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# UNIVERSITÀ DEGLI STUDI DI PADOVA

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DOTTORATO DI RICERCA IN ECONOMIA E MANAGEMENT

CICLO XXIV

## ESSAYS ON VULNERABILITY TO POVERTY AND INEQUALITY

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*“If we knew what it was we were doing,  
it would not be called research, would it?”*

*Albert Einstein (1879-1955)*

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# Contents

<b>Acknowledgements</b>	<b>vii</b>
<b>Introduction</b>	<b>ix</b>
<b>Introduzione</b>	<b>xi</b>
<b>1 Vulnerability as Predictor of Poverty</b>	<b>1</b>
1.1 Introduction . . . . .	1
1.2 Literature review . . . . .	2
1.2.1 <i>Vulnerability to Poverty</i> . . . . .	2
1.2.2 <i>Measuring Vulnerability as Expected Poverty</i> . . . . .	4
1.3 Data . . . . .	8
1.4 Empirical strategy . . . . .	11
1.5 Results . . . . .	14
1.6 Conclusions . . . . .	21
<b>2 Vulnerability: A Non-Parametric Decomposition</b>	<b>23</b>
2.1 Introduction . . . . .	23
2.2 The Three Vulnerability Contributing Factors . . . . .	25
2.3 Data . . . . .	28
2.4 Empirical Illustrations . . . . .	31
2.5 Conclusions . . . . .	38
<b>3 Age-Adjustments in Inequality Dynamics</b>	<b>43</b>
3.1 Introduction . . . . .	43
3.2 Stylised Facts . . . . .	44
3.3 Literature review . . . . .	46
3.4 Data . . . . .	51

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3.5	Results . . . . .	53
3.6	Conclusions . . . . .	56
<b>Appendices</b>		<b>65</b>
A	ROC curve analysis . . . . .	65
B	The Shapley decomposition . . . . .	66
C	The non-parametric decomposition of Vulnerability to Poverty . . . . .	67
D	Italy - Decomposition I . . . . .	68
E	Italy - Decomposition II . . . . .	69

# List of Tables

1.1	Sample Characteristics . . . . .	10
1.2	Vulnerability to poverty - UK and Germany . . . . .	15
1.3	Vulnerability to poverty and Income poverty correlation - UK and Germany . . . . .	16
1.4	Equality tests among areas under the ROC curves - UK and Germany . . . . .	18
1.5	Specificity for given values of sensitivity (85%, 80%, 75%) - UK and Germany . . . . .	19
1.6	Vulnerability to poverty - Italy . . . . .	20
1.7	Vulnerability to poverty and Income poverty correlation - Italy . . . . .	21
1.8	Equality tests among areas under the ROC curves - Italy . . . . .	22
2.1	An example of Vulnerability to Poverty and the proposed decomposition . . . . .	27
2.2	Sample Characteristics . . . . .	29
2.3	Vulnerability to Poverty and its contributing factors - England . . . . .	33
2.4	England - Paired t-tests . . . . .	34
2.5	Vulnerability to Poverty decomposition - England . . . . .	34
2.6	Vulnerability among households with children - England - Paired t-tests . . . . .	35
2.7	Vulnerability among households whose head is retired - England - Paired t-tests . . . . .	36
2.8	Vulnerability among low-income households - England - Paired t-tests . . . . .	37
2.9	Vulnerability to Poverty and its contribution factors - Italy . . . . .	37
2.10	Vulnerability to Poverty - Italy - T-tests . . . . .	38
2.11	Expected Incidence - Italy - T-tests . . . . .	39
2.12	Expected Intensity - Italy - T-tests . . . . .	40
2.13	Expected Downward Variability - Italy - T-tests . . . . .	41
3.1	Sample Characteristics . . . . .	53
3.2	Inequality Indexes . . . . .	54
A.1	Vulnerability to poverty and Income poverty correlation . . . . .	65

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# List of Figures

1.1	The ROC curve . . . . .	12
2.1	Example of Poverty Gaps Pattern - Individual 1 . . . . .	26
2.2	Example of Poverty Gaps Pattern - Individual 2 . . . . .	26
2.3	Example of Poverty Gaps Pattern - Individual 3 . . . . .	26
2.4	Average log income values by age group and Age-adjustment (UK) . . . . .	29
3.1	Example of Age-Wealth Profile . . . . .	45
3.2	Changes in the age structure over time in Italy . . . . .	45
3.3	Wealth Inequality (Bank of Italy) - Share 65–74 years old . . . . .	46
3.4	Income Inequality (Bank of Italy) - Share 65–74 years old . . . . .	46
3.5	Average Age-Income profile . . . . .	48
3.6	Dynamic of wealth inequality indexes I . . . . .	55
3.7	Dynamic of wealth inequality indexes II . . . . .	55

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# Introduction

According to the recent report of *The Commission on the Measurement of Economic Performance and Social Progress* (CMEPSP), whose members are also Joseph Stiglitz, Amartya Sen and Jean Paul Fitoussi, *statistical indicators are important for designing and assessing policies aiming at advancing the progress of society* (Stiglitz, Sen, and Fitoussi, 2009).

The main objective of the present work is to shed light on some aspects concerning the information provided by vulnerability to poverty and inequality indexes.

The first chapter compares empirically the several measures of individual vulnerability to poverty proposed in the literature, in order to understand which is the best signal of poverty that can be used for policies purposes. To this aim the Receiver Operating Characteristic (ROC) curve, the Pearson and Spearman correlation coefficients are used as precision criteria. Using data from the British Household Panel Survey (BHPS), the German Socio-Economic Panel (SOEP) and the Survey on Household Income and Wealth (SHIW), the results show that two groups of indexes can be identified, *high-* and *low-performers*, and, among the former, that proposed by Dutta, Foster, and Mishra (2011) is the most precise.

The second chapter applies a non-parametric decomposition of the Foster-Greer-Thorbecke poverty index to the measurement of individual vulnerability to poverty. I highlight that poverty risk can be expressed as a function of three components expected incidence, expected intensity and expected downward variability. This decomposition is useful for risk management purposes since it describes the characteristics of the poverty risk faced by individuals. An empirical illustration is provided using the British Household Panel Survey and the Survey on Household Income and Wealth.

The third chapter focuses on inequality. According to Atkinson (1971), inequality attributable to age should be of little concern for policymakers because it is irrelevant for the distribution of lifetime income or wealth. Concerning that I provide age-adjusted measures of wealth inequality to understand the role of demographic changes in Italy in determining the trends in disparities. Using the Survey on Household Income and Wealth from 1991 to 2008, the results confirm previous findings: age-adjustments are not very important in terms

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of dynamics.

# Introduzione

Alla luce del recente rapporto della *Commissione sulla Misura della Performance Economica e del progresso Sociale* (CMEPSP), composta anche da Joseph Stiglitz, Amartya Sen e Jean Paul Fitoussi, gli indicatori statistici sono importanti per il design e la valutazione delle politiche pubbliche in termini di progresso sociale (Stiglitz, Sen, and Fitoussi, 2009).

L'obiettivo principale della tesi in oggetto é l'analisi dell'informazione fornita dagli indici di vulnerabilit  alla povert  e disuguaglianza.

Il primo capitolo confronta in termini empirici le misure individuali di vulnerabilit  alla povert  proposte in letteratura. Lo scopo é capire quale sia l'indice pi  preciso nel predire la povert , affinche questo possa essere utilizzato come fonte di informazione per le politiche pubbliche. La Receiver Operating Characteristic (ROC) curve, i coefficienti di correlazione di Pearson e Spearman sono utilizzati come criteri per la valutazione della precisione. Usando dati del British Household Panel Survey (BHPS), del German Socio-Economic Panel (SOEP) e della Survey on Household Income and Wealth (SHIW), i risultati mostrano che possono essere identificate due categorie di indici, *high-* e *low-performers*; fra i primi, l'indice proposto da Dutta, Foster, and Mishra (2011) é il pi  preciso nell'identificare i *futuri poveri*.

Il secondo capitolo applica una scomposizione non parametrica dell'indice di povert  Foster-Greer-Thorbecke alla vulnerabilit  alla povert  individuale. Questo approccio mostra come il rischio di povert  pu  essere espresso come funzione di incidenza attesa, intensit  attesa e variabilit  negativa attesa. La scomposizione proposta é utile in termini di politiche di risk management per le informazioni circa le caratteristiche del rischio di povert . Il capitolo prevede due illustrazioni empiriche con dati del British Household Panel Survey e della Survey on Household Income and Wealth.

Il terzo capitolo di focalizza sugli indici di disuguaglianza. Secondo Atkinson (1971), la disuguaglianza attribuibile all'et  é irrilevante se l'interesse é concentrato nella distribuzione di reddito e ricchezza di lungo periodo (lifetime perspective). Riguardo ci , il terzo capitolo propone delle misure di disuguaglianza basate sulla ricchezza netta e corrette per l'effetto dei cambiamenti demografici nella popolazione italiana fra il 1991 ed il 2008. Utilizzando i dati

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della Survey on Household Income and Wealth della Banca d'Italia, i risultati confermano quanto già osservato in letteratura: gli aggiustamenti demografici non risultano determinanti nella dinamica della disuguaglianza in termini di ricchezza netta.



# Chapter 1

# Vulnerability as Predictor of Poverty

## 1.1 Introduction

The recent financial crisis and the increasing recognition that there are considerable flows into and out of poverty (Baulch and Hoddinott, 2000) gained the attention of governments, researchers and foundations in several countries on economic risk and its role as *threat*.

In the economic literature we find two concepts related to economic hazard: *economic insecurity* and *vulnerability to poverty*. These concepts have evolved quite independently, but a common basis exists between them. Both concepts deal with an economic risk that produces anxiety (Osberg, 1998) and represents a threat (Dercon, 2006), but, according to Osberg (2010), they differ in terms of countries analysed, perspective and risk exposure consequences.

The main difference, in my opinion, is that economic insecurity, unlike vulnerability to poverty, concerns more the *ex post* subjective measurement of the lack of safety rather than an objective poverty danger. Everyone could feel economically insecure but only a part of the population, those vulnerable, are likely to become poor in the future.

Therefore, if the interest is to provide information for anti-poverty protection strategies, vulnerability to poverty is the concern. Vulnerability aims to identify the poor in advance representing an *ex ante* information source for policies design. Chaudhuri, Jalan, and Suryahadi (2002), for example, write that what really matters for forward-looking anti-poverty interventions is vulnerability to poverty. Zhang and Guanghai (2008) argue that measuring

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vulnerability is important because it allows the identification of those who are not currently poor but may fall into poverty. Vulnerability therefore can be used, once those vulnerable to poverty are identified, to design appropriate policies to prevent them from falling into poverty. Also Jamal (2009), by highlighting the distinction between *ex ante* poverty prevention and *ex post* poverty alleviation interventions, considers vulnerability assessments as a way to improve risk-management policies.

This paper tries to understand which index, among those proposed in literature, can detect with more precision the individuals at risk of poverty in the next year. I believe that this exercise is useful since it identifies the most precise *ex ante* information source for policies purposes.

## 1.2 Literature review

### 1.2.1 *Vulnerability to Poverty*

According to The World Bank definition, vulnerability to poverty is the probability, today, of being in poverty or to fall into deeper poverty in the future. Vulnerability is very different from the standard analysis of poverty because it recalls a forward-looking perspective rather than an *ex post* assessment, allowing the design of protection policies that can prevent households and individuals from experiencing welfare losses.

The concept of vulnerability to poverty stems its roots in a seminal article by Jalan and Ravallion (1998) on transient and chronic poverty. Here the authors noticed how in rural China variability in consumption accounts for a large part of the observed poverty: half of the mean squared poverty gap and over a third of the mean poverty gap is transient and directly attributable to year-to-year consumption fluctuations.

While theoretically vulnerability to poverty is almost well-defined as the risk of experiencing poverty, three different definitions can be recognized empirically: vulnerability as expected poverty (VEP), vulnerability as low expected utility (VEU) and vulnerability as uninsured exposure to risk (VER). These definitions are all equally used in literature, since they describe the poverty risk according to three different perspectives.

The very first VEP version translates vulnerability into a probability measure of facing poverty in the future. More precisely, when welfare is defined in terms of consumption or income, then vulnerability of the  $h^{th}$  household (or individual), at time  $t$ , is  $V_{ht}$ , the probability that consumption or income tomorrow,  $y_{h,t+1}$ , falls below the poverty line,  $z$ , that is

$$V_{ht} = Pr(y_{h,t+1} < z). \tag{1.1}$$

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Ligon and Schechter (2003) proposed a different measure, based on utility, to take properly into account risk sensitivity. They pointed out that a policy-maker, who allocates resources to minimize the expected value of one of the Foster, Greer, and Thorbecke (1984) (FGT) indexes, would tend to assign too much risk to poorer households. Therefore they defined vulnerability as the difference between the utility derived from some level of certainty-equivalent,  $z_{CE}$ , at and above which the household  $h$  would not be considered vulnerable, and the expected utility of consumption,  $c_h$ ,

$$V_h = U_h(z_{CE}) - EU_h(c_h). \quad (1.2)$$

This approach, while appealing in terms of risk considerations, has some drawbacks since it is necessary to specify a utility functional form for  $U_h$  and a value for the risk aversion parameter. VEU has been used less extensively compared to VEP because it measures vulnerability in terms of utility units, with a less straightforward interpretation of the results.

The third approach, VER, even if based on intertemporal variability of consumption as VEP and VEU, is very different in terms of perspective. Vulnerability as uninsured exposure to risk is backward-looking, while the former methods are forward-looking. VER is in fact an *ex post* assessment of the extent to which a negative income shock caused a welfare loss in terms of consumption. This third approach is based on the consumption smoothing and risk sharing literature, where the degree of vulnerability is defined by the extent to which the growth rate of household consumption covaries with the household income growth rate (Gerry and Li 2010, Skoufias and Quisumbing 2003). VER aims to understand if households are able to spread the effects of income shocks through formal or informal insurance strategies, with the following interpretation in terms of vulnerability: if consumption and income are correlated, then the households use not so effective risk management instruments, increasing their vulnerability to negative income shocks. Using the following equation

$$\Delta c_{h,t,v} = \beta \Delta \ln y_{h,t,v} + \delta X_{h,t,v} + \sum_{t,v} \delta_{t,v} D_{t,v} + \Delta \epsilon_{h,t,v}, \quad (1.3)$$

where  $\Delta c_{h,t,v}$  denotes the growth rate from  $t-1$  to  $t$  of the total consumption of household  $h$  in the community  $v$ ,  $\Delta \ln y_{h,t,v}$  is the growth rate of income,  $X$  is a vector of household characteristics,  $D_{t,v}$  are other controls and  $\Delta \epsilon_{h,t,v}$  is a household-specific error term, the parameter of interest for VER is  $\beta$ .

For this analysis, since I am interested in the ability of vulnerability measures to identify in advance the future poor, I will focus on the first approach mentioned, VEP, that has a forward-looking perspective and is easier to interpret as its value is expressed in monetary

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terms.

Vulnerability to poverty has been often studied in developing countries (see among others Gaiha and Imai 2008, Gaiha, Imai, and Kang 2011, Imai, Wang, and Kang 2009, Jha, Dang, and Sharma 2009) because poverty risk is in relative terms quantitatively more important, but volatile incomes are commonly encountered also in developed countries and are, under certain conditions, symptoms of being prone to poverty. Bandyopadhyay and Cowell (2007) for instance estimate vulnerability to poverty for the United Kingdom using the VER approach and quantile regressions. They found that, apart from those households around the poverty line, there are some, well away from the poverty zone, that are susceptible to be income shocks vulnerable.

In this paper different data sources on some EU countries, UK, Germany and Italy are used. The choice is driven mainly by the quality of data available necessary to estimate properly vulnerability and to highlight the differences among measures. To emphasize the features of each index a sufficiently long longitudinal component is needed and information on the household disposable income has to be collected accurately. I will estimate therefore vulnerability exploiting the British Household Panel Survey (BHPS), the German Socio-Economic Panel Study (SOEP) and the Survey on Household Income and Wealth (SHIW) for Italy, datasets that meet all the requirements.

### 1.2.2 *Measuring Vulnerability as Expected Poverty*

One of the first papers formalizing the idea of a measure that can anticipate the poverty status is Pritchett, Suryahadi, and Sumarto (2000), where the authors point out how many households, while not currently in poverty, are vulnerable to events such as jobloss, or unexpected expenditures due to illnesses or economic downturns. As vulnerability to poverty is intended to be an expected poverty, the authors propose an expansion of the traditional poverty measures to quantify an *ex ante* vulnerability to poverty and to measure the risk for a household of falling into poverty in the future,

$$V_t^h(p, n, z) = I \left[ R_t^h(n, z) > p \right] \quad (1.4)$$

$$R_t^h(n, z) = 1 - [(1 - Pr(y_{h,t+1} < z)) * \dots * (1 - Pr(y_{h,t+n} < z))]. \quad (1.5)$$

The general definition that they state is then clarified in terms of risk and time, *falling into poverty at least once in the next few years*. Therefore the vulnerability of the household  $h$  for  $n$  periods is the probability of observing, in the time span considered, at least one episode of poverty, i.e. the complementary probability of observing no episodes of poverty,

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see Equations 1.4 and 1.5.

According to Pritchett, Suryahadi, and Sumarto (2000), vulnerability is a risk measured in terms of probability,  $R_t^h(n, z)$ , that depends on the time horizon,  $n$ , and the poverty line,  $z$ ;  $I[\cdot]$  is an indicator function that translates vulnerability into a state variable, by defining a probability threshold,  $p$ . The authors observe that everybody face a certain degree of poverty risk, also the richest individuals, therefore, to have a more reliable aggregate measure of poverty risk, called *Headcount Vulnerable to Poverty Rate*, they introduce the function,  $I[\cdot]$ , that takes value 1 if the probability computed is higher than the chosen threshold level, 0.5, and zero otherwise. As already noticed in literature this approach fails to consider explicitly the depth of poverty (Hoddinott and Quisumbing, 2003), but this issue is relatively straightforward to redress by writing the following

$$V_t^h = \sum_s p_s P(y_{h,t+1}, z) = \sum_s p_s I[y_{h,t+1}, z] \cdot [(z - y_{h,t+1}) / z]^\alpha. \quad (1.6)$$

Expression 1.6 echoes the FGT index of poverty, where  $\alpha$  is the relative weight attached to extreme poverty,  $S$  are the *possible states of the world*,  $p_s$  is the probability that the  $s^{th}$  state occurs and  $I[\cdot]$  is a function that allows to consider only those states in which the expected income,  $y_{h,t+1}$ , falls below the chosen poverty line  $z$ . The drawback of the adoption of this index, based on income or consumption standard deviation, is to fail to consider the persistence of the phenomenon.<sup>1</sup>

Despite the discussed drawbacks, vulnerability expressed in terms of probability has been extensively used because easy to interpret, even if very demanding in terms of data when it is translated empirically. When estimating Pritchett, Suryahadi, and Sumarto (2000)'s vulnerability in fact some assumptions have necessarily to be made: to compute probabilities information about the distribution of the welfare measure, either consumption or income, is needed not only at the aggregate level but also at the household (or individual) level. This is the reason why in the empirical applications, to preserve computational simplicity, the distribution of the welfare measure is always assumed to be normal (see among others Azam and Imai 2009, Chaudhuri, Jalan, and Suryahadi 2002, Gaiha, Imai, and Kang 2011, Jha, Dang, and Sharma 2009, Zhang and Guanghua 2008). Nevertheless, in some cases other problems related to the quality of data could arise: measurement errors for instance are something to account for in this type of analysis; when this problem contaminates data, in

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<sup>1</sup>Kamanou and Morduch (2002) propose a simple example on this problem. Let us suppose to observe two household consumption patterns over 8 periods, the former is (1, 2, 3, 4, 5, 6, 7, 8), while the latter is (7, 5, 2, 6, 3, 1, 4, 8). If we base our analysis on standard deviations, both series are identical, but the trend of consumption is very different, in the latter case we notice a steady upward path. As highlighted by Kamanou and Morduch (2002, p. 9), "labeling them both as *identically vulnerable misses the key part of their stories*".

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fact, it is important to consider the part of the measured shocks which is not true risk, that leads to a potential over-estimation of the poverty danger.

