{6.24c}$$

$$\overline{q}_{ijk}^{\mathrm{DW}} = \sum_{j'} M_{ijj'}^{\mathrm{DW}} \, o_{ij'k}^{\mathrm{DW}}, \quad \forall i, \forall j, \forall k, \tag{6.24d}$$

$$o_{ij'k}^{\mathrm{UP}} = 0 \quad \text{if} \quad \lambda_{ij'k}^{\mathrm{BA}} \leq \lambda_{ik}^{\mathrm{DA}}, \quad \forall i, \forall j', \forall k, \tag{6.24e}$$

$$o_{ij'k}^{\mathrm{DW}} = 0 \quad \text{if} \quad \lambda_{ij'k}^{\mathrm{BA}} \geq \lambda_{ik}^{\mathrm{DA}}, \quad \forall i, \forall j', \forall k, \tag{6.24f}$$

$$o_{ij'k}^{\mathrm{UP}}, o_{ij'k}^{\mathrm{DW}} \geq 0, \quad \forall i, \forall j', \forall k. \tag{6.24g}$$

where the matrices M^{UP} and M^{DW} are defined as:

$$M_{ijj'}^{\mathrm{UP}} = \begin{cases} 1, & \lambda_{ij}^{\mathrm{BA}} \geq \lambda_{ij'}^{\mathrm{BA}} \\ 0, & \text{otherwise.} \end{cases} \tag{6.25}$$

$$M_{ijj'}^{\mathrm{DW}} = \begin{cases} 1, & \lambda_{ij}^{\mathrm{BA}} \leq \lambda_{ij'}^{\mathrm{BA}} \\ 0, & \text{otherwise.} \end{cases} \tag{6.26}$$

Constraints (6.24a) and (6.24b) compute the expected market revenues from submitting up-regulation and down-regulation offer curves in a pay-as-bid balancing market, respectively. Constraints (6.24c) and (6.24d) yield the total amount of energy $\overline{q}_{ij}^{\mathrm{UP}}$ and $\overline{q}_{ij}^{\mathrm{DW}}$ scheduled provided that scenario (ij) realizes at k. Constraint

(6.24e) restricts the offering of up-regulation energy to the scenarios in which it is required. Similarly, constraint (6.24f) does for the down-regulation offers. Finally, constraint (6.24g) forces $o_{ij'}^{\mathrm{UP}}$ and $o_{ij'}^{\mathrm{DW}}$ to be non-negative.

Linear Formulation of the Cost Function

Constraint (6.21e) is replaced with the formulation provided in Section 6.3.2, i.e.,

$$\hat{c}_k = \sum_{ij} \pi_i^{\mathrm{DA}} \pi_{ij}^{\mathrm{BA}} c_{ijk}, \quad \forall k, \tag{6.27a}$$

$$c_{ijk} = C_0 u_{ijk} + \sum_s C_s x_{ijsk} + C^{\mathrm{UP}} y_{ijk} + C^{\mathrm{DW}} z_{ijk}, \quad \forall i, \forall j, \forall k, \tag{6.27b}$$

$$d_{ijk} = \underline{D} u_{ijk} + \sum_s x_{ijsk}, \quad \forall i, \forall j, \forall k, \tag{6.27c}$$

$$0 \leq x_{ijsk} \leq \overline{X}_s, \quad \forall i, \forall j, \forall s, \forall k, \tag{6.27d}$$

$$u_{ijk} - u_{ij(k-1)} \leq y_{ijk}, \quad \forall i, \forall j, \forall k, \tag{6.27e}$$

$$u_{ij(k-1)} - u_{ijk} \leq z_{ijk}, \quad \forall i, \forall j, \forall k, \tag{6.27f}$$

$$y_{ijk}, z_{ijk} \in \{0, 1\}, \quad \forall i, \forall j, \forall k. \tag{6.27g}$$

Constraint (6.27a) yields the expected total cost $\hat{c}_k$ per each interval k. Constraint (6.27b) evaluates the total production cost c_{ijk} of the unit when scenario (ij) occurs at k. Constraints (6.27c) and (6.27d) are auxiliary constraints to the piecewise linear formulation of the quadratic cost function. Constraints (6.27e) and (6.27f) compute the start-up (y_{ijk}) and shut-down (z_{ijk}) status of the unit. Finally, constraint (6.27g) imposes y_{ijk} and z_{ijk} to be binary variables.

Linear Formulation of Ω

The set Ω in constraint (6.21f) forces the power unit to operate in its feasible region. We replace it with the set of constraints (6.6) presented in Section 6.3.1. Its formulation adapted to the actual stochastic programming framework leads to

$$u_{ijk} \underline{D} \leq d_{ijk} \leq u_{ijk} \overline{D}, \quad \forall i, \forall j, \forall k, \tag{6.28a}$$

$$d_{ijk} - d_{i(k-1)} \leq R^{\mathrm{UP}}, \quad \forall i, \forall j, \forall k, \tag{6.28b}$$

$$d_{i(k-1)} - d_{ijk} \leq R^{\mathrm{DW}}, \quad \forall i, \forall j, \forall k, \tag{6.28c}$$

$$u_{ijk} \in \{0, 1\}, \quad \forall i, \forall j, \forall k. \tag{6.28d}$$

Constraint (6.28a) forces d_{ijk} to operate between $\underline{D}$ and $\overline{D}$ when on-line, and to be equal to 0 when off-line. Constraints (6.28b) and (6.28c) limits the ramp up and down of the production unit. Finally, constraint (6.28d) imposes that u_{ijk} is a binary variable.

6.5 Case Study

We test the two offering models (6.16) and (6.21) in a realistic case study. We generate market price scenarios according to the methodology presented in Chapter 4. The input parameters of the fundamental market model of Section 4.3 are shown in Tables 6.1 and 6.2. First, we generate 300 scenarios for $\lambda_{ik}^{\mathrm{DA}}$ and we select the 20 most representative ones. Then, for each scenario $\lambda_{ik}^{\mathrm{DA}}$, we generate 300 scenarios of $\lambda_{ijk}^{\mathrm{BA}}$ and keep the 20 most representative ones. This procedure results in a scenario tree with 400 branches.

TABLE 6.1: Parameters of the market price generation model

δ (€/MWh^2)	β (€/MWh^2)	μ_γ (€/MWh^3)	σ_γ^2 (€/MWh^3)	λ^0 (€/MWh)
-6.67×10^{-3}	1×10^{-4}	2×10^{-8}	3×10^{-9}	-20

TABLE 6.2: Values of parameter α_k

k	1	2	3	4	5	6	7	8
α_k (€/MWh)	322	312	315	317	340	349	353	369
k	9	10	11	12	13	14	15	16
α_k (€/MWh)	394	424	444	445	440	429	437	458
k	17	18	19	20	21	22	23	24
α_k (€/MWh)	446	423	408	383	373	346	331	332

We consider a thermal unit of capacity $\overline{D}$ = 120 MWh and a minimum production level of $\underline{D}$ = 40 MWh. Ramping limits are 40 MWh for both R^{UP} and R^{DW}. The quadratic cost function is approximated by a piecewise linear function of four generation blocks of equal size, i.e. X_s = 20 MWh $\forall s$. Table 6.3 shows the marginal cost C_s of each block, the cost C_0, the start-up cost C^{UP} and the shut-down cost C^{DW}. The optimization model is implemented using GUROBI in PYTHON environment.

TABLE 6.3: Parameters of the cost function

C_0 (€)	C_{s_1} (€/MWh)	C_{s_2} (€/MWh)	C_{s_3} (€/MWh)	C_{s_4} (€/MWh)	C^{UP} (€)	C^{DW} (€)
2860	23.5	31.5	45.6	72.3	800	100

We compare the two-stage co-optimization model with a sequential offering approach. As the sequential offering model only considers the day-ahead market offers, the results of the two models can not be compared. However, we use the co-optimized model to estimate the future balancing revenues when offering in the day-ahead with the sequential one. Given the optimal day-ahead offers $\widetilde{Q}_{ik}^{\mathrm{DA}*}$ obtained with the sequential model (6.16), we solve the co-optimized model (6.21) by replacing constraint (6.21c) with

$$\hat{\rho}_k^{\mathrm{DA}} = \sum_i \pi_i^{\mathrm{DA}} \, \lambda_{ik}^{\mathrm{DA}} \, \overline{q}_{ik}^{\mathrm{DA}}, \quad \forall k, \tag{6.29a}$$

$$\overline{q}_{ik}^{\mathrm{DA}} = \widetilde{Q}_{ik}^{\mathrm{DA}*}, \quad \forall i, \forall k. \tag{6.29b}$$

This translates in imposing that the day-ahead offer curves are the ones obtained with the sequential approach. By solving such model, we evaluate the total expected profit (day-ahead and balancing) when offering in the day-ahead market with a sequential approach. The remaining of the section is organized as follows. Sections 6.5.1, 6.5.2, and 6.5.3, analyze the optimal offering strategies (i.e., sequential and co-optimized) of the conventional power producer in an electricity market where the installed renewable energy capacity is 10, 20, and 30 GW, respectively. Finally, Section 6.5.4 compares the producer's profit based on the sequential and the co-optimized approach for increasing values of the installed wind capacity in the electricity market model.

6.5.1 Simulation with 10 GW of installed wind

Figure 6.9 illustrates the 20 selected trajectories $\lambda_{ik}^{\mathrm{DA}}$ when the installed wind power $\overline{W}$ in the market model is 10 GW. Similarly Figure 6.10 does for the 20 scenarios $\lambda_{ijk}^{\mathrm{BA}}$, for a given realization λ_k^{DA}. These scenarios are used to derive the optimal day-ahead market offer curves.

As an example, Table 6.4 shows the result of the day-ahead production variables $\overline{q}_{ik}^{\mathrm{DA}}$ for $k = 7$ obtained with the co-optimized and the sequential approach. Note that

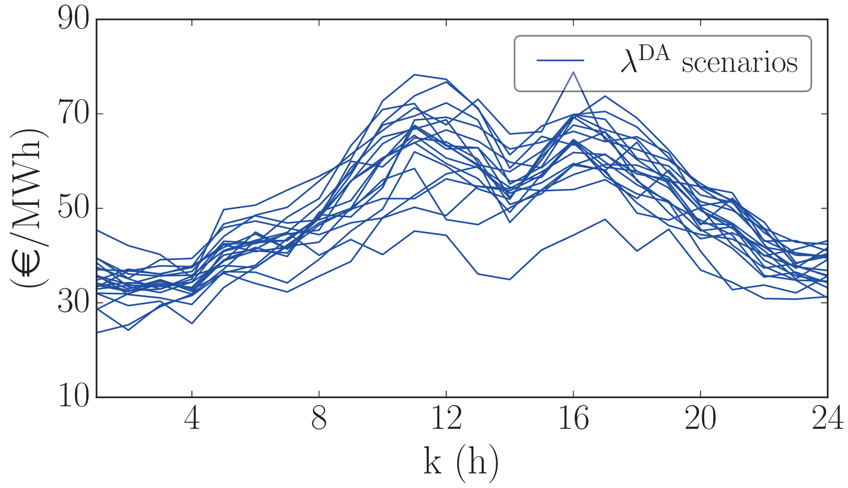


FIGURE 6.9: Day-ahead market price scenarios for $\overline{W} = 10$ GW in the market model.

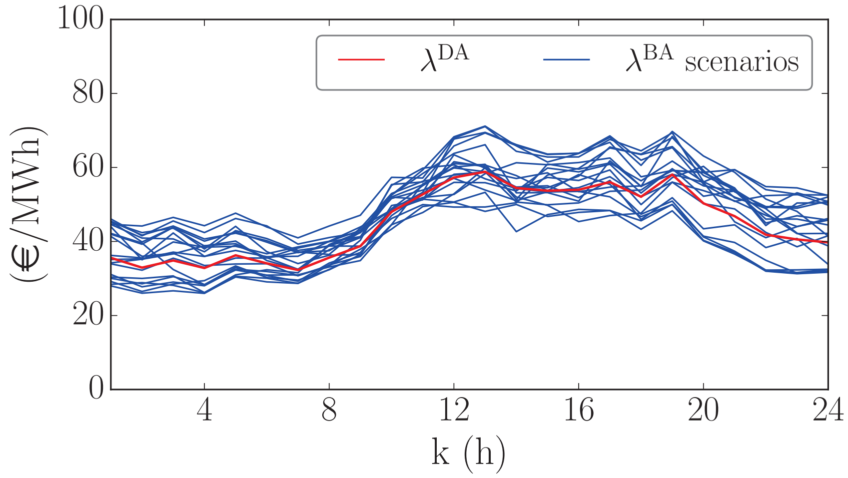


FIGURE 6.10: Balancing market price scenarios for $\overline{W} = 10$ GW in the market model, for a given realization of λ_k^{DA} (red).

$\{\lambda_{i7}^{\mathrm{DA}}, i \in I\}$, is the set of the day-ahead price scenarios, and each member of this set is viewed as a potential price offer. The results of Table 6.4 can be summarized, for

TABLE 6.4: Optimal values of $\overline{q}_{i7}^{\mathrm{DA}}$ for the sequential and the co-optimized approach ($\overline{W} = 10$ GW).

i	$\lambda_{i7}^{\mathrm{DA}}$ (€/MWh)	q_{i7}^{DA} (MWh) coop	seq	i	$\lambda_{i7}^{\mathrm{DA}}$ (€/MWh)	q_{i7}^{DA} (MWh) coop	seq
1	47.6	120	120	11	39.2	40	80
2	35.6	0	40	12	48.0	120	120
3	48.7	120	120	13	48.2	120	120
4	49.2	120	120	14	46.9	120	120
5	42.8	80	120	15	56.8	120	120
6	40.1	40	120	16	48.2	120	120
7	52.1	120	120	17	45.4	120	120
8	48.5	120	120	18	44.4	120	120
9	55.8	1120	120	19	47.9	120	120
10	46.5	120	120	20	50.6	120	120

the sequential approach, as

$$\overline{q}_7^{\mathrm{DA,seq}} = \begin{cases} 0, & \text{if } \lambda_7^{\mathrm{DA}} < 35.6, \\ 40, & \text{if } 35.6 \leq \lambda_7^{\mathrm{DA}} < 39.2, \\ 80, & \text{if } 39.2 \leq \lambda_7^{\mathrm{DA}} < 40.1, \\ 120, & \text{if } \lambda_7^{\mathrm{DA}} \geq 40.1, \end{cases} \tag{6.30}$$

while for the co-optimized offering strategy as

$$\overline{q}_7^{\mathrm{DA,coop}} = \begin{cases} 0, & \text{if } \lambda_7^{\mathrm{DA}} < 39.2, \\ 40, & \text{if } 39.2 \leq \lambda_7^{\mathrm{DA}} < 42.8, \\ 80, & \text{if } 42.8 \leq \lambda_7^{\mathrm{DA}} < 44.4, \\ 120, & \text{if } \lambda_7^{\mathrm{DA}} \geq 44.4. \end{cases} \tag{6.31}$$

Notice that $\overline{q}_7^{\mathrm{DA}}$ is expressed in MWh and λ_7^{DA} in €/MWh.

Following (6.30), a scenario-independent offer curve, using the sequential approach, can be built using three price-quantity offer points, i.e., (€35.6/MWh, 40 MWh), (€39.2/MWh, 80 MWh) and (€40.1/MWh, 120 MWh). Similarly, using the co-optimized approach, the day-ahead offer curve is created using three price-quantity offer points, i.e., (€39.2/MWh, 40 MWh), (€42.8/MWh, 80 MWh) and (€44.4/MWh, 120 MWh).

Figure 6.12 illustrates the two offer curves, the co-optimized curve in blue and the sequential one in red. Note that, based on the sequential approach, the producer is not willing to produce if $\lambda_7^{\mathrm{DA}} \leq 35.6$, while it wants to produce at full capacity if $\lambda_7^{\mathrm{DA}} \geq 40.1$. Differently, under the co-optimized strategy, the producer desires to produce 40 MWh if $39.2 \leq \lambda_7^{\mathrm{DA}} < 42.8$. For the same price interval, the sequential approach suggests operating at 80 MWh or even 120 MWh.

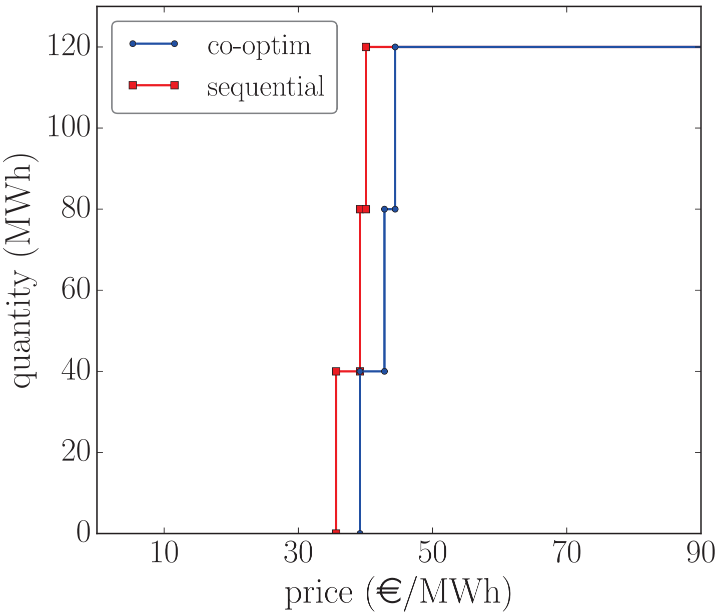


FIGURE 6.11: Illustration of the day-ahead market offer curves obtained with the sequential (red) and the co-optimized (blue) approach, for the time interval $k = 7$ ($\overline{W} = 10$ GW).

For $k = 7$, we also compute the balancing market offer curves. When the day-ahead market price λ_7^{DA} realization is €42.1/MWh, the offer curves in the balancing market under the co-optimized approach are

$$\overline{q}_7^{\mathrm{BA,coop}} = \begin{cases} 0, & \text{if } \lambda_7^{\mathrm{BA}} < 48.1 \\ 40, & \text{if } \lambda_7^{\mathrm{BA}} \geq 48.1, \end{cases} \tag{6.32}$$

Based on the co-optimized approach, the producer schedules to produce 80 MWh in the day-ahead market, following Equation (6.31). Then, it offers to increase its production of additional 40 MWh, provided that $\lambda_7^{\mathrm{BA}} \geq 48.1$. Differently, the sequential approach schedules 120 MWh for the same day-ahead market realization. In this case, the producer does not offer any regulating energy in the balancing market.

Figure 6.12 illustrates the offer curves submitted in the balancing market.

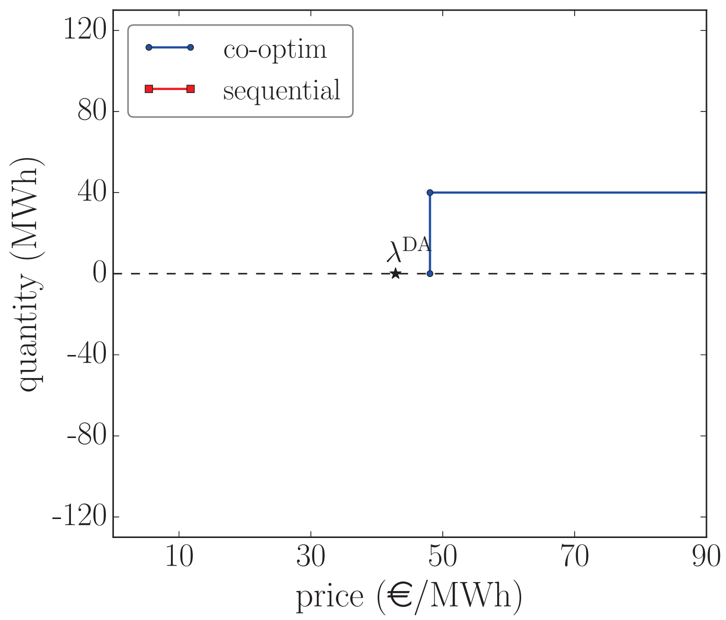


FIGURE 6.12: Illustration of the balancing market offer curves obtained with the sequential (red) and the co-optimized (blue) approach, for the time interval $k = 7$ ($\overline{W} = 10$ GW).

6.5.2 Simulation with 20 GW of installed wind

Figure 6.13 shows the 20 scenarios of λ^{DA}_{ik} when $\overline{W}$ is set to 20 GW. Then, Figure 6.14 shows to 20 scenarios λ^{BA}_{ijk}, for a given realization λ^{DA}_{k}.

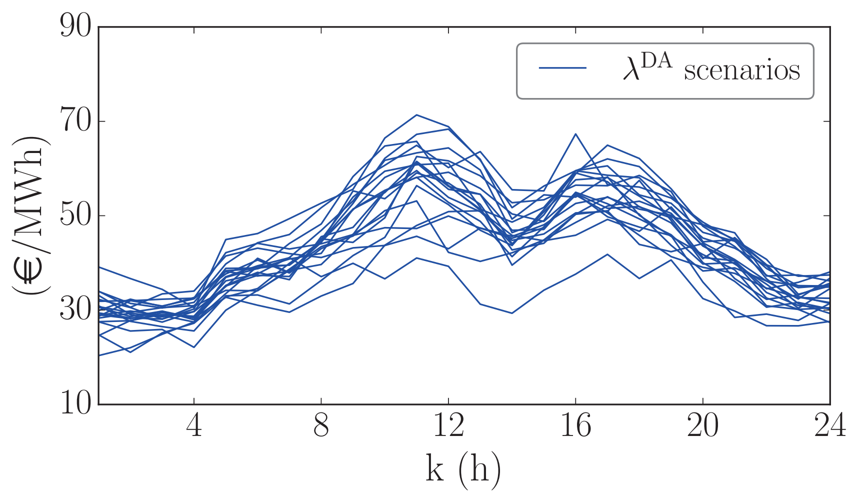


FIGURE 6.13: Day-ahead market price scenarios for $\overline{W} = 20$ GW in the market model.

Table 6.5 reports the optimal value of the day-ahead production variables $\overline{q}^{DA}_{ik}$ at $k = 7$ obtained from both the co-optimized and the sequential approach. For the two

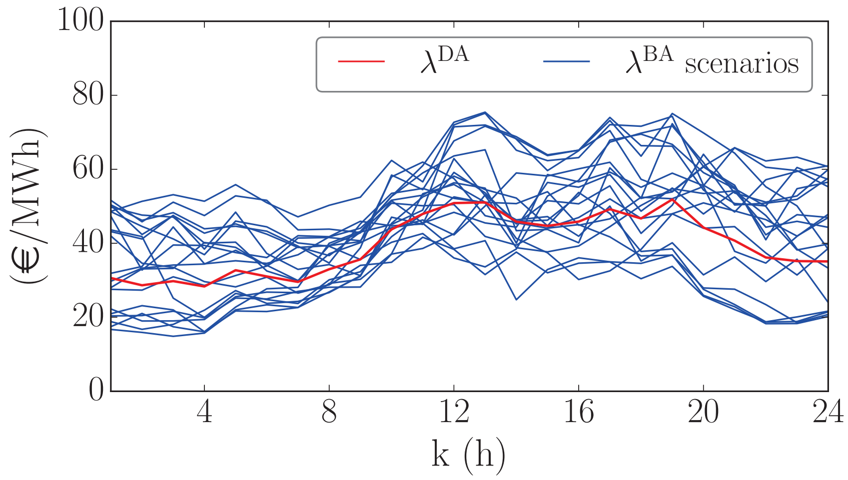


FIGURE 6.14: Balancing market price scenarios for $\overline{W} = 20$ GW in the market model, for a given realization of λ_k^{DA} (red).

