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## REALIZED VOLATILITY: Macroeconomic determinants, Forecasting and Option trading

Direttore della Scuola: Ch.mo Prof. Guglielmo Weber
Supervisore: Ch.mo Prof. Massimiliano Caporin

Dottorando: Gabriel G. Velo

Para Bettina y Luis

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## Introduzione

La stima e la modellazione della volatilità finanziaria è stata una delle aree di ricerca più attive degli ultimi 20 anni vista la sua cruciale importanza nell' asset pricing, nel risk management e nell'allocazione del portafoglio. Tuttavia, il problema principale è che la volatilità non è direttamente osservabile. Gli approcci più comuni per affrontare la latenza della volatilità sono i modelli parametrici (Generalized) Autoregressive Conditional Heteroskedasticity ((G)ARCH) introdotti da Engle (1982) e Bollerslev (1986) e i modelli di volatilità stochastica (SV) (vedere per esempio, Taylor (1986)).

Con la crescente disponibilità di dati ad alta frequenza e utilizzando gli sviluppi teorici legati alla teoria della variazione quadratica, la stima della volatilità si è evoluta dai modelli latenti alle misure realizzate non parametriche. Andersen, Bollerslev, Diebold, and Labys (2001) e Barndorff-Nielsen and Shephard (2002) hanno introdotto la varianza realizzata che è definita come la somma dei rendimenti al quadrato all'interno di un intervallo di tempo. In teoria, la varianza realizzata è uno stimatore non distorto e altamente efficiente che converge alla varianza integrata quando la lunghezza dell'intervallo di campionamento intragiornaliero tende a zero. Inoltre, Christensen and Podolskij (2007) e Martens and van Dijk (2007) hanno proposto una diversa misura della varianza realizzata basata sul range. Lo stimatore è definito come la somma dei range al quadrato (dove il range è la differenza tra il massimo e prezzo minimo osservato nel corso di un determinato intervallo). Si tratta di uno stimatore consistente e altamente efficiente della volatilità integrata ed è circa cinque volte più efficiente della varianza realizzata. In seguito a quest'operazione, la volatilità diventa osservabile ed è possibile modellizzarla in modo da ottenere previsioni che sono necessarie in molte applicazioni finanziarie. L'implementazione degli stimatori realizzati utilizzando dati ad alta frequenza incontra tuttavia diverse difficoltà. Infatti, tali stimatori sono basati sulle ipotesi secondo le quali i prezzi seguono un processo continuo e sono inoltre sensibili agli effetti microstrutturali, elementi che possono provocare gravi problemi riguardo la consistenza dello stimatore.

Questa tesi di dottorato di ricerca si propone di modellizzare e prevedere la volatilità, e contribuisce alla letteratura dell'econometria per la finanza in tre direzioni diverse. Il primo capitolo, un lavoro co-autorato con il Professore Massimiliano Caporin, considera la modellizzazione e la previsioni della volatilità sulla base dello stimatore realizzato basato sul range ed esplora le performance di specificazioni alternative che tengano conto del comportamento specifico e dei fatti stilizzati delle serie storiche della volatilità finanziaria. Il secondo capitolo si con-
centra sulle determinanti economiche della volatilità e analizza la capacità predittiva delle variabili macroeconomiche e finanziarie. Infine, il terzo capitolo studia la valutazione di stimatori alternativi della volatilità e di un set di modelli di previsione della volatilità da un punto di vista economico.

Più in dettaglio, il primo capitolo ${ }^{1}$ si focalizza nella stima della volatilità realizzata attraverso lo stimatore basato sul range in 16 azioni scambiate al New York Stock Exchange (NYSE), si considera l'effetto microstrutturale dei dati ad alta frequenza e si correggono le stime seguendo la procedura di Martens and van Dijk (2007). Inoltre, il modello per le serie di volatilità realizzata prende in considerazione l'effetto asimmetrico sulla volatilità causata dai rendimenti ritardati e la dipendenza di lungo termine, la volatility clustering della volatilità e la non Gaussianità delle innovazioni del modello. In particolare, si specficifa un modello Autoregressive Heterogenuos (HAR) introdotto da Corsi (2009), effetti leverage riguardo ai rendimenti e presenza di innovazioni caratterizzate da varianze GARCH o GJR-GARCH e con una distribuzione Normal Inverse Gaussian (NIG). L'analisi empirica dell' abilità predittiva nelle 16 azioni considerate suggerisce che l'introduzione degli effetti asimmetrici rispetto ai rendimenti e la volatilità nel modello HAR portano ad un significativo miglioramento della performance delle previsioni.

Nel secondo capitolo, si esamina il ruolo che le variabili macroeconomiche e finanziarie hanno nella modellazione e nella previsione della volatilità giornaliera. Il punto di partenza sono le serie stimate nel primo capitolo e si estende il modello includendo variabili economiche e finanziarie che contengono informazioni relative allo stato presente e futuro dell'economia. È stato eseguita un'analisi empirica dentro e fuori campione in 16 serie di volatilità di titoli azionari. I risultati suggeriscono che le variabili macroeconomiche e finanziarie, in particolare le proxy per le aspettative di rischi di mercato e rischi di credito, sono significativamente correlate con il primo componente principale delle serie di volatilità e hanno un potere altamente esplicativo. Successivamente, si considera un esercizio di previsione fuori campione per analizzare il miglioramento che deriva dall'introduzione delle variabili macroeconomiche che hanno avuto più potere esplicativo nel'analisi nel campione. Variabili come il VIX e credit default swap per il settore bancario statunitense producono miglioramenti significativi nella accuratezza delle previsioni. Infine, esplorando l'impatto durante la crisi finanziaria 2008-2009, si osserva una maggiore correlazione tra i credit default swap e la volatilità che riflette un aumento del rischio di credito percepito, mentre non si ottiene alcun miglioramento significativo nella previsioni fuori campione .

Nell'ultimo capitolo, si esamina e confronta la prestazione di stimatori alternativi della volatilità realizzata e di diversi modelli di previsione di serie storiche dal punto di vista economico. Nello specifico, si considera un investitore che specula sul futuro livello della volatilità e investe in un strategia buy-and-hold su opzione che dipende dal livello atteso per la volatilità, e di conseguenza risente delle differenze tra i diversi stimatori e modelli di previsione utilizzati. La strategia di

[^0]trading viene implementata settimanalmente su opzioni dove il sottostante è S\&P 500 Index e la volatilità viene stimata da una serie ad alta frequenza relativa allo S\&P 500 Futures. Gli stimatori considerati sono la volatilità realizzata e la volatilità realizzata basata sul range in entrambi i casi nelle versioni robuste all'effetto microstrutturale ed alla presenza di salti. Le previsioni si ottengono con modelli Autoregressive Fractional Integrated Moving Average (ARFIMA). Inoltre, per tenere conto della possibilità di confondere memoria lunga con processi con memoria corta e cambiamenti strutturali nei livelli, si considera il modello ARFIMA con break strutturali, recentemente introdotti da Grassi and Santucci de Magistris (2011) e Varneskov and Perron (2011). I principali risultati mostrano che le stime ottenute con il metodo basato sul range e corretti per la presenza di effetti microstrutturali e discontinuità nel processo del prezzo producono i rendimenti più elevati. Inoltre, la scelta dello stimatore realizzato per la volatilità sembra essere più importante rispetto ai modelli di previsione. Infine, si osserva come sia possibile ottenere profitti positivi annualizzati fino ad oltre il $60 \%$ nel periodo di trading considerato e che parte da Ottobre 2005 fino ad arrivare alla crisi finanziaria del 2008-2009.

## Introduction

The estimation and modeling of financial volatility asset has been one of the most active research areas because of its crucial importance in asset pricing, risk management and portfolio allocation. However, the main problem is that volatility is not directly observable. Common approaches to deal with the latency of the volatility are the parametric (Generalized) Autoregressive Conditional Heteroskedasticity ((G)ARCH) family of models introduced by Engle (1982) and Bollerslev (1986) or the stochastic volatility (SV) models (see, for example, Taylor (1986)).

With the growing availability of high frequency price data and based on the theory of quadratic variation, the estimation of volatility has moved from latent volatility model to non-parametric realized measures. Andersen, Bollerslev, Diebold, and Labys (2001) and Barndorff-Nielsen and Shephard (2002) introduced the realized variance as the sum of squared returns over non-overlapping intervals within a sampling period. In theory, the realized variance is an unbiased and highly efficient estimator that converges to the integrated variance when the length of the intra-day intervals goes to zero. In addition, Christensen and Podolskij (2007) and Martens and van Dijk (2007) proposed the realized range-based variance. This estimator is defined as the sum of the squared ranges (i.e. the difference between the maximum and minimum prices observed during a period) over non-overlapping intervals. It is a consistent and highly efficient estimator of the integrated volatility and it is about five times more efficient than realized variance. Then, volatility becomes observable and it is possible to model it in order to obtain forecasts that are needed in many financial applications. In practice, the implementation of realized estimation measures in the reality of high frequency data faces various difficulties. These estimators are based on the assumptions that the prices follow a continuous sample path and they are sensitive to microstructure noise, which can cause severe problems for the estimator's consistency.

This PhD thesis aims at modeling and forecasting volatility, and contributes to the financial econometrics literature in three different directions. The first chapter, a joint work with Professor Massimiliano Caporin, considers the modeling and forecasting of volatility based on the realized range estimator and explores the performance of alternative specifications that take into account the distinct behavior and stylized facts of financial volatility time series. The second chapter focuses on the economic determinants of financial volatility and analyzes the predictive ability that macroeconomic and financial variables have when forecasting volatility. Finally, the third chapter deals with the evaluation of alternative estimators and
forecasting models of volatility from an economic point of view.
More in detail, in the first chapter ${ }^{2}$, we concentrate on the estimation of volatility through the realized range-based for 16 stocks traded at the New York Stock Exchange (NYSE) and we consider the impact of the microstructure noise in high frequency data and correct our estimations following the procedure of Martens and van Dijk (2007). In addition, we model the volatility series accounting for the asymmetric effect on volatility caused by lagged returns and the long-range dependence, the volatility clustering in the volatility and non-Gaussianity of the innovations of the model. In particular, we specify an Heterogenuos Autoregressive (HAR) model introduced by Corsi (2009), leverage effects with respect to the return and the volatility, GARCH and GJR-GARCH variance and the Normal Inverse Gaussian (NIG) distribution. The empirical analysis of the forecast performance in 16 stocks suggests that the introduction of asymmetric effects with respect to the returns and the volatility in the HAR model results in significant improvement in the point forecasting accuracy.

In the second chapter, we investigate the role that macroeconomic and financial variables play when modeling and forecasting daily stocks volatility. We depart from the estimated series in chapter 1 and we extend the model including economic and financial variables that capture the present and the future state of the economy. We perform an in-sample and out-of-sample empirical analysis in 16 series of stock volatility. We find that macroeconomic and financial variables, in particular proxies for market risk expectation and credit risk, are significantly correlated with the first principal component of the volatility series and they have a highly in-sample explanatory power. Then, we consider an out-of-sample forecasting exercise to analyze the improvement that results from the introduction of the macroeconomic variables that better perform in our in-sample analysis. Variables such as the VIX and the credit default swap index for the US bank sector produce significant improvements in the forecasting accuracy. Last, exploring the impact during the 2008-2009 financial crisis, we observe a higher correlation between the credit default swap index and the volatility reflecting an increase in the perceived credit risk, while there is no significant improvement in the out-of-sample predictions.

In the last chapter, we examine and compare the performance of alternative realized estimators of volatility and of different time series forecasting models from an economic point of view. We consider an investor that speculates on the future level of the volatility and invests in a buy-and-hold option strategy depending on his expected level of volatility, that is obtained from different estimators and forecasting models. We implement the trading strategy with weekly S\&P 500 Index options and compute volatility from a high frequency series for S\&P 500 futures. The volatility estimators are based on the realized volatility and the realized range and they are robust to microstructure noise and jumps while out-of-sample forecast are obtained with Autoregressive Fractional Integrated Moving Average (ARFIMA) models. Additionally, to account for the possibility of confusing long

[^1]memory with process with short memory and structural change in levels we take into consideration the ARFIMA model with random level shift component recently introduced by Grassi and Santucci de Magistris (2011) and Varneskov and Perron (2011). The main results show that realized range based estimator corrected for the presence of microstructure noise and discontinuities in the price process produces the highest returns. Moreover, the choice of the realized estimator for the volatility seems to be more important than the forecasting models. Finally, positive annualized profits of up to more than $60 \%$ are obtained for the trading period before the financial 2008-2009 crisis.

## Chapter 1

## Modeling and Forecasting Realized Range Volatility

### 1.1 Introduction

In the last years, realized volatility measures, constructed from high frequency financial data and modeled with standard time series techniques, have shown to perform much better than traditional generalized autoregressive conditional heteroskedasticity (GARCH) and stochastic volatility models, when forecasting conditional second order moments. Most of the works that forecast volatility through realized measures, have concentrated on the realized variance ( $R V$ ) introduced by Andersen, Bollerslev, Diebold, and Labys (2001) and Barndorff-Nielsen and Shephard (2002). The $R V$ is based on the continuous time price theory and it is defined as a function of the sum of squared intra-day returns. The $R V$ is a highly efficient and unbiased estimator of the quadratic variation and converges to it when the intraday measurement period goes to zero. Later on, Martens and van Dijk (2007) and Christensen and Podolskij (2007) introduced the realized range volatility $(R R V)$, another realized estimator consistent for the quadratic variation. The $R R V$ is based on the difference between the minimum and maximum prices observed during a certain time interval. This new estimator tries to exploit the higher efficiency of the range relatively to that of the squared daily close-to-close return in the estimation of quadratic variation.

When dealing with high frequency financial market data, the asymptotic properties of the simple estimators are highly affected by the microstructure noise (non continuous trading, infrequent trade, bid ask bounce). As a result, an important part of the literature has presented different corrections to restore the efficiency of realized estimators for the volatility. These studies aimed at improving over the first generation of models, whose purpose was to construct estimates of realized variances by using series at a moderate frequency (see Andersen, Bollerslev, Diebold, and Labys (2003)). Some of the corrections presented to the $R V$ are the Two Time Scale Estimator (TTSE), the sub-sampling method of Zhang, Mykland, and Ait-Sahalia (2005), the generalization introduced by Zhang (2006). We also mention the approach for identifying the optimal sampling frequency by Bandi
and Russell (2008), through a minimization of the MSE.
Furthermore, kernel estimation was introduced by Hansen and Lunde (2006), while Barndorff-Nielsen, Hansen, Lunde, and Shephard (2008) provide a generalization of this approach. Differently, Martens and van Dijk (2007) proposed a correction for the $R R V$ based on scaling the range with the daily range, while Christensen, Podolskij, and Vetter (2009) presented another approach based on an adjustment by the variance of the noise and a constant which has to be estimated by simulation methods.

With the availability of new observable series for the volatility, many authors have applied traditional discrete time series models for their forecast (and implicitly for the forecast of returns volatility). Financial data is characterized by a series of well-known stylized facts. Being able to capture them, will result in a more accurate prevision of our variable of interest. These stylized facts are also observable over realized variance series and require appropriate modeling strategies. The presence of long-memory in volatility, documented in several studies, has been modeled through different specifications: Andersen, Bollerslev, Diebold, and Labys (2003) introduced an ARFIMA model, and their forecasts for the $R V$ generally dominate those obtained through GARCH models; Corsi (2009) presented the Heterogenous Autoregressive (HAR) model, that reproduces the hyperbolic decay of the autocorrelation function by including the sums of $R V$ over different horizons in order to capture the time strategies of the agents in the market. The second model has the advantage to be much simpler to estimate. Additionally, asymmetry, leverage effects, and fat tails should also be taken into account. Martens, van Dijk, and de Pooter (2009) specified a flexible unrestricted high-order AR model. They also considered leverage effects, days of the week effects and macroeconomics announcement. Differently, Corsi, Mittnik, Pigorsch, and Pigorsch (2008) introduced two important extensions specifying a GARCH component modeling the volatility of volatility and assuming non-Gaussian errors. Their results suggested an improvement in the accuracy in the point forecasting and a better density forecast.

In this chapter, we model and forecast volatility through the realized range volatility with data on 16 stocks quoted at the New York Stock Exchange (NYSE). Our main objective is to study the prediction performance of the range as a proxy of the volatility. An accurate forecast of financial variability should have important implications in asset and derivative pricing, asset allocation, and risk management. In the first part of this chapter, we construct and analyze the realized range series, and correct it from the microstructure noise following Martens and van Dijk (2007). In the second part, we implement time series techniques to model and capture the stylized facts within the volatility equation to gain in forecasting accuracy. In details, we consider an HAR model, we introduce leverage effects with respect to the return and the volatility, a GARCH and a GJR-GARCH specification for the volatility of volatility, introduced by Bollerslev (1986) and Glosten, Jagannathan, and Runkle (1993), respectively. Furthermore, in order to capture the statistical feature of the residuals of our model, we also consider a Normal Inverse Gaussian (NIG) distribution.

The remainder of this chapter is structured as follows. In section 1.2, we present the data and the correction procedure. In section 1.3 and 1.4, we present the model and the forecast evaluation framework. We discuss the results for the estimation and forecast in section 1.5 and 1.6. Finally, section 1.7 presents the results and futures steps.

### 1.2 Data and correction procedure

Let us consider a price process at time $t$ that follows a geometric Brownian motion

$$
\begin{equation*}
d p_{t}=\mu p_{t} d t+\sigma p_{t} d W_{t} \tag{1.1}
\end{equation*}
$$

where $\mu$ is the drift and $\sigma$ is the volatility, and $W_{t}$ is a standard Brownian motion. Then, the logarithmic price follows a Brownian motion with drift ( $\mu^{*}$ ) and volatility $(\sigma)$. The objective is to define a measure of the return variation over a subinterval that is assume to be a trading day and it is define between $[0,1]$. Given a high frequency series of prices, we define the $i$ th interval of length $\Delta$ of an equidistant partition on day $t$, for $i=1, \ldots, n$, where $n=1 / \Delta$, the last price $p_{t, i}^{c l}=p_{t-1+i \Delta}$, the low price $p_{t, i}^{l o}=\min _{(i-1) \Delta<j<i \Delta} p_{t-1+j}$ and the high price $p_{t, i}^{h i}=\max _{(i-1) \Delta<j<i \Delta} p_{t-1+j}$ observed in the $i$ th interval of length $\Delta$ on day $t$. The realized range estimator $R R V_{t}^{\Delta}$ is defined as

$$
\begin{equation*}
R R V_{t}^{\Delta}=\frac{1}{\lambda^{2}} \sum_{i=1}^{n}\left(\ln p_{t, i}^{h g}-\ln p_{t, i}^{l o}\right)^{2} \tag{1.2}
\end{equation*}
$$

$\lambda^{2}$ is the $2^{\text {nd }}$ moment of the range of a standard Brownian motion over the unit interval and equal to $4 \log (2)$ in this case. If the logarithmic price process follows a drift-less process then

$$
\begin{equation*}
\sqrt{n}\left(R R V^{\Delta}-I V\right) \rightarrow_{d} N(0, \Lambda I Q) \tag{1.3}
\end{equation*}
$$

where $I V$ is the integrated volatility, $I Q$ is the integrated quarticity and $\Lambda \simeq$ 0.4073 . As a consequence, under the assumption that there are no market frictions and there is continuous trading, the $R R V_{t}^{\Delta}$ is an unbiased and consistent estimator of the integrated variance and it is five time more efficient than $R V$, see for example Christensen and Podolskij (2007).

In the reality, there are evidences against these assumptions and the realized estimators become inconsistent and biased (see for example McAleer and Medeiros (2008)). Hence, a corrected version for the $R R V$ should restore the efficiency of this estimator over the $R V$. In this paper, we follow Martens and van Dijk (2007) that proposed a correction based on scaling the realized range with a ratio involving the daily range and the realized range over the previous trading days. ${ }^{1}$ Basically, the

[^2]scaling bias correction is not difficult to implement because it does not require the availability of tick by tick data. The idea of Martens and van Dijk (2007) is based on the fact that the daily range is almost not contaminated by market frictions. The simulation results of Martens and van Dijk (2007) confirm the theory that the range is more efficient than the $R V$, and in the presence of market frictions the scaling correction removes the bias and restores the efficiency of the realized range estimator over the realized volatility. Therefore, the scaled $R R V_{t}^{\Delta}$ is defined as:
\[

$$
\begin{equation*}
R R V_{\text {scaled }, t}^{\Delta}=\left(\frac{\sum_{l=1}^{q} R R V_{t-l}}{\sum_{l=1}^{q} R R V_{t-l}^{\Delta}}\right) R R V_{t}^{\Delta} \tag{1.4}
\end{equation*}
$$

\]

where $R R V_{t} \equiv R R V_{t}^{\Delta}$ (with $\Delta=1$ day) is the daily range and $q$ is the number of previous trading days used to compute the scaling factor. If the trading intensity and the spread do not change, $q$ must be set as large as possible. However, in the reality only recent history should be taken into consideration.

Our database includes more than seven years of 1 minute high, low, open and close prices for 16 stocks quoted at the NYSE: Boeing $(B A)$, Bank of America $(B A C)$, Citigroup Inc. ( $C$ ), Caterpillar Inc. ( $C A T$ ), FedEx Corporation (FDX), Honeywell International Inc. (HON), Hewlett-Packard Company (HPQ), International Business Machines Corp. (IBM), JPMorgan Chase \& Co. (JPM), Kraft Foods Inc. ( $K F T$ ), PepsiCo, Inc. (PEP), The Procter \& Gamble Company $(P G)$, AT\&T Inc. $(T)$, Time Warner Inc. $(T W X)$, Texas Instruments Incorporated $(T X N)$ and Wells Fargo \& Company (WFC).

The original sample covers the period from January 2, 2003 to March 30, 2010, from 09:30 trough 16:00 for a total of 1823 trading days. We construct the series for the range for the one, five, thirty minutes and daily sampling frequency (i.e. $\Delta=1,5,30$ minutes or 1 day). We correct them using one, two and three previous months (i.e. $q=22,44$ or 66 ). The results of the corrections show that, after scaling with different $q$, the mean volatility stabilized across the different sampling frequencies. We choose to sample every five minutes and to correct for the presence of noise with the 66 previous days, as in Martens and van Dijk (2007).

Table 1.1 reports descriptive statistics for the logarithm of the range based volatility estimators and for the returns. A statistical analysis of the return and the volatility series confirms the presence of the stylized facts vastly documented in the literature. The annualized volatility range from $15.5205 \%$ to $33.8154 \%$ and the mean of the log series range from -4.8362 to -3.4884 . The skewness parameter ranges from 0.5266 to 1.1342 and the kurtosis ranges from 3.0242 to 5.6782 . Moreover, the returns exhibit excess of kurtosis ranging from 6.0939 to 25.094. The summary statistics seems to be similar across the different series.

Plots of the volatility series and the sample autocorrelation functions (ACF) of the logarithmic volatility series for some of the assets are presented in figure 1.1. We can observe a common period of high volatility that corresponds to the 20082009 financial crisis and the long-memory pattern in the hyperbolic and slowly decay of the ACF that is present in the logarithmic volatility series.

### 1.3 The models for the observed volatility sequences

Different models have been presented to capture the stylized facts that financial series exhibit. Based on the statistical features briefly mentioned before, we consider the HAR model of Corsi (2009) to capture the long-memory pattern. The long-range dependence is captured by aggregating the volatility over different periods, on a daily, weekly and monthly basis. Each of whom represents the different preferences of the individuals in the market. This representation can be seen as a long and restricted lagged autoregressive process, with the advantages of a simple estimation that regress today's volatility in its past averages (daily, weekly, monthly). We account for asymmetric effects with respect to the volatility and the returns. Moreover, following Corsi, Mittnik, Pigorsch, and Pigorsch (2008) we also include a GARCH specification to account for heteroskedasticity in observed volatility sequences and a standardized Normal Inverse Gaussian (NIG) distribution to deal with the observed skewness of the residuals. Finally, to account for asymmetric effects in the variance equation or volatility of the volatility we consider a GJR-GARCH specification of Glosten, Jagannathan, and Runkle (1993).

We thus estimate the following model:

$$
\begin{gather*}
h_{t}=\alpha+\delta_{s} I_{s}\left(h_{t-1}\right) h_{t-1}+\beta_{d} h_{t-1}+\beta_{w} h_{(t-1: t-5)}+\beta_{m} h_{(t-1: t-22)}+  \tag{1.5}\\
+\gamma_{R} R_{t-1}+\gamma_{I R} I\left(R_{t-1}\right) R_{t-1}+\sqrt{\sigma_{t}} \epsilon_{t} \\
\sigma_{t}=\omega+\beta_{1} \sigma_{t-1}+\alpha_{1} u_{t-1}^{2}+\phi_{1} u_{t-1}^{2} I\left(u_{t-1}\right) \\
\epsilon_{t} \mid \Omega_{t-1} \sim d(0,1)
\end{gather*}
$$

where $h_{t}$ is the $\log R R V_{\text {scaled }, t}^{\Delta}, h_{(t-1: t-j)}$ is the HAR component defined as

$$
\begin{equation*}
h_{(t-1: t-j)}=\frac{1}{j} \sum_{k=1}^{j} h_{t-k} \tag{1.6}
\end{equation*}
$$

with $j=5$ and 22 in order to capture the weekly and monthly components. $I_{s}\left(h_{t-1}\right)$ is an indicator for $R R V_{\text {scaled }, t-1}^{\Delta}$ bigger than the mean over $s=5,10,22,44$ and 66 previous days and the unconditional mean (full) up to $t-1$. These variables capture the asymmetric effects with respect to the volatility. $R_{t}=\ln \left(p_{t}^{c l} / p_{t-1}^{c l}\right)$ is the return, with $p_{t}^{c l}$ the closing price for the day $t$ and $I\left(R_{t-1}\right)$ is an indicator for negative returns in $t-1$, that captures the asymmetric effects with regard to the lagged return. $u_{t}=\sqrt{\sigma_{t}} \epsilon_{t}$ is the error term. The full specification for $\sigma_{t}$ is a GJR-GARCH model to account for the asymmetric effect in the volatility of the volatility, where $I\left(u_{t-1}\right)$ is an indicator for $u_{t-1}<0, \omega \geq 0, \beta_{1} \geq 0$, $\alpha_{1} \geq 0$ to guarantee the conditional variance to be positive and $\beta_{1}+\alpha_{1} \leq 1$ and $\beta_{1}+\alpha_{1}+\frac{1}{2} \phi_{1} \leq 1$ to guarantee stationarity under symmetry of the density of the standardized residuals.

Initially, we consider 21 alternative specifications for the mean equation, three possible variance equations and two different distributions for the innovations. Combining those three elements, we have a total of 126 models. After an in-sample selection, we decide to concentrate on six different specifications for the mean equation that account for all the discussed stylized facts. The specifications for the mean are the $\mathrm{AR}(1)(I)$, the HAR model of Corsi (2009) (II), the HAR model with asymmetric effects with respect to the historical volatility and the returns and the lagged returns ( $I I I$ ), the HAR with asymmetric effects with respect to the returns and lagged returns ( $I V$ ), the HAR model with only asymmetric effects with respect to the returns $(V)$ and finally the HAR model with symmetric effects for the past weekly volatility and the returns (VI). The last five models also include an $\mathrm{AR}(1)$ term. Finally, we have 36 models that are used to compute one-step-ahead rolling forecast for the volatility series and to assess the importance of the variable. Because of a space restriction, we only present estimation results for some of the considered models.

### 1.4 Forecast evaluation

In order to test the out-of-sample ability of the different models we implement two different procedures, the Diebold and Mariano (1995) test and the Models Confidence Set approach of Hansen, Lunde, and Nason (2010) based on two loss functions the Mean absolute error (MAE) and the Mean squared error (MSE):

$$
\begin{align*}
& M A E(i)=\frac{1}{m} \sum_{t=i}^{m}\left|\sigma_{t}^{2}-\hat{\sigma}_{i, t}^{2}\right|  \tag{1.7}\\
& M S E(i)=\frac{1}{m} \sum_{t=i}^{m}\left(\sigma_{t}^{2}-\hat{\sigma}_{i, t}^{2}\right)^{2} \tag{1.8}
\end{align*}
$$

where $\sigma_{t}^{2}$ is the true and unknown variance at time $t$ and $\hat{\sigma}_{i, t}^{2}$ is the forecast volatility for the model $i$ for period $t$. Since $\sigma_{t}^{2}$ is not observable, we replace it by $h_{t}$, our volatility series and a proxy for the volatility $\sigma_{t}^{2}$. As pointed out by Patton (2011), the MSE loss function is robust to the presence of noise in the volatility proxies, resulting in an unbiased model ranking. As the author states, the introduction of proxies for the true volatility generates distortions in the rankings of competing forecasts based in common used loss functions.

The Diebold and Mariano (1995) test (DM) is a test for the equal predictive ability between two competing models. Under the null hypothesis of equal expected loss differential we have:

$$
\begin{equation*}
H_{0}=E[L(i)]-E[L(j)]=E[L(i)-L(j)]=E[d L(i, j)]=0 \tag{1.9}
\end{equation*}
$$

where $L(i)$ represents a loss function for the model i and $d L(i, j)$ is the loss differential equation between the two models $i$ and $j$, for a given loss function. The test statistic is given by

$$
\begin{equation*}
t_{L}(i, j)=\frac{D L(i, j)}{\sqrt{V(D L(i, j))}} \tag{1.10}
\end{equation*}
$$

where $D L(i, j)=1 / m \sum_{t=1}^{m} d L(i, j)$ and $V(D L(i, j))$ is an heteroskedasticity and autocorrelation consistent variance estimator and it is asymptotically distributed as a standardized normal and allows for a simple evaluation of the null hypothesis. As pointed out by Caporin and McAleer (2011) a limitation of the forecast evaluation based on Diebold and Mariano (1995) type tests is that they represent pairwise comparisons, so that it is not possible to exclude a priori the possibility of having different model rankings associated with different robust loss functions.

Then, we also consider the Model Confidence Set approach (MCS) of Hansen, Lunde, and Nason (2010). The goal of the MCS is to identify the set of models that contains the best out-of-sample forecasting models given a level of confidence. The procedure sequentially eliminates the worst performing models from an initial set. Starting from M, a set that contains all the original models that produced out-of-sample forecasting and $d L(i, j)$ the loss differential equation between two models for a given loss function, the null hypothesis of the MCS is defined as

$$
\begin{equation*}
H_{0}=E[d L(i, j)]=0 \tag{1.11}
\end{equation*}
$$

for $i>j$, and $\forall i j \in M$, the original set. $H_{0}$ can be tested with two test statistic based on different deviation methods:

$$
\begin{align*}
t_{R} & =\max _{i, j \epsilon M}\left|\frac{D L(i, j)}{\sqrt{V(D L(i, j))}}\right|  \tag{1.12}\\
t_{S Q} & =\sum_{i, j \in M, j>1}\left(\frac{D L(i, j)}{\sqrt{V(D L(i, j))}}\right)^{2} \tag{1.13}
\end{align*}
$$

When the null hypothesis is rejected for a given $\alpha$, the procedure excludes the worst performing models. The procedure is repeated until the null hypothesis of equal predictive ability for the remaining models is not rejected, resulting in the $M_{1-\alpha}^{*}$ set containing the best models.

### 1.5 Estimation results

We estimate the models with a Quasi-Maximum Likelihood procedure for the 16 stocks and for the entire sample from January 2003 to March 2010. The aim is to assess the impact and significance of our different variables in our models. Tables $1.2,1.4,1.3$ and 1.5 present the estimation result for the 2003-2010 period. In particular, table 1.2 reports the estimated parameters of the HAR model with constant variance and Normal distribution and the table 1.3 shows the same specification for the mean equation model with GJR-GARCH variance and NIG distribution (model $I I$ ) for the 16 stocks. Moreover, tables 1.4 and 1.5 provide the estimated parameters of the HAR with lagged and asymmetry over the return (model $I V$ ) with constant variance and Normal distribution and GJR-GARCH variance and NIG distribution respectively.

Estimation results suggest that the HAR components are highly significant for the three variance specifications and the two different distributions for all the
assets. The sum of the estimated parameters $\left(\beta_{d}, \beta_{w}\right.$ and $\left.\beta_{m}\right)$, range from 0.85 to 0.97 , suggesting a highly persistent process. The lagged and asymmetric effects with respect to the return improve the goodness of fit of the model. The first one is highly significant in most of the 16 stocks and it increases the volatility after a negative return while the second one is highly significant in the 16 series and negative, which implies that a negative return tends to increase the volatility more than a positive return.

On the contrary, the asymmetric effects with respect to the volatility, in the different horizons tend to be not significant. The asymmetric effect on the previous five days is marginally significant for some models ${ }^{2}$. The sign and significance of the coefficients in the mean equation remain stable for the different specifications in the variance equation. The inclusion of the GARCH and GJR specifications improve the fitting of the models. The models that best fit the series are the ones that include the HAR and leverage effects, with GARCH and GJR variances.

Moreover, we perform three different Normality tests for the residuals: the Jarque-Bera test $(J B)$, the Kolmogorov-Smirnov test $(K S)$ and the Lilliefors test $L L$. Under the null hypothesis, in the three tests, the residuals should be normal. P-values reported in tables 1.2 and 1.4 reject the null hypothesis of the three test in all the assets. This is an argument to introduce a non Gaussian distribution. As we said, the estimated parameters of the mean equation for the models with NIG distribution are similar to the models with Normal distribution. The estimated parameters of the NIG distribution ( $\alpha_{\text {NIG }}$ and $\beta_{N I G}$ ) capture the right skewness and excess of kurtosis displayed in the residuals.

Tables 1.6, 1.7, 1.8 and 1.9 report three different Likelihood ratio test for the 16 stocks. In the first test ( $L R 1$ ), we want to analyze the introduction of asymmetric effects in the mean equation with respect to the HAR model for the three variance specifications and the two distributions for the innovations. Whereas in the second and third tests ((LR2) and (LR3) respectively), we examine the introduction of the different variance specifications for the same mean equation and distribution for the residuals. In the $L R 2$ test the restricted model has constant variance, and the unrestricted models GARCH and GJR-GARCH variances, while in the LR3 test the restricted model has GARCH variance and the unrestricted models GJRGARCH variance. P-values for the first and the second test favor the introduction of asymmetric effects with respect to the return and the volatility in the mean equation and GARCH and GJR-GARCH specifications for the variance equation in all the series. P-values for the last test reject the null in most of the cases.

### 1.6 Forecast results

To examine the predictive accuracy of the different models, we compute one-day-ahead out-of-sample rolling forecast from January 3, 2006 to March 30, 2010 for the 16 stocks. We estimate the models until December 30, 2005 and then we reestimate each model at each recursion expanding the data set by one observation.

[^3]We analyze the results for the out-of-sample forecast in two different subperiods. In particular, we study the performance of the alternative models for the full out-of-sample ( 1067 days) and during the financial crisis, from September 15, 2008 to July 30, 2009 (200 days). As we said, after the in-sample analysis, we decide to compute forecasts with six different specifications for the mean equation with three alternative variance equations and two distributions for the innovations. The six specifications for the mean are: the simple $\operatorname{AR}(1)(I)$, the HAR model of Corsi (2009) (II), the HAR model with symmetric effects with respect to the historical volatility the returns and the lagged returns (III), the HAR with asymmetric effects with respect to the returns and lagged returns $(I V)$, the HAR model with only asymmetric effects with respect to the returns $(V)$ and the HAR model with symmetric effects for the past weekly volatility and the returns ( $V I$ ). In particular, the last four models include the different effects that resulted highly significant and relevant in the previous analysis for all the series.

In order to evaluate and compare the performance of the different volatility models, we implement the Diebold and Mariano (1995) test based on the MAE and the MSE, a robust loss function in the sense of Patton (2011) and we consider the Model Confidence Set (MCS) approach of Hansen, Lunde, and Nason (2010) based on the same two loss function ${ }^{3}$. Tables 1.10 and 1.11 report the results for the Diebold and Mariano (1995) test and MCS for the HAR model with the three different variance equations and the two distributions for the residuals for the 16 stocks for the full sample. The goal is to analyze the performance of the different variance and distributional assumption leaving the mean equation constant. Although the results of pairwise comparison with the Diebold and Mariano (1995) test vary across different stocks, the MCS procedure allows for a comparison between all the models.

In particular, the HAR model with GJR-GARCH specification for the variance and the NIG distribution for the innovations belongs to the set of best models in 15 out of 16 stocks. Table 1.12 provides the Diebold and Mariano (1995) test for equal predictive accuracy between the HAR model and the five competing specifications for the mean equation with same variance specification and distribution for the residual of the 16 stocks and the full out-of-sample forecasting. As an example, consider the first t -statistic of the first column for $(B A)$ and equal to -4.51 . This t-statistic is the result of the Diebold and Mariano (1995) test of equal predictive accuracy between the HAR model and the $\operatorname{AR}(1)$ model with constant variance and Normal distribution. As we expected, the t-statistic strongly rejects null hypothesis in favor of the HAR model.

Results show that in 12 out of 16 stocks, the models with asymmetric effects with respect to the return and the volatility $(I I I, I V, V$ and $V I)$ perform significantly better than the HAR model (II) in at least one of the different specifications for the variance and the distribution for the innovations. In particular, in 8 out of 16 series there is at least one model with asymmetric effects with respect to the return and the volatility for each specification for the variance and the distribution for the residuals that performs better than the HAR models. At the

[^4]same time, the HAR model outperforms the simple $\operatorname{AR}(1)$ model in all the assets and three variances and distribution assumptions. Consider the seventh column for $H P Q$. Each of the four models with asymmetric effects perform significantly better than the HAR model at $1 \%$ level. On the contrary, in four assets none of the models perform better than the HAR. Although these four assets belong to different sectors, it seems that the long memory component of the series dominates the asymmetric effects.

The results for the MCS procedure when the 36 models are included in the initial set for the 16 stocks and the full out-of-sample forecasting are reported in table 1.14. In 13 out of 16 stocks, the p-values associated with the HAR model with different specifications for the variance and distribution for the residuals range from 0 to 0.22 . Consequently, the null hypothesis of equal predictive performance cannot be rejected at lower significance levels. However, at $25 \%$ level, the HAR model remains out of the set of best models in these 13 series and at the $15 \%$ level, in six stocks we exclude the six different HAR models from the best set. Regarding the four series where the HAR models display the same performance than the competitors, they are included in the best set at the $10 \%$. Last, the set of best models for the different assets includes models with asymmetric effects with respect to the volatility and the return and with different specifications for the variance and the distribution of the innovations while, as we expected, the $\mathrm{AR}(1)$ model for the mean equation is always excluded at the lowest significant levels. In contrast to the estimation results, the performance of the different models varies across the stocks. However, the introduction of different effects in the mean equation of the volatility provides a significant improvement in the forecast accuracy.

Since the full out-of-sample forecasting includes the 2008-2009 financial crisis, we also analyze the performance of the models during this period (200 days). Table 1.13 and 1.15 report the Diebold and Mariano (1995) test and MCS for the same 36 models considered before for the 16 stocks and the out-of-sample period during financial crisis. Contrary to the results of the analysis of the full out-ofsample forecasting, asymmetric effects in the mean equation do not improve the performance of the models during periods of high volatility. T-statistics for the Diebold and Mariano (1995) test (in table 1.13) do not reject the null hypothesis of equal performance between HAR and the four models with asymmetric effects in 12 out of 16 series. This result is confirmed by the MCS approach were most of the best set includes the HAR models. However, an interesting result emerges from table 1.15. P-values associated to HAR models for the bank's stocks (BAC, C, JPM and WFC) exclude them from the best models at reasonable levels.