As the majority of the restrictions are imposed by the empirical analysis, part of the literature on vulnerability as expected poverty has focused on overcoming the limitations of the data by improving the estimates of income or consumption variability. Chaudhuri, Jalan, and Suryahadi (2002) and Chaudhuri (2003), for instance, using the same measure of Pritchett, Suryahadi, and Sumarto (2000), compute vulnerability when only cross-sectional data are available, with a consistent estimate of the variance, while Kamanou and Morduch (2002) propose a non-parametric approach, based on a bootstrap technique, to compute an aggregate index of vulnerability.

As it is difficult to find a broadly recognized procedure for vulnerability, Hoddinott and Quisumbing (2003) summarized all the attempts used in the literature, highlighting the drawbacks of each approach. About the expected poverty version, they criticize mainly the fact that downside and upside risks are weighted the same way.

After the first empirical focus, the literature has developed towards a more formal attempt to test if some desirable properties were satisfied by the measures proposed, this is what is called *axiomatic approach*. In their definition, Calvo and Dercon (2005) consider as vulnerability the *magnitude of the threat of poverty* and the *sense of insecurity*, they clarify how their view of vulnerability is not simply low expected welfare, as often turns out from previous studies, but is related to dangers or threats, as opposed to uncertainties in general. The two authors formally require that their measure of vulnerability satisfies the following properties:

- Symmetry: This axiom ensures that the measure used for vulnerability does not consider differently two possible states of the world, if they do not differ in terms of probabilities and outcomes. An illness or a bad harvest are equivalent if they occur with equal probability and have the same effect on the outcome.
- Focus: Changes in outcomes of good states of the world do not affect individuals' vulnerability to poverty. This axiom clarifies that the threat of future poverty will not be mitigated by simultaneous (*ex ante*) possibilities of being well-off.
- Probability-dependent effect on outcomes: If the outcome in one state of the world improves, the consequent effect on vulnerability does not depend on the outcomes or probabilities of other states of the world but on the likelihood of that particular state of the world.
- Probability transfer: Vulnerability is linear in probabilities and, as long as outcomes are below the poverty line, its increases are monotonically related to decreases in

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outcomes.

- Risk sensitivity: Risk leads to higher vulnerability.
- Scale invariance: This axiom requires that the index does not depend on the unit of measurement because what matters is only the relative distance from the poverty line.

Even if the often used vulnerability version of the Foster-Greer-Thorbecke poverty index (1.6) satisfies the desiderata listed by the two authors, it fails to meet the *Probability transfer* and *Risk sensitivity* axioms under the most frequently used values of  $\alpha$ , i.e. with  $\alpha = 0$  or  $\alpha = 1$ . Moreover, even if we consider  $\alpha > 1$ , satisfying all properties, the risk sensitivity axiom implies that better outcomes will exacerbate the extent to which the individual fears an increase in risk exposure, against empirical evidences. Therefore to have alternative risk aversion attitudes, more consistent with data, Calvo and Dercon (2005) propose two other classes of measures that satisfy additional properties, not imposed as forcefully:

$$V_\alpha = 1 - E \left[ \left( \frac{\min(y_{h,t+1}, z)}{z} \right)^\alpha \right] \quad 0 < \alpha < 1, \quad (1.7)$$

$$V_\beta = E \left[ \frac{e^{\beta(1-x_h, t+n)} - 1}{e^\beta - 1} \right] \quad \beta > 0, \quad x_h = \frac{\min(y_{h,t+1}, z)}{z}. \quad (1.8)$$

The former class, 1.7, satisfies the constant relative risk sensitivity, i.e. the *efficiency loss* due to risk is determined as a constant proportion of expected outcome,  $E[\cdot]$  denotes expectations in the formula. The latter, 1.8, meets the constant absolute risk sensitivity, i.e. the *efficiency loss* is a constant value of  $y^{ce} - \hat{y}_{t+1}$ , where  $y^{ce}$  is the certainty-equivalent outcome. While different risk attitudes are the main innovation proposed by Calvo and Dercon (2005) in measuring vulnerability, Dutta, Foster, and Mishra (2011) have recently highlighted the importance of current living standard in this context, by proposing the following measures

$$V(L) = \sum_{s=1}^S p_s (R(z, y_t) - y_{t+1}^s)^\gamma, \quad \gamma > 1, \quad (1.9)$$

$$R(z, y_t) = z^{1-\alpha} y_t^\alpha, \quad 0 \geq \alpha \geq 1, \quad (1.10)$$

$$R(z, y_t) = z^{1+\alpha} \setminus y_t^\alpha, \quad 0 \geq \alpha \geq 1. \quad (1.11)$$

Dutta, Foster, and Mishra (2011) argue that the threat of poverty depends not only on the poverty line, but also on the current living standard that can exacerbate or mitigate against the welfare loss; they propose therefore an index of vulnerability based on an individual reference line  $R(z, y_t)$  rather than a general poverty line  $z$ , as all the previous studies have done, that depends also the current income or consumption level,  $y_t$ . Moreover, their measure is flexible enough to catch two opposite effects of the current living standard on the individual

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vulnerability, positive or negative. The index 1.10 considers a reference line  $R(z, y_t)$  that reflects the idea of worse consequences in term of vulnerability for those with higher current living standard, while 1.11, on the contrary, says that low current income exacerbates the potential drops in welfare.

In this analysis, the index proposed by Kamanou and Morduch (2002) is not considered, even if it is an *ex ante* poverty risk measure. The reason for this is that they define vulnerability directly at the society level, as difference between the expected value of a poverty measure, the poverty head count ratio, and its current value rather than estimating a degree of poverty risk for each household or individual. Their approach therefore does not aim at identifying the vulnerable, but has the purpose to estimate poverty indexes using a non-parametric technique based on a large number of bootstrap samples.

All these indexes are rich in terms of information summarized and they focus on different and equally relevant aspects of poverty risk. The index proposed by Pritchett, Suryahadi, and Sumarto (2000) or Chaudhuri (2003) for instance summarizes upward and downward variability of income, stressing the role of fluctuations in general to forecast poverty; the FGT version instead focuses especially on the downward variability and accounts for different types of weights that can be attached to extreme poverty, highlighting implicitly that not only the number of cases in which poverty is experienced matters but also the magnitude of the shock could be relevant in predicting the poverty status. Calvo and Dercon (2005) consider instead the risk attitude important, they stress therefore the role of risk sensitivity as key element in their measure; finally Dutta, Foster, and Mishra (2011)'s measures are different from the others because they suggest that the current financial situation affects, in two opposite ways, the importance of the potential drops in income. It is not possible to distinguish *a priori* which is the best signal of poverty, since they favor different sides of the same phenomenon. Therefore I try to evaluate their effectiveness empirically and classify them according to precision criteria.

### 1.3 Data

In this paper different data sources are used to evaluate if the predictive power of the vulnerability indexes is data dependent. The choice of the three countries analysed, UK, Germany and Italy, is driven mainly by the quality of data available necessary to estimate properly vulnerability and to highlight the differences among measures. To emphasize the features of each index a sufficiently long longitudinal component is needed and information on the household disposable income has to be collected accurately. I will estimate therefore vulnerability exploiting the British Household Panel Survey (BHPS), the German Socio-



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Economic Panel Study (SOEP) and the Survey on Household Income and Wealth (SHIW) for Italy, datasets that meet all the requirements.

The BHPS follows a representative sample of British individuals over the period 1991-2005; it was designed as an annual survey of each adult member for a nationally representative sample of about 5000 households, making a total of approximately 10000 individual interviews. The same individuals are re-interviewed in successive waves and, in case of split-off from the original household, all adults of the new household are also interviewed, preserving the representativeness of the British population. Additional sub-samples were added in 1997 and 1999, respectively Scotland-Wales and Northern Ireland. The aims of the extensions were to increase the relative small Scottish and Welsh samples size and to cover Northern Ireland properly, for a UK analysis rather than England only.<sup>2</sup> It must be kept in mind that in this analysis sample weights are not used, even if that is the conventional way to mitigate against potential attrition biases and new sub-samples effects. This is because the longitudinal weights supplied in the BHPS refer to a rather specific sample. The results therefore can be sensitive to the characteristics of the data used, especially to information on the net annual equivalized households income,<sup>3</sup> provided for those households in which all eligible adults gave a full interview.

The final sample is composed by 5735 households,<sup>4</sup> whose characteristics are summarized in Table 1. Missing information on education or region was retrieved from the previous waves. For Pritchett, Suryahadi, and Sumarto (2000)'s and Chaudhuri (2003)'s approach, information on the age of the household head, the percentage of household members respectively with O-level of education or lower, A-level or equivalent and with a degree or higher education, is exploited as well as the percentage of children and earners.

In order to understand if the rank of vulnerability measures, estimated using the BHPS, is stable and reliable, another database is used, the SOEP, that is very similar to the British one. The German survey was started in 1984 as a longitudinal survey of private households and individuals in the Federal Republic of Germany, then it was extended to the territory of the German Democratic Republic in June 1990. The analysis is restricted to the period 1991-2005 in order to have a representation of the whole German residential population. In the SOEP there are several sub-samples for households whose head does not belong to the main foreigner groups, for households with a Turkish, Greek, Yugoslavian, Spanish or Italian

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<sup>2</sup>For a more detailed description of the data see <http://www.iser.essex.ac.uk/bhps>.

<sup>3</sup>The equivalence scales used are the square root of the household size, as well as the Oxford scale and the OECD-modified scale, and all values have been expressed in real terms (deflated to January 1998 prices).

<sup>4</sup>I selected those households that were present in the panel for at least three periods, with observations in the years 2004 and 2005, since I compare the different vulnerability measures computed for the year 2004 with the poverty status in 2005.

**Table 1.1.** Sample Characteristics

	UK - BHPS (1991-2005)		Germany - SOEP (1991-2005)		Italy - SHIW (1989-2004)	
	Obs	%	Obs	%	Obs	%
<b>Household Head's age:</b>						
≤ 34	827	14.42	1290	13.44	66	2.62
35-44	1184	20.65	2073	21.60	373	14.81
≥ 45	3724	64.93	6234	64.96	2080	82.57
<b>Education:</b>	Mean	S. D.	Mean	S. D.	Mean	S. D.
% O-level or lower in HH	0.35	0.41	0.20	0.33	0.70	0.36
% A-level or equivalent in HH	0.21	0.31	0.52	0.42	0.22	0.30
% Degree or higher in HH	0.12	0.27	0.28	0.39	0.08	0.21
% Earners in HH	0.44	0.40	0.77	0.34	0.32	0.32
% Children in HH	0.13	0.22	0.05	0.14	0.09	0.17

household head, and for immigrants which started in 1994-1995. In 1998 and 2000 also new samples as refreshments were added from the population of private households in Germany.<sup>5</sup> As in the BHPS case, information on the household disposable income,<sup>6</sup> the education level and the employment status of each member is used; the final sample size is 9597.

I use also the SHIW, that gathers information for a representative sample of the Italian population on the households disposable income<sup>7</sup> and its sources, as well as the characteristics of the individuals and their occupational status. Even if it is possible to find the same data in the SHIW, the questionnaire is slightly different from the BHPS and the SOEP because conducted every two years instead of yearly.<sup>8</sup> The time period considered for the analysis is 1989-2004.<sup>9</sup> For the SHIW, the final sample size is 2519 households, imposing the same restrictions for the sample selection in the two previous cases; in Table 1.1 I describe also the Italian sample.<sup>10</sup> As for the BHPS and the SOEP, sample weights are not used.

For each database I will compute the different vulnerability measures that aim to anticipate who will be poor in the last period of time observed, that will be respectively 2004 for the SHIW and 2005 for the BHPS and SOEP. More precisely the estimated indexes are those proposed by Pritchett, Suryahadi, and Sumarto (2000), Chaudhuri (2003), Hoddinott

<sup>5</sup>For a more detailed description of the data see <http://www.diw.de/en/soep>.

<sup>6</sup>Also in this case three different equivalence scales are used, to take into account different degree of *equivalence elasticity*, i.e. different economies of scale within the household. Real income is deflated to 2005 prices.

<sup>7</sup>Real equivalized net income is deflated to 1991 prices.

<sup>8</sup>The data are collected every two years from 1987, with an exception for the year 1998 when information was gathered three years after 1995.

<sup>9</sup>Even if the Bank of Italy provides data from 1977, the longitudinal component starts only from 1987, but I restrict the time period analyzed to 1989-2004 because, as already pointed out in literature (Biagi, Giraldo, and Rettore, 2009), two few households remain in the panel from 1987 to 1989.

<sup>10</sup>See <http://www.bancaditalia.it/statistiche/indcamp/bilfait> for a more detailed description of the data.

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and Quisumbing (2003), Calvo and Dercon (2005) and Dutta, Foster, and Mishra (2011).<sup>11</sup>

## 1.4 Empirical strategy

Focusing on Chaudhuri (2003) and Pritchett, Suryahadi, and Sumarto (2000)'s approach, if a panel dataset is available, an income generating function can be defined as follows

$$y_{h,t} = y(X_h, \beta_t \alpha_h, e_{h,t}) \quad (1.12)$$

$$v_{h,t} = E[p_{\alpha,h,t+1}(y_{h,t+1}) | F(y_{h,t+1} | X_h, \beta_t \alpha_h, e_{h,t})], \quad (1.13)$$

where  $X_h$  represents the observable household characteristics,  $\beta_h$  is a vector of parameters describing the state of the economy at time  $t$ ,  $\alpha_h$  is an unobserved time-invariant household-level effect and  $e_{h,t}$  represents any idiosyncratic factors (shocks) that determines the variability of household income. This function will allow us to predict not only the income level at  $t + 1$ , given the information up to time  $t$ , but also its variability in the period considered, using the residuals of the model specified.

According to this first method of assessing vulnerability as stated in expression 1.13, it is possible to estimate the *conditional probability that the household income falls below the poverty line in the next period of time*. Differently from Chaudhuri (2003), income is used as measure of welfare, rather than consumption, simplifying the econometric issues related to *predetermined*, rather than *strictly exogenous* variables<sup>12</sup> and the parameters are estimated using a fixed-effect model, where education, demographics, geographical location and time dummies are the explanatories.

The econometric strategy is slightly different if the data considered are cross-sectional: it is not possible to observe a series of shocks for each household, but the heteroskedasticity in the data can be exploited to describe how the variability in income changes according to some characteristics. This is the strategy used by Chaudhuri (2003) who estimates the parameters of the specified model through a three-step feasible generalized least squares (FGLS) procedure, suggested by Amemiya (1977)

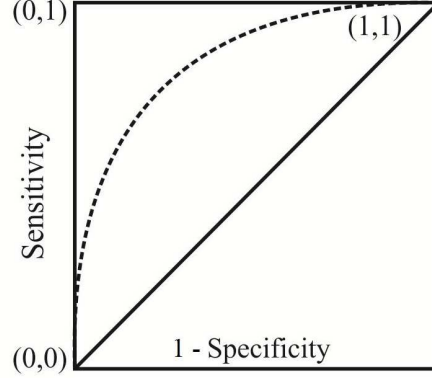
It must be noticed that using cross-sectional data forces to assume that households with

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<sup>11</sup>I will use the following notation: PC = Pritchett, Suryahadi and Sumarto (2000) and Chaudhuri (2003); C = Chaudhuri (2003); FGT = Foster, Greer and Thorbecke; CD=Calvo and Dercon (2005); DFM = Dutta, Foster and Mishra (2010).

<sup>12</sup>In his consumption generating function, Chaudhuri (2003) assumes that the elements of  $X_{h,t}$  are contemporaneously uncorrelated with  $e_{h,t}$  but allows for potential correlation between  $X_{h,t}$  and lagged consumption shocks. If this is the case, the standard within-estimator cannot be used, that is the reason why Chaudhuri (2003) uses first differences of consumption and instruments the changes in the predetermined variables using lagged changes and levels of the same variables. In this case, if income is used rather than consumption, the correlation between  $X_{h,t}$  and lagged shocks should not be an issue.

**Figure 1.1.** The ROC curve



similar characteristics are subjected to the same variability in income, while panel analysis can give a more idiosyncratic idea of shocks, the more correct is the model and the longer the dataset. In both cross-sectional and panel analysis, normality is assumed to compute probabilities. As a consequence, in expression 1.14,  $\Theta(\cdot)$  denotes the cumulative density of the standard normal,  $\hat{\mu}$  and  $\hat{\sigma}$  are respectively the estimated expected equivalized disposable income at  $t + 1$  and the standard deviation;

$$\hat{v}_{h,t} = \hat{pr}(\ln y_{h,t+1} < \ln z \mid \hat{\mu}_{\ln y_{h,t+1}}, \hat{\sigma}_{\ln y_{h,t+1}}) = \Theta\left(\frac{\ln z - \hat{\mu}_{\ln y_{h,t+1}}}{\hat{\sigma}_{\ln y_{h,t+1}}}\right). \quad (1.14)$$

For the FGT version of vulnerability to poverty (Hoddinott and Quisumbing, 2003), for Calvo and Dercon (2005) and Dutta, Foster, and Mishra (2011) that do not explicitly define an income generating function as in the previous cases, I decided to use as possible income values those already experienced by the household in the past, assuming that the data are informative about all the possible idiosyncratic shocks. The probabilities attached to each income drops below the poverty line is given by  $1/d$ , where  $d$  is the number of observations for each household.

In order to understand which vulnerability measure can detect with more precision poor individuals in advance, I use Receiver Operating Characteristic curve (ROC), which can provide a summary of the degree to which vulnerability acts as a signal for poverty. This method was originally used in the field of engineering or disease diagnosis, to measure the extent to which a given signal can detect an underlying condition. This approach has been then proposed by several authors, among others Madden (2008), also to assess the degree of overlapping between dimensions in the multidimensional poverty framework.

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In this context the underlying condition is income poverty in the last year observed,  $t + 1$ , while the vulnerability indexes, computed on information up to time  $t$ , are the *symptoms* of poverty; by analyzing the ROC curves of each vulnerability measure is therefore possible to understand which index is the most reliable signal of poverty.

To draw the ROC curve, I first define *poor* those households with equivalized disposable income in the last period observed lower than the traditional relative poverty threshold (60% of the median equivalized income) and *non-poor* otherwise. Given the two groups, it is possible to understand, for each index, to what extent the distinction between *vulnerable* and *non-vulnerable* households produces the *same* partition of the poverty status. For each vulnerability threshold, those individuals that are poor in income and vulnerable are called true positive (TP), those who are classified as non-poor and non-vulnerable are called true negative (TN). If the vulnerability threshold identifies as vulnerable someone who is not poor according to income, he or she will be a false positive (FP), while false negative (FN) is someone poor in income but non-vulnerable. The ROC curve exploits this classification to plot, on the vertical axis, the sensitivity or TP rate,  $TP/(TP+FN)$ , against 1-the specificity or TN rate,  $1-TN/(FP+TN)$ , on the horizontal axis, for all possible values of the vulnerability threshold. The more correlated are vulnerability and poverty, the higher will be the sensitivity and the specificity, the more vulnerability acts as a signal of poverty and, in graphical terms (Fig. 1.1), the nearer will be the curve to the point (0,1). For a more intuitive summary of the extent to which vulnerability is correlated with income poverty, in the sense of identifying the poor, the area under the ROC curve is reported: the higher is this area the better the signaling.

Even if the area under the ROC curve is a measure of association specifically designed to deal with dichotomous variables, to assess the signaling power, other two alternative criteria are used: the Pearson and the Spearman correlation coefficients.

While the ROC curve is appropriate for binary variables, the correlation coefficients reflect the correlation between individuals across the complete distribution of vulnerability and income. Especially, the Pearson coefficient assumes a linear relationship between two variables and estimates their linear dependence, while the Spearman correlation coefficient is a non-parametric measure of statistical dependence and evaluates how well the relation between two variables can be described using a monotonic function. The latter is different from the former coefficient, because less sensitive to strong outliers that are in the tails of both samples.