TABLE 6.5: Optimal values of $\overline{q}_{i7}^{\mathrm{DA}}$ for the sequential and the co-optimized approach ($\overline{W} = 20$ GW).

i	$\lambda_{i7}^{\mathrm{DA}}$ (€/MWh)	$\overline{q}_{i7}^{\mathrm{DA}}$ (MWh)		i	$\lambda_{i7}^{\mathrm{DA}}$ (€/MWh)	$\overline{q}_{i7}^{\mathrm{DA}}$ (MWh)	
		coop	seq			coop	seq
1	44.1	80	40	11	36.3	0	0
2	33.0	0	0	12	44.4	80	40
3	45.1	80	40	13	44.6	80	40
4	45.6	80	40	14	43.4	80	40
5	39.7	40	40	15	52.7	120	120
6	37.1	0	0	16	44.7	80	40
7	48.2	80	80	17	42.0	80	40
8	44.9	80	40	18	41.1	40	40
9	51.7	120	120	19	44.3	80	40
10	43.0	80	40	20	46.9	80	40

approaches, the results of Table 6.5 are summarized in Equations (6.33) (sequential) and (6.34) (co-optimized) below:

$$\overline{q}_7^{\text{DA,seq}} = \begin{cases} 0, & \text{if } \lambda_7^{\text{DA}} < 39.7 \\ 40, & \text{if } 39.7 \leq \lambda_7^{\text{DA}} < 48.2 \\ 80, & \text{if } 48.2 \leq \lambda_7^{\text{DA}} < 51.7 \\ 120, & \text{if } \lambda_7^{\text{DA}} \geq 51.7, \end{cases} \tag{6.33}$$

$$\overline{q}_7^{\text{DA,coop}} = \begin{cases} 0, & \text{if } \lambda_7^{\text{DA}} < 39.7 \\ 40, & \text{if } 39.7 \leq \lambda_7^{\text{DA}} < 42.0 \\ 80, & \text{if } 42.0 \leq \lambda_7^{\text{DA}} < 51.7 \\ 120, & \text{if } \lambda_7^{\text{DA}} \geq 51.7, \end{cases} \tag{6.34}$$

where $\overline{q}_7^{\text{DA}}$ is expressed in MWh and λ_7^{DA} in €/MWh. Following Equation (6.33), the day-ahead offer curve, using the sequential approach, is built with three price-quantity offer points, i.e., (€39.7/MWh, 40 MWh), (€48.2/MWh, 80 MWh) and (€51.7/MWh, 120 MWh). Similarly, using the co-optimized approach, the day-ahead offer curve is identified by three price-quantity offer points, i.e., (€39.7/MWh, 40 MWh), (€42.0/MWh, 80 MWh) and (€51.7/MWh, 120 MWh). A graphic representation of the offer curves is given in Figure 6.15, where the co-optimized approach is shown in blue and the sequential one in red. Note that, in both cases, the producer is not willing to produce if $\lambda_7^{\text{DA}} \leq 39.7$ while desires to operate at its full capacity if $\lambda_7^{\text{DA}} \geq 51.7$. However, when $42.0 \leq \lambda_7^{\text{DA}} \leq 48.2$, the co-optimized approach suggests to produce 80 MWh, while the sequential approach does 40 MWh only.

For the same time interval, i.e., $k = 7$, we also derive the possible offer curves in the balancing market, for a given day-ahead market realization. E.g., provided that the realized day-ahead market price λ_7^{DA} is €44.1/MWh, the offer curve in the balancing market under the sequential approach is

$$\overline{q}_7^{\text{BA,seq}} = \begin{cases} 0, & \text{if } \lambda_7^{\text{BA}} < 56.0 \\ 40, & \text{if } 56.0 \leq \lambda_7^{\text{BA}} < 58.7 \\ 120, & \text{if } \lambda_7^{\text{BA}} \geq 58.7, \end{cases} \tag{6.35}$$

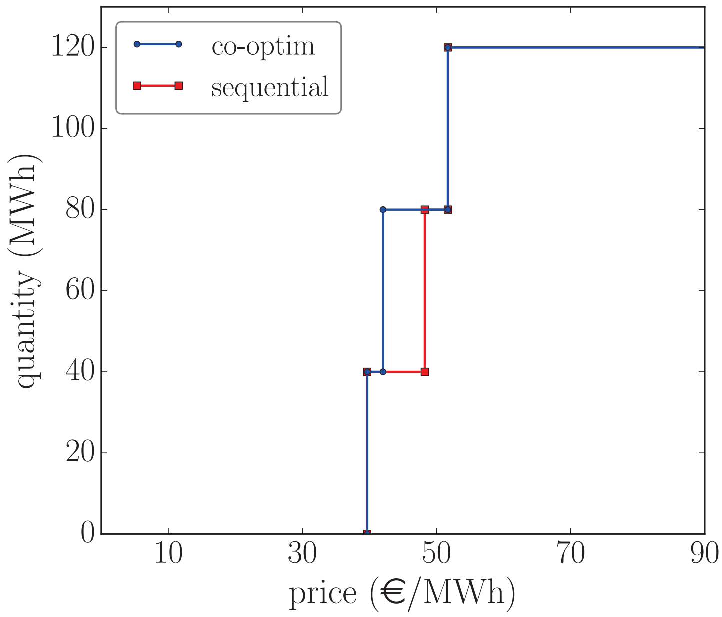


FIGURE 6.15: Illustration of the day-ahead market offer curves obtained with the sequential (red) and the co-optimized (blue) approach, for the time interval $k = 7$ ($\overline{W} = 20$ GW).

while based on the co-optimized one is

$$\overline{q}_7^{\text{BA,coop}} = \begin{cases} -40, & \text{if } \lambda_7^{\text{BA}} \leq 35.6 \\ 0, & \text{if } 35.6 < \lambda_7^{\text{BA}} < 55.7 \\ 40, & \text{if } \lambda_7^{\text{BA}} \geq 55.7, \end{cases} \tag{6.36}$$

Figure 6.16 illustrates the offer curves in the balancing market for the sequential (red) and the co-optimized approach (blue). Based on the co-optimized strategy, the producer schedules to produce 80 MWh (in the day-ahead market) and then reduces its production level to 40 MWh if $\lambda_7^{\text{BA}} \leq 35.6$, or increases it to 120 MWh in case $\lambda_7^{\text{BA}} \geq 55.7$, in the balancing market. Unlike the co-optimized approach, the sequential one schedules 40 MWh in the day-ahead market and then offers for up-regulation only in the balancing market. For instance, its production increases of 40 MWh if $\lambda_7^{\text{BA}} \geq 56.0$ while the increase is even more (80 MWh) in case $\lambda_7^{\text{BA}} \geq 58.7$.

6.5.3 Simulation with 30 GW of installed wind

Figure 6.17 shows the 20 scenarios of λ_{ik}^{DA} for $\overline{W}$ of 30 GW. Then, Figure 6.18 shows to 20 scenarios $\lambda_{ijk}^{\text{BA}}$, for a given realization λ_k^{DA}. Those scenarios are provided as input to the offering models to derive the optimal day-ahead market offers.

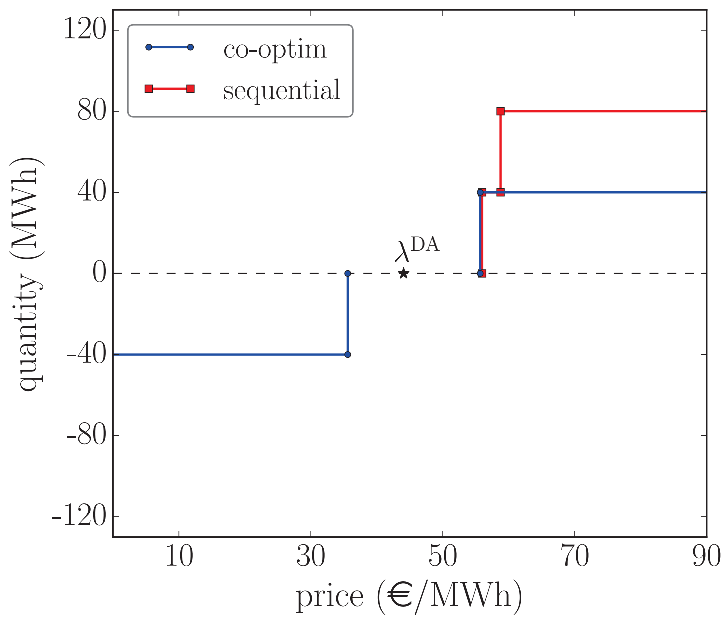


FIGURE 6.16: Illustration of the balancing market offer curves obtained with the sequential (red) and the co-optimized (blue) approach, for the time interval $k = 7$ ($\overline{W} = 20$ GW).

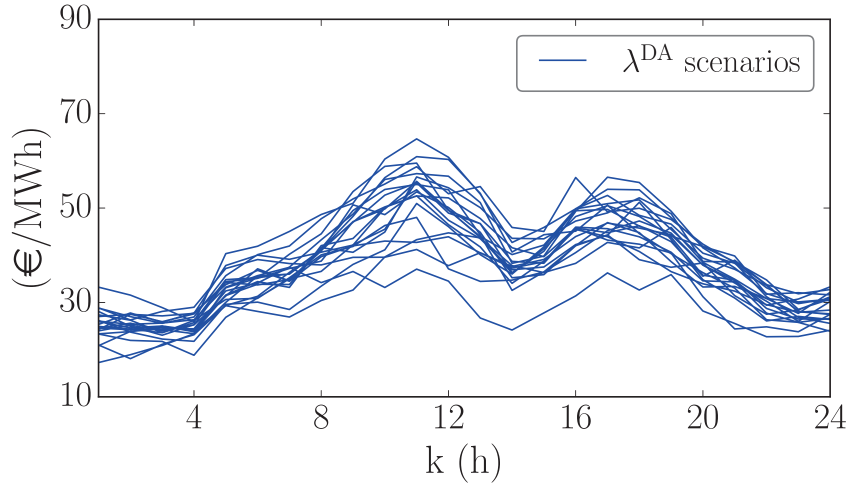


FIGURE 6.17: Day-ahead market price scenarios for $\overline{W} = 30$ GW in the market model.

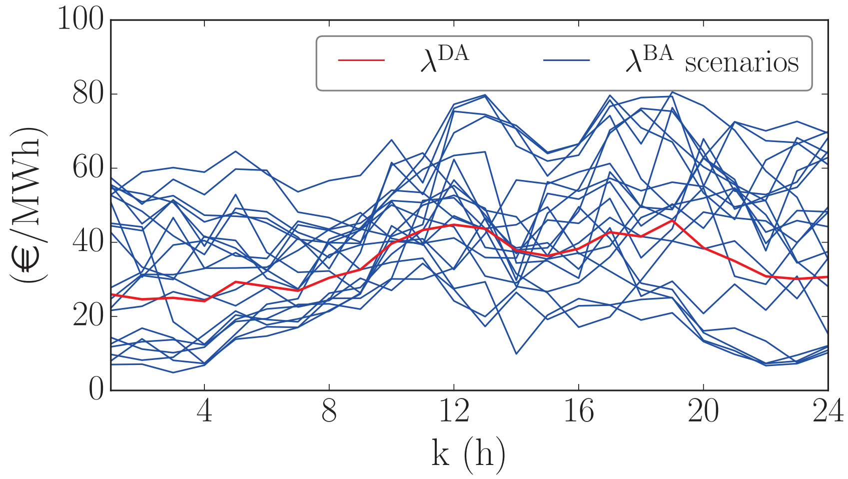


FIGURE 6.18: Balancing market price scenarios for $\overline{W} = 30$ GW in the market model, for a given realization of λ_k^{DA} (red).

Table 6.6 reports the optimal value of the day-ahead production variables $\overline{q}_{ik}^{DA}$ for $k = 7$ obtained with both the co-optimized and the sequential approach. Based on

TABLE 6.6: Optimal values of $\overline{q}_{i7}^{DA}$ for the sequential and the co-optimized approach ($\overline{W} = 30$ GW).

i	λ_{i7}^{DA} (€/MWh)	$\overline{q}_{i7}^{DA}$ (MWh) coop	$\overline{q}_{i7}^{DA}$ (MWh) seq	i	λ_{i7}^{DA} (€/MWh)	$\overline{q}_{i7}^{DA}$ (MWh) coop	$\overline{q}_{i7}^{DA}$ (MWh) seq
1	40.6	80	0	11	33.5	0	0
2	30.4	0	0	12	41.0	80	0
3	41.6	80	0	13	41.2	80	0
4	42.0	80	0	14	40.1	80	0
5	36.6	40	0	15	48.6	80	0
6	34.2	0	0	16	41.2	80	0
7	44.5	80	0	17	38.8	40	0
8	41.5	80	0	18	38.0	40	0
9	47.7	80	0	19	40.9	80	0
10	39.7	40	0	20	43.4	80	0

the co-optimized approach, the results of Table 6.6 are summarized as

$$\overline{q}_7^{DA,coop} = \begin{cases} 0, & \text{if } \lambda_7^{DA} < 36.6 \\ 40, & \text{if } 36.6 \leq \lambda_7^{DA} < 40.1 \\ 80, & \text{if } \lambda_7^{DA} \geq 40.1, \end{cases} \tag{6.37}$$

where $\overline{q}_7^{DA}$ is expressed in MWh and λ_7^{DA} in €/MWh. Following (6.37), the day-ahead offer curve is built with two price-quantity offer points, i.e., (€36.6/MWh, 40

MWh) and (€40.1/MWh, 80 MWh). Differently, using the sequential approach, the producer does not submit any offer in the day-ahead market. Figure 6.15 shows a graphic representation of the offer curve of the co-optimized approach (blue). The producer, based on the co-optimized approach, is not willing to produce if $\lambda_7^{\mathrm{DA}} \leq 36.6$ while desires to operate at 80 MWh if $\lambda_7^{\mathrm{DA}} \geq 40.1$.

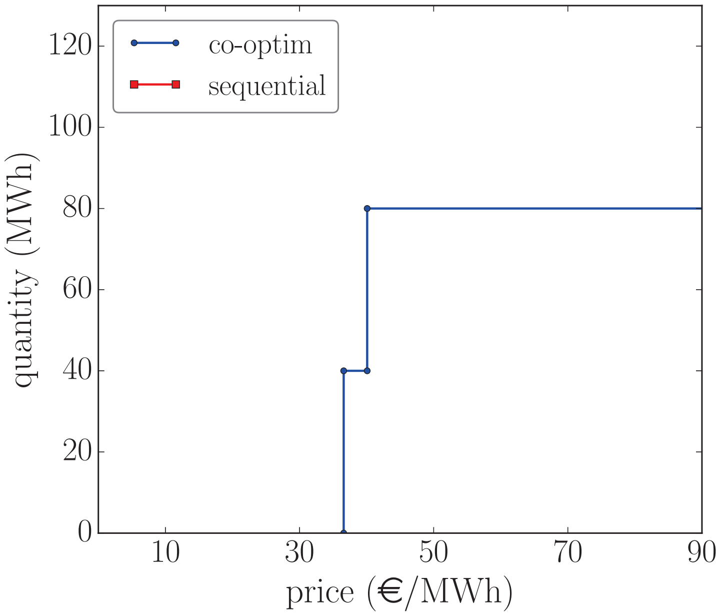


FIGURE 6.19: Illustration of the day-ahead market offer curves obtained with the sequential (red) and the co-optimized (blue) approach, for the time interval $k = 7$ ($\overline{W} = 30$ GW).

For the same time interval, i.e., $k = 7$, we also derive the offer curves in the balancing market, for a given day-ahead market outcome. When the realized day-ahead market price λ_7^{DA} is €40.7/MWh, the offer curve in the balancing market under the sequential approach is

$$\overline{q}_7^{\mathrm{BA,seq}} = \begin{cases} 0, & \text{if } \lambda_7^{\mathrm{BA}} < 62.7 \\ 40, & \text{if } 62.7 \leq \lambda_7^{\mathrm{BA}} < 76.4 \\ 80, & \text{if } \lambda_7^{\mathrm{BA}} \geq 76.4, \end{cases} \tag{6.38}$$

while under the co-optimized one is

$$\overline{q}_7^{\text{BA,coop}} = \begin{cases} -80, & \text{if } \lambda_7^{\text{BA}} \leq 28.5 \\ -40, & \text{if } 28.5 < \lambda_7^{\text{BA}} \leq 32.6 \\ 0, & \text{if } 32.6 < \lambda_7^{\text{BA}} < 58.0 \\ 40, & \text{if } \lambda_7^{\text{BA}} \geq 58.0. \end{cases} \tag{6.39}$$

Based on the sequential approach, the producer is not scheduled to produce in the day-ahead market. Then, it is willing to produce 40 MWh if $\lambda_7^{\text{BA}} \geq 62.6$, and 80 MWh if $\lambda_7^{\text{BA}} \geq 76.4$. Of course, it cannot offer down-regulation as it is not scheduled to operate in the day-ahead market. Differently, under the co-optimized approach, the producer operates at 80 MWh after the day-ahead market clearing. Then, it offers to adjust its production of additional 40 MWh upward (i.e., to produce 120 MWh) if $\lambda_7^{\text{BA}} \geq 58.0$. In this case, the producer can offer down-regulation energy in the balancing market. Indeed, it decreases its production to 40 MWh if $\lambda_7^{\text{BA}} \leq 32.6$, and to 0 MWh if $\lambda_7^{\text{BA}} \leq 28.5$. Figure 6.20 illustrates the offer curves in the balancing market for the two approaches.

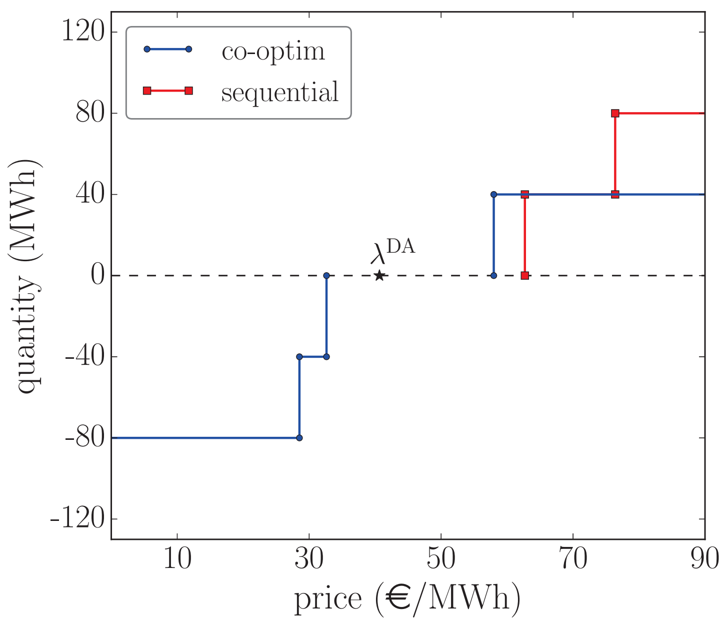


FIGURE 6.20: Illustration of the balancing market offer curves obtained with the sequential (red) and the co-optimized (blue) approach, for the time interval $k = 7$ ($\overline{W} = 30$ GW).

6.5.4 Comparative Analysis

This section compares the expected profits obtained by the sequential and the co-optimized offering strategy. The expected producer's profit gained based on the two alternative approaches under different conditions is shown in Table 6.7. In the case with $\overline{W} = 10$ GW, the expected profit loss of the sequential approach is around 2%. The power producer obtains a lower expected profit in the day-ahead market while earning more in the balancing stage, such that its total expected profit (including both markets) increases as well. This behavior is more observable in the simulations with higher values of installed wind capacity. For instance, the loss of profit is 22% and 91% in cases in which $\overline{W}$ is equal to 20 and 30 GW, respectively. The last case ($\overline{W}$=30 GW) gives more insight: based on the sequential approach, the producer does not participate in the day-ahead market and earns a low profit in the balancing stage only. In contrast, the producer gains a significant profit in the co-optimized approach, though it loses money in the day-ahead stage. In fact, it takes such a losing position in the day-ahead market to be able to produce profitable regulation services at the balancing stage.

TABLE 6.7: Expected profit of the producer

$\overline{W}$ (GW)	Approach	Profit in DA (10^3€)	Profit in BA (10^3€)	Total profit (10^3€)
10	co-optimized	16.82	3.25	20.09
	sequential	18.08	1.59	19.68
20	co-optimized	4.45	8.27	12.27
	sequential	7.68	2.27	9.95
30	co-optimized	-8.45	18.44	9.99
	sequential	0.00	0.87	0.87

Figures 6.21 illustrates the results of Table 6.7, where the sequential approach is shown in red and the co-optimized one in blue. Note how the difference between the expected profits grows as the share of renewable energy generation increases. The figure clarifies the strategy of the co-optimized approach of accepting producing in the day-ahead market even if the prices are below the marginal cost of the producer. This leads the producer in a profitable position for offering regulating energy in the balancing market and compensate the losses in the day-ahead market. Differently, based on the sequential approach, the producer does not include the future balancing market revenue in its objective function and accordingly does not schedule the unit if the day-ahead market prices are lower than its marginal cost.

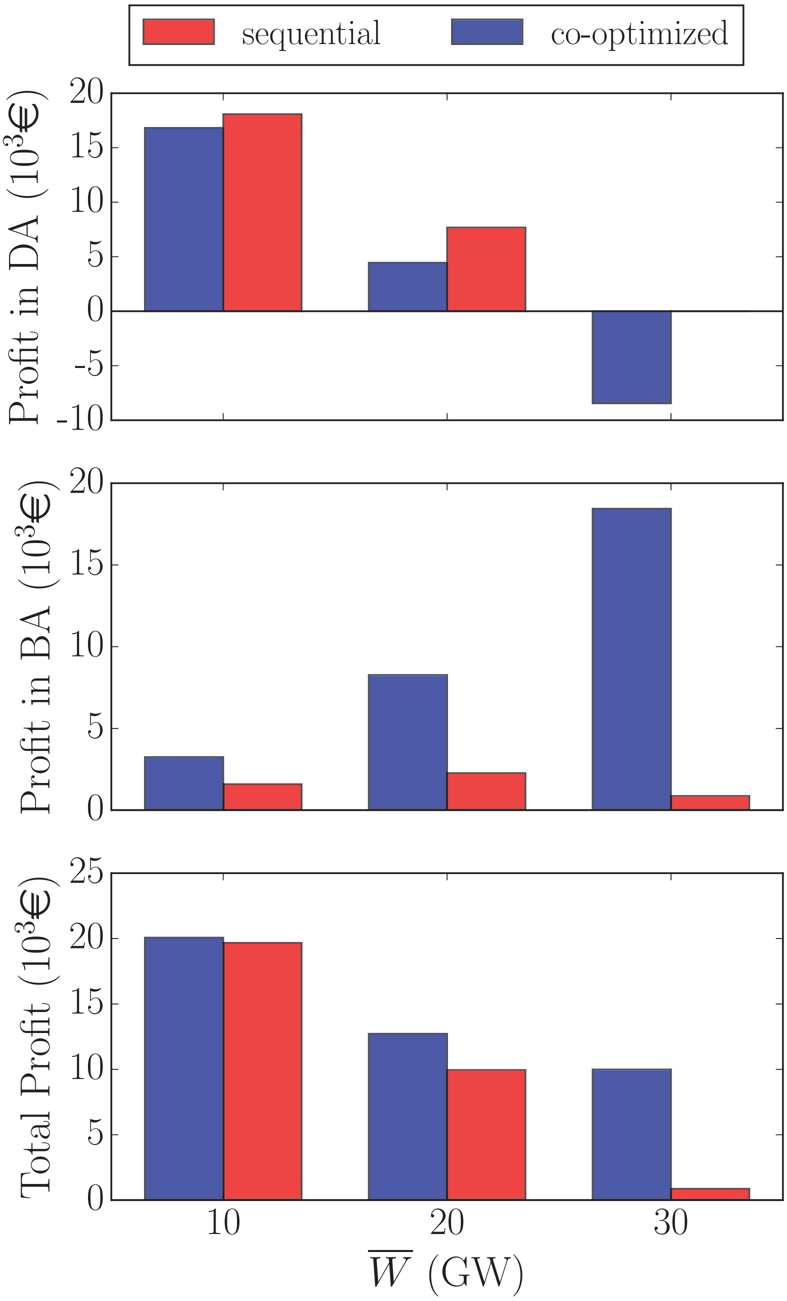


FIGURE 6.21: Day-ahead market, balancing market, and total expected profits of the co-optimized and the sequential approach.

6.6 Summary

This chapter takes the perspective of a price-taker and risk-neutral conventional power producer offering in a two-settlement electricity market having the balancing market settled under a pay-as-bid pricing scheme. First, we present a MILP formulation to approximate the non-convex feasible region and the non-convex cost function of the production unit. Then, we adapt the general offering strategy proposed in Section 3.4 to the case of the conventional producer. We develop two alternative offering strategies. The first is a sequential offering model, where the producer only looks at the upcoming trading floor when offering in the electricity market, without considering the following stages. The second co-optimizes the producer's offers in the day-ahead and the balancing market, these as recourse decisions as they do not need to be taken at the day-ahead stage. In this context, it is essential the LP formulation for the trading problem in pay-as-bid electricity market presented in Section 3.4.6. Indeed, this innovative formulation allows building the two-stage stochastic programming problem (co-optimized approach) as a MILP problem, where the non-linearities (i.e., the binary variables) arise from the operating region of the unit. Differently, using the NLP models available in the literature would have resulted in a MINLP problem, which may have high computational cost and, usually, does not ensure the optimality of the solution obtained.

We test the two offering approaches in a case study, using the electricity market model of Section 4.3 to generate the day-ahead and the balancing market price scenarios. Thanks to the fundamental market model, we simulate the effect of an increasing penetration of renewable energy generation on the day-ahead and the balancing market prices. Accordingly, we test the two approaches for 10, 20, and 30 GW of renewable generation capacity installed in the market model, analyzing both the optimal offer curves and the expected profits obtained with the two offering strategies. We show that for a low penetration of renewable energy generation, the difference between the two approaches is minimal. Indeed, the high prices in the day-ahead market lead the producer's to schedule most of its capacity in the day-ahead stage while having few possibilities of offering in the balancing stage. Differently, in the case with 30 GW of renewable energy generation installed, the day-ahead market prices are below the marginal cost of the producer which would not schedule its unit in the day-ahead market based on a sequential offering strategy.