### 1.7 Conclusions

In this chapter, we have modeled and forecasted price variation through the realized range volatility introduced by Martens and van Dijk (2007) and Christensen and Podolskij (2007). We have estimated the series for 16 stocks traded at NYSE for different sampling frequencies and corrected them with the scaling procedure of Martens and van Dijk (2007). After the corrections, the volatility
stabilizes across different sampling frequencies and scaling factors, which suggest that the bias caused by the microstructure friction was removed, restoring the efficiency of the estimator. We have considered a model which approximates long memory, has asymmetric effects with respect to the return and the volatility in the mean equation, and includes GARCH and GJR-GARCH specifications for the variance equation (which models the volatility of the volatility). A non Gaussian distribution was also considered for the innovations.

The estimation results of the different considered models in the 16 stocks suggest that the introduction of asymmetric effects with respect to the returns and lagged returns induces a significant improvement in the in-sample fit. We document a strongly significant increment of the volatility after negatives returns, or leverage effects. Moreover, GARCH and GJR-GARCH specification for the variance equation improve the fitness of the models while NIG coefficients are able to capture the right skewness and excess of kurtosis of the series.

In the last part of this work, we have computed one-step-ahead rolling forecast for the 16 assets. The forecast comparison analysis reflects the findings of the in-sample analysis. Taking into consideration the asymmetric effects in the mean equation produces more accurate and statistically significant predictions than the HAR model. As we expected, models with GARCH and GJR-GARCH specifications and the two different distributions for the innovations do not lead to more accurate point forecasts than models with constant variance.

Finally, the HAR components with asymmetric effects are able to capture most of the variability during the out-of-sample prevision. In order to improve this performance, the introduction of financial and macroeconomics variables should be considered. Other future steps are the possible correction for jumps in the volatility series and an economic analysis of the performances of the models.

### 1.8 Tables and Figures

Table 1.1: Descriptive statistic

|  |  |  |  |  |  |  | Volatility |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Mean | St.Dev | Skew | Kurt | Mean | St.Dev | Skew | Kurt | $\sigma^{\text {annual }}$ |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
| BA | 0.0553 | 0.0191 | 0.2344 | 7.9256 | -3.9939 | 0.7969 | 0.9001 | 4.3752 | $23.5871 \%$ |  |  |
| BAC | -0.0421 | 0.0382 | -0.3529 | 25.094 | -4.1253 | 1.5707 | 1.085 | 3.4125 | $29.8134 \%$ |  |  |
| C | -0.0989 | 0.0395 | 0.4240 | 27.401 | -3.8379 | 1.5529 | 0.9733 | 3.2486 | $33.8154 \%$ |  |  |
| CAT | 0.0498 | 0.0220 | -0.1815 | 8.8372 | -3.7869 | 0.8657 | 1.0246 | 4.3198 | $26.6612 \%$ |  |  |
| FDX | 0.0244 | 0.0198 | -0.2691 | 8.2731 | -4.004 | 0.8678 | 0.6636 | 3.3228 | $23.7768 \%$ |  |  |
| HON | 0.0369 | 0.0183 | -0.0622 | 6.9394 | -3.9711 | 0.7762 | 0.9930 | 4.8865 | $23.7787 \%$ |  |  |
| HPQ | 0.0655 | 0.0196 | 0.1723 | 9.8753 | -3.9128 | 0.7815 | 0.6730 | 4.1872 | $24.4143 \%$ |  |  |
| IBM | 0.0231 | 0.0143 | 0.0016 | 9.005 | -4.5841 | 0.8338 | 1.1342 | 5.0097 | $17.7876 \%$ |  |  |
| JPM | 0.0229 | 0.0291 | 0.3502 | 17.860 | -3.9327 | 1.2644 | 0.9281 | 3.3023 | $28.1828 \%$ |  |  |
| KFT | -0.0012 | 0.0131 | -0.1650 | 8.1409 | -4.4415 | 0.7469 | 0.8114 | 4.2079 | $18.6216 \%$ |  |  |
| PEP | 0.0252 | 0.0117 | -0.5823 | 19.314 | -4.773 | 0.7775 | 1.1291 | 5.4579 | $15.9629 \%$ |  |  |
| PG | 0.0201 | 0.0117 | -0.0886 | 10.540 | -4.8362 | 0.7868 | 1.227 | 5.6782 | $15.5205 \%$ |  |  |
| T | 0.0068 | 0.0159 | 0.4430 | 10.769 | -4.2059 | 0.8508 | 0.8970 | 4.3824 | $21.5056 \%$ |  |  |
| TWX | 0.0057 | 0.0196 | 0.0910 | 11.832 | -3.9856 | 0.9131 | 1.0217 | 4.1886 | $24.4534 \%$ |  |  |
| TXN | 0.0154 | 0.0217 | -0.2859 | 6.0939 | -3.4884 | 0.7079 | 0.5266 | 3.7832 | $29.6711 \%$ |  |  |
| WFC | 0.0158 | 0.0322 | 0.7732 | 23.786 | -4.1098 | 1.5129 | 0.9280 | 3.0242 | $28.7103 \%$ |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |

Note: Descriptive statistics for the returns and the of the $\log (\mathrm{RRV})$ series for the 16 stocks: Boeing ( $B A$ ), Bank of America $(B A C)$, Citigroup Inc. ( $C$ ), Caterpillar Inc. ( $C A T$ ), FedEx Corporation ( $F D X$ ), Honeywell International Inc. $(H O N)$, Hewlett-Packard Company ( $H P Q$ ), International Business Machines Corp. (IBM), JPMorgan Chase \& Co. $(J P M)$, Kraft Foods Inc. $(K F T)$, PepsiCo, Inc. $(P E P)$, The Procter \& Gamble Company ( $P G$ ), AT\&T Inc. $(T)$, Time Warner Inc. ( $T W X$ ), Texas Instruments Incorporated ( $T X N$ ) and Wells Fargo \& Company (WFC). $\sigma^{\text {annual }}$ is the mean annualized volatility in percentage terms.


Figure 1.1: Plot and ACF for $B A, I B M$ and $P E P$

| L00 0 | LZ0．0 | L00．0 | L00．0 | 100．0 | L00．0 | L00＇0 | L00．0 | L00．0 | L00．0 | L00＇0 | L00＇0 | L00．0 | L00．0 | L00＇0 | L00＇0 | 77 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $000^{\circ} 0$ | $000^{\circ} 0$ | $000^{\circ} 0$ | $000^{\circ} 0$ | $000^{\circ} 0$ | $000^{\circ} 0$ | $000^{\circ} 0$ | $000^{\circ} 0$ | $000^{\circ} 0$ | $000^{\circ} 0$ | $000^{\circ} 0$ | $000^{\circ} 0$ | $000^{\circ} 0$ | $000^{\circ} 0$ | $000^{\circ} 0$ | $000^{\circ} 0$ | SY |
| L00．0 | L00．0 | L00．0 | L00．0 | L00．0 | L00．0 | L00．0 | L00．0 | L00．0 | L00．0 | $100{ }^{\circ}$ | L00．0 | L00．0 | L00．0 | L00＇0 | L00．0 | gr |
| OtI「0 | 20000 | ¢¢000 | $\angle 900$ | 00 ＇$^{\circ}$ | $900^{\circ} 0$ | ¢LE\％ | 080.0 | L10．0 | 207：0 | ${ }^{9} 00^{\circ} 0$ | gtio | 288．0 | $\angle 70^{\circ} 0$ | $60 \mathrm{I}^{\circ} 0$ | 180\％ 0 | ${ }^{0+} C_{T}$ |
| ヵ6T＇0 | L00 0 | $980{ }^{\circ}$ | DL0 0 | $98 \mathrm{~T}^{\circ} 0$ | \＆10＇0 | DLE 0 | 770．0 | 9L0 0 | $897^{\circ} 0$ | $800^{\circ} 0$ | $690{ }^{\circ}$ | でT＇0 | L10．0 | $810^{\circ} 0$ | $810{ }^{\circ} 0$ | ${ }^{08} C^{\prime} T$ |
| 8．86LZ | $6 \mathrm{Cz8T}$ | 86181 | 6.2807 | \＆L00］ | て＇890\％ | L＇6997 | 6.9907 | 7：872I | ¢＇18Ľ | でも¢LZ | ¢98LZ | て．7LIz | 6． 1807 | $6.798 \%$ | ¢．000\％ | गIG |
| 9．99LZ | 9.962 L | 9762 L | 9．010\％ | 0086 ［ | $6 \cdot 9807$ | 8＇TE92 | $9 \cdot 670{ }^{\text {c }}$ | 6．002I | 0 －tctz | 0＊2\％LZ | 069 Lz | 6 ＇180\％ | 9 －$¢ 00$ | 9 L\＆\＆Z | T＇8L6 | OIV |
| て＇8L01－ | t¢ 868 － | 08＇ L68 $^{-}$ | \＆000 ${ }^{-}$ | 80＇986－ | 6．7101－ | 60 IEL ${ }^{-}$ | 8600I－ | $97^{\text {¢ }}$ ¢78－ | 0＇ZLOL－ | 9．8901－ | G．ELOL－ | も． 280 I－$^{-}$ | £＇\％て0L－ | $8{ }^{\text {8 }}$ 9 LI－$^{\text {－}}$ | 99 ${ }^{\text {＇186 }}{ }^{-}$ | HTT |
| － | － | － | － | － | － | － | － | － | － | － | － | － | － | － | － | $\operatorname{SIN}^{\text {g }}$ |
| － | － | － | － | － | － | － | － | － | － | － | － | － | － | － | － | DINO |
| － | － | － | － | － | － | － | － | － | － | － | － | － | － | － | － | ${ }^{1} \phi$ |
| － | － | － | － | － | － | － | － | － | － | － | － | － | － | － | － | ${ }_{5}$ |
| － | － | － | － | － | － | － | － | － | － | － | － | － | － | － | － | ${ }^{1} g$ |
| （ $900 \cdot 0$ ） | （ 700.0 ） | （800．0） | （ 000 ） | （ 700.0 ） | （ $700 \%$ ） | （900．0） | （ 000.0 ） | （800．0） | （ 700.0 ） | （ $700 \cdot 0$ ） | （900．0） | （ $700 \cdot 0$ ） | （ $000 \cdot 0$ ） | （ 500.0 ） | （ 700.0 ） |  |
| ャ $707^{\circ} 0$ | e¢9T ${ }^{\circ} 0$ | e\＆9T 0 | e98L 0 | ャ781＇0 | －885 0 | e 9970 | в 28 L＇0 $^{\circ}$ | egcto | ${ }_{\text {e }}$ L0\％＇0 | －865＇0 | в $207 \% 0$ | セ $865^{\circ} 0$ | ¢06I＇0 | ャ¢7\％＇0 | ${ }_{\text {e }}$［85\％ 0 | m |
| － | － | － | － | － | － | － | － | － | － | － | － | － | － | － | － | LYIL |
| － | － | － | － | － | － | － | － | － | － | － | － | － | － | － | － | L2 |
| （670．0） | （880．0） | （880 0） | （z80\％0） | （980．0） | （280．0） | （8゙0．0） | （270．0） | （670＇0） | （L80＇0） | （980 0） | （680＇0） | （980＊0） | （870．0） | （870．0） | （ $\ddagger 80^{\circ} 0$ ） |  |
| eでT「0 | e7ヵI ${ }^{\text {¢ }} 0$ | ${ }^{\text {e } 981.0}$ | в995 0 |  | ${ }^{\text {e } 865}{ }^{\circ} 0$ | ${ }^{\text {e }}$［07．0 | －LZİ0 | e901．0 | egctio | e\＆tio | в $8887^{\circ} 0$ | 887T 0 | egzT 0 | ャ691．0 | ィ $707^{\circ} 0$ | ${ }^{u} g$ |
| （680\％0） | （ $770 \cdot 0$ ） | （8̇0．0） | （070．0） | （ $770 \cdot 0$ ） | （E®0．0） | （9E0．0） | （ $2800^{\circ} 0$ ） | （680＇0） | （750．0） | （2T0．0） | （270\％0） | （270．0） | （980．0） | （280．0） | （ti0 0） |  |
| еLZヤ＊0 | в¢97＊0 | e¢LD＊0 | eTLE 0 | ャ097＊0 | 889\％0 | －20才 0 | ${ }_{\text {в } 6980} 0$ | ${ }^{0} 08 \square^{\circ} 0$ | e $628^{\circ} 0$ | －6Z®＊0 | ${ }_{\text {в } 88700}$ | e917＊0 | ¢ 7980 | ¢ ¢08．0 | セ888．0 | ${ }^{m} g$ |
| （8Z0．0） | （9z00） | （z70．0） | （\％て0．0） | （7\％0．0） | （七70．0） | （970＊0） | （z70．0） | （970．0） | （z70．0） | （z70．0） | （970＇0） | （870＇0） | （ $7 \mathrm{Z} 0^{\circ} 0$ ） | （070．0） | $\left(970^{\circ} 0\right)$ |  |
|  |  |  | －リL゙o | «098．0 | ¢ $667^{\circ} 0$ | e\＆67．0 | ャ\＆67．0 | 88L゙0 | とLOt＇0 | ¢0980 | ${ }_{\text {e } 8680} 0$ | e988．0 | －2090 | eLZ9．0 | マ698．0 | ${ }^{p} g$ |
| （L80．0） | （990．0） | （970．0） | （990．0） | （690．0） | （ 220.0 ） | （z01．0） | （ $¢ 80 \cdot 0$ ） | （8¢0．0） | （990．0） | （990．0） | （ 790.0 ） | （9900） | （†70．0） | （L80．0） | （090＊0） |  |
| ง690 0 － | ョ¢87．0－ | egzI．0－ | ¢007．0－ | －6270－ | セ89z．0－ | ョ8¢F＊－ | q980 ${ }^{\circ}$－ | ๕907．0－ | セ89\％．0－ | ะ997．0－ | qLSEO－ | e¢850－ |  | q［90 ${ }^{-}$ | e965：0－ | $\bigcirc$ |
| ОНМ | NXL | XMSL | L | Dd | d＇Ad | LAY | NdS | N＇II | OdH | NOH | ХСง | LVD | 0 | DVG | VG |  |

Table 1.3: HAR model - NIG distribution and GJR-GARCH variance

|  | BA | BAC | C | CAT | FDX | HON | HPQ | IBM | JPM | KFT | PEP | PG | T | TWX | TXN | WFC |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\alpha$ | $\begin{gathered} -0.178^{\mathbf{a}} \\ (0.059) \end{gathered}$ | $\begin{gathered} -0.087^{\mathrm{a}} \\ (0.031) \end{gathered}$ | $\begin{gathered} -0.084^{\mathrm{a}} \\ (0.030) \end{gathered}$ | $\begin{gathered} -0.147^{a} \\ (0.050) \end{gathered}$ | $\begin{aligned} & -0.085 \\ & (0.053) \end{aligned}$ | $\begin{gathered} -0.190^{\mathrm{a}} \\ (0.059) \end{gathered}$ | $\begin{gathered} -0.230^{\mathrm{a}} \\ (0.061) \end{gathered}$ | $\begin{gathered} -0.206^{\mathrm{a}} \\ (0.063) \end{gathered}$ | $\begin{gathered} -0.110^{\mathrm{a}} \\ (0.032) \end{gathered}$ | $\begin{gathered} -0.441^{a} \\ (0.088) \end{gathered}$ | $\begin{gathered} -0.243^{\mathrm{a}} \\ (0.069) \end{gathered}$ | $\begin{gathered} -0.263^{a} \\ (0.076) \end{gathered}$ | $\begin{gathered} -0.204^{\mathrm{a}} \\ (0.056) \end{gathered}$ | $\begin{gathered} -0.144^{\mathrm{a}} \\ (0.042) \end{gathered}$ | $\begin{gathered} -0.192^{\mathrm{a}} \\ (0.061) \end{gathered}$ | $\begin{gathered} -0.107^{a} \\ (0.028) \end{gathered}$ |
| $\beta_{d}$ | $0.357^{\text {a }}$ | $0.498^{\text {a }}$ | $0.447^{\text {a }}$ | $0.403^{\text {a }}$ | $0.381^{\text {a }}$ | $0.337^{\text {a }}$ | $0.410^{\text {a }}$ | $0.380^{\text {a }}$ | $0.496^{\text {a }}$ | $0.281^{\text {a }}$ | $0.285^{\text {a }}$ | $0.334^{\text {a }}$ | $0.388^{\text {a }}$ | $0.396^{\text {a }}$ | $0.337^{\text {a }}$ | $0.432^{\text {a }}$ |
|  | (0.029) | (0.026) | (0.028) | (0.029) | (0.027) | (0.027) | (0.028) | (0.028) | (0.028) | (0.024) | (0.028) | (0.027) | (0.026) | (0.030) | (0.028) | (0.030) |
| $\beta_{w}$ | $0.368^{\text {a }}$ | $0.334^{\text {a }}$ | $0.403^{\text {a }}$ | $0.369^{\text {a }}$ | $0.334^{\text {a }}$ | $0.437^{\text {a }}$ | $0.374^{\text {a }}$ | $0.455^{\text {a }}$ | $0.318^{\text {a }}$ | $0.434^{\text {a }}$ | $0.468^{\text {a }}$ | $0.462^{\text {a }}$ | $0^{0.363}{ }^{\text {a }}$ | $0.359^{\text {a }}$ | $0.446^{\text {a }}$ | $0.367^{\text {a }}$ |
|  | (0.042) | (0.037) | (0.041) | (0.040) | (0.042) | (0.042) | (0.041) | (0.042) | (0.038) | (0.042) | (0.045) | (0.042) | (0.040) | (0.041) | (0.044) | (0.037) |
| $\beta_{m}$ | $0.228^{a}$ (0.034) | $0.145^{a}$ $(0026)$ | $0.127^{a}$ | $0.187^{a}$ | $0.260^{\mathrm{a}}$ | $0.176^{a}$ $(0.033)$ | $0.155^{a}$ $(0,034)$ | $0.120^{\mathrm{a}}$ (0.031) | $0.157^{a}$ | $0.184^{a}$ <br> (0.038) | $0.194^{a}$ $(0.035)$ | $0.148^{a}$ $(0.034)$ | $0.200^{a}$ | $0.206^{\mathrm{a}}$ (0.030) | $0.162^{a}$ | $0.173^{a}$ |
| $\gamma_{R T}$ | ( | ) |  |  |  | (0.033) |  |  |  |  |  |  |  | (0.030) | (0.055) |  |
| $\gamma_{I R T}$ | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| $\omega$ | $\begin{aligned} & 0.063^{\text {b }} \\ & (0.024) \end{aligned}$ | $\begin{aligned} & 0.006^{\mathrm{a}} \\ & (0.002) \end{aligned}$ | $\begin{gathered} 0.006^{a} \\ (0.001) \end{gathered}$ | $\begin{aligned} & 0.082^{\mathrm{a}} \\ & (0.026) \end{aligned}$ | $\begin{aligned} & 0.014^{\mathrm{b}} \\ & (0.005) \end{aligned}$ | $\begin{aligned} & 0.006^{\mathrm{a}} \\ & (0.002) \end{aligned}$ | $\begin{gathered} 0.037^{\mathrm{a}} \\ (0.012) \end{gathered}$ | $\begin{aligned} & 0.004^{\text {b }} \\ & (0.001) \end{aligned}$ | $\begin{aligned} & 0.105^{\mathrm{a}} \\ & (0.025) \end{aligned}$ | $\begin{gathered} 0.000 \\ (0.000) \end{gathered}$ | $\begin{aligned} & 0.013^{a} \\ & (0.004) \end{aligned}$ | $\begin{aligned} & 0.009^{\mathrm{a}} \\ & (0.003) \end{aligned}$ | $\begin{gathered} 0.006^{a} \\ (0.002) \end{gathered}$ | $\begin{aligned} & 0.072^{\mathrm{a}} \\ & (0.021) \end{aligned}$ | $\begin{aligned} & 0.003^{b} \\ & (0.001) \end{aligned}$ | $\begin{aligned} & 0.135^{\mathrm{a}} \\ & (0.024) \end{aligned}$ |
| $\beta_{1}$ | $\begin{aligned} & 0.562^{\mathrm{a}} \\ & (0.150) \end{aligned}$ | $\begin{aligned} & 0.940^{a} \\ & (0.016) \end{aligned}$ | $\begin{gathered} 0.932^{a} \\ (0.016) \end{gathered}$ | $\begin{aligned} & 0.467^{\mathrm{a}} \\ & (0.155) \end{aligned}$ | $\begin{aligned} & 0.878^{a} \\ & (0.039) \end{aligned}$ | $\begin{aligned} & 0.915^{a} \\ & (0.020) \end{aligned}$ | $\begin{aligned} & 0.753^{a} \\ & (0.073) \end{aligned}$ | $\begin{aligned} & 0.946^{\mathrm{a}} \\ & (0.015) \end{aligned}$ | $\begin{aligned} & 0.318^{b} \\ & (0.148) \end{aligned}$ | $\begin{aligned} & 0.983^{a} \\ & (0.006) \end{aligned}$ | $\begin{aligned} & 0.876^{\mathbf{a}} \\ & (0.033) \end{aligned}$ | $\begin{aligned} & 0.898^{\mathbf{a}} \\ & (0.029) \end{aligned}$ | $\begin{aligned} & 0.933^{a} \\ & (0.019) \end{aligned}$ | $\begin{aligned} & 0.443^{a} \\ & (0.147) \end{aligned}$ | $\begin{aligned} & 0.951^{1} \\ & (0.015) \end{aligned}$ | $\begin{gathered} 0.180 \\ (0.123) \end{gathered}$ |
| $\alpha_{1}$ | $0.110^{\text {b }}$ | $0.040^{\mathrm{a}}$ | $0.061^{\text {a }}$ | $0.134^{a}$ | $0.056^{a}$ | $0.070^{a}$ | $0.109^{a}$ $(0,032)$ | $0.042^{\text {a }}$ | $0.192^{\mathrm{a}}$ | $0.008^{\text {b }}$ | $0.086^{\text {a }}$ | $0.063^{\text {a }}$ | $0.049^{a}$ | $0.154^{\text {a }}$ | $0.032^{\text {a }}$ | $0.255^{\text {a }}$ |
| $\phi_{1}$ | $\begin{gathered} (0.043) \\ -0.0 .026 \\ (0.053) \end{gathered}$ | $\begin{aligned} & -0.026 \\ & (0.020) \end{aligned}$ | $\begin{gathered} (0.014) \\ -0.061^{\mathrm{a}} \\ (0.021) \end{gathered}$ | $\begin{gathered} (0.050) \\ -0.072 \\ (0.060) \end{gathered}$ | $\begin{gathered} (0.018) \\ -0.013 \\ (0.028) \end{gathered}$ | $\begin{gathered} (0.017) \\ -0.046^{\mathrm{b}} \\ (0.021) \end{gathered}$ | $\begin{gathered} (0.032) \\ -0.105^{a} \\ (0.037) \end{gathered}$ | $\begin{gathered} (0.012) \\ -0.036^{\mathrm{b}} \\ (0.015) \end{gathered}$ | $\begin{gathered} (0.061) \\ -0.165^{\text {b }} \\ (0.066) \end{gathered}$ | $\begin{gathered} (0.004) \\ 0.013 \\ (0.008) \end{gathered}$ | $\begin{gathered} (0.022) \\ -0.002^{\mathrm{a}} \\ (0.026) \end{gathered}$ | $\begin{gathered} (0.017) \\ -0.036^{\mathrm{c}} \\ (0.022) \end{gathered}$ | $\begin{gathered} (0.013) \\ -0.038^{\mathrm{b}} \\ (0.017) \end{gathered}$ | $\begin{gathered} (0.046) \\ -0.131^{\mathrm{b}} \\ (0.056) \end{gathered}$ | $\begin{aligned} & (0.010) \\ & -0.010 \\ & (0.014) \end{aligned}$ | $\begin{gathered} (0.063) \\ -0.2^{\mathrm{a}} \\ (0.074) \end{gathered}$ |
| $\alpha_{\text {NIG }}$ | $1.591^{\text {a }}$ | $1.779^{\text {a }}$ | $2.225^{\text {a }}$ | $1.640^{\text {a }}$ | $1.727^{\text {a }}$ | $1.578^{\text {a }}$ | $1.560^{\text {a }}$ | $1.76{ }^{\text {a }}$ | $1.859^{\text {a }}$ | $1.523^{\text {a }}$ | $1.788^{\text {a }}$ | 1.584 ${ }^{\text {a }}$ | $1.644^{\text {a }}$ | $1.558^{\text {a }}$ | $2.155^{\text {a }}$ | $1.917^{\text {a }}$ |
|  | (0.178) | (0.243) | (0.350) | (0.180) | (0.236) | (0.166) | (0.164) | (0.216) | (0.239) | (0.170) | (0.223) | (0.181) | (0.194) | (0.165) | (0.328) | (0.264) |
| $\beta_{\text {NIG }}$ | $0.488^{\text {a }}$ | 0.709 ${ }^{\text {a }}$ | $0.785^{\text {a }}$ | $0.566^{\text {a }}$ | $0.599^{\text {a }}$ | $0.556^{\text {a }}$ | $0.406^{\text {a }}$ | $0.278^{\text {a }}$ | $0.624^{\text {a }}$ | $0.526^{\text {a }}$ | $0.548^{\text {a }}$ | $0.429^{\text {a }}$ | 0.485 ${ }^{\text {a }}$ | $0.464^{\text {a }}$ | $0.394^{\text {b }}$ | $0.796^{\text {a }}$ |
|  | (0.103) | (0.188) | (0.225) | (0.121) | (0.153) | (0.116) | (0.092) | (0.107) | (0.152) | (0.110) | (0.140) | (0.112) | (0.121) | (0.110) | (0.172) | (0.176) |
| LLF | -913.93 | -1023.4 | -912.08 | -933.21 | -973.66 | -921.22 | -990.72 | -776.71 | -927.60 | -1185.0 | -905.29 | -884.67 | -901.95 | -768.55 | -841.14 | -985.00 |
| AIC | 1847.8 | 2066.8 | 1844.1 | 1886.4 | 1967.3 | 1862.4 | 2001.4 | 1573.4 | 1875.2 | 2390.1 | 1830.5 | 1789.3 | 1823.9 | 1557.1 | 1702.2 | 1990.0 |
| BIC | 1902.4 | 2121.4 | 1898.7 | 1941.0 | 2021.9 | 1917.0 | 2056.0 | 1628.0 | 1929.8 | 2444.7 | 1885.1 | 1843.9 | 1878.5 | 1611.6 | 1756.8 | 2044.6 |
| $L j_{30}$ | 0.014 | 0.116 | 0.075 | 0.171 | 0.072 | 0.005 | 0.318 | 0.149 | 0.021 | 0.237 | 0.051 | 0.164 | 0.075 | 0.193 | 0.008 | 0.170 |
| $L_{j}{ }_{0}$ | 0.027 | 0.238 | 0.204 | 0.368 | 0.124 | 0.006 | 0.226 | 0.195 | 0.054 | 0.247 | 0.012 | 0.388 | 0.247 | 0.128 | 0.011 | 0.428 |
| $J B$ | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| KS | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| LL | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |

[^5] and $L L$ is the Lilliefors test. Standard errors in bracket. "a", "b" and "c" indicate significance at the $1 \%, 5 \%$ and $10 \%$.
and $B I C$ is the Bayesian information criterion．$L J_{30}$ and $L J_{40}$ are the Ljung Box test for 30 and 40 lags．$J B$ is the Jarq
and $L L$ is the Lilliefors test．Standard errors in bracket．＂a＂，＂b＂and＂c＂indicate significance at the $1 \%, 5 \%$ and $10 \%$ ． Note：Estimation results for the 16 series of stock volatility for the period January 2003 to March 2010．LLF is the Log－likelihood function，AIC is the Akaike Information Criteria

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[^6]Table 1.5: HAR model with asymmetric effects with respect to the returns - NIG distribution and GJR-GARCH variance

|  | BA | BAC | C | CAT | FDX | HON | HPQ | IBM | JPM | KFT | PEP | PG | T | TWX | TXN | WFC |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\alpha$ | $\begin{gathered} -0.386^{a} \\ (0.070) \end{gathered}$ | $\begin{gathered} -0.341^{\mathrm{a}} \\ (0.040) \end{gathered}$ | $\begin{gathered} -0.300^{\mathbf{a}} \\ (0.040) \end{gathered}$ | $\begin{gathered} -0.373^{a} \\ (0.063) \end{gathered}$ | $\begin{gathered} -0.219^{\mathrm{a}} \\ (0.069) \end{gathered}$ | $\begin{gathered} -0.383^{\mathrm{a}} \\ (0.071) \end{gathered}$ | $\begin{gathered} -0.447^{\mathrm{a}} \\ (0.069) \end{gathered}$ | $\begin{gathered} -0.400^{\mathrm{a}} \\ (0.075) \end{gathered}$ | $\begin{gathered} -0.386^{\mathrm{a}} \\ (0.045) \end{gathered}$ | $\begin{gathered} -0.619^{\mathrm{a}} \\ (0.100) \end{gathered}$ | $\begin{gathered} -0.516^{\mathrm{a}} \\ (0.080) \end{gathered}$ | $\begin{gathered} -0.449^{a} \\ (0.084) \end{gathered}$ | $\begin{gathered} -0.418^{\mathbf{a}} \\ (0.067) \end{gathered}$ | $\begin{gathered} -0.340^{\mathrm{a}} \\ (0.058) \end{gathered}$ | $\begin{gathered} -0.304^{\mathrm{a}} \\ (0.067) \end{gathered}$ | $\begin{gathered} -0.287^{\mathrm{a}} \\ (0.042) \end{gathered}$ |
| $\beta_{d}$ | $\begin{aligned} & 0.317^{a} \\ & (0.030) \end{aligned}$ | $\begin{aligned} & 0.498^{a} \\ & (0.026) \end{aligned}$ | $\begin{aligned} & 0.397^{a} \\ & (0.028) \end{aligned}$ | $\begin{aligned} & 0.377^{\mathrm{a}} \\ & (0.029) \end{aligned}$ | $\begin{aligned} & 0.355^{\mathrm{a}} \\ & (0.027) \end{aligned}$ | $\begin{aligned} & 0.307^{\mathrm{a}} \\ & (0.027) \end{aligned}$ | $\begin{aligned} & 0.351^{\mathrm{a}} \\ & (0.029) \end{aligned}$ | $\begin{aligned} & 0.338^{a} \\ & (0.028) \end{aligned}$ | $\begin{aligned} & 0.454^{\mathrm{a}} \\ & (0.028) \end{aligned}$ | $\begin{aligned} & 0.279^{a} \\ & (0.029) \end{aligned}$ | $\begin{aligned} & 0.253^{a} \\ & (0.029) \end{aligned}$ | $\begin{aligned} & 0.309^{\mathrm{a}} \\ & (0.028) \end{aligned}$ | $\begin{aligned} & 0.353^{\mathrm{a}} \\ & (0.027) \end{aligned}$ | $\begin{aligned} & 0.370^{a} \\ & (0.030) \end{aligned}$ | $\begin{aligned} & 0.307^{\mathrm{a}} \\ & (0.029) \end{aligned}$ | $\begin{aligned} & 0.410^{\mathrm{a}} \\ & (0.029) \end{aligned}$ |
| $\beta_{w}$ | $\begin{aligned} & 0.383^{a} \\ & (0.042) \end{aligned}$ | $\begin{aligned} & 0.259^{a} \\ & (0.034) \end{aligned}$ | $\begin{aligned} & 0.397^{\mathrm{a}} \\ & (0.040) \end{aligned}$ | $\begin{aligned} & 0.353^{a} \\ & (0.039) \end{aligned}$ | $\begin{gathered} 0.34^{\mathrm{a}} \\ (0.042) \end{gathered}$ | $\begin{aligned} & 0.451^{\mathrm{a}} \\ & (0.041) \end{aligned}$ | $\begin{aligned} & 0.398^{a} \\ & (0.040) \end{aligned}$ | $\begin{gathered} 0.467^{\mathrm{a}} \\ (0.041) \end{gathered}$ | $\begin{aligned} & 0.324^{a} \\ & (0.036) \end{aligned}$ | $\begin{aligned} & 0.409^{\mathbf{a}} \\ & (0.042) \end{aligned}$ | $\begin{aligned} & 0.470^{\mathrm{a}} \\ & (0.044) \end{aligned}$ | $0.466^{a}$ <br> (0.041) | $\begin{gathered} 0.377^{\mathrm{a}} \\ (0.040) \end{gathered}$ | $\begin{aligned} & 0.353^{a} \\ & (0.040) \end{aligned}$ | $\begin{aligned} & 0.453^{\mathrm{a}} \\ & (0.044) \end{aligned}$ | $\begin{aligned} & 0.364^{\mathrm{a}} \\ & (0.035) \end{aligned}$ |
| $\beta_{m}$ | $\begin{aligned} & 0.216^{a} \\ & (0.033) \end{aligned}$ | $\begin{aligned} & 0.173^{a} \\ & (0.022) \end{aligned}$ | $\begin{aligned} & 0.143^{a} \\ & (0.027) \end{aligned}$ | $\begin{aligned} & 0.186^{a} \\ & (0.031) \end{aligned}$ | $\begin{aligned} & 0.256^{a} \\ & (0.036) \end{aligned}$ | $\begin{aligned} & 0.154^{a} \\ & (0.032) \end{aligned}$ | $\begin{aligned} & 0.150^{a} \\ & (0.033) \end{aligned}$ | $\begin{aligned} & 0.117^{a} \\ & (0.030) \end{aligned}$ | $\begin{aligned} & 0.142^{a} \\ & (0.024) \end{aligned}$ | $\begin{aligned} & 0.183^{a} \\ & (0.037) \end{aligned}$ | $\begin{aligned} & 0.180^{a} \\ & (0.034) \end{aligned}$ | $\begin{aligned} & 0.140^{a} \\ & (0.033) \end{aligned}$ | $\begin{gathered} 0.183^{a} \\ (0.031) \end{gathered}$ | $\begin{aligned} & 0.201^{\mathrm{a}} \\ & (0.030) \end{aligned}$ | $\begin{aligned} & 0.164^{\mathrm{a}} \\ & (0.034) \end{aligned}$ | $\begin{aligned} & 0.166^{\mathbf{a}} \\ & (0.025) \end{aligned}$ |
| $\gamma_{R T}$ | $\begin{aligned} & 2.420^{b} \\ & (1.026) \end{aligned}$ | $\begin{aligned} & 2.231^{a} \\ & (0.426) \end{aligned}$ | $\begin{aligned} & 1.244^{a} \\ & (0.403) \end{aligned}$ | $\begin{aligned} & 2.862^{a} \\ & (0.957) \end{aligned}$ | $\begin{gathered} 0.626 \\ (0.908) \end{gathered}$ | $\begin{gathered} 1.536 \\ (1.027) \end{gathered}$ | $\begin{aligned} & 1.358 \\ & (0.931) \end{aligned}$ | $\begin{gathered} 0.952 \\ (1.332) \end{gathered}$ | $\begin{aligned} & 2.634^{a} \\ & (0.594) \end{aligned}$ | $\begin{aligned} & 5.040^{a} \\ & (1.811) \end{aligned}$ | $\begin{aligned} & 2.686^{c} \\ & (1.580) \end{aligned}$ | $\begin{gathered} 2.009 \\ (1.609) \end{gathered}$ | $\begin{aligned} & 3.505^{a} \\ & (1.219) \end{aligned}$ | $\begin{aligned} & 2.614^{a} \\ & (0.818) \end{aligned}$ | $\begin{gathered} 0.968 \\ (0.841) \end{gathered}$ | $\begin{aligned} & 1.399^{b} \\ & (0.572) \end{aligned}$ |
| $\gamma_{I R T}$ | $\begin{gathered} -8.176^{\mathrm{a}} \\ (1.773) \end{gathered}$ | $\begin{gathered} -6.925^{\mathrm{a}} \\ (0.661) \end{gathered}$ | $\begin{gathered} -6.407^{a} \\ (0.683) \end{gathered}$ | $\begin{aligned} & -8.469^{\mathrm{a}} \\ & (1.526) \end{aligned}$ | $\begin{aligned} & -5.285^{a} \\ & (1.525) \end{aligned}$ | $\begin{gathered} -7.343^{\mathrm{a}} \\ (1.755) \end{gathered}$ | $\begin{gathered} -9.728^{a} \\ (1.646) \end{gathered}$ | $\begin{gathered} -10.25^{\mathrm{a}} \\ (2.167) \end{gathered}$ | $\begin{gathered} -9.333^{a} \\ (1.012) \end{gathered}$ | $\begin{gathered} -12.42^{\mathrm{a}} \\ (2.862) \end{gathered}$ | $\begin{gathered} -15.89^{\mathrm{a}} \\ (2.630) \end{gathered}$ | $\begin{gathered} -10.87^{\mathrm{a}} \\ (2.584) \end{gathered}$ | $\begin{gathered} -10.41^{\mathrm{a}} \\ (2.034) \end{gathered}$ | $\begin{gathered} -8.054^{\mathrm{a}} \\ (1.437) \end{gathered}$ | $\begin{gathered} -5.198^{a} \\ (1.365) \end{gathered}$ | $\begin{gathered} -6.541^{\mathrm{a}} \\ (0.943) \end{gathered}$ |
| $\omega$ | $\begin{aligned} & 0.062^{\text {b }} \\ & (0.026) \end{aligned}$ | $\begin{aligned} & 0.184^{\mathrm{a}} \\ & (0.009) \end{aligned}$ | $\begin{aligned} & 0.006^{\mathrm{a}} \\ & (0.002) \end{aligned}$ | $\begin{aligned} & 0.119^{\mathrm{a}} \\ & (0.025) \end{aligned}$ | $\begin{gathered} 0.016^{\mathrm{b}} \\ (0.006) \end{gathered}$ | $\begin{aligned} & 0.008^{\mathrm{a}} \\ & (0.002) \end{aligned}$ | $\begin{aligned} & 0.049^{a} \\ & (0.015) \end{aligned}$ | $\begin{aligned} & 0.004^{\mathrm{b}} \\ & (0.001) \end{aligned}$ | $\begin{aligned} & 0.097^{\mathrm{a}} \\ & (0.025) \end{aligned}$ | $\begin{aligned} & 0.149^{\mathrm{a}} \\ & (0.039) \end{aligned}$ | $\begin{gathered} 0.023^{\mathrm{a}} \\ (0.006) \end{gathered}$ | $\begin{gathered} 0.011^{\mathrm{a}} \\ (0.004) \end{gathered}$ | $\begin{aligned} & 0.009^{\mathrm{b}} \\ & (0.003) \end{aligned}$ | $\begin{aligned} & 0.090^{\mathrm{a}} \\ & (0.022) \end{aligned}$ | $\begin{gathered} 0.003^{\text {b }} \\ (0.001) \end{gathered}$ | $\begin{aligned} & 0.133^{a} \\ & (0.022) \end{aligned}$ |
| $\beta_{1}$ | $\begin{aligned} & 0.572^{a} \\ & (0.160) \end{aligned}$ | $\begin{gathered} 4.819 \\ (0.001) \end{gathered}$ | $\begin{aligned} & 0.922^{\mathrm{a}} \\ & (0.021) \end{aligned}$ | $\begin{gathered} 0.229 \\ (0.148) \end{gathered}$ | $\begin{aligned} & 0.866^{a} \\ & (0.044) \end{aligned}$ | $\begin{aligned} & 0.905^{\mathrm{a}} \\ & (0.023) \end{aligned}$ | $\begin{aligned} & 0.666^{a} \\ & (0.089) \end{aligned}$ | $\begin{aligned} & 0.946^{a} \\ & (0.016) \end{aligned}$ | $\begin{aligned} & 0.335^{b} \\ & (0.160) \end{aligned}$ | $\begin{aligned} & 0.342^{\mathrm{b}} \\ & (0.162) \end{aligned}$ | $\begin{aligned} & 0.797^{a} \\ & (0.050) \end{aligned}$ | $\begin{aligned} & 0.884^{a} \\ & (0.035) \end{aligned}$ | $\begin{aligned} & 0.910^{a} \\ & (0.027) \end{aligned}$ | $\begin{aligned} & 0.310^{\mathrm{b}} \\ & (0.155) \end{aligned}$ | $\begin{aligned} & 0.949^{a} \\ & (0.015) \end{aligned}$ | $\begin{gathered} 0.159 \\ (0.118) \end{gathered}$ |
| $\alpha_{1}$ | $\begin{aligned} & 0.103^{\text {b }} \\ & (0.042) \end{aligned}$ | $\begin{aligned} & 0.205^{\mathrm{a}} \\ & (0.058) \end{aligned}$ | $\begin{aligned} & 0.064^{\mathrm{a}} \\ & (0.015) \end{aligned}$ | $\begin{aligned} & 0.198^{\mathbf{a}} \\ & (0.064) \end{aligned}$ | $\begin{aligned} & 0.059^{a} \\ & (0.019) \end{aligned}$ | $\begin{aligned} & 0.075^{a} \\ & (0.019) \end{aligned}$ | $\begin{aligned} & 0.140^{\mathrm{a}} \\ & (0.038) \end{aligned}$ | $\begin{aligned} & 0.041^{a} \\ & (0.012) \end{aligned}$ | $\begin{aligned} & 0.184^{\mathrm{a}} \\ & (0.060) \end{aligned}$ | $\begin{aligned} & 0.158^{\mathrm{a}} \\ & (0.053) \end{aligned}$ | $\begin{aligned} & 0.120^{\mathrm{a}} \\ & (0.031) \end{aligned}$ | $\begin{aligned} & 0.067^{\mathrm{a}} \\ & (0.019) \end{aligned}$ | $\begin{aligned} & 0.058^{\mathrm{a}} \\ & (0.016) \end{aligned}$ | $\begin{aligned} & 0.173^{\mathrm{a}} \\ & (0.054) \end{aligned}$ | $\begin{aligned} & 0.033^{a} \\ & (0.010) \end{aligned}$ | $\begin{aligned} & 0.266^{\mathrm{a}} \\ & (0.064) \end{aligned}$ |
| $\phi_{1}$ | $\begin{aligned} & -0.041 \\ & (0.051) \end{aligned}$ | $\begin{aligned} & -0.205^{a} \\ & (0.065) \end{aligned}$ | $\begin{gathered} -0.045^{\mathrm{b}} \\ (0.018) \end{gathered}$ | $\begin{aligned} & -0.128^{c} \\ & (0.075) \end{aligned}$ | $\begin{aligned} & -0.018 \\ & (0.030) \end{aligned}$ | $\begin{gathered} -0.052^{\mathrm{b}} \\ (0.023) \end{gathered}$ | $\begin{gathered} -0.122^{\mathrm{a}} \\ (0.045) \end{gathered}$ | $\begin{aligned} & -0.033^{\mathrm{b}} \\ & (0.015) \end{aligned}$ | $\begin{aligned} & -0.152^{\mathrm{b}} \\ & (0.065) \end{aligned}$ | $\begin{gathered} -0.158^{\mathrm{b}} \\ (0.069) \end{gathered}$ | $\begin{gathered} -0.106^{\mathrm{a}} \\ (0.036) \end{gathered}$ | $\begin{gathered} -0.045^{\mathbf{c}} \\ (0.024) \end{gathered}$ | $\begin{gathered} -0.046^{\mathrm{b}} \\ (0.020) \end{gathered}$ | $\begin{gathered} -0.142^{\mathrm{b}} \\ (0.065) \end{gathered}$ | $\begin{aligned} & -0.011 \\ & (0.015) \end{aligned}$ | $\begin{aligned} & -0.193^{\mathrm{b}} \\ & (0.076) \end{aligned}$ |
| $\alpha_{\text {NIG }}$ | $\begin{aligned} & 1.565^{\mathrm{a}} \\ & (0.173) \end{aligned}$ | $\begin{aligned} & 1.534^{\mathrm{a}} \\ & (0.165) \end{aligned}$ | $\begin{aligned} & 2.003^{a} \\ & (0.285) \end{aligned}$ | $\begin{aligned} & 1.631^{\mathrm{a}} \\ & (0.178) \end{aligned}$ | $\begin{aligned} & 1.649^{\mathbf{a}} \\ & (0.215) \end{aligned}$ | $\begin{aligned} & 1.541^{\mathrm{a}} \\ & (0.161) \end{aligned}$ | $\begin{aligned} & 1.566^{\mathrm{a}} \\ & (0.160) \end{aligned}$ | $\begin{aligned} & 1.725^{\mathrm{a}} \\ & (0.206) \end{aligned}$ | $\begin{aligned} & 1.680^{\mathbf{a}} \\ & (0.197) \end{aligned}$ | $\begin{aligned} & 1.456^{\mathrm{a}} \\ & (0.165) \end{aligned}$ | $\begin{aligned} & 1.713^{\mathrm{a}} \\ & (0.207) \end{aligned}$ | $\begin{aligned} & 1.528^{\mathrm{a}} \\ & (0.170) \end{aligned}$ | $\begin{aligned} & 1.643^{\mathrm{a}} \\ & (0.186) \end{aligned}$ | $\begin{aligned} & 1.545^{\mathrm{a}} \\ & (0.167) \end{aligned}$ | $\begin{gathered} 2.031^{\mathrm{a}} \\ (0.290) \end{gathered}$ | $\begin{aligned} & 1.801^{\mathrm{a}} \\ & (0.242) \end{aligned}$ |
| $\beta_{\text {NIG }}$ | $\begin{aligned} & 0.476^{a} \\ & (0.100) \end{aligned}$ | $\begin{aligned} & 0.524^{a} \\ & (0.120) \end{aligned}$ | $\begin{aligned} & 0.607^{a} \\ & (0.172) \end{aligned}$ | $\begin{aligned} & 0.565^{a} \\ & (0.120) \end{aligned}$ | $\begin{aligned} & 0.544^{a} \\ & (0.137) \end{aligned}$ | $\begin{aligned} & 0.520^{\mathrm{a}} \\ & (0.111) \end{aligned}$ | $\begin{aligned} & 0.416^{a} \\ & (0.091) \end{aligned}$ | $\begin{aligned} & 0.227^{7} \\ & (0.100) \end{aligned}$ | $\begin{aligned} & 0.528^{a} \\ & (0.124) \end{aligned}$ | $\begin{aligned} & 0.497^{a} \\ & (0.107) \end{aligned}$ | $\begin{aligned} & 0.504^{a} \\ & (0.131) \end{aligned}$ | $\begin{aligned} & 0.368^{\mathrm{a}} \\ & (0.103) \end{aligned}$ | $\begin{aligned} & 0.469^{a} \\ & (0.115) \end{aligned}$ | $\begin{aligned} & 0.452^{\mathrm{a}} \\ & (0.112) \end{aligned}$ | $\begin{aligned} & 0.329^{\mathrm{b}} \\ & (0.149) \end{aligned}$ | $\begin{aligned} & 0.713^{a} \\ & (0.159) \end{aligned}$ |
| LLF | -897.49 | -990.97 | -874.13 | -913.25 | -960.65 | -905.04 | -960.58 | -748.61 | -877.06 | -1188.1 | -876.61 | -868.96 | -887.70 | -751.91 | -827.93 | -951.47 |
| AIC | 1818.9 | 2005.9 | 1772.2 | 1850.5 | 1945.3 | 1834.0 | 1945.1 | 1521.2 | 1778.1 | 2400.3 | 1777.2 | 1761.9 | 1799.4 | 1527.8 | 1679.8 | 1926.9 |
| BIC | 1884.4 | 2071.4 | 1837.7 | 1916.0 | 2010.8 | 1899.6 | 2010.6 | 1586.7 | 1843.6 | 2465.8 | 1842.7 | 1827.4 | 1864.9 | 1593.3 | 1745.3 | 1992.4 |
| $L j_{30}$ | 0.025 | 0.046 | 0.04 | 0.164 | 0.050 | 0.005 | 0.107 | 0.074 | 0.070 | 0.326 | 0.050 | 0.271 | 0.094 | 0.167 | 0.010 | 0.170 |
| $L j_{40}$ | 0.038 | 0.097 | 0.134 | 0.334 | 0.075 | 0.007 | 0.072 | 0.094 | 0.180 | 0.289 | 0.007 | 0.577 | 0.241 | 0.118 | 0.022 | 0.442 |
| JB | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| KS | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| LL | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |