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## 1.5 Results

Before looking at the vulnerability measures, I ensured that the samples are representative for the poverty phenomenon in the countries analyzed. The poverty headcount ratio is computed for the UK and Germany in the year 2005 and for Italy in 2004 and then compared with the official data. According to Brewer, Goodman, Shaw, and Sibieta (2006), I find that about 16% of the households are poor in 2005 in the United Kingdom if the relative poverty line is set to the 60% of the median equivalized disposable income, for Germany the percentage of poor households in 2005 is about 12% and for Italy in 2004 the 19% of households is poor as the official Eurostat statistics report.<sup>13</sup> Moreover, since vulnerables include also the *permanently* poor, that are those who stay poor over long periods of time, it is useful also to have an idea of the permanent poverty phenomenon, in this case those households that are poor in both years considered. In Italy among those that are poor in 2004, the 60% of them were poor also in 2002, while in the UK this percentage is 63%, in Germany the persistence of poverty is the highest compared to the other two countries, about 73% of households poor in 2004 remain in the same condition the next year.

In Table 1.2 a summary of the discussed vulnerability indexes is reported: the mean value, the standard deviation, the average vulnerability for the two categories of households, poor or not, and also the ratio between the mean value of the these two groups.

Starting from the UK and Germany and focusing on the estimated indexes of vulnerability, it is possible to notice that those households poor in 2005 are, on average, more vulnerable than those non poor and this for each measure. The ratio of vulnerability between the two categories is different among indexes and depends also on their functional form. For instance in the UK, according to Pritchett, Suryahadi, and Sumarto (2000)'s index, those households poor in 2005 are twice more vulnerable, in terms of probability, than those who turned out to be non poor, while for the index proposed by Dutta, Foster, and Mishra (2011), when the low current living standard exacerbates the potential drops in welfare, this ratio is seven times higher.

In order to assess which index is the best signal of poverty, I focus on the area under the ROC curve reported in Table 1.3 as precision criterion.

Comparing the UK and Germany, it is possible to distinguish in both countries two groups of measures, those with an area larger than 0.8, that can be labelled *high-performers*, and those with lower values (*low-performers*). According to the ROC area point estimates, the indexes that belong to the high-performers group, in both countries, are the FGT version of vulnerability, regardless of the alpha value, the Calvo and Dercon's index that accounts

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<sup>13</sup>As in Brandolini, Magri, and Smeeding (2010), if the 50% of the median equivalized disposable income poverty threshold is used, I find for Italy that about the 12% of households is poor.

**Table 1.2.** Vulnerability to poverty - UK and Germany

UK - BHPS (1991-2004)				
Author(s)	PC	C	FGT $\alpha = 1$	FGT $\alpha = 2$
Mean	0.134	0.125	0.047	0.024
Std Dev.	0.205	0.176	0.095	0.066
Poor (1)	0.230	0.260	0.142	0.072
Non-poor (2)	0.115	0.098	0.029	0.015
(1)/(2)	2.000	2.653	4.896	4.800
Author(s)	CD rel.	CD abs.	DFM1	DFM2
Mean	0.174	0.042	$8.8 \cdot 10^5$	$4.6 \cdot 10^8$
Std Dev.	0.105	0.087	$2.6 \cdot 10^6$	$7.3 \cdot 10^9$
Poor (1)	0.245	0.125	$10.5 \cdot 10^5$	$20.5 \cdot 10^8$
Non-poor (2)	0.160	0.025	$8.5 \cdot 10^5$	$1.44 \cdot 10^8$
(1)/(2)	1.531	5.000	1.235	14.236

Germany - SOEP (1991-2004)				
Author(s)	PC	C	FGT $\alpha = 1$	FGT $\alpha = 2$
Mean	0.04693	0.0718	0.0243	0.0088
Std Dev.	0.1049	0.1546	0.0641	0.0300
Poor (1)	0.1100	0.1873	0.1120	0.0412
Non-poor (2)	0.0362	0.0522	0.0094	0.0033
(1)/(2)	3.0387	3.5881	11.9042	12.4848
Author(s)	CD relative	CD abs.	DFM1	DFM2
Mean	0.1722	0.0206	$2.79 \cdot 10^6$	$6.81 \cdot 10^7$
Std Dev.	0.0845	0.0554	$12.6 \cdot 10^6$	$284.4 \cdot 10^6$
Poor (1)	0.2229	0.0948	$4.41 \cdot 10^6$	$21 \cdot 10^7$
Non-poor (2)	0.1636	0.0080	$2.51 \cdot 10^6$	$4.4 \cdot 10^7$
(1)/(2)	1.3625	11.8500	1.7522	4.7623

Notes: PC = Pritchett, Suryahadi and Sumarto (2000) and Chaudhuri (2003); C = Chaudhuri (2003); FGT = Foster, Greer and Thorbecke (2008); CD=Calvo and Dercon (2005); DFM = Dutta, Foster and Mishra (2010). Square root of household size as equivalence scale.

**Table 1.3.** Vulnerability to poverty and Income poverty correlation - UK and Germany

UK - BHPS (1991-2005)				
Index	Area under the ROC (SE)	95% Conf. Interval	Pearson coefficient	Spearman coefficient
PC	0.6758 (0.0093)	0.6576-0.6940	-0.2429	-0.3724
C	0.7480 (0.0088)	0.7307-0.7652	-0.3653	-0.5433
FGT $\alpha = 1$	0.8272 (0.0072)	0.8130-0.8413	-0.3192	-0.5537
FGT $\alpha = 2$	0.8147 (0.0072)	0.8006-0.8289	-0.2284	-0.5398
CD (rel.)	0.7118 (0.0092)	0.6938-0.7298	-0.2411	-0.3066
CD (abs.)	0.8256 (0.0072)	0.8114-0.8397	-0.3063	-0.5518
DFM1	0.6809 (0.0078)	0.6656-0.6961	0.0458	-0.3172
DFM2	0.8432 (0.0072)	0.8291-0.8573	-0.0679	-0.5977

Germany - SOEP (1991-2005)				
Index	Area under the ROC (SE)	95% Conf. Interval	Pearson coefficient	Spearman coefficient
PC	0.6933 (0.0080)	0.6776-0.7089	-0.1449	-0.2723
C	0.7702 (0.0067)	0.7571-0.7832	-0.2490	-0.5255
FGT $\alpha = 1$	0.8883 (0.0050)	0.8784-0.8981	-0.2850	-0.5598
FGT $\alpha = 2$	0.8826 (0.0051)	0.8727-0.8925	-0.2234	-0.5547
CD (rel.)	0.6762 (0.0081)	0.6603-0.6920	0.0163	-0.0736
CD (abs.)	0.8878 (0.0050)	0.8779-0.8977	-0.2793	-0.5593
DFM1	0.7624 (0.0056)	0.7516-0.7733	0.0089	-0.3796
DFM2	0.8937 (0.0052)	0.8835-0.9038	-0.0169	-0.5925

Notes: PC = Pritchett, Suryahadi and Sumarto (2000) and Chaudhuri (2003); C = Chaudhuri (2003); FGT = Foster, Greer and Thorbecke (2008); CD=Calvo and Dercon (2005); DFM = Dutta, Foster and Mishra (2010). Square root of household size as equivalence scale.



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for the absolute risk sensitivity, and the second version of Dutta, Foster, and Mishra (2011)'s measure of poverty risk. In the Appendix A it is shown that these results are insensitive to the choice of the equivalence scale.

Ranking further the indexes of vulnerability is not so straightforward. Looking at the 95% confidence intervals it is possible to notice in fact that they always overlap, meaning that it is not certain that the estimated areas are statistically different among them; a possible strategy that could help in distinguishing the most precise index of vulnerability could be testing the equality among areas, in order to understand if the difference in terms of point estimates is really significant or if, on the contrary, we are dealing with measures of poverty risk equally precise.

I use therefore a non-parametric comparison of the areas under correlated ROC curves proposed by DeLong, DeLong, and Clarke-Pearson (1988) that exploits the theory on generalized  $U$ -statistics to generate an estimated covariance matrix and a test statistic with an asymptotically chi-square distribution. In Table 1.4 the tests for the following null hypothesis are reported: equality among the ROC areas of the high-performer indexes and pairwise equality between each high-performer index with that one which records the highest area (for the UK and Germany the highest value is estimated for the second version of Dutta, Foster, and Mishra (2011)'s measure of poverty risk).

Focusing on the UK, the equality in all cases can be strongly rejected, among all the areas and for each pairwise test, meaning that the estimated ROC areas are statistically different. These results lead to the conclusion that the second version of the index proposed by Dutta, Foster, and Mishra (2011) can be considered the best signal of poverty if the identification of poors is the concern. The same tests are repeated for Germany with the same results, rejection of equality in all cases.

The area under the ROC curve can be seen as a criterion of the overall signal precision: all the false positive-false negative combinations are chosen by varying the threshold that divides vulnerable and non vulnerable households. But in this context the two types of errors that can be made could have a different relevance for assessing the signal precision: identifying as non-vulnerable households that will be poor is worse than defining as vulnerable someone who will not be poor.

Both errors anyhow cannot be reduced at the same time: if few false negatives are preferred, a higher error in terms of false positive has to be tolerated and viceversa. What can be done, for taking into account the different weight attached to the two types of errors, is to choose a specific, high and fixed level of sensitivity (that means few false negative cases) and classify the measure in terms of specificity: the raking will give an idea of identification precision when we tolerate only a certain percentage of false negatives. The overall rank

**Table 1.4.** Equality tests among areas under the ROC curves - UK and Germany

UK - BHPS (1991-2005)			
Index	Area under the ROC	Std. Err.	95% Conf. Interval
FGT $\alpha = 1$	0.8272	0.0072	0.81304-0.84131
FGT $\alpha = 2$	0.8147	0.0072	0.80060-0.82888
CD (abs.)	0.8256	0.0072	0.81145-0.83972
DFM2	0.8432	0.0072	0.82910-0.85734
$H_0$ : area(FGT $\alpha = 1$ ) = area(FGT $\alpha = 2$ ) = area(CD (abs.)) = area(DFM2)			
chi2(3) = 220.79		Prob>chi2 = 0.0000 ***	
$H_0$ : area(FGT $\alpha = 1$ ) = area(DFM2)			
chi2(1) = 31.64		Prob>chi2 = 0.0000 ***	
$H_0$ : area(FGT $\alpha = 2$ ) = area(DFM2)			
chi2(1) = 89.84		Prob>chi2 = 0.0000 ***	
$H_0$ : area(CD (abs.)) = area(DFM2)			
chi2(1) = 38.11		Prob>chi2 = 0.0000 ***	

Germany - SOEP (1991-2005)			
Index	Area under the ROC	Std. Err.	95% Conf. Interval
FGT $\alpha = 1$	0.8883	0.005	0.8784-0.8981
FGT $\alpha = 2$	0.8826	0.005	0.8727-0.8925
CD (abs.)	0.8878	0.005	0.8779-0.8977
DFM2	0.8937	0.005	0.8835-0.9038
$H_0$ : area(FGT $\alpha = 1$ ) = area(FGT $\alpha = 2$ ) = area(CD (abs.)) = area(DFM2)			
chi2(3) = 201.33		Prob>chi2 = 0.0000 ***	
$H_0$ : area(FGT $\alpha = 1$ ) = area(DFM2)			
chi2(1) = 5.95		Prob>chi2 = 0.0147 **	
$H_0$ : area(FGT $\alpha = 2$ ) = area(DFM2)			
chi2(1) = 23.76		Prob>chi2 = 0.0000 ***	
$H_0$ : area(CD (abs.)) = area(DFM2)			
chi2(1) = 7.06		Prob>chi2 = 0.0079 ***	

Notes: FGT = Foster, Greer and Thorbecke (2008); CD=Calvo and Dercon (2005); DFM = Dutta, Foster and Mishra (2010). Square root of household size as equivalence scale. Confidence levels are reported with the following notation: p-value \*\*\*  $\leq 0.01$ , \*\*  $\leq 0.05$ , \*  $\leq 0.1$ .

**Table 1.5.** Specificity for given values of sensitivity (85%, 80%, 75%) - UK and Germany

UK - BHPS (1991-2005)						
Sensitivity	85%		80%		75%	
	Specificity	Rank	Specificity	Rank	Specificity	Rank
PC	37.69	6	44.92	7	51.27	8
C	48.02	5	58.40	5	65.57	5
FGT $\alpha = 1$	70.27	2	74.86	3	78.00	2
FGT $\alpha = 2$	70.14	3	73.86	4	76.83	4
CD (rel.)	34.79	7	41.35	8	54.43	7
CD (abs.)	70.27	2	75.05	2	77.94	3
DFM1	50.54	4	54.16	6	57.86	6
DFM2	72.77	1	78.21	1	82.16	1

Germany - SOEP (1991-2005)						
Sensitivity	85%		80%		75%	
	Specificity	Rank	Specificity	Rank	Specificity	Rank
PC	66.18	5	43.36	7	52.94	7
C	52.89	7	61.08	6	66.19	6
FGT $\alpha = 1$	83.70	3	86.79	3	89.32	2
FGT $\alpha = 2$	83.52	4	86.52	4	88.52	4
CD (rel.)	32.61	8	37.64	8	40.25	8
CD (abs.)	83.75	2	86.80	2	89.29	3
DFM1	63.68	6	66.97	5	69.20	5
DFM2	86.87	1	89.48	1	92.14	1

Notes: PC = Pritchett, Suryahadi and Sumarto (2000) and Chaudhuri (2003); C = Chaudhuri (2003); FGT = Foster, Greer and Thorbecke (2008); CD=Calvo and Dercon (2005); DFM = Dutta, Foster and Mishra (2010). Square root of household size as equivalence scale.

based on the area under the ROC curve may not be the same if we focus only on a specific partition of vulnerables.

Table 1.5 reports for given values of sensitivity (85%, 80%, 75%)<sup>14</sup> the corresponding specificity that allows to rank the measures: the higher the specificity, for a certain sensitivity level, the lower the fraction of false positives and the better the signal.

The results show how the second version of Dutta, Foster, and Mishra (2011)'s index of vulnerability minimizes the false positives for each sensitivity value both in the UK and Germany, this allows to say that also controlling for a specific type of error DFM2 remains the most precise.

The Italian case is only partly comparable with the other two countries and this is due

<sup>14</sup>The corresponding false negatives are respectively 15%, 20% and 25%.

**Table 1.6.** Vulnerability to poverty - Italy

Italy - SHIW (1989-2002)				
Author(s)	PC	C	FGT $\alpha = 1$	FGT $\alpha = 2$
Mean	0.0765	0.1305	0.0395	0.0193
Std Dev.	0.1169	0.2223	0.0957	0.0643
Poor (1)	0.1487	0.3626	0.1456	0.0754
Non-poor (2)	0.0602	0.0784	0.0156	0.0067
(1)/(2)	2.4701	4.6250	9.333	11.2573
Author(s)	CD rel.	CD abs.	DFM1	DFM2
Mean	0.2564	0.0346	$9.7 \cdot 10^5$	$1.1 \cdot 10^9$
Std Dev.	0.1015	0.0872	$28.4 \cdot 10^5$	$2.39 \cdot 10^{10}$
Poor (1)	0.3318	0.1285	$15.7 \cdot 10^5$	$45.1 \cdot 10^8$
Non-poor (2)	0.2395	0.0135	$8.4 \cdot 10^5$	$3.28 \cdot 10^8$
(1)/(2)	1.3854	9.5185	1.87	14.09

Notes: PC = Pritchett, Suryahadi and Sumarto (2000) and Chaudhuri (2003); C = Chaudhuri (2003); FGT = Foster, Greer and Thorbecke (2008); CD=Calvo and Dercon (2005); DFM = Dutta, Foster and Mishra (2010). Square root of household size as equivalence scale.

to the data available: the information in the SHIW for each household is different compared to the BHPS and SOEP in terms of frequency of observations and freshness. In fact while in the UK and Germany I aim to forecast the poverty status in the next year (from 2004 to 2005), in the Italian case I aim to anticipate poverty two years later (from 2002 to 2004), therefore the performance of indexes may change. For instance those based on the *current living standard* could be penalized by not so up-to-date information about income in terms of forecasting power.

The summary statistics in Table 1.6 show how, on average, even if information is not as rich as in the other two countries, those households poor in 2004 were more vulnerable in 2002 than those non poor. According to the area under the ROC it is still possible to notice two groups of measures: among the most precise already mentioned there is also vulnerability computed with cross-sectional data, but the associated area (0.82) remains anyhow lower than 0.85 estimated for all the other high-performers. In this case the estimated precision of the two FGT indexes, the CD (absolute) and DFM2 is very similar, the areas range from 0.8507 to 0.8551 meaning that frequency and freshness of information affect the precision of indexes in terms of equalizing the identification power, especially when current living standard condition plays a role in explaining poverty two periods later. Similar results are obtained also using other equivalence scales (see Appendix A).

In order to understand if, in this case, the two FGT indexes, the CD (abs.) and DFM2 are equally precise, the results of the equality test between the FGT index with  $\alpha = 1$  and DFM2, the two indexes that register the larger difference in terms of areas point estimates (area larger than 0.85) are reported, if the null hypothesis is accepted, it is possible to

**Table 1.7.** Vulnerability to poverty and Income poverty correlation - Italy

Italy - SHIW (1989-2004)								
Index	Area under the ROC (SE)	95% Conf. Interval	Pearson coeff.	Spearman coeff.	Se = 0.8		Se = 0.7	
					Sp	Rank	Sp	Rank
PC	0.6996 (0.0138)	0.6725-0.7267	-0.1530	-0.3097	45.11	7	61.01	8
C	0.8298 (0.0104)	0.8096-0.8501	-0.2510	-0.6117	69.18	5	79.87	5
FGT $\alpha = 1$	0.8551 (0.0101)	0.8353-0.8749	-0.2540	-0.5882	84.83	2	87.85	2
FGT $\alpha = 2$	0.8510 (0.0101)	0.8312-0.8708	-0.1962	-0.5842	84.05	4	87.21	4
CD (rel.)	0.7294 (0.0138)	0.7024-0.7564	-0.1394	-0.2728	40.01	8	68.16	7
CD (abs.)	0.8546 (0.0101)	0.8348-0.8745	-0.2461	-0.5878	84.78	3	87.75	3
DFM1	0.7606 (0.0105)	0.7400-0.7811	-0.0620	-0.4156	66.60	6	71.03	6
DFM2	0.8507 (0.0104)	0.8302-0.8711	-0.0336	-0.5922	85.66	1	89.30	1

Notes: PC = Pritchett et al. (2000) and Chaudhuri (2003); C = Chaudhuri (2003); FGT = Foster, Greer and Thorbecke (2008); CD=Calvo and Dercon(2005); DFM=Dutta,Foster and Mishra (2010). Square root of household size as equivalence scale. Sp=Specificity, Se=Sensitivity.

conclude that the indexes are equally precise in terms of identification of future poor. As shown in Table 1.8 the null hypothesis of equality is accepted.

As the overall test of precision based on the area under ROC does not allow to rank the measures in this case, I try to understand, for given values of sensitivity, if there is some index that performs better controlling for a certain type of error. By setting the sensitivity at 80% and 70%, i.e. tolerating respectively 20% and 30% of false negative, it is possible to rank the measures according to the specificity rate. The last columns of Table 1.7 show that even if the Dutta, Foster, and Mishra (2011)'s index in general is as precise as some other index of vulnerability, nevertheless it minimizes the false positives when controlling for specific high sensitivity rates.

## 1.6 Conclusions

According to the chosen correlation criterion, the Receiver Operating Characteristic curve, which is specifically designed for binary variables, I found that among those indexes proposed in literature to anticipate poverty risk, some are more precise than other in identifying the future poor, i.e. the FGT indexes of vulnerability independently of the  $\alpha$  value, the Calvo and Dercon (2005) version when absolute risk sensitivity is taken into account and the Dutta, Foster, and Mishra (2011)'s measure, that accounts for the role of the current living standard in mitigating the potential drop in income. These indexes, as more accurate in anticipating the poverty status, can be used as operational measures or *ex ante* information instruments for improving anti-poverty policies design. Moreover, if there is particular

**Table 1.8.** Equality tests among areas under the ROC curves - Italy

Italy - SHIW (1989-2004)			
Index	Area under the ROC	Std. Err.	95% Conf. Interval
C	0.8298	0.0104	0.80956-0.85013
FGT $\alpha = 1$	0.8551	0.0101	0.83526-0.87490
FGT $\alpha = 2$	0.8510	0.0101	0.83120-0.87085
CD (abs.)	0.8546	0.0101	0.83482-0.87446
DFM2	0.8507	0.0104	0.83024-0.87114
$H_0$ : area(DFM2) = area(FGT $\alpha = 1$ )			
chi2(1) = 1.08		Prob>chi2 = 0.2995	

Notes: C = Chaudhuri (2003); FGT = Foster, Greer and Thorbecke (2008); CD=Calvo and Dercon (2005); DFM = Dutta, Foster and Mishra (2010). Square root of household size as equivalence scale. Confidence levels are reported with the following notation: p-value \*\*\*  $\leq 0.01$ , \*\*  $\leq 0.05$ , \*  $\leq 0.1$ .

interest in limiting a certain type of identification error, i.e. avoiding too much cases in which households labelled as non-vulnerable turn out to be poor, the index proposed by Dutta, Foster, and Mishra (2011) among the high-performers behaves better than the others, even if frequency of observations and freshness of information are different.