Subsequently, even if the prices are profitable at the balancing stage (due to potential significant imbalances in the real-time), the producer is not in the position of offering regulating energy efficiently. Differently, co-optimizing the offering strategy in the two market stages, the producer decides to operate in the day-ahead market even if the prices are below its marginal cost. However, it knows that it will be in a convenient position for the balancing market where the large imbalances of the stochastic generation may lead to very lucrative prices. Co-optimizing its offering strategy, the producer can, at least partially, compensate the decrease of profit in the day-ahead market due to the increasing share of renewable energy generation in the electricity market.

The proposed test case shows that a co-optimized approach (where our innovative LP trading problem formulation is essential) may be increasingly important as the penetration of renewable capacity in the electricity market increases. Differently, the power producer would not schedule its unit in the day-ahead market due to the low market prices. However, this affects the producer, which incur in lower market profits, as well as the System Operator who has less available flexibility in the balancing stage.

Chapter 7

Trading of Virtual Power Plants

7.1 Introduction

This chapter considers the optimal offering strategy a Virtual Power Plant (VPP). A VPP is defined as a cluster of combined generating units (either stochastic or controllable), storage systems and flexible loads (Morales et al., 2013), which together act as a single participant in the electricity market. The key idea is to exploit the characteristics of the different technologies within the cluster and being more competitive in the electricity market. E.g., the real-time deviations of a stochastic production unit, such as wind and solar power unit, could be handled internally to a VPP thanks to the flexibility of other controllable technologies, such as an electric storage system or a conventional generation unit.

The optimal offering strategy of a renewable energy source has already been widely studied in the literature. Pinson, Chevallier, and Kariniotakis (2007) derive the optimal day-ahead market offer that maximizes the expected profit of a price-taker wind power producer provided with probabilistic forecasts of its future power production. They show that the optimal day-ahead market offer is a quantile of the probability distribution of the wind power production of the unit. Then, Morales, Conejo, and Pérez-Ruiz (2010) attach a similar problem through a stochastic programming approach. They consider an electricity market that includes a day-ahead, an intra-day, and a balancing market. They formulate the optimal offering strategy as a LP problem, while accurately modeling the future decisions of the producer in the intra-day and the balancing market within the optimization problem as recourse decisions. These studies model the stochastic producer as a passive actor in the balancing market. Indeed, it accesses the balancing stage for compensating its real-time deviations from the day-ahead contracted schedule. We refer the interested reader to Chapter 5 for a more extensive literature review on the topic of optimal trading of renewable energy generation.

Similarly, several optimal offering strategies for a conventional power generator are available in the literature. References (Arroyo and Conejo, 2000; Arroyo and Conejo, 2004; Conejo et al., 2004; Jabr, 2005) show how the feasibility region of a conventional production unit, e.g., coal- or gas-fired power unit, can be successfully modeled through a MILP formulation. Then, the trading problem in an electricity market settled under a uniform pricing scheme can be cast as an LP problem (Conejo, Carrión, and Morales, 2010), under the assumption of a price-taker producer. Differently than a stochastic producer, a conventional one is modeled as an active participant in the balancing market. Indeed, it accesses the balancing stage to offer its available regulating energy to the System Operator. We refer the interested reader to Chapter 6 for a broader coverage of this topic.

The optimal market participation of a VPP is less investigated in the literature. Ruiz, Cobelo, and Oyarzabal (2009) present an optimization algorithm to manage an aggregate of controllable loads, based on a direct load control. References (Mashhour and Moghaddas-Tafreshi, 2011a; Mashhour and Moghaddas-Tafreshi, 2011b) consider the bidding problem of a VPP in a joint electricity market for energy and reserve. In a deterministic environment, the authors provide a MINLP problem, solved by a genetic algorithm. Morales et al. (2013) analyze different combinations of generating units, flexible loads and storage systems while showing a general modeling approach of VPPs. Peik-Herfeh, Seifi, and Sheikh-El-Eslami (2013) formulate a price-based unit commitment aimed at obtaining the optimal day-ahead market offering of a VPP. They use a point estimate method to represent the uncertainty in the market prices and the generation sources. Differently, Pandžić, Kuzle, and Capuder (2013) analyze the optimal self-scheduling of a VPP, considering a weekly time horizon. They include long-term bilateral contracts and technical constraints of the units while deriving the optimal dispatch as a MILP stochastic optimization problem. Reference (Pandžić et al., 2013) proposes a decision-making tool for the optimal offering of a VPP under price and production uncertainty. The authors develop a two-stage stochastic offering model that aims at maximizing the expectation of the profit using a MILP approach. Other works, e.g., (Kardakos, Simoglou, and Bakirtzis, 2016), include the electricity market clearing process within the offering model, resulting in a bi-level stochastic optimization model. By doing that, the authors can endogenously model the effect of the VPP decisions on the market price formation.

The offering models for a VPP participating in an electricity market mentioned above, consider the VPP as a passive actor in the balancing market. They assume that the VPP uses the balancing stage to compensate the deviations from the day-ahead schedule that it can not handle within the cluster. However, the VPP, being a mix of stochastic and controllable technologies, may have some flexibility to offer in the balancing market in some trading intervals while needing to deviate from its schedule in other intervals. State-of-art electricity markets, usually, allow the offering of regulating energy in the balancing market only to production units that can always (except for unpredictable unit's failure) guarantee to respect their day-ahead schedule and being able to offer additional regulating energy to the System Operator. In this context, a VPP that includes stochastic generation units can hardly fulfill such requirements. This chapter introduces an innovative balancing market regulation that allows a more flexible participation of the VPP, denoted as *Active/Passive* participation. Indeed, the VPP can actively offer its flexibility in some trading intervals, while passively deviate in the remaining ones. However, in the trading intervals in which the VPP is an active participant, it cannot create uncontracted imbalances.

We consider two VPPs, composed of a stochastic generation unit, a conventional production unit, and an electric energy storage system. The first includes a wind farm and the second a PV solar unit. We formulate the optimal offering model of a VPP participating in an electricity market where the balancing stage allows an *Active/Passive* participation. A multi-stage stochastic optimization problem with recourse is formulated as a MILP problem. We analyze when the VPP may prefer to be an active participant and when a passive one. Active participation means more convenient prices in the balancing market, as well as a more constrained operation of the units. Indeed, the VPP needs to handle the wind (or solar) power fluctuations internally and cannot settle them in the balancing market. Differently, a passive participation translates into penalized balancing market prices but more flexibility of operation, as the VPP can deviate from its day-ahead production schedule. We compare the proposed *Passive/Active* participation strategy against a passive-only (i.e., *Passive*) and an active-only (i.e., *Active*) strategy. We investigate the increment in the expected VPP profit adopting the *Passive/Active*, with respect to the *Active* and *Passive* ones, for increasing values of the stochastic generation unit (either wind or PV solar) capacity.

The remaining of the chapter is organized as follows. Section 7.2 introduces the electricity market framework and the assumptions needed to formulate the offering model. Section 7.3 presents the structure of the VPP and formulates the feasible operating region of the different units in the cluster. Then, Section 7.4 derives the offering models that the VPP can decide to use while trading in the day-ahead market. Section 7.5 presents a case study, while Section 7.6 concludes the chapter with a summary.

7.2 Electricity Market Framework and Assumptions

We take the perspective a VPP offering in a two-settlement electricity market, similar to the one presented in Section 2.3. The day-ahead market is cleared once a day, at noon, simultaneously for the whole 24 hourly trading periods of the following day. The day-ahead market is settled under a uniform pricing scheme. Then, a balancing market is cleared separately per each hourly interval, one hour before the real-time operation. The provision of balancing energy is remunerated under a uniform pricing scheme. Both the day-ahead and balancing market give the possibility to submit non-decreasing offer curves. Then, deviations from the day-ahead schedule are settled in the balancing market under a dual-price imbalance settlement scheme. The VPP can offer its regulating energy as an active actor in the balancing market for some trading intervals, while "passively" deviating in remaining ones. However, in the intervals in which it decides to operate "actively," it cannot deviate from its production schedule. The intra-day trading floor is neglected for the sake of simplicity. Figure 7.1 shows a schematic representation of the electricity market submission process.

The power producer is assumed to be price-taker in both day-ahead and the balancing market. Hence, the market prices within the offering strategy of that producer are exogenous and uncertain. A set of scenarios models those uncertainties. The fundamental market model presented in Section 4.3 generates the market price scenarios, instead of using real market data.

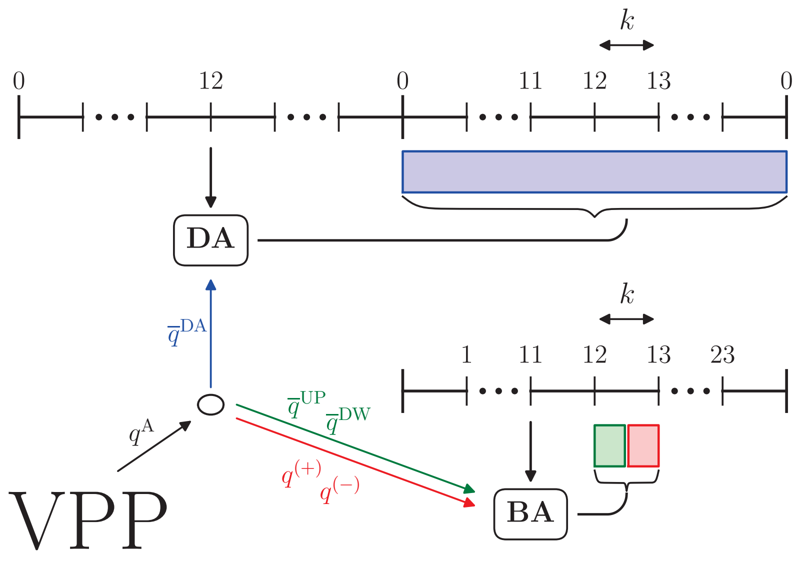


FIGURE 7.1: Schematic representation of the electricity market framework. The VPP submits the quantity offer $\overline{q}^{\mathrm{DA}}$ in the day-ahead (DA) market, while actively submitting regulation offers ($\overline{q}^{\mathrm{UP}}$, $\overline{q}^{\mathrm{DW}}$) or passively deviating from its day-ahead schedule ($q^{(+)}$, $q^{(-)}$) in the balancing (BA) market.

The scenario generation and reduction algorithm of Chapter 4 gives a a set of day-ahead market price trajectories $\{\lambda_{ik}^{\mathrm{DA}} : \forall i \in I, \forall k \in K\}$, where i is the index of the day-ahead market price scenarios and k the index of the time intervals. Each scenario i is associated with a probability π_i^{DA} of occurrence. Then, for each possible day-ahead realization i, it generates a set of balancing market price trajectories $\{\lambda_{ijk}^{\mathrm{BA}} : \forall i \in I, \forall j \in J, \forall k \in K\}$, where j is the index of the balancing market price scenarios. The scenario (ij) occurs with a probability $\pi_i^{\mathrm{DA}}\pi_{ij}^{\mathrm{BA}}$. Finally, the VPP has a set W of forecasts of the power production of the stochastic source (either solar or wind) $\{E_{\omega k} : \forall \omega \in W, \forall k \in K\}$. Each scenario ω realizes with a probability π_ω^E.

7.3 Virtual Power Plant model

This section presents the approach for modeling the feasible operating region and the production cost function of the VPP. Section 7.3.1 presents the structure of the VPP. Section 7.3.2 models the feasible operating region of the VPP. It replaces constraint (3.18e) (i.e., the set Ω) in the general offering strategy formulation (3.18) in Section 3.4. Finally, Section 7.3.3 provides a mathematical formulation of the cost function of the VPP. Such formulation substitutes constraint (3.18f) (i.e., the function $h(\cdot)$) in the general offering strategy (3.18).

7.3.1 Virtual Power Plant description

The VPP is composed of a conventional production unit, a stochastic power unit, and an electric energy storage. Let d_k (MWh) be the power production of the conventional unit, while E_k (MWh) is the power output of the stochastic energy source at k. Then, we denote with $p_k^{(\uparrow)}$ (MWh) and $p_k^{(\downarrow)}$ (MWh) the charging and discharging power of the electric energy storage. Accordingly, the total power production q_k^{A} of the VPP is given by

$$q_k^{\mathrm{A}} = d_k + p_k^{(\downarrow)} - p_k^{(\uparrow)} + E_k, \quad \forall k. \tag{7.1}$$

Notice that the term $p_k^{(\uparrow)}$ in Equation (7.1) is negative, since when the electric storage is charging it consumes electricity. We consider two alternative VPP structures. The first includes a wind turbine as the stochastic energy source. Figure 7.2 shows a schematic representation of the VPP. In the second the wind turbine is replaced by a PV solar unit. Figure 7.3 gives a representation of the composition of the second structure of the VPP.

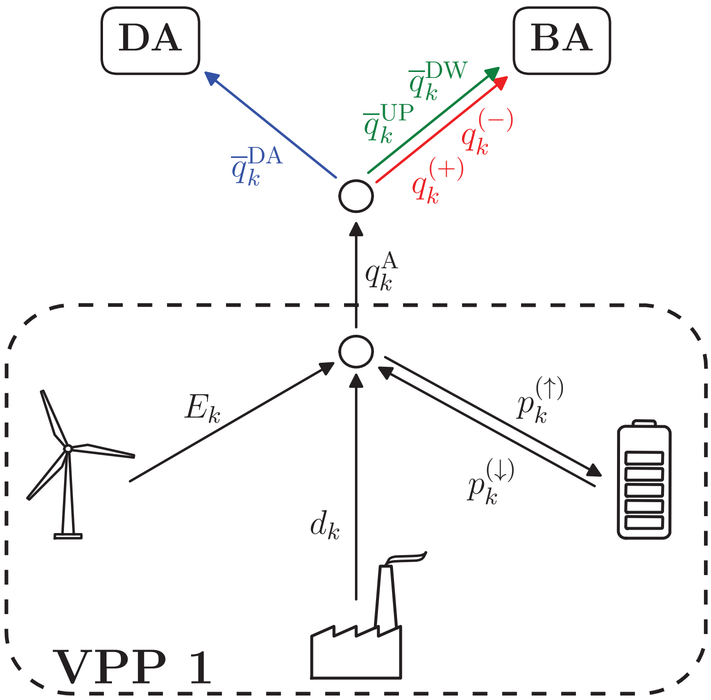


FIGURE 7.2: Schematic representation of the VPP1. It includes a wind farm, a conventional unit, and an electric storage.

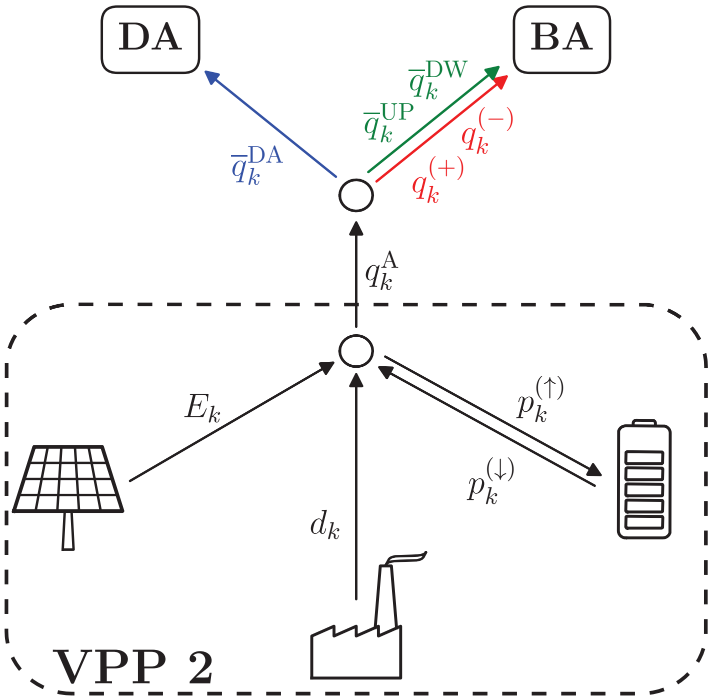


FIGURE 7.3: Schematic representation of the VPP2. It includes a PV solar unit, a conventional unit, and an electric storage.

7.3.2 Feasible Operating Region

The aim of this section is to obtain a set of constraints to replace constraint (3.18e) in the general offering strategy formulation (3.18), i.e.,

$$q_k^{\mathrm{A}} \in \Omega, \ \forall k, \tag{7.2}$$

The VPP is composed of three production units, i.e., a conventional unit, a stochastic energy source, and an electric storage. The uncertain power production E_k of the stochastic energy source is represented using a set W of scenarios. Consequently, it is seen as an input parameter and not as a variable. Differently, the conventional and storage units (dispatchable units) have a feasible region associated with their operational variables d_k, $p_k^{(\uparrow)}$ and $p_k^{(\downarrow)}$.

Storage Unit

Let denote with l_k the level of the energy stored in the battery at k. The energy balance within the storage is imposed as

$$l_k = l_{(k-1)} + \eta p_k^{(\uparrow)} - \frac{1}{\eta} p_k^{(\downarrow)}, \quad \forall k, \tag{7.3}$$

where $\eta \in [0, 1]$ is the charging/discharging efficiency of the storage unit. Note that Equation (7.3) needs the initial level L_0 of the storage to be implemented. Then, additional constraints model the feasible operation of the battery. Indeed, the level l_k of the storage has to lie between its minimum $\underline{L}$ (MWh) and maximum $\overline{L}$ (MWh) level. This leads to

$$\underline{L} \leq l_k \leq \overline{L}, \quad \forall k, \tag{7.4}$$

Also the charging/discharging powers are limited by their maximum capacity, denoted as $\overline{P}^{(\uparrow)}$ (MWh) and $\overline{P}^{(\downarrow)}$ (MWh), respectively. This writes

$$0 \leq p_k^{(\uparrow)} \leq \overline{P}^{(\uparrow)}, \quad \forall k, \tag{7.5}$$

$$0 \leq p_k^{(\uparrow)} \leq \overline{P}^{(\downarrow)}, \quad \forall k, \tag{7.6}$$

Note that $\overline{P}^{(\uparrow)}$ and $\overline{P}^{(\downarrow)}$ are expressed in MWh as we consider hourly intervals. Lastly, we impose the level of the storage in the last interval of the time horizon considered, i.e.,

$$l_K = L_k, \tag{7.7}$$

where L_k is expressed in MWh.

Conventional Unit

The feasible operating region of the conventional unit can be modeled using the following set of constraints:

$$u_k \underline{D} \leq d_k \leq u_k \overline{D}, \quad \forall k, \tag{7.8a}$$

$$d_k - d_{(k-1)} \leq R^{\mathrm{UP}}, \quad \forall k, \tag{7.8b}$$

$$d_{(k-1)} - d_k \leq R^{\mathrm{DW}}, \quad \forall k, \tag{7.8c}$$

$$u_k \in \{0, 1\}, \quad \forall k, \tag{7.8d}$$

where $\underline{D}$ (MWh) and $\overline{D}$ (MWh) are the minimum and the maximum power output of the unit, respectively. R^{UP} (MWh) and R^{DW} (MWh) are the upward and downward ramping limits of the unit, while u_k is the commitment (binary) status of the unit. Note that $\underline{D}$, $\overline{D}$, R^{UP}, and R^{DW} are all expressed in MWh as we consider hourly intervals. Constraint (7.8a) forces d_k to lie between $\underline{D}$ and $\overline{D}$ when the unit is on-line, i.e., when $u_k = 1$. Constraints (7.8b) and (7.8c) impose the ramping limitations, in upward and downward direction, respectively. Finally, constraint (7.8d) imposes that u_k is a binary variable.

Formulation of Ω

The set Ω representing the feasible region of the VPP can be replaced by

$$q_k^{\mathrm{A}} = d_k + p_k^{(\downarrow)} - p_k^{(\uparrow)} + E_k, \quad \forall k, \tag{7.9a}$$

$$l_k = l_{(k-1)} + \eta p_k^{(\uparrow)} - \frac{1}{\eta} p_k^{(\downarrow)}, \quad \forall k, \tag{7.9b}$$

$$\underline{L} \leq l_k \leq \overline{L}, \quad \forall k, \tag{7.9c}$$

$$0 \leq p_k^{(\uparrow)} \leq \overline{P}^{(\uparrow)}, \quad \forall k, \tag{7.9d}$$

$$0 \leq p_k^{(\downarrow)} \leq \overline{P}^{(\downarrow)}, \quad \forall k, \tag{7.9e}$$

$$u_k \underline{D} \leq d_k \leq u_k \overline{D}, \quad \forall k, \tag{7.9f}$$

$$d_k - d_{(k-1)} \leq R^{\mathrm{UP}}, \quad \forall k, \tag{7.9g}$$

$$d_{(k-1)} - d_k \leq R^{\mathrm{DW}}, \quad \forall k, \tag{7.9h}$$

$$u_k \in \{0, 1\}, \quad \forall k, \tag{7.9i}$$

Constraint (7.9a) imposes the energy balance between the total energy produced by the VPP, i.e., q_k^{A}, and the sum of the power injection of each component of the cluster. Constraints from (7.9b) to (7.9e) are associated with the storage unit, while constraints from (7.9f) to (7.9i) with the conventional production unit.

7.3.3 Production Cost Function

This section gives a formulation to compute the production cost function of the VPP. We derive a formulation aimed at replacing constrain (3.18f) in the general offering strategy formulation (3.18), i.e.,

$$\hat{c}_k = h\big(q_k^{\mathrm{A}}\big), \ \forall k \tag{7.10}$$

The stochastic energy source, i.e., wind or solar power, is assumed to produce energy at zero marginal cost. Similarly, the battery does not have a production cost associated with its charging and discharging power. Therefore, the production cost of the VPP is solely linked to the conventional unit power production d_k. In this case, we consider a linear cost function, neglecting non-convex costs such as start-up and shut-down costs. Accordingly, the cost c_k for producing the energy d_k at interval k can be evaluated as

$$c_k = C_0\, u_k + C\, d_k, \quad \forall k, \tag{7.11}$$

where C_0 (€) and C (€/MWh) control the cost function. Finally, Equation (7.10) can be reformulated as

$$\hat{c}_k = \mathbb{E}[c_k] \quad \forall k, \tag{7.12a}$$

$$c_k = C_0\, u_k + C\, d_k, \quad \forall k, \tag{7.12b}$$

7.4 Optimal Offering Strategy

This section derives different offering strategies that the VPP may use to obtain its best day-ahead market offers. We start from the generic formulation for the price-taker and risk-neutral producer in Equations (3.18), extended to the 24 trading intervals. We replace q_k^{A} with its formulation in Equation (7.1), thus leading to

$$\underset{\Gamma}{\text{Max}} \quad \sum_k \hat{\rho}_k^{\mathrm{DA}} + \hat{\rho}_k^{\mathrm{UP}} + \hat{\rho}_k^{\mathrm{DW}} + \hat{\rho}_k^{\mathrm{BA}} - \hat{c}_k \tag{7.13a}$$

$$\text{s.t.} \quad \overline{q}_k^{\mathrm{DA}} + \overline{q}_k^{\mathrm{UP}} - \overline{q}_k^{\mathrm{DW}} + q_k^{(+)} - q_k^{(-)} = d_k + p_k^{(\downarrow)} - p_k^{(\uparrow)} + E_k, \quad \forall k, \tag{7.13b}$$

$$\overline{q}_k^{\mathrm{DA}}, \hat{\rho}_k^{\mathrm{DA}} \in \Pi^{\mathrm{DA}}, \quad \forall k, \tag{7.13c}$$

$$\overline{q}_k^{\mathrm{UP}}, \overline{q}_k^{\mathrm{DW}}, \hat{\rho}_k^{\mathrm{UP}}, \hat{\rho}_k^{\mathrm{DW}} \in \Pi_{\mathrm{Act}}^{\mathrm{BA}}, \quad \forall k, \tag{7.13d}$$

$$q_k^{(+)}, q_k^{(-)}, \hat{\rho}_k^{\mathrm{BA}} \in \Pi_{\mathrm{Pas}}^{\mathrm{BA}}, \quad \forall k, \tag{7.13e}$$

$$\hat{c} = h(d_k)\,, \quad \forall k, \tag{7.13f}$$

$$d_k, p_k^{(\downarrow)}, p_k^{(\uparrow)} \in \Omega, \quad \forall k, \tag{7.13g}$$

where

$$\Gamma = \{d_k, p_k^{(\downarrow)}, p_k^{(\uparrow)}, \overline{q}_k^{\mathrm{DA}}, \overline{q}_k^{\mathrm{UP}}, \overline{q}_k^{\mathrm{DW}}, q_k^{(+)}, q_k^{(-)}, \hat{\rho}_k^{\mathrm{DA}}, \hat{\rho}_k^{\mathrm{UP}}, \hat{\rho}_k^{\mathrm{DW}}, \hat{\rho}_k^{\mathrm{BA}}, \hat{c}_k\}. \tag{7.14}$$

Section 7.4.1 presents a *Passive* offering strategy, i.e., when the VPP passively deviates from its schedule in the balancing market. Then, Section 7.4.2 formulates an *Active* offering strategy, i.e., when the producer actively offers the available regulating energy in the balancing market. Finally, Section 7.4.3 presents a *Active/Passive* strategy, where the VPP is allowed to actively offer regulating energy in some trading intervals and passively deviate from its schedule in the remaining ones.