Note: Estimation results for the 16 series of stock volatility for the period January 2003 to March 2010. $L L F$ is the Log-likelihood function, $A I C$ is the Akaike Information Criteria
and BIC is the Bayesian information criterion. $L J_{30}$ and $L J_{40}$ are the Ljung Box test for 30 and 40 lags. $J B$ is the Jarque-Bera test for Normality, $K S$ is the Kolmogorov-Smirnov and $L L$ is the Lilliefors test. Standard errors in bracket. "a", "b" and "c" indicate significance at the $1 \%, 5 \%$ and $10 \%$.
 heteroskedasticity specifications．See the note in table 1.12 for the description of the models．The $L R 1$ is a likelihood ratio test to asses the significance of the introduction of


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| ！ $\mathrm{n}-\mathrm{I} \Lambda$ | ！$\dagger$－ | ！ $\mathrm{D}-\mathrm{NI}$ | ！ $\mathrm{D}-\mathrm{III}$ | ［D－II | ！D－I | セサーİ | ロワー | ¢刀－$\wedge$ I | セわ－III | セわ－II | セワ－I | $0 \mathrm{O}-\mathrm{I} \Lambda$ | $0 \sim-\Lambda$ | $0{ }^{0}-1$. | O－D－III | O－D－II | O，－I | LVP |
| LIO．0 | 210\％ 0 | 970 0 | 090\％ 0 | $000 \cdot 0$ | $000 \cdot 0$ | － | － | － | － | － | － | － | － | － | － | － | － | £ЧТ |
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| ！ $\mathrm{n}-\mathrm{I} \Lambda$ | ！ | ！ $\mathrm{D}^{-1}$ | ！ $\mathrm{D}-\mathrm{III}$ | ！n－II | ！n－I | セサーİ | セサー $\Lambda$ | ¢ヵ－$\wedge$ I | セサーIII | en－II | セサーI | O－D－I | $0-\mathrm{O}$ | $0{ }_{0}-\Lambda \mathrm{I}$ | O－D－III | $0 \mathrm{O}-\mathrm{II}$ | OD－I | OVG |
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| ！ $\mathrm{D}-\mathrm{I} \Lambda$ | ！n－ |  | ！ $\mathrm{D}-\mathrm{III}$ | ！n－II | ！n－I | ロサーİ | ロŋ－ | $\cdots \mathrm{g}-\Lambda \mathrm{I}$ | セわ－III | セわ－II | ロソ－I | O－D－İ | $0 \sim-\Lambda$ | $0{ }^{0}-1$. | OD－III | o－D－II | OD－I | vg |


Table 1.7: Likelihood-ratio test (cont.)

| FDX | I-Co | II-Co | III-Co | IV-Co | V-Co | VI-Co | I-Ga | II-Ga | III-Ga <br> Normal | IV-Ga | V-Ga | VI-Ga | I-Gj | II-Gj | III-Gj | IV-Gj | V-Gj | VI-Gj |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| LR1 | - | - | 0.000 | 0.000 | 0.000 | 0.000 | - | - | 0.000 | 0.000 | 0.000 | 0.000 | - | - | 0.000 | 0.000 | 0.000 | 0.000 |
| LR2 | - | - | - | - | - | - | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| LR3 | - | - | - | - | - | - | - | - | - | - | - | - | 0.525 | 0.211 | 0.182 | 0.197 | 0.197 | 0.223 |
| LR1 | - | - | 0.000 | 0.000 | 0.000 | 0.000 | - | - | $\begin{gathered} \text { NIG } \\ 0.000 \end{gathered}$ | 0.000 | 0.000 | 0.000 | - | - | 0.000 | 0.000 | 0.000 | 0.000 |
| LR2 | - | - | - | - | - | - | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| LR3 | - | - | - | - | - | - | - | - | - | - | - | - | 0.001 | 0.648 | 0.593 | 0.551 | 0.551 | 0.522 |
| HON | I-Co | II-Co | III-Co | IV-Co | V-Co | VI-Co | I-Ga | II-Ga | III-Ga <br> Normal | IV-Ga | V-Ga | VI-Ga | I-Gj | II-Gj | III-Gj | IV-Gj | V-Gj | VI-Gj |
| LR1 | - | - | 0.000 | 0.000 | 0.000 | 0.000 | - | - | 0.000 | 0.000 | 0.000 | 0.000 | - | - | 0.000 | 0.000 | 0.000 | 0.000 |
| LR2 | - | - | - | - | - | - | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| LR3 | - | - | - | - | - | - | - | - | - | - | - | - | 0.002 | 0.011 | 0.006 | 0.006 | 0.005 | 0.007 |
| LR1 | - | - | 0.000 | 0.000 | 0.000 | 0.000 | - | - | $\begin{gathered} \text { NIG } \\ 0.000 \end{gathered}$ | 0.000 | 0.000 | 0.000 | - | - | 0.000 | 0.000 | 0.000 | 0.000 |
| LR2 | - | - | - | - | - | - | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| LR3 | - | - | - | - | - | - | - | - | - | - | - | - | 0.000 | 0.036 | 0.027 | 0.026 | 0.023 | 0.025 |
| HPQ | $\mathrm{I}-\mathrm{Co}$ | II-Co | III-Co | IV-Co | V-Co | VI-Co | I-Ga | II-Ga | III-Ga <br> Normal | IV-Ga | V-Ga | VI-Ga | I-Gj | II-Gj | III-Gj | IV-Gj | V-Gj | VI-Gj |
| LR1 | - | - | 0.000 | 0.000 | 0.000 | 0.000 | - | - | 0.000 | 0.000 | 0.000 | 0.000 | - | - | 0.000 | 0.000 | 0.000 | 0.000 |
| LR2 | - | - | - | - | - | - | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| LR3 | - | - | - | - | - | - | - | - | - | - | - | - | 0.000 | 0.000 | 0.004 | 0.002 | 0.002 | 0.002 |
| LR1 | - | - | 0.000 | 0.000 | 0.000 | 0.000 | - | - | $\begin{gathered} \text { NIG } \\ 0.000 \end{gathered}$ | 0.000 | 0.000 | 0.000 | - | - | 0.000 | 0.000 | 0.000 | 0.000 |
| LR2 | - | - | - | - | - | - | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| LR3 | - | - | - | - | - | - | - | - | - | - | - | - | 0.000 | 0.001 | 0.006 | 0.005 | 0.005 | 0.005 |
| IBM | I-Co | II-Co | III-Co | IV-Co | V-Co | VI-Co | I-Ga | II-Ga | III-Ga <br> Normal | IV-Ga | V-Ga | VI-Ga | I-Gj | II-Gj | III-Gj | IV-Gj | V-Gj | VI-Gj |
| LR1 | - | - | 0.000 | 0.000 | 0.000 | 0.000 | - | - | 0.000 | 0.000 | 0.000 | 0.000 | - | - | 0.000 | 0.000 | 0.000 | 0.000 |
| LR2 | - | - | - | - | - | - | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| LR3 | - | - | - | - | - | - | - | - | - | - | - | - | 0.000 | 0.020 | 0.234 | 0.000 | 0.000 | 0.153 |
| LR1 | - | - | 0.000 | 0.000 | 0.000 | 0.000 | - | - | $\begin{gathered} \text { NIG } \\ 0.000 \end{gathered}$ | 0.000 | 0.000 | 0.000 | - | - | 0.000 | 0.000 | 0.000 | 0.000 |
| LR2 | - | - | - | - | - | - | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| LR3 | - | - | - | - | - | - | - | - | - | - | - | - | 0.002 | 0.014 | 0.035 | 0.028 | 0.000 | 0.000 |





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| $000{ }^{\circ}$ | 000＇0 | $000{ }^{\circ}$ | 000＇0 | 000\％ | 0000 | $000 \cdot 0$ | 000＇0 | $000 \cdot 0$ | $000^{\circ} 0$ | 000＇0 | 000＊0 | － | － | － | － | － | － | マบT |
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| ！n－I＾ | ！n－ | ！n－ | ！n－III | ！n－II | ！n－I | セŋ－I $\Lambda$ | eが八 | ¢サ－$\wedge$－ | セワーIII | eサ－II | セわ－I | o $\mathrm{D}^{-\mathrm{I}}$－ | $0{ }^{\circ} \mathrm{O}-\Lambda$ | ${ }^{0} \mathrm{D}^{-}$－ I | op－III | o，－II | O－I | Wdr |


Table 1.9: Likelihood-ratio test (cont.)

| T | I-Co | II-Co | III-Co | IV-Co | V-Co | VI-Co | I-Ga | II-Ga | III-Ga | IV-Ga | V-Ga | VI-Ga | I-Gj | II-Gj | III-Gj | IV-Gj | V-Gj | VI-Gj |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| LR1 | - | - | 0.000 | 0.000 | 0.000 | 0.000 | - | - | Normal | 0.000 | 0.000 | 0.000 | - | - | 0.000 | 0.000 | 0.000 | 0.000 |
| LR2 | - | - | - | - | - | - | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| LR3 | - | - | - | - | - | - | - | - | - | - | - | - | 0.000 | 0.014 | 0.019 | 0.018 | 0.014 | 0.015 |
|  |  |  |  |  |  |  |  |  | NIG |  |  |  |  |  |  |  |  |  |
| LR1 | - | - | 0.000 | 0.000 | 0.000 | 0.000 | - | - | 0.000 | 0.000 | 0.000 | 0.000 | - | - | 0.000 | 0.000 | 0.000 | 0.000 |
| LR2 | - | - | - | - | - | - | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| LR3 | - | - | - | - | - | - | - | - | - | - | - | - | 0.000 | 0.018 | 0.015 | 0.015 | 0.011 | 0.012 |
| TWX | I-Co | II-Co | III-Co | IV-Co | V-Co | VI-Co | I-Ga | II-Ga | III-Ga Normal | IV-Ga | V-Ga | VI-Ga | I-Gj | II-Gj | III-Gj | IV-Gj | V-Gj | VI-Gj |
| LR1 | - | - | 0.000 | 0.000 | 0.000 | 0.000 | - | - | 0.000 | 0.000 | 0.000 | 0.000 | - | - | 0.000 | 0.000 | 0.000 | 0.000 |
| LR2 | - | - | - | - | - | - | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| LR3 | - | - | - | - | - | - | - | - | - | - | - | - | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
|  |  |  |  |  |  |  |  |  | NIG |  |  |  |  |  |  |  |  |  |
| LR1 | - | - | 0.000 | 0.000 | 0.000 | 0.000 | - | - | 0.000 | 0.000 | 0.000 | 0.000 | - | - | 0.000 | 0.000 | 0.000 | 0.000 |
| LR2 | - | - | - | - | - | - | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| LR3 | - | - | - | - | - | - | - | - | - | - | - | - | 0.000 | 0.009 | 0.014 | 0.013 | 0.007 | 0.008 |
| TXN | I-Co | II-Co | III-Co | IV-Co | V-Co | VI-Co | I-Ga | II-Ga | III-Ga Normal | IV-Ga | V-Ga | VI-Ga | I-Gj | II-Gj | III-Gj | IV-Gj | V-Gj | VI-Gj |
| LR1 | - | - | 0.000 | 0.000 | 0.000 | 0.000 | - |  | 0.000 | 0.000 | 0.000 | 0.000 | - | - | 0.000 | 0.000 | 0.000 | 0.000 |
| LR2 | - | - | - | - | - | - | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| LR3 | - | - | - | - | - | - | - | - | - | - | - | - | 0.244 | 0.487 | 0.922 | 0.886 | 0.970 | 0.935 |
|  |  |  |  |  |  |  |  |  | NIG |  |  |  |  |  |  |  |  |  |
| LR1 | - | - | 0.000 | 0.000 | 0.000 | 0.000 | - | - | 0.000 | 0.000 | 0.000 | 0.000 | - | - | 0.000 | 0.000 | 0.000 | 0.000 |
| LR2 | - | - | - | - | - | - | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| LR3 | - | - | - | - | - | - | - | - | - | - | - | - | 0.046 | 0.466 | 0.452 | 0.460 | 0.452 | 0.474 |
| WFC | I-Co | II-Co | III-Co | IV-Co | V-Co | VI-Co | I-Ga | II-Ga | III-Ga | IV-Ga | V-Ga | VI-Ga | I-Gj | II-Gj | III-Gj | IV-Gj | V-Gj | VI-Gj |
| LR1 | - | - | 0.000 | 0.000 | 0.000 | 0.000 | - | - | Normal 0.000 | 0.000 | 0.000 | 0.000 | - | - | 0.000 | 0.000 | 0.000 | 0.000 |
| LR2 | - | - | - | - | - | - | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| LR3 | - | - | - | - | - | - | - | - | - | - | - | - | 0.000 | 0.253 | 0.045 | 0.025 | 0.021 | 0.014 |
|  |  |  |  |  |  |  |  |  | NIG |  |  |  |  |  |  |  |  |  |
| LR1 | - | - | 0.000 | 0.000 | 0.000 | 0.000 | - | - | 0.000 | 0.000 | 0.000 | 0.000 | - | - | 0.000 | 0.000 | 0.000 | 0.000 |
| LR2 | - | - | - | - | - | - | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| LR3 | - | - | - | - | - | - | - | - | - | - | - | - | 0.000 | 0.014 | 0.016 | 0.008 | 0.007 | 0.003 |






| 800 | LZ＇0 | － | EI＇I | 6ヵ0 0 | 68：0 | で0 | ${ }_{\text {q } 61 .}{ }^{\circ} \mathrm{Z}$－ |  | q $00{ }^{\circ} \mathrm{I}$ | 7\％ 0 | － | q9I＇\％ | ${ }^{\text {e } 08}{ }^{\circ} \mathrm{E}$ | 66.0 | 76.0 | $69^{\circ} 0$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $00 \cdot 0$ | LZ＇0 |  | － | q $\ddagger 0 \cdot \square-$ | $68^{\circ} 0$ | 汿0－ |  | IN－en－II | $80 \cdot 0$ | $87^{\circ} 0$ |  | － | ع99 ${ }^{\circ} \mathrm{E}$ | $90^{\circ}{ }^{-}$ | $90^{\circ}{ }^{-}$ | 970 | IN－eŋ－II |
| 800 | LZ＇0 |  |  | － | 70． | $20^{\circ} \mathrm{I}$ | qLE＇ $\mathrm{C}^{-}$ | IN－0，${ }^{-1 / 2}$ | $00^{\circ}$ | $87^{\circ} 0$ |  |  | － | q $7 \square^{\prime} \square^{-}$ | q $8^{\circ} \square^{-}$ | －g $L^{\prime}$ \％－ | IN－o $\mathrm{D}^{\text {－II }}$ |
| $00 \cdot 0$ | LZ＊0 |  |  |  | － | E\＆ 0 | ¢98＇\％－ | ON－¢0－II | 800 | $87^{\circ} 0$ |  |  |  | － | 80：0－ | ［Z：0－ | ON－¢0－II |
| $80 \cdot 0$ | LZ．0 |  |  |  |  | － | qで「 ${ }^{\text {－}}$ | ON－Pか－II | $00^{\circ} 0$ | $87^{\circ} 0$ |  |  |  |  | － | 07．0－ | ON－セサ－II |
| $\mathrm{q}^{0} 0^{\circ} \mathrm{I}$ | 07\％ |  |  |  |  |  | － | ON－O－ $\mathrm{O}^{\text {－II }}$ | $00^{\circ}$ | ¢ $z^{\circ}$ |  |  |  |  |  | － | ON－O－－II |
| $\begin{gathered} \quad \text { and } \\ \text { SON } \end{gathered}$ | ASN | IN－Con－il | $1 N^{-r 9}$－II | $\mathrm{IN}^{-0} \mathrm{O}-\mathrm{II}$ | $\begin{gathered} \text { ON-CD-II } \\ \text { NG } \end{gathered}$ | ON－ry－II | $\mathrm{ON}^{-0} \mathrm{O}-\mathrm{II}$ | NSII | $\begin{gathered} u_{\Omega d} \\ \text { SDN } \end{gathered}$ | GSN | IN－－ n －II | IN－eか－II | $\mathrm{IN}^{-\mathrm{o}} \mathrm{D}-\mathrm{II}$ | $\begin{gathered} \text { ON-CD-II } \\ \text { NU } \end{gathered}$ | ON－セŋ－II | $\mathrm{ON}^{-0} \mathrm{O}-\mathrm{II}$ | LVO |
| q $00 . \mathrm{I}$ | $87^{\circ} 0$ | － | ${ }^{\text {e }}$ ¢ $0 \cdot 8$ | 2． $28^{\circ} \mathrm{I}$ | $87^{\circ} 0$ | $29^{\circ} 0$ | L0＇ I | IN－- － $\mathrm{D}-\mathrm{II}$ | q $00{ }^{\circ} \mathrm{I}$ | $27^{\circ} 0$ | － | q $6 I^{\prime} \%$ | 77\％ $0-$ | ${ }_{\text {q }} \ddagger 0 \cdot \square-$ | ง $\mathcal{L}$＇ I － | q79．7－ | In－- ¢－II |
| eで＇0 | 8 $7^{\circ} 0$ |  | － | $28^{\circ} 0$ | T9．L－ | $0 \mathrm{C}^{\prime} \mathrm{L}-$ | 62．0－ | IN－br－II | $00^{\circ} 0$ | $27^{\circ} 0$ |  | － | q $70{ }^{\circ}{ }^{-}$ | ャ\＆L＇\％－ | ャ6L＇\％－ | eโ9\％${ }^{-}$ | IN－en－II |
| eで＇0 | \＆$\%^{\circ}$ |  |  | － | っ91．L－ | ${ }^{\text {®6 }}$ ． $\mathrm{I}-$ | \％$\varepsilon^{\circ} \mathrm{I}-$ | IN－op－II | L0．0 | $27^{\circ} 0$ |  |  | － | $860-$ | $68^{\circ} 0$ | qLI＇${ }^{\text {－}}$ | IN－o $\mathrm{D}^{\text {－II }}$ |
| eZI＇0 | ¢ $\chi^{\circ} 0$ |  |  |  | － | 79．0 | $80^{\circ} \mathrm{I}$ | On－¢0－II | $00^{\circ} 0$ | $87^{\circ} 0$ |  |  |  | － | 07．0－ | ${ }_{9} \ddagger Z^{\prime} \sigma^{-}$ | ON－T－II |
| eで＇0 | 8 $\%^{\circ} 0$ |  |  |  |  | － | 81＇T | ON－8ワ－II | $00^{\circ} 0$ | $87^{\circ} 0$ |  |  |  |  | － | ${ }_{q} \ddagger z^{\prime} \square^{-}$ | ON－セサ－II |
|  | $8 z^{\circ} 0$ |  |  |  |  |  | － | $\mathrm{ON}^{-0} \mathrm{O}-\mathrm{II}$ | qTE．0 | $27^{\circ} 0$ |  |  |  |  |  | － | ON－O－${ }^{-1 I}$ |
| $\begin{gathered} u_{a d} \\ \text { SON } \end{gathered}$ | ASN | IN－Con－il | $1 N^{-r 9}$－ | $\mathrm{IN}^{-0} \mathrm{O}-\mathrm{II}$ | $\begin{gathered} \text { ON-โ?-II } \\ \text { WU } \end{gathered}$ | ON－ry－II | $\mathrm{ON}^{-0} \mathrm{O}-\mathrm{II}$ | OdH | $\begin{gathered} y_{\Omega d} \\ \text { SDN } \end{gathered}$ | GSN | IN－${ }^{\text {－}}$ D－II |  | $\mathrm{IN}^{-\mathrm{o}} \mathrm{D}-\mathrm{II}$ | $\begin{gathered} \text { ON-โ?-II } \\ \text { WU } \end{gathered}$ | ON－rワ－II | $\mathrm{ON}^{-0} \mathrm{O}-\mathrm{II}$ | 0 |
| $\mathrm{q}^{0} 0^{\text { }}$ I | $77^{\circ} 0$ | － | $\mathrm{q}^{0} \mathcal{Z}^{\prime} 7$ | ${ }_{\text {q }} \mathrm{C} \mathrm{c}^{\circ} \mathrm{Z}$ | ${ }_{q}{ }^{\text {E }}{ }^{\circ} 7^{-}$ | 9才＇${ }^{\text {－}}$ | Eq．${ }^{\text {－}}$ |  | q $00{ }^{\circ} \mathrm{I}$ | $08^{\circ} 0$ | － | $Z^{\prime} \mathrm{I}$ |  |  |  |  |  |
| L0．0 | $8 z^{\circ} 0$ |  | － | q T ＇$\%$ | ${ }_{\text {e } 20 ~}^{1}$－ | q $8 \mathrm{~T}^{\prime} 7-$ | 8LL＇\％－ | IN－eか－II | $00 \cdot 0$ | L\＆ 0 |  | － | ${ }^{\text {¢ } 6 . ~}{ }^{\text {I }}$ | $78.0-$ | ${ }^{\text {¢ }}$ ． $\mathrm{I}-$ | $98.0-$ | IN－en－II |
| 10.0 | £ $\mathcal{Z}^{\circ} 0$ |  |  | － |  | etL＇${ }^{\text {\％}}$－ | ะ99．8－ | IN－OP－II | $00^{\circ} 0$ | LE0 |  |  | － | q86．${ }^{\text {－}}$ | qL゙でて－ | q26．${ }^{\text {－}}$ | IN－OP－II |
| 10．0 | ¢ $7^{\circ} 0$ |  |  |  | － | L6．0 | LI＇t | ON－¢0－II | $00^{\circ} 0$ | 080 |  |  |  |  | 69． $\mathrm{L}-$ | $90^{\circ}{ }^{-}$ | ON－T－- II |
| L0＇0 | \＆$z^{\circ} 0$ |  |  |  |  | － | 68.0 | ON－eか－II | $00^{\circ} 0$ | LE0 |  |  |  |  | － | 98.0 | ON－セサ－II |
| L0．0 | 8 $\%^{\circ} 0$ |  |  |  |  |  | － | ON－O－II | $00 \cdot 0$ | L80 |  |  |  |  |  | － | ON－O－${ }^{-1 I}$ |
| $\begin{gathered} u_{a d} \\ \text { SON } \end{gathered}$ | ASN | IN－Comeril | $1 N^{-r 9} \mathrm{D}-\mathrm{II}$ | $\mathrm{IN}^{-0} \mathrm{D}-\mathrm{II}$ | $\begin{gathered} \text { ON-โn-II } \\ \text { NG } \end{gathered}$ | On－rn－II | $\mathrm{ON}^{-0} \mathrm{O}-\mathrm{II}$ | NOH | $\begin{gathered} u_{\Omega d} \\ \text { SDN } \end{gathered}$ | HSN | IN－- ¢－II | IN－eか－II | IN－op－II | $\begin{gathered} \text { ON-؟n-II } \\ \text { WU } \end{gathered}$ | ON－rワ－II | $\mathrm{ON}^{-0} \mathrm{O}-\mathrm{II}$ | OVG |
| q $00^{\circ} \mathrm{I}$ | $87^{\circ} 0$ | － | っSL＇L－ | ${ }^{2} 06.1$ | ${ }^{\text {¢ }} 908$ | q $6 \chi^{\circ} \mathrm{Z}$ | » 70.8 |  | q 00.1 | 77.0 | － | ¢ 90 | ${ }^{\text {e }} 16.7$ | q $00 \cdot \square-$ | $\mathrm{q}^{\text {S }} \mathrm{T}^{\prime} \mathrm{Z}^{-}$ | $08^{\circ} \mathrm{I}-$ |  |
| 90.0 | も\％ 0 |  | － | q9 ${ }^{\prime}$＇\％ | eLIE | e CL L \％ | － 00.8 | IN－en－II | L0．0 | $77^{\circ} 0$ |  | － | っ 78.1 | $09.1-$ | งGL．${ }^{\text {c－}}$ | 90．${ }^{\text {－}}$ | IN－en－il |
| $00^{\circ} 0$ | $9 \mathrm{c}^{\circ} 0$ |  |  | － | q 97.7 | で． I | e 29.6 | IN－0，${ }^{-1 / 2}$ | L0．0 | \＆ $7^{\circ} 0$ |  |  | － | q $8^{\circ}{ }^{\circ} 7-$ | в 89.7 － | q87 ${ }^{\text {\％}}$－ | IN－OP－II |
| $00 \cdot 0$ | もて\％ |  |  |  | － | е06 ¢ $^{-}$ | 90 － | ON－¢0－II | z0＇0 | 77\％ |  |  |  | － | て7＇I－ | ${ }^{2} 08^{\prime}$ I | ON－¢¢－II |
| $00 \cdot 0$ | $77^{\circ} 0$ |  |  |  |  | － | $07^{\circ} \mathrm{L}$ | ON－rか－II | L0．0 | $77^{\circ} 0$ |  |  |  |  | － | q $¢ \in \cdot \square$ | ON－m－II |
| $00 \%$ $q_{\Omega}{ }^{\circ} \mathrm{d}$ | 9\％＊ |  |  |  |  |  | ON－O－－II | ON－O－II | $10 \cdot 0$ $y_{\Omega}{ }^{\text {d }}$ | ¢ $8^{\circ} 0$ |  |  |  |  |  | $\stackrel{-}{\text {－}}$ | ON－O－${ }^{-1 I}$ |
| SON | ASN | IN－Th－II | IN－e $\mathrm{N}^{-1 I}$ | IN－O－P－II | $\begin{gathered} \text { ON-؟’-II } \\ \text { WG } \end{gathered}$ | ON－en－II | ON－O－II | XG丹 | $\begin{gathered} u_{\Omega d} \\ \text { SDN } \end{gathered}$ | 田SN | IN－${ }^{\text {－}}$－ | IN－eか－II | $\mathrm{IN}^{-0} \mathrm{O}-\mathrm{II}$ | $\begin{gathered} \text { ON-?N-II } \\ \text { NG } \end{gathered}$ | ON－Pか－II | $\mathrm{ON}^{-0} \mathrm{O}-\mathrm{II}$ | Vg |

Table 1．10：Out－of－sample forecast evaluation DM test and MCS－HAR model－Full sample
Table 1.11: Out-of-sample forecast evaluation DM test and MCS - HAR model (cont.) - Full sample

| JPM |  |  | DM |  |  |  |  | MCS | T |  |  | DM |  |  |  |  | MCS |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | II-Co-NO | II-Ga-NO | II-Gj-NO | II-Co-NI | II-Ga-NI | II-Gj-NI | MSE | $p v_{R}$ |  | II-Co-NO | II-Ga-NO | II-Gj-NO | II-Co-NI | II-Ga-NI | II-Gj-NI | MSE | $p v_{R}$ |
| II-Co-NO | - |  |  |  |  |  | 0.23 | 0.00 | II-Co-NO | - |  |  |  |  |  | 0.23 | 0.07 |
| II-Ga-NO | $-1.91^{\text {c }}$ | - |  |  |  |  | 0.23 | 0.00 | II-Ga-NO | -0.74 | - |  |  |  |  | 0.23 | 0.00 |
| II-Gj-NO | $-1.65{ }^{\text {c }}$ | 0.16 | - |  |  |  | 0.23 | 0.00 | II-Gj-NO | -0.70 | -0.34 | - |  |  |  | 0.23 | 0.07 |
| II-Co-NI | $-3.55{ }^{\text {a }}$ | $-3.56{ }^{\text {a }}$ | $-3.68{ }^{\text {a }}$ | - |  |  | 0.23 | 0.00 | II-Co-NI | $-2.91{ }^{\text {a }}$ | $-3.04{ }^{\text {a }}$ | $-2.87^{\text {a }}$ | - |  |  | 0.24 | 0.07 |
| II-Ga-NI | $-3.03^{\text {a }}$ | $-3.36{ }^{\text {a }}$ | $-3.56{ }^{\text {a }}$ | 0.70 | - |  | 0.23 | 0.00 | II-Ga-NI | $-2.54{ }^{\text {b }}$ | $-2.82^{\text {a }}$ | $-2.52^{\text {b }}$ | $3.06{ }^{\text {a }}$ | - |  | 0.23 | 0.07 |
| II-Gj-NI | $-1.94{ }^{\text {c }}$ | $-1.67^{\text {c }}$ | $-2.04{ }^{\text {b }}$ | $3.13{ }^{\text {a }}$ | $4.02^{\text {a }}$ | - | 0.22 | $1.00{ }^{\text {b }}$ | II-Gj-NI | $-1.64{ }^{\text {c }}$ | $-1.37$ | -1.02 | $3.12^{\text {a }}$ | $2.80^{\text {a }}$ | - | 0.23 | $1.00^{\text {b }}$ |
| KFT |  |  | DM |  |  |  |  | MCS | TWX |  |  | DM |  |  |  |  | MCS |
|  | II-Co-NO | II-Ga-NO | II-Gj-NO | II-Co-NI | II-Ga-NI | II-Gj-NI | MSE | $p v_{R}$ |  | II-Co-NO | II-Ga-NO | II-Gj-NO | II-Co-NI | II-Ga-NI | II-Gj-NI | MSE | $p v_{R}$ |
| II-Co-NO | - |  |  |  |  |  | 0.33 | 0.01 | $\mathrm{II}-\mathrm{Co}-\mathrm{NO}$ | - |  |  |  |  |  | 0.20 | 0.01 |
| II-Ga-NO | 0.67 | - |  |  |  |  | 0.34 | 0.01 | II-Ga-NO | $2.33{ }^{\text {b }}$ | - |  |  |  |  | 0.19 | $0.96{ }^{\text {b }}$ |
| II-Gj-NO | 0.68 | 0.42 | - |  |  |  | 0.32 | 0.02 | II-Gj-NO | 1.55 | -1.33 | - |  |  |  | 0.19 | $0.23{ }^{\text {a }}$ |
| II-Co-NI | $-2.96{ }^{\text {a }}$ | $-3.08^{\text {a }}$ | $-3.31{ }^{\text {a }}$ | - |  |  | 0.34 | 0.01 | II-Co-NI | $-2.41^{\text {b }}$ | $-3.11^{\text {a }}$ | $-2.86{ }^{\text {a }}$ | - |  |  | 0.20 | 0.01 |
| II-Ga-NI | $-2.28^{\text {b }}$ | $-2.51^{\text {b }}$ | $-2.73^{\text {a }}$ | $2.86{ }^{\text {a }}$ | - |  | 0.33 | 0.01 | II-Ga-NI | -0.45 | $-2.89{ }^{\text {a }}$ | $-2.49^{\text {b }}$ | $2.90{ }^{\text {a }}$ | - |  | 0.19 | 0.01 |
| II-Gj-NI | $-1.23$ | $-1.78^{\text {c }}$ | $-2.42^{\text {b }}$ | $3.19{ }^{\text {a }}$ | 1.49 | - | 0.32 | $1.00^{\text {b }}$ | II-Gj-NI | 0.51 | $-2.65{ }^{\text {a }}$ | $-2.11^{\text {b }}$ | $2.95{ }^{\text {a }}$ | $2.84{ }^{\text {a }}$ | - | 0.19 | $1.00^{\text {b }}$ |
| PEP |  |  | DM |  |  |  |  | MCS | TXN |  |  | ${ }_{\text {DM }}^{\text {D }}$ |  |  |  |  | MCS |
|  | II-Co-NO | II-Ga-NO | II-Gj-NO | II-Co-NI | II-Ga-NI | II-Gj-NI | MSE | $p v_{R}$ |  | II-Co-NO | II-Ga-NO | II-Gj-NO | II-Co-NI | II-Ga-NI | II-Gj-NI | MSE | $p v_{R}$ |
| II-Co-NO | - |  |  |  |  |  | 0.26 | 0.05 | II-Co-NO | - |  |  |  |  |  | 0.21 | 0.00 |
| II-Ga-NO | -0.56 | - |  |  |  |  | 0.26 | 0.05 | II-Ga-NO | $1.74{ }^{\text {c }}$ | - |  |  |  |  | 0.20 | 0.06 |
| II-Gj-NO | 0.14 | 1.13 | - |  |  |  | 0.26 | 0.05 | II-Gj-NO | 1.59 | -1.09 | - |  |  |  | 0.20 | 0.00 |
| II-Co-NI | $-2.20^{\text {b }}$ | $-2.49^{\text {b }}$ | $-2.69^{\text {a }}$ | - |  |  | 0.26 | 0.05 | II-Co-NI | -0.02 | $-1.87^{\text {c }}$ | -1.62 | - |  |  | 0.20 | $0.10^{\text {a }}$ |
| II-Ga-NI | -1.01 | -1.06 | $-2.18{ }^{\text {b }}$ | $2.67{ }^{\text {a }}$ |  |  | 0.26 | 0.05 | II-Ga-NI | 1.15 | -0.27 | 0.01 | $2.04{ }^{\text {b }}$ | - |  | 0.20 | $0.56^{\text {b }}$ |
| II-Gj-NI | -0.10 | 0.63 | -0.69 | $2.57{ }^{\text {a }}$ | $1.95{ }^{\text {c }}$ | - | 0.25 | $1.00{ }^{\text {b }}$ | II-Gj-NI | 1.07 | -0.54 | -0.23 | $2.26{ }^{\text {b }}$ | -0.62 | - | 0.20 | $1.00^{\text {b }}$ |
| PG |  |  | DM |  |  |  |  | MCS | WFC |  |  | DM |  |  |  |  | MCS |
|  | II-Co-NO | II-Ga-NO | II-Gj-NO | II-Co-NI | II-Ga-NI | II-Gj-NI | MSE | $p v_{R}$ |  | II-Co-NO | II-Ga-NO | II-Gj-NO | II-Co-NI | II-Ga-NI | II-Gj-NI | MSE | $p v_{R}$ |
| II-Co-NO | 208 b |  |  |  |  |  | 0.24 | $1.00^{\text {b }}$ | II-Co-NO | , |  |  |  |  |  | 0.27 | 0.00 |
| II-Ga-NO | $-2.08{ }^{\text {b }}$ | - |  |  |  |  | 0.26 | 0.01 | II-Ga-NO | -1.45 | - |  |  |  |  | 0.29 | 0.00 |
| II-Gj-NO | $-1.98{ }^{\text {b }}$ | $2.23{ }^{\text {b }}$ | - |  |  |  | 0.25 | 0.05 | II-Gj-NO | -1.24 | 0.00 | - |  |  |  | 0.26 | 0.08 |
| II-Co-NI | $-2.16^{\text {b }}$ | -0.08 | -1.04 | - |  |  | 0.25 | 0.03 | II-Co-NI | $-3.54{ }^{\text {a }}$ | $-3.95{ }^{\text {a }}$ | $-3.87^{\text {a }}$ | - |  |  | 0.28 | 0.00 |
| II-Ga-NI | $-2.17^{\text {b }}$ | -0.77 | $-1.79^{\text {c }}$ | -0.78 | - |  | 0.25 | 0.03 | II-Ga-NI | $-3.25{ }^{\text {a }}$ | $-3.77^{\text {a }}$ | $-4.04{ }^{\text {a }}$ | 1.16 | - |  | 0.27 | 0.00 |
| II-Gj-NI | $-2.19^{\text {b }}$ | 0.46 | -1.27 | 0.64 | $1.81{ }^{\text {c }}$ | - | 0.24 | $0.34{ }^{\text {b }}$ | II-Gj-NI | $-2.29^{\text {b }}$ | $-2.37^{\text {b }}$ | $-2.76{ }^{\text {a }}$ | $2.75{ }^{\text {a }}$ | $3.95{ }^{\text {a }}$ | - | 0.25 | $1.00^{\text {b }}$ |