Individual vulnerability assessments can be useful for understanding the characteristics of households that are on average more exposed to income shocks to design better risk-management and anti-poverty policies, but also moving this analysis at the aggregate level could be interesting. Aggregate indexes of vulnerability could be important not only for evaluating the economic performance and the social progress in a country as the Commission on the Measurement of Economic Performance and Social Progress (CMEPSP) suggested, but also in terms of how relevant could be this exposure to risk in case of crisis and financial downturns. We can expect that countries where a larger part of the population is vulnerable could suffer more severe negative consequences in case of aggregate shocks, leading to higher costs not only in terms of welfare drop but also in recovering from such situations. Kamanou and Morduch (2002) took a step towards this direction, proposing a version of aggregate vulnerability: they computed their measure using a statistical method to generate a possible distribution of aggregate poverty indexes. Alternatively it would be also interesting understanding how we can aggregate vulnerability, starting from the individual or household level, and relate this exposure to risk with other macroeconomic variables.

## Chapter 2

# Vulnerability: A Non-Parametric Decomposition

### 2.1 Introduction

Poverty analysis usually focuses on indexes that are sensitive to the number of people below the poverty line, the poverty gap and the distribution of income among the poor; these three poverty aspects are usually defined in literature as the three *Is* of poverty (Jenkins and Lambert, 1997). The description of the phenomenon based on these three components has been widely used because it helps in disentangling different sources of changes in poverty, allowing richer inter-temporal, inter-regional, cross-national or inter-group comparisons.

I propose to adopt the same approach to vulnerability to poverty, that is the probability, today, of being in poverty or to fall into deeper poverty in the future. Differently from the standard analysis of poverty, vulnerability is related to poverty risk with a more forward-looking perspective rather than an ex post lowness of income assessment. This concept is important because it can be considered an ex ante information source that allows the design of better protection policies to prevent households and individuals from experiencing severe welfare losses, rather than cure them when they are already poor (Chaudhuri, Jalan, and Suryahadi 2002, Zhang and Guanghua 2008, Jamal 2009).

Similarly to decomposing poverty as a function of incidence, intensity and inequality

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of income among the poor people, individual vulnerability to poverty in its Foster-Greer-Thorbecke (FGT) version, can be rewritten in terms of three potential sources of risk: the possible states of the world in which poverty is experienced (expected incidence), the expected poverty gap and a measure of the downward income volatility.

Each of these three components describes a particular aspect of poverty risk that can potentially lead to different risk-management policies. Being prone to poverty can increase because there are more possibilities that the income falls below a chosen poverty threshold, independently from the magnitude of the negative income shock. This source of risk recalls in some sense the incidence in the poverty decomposition framework, where the number of poor is substituted by the possible *contingencies* that an individual faces. Very close to intensity there is instead the expected poverty gap. If the latter increases also vulnerability is higher. The third contributing factor is downward variability of income: the higher this volatility the more unpredictable is the risk faced by the individual. The focus especially on negative shocks aims at separating out threats from the overall expectations, i.e. downward risks from uncertainty in general.

This view in terms of contributing factors that I propose meets the need, highlighted by Dercon (2001), of describing the different types of risk faced by individuals. He argues that risk is quite different in size, likelihood and frequency over time and different features correspond to different implications for the ability to cope with them as well as for policy purposes. Also Morduch (2000) says that it is important considering some of the patterns related to risk, since they have quite different impacts on the ability to cope with them for individuals, households, communities and other institutions. For instance it is possible to distinguish between *catastrophic versus non-catastrophic risks* according to the size of the shock. The former could be very unlikely with nonetheless a large impact so that it takes a long time before recovering from them. Different patterns of risk could also have different effects on the decision-making of individuals about investments in education or health.

This approach to vulnerability to poverty provides information that could be useful for policy makers who follow especially the World Development Report 2000/01's directions, where it is argued how optimal design should aim to strengthen, complement and replace existing coping strategies. It is stressed also the importance of overcoming the traditional *safety net policies*, which allow households to survive the consequences of poor outcomes in favor of welfare drops prevention. From this point of view therefore it is worthwhile examining poverty risk measures also in terms of their contributing components, to provide more accurate information about the ex ante risk faced by households.<sup>1</sup> If for instance poverty risk

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<sup>1</sup>In the process proposed by Dercon (2001) for optimal policy design, this analysis is related especially to the first step about understanding the poverty risk.



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is due mostly to volatility and the inability of smoothing consumption (i.e. large expected downward volatility), risk-insurance programs or incentives for self-protecting savings are the candidates for helping households avoiding poverty. If instead rare catastrophic events are poverty trigger (i.e. large expected intensity), adequate financial support is needed to recover faster from them. When, on the contrary, there are several poverty episodes (i.e. large expected incidence) and the phenomenon becomes *structural*, the solution cannot be only financial but also based on non-monetary strategies. In this paper I will also present two empirical applications using British and Italian data.

## 2.2 The Three Vulnerability Contributing Factors

In poverty analysis the FGT family of poverty indexes (2.1) includes the *headcount ratio*,  $H$ , if  $\alpha = 0$ , the *poverty gap ratio*,  $I$ , if  $\alpha = 1$ . When  $\alpha = 2$ , (2.1) can be expressed as a function of headcount ratio, the poverty gap ratio and the squared *coefficient of variation* of income among the poor,  $CV_p^2$ , as inequality index<sup>2</sup>

$$P_\alpha(\mathbf{y}; \mathbf{z}) = \frac{1}{N} \sum_{h=1}^Q \left[ \frac{z - y_h}{z} \right]^\alpha, \quad (2.1)$$

$$P_{\alpha=2}(\mathbf{y}; \mathbf{z}) = H [I^2 + (1 - I)^2 CV_p^2], \quad (2.2)$$

$$H = Q/N, \quad (2.3)$$

$$I = \frac{1}{Q} \sum_{h=1}^Q \left[ \frac{z - y_h}{z} \right], \quad (2.4)$$

$$CV_p^2 = \frac{1}{Q} \sum_{h=1}^Q \frac{(\mu_p - y_h)^2}{\mu_p^2}. \quad (2.5)$$

In the expressions (1)-(5),  $Q$  represents the number of households whose income  $y_h$  is below the chosen poverty line,  $z$ ,  $N$  is the dimension of the society and  $\mu_p$  is the average income of poor households. The parameter  $\alpha$  can be considered the weight attached to extreme poverty, the higher this value the greater the aversion for deep poverty.

$$V_{\alpha=2,h}(\tilde{\mathbf{y}}; \mathbf{z}) = \sum_{s=1}^{S_h} p_s \left[ \frac{z - \tilde{y}_s^h}{z} \right]^2. \quad (2.6)$$

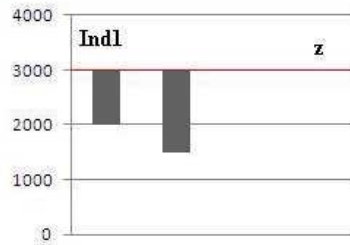
The analogous in the vulnerability framework when  $\alpha = 2$  is contained in (2.6). Differently from the poverty context, it focuses on the individual level rather than on the society. Instead of considering a vector of actual household incomes,  $\mathbf{y} = (y_1, y_2, \dots, y_N)$ , as

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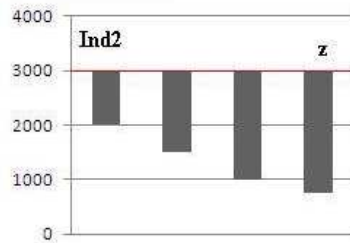
<sup>2</sup>An alternative decomposition is described in Aristondo, De la Vega, and Urrutia (2010).

the poverty index does, in the vulnerability analysis there is a vector of possible income values at  $t + 1$  for the household  $h$ ,  $\mathbf{y}_s^h = (y_1^h, y_2^h, \dots, y_N^h)$ , where  $N$  are the *possible states of the world* that the household could face.<sup>3</sup> Let us consider a new vector  $\tilde{\mathbf{y}}_s^h$ , which represents a permutation of  $\mathbf{y}_s^h$ , so that the elements are non-decreasingly ranked, i.e. for all  $\tilde{y}_s^h$ ,  $\tilde{y}_1^h \leq \tilde{y}_2^h \leq \dots \leq \tilde{y}_{S_h}^h \dots \leq \tilde{y}_N^h$ . I denote  $S_h$  the number of states in which the welfare measure is expected to fall below the poverty threshold,  $z$ , and  $p_s$  the probability that the  $s^{th}$  state occurs. The FGT index of vulnerability for the household  $h$  will be a sum of possible poverty gaps in  $t + 1$ , weighted by the their probability.

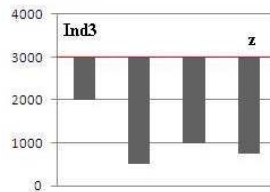
**Figure 2.1.** Example of Poverty Gaps Pattern - Individual 1



**Figure 2.2.** Example of Poverty Gaps Pattern - Individual 2



**Figure 2.3.** Example of Poverty Gaps Pattern - Individual 3



The decomposition proposed by Foster, Greer, and Thorbecke (1984), applied to vulnerability to poverty, can be performed as follows:  $EH$  is the *expected incidence*, i.e. the

<sup>3</sup>For expositional convenience, I assume that the number of possible states of the world for each household is the same, but nothing changes if  $N$  is substituted by  $N_h$ .

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**Table 2.1.** An example of Vulnerability to Poverty and the proposed decomposition

	$V_{\alpha=2,h}$	$EH_h$	$EI_h$	$ECV_h^2$
Ind1	0.3611	0.5	0.83	0.0408
Ind2	1.3681	1.0	2.25	0.5352
Ind3	1.8125	1.0	2.58	1.1488

number of states in which the household is expected to be poor; the aggregate poverty gap is substituted by  $EI$ , the *expected intensity* or expected poverty gap, and finally  $ECV^2$  replaces the inequality among the poor and describes in this context the *expected downward variability* for the household income, where  $\mu_h$  is the expected average income for the household  $h$  during poverty,

$$V_{\alpha=2,h}(\tilde{\mathbf{y}}; \mathbf{z}) = EH_h [EI_h^2 + (1 - EI_h)^2 ECV_h^2] \quad (2.7)$$

$$EH_h = \frac{S_h}{N} = p(\tilde{y} < z) \quad (2.8)$$

$$EI_h = \sum_{s=1}^{S_h} \frac{1}{S_h} \frac{(z - \tilde{y}_s^h)}{z}, \quad \frac{1}{S_h} = p(s | \tilde{y} < z) \quad (2.9)$$

$$ECV_h^2 = \sum_{s=1}^{S_h} \frac{1}{S_h} \frac{(\mu_h - \tilde{y}_s^h)^2}{\mu_h^2}, \quad \frac{1}{S_h} = p(s | \tilde{y} < z). \quad (2.10)$$

Figure 2.1, 2.2 and 2.3 provide a representation for some examples of poverty gaps pattern; they are described in Table 2.1 in terms of *expected incidence*, *expected intensity* and *expected downward variability*. Comparing Individual 1 with the other two, it is possible to observe that he or she faces a lower number of possible poverty episodes, this translates into a lower *expected incidence*, 0.5 compared to 1 (Table 2.1). If the focus is on the *expected intensity*, it is possible to notice that Individual 1 again is in a relative better position compared to the other individuals: the expected poverty episodes are less severe. About the *expected downward variability*, Individual 3 registers the highest value, meaning that the possible negative shocks are more unpredictable in his or her case than those potentially experinced by the other two individuals.

It is possible to derive also an expression for the change of the FGT vulnerability index, which will depend on the variations of its three contributing factors. To show this more explicitly, the subscripts 1 and 0 are used referring to the period in which vulnerability is measured. The change of  $V_{\alpha=2,h,t}$  between the values at times 0 and 1 can then be expressed as

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$$\Delta V_{\alpha=2,h} = EH_{h,1} [EI_{h,1}^2 + (1 - EI_{h,1})^2 ECV_{h,1}^2] - \quad (2.11)$$

$$- EH_{h,0} [EI_{h,0}^2 + (1 - EI_{h,0})^2 ECV_{h,0}^2],$$

$$\Delta V_{\alpha=2,h} = f(\Delta EH_h, \Delta EI_h, \Delta ECV_h^2) \quad (2.12)$$

where the operator  $\Delta$  denotes the variation between times 0 and 1 of  $V_{\alpha=2,h}$  and the three factors that appear in (2.12). In Appendix A I describe the Shapley decomposition of (2.11) to derive the contributions of  $\Delta EH_h$ ,  $\Delta EI_h$  and  $\Delta ECV_h^2$  to the overall change in the FGT vulnerability index,  $V_{\alpha=2,h}$ , as suggested by Chakravarty, Deutsch, and Silber (2008).

## 2.3 Data

I will estimate vulnerability to poverty and its three components using data of the British Household Panel Survey (BHPS) to show an inter-temporal comparison and the Italian Survey on Household Income and Wealth (SHIW) for an inter-regional empirical illustration.

The BHPS follows a representative sample of British households yearly; I consider especially the period 1991-2004. Additional sub-samples were added in 1997 and 1999, respectively Scotland-Wales and Northern Ireland, to increase the relative small Scottish and Welsh samples size and to cover Northern Ireland properly, for a UK analysis rather than England only.<sup>4</sup> In the empirical application I do not include those sub-samples in order to allow a more straightforward inter-temporal comparison, therefore the focus will be on England only. The disposable annual equivalized household income is used as welfare measure; this information is provided in the survey for those households in which all eligible adults gave a full interview. The final sample is composed by 1973 households,<sup>5</sup> whose characteristics are summarized in Table 1. The disposable income has been equivalised using the square root of the household size and regressed on a polynomial in age and time effects, then the residuals have been used to compute the vulnerability indexes.

As shown in Figure 2.4 this procedure allows to eliminate the age effects and to compare properly the vulnerability especially between the two periods considered.<sup>6</sup>

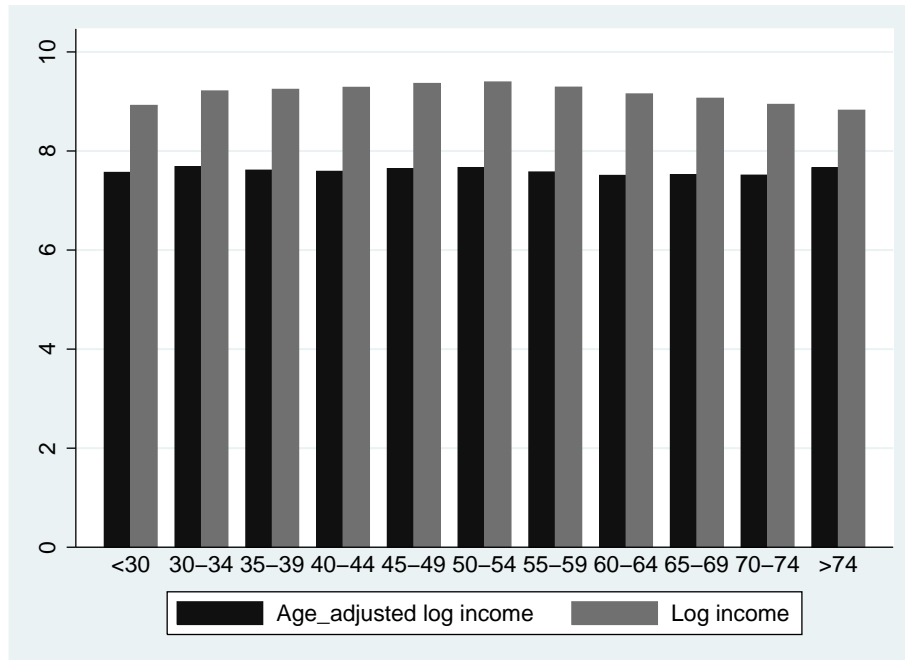
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<sup>4</sup>For a more detailed description of the data see <http://www.iser.essex.ac.uk/bhps>.

<sup>5</sup>I selected those households that were present in the panel for at least three times in the periods 1991-1997 and 1998-2004, to have sufficient observations for the vulnerability computation and the inter-temporal comparison. Moreover, I do not use sample weights provided in the BHPS because related to a rather special sample in the dataset.

<sup>6</sup>Due to the positive relation between income and age, if this is not taken into account, there could be an improvement when comparing vulnerability inter-temporally just because of an age effect. The results shown are based on a quadratic polynomial; they do not change if additional terms are included.

**Figure 2.4.** Average log income values by age group and Age-adjustment (UK)



**Table 2.2.** Sample Characteristics

	UK - BHPS (1991-2004)		Italy - SHIW (1989-2004)	
	Obs	%	Obs	%
<b>Household Head's age:</b>				
≤ 34	89	4.51	66	2.62
35-44	392	19.87	373	14.81
≥ 45	1492	75.62	2080	82.57
<b>Education:</b>	Mean	S. D.	Mean	S. D.
% O-level or lower in HH	0.36	0.42	0.70	0.36
% A-level or equivalent in HH	0.20	0.31	0.22	0.30
% Degree or higher in HH	0.12	0.27	0.08	0.21
% Earners in HH	0.45	0.40	0.32	0.32
% Children in HH	0.10	0.20	0.09	0.17

For an inter-regional illustration, the SHIW is used; it collects information for a representative sample of the Italian population about the households disposable income and consumption.<sup>7</sup> In this case in which both income and consumption are available, I use the

<sup>7</sup>See <http://www.bancaditalia.it/statistiche/indcamp/bilfait> for a more detailed description of the data.

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latter as welfare measure since it incorporates the risk-management strategies of the household.<sup>8</sup> The Italian survey is slightly different from the BHPS because it is conducted every two years;<sup>9</sup> the time period that I will consider for the analysis is 1989-2004.<sup>10</sup> For the SHIW, the final sample size is 2519 households<sup>11</sup> and it is described in Table 2.2.

For England the FGT vulnerability index will be computed in two periods of time, splitting the dataset in two parts with equal number of waves, 1991-1997 and 1998-2004, then vulnerability will be computed using data up to 1997 and compared with that of the second period, for each household. By doing this, I assume implicitly that, within the period, I observe for each household income values drawn from the same distribution. The poverty lines used are the 60% of the median values respectively in 1997 and 2004. For England I propose also the Shapley decomposition, in order to understand which factor, among the three listed (2.8, 2.9 and 2.10), contributed the most in explaining the changes in poverty risk.

The FGT version of vulnerability to poverty is computed using as possible income values those already experienced by the household in the past, assuming that the data are informative about all the possible idiosyncratic shocks.

Translating the vulnerability concept into an empirical index is not so straightforward, because it incorporates a forward looking perspective and the idea of the future distribution of households net income. Some strategies have been proposed in the literature to overcome this problem mostly based on past values. Pritchett, Suryahadi, and Sumarto (2000) for instance used panel data to compute vulnerability to poverty as the probability that the income falls below a chosen poverty threshold,  $z$ . Very similar is the poverty risk estimated in Chaudhuri (2003) where the heteroskedasticity of cross-sectional households data is exploited. Other examples which aim at measuring vulnerability to poverty adopt the same implicit assumption, i.e. the past can be used to have an idea of our welfare measure future distribution.

One can criticize this approach by saying that past and future could be very different between them and not necessary what we observe is fully informative about all the possible realizations, but there are at least two reasons that support the strategy used in the literature so far. The first argument is related to the index predictive power. It is shown in the first

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<sup>8</sup>Consumption is deflated to 1991 prices.