7.4.1 Passive-only Offering Strategy

This section considers the VPP as a passive actor in the balancing market. Accordingly, we remove from model (7.13) the variables and constraints associated with an active participation. We eliminate constraint (7.13d), we remove the variables $\overline{q}_k^{\mathrm{UP}}$ and $\overline{q}_k^{\mathrm{DW}}$ from constraint (7.13b), and we remove $\hat{\rho}_k^{\mathrm{UP}}$ and $\hat{\rho}_k^{\mathrm{DW}}$ from the objective function (7.13a). The day-ahead market offers $\overline{q}_k^{\mathrm{DA}}$ are modeled as first stage decisions, while the real time deviations $q_k^{(+)}$ and $q_k^{(-)}$ as recourse decisions. Accordingly, we make them scenario-dependent, i.e., $q_k^{(+)} \rightarrow q_{i\omega k}^{(+)}$ and $q_k^{(-)} \rightarrow q_{i\omega k}^{(-)}$. Note that they are independent from the scenario j, as their value is fixed before that the balancing market prices are revealed. Figure 6.7 illustrates the stochastic programming framework of the *Passive* offering strategy. Besides, we render scenario-dependent

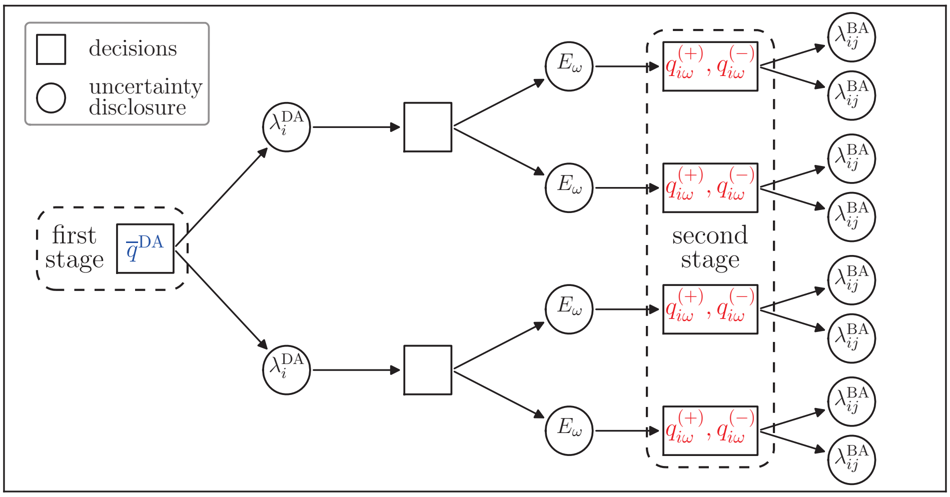


FIGURE 7.4: Schematic illustration of the stochastic programming framework of the *Passive* offering strategy.

the operational variables of the VPP, e.g., $d_k \rightarrow d_{i\omega k}$. Finally, we make the day-ahead market offer scenario-dependent to derive the offer curves, i.e., $\overline{q}_k^{\mathrm{DA}} \rightarrow \overline{q}_{ik}^{\mathrm{DA}}$. This leads to the following formulation:

$$\underset{\Gamma^{\text{Pas}}}{\text{Max}} \quad \sum_{k} \hat{\rho}_k^{\text{DA}} + \hat{\rho}_k^{\text{BA}} - \hat{c}_k \tag{7.15a}$$

$$\text{s.t.} \quad \overline{q}_{ik}^{\text{DA}} + q_{i\omega k}^{(+)} - q_{i\omega k}^{(-)} = d_{i\omega k} + p_{i\omega k}^{(\downarrow)} - p_{i\omega k}^{(\uparrow)} + E_{\omega k}, \quad \forall i, \forall \omega, \forall k \tag{7.15b}$$

$$\overline{q}_{ik}^{\text{DA}}, \hat{\rho}_k^{\text{DA}} \in \Pi^{\text{DA}}, \quad \forall i, \forall k \tag{7.15c}$$

$$q_{i\omega k}^{(+)}, q_{i\omega k}^{(-)}, \hat{\rho}_k^{\text{BA}} \in \Pi_{\text{Pas}}^{\text{BA}}, \quad \forall i, \forall \omega, \forall k \tag{7.15d}$$

$$\hat{c} = h(d_{i\omega k}), \quad \forall k \tag{7.15e}$$

$$d_{i\omega k}, p_{i\omega k}^{(\downarrow)}, p_{i\omega k}^{(\uparrow)} \in \Omega. \quad \forall i, \forall \omega, \forall k \tag{7.15f}$$

where

$$\Gamma^{\text{Pas}} = \{d_{i\omega k}, p_{i\omega k}^{(\downarrow)}, p_{i\omega k}^{(\uparrow)}, \overline{q}_{ik}^{\text{DA}}, q_{i\omega k}^{(+)}, q_{i\omega k}^{(-)}, \hat{\rho}_k^{\text{DA}}, \hat{\rho}_k^{\text{BA}}, \hat{c}_k\}. \tag{7.16}$$

The objective function (7.15a) maximizes the producer's expected profit, including both the day-ahead market revenue and the one linked to a passive balancing market participation. Constraint (7.15b) imposes the energy balance between the energy produced by the VPP and the energy exchanged with the electricity market. Constraint (7.15c) is a set of constraints associated with the day-ahead market offer curves. Similarly, constraint (7.32e) is a set of constraints related to the passive participation in the balancing market. Constraint (7.15e) yields the expected production cost of the VPP, while constraint (7.15f) imposes the feasible operating region of the VPP.

Linear Formulation of Π^{DA}

The set Π^{DA} in Equation (7.15c) is a set of constraints associated with the day-ahead market offer curves. The formulation of the set Π^{DA} is presented in Equations (3.44) in Section 3.4.3. Its formulation is adapted to the VPP, i.e.,

$$\underline{Q} = -\overline{P}^{(\uparrow)}, \tag{7.17}$$

$$\overline{Q} = \overline{D} + \overline{E} + \overline{P}^{(\downarrow)}, \tag{7.18}$$

where $\overline{E}$ (MW) is the capacity of the stochastic energy source. This leads to

$$\hat{\rho}_k^{\mathrm{DA}} = \sum_i \pi_i^{\mathrm{DA}} \, \lambda_{ik}^{\mathrm{DA}} \, \overline{q}_{ik}^{\mathrm{DA}}, \quad \forall k, \tag{7.19a}$$

$$\overline{q}_{ik}^{\mathrm{DA}} \geq \overline{q}_{i'k}^{\mathrm{DA}} \quad \text{if} \quad \lambda_{ik}^{\mathrm{DA}} \geq \lambda_{i'k}^{\mathrm{DA}}, \quad \forall i, i', \forall k \tag{7.19b}$$

$$\overline{q}_{ik}^{\mathrm{DA}} = \overline{q}_{i'k}^{\mathrm{DA}} \quad \text{if} \quad \lambda_{ik}^{\mathrm{DA}} = \lambda_{i'k}^{\mathrm{DA}}, \quad \forall i, i', \forall k \tag{7.19c}$$

$$-\overline{P}^{(\uparrow)} \leq \overline{q}_{ik}^{\mathrm{DA}} \leq \overline{D} + \overline{E} + \overline{P}^{(\downarrow)}, \quad \forall i, \forall k \tag{7.19d}$$

Constraint (7.25a) yields the expected day-ahead market income of the power producer. Constraints (7.25b) and (7.25c) impose the non-decreasing and non-anticipativity conditions of the offer curves, respectively. Finally, constraint (7.25d) limits the day-ahead market offers $\overline{q}_{ik}^{\mathrm{DA}}$.

Linear Formulation of $\Pi_{\mathrm{Pas}}^{\mathrm{BA}}$

The set $\Pi_{\mathrm{Pas}}^{\mathrm{BA}}$ in Equation (7.32e) is a set of constraint associated with the passive participation in the balancing market. Section 3.4.8 provides a formulation of $\Pi_{\mathrm{Pas}}^{\mathrm{BA}}$ for a price-taker and risk-neutral producer participating in a balancing market settled under a dual-price imbalance settlement scheme, i.e., Equations (3.106). Adapting it the VPP leads to

$$\hat{\rho}_k^{\mathrm{BA}} = \sum_{ij\omega} \pi_i^{\mathrm{DA}} \pi_{ij}^{\mathrm{BA}} \pi_\omega^{\mathrm{E}} \left(\lambda_{ij}^{(+)} q_{ij\omega}^{(+)} - \lambda_{ij}^{(-)} q_{ij\omega}^{(-)} \right), \quad \forall k \tag{7.20a}$$

$$q_{ij\omega}^{(+)}, q_{ij\omega}^{(-)} \geq 0, \quad \forall i, \forall j, \forall \omega. \tag{7.20b}$$

where $\lambda_{ij}^{(+)} = \min\left(\lambda_i^{\mathrm{DA}}, \lambda_{ij}^{\mathrm{BA}}\right)$ and $\lambda_{ij}^{(-)} = \max\left(\lambda_i^{\mathrm{DA}}, \lambda_{ij}^{\mathrm{BA}}\right)$. Constraint (7.20a) yields the expected balancing market income $\hat{\rho}_k^{\mathrm{BA}}$, while constraint (7.20b) forces $q_{ij\omega}^{(+)}$ and $q_{ij\omega}^{(-)}$ to be non negative.

Linear Formulation of the Cost Function

The formulation of the cost function presented in Section 7.3.3 is used to replace constraint (7.15e), i.e.,

$$\hat{c}_k = \sum_{i\omega} \pi_i^{\mathrm{DA}} \pi_\omega^{\mathrm{E}} c_{i\omega k}, \quad \forall k, \tag{7.21a}$$

$$c_{i\omega k} = C_0 \, u_{i\omega k} + C \, d_{i\omega k}, \quad \forall i, \forall \omega, \forall k, \tag{7.21b}$$

Linear Formulation of Ω

The formulation of the Ω shown in Section 7.3.2. Readjusting it according to the stochastic programming framework, constraint (7.15f) ca be replaced by

$$l_{i\omega k} = l_{i\omega(k-1)} + \eta p^{(\uparrow)}_{i\omega k} - \frac{1}{\eta} p^{(\downarrow)}_{i\omega k}, \quad \forall i, \forall \omega, \forall k, \tag{7.22a}$$

$$\underline{L} \leq l_{i\omega k} \leq \overline{L}, \quad \forall i, \forall \omega, \forall k, \tag{7.22b}$$

$$0 \leq p^{(\uparrow)}_{i\omega k} \leq \overline{P}^{(\uparrow)}, \quad \forall i, \forall \omega, \forall k, \tag{7.22c}$$

$$0 \leq p^{(\downarrow)}_{i\omega k} \leq \overline{P}^{(\downarrow)}, \quad \forall i, \forall \omega, \forall k, \tag{7.22d}$$

$$u_{i\omega k}\underline{D} \leq d_{i\omega k} \leq u_{i\omega k}\overline{D}, \quad \forall i, \forall \omega, \forall k, \tag{7.22e}$$

$$d_{i\omega k} - d_{i\omega(k-1)} \leq R^{\mathrm{UP}}, \quad \forall i, \forall \omega, \forall k, \tag{7.22f}$$

$$d_{i\omega(k-1)} - d_{i\omega k} \leq R^{\mathrm{DW}}, \quad \forall i, \forall \omega, \forall k, \tag{7.22g}$$

$$u_{i\omega k} \in \{0, 1\}, \quad \forall i, \forall \omega, \forall k. \tag{7.22h}$$

7.4.2 Active-only Offering Strategy

This section considers the VPP as an active participant in the balancing market. Consequently, we remove from model (7.13) the variables and constraints associated with a passive behavior. Indeed, we eliminate constraint (7.13e), we remove the variables $q^{(+)}_k$ and $q^{(-)}_k$ from the energy balance in constraint (7.13b), and we remove $\hat{\rho}^{\mathrm{BA}}_k$ from the objective function (7.13a). The day-ahead market offers $\overline{q}^{\mathrm{DA}}_k$ are modeled as first stage decisions, while the upward and downward regulation adjustments $\overline{q}^{\mathrm{UP}}_k$ and $\overline{q}^{\mathrm{DW}}_k$ as recourse decisions. Accordingly, they are made dependent to the day-ahead market realization, i.e., $\overline{q}^{\mathrm{UP}}_k \rightarrow \overline{q}^{\mathrm{UP}}_{ik}$ and $\overline{q}^{\mathrm{DW}}_k \rightarrow \overline{q}^{\mathrm{DW}}_{ik}$. Figure 6.7 illustrates the stochastic programming framework of the *Active* offering strategy. Then, we make the day-ahead market offers scenario-dependent in order to obtain non-decreasing offering curves, i.e., $\overline{q}^{\mathrm{DA}}_k \rightarrow \overline{q}^{\mathrm{DA}}_{ik}$. We do the same for the balancing market offers, i.e., $\overline{q}^{\mathrm{UP}}_{ik} \rightarrow \overline{q}^{\mathrm{UP}}_{ijk}$ and $\overline{q}^{\mathrm{DW}}_{ik} \rightarrow \overline{q}^{\mathrm{DW}}_{ijk}$. Additionally, we render scenario-dependent the operational variables of the VPP, e.g., $d_k \rightarrow d_{ij\omega k}$. This leads to the following offering

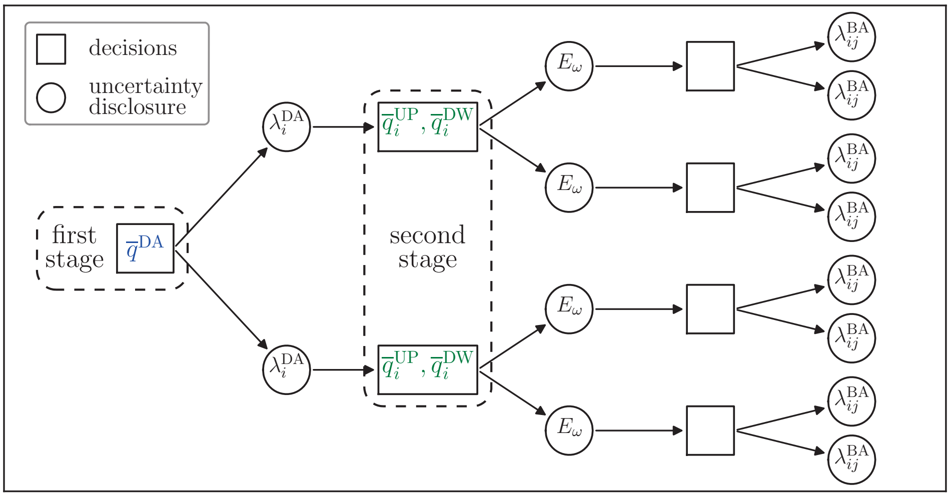


FIGURE 7.5: Schematic illustration of the stochastic programming framework of the *Active* offering strategy.

strategy:

$$\underset{\Gamma^{\text{Act}}}{\text{Max}} \quad \sum_{k} \hat{\rho}_k^{\text{DA}} + \hat{\rho}_k^{\text{UP}} + \hat{\rho}_k^{\text{DW}} - \hat{c}_k \tag{7.23a}$$

$$\text{s.t.} \quad \overline{q}_{ik}^{\text{DA}} + \overline{q}_{ijk}^{\text{UP}} - \overline{q}_{ijk}^{\text{DW}} = d_{ij\omega k} + p_{ij\omega k}^{(\downarrow)} - p_{ij\omega k}^{(\uparrow)} + E_{\omega k}, \quad \forall k, \tag{7.23b}$$

$$\overline{q}_{ik}^{\text{DA}}, \hat{\rho}_k^{\text{DA}} \in \Pi^{\text{DA}}, \quad \forall k, \tag{7.23c}$$

$$\overline{q}_{ijk}^{\text{UP}}, \overline{q}_{ijk}^{\text{DW}}, \hat{\rho}_k^{\text{UP}}, \hat{\rho}_k^{\text{DW}} \in \Pi_{\text{Act}}^{\text{BA}}, \quad \forall k, \tag{7.23d}$$

$$\hat{c} = h(d_{ij\omega k}), \quad \forall k, \tag{7.23e}$$

$$d_{ij\omega k}, p_{ij\omega k}^{(\downarrow)}, p_{ij\omega k}^{(\uparrow)} \in \Omega, \quad \forall k, \tag{7.23f}$$

where

$$\Gamma^{\text{Act}} = \{d_{ij\omega k}, p_{ij\omega k}^{(\downarrow)}, p_{ij\omega k}^{(\uparrow)}, \overline{q}_{ik}^{\text{DA}}, \overline{q}_{ijk}^{\text{UP}}, \overline{q}_{ijk}^{\text{DW}}, \hat{\rho}_k^{\text{DA}}, \hat{\rho}_k^{\text{UP}}, \hat{\rho}_k^{\text{DW}}, \hat{c}_k\}. \tag{7.24}$$

The objective function maximizes the expected producer's profit, including the revenues from both the day-ahead and the balancing market. Constraint (7.23b) imposes the energy balance between the energy production of the VPP and the amount of energy exchanged with the electricity market platform. Constraints (7.23c) and (7.23d) are two set of constraints associated with the day-ahead and balancing market offer curves, respectively. Finally, constraint (7.23e) yields the expected production cost of the VPP, while constraint (7.23f) imposes the feasible operating region of the units composing the VPP. Notice that the *Active* offering strategy may be infeasible. It occurs when the VPP cannot compensate the imbalances of the stochastic energy source with the dispatchable units.

Linear Formulation of Π^{DA}

The set Π^{DA} in Equation (7.15c) is the same one seen in Section 7.4.1, i.e.,

$$\hat{\rho}_k^{\text{DA}} = \sum_i \pi_i^{\text{DA}} \lambda_{ik}^{\text{DA}} \overline{q}_{ik}^{\text{DA}}, \quad \forall k, \tag{7.25a}$$

$$\overline{q}_{ik}^{\text{DA}} \geq \overline{q}_{i'k}^{\text{DA}} \quad \text{if} \quad \lambda_{ik}^{\text{DA}} \geq \lambda_{i'k}^{\text{DA}}, \quad \forall i, i', \forall k \tag{7.25b}$$

$$\overline{q}_{ik}^{\text{DA}} = \overline{q}_{i'k}^{\text{DA}} \quad \text{if} \quad \lambda_{ik}^{\text{DA}} = \lambda_{i'k}^{\text{DA}}, \quad \forall i, i', \forall k \tag{7.25c}$$

$$-\overline{P}^{(\uparrow)} \leq \overline{q}_{ik}^{\text{DA}} \leq \overline{D} + \overline{E} + \overline{P}^{(\downarrow)}, \quad \forall i, \forall k \tag{7.25d}$$

Linear Formulation of $\Pi_{\text{Act}}^{\text{BA}}$

The set $\Pi_{\text{Act}}^{\text{BA}}$ imposing a set of constraints associated with the balancing market offer curves is presented in Section 3.4.6. As the balancing market is settled under a uniform pricing scheme, we use the formulation in Equations (3.81). Adapting it the stochastic framework leads to:

$$\hat{\rho}_k^{\text{UP}} = \sum_{ij} \pi_i^{\text{DA}} \pi_{ji}^{\text{BA}} \lambda_{ijk}^{\text{BA}} \overline{q}_{ijk}^{\text{UP}}, \quad \forall k, \tag{7.26a}$$

$$\hat{\rho}_k^{\text{DW}} = -\sum_{ij} \pi_i^{\text{DA}} \pi_{ij}^{\text{BA}} \lambda_{ijk}^{\text{BA}} \overline{q}_{ijk}^{\text{DW}}, \quad \forall k, \tag{7.26b}$$

$$\overline{q}_{ijk}^{\text{UP}} \geq \overline{q}_{ij'k}^{\text{UP}} \quad \text{if} \quad \lambda_{ijk}^{\text{BA}} \geq \lambda_{ij'k}^{\text{BA}}, \quad \forall i, \forall j, \forall j', \forall k \tag{7.26c}$$

$$\overline{q}_{ijk}^{\text{UP}} = \overline{q}_{ij'k}^{\text{UP}} \quad \text{if} \quad \lambda_{ijk}^{\text{BA}} = \lambda_{ij'k}^{\text{BA}}, \quad \forall i, \forall j, \forall j', \forall k \tag{7.26d}$$

$$\overline{q}_{ijk}^{\text{DW}} \leq \overline{q}_{ij'k}^{\text{DW}} \quad \text{if} \quad \lambda_{ijk}^{\text{BA}} \geq \lambda_{ij'k}^{\text{BA}}, \quad \forall i, \forall j, \forall j', \forall k \tag{7.26e}$$

$$\overline{q}_{ijk}^{\text{DW}} = \overline{q}_{ij'k}^{\text{DW}} \quad \text{if} \quad \lambda_{ijk}^{\text{BA}} = \lambda_{ij'k}^{\text{BA}}, \quad \forall i, \forall j, \forall j', \forall k \tag{7.26f}$$

$$\overline{q}_{ijk}^{\text{UP}} = 0 \quad \text{if} \quad \lambda_{ijk}^{\text{BA}} \leq \lambda_{ik}^{\text{DA}}, \quad \forall i, \forall j, \forall k, \tag{7.26g}$$

$$\overline{q}_{ijk}^{\text{DW}} = 0 \quad \text{if} \quad \lambda_{ijk}^{\text{BA}} \geq \lambda_{ik}^{\text{DA}}, \quad \forall i, \forall j, \forall k, \tag{7.26h}$$

$$\overline{q}_{ijk}^{\text{UP}}, \overline{q}_{ijk}^{\text{DW}} \geq 0, \quad \forall i, \forall j, \forall k. \tag{7.26i}$$

Constraints (7.26a) and (7.26b) yield the expected revenue from submitting offer curves in the balancing market for up- and down-regulation, respectively. Constraints (7.26c) and (7.26e) impose the non-decreasing and non-increasing requirements of the up- and down-regulation offer curves, respectively. Constraints (7.26d) and (7.26f) enforce the non-anticipativity condition of the offer curves, while constraints (7.26g) and (7.26h) prevent from offering of up- and down-regulation energy when not required by the system. Finally, constraint (7.26i) sets $\overline{q}^{\text{UP}}_{ijk}$ and $\overline{q}^{\text{DW}}_{ijk}$ as non-negative variables.

Linear Formulation of the Cost Function

The formulation of the cost function presented in Section 7.3.3 replaces constraint (7.15e), i.e.,

$$\hat{c}_k = \sum_{ij\omega} \pi^{\text{DA}}_i \pi^{\text{BA}}_{ij} \pi^{\text{E}}_\omega c_{ij\omega k}, \quad \forall k, \tag{7.27a}$$

$$c_{ij\omega k} = C_0 u_{ij\omega k} + C d_{ij\omega k}, \quad \forall i, \forall j, \forall \omega, \forall k, \tag{7.27b}$$

Linear Formulation of Ω

The formulation of Ω given in Section 7.3.2 substitutes constraint (7.15f) as

$$l_{ij\omega k} = l_{ij\omega(k-1)} + \eta p^{(\uparrow)}_{ij\omega k} - \frac{1}{\eta} p^{(\downarrow)}_{ij\omega k}, \quad \forall i, \forall j, \forall \omega, \forall k, \tag{7.28a}$$

$$\underline{L} \leq l_{ij\omega k} \leq \overline{L}, \quad \forall i, \forall j, \forall \omega, \forall k, \tag{7.28b}$$

$$0 \leq p^{(\uparrow)}_{ij\omega k} \leq \overline{P}^{(\uparrow)}, \quad \forall i, \forall j, \forall \omega, \forall k, \tag{7.28c}$$

$$0 \leq p^{(\downarrow)}_{ij\omega k} \leq \overline{P}^{(\downarrow)}, \quad \forall i, \forall j, \forall \omega, \forall k, \tag{7.28d}$$

$$u_{ij\omega k} \underline{D} \leq d_{ij\omega k} \leq u_{ij\omega k} \overline{D}, \quad \forall i, \forall j, \forall \omega, \forall k, \tag{7.28e}$$

$$d_{ij\omega k} - d_{ij\omega(k-1)} \leq R^{\text{UP}}, \quad \forall i, \forall j, \forall \omega, \forall k, \tag{7.28f}$$

$$d_{ij\omega(k-1)} - d_{ij\omega k} \leq R^{\text{DW}}, \quad \forall i, \forall j, \forall \omega, \forall k, \tag{7.28g}$$

$$u_{ij\omega k} \in \{0, 1\}, \quad \forall i, \forall j, \forall \omega, \forall k. \tag{7.28h}$$

7.4.3 Active-Passive Offering Strategy

This section considers the novel *Active/Passive* participation in the balancing market. This offering strategy is a trade-off between the *Passive* strategy shown in Section 7.4.1 and the *Active* one of Section 7.4.2. The day-ahead market offers $\overline{q}^{\text{DA}}_k$ are modeled as first stage decisions, while the upward and downward regulation adjustments $\overline{q}^{\text{UP}}_k$ and $\overline{q}^{\text{DW}}_k$ and the real-time deviations $q^{(+)}_k$ and $q^{(-)}_k$ as recourse decisions.