[^7]

 Note：Diebold and Mariano（1995）test for the row model vs．the HAR model with same variance equation and distribution assumption．Forecast performance evaluation for the 16
series of stock volatility for the full out－of－sample period（1067 observation）．Model $I$ is an $A R(1)$ specification，$I I$ is an $H A R, I I I$ is an $H A R+I_{u m}\left(h_{t-1}\right) h_{t-1}+R_{t-1}+I\left(R_{t-1}\right) R_{t-1}$,

| q $6 \downarrow$＇ | $\angle D^{\circ} 0$ | 20＇ | q $90^{\circ} \mathrm{Z}$ | $\mathrm{q}^{8} 00^{\circ} \mathrm{z}$ | 78.0 | $8 L^{\circ} 0$ | ${ }_{\text {e }} 8^{\circ} \mathrm{Z}$ | ¢98．${ }^{\text {\％}}$ | ${ }_{\text {e }} \chi^{\prime}$ ¢ | $\mathrm{q}^{\text {O }}{ }^{\circ} \mathrm{C}$ | 09.0 | $80^{\circ} \mathrm{I}$ | 2． 28.1 | $9 \square^{\text { }}$ I | ¢99 ${ }^{\text {I }}$ | IN－ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| q 66.1 | $88^{\circ} 0$ | 72＇1 | q\＆I＇$\quad$ \％ | q 90.7 | $\pm 8^{\circ} 0$ | L $\varepsilon^{\prime}$ I | －84＇${ }^{\text {c }}$ | ${ }_{\text {e }}^{\text {¢ } 2.7}$ | ${ }^{\text {e }}$ LE\％ | qTS． Z | 790 | ¢98． | ${ }^{2} 06{ }^{\text {I }}$ | 89＇I | ${ }^{0} 0 L^{\circ} \mathrm{I}$ | IN－-1 |
| q 80.7 | $98^{\circ} 0$ | $9 \mathrm{SF}^{\circ}$ | －98 ${ }^{\text {I }}$ | ${ }^{2} 08{ }^{\text {I }}$ | 080 | ${ }^{8} 88^{\prime} \mathrm{I}$ | ${ }^{\text {e } 60}$ ¢ | е69\％ | ${ }_{\text {e }} 00^{\circ} 8$ | ${ }^{\text {8 }} 8{ }^{\circ} \mathrm{F}$ | L\＆ 0 | د 28.1 | ， $76 \cdot$ I | oLL＇I | ，88．I | IN－$-\mathrm{D}-\mathrm{NI}$ |
| $\mathrm{q}^{\text {L }} \mathrm{C}^{\circ} \mathrm{Z}$ | LZ：0 | L゙0 | 2． $28^{\circ} \mathrm{I}$ | 。89 ${ }^{\text {I }}$ | \＆ 20 | ， $\mathrm{ZL}^{\prime}$＇I | e 10 \％ | ¢08． 7 | ะ66．${ }^{\text {\％}}$ | q $¢ 9.7$ | 080 | 89＇ | د28．I | ， $78 . \mathrm{I}$ | ${ }^{2} 88 . \mathrm{I}$ | IN－？${ }^{\text {de－III }}$ |
|  | － | － | － | － |  | ＊ |  | － | － |  | － | － | － | － | － | IN－！n－II |
| q98．${ }^{\text {c }}$ | $\angle F^{\circ} 0$ | II＇I | qZI＇Z | $\mathrm{q}^{\square 0} 0^{\circ} \mathrm{z}$ | 78.0 | L9．0 | ${ }^{\text {－} 68 .}{ }^{\circ}$ | ${ }_{\text {e } 88}{ }^{\text {\％}}$ | ${ }^{8} 878$ | ${ }_{9} \mathrm{LF}^{\text {\％}}$ | 790 | ¢0＇ | ${ }^{\text {¢ \％}} 8 \times$ I | LG．I | $0 z^{\prime}$ I | IN－en－I |
| q96．${ }^{\text {I }}$ | $97^{\circ} 0$ | $97^{\prime}$ I | q61＇z | q90 ${ }^{\text {\％}}$ | 280 | LZ＇I | e 58.7 | egL ${ }^{\text {c }}$ | е98\％ | ${ }_{9} 0 S^{\prime} \%$ | $89^{\circ} 0$ | 281． | －98．I | ${ }^{\text {¢ }} 8^{\prime} \mathrm{I}$ | ¢G＇I | IN－rが八 |
| q 66.1 | $98 \cdot 0$ | $09^{\circ} 0$ | －98．I | 282．I | 290 | 2 28.1 | ${ }^{\text {e }} 8 \mathrm{I}^{\circ} \mathrm{E}$ | ${ }_{\text {e }}$ L2＇Z | ${ }^{\text {e } 80} 0^{\circ}$ | －89．7 | E80 | ${ }^{2} 82^{\circ} \mathrm{I}$ | ${ }^{2} 06{ }^{\text {I }}$ | q96．I | $9 \square^{\circ} \mathrm{I}$ | IN－¢か－ |
| $\mathrm{q} 97 \cdot \mathrm{Z}$ | LZ＇0 | 680 | －98 ${ }^{\text {I }}$ | ，89＇I | 620 | ${ }^{2} 0 L^{\prime} \mathrm{I}$ | ャ60＇8 | ${ }_{\text {e } 82 .} \mathrm{Z}$ | ${ }^{\text {e }} 0.8$ | ${ }_{q} \ddagger \underline{\text { c }}$ | 780 | $09^{\text { }}$ | っf8＇I | $\mathrm{q}^{7} 0^{\circ} \mathrm{Z}$ | $99^{\prime} \mathrm{I}$ | IN－eがIII |
| － | － | － | － | － |  | － | － |  |  |  |  |  |  |  |  | IN－セか－II |
| ${ }_{\text {q }}{ }^{\circ} \mathrm{Z}$ | $99^{\circ} 0$ | 76.0 | q $00^{\circ} \mathrm{Z}$ | q26．I | DL\％ | $79^{\circ} 0$ | ${ }^{\text {e }}$ L C \％ | ${ }_{\text {el }}$ 8＇$^{\text {\％}}$ | ${ }^{4} 76 \%$ | qLI＇$\%$ | $\pm 9^{\circ} 0$ | L20 | －92．I | oLL＇I | ${ }^{\circ} \mathrm{F} 9{ }^{\text { }}$ | IN－0－9－I |
| っ\＆$L^{\circ} \mathrm{L}$ | EF\％ | L $\%^{\prime}$ I | q01＇ 7 | ${ }_{\text {q }} \mathrm{IO}^{\circ} \mathrm{Z}$ | $92^{\circ} 0$ | 81＇I | eqL．${ }^{\text {c }}$ |  | е98．8 | q $\delta^{\circ}{ }^{\circ} \mathrm{Z}$ | $99^{\circ} 0$ | q $80{ }^{\circ} \mathrm{z}$ | っ78．I | っち6．I | ，89 ${ }^{\text {I }}$ | $\mathrm{IN}^{-0} \mathrm{D}^{-} \Lambda$ |
| ง $2 L^{\prime}$ T | 07．0 | $67^{\circ} 0$ | っI6．I | ${ }^{2} 02 \cdot \mathrm{~L}$ | $\square 90$ | ${ }^{2} 6 L^{\prime} \mathrm{I}$ | e 80.8 | ${ }_{\text {e } 02} L^{\text {\％}}$ | ${ }^{\text {e }} \mathrm{T}^{\circ} \mathrm{E}$ | q99． 7 | LZ\％ 0 | q 80.7 | ${ }^{2} 88 \cdot \mathrm{I}$ | q $20 \%$ | 998．I | IN－0，${ }^{-1}$ |
| っ\＃6 ${ }^{\text {I }}$ | $08^{\circ} 0$ | LI＇0 | 288 ${ }^{\text {I }}$ | G9．I | $69^{0}$ | $89^{\prime}$ I | в 26.7 | ＊ 82.7 | 886．${ }^{\text {\％}}$ | $\mathrm{q}^{0} \mathrm{C} \%$ | 07．0 | ¢ $\mathrm{Z}^{\prime}$ I | －98． | q90\％ | ${ }^{\text {J }} 8 . \mathrm{I}$ | IN－0，${ }^{-1 / 2}$ |
|  |  |  | － |  |  |  |  |  |  |  |  |  |  |  | － | $\mathrm{IN}^{-} \mathrm{O} \mathrm{D}^{-\mathrm{II}}$ |
| q 87.7 | $67^{\circ} 0$ | $78^{\circ} 0$ | ${ }^{2} 06{ }^{\text {I }}$ | ${ }^{\text {¢ } 76.1}$ | $00^{\circ} \mathrm{I}$ | $6 L^{\circ} 0$ | ${ }^{\text {® } 62 .} 7$ | ${ }^{\text {e }} 08.7$ | ${ }^{2} 20{ }^{\circ} \mathrm{E}$ | q $90{ }^{\circ} \mathrm{Z}$ | $99^{\circ} 0$ | LZ＇I | ． $788^{\text {I }}$ | L9．I | $87^{\circ} \mathrm{I}$ |  |
| ง $88{ }^{\circ} \mathrm{L}$ | $\angle \mathrm{F}^{0} 0$ | $68^{\circ}$ | q66．${ }^{\text {I }}$ | $\mathrm{q}^{0} 0{ }^{\circ} \mathrm{z}$ | 70＇ | 96.0 | ${ }^{\text {e }}$ \＆$L^{\prime}$ \％ |  | e9t＇ | q $09^{\prime}$ \％ | $09^{\circ}$ | $09^{\prime}$ I | 288＇I | もて＇I | $98^{\circ} \mathrm{I}$ | ON－-1 |
| ${ }^{\text {\％}} 78 . \mathrm{L}$ | 070 | 08：0 | ［9＊ | ${ }^{\text {¢ }} 8^{\circ} \mathrm{I}$ | 66.0 | $\angle 7^{\circ} \mathrm{I}$ | e $20 \%$ | ${ }_{\text {e }}$ ¢9\％ 7 | ${ }^{\text {e } 86.7}$ | q 87.7 | $09^{\circ} 0$ | 2 29.1 | ¢96．I | \％\％＇I | 閏I | ON－-1.0 I |
| $\mathrm{q}^{9} 0^{\circ} \mathrm{Z}$ | 78.0 | $6 \mathrm{I}^{\circ} 0$ | $99^{\circ} \mathrm{I}$ | ${ }^{\circ} \mathrm{L}$ L＇I | 760 | 9\％＇I | －66． 7 | ${ }_{\text {e }} \mathrm{I}^{\circ} \mathrm{Z}$ | ¢06．${ }^{\text {\％}}$ | q9 ${ }^{\prime}$＇ 7 | LS 0 | 。89 ${ }^{\text {I }}$ | ， $688^{\text {I }}$ | じ＇ | $07^{\circ} \mathrm{I}$ | ON－？－III |
| － | － | － | － | － | － | － | － | － | － | － | － | － | － |  |  | ON－？－Til |
| q $¢ L^{\prime} \%$ | EF－0 | 78.0 | ${ }^{2} 06{ }^{\text {I }}$ | ${ }_{q}{ }^{\text {I }}$ \％$\%$ | ELO | LZ＇0 | ${ }^{8} 8 L^{\circ} \mathrm{Z}$ | e92．7 | ${ }^{\text {e } 60}$ ¢ | q6［ $\%$ | 990 | $9 \mathrm{q}^{\circ} \mathrm{I}$ | ったじI | $97^{\prime}$ I | $87^{\prime}$ I | ON－br－İ |
| ง62＇L | 07．0 | \＆60 | q96．［ | q $80{ }^{\circ} \mathrm{z}$ | $9 L^{\circ} 0$ | $68^{\circ} 0$ | ャ $72 \cdot \square$ | ${ }_{\text {e } 79}{ }^{\text {¢ }}$ | ${ }^{\text {e } 2 \text { L＇}}$ ¢ | q0才，$冖$ | $89^{\circ} 0$ | $89^{\circ} \mathrm{L}$ | っ $78 . \mathrm{I}$ | 弫 1 | $2 E^{\circ} \mathrm{I}$ | ON－¢ワ |
| っち9 ${ }^{\text {I }}$ | 98.0 | \＆6：0 | ¢G．${ }^{\text {L }}$ | ， $76 \cdot$ I | 290 | 77．${ }^{\text {L }}$ | е90 $¢$ | e89．7 | ${ }^{2} 26.7$ | q 98.7 | $86^{\circ} 0$ | ったL＇I | ， 56.1 | $99^{\prime} \mathrm{I}$ | Sti | ON－er－ $\mathrm{OL}^{-1}$ |
| 2 $28^{\circ} \mathrm{I}$ | $97 \%$ | 200 | $87^{\circ} \mathrm{I}$ | 982＇L | 790 | 07＇I | в 26.7 | ${ }_{\text {e }} 09.7$ | ${ }^{6} 68^{\circ}$ | q J ＇$\%$ | $86^{\circ} 0$ | 992．L | ，I8．I | ${ }^{\text {¢ }} 9 . \mathrm{I}$ | $68^{\circ} \mathrm{I}$ | ON－Pŋ－III |
|  | － | － | － | － | － | － | － | － | － | － | － |  |  |  |  | On－by－II |
| ${ }^{6} 68^{\circ} \mathrm{I}$ | $28^{\circ} 0$ | 89.0 | 284．${ }^{\circ}$ | o $\mathrm{SL}^{\prime}$ I | $97^{\circ} 0$ | \＆\％ 0 | q $00^{\circ} \mathrm{Z}$ | ${ }_{\text {e }} \mathrm{T}^{\circ} \mathrm{Z}$ | ${ }^{\text {e } 67.8}$ | ${ }_{q} 80{ }^{\circ}$ | Et 0 | LI＇I | ${ }^{2} 89^{\text { }}$ | ，TL＇I | 07＇I | ON－O－I |
| ¢IL＇I | $68^{\circ} 0$ | $8 L^{\circ} 0$ | ．98． I | ¢88 ${ }^{\text {I }}$ | $27^{\circ} 0$ | \＆60 | q $79^{\circ} 7$ | q $7 \mathrm{c}^{\circ} \mathrm{Z}$ | － $17{ }^{\text {¢ }}$ | q $0 z^{\prime} \%$ | $\angle \overbrace{}^{\circ} 0$ | ${ }^{2} 28^{\circ} \mathrm{I}$ | دLL＇I | q66．I | gr＇I | $\mathrm{ON}^{-0} \mathrm{O}-\Lambda$ |
| $69^{\prime}$ I | E\＆： 0 | $9 \mathrm{c}^{\circ} 0-$ | 09 ${ }^{\text {I }}$ | 09.1 | 01．0 | Et＇ L | e78．7 | q $7 \uparrow$ \％ | ${ }^{0} 0 \varepsilon^{\circ} \mathrm{E}$ | ${ }_{q} I Z .7$ | 280 | q 86.1 | ， $788^{\text {I }}$ | q $77 \%$ | L\＆ | $\mathrm{ON}^{-0} \mathrm{O}^{-\Lambda I}$ |
| ง\＆8＇ I | 78.0 | $97^{\circ} 0-$ | $09^{\prime} \mathrm{I}$ | Ef＇I | $\pm 0.0$ | EI＇I | － EL＇$^{\text {\％}}$ | q T＇$^{\text {c }}$ | －85＇ 8 | q 26.1 | EE0 | ［9＇I | ったL＇I | q9 ${ }^{\text {c }}$ \％ | $97^{\circ} \mathrm{I}$ | ON－O－III |
| － | － | － | － | － | － | － | － | － | － | － | － | － | － | － | － | On－on－II |
| OHM | NXL | XML | L | Dd | d＇Hd | LAY | NdS | WgI | OdH | NOH | ХСл | LVD | 0 | DVG | Vg |  |


Table 1.13: Out-of-sample forecast evaluation DM test - Crisis

|  | BA | BAC | C | CAT | FDX | HON | HPQ | IBM | JPM | KFT | PEP | PG | T | TWX | TXN | WFC |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| II-Co-NO | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| III-Co-NO | $-0.60$ | 0.69 | 0.55 | 1.58 | -0.43 | 0.76 | $1.86{ }^{\text {c }}$ | $1.88{ }^{\text {c }}$ | 1.10 | 0.84 | -0.40 | $1.74{ }^{\text {c }}$ | 1.12 | -0.54 | 0.61 | 1.03 |
| IV-Co-NO | -0.64 | 0.88 | 0.74 | 1.49 | $-0.51$ | 0.82 | $1.93{ }^{\text {c }}$ | $1.85{ }^{\text {c }}$ | 1.20 | 0.90 | -0.40 | $1.79{ }^{\text {c }}$ | 1.16 | -0.38 | 0.64 | 0.99 |
| V-Co-NO | $-0.61$ | 0.58 | 0.75 | 1.50 | -0.46 | 0.77 | $1.97{ }^{\text {b }}$ | $1.97{ }^{\text {b }}$ | 1.11 | 1.00 | -0.36 | $1.75{ }^{\text {c }}$ | 1.10 | 0.19 | 0.83 | 1.14 |
| VI-Co-NO | $-0.68$ | 0.51 | 0.76 | 1.35 | $-0.45$ | 0.74 | $1.98{ }^{\text {b }}$ | $1.93{ }^{\text {c }}$ | 1.15 | 0.92 | -0.35 | $1.78{ }^{\text {c }}$ | 1.08 | 0.09 | 0.42 | 1.31 |
| II-Ga-NO | - | - | - | - |  | - | - |  | - | - |  |  | - |  | - | - |
| III-Ga-NO | $-0.47$ | -0.14 | 0.55 | 1.60 | -0.34 | 1.09 | 1.51 | $2.17{ }^{\text {b }}$ | 1.24 | 0.88 | -0.09 | $2.13{ }^{\text {b }}$ | 1.19 | -0.13 | 0.80 | 0.97 |
| IV-Ga-NO | -0.49 | 0.09 | 0.79 | 1.39 | -0.44 | 1.10 | 1.59 | $2.08{ }^{\text {b }}$ | 1.34 | 0.90 | -0.10 | $2.15{ }^{\text {b }}$ | 1.21 | -0.05 | 0.81 | 0.97 |
| V-Ga-NO | -0.48 | -0.02 | 0.78 | 1.37 | -0.40 | 1.03 | 1.62 | $2.19{ }^{\text {b }}$ | 1.22 | 0.99 | -0.10 | $2.11^{\text {b }}$ | 1.20 | 0.32 | 0.74 | 1.11 |
| VI-Ga-NO | $-0.57$ | -0.06 | 0.78 | 1.30 | -0.38 | 1.00 | 1.62 | $2.16{ }^{\text {b }}$ | 1.28 | 0.92 | -0.11 | $2.16{ }^{\text {b }}$ | 1.20 | 0.24 | 0.48 | 1.32 |
| II-Gj-NO | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| III-Gj-NO | -0.42 | -0.35 | 0.60 | 1.54 | $-0.30$ | 1.02 | 1.46 | $2.13{ }^{\text {b }}$ | 1.26 | 0.96 | 0.01 | $2.10{ }^{\text {b }}$ | 1.22 | -0.09 | 0.81 | 0.99 |
| IV-Gj-NO | -0.44 | -0.17 | 0.80 | 1.33 | -0.41 | 1.02 | 1.55 | $2.05^{\text {b }}$ | 1.36 | 0.98 | 0.00 | $2.09^{\text {b }}$ | 1.23 | -0.04 | 0.82 | 0.97 |
| V-Gj-NO | -0.47 | -0.22 | 0.79 | 1.30 | $-0.37$ | 0.94 | 1.57 | $2.12{ }^{\text {b }}$ | 1.23 | 1.07 | -0.02 | $2.05^{\text {b }}$ | 1.25 | 0.29 | 0.75 | 1.10 |
| VI-Gj-NO | $-0.55$ | -0.01 | 0.81 | 1.24 | $-0.36$ | 0.95 | 1.57 | $2.13{ }^{\text {b }}$ | 1.28 | 0.99 | -0.02 | $2.09^{\text {b }}$ | 1.24 | 0.29 | 0.48 | 1.37 |
| II-Co-NI | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| III-Co-NI | 0.03 | 0.17 | 0.61 | 1.52 | -0.58 | 1.12 | $1.78{ }^{\text {c }}$ | $2.26{ }^{\text {b }}$ | 1.18 | 1.08 | -0.11 | $1.96{ }^{\text {b }}$ | 1.40 | -0.09 | 0.79 | 1.03 |
| IV-Co-NI | -0.08 | 0.35 | 0.77 | $1.711^{\text {c }}$ | -0.61 | 1.06 | $1.83{ }^{\text {c }}$ | $2.19{ }^{\text {b }}$ | 1.36 | 1.11 | -0.12 | $1.98{ }^{\text {b }}$ | 1.42 | 0.04 | 0.81 | 0.99 |
| V-Co-NI | -0.15 | 0.25 | 0.82 | $1.73{ }^{\text {c }}$ | -0.48 | 1.05 | $1.86{ }^{\text {c }}$ | $2.19{ }^{\text {b }}$ | 1.24 | 1.22 | -0.12 | $2.00^{\text {b }}$ | 1.36 | 0.57 | 0.84 | 1.09 |
| VI-Co-NI | $-0.22$ | 0.24 | 0.84 | 1.21 | -0.46 | 0.96 | $1.85{ }^{\text {c }}$ | $2.16{ }^{\text {b }}$ | 1.26 | 1.11 | -0.11 | $2.01{ }^{\text {b }}$ | 1.32 | 0.45 | 0.54 | 1.28 |
| II-Ga-NI | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| III-Ga-NI | -0.08 | 0.08 | 0.52 | $1.66{ }^{\text {c }}$ | -0.53 | 1.30 | $1.70{ }^{\text {c }}$ | $2.33{ }^{\text {b }}$ | 1.27 | 1.10 | -0.01 | $2.05{ }^{\text {b }}$ | 1.39 | 0.05 | 0.87 | 1.14 |
| IV-Ga-NI | -0.24 | 0.22 | 0.75 | 1.54 | -0.59 | 1.18 | $1.74{ }^{\text {c }}$ | $2.26{ }^{\text {b }}$ | 1.41 | 1.11 | -0.09 | $2.04{ }^{\text {b }}$ | 1.39 | 0.11 | 0.86 | 1.13 |
| V-Ga-NI | -0.19 | 0.14 | 0.77 | 1.50 | -0.47 | 1.14 | $1.78{ }^{\text {c }}$ | $2.25{ }^{\text {b }}$ | 1.28 | 1.19 | -0.04 | $2.05^{\text {b }}$ | 1.42 | 0.59 | 0.78 | 1.21 |
| VI-Ga-NI | -0.58 | 0.10 | 0.80 | 1.29 | -0.43 | 1.13 | $1.77^{\text {c }}$ | $2.24{ }^{\text {b }}$ | 1.34 | 1.11 | -0.06 | $2.09^{\text {b }}$ | 1.39 | 0.55 | 0.58 | 1.43 |
| II-Gj-NI | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| III-Gj-NI | 0.10 | -0.07 | 0.53 | 1.51 | -0.54 | 1.21 | $1.65{ }^{\text {c }}$ | $2.31{ }^{\text {b }}$ | 1.20 | 1.09 | -0.05 | $2.06{ }^{\text {b }}$ | 1.34 | 0.08 | 0.87 | 1.19 |
| IV-Gj-NI | -0.01 | 0.08 | 0.73 | 1.47 | -0.61 | 1.13 | $1.70{ }^{\text {c }}$ | $2.26{ }^{\text {b }}$ | 1.35 | 1.07 | -0.03 | $2.07^{\text {b }}$ | 1.34 | 0.07 | 0.87 | 1.20 |
| V-Gj-NI | -0.11 | 0.00 | 0.78 | 1.42 | $-0.47$ | 1.10 | $1.74{ }^{\text {c }}$ | $2.26{ }^{\text {b }}$ | 1.22 | 1.16 | -0.08 | $2.07^{\text {b }}$ | 1.34 | 0.57 | 0.80 | 1.24 |
| VI-Gj-NI | -0.18 | -0.01 | 0.80 | 1.22 | -0.42 | 1.09 | $1.72^{\text {c }}$ | $2.26{ }^{\text {b }}$ | 1.25 | 1.11 | -0.08 | $2.07^{\text {b }}$ | 1.33 | 0.52 | 0.59 | 1.45 |

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Table 1.15: Out-of-sample forecast evaluation MCS - Crisis

|  | BA | BAC | C | CAT | FDX | HON | H | IBM | JPM | KFT | PEP | PG | T | TWX | TXN | WFC |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Co-NO | 0.53 | 0.11 | 0.38 | 0.56 | 0.39 | 0. | 0.26 |  | 0.16 | 0.67 |  | $0.46{ }^{\text {b }}$ | $0.18^{\text {a }}$ | 0.40 | $0.29{ }^{\text {b }}$ | 0.01 |
| Co-NO | . 00 | $0.21^{\text {a }}$ | $0.20^{\text {a }}$ | $0.94{ }^{\text {b }}$ | . 08 | $0.11{ }^{\text {a }}$ | $0.26{ }^{\text {b }}$ | $0.79{ }^{\text {b }}$ | $0.19^{\text {a }}$ | $0.67{ }^{\text {b }}$ | $0.31{ }^{\text {b }}$ | $0.82{ }^{\text {b }}$ | $0.79{ }^{\text {b }}$ | $0.18{ }^{\text {a }}$ | $0.29{ }^{\text {b }}$ | $0.12^{\text {a }}$ |
| IV-Co-NO | 0.00 | $1.00{ }^{\text {b }}$ | $0.76{ }^{\text {b }}$ | $0.75{ }^{\text {b }}$ | . 02 | $0.18^{\text {a }}$ | $0.26{ }^{\text {b }}$ | $0.68^{\text {b }}$ | $0.19^{\text {a }}$ | $0.86{ }^{\text {b }}$ | $0.31{ }^{\text {b }}$ | $1.00^{\text {b }}$ | $0.79{ }^{\text {b }}$ | $0.18{ }^{\text {a }}$ | $0.29{ }^{\text {b }}$ | 0.02 |
| Co-NO | 0.00 | $0.21^{\text {a }}$ | $0.68{ }^{\text {b }}$ | $0.75{ }^{\text {b }}$ | . 02 | $0.12^{\text {a }}$ | 0.26 | $0.88^{\text {b }}$ | 0.19 | 0.67 | $0.31{ }^{\text {b }}$ | $0.82{ }^{\text {b }}$ | 0.79 | $0.31{ }^{\text {b }}$ | $0.29{ }^{\text {b }}$ | $0.12^{\text {a }}$ |
| VI-Co-NO | 0.00 | $0.21^{\text {a }}$ | $0.76{ }^{\text {b }}$ | $0.56{ }^{\text {b }}$ | 0.03 | $0.11^{\text {a }}$ | $0.26{ }^{\text {b }}$ | $0.88{ }^{\text {b }}$ | $0.20^{\text {a }}$ | $0.67{ }^{\text {b }}$ | $0.31{ }^{\text {b }}$ | $0.82{ }^{\text {b }}$ | $0.64{ }^{\text {b }}$ | $0.28{ }^{\text {b }}$ | $0.29{ }^{\text {b }}$ | $0.42^{\text {b }}$ |
| Ga-NO | $1.00^{\text {b }}$ | $0.21^{\text {a }}$ | . 03 | $0.25{ }^{\text {b }}$ | $0.39^{\text {b }}$ | $0.11^{\text {a }}$ | $0.26{ }^{\text {b }}$ | $0.55{ }^{\text {b }}$ | $0.16^{\text {a }}$ | $0.67{ }^{\text {b }}$ | $0.31{ }^{\text {b }}$ | $0.30^{\text {b }}$ | $0.18^{\text {a }}$ | $0.40{ }^{\text {b }}$ | $0.29{ }^{\text {b }}$ | 0.01 |
| -Ga-NO | $0.20^{\text {a }}$ | 0.04 | 0.04 | $0.94{ }^{\text {b }}$ | $0.29^{\text {b }}$ | $0.18^{\text {a }}$ | 0.26 | $0.79{ }^{\text {b }}$ | $0.20^{\text {a }}$ | 0.86 | $0.31{ }^{\text {b }}$ | $0.46{ }^{\text {b }}$ | $0.91{ }^{\text {b }}$ | 0.31 | $0.58{ }^{\text {b }}$ | 0.02 |
| IV-Ga-NO | $0.10^{\text {a }}$ | $0.11^{\text {a }}$ | $0.38^{\text {b }}$ | $0.56{ }^{\text {b }}$ | $0.12^{\text {a }}$ | $0.18^{\text {a }}$ | $0.26{ }^{\text {b }}$ | $0.55{ }^{\text {b }}$ | $0.63{ }^{\text {b }}$ | $0.86{ }^{\text {b }}$ | $0.31{ }^{\text {b }}$ | $0.46{ }^{\text {b }}$ | $0.91{ }^{\text {b }}$ | $0.40{ }^{\text {b }}$ | $0.58{ }^{\text {b }}$ | 0.02 |
| V-Ga-NO | $0.20^{\text {a }}$ | 0.05 | $0.53{ }^{\text {b }}$ | $0.56{ }^{\text {b }}$ | $0.12^{\text {a }}$ | $0.12^{\text {a }}$ | $0.26{ }^{\text {b }}$ | $0.79{ }^{\text {b }}$ | $0.63{ }^{\text {b }}$ | $0.67{ }^{\text {b }}$ | $0.31{ }^{\text {b }}$ | $0.31{ }^{\text {b }}$ | $0.79{ }^{\text {b }}$ | $1.00^{\text {b }}$ | $0.29{ }^{\text {b }}$ | 0.02 |
| VI-Ga-NO | $0.10^{\text {a }}$ | 0.05 | $0.38{ }^{\text {b }}$ | $0.45{ }^{\text {b }}$ | $0.12{ }^{\text {a }}$ | $0.11{ }^{\text {a }}$ | $0.26^{\text {b }}$ | $0.79^{\text {b }}$ | 1.00 | 0.67 | $0.31{ }^{\text {b }}$ | $0.46{ }^{\text {b }}$ | 0.79 | $0.40{ }^{\text {b }}$ | $0.29{ }^{\text {b }}$ | $0.42^{\text {b }}$ |
| II-Gj-NO | $0.53{ }^{\text {b }}$ | $0.21^{\text {a }}$ | 06 | $0.25^{\text {b }}$ | $0.39^{\text {b }}$ | $0.47^{\text {b }}$ | $0.26{ }^{\text {b }}$ | $0.55^{\text {b }}$ | $0.16^{\text {a }}$ | $0.47{ }^{\text {b }}$ | $0.84{ }^{\text {b }}$ | $0.31{ }^{\text {b }}$ | 0.03 | $0.40^{\text {b }}$ | $0.29{ }^{\text {b }}$ | 0.01 |
| III-Gj-NO | $0.10^{\text {a }}$ | 0.02 | 0.06 | $1.00^{\text {b }}$ | $0.12^{\text {a }}$ | $0.47^{\text {b }}$ | $0.26{ }^{\text {b }}$ | $0.79{ }^{\text {b }}$ | $0.63{ }^{\text {b }}$ | $0.86{ }^{\text {b }}$ | $0.96{ }^{\text {b }}$ | $0.82{ }^{\text {b }}$ | $1.00^{\text {b }}$ | $0.28{ }^{\text {b }}$ | $0.58{ }^{\text {b }}$ | 0.02 |
| IV-Gj-NO | $0.10^{\text {a }}$ | 0.05 | $0.53{ }^{\text {b }}$ | $0.56^{\text {b }}$ | . 03 | $0.47{ }^{\text {b }}$ | $0.26{ }^{\text {b }}$ | $0.55{ }^{\text {b }}$ | $0.95{ }^{\text {b }}$ | $1.00^{\text {b }}$ | $0.96{ }^{\text {b }}$ | $0.82{ }^{\text {b }}$ | 0.79 | $0.31{ }^{\text {b }}$ | $0.58{ }^{\text {b }}$ | 0.02 |
| V-Gj-NO | $0.10^{\text {a }}$ | 02 | $0.59^{\text {b }}$ | $0.54{ }^{\text {b }}$ | 0.09 | $0.47^{\text {b }}$ | $0.26{ }^{\text {b }}$ | $0.79{ }^{\text {b }}$ | $0.63{ }^{\text {b }}$ | $0.86{ }^{\text {b }}$ | $0.84{ }^{\text {b }}$ | $0.78{ }^{\text {b }}$ | $0.79{ }^{\text {b }}$ | $0.40^{\text {b }}$ | $0.29{ }^{\text {b }}$ | 0.02 |
| VI-Gj-NO | 0.04 | 0.05 | $0.66^{\text {b }}$ | $0.25{ }^{\text {b }}$ | $0.11^{\text {a }}$ | $0.47^{\text {b }}$ | $0.26{ }^{\text {b }}$ | $0.88{ }^{\text {b }}$ | $0.95{ }^{\text {b }}$ | $0.67{ }^{\text {b }}$ | $0.54{ }^{\text {b }}$ | $0.82{ }^{\text {b }}$ | $0.79{ }^{\text {b }}$ | $0.40{ }^{\text {b }}$ | $0.29{ }^{\text {b }}$ | $0.66{ }^{\text {b }}$ |
| II-Co-NI | $0.10^{\text {a }}$ | $0.11^{\text {a }}$ | $0.20^{\text {a }}$ | $0.25{ }^{\text {b }}$ | $0.39^{\text {b }}$ | $0.11^{\text {a }}$ | $0.26^{\text {b }}$ | $0.55^{\text {b }}$ | $0.16^{\text {a }}$ | $0.47{ }^{\text {b }}$ | $0.31{ }^{\text {b }}$ | $0.46{ }^{\text {b }}$ | 0.03 | $0.28{ }^{\text {b }}$ | $0.29{ }^{\text {b }}$ | 0.00 |
| III-Co-NI | 0.04 | $0.11^{\text {a }}$ | $0.20^{\text {a }}$ | $0.25{ }^{\text {b }}$ | $0.11^{\text {a }}$ | $0.12^{\text {a }}$ | $0.26{ }^{\text {b }}$ | $0.88^{\text {b }}$ | $0.16^{\text {a }}$ | $0.47{ }^{\text {b }}$ | $0.31{ }^{\text {b }}$ | $0.78{ }^{\text {b }}$ | $0.64{ }^{\text {b }}$ | $0.18{ }^{\text {a }}$ | $0.58{ }^{\text {b }}$ | 0.02 |
| IV-Co-NI | 0.00 | $0.21^{\text {a }}$ | $0.59{ }^{\text {b }}$ | $0.54{ }^{\text {b }}$ | 0.09 | $0.11^{\text {a }}$ | $0.26{ }^{\text {b }}$ | $0.68{ }^{\text {b }}$ | $0.19^{\text {a }}$ | $0.47{ }^{\text {b }}$ | $0.31{ }^{\text {b }}$ | $0.81{ }^{\text {b }}$ | $0.64{ }^{\text {b }}$ | $0.18{ }^{\text {a }}$ | $0.58{ }^{\text {b }}$ | 0.02 |
| V-Co-NI | . 00 | $0.11^{\text {a }}$ | $0.76{ }^{\text {b }}$ | $0.54{ }^{\text {b }}$ | $0.12^{\text {a }}$ | $0.11^{\text {a }}$ | $0.26{ }^{\text {b }}$ | $0.75{ }^{\text {b }}$ | $0.19^{\text {a }}$ | $0.47^{\text {b }}$ | $0.31{ }^{\text {b }}$ | $0.68{ }^{\text {b }}$ | $0.18^{\text {a }}$ | $0.28{ }^{\text {b }}$ | $0.58{ }^{\text {b }}$ | 0.02 |
| VI-Co-NI | 0.00 | $0.11^{\text {a }}$ | $1.00^{\text {b }}$ | $0.25^{\text {b }}$ | $0.13^{\text {a }}$ | $0.11^{\text {a }}$ | $0.26{ }^{\text {b }}$ | $0.79{ }^{\text {b }}$ | $0.19{ }^{\text {a }}$ | $0.44{ }^{\text {b }}$ | $0.31{ }^{\text {b }}$ | $0.78{ }^{\text {b }}$ | 0.03 | $0.24{ }^{\text {a }}$ | $0.29{ }^{\text {b }}$ | $0.20^{\text {a }}$ |
| II-Ga-NI | $0.53{ }^{\text {b }}$ | $0.11^{\text {a }}$ | . 03 | $0.25{ }^{\text {b }}$ | $1.00^{\text {b }}$ | $0.11^{\text {a }}$ | $0.26{ }^{\text {b }}$ | $0.55{ }^{\text {b }}$ | $0.16^{\text {a }}$ | $0.47{ }^{\text {b }}$ | $0.31{ }^{\text {b }}$ | $0.31{ }^{\text {b }}$ | 0.03 | $0.28{ }^{\text {b }}$ | $0.29{ }^{\text {b }}$ | 0.00 |
| III-Ga-NI | $0.10^{\text {a }}$ | 0.05 | 0.03 | $0.75{ }^{\text {b }}$ | $0.39^{\text {b }}$ | $0.47^{\text {b }}$ | $0.26{ }^{\text {b }}$ | $0.88{ }^{\text {b }}$ | $0.19^{\text {a }}$ | $0.67{ }^{\text {b }}$ | $0.31{ }^{\text {b }}$ | $0.56{ }^{\text {b }}$ | $0.79^{\text {b }}$ | $0.24{ }^{\text {a }}$ | $1.00^{\text {b }}$ | 0.02 |
| IV-Ga-NI | 0.04 | $0.21^{\text {a }}$ | $0.20^{\text {a }}$ | $0.56{ }^{\text {b }}$ | $0.13^{\text {a }}$ | $0.29{ }^{\text {b }}$ | $0.26{ }^{\text {b }}$ | $0.75{ }^{\text {b }}$ | $0.20^{\text {a }}$ | $0.67{ }^{\text {b }}$ | $0.31{ }^{\text {b }}$ | $0.56{ }^{\text {b }}$ | $0.79{ }^{\text {b }}$ | $0.24{ }^{\text {a }}$ | $0.87^{\text {b }}$ | 0.02 |
| V-Ga-NI | 0.04 | $0.11^{\text {a }}$ | $0.38^{\text {b }}$ | $0.56{ }^{\text {b }}$ | $0.39^{\text {b }}$ | $0.18^{\text {a }}$ | $0.26{ }^{\text {b }}$ | $0.75{ }^{\text {b }}$ | $0.55{ }^{\text {b }}$ | $0.67{ }^{\text {b }}$ | $0.31{ }^{\text {b }}$ | $0.56{ }^{\text {b }}$ | $0.79{ }^{\text {b }}$ | $0.40{ }^{\text {b }}$ | $0.60{ }^{\text {b }}$ | 0.02 |
| VI-Ga-NI | 0.00 | $0.11^{\text {a }}$ | $0.53{ }^{\text {b }}$ | $0.25{ }^{\text {b }}$ | $0.39^{\text {b }}$ | $0.18^{\text {a }}$ | $0.26{ }^{\text {b }}$ | $0.79{ }^{\text {b }}$ | $0.95{ }^{\text {b }}$ | $0.67{ }^{\text {b }}$ | $0.31{ }^{\text {b }}$ | $0.68{ }^{\text {b }}$ | $0.62^{\text {b }}$ | $0.40{ }^{\text {b }}$ | $0.58{ }^{\text {b }}$ | $0.42{ }^{\text {b }}$ |
| II-Gj-NI | $0.53{ }^{\text {b }}$ | $0.11^{\text {a }}$ | 03 | $0.54{ }^{\text {b }}$ | $0.58{ }^{\text {b }}$ | $0.11^{\text {a }}$ | $0.26{ }^{\text {b }}$ | $0.55^{\text {b }}$ | $0.16^{\text {a }}$ | $0.47{ }^{\text {b }}$ | $0.54{ }^{\text {b }}$ | $0.31^{\text {b }}$ | $0.18^{\text {a }}$ | $0.31{ }^{\text {b }}$ | $0.29{ }^{\text {b }}$ | 0.00 |
| III-Gj-NI | $0.53{ }^{\text {b }}$ | 0.02 | 0.03 | $0.96{ }^{\text {b }}$ | $0.29{ }^{\text {b }}$ | $1.00^{\text {b }}$ | $1.00{ }^{\text {b }}$ | $1.00^{\text {b }}$ | $0.19^{\text {a }}$ | $0.86{ }^{\text {b }}$ | $0.31{ }^{\text {b }}$ | $0.82{ }^{\text {b }}$ | $0.91{ }^{\text {b }}$ | $0.28{ }^{\text {b }}$ | $0.82^{\text {b }}$ | $0.42{ }^{\text {b }}$ |
| IV-Gj-NI | $0.53{ }^{\text {b }}$ | $0.11^{\text {a }}$ | $0.20^{\text {a }}$ | $0.75{ }^{\text {b }}$ | $0.12^{\text {a }}$ | $0.47^{\text {b }}$ | $0.94{ }^{\text {b }}$ | $0.88{ }^{\text {b }}$ | $0.95{ }^{\text {b }}$ | $0.67{ }^{\text {b }}$ | $0.54{ }^{\text {b }}$ | $0.82^{\text {b }}$ | $0.79{ }^{\text {b }}$ | $0.28{ }^{\text {b }}$ | $0.82^{\text {b }}$ | $0.12^{\text {a }}$ |
| V-Gj-NI | $0.53{ }^{\text {b }}$ | 0.05 | $0.53{ }^{\text {b }}$ | $0.75{ }^{\text {b }}$ | $0.39^{\text {b }}$ | $0.47^{\text {b }}$ | $0.94{ }^{\text {b }}$ | $0.88{ }^{\text {b }}$ | $0.63{ }^{\text {b }}$ | $0.67{ }^{\text {b }}$ | $0.31{ }^{\text {b }}$ | $0.81{ }^{\text {b }}$ | $0.79{ }^{\text {b }}$ | $0.61{ }^{\text {b }}$ | $0.60{ }^{\text {b }}$ | $0.12^{\text {a }}$ |
| VI-Gj-NI | $0.10^{\text {a }}$ | 0.04 | $0.53{ }^{\text {b }}$ | $0.56{ }^{\text {b }}$ | $0.39^{\text {b }}$ | $0.47^{\text {b }}$ | $0.26{ }^{\text {b }}$ | $0.88{ }^{\text {b }}$ | $0.95{ }^{\text {b }}$ | $0.67{ }^{\text {b }}$ | $0.31{ }^{\text {b }}$ | $0.82^{\text {b }}$ | $0.79{ }^{\text {b }}$ | $0.40{ }^{\text {b }}$ | $0.58{ }^{\text {b }}$ | $1.00^{\text {b }}$ |