<sup>9</sup>The data are collected every two years from 1987, with an exception for the year 1998 when information was gathered three years after 1995.

<sup>10</sup>Even if the Bank of Italy provides data from 1977, the longitudinal component starts only from 1987, but I restrict the time period analyzed to 1989-2004 because, as already pointed out in literature (Biagi, Giraldo, and Rettore, 2009), too few households remain in the panel from 1987 to 1989.

<sup>11</sup>The sample selection in this case is different from the previous case, since I am interested only in comparing vulnerability across regions, I therefore selected those households that were present in the year 2004 and observed for at least three times.

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chapter that vulnerability to poverty, in its FGT version, based on past income values, is among the best predictors of the short-term poverty status. A second argument in favor of past information as proxy for future is given by what has been largely documented in poverty literature using transition probabilities: having experienced poverty makes you more likely to experience it again (see for instance Jenkins 2011). This can be seen as the importance of past episodes in shaping future events, especially in the poverty context. Using past values seems a plausible and informative strategy for translating vulnerability to poverty into an empirical measure to analyse the poverty risk characteristics for policies purposes.

The probabilities,  $p_{s_h}$ , are given by  $1/d$ , where  $d$  is the number of observations for each household.

Very similar to the England case is the computation of vulnerability for Italy, with the only difference that I consider only one period, because I am interested in comparing the poverty risk across regions. The poverty line is computed as the 60% of the median equivalised household consumption in 2004 at the national level (in Appendix E the different cost of living across regions is taken into consideration).

## 2.4 Empirical Illustrations

The decomposition described is now applied to England and Italy as illustrative examples respectively for an inter-temporal and inter-regional comparison of the poverty risk and its contributing factors.

This type of analysis is interesting in the British case because of the welfare reform implemented in the late 1990s. According to Gregg (2008), the objective of the government in 1996/1997 was to increase economic activity, limit welfare dependency and, at the same time, reduce poverty. To meet these goals, the government proposed a strategy based on the following measures: incentives to work, welfare payments conditional on behavioral requirements, minimum income secure for vulnerable groups and incentives for self-protecting savings among low income groups. Also ? report that the reduction of poverty amongst pensioners and households with children has formed an important part of the Labour government's agenda, especially during its second term in office (2000/01-2004/05). Poverty, measured as the number of families whose income is below the 60% of the median equivalized income, fell by 2.1 %, considering incomes after housing costs, during the Labour's first term (1996/97-2000/01), and slightly faster during the second term (2.5%).

In more details, a particularly relevant measure was the introduction of and, later increases in, the National Minimum Wage (NMW). The previous industry specific minimum wage system, set by the Wages Councils, was introduced in 1917 and abolished in 1993. In

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1998 a new NMW was proposed by the Low Pay Commission for the whole country. The minimum level was not raised much above prices until 2001, after which a sharp increase occurred until 2006. The effects of this measure can be noticed, according to Gregg (2008), looking at the growth by decile of the earnings distribution. Prior to the introduction of the NMW, the growth in earnings was slower in lowest decile and faster at the top of the distribution. By contrast, after the introduction, the most rapid growth in earnings was registered at the lowest paid part of the distribution, while the upper part has continued in a very similar fashion as before.

While the NMW focused especially on the pay of all low paid workers, independently from the family structure, the innovations in the Tax and Benefit System tried to account for families with dependent children. The government proposed an expansion of the Tax Credit system (then called Family Credit) in two directions: the Working Tax Credit and the Child Tax Credit. Before 1998 support for children came from four sources whose generosity was increased starting with the March 1998 budget. According to Gregg (2008), this reform partly reflects the Government thought that poverty was concentrated among families with younger children. The overall impact of the new Childrens Tax Credit was that families with children, independently from their marital status, received around twice as much as before while married childless couples lost an extra tax allowance.

At the same time The Working Families Tax Credit (WFTC) was announced, and became available to claimants from October 1999. Compared to its predecessor, it increased support for those in full-time or better paid part-time work (i.e. earning more than £92.90) and extended eligibility to in-work support to a large number of families. For a detailed description, see Gregg (2008), who reports that for lower earnings individuals there was also a significant reduction in income tax and National Insurance (NI) contributions.

Specifically targeted for vulnerable groups, the government introduced also the so-called *Personalized Welfare-to-work Support* that is the delivering of a support services package tailored to the individual's needs of lone parents, sick and disabled. For pensioners instead, the Labour government chose to support the poorest individuals by increasing the value of means-tested benefits. The Minimum Income Guarantee was introduced in 1999, then changed to Pension Credit in 2003. These reforms have had relatively good outcomes in terms of a lower pensioner poverty and higher replacement rates at the bottom of the income distribution (Gregg, 2008).

Given all these innovations in the British welfare system in favour of low-pay workers, families with children, vulnerable groups and pensioners, England offers an interesting illustrative example for the inter-temporal analysis of poverty risk and its factors.

The aim of this empirical application is not to test causal effects or to evaluate the



effectiveness of these policies, but to describe how the poverty risk has evolved in a period of relevant changes.<sup>12</sup>

**Table 2.3.** Vulnerability to Poverty and its contributing factors - England

BHPS (1997-2004)				
	$V_{\alpha=2,h}$ (s.d.)	$EH_h$ (s.d.)	$EI_h$ (s.d.)	$ECV_h^2$ (s.d.)
t = I	0.0216 (0.065)	0.1657 (0.293)	0.0869 (0.158)	0.0171 (0.094)
t = II	0.0169 (0.054)	0.1355 (0.255)	0.0773 (0.161)	0.0228 (0.158)

Notes:  $V_{\alpha=2,h}$  is the average vulnerability. Period I: 1991-1997. Period II: 1998-2004

Looking at Table 2.3 where the averages of the whole index and its contributing factors are reported, it is possible to observe that vulnerability to poverty has decreased between the two periods, from 0.0216 to 0.0169 on average. This difference is statistically different from zero according to the paired t-test<sup>13</sup> in Table 2.4 where it is shown the rejection of the null hypothesis, i.e. equality in poverty risk between the two periods analyzed.

After having decomposed the vulnerability index, it is possible to notice that the reduction in poverty risk is driven by the expected incidence that decreases from 0.1657 to 0.1355 and expected intensity (0.0869 in the first period 0.0773 in the second). Downward variability stays quite constant between the two periods, in fact in Table 2.4 we accept the null hypothesis of equality in the paired t-test if the confidence value is set to 5%. Looking at Table 2.5, where the contributions of each factor variation have been estimated using the Shapley decomposition, it can be noticed that downward variability explains a small part of the inter-temporal variation measured with the FGT vulnerability index. The whole index has decreased because of a reduction in the possible states in which the household experiences poverty and in the expected poverty gap but understanding which policy has especially driven this result remains to be explored. Even if the causal effect must be documented, the attempt to favor work participation or to condition financial support to active job search seems to be a possible successful strategy for reducing expected incidence through earnings.

Since some welfare reforms were particularly targeted for specific groups, it is interesting looking more in details at those. I consider therefore families with children, pensioners and low-income households.

<sup>12</sup>Piachaud, Sutherland, and Centre (2000) attempt to evaluate the potential impacts of the government initiatives on child poverty. Using micro-simulation modeling, they estimated an increase in incomes of the poorest more than those better-off and of households with children more than others. They also simulated a decrease in the proportion of children in poverty (living in households with equalized disposable income below 50% of mean value) from 26% to 20% and a reduction in the size of the poverty gap. Moreover Gregg (2008) argues that there has been a decline in poverty among families with children which came about partly through increased employment and partly through the increased generosity of benefits.

<sup>13</sup>The test takes into account that the two samples are not independent.

**Table 2.4.** England - Paired t-tests

<b>Vulnerability to poverty</b>			
	<b>Obs</b>	<b>Mean</b>	<b>SD</b>
fgtI	1973	0.0216096	0.0649725
fgtII	1973	0.0169134	0.0543854
diff	1973	0.0046962	0.063896
Ho: mean(diff) = 0 Ha: mean(diff) $\neq$ 0 Pr(  T   >   t  ) = <b>0.0011</b>			
<b>Expected Incidence</b>			
	<b>Obs</b>	<b>Mean</b>	<b>SD</b>
EIncidenceI	1973	0.165653	0.2926451
EIncidenceII	1973	0.1355345	0.2552977
diff	1973	0.0301185	0.2427228
Ho: mean(diff) = 0 Ha: mean(diff) $\neq$ 0 Pr(  T   >   t  ) = <b>0.0000</b>			
<b>Expected Intensity</b>			
	<b>Obs</b>	<b>Mean</b>	<b>SD</b>
EIntensityI	1973	0.086887	0.1584452
EIntensityII	1973	0.0772852	0.1614714
diff	1973	0.0096018	0.1776297
Ho: mean(diff) = 0 Ha: mean(diff) $\neq$ 0 Pr(  T   >   t  ) = <b>0.0164</b>			
<b>Expected downward variability</b>			
	<b>Obs</b>	<b>Mean</b>	<b>SD</b>
EDownVariabilityI	1973	0.0171414	0.0939325
EDownVariabilityII	1973	0.022757	0.1581905
diff	1973	-0.0056155	0.1447741
Ho: mean(diff) = 0 Ha: mean(diff) $\neq$ 0 Pr(  T   >   t  ) = 0.0851			

**Table 2.5.** Vulnerability to Poverty decomposition - England

<b>BHPS - Contributing factors</b>			
$\Delta V_{\alpha=2,h}$ (s.d.)	$C(\Delta EH_h)$ (s.d.)	$C(\Delta EI_h)$ (s.d.)	$C(\Delta ECV_h^2)$ (s.d.)
-0.0047 (0.064)	-0.0016 (0.037)	-0.0035 (0.032)	0.00033 (0.023)

Table 2.6 reports the vulnerability index and its contributing factors in the two periods for households with at least one child. If the paired t-test are performed, it is possible to notice how the reduction is statistically significant on average for the overall vulnerability index and the expected intensity. Expected incidence and expected downward variability has not changed for the chosen level of confidence.

If the focus is on households whose head is retired, there is a statistically significant

**Table 2.6.** Vulnerability among households with children - England - Paired t-tests

	<b>Obs</b>	<b>Mean</b>	<b>SD</b>
<b>Vulnerability to poverty</b>			
fgtI	480	0.0208104	0.057556
fgtII	480	0.0135002	0.0406495
diff	480	0.0073102	0.058383
Ho: mean(diff) = 0 Ha: mean(diff) $\neq$ 0 Pr(  T   >   t  ) = <b>0.0063</b>			
<b>Expected Incidence</b>			
EIncidenceI	480	0.1634226	0.291657
EIncidenceII	480	0.1428919	0.26119
diff	480	0.0205308	0.2406946
Ho: mean(diff) = 0 Ha: mean(diff) $\neq$ 0 Pr(  T   >   t  ) = 0.0623			
<b>Expected Intensity</b>			
	<b>Obs</b>	<b>Mean</b>	<b>SD</b>
EIntensityI	480	0.0837655	0.1522128
EIntensityII	480	0.0672556	0.1303707
diff	480	0.0165099	0.1526479
Ho: mean(diff) = 0 Ha: mean(diff) $\neq$ 0 Pr(  T   >   t  ) = <b>0.0182</b>			
<b>Expected downward variability</b>			
	<b>Obs</b>	<b>Mean</b>	<b>SD</b>
EDownVariabilityI	480	0.0120265	0.0457116
EDownVariabilityII	480	0.0106434	0.0457703
diff	480	0.0013831	0.0609902
Ho: mean(diff) = 0 Ha: mean(diff) $\neq$ 0 Pr(  T   >   t  ) = 0.6195			

change in the overall index explained by a lower expected incidence and intensity (Table 2.7).

Table 2.8 reports the poverty risk indexes for those households that were in the lowest<sup>14</sup> part of the income distribution in both periods analysed. The t-tests suggest a statistically significant decrease in the overall vulnerability index, driven by the expected intensity.

I propose also a second example: the inter-regional comparison of vulnerability to poverty using Italian data. According to the Italian National Institute of Statistics (ISTAT), Italy is characterized by a strong territorial difference in poverty rates; from 1997 to 2006 in the South the incidence of poverty is about five times higher than the North. Italy therefore represents an interesting example for an inter-regional comparison to highlight how risk changes according to regions or groups of regions. In this case I consider three groups of

<sup>14</sup>I define as the lowest part of the income distribution up to the 25<sup>th</sup> percentile at the end of the two periods.

**Table 2.7.** Vulnerability among households whose head is retired - England - Paired t-tests

	<b>Obs</b>	<b>Mean</b>	<b>SD</b>
<b>Vulnerability to poverty</b>			
fgtI	715	0.0209303	0.0558268
fgtII	715	0.0148155	0.0460949
diff	715	0.0061148	0.0595544
Ho: mean(diff) = 0 Ha: mean(diff) $\neq$ 0 Pr(  T   >   t  ) = <b>0.0062</b>			
<b>Expected Incidence</b>			
EIncidenceI	715	0.195651	0.3129282
EIncidenceII	715	0.1449684	0.2550703
diff	715	0.0506827	0.2605915
Ho: mean(diff) = 0 Ha: mean(diff) $\neq$ 0 Pr(  T   >   t  ) = <b>0.0000</b>			
<b>Expected Intensity</b>			
	<b>Obs</b>	<b>Mean</b>	<b>SD</b>
EIntensityI	715	0.0949148	0.1583907
EIntensityII	715	0.0785798	0.1573525
diff	715	0.016335	0.1817915
Ho: mean(diff) = 0 Ha: mean(diff) $\neq$ 0 Pr(  T   >   t  ) = <b>0.0165</b>			
<b>Expected downward variability</b>			
	<b>Obs</b>	<b>Mean</b>	<b>SD</b>
EDownVariabilityI	715	0.0164265	0.0773791
EDownVariabilityII	715	0.0210418	0.1584906
diff	715	-0.0046153	0.1620583
Ho: mean(diff) = 0 Ha: mean(diff) $\neq$ 0 Pr(  T   >   t  ) = 0.4466			

regions: those in the North-, Centre- and South-Italy.<sup>15</sup>

As expected, Table 2.9 shows how the poverty risk in the sample is mainly concentrated in the South-regions, the index is in fact higher than in the North- and Centre-Italy. In Table 2.10, the t-tests suggest that the poverty risk between North- and Centre-Italy is not statistically different, while it does increase if we compare the South with them.

For a more detailed description of poverty risk, it is possible to look at the three contributing factors: expected incidence is on average 0.11 in the South while about 0.08 in the other Italian regions, the average expected poverty gap is about 0.0230 compared to 0.0155 and 0.0158 respectively in the North and in the Centre and finally also the downward variability is much larger in the South. See Appendix D and E for a more detailed regional breakdown. By performing the equality tests, the null hypothesis is accepted always when

<sup>15</sup>I include the islands in the South-Italy category.

**Table 2.8.** Vulnerability among low-income households - England - Paired t-tests

	<b>Obs</b>	<b>Mean</b>	<b>SD</b>
<b>Vulnerability to poverty</b>			
fgtI	251	0.0791058	0.1044282
fgtII	251	0.0596897	0.0872977
diff	251	0.0194161	0.0982142
Ho: mean(diff) = 0 Ha: mean(diff) $\neq$ 0 Pr(  T   >   t  ) = <b>0.0019</b>			
<b>Expected Incidence</b>			
EIncidenceI	251	0.5791785	0.343182
EIncidenceII	251	0.5605483	0.3265046
diff	251	0.0186302	0.3726685
Ho: mean(diff) = 0 Ha: mean(diff) $\neq$ 0 Pr(  T   >   t  ) = 0.4291			
<b>Expected Intensity</b>			
	<b>Obs</b>	<b>Mean</b>	<b>SD</b>
EIntensityI	251	0.2546807	0.1715719
EIntensityII	251	0.2028458	0.1570548
diff	251	0.0518349	0.1731244
Ho: mean(diff) = 0 Ha: mean(diff) $\neq$ 0 Pr(  T   >   t  ) = <b>0.0000</b>			
<b>Expected downward variability</b>			
	<b>Obs</b>	<b>Mean</b>	<b>SD</b>
EDownVariabilityI	251	0.0523432	0.1344603
EDownVariabilityII	251	0.0607155	0.1791544
diff	251	-0.0083723	0.1866036
Ho: mean(diff) = 0 Ha: mean(diff) $\neq$ 0 Pr(  T   >   t  ) = 0.4779			

**Table 2.9.** Vulnerability to Poverty and its contribution factors - Italy

ITALY (1989-2004)					
	<b>Obs.</b>	$V_{\alpha=2,h}$ (s.d.)	$EH_h$ (s.d.)	$EI_h$ (s.d.)	$ECV_h^2$ (s.d.)
North	1155	0.0050 (0.024)	0.0844 (0.186)	0.0155 (0.049)	0.0015 (0.013)
Centre	564	0.0048 (0.020)	0.0879 (0.189)	0.0158 (0.047)	0.0012 (0.006)
South	803	0.0076 (0.024)	0.1112 (0.218)	0.0230 (0.057)	0.0026 (0.014)

comparing North- and Centre-Italy while the South almost always registers higher statistically significant values (Table 2.11, 2.12 and 2.13).

This picture of vulnerability in Italy confirms the strong territorial component of the poverty phenomenon, characterized by a persistent large gap between poverty risk in the North-/Centre-Italy and the South. In this illustration I adopted a different relative poverty line for the three areas to account for the differences in the cost of living across regions.

**Table 2.10.** Vulnerability to Poverty - Italy - T-tests

<b>North- and Centre-</b>			
	<b>Obs</b>	<b>Mean</b>	<b>SD</b>
North	1155	0.0050149	0.0241746
Centre	564	0.0048001	0.0200352
		<b>Mean</b>	<b>SE</b>
diff		0.0002148	0.0011035
Ho: mean(diff) = 0 Ha: mean(diff) $\neq$ 0 Pr(  T  >  t  ) = 0.8457			
<b>North- and South-</b>			
	<b>Obs</b>	<b>Mean</b>	<b>SD</b>
North	1155	0.0050149	0.0241746
South	803	0.0075568	0.0238242
		<b>Mean</b>	<b>SE</b>
diff		-0.0025419	0.0011013
Ho: mean(diff) = 0 Ha: mean(diff) $\neq$ 0 Pr(  T  >  t  ) = <b>0.0211</b>			
<b>Centre- and South-</b>			
	<b>Obs</b>	<b>Mean</b>	<b>SD</b>
Centre	564	0.0048001	0.0200352
South	803	0.0075568	0.0238242
		<b>Mean</b>	<b>SE</b>
diff		-0.0027567	0.001191
Ho: mean(diff) = 0 Ha: mean(diff) $\neq$ 0 Pr(  T  >  t  ) = <b>0.0208</b>			

## 2.5 Conclusions

For a more complete description of the phenomenon, poverty is usually described in terms of the number of people below the poverty line, the poverty gap and the distribution of income among the poor, as Sen (1976) proposed.

Using the decomposition of one of the FGT poverty index ( $\alpha = 2$ ) (Foster, Greer, and Thorbecke, 1984), I suggest to express also individual vulnerability to poverty as function of three contributing factors, expected incidence, expected intensity and downward variability. This approach to poverty risk can be useful as information source for policies design, since different patterns of risk faced by individuals could lead to different risk management policies (Dercon, 2001).