Accordingly, the regulation adjustments are made dependent on the day-ahead scenarios, i.e., $\overline{q}_k^{\mathrm{UP}} \to \overline{q}_{ik}^{\mathrm{UP}}$ and $\overline{q}_k^{\mathrm{DW}} \to \overline{q}_{ik}^{\mathrm{DW}}$, while the real time deviations dependent on the scenario $(i\omega)$, as their value is conditional to the realization of the stochastic energy source production, i.e., $q_k^{(+)} \to q_{i\omega k}^{(+)}$ and $q_k^{(-)} \to q_{i\omega k}^{(-)}$. Figure 6.7 illustrates the stochastic programming framework of the *Active/Passive* offering strategy. Then,

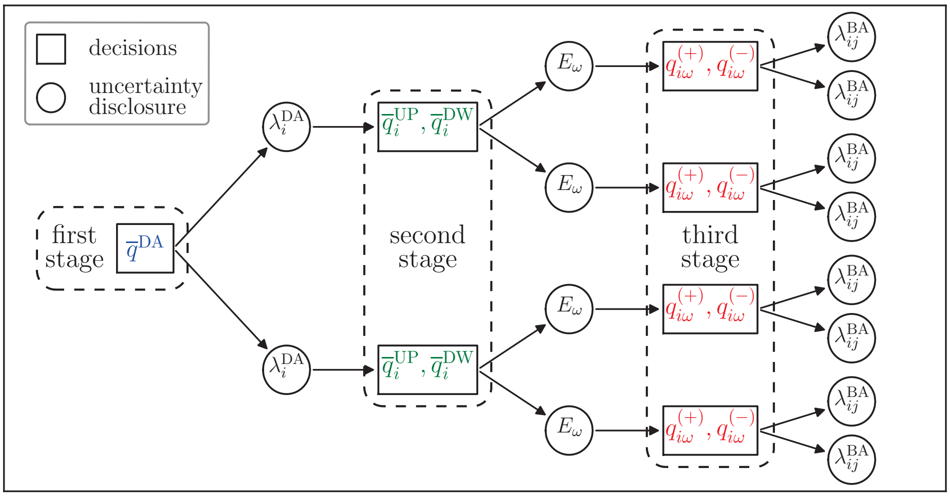


FIGURE 7.6: Schematic illustration of the stochastic programming framework of the *Active/Passive* offering strategy.

we make the day-ahead market offers scenario-dependent to obtain offer curves, i.e., $\overline{q}_k^{\mathrm{DA}} \to \overline{q}_{ik}^{\mathrm{DA}}$. We do the same for the balancing market regulation offers, i.e., $\overline{q}_{ik}^{\mathrm{UP}} \to \overline{q}_{ijk}^{\mathrm{UP}}$ and $\overline{q}_{ik}^{\mathrm{DW}} \to \overline{q}_{ijk}^{\mathrm{DW}}$. Additionally, we prevent the VPP from operating actively and passively simultaneously in the same trading interval k. Accordingly, for each k, we impose

$$(\overline{q}_{ijk}^{\mathrm{UP}} + \overline{q}_{ijk}^{\mathrm{DW}}) \perp \left(q_{i\omega k}^{(+)} + q_{i\omega k}^{(-)}\right), \quad \forall i, \forall j, \forall \omega. \tag{7.29}$$

Equation (7.29) imposes that, at k, if $\overline{q}_{ijk}^{\mathrm{UP}}$ and $\overline{q}_{ijk}^{\mathrm{DW}}$ are greater than 0, then $q_{i\omega k}^{(+)}$ and $q_{i\omega k}^{(-)}$ are forced to be null. This translates in being active at k. Conversely, if $q_{i\omega k}^{(+)}$ and $q_{i\omega k}^{(-)}$ are greater than 0, then $\overline{q}_{ijk}^{\mathrm{UP}}$ and $\overline{q}_{ijk}^{\mathrm{DW}}$ are constrained to be null. Indeed, the VPP is a passive participant for that interval. However, the formulation (7.29) is not suitable to be implemented in conventional optimization solver as GAMS or GUROBI. To reformulate it, we use the so-called *big-M* approach, generally used for complementarity constraints as Equation (7.29). To do so, we introduce the binary variable $\epsilon_{ik} \in \{0, 1\}$, and the parameter $\overline{M}$. Then, the complementarity between the active and the passive participation is imposed by

$$\overline{q}_{ijk}^{\mathrm{UP}} + \overline{q}_{ijk}^{\mathrm{DW}} \leq \epsilon_{ik} \overline{M}, \quad \forall i, \forall j, \forall \omega, \forall k,$$
$$q_{i\omega k}^{(+)} + q_{i\omega k}^{(-)} \leq (1 - \epsilon_{ik})\, \overline{M}, \quad \forall i, \forall j, \forall \omega, \forall k,$$
$$\epsilon_{ik} \in \{0, 1\}, \quad \forall k.$$

Note that if $\epsilon_{ik} = 1$, then $\overline{q}_{ijk}^{\mathrm{UP}} + \overline{q}_{ijk}^{\mathrm{DW}} \leq \overline{M}$ and $q_{i\omega k}^{(+)} + q_{i\omega k}^{(-)} \leq 0$, i.e., the VPP is active at k, provided that scenario i realizes. Conversely, if $\epsilon_{ik} = 0$, then $\overline{q}_{ijk}^{\mathrm{UP}} + \overline{q}_{ijk}^{\mathrm{DW}} \leq 0$ and $q_{i\omega k}^{(+)} + q_{i\omega k}^{(-)} \leq \overline{M}$, i.e., the VPP is passive at k. An appropriate choice of $\overline{M}$ is important as a small value of $\overline{M}$ may impose an undesired bound to the balancing market decisions. Differently, a too high value of $\overline{M}$ may lead to approximation errors in the optimization solver. In this case, $\overline{q}_{ijk}^{\mathrm{UP}}, \overline{q}_{ijk}^{\mathrm{DW}}, q_{i\omega k}^{(+)}, q_{i\omega k}^{(-)}$ are naturally bounded by the VPP capacity. Indeed, we set

$$\overline{M} = \overline{E} + \overline{D} + \overline{P}^{(\uparrow)} + \overline{P}^{(\downarrow)} \tag{7.31}$$

The *Active/Passive* offering strategy is formulated as

$$\underset{\Gamma^{\mathrm{ActPas}}}{\mathrm{Max}} \quad \sum_{k} \hat{\rho}_k^{\mathrm{DA}} + \hat{\rho}_k^{\mathrm{UP}} + \hat{\rho}_k^{\mathrm{DW}} + \hat{\rho}_k^{\mathrm{BA}} - \hat{c}_k \tag{7.32a}$$

$$\text{s.t.} \quad \overline{q}_{ik}^{\mathrm{DA}} + \overline{q}_{ijk}^{\mathrm{UP}} - \overline{q}_{ijk}^{\mathrm{DW}} + q_{i\omega k}^{(+)} - q_{i\omega k}^{(-)} = d_{ij\omega k} + p_{ij\omega k}^{(\downarrow)} - p_{ij\omega k}^{(\uparrow)} + E_{\omega k}, \quad \forall k, \tag{7.32b}$$

$$\overline{q}_{ik}^{\mathrm{DA}}, \hat{\rho}_k^{\mathrm{DA}} \in \Pi^{\mathrm{DA}}, \quad \forall k, \tag{7.32c}$$

$$\overline{q}_{ijk}^{\mathrm{UP}}, \overline{q}_{ijk}^{\mathrm{DW}}, \hat{\rho}_k^{\mathrm{UP}}, \hat{\rho}_k^{\mathrm{DW}} \in \Pi_{\mathrm{ActPass}}^{\mathrm{BA}}, \quad \forall k, \tag{7.32d}$$

$$q_{i\omega k}^{(+)}, q_{i\omega k}^{(-)}, \hat{\rho}_k^{\mathrm{BA}} \in \Pi_{\mathrm{Pas}}^{\mathrm{BA}}, \quad \forall i, \forall \omega, \forall k \tag{7.32e}$$

$$\hat{c} = h(d_{ij\omega k}), \quad \forall k, \tag{7.32f}$$

$$d_{ij\omega k}, p_{ij\omega k}^{(\downarrow)}, p_{ij\omega k}^{(\uparrow)} \in \Omega, \quad \forall k, \tag{7.32g}$$

$$\overline{q}_{ijk}^{\mathrm{UP}} + \overline{q}_{ijk}^{\mathrm{DW}} \leq \epsilon_{ik} \overline{M}, \quad \forall i, \forall j, \forall \omega, \forall k, \tag{7.32h}$$

$$q_{i\omega k}^{(+)} + q_{i\omega k}^{(-)} \leq (1 - \epsilon_{ik})\, \overline{M}, \quad \forall i, \forall j, \forall \omega, \forall k, \tag{7.32i}$$

$$\epsilon_{ik} \in \{0, 1\}, \quad \forall i, \forall k. \tag{7.32j}$$

where

$$\Gamma^{\text{ActPas}} = \{d_{ij\omega k}, p^{(\downarrow)}_{ij\omega k}, p^{(\uparrow)}_{ij\omega k}, \overline{q}^{\text{DA}}_{ik}, \overline{q}^{\text{UP}}_{ijk}, \overline{q}^{\text{DW}}_{ijk}, q^{(+)}_{i\omega k}, q^{(-)}_{i\omega k}, \epsilon_{ik}, \hat{\rho}^{\text{DA}}_{k}, \hat{\rho}^{\text{UP}}_{k}, \hat{\rho}^{\text{DW}}_{k}, \hat{\rho}^{\text{BA}}_{k}, \hat{c}_k\}. \tag{7.33}$$

The objective function (7.32a) maximizes the producer's expected profit, including both an active and a passive balancing market participation. The energy balance between the VPP power production and the amount of energy exchanged with the market is enforced by constraint (7.32b). Constraints (7.23c) and (7.23d) are two set of constraints associated with the day-ahead and balancing market offer curves, respectively. Similarly, constraint (7.32e) is a set of constraints related to the passive participation in the balancing market. Constraint (7.32f) yields the expected production cost of the VPP, while constraint (7.32g) imposes the feasible operating region of the VPP. Finally, constraints (7.32h), (7.32i) and (7.32j) impose the complementarity between the active and passive participation in the balancing market.

Linear Formulation of Π^{DA}

The set Π^{DA} in Equation (7.15c) is the same of Section 7.4.1, i.e.,

$$\hat{\rho}^{\text{DA}}_{k} = \sum_{i} \pi^{\text{DA}}_{i} \, \lambda^{\text{DA}}_{ik} \, \overline{q}^{\text{DA}}_{ik}, \quad \forall k, \tag{7.34a}$$

$$\overline{q}^{\text{DA}}_{ik} \geq \overline{q}^{\text{DA}}_{i'k} \quad \text{if} \quad \lambda^{\text{DA}}_{ik} \geq \lambda^{\text{DA}}_{i'k}, \quad \forall i, i', \forall k \tag{7.34b}$$

$$\overline{q}^{\text{DA}}_{ik} = \overline{q}^{\text{DA}}_{i'k} \quad \text{if} \quad \lambda^{\text{DA}}_{ik} = \lambda^{\text{DA}}_{i'k}, \quad \forall i, i', \forall k \tag{7.34c}$$

$$-\overline{P}^{(\uparrow)} \leq \overline{q}^{\text{DA}}_{ik} \leq \overline{D} + \overline{E} + \overline{P}^{(\downarrow)}, \quad \forall i, \forall k \tag{7.34d}$$

Linear Formulation of $\Pi^{\text{BA}}_{\text{Act}}$

The set $\Pi^{\text{BA}}_{\text{Act}}$ is the same of the *Active* offering strategy in Section 7.4.2, i.e.,

$$\hat{\rho}^{\text{UP}}_k = \sum_{ij} \pi^{\text{DA}}_i \pi^{\text{BA}}_{ji}\, \lambda^{\text{BA}}_{ijk}\, \overline{q}^{\text{UP}}_{ijk}, \quad \forall k, \tag{7.35a}$$

$$\hat{\rho}^{\text{DW}}_k = -\sum_{ij} \pi^{\text{DA}}_i \pi^{\text{BA}}_{ij}\, \lambda^{\text{BA}}_{ijk}\, \overline{q}^{\text{DW}}_{ijk}, \quad \forall k, \tag{7.35b}$$

$$\overline{q}^{\text{UP}}_{ijk} \geq \overline{q}^{\text{UP}}_{ij'k} \quad \text{if} \quad \lambda^{\text{BA}}_{ijk} \geq \lambda^{\text{BA}}_{ij'k}, \quad \forall i, \forall j, \forall j', \forall k \tag{7.35c}$$

$$\overline{q}^{\text{UP}}_{ijk} = \overline{q}^{\text{UP}}_{ij'k} \quad \text{if} \quad \lambda^{\text{BA}}_{ijk} = \lambda^{\text{BA}}_{ij'k}, \quad \forall i, \forall j, \forall j', \forall k \tag{7.35d}$$

$$\overline{q}^{\text{DW}}_{ijk} \leq \overline{q}^{\text{DW}}_{ij'k} \quad \text{if} \quad \lambda^{\text{BA}}_{ijk} \geq \lambda^{\text{BA}}_{ij'k}, \quad \forall i, \forall j, \forall j', \forall k \tag{7.35e}$$

$$\overline{q}^{\text{DW}}_{ijk} = \overline{q}^{\text{DW}}_{ij'k} \quad \text{if} \quad \lambda^{\text{BA}}_{ijk} = \lambda^{\text{BA}}_{ij'k}, \quad \forall i, \forall j, \forall j', \forall k \tag{7.35f}$$

$$\overline{q}^{\text{UP}}_{ijk} = 0 \quad \text{if} \quad \lambda^{\text{BA}}_{ijk} \leq \lambda^{\text{DA}}_{ik}, \quad \forall i, \forall j, \forall k, \tag{7.35g}$$

$$\overline{q}^{\text{DW}}_{ijk} = 0 \quad \text{if} \quad \lambda^{\text{BA}}_{ijk} \geq \lambda^{\text{DA}}_{ik}, \quad \forall i, \forall j, \forall k, \tag{7.35h}$$

$$\overline{q}^{\text{UP}}_{ijk}, \overline{q}^{\text{DW}}_{ijk} \geq 0, \quad \forall i, \forall j, \forall k. \tag{7.35i}$$

Linear Formulation of $\Pi^{\text{BA}}_{\text{Pas}}$

The set $\Pi^{\text{BA}}_{\text{Pas}}$ is the same of the *Passive* offering strategy in Section 7.4.1, i.e.,

$$\hat{\rho}^{\text{BA}}_k = \sum_{ij\omega} \pi^{\text{DA}}_i \pi^{\text{BA}}_{ij} \pi^{\text{E}}_{\omega} \left(\lambda^{(+)}_{ij} q^{(+)}_{ij\omega} - \lambda^{(-)}_{ij} q^{(-)}_{ij\omega} \right), \quad \forall k \tag{7.36a}$$

$$q^{(+)}_{ij\omega}, q^{(-)}_{ij\omega} \geq 0, \quad \forall i, \forall j, \forall \omega. \tag{7.36b}$$

Linear Formulation of the Cost Function

The formulation of the cost function presented in Section 7.3.3 replaces constraint (7.15e), i.e.,

$$\hat{c}_k = \sum_{ij\omega} \pi^{\text{DA}}_i \pi^{\text{BA}}_{ij} \pi^{\text{E}}_{\omega} c_{ij\omega k}, \quad \forall k, \tag{7.37a}$$

$$c_{ij\omega k} = C_0\, u_{ij\omega k} + C\, d_{ij\omega k}, \quad \forall i, \forall j, \forall \omega, \forall k, \tag{7.37b}$$

Linear Formulation of Ω

The formulation of the Ω Section 7.3.2 substitutes constraint (7.15f), i.e.,

$$l_{ij\omega k} = l_{ij\omega(k-1)} + \eta p^{(\uparrow)}_{ij\omega k} - \frac{1}{\eta} p^{(\downarrow)}_{ij\omega k}, \quad \forall i, \forall j, \forall \omega, \forall k, \tag{7.38a}$$

$$\underline{L} \leq l_{ij\omega k} \leq \overline{L}, \quad \forall i, \forall j, \forall \omega, \forall k, \tag{7.38b}$$

$$0 \leq p^{(\uparrow)}_{ij\omega k} \leq \overline{P}^{(\uparrow)}, \quad \forall i, \forall j, \forall \omega, \forall k, \tag{7.38c}$$

$$0 \leq p^{(\downarrow)}_{ij\omega k} \leq \overline{P}^{(\downarrow)}, \quad \forall i, \forall j, \forall \omega, \forall k, \tag{7.38d}$$

$$u_{ij\omega k}\underline{D} \leq d_{ij\omega k} \leq u_{ij\omega k}\overline{D}, \quad \forall i, \forall j, \forall \omega, \forall k, \tag{7.38e}$$

$$d_{ij\omega k} - d_{ij\omega(k-1)} \leq R^{\mathrm{UP}}, \quad \forall i, \forall j, \forall \omega, \forall k, \tag{7.38f}$$

$$d_{ij\omega(k-1)} - d_{ij\omega k} \leq R^{\mathrm{DW}}, \quad \forall i, \forall j, \forall \omega, \forall k, \tag{7.38g}$$

$$u_{ij\omega k} \in \{0, 1\}, \quad \forall i, \forall j, \forall \omega, \forall k. \tag{7.38h}$$

7.5 Case Study

This section presents a case study to test the offering models of Section 7.4. The scenarios to be used as input to the stochastic offering models are generated following the methodology presented in Chapter 4. Tables 7.1 and 7.2 show the parameters used to characterize the fundamental market model of Section 4.3. First, we generate 300 scenarios for $\lambda^{\mathrm{DA}}_{ik}$ and we select the 10 most representative ones. Then, for each scenario $\lambda^{\mathrm{DA}}_{ik}$, we generate 300 scenarios of $\lambda^{\mathrm{BA}}_{ijk}$ and keep the 6 most representative ones.

TABLE 7.1: Parameters of the market price generation model

δ (€/MWh^2)	β (€/MWh^2)	μ_γ (€/MWh^3)	σ^2_γ (€/MWh^3)	λ^0 (€/MWh)
-6.67×10^{-3}	1×10^{-4}	2×10^{-8}	3×10^{-9}	-20

TABLE 7.2: Values of parameter α_k

k	1	2	3	4	5	6	7	8
α_k (€/MWh)	322	312	315	317	340	349	353	369
k	9	10	11	12	13	14	15	16
α_k (€/MWh)	394	424	444	445	440	429	437	458
k	17	18	19	20	21	22	23	24
α_k (€/MWh)	446	423	408	383	373	346	331	332

The data of the conventional production unit are shown in Table 7.3. Similarly, the characteristics of the storage unit are presented in Table 7.4. The optimization models are implemented using GUROBI in PYTHON environment.

TABLE 7.3: Parameters of the conventional generation unit

$\underline{D}$ (MWh)	$\overline{D}$ (MWh)	R^{UP} (MWh)	R^{DW} (MWh)	C_0 (€)	C (€/MWh)
0	70	30	30	0	45

TABLE 7.4: Parameters of the electric storage unit

$\underline{L}$ (MWh)	$\overline{L}$ (MWh)	$\overline{P}^{(\uparrow)}$ (MWh)	$\overline{P}^{(\downarrow)}$ (MWh)	η
0	80	30	30	0.90

7.5.1 VPP with wind farm

Figure 7.7 illustrates the 10 selected trajectories λ_{ik}^{DA}. Similarly, Figure 7.8 does for the 6 scenarios $\lambda_{ijk}^{\text{BA}}$, for a given realization λ_{k}^{DA}. Note that the balancing market shows high uncertainty up to 10 a.m. and from 4 p.m. to midnight. Differently, the hourly intervals between 10 a.m. and 4 p.m. show less uncertainty in the balancing market price. Together with the market price scenarios, we generate trajectories of

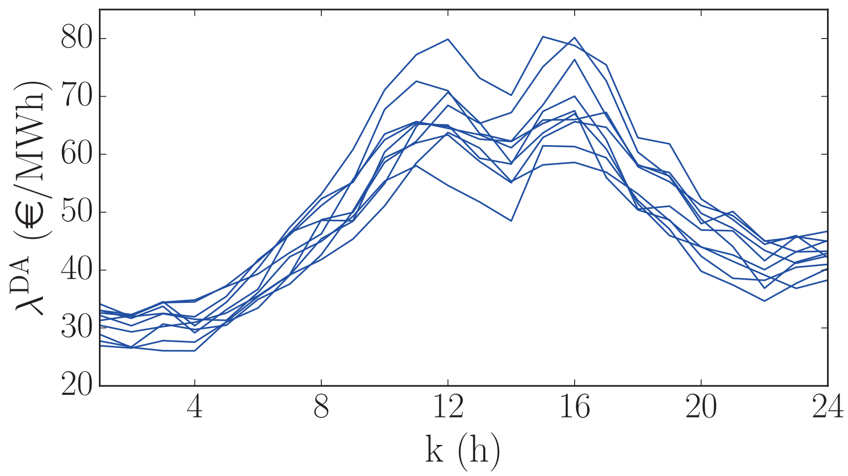


FIGURE 7.7: Day-ahead market price scenarios.

the power production of the wind farm. We randomly sample 300 scenarios and we reduce to the 5 more significant ones. Figure 7.9 shows the 5 selected trajectories for the wind power production, in p.u. The result is a scenario tree with 300 branches $(10 \times 6 \times 5)$.

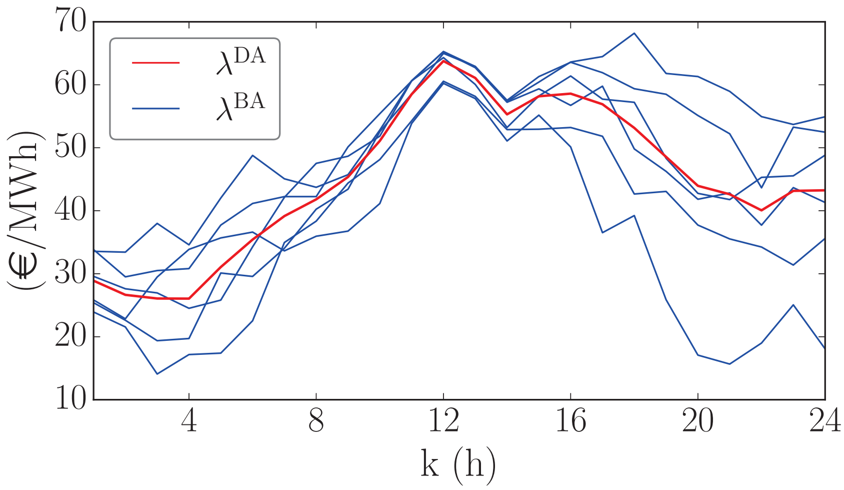


FIGURE 7.8: Balancing market price scenarios for a given realization of λ_k^{DA} (red).

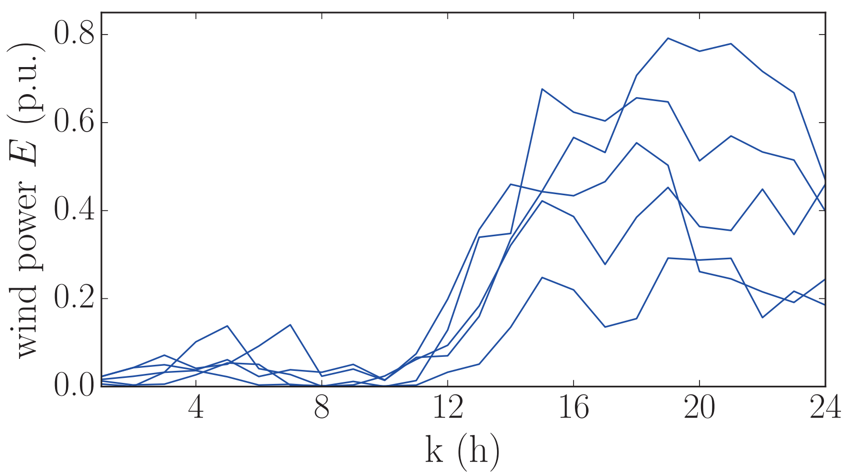


FIGURE 7.9: Wind power production trajectories, in p.u.