[^9]
## Chapter 2

## The predictability of Realized Range: the role of macroeconomic and financial variables

### 2.1 Introduction

During the last decades an increasing attention has been paid to the estimation and modeling of volatility asset because of its crucial importance in risk management, asset pricing and portfolio allocation. With the availability of high frequency data Andersen, Bollerslev, Diebold, and Labys (2001) and Barndorff-Nielsen and Shephard (2002) introduced a new approach for measuring and estimating daily volatility. The realized variance, defined as the sum of squared intra-daily returns, provides a more accurate measure of price variation. Christensen and Podolskij (2007) and Martens and van Dijk (2007) proposed the realized range-based variance an estimator that is defined as the sum of the squared ranges. What is more, an observable volatility series modeled with time series techniques can be used to obtain forecast of the realized estimation of the volatility. While a large part of the literature has focused on the estimation and on the correction of the bias introduced by the microstructure noise present in the high frequency price series ${ }^{1}$, another part has centered on modeling and forecasting realized volatility.

The long memory in financial volatility has been documented by many authors, it is associated with the slow decay of its autocorrelation function and it has been modeled with different approaches. Andersen, Bollerslev, Diebold, and Labys (2003) presented an Autoregressive Fractionally Integrated Moving Average (ARFIMA) model and Corsi (2009) introduced the Heterogenous Autoregressive (HAR) model, that approximates the hyperbolic decay of the autocorrelation

[^10]function by including the sums of RV over different periods (that is, by mean of a restricted autoregressive process with a large number of lags). Moreover, different models have been presented to tackle the stylized facts present in financial data in order to obtain more accurate forecasts. For instance, Corsi, Mittnik, Pigorsch, and Pigorsch (2008) proposed extensions to explicitly account for non-Gaussianity and volatility cluster in the volatility. Andersen, Bollerslev, and Diebold (2007) and Bollerslev, Kretschmer, Pigorsch, and Tauchen (2009) modeled and forecasted realized volatility considering jump component. Andersen, Bollerslev, and Diebold (2007) provide a practical framework for measuring significant jumps in financial asset prices. They found that jumps dynamics are much less persistent (and predictable) than continuous sample path dynamics. Moreover, they demonstrated important gains in terms of volatility forecast accuracy by explicitly differentiating the jump from the continuous sample path component. Bollerslev, Kretschmer, Pigorsch, and Tauchen (2009) presented a multivariate discrete-time volatility model that jointly incorporates the returns, the realized volatility and jumps.

In addition, the predictive power of macroeconomic and financial variable has been also studied with alternative approaches, in different markets and instrument, considering different sample periods and predictor variables that are related to the sampling frequency of the studied variable. While financial data is available on a daily basis, most of macroeconomic data is sampled at least at monthly frequency. Among others, recent contributions in the literature are Christiansen, Schmeling, and Schrimpf (2010), Paye (2010) for monthly series, while Fernandes, Medeiros, and Scharth (2009) and Martens, van Dijk, and de Pooter (2009) considered daily series of volatility and macroeconomic and financial variables. Christiansen, Schmeling, and Schrimpf (2010) studied the predictability by macroeconomic fundamentals and financial variables of monthly volatility in foreign exchange, bond, equity, and commodity markets. They considered a large number of potential predictors from the return predictability literature ${ }^{2}$ as well as proxies for the macroeconomic activity as inflation and industrial production growth. They found that economic variables provide information about future volatility in foreign exchange, bond, and commodity markets, both from an in-sample and out-of-sample perspective while their results for stocks are less significatives. Paye (2010) analyzed the ability of financial and economic variables to forecast monthly aggregate stock return volatility and rarely found statistical difference between the performance of macroeconomic fundamentals and univariate benchmarks.

At the daily frequency, Fernandes, Medeiros, and Scharth (2009) modeled the VIX with a parametric and semi parametric long memory specifications, controlling for macroeconomic and financial market conditions and including proxies for the present and future real economic and macroeconomic activity and liquidity in the market. They found an in-sample negative relationship between the VIX and contemporaneous and lagged S\&P 500 index returns, positive correlation between the VIX and the S\&P 500 index volume and a decreasing relation with lagged oil futures returns. In the out-of-sample analysis they remarked the importance of the

[^11]persistence of the VIX while their semi parametric model with economic variables presented a good performance across different forecasting horizons. An alternative approach is to consider macroeconomic announcements. Martens, van Dijk, and de Pooter (2009) analyzed the forecasting performance for different horizons of time series models for daily realized volatility for the S\&P 500 index including macroeconomic news announcements. They concluded that the introduction of these effects results in a more accurate prevision of the volatility. Finally, Caporin, Rossi, and Santucci de Magistris (2011) presented an extension of the HAR model for estimating the presence of jumps in volatility and incorporated the creditdefault swap in the dynamics of the jump size and intensity. Moreover, they studied the relation between the first principal component of the volatility jumps estimated from a set of 36 stocks quoted at the New York Stock Exchange (NYSE) and the same set of financial and policy daily variables of this work. Their results suggest that the credit deterioration of US bank sector, proxied by the credit default swap index of the US bank sector, impacts on an increase of the volatility over some days during the 2008-2009 financial crisis.

In this chapter, we wish to disentangle the role that related macroeconomic and financial variables play when modeling and forecasting daily stocks volatility. We analyze in-sample the effect of potential predictors that capture the present and the future state of the economy as well as the present and the future stock market condition. Then, we consider an out-of-sample forecasting exercise to analyze the improvement that results from the introduction of the macroeconomic variables that better perform in our in-sample analysis. Our empirical study is based on seven years of volatility series estimated from our database of high frequency prices for 16 stocks quoted on the New York Stock Exchange (NYSE). Following Caporin and Velo (2011) we construct volatility series through the realized range volatility, introduced by Martens and van Dijk (2007) and Christensen and Podolskij (2007) and consider the correction procedure of Martens and van Dijk (2007) to account for the presence of Microstructure noise in high frequency data. Moreover, we implement a model that accounts for long memory, leverage effects with respect to the return and the volatility, GARCH and GJR-GARCH specifications for the volatility of volatility and the Normal Inverse Gaussian (NIG) distribution to deal with the non-Gaussianity of the innovation.

To study the relationship and the forecasting predictive power of macroeconomic and financial information, we consider nine different daily variables. In particular, we use proxies for the U.S stock market performance and the trading activity on S\&P 500, the expectation about future conditions of the market or volatility through the VIX. Moreover, we include variables to account for the real economic activity through commodity prices, macroeconomic condition in the US by means of a US dollar exchange rate index and the Federal Fund deviation, measure of liquidity and proxies for the credit risk represented by the credit spread and the credit default swaps index for the US bank sector; many of them introduced by Fernandes, Medeiros, and Scharth (2009). The results of the empirical analysis suggest that macroeconomic and financial fundamentals have a significant explanatory power when we use them as regressors on the first common factor of
the 16 series of volatility. The effects of the different variables are the expected and indicate a link between volatility and financial activity and the perceived credit risk in the market. When we use the variables to improve the predictive accuracy of the 16 volatility series we obtain different results. The variables related with the U.S stock market performance and proxies for the credit risk produce a significant improvement in the out-of-sample accuracy in 8 out of 16 stocks.

The rest of the chapter is structured as follows. In section 2.2, we present the volatility estimators and the macroeconomic and financial variables. In section 2.3 we discuss the in-sample performance of the macroeconomic and financial variables while in section 2.4 we present our forecasting framework. Section 2.5 shows the result for the forecasting analysis and we present the conclusions in section 2.6.

### 2.2 Volatility estimation

To obtain our volatility series we follow Caporin and Velo (2011) and estimate the realized range volatility ( $R R V$ ) introduced by Martens and van Dijk (2007) and Christensen and Podolskij (2007). The $R R V$ is an unbiased and consistent estimator of the integrated variance and in theory, it is five time more efficient than $R V$. In the reality, high frequency data is contaminated by microstructure noise that result in a realized estimator that is inconsistent and biased. To remove this bias and restore the efficiency of the estimator, we implement Martens and van Dijk (2007) correction, based on scaling the range with the daily range. ${ }^{3}$ The idea behind this correction is the fact that the daily range is almost not contaminated by market frictions. The $R R V_{t}^{\Delta}$ is defined as

$$
\begin{equation*}
R R V_{t}^{\Delta}=\frac{1}{\lambda^{2}} \sum_{i=1}^{n}\left(\ln p_{t, i}^{h g}-\ln p_{t, i}^{l o}\right)^{2} \tag{2.1}
\end{equation*}
$$

where $p_{t, i}^{h g}$ and $p_{t, i}^{l o}$ are the high and low prices observed in the $i$ th interval of length $\Delta$ of an equidistant partition of day $t$, and $\lambda$ is the scaling factor. Therefore, the scaled $R R V_{t}^{\Delta}$ is defined as:

$$
\begin{equation*}
R R V_{\text {scaled }, t}^{\Delta}=\left(\frac{\sum_{l=1}^{q} R R V_{t-l}}{\sum_{l=1}^{q} R R V_{t-l}^{\Delta}}\right) R R V_{t}^{\Delta} \tag{2.2}
\end{equation*}
$$

where $R R V_{t} \equiv R R V_{t}^{\Delta}$ (with $\Delta=1$ day) is the daily range and $q$ is the number of previous trading days used to compute the scaling factor. If the trading intensity and the spread do not change, $q$ must be set as large as possible. However, in the reality only recent history should be taken into consideration.

[^12]Our empirical analysis in based on 16 stocks quoted on the NYSE: Boeing $(B A)$, Bank of America ( $B A C$ ), Citigroup Inc. ( $C$ ), Caterpillar Inc. ( $C A T$ ), FedEx Corporation (FDX), Honeywell International Inc. (HON), Hewlett-Packard Company ( $H P Q$ ), International Business Machines Corp. (IBM), JPMorgan Chase \& Co. ( $J P M$ ), Kraft Foods Inc. ( $K F T$ ), PepsiCo, Inc. (PEP), The Procter \& Gamble Company $(P G)$, AT\&T Inc. ( $T$ ), Time Warner Inc. ( $T W X$ ), Texas Instruments Incorporated (TXN) and Wells Fargo \& Company (WFC). We construct and correct ${ }^{4}$ the volatility series from a high frequency database with prices sampled at one minute frequency from January 2, 2003 to March 30, 2010, from 09:30 trough 16:00 and a total of 1887 trading days.

### 2.3 Macroeconomic determinants of volatility

To analyze the impact that macroeconomic and financial information has on the stock volatility, we extract the first common factor of the 16 series of volatility performing a principal components analysis and we regress it on the different variables. In particular, we consider the following proxies. The return on S\&P 500 that reflects the condition and the performance of the US stock market and the financial sector in general. We expect that a positive performance of the market will produce a reduction in the volatility of the stocks. We consider the returns over three different periods (daily, weekly and monthly); the volume on S\&P 500 which reflects the level of activity in the market. As early documented in the literature, an increase in the volume tends to increase the volatility. We also include the VIX, a model free implied volatility index based on the S\&P 500 monthly options that represents the expected future of volatility over the next month. As Fernandes, Medeiros, and Scharth (2009) have stated high VIX levels typically reflect pessimism while low VIX levels would mirror complacency among market participants. Therefore, we expect the VIX to be positively correlated with the realized range volatility. We include the UBS commodity index that is composed of a set of 19 commodity futures from seven different sectors to capture the evolution of the real economic activity. The index should be negatively correlated with the stock volatility. The US dollar exchange rate index reflects the macroeconomic condition of the US economy as well as the Federal Fund rate deviation between the target and effective rates. We expect the appreciation of the dollar to be negatively correlated with the volatility. We also consider the credit spread and the credit default swap index for the US banks sector that represent the credit risk and they should be positively correlated with stock volatility. Finally, the term spread is a measure of liquidity.

Our linear model for the first component is

$$
\begin{equation*}
f c_{t}=\alpha+\lambda^{\prime} x_{t-1}+\epsilon_{t} \tag{2.3}
\end{equation*}
$$

[^13]where $x_{t}$ is a k-dimensional vector that contains the macroeconomic explanatory variables: the daily, weekly and monthly S\&P 500 logarithmic returns ( $\Delta S P_{d}$, $\Delta S P_{w}$ and $\left.\Delta S P_{m}\right)$, the S\&P 500 volume change ( $\Delta V_{d}^{S P}$ ), the logarithmic of the VIX $\left(V I X_{d}\right)$, the daily, weekly and monthly DJ-UBS Commodity Index log return $\left(U B S_{d}, U B S_{w}\right.$ and $\left.U B S_{m}\right)$, the first difference of the logarithmic of the US dollar foreign exchange index value $\left(\Delta U S d_{d}\right)$, the credit spread defined as the excess yield on Moody's seasoned Baa corporate bond over the Moody's seasoned Aaa corporate bond $\left(C S_{d}\right)$, the term spread $\left(T S_{d}\right)$ defined as the difference between the 3 -month and 10 -Year treasury constant maturity rates, the difference between the effective and the target Federal Funds rates $\left(F F_{d}\right)$ and the credit default swap index (CDS) for the US bank sector $\left(C D S_{d}\right)$.

Since the aim of this work is to concentrate on the forecasting ability of economic and finance variable, we present and discuss the result for the regression of the first component of the volatilities in the lagged variables. A contemporaneous analysis is performed and presented in table 2.1. We regress the first component in three different periods. The objective is to analyze the impact of the economic and financial proxies in the full sample as well as to the period before and during the 2008-2009 financial crisis. The first component accounts for the $80 \%$ of the variability of the 16 volatility series.

The outcomes are presented in table 2.2. We first consider the result of the regression of all the variables on the full sample. On the one side, the returns on S\&P 500 are only significant at the monthly level and negatively correlated. On the other side, the S\&P 500 changes in volume are significant and positive. Moreover, the VIX is significant and positively correlated. Regarding the effect of the commodity index and the exchange rate index both of them are not significant in any of the considered periods (full, precrisis and crisis). The last four remaining variables are significant. The credit spread is positively correlated while the term spread is negatively correlated. Finally, the difference between the effective and target Federal Fund rates, and the credit default index for the bank sector in the US are positively correlated with the first component of the log volatility. The Rsquared for the regression of all the variables is $88 \%$, suggesting a high predictive power of this variable on the volatility. The effect of the variables in the periods corresponding to the 2008-2009 precrisis and the crisis are similar. Finally, we regress the first component in some of the variables. We consider the returns and the volume on S\&P 500, the VIX, the credit spread and the credit default spread. These variables have also an $88 \%$ of R -squared, which suggest a large predictive power. They are significant and have the same effect as before. During the crisis most of the variable maintain their effects, except for the credit spread that is negatively correlated but insignificant. The partial $r^{2}$ displays nonzero correlation only for the VIX and the credit default swap index. The $r^{2}$ for the VIX is lower during the 2008-2009 financial crisis while the $r^{2}$ of the CDS increases considerably during the crisis. As Caporin, Rossi, and Santucci de Magistris (2011) suggest, the effect of the credit default swaps is an evidence of the perceived credit risk during the financial crisis.

To analyze the improvement in the performance accuracy that results when con-
sidering macroeconomic and financial variables in volatility forecasting, we propose to include the last set of five variables in our model for the logarithmic volatility series of 16 stocks.

### 2.4 The models for the volatility with macroeconomic variables

In order to forecast volatility we present a model for the series that take into consideration the stylized facts documented in the literature for financial series. We follow Caporin and Velo (2011) and consider the HAR model of Corsi (2009) to capture the long range pattern aggregating the volatility over different periods. Moreover, we include asymmetric effects with respect to the returns and the volatility as well as lagged returns. We specify two different equations for the volatility of the volatility, early introduced by Corsi, Mittnik, Pigorsch, and Pigorsch (2008), with a GARCH and a GJR-GARCH specification, where the last one accounts for the asymmetric effects in the variance term. Finally, we consider two different distributions for the innovations the Normal and the Normal Inverse Gaussian (NIG), where the former is able to capture the observed skewness of the errors. Then, we extend this model including a set of lagged macroeconomic and financial variables that reflect the market expectations of economic and financial activity. We estimate the following model:

$$
\begin{gather*}
h_{t}=\alpha+\delta_{s} I_{s}\left(h_{t-1}\right) h_{t-1}+\beta_{d} h_{t-1}+\beta_{w} h_{(t-1: t-5)}+\beta_{m} h_{(t-1: t-22)}+  \tag{2.4}\\
+\gamma_{R} R_{t-1}+\gamma_{I R} I\left(R_{t-1}\right) R_{t-1}+\lambda^{\prime} x_{t-1}^{*}+\sqrt{\sigma_{t} \epsilon_{t}} \\
\sigma_{t}=\omega+\beta_{1} \sigma_{t-1}+\alpha_{1} u_{t-1}^{2}+\phi_{1} u_{t-1}^{2} I\left(u_{t-1}\right) \\
\epsilon_{t} \mid \Omega_{t-1} \sim d(0,1)
\end{gather*}
$$

where $h_{t}$ is the $\log R R V_{\text {scaled,t }}, h_{(t-1: t-j)}$ is the HAR component defined as

$$
\begin{equation*}
h_{(t-1: t-j)}=\frac{1}{j} \sum_{k=1}^{j} h_{t-k} \tag{2.5}
\end{equation*}
$$

where $x_{t}^{*}$ is a k -dimensional vector of five macroeconomic explanatory variables: the returns over three different periods and the volume on S\&P $500\left(\Delta S P_{d}, \Delta S P_{w}\right.$, $\Delta S P_{m}$ and $\left.\Delta V_{d}^{S P}\right)$, the VIX $\left(V I X_{d}\right)$, the credit spread $\left(C S_{d}\right)$ and the credit default spread $\left(C D S_{d}\right) . j=5$ and 22 are the weekly and monthly HAR components. $I_{s}\left(h_{t-1}\right)$ is an indicator for $R R V_{\text {scaled }, t-1}$ bigger than the mean over $s=5$ previous days and the unconditional mean ( $s=$ full $)$ up to $t-1$. These variables capture the asymmetric effects with respect to the volatility. $R_{t}=\ln \left(p_{t}^{c l} / p_{t-1}^{c l}\right)$ is the return, with $p^{c l}$ the closure price for the day $t$ and $I\left(R_{t-1}\right)$ is an indicator for negative returns in $t-1$, that captures the asymmetric effects with regard to the lagged return. $u_{t}=\sqrt{\sigma_{t}} \epsilon_{t}$ is the error term. The full specification for $\sigma_{t}$ is a GJRGARCH to account for the asymmetric effect in the volatility of the volatility,
where $I\left(u_{t-1}\right)$ is an indicator for $u_{t-1}<0, \omega \geq 0, \beta_{1} \geq 0, \alpha_{1} \geq 0$ to guarantee the conditional variance to be positive and $\beta_{1}+\alpha_{1} \leq 1$ and $\beta_{1}+\alpha_{1}+\frac{1}{2} \phi_{1} \leq 1$ to guarantee stationary under symmetry of the density of the standardized residuals.

We concentrate on the six different specifications for the mean equation considered in Caporin and Velo (2011) and that account for all the discussed stylized facts. Then, we compute one-step-ahead rolling forecast for the volatility series with these 36 models and the macroeconomic variables. The models are the $\operatorname{AR}(1)$ (I), the HAR model of Corsi (2009) (II), the HAR model with asymmetric effects with respect to the historical volatility and the returns and the lagged returns (III), the HAR with asymmetric effects with respect to the returns and lagged returns(IV), the HAR model with symmetric effects with respect to the returns (V) and finally the HAR model with symmetric effects for the past weekly volatility and the returns (VI).

Estimation results for the HAR model with asymmetric effects with respect to the returns and lagged returns with GJR-GARCH variance and NIG distribution (IV) are presented in tables 2.5 and 2.6 for 16 stocks. The estimated coefficients for the HAR are significant with the same sign that for the model estimated without the macroeconomic variables. The sum of the estimated parameters $\left(\beta_{d}, \beta_{w}\right.$ and $\left.\beta_{m}\right)$ ranges from 0.68 to 0.85 . Although they suggest a persistent process, they are smaller than the sum of the coefficients without the macro variables. Variables such the VIX may capture some of this persistence. Lagged returns are positively correlated while the asymmetric effects for the return are highly significant and they are still negative, inducing an increment of the volatility after a negative return.

We briefly discuss the effect of the macroeconomic and financial variables. The estimated coefficients and their significance are similar to the results presented for the OLS regression for the first principal component. Positive daily, weekly and monthly S\&P 500 index returns decrease the volatility while the S\&P 500 volume has the opposite effect. As we expected, the VIX, a measure of the monthly implied volatility in S\&P 500 options is positively correlated. Finally, the credit spread and credit default swap, which capture the credit risk, are significant in most of the series and they have a positive impact.

### 2.5 Forecast results

To analyze the effect of the introduction of macroeconomic variables in the out-of-sample forecasting accuracy of the different models, we compute one-dayahead out-of-sample rolling forecast from January 3, 2006 to March 30, 2010 for 16 stocks for a total of 1067 days period. We estimate the models with and without the economic and financial variables until December 30, 2005 and then we re-estimate each model at each recursion expanding the data set by one observation.

Based on the results discussed in Caporin and Velo (2011) we compute forecasts with six different models for the mean equation with the economic variables. Those are combined with three variance specification and two distributions for the innovations. Overall, we have a total of 72 models.

These models include the variables that capture the stylized effects vastly documented in the literature and five lagged macroeconomic variables: the S\&P 500 returns over different horizons, S\&P 500 volume, the VIX, the credit spread and the credit default swaps for the banking sector, which display a highly predictive in-sample ability. To evaluate the performance of the different volatility models, we implement the Diebold and Mariano (1995) test based on the Mean Absolute error (MAE) and the mean square error (MSE). As Patton (2011) state, the MSE is a robust loss function to the presence of noise in the volatility proxies that results in an unbiased model ranking. Moreover, we perform the Model Confidence Set (MCS) procedure of Hansen, Lunde, and Nason (2010) based on the same two loss functions.

First, consider the Diebold and Mariano (1995) tests for the full out-of-sample period in tables 2.7 and 2.8. The first table (2.7) displays $t$-statistics for the Diebold and Mariano (1995) test for 36 models estimated with the economic and financial variables versus the HAR model for the same variance specification and distributions of the error terms. As an example, the second row of the table presents the t-statistic for the test of equal performance between the HAR model with normal distribution and constant variance versus the HAR model with macroeconomic variables and the same specification for the variance and distribution. The test rejects the null hypothesis in 12 out of 16 series in favor of the model that includes the macro variables with constant variance and normal distribution. Moreover, when we consider different specifications for the variance and different distributions the test tends to reject the equal performance. For example, the test also rejects in 14 out of 16 stocks the null hypothesis in favor of the model with macroeconomic fundamentals for the NIG distribution and constant and GARCH variance. Only for Time Warner Inc. (TWX), the introduction of asymmetric effects and macroeconomic variables does not improve the performance of the HAR model for any of the variance and distribution assumptions.

Table 2.8 presents the result for the Diebold and Mariano (1995) test between the six specification for the mean equation with and without the introduction of the macroeconomic variable (and the different variance and distribution assumptions). For example, the third row of the first column is the Diebold and Mariano (1995) test for the HAR model with symmetric effects with respect to the volatility and the returns and lagged returns versus the same specification for the mean equation with macroeconomic and financial variables for Boeing Co (BA). In 8 out of 16 stocks, the introduction of the macroeconomic variables produces an improvement in the forecast accuracy of most of the models. For instance, for Bank of America (BAC), any model with the economic variables perform significatively better than the competitor without the explanatory variables, while for Texas Instruments Incorporated (TXN) the introduction of the macroeconomic variables produces a more accurate forecast only for the case of the $\operatorname{AR}(1)$ model.

Tables 2.10 and 2.11 report the result for the Model Confidence Set approach (MCS) of Hansen, Lunde, and Nason (2010). In contrast to the Diebold and Mariano (1995) test, the MCS procedure defines, for a given level of confidence, the set of models that contains the best out-of-sample forecasts. This is a procedure
that allows for a comparison of all the models at the same time, eliminating the worst performing specifications that belong to an initial set. Our initial set of models covers 72 specifications, 36 models without economic and financial variables and 36 models which include them. In 8 out of 16 stocks, the models without macroeconomic variables are excluded from the best set at the $25 \%$ level, with 7 of them excluding the models at the $10 \%$ level. Moreover, the models with macroeconomic variables always belong to the best set of each stock.

We now consider the forecast result for the 2008-2009 financial crisis. Table 2.9 displays the t -statistics for the Diebold and Mariano (1995) test. Only in one stock, Kraft (KFT) the models that include macroeconomic variables perform better than the models without economic variables. The introduction of financial and economic variables seems to be irrelevant in the pairwise comparison, since they are not able to produce any improvement in the forecast accuracy. Table 2.12 and 2.13 present the result of the MCS procedure. The models without economic variables are excluded from the best set in 4 out of 16 stocks at the $10 \%$ and in 5 out of 16 at the $25 \%$. These results seem to be counterintuitive and they are opposite to the results of the in-sample analysis for the 2008-2009 financial crisis of the first principal component, where the macroeconomic variables are significant and seem to have a large predictive power with an R-squared of almost $70 \%$. During the financial crisis, the models with these variables do not produce better forecasts than simpler models.

### 2.6 Conclusions

In this Chapter, we have examined the importance of introducing macroeconomic variables when forecasting financial volatility. Following Caporin and Velo (2011), we have estimated and forecasted price variation through the realized range volatility proposed by Martens and van Dijk (2007) and Christensen and Podolskij (2007). We have estimated and corrected due to the presence of microstructure noise 16 volatility series of stocks traded at NYSE. Moreover, we have considered nine macroeconomic and financial variables that contain information of the economic activity and financial market and that were included in our model to produce one-step-ahead volatility forecast. The model has taken into consideration long memory, asymmetric effects with respect to the return and the volatility in the mean equation, and we have included a GARCH and GJR-GARCH specifications for the variance equation (which models the volatility of the volatility) as well as a non Gaussian distribution for the innovations.

To evaluate the explanatory power of the macroeconomic variables, we use them as regressors on the first principal component of 16 series of volatility. We find that S\&P 500 changes in volume and the VIX are significant and positively correlated, which is expected. The value of the US dollar and our commodity index does not affect the volatility. Finally, the credit spread and the credit default swap of the US banking sector, two indicators of the credit risk, impact positively in the first component of the log volatility. The effects of these variables do not change during the crisis, while the high R-squared obtained for the different periods
suggests a high predictive power of this variable on the volatility. The same results are obtained when considering the estimation of the single series of volatility with different specifications for the variance and distribution assumptions.

Finally, we have computed one-step-ahead rolling forecast including the most representative macroeconomic variables in the models for the 16 stocks. Pairwise comparisons, through the Diebold and Mariano (1995), favor the introduction of the macroeconomic variable in 8 out of 16 assets, where these 8 stocks belong to different sectors. When testing for the improvement of the accuracy through the Model Confidence Set approach of Hansen, Lunde, and Nason (2010), in 8 out of 16 stocks, models without macroeconomic variables are excluded from the best set. On the contrary, taking into consideration the macroeconomic variables during the financial crisis does not result in a better performance.

In conclusion, although the strong correlation found between stock volatility and different economic and financial indicators, the role of these variables depends on each stock when forecasting. Future research may consider the introduction of non-linearities in the models as well as the analysis of alternative class of instrument such as bond, commodity or foreign exchange markets in different forecasting horizons.

### 2.7 Tables and Figures

Table 2.1: The effects of contemporaneous macroeconomic and financial variables

|  | Full |  | Pre crisis |  | Crisis |  | Full |  | Pre crisis |  | Crisis |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\beta$ | $r^{2}$ | $\beta$ | $r^{2}$ | $\beta$ | $r^{2}$ | $\beta$ | $r^{2}$ | $\beta$ | $r^{2}$ | $\beta$ | $r^{2}$ |
| Cons | $\begin{gathered} -14.26^{\mathbf{a}} \\ (3.322) \end{gathered}$ | - | $\begin{gathered} -15.08^{\mathrm{a}} \\ (1.577) \end{gathered}$ | - | $\begin{aligned} & -24.43 \\ & (19.27) \end{aligned}$ | - | $\begin{gathered} -13.44^{\mathrm{a}} \\ (1.858) \end{gathered}$ | - | $\begin{gathered} -14.61^{\mathrm{a}} \\ (1.508) \end{gathered}$ | - | $\begin{gathered} -20.50^{\mathbf{b}} \\ (9.558) \end{gathered}$ | - |
| $\Delta S P_{d}$ | $\begin{gathered} 13.45 \\ (10.44) \end{gathered}$ | 0.016 | $\begin{gathered} 20.94 \\ (14.87) \end{gathered}$ | 0.022 | $\begin{gathered} 12.94 \\ (24.07) \end{gathered}$ | 0.051 | $\begin{gathered} 12.58 \\ (10.37) \end{gathered}$ | 0.015 | $\begin{gathered} 21.21 \\ (15.85) \end{gathered}$ | 0.022 | $\begin{gathered} 11.25 \\ (18.43) \end{gathered}$ | 0.049 |
| $\Delta S P_{w}$ | $\begin{aligned} & -7.608 \\ & (5.565) \end{aligned}$ | 0.015 | $\begin{aligned} & -10.97 \\ & (13.75) \end{aligned}$ | 0.018 | $\begin{aligned} & -0.779 \\ & (8.168) \end{aligned}$ | 0.000 | $\begin{gathered} -9.244^{\mathrm{c}} \\ (4.856) \end{gathered}$ | 0.024 | $\begin{aligned} & -11.97 \\ & (16.84) \end{aligned}$ | 0.021 | $\begin{aligned} & -2.663 \\ & (6.867) \end{aligned}$ | 0.007 |
| $\Delta S P_{m}$ | $\begin{gathered} -10.46^{\mathbf{a}} \\ (3.675) \end{gathered}$ | 0.093 | $\begin{gathered} -14.51^{\mathrm{c}} \\ (7.439) \end{gathered}$ | 0.096 | $\begin{aligned} & -4.089 \\ & (4.795) \end{aligned}$ | 0.026 | $\begin{gathered} -10.30^{\mathrm{a}} \\ (3.012) \end{gathered}$ | 0.110 | $\begin{gathered} -15.85^{\mathrm{b}} \\ (7.256) \end{gathered}$ | 0.114 | $\begin{aligned} & -4.009 \\ & (3.998) \end{aligned}$ | 0.029 |
| $\Delta V_{d}^{S P}$ | $\begin{aligned} & 1.539^{a} \\ & (0.017) \end{aligned}$ | 0.052 | $\begin{aligned} & 1.611^{\mathrm{a}} \\ & (0.025) \end{aligned}$ | 0.067 | $\begin{aligned} & 1.691^{\mathrm{a}} \\ & (0.189) \end{aligned}$ | 0.064 | $\begin{aligned} & 1.572^{a} \\ & (0.019) \end{aligned}$ | 0.054 | $\begin{aligned} & 1.643^{\mathrm{a}} \\ & (0.026) \end{aligned}$ | 0.069 | $\begin{aligned} & 1.759^{a} \\ & (0.207) \end{aligned}$ | 0.065 |
| $V I X_{d}$ | $\begin{aligned} & 4.217^{\mathrm{a}} \\ & (0.612) \end{aligned}$ | 0.248 | $\begin{aligned} & 4.404^{\mathbf{a}} \\ & (0.220) \end{aligned}$ | 0.296 | $\begin{aligned} & 9.541^{a} \\ & (1.492) \end{aligned}$ | 0.321 | $\begin{aligned} & 3.846^{\mathbf{a}} \\ & (0.329) \end{aligned}$ | 0.275 | $\begin{aligned} & 4.034^{\mathrm{a}} \\ & (0.190) \end{aligned}$ | 0.290 | $\begin{aligned} & 7.079^{a} \\ & (0.895) \end{aligned}$ | 0.283 |
| $U B S_{d}$ | $\begin{aligned} & -5.517 \\ & (5.895) \end{aligned}$ | 0.002 | $\begin{aligned} & -3.010 \\ & (9.403) \end{aligned}$ | 0.000 | $\begin{aligned} & -4.617 \\ & (25.63) \end{aligned}$ | 0.002 | - | - | - | - | - | - |
| $U B S_{w}$ | $\begin{aligned} & -1.797 \\ & (5.612) \end{aligned}$ | 0.000 | $\begin{aligned} & -3.261 \\ & (7.414) \end{aligned}$ | 0.003 | $\begin{gathered} 2.180 \\ (14.61) \end{gathered}$ | 0.002 | - | - | - | - | - | - |
| $U B S_{m}$ | $\begin{gathered} 1.704 \\ (1.448) \end{gathered}$ | 0.004 | $\begin{gathered} 0.301 \\ (1.927) \end{gathered}$ | 0.000 | $\begin{gathered} 4.015 \\ (10.39) \end{gathered}$ | 0.016 | - | - | - | - | - | - |
| $\Delta U S d_{d}$ | $\begin{gathered} 6.367 \\ (25.45) \end{gathered}$ | 0.000 | $\begin{gathered} 6.941 \\ (34.62) \end{gathered}$ | 0.000 | $\begin{gathered} 15.92 \\ (107.4) \end{gathered}$ | 0.008 | ${ }^{-}$ | - | ${ }^{-}$ | - | ${ }^{-}$ | - |
| $C S_{d}$ | $\begin{aligned} & 0.963^{\mathbf{a}} \\ & (0.090) \end{aligned}$ | 0.057 | $\begin{aligned} & 1.498^{\mathbf{a}} \\ & (0.417) \end{aligned}$ | 0.020 | $\begin{gathered} -1.788^{\mathbf{a}} \\ (0.398) \end{gathered}$ | 0.085 | $\begin{aligned} & 1.027^{\mathrm{a}} \\ & (0.067) \end{aligned}$ | 0.072 | $\begin{gathered} 1.968^{\mathbf{a}} \\ (0.299) \end{gathered}$ | 0.041 | $\begin{gathered} -0.833^{\mathrm{a}} \\ (0.288) \end{gathered}$ | 0.032 |
| $T S_{d}$ | $\begin{gathered} -0.114^{\mathrm{a}} \\ (0.006) \end{gathered}$ | 0.009 | $\begin{gathered} -0.133^{a} \\ (0.006) \end{gathered}$ | 0.014 | $\begin{gathered} -0.621^{\mathbf{a}} \\ (0.216) \end{gathered}$ | 0.020 | - | - | - | - | - | - |
| $F F_{d}$ | $\begin{aligned} & 0.333^{c} \\ & (0.189) \end{aligned}$ | 0.001 | $\begin{aligned} & 0.891^{a} \\ & (0.226) \end{aligned}$ | 0.005 | $\begin{aligned} & 0.763^{a} \\ & (0.109) \end{aligned}$ | 0.016 | ${ }^{-}$ | - | ${ }^{-}$ | - | ${ }^{-}$ | - |
| $C D S_{d}$ | $\begin{aligned} & 1.288^{a} \\ & (0.037) \end{aligned}$ | 0.236 | $\begin{aligned} & 0.961^{a} \\ & (0.069) \end{aligned}$ | 0.080 | $\begin{aligned} & 1.151^{a} \\ & (0.069) \end{aligned}$ | 0.186 | $\begin{aligned} & 1.278^{\mathrm{a}} \\ & (0.034) \end{aligned}$ | 0.246 | $\begin{aligned} & 0.865^{\mathrm{a}} \\ & (0.054) \end{aligned}$ | 0.075 | $\begin{aligned} & 1.437^{\mathrm{a}} \\ & (0.056) \end{aligned}$ | 0.286 |
| $R^{2}$ | 0.89 | - | 0.81 | - | 0.72 | - | 0.88 | - | 0.80 | - | 0.70 | - |

Note: The first principal component of the logarithmic RRV of the 16 stocks is regressed on 13 contemporaneous macroeconomics and financial variables. Newey-West errors in bracket. $r^{2}$ is the partial $r^{2}$. $R^{2}$ is the adjusted R-squared. "a", "b" and "c" indicate significance at the $1 \%, 5 \%$ and $10 \%$. Full indicates the full in-sample period (1580 observations), Pre crisis indicates the period before the financial crisis (1193 observations) and Crisis indicates the 2008-2009 financial crisis period (200 observations).