**Table 2.11.** Expected Incidence - Italy - T-tests

<b>North- and Centre-</b>			
	<b>Obs</b>	<b>Mean</b>	<b>SD</b>
North	1155	0.0844362	0.1856396
Centre	564	0.0878462	0.1886659
		<b>Mean</b>	<b>SE</b>
diff		-0.00341	0.009641
Ho: mean(diff) = 0 Ha: mean(diff) $\neq$ 0 Pr(  T  >  t  ) = 0.7236			
<b>North- and South-</b>			
	<b>Obs</b>	<b>Mean</b>	<b>SD</b>
North	1155	0.0844362	0.1856396
South	803	0.1111531	0.2180158
		<b>Mean</b>	<b>SE</b>
diff		-0.0267169	0.0094355
Ho: mean(diff) = 0 Ha: mean(diff) $\neq$ 0 Pr(  T  >  t  ) = <b>0.0047</b>			
<b>Centre- and South-</b>			
	<b>Obs</b>	<b>Mean</b>	<b>SD</b>
Centre	564	0.0878462	0.1886659
South	803	0.1111531	0.2180158
		<b>Mean</b>	<b>SE</b>
diff		-0.0233069	0.0110591
Ho: mean(diff) = 0 Ha: mean(diff) $\neq$ 0 Pr(  T  >  t  ) = <b>0.0353</b>			

**Table 2.12.** Expected Intensity - Italy - T-tests

<b>North- and Centre-</b>			
	<b>Obs</b>	<b>Mean</b>	<b>SD</b>
North	1155	0.0155009	0.0490031
Centre	564	0.0157766	0.0471787
		<b>Mean</b>	<b>SE</b>
diff	-0.0002758	0.0024547	
Ho: mean(diff) = 0 Ha: mean(diff) $\neq$ 0 Pr(  T  >  t  ) = 0.9106			
<b>North- and South-</b>			
	<b>Obs</b>	<b>Mean</b>	<b>SD</b>
North	1155	0.0155009	0.0490031
South	803	0.0229566	0.0566516
		<b>Mean</b>	<b>SE</b>
diff		-0.0074557	0.0024649
Ho: mean(diff) = 0 Ha: mean(diff) $\neq$ 0 Pr(  T  >  t  ) = <b>0.0025</b>			
<b>Centre- and South-</b>			
	<b>Obs</b>	<b>Mean</b>	<b>SD</b>
Centre	564	0.0157766	0.0471787
South	803	0.0229566	0.0566516
		<b>Mean</b>	<b>SE</b>
diff		-0.00718	0.0028184
Ho: mean(diff) = 0 Ha: mean(diff) $\neq$ 0 Pr(  T  >  t  ) = <b>0.0110</b>			



**Table 2.13.** Expected Downward Variability - Italy - T-tests

<b>North- and Centre-</b>			
	<b>Obs</b>	<b>Mean</b>	<b>SD</b>
North	1155	0.0014795	0.0137492
Centre	564	0.0011448	0.0063794
		<b>Mean</b>	<b>SE</b>
diff		0.0003347	0.0004856
Ho: mean(diff) = 0 Ha: mean(diff) $\neq$ 0 Pr(  T  >  t  ) = 0.4908			
<b>North- and South-</b>			
	<b>Obs</b>	<b>Mean</b>	<b>SD</b>
North	1155	0.0014795	0.0137492
South	803	0.002615	0.0144028
		<b>Mean</b>	<b>SE</b>
diff		-0.0011355	0.0006496
Ho: mean(diff) = 0 Ha: mean(diff) $\neq$ 0 Pr(  T  >  t  ) = 0.0807			
<b>Centre- and South-</b>			
	<b>Obs</b>	<b>Mean</b>	<b>SD</b>
Centre	564	0.0011448	0.0063794
South	803	0.002615	0.0144028
		<b>Mean</b>	<b>SE</b>
diff		-0.0014702	0.0005749
Ho: mean(diff) = 0 Ha: mean(diff) $\neq$ 0 Pr(  T  >  t  ) = <b>0.0107</b>			

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## Chapter 3

# Age-Adjustments in Inequality Dynamics

### 3.1 Introduction

In the last decades since the mid-1980s, Italy has been characterised by important economic and demographic changes: financial, fiscal, pensions and labour market reforms as well as a progressive aging of the population and a low fertility rate. All these facts could have influenced the dynamic of economic inequality and understanding how they have contributed in explaining the trends is necessary for a complete analysis of the phenomenon. According to Atkinson (1971) some sources of inequality are less worrying than other, if for instance the perspective is in terms of life-time income or wealth, inequalities attributable to age should be of little concern for policymakers because they do not affect the overall life-time distribution. If this is the case, when assessing the dynamic of inequality through time, age should be taken into account.

The demographic aspect is not taken into account when inequality is based on cross-sectional information, the dynamic that appears in comparing the inequality indexes is in fact influenced by the changes in the age structure of the population, taking therefore the risk of confusing demographic with economic effects. Is the population *older or wealthier*? To answer this question Almås and Mogstad (2009) propose a new age-adjusted inequality measure, to properly eliminate the inequality due to age effects.

This paper provides age-adjusted measures of wealth inequality for Italy to integrate the existing analysis, whose Jappelli and Pistaferri (2010) represents one of the most recent

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study. The aim is to understand the role of demographic changes in explaining the wealth inequality trends.

Italy represents an interesting case among the OECD countries for the inequality analysis, since it ranks very high in terms of income disparities, second only to the US and the UK (Jappelli and Pistaferri, 2010). This increase in inequality has been recorded especially during the 1990s, apparently not justified by a widening in the wage structure (Jappelli and Pistaferri, 2010), often identified as potential source of disparities. Also wealth inequality has evolved in a similar fashion, this is particularly worrying because wealth is widely considered a measure of long-term inequality and a driver of economic disparities through generations (Almås and Mogstad, 2009).

After some basic facts and review of the literature on inequality in Italy, I presents the method used in the paper (Section 3). The data are described in Section 4. Section 5 reports the results and the comparisons of inequality measures. Section 6 concludes.

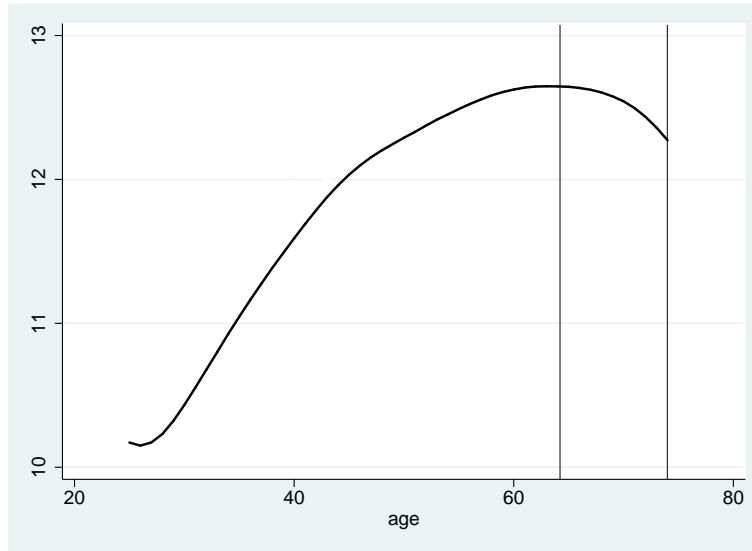
## 3.2 Stylised Facts

The life-cycle model in its simplest version, with no uncertainty, theorizes that the wealth of an individual increases up to retirement and declines afterwards. If the earning profile is upward sloping, the model predicts borrowing in the early stages of the life-course, but this is not always observed due to possible credit market imperfections. It is also possible to introduce life-time uncertainty and non-insurable health hazard, so that the model will predict the elderly to hold assets for precautionary purposes with a reduced decumulation rate. Moreover if the sole purpose of saving is to leave bequests to children, according to the model, individuals behave as if their horizons were infinite and wealth does not decline with age.

In Italy, looking at Figure 3.1 where it is represented a simple estimation of the age-wealth profile in 1995, we can see that individuals start decumulating wealth upon retirement from 65 years old. At the same time, it must be notice also that, in the period considered, 1991-2008, the share of elderly people in the Italian population has increased. Figure 3.2 reports the changes in the age structure in 1991, 2000, 2008, respectively at the beginning, in the middle and at the end of the period: the distribution moves to the right over time, highlighting the aging process of the population. More precisely there is an increase in the share where the decumulation behaviour starts, suggesting a possible connection between inequality dynamics and demographic composition.

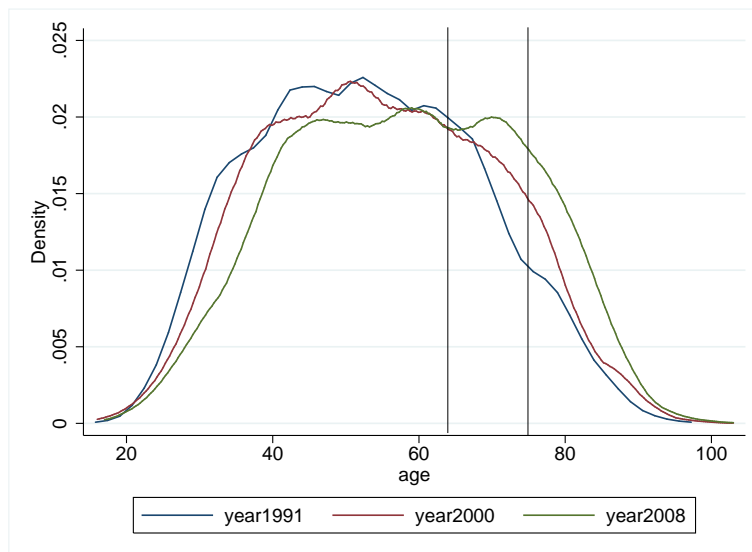
The possible influence of changes in the age structure on inequality is clearer looking at Figure 3.3 and 3.4. Figure 3.3 represents the dynamic of wealth inequality, measured in

**Figure 3.1.** Example of Age-Wealth Profile



Profile estimated using SHIW data, year 1995

**Figure 3.2.** Changes in the age structure over time in Italy



terms of Gini-index, that can be compared with the evolution in the share of households whose head is aged 65-74 (stage where the decumulating behaviour stars). Figure 3.3b proposes the same comparison but looking at income. Figure 3.3 and 3.4 suggest a negative correlation between the series, both for wealth and income, justifying the analysis in terms

Figure 3.3. Wealth Inequality (Bank of Italy) - Share 65–74 years old

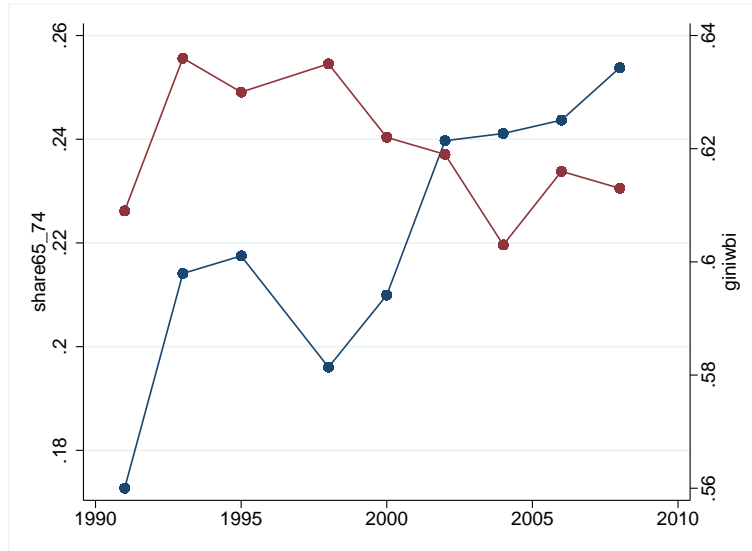
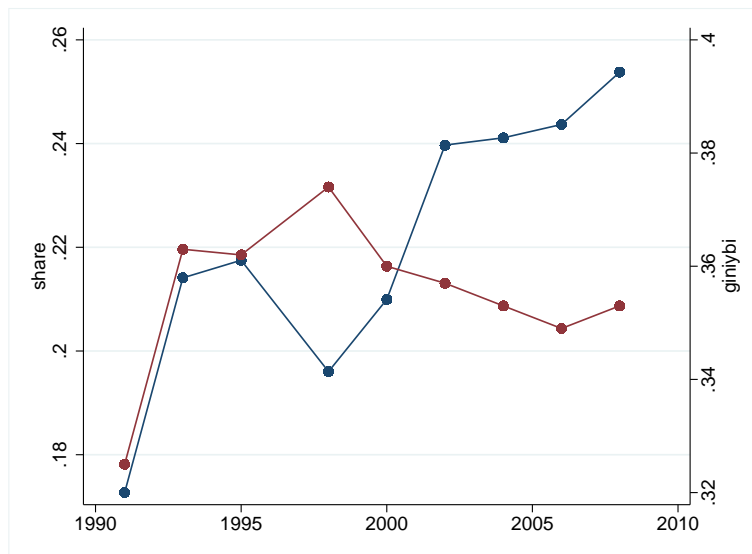


Figure 3.4. Income Inequality (Bank of Italy) - Share 65–74 years old



od age-adjustment.

### 3.3 Literature review

A recent paper of Jappelli and Pistaferri (2010) presents the dynamics of inequality in Italy using several welfare measures: earnings, income and wealth. They try to understand

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how economic changes in Italy have influenced the inequality trends.

In their analysis, the two authors describe the specific macroeconomic context stressing the fact that *the early 1990s represents a turning point away from policies favoring wage compression and reduction of inequalities towards policies associated with widening income disparities and greater wage instability* (Jappelli and Pistaferri, 2010, p. 139). Before labour market reforms in fact wage indexation granted the same absolute wage increase to all employees in response to price changes, inducing wage compressions (Manacorda, 2004) and a decline in wage inequality between the late 1970s and late 1980s (Erickson and Ichino, 1995).

After that period started the labor market reforms with the abolition of the indexation system in 1992 and the fixed-term contracts deregulation. The aim was increasing the flexibility, which is considered in Jappelli and Pistaferri (2010) the most plausible explanation for the inequality dynamics during the sample period. Franzini (2010) says that a relevant driver of inequality in 1992 was also the financial crisis with the lack of social protection policies, that affected mostly those at the bottom of the income distribution. In that period the politics attention was not particularly focused on inequalities and this led to negative consequences that could have been probably more contained nowadays (Franzini, 2010).

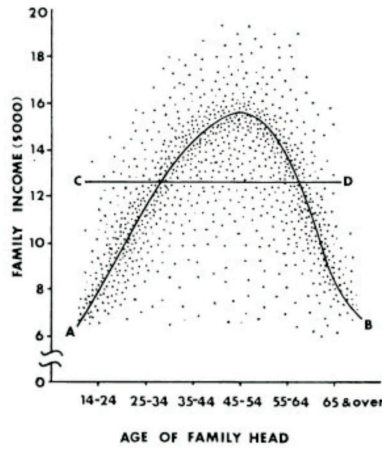
In their analysis Jappelli and Pistaferri (2010) observe that also demographic changes could have affected economic inequality. Taxes and deductions as well as economies of scale in consumption are some channels through which demographic changes could have influenced inequality and to take them into account, they use inequality measures expressed in terms of adult equivalent.

While the equivalized income (or wealth) is the standard approach to compare households with different compositions, it does not take into account explicitly the hump-shaped age-wealth profile theorized by the life-cycle model. In this paper I address this issue, by considering the recent age-adjusted measure proposed by Almås and Mogstad (2009) that eliminates inequality due to age and preserves inequality arising from other sources.

The very first attempt to isolate inequalities not due to age can be found in Paglin (1975) where the equality benchmark that underlies the standard Gini-index is criticised, more recently Almås, Cappelen, Lind, Sørensen, and Tungodden (2011) reports that most people view a strict egalitarian income distribution as unfair looking at political debate, surveys, economic experiments and contemporary theories of justice.

Considering the very first criticism, in Figure 3.1 Paglin (1975) reports an illustrative example for the distribution of income over the life-cycle, he highlights how the perfect equality represented as the CD line has some weaknesses in terms of normative implications. In his opinion the 45-degree line overspecifies the conditions of equality when used with

**Figure 3.5.** Average Age-Income profile



Source: Paglin (1975).

annual income data. Assuming no economic growth, it requires in fact not only equality in life-time incomes, but also equality among different stages of the life-course ignoring the fact that for instance families during the period of child rearing have maximum income needs. The conditions imposed by the Gini-index seem to be too restrictive, using the author's words, *it would be difficult to argue that a flat age-income profile is essential to equality.*

$$PG = \frac{\sum_j \sum_i (|w_i - w_j| - |\mu_i - \mu_j|)}{2\mu n^2} \quad (3.1)$$

It would be preferable to *generate new reference lines corresponding to explicit and reasonable definitions of equality, equity, or Pareto optimality* (Paglin, 1975, p. 599). He therefore proposes a new equality benchmark that depends on age to represent an *equal lifetime income* concept, 3.1. In Expression 3.1  $\mu_i$  and  $\mu_j$  are the average income (or wealth) in the age category respectively of individual  $i$  and  $j$ .

Paglin (1975) studied the effect of age adjustment on the distribution of income and wealth in the United States. Later Pudney (1993) proposed the same analysis for China. Other studies that have focused on adjustments for age effects are Mookherjee and Shorrocks (1982) for the United Kingdom as well as Danziger, Haveman, and Smolensky (1977), Minarik (1977), Nelson (1977), Friesen and Miller (1983), Formby, Seaks, and Smith (1989) and Bishop, Formby, and Smith (1997) for the United States. Also Wertz (1979) proposed an adjustment for age very similar to what is found in Paglin (1975), as follows



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$$WG = \frac{\sum_j \sum_i | (w_i - \mu_i) - (w_j - \mu_j) |}{2\mu n^2}. \quad (3.2)$$

While Paglin (1975) compares differences between actual values with differences between averages by age groups, ( $| w_i - w_j | - | \mu_i - \mu_j |$ ), Wertz (1979) compares differences between each actual income/wealth value with an average level in the individual's age group,  $| (w_i - \mu_i) - (w_j - \mu_j) |$ . The two expressions are not equivalent, in fact when  $| (w_i - \mu_i) - (w_j - \mu_j) | = 0$  this does not necessarily imply that  $(| w_i - w_j | - | \mu_i - \mu_j |) = 0$ , see Almås and Mogstad (2009) for an extensive discussion. According to Almås and Mogstad (2009) all these studies have used methods which fail to adjust properly for age effects net of other drivers.

The age-adjusted index used in Almås, Cappelen, Lind, Sørensen, and Tungodden (2011) instead isolates the net age effect through a multivariate regression model. They consider how much each individual's actual income or wealth deviates from what would be his or her age-related (or *fair*<sup>1</sup>) level,  $\tilde{w}$ .

$$AG = \frac{\sum_j \sum_i | (w_i - \tilde{w}_i) - (w_j - \tilde{w}_j) |}{2\mu n^2} \quad (3.3)$$

$$G = \frac{\sum_j \sum_i | (w_i - \mu) - (w_j - \mu) |}{2\mu n^2} \quad (3.4)$$

The index satisfies conditions that are similar to those underlying the classical Gini coefficient (3.4) in all respects but one: the equalizing income/wealth is not given by the mean value in the society as a whole, but depends on the age of individuals.

The *fair* level for the individual  $i$  depends on his or her age and is formally defined in 3.5, where  $\delta_i$  is the estimated age effect for the individual  $i$  who belongs to one of the  $k$  age classes. The *fair* level,  $\tilde{w}_i$ , can be viewed as the share of total income/wealth that an individual from his or her age group would hold if all the generating factors, except age, were the same for everyone in the population. To compute  $\tilde{w}_i$ , Almås, Cappelen, Lind, Sørensen, and Tungodden (2011) use the generalised classical proportionality principle as developed in Cappelen and Tungodden (2010). Empirically  $\delta_i$  is estimated using a log-linear specification, as in 3.7.

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<sup>1</sup>The underlying idea of Almås, Cappelen, Lind, Sørensen, and Tungodden (2011)'s approach is that individual outcomes are determined by responsibility and non-responsibility factors, that generate respectively the fair and unfair inequality. Since I am interested in which is the role of the aging process in explaining the dynamic of inequality and what would have been the temporal pattern without the age effect, *fair* is used only to recall the original framework of analysis and does not claim to identify all the *fair* components of inequality.

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$$\tilde{w}_i = \frac{n\mu \sum_j f(a_i)h(X_j)}{\sum_k \sum_j f(a_k)h(X_j)} = \frac{n\mu e^{\delta_i}}{\sum_k e^{\delta_k}}, \quad (3.5)$$

$$w_i = f(a_i)h(X_i), \quad (3.6)$$

$$\ln w_i = \ln f(a_i) + \ln h(X_i) = \delta_i + X_i' B. \quad (3.7)$$

There are three articles in literature that adopt this strategy to evaluate inequality. The first paper, written by Almås, Cappelen, Lind, Srensen, and Tungodden (2011), compares standard inequality measures to adjusted indexes. The authors aim at understanding if unfairness has increased in the period analysed. They use data about pre-tax and post-tax income in Norway from 1986 to 2005 and find that while the inequality according to the standard Gini-index has decreased, the inequality not explained by responsibility factors has increased. They observe that two facts could explain these divergent dynamics between inequality measures: the role of top incomes share and the female condition in the labour market. In fact, while the increase in the share of top incomes has fostered unfairness and the overall inequality, in the meanwhile the improvement in the labour market conditions of females has mitigated disparities. Therefore the decrease of the Gini-index is the result of two effects going in two different directions, the top incomes and the female labour market better conditions, with the latter more than compensating the former. Unfairness instead has increased because only the former plays a role.