We initially consider a wind farm of capacity $\overline{E}$ of 50 MW. With the *Active* strategy the VPP offers its regulating energy in the balancing market, but it is not allowed to deviate from its production schedule. Differently, with the *Passive* strategy, the VPP deviates from its day-ahead contracted schedule, but it is prevented from offering regulating power at the balancing stage. Finally, the *Active/Passive* strategy is a combination of the *Active* and *Passive* ones. Indeed, the VPP can offer regulating energy for some trading intervals and passively deviate in the remaining ones. The complementarity between the active/passive choice is imposed by the binary variable ϵ_{ik}. If $\epsilon_{ik} = 1$, the VPP is predicting to act as an active participant at the balancing stage during the interval k, provided that the day-ahead scenario i realizes. For the same scenario, if $\epsilon_{ik} = 0$, the VPP is considering to behave passively and deviate from its contracted schedule. Being ϵ^*_{ik} its optimal solution, we compute the probability that the VPP will be an active participant during the interval k as $\sum_i \pi_i^{\mathrm{DA}} \epsilon^*_{ik}$. Similarly, the probability of being passive is $\sum_i \pi_i^{\mathrm{DA}}(1 - \epsilon^*_{ik})$. Those probabilities are illustrated in Figure 7.10, the active one in green and the passive one in red.

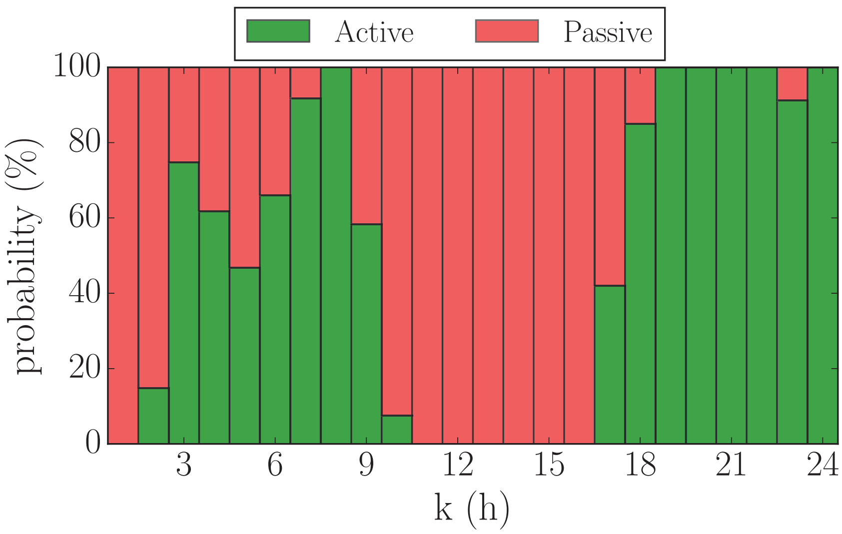


FIGURE 7.10: Probability that the VPP is going to be active (blue) and passive (red).

From midnight to 10 a.m., the VPP will decide to be either active or passive based on the day-ahead market scenario realization. Indeed, in this time horizon, the balancing market price (see Figure 7.8) is quite uncertain, i.e., it is hard to guess if the system is going to require upward or downward regulation. This makes a passive approach riskier, due to the possibility of incurring in penalties when the producer's deviation and the system imbalance have the same sign. However, even if uncertain, the spread between the balancing price scenarios and the day-ahead realization is narrow (both in upward and downward regulation), thus resulting in a small penalization when occurs and, consequently, low profits from actively selling regulating energy. For the same trading intervals, the uncertainty in the wind power production is limited (see Figure 7.9) as likely occurs a low wind production. This would benefit an active approach as the flexibility of the storage and the conventional unit could be used to offer regulating energy and not to compensate the wind power fluctuations, which are likely to be small. This trade-off is clear in Figure 7.10, as the VPP decides to be active or passive depending on the amount of energy scheduled in the day-ahead market. Then, from 10 a.m. to 4 p.m. the VPP decides that it is going to passively deviate from its day-ahead schedule, disregarding the day-ahead market scenario realization. In this interval, the uncertainty in the balancing market prices is limited as the realizations are likely to be close to the day-ahead market price. This, together with an uncertain wind power production, makes a passive approach more attractive in this time horizon. Finally, from 6 p.m. to midnight the VPP is almost sure to sell regulating energy in the balancing market. This translates in internally handling the wind power production fluctuations, which is extremely uncertain in this time interval. Indeed, it can vary from around 20% to almost 80% of the wind farm capacity. However, the balancing market prices are going to be "far" from the day-ahead market ones with high probability. Accordingly, passive deviations from the day-ahead schedule may incur in high penalties, while the sell of regulating energy can be very profitable.

Table 7.5 shows the optimal day-ahead quantity offers $\overline{q}_{ik}^{\mathrm{DA}}$ for the hourly interval $k = 6$ of the three offering strategy (i.e., *Active/Passive*, *Active*, and *Passive*). With

TABLE 7.5: Optimal values of $\overline{q}_{i6}^{\mathrm{DA}}$ for the three offering strategy (i.e., *Active/Passive*, *Active*, and *Passive*). The capacity of wind farm $\overline{E}$ is 50 MW).

i	$\lambda_{i6}^{\mathrm{DA}}$ (€/MWh)	q_{i6}^{DA} (MWh)		
		Active/Passive	*Active*	*Passive*
1	36.7	8.0	2.0	2.5
2	35.4	2.0	2.0	2.0
3	41.8	13.0	5.8	12.4
4	39.3	8.0	2.0	2.5
5	35.6	2.0	2.0	2.0
6	36.1	2.0	2.0	2.0
7	35.0	2.0	2.0	2.0
8	41.5	12.4	2.0	12.4
9	33.5	2.0	2.0	2.0
10	40.3	12.4	2.0	12.4

these results, the VPP builds its day-ahead offer curve. For the *Active/Passive* approach, they the amount of energy $q_6^{\mathrm{DA,Act/Pas}}$ scheduled by the offer curve is

$$q_6^{\mathrm{DA,Act/Pas}} = \begin{cases} 0, & \text{if } \lambda_6^{\mathrm{DA}} < 33.5 \\ 2.0, & \text{if } 33.5 \leq \lambda_6^{\mathrm{DA}} < 36.7 \\ 8.0, & \text{if } 36.7 \leq \lambda_6^{\mathrm{DA}} < 40.3 \\ 12.4, & \text{if } 40.3 \leq \lambda_6^{\mathrm{DA}} < 41.8 \\ 13.0, & \text{if } \lambda_6^{\mathrm{DA}} \geq 41.8, \end{cases} \tag{7.39}$$

and its associated offer curve is built with four price-quantity offer points, i.e., (€33.5/MWh, 2.0 MWh), (€36.7/MWh, 8.0 MWh), (€40.3/MWh, 12.4 MWh), and (€41.8/MWh, 13.0 MWh). Differently, for the *Active* strategy the quantity $q_6^{\mathrm{DA,Act}}$ is given by

$$q_6^{\mathrm{DA,}} = \begin{cases} 0, & \text{if } \lambda_6^{\mathrm{DA}} < 33.5 \\ 2.0, & \text{if } 33.5 \leq \lambda_6^{\mathrm{DA}} < 41.8 \\ 5.8, & \text{if } \lambda_6^{\mathrm{DA}} \geq 41.8, \end{cases} \tag{7.40}$$

and the related offer curve is made by the two offer points (€33.5/MWh, 2.0 MWh), and (€5.8/MWh, 5.8 MWh). Finally, for the *Passive* strategy $q_6^{\text{DA,Pas}}$ is evaluated as

$$q_6^{\text{DA,Pas}} = \begin{cases} 0, & \text{if } \lambda_6^{\text{DA}} < 33.5 \\ 2.0, & \text{if } 33.5 \leq \lambda_6^{\text{DA}} < 36.7 \\ 2.5, & \text{if } 36.7 \leq \lambda_6^{\text{DA}} < 40.3 \\ 12.4, & \text{if } \lambda_6^{\text{DA}} \geq 40.3, \end{cases} \tag{7.41}$$

and its offer curve is generated by the price-quantity points (€33.5/MWh, 2.0 MWh), (€36.7/MWh, 2.5 MWh), and (€40.3/MWh, 12.4 MWh).

A graphic representation of the offer curves of the three strategy is shown in Figure 7.11. Under all the three approaches the VPP is willing to produce 2.0 MWh pro-

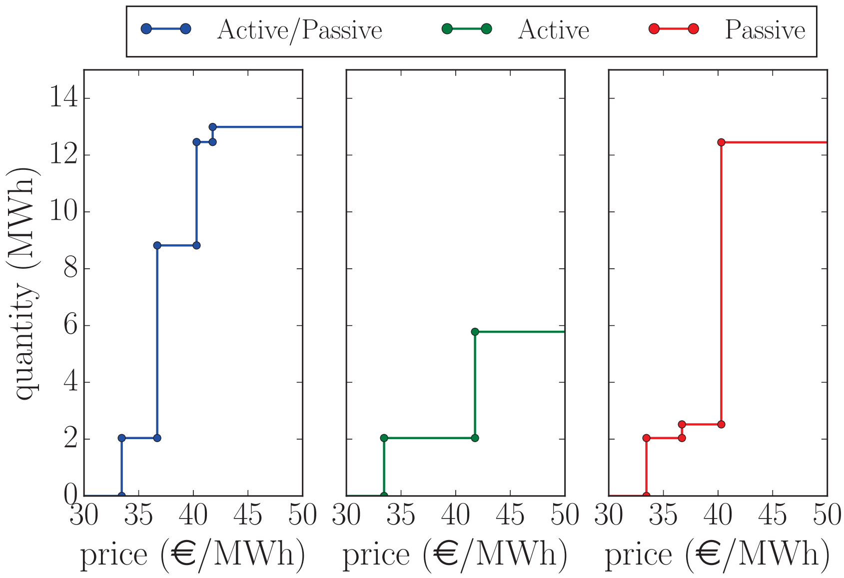


FIGURE 7.11: Illustration of the offer curves of the three strategies, i.e., *Active/Passive* (blue), *Active* (green), and *Passive* (red), for the hourly interval $k = 6$.

vided that $\lambda_6^{\text{DA}} \geq 33.5$. Then, if $\lambda_6^{\text{DA}} \geq 36.7$ the *Active/Passive* and *Passive* strategies schedule additional 6.0 MWh and 0.5 MWh, respectively. Differently, the *Active* strategy increases its production from 2.0 to 5.8 MWh only if $\lambda_6^{\text{DA}} \geq 41.8$. For the same price interval (i.e., $\lambda_6^{\text{DA}} \geq 41.8$) the *Active/Passive* approach schedules 13.0 MWh and the *Passive* one 12.4 MWh. The *Active* strategy contracts a lower quantity in the day-ahead compared to the *Passive* one. In this way, it can subsequently submits upward regulation offers in the balancing stage. Consequently, in the *Active/Passive* strategy the VPP will be an active participant if the amount of energy scheduled in the day-ahead market is low, and a passive one otherwise. Table 7.6 shows the optimal day-ahead quantity offers $\overline{q}_{ik}^{\text{DA}}$ for the hourly interval $k = 15$ of the three offering strategy (i.e., *Active/Passive*, *Active*, and *Passive*). Under the *Passive/Active*

TABLE 7.6: Optimal values of $\overline{q}_{i15}^{\text{DA}}$ for the three offering strategy (i.e., *Active/Passive*, *Active*, and *Passive*). The capacity of wind farm $\overline{E}$ is 50 MW).

i	$\lambda_{i15}^{\text{DA}}$ (€/MWh)	q_{i15}^{DA} (MWh)		
		Active/Passive	*Active*	*Passive*
1	67.4	97.9	105.4	112.4
2	58.2	91.1	92.1	91.1
3	68.5	97.9	105.4	112.4
4	65.3	97.9	105.4	106.5
5	75.1	121.1	105.4	121.1
6	61.5	91.1	92.1	106.5
7	62.9	91.1	92.1	106.5
8	65.9	97.9	105.4	106.5
9	63.7	91.1	92.1	106.5
10	80.3	121.1	105.4	121.1

strategy, the VPP builds its day-ahead offer curves with three price-quantity offer points, i.e., (€58.2/MWh, 91.1 MWh), (€65.2/MWh, 97.9 MWh), and (€75.1/MWh, 121.1 MWh). Therefore, it schedules the following amount of energy $q_{15}^{\text{DA,Act/Pas}}$:

$$q_{15}^{\text{DA,Act/Pas}} = \begin{cases} 0, & \text{if } \lambda_{15}^{\text{DA}} < 58.2 \\ 91.1, & \text{if } 58.2 \leq \lambda_{15}^{\text{DA}} < 65.3 \\ 97.9, & \text{if } 65.3 \leq \lambda_{15}^{\text{DA}} < 75.1 \\ 121.1, & \text{if } \lambda_{15}^{\text{DA}} \geq 75.1. \end{cases} \tag{7.42}$$

Differently, the *Active* strategy offer curve is built with two offer points, i.e., (€58.2/MWh, 92.1 MWh), and (€65.3/MWh, 105.4 MWh). Accordingly, the quantity $q_{15}^{\text{DA,Act}}$ is given by

$$q_{15}^{\text{DA,Act}} = \begin{cases} 0, & \text{if } \lambda_{15}^{\text{DA}} < 58.2 \\ 92.1, & \text{if } 58.2 \leq \lambda_{15}^{\text{DA}} < 65.3 \\ 105.4, & \text{if } \lambda_{15}^{\text{DA}} \geq 65.3. \end{cases} \tag{7.43}$$

Finally, for the *Passive* strategy offer curve is composed by (€58.2/MWh, 91.1 MWh), (€61.5/MWh, 106.5 MWh), (€67.4/MWh, 112.4 MWh), and (€75.1/MWh, 121.1

MWh) and its related $q_{15}^{\text{DA,Pas}}$ is

$$q_{15}^{\text{DA,Pas}} = \begin{cases} 0, & \text{if } \lambda_{15}^{\text{DA}} < 58.2 \\ 91.1, & \text{if } 58.2 \leq \lambda_{15}^{\text{DA}} < 61.5 \\ 106.5, & \text{if } 61.5 \leq \lambda_{15}^{\text{DA}} < 67.4 \\ 112.4, & \text{if } 67.4 \leq \lambda_{15}^{\text{DA}} < 75.1 \\ 121.1, & \text{if } \lambda_{15}^{\text{DA}} \geq 75.1. \end{cases} \tag{7.44}$$

A graphic representation of the offer curves for the three strategy is shown in Figure 7.12. The VPP is willing to produce 91.1 MWh (*Active/Passive* and *Active*) or

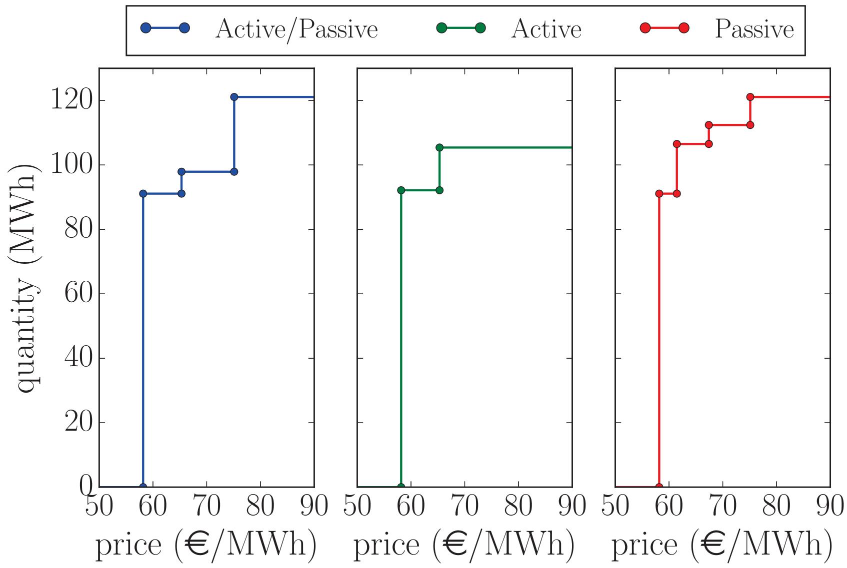


FIGURE 7.12: Illustration of the offer curves of the three strategies, i.e., *Active/Passive* (blue), *Active* (green), and *Passive* (red), for the hourly interval $k = 15$.

92.1 MWh (*Active*) provided that $\lambda_{15}^{\text{DA}} \geq 33.5$. Then, if $\lambda_{15}^{\text{DA}} \geq 65.3$ the *Active/Passive* and the *Active* approach contract additional 6.8 MWh and 13.3 MWh, respectively. Differently, the *Active* approach increases its production to 106.5 MWh if $\lambda_{15}^{\text{DA}} \geq 67.4$. Finally, if $\lambda_{15}^{\text{DA}} \geq 75.1$ the *Active/Passive* and the *Passive* approach further increase the production level to 121.1 MWh. From Figures 7.11 and 7.12 we note that the *Active* approach is less "reactive" to the day-ahead market price compared to the alternative strategies and it is not willing to schedule an additional quantity for high values of the day-ahead market price. Indeed, the position of the VPP after the day-ahead market affects the capability of the VPP of internally compensate the wind power fluctuations. Therefore, its position is more constrained and driven by feasibility limitations compared to the alternative strategies.

We tested and compared the expected profit of the three offering models, for increasing values of the wind farm capacity $\overline{E}$, i.e., 10, 30, 50, 70, and 90 MW. The results are shown in Table 7.7. It can be noticed that the *Active/Passive* strategy is always able to increase the expected profit of the VPP.

TABLE 7.7: Expected profit for the three offering strategy (i.e., *Active/Passive*, *Active*, and *Passive*), for different values of the capacity of wind farm $\overline{E}$.

$\overline{E}$ (MW)	Expected profit $\hat{\mu}$ (10^3 €)		
	Active/Passive	*Active*	*Passive*
10	18.16	18.06	16.81
30	22.89	22.48	21.69
50	27.59	26.85	26.57
70	32.35	31.16	31.45
90	37.07	35.44	36.32

Then, Figure 7.13 illustrates the increase in the expected profit obtained using the *Active/Passive* strategy compared to the *Active* and the *Passive* one. Figure 7.14 does the same in percentage terms. When $\overline{E} = 10$ MW, the increase of profit is of 0.6 %

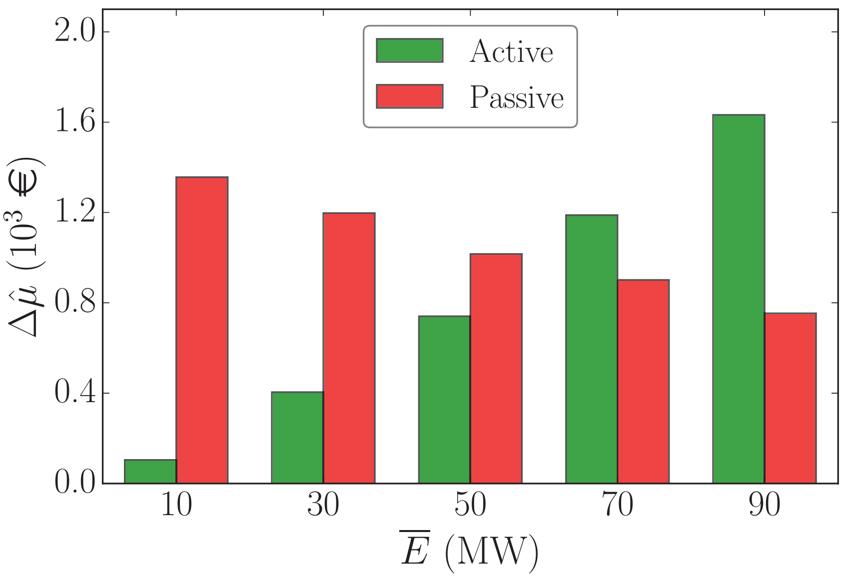


FIGURE 7.13: Difference between the profit with the *Active/Passive* strategy and the profit with the alternative ones (i.e., *Active* and *Passive*), for values of $\overline{E}$ of 10, 30, 50, 70, and 90 MW.

with the respect to the *Active* strategy and of 8.1 % to the *Passive* one. Hence, when

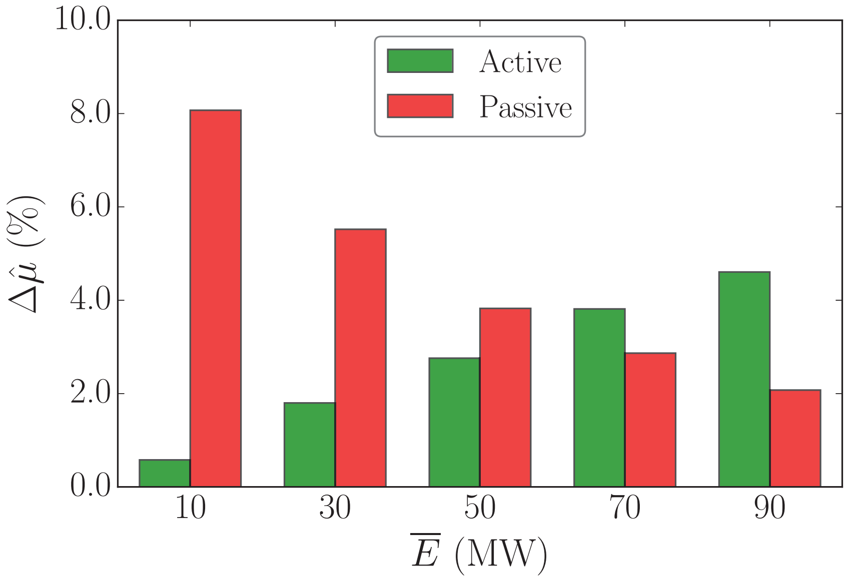


FIGURE 7.14: Percentage difference between the profit with the *Active/Passive* strategy and the profit with the alternative ones (i.e., *Active* and *Passive*), for values of $\overline{E}$ of 10, 30, 50, 70, and 90 MW.

the capacity is small, the VPP can internally handle most of the wind power deviations and offers its regulating energy into the balancing market. Accordingly, the increase in profit compared to the *Active* strategy is limited. For the same reason, the *Active/Passive* strategy strongly outperforms the *Passive* one, as the last one can not exploit the balancing stage for additional profits. This trend progressively changes as the wind farm capacity $\overline{E}$ increases. As $\overline{E}$ grows, the VPP is more likely to settle deviations in the balancing stage and has less flexibility to offer in the balancing market as it is partly allocated for balancing the wind power fluctuations. Indeed, when $\overline{E} = 90$ MW the increase in profit of 4.6 % compared to the *Active* strategy and of 2.1 % to the *Passive* one.

7.5.2 VPP with PV solar

This section considers the VPP composed of a conventional generation unit, the electric storage, and a PV solar unit. Figure 7.15 shows the 10 selected trajectories of the day-ahead market price $\lambda^{\mathrm{DA}}_{ik}$. Similarly, Figure 7.16 illustrates the 6 scenarios of the balancing market price $\lambda^{\mathrm{BA}}_{ijk}$, for a given day-ahead realization $\lambda^{\mathrm{DA}}_{k}$. Together with the market price scenarios, we generate trajectories of the power production of the wind unit. We randomly sample 300 scenarios and we reduce to the 5 more significant ones. Figure 7.17 shows the 5 selected trajectories for the PV solar power production, in p.u. The result is a scenario tree with 300 branches ($10 \times 6 \times 5$).

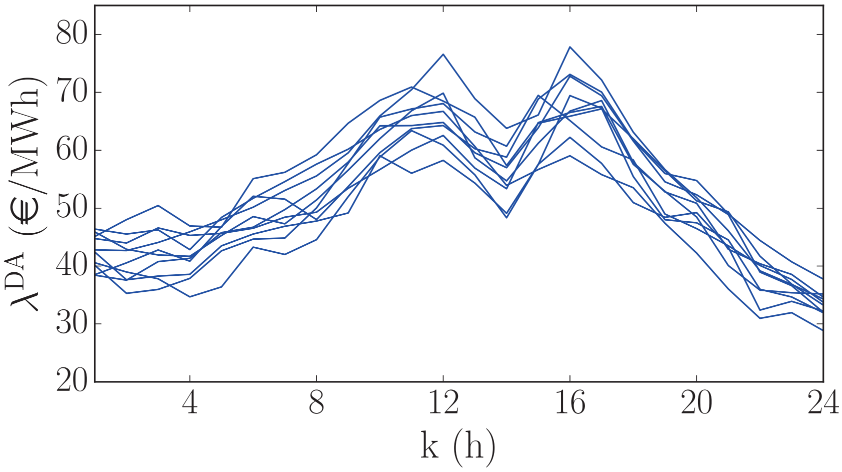


FIGURE 7.15: Day-ahead market price scenarios.

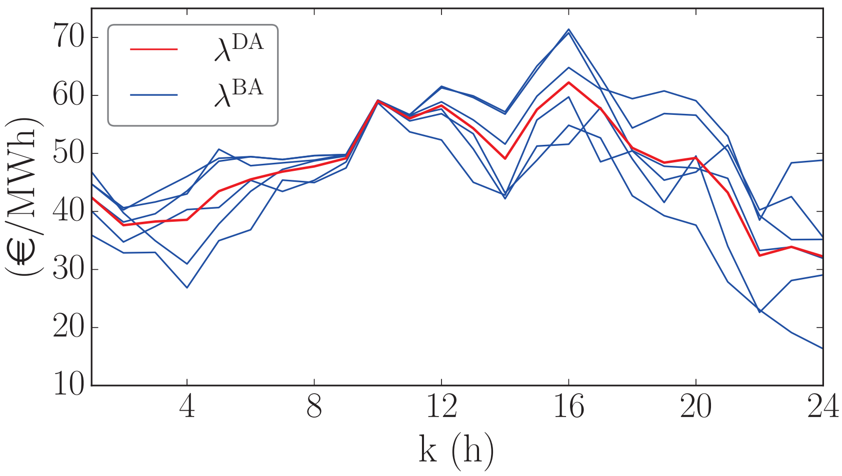


FIGURE 7.16: Balancing market price scenarios for a given realization of λ_k^{DA} (red).