Table 2.2: The effects of lagged macroeconomic and financial variables

|  | Full |  | Pre crisis |  | Crisis |  | Full |  | Pre crisis |  | Crisis |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\beta$ | $r^{2}$ | $\beta$ | $r^{2}$ | $\beta$ | $r^{2}$ | $\beta$ | $r^{2}$ | $\beta$ | $r^{2}$ | $\beta$ | $r^{2}$ |
| Cons | $\begin{gathered} -14.85^{\mathrm{a}} \\ (1.172) \end{gathered}$ | - | $\begin{gathered} -14.04^{\mathrm{a}} \\ (1.409) \end{gathered}$ | - | $\begin{aligned} & -19.19 \\ & (13.94) \end{aligned}$ | - | $\begin{gathered} -13.75^{\mathrm{a}} \\ (0.727) \end{gathered}$ | - | $\begin{gathered} -13.82^{\mathrm{a}} \\ (1.328) \end{gathered}$ | - | $\begin{gathered} -16.07^{\mathrm{c}} \\ (8.978) \end{gathered}$ | - |
| $\Delta S P_{d}$ | $\begin{aligned} & -0.298 \\ & (5.650) \end{aligned}$ | 0.001 | $\begin{aligned} & -0.521 \\ & (12.52) \end{aligned}$ | 0.001 | $\begin{aligned} & -1.392 \\ & (17.22) \end{aligned}$ | 0.000 | $\begin{aligned} & -0.605 \\ & (4.896) \end{aligned}$ | 0.001 | $\begin{aligned} & -0.269 \\ & (13.05) \end{aligned}$ | 0.001 | $\begin{gathered} 0.669 \\ (8.226) \end{gathered}$ | 0.000 |
| $\Delta S P_{w}$ | $\begin{aligned} & -6.985 \\ & (6.251) \end{aligned}$ | 0.012 | $\begin{aligned} & -9.795 \\ & (13.20) \end{aligned}$ | 0.012 | $\begin{aligned} & -2.174 \\ & (11.27) \end{aligned}$ | 0.003 | $\begin{aligned} & -8.105 \\ & (6.061) \end{aligned}$ | 0.018 | $\begin{aligned} & -11.23 \\ & (15.09) \end{aligned}$ | 0.016 | $\begin{aligned} & -2.856 \\ & (10.42) \end{aligned}$ | 0.007 |
| $\Delta S P_{m}$ | $\begin{gathered} -9.925^{\mathbf{a}} \\ (2.890) \end{gathered}$ | 0.081 | $\begin{gathered} -13.59^{\mathrm{c}} \\ (7.184) \end{gathered}$ | 0.075 | $\begin{aligned} & -7.146 \\ & (8.435) \end{aligned}$ | 0.074 | $\begin{gathered} -9.713^{\mathrm{a}} \\ (2.479) \end{gathered}$ | 0.095 | $\begin{gathered} -14.73^{\mathbf{b}} \\ (6.909) \end{gathered}$ | 0.090 | $\begin{aligned} & -7.037 \\ & (4.801) \end{aligned}$ | 0.080 |
| $\Delta V_{d}^{S P}$ | $\begin{aligned} & 0.853^{\mathrm{a}} \\ & (0.023) \end{aligned}$ | 0.016 | $\begin{aligned} & 0.887^{\mathrm{a}} \\ & (0.027) \end{aligned}$ | 0.019 | $\begin{aligned} & 0.927^{\mathrm{a}} \\ & (0.297) \end{aligned}$ | 0.019 | $\begin{aligned} & 0.879^{a} \\ & (0.024) \end{aligned}$ | 0.017 | $\begin{aligned} & 0.888^{\mathrm{a}} \\ & (0.028) \end{aligned}$ | 0.018 | $\begin{aligned} & 0.981^{\mathrm{a}} \\ & (0.317) \end{aligned}$ | 0.019 |
| $V I X_{d}$ | $\begin{aligned} & 4.479^{\mathrm{a}} \\ & (0.226) \end{aligned}$ | 0.243 | $\begin{aligned} & 4.064^{\mathrm{a}} \\ & (0.213) \end{aligned}$ | 0.239 | $\begin{aligned} & 7.423^{\mathrm{a}} \\ & (1.306) \end{aligned}$ | 0.216 | $\begin{aligned} & 3.977^{\mathrm{a}} \\ & (0.134) \end{aligned}$ | 0.264 | $\begin{aligned} & 3.770^{\mathrm{a}} \\ & (0.173) \end{aligned}$ | 0.241 | $\begin{aligned} & 5.475^{\mathrm{a}} \\ & (0.924) \end{aligned}$ | 0.181 |
| $U B S_{d}$ | $\begin{aligned} & -0.716 \\ & (8.246) \end{aligned}$ | 0.001 | $\begin{aligned} & -1.234 \\ & (9.816) \end{aligned}$ | 0.001 | $\begin{gathered} 0.875 \\ (48.04) \end{gathered}$ | 0.001 | - | - | - | - | - | - |
| $U B S_{w}$ | $\begin{gathered} 0.151 \\ (6.116) \end{gathered}$ | 0.001 | $\begin{aligned} & -0.697 \\ & (7.474) \end{aligned}$ | 0.000 | $\begin{gathered} 3.298 \\ (18.30) \end{gathered}$ | 0.004 | - | - | - | - | - | - |
| $U B S_{m}$ | $\begin{gathered} 1.745 \\ (1.527) \end{gathered}$ | 0.004 | $\begin{gathered} 1.019 \\ (1.793) \end{gathered}$ | 0.001 | $\begin{gathered} 2.727 \\ (12.40) \end{gathered}$ | 0.007 | - | - | - | - | - | - |
| $\Delta U S d_{d}$ | $\begin{gathered} 4.668 \\ (37.77) \end{gathered}$ | 0.000 | $\begin{aligned} & -7.881 \\ & (41.28) \end{aligned}$ | 0.000 | $\begin{gathered} 31.48 \\ (83.94) \end{gathered}$ | 0.032 | ${ }^{-}$ | - | ${ }^{-}$ | - | ${ }^{-}$ | - |
| $C S_{d}$ | $\begin{aligned} & 0.832^{\mathrm{a}} \\ & (0.057) \end{aligned}$ | 0.040 | $\begin{aligned} & 1.273^{\mathrm{a}} \\ & (0.353) \end{aligned}$ | 0.013 | $\begin{gathered} -1.110^{\mathrm{a}} \\ (0.302) \end{gathered}$ | 0.032 | $\begin{aligned} & 0.943^{\mathrm{a}} \\ & (0.047) \end{aligned}$ | 0.058 | $\begin{gathered} 1.829^{a} \\ (0.269) \end{gathered}$ | 0.031 | $\begin{aligned} & -0.345 \\ & (0.257) \end{aligned}$ | 0.005 |
| $T S_{d}$ | $\begin{gathered} -0.148^{\mathrm{a}} \\ (0.005) \end{gathered}$ | 0.015 | $\begin{gathered} -0.141^{\mathrm{a}} \\ (0.006) \end{gathered}$ | 0.014 | $\begin{gathered} -0.497^{a} \\ (0.191) \end{gathered}$ | 0.012 |  | - | - | - | - | - |
| $F F_{d}$ | $\begin{aligned} & 0.344^{\text {b }} \\ & (0.165) \end{aligned}$ | 0.001 | $\begin{gathered} 0.323 \\ (0.198) \end{gathered}$ | 0.000 | $\begin{aligned} & 0.592^{\mathrm{b}} \\ & (0.254) \end{aligned}$ | 0.009 | ${ }^{-}$ | - | ${ }^{-}$ | - | ${ }^{-}$ | - |
| $C D S_{d}$ | $\begin{aligned} & 1.321^{\mathrm{a}} \\ & (0.029) \end{aligned}$ | 0.237 | $\begin{aligned} & 1.147^{\mathrm{a}} \\ & (0.057) \end{aligned}$ | 0.099 | $\begin{aligned} & 1.286^{\mathbf{a}} \\ & (0.070) \end{aligned}$ | 0.235 | $\begin{aligned} & 1.298^{a} \\ & (0.027) \end{aligned}$ | 0.242 | $\begin{aligned} & 0.993^{\mathrm{a}} \\ & (0.046) \end{aligned}$ | 0.086 | $\begin{aligned} & 1.500^{\mathrm{a}} \\ & (0.048) \end{aligned}$ | 0.320 |
| $R^{2}$ | 0.88 | - | 0.78 | - | 0.71 | - | 0.88 | - | 0.78 | - | 0.68 | - |

Note: The first principal component of the logarithmic RRV of the 16 stocks is regressed on 13 lagged macroeconomics and financial variables. Newey-West errors in bracket. $r^{2}$ is the partial $r^{2} . R^{2}$ is the adjusted R-squared. "a", "b" and "c" indicate significance at the $1 \%, 5 \%$ and $10 \%$. Full indicates the full in-sample period (1580 observations), Pre crisis indicates the period before the financial crisis (1193 observations) and Crisis indicates the 2008-2009 financial crisis period (200 observations).

Table 2.3: HAR model with macroeconomic and financial variables - Normal distribution and Constant variance

|  | BA | BAC | C | CAT | FDX | HON | HPQ | IBM |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\alpha$ | $\begin{gathered} -1.638^{\mathrm{a}} \\ (0.293) \end{gathered}$ | $\begin{gathered} -2.268^{\mathbf{a}} \\ (0.292) \end{gathered}$ | $\begin{gathered} -1.916^{\mathbf{a}} \\ (0.308) \end{gathered}$ | $\begin{gathered} -1.221^{\mathrm{a}} \\ (0.272) \end{gathered}$ | $\begin{gathered} -1.305^{a} \\ (0.290) \end{gathered}$ | $\begin{gathered} -1.899^{\mathrm{a}} \\ (0.310) \end{gathered}$ | $\begin{gathered} -0.670^{\mathbf{a}} \\ (0.220) \end{gathered}$ | $\begin{gathered} -1.131^{\mathrm{a}} \\ (0.248) \end{gathered}$ |
| $\beta_{d}$ | $\begin{aligned} & 0.277^{a} \\ & (0.027) \end{aligned}$ | $\begin{aligned} & 0.433^{\mathrm{a}} \\ & (0.025) \end{aligned}$ | $\begin{aligned} & 0.408^{\mathrm{a}} \\ & (0.027) \end{aligned}$ | $\begin{aligned} & 0.331^{\mathbf{a}} \\ & (0.026) \end{aligned}$ | $\begin{aligned} & 0.332^{\mathrm{a}} \\ & (0.028) \end{aligned}$ | $\begin{aligned} & 0.305^{\mathrm{a}} \\ & (0.026) \end{aligned}$ | $\begin{aligned} & 0.328^{a} \\ & (0.028) \end{aligned}$ | $\begin{aligned} & 0.331^{\mathrm{a}} \\ & (0.028) \end{aligned}$ |
| $\beta_{w}$ | $\begin{aligned} & 0.360^{a} \\ & (0.044) \end{aligned}$ | $\begin{aligned} & 0.291^{a} \\ & (0.040) \end{aligned}$ | $\begin{aligned} & 0.371^{a} \\ & (0.040) \end{aligned}$ | $\begin{aligned} & 0.394^{a} \\ & (0.046) \end{aligned}$ | $\begin{aligned} & 0.233^{a} \\ & (0.047) \end{aligned}$ | $\begin{aligned} & 0.379^{a} \\ & (0.043) \end{aligned}$ | $\begin{aligned} & 0.372^{a} \\ & (0.047) \end{aligned}$ | $\begin{aligned} & 0.418^{\mathbf{a}} \\ & (0.042) \end{aligned}$ |
| $\beta_{m}$ | $\begin{aligned} & 0.098^{\text {b }} \\ & (0.047) \end{aligned}$ | $\begin{gathered} 0.003 \\ (0.035) \end{gathered}$ | $\begin{gathered} 0.013 \\ (0.033) \end{gathered}$ | $\begin{gathered} 0.068 \\ (0.043) \end{gathered}$ | $\begin{aligned} & 0.205^{a} \\ & (0.054) \end{aligned}$ | $\begin{aligned} & -0.007 \\ & (0.048) \end{aligned}$ | $\begin{aligned} & 0.131^{\mathrm{a}} \\ & (0.045) \end{aligned}$ | $\begin{aligned} & 0.060^{c} \\ & (0.036) \end{aligned}$ |
| $\gamma_{R T}$ | - | - | - | - | - | - | - | - |
| $\gamma_{I R T}$ | - | - | - | - | - | - | - | - |
| $\Delta S P_{d}$ | $\begin{aligned} & -1.63^{\mathbf{b}} \\ & (0.819) \end{aligned}$ | $\begin{gathered} -3.961^{\mathrm{a}} \\ (0.772) \end{gathered}$ | $\begin{gathered} -3.893^{\mathrm{a}} \\ (0.680) \end{gathered}$ | $\begin{gathered} -1.991^{\mathrm{a}} \\ (0.734) \end{gathered}$ | $\begin{gathered} -2.720^{\mathbf{a}} \\ (0.904) \end{gathered}$ | $\begin{gathered} -2.888^{\mathbf{a}} \\ (0.803) \end{gathered}$ | $\begin{aligned} & -2.74^{\mathrm{a}} \\ & (0.825) \end{aligned}$ | $\begin{gathered} -3.460^{\mathrm{a}} \\ (0.703) \end{gathered}$ |
| $\Delta S P_{w}$ | $\begin{gathered} -1.855^{\mathbf{a}} \\ (0.441) \end{gathered}$ | $\begin{gathered} -1.605^{\mathrm{a}} \\ (0.455) \end{gathered}$ | $\begin{gathered} -1.091^{\mathrm{a}} \\ (0.382) \end{gathered}$ | $\begin{gathered} -2.219^{\mathbf{a}} \\ (0.441) \end{gathered}$ | $\begin{gathered} -1.544^{\mathbf{a}} \\ (0.470) \end{gathered}$ | $\begin{gathered} -2.235^{\mathbf{a}} \\ (0.437) \end{gathered}$ | $\begin{gathered} -1.941^{\mathrm{a}} \\ (0.431) \end{gathered}$ | $\begin{gathered} -1.701^{\mathrm{a}} \\ (0.419) \end{gathered}$ |
| $\Delta S P_{m}$ | $\begin{gathered} -0.854^{\mathbf{a}} \\ (0.256) \end{gathered}$ | $\begin{gathered} -0.423^{\mathrm{c}} \\ (0.241) \end{gathered}$ | $\begin{gathered} 0.032 \\ (0.206) \end{gathered}$ | $\begin{aligned} & -0.354 \\ & (0.261) \end{aligned}$ | $\begin{gathered} -0.825^{\mathbf{a}} \\ (0.284) \end{gathered}$ | $\begin{gathered} -0.912^{\text {a }} \\ (0.279) \end{gathered}$ | $\begin{gathered} -1.123^{a} \\ (0.280) \end{gathered}$ | $\begin{gathered} -0.853^{\text {a }} \\ (0.243) \end{gathered}$ |
| $\Delta V_{d}^{S P}$ | $\begin{aligned} & 0.226^{\mathbf{a}} \\ & (0.051) \end{aligned}$ | $\begin{aligned} & 0.171^{\mathrm{a}} \\ & (0.060) \end{aligned}$ | $\begin{aligned} & 0.170^{\mathrm{a}} \\ & (0.057) \end{aligned}$ | $\begin{aligned} & 0.198^{\mathrm{a}} \\ & (0.054) \end{aligned}$ | $\begin{aligned} & 0.121^{b} \\ & (0.058) \end{aligned}$ | $\begin{aligned} & 0.111^{\text {c }} \\ & (0.058) \end{aligned}$ | $\begin{aligned} & 0.133^{b} \\ & (0.063) \end{aligned}$ | $\begin{aligned} & 0.108^{b} \\ & (0.051) \end{aligned}$ |
| $V I X_{d}$ | $\begin{aligned} & 0.167^{a} \\ & (0.061) \end{aligned}$ | $\begin{aligned} & 0.283^{a} \\ & (0.067) \end{aligned}$ | $\begin{aligned} & 0.328^{a} \\ & (0.077) \end{aligned}$ | $\begin{aligned} & 0.116^{\text {c }} \\ & (0.065) \end{aligned}$ | $\begin{gathered} 0.087 \\ (0.063) \end{gathered}$ | $\begin{aligned} & 0.160^{b} \\ & (0.065) \end{aligned}$ | $\begin{aligned} & -0.029 \\ & (0.059) \end{aligned}$ | $\begin{gathered} 0.058 \\ (0.055) \end{gathered}$ |
| $C S_{d}$ | $\begin{gathered} 0.043 \\ (0.034) \end{gathered}$ | $\begin{aligned} & 0.115^{a} \\ & (0.035) \end{aligned}$ | $\begin{gathered} 0.022 \\ (0.030) \end{gathered}$ | $\begin{aligned} & 0.060^{c} \\ & (0.035) \end{aligned}$ | $\begin{aligned} & 0.070^{c} \\ & (0.041) \end{aligned}$ | $\begin{aligned} & 0.099^{a} \\ & (0.037) \end{aligned}$ | $\begin{aligned} & 0.063^{c} \\ & (0.036) \end{aligned}$ | $\begin{gathered} 0.039 \\ (0.030) \end{gathered}$ |
| $C D S_{d}$ | $\begin{aligned} & 0.054^{\mathrm{a}} \\ & (0.019) \end{aligned}$ | $\begin{aligned} & 0.248^{a} \\ & (0.027) \end{aligned}$ | $\begin{aligned} & 0.184^{\mathrm{a}} \\ & (0.020) \end{aligned}$ | $\begin{aligned} & 0.048^{\mathrm{a}} \\ & (0.018) \end{aligned}$ | $\begin{aligned} & 0.076^{\text {a }} \\ & (0.025) \end{aligned}$ | $\begin{aligned} & 0.033^{\mathrm{c}} \\ & (0.019) \end{aligned}$ | $\begin{gathered} 0.021 \\ (0.018) \end{gathered}$ | $\begin{aligned} & 0.052^{a} \\ & (0.016) \end{aligned}$ |
| $\omega$ | $\begin{aligned} & 0.168^{\mathbf{a}} \\ & (0.005) \end{aligned}$ | $\begin{aligned} & 0.194^{a} \\ & (0.004) \end{aligned}$ | $\begin{aligned} & 0.182^{\mathrm{a}} \\ & (0.005) \end{aligned}$ | $\begin{aligned} & 0.174^{a} \\ & (0.004) \end{aligned}$ | $\begin{aligned} & 0.188^{\mathbf{a}} \\ & (0.004) \end{aligned}$ | $\begin{aligned} & 0.180^{\mathrm{a}} \\ & (0.004) \end{aligned}$ | $\begin{aligned} & 0.183^{\mathrm{a}} \\ & (0.004) \end{aligned}$ | $\begin{aligned} & 0.147^{\mathrm{a}} \\ & (0.003) \end{aligned}$ |
| $\beta_{1}$ | - | - | - | - | - | - | - | - |
| $\alpha_{1}$ | - | - | - | - | - | - | - | - |
| $\phi_{1}$ | - | - | - | - | - | - | - | - |
| $\alpha_{\text {NIG }}$ | - | - | - | - | - | - | - | - |
| $\beta_{N I G}$ | - | - | - | - | - | - | - | - |
| LLF | -834.48 | -946.41 | -899.57 | -862.36 | -925.43 | -888.16 | -901.67 | -730.34 |
| AIC | 1692.9 | 1916.8 | 1823.1 | 1748.7 | 1874.8 | 1800.3 | 1827.3 | 1484.6 |
| $B I C$ | 1757.3 | 1981.2 | 1887.5 | 1813.1 | 1939.2 | 1864.7 | 1891.7 | 1549.0 |
| $L j_{30}$ | 0.001 | 0.008 | 0.025 | 0.075 | 0.034 | 0.191 | 0.184 | 0.002 |
| $L j_{40}$ | 0.001 | 0.031 | 0.023 | 0.093 | 0.084 | 0.126 | 0.135 | 0.004 |
| $J B$ | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 |
| KS | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| LL | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 |

Note: Estimation results for the 16 series of stock volatility for the period January 2004 to March 2010. LLF is the Log-likelihood function, $A I C$ is the Akaike Information Criteria and BIC is the Bayesian information criterion. $L J_{30}$ and $L J_{40}$ are the Ljung Box test for 30 and 40 lags. $J B$ is the Jarque-Bera test for Normality, $K S$ is the Kolmogorov-Smirnov and $L L$ is the Lilliefors test. Standard errors in bracket. "a", "b" and "c" indicate significance at the $1 \%, 5 \%$ and $10 \%$.

Table 2.4: HAR model with macroeconomic and financial variables - Normal distribution and Constant variance (cont.)

|  | JPM | KFT | PEP | PG | T | TWX | TXN | WFC |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\alpha$ | $\begin{gathered} -2.643^{a} \\ (0.331) \end{gathered}$ | $\begin{gathered} -1.007^{\mathrm{a}} \\ (0.280) \end{gathered}$ | $\begin{gathered} -2.049^{\mathrm{a}} \\ (0.368) \end{gathered}$ | $\begin{gathered} -1.417^{\mathrm{a}} \\ (0.274) \end{gathered}$ | $\begin{gathered} -0.721^{\mathrm{a}} \\ (0.235) \end{gathered}$ | $\begin{gathered} -1.897^{\mathrm{a}} \\ (0.327) \end{gathered}$ | $\begin{gathered} -0.583^{\mathrm{a}} \\ (0.179) \end{gathered}$ | $\begin{gathered} -1.715^{a} \\ (0.317) \end{gathered}$ |
| $\beta_{d}$ | $\begin{aligned} & 0.372^{\mathrm{a}} \\ & (0.027) \end{aligned}$ | $\begin{aligned} & 0.252^{\mathrm{a}} \\ & (0.027) \end{aligned}$ | $\begin{aligned} & 0.201^{\mathrm{a}} \\ & (0.027) \end{aligned}$ | $\begin{aligned} & 0.266^{\mathrm{a}} \\ & (0.028) \end{aligned}$ | $\begin{aligned} & 0.343^{a} \\ & (0.028) \end{aligned}$ | $\begin{aligned} & 0.296^{a} \\ & (0.024) \end{aligned}$ | $\begin{aligned} & 0.279^{a} \\ & (0.028) \end{aligned}$ | $\begin{aligned} & 0.363^{a} \\ & (0.028) \end{aligned}$ |
| $\beta_{w}$ | $\begin{aligned} & 0.350^{a} \\ & (0.041) \end{aligned}$ | $\begin{aligned} & 0.398^{a} \\ & (0.048) \end{aligned}$ | $\begin{aligned} & 0.421^{a} \\ & (0.048) \end{aligned}$ | $\begin{aligned} & 0.453^{a} \\ & (0.046) \end{aligned}$ | $\begin{aligned} & 0.353^{a} \\ & (0.046) \end{aligned}$ | $\begin{aligned} & 0.379^{a} \\ & (0.047) \end{aligned}$ | $\begin{aligned} & 0.458^{\mathbf{a}} \\ & (0.047) \end{aligned}$ | $\begin{aligned} & 0.387^{a} \\ & (0.041) \end{aligned}$ |
| $\beta_{m}$ | $\begin{aligned} & -0.054 \\ & (0.037) \end{aligned}$ | $\begin{aligned} & 0.133^{a} \\ & (0.048) \end{aligned}$ | $\begin{gathered} 0.083 \\ (0.051) \end{gathered}$ | $\begin{gathered} 0.043 \\ (0.045) \end{gathered}$ | $\begin{aligned} & 0.152^{\mathrm{a}} \\ & (0.039) \end{aligned}$ | $\begin{gathered} 0.056 \\ (0.046) \end{gathered}$ | $\begin{aligned} & 0.110^{a} \\ & (0.039) \end{aligned}$ | $\begin{aligned} & 0.058^{\mathrm{c}} \\ & (0.034) \end{aligned}$ |
| $\gamma_{R T}$ | - | - | - | - | - | - | - | - |
| $\gamma_{I R T}$ | - | - | - | - | - | - | - | - |
| $\Delta S P_{d}$ | $\begin{gathered} -2.688^{\mathrm{a}} \\ (0.768) \end{gathered}$ | $\begin{aligned} & -1.112 \\ & (1.114) \end{aligned}$ | $\begin{gathered} -1.612^{\mathrm{a}} \\ (0.601) \end{gathered}$ | $\begin{gathered} -2.296^{\mathrm{a}} \\ (0.750) \end{gathered}$ | $\begin{gathered} -1.354^{\mathrm{c}} \\ (0.819) \end{gathered}$ | $\begin{aligned} & -1.088 \\ & (0.763) \end{aligned}$ | $\begin{gathered} -3.153^{\mathrm{a}} \\ (0.661) \end{gathered}$ | $\begin{gathered} -3.482^{\mathrm{a}} \\ (0.833) \end{gathered}$ |
| $\Delta S P_{w}$ | $\begin{gathered} -1.923^{\mathrm{a}} \\ (0.422) \end{gathered}$ | $\begin{gathered} -1.907^{a} \\ (0.651) \end{gathered}$ | $\begin{gathered} -2.732^{\mathrm{a}} \\ (0.441) \end{gathered}$ | $\begin{gathered} -2.138^{\mathrm{a}} \\ (0.450) \end{gathered}$ | $\begin{gathered} -2.229^{\mathrm{a}} \\ (0.444) \end{gathered}$ | $\begin{gathered} -1.746^{\mathbf{a}} \\ (0.392) \end{gathered}$ | $\begin{gathered} -1.751^{\mathbf{a}} \\ (0.386) \end{gathered}$ | $\begin{gathered} -2.018^{\mathbf{a}} \\ (0.432) \end{gathered}$ |
| $\Delta S P_{m}$ | $\begin{gathered} -0.682^{\mathrm{a}} \\ (0.237) \end{gathered}$ | $\begin{gathered} -1.057^{\mathrm{a}} \\ (0.352) \end{gathered}$ | $\begin{gathered} -0.971^{\mathrm{a}} \\ (0.275) \end{gathered}$ | $\begin{gathered} -0.913^{\mathrm{a}} \\ (0.291) \end{gathered}$ | $\begin{gathered} -0.940^{\mathrm{a}} \\ (0.270) \end{gathered}$ | $\begin{gathered} -0.638^{\mathbf{a}} \\ (0.231) \end{gathered}$ | $\begin{gathered} -0.450^{\mathbf{c}} \\ (0.235) \end{gathered}$ | $\begin{aligned} & -0.160 \\ & (0.227) \end{aligned}$ |
| $\Delta V_{d}^{S P}$ | $\begin{aligned} & 0.185^{\mathrm{a}} \\ & (0.054) \end{aligned}$ | $\begin{aligned} & 0.227^{\mathrm{a}} \\ & (0.064) \end{aligned}$ | $\begin{aligned} & 0.156^{\mathbf{b}} \\ & (0.063) \end{aligned}$ | $\begin{aligned} & 0.134^{\mathbf{b}} \\ & (0.058) \end{aligned}$ | $\begin{aligned} & 0.119^{c} \\ & (0.062) \end{aligned}$ | $\begin{aligned} & 0.155^{\mathrm{a}} \\ & (0.053) \end{aligned}$ | $\begin{aligned} & 0.166^{\mathrm{a}} \\ & (0.053) \end{aligned}$ | $\begin{aligned} & 0.123^{\mathrm{b}} \\ & (0.062) \end{aligned}$ |
| $V I X_{d}$ | $\begin{aligned} & 0.379^{\mathrm{a}} \\ & (0.073) \end{aligned}$ | $\begin{aligned} & -0.018 \\ & (0.073) \end{aligned}$ | $\begin{aligned} & 0.186^{\mathrm{a}} \\ & (0.068) \end{aligned}$ | $\begin{gathered} 0.048 \\ (0.059) \end{gathered}$ | $\begin{gathered} 0.000 \\ (0.061) \end{gathered}$ | $\begin{aligned} & 0.229^{\mathrm{a}} \\ & (0.067) \end{aligned}$ | $\begin{aligned} & -0.015 \\ & (0.049) \end{aligned}$ | $\begin{aligned} & 0.271^{\mathrm{a}} \\ & (0.076) \end{aligned}$ |
| $C S_{d}$ | $\begin{gathered} 0.049 \\ (0.032) \end{gathered}$ | $\begin{gathered} 0.063 \\ (0.045) \end{gathered}$ | $\begin{aligned} & 0.062^{\mathrm{c}} \\ & (0.036) \end{aligned}$ | $\begin{aligned} & 0.093^{\mathbf{b}} \\ & (0.036) \end{aligned}$ | $\begin{gathered} 0.034 \\ (0.034) \end{gathered}$ | $\begin{aligned} & 0.093^{\mathrm{a}} \\ & (0.035) \end{aligned}$ | $\begin{aligned} & 0.075^{\mathbf{b}} \\ & (0.033) \end{aligned}$ | $\begin{gathered} 0.020 \\ (0.030) \end{gathered}$ |
| $C D S_{d}$ | $\begin{aligned} & 0.221^{\mathrm{a}} \\ & (0.024) \end{aligned}$ | $\begin{gathered} 0.035 \\ (0.024) \end{gathered}$ | $\begin{aligned} & 0.035^{\text {c }} \\ & (0.018) \end{aligned}$ | $\begin{aligned} & 0.033^{\mathrm{c}} \\ & (0.017) \end{aligned}$ | $\begin{aligned} & 0.047^{a} \\ & (0.017) \end{aligned}$ | $\begin{aligned} & 0.051^{\mathrm{a}} \\ & (0.018) \end{aligned}$ | $\begin{gathered} 0.008 \\ (0.015) \end{gathered}$ | $\begin{aligned} & 0.163^{a} \\ & (0.024) \end{aligned}$ |
| $\omega$ | $\begin{aligned} & 0.169^{\mathrm{a}} \\ & (0.004) \end{aligned}$ | $\begin{aligned} & 0.250^{\mathbf{a}} \\ & (0.006) \end{aligned}$ | $\begin{aligned} & 0.176^{\mathrm{a}} \\ & (0.004) \end{aligned}$ | $\begin{aligned} & 0.176^{\mathbf{a}} \\ & (0.004) \end{aligned}$ | $\begin{aligned} & 0.180^{\mathrm{a}} \\ & (0.004) \end{aligned}$ | $\begin{aligned} & 0.152^{a} \\ & (0.003) \end{aligned}$ | $\begin{aligned} & 0.151^{\mathrm{a}} \\ & (0.004) \end{aligned}$ | $\begin{aligned} & 0.177^{a} \\ & (0.005) \end{aligned}$ |
| $\beta_{1}$ | - | - | - | - | - | - | - | - |
| $\alpha_{1}$ | - | - | - | - | - | - | - | - |
| $\phi_{1}$ | - | - | - | - | - | - | - | - |
| $\alpha_{\text {NIG }}$ | - | - | - | - | - | - | - | - |
| $\beta_{N I G}$ | - | - | - | - | - | - | - | - |
| LLF | -837.60 | -1147.8 | -869.97 | -872.21 | -887.93 | -758.14 | -752.11 | -877.97 |
| AIC | 1699.2 | 2319.7 | 1763.9 | 1768.4 | 1799.8 | 1540.2 | 1528.2 | 1779.9 |
| BIC | 1763.5 | 2384.1 | 1828.3 | 1832.8 | 1864.2 | 1604.6 | 1592.6 | 1844.3 |
| $L j_{30}$ | 0.227 | 0.285 | 0.004 | 0.291 | 0.001 | 0.087 | 0.000 | 0.158 |
| $L j_{40}$ | 0.169 | 0.198 | 0.004 | 0.299 | 0.004 | 0.014 | 0.000 | 0.277 |
| $J B$ | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 |
| KS | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| LL | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.025 | 0.001 |

Note: Estimation results for the 16 series of stock volatility for the period January 2004 to March 2010. LLF is the Log-likelihood function, $A I C$ is the Akaike Information Criteria and BIC is the Bayesian information criterion. $L J_{30}$ and $L J_{40}$ are the Ljung Box test for 30 and 40 lags. $J B$ is the Jarque-Bera test for Normality, $K S$ is the Kolmogorov-Smirnov and $L L$ is the Lilliefors test. Standard errors in bracket. "a", "b" and "c" indicate significance at the $1 \%, 5 \%$ and $10 \%$.