The second article focuses especially on age adjustments: the authors, Almås and Mogstad (2009), compare wealth inequality according to standard measures and adjusted indexes among the following countries, Canada, Germany, Italy, Sweden, UK, US, Finland. The aim is to understand the role of the population demographic structure in determining the raking among them. Using the Luxemburg Wealth Study (LWS), they find that age-adjustments are less important than previous studies have suggested.

The last paper that uses the illustrated approach for age adjustments is Almås, Havnes, and Mogstad (2011), the authors are interested in earnings inequality trends and how they are affected by some demographic changes. By comparing the several indexes over a rather long period, 1967-2004, in Norway, they cannot find substantial impact of the population age structure, despite a divergence at the very end of the period.

Even if there seems to be little evidence about the role the age structure, the authors observe that *this conclusion may not necessarily hold true for other applications* (Almås and Mogstad, 2009, p. 20). This chapter considers wealth inequality in Italy in a dynamic perspective taking explicitly into account the aging process of the population to integrate the

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existing analysis in terms of inequality in Italy and to provide another possible application for assessing the role of demographic changes in this context.

Therefore, while Almås and Mogstad (2009) propose a comparison of age-adjusted wealth inequality across countries, in this paper I look at the evolution of wealth inequality in Italy, recently described by Jappelli and Pistaferri (2010).

### 3.4 Data

As in Jappelli and Pistaferri (2010) I use the Survey on Household Income and Wealth (SHIW), the main source of microeconomic data on Italy. The survey gathers information for a representative sample of the Italian population on the households disposable income, net wealth and its sources, as well as the characteristics of the individuals and their occupational status.<sup>2</sup> In the period considered, 1991-2008, the questionnaire is conducted every two years.<sup>3</sup>

The inequality indexes are computed for households whose head has age between 25 and 74. This sample selection is driven by potential biases in the estimation of the age effects: due to survival probabilities, rich households are over-represented in the oldest cohorts, implying a low decumulation rate. Too young household heads are also excluded because in Italy independent young working adults tend to be wealthier than average. The majority of young working adults live with their parents due to mortgage market imperfections, which prevent them from borrowing, and the characteristics of the rental market for housing (Jappelli, 1999).

Since the dependent variable is the logarithm of net wealth, households with negative or zero wealth are excluded. The sample truncation may lead to biased estimates of the age-wealth profile, however those households represent at most the 5% of the original sample.

The net wealth considered in the analysis is computed as the sum of real assets and financial assets less financial liabilities<sup>4</sup>. Net wealth information is available from 1987 to 2008, but I selected a shorter period because of comparability issues, before 1991 in fact it is estimated differently.

According to the Bank of Italy, the net wealth is highly concentrated. Official statistics on its distribution of wealth based on SHIW indicate that in 2008 the bottom 50 % of Italian households owned 10 % of total wealth, while the richest 10 % owned almost 45 %. The

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<sup>2</sup>The dataset is publicly available at the Bank of Italy's website, see <http://www.bancaditalia.it/statistiche/indcamp/bilfait/dismicro>.

<sup>3</sup>The data are collected every two years from 1991, with an exception for the year 1998 when information was gathered three years after 1995.

<sup>4</sup>See the additional material provided by the Bank of Italy for a detailed description of the estimation procedure for the various components of wealth, especially [http://www.bancaditalia.it/studiricerche/convegni/atti/ric\\_fam\\_it/Household\\\_wealth\\\_Italy.pdf](http://www.bancaditalia.it/studiricerche/convegni/atti/ric_fam_it/Household\_wealth\_Italy.pdf)

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Gini-index computed using the survey sample weights is equal to 0.613 in 2008, not very different from the value in 2006. This high concentration could be a worrying condition when the inequality of opportunity is the concern, since wealth inequality can be seen as a *long term* measure of disparities and a possible driver for the transmission of such inequality through generations.

When net wealth is considered as welfare measure, some issues must be highlighted. First of all dwellings owned by individuals represent a large fraction of wealth, meaning that price movements in the housing market can be determinant in the net wealth estimation. In Italy dwellings account for more than the 82% of households' real assets in 2008 and this proportion has changed little since 1995, so that changes in wealth could be explained mainly by the property price trends. The Bank of Italy clearly shows that the dynamic of the changes in housing wealth reflects the increasing trend of the changes in the average price per squared metre in the property market.<sup>5</sup> On the other hand, if dwellings are not considered in the net wealth, to avoid the problems related to their market value, there is the risk of depicting a very incomplete picture of households wealth, I therefore include in the analysis also dwellings.

A second issue in this context is the problem of under-reporting. It is well known in fact that usually in surveys there is an under-representation in the top of the wealth/income distribution, and this becomes relevant in terms of estimates for the age-wealth profile. Because of the under-reporting issue, it could be that the estimated age-wealth profile is flatter compared to the true one with the result of underestimating the role of age in terms of inequality.

Table 3.1 reports the characteristics of the sample used: the average wealth by year, average age and the composition in terms of education and geographical location. Looking at the fifth column, it is possible to notice an increasing average value of age reflecting the progressive aging of the population.

With respect to the other age-adjusted indexes, Almås and Mogstad (2009) try to estimate the net age effect on inequality. To do that they make an effort also to separate age effects from cohort specific effects that could arise through productivity growth (Jappelli, 1999). Following Masson (1986) they assume that the age cross-sections and the cohort profiles of wealth (in constant prices)<sup>6</sup> coincide except for a constant state of real growth,  $g$ . If this is the case, by inflating each household's wealth value in the cross-section by the factor  $(1 + g)^{age}$ , it is possible to interpret the estimated age-profile as a pure age effect. I use an annual growth rate of 2%, but different values do not affect results (see Almås and

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<sup>5</sup>See [http://www.bancaditalia.it/statistiche/stat\\_mon\\_cred\\_fin/banc\\_fin/ricfamit/2010/en\\\_suppl67\\\_10.pdf](http://www.bancaditalia.it/statistiche/stat_mon_cred_fin/banc_fin/ricfamit/2010/en\_suppl67\_10.pdf) for a more detailed description.

<sup>6</sup>Real wealth is deflated to 1991 prices.

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**Table 3.1.** Sample Characteristics

year	Obs	Real Wealth	Age	Geographical location			Education		
		Average	Average	North	Centre	South	Prim.	Sec.	High
1991	7186	289212	51	0.42	0.20	0.38	0.68	0.24	0.08
1993	6903	366854	52	0.44	0.22	0.34	0.69	0.24	0.07
1995	6970	367712	52	0.45	0.21	0.34	0.70	0.22	0.08
1998	6101	387755	51	0.42	0.22	0.36	0.64	0.27	0.09
2000	6678	400695	52	0.45	0.20	0.35	0.66	0.26	0.08
2002	6512	415322	53	0.46	0.21	0.33	0.67	0.24	0.09
2004	6412	470525	53	0.46	0.22	0.32	0.64	0.26	0.10
2006	6139	544399	53	0.49	0.20	0.31	0.62	0.28	0.10
2008	6135	533864	54	0.48	0.20	0.32	0.62	0.28	0.10

Mogstad 2009). This approach implicitly assume also that there is not intracohort mobility in individual wealth holdings, aspect criticised by Johnson (1977) and Friesen and Miller (1983) but conditioning on individual characteristics, the assumption of parallel age-wealth profiles may be more reasonable for Almås and Mogstad (2009)'s measure than for existing age-adjusted inequality measures.

As robustness analysis, I computed three versions of Almås and Mogstad (2009)'s measure by including additional controls in the regressions and using equivalised household wealth. The baseline version, AG1, is based on regressions for each year with the following control variables: age, education and gender. The AG2 index considers also geographical location (North, Centre and South-Italy) and occupation (employed, self-employed, retired, non-occupied) as controls, while AG3 uses equivalised wealth with age, education, gender, geographical location and occupation as control variables.

### 3.5 Results

In Table 3.2 I report the computed inequality indexes: the standard Gini Index, overall and between age groups, The Paglin and Wertz adjusted indexes of inequality and the new measures proposed by Almås and Mogstad (2009). The two last columns report the Gini index and Almås and Mogstad (2009)'s measure on the bases of equivalised household wealth.<sup>7</sup>

Focusing the attention on the second column, it is possible to recognize the following temporal patter of inequality: the Gini index increases at the beginning of the period, reflecting the changes of the early 1990s in terms of policies directions. After that it decreases

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<sup>7</sup>The square root of the household size is used as equivalence scale.

**Table 3.2.** Inequality Indexes

Year	Gini		Paglin	Wertz	AG1	AG2	Gini		AG3
	Total	Due to age					Equivalised	Wealth	
1991	0.586	0.304	0.436	0.586	0.595	0.602	0.604	0.610	
1993	0.600	0.311	0.475	0.602	0.617	0.624	0.617	0.628	
1995	0.589	0.273	0.449	0.591	0.600	0.600	0.608	0.600	
1998	0.594	0.285	0.446	0.605	0.605	0.604	0.621	0.619	
2000	0.596	0.326	0.466	0.599	0.624	0.628	0.618	0.631	
2002	0.584	0.280	0.464	0.584	0.614	0.608	0.599	0.609	
2004	0.572	0.287	0.454	0.570	0.587	0.578	0.592	0.585	
2006	0.590	0.298	0.439	0.597	0.617	0.605	0.618	0.609	
2008	0.570	0.326	0.420	0.570	0.611	0.598	0.593	0.594	

*Notes: Gini due to age represents the inequality between age groups. AG1 based on regressions with the following control variables: age, education and gender. AG2 based on regressions with the following control variables: age, education, gender, geographical location (North, Centre and South-Italy) and occupation (employed, self-employed, retired, non-occupied). AG3 uses equivalised wealth with age, education, gender, location and occupation as controls.*

with the exception of the year 2006, probably due to the financial crisis. If we look at third column instead, where the inequality between age-groups is reported, we recognise an upward trend from 1993, suggesting an increasing role of age in explaining inequality. This measure of inequality however is based on the distances between averages by age groups without taking into account the net effect. To understand the role of age in terms of inequality, the Almås, Cappelen, Lind, Srensen, and Tungodden (2011)'s has to be considered, since it is based on the effect of age after integrating out other wealth generating factors.

Looking at Figure 3.6 that represents the inequality dynamic of the Gini index, Adjusted Gini and Wertz index in terms of household net wealth not equivalised, it is possible to notice that adjustments for age structure do not determine a different temporal dynamic. The same result can be found also in Figure 3.7, where equivalised net wealth is considered.

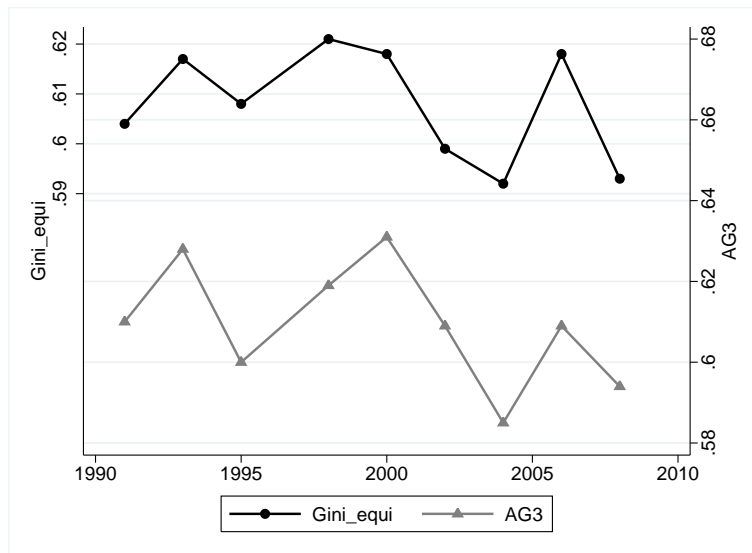
This finding is in line with what Almås and Mogstad (2009) observe by ranking the countries according to the different inequality indexes: age effects do not determine the upward and downward movements. They say that a possible interpretation is that *age adjustments are less important than previous studies have suggested, albeit this conclusion may not necessarily hold true for other applications* (Almås and Mogstad, 2009, p. 20).

Italy could represent a confirmation of the role of demographic changes, but it must be kept in mind that two main problems could affect the estimates of the age-wealth profile in this context. The first issue, already discussed, refers to the underreporting of high levels of wealth that translates into a flatter age-wealth profile. The second issue is related to one of the assumption that undelies Almås and Mogstad (2009)'s approach: there is no intra-cohort

Figure 3.6. Dynamic of wealth inequality indexes I



Figure 3.7. Dynamic of wealth inequality indexes II



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mobility of wealth. Transfers from parents to sons has been recently analysed by Alessie, Angelini, and Pasini (2011), who investigate empirically what motivates them. Comparing financial transfers characteristics in Italy, with respect to other European Countries, it can be noticed that in the sample considered the percentage of children who receive a transfer is not very high, 9.9% (the value ranges from 3.6% in Spain to 22.2% in Sweden). However, conditional on receiving, children in Italy register the fourth largest amount. This means that even if the phenomenon is less frequent than in other countries, it is more relevant in terms of amount transferred. This could affect in some extent the age-wealth profile which is estimated flatter than the case in which there are no transfers.

### 3.6 Conclusions

Inequality is often described using indexes that do not distinguish between *age* and *non-age-related* disparities, taking the risk of confounding demographic with economic effects. Almås and Mogstad (2009)'s framework can be used to analyse inequality taking into account this aspect.

The aim of this paper was to understand if the Italian inequality dynamic of wealth changes once the aging process is considered with respect to the standard description with the Gini index.

The results suggest that increasing and decreasing pattern is the same for the age-adjusted indexes used with the exception of Paglin (1975)'s measure. This suggests that in Italy for the sample-period considered the aging process does not affect increasing and decreasing overall dynamic, that is determined by non-age-related drivers.

These results are in line with those in Almås and Mogstad (2009), that observe how the Gini coefficient provides the same wealth inequality ranking among countries of the age-adjusted measures.

Even if in this case demographic changes do not affect the upward and downward movements, this may not necessarily hold true for other applications. In fact Almås, Havnes, and Mogstad (2011) find that age adjustments affect the trend in earnings inequality in Norway between 1967 and 2004, it could be interesting looking at age-adjusted earnings inequality in Italy, to understand whether that is the case.

This type of analysis than can be also generalised to disentangle *fair* and *unfair* inequalities dynamics, as in Almås, Cappelen, Lind, Srensen, and Tungodden (2011), who find that the Norwegian tax system contributes to a reduction in overall unfairness.



# Bibliography

- ALESSIE, R., V. ANGELINI, AND G. PASINI (2011): “Is it true love? Altruism versus exchange in time and money transfers,” *Netspar Discussion paper*, 02/2011-012.
- ALMÅS, I., A. W. CAPPELEN, J. T. LIND, E. . SRENSSEN, AND B. TUNGODDEN (2011): “Measuring unfair (in)equality,” *Journal of Public Economics*, 95, 488–499.
- ALMÅS, I., T. HAVNES, AND M. MOGSTAD (2011): “Baby booming inequality? Demographic change and earnings inequality in Norway, 1967–2000,” *Journal of Economic Inequality*, 9, 629–650.
- ALMÅS, I., AND M. MOGSTAD (2009): “Older or wealthier? The impact of age adjustments on the wealth inequality ranking of countries,” Working Papers 113, ECINEQ, Society for the Study of Economic Inequality.
- AMEMIYA, T. (1977): “The maximum likelihood estimator and the non-linear three stage least squares estimator in the general nonlinear simultaneous equation model,” *Econometrica*, 45, 955–968.
- ARISTONDO, O., C. L. DE LA VEGA, AND A. URRUTIA (2010): “A new multiplicative decomposition for the Foster-Greer-Thorbecke Poverty Indices,” *Bulletin of Economic Research*, 62, 259–267.
- ATKINSON, A. (1971): “The Distribution of Wealth and the Individual Life-Cycle,” *Oxford Economic Papers*, 23, 239–254.
- AZAM, M., AND K. IMAI (2009): “Vulnerability and Poverty in Bangladesh,” The University of Manchester: The School of Economics Discussion Paper Series 0905.
- BANDYOPADHYAY, S., AND F. COWELL (2007): “Modelling Vulnerability in the UK,” LSE STICERD Research Paper 89.

- 
- BAULCH, B., AND J. HODDINOTT (2000): "Economic mobility and poverty dynamics in developing countries," *Journal of Development Studies*, 36, 1–24.
- BIAGI, F., A. GIRALDO, AND E. RETTORE (2009): "Gli effetti dell'attrito sulla stima della disuguaglianza in Italia," in *Dimensioni della disuguaglianza in Italia: povertà, salute, abitazione*, ed. by B. A. S. C., and A. Schizzerotto. Il Mulino, Bologna.
- BISHOP, J., J. P. FORMBY, AND J. W. SMITH (1997): "Demographic Change and Income Inequality in the United States, 1976-1989," *Southern Economic Journal*, 64, 34–44.
- BRANDOLINI, A., S. MAGRI, AND T. M. SMEEDING (2010): "Asset-based measurement of poverty," Bank of Italy, Economic Research Department: Temi di discussione (Economic working papers) 755.
- BREWER, M., A. GOODMAN, J. SHAW, AND L. SIBIETA (2006): "Poverty and Inequality in Britain: 2006," Institute for Fiscal Studies: IFS Commentary 101.
- CALVO, C., AND S. DERCON (2005): "Measuring Individual Vulnerability," University of Oxford, Department of Economics: Economics Series Working Papers 229.
- CAPPELEN, A., AND B. TUNGODDEN (2010): "Fairness and the Proportionality Principle," Discussion paper, Department of Economics, Norwegian School of Economics and Business Administration.
- CHAKRAVARTY, S. R., J. DEUTSCH, AND J. SILBER (2008): "On the Watts Multidimensional Poverty Index and its Decomposition," *World Development*, 36, 1067–1077.
- CHAUDHURI, S. (2003): "Assessing household vulnerability to poverty: concepts, empirical methods and illustrative examples," Columbia University. Mimeo.
- CHAUDHURI, S., J. JALAN, AND A. SURYAHADI (2002): "Assessing household vulnerability to poverty from cross-sectional data: a methodology and estimates from Indonesia," Columbia University, Department of Economics: Discussion Paper 0102-52.
- DANZIGER, S., R. HAVEMAN, AND E. SMOLENSKY (1977): "The Measurement and Trend of Inequality: Comment," *American Economic Review*, 67, 505–512.
- DELONG, E., D. DELONG, AND D. CLARKE-PEARSON (1988): "Comparing the Areas Under Two or More Correlated Receiver Operating Characteristic Curves: A Nonparametric Approach," *Biometrics*, 44, 837–845.

- 
- DERCON, S. (2001): "Assessing Vulnerability to Poverty.," Jesus College, Oxford and Centre for the Study of African Economies (CSAE), Department of Economics, Oxford University. Mimeo.
- (2006): "Vulnerability: a micro perspective," QEH Working Papers QEHWPS149.
- DUTTA, I., J. FOSTER, AND A. MISHRA (2011): "On measuring Vulnerability to Poverty," *Social Choice and Welfare*, forthcoming.
- ERICKSON, C., AND A. ICHINO (1995): "Wage Differentials in Italy: Market Forces, Institutions, and Inflation," in *Differences and Changes in Wage Structures*, NBER Chapters, pp. 265–306. National Bureau of Economic Research, Inc.
- FORMBY, J. P., T. G. SEAKS, AND J. W. SMITH (1989): "On the Measurement and Trend of Inequality: A Reconsideration," *American Economic Review*, 79, 256–64.
- FOSTER, J., J. GREER, AND E. THORBECKE (1984): "A class of decomposable poverty measures," *Econometrica*, 52, 761–766.
- FRANZINI, M. (2010): *Ricchi e poveri. L'Italia e le disuguaglianze (in)accettabili*. EGEA.
- FRIESEN, P. H., AND D. MILLER (1983): "Annual Inequality and Lifetime Inequality," *The Quarterly Journal of Economics*, 98, 139–55.
- GAIHA, R., AND K. IMAI (2008): "Measuring Vulnerability and Poverty: Estimates for Rural India," World Institute for Development Economic Research (UNU-WIDER): Working Papers RP2008/40.
- GAIHA, R., K. IMAI, AND W. KANG (2011): "Vulnerability and poverty dynamics in Vietnam," *Applied Economics*, 43, 3603–3618.
- GERRY, C., AND C. LI (2010): "Consumption smoothing and vulnerability in Russia," *Applied Economics*, 42, 1995–2007.
- GREGG, P. (2008): "UK Welfare Reform 1996 to 2008 and beyond: A personalised and responsive welfare system?," The Centre for Market and Public Organisation 08/196, Department of Economics, University of Bristol, UK.
- HODDINOTT, J., AND A. R. QUISUMBING (2003): "Methods for microeconomic risk and vulnerability assessments," The World Bank: Social Protection Discussion Paper Series 0324.