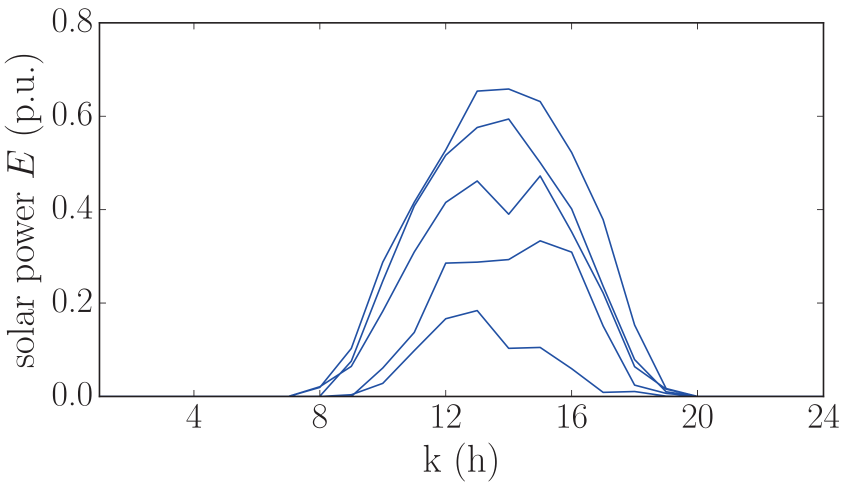


FIGURE 7.17: PV solar power production trajectories, in per unit.

The capacity $\overline{E}$ of the PV solar unit is initially set to 50 MW. We run the three optimal offering strategies, i.e., *Active/Passive*, *Active* and *Passive*. In the *Active/Passive* approach, the complementarity between the active and passive participation is enforced by mean of the binary variable ϵ_{ik} (i.e., through the so-called *Big-M* approach). Being $\epsilon^*_{i,k}$ its optimal solution, the probability of the VPP to actively offer in the market at k is evaluated as $\sum_i \pi_i^{\mathrm{DA}} \epsilon^*_{ik}$. Consequently, its probability of being a passive participant as $\sum_i \pi_i^{\mathrm{DA}} (1 - \epsilon^*_{ik})$.

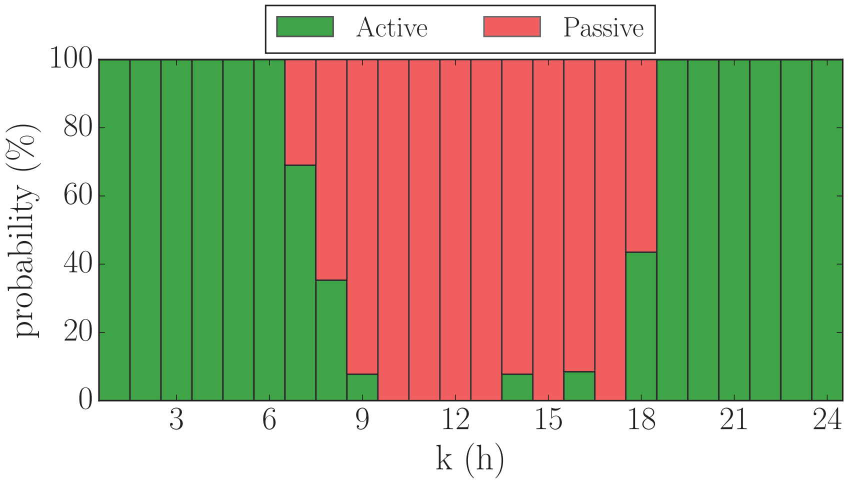


FIGURE 7.18: Probability that the VPP is going to be active (green) and passive (red) in the balancing market.

Those probabilities, for the 24 hourly trading intervals, are shown in Figure 7.18. The probability of being active is plotted in green, while the one of being passive in red. From midnight to 6 a.m. and from 8 p.m. to midnight, the VPP decides that it is going to participate in the balancing market by actively offering its available regulating energy to the System Operator. Indeed, in these time intervals, the production of the PV solar unit is certain (see Figure 7.17). These intervals correspond to the time before the sunrise (i.e., from midnight to 6 a.m.) and after the sunset (i.e., from 8 p.m. to midnight), where the VPP is certain that its PV power production is going to be null. Differently, from 10 a.m. to 5 p.m. the VPP is almost sure that it is going to passively deviate from its contracted schedule to compensate the fluctuations of the PV solar unit. In this time horizon, the PV power production is very uncertain (e.g., at 2 p.m. it can vary from 20% to 70% of the unit capacity) and it is more convenient for the VPP to settle deviations in the balancing market. Finally, from 6 a.m. to 9 a.m. the VPP will decide to be active or passive depending on the day-ahead market price realization. Indeed, based on the amount of energy contracted in the day-ahead, it would decide which of the two approaches is more profitable. In this time interval, the uncertainty of the PV solar production is limited, as those are the first hours after the sunrise. This would suggest that an active participation may be preferable as the VPP would easily handle the PV fluctuations within the cluster. However, in this time horizon, the possibility of gaining extra profits from the balancing market is low as the balancing market price scenarios are very close to the day-ahead one (see Figure 7.16). Differently, from 6 p.m. to 8 p.m., the active participation is more attractive since the balancing market price scenarios give the opportunity for extra profits.

Table 7.8 shows the optimal day-ahead quantity offers $\overline{q}_{ik}^{\mathrm{DA}}$ for the hourly interval $k = 5$ of the three offering strategy (i.e., *Active/Passive*, *Active*, and *Passive*). With these results the VPP derives the offer curves submitted in the day-ahead market.

TABLE 7.8: Optimal values of $\overline{q}_{i5}^{\mathrm{DA}}$ for the three offering strategy (i.e., *Active/Passive*, *Active*, and *Passive*). The capacity of PV solar unit $\overline{E}$ is 50 MW).

i	$\lambda_{i5}^{\mathrm{DA}}$ (€/MWh)	q_{i5}^{DA} (MWh)		
		Active/Passive	*Active*	*Passive*
1	45.3	0.0	0.0	40.0
2	43.5	0.0	0.0	40.0
3	48.0	40.0	40.0	70.0
4	48.4	40.0	40.0	70.0
5	46.8	40.0	40.0	40.0
6	42.6	0.0	0.0	0.0
7	45.6	0.0	0.0	40.0
8	46.7	40.0	40.0	40.0
9	45.7	0.0	0.0	40.0
10	36.4	-14.4	-14.4	0.0

Under the *Active/Passive* approach the quantity $q_5^{\mathrm{DA,Act/Pas}}$ scheduled as function of the realization of λ_5^{DA} is

$$q_5^{\mathrm{DA,Act/Pas}} = \begin{cases} -14.4, & \text{if } \lambda_5^{\mathrm{DA}} \leq 36.4 \\ 0.0, & \text{if } 36.4 < \lambda_5^{\mathrm{DA}} < 46.7 \\ 40.0, & \text{if } \lambda_5^{\mathrm{DA}} \geq 46.7, \end{cases} \tag{7.45}$$

and its associated offer curve is built with two price-quantity offer points, i.e., (€36.4/MWh, -14.4 MWh) and (€46.7/MWh, 40.0 MWh). Note that (€36.4/MWh, -14.4 MWh) is a buy and not a sell offer. The amount of energy $q_5^{\text{DA,Act}}$ contracted with the *Active* strategy is equivalent to $q_5^{\text{DA,Act/Pas}}$ in Equation (7.45), according to Table 7.8. Finally, the quantity $q_5^{\text{DA,Pas}}$ scheduled by the *Passive* strategy is given by

$$q_5^{\text{DA,Pas}} = \begin{cases} 0.0, & \text{if } \lambda_5^{\text{DA}} < 43.5 \\ 40.0, & \text{if } 43.5 \leq \lambda_5^{\text{DA}} < 48.0 \\ 70.0, & \text{if } \lambda_5^{\text{DA}} \geq 48.0, \end{cases} \tag{7.46}$$

while the two offer points (€43.5/MWh, 40.0 MWh) and (€48.0/MWh, 70.0 MWh) build the associated offer curve.

Figure 7.19 provides a graphical interpretation of the offer curves derived by the three strategy. The *Active/Passive* approach is shown in blue, while the *Active* and the *Passive* one in green and red, respectively. The VPP, under the *Active/Passive* and

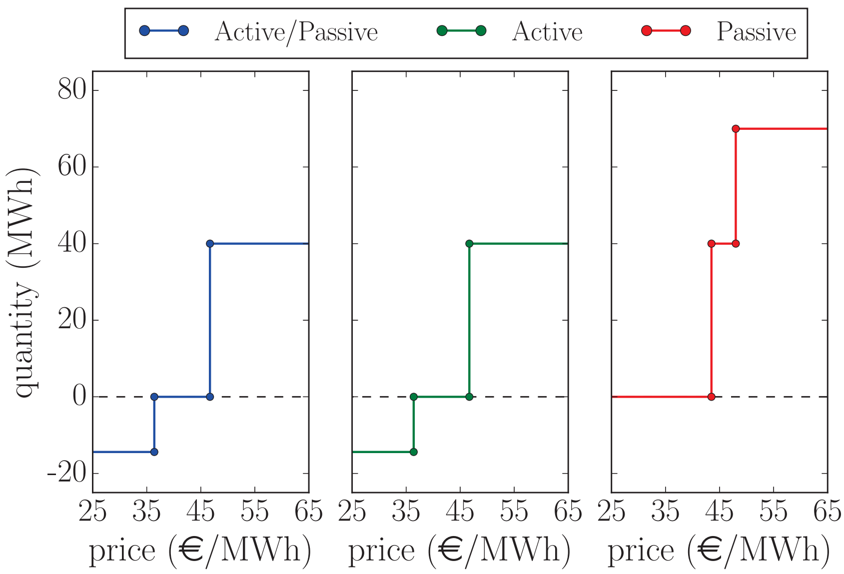


FIGURE 7.19: Illustration of the offer curves of the three strategies, i.e., *Active/Passive* (blue), *Active* (green), and *Passive* (red), for the hourly interval $k = 5$.

the *Active* strategy, is willing to consume (buy) 14.4 MWh provided that $\lambda_5^{\text{DA}} \leq 36.4$. This energy is used to charge the electric storage unit. Then, with these strategies, the VPP schedules to produce 40.0 MWh when $\lambda_5^{\text{DA}} \geq 46.7$ and it does not produce if $36.4 < \lambda_5^{\text{DA}} < 46.7$. The offer curves derived with these two strategies are equivalent in accordance with the results shown in Figure 7.18. Indeed, the VPP decides to be an active participant from midnight to 6 a.m.. Differently, the *Passive* strategy suggests to schedule 40.0 MWh when $\lambda_5^{\text{DA}} \geq 43.5$ and to increase the production of additional 30.0 MWh if $\lambda_5^{\text{DA}} \geq 48.0$.

Table 7.9 presents the optimal day-ahead quantity offers $\overline{q}_{ik}^{\text{DA}}$ for the hourly interval $k = 17$ of the three offering strategy (i.e., *Active/Passive*, *Active*, and *Passive*). With these results the VPP builds the offer curves to be submitted in the day-ahead market.

TABLE 7.9: Optimal values of $\overline{q}_{i17}^{\text{DA}}$ for the three offering strategy (i.e., *Active/Passive*, *Active*, and *Passive*). The capacity of PV solar unit $\overline{E}$ is 50 MW).

i	$\lambda_{i17}^{\text{DA}}$ (€/MWh)	q_{i17}^{DA} (MWh) *Active/Passive*	*Active*	*Passive*
1	68.6	116.1	98.9	116.1
2	57.7	90.0	70.8	90.0
3	72.1	116.1	98.9	116.1
4	55.8	90.0	70.8	90.0
5	70.0	116.1	98.9	116.1
6	60.6	90.0	70.8	90.0
7	67.6	116.1	98.9	116.1
8	67.1	90.0	70.8	90.0
9	69.4	116.1	98.9	116.1
10	67.3	116.1	98.9	116.1

The *Active/Passive* approach schedules an amount of energy $q_{17}^{\text{DA,Act/Pas}}$, depending on the realization of λ_{17}^{DA} of

$$q_{17}^{\text{DA,Act/Pas}} = \begin{cases} 0.0, & \text{if } \lambda_{17}^{\text{DA}} \leq 55.8 \\ 90.0, & \text{if } 55.8 < \lambda_{17}^{\text{DA}} < 67.3 \\ 116.1, & \text{if } \lambda_{17}^{\text{DA}} \geq 67.3. \end{cases} \tag{7.47}$$

The offer curve composed of two price-quantity offer points, i.e., (€55.8/MWh, 90.0 MWh) and (€67.3/MWh, 116.1 MWh), summarizes Equation 7.47. Then, the amount of energy $q_{17}^{\text{DA,Act}}$ contracted with the *Active* strategy is

$$q_{17}^{\text{DA,Pas}} = \begin{cases} 0.0, & \text{if } \lambda_{17}^{\text{DA}} \leq 55.8 \\ 70.8, & \text{if } 55.8 < \lambda_{17}^{\text{DA}} < 67.3 \\ 98.9, & \text{if } \lambda_{17}^{\text{DA}} \geq 67.3. \end{cases} \tag{7.48}$$

and its associated offer curve is built with two price-quantity offer points, i.e., (€55.8/MWh, 70.8 MWh) and (€67.3/MWh, 98.9 MWh). Finally, the offer curve obtained with the *Passive* strategy is equivalent to the one obtained with the *Active/Passive* approach, according the results in Table 7.9.

Figure 7.20 gives a graphical representation of the offer curves obtained with the three strategies. The *Active/Passive* approach is shown in blue, while the *Active* and the *Passive* ones in green and red, respectively.

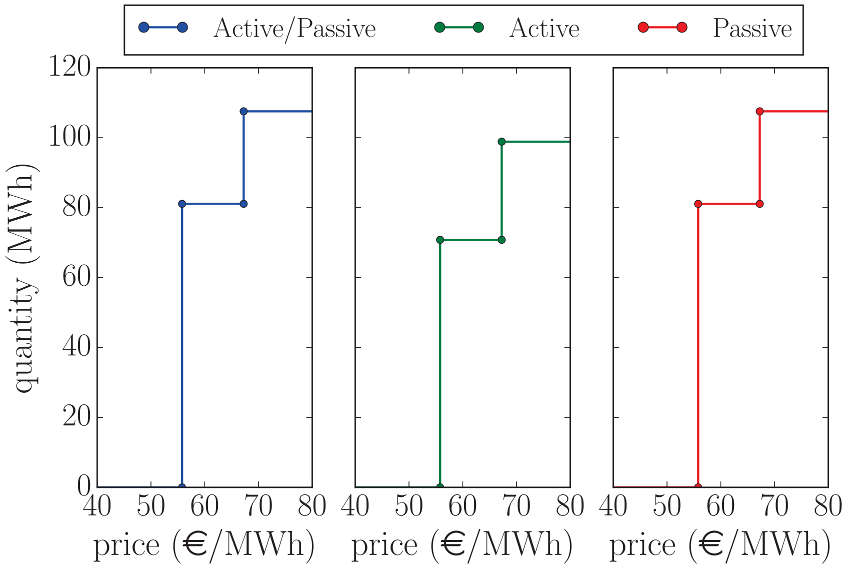


FIGURE 7.20: Illustration of the offer curves of the three strategies, i.e., *Active/Passive* (blue), *Active* (green), and *Passive* (red), for the hourly interval $k = 17$.

Based on the *Active/Passive* and the *Passive* strategy, schedules to produce 90.0 MWh when that $\lambda_{17}^{\mathrm{DA}} \leq 55.8$. Differently, the *Active* approach decides to operate at 70.8 MWh when $\lambda_{17}^{\mathrm{DA}} \leq 55.8$. Then, the *Active/Passive* and the *Passive* strategy increase the power production of additional 26.1 MWh, and the *Active* one of 28.1 MWh, provided that $\lambda_{17}^{\mathrm{DA}} \leq 67.3$. The offer curves obtained with the *Active/Passive* and the *Passive* approach are equivalent in accordance with the results shown in Figure 7.18. Indeed, the VPP decides to be a passive participant in this time period, disregarding the day-ahead market price realization.

We test the three offering, for increasing values of the wind farm capacity $\overline{E}$, i.e., 10, 30, 50, 70, and 90 MW. The results are shown in Table 7.10. Note that the *Active/Passive* strategy is always able to increase the expected profit of the VPP. Then, Figure 7.21 illustrates the increase in the expected profit obtained using the *Active/Passive* strategy compared to the *Active* and the *Passive* one. Figure 7.22 does

TABLE 7.10: Expected profit for the three offering strategy (i.e., *Active/Passive*, *Active*, and *Passive*), for different values of the capacity $\overline{E}$ of the PV solar unit.

$\overline{E}$ (MW)	Expected profit $\hat{\mu}$ (10^3 €)		
	Active/Passive	*Active*	*Passive*
10	18.59	18.27	17.54
30	22.19	21.28	21.15
50	25.79	24.24	24.76
70	29.39	27.17	28.36
90	32.98	30.08	31.97

the same in percentage terms. When the PV solar unit in the VPP is of capacity

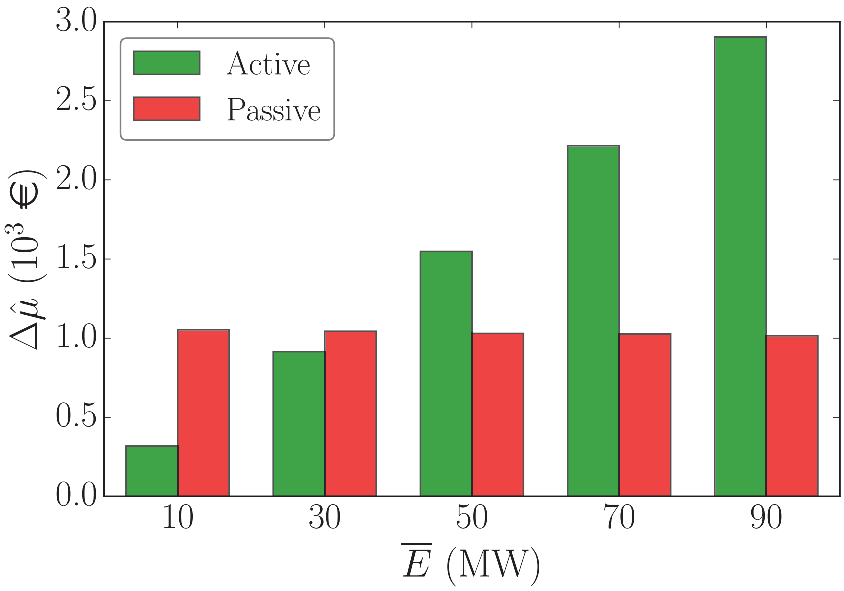


FIGURE 7.21: Difference between the profit with the *Active/Passive* strategy and the profit with the alternative ones (i.e., *Active* and *Passive*), for values of $\overline{E}$ of 10, 30, 50, 70, and 90 MW.

$\overline{E}$ of 10 MW, the expected profit of the *Active/Passive* and the *Active* approach are similar. Indeed, as the capacity of the PV unit is small, the VPP can easily handle the PV solar fluctuations internally and offer the remaining flexibility in the balancing market. The increase of profit due to the innovative *Active/Passive* approach is of 1.7% with respect to the *Active* strategy and of 6.0 % compared to the *Passive* one. As the capacity of the unit increases, the *Passive* strategy gets more competitive than the *Active* one, as it can exploit the flexibility of operation and contracts a more profitable position in the day-ahead market. Differently, as $\overline{E}$ grows, the *Active* approach is increasingly constrained in its operation. First, it has less flexibility to offer in the balancing stage as it needs to allocate it to balance the PV unit fluctuations. Second,

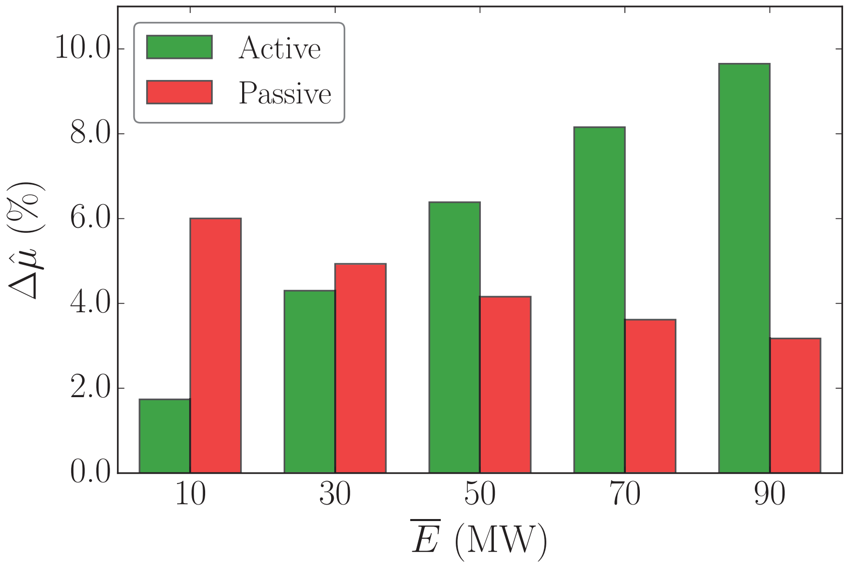


FIGURE 7.22: Percentage difference between the profit with the *Active/Passive* strategy and the profit with the alternative ones (i.e., *Active* and *Passive*), for values of $\overline{E}$ of 10, 30, 50, 70, and 90 MW.

the day-ahead position is more constrained by the feasibility of operation and not driven by the market prices.

7.6 Summary

This chapter takes the perspective of a VPP offering in a two-settlement electricity market where the balancing stage allows an *Active/Passive* participation of the VPP. Indeed, the VPP can decide to actively offer its regulating energy in some trading intervals while passively deviate from its schedule in the remaining ones. We start defining the structure of the VPP, which is composed of a conventional production unit, an electric energy storage, and a renewable energy generation unit (either a wind farm or a PV solar unit). We present a MILP approach to represent the feasible operating region and the cost function of the VPP. Then, the general price-taker and risk-neutral offering strategy formulation of Section 3.4 is adapted to the VPP. We consider three alternative offering strategies that the VPP may choose:

- An *Active* strategy, under which the VPP can only be an active participant in the balancing stage but can not deviate from its schedule.
- A *Passive* strategy, based on a solely passive participation in the balancing market.

- An *Active/Passive* strategy where the VPP can decide whether to be active or passive in each trading interval.

We run a case study for the two VPP configurations, i.e., one with a wind farm and the other with a PV solar unit. We show that the VPP has advantages by adopting the *Active/Passive* approach. Indeed, it can decide to be *Active* in the hourly intervals with a limited uncertainty in the renewable energy production and profitable balancing market price scenarios. Differently, a *Passive* approach is more attractive in the intervals where the balancing market price scenarios are "close" to the day-ahead price realization, and the future real-time production of the stochastic sources shows a high level of uncertainty. We test the three approaches for increasing values of the renewable energy unit capacity. When the stochastic unit capacity is small, the *Active* and the *Active/Passive* strategy bring comparable expected profits. Indeed, the VPP can easily handle the wind or solar power fluctuations internally and offer the remaining capacity in the balancing market. Differently, a *Passive* approach is not profitable as the wind and solar real-time deviations are easy to compensate within the VPP and there is no need of deviating in the real-time. Increasing the installed capacity of the stochastic source, the *Passive* strategy becomes more attractive as it allows a more flexible operation of the VPP. Differently, under the *Active* approach, the offering strategy is driven by ensuring a feasible operation since the deviations need to be handled internally. This translates into being less focused on maximizing the profit and more on guaranteeing a feasible solution. In this context, the *Active/Passive* gives the possibility to always reach higher expected profits as it can choose when it is more profitable to be active and when to be passive. This novel participation strategy can be also interesting from a system perspective. Indeed, the actual structure of electricity markets only allows a *Passive* participation of a VPP including stochastic energy sources. With the proposed *Active/Passive* participation the System Operator may have more flexible regulating energy to schedule in the real-time.

The case study proposed in the thesis uses a limited number of scenarios and a poor representation of the conventional production unit. Indeed, the complementarity between the active and passive participation in *Active/Passive* strategy strongly increases the computational time to solve the stochastic optimization model. Accordingly, we limit the size of the model to solve it in around 30 minutes. The extension to a more realistic case study (e.g., with 1000 scenarios) may require developing a heuristic algorithm able to guess a solution to be used as starting point for the MILP solver. Differently, the computational time to solve the model may restrict its applicability. Moreover, the inclusion of the intra-day trading floor may bring an additional flexibility of operation to the *Active/Passive* strategy. Indeed, the VPP could choose between the active and the passive participation closer to the real-time, when more accurate forecasts are available.