Table 2.5: HAR model with asymmetric effects with respect to the returns and macroeconomic and financial variables - NIG distribution and GJR-GARCH variance

|  | BA | BAC | C | CAT | FDX | HON | HPQ | IBM |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\alpha$ | $\begin{gathered} -1.425^{\mathbf{a}} \\ (0.269) \end{gathered}$ | $\begin{gathered} -1.929^{\mathrm{a}} \\ (0.269) \end{gathered}$ | $\begin{gathered} -1.485^{\mathrm{a}} \\ (0.309) \end{gathered}$ | $\begin{gathered} -1.176^{\mathrm{a}} \\ (0.234) \end{gathered}$ | $\begin{gathered} -1.400^{\mathrm{a}} \\ (0.248) \end{gathered}$ | $\begin{gathered} -1.695^{a} \\ (0.271) \end{gathered}$ | $\begin{gathered} -0.797^{a} \\ (0.184) \end{gathered}$ | $\begin{gathered} -1.166^{\mathbf{a}} \\ (0.219) \end{gathered}$ |
| $\beta_{d}$ | $\begin{aligned} & 0.268^{\mathrm{a}} \\ & (0.033) \end{aligned}$ | $\begin{aligned} & 0.413^{\mathrm{a}} \\ & (0.028) \end{aligned}$ | $\begin{aligned} & 0.335^{\mathrm{a}} \\ & (0.029) \end{aligned}$ | $\begin{aligned} & 0.323^{\mathrm{a}} \\ & (0.031) \end{aligned}$ | $\begin{aligned} & 0.301^{\mathrm{a}} \\ & (0.029) \end{aligned}$ | $\begin{aligned} & 0.268^{\mathrm{a}} \\ & (0.029) \end{aligned}$ | $\begin{aligned} & 0.307^{\mathrm{a}} \\ & (0.031) \end{aligned}$ | $\begin{aligned} & 0.287^{\mathrm{a}} \\ & (0.031) \end{aligned}$ |
| $\beta_{w}$ | $\begin{aligned} & 0.361^{\mathrm{a}} \\ & (0.044) \end{aligned}$ | $\begin{aligned} & 0.287^{a} \\ & (0.035) \end{aligned}$ | $\begin{aligned} & 0.403^{a} \\ & (0.042) \end{aligned}$ | $\begin{aligned} & 0.369^{a} \\ & (0.041) \end{aligned}$ | $\begin{aligned} & 0.287^{a} \\ & (0.045) \end{aligned}$ | $\begin{aligned} & 0.412^{a} \\ & (0.043) \end{aligned}$ | $\begin{aligned} & 0.387^{a} \\ & (0.043) \end{aligned}$ | $\begin{aligned} & 0.455^{a} \\ & (0.043) \end{aligned}$ |
| $\beta_{m}$ | $\begin{aligned} & 0.127^{a} \\ & (0.042) \end{aligned}$ | $\begin{aligned} & 0.055^{\mathbf{c}} \\ & (0.030) \end{aligned}$ | $\begin{aligned} & 0.084^{b} \\ & (0.034) \end{aligned}$ | $\begin{aligned} & 0.110^{a} \\ & (0.037) \end{aligned}$ | $\begin{aligned} & 0.184^{a} \\ & (0.046) \end{aligned}$ | $\begin{gathered} 0.036 \\ (0.041) \end{gathered}$ | $\begin{aligned} & 0.115^{a} \\ & (0.037) \end{aligned}$ | $\begin{gathered} 0.058 \\ (0.037) \end{gathered}$ |
| $\gamma_{R T}$ | $\begin{aligned} & 3.503^{a} \\ & (1.132) \end{aligned}$ | $\begin{aligned} & 3.324^{\mathbf{a}} \\ & (0.492) \end{aligned}$ | $\begin{aligned} & 2.065^{a} \\ & (0.434) \end{aligned}$ | $\begin{aligned} & 3.942^{a} \\ & (0.971) \end{aligned}$ | $\begin{aligned} & 1.745^{\mathrm{c}} \\ & (1.032) \end{aligned}$ | $\begin{aligned} & 3.589^{\mathbf{a}} \\ & (1.124) \end{aligned}$ | $\begin{aligned} & 1.829^{\mathrm{c}} \\ & (1.051) \end{aligned}$ | $\begin{gathered} 1.817 \\ (1.435) \end{gathered}$ |
| $\gamma_{I R T}$ | $\begin{gathered} -6.139^{a} \\ (1.858) \end{gathered}$ | $\begin{gathered} -5.849^{a} \\ (0.737) \end{gathered}$ | $\begin{gathered} -5.832^{\mathrm{a}} \\ (0.753) \end{gathered}$ | $\begin{gathered} -6.232^{\mathrm{a}} \\ (1.494) \end{gathered}$ | $\begin{aligned} & -2.844^{\mathrm{c}} \\ & (1.602) \end{aligned}$ | $\begin{gathered} -3.937^{\mathrm{b}} \\ (1.738) \end{gathered}$ | $\begin{gathered} -8.458^{\mathrm{a}} \\ (1.724) \end{gathered}$ | $\begin{gathered} -7.284^{\mathbf{a}} \\ (2.206) \end{gathered}$ |
| $\Delta S P_{d}$ | $\begin{gathered} -1.826^{\mathbf{c}} \\ (1.054) \end{gathered}$ | $\begin{gathered} -4.623^{\mathrm{a}} \\ (1.006) \end{gathered}$ | $\begin{gathered} -2.975^{\mathrm{a}} \\ (0.992) \end{gathered}$ | $\begin{gathered} -2.730^{\mathrm{a}} \\ (1.020) \end{gathered}$ | $\begin{gathered} -2.730^{\mathrm{b}} \\ (1.070) \end{gathered}$ | $\begin{gathered} -3.929^{\mathrm{a}} \\ (1.095) \end{gathered}$ | $\begin{aligned} & -0.907 \\ & (1.020) \end{aligned}$ | $\begin{gathered} -1.992^{\text {b }} \\ (1.000) \end{gathered}$ |
| $\Delta S P_{w}$ | $\begin{gathered} -1.923^{a} \\ (0.443) \end{gathered}$ | $\begin{gathered} -1.554^{\mathbf{a}} \\ (0.443) \end{gathered}$ | $\begin{gathered} -1.433^{a} \\ (0.509) \end{gathered}$ | $\begin{gathered} -2.363^{\mathrm{a}} \\ (0.437) \end{gathered}$ | $\begin{gathered} -1.575^{a} \\ (0.423) \end{gathered}$ | $\begin{gathered} -2.317^{\mathrm{a}} \\ (0.440) \end{gathered}$ | $\begin{gathered} -1.725^{\mathbf{a}} \\ (0.453) \end{gathered}$ | $\begin{gathered} -1.517^{\mathrm{a}} \\ (0.450) \end{gathered}$ |
| $\Delta S P_{m}$ | $\begin{gathered} -0.676^{\mathbf{a}} \\ (0.238) \end{gathered}$ | $\begin{aligned} & -0.279 \\ & (0.217) \end{aligned}$ | $\begin{gathered} 0.202 \\ (0.260) \end{gathered}$ | $\begin{aligned} & -0.080 \\ & (0.230) \end{aligned}$ | $\begin{gathered} -0.550^{b} \\ (0.241) \end{gathered}$ | $\begin{gathered} -0.439^{\mathrm{c}} \\ (0.237) \end{gathered}$ | $\begin{gathered} -0.853^{a} \\ (0.260) \end{gathered}$ | $\begin{gathered} -0.747^{a} \\ (0.245) \end{gathered}$ |
| $\Delta V_{d}^{S P}$ | $\begin{aligned} & 0.191^{\mathrm{a}} \\ & (0.052) \end{aligned}$ | $\begin{aligned} & 0.158^{\mathrm{a}} \\ & (0.052) \end{aligned}$ | $\begin{aligned} & 0.154^{\mathrm{a}} \\ & (0.054) \end{aligned}$ | $\begin{aligned} & 0.187^{a} \\ & (0.049) \end{aligned}$ | $\begin{aligned} & 0.111^{\mathbf{b}} \\ & (0.052) \end{aligned}$ | $\begin{aligned} & 0.114^{b} \\ & (0.051) \end{aligned}$ | $\begin{aligned} & 0.104^{c} \\ & (0.055) \end{aligned}$ | $\begin{aligned} & 0.102^{\mathrm{b}} \\ & (0.049) \end{aligned}$ |
| $V I X_{d}$ | $\begin{aligned} & 0.108^{c} \\ & (0.058) \end{aligned}$ | $\begin{aligned} & 0.234^{\mathrm{a}} \\ & (0.059) \end{aligned}$ | $\begin{aligned} & 0.236^{\mathrm{a}} \\ & (0.074) \end{aligned}$ | $\begin{aligned} & 0.103^{\text {c }} \\ & (0.055) \end{aligned}$ | $\begin{aligned} & 0.124^{\mathbf{b}} \\ & (0.054) \end{aligned}$ | $\begin{aligned} & 0.144^{\mathrm{a}} \\ & (0.055) \end{aligned}$ | $\begin{aligned} & -0.039 \\ & (0.050) \end{aligned}$ | $\begin{gathered} 0.037 \\ (0.046) \end{gathered}$ |
| $C S_{d}$ | $\begin{aligned} & 0.057^{\mathrm{c}} \\ & (0.030) \end{aligned}$ | $\begin{gathered} 0.030 \\ (0.032) \end{gathered}$ | $\begin{aligned} & -0.037 \\ & (0.033) \end{aligned}$ | $\begin{aligned} & 0.055^{c} \\ & (0.029) \end{aligned}$ | $\begin{aligned} & 0.062^{\text {c }} \\ & (0.035) \end{aligned}$ | $\begin{aligned} & 0.098^{a} \\ & (0.030) \end{aligned}$ | $\begin{aligned} & 0.080^{b} \\ & (0.032) \end{aligned}$ | $\begin{aligned} & 0.068^{b} \\ & (0.029) \end{aligned}$ |
| $C D S_{d}$ | $\begin{aligned} & 0.039^{\text {b }} \\ & (0.018) \end{aligned}$ | $\begin{aligned} & 0.204^{\mathrm{a}} \\ & (0.024) \end{aligned}$ | $\begin{aligned} & 0.136^{\mathrm{a}} \\ & (0.021) \end{aligned}$ | $\begin{aligned} & 0.034^{\mathrm{c}} \\ & (0.017) \end{aligned}$ | $\begin{aligned} & 0.064^{a} \\ & (0.020) \end{aligned}$ | $\begin{gathered} 0.011 \\ (0.016) \end{gathered}$ | $\begin{gathered} 0.016 \\ (0.017) \end{gathered}$ | $\begin{aligned} & 0.031^{\mathrm{b}} \\ & (0.015) \end{aligned}$ |
| $\omega$ | $\begin{aligned} & 0.152^{\mathrm{a}} \\ & (0.010) \end{aligned}$ | $\begin{aligned} & 0.154^{\mathrm{a}} \\ & (0.036) \end{aligned}$ | $\begin{aligned} & 0.004^{\mathbf{b}} \\ & (0.001) \end{aligned}$ | $\begin{aligned} & 0.104^{\mathrm{a}} \\ & (0.027) \end{aligned}$ | $\begin{aligned} & 0.014^{\mathbf{b}} \\ & (0.006) \end{aligned}$ | $\begin{aligned} & 0.008^{\mathbf{b}} \\ & (0.003) \end{aligned}$ | $\begin{aligned} & 0.163^{\mathbf{a}} \\ & (0.036) \end{aligned}$ | $\begin{aligned} & 0.003^{\mathbf{b}} \\ & (0.001) \end{aligned}$ |
| $\beta_{1}$ | $\begin{gathered} 8.279 \\ (0.033) \end{gathered}$ | $\begin{gathered} 0.094 \\ (0.197) \end{gathered}$ | $\begin{aligned} & 0.935^{\mathrm{a}} \\ & (0.017) \end{aligned}$ | $\begin{gathered} 0.278 \\ (0.170) \end{gathered}$ | $\begin{aligned} & 0.859^{a} \\ & (0.048) \end{aligned}$ | $\begin{aligned} & 0.894^{\mathrm{a}} \\ & (0.030) \end{aligned}$ | $\begin{gathered} 0.000 \\ (0.198) \end{gathered}$ | $\begin{aligned} & 0.946^{a} \\ & (0.017) \end{aligned}$ |
| $\alpha_{1}$ | $\begin{aligned} & 0.112^{\text {b }} \\ & (0.052) \end{aligned}$ | $\begin{aligned} & 0.122^{\mathrm{a}} \\ & (0.042) \end{aligned}$ | $\begin{aligned} & 0.049^{\mathrm{a}} \\ & (0.013) \end{aligned}$ | $\begin{aligned} & 0.158^{\mathrm{a}} \\ & (0.055) \end{aligned}$ | $\begin{aligned} & 0.046^{a} \\ & (0.018) \end{aligned}$ | $\begin{aligned} & 0.056^{\mathbf{a}} \\ & (0.017) \end{aligned}$ | $\begin{aligned} & 0.145^{\mathrm{b}} \\ & (0.061) \end{aligned}$ | $\begin{aligned} & 0.038^{\mathrm{a}} \\ & (0.012) \end{aligned}$ |
| $\phi_{1}$ | $\begin{aligned} & -0.015 \\ & (0.072) \end{aligned}$ | $\begin{gathered} -0.122^{\mathbf{b}} \\ (0.058) \end{gathered}$ | $\begin{aligned} & -0.018 \\ & (0.018) \end{aligned}$ | $\begin{aligned} & -0.095 \\ & (0.068) \end{aligned}$ | $\begin{gathered} 0.030 \\ (0.033) \end{gathered}$ | $\begin{gathered} 0.003 \\ (0.023) \end{gathered}$ | $\begin{aligned} & -0.092 \\ & (0.072) \end{aligned}$ | $\begin{gathered} -0.029^{\mathbf{c}} \\ (0.015) \end{gathered}$ |
| $\alpha_{\text {NIG }}$ | $\begin{aligned} & 1.708^{\mathrm{a}} \\ & (0.219) \end{aligned}$ | $\begin{aligned} & 1.454^{\mathrm{a}} \\ & (0.160) \end{aligned}$ | $\begin{aligned} & 2.025^{\mathrm{a}} \\ & (0.330) \end{aligned}$ | $\begin{aligned} & 1.902^{\mathrm{a}} \\ & (0.229) \end{aligned}$ | $\begin{aligned} & 1.737^{a} \\ & (0.237) \end{aligned}$ | $\begin{gathered} 1.513^{\mathrm{a}} \\ (0.158) \end{gathered}$ | $\begin{aligned} & 1.464^{\mathrm{a}} \\ & (0.154) \end{aligned}$ | $\begin{aligned} & 1.699^{\mathrm{a}} \end{aligned}$ |
| $\beta_{N I G}$ | $\begin{aligned} & 0.518^{\mathrm{a}} \\ & (0.123) \end{aligned}$ | $\begin{aligned} & 0.405^{\mathrm{a}} \\ & (0.105) \end{aligned}$ | $\begin{aligned} & 0.615^{\mathrm{a}} \\ & (0.204) \end{aligned}$ | $\begin{aligned} & 0.767^{\mathrm{a}} \\ & (0.158) \end{aligned}$ | $\begin{aligned} & 0.603^{a} \\ & (0.154) \end{aligned}$ | $\begin{aligned} & 0.506^{\mathbf{a}} \\ & (0.108) \end{aligned}$ | $\begin{aligned} & 0.341^{a} \\ & (0.086) \end{aligned}$ | $\begin{aligned} & 0.282^{\mathrm{a}} \\ & (0.106) \end{aligned}$ |
| LLF | -780.79 | -833.32 | -791.01 | -769.19 | -828.94 | -770.41 | -831.41 | -658.65 |
| AIC | 1599.5 | 1704.6 | 1620.0 | 1576.3 | 1695.8 | 1578.8 | 1700.8 | 1355.3 |
| BIC | 1701.5 | 1806.5 | 1721.9 | 1678.3 | 1797.8 | 1680.7 | 1802.7 | 1457.2 |
| $L j_{30}$ | 0.011 | 0.027 | 0.111 | 0.202 | 0.078 | 0.409 | 0.129 | 0.051 |
| $L j_{40}$ | 0.012 | 0.082 | 0.189 | 0.196 | 0.149 | 0.377 | 0.098 | 0.104 |
| $J B$ | - | - | - | - | - | - | - | - |
| KS | - | - | - | - | - | - | - | - |
| LL | - | - | - | - | - | - | - | - |

Note: Estimation results for the 16 series of stock volatility for the period January 2004 to March 2010. LLF is the Log-likelihood function, $A I C$ is the Akaike Information Criteria and BIC is the Bayesian information criterion. $L J_{30}$ and $L J_{40}$ are the Ljung Box test for 30 and 40 lags. $J B$ is the Jarque-Bera test for Normality, $K S$ is the Kolmogorov-Smirnov and $L L$ is the Lilliefors test. Standard errors in bracket. "a", "b" and "c" indicate significance at the $1 \%, 5 \%$ and $10 \%$.

Table 2.6: HAR model with asymmetric effects with respect to the returns and macroeconomic and financial variables - NIG distribution and GJR-GARCH variance (cont.)

|  | JPM | KFT | PEP | PG | T | TWX | TXN | WFC |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\alpha$ | $\begin{gathered} -2.398^{\mathbf{a}} \\ (0.297) \end{gathered}$ | $\begin{gathered} -1.787^{\mathrm{a}} \\ (0.243) \end{gathered}$ | $\begin{gathered} -1.901^{\mathrm{a}} \\ (0.315) \end{gathered}$ | $\begin{gathered} -1.377^{a} \\ (0.246) \end{gathered}$ | $\begin{gathered} -0.803^{\mathrm{a}} \\ (0.203) \end{gathered}$ | $\begin{gathered} -1.657^{a} \\ (0.288) \end{gathered}$ | $\begin{gathered} -0.587^{a} \\ (0.172) \end{gathered}$ | $\begin{gathered} -1.589^{\mathbf{a}} \\ (0.277) \end{gathered}$ |
| $\beta_{d}$ | $\begin{aligned} & 0.359^{\mathrm{a}} \\ & (0.030) \end{aligned}$ | $\begin{aligned} & 0.242^{\mathrm{a}} \\ & (0.031) \end{aligned}$ | $\begin{aligned} & 0.181^{a} \\ & (0.032) \end{aligned}$ | $\begin{aligned} & 0.238^{a} \\ & (0.030) \end{aligned}$ | $\begin{aligned} & 0.309^{\mathbf{a}} \\ & (0.030) \end{aligned}$ | $\begin{aligned} & 0.324^{\mathrm{a}} \\ & (0.032) \end{aligned}$ | $\begin{aligned} & 0.273^{\mathrm{a}} \\ & (0.034) \end{aligned}$ | $\begin{aligned} & 0.337^{\mathrm{a}} \\ & (0.031) \end{aligned}$ |
| $\beta_{w}$ | $\begin{aligned} & 0.316^{\mathrm{a}} \\ & (0.038) \end{aligned}$ | $\begin{aligned} & 0.416^{a} \\ & (0.043) \end{aligned}$ | $\begin{aligned} & 0.453^{a} \\ & (0.047) \end{aligned}$ | $\begin{aligned} & 0.482^{\mathrm{a}} \\ & (0.044) \end{aligned}$ | $\begin{aligned} & 0.362^{\mathrm{a}} \\ & (0.044) \end{aligned}$ | $\begin{aligned} & 0.349^{a} \\ & (0.043) \end{aligned}$ | $\begin{aligned} & 0.462^{a} \\ & (0.048) \end{aligned}$ | $\begin{aligned} & 0.359^{a} \\ & (0.038) \end{aligned}$ |
| $\beta_{m}$ | $\begin{gathered} 0.013 \\ (0.032) \end{gathered}$ | $\begin{gathered} 0.048 \\ (0.042) \end{gathered}$ | $\begin{aligned} & 0.095^{b} \\ & (0.043) \end{aligned}$ | $\begin{gathered} 0.053 \\ (0.039) \end{gathered}$ | $\begin{aligned} & 0.164^{\mathrm{a}} \\ & (0.037) \end{aligned}$ | $\begin{aligned} & 0.083^{b} \\ & (0.040) \end{aligned}$ | $\begin{aligned} & 0.116^{\mathbf{a}} \\ & (0.038) \end{aligned}$ | $\begin{aligned} & 0.117^{\mathrm{a}} \\ & (0.031) \end{aligned}$ |
| $\gamma_{R T}$ | $\begin{aligned} & 2.589^{\text {a }} \\ & (0.688) \end{aligned}$ | $\begin{aligned} & 5.280^{a} \\ & (1.814) \end{aligned}$ | $\begin{aligned} & 3.309^{\text {c }} \\ & (1.702) \end{aligned}$ | $\begin{aligned} & 3.199^{c} \\ & (1.718) \end{aligned}$ | $\begin{aligned} & 5.034^{\mathrm{a}} \\ & (1.426) \end{aligned}$ | $\begin{aligned} & 3.321^{\mathrm{a}} \\ & (0.936) \end{aligned}$ | $\begin{aligned} & 2.544^{a} \\ & (0.931) \end{aligned}$ | $\begin{aligned} & 2.229^{a} \\ & (0.618) \end{aligned}$ |
| $\gamma_{I R T}$ | $\begin{gathered} -7.684^{\mathrm{a}} \\ (1.029) \end{gathered}$ | $\begin{gathered} -8.964^{\mathrm{a}} \\ (2.809) \end{gathered}$ | $\begin{gathered} -11.37^{\mathrm{a}} \\ (2.687) \end{gathered}$ | $\begin{gathered} -8.978^{\mathbf{a}} \\ (2.629) \end{gathered}$ | $\begin{gathered} -10.35^{\mathbf{a}} \\ (2.174) \end{gathered}$ | $\begin{gathered} -4.372^{\mathrm{a}} \\ (1.491) \end{gathered}$ | $\begin{gathered} -5.307^{a} \\ (1.447) \end{gathered}$ | $\begin{gathered} -5.944^{\mathrm{a}} \\ (0.972) \end{gathered}$ |
| $\Delta S P_{d}$ | $\begin{aligned} & -0.574 \\ & (1.105) \end{aligned}$ | $\begin{aligned} & -1.308 \\ & (1.055) \end{aligned}$ | $\begin{aligned} & -1.048 \\ & (0.932) \end{aligned}$ | $\begin{aligned} & -1.510 \\ & (1.037) \end{aligned}$ | $\begin{aligned} & -1.472 \\ & (1.143) \end{aligned}$ | $\begin{gathered} -2.734^{\mathrm{a}} \\ (1.013) \end{gathered}$ | $\begin{gathered} -2.857^{a} \\ (0.930) \end{gathered}$ | $\begin{gathered} -2.340^{\text {b }} \\ (1.064) \end{gathered}$ |
| $\Delta S P_{w}$ | $\begin{gathered} -2.124^{\mathbf{a}} \\ (0.428) \end{gathered}$ | $\begin{gathered} -1.601^{\mathrm{a}} \\ (0.533) \end{gathered}$ | $\begin{gathered} -2.267^{\mathrm{a}} \\ (0.442) \end{gathered}$ | $\begin{gathered} -1.788^{\mathrm{a}} \\ (0.455) \end{gathered}$ | $\begin{gathered} -1.967^{a} \\ (0.466) \end{gathered}$ | $\begin{gathered} -1.833^{\mathbf{a}} \\ (0.393) \end{gathered}$ | $\begin{gathered} -1.761^{\mathrm{a}} \\ (0.428) \end{gathered}$ | $\begin{gathered} -2.493^{\mathrm{a}} \\ (0.441) \end{gathered}$ |
| $\Delta S P_{m}$ | $\begin{aligned} & -0.341 \\ & (0.215) \end{aligned}$ | $\begin{gathered} -0.668^{\text {b }} \\ (0.261) \end{gathered}$ | $\begin{gathered} -0.532^{\text {b }} \\ (0.247) \end{gathered}$ | $\begin{gathered} -0.491^{\mathrm{c}} \\ (0.251) \end{gathered}$ | $\begin{gathered} -0.611^{\mathrm{b}} \\ (0.243) \end{gathered}$ | $\begin{aligned} & -0.305 \\ & (0.213) \end{aligned}$ | $\begin{aligned} & -0.217 \\ & (0.237) \end{aligned}$ | $\begin{gathered} 0.021 \\ (0.208) \end{gathered}$ |
| $\Delta V_{d}^{S P}$ | $\begin{aligned} & 0.161^{\mathrm{a}} \\ & (0.053) \end{aligned}$ | $\begin{aligned} & 0.175^{a} \\ & (0.054) \end{aligned}$ | $\begin{aligned} & 0.145^{a} \\ & (0.053) \end{aligned}$ | $\begin{aligned} & 0.124^{\mathrm{b}} \\ & (0.050) \end{aligned}$ | $\begin{aligned} & 0.110^{\mathrm{b}} \\ & (0.053) \end{aligned}$ | $\begin{aligned} & 0.104^{\mathrm{b}} \\ & (0.046) \end{aligned}$ | $\begin{aligned} & 0.161^{a} \\ & (0.055) \end{aligned}$ | $\begin{gathered} 0.058 \\ (0.055) \end{gathered}$ |
| $V I X_{d}$ | $\begin{aligned} & 0.327^{\mathrm{a}} \\ & (0.065) \end{aligned}$ | $\begin{aligned} & 0.120^{\mathrm{b}} \\ & (0.057) \end{aligned}$ | $\begin{aligned} & 0.173^{\mathrm{a}} \\ & (0.060) \end{aligned}$ | $\begin{gathered} 0.043 \\ (0.051) \end{gathered}$ | $\begin{aligned} & -0.008 \\ & (0.051) \end{aligned}$ | $\begin{aligned} & 0.181^{\mathrm{a}} \\ & (0.058) \end{aligned}$ | $\begin{aligned} & -0.019 \\ & (0.048) \end{aligned}$ | $\begin{aligned} & 0.243^{\mathrm{a}} \\ & (0.066) \end{aligned}$ |
| $C S_{d}$ | $\begin{gathered} 0.010 \\ (0.029) \end{gathered}$ | $\begin{aligned} & 0.064^{\mathrm{b}} \\ & (0.032) \end{aligned}$ | $\begin{gathered} 0.042 \\ (0.029) \end{gathered}$ | $\begin{aligned} & 0.100^{a} \\ & (0.031) \end{aligned}$ | $\begin{gathered} 0.042 \\ (0.028) \end{gathered}$ | $\begin{aligned} & 0.078^{\mathbf{a}} \\ & (0.029) \end{aligned}$ | $\begin{aligned} & 0.062^{\text {b }} \\ & (0.031) \end{aligned}$ | $\begin{aligned} & -0.026 \\ & (0.029) \end{aligned}$ |
| $C D S_{d}$ | $\begin{aligned} & 0.186^{\mathrm{a}} \\ & (0.021) \end{aligned}$ | $\begin{gathered} 0.026 \\ (0.019) \end{gathered}$ | $\begin{gathered} 0.023 \\ (0.017) \end{gathered}$ | $\begin{gathered} 0.016 \\ (0.017) \end{gathered}$ | $\begin{aligned} & 0.032^{\mathrm{c}} \\ & (0.016) \end{aligned}$ | $\begin{aligned} & 0.045^{\mathrm{a}} \\ & (0.016) \end{aligned}$ | $\begin{gathered} 0.010 \\ (0.016) \end{gathered}$ | $\begin{aligned} & 0.138^{a} \\ & (0.021) \end{aligned}$ |
| $\omega$ | $\begin{aligned} & 0.099^{a} \\ & (0.025) \end{aligned}$ | $\begin{aligned} & 0.142^{\mathrm{a}} \\ & (0.040) \end{aligned}$ | $\begin{aligned} & 0.045^{\mathrm{a}} \\ & (0.013) \end{aligned}$ | $\begin{aligned} & 0.015^{\text {b }} \\ & (0.007) \end{aligned}$ | $\begin{aligned} & 0.012^{\mathrm{b}} \\ & (0.005) \end{aligned}$ | $\begin{aligned} & 0.085^{a} \\ & (0.025) \end{aligned}$ | $\begin{aligned} & 0.042^{\text {b }} \\ & (0.016) \end{aligned}$ | $\begin{gathered} 0.144^{\mathrm{a}} \\ (0.029) \end{gathered}$ |
| $\beta_{1}$ | $\begin{gathered} 0.275 \\ (0.172) \end{gathered}$ | $\begin{aligned} & 0.341^{\mathrm{c}} \\ & (0.174) \end{aligned}$ | $\begin{aligned} & 0.630^{a} \\ & (0.093) \end{aligned}$ | $\begin{aligned} & 0.870^{\mathrm{a}} \\ & (0.053) \end{aligned}$ | $\begin{aligned} & 0.893^{\mathbf{a}} \\ & (0.040) \end{aligned}$ | $\begin{aligned} & 0.326^{\mathrm{c}} \\ & (0.185) \end{aligned}$ | $\begin{aligned} & 0.611^{a} \\ & (0.130) \end{aligned}$ | $\begin{gathered} 0.075 \\ (0.171) \end{gathered}$ |
| $\alpha_{1}$ | $\begin{aligned} & 0.163^{a} \\ & (0.056) \end{aligned}$ | $\begin{aligned} & 0.144^{a} \\ & (0.051) \end{aligned}$ | $\begin{aligned} & 0.132^{a} \\ & (0.041) \end{aligned}$ | $\begin{aligned} & 0.052^{a} \\ & (0.019) \end{aligned}$ | $\begin{aligned} & 0.055^{\mathbf{a}} \\ & (0.019) \end{aligned}$ | $\begin{aligned} & 0.140^{\mathrm{a}} \\ & (0.049) \end{aligned}$ | $\begin{aligned} & 0.084^{\text {b }} \\ & (0.032) \end{aligned}$ | $\begin{aligned} & 0.162^{\mathrm{a}} \\ & (0.058) \end{aligned}$ |
| $\phi_{1}$ | $\begin{aligned} & -0.094 \\ & (0.065) \end{aligned}$ | $\begin{gathered} -0.144^{\text {b }} \\ (0.070) \end{gathered}$ | $\begin{aligned} & -0.053 \\ & (0.052) \end{aligned}$ | $\begin{aligned} & -0.029 \\ & (0.024) \end{aligned}$ | $\begin{gathered} -0.042^{\mathrm{c}} \\ (0.023) \end{gathered}$ | $\begin{gathered} -0.111^{\mathbf{c}} \\ (0.062) \end{gathered}$ | $\begin{gathered} 0.036 \\ (0.046) \end{gathered}$ | $\begin{aligned} & -0.108 \\ & (0.070) \end{aligned}$ |
| $\alpha_{\text {NIG }}$ | $1.654^{\mathrm{a}}$ | $\begin{gathered} 1.51^{\mathrm{a}} \\ (0.169) \end{gathered}$ | $\begin{aligned} & 1.794^{\mathrm{a}} \\ & (0.256) \end{aligned}$ | $\begin{aligned} & 1.623^{a} \\ & (0.214) \end{aligned}$ | $\begin{aligned} & 1.607^{\mathrm{a}} \\ & (0.193) \end{aligned}$ | $\begin{aligned} & 1.595^{\mathrm{a}} \\ & (0.181) \end{aligned}$ | $\begin{aligned} & 2.078^{\mathbf{a}} \\ & (0.311) \end{aligned}$ | $1.647^{a}$ |
| $\beta_{\text {NIG }}$ | $\begin{aligned} & 0.480^{\mathrm{a}} \\ & (0.121) \end{aligned}$ | $\begin{aligned} & 0.637^{a} \\ & (0.121) \end{aligned}$ | $\begin{aligned} & 0.643^{a} \\ & (0.177) \end{aligned}$ | $\begin{aligned} & 0.434^{\mathrm{a}} \\ & (0.135) \end{aligned}$ | $\begin{aligned} & 0.439^{\mathrm{a}} \\ & (0.119) \end{aligned}$ | $\begin{aligned} & 0.482^{\mathrm{a}} \\ & (0.123) \end{aligned}$ | $\begin{aligned} & 0.260^{\text {c }} \\ & (0.149) \end{aligned}$ | $\begin{aligned} & 0.552^{\mathrm{a}} \\ & (0.131) \end{aligned}$ |
| LLF | -746.75 | -1014.7 | -769.36 | -787.92 | -797.23 | -647.91 | -709.50 | -801.38 |
| AIC | 1531.5 | 2067.5 | 1576.7 | 1613.8 | 1632.4 | 1333.8 | 1457.0 | 1640.7 |
| BIC | 1633.4 | 2169.5 | 1678.6 | 1715.7 | 1734.4 | 1435.7 | 1558.9 | 1742.7 |
| $L j_{30}$ | 0.296 | 0.060 | 0.054 | 0.463 | 0.021 | 0.274 | 0.004 | 0.200 |
| $L j_{40}$ | 0.363 | 0.049 | 0.026 | 0.575 | 0.058 | 0.050 | 0.012 | 0.444 |
| $J B$ | - | - | - | - | - | - | - | - |
| KS | - | - | - | - | - | - | - | - |
| $L L$ | - | - | - | - | - | - | - | - |

Note: Estimation results for the 16 series of stock volatility for the period January 2004 to March 2010. LLF is the Log-likelihood function, AIC is the Akaike Information Criteria and BIC is the Bayesian information criterion. $L J_{30}$ and $L J_{40}$ are the Ljung Box test for 30 and 40 lags. $J B$ is the Jarque-Bera test for Normality, $K S$ is the Kolmogorov-Smirnov and $L L$ is the Lilliefors test. Standard errors in bracket. "a", "b" and "c" indicate significance at the $1 \%, 5 \%$ and $10 \%$.












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Table 2．8：Out－of－sample forecast evaluation DM test II－Full sample

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Table 2.10: Out-of-sample forecast evaluation MCS - Full sample

|  | BA | BAC | C | CAT | FDX | HON | HPQ | IBM |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| II-Co-NO | 0.01 | 0.03 | $0.36{ }^{\text {b }}$ | 0.02 | 0.01 | 0.02 | $0.13{ }^{\text {a }}$ | $0.28{ }^{\text {b }}$ |
| III-Co-NO | 0.06 | 0.06 | $0.72{ }^{\text {b }}$ | 0.02 | 0.01 | 0.04 | $0.13^{\text {a }}$ | $0.28{ }^{\text {b }}$ |
| IV-Co-NO | 0.08 | 0.08 | $0.72^{\text {b }}$ | 0.02 | 0.01 | 0.04 | $0.13^{\text {a }}$ | $0.28{ }^{\text {b }}$ |
| V-Co-NO | 0.08 | 0.06 | $0.72{ }^{\text {b }}$ | 0.02 | 0.01 | 0.04 | $0.13{ }^{\text {a }}$ | $0.29{ }^{\text {b }}$ |
| VI-Co-NO | 0.06 | 0.06 | $0.65{ }^{\text {b }}$ | 0.02 | 0.01 | 0.04 | $0.13^{\text {a }}$ | $0.56{ }^{\text {b }}$ |
| II-Ga-NO | 0.06 | 0.03 | $0.36{ }^{\text {b }}$ | 0.02 | 0.01 | 0.02 | $0.13^{\text {a }}$ | $0.28{ }^{\text {b }}$ |
| III-Ga-NO | 0.08 | 0.08 | $0.58{ }^{\text {b }}$ | 0.02 | 0.01 | 0.04 | $0.13^{\text {a }}$ | $0.28{ }^{\text {b }}$ |
| IV-Ga-NO | 0.08 | 0.08 | $0.72{ }^{\text {b }}$ | 0.02 | 0.01 | 0.04 | $0.13{ }^{\text {a }}$ | $0.28{ }^{\text {b }}$ |
| V-Ga-NO | 0.08 | 0.06 | $0.65{ }^{\text {b }}$ | 0.02 | 0.01 | 0.04 | $0.13^{\text {a }}$ | $0.28{ }^{\text {b }}$ |
| VI-Ga-NO | 0.08 | 0.06 | $0.38{ }^{\text {b }}$ | 0.02 | 0.01 | 0.04 | $0.13{ }^{\text {a }}$ | $0.28{ }^{\text {b }}$ |
| II-Gj-NO | 0.06 | 0.03 | $0.36{ }^{\text {b }}$ | 0.02 | 0.01 | 0.02 | $0.13{ }^{\text {a }}$ | $0.28{ }^{\text {b }}$ |
| III-Gj-NO | 0.08 | 0.09 | $0.72{ }^{\text {b }}$ | 0.02 | 0.01 | 0.04 | $0.13{ }^{\text {a }}$ | $0.28{ }^{\text {b }}$ |
| IV-Gj-NO | 0.08 | 0.08 | $0.72{ }^{\text {b }}$ | 0.02 | 0.01 | 0.04 | $0.13{ }^{\text {a }}$ | $0.28{ }^{\text {b }}$ |
| V-Gj-NO | 0.08 | 0.06 | $0.65{ }^{\text {b }}$ | 0.02 | 0.01 | 0.08 | $0.13{ }^{\text {a }}$ | $0.28{ }^{\text {b }}$ |
| VI-Gj-NO | 0.08 | 0.06 | $0.58{ }^{\text {b }}$ | 0.02 | 0.01 | 0.04 | $0.13^{\text {a }}$ | $0.31{ }^{\text {b }}$ |
| II-Co-NI | 0.01 | 0.03 | $0.36{ }^{\text {b }}$ | 0.02 | 0.01 | 0.02 | $0.13{ }^{\text {a }}$ | $0.28{ }^{\text {b }}$ |
| III-Co-NI | 0.06 | 0.06 | $0.58{ }^{\text {b }}$ | 0.02 | 0.01 | 0.02 | $0.13^{\text {a }}$ | $0.28{ }^{\text {b }}$ |
| IV-Co-NI | 0.06 | 0.08 | $0.49{ }^{\text {b }}$ | 0.02 | 0.01 | 0.02 | $0.13{ }^{\text {a }}$ | $0.28{ }^{\text {b }}$ |
| V-Co-NI | 0.06 | 0.06 | $0.58{ }^{\text {b }}$ | 0.02 | 0.01 | 0.02 | $0.13^{\text {a }}$ | $0.28{ }^{\text {b }}$ |
| VI-Co-NI | 0.01 | 0.06 | $0.38{ }^{\text {b }}$ | 0.02 | 0.01 | 0.02 | $0.13{ }^{\text {a }}$ | $0.28{ }^{\text {b }}$ |
| II-Ga-NI | 0.01 | 0.03 | $0.26{ }^{\text {b }}$ | 0.02 | 0.01 | 0.02 | $0.13{ }^{\text {a }}$ | $0.28{ }^{\text {b }}$ |
| III-Ga-NI | 0.08 | 0.08 | $0.38^{\text {b }}$ | 0.02 | 0.01 | 0.04 | $0.13{ }^{\text {a }}$ | $0.28{ }^{\text {b }}$ |
| IV-Ga-NI | 0.08 | 0.09 | $0.38{ }^{\text {b }}$ | 0.02 | 0.01 | 0.04 | $0.13^{\text {a }}$ | $0.28{ }^{\text {b }}$ |
| V-Ga-NI | 0.08 | 0.06 | $0.38{ }^{\text {b }}$ | 0.02 | 0.06 | 0.04 | $0.13^{\text {a }}$ | $0.28{ }^{\text {b }}$ |
| VI-Ga-NI | 0.01 | 0.06 | $0.36{ }^{\text {b }}$ | 0.02 | 0.06 | 0.04 | $0.13{ }^{\text {a }}$ | $0.28{ }^{\text {b }}$ |
| II-Gj-NI | 0.01 | 0.03 | $0.32^{\text {b }}$ | 0.02 | 0.01 | 0.02 | $0.13{ }^{\text {a }}$ | $0.28{ }^{\text {b }}$ |
| III-Gj-NI | 0.08 | 0.08 | $0.58{ }^{\text {b }}$ | 0.02 | 0.01 | 0.04 | $0.13^{\text {a }}$ | $0.52^{\text {b }}$ |
| IV-Gj-NI | 0.08 | 0.08 | $0.58{ }^{\text {b }}$ | 0.02 | 0.01 | 0.04 | $0.13{ }^{\text {a }}$ | $0.28{ }^{\text {b }}$ |
| V-Gj-NI | 0.08 | 0.06 | $0.58{ }^{\text {b }}$ | 0.02 | 0.01 | 0.04 | $0.93{ }^{\text {b }}$ | $0.28{ }^{\text {b }}$ |
| VI-Gj-NI | 0.08 | 0.06 | $0.49{ }^{\text {b }}$ | 0.02 | 0.01 | 0.04 | $0.13{ }^{\text {a }}$ | $0.56{ }^{\text {b }}$ |
| II-Co-NO-MS | $0.40{ }^{\text {b }}$ | $0.35{ }^{\text {b }}$ | $0.38{ }^{\text {b }}$ | 0.02 | 0.09 | $0.74{ }^{\text {b }}$ | $0.13{ }^{\text {a }}$ | $0.28{ }^{\text {b }}$ |
| III-Co-NO-MS | $0.40{ }^{\text {b }}$ | $0.81{ }^{\text {b }}$ | $0.72{ }^{\text {b }}$ | $0.11^{\text {a }}$ | 0.01 | $0.74{ }^{\text {b }}$ | $0.13{ }^{\text {a }}$ | $0.28{ }^{\text {b }}$ |
| IV-Co-NO-MS | $1.00^{\text {b }}$ | $0.83{ }^{\text {b }}$ | $1.00^{\text {b }}$ | $0.11^{\text {a }}$ | 0.01 | $0.74{ }^{\text {b }}$ | $0.13{ }^{\text {a }}$ | $0.29{ }^{\text {b }}$ |
| V-Co-NO-MS | $0.40{ }^{\text {b }}$ | $0.35{ }^{\text {b }}$ | $0.72{ }^{\text {b }}$ | 0.02 | 0.06 | $0.74{ }^{\text {b }}$ | $0.93{ }^{\text {b }}$ | $0.56{ }^{\text {b }}$ |
| VI-Co-NO-MS | $0.21{ }^{\text {a }}$ | $0.35^{\text {b }}$ | $0.72{ }^{\text {b }}$ | 0.02 | 0.01 | $0.74{ }^{\text {b }}$ | $0.13^{\text {a }}$ | $0.56{ }^{\text {b }}$ |
| II-Ga-NO-MS | $0.16^{\text {a }}$ | $0.35{ }^{\text {b }}$ | $0.36{ }^{\text {b }}$ | 0.02 | $0.87^{\text {b }}$ | $0.74{ }^{\text {b }}$ | $0.13^{\text {a }}$ | $0.28{ }^{\text {b }}$ |
| III-Ga-NO-MS | $0.16^{\text {a }}$ | $0.91{ }^{\text {b }}$ | $0.72{ }^{\text {b }}$ | $0.18^{\text {a }}$ | 0.06 | $0.74{ }^{\text {b }}$ | $0.13^{\text {a }}$ | $0.28{ }^{\text {b }}$ |
| IV-Ga-NO-MS | $0.40{ }^{\text {b }}$ | $0.91{ }^{\text {b }}$ | $0.72{ }^{\text {b }}$ | $0.18{ }^{\text {a }}$ | 0.06 | $1.00^{\text {b }}$ | $0.93{ }^{\text {b }}$ | $0.31{ }^{\text {b }}$ |
| V-Ga-NO-MS | $0.16^{\text {a }}$ | $0.35{ }^{\text {b }}$ | $0.72{ }^{\text {b }}$ | 0.02 | 0.09 | $0.74{ }^{\text {b }}$ | $0.93{ }^{\text {b }}$ | $0.52^{\text {b }}$ |
| VI-Ga-NO-MS | 0.08 | $0.35^{\text {b }}$ | $0.72{ }^{\text {b }}$ | 0.02 | 0.06 | $0.74{ }^{\text {b }}$ | $0.13^{\text {a }}$ | $0.58{ }^{\text {b }}$ |
| II-Gj-NO-MS | 0.08 | $0.35{ }^{\text {b }}$ | $0.36{ }^{\text {b }}$ | 0.02 | 0.09 | 0.04 | $0.13^{\text {a }}$ | $0.28{ }^{\text {b }}$ |
| III-Gj-NO-MS | 0.08 | $0.35{ }^{\text {b }}$ | $0.72{ }^{\text {b }}$ | $0.18^{\text {a }}$ | 0.01 | 0.04 | $0.13^{\text {a }}$ | $0.28{ }^{\text {b }}$ |
| IV-Gj-NO-MS | 0.08 | $0.35{ }^{\text {b }}$ | $0.72{ }^{\text {b }}$ | $0.43{ }^{\text {b }}$ | 0.06 | 0.06 | $0.13^{\text {a }}$ | $0.28{ }^{\text {b }}$ |
| V-Gj-NO-MS | 0.08 | $0.35{ }^{\text {b }}$ | $0.72{ }^{\text {b }}$ | 0.02 | 0.06 | 0.04 | $0.13^{\text {a }}$ | $0.28{ }^{\text {b }}$ |
| VI-Gj-NO-MS | 0.06 | $0.35{ }^{\text {b }}$ | $0.72{ }^{\text {b }}$ | 0.02 | 0.06 | 0.04 | $0.13^{\text {a }}$ | $0.52^{\text {b }}$ |
| II-Co-NI-MS | $0.40{ }^{\text {b }}$ | $0.35{ }^{\text {b }}$ | $0.36{ }^{\text {b }}$ | 0.02 | 0.06 | 0.08 | $0.13^{\text {a }}$ | $0.56{ }^{\text {b }}$ |
| III-Co-NI-MS | $0.40^{\text {b }}$ | $0.35{ }^{\text {b }}$ | $0.72{ }^{\text {b }}$ | $0.11^{\text {a }}$ | 0.01 | 0.08 | $0.13{ }^{\text {a }}$ | $0.73{ }^{\text {b }}$ |
| IV-Co-NI-MS | $0.40{ }^{\text {b }}$ | $0.35{ }^{\text {b }}$ | $0.72{ }^{\text {b }}$ | $0.18^{\text {a }}$ | 0.06 | 0.08 | $0.97{ }^{\text {b }}$ | $0.87{ }^{\text {b }}$ |
| V-Co-NI-MS | $0.16^{\text {a }}$ | $0.35{ }^{\text {b }}$ | $0.72{ }^{\text {b }}$ | 0.02 | 0.06 | 0.06 | $0.97{ }^{\text {b }}$ | $0.87^{\text {b }}$ |
| VI-Co-NI-MS | 0.08 | $0.35{ }^{\text {b }}$ | $0.65{ }^{\text {b }}$ | 0.02 | 0.06 | 0.04 | $0.93{ }^{\text {b }}$ | $1.00^{\text {b }}$ |
| II-Ga-NI-MS | $0.16^{\text {a }}$ | $0.35{ }^{\text {b }}$ | $0.36{ }^{\text {b }}$ | $0.43{ }^{\text {b }}$ | $1.00^{\text {b }}$ | $0.74{ }^{\text {b }}$ | $0.13{ }^{\text {a }}$ | $0.56{ }^{\text {b }}$ |
| III-Ga-NI-MS | 0.08 | $0.83{ }^{\text {b }}$ | $0.58^{\text {b }}$ | $0.43{ }^{\text {b }}$ | 0.09 | $0.74{ }^{\text {b }}$ | $0.93{ }^{\text {b }}$ | $0.58{ }^{\text {b }}$ |
| IV-Ga-NI-MS | 0.08 | $1.00{ }^{\text {b }}$ | $0.72{ }^{\text {b }}$ | $0.43{ }^{\text {b }}$ | 0.09 | $0.74{ }^{\text {b }}$ | $0.97{ }^{\text {b }}$ | $0.87{ }^{\text {b }}$ |
| V-Ga-NI-MS | 0.01 | $0.35{ }^{\text {b }}$ | $0.58{ }^{\text {b }}$ | $0.43{ }^{\text {b }}$ | $0.79{ }^{\text {b }}$ | $0.74{ }^{\text {b }}$ | $1.00{ }^{\text {b }}$ | $0.87{ }^{\text {b }}$ |
| VI-Ga-NI-MS | 0.01 | $0.35{ }^{\text {b }}$ | $0.65{ }^{\text {b }}$ | $0.18^{\text {a }}$ | $0.79{ }^{\text {b }}$ | $0.64{ }^{\text {b }}$ | $0.93{ }^{\text {b }}$ | $1.00{ }^{\text {b }}$ |
| II-Gj-NI-MS | 0.06 | $0.35{ }^{\text {b }}$ | $0.36{ }^{\text {b }}$ | $0.43{ }^{\text {b }}$ | $0.79{ }^{\text {b }}$ | $0.74{ }^{\text {b }}$ | $0.13^{\text {a }}$ | $0.56{ }^{\text {b }}$ |
| III-Gj-NI-MS | 0.08 | $0.91{ }^{\text {b }}$ | $0.58{ }^{\text {b }}$ | $0.43{ }^{\text {b }}$ | 0.06 | $0.74{ }^{\text {b }}$ | $0.13{ }^{\text {a }}$ | $0.56{ }^{\text {b }}$ |
| IV-Gj-NI-MS | 0.08 | $0.91{ }^{\text {b }}$ | $0.72{ }^{\text {b }}$ | $1.00^{\text {b }}$ | 0.06 | $0.74{ }^{\text {b }}$ | $0.13{ }^{\text {a }}$ | $0.85{ }^{\text {b }}$ |
| V-Gj-NI-MS | 0.08 | $0.35{ }^{\text {b }}$ | $0.58{ }^{\text {b }}$ | $0.43{ }^{\text {b }}$ | $0.79{ }^{\text {b }}$ | $0.64{ }^{\text {b }}$ | $0.97{ }^{\text {b }}$ | $0.87^{\text {b }}$ |
| VI-Gj-NI-MS | 0.01 | $0.35{ }^{\text {b }}$ | $0.58{ }^{\text {b }}$ | $0.18{ }^{\text {a }}$ | $0.48{ }^{\text {b }}$ | 0.08 | $0.13{ }^{\text {a }}$ | $1.00{ }^{\text {b }}$ |