- 
- IMAI, K., X. WANG, AND W. KANG (2009): "Poverty and Vulnerability in Rural China: Effects of Taxation," The University of Manchester, The School of Economics: Discussion Paper Series 0913.
- JALAN, J., AND M. RAVAILLON (1998): "Determinants of Transient and Chronic Poverty: Evidence from Rural China," World Bank Policy Research Working Paper No. 1936.
- JAMAL, H. (2009): "Assessing Vulnerability to Poverty: Evidence from Pakistan," Social Policy and Development Centre (SPDC): Research Report N.80.
- JAPPELLI, T. (1999): "The Age-Wealth Profile and the Life-Cycle Hypothesis: A Cohort Analysis with Time Series of Cross-Sections of Italian Households," *Review of Income and Wealth*, 45, 57–75.
- JAPPELLI, T., AND L. PISTAFERRI (2010): "Does Consumption Inequality Track Income Inequality in Italy?," *Review of Economic Dynamics*, 13, 133–153.
- JENKINS, S. (2011): *Changing Fortunes. Income Mobility and Poverty Dynamics in Britain*. Oxford University Press.
- JENKINS, S. P., AND P. J. LAMBERT (1997): "Three 'I's of Poverty Curves, with an Analysis of UK Poverty Trends," *Oxford Economic Papers*, 49, 317–327.
- JHA, R., T. DANG, AND K. SHARMA (2009): "Vulnerability to poverty in Fiji," *International Journal of Applied Econometrics and Quantitative Studies*, 6, 51–68.
- JOHNSON, W. (1977): "The Measurement and Trend of Inequality: Comment," *American Economic Review*, 67, 502–504.
- KAMANOU, G., AND J. MORDUCH (2002): "Measuring Vulnerability to Poverty," World Institute for Development Economic Research: Working Papers UNU-WIDER Research Paper, 2002/58.
- LIGON, E., AND L. SCHECHTER (2003): "Measuring Vulnerability," *Economic Journal*, 113, C95–C102.
- MADDEN, D. (2008): "Health and Income Poverty in Ireland, 2003-2006," University College Dublin, School Of Economics: Working Paper 200815.
- MANACORDA, M. (2004): "Can the Scala Mobile Explain the Fall and Rise of Earnings Inequality in Italy? A Semiparametric Analysis, 1977-1993," *Journal of Labor Economics*, 22, 585–614.

- 
- MASSON, A. (1986): "A Cohort Analysis of Wealth-Age Profiles Generated by a Simulation Model in France (1949-75)," *Economic Journal*, 96, 173-90.
- MINARIK, J. (1977): "The Measurement and Trend of Inequality: Comment," *American Economic Review*, 67, 513-516.
- MOOKHERJEE, D., AND A. F. SHORROCKS (1982): "A Decomposition Analysis of the Trend in UK Income Inequality," *Economic Journal*, 92, 886-902.
- MORDUCH, J. (2000): "Between the State and the Market: Can Informal Insurance Patch the Safety Net?," *World Bank Research Observer*, 14, 187-207.
- NELSON, E. (1977): "The Measurement and Trend of Inequality: Comment," *American Economic Review*, 67, 497-501.
- OSBERG, L. (1998): "Economic Insecurity," University of New South Wales, Social Policy Research Centre: SPRC Discussion Paper 88.
- (2010): "Measuring Economic Insecurity and Vulnerability as part of Economic Well-being: Concepts and Context," in *IARIW 31st General Conference*, St. Gallen, Switzerland.
- PAGLIN, M. (1975): "The Measurement and Trend of Inequality: A basic Revision," *American Economic Review*, 65, 598-609.
- PIACHAUD, D., H. SUTHERLAND, AND U. I. R. CENTRE (2000): "How Effective is the British Government's Attempt to Reduce Child Poverty?," Discussion paper.
- PRITCHETT, L., A. SURYAHADI, AND S. SUMARTO (2000): "Quantifying Vulnerability to Poverty: A proposed Measure Applied to Indonesia," The World Bank: Working Paper WPS 2437.
- PUDNEY, S. (1993): "Income and Wealth Inequality and the Life Cycle: A Non Parametric Analysis for China," *Journal of Applied Econometrics*, 8, 249-276.
- SEN, A. K. (1976): "Poverty: an ordinal approach to measurement," *Econometrica*, 44, 219-231.
- SHAPLEY, L. S. (1953): "A value for n-person games," in *Contributions to the theory of games II, Annals of mathematics studies*, ed. by H. W. Kuhn, and A. W. Tucker. Princeton University Press, Princeton.

- 
- SHORROCKS, A. F. (1999): "Decomposition procedures for distributional analysis. A unified framework based on the Shapley value," University of Essex: mimeo.
- SKOUFIAS, E., AND A. R. QUISUMBING (2003): "Consumption Insurance and Vulnerability to Poverty: A Synthesis of the Evidence from Bangladesh, Ethiopia, Mali, Mexico and Russia," International Food Policy Research Institute: FCND Discussion paper 155.
- STIGLITZ, J., A. SEN, AND J. FITOUSSI (2009): "Report by the Commission on the Measurement of Economic Performance and Social Progress," .
- THE WORLD BANK (2011): "Measuring Vulnerability," Available at: <http://web.worldbank.org/WBSITE/EXTERNAL/TOPICS/EXTPOVERTY/EXTPA/0,,contentMDK:20238993~menuPK:492141~pagePK:148956~piPK:216618~theSitePK:430367,00.html>[2011, February 10].
- WERTZ, K. (1979): "The Measurement of Inequality: Comment," *American Economic Review*, 69, 670–672.
- ZHANG, Y., AND W. GUANGHUA (2008): "Can We Predict Vulnerability to Poverty?," WIDER Research Paper N. 2008/82.

# Appendices





## A ROC curve analysis

Robustness check: different equivalence scales

**Table A.1.** Vulnerability to poverty and Income poverty correlation

UK - BHPS (1991-2005)				
Index	OECD equivalence scale		OECD-modified scale	
	Area under the ROC (SE)	95% Conf. Interval	Area under the ROC (SE)	95% Conf. Interval
PC	0.675 (0.0094)	0.6566-0.6935	0.672 (0.0091)	0.6546-0.6902
C	0.743 (0.0089)	0.7255-0.7604	0.749 (0.0088)	0.7323-0.7666
FGT $\alpha = 1$	0.823 (0.0074)	0.8088-0.8378	0.838 (0.0068)	0.8250-0.8518
FGT $\alpha = 2$	0.811 (0.0074)	0.7963-0.8253	0.826 (0.0068)	0.8126-0.8394
CD (rel.)	0.706 (0.0093)	0.6874-0.7240	0.706 (0.0091)	0.6877-0.7236
CD (abs.)	0.822 (0.0074)	0.8072-0.8363	0.837 (0.0068)	0.8235-0.8502
DFM1	0.679 (0.0079)	0.6639-0.6947	0.689 (0.0076)	0.6740-0.7040
DFM2	0.837 (0.0075)	0.8221-0.8513	0.851 (0.0070)	0.8371-0.8643
Germany - SOEP (1991-2005)				
Index	OECD equivalence scale		OECD-modified scale	
	Area under the ROC (SE)	95% Conf. Interval	Area under the ROC (SE)	95% Conf. Interval
PC	0.705 (0.0078)	0.6897-0.7204	0.690 (0.0081)	0.6744-0.7061
C	0.768 (0.0067)	0.7551-0.7815	0.767 (0.0067)	0.7535-0.7798
FGT $\alpha = 1$	0.874 (0.0056)	0.8626-0.8846	0.881 (0.0054)	0.8705-0.8916
FGT $\alpha = 2$	0.868 (0.0056)	0.8574-0.8794	0.875 (0.0054)	0.8647-0.8858
CD (rel.)	0.646 (0.0084)	0.6295-0.6626	0.664 (0.0083)	0.6482-0.6807
CD (abs.)	0.873 (0.0056)	0.8622-0.8841	0.880 (0.0054)	0.8700-0.8911
DFM1	0.739 (0.0059)	0.7273-0.7505	0.752 (0.0059)	0.7402-0.7632
DFM2	0.884 (0.0056)	0.8734-0.8953	0.883 (0.0056)	0.8721-0.8940
Italy - SHIW (1989-2004)				
Index	OECD equivalence scale		OECD-modified scale	
	Area under the ROC (SE)	95% Conf. Interval	Area under the ROC (SE)	95% Conf. Interval
PC	0.681 (0.0142)	0.6534-0.7089	0.670 (0.0136)	0.6631-0.7166
C	0.824 (0.0108)	0.8025-0.8447	0.827 (0.0106)	0.8060-0.8474
FGT $\alpha = 1$	0.853 (0.0104)	0.8329-0.8736	0.857 (0.0100)	0.8379-0.8772
FGT $\alpha = 2$	0.849 (0.0104)	0.8291-0.8697	0.854 (0.0100)	0.8344-0.8738
CD (rel.)	0.729 (0.0139)	0.7019-0.7562	0.728 (0.0139)	0.7008-0.7551
CD (abs.)	0.853 (0.0104)	0.8324-0.8731	0.857 (0.0100)	0.8375-0.8768
DFM1	0.756 (0.0108)	0.7382-0.7807	0.764 (0.0107)	0.7433-0.7852
DFM2	0.846 (0.0108)	0.8244-0.8668	0.851 (0.0105)	0.8304-0.8715

Notes: PC = Pritchett, Suryahadi and Sumarto (2000) and Chaudhuri (2003); C = Chaudhuri (2003); FGT = Foster and Thorbecke (2008); CD=Calvo and Dercon (2005); DFM = Dutta, Foster and Mishra (2010).

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## B The Shapley decomposition

The Shapley decomposition technique (Shapley, 1953) was for the first time applied in game theory, then Shorrocks (1999) used this method in distributional analysis to decompose also income inequality indexes. In this paper I propose, as in Chakravarty, Deutsch, and Silber (2008), the Shapley decomposition to understand the factors contributions to the change over time in the value of the indicator  $V_{\alpha=2,h,t}$ . I denote  $\Delta V = I$  the change of  $V_{\alpha=2,h,t}$  and  $\Delta EH$ ,  $\Delta EI$ , and  $\Delta ECV^2$  represent respectively the variations over time of the three determinants  $EH$ ,  $EI$ , and  $ECV^2$ . Since the change in the vulnerability index,  $I$ , can be expressed as a function of three variables  $\Delta EH = a$ ,  $\Delta EI = b$ , and  $\Delta ECV^2 = c$ , the contribution  $C(a)$  of  $a$  in explaining  $I$ , can be expressed by the following

$$C(a) = \frac{2}{6} [I(a, b, c) - I(b, c)] + \frac{1}{6} [I(a, c) - I(c)] + \frac{1}{6} [I(a, b) - I(b)] + \frac{2}{6} [I(a)], \quad (\text{B.1})$$

where the order in which  $a, b$  and  $c$  are eliminated is taken into account. Similarly it is possible to determine the marginal contribution  $C(b)$  of  $b$  and  $C(c)$  of  $c$  and then find out that

$$I(a, b, c) = C(a) + C(b) + C(c). \quad (\text{B.2})$$

In order to clarify that in case analysed  $a$ ,  $b$  and  $c$  represent changes in the contributing factors, I rewrite the marginal contribution of  $a$  as follows

$$\begin{aligned} C(\Delta EH) = & \frac{2}{6} [\Delta V(\Delta EH \neq 0, \Delta EI \neq 0, \Delta ECV^2 \neq 0) - \Delta V(\Delta EH = 0, \Delta EI \neq 0, \Delta ECV^2 \neq 0)] + \\ & + \frac{1}{6} [\Delta V(\Delta EH \neq 0, \Delta EI = 0, \Delta ECV^2 \neq 0) - \Delta V(\Delta EH = 0, \Delta EI = 0, \Delta ECV^2 \neq 0)] + \\ & + \frac{1}{6} [\Delta V(\Delta EH \neq 0, \Delta EI \neq 0, \Delta ECV^2 = 0) - \Delta V(\Delta EH = 0, \Delta EI \neq 0, \Delta ECV^2 = 0)] + \\ & + \frac{2}{6} [\Delta V(\Delta EH \neq 0, \Delta EI = 0, \Delta ECV^2 = 0) - \Delta V(\Delta EH = 0, \Delta EI = 0, \Delta ECV^2 = 0)], \end{aligned} \quad (\text{B.3})$$

where  $\Delta EH = 0$  means that, when the change in  $V_{\alpha=2,h,t}$  is computed, I assume that the expected incidence did not change between time 0 and 1, whereas  $\Delta EH \neq 0$  will mean that the expected incidence changed. Similar interpretations hold for  $\Delta EI$  and  $\Delta ECV^2$ .

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## C The non-parametric decomposition of Vulnerability to Poverty

$$\begin{aligned}
V_{\alpha=2,h}(\tilde{\mathbf{Y}}; \mathbf{z}) &= \sum_{s=1}^{S_h} p_s \left[ \frac{z - \tilde{y}_s^h}{z} \right]^2, \quad p_s = p(\tilde{y} < z)p(s | \tilde{y} < z) \\
&= p(\tilde{y} < z) \sum_s^{S_h} p(s | \tilde{y} < z) \left[ \frac{z - \mu + \mu - \tilde{y}_s^h}{z} \right]^2 \\
&= p(\tilde{y} < z) \sum_{s=1}^{S_h} p(s | \tilde{y} < z) \left[ \frac{(z - \mu)^2}{z^2} + \frac{2(z - \mu)(\mu - \tilde{y}_s^h)}{z^2} + \frac{(\mu - \tilde{y}_s^h)^2}{z^2} \right] \\
&= p(\tilde{y} < z) \left[ \sum_{s=1}^{S_h} p(s | \tilde{y} < z) \left( \frac{(z - \mu)^2}{z^2} \right) + \sum_{s=1}^{S_h} p(s | \tilde{y} < z) \frac{(\tilde{y}_s^h)^2}{z^2} - \frac{\mu^2}{z^2} \right] \\
&= p(\tilde{y} < z) \left[ \left( \frac{(z - \mu)^2}{z^2} \right) + \sum_{s=1}^{S_h} p(s | \tilde{y} < z) \frac{(\tilde{y}_s^h - \mu)^2}{z^2} \right] \\
&= p(\tilde{y} < z) \left[ \left( \frac{(z - \mu)^2}{z^2} \right) + \sum_{s=1}^{S_h} p(s | \tilde{y} < z) \frac{(\tilde{y}_s^h - \mu)^2 \mu^2}{z^2 \mu^2} \right] \\
&= p(\tilde{y} < z) \left[ \left( \frac{(z - \mu)^2}{z^2} \right) + \sum_{s=1}^{S_h} p(s | \tilde{y} < z) \left( \frac{-2z^2 + 2z\mu + 2z^2 - 2z\mu + \mu^2}{z^2} \right) \frac{(\tilde{y}_s^h - \mu)^2}{\mu^2} \right] \\
&= p(\tilde{y} < z) \left[ \left( \frac{(z - \mu)^2}{z^2} \right) + \left( -2 \frac{z - \mu}{z} + 1 + \frac{z^2 - 2z\mu + \mu^2}{z^2} \right) \sum_{s=1}^{S_h} p(s | \tilde{y} < z) \frac{(\tilde{y}_s^h - \mu)^2}{\mu^2} \right]
\end{aligned} \tag{C.4}$$

$$\begin{aligned}
V_{\alpha=2,h}(\tilde{\mathbf{Y}}; \mathbf{z}) &= p(\tilde{y} < z) \left[ \left( \frac{(z - \mu)^2}{z^2} \right) + \left( -2 \frac{z - \mu}{z} + 1 + \frac{z^2 - 2z\mu + \mu^2}{z^2} \right) \sum_{s=1}^{S_h} p(s | \tilde{y} < z) \frac{(\tilde{y}_s^h - \mu)^2}{\mu^2} \right] \\
&= EH_h [EI_h^2 + (1 - EI_h)^2 ECV_h^2]
\end{aligned} \tag{C.5}$$

$$EH_h = \frac{S_h}{N} = p(\tilde{y} < z) \tag{C.6}$$

$$EI_h = \sum_{s=1}^{S_h} \frac{1}{S_h} \frac{(z - \tilde{y}_s^h)}{z}, \quad \frac{1}{S_h} = p(s | \tilde{y} < z) \tag{C.7}$$

$$ECV_h^2 = \sum_{s=1}^{S_h} \frac{1}{S_h} \frac{(\mu - \tilde{y}_s^h)^2}{\mu^2}, \quad \frac{1}{S_h} = p(s | \tilde{y} < z) \tag{C.8}$$

## D Italy - Decomposition I

Decomposition of Vulnerability to Poverty (by regions, national poverty line)

Italy - FGT vulnerability index  $\alpha = 2$



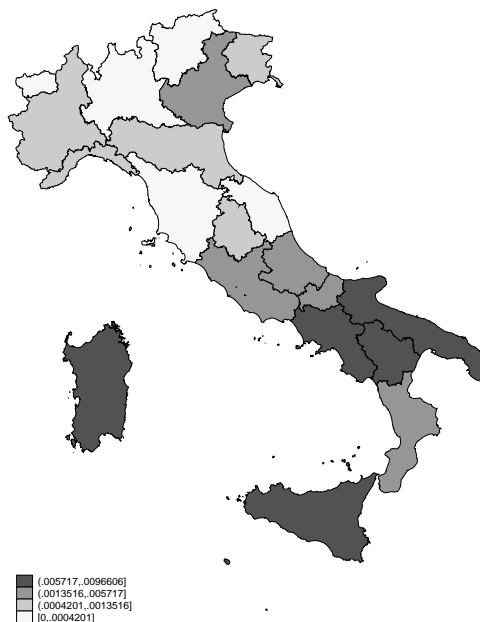
Italy - Expected Incidence,  $EH$



Italy - Expected Intensity,  $EI$



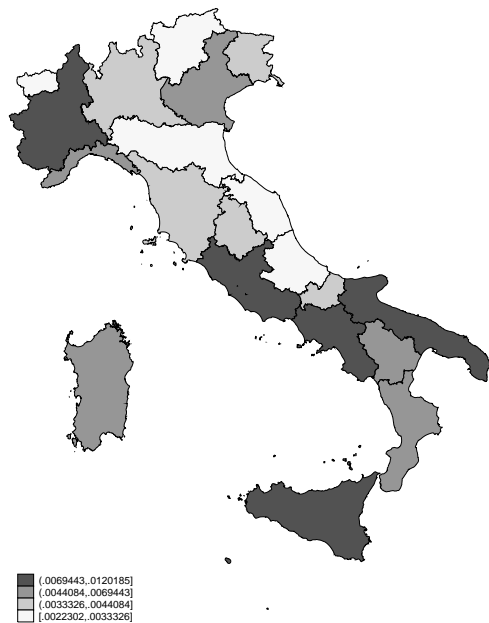
Italy - Expected Downward variability,  $ECV^2$



## E Italy - Decomposition II

Decomposition of Vulnerability to Poverty (by regions, regional poverty line)

Italy - FGT vulnerability index  $\alpha = 2$



Italy - Expected Incidence,  $EH$



Italy - Expected Intensity,  $EI$



Italy - Expected Downward variability,  $ECV^2$