Chapter 8

Conclusion

This chapter concludes the thesis. Section 8.1 provides a summary of the work while Section 8.2 gives some perspective for further research.

8.1 Summary

The thesis presents a comprehensive methodology to derive optimal offering and operating strategies in electricity markets. Differently than most of the optimization models available in the literature, which is tailored to a specific market structure or a particular technology, the thesis proposes to build the optimal offering strategy of a power producer with a modular approach. Indeed, we give the general formulation of an offering strategy, which includes set of constraints associated with the trading problem in the different market stages and with the feasible operating region and the cost function of the power production unit. These sets of constraints are included as "blocks" in the general offering strategy. The power producer can replace them according to the electricity market structure considered or the specific production unit.

Together with the general offering strategy, the thesis provides several formulations of the trading problem for a price-taker and risk-neutral power producer in the different market stages. For the day-ahead market trading problem, we start from single price-quantity offers, and we extend it to non-decreasing offer curves. Moreover, we show how a stochastic programming approach can be used to formulate the day-ahead market trading problem with offer curves as an LP model. We proceed similarly for the trading problem in the balancing market of an active participant that submits regulating offers to the System Operator. For the balancing market, we consider both a uniform and a pay-as-bid pricing scheme as some European electricity markets (e.g., in Italy and in Germany) are settled under a pay-as-bid pricing scheme. In this context, an LP formulation for the trading problem in a pay-as-bid balancing market under price uncertainty is novel in the literature, and it is a contribution. Finally, we formulate the trading problem in the balancing market for a passive participant that deviates from its contracted schedule. We consider a single- and a dual-price imbalance settlement scheme.

Starting from the general formulation of the offering strategy, we take the perspective of different power producers, showing how they can build their offering strategy from the general one. Initially, we consider a power producer that manages a stochastic energy source, e.g., wind or solar power. We give an analytical formulation of the quantity offer that maximizes the expected profit of the power producer, for both the single- and the dual-price imbalance settlement scheme. Then, we extend the analysis to two alternative settlement schemes of the Italian electricity market. Such schemes, introduce a tolerance band that differentiates the penalization of the portion of the imbalance within the band from the part that exceeds it. We compute the optimal quantity offer maximizing its expected profit under those alternative settlement schemes. Moreover, the thesis links the optimal market quantity submitted in the day-ahead market with the expected real-time imbalance of the producer, differentiating "helpful" imbalances (i.e., that reduce the system's imbalance) from "harmful" imbalances (i.e., that increase the system's imbalance). We conclude that the imbalance settlement schemes with tolerance margins introduced by the Italian electricity market may distort the optimal offering strategy of the power producer. E.g., the tolerance band pushes the power producer to overestimate its day-ahead market offer when the expected power production is low, compared to a dual-price settlement scheme. Indeed, the market participant gains a wider tolerance band, which is proportional to the day-ahead quantity offer. This market distortion leads to more significant imbalances of the stochastic producer in the real-time (in expectation), which may lead to a higher balancing cost for the system.

Then, we take the perspective of a conventional power producer offering in a two-settlement electricity market having the balancing market settled under a pay-as-bid pricing scheme. We tailor the general offering strategy on the characteristics of the conventional production unit. To our best knowledge, a MILP two-stage stochastic programming program for offering in an electricity market with a pay-as-bid scheme at the balancing stage is not available in the literature. Indeed, we exploit the novel LP formulation for the trading problem in pay-as-bid electricity markets to cast the optimization problem with a MILP formulation, where the binary variables are associated with the feasible operating region of the thermal unit. We test the innovative offering model in a realistic case study against a sequential offering strategy, which does not consider the balancing market when deriving the day-ahead market offers. Oppositely, the two-stage stochastic offering model includes the future decisions in the balancing market as recourse variables, thus co-optimizing the trading in the two market stages. We show that a co-optimized strategy may be increasingly important as the penetration of renewable energy generation in the electricity market increases. Indeed, lower market prices in the day-ahead may lead a conventional producer that uses a sequential approach not to operate its production unit in the day-ahead market (if the prices are lower than its marginal cost). However, it translates into few profits in the balancing market, e.g., it can not submit down-regulation offers. Differently, based on a co-optimized approach the producer may accept to produce in the day-ahead market in a "losing" position. Then, in the balancing market, it can submit both up- and down-regulation offers and gain high total profits.

Finally, we consider the optimal market participation of a VPP, i.e., a cluster of conventional production units, renewable energy generation, and energy storages. We propose an innovative *Active/Passive* offering strategy, assuming that the VPP can offer regulating energy in the balancing market in some trading intervals and passively deviate in the remaining ones. Differently, the offering models in the literature solely consider the VPP as a passive actor in the balancing stage. We formulate the offering strategy as a three-stage stochastic programming problem with recourse while imposing the complementarity between the active and the passive participation in the balancing market. We test the proposed model in a case study against an *Active* (i.e., active-only participation) and a *Passive* (i.e., passive-only participation) approach. We formulate those alternative models as a two-stage stochastic programming problem with recourse. We analyze when the VPP may be interested in being an active actor and when a passive one. Indeed, as an active participant, the VPP has more favorable prices at the balancing stage while it is more constrained in the operation of the units as it needs to compensate the fluctuations of the stochastic energy source internally. Differently, as a passive actor, the VPP has penalized prices in the balancing market, but its operation is flexible, and it can deviate from the day-ahead contracted schedule. This innovative *Active/Passive* participation is interesting for both the VPP, that increases its expected profit, and the System Operator, that may have more regulating power available in the balancing market.

8.2 Perspectives for future research

Based on the methods and results provided in the thesis, continuing research in this field is significant for ensuring a practical application of the presented models. First, an extension of the work including a price-maker power producer and risk-analysis would complete the work, as the thesis focuses on a price-taker and risk-neutral power producer. Second, several alternative case studies can be generated starting from the general formulation of the offering strategy. E.g., the proposed co-optimized offering model in electricity markets having a pay-as-bid balancing market could be tested on production units different than the conventional ones. E.g., an efficient balancing market participation is significant for flexible production units as electric storages. Consequently, an assessment of the effectiveness of the proposed co-optimized offering strategy applied to an energy storage unit may be of interest. Third, it may be interesting to use different uncertainty characterization techniques to model the stochastic processes involved in the decision-making process. E.g., linear-decision rules or interval optimization may extend the applicability of the proposed offering models. Finally, an extension to the consumption side may be timely. The rising amount of prosumers (i.e., a consumer/producer of electric energy) may, indeed, lead to a re-organization of electricity markets towards more consumer-centric structures.

Appendix A

Stochastic Programming

A.1 Introduction

This Appendix presents the basic concepts of mathematical optimization and stochastic programming, which are used and referred several times within the thesis. We refer the interested reader to CITE for a more extensive and detailed coverage of the topic.

A.2 Mathematical Optimization

The general form of a linear optimization problem is the following:

$$\underset{\mathbf{x}}{\text{Max}} \quad \mathbf{c}^\top \mathbf{x} \tag{A.1a}$$

$$\text{s.t.} \quad \mathbf{A}\,\mathbf{x} = \mathbf{b}, \tag{A.1b}$$

$$\mathbf{D}\,\mathbf{x} \leq \mathbf{f}, \tag{A.1c}$$

where $\mathbf{x}$ is the vector of decision variables. The matrices $\mathbf{A}$ and $\mathbf{D}$ and the vectors $\mathbf{c}$, $\mathbf{b}$, and $\mathbf{f}$ are input parameters of the problem. Any vector $\mathbf{x}'$ that satisfy constraints (A.1b) and (A.1c) is a feasible solution of the optimization problem (A.1). The decision vector $\mathbf{x}^*$ that maximizes the objective function (A.1a) is called optimal solution, i.e., $\mathbf{c}^\top\mathbf{x}^* \geq \mathbf{c}^\top\mathbf{x}'$, for all the feasible solutions $\mathbf{x}'$. In case the parameters $\mathbf{A}$, $\mathbf{D}$, $\mathbf{c}$, $\mathbf{b}$, and $\mathbf{f}$ are perfectly known, the optimal solution $\mathbf{x}^*$ can be obtain with a solution algorithm for linear problems, e..g, the simplex method.

A.3 Stochastic Programming

A.4 Single-Stage Stochastic Programming

When the input parameters of problem (A.1) are conditional to the realization σ_ω of a random parameter σ, we can write $\mathbf{A}_\omega = \mathbf{A}(\sigma_\omega)$, $\mathbf{D}_\omega = \mathbf{D}(\sigma_\omega)$, ..., $\mathbf{c}_\omega = \mathbf{c}(\sigma_\omega)$. When the decision vector $\mathbf{x}$ has to be decide before the realization of the uncertain parameter σ, the optimization problem (A.1) is called stochastic linear program. A feasible solution $\mathbf{x}'$ has now to satisfy (A.1b) and (A.1c) for each possible realization of σ. In addition, the objective function (A.1a) becomes an uncertain variable itself as $\mathbf{c}_\omega$ is conditional upon the realization σ_ω.

It is necessary to recast the optimization problem in a form so that it can be solved by a solution algorithm for linear programming problems. Under a stochastic programming approach, the continuous random parameter σ is approximated though a discrete set Ω of possible scenarios, i.e., $\sigma \in \{\sigma_\omega, \ \forall \omega \in \Omega\}$, where each scenario ω has an associated probability π_ω of occurrence such that $\sum_\omega \pi_\omega = 1$. Finally, we need transform a random variable (i.e., the objective function) into a deterministic one. A typical approach is to use the expectation, i.e., $\mathbb{E}[\mathbf{c}_\omega^\top \mathbf{x}] = \sum_\omega \pi_\omega \mathbf{c}_\omega^\top \mathbf{x}$. However, other rankings are valid, e.g., the worst case realization is a valid alternative when considering a risk-averse approach. Accordingly, a single-stage stochastic programming problem can be formulated as follows:

$$\underset{\mathbf{x}}{\text{Max}} \quad \sum_\omega \pi_\omega \mathbf{c}_\omega^\top \mathbf{x} \tag{A.2a}$$

$$\text{s.t.} \quad \mathbf{A}_\omega \mathbf{x} = \mathbf{b}_\omega, \forall \omega, \tag{A.2b}$$

$$\mathbf{D}_\omega \mathbf{x} \leq \mathbf{f}_\omega, \forall \omega. \tag{A.2c}$$

Note that the way (A.2) is formulated forces the decision vector $\mathbf{x}$ to be independent from the the scenario realization ω.

A.5 Two-Stage Stochastic Programming

The idea of Section A.4 can be extended to model multi-stage decision problems under uncertainty. Indeed, we can model the sequence of events and decisions by distinguishing between two types of decisions:

- *Here-and-now* or first-stage decisions need to be taken before the realization of the uncertainty, and we denote them with $\mathbf{x}$.
- *Wait-and-see* or second-stage decisions can be fixed after the uncertainty realization, and we denote them with $\mathbf{y}(\omega)$ to highlight the dependency from the realization ω.

A mathematical formulation of a two-stage stochastic linear programming problem with recourse is the following:

$$\underset{\mathbf{x}}{\text{Max}} \quad \mathbf{c}^\top \mathbf{x} + \mathbb{E}[\mathcal{Q}(\mathbf{x}, \omega)] \tag{A.3a}$$

$$\text{s.t.} \quad \mathbf{A}\mathbf{x} = \mathbf{b}, \tag{A.3b}$$

where

$$\mathcal{Q}(\mathbf{x}, \omega) = \left\{ \underset{\mathbf{y}_\omega}{\text{Max}} \quad \mathbf{q}_\omega^\top \mathbf{y}(\omega) \right. \tag{A.4a}$$

$$\left. \text{s.t.} \quad \mathbf{T}_\omega \mathbf{x} + \mathbf{W}_\omega \mathbf{y}(\omega) = \mathbf{h}_\omega \right\}, \forall \omega \in \Omega. \tag{A.4b}$$

The objective function (A.3a) maximizes the sum of the first-stage cost, i.e., $\mathbf{c}^\top \mathbf{x}$ and the expectation of the recourse cost, i.e., $[\mathcal{Q}(\mathbf{x}, \omega)]$. The subproblem (A.4) is to be solved after the uncertainty realization and it is accordingly named *recourse* problem. Alternatively, problem (A.3) can be expressed using stochastic programming in a more compact formulation, i.e.,

$$\underset{\mathbf{x}, \mathbf{y}_\omega}{\text{Max}} \quad \mathbf{c}^\top \mathbf{x} + \sum_\omega \pi_\omega \mathbf{q}_\omega^\top \mathbf{y}_\omega \tag{A.5a}$$

$$\text{s.t.} \quad \mathbf{A}\mathbf{x} = \mathbf{b}, \tag{A.5b}$$

$$\mathbf{T}_\omega \mathbf{x} + \mathbf{W}_\omega \mathbf{y}_\omega = \mathbf{h}_\omega, \quad \forall \omega. \tag{A.5c}$$

The same idea can be used to model multistage (i.e., more than two) problems with recourse. In this case it is important to impose the *non-anticipativity* constraints aimed at ensuring that the decision sequence is respected. E.g., in case of a three-stage problem with recourse, a first stage decision $\mathbf{x}$ will independent from both the second-stage uncertainty realization ω^1 and the third-stage one ω^2. Differently, a second stage decision $\mathbf{y}_{\omega^1}$ will be dependent on ω^1 but independent to ω^2. Finally, a third stage decision $\mathbf{z}_{\omega^1 \omega^2}$ can be chosen after the realization of both ω^1 and ω^2.

Bibliography

Arroyo, José M and Antonio J Conejo (2000). “Optimal response of a thermal unit to an electricity spot market”. In: *IEEE Trans. Power Syst.* 15.3, pp. 1098–1104.

— (2004). “Modeling of start-up and shut-down power trajectories of thermal units”. In: *IEEE Trans. Power Syst.* 19.3, pp. 1562–1568.

Baillo, Alvaro et al. (2004). “Optimal offering strategies for generation companies operating in electricity spot markets”. In: *IEEE Trans. Power Syst.* 19.2, pp. 745–753.

Bakirtzis, Anastasios G et al. (2007). “Electricity producer offering strategies in day-ahead energy market with step-wise offers”. In: *IEEE Trans. Power Syst.* 22.4, pp. 1804–1818.

Baringo, Luis and Antonio J Conejo (2013). “Strategic offering for a wind power producer”. In: *IEEE Transactions on Power Systems* 28.4, pp. 4645–4654.

Batalla-Bejerano, Joan and Elisa Trujillo-Baute (2016). “Impacts of intermittent renewable generation on electricity system costs”. In: *Energy Policy* 94, pp. 411–420.

Bathurst, Graeme N, Jennie Weatherill, and Goran Strbac (2002). “Trading wind generation in short term energy markets”. In: *IEEE Transactions on Power Systems* 17.3, pp. 782–789.

Batlle, Carlos, Ignacio J Pérez-Arriaga, and Patricio Zambrano-Barragán (2012). “Regulatory design for RES-E support mechanisms: Learning curves, market structure, and burden-sharing”. In: *Energy Policy* 41, pp. 212–220.

Bitar, Eilyan Y et al. (2012). “Bringing wind energy to market”. In: *IEEE Transactions on Power Systems* 27.3, pp. 1225–1235.

Bremnes, John Bjørnar (2004). “Probabilistic wind power forecasts using local quantile regression”. In: *Wind Energy* 7.1, pp. 47–54.

— (2006). “A comparison of a few statistical models for making quantile wind power forecasts”. In: *Wind Energy* 9.1-2, pp. 3–11.

Bueno-Lorenzo, Miriam, M Ángeles Moreno, and Julio Usaola (2013). "Analysis of the imbalance price scheme in the Spanish electricity market: A wind power test case". In: *Energy policy* 62, pp. 1010–1019.

Chaves-Ávila, José Pablo, Rudi A Hakvoort, and A Ramos (2014). "The impact of European balancing rules on wind power economics and on short-term bidding strategies". In: *Energy Policy* 68, pp. 383–393.

Clò, Stefano, Alessandra Cataldi, and Pietro Zoppoli (2015). "The merit-order effect in the Italian power market: The impact of solar and wind generation on national wholesale electricity prices". In: *Energy Policy* 77, pp. 79–88.

Conejo, Antonio J, Miguel Carrión, and Juan M Morales (2010). *Decision Making Under Uncertainty in Electricity Markets*. Springer.

Conejo, Antonio J, Francisco Javier Nogales, and José Manuel Arroyo (2002). "Price-taker bidding strategy under price uncertainty". In: *IEEE Trans. Power Syst.* 17.4, pp. 1081–1088.

Conejo, Antonio J et al. (2004). "Risk-constrained self-scheduling of a thermal power producer". In: *IEEE Trans. Power Syst.* 19.3, pp. 1569–1574.

De Vos, Kristof and Johan Driesen (2009). "Balancing management mechanisms for intermittent power sources—A case study for wind power in Belgium". In: *2009 6th International Conference on the European Energy Market*. IEEE, pp. 1–6.

De Vos, Kristof, Johan Driesen, and Ronnie Belmans (2011). "Assessment of imbalance settlement exemptions for offshore wind power generation in Belgium". In: *Energy policy* 39.3, pp. 1486–1494.

Giannitrapani, Antonio et al. (2013). "Wind power bidding in a soft penalty market". In: *52nd IEEE Conference on Decision and Control*. IEEE, pp. 1013–1018.

Gneiting, Tilmann et al. (2006). "Calibrated probabilistic forecasting at the stateline wind energy center: The regime-switching space–time method". In: *Journal of the American Statistical Association* 101.475, pp. 968–979.

Gountis, Vasileios P and Anastasios G Bakirtzis (2004). "Bidding strategies for electricity producers in a competitive electricity marketplace". In: *IEEE Trans. Power Syst.* 19.1, pp. 356–365.

Growe-Kuska, Nicole, Holger Heitsch, and Werner Romisch (2003). "Scenario reduction and scenario tree construction for power management problems". In: *IEEE Power Tech Conference Proceedings*, pp. 1–7.

Italian Regulatory Authority for Electricity Gas and Water (2014). *Resolution 800/2016/R/eel*. Tech. rep.

— (2016). *Resolution 800/2016/R/eel*. Tech. rep.

Jabr, Rabih A (2005). "Robust self-scheduling under price uncertainty using conditional value-at-risk". In: *IEEE Trans. Power Syst.* 20.4, pp. 1852–1858.

Juban, Jeremie, Nils Siebert, and George N Kariniotakis (2007). "Probabilistic short-term wind power forecasting for the optimal management of wind generation". In: *Power Tech, 2007 IEEE Lausanne*. IEEE, pp. 683–688.

Kardakos, Evaggelos G, Christos K Simoglou, and Anastasios G Bakirtzis (2016). "Optimal offering strategy of a virtual power plant: A stochastic bi-level approach". In: *IEEE Transactions on Smart Grid* 7.2, pp. 794–806.

Khorasani, J and H Rajabi Mashhadi (2012). "Bidding analysis in joint energy and spinning reserve markets based on pay-as-bid pricing". In: *IET Generation, Transmission & Distribution* 6.1, pp. 79–87.

Lange, Matthias and Ulrich Focken (2006). *Physical approach to short-term wind power prediction*. Springer.

Liu, Xian (2011). "Impact of beta-distributed wind power on economic load dispatch". In: *Electric Power Components and Systems* 39.8, pp. 768–779.

Maenhoudt, Marijn and Geert Deconinck (2014). "Strategic offering to maximize day-ahead profit by hedging against an infeasible market clearing result". In: *IEEE Trans. Power Syst.* 29.2, pp. 854–862.

Mashhour, Elaheh and Seyed Masoud Moghaddas-Tafreshi (2011a). "Bidding strategy of virtual power plant for participating in energy and spinning reserve markets—Part I: Problem formulation". In: *IEEE Transactions on Power Systems* 26.2, pp. 949–956.

— (2011b). "Bidding strategy of virtual power plant for participating in energy and spinning reserve markets—Part II: Numerical analysis". In: *IEEE Transactions on Power Systems* 26.2, pp. 957–964.

Møller, Jan Kloppenborg, Henrik Aalborg Nielsen, and Henrik Madsen (2008). "Time-adaptive quantile regression". In: *Computational Statistics & Data Analysis* 52.3, pp. 1292–1303.

Morales, Juan M, Antonio J Conejo, and Juan Pérez-Ruiz (2010). "Short-term trading for a wind power producer". In: *IEEE Transactions on Power Systems* 25.1, pp. 554–564.

Morales, Juan M et al. (2009). "Scenario reduction for futures market trading in electricity markets". In: *IEEE Transactions on Power Systems* 24.2, pp. 878–888.

Morales, Juan M et al. (2013). *Integrating renewables in electricity markets: operational problems*. Vol. 205. Springer Science & Business Media.

Morales, Juan M et al. (2014). "Electricity market clearing with improved scheduling of stochastic production". In: *European Journal of Operational Research* 235.3, pp. 765–774.

Ni, Ernan, Peter B Luh, and Stephen Rourke (2004). "Optimal integrated generation bidding and scheduling with risk management under a deregulated power market". In: *IEEE Trans. Power Syst.* 19.1, pp. 600–609.

Nielsen, Henrik Aalborg et al. (2006). "From wind ensembles to probabilistic information about future wind power production–results from an actual application". In: *Probabilistic Methods Applied to Power Systems, 2006. PMAPS 2006. International Conference on*. IEEE, pp. 1–8.

Pandžić, Hrvoje, Igor Kuzle, and Tomislav Capuder (2013). "Virtual power plant mid-term dispatch optimization". In: *Applied Energy* 101, pp. 134–141.

Pandžić, Hrvoje et al. (2013). "Offering model for a virtual power plant based on stochastic programming". In: *Applied Energy* 105, pp. 282–292.

Peik-Herfeh, Malahat, H Seifi, and MK Sheikh-El-Eslami (2013). "Decision making of a virtual power plant under uncertainties for bidding in a day-ahead market using point estimate method". In: *International Journal of Electrical Power & Energy Systems* 44.1, pp. 88–98.

Pinson, Pierre, Christophe Chevallier, and George N Kariniotakis (2007). "Trading wind generation from short-term probabilistic forecasts of wind power". In: *IEEE Transactions on Power Systems* 22.3, pp. 1148–1156.

Pinson, Pierre and Robin Girard (2012). "Evaluating the quality of scenarios of short-term wind power generation". In: *Applied Energy* 96, pp. 12–20.

Pinson, Pierre et al. (2009). "From probabilistic forecasts to statistical scenarios of short-term wind power production". In: *Wind Energy* 12.1, pp. 51–62.

Ren, Yongjun and Francisco D Galiana (2004a). "Pay-as-bid versus marginal pricing - Part I: Strategic generator offers". In: *IEEE Trans. Power Syst.* 19.4, pp. 1771–1776.

— (2004b). "Pay-as-bid versus marginal pricing - Part II: Market behavior under strategic generator offers". In: *IEEE Trans. Power Syst.* 19.4, pp. 1777–1783.

Ruiz, Carlos and Antonio J Conejo (2009). "Pool strategy of a producer with endogenous formation of locational marginal prices". In: *IEEE Trans. Power Syst.* 24.4, pp. 1855–1866.

Ruiz, Nerea, Iñigo Cobelo, and José Oyarzabal (2009). "A direct load control model for virtual power plant management". In: *IEEE Transactions on Power Systems* 24.2, pp. 959–966.

Sadeh, Javad, Habib Rajabi Mashhadi, and Mohammad Amin Latifi (2009). "A risk-based approach for bidding strategy in an electricity pay-as-bid auction". In: *European Transactions on Electrical Power* 19.1, pp. 39–55.

Sensfuss, Frank, Mario Ragwitz, and Massimo Genoese (2008). "The merit-order effect: A detailed analysis of the price effect of renewable electricity generation on spot market prices in Germany". In: *Energy Policy* 36.8, pp. 3086–3094.

Swider, Derk J (2007). "Simultaneous bidding in day-ahead auctions for spot energy and power systems reserve". In: *International Journal of Electrical Power & Energy Systems* 29.6, pp. 470–479.

Swider, Derk J and Christoph Weber (2007). "Bidding under price uncertainty in multi-unit pay-as-bid procurement auctions for power systems reserve". In: *European Journal of Operational Research* 181.3, pp. 1297–1308.

Vandezande, Leen et al. (2010). "Well-functioning balancing markets: A prerequisite for wind power integration". In: *Energy Policy* 38.7, pp. 3146–3154.

Wang, Qi et al. (2015). "Review of real-time electricity markets for integrating distributed energy resources and demand response". In: *Applied Energy* 138, pp. 695–706.

Winkler, Jenny et al. (2016). "Impact of renewables on electricity markets–Do support schemes matter?" In: *Energy Policy* 93, pp. 157–167.

Zugno, Marco et al. (2013). "Pool strategy of a price-maker wind power producer". In: *IEEE Transactions on Power Systems* 28.3, pp. 3440–3450.