Note: Forecast performance evaluation for the 16 series of stock volatility for the full out-of-sample period (1067 observation). See the note in table 2.7 for the description of the models. The $M C S$ is a procedure to determine the "best" models from a collection of models based on the $M S E$ loss function. p-values for the range deviation method in the table. "a" and "b" denote that the model belongs to the $10 \%$ and $25 \% \mathrm{MCS}$. Results for the model $I$ not reported to save space. Results for the model $I$ not reported to save space.

Table 2.11: Out-of-sample forecast evaluation MCS - Full sample (cont.)

|  | JPM | KFT | PEP | PG | T | TWX | TXN | WFC |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| II-Co-NO | 0.05 | $0.15{ }^{\text {a }}$ | $0.21^{\text {a }}$ | $0.15{ }^{\text {a }}$ | 0.08 | $0.30{ }^{\text {b }}$ | $0.49{ }^{\text {b }}$ | 0.00 |
| III-Co-NO | $0.18^{\text {a }}$ | $0.15{ }^{\text {a }}$ | $0.86{ }^{\text {b }}$ | $0.66{ }^{\text {b }}$ | $0.42{ }^{\text {b }}$ | 0.02 | $0.49{ }^{\text {b }}$ | 0.07 |
| IV-Co-NO | $0.39{ }^{\text {b }}$ | $0.22^{\text {a }}$ | $0.86{ }^{\text {b }}$ | $0.77^{\text {b }}$ | $0.42{ }^{\text {b }}$ | 0.02 | $0.49{ }^{\text {b }}$ | 0.00 |
| V-Co-NO | $0.30^{\text {b }}$ | $0.15^{\text {a }}$ | $0.84{ }^{\text {b }}$ | $0.77^{\text {b }}$ | $0.42{ }^{\text {b }}$ | $0.30^{\text {b }}$ | $0.49{ }^{\text {b }}$ | 0.07 |
| VI-Co-NO | $0.18{ }^{\text {a }}$ | $0.15{ }^{\text {a }}$ | $0.84{ }^{\text {b }}$ | $0.77^{\text {b }}$ | $0.42{ }^{\text {b }}$ | $0.23{ }^{\text {a }}$ | $0.49{ }^{\text {b }}$ | 0.07 |
| II-Ga-NO | 0.05 | $0.15{ }^{\text {a }}$ | $0.21^{\text {a }}$ | $0.15{ }^{\text {a }}$ | 0.08 | $0.30^{\text {b }}$ | $0.49{ }^{\text {b }}$ | 0.00 |
| III-Ga-NO | $0.39{ }^{\text {b }}$ | $0.22^{\text {a }}$ | $0.84{ }^{\text {b }}$ | $0.15{ }^{\text {a }}$ | $0.42{ }^{\text {b }}$ | $0.30{ }^{\text {b }}$ | $0.49{ }^{\text {b }}$ | 0.07 |
| IV-Ga-NO | $0.42{ }^{\text {b }}$ | $0.22^{\text {a }}$ | $0.84{ }^{\text {b }}$ | $0.15{ }^{\text {a }}$ | $0.42{ }^{\text {b }}$ | $0.30^{\text {b }}$ | $0.49{ }^{\text {b }}$ | 0.00 |
| V-Ga-NO | $0.39{ }^{\text {b }}$ | $0.15{ }^{\text {a }}$ | $0.29{ }^{\text {b }}$ | $0.48^{\text {b }}$ | $0.42{ }^{\text {b }}$ | $0.30^{\text {b }}$ | $0.49{ }^{\text {b }}$ | 0.00 |
| VI-Ga-NO | $0.48^{\text {b }}$ | $0.15{ }^{\text {a }}$ | $0.29{ }^{\text {b }}$ | $0.15{ }^{\text {a }}$ | $0.42^{\text {b }}$ | $0.30^{\text {b }}$ | $0.49{ }^{\text {b }}$ | 0.07 |
| II-Gj-NO | 0.05 | $0.15{ }^{\text {a }}$ | $0.21^{\text {a }}$ | $0.15{ }^{\text {a }}$ | 0.08 | $0.30^{\text {b }}$ | $0.49{ }^{\text {b }}$ | 0.00 |
| III-Gj-NO | $0.42{ }^{\text {b }}$ | $0.22^{\text {a }}$ | $0.84{ }^{\text {b }}$ | $0.15{ }^{\text {a }}$ | $0.42{ }^{\text {b }}$ | $0.30^{\text {b }}$ | $0.49{ }^{\text {b }}$ | 0.07 |
| IV-Gj-NO | $0.42{ }^{\text {b }}$ | $0.44{ }^{\text {b }}$ | $0.84{ }^{\text {b }}$ | $0.48{ }^{\text {b }}$ | $0.42{ }^{\text {b }}$ | $0.30^{\text {b }}$ | $0.49{ }^{\text {b }}$ | 0.07 |
| V-Gj-NO | $0.42{ }^{\text {b }}$ | $0.15^{\text {a }}$ | $0.84{ }^{\text {b }}$ | $0.66{ }^{\text {b }}$ | $0.42{ }^{\text {b }}$ | $0.30^{\text {b }}$ | $0.49{ }^{\text {b }}$ | 0.07 |
| VI-Gj-NO | $0.42{ }^{\text {b }}$ | $0.15{ }^{\text {a }}$ | $0.84{ }^{\text {b }}$ | $0.48{ }^{\text {b }}$ | $0.42^{\text {b }}$ | $0.30^{\text {b }}$ | $0.49{ }^{\text {b }}$ | 0.07 |
| II-Co-NI | 0.02 | $0.15^{\text {a }}$ | $0.21^{\text {a }}$ | $0.15{ }^{\text {a }}$ | 0.08 | 0.02 | $0.49{ }^{\text {b }}$ | 0.00 |
| III-Co-NI | $0.11^{\text {a }}$ | $0.15{ }^{\text {a }}$ | $0.29{ }^{\text {b }}$ | $0.15{ }^{\text {a }}$ | $0.29{ }^{\text {b }}$ | 0.02 | $0.49{ }^{\text {b }}$ | 0.00 |
| IV-Co-NI | $0.11^{\text {a }}$ | $0.15^{\text {a }}$ | $0.29{ }^{\text {b }}$ | $0.15{ }^{\text {a }}$ | $0.32^{\text {b }}$ | 0.02 | $0.49{ }^{\text {b }}$ | 0.00 |
| V-Co-NI | 0.09 | $0.15^{\text {a }}$ | $0.21^{\text {a }}$ | $0.66{ }^{\text {b }}$ | 0.08 | $0.23{ }^{\text {a }}$ | $0.49{ }^{\text {b }}$ | 0.00 |
| VI-Co-NI | 0.09 | $0.15{ }^{\text {a }}$ | $0.21^{\text {a }}$ | $0.48{ }^{\text {b }}$ | 0.08 | 0.02 | $0.49{ }^{\text {b }}$ | 0.00 |
| II-Ga-NI | 0.02 | $0.15{ }^{\text {a }}$ | $0.21^{\text {a }}$ | $0.15{ }^{\text {a }}$ | 0.08 | $0.29{ }^{\text {b }}$ | $0.49{ }^{\text {b }}$ | 0.00 |
| III-Ga-NI | $0.18^{\text {a }}$ | $0.15{ }^{\text {a }}$ | $0.84{ }^{\text {b }}$ | $0.15^{\text {a }}$ | $0.42{ }^{\text {b }}$ | $0.29{ }^{\text {b }}$ | $0.49{ }^{\text {b }}$ | 0.00 |
| IV-Ga-NI | $0.18^{\text {a }}$ | $0.22^{\text {a }}$ | $0.84{ }^{\text {b }}$ | $0.15{ }^{\text {a }}$ | $0.42^{\text {b }}$ | $0.30^{\text {b }}$ | $0.49{ }^{\text {b }}$ | 0.00 |
| V-Ga-NI | $0.11^{\text {a }}$ | $0.15^{\text {a }}$ | $0.29{ }^{\text {b }}$ | $0.41{ }^{\text {b }}$ | $0.42{ }^{\text {b }}$ | $0.30^{\text {b }}$ | $0.49{ }^{\text {b }}$ | 0.00 |
| VI-Ga-NI | $0.30^{\text {b }}$ | $0.15{ }^{\text {a }}$ | $0.29{ }^{\text {b }}$ | $0.41{ }^{\text {b }}$ | $0.42^{\text {b }}$ | $0.30^{\text {b }}$ | $0.49{ }^{\text {b }}$ | 0.00 |
| II-Gj-NI | 0.05 | $0.15^{\text {a }}$ | $0.21^{\text {a }}$ | $0.15{ }^{\text {a }}$ | 0.08 | $0.30^{\text {b }}$ | $0.49{ }^{\text {b }}$ | 0.00 |
| III-Gj-NI | $0.42{ }^{\text {b }}$ | $0.22^{\text {a }}$ | $0.84{ }^{\text {b }}$ | $0.15{ }^{\text {a }}$ | $0.42{ }^{\text {b }}$ | $0.30^{\text {b }}$ | $0.49{ }^{\text {b }}$ | 0.07 |
| IV-Gj-NI | $0.42{ }^{\text {b }}$ | $0.61{ }^{\text {b }}$ | $0.84{ }^{\text {b }}$ | $0.41{ }^{\text {b }}$ | $0.42{ }^{\text {b }}$ | $0.30^{\text {b }}$ | $0.49{ }^{\text {b }}$ | 0.00 |
| V-Gj-NI | $0.42{ }^{\text {b }}$ | $0.15^{\text {a }}$ | $0.29{ }^{\text {b }}$ | $0.66{ }^{\text {b }}$ | $0.42{ }^{\text {b }}$ | $0.30^{\text {b }}$ | $0.49{ }^{\text {b }}$ | 0.00 |
| VI-Gj-NI | $0.48{ }^{\text {b }}$ | $0.15{ }^{\text {a }}$ | $0.29{ }^{\text {b }}$ | $0.66^{\text {b }}$ | $0.42{ }^{\text {b }}$ | $0.30^{\text {b }}$ | $0.49{ }^{\text {b }}$ | 0.07 |
| II-Co-NO-MS | $0.48{ }^{\text {b }}$ | $0.61{ }^{\text {b }}$ | $0.92{ }^{\text {b }}$ | $0.77^{\text {b }}$ | $0.42{ }^{\text {b }}$ | $0.48{ }^{\text {b }}$ | $0.83{ }^{\text {b }}$ | $0.24{ }^{\text {a }}$ |
| III-Co-NO-MS | $0.48{ }^{\text {b }}$ | $0.72{ }^{\text {b }}$ | $0.92{ }^{\text {b }}$ | $0.77^{\text {b }}$ | $0.71{ }^{\text {b }}$ | $0.30^{\text {b }}$ | $0.49{ }^{\text {b }}$ | $0.93{ }^{\text {b }}$ |
| IV-Co-NO-MS | $0.48{ }^{\text {b }}$ | $0.72{ }^{\text {b }}$ | $0.92{ }^{\text {b }}$ | $0.77^{\text {b }}$ | $0.77^{\text {b }}$ | $0.30^{\text {b }}$ | $0.49{ }^{\text {b }}$ | $0.74{ }^{\text {b }}$ |
| V-Co-NO-MS | $0.48{ }^{\text {b }}$ | $0.61{ }^{\text {b }}$ | $0.92{ }^{\text {b }}$ | $0.86{ }^{\text {b }}$ | $0.71{ }^{\text {b }}$ | $0.30^{\text {b }}$ | $0.49{ }^{\text {b }}$ | $0.69{ }^{\text {b }}$ |
| VI-Co-NO-MS | $0.48^{\text {b }}$ | $0.22^{\text {a }}$ | $0.92{ }^{\text {b }}$ | $0.77^{\text {b }}$ | $0.71{ }^{\text {b }}$ | $0.30^{\text {b }}$ | $0.83{ }^{\text {b }}$ | $0.95{ }^{\text {b }}$ |
| II-Ga-NO-MS | $0.48^{\text {b }}$ | $0.61{ }^{\text {b }}$ | $0.92{ }^{\text {b }}$ | $0.77^{\text {b }}$ | $0.52^{\text {b }}$ | $0.48^{\text {b }}$ | $1.00{ }^{\text {b }}$ | $0.20^{\text {a }}$ |
| III-Ga-NO-MS | $0.77^{\text {b }}$ | $0.72{ }^{\text {b }}$ | $0.92{ }^{\text {b }}$ | $0.77{ }^{\text {b }}$ | $0.52^{\text {b }}$ | $0.30^{\text {b }}$ | $0.95{ }^{\text {b }}$ | $0.69{ }^{\text {b }}$ |
| IV-Ga-NO-MS | $0.99{ }^{\text {b }}$ | $0.72{ }^{\text {b }}$ | $0.92{ }^{\text {b }}$ | $0.77^{\text {b }}$ | $0.71{ }^{\text {b }}$ | $0.34^{\text {b }}$ | $0.99{ }^{\text {b }}$ | $0.61{ }^{\text {b }}$ |
| V-Ga-NO-MS | $0.77{ }^{\text {b }}$ | $0.61{ }^{\text {b }}$ | $0.92{ }^{\text {b }}$ | $0.89{ }^{\text {b }}$ | $0.76{ }^{\text {b }}$ | $0.48^{\text {b }}$ | $0.99{ }^{\text {b }}$ | $0.24{ }^{\text {a }}$ |
| VI-Ga-NO-MS | $0.73{ }^{\text {b }}$ | $0.22^{\text {a }}$ | $0.92{ }^{\text {b }}$ | $0.89{ }^{\text {b }}$ | $0.76{ }^{\text {b }}$ | $0.48^{\text {b }}$ | $0.99{ }^{\text {b }}$ | $0.74{ }^{\text {b }}$ |
| II-Gj-NO-MS | $0.48{ }^{\text {b }}$ | $0.72{ }^{\text {b }}$ | $1.00^{\text {b }}$ | $0.48^{\text {b }}$ | $0.42^{\text {b }}$ | $0.97{ }^{\text {b }}$ | $0.99{ }^{\text {b }}$ | $0.24{ }^{\text {a }}$ |
| III-Gj-NO-MS | $0.77^{\text {b }}$ | $0.22^{\text {a }}$ | $0.92{ }^{\text {b }}$ | $0.15{ }^{\text {a }}$ | $0.42{ }^{\text {b }}$ | $0.30^{\text {b }}$ | $0.83{ }^{\text {b }}$ | $0.74{ }^{\text {b }}$ |
| IV-Gj-NO-MS | $0.99{ }^{\text {b }}$ | $1.00^{\text {b }}$ | $0.93{ }^{\text {b }}$ | $0.66{ }^{\text {b }}$ | $0.55{ }^{\text {b }}$ | $0.46{ }^{\text {b }}$ | $0.99{ }^{\text {b }}$ | $0.74{ }^{\text {b }}$ |
| V-Gj-NO-MS | $0.73{ }^{\text {b }}$ | $0.61{ }^{\text {b }}$ | $0.93{ }^{\text {b }}$ | $0.77^{\text {b }}$ | $0.42{ }^{\text {b }}$ | $0.48{ }^{\text {b }}$ | $0.99{ }^{\text {b }}$ | $0.61{ }^{\text {b }}$ |
| VI-Gj-NO-MS | $0.73{ }^{\text {b }}$ | $0.22^{\text {a }}$ | $0.92{ }^{\text {b }}$ | $0.77^{\text {b }}$ | $0.42{ }^{\text {b }}$ | $0.48^{\text {b }}$ | $0.99{ }^{\text {b }}$ | $1.00^{\text {b }}$ |
| II-Co-NI-MS | $0.48^{\text {b }}$ | $0.22^{\text {a }}$ | $0.90^{\text {b }}$ | $0.89{ }^{\text {b }}$ | $0.42{ }^{\text {b }}$ | $0.48^{\text {b }}$ | $0.49{ }^{\text {b }}$ | 0.07 |
| III-Co-NI-MS | $0.48{ }^{\text {b }}$ | $0.61{ }^{\text {b }}$ | $0.92{ }^{\text {b }}$ | $0.96{ }^{\text {b }}$ | $0.76{ }^{\text {b }}$ | $0.30^{\text {b }}$ | $0.49{ }^{\text {b }}$ | 0.08 |
| IV-Co-NI-MS | $0.50{ }^{\text {b }}$ | $0.72{ }^{\text {b }}$ | $0.92{ }^{\text {b }}$ | $0.96{ }^{\text {b }}$ | $0.77^{\text {b }}$ | $0.30^{\text {b }}$ | $0.49{ }^{\text {b }}$ | 0.07 |
| V-Co-NI-MS | $0.48{ }^{\text {b }}$ | $0.22^{\text {a }}$ | $0.90^{\text {b }}$ | $1.00{ }^{\text {b }}$ | $0.55{ }^{\text {b }}$ | $0.46{ }^{\text {b }}$ | $0.49{ }^{\text {b }}$ | 0.07 |
| VI-Co-NI-MS | $0.48{ }^{\text {b }}$ | $0.15{ }^{\text {a }}$ | $0.90{ }^{\text {b }}$ | $0.99{ }^{\text {b }}$ | $0.52{ }^{\text {b }}$ | $0.34{ }^{\text {b }}$ | $0.49{ }^{\text {b }}$ | 0.08 |
| II-Ga-NI-MS | $0.48{ }^{\text {b }}$ | $0.61{ }^{\text {b }}$ | $0.92{ }^{\text {b }}$ | $0.89{ }^{\text {b }}$ | $0.55{ }^{\text {b }}$ | $0.60^{\text {b }}$ | $0.95{ }^{\text {b }}$ | 0.07 |
| III-Ga-NI-MS | $0.73{ }^{\text {b }}$ | $0.72{ }^{\text {b }}$ | $0.92{ }^{\text {b }}$ | $0.89{ }^{\text {b }}$ | $0.92{ }^{\text {b }}$ | $0.46^{\text {b }}$ | $0.83{ }^{\text {b }}$ | 0.08 |
| IV-Ga-NI-MS | $0.77^{\text {b }}$ | $0.9{ }^{\text {b }}$ | $0.92{ }^{\text {b }}$ | $0.94{ }^{\text {b }}$ | $0.92{ }^{\text {b }}$ | $0.48^{\text {b }}$ | $0.83{ }^{\text {b }}$ | 0.08 |
| V-Ga-NI-MS | $0.73{ }^{\text {b }}$ | $0.44{ }^{\text {b }}$ | $0.92{ }^{\text {b }}$ | $0.94{ }^{\text {b }}$ | $0.90{ }^{\text {b }}$ | $0.48^{\text {b }}$ | $0.83{ }^{\text {b }}$ | 0.07 |
| VI-Ga-NI-MS | $0.73{ }^{\text {b }}$ | $0.22^{\text {a }}$ | $0.92{ }^{\text {b }}$ | $0.96{ }^{\text {b }}$ | $0.77^{\text {b }}$ | $0.48{ }^{\text {b }}$ | $0.83{ }^{\text {b }}$ | $0.20{ }^{\text {a }}$ |
| II-Gj-NI-MS | $0.48{ }^{\text {b }}$ | $0.61{ }^{\text {b }}$ | $0.92{ }^{\text {b }}$ | $0.89{ }^{\text {b }}$ | $0.71{ }^{\text {b }}$ | $1.00{ }^{\text {b }}$ | $0.95{ }^{\text {b }}$ | 0.07 |
| III-Gj-NI-MS | $0.77^{\text {b }}$ | $0.72{ }^{\text {b }}$ | $0.92{ }^{\text {b }}$ | $1.00{ }^{\text {b }}$ | $0.92{ }^{\text {b }}$ | $0.48{ }^{\text {b }}$ | $0.49{ }^{\text {b }}$ | $0.20^{\text {a }}$ |
| IV-Gj-NI-MS | $1.00{ }^{\text {b }}$ | $0.9{ }^{\text {b }}$ | $0.92{ }^{\text {b }}$ | $0.96{ }^{\text {b }}$ | $1.00^{\text {b }}$ | $0.60{ }^{\text {b }}$ | $0.83{ }^{\text {b }}$ | 0.08 |
| V-Gj-NI-MS | $0.77^{\text {b }}$ | $0.61{ }^{\text {b }}$ | $0.92{ }^{\text {b }}$ | $0.89{ }^{\text {b }}$ | $0.92{ }^{\text {b }}$ | $0.94{ }^{\text {b }}$ | $0.83{ }^{\text {b }}$ | $0.20^{\text {a }}$ |
| VI-Gj-NI-MS | $0.73{ }^{\text {b }}$ | $0.22^{\text {a }}$ | $0.92{ }^{\text {b }}$ | $1.00{ }^{\text {b }}$ | $0.92{ }^{\text {b }}$ | $0.97{ }^{\text {b }}$ | $0.83{ }^{\text {b }}$ | $0.24{ }^{\text {a }}$ |

Note: Forecast performance evaluation for the 16 series of stock volatility for the full out-of-sample period (1067 observation). See the note in table 2.7 for the description of the models. The $M C S$ is a procedure to determine the "best" models from a collection of models based on the MSE loss function. p-values for the range deviation method in the table. "a" and "b" denote that the model belongs to the $10 \%$ and $25 \% \mathrm{MCS}$. Results for the model $I$ not reported to save space. Results for the model $I$ not reported to save space.

Table 2.12: Out-of-sample forecast evaluation MCS - Crisis

|  | BA | BAC | C | CAT | FDX | HON | HPQ | IBM |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| II-Co-NO | 0.05 | $0.22^{\text {a }}$ | $0.32{ }^{\text {b }}$ | $0.19^{\text {a }}$ | 0.00 | 0.02 | $0.39{ }^{\text {b }}$ | $0.62{ }^{\text {b }}$ |
| III-Co-NO | 0.00 | $0.43{ }^{\text {b }}$ | $0.32^{\text {b }}$ | $0.19^{\text {a }}$ | 0.00 | 0.02 | $0.39{ }^{\text {b }}$ | $0.70^{\text {b }}$ |
| IV-Co-NO | 0.00 | $0.54{ }^{\text {b }}$ | $0.75{ }^{\text {b }}$ | $0.19^{\text {a }}$ | 0.00 | 0.02 | $0.39{ }^{\text {b }}$ | $0.70^{\text {b }}$ |
| V-Co-NO | 0.00 | $0.41^{\text {b }}$ | $0.75{ }^{\text {b }}$ | $0.19^{\text {a }}$ | 0.00 | 0.02 | $0.39{ }^{\text {b }}$ | $0.84{ }^{\text {b }}$ |
| VI-Co-NO | 0.00 | $0.41^{\text {b }}$ | $0.75{ }^{\text {b }}$ | $0.19^{\text {a }}$ | 0.00 | 0.02 | $0.39{ }^{\text {b }}$ | $0.84{ }^{\text {b }}$ |
| II-Ga-NO | $0.16^{\text {a }}$ | $0.23{ }^{\text {a }}$ | 0.04 | $0.19^{\text {a }}$ | 0.00 | 0.02 | $0.39{ }^{\text {b }}$ | $0.62{ }^{\text {b }}$ |
| III-Ga-NO | 0.01 | $0.10^{\text {a }}$ | 0.08 | $0.19^{\text {a }}$ | 0.00 | 0.02 | $0.39{ }^{\text {b }}$ | $0.84{ }^{\text {b }}$ |
| IV-Ga-NO | 0.01 | $0.23{ }^{\text {a }}$ | $0.58{ }^{\text {b }}$ | $0.19^{\text {a }}$ | 0.00 | 0.02 | $0.39^{\text {b }}$ | $0.70^{\text {b }}$ |
| V-Ga-NO | 0.01 | $0.10^{\text {a }}$ | $0.58{ }^{\text {b }}$ | $0.19^{\text {a }}$ | 0.00 | 0.02 | $0.39{ }^{\text {b }}$ | $0.84{ }^{\text {b }}$ |
| VI-Ga-NO | 0.01 | 0.03 | $0.58{ }^{\text {b }}$ | $0.19^{\text {a }}$ | 0.00 | 0.02 | $0.39{ }^{\text {b }}$ | $0.84{ }^{\text {b }}$ |
| II-Gj-NO | 0.05 | $0.23{ }^{\text {a }}$ | $0.12^{\text {a }}$ | $0.19^{\text {a }}$ | 0.00 | 0.02 | $0.39^{\text {b }}$ | $0.62{ }^{\text {b }}$ |
| III-Gj-NO | 0.01 | 0.03 | $0.12^{\text {a }}$ | $0.19^{\text {a }}$ | 0.00 | 0.02 | $0.39{ }^{\text {b }}$ | $0.84{ }^{\text {b }}$ |
| IV-Gj-NO | 0.01 | $0.10^{\text {a }}$ | $0.58{ }^{\text {b }}$ | $0.19^{\text {a }}$ | 0.00 | 0.02 | $0.39{ }^{\text {b }}$ | $0.70^{\text {b }}$ |
| V-Gj-NO | 0.01 | 0.03 | $0.58^{\text {b }}$ | $0.19^{\text {a }}$ | 0.00 | 0.02 | $0.39{ }^{\text {b }}$ | $0.84{ }^{\text {b }}$ |
| VI-Gj-NO | 0.00 | 0.03 | $0.75{ }^{\text {b }}$ | $0.19^{\text {a }}$ | 0.00 | 0.02 | $0.39^{\text {b }}$ | $0.86{ }^{\text {b }}$ |
| II-Co-NI | 0.05 | $0.23{ }^{\text {a }}$ | $0.12^{\text {a }}$ | $0.19^{\text {a }}$ | 0.00 | 0.02 | $0.39{ }^{\text {b }}$ | $0.62^{\text {b }}$ |
| III-Co-NI | 0.01 | $0.23{ }^{\text {a }}$ | $0.32^{\text {b }}$ | $0.19^{\text {a }}$ | 0.00 | 0.02 | $0.39{ }^{\text {b }}$ | $0.84{ }^{\text {b }}$ |
| IV-Co-NI | 0.01 | $0.41^{\text {b }}$ | $0.75{ }^{\text {b }}$ | $0.19^{\text {a }}$ | 0.00 | 0.02 | $0.39{ }^{\text {b }}$ | $0.84{ }^{\text {b }}$ |
| V-Co-NI | 0.00 | $0.23{ }^{\text {a }}$ | $0.75{ }^{\text {b }}$ | $0.19^{\text {a }}$ | 0.00 | 0.02 | $0.39{ }^{\text {b }}$ | $0.84{ }^{\text {b }}$ |
| VI-Co-NI | 0.00 | $0.22^{\text {a }}$ | $0.75{ }^{\text {b }}$ | $0.19^{\text {a }}$ | 0.00 | 0.02 | $0.39{ }^{\text {b }}$ | $0.84{ }^{\text {b }}$ |
| II-Ga-NI | 0.05 | $0.22^{\text {a }}$ | 0.04 | $0.19^{\text {a }}$ | 0.00 | 0.02 | $0.39{ }^{\text {b }}$ | $0.62{ }^{\text {b }}$ |
| III-Ga-NI | 0.01 | $0.10^{\text {a }}$ | 0.04 | $0.19^{\text {a }}$ | 0.00 | 0.02 | $0.39{ }^{\text {b }}$ | $0.86{ }^{\text {b }}$ |
| IV-Ga-NI | 0.01 | $0.26^{\text {b }}$ | $0.32^{\text {b }}$ | $0.19^{\text {a }}$ | 0.00 | 0.02 | $0.39{ }^{\text {b }}$ | $0.84{ }^{\text {b }}$ |
| V-Ga-NI | 0.01 | $0.10^{\text {a }}$ | $0.58{ }^{\text {b }}$ | $0.19^{\text {a }}$ | 0.00 | 0.02 | $0.39{ }^{\text {b }}$ | $0.84{ }^{\text {b }}$ |
| VI-Ga-NI | 0.00 | $0.10^{\text {a }}$ | $0.58^{\text {b }}$ | $0.19^{\text {a }}$ | 0.00 | 0.02 | $0.39{ }^{\text {b }}$ | $0.84{ }^{\text {b }}$ |
| II-Gj-NI | 0.05 | $0.22^{\text {a }}$ | 0.08 | $0.19^{\text {a }}$ | 0.00 | 0.02 | $0.39{ }^{\text {b }}$ | $0.62{ }^{\text {b }}$ |
| III-Gj-NI | 0.01 | 0.03 | 0.04 | $0.19^{\text {a }}$ | 0.00 | 0.02 | $0.39{ }^{\text {b }}$ | $0.86{ }^{\text {b }}$ |
| IV-Gj-NI | 0.01 | $0.22^{\text {a }}$ | $0.32^{\text {b }}$ | $0.19^{\text {a }}$ | 0.00 | 0.02 | $0.39{ }^{\text {b }}$ | $0.84{ }^{\text {b }}$ |
| V-Gj-NI | 0.01 | 0.03 | $0.58{ }^{\text {b }}$ | $0.19^{\text {a }}$ | 0.00 | 0.02 | $0.39{ }^{\text {b }}$ | $0.84{ }^{\text {b }}$ |
| VI-Gj-NI | 0.01 | 0.03 | $0.75{ }^{\text {b }}$ | $0.19^{\text {a }}$ | 0.00 | 0.02 | $0.39{ }^{\text {b }}$ | $0.86{ }^{\text {b }}$ |
| II-Co-NO-MS | $1.00^{\text {b }}$ | $0.43{ }^{\text {b }}$ | $0.32^{\text {b }}$ | $0.19^{\text {a }}$ | 0.00 | 0.02 | $0.39{ }^{\text {b }}$ | $0.86{ }^{\text {b }}$ |
| III-Co-NO-MS | $0.39^{\text {b }}$ | $0.54{ }^{\text {b }}$ | $0.75{ }^{\text {b }}$ | $0.19^{\text {a }}$ | 0.00 | 0.02 | $0.39{ }^{\text {b }}$ | $0.93{ }^{\text {b }}$ |
| IV-Co-NO-MS | $0.41^{\text {b }}$ | $0.54{ }^{\text {b }}$ | $1.00^{\text {b }}$ | $0.19^{\text {a }}$ | 0.00 | 0.02 | $0.46{ }^{\text {b }}$ | $0.93{ }^{\text {b }}$ |
| V-Co-NO-MS | $0.16^{\text {a }}$ | $0.41^{\text {b }}$ | $0.75{ }^{\text {b }}$ | $0.19^{\text {a }}$ | 0.00 | 0.02 | $0.52^{\text {b }}$ | $0.99{ }^{\text {b }}$ |
| VI-Co-NO-MS | $0.16^{\text {a }}$ | $0.41^{\text {b }}$ | $0.75{ }^{\text {b }}$ | $0.19^{\text {a }}$ | 0.00 | 0.02 | $0.46{ }^{\text {b }}$ | $0.93{ }^{\text {b }}$ |
| II-Ga-NO-MS | $0.39^{\text {b }}$ | $0.41^{\text {b }}$ | $0.32^{\text {b }}$ | $0.19^{\text {a }}$ | $1.00^{\text {b }}$ | $0.75{ }^{\text {b }}$ | $0.39^{\text {b }}$ | $0.86{ }^{\text {b }}$ |
| III-Ga-NO-MS | $0.16^{\text {a }}$ | $0.54{ }^{\text {b }}$ | $0.58{ }^{\text {b }}$ | $0.19^{\text {a }}$ | 0.00 | $0.75{ }^{\text {b }}$ | $0.46{ }^{\text {b }}$ | $0.86{ }^{\text {b }}$ |
| IV-Ga-NO-MS | $0.28{ }^{\text {b }}$ | $0.54{ }^{\text {b }}$ | $0.75{ }^{\text {b }}$ | $0.19^{\text {a }}$ | 0.00 | $0.75{ }^{\text {b }}$ | $0.52^{\text {b }}$ | $0.86{ }^{\text {b }}$ |
| V-Ga-NO-MS | 0.05 | $0.26^{\text {b }}$ | $0.75{ }^{\text {b }}$ | $0.19^{\text {a }}$ | 0.00 | 0.02 | $0.39{ }^{\text {b }}$ | $0.93{ }^{\text {b }}$ |
| VI-Ga-NO-MS | 0.01 | $0.41^{\text {b }}$ | $0.75{ }^{\text {b }}$ | $0.19^{\text {a }}$ | 0.00 | 0.02 | $0.39{ }^{\text {b }}$ | $0.86{ }^{\text {b }}$ |
| II-Gj-NO-MS | $0.53{ }^{\text {b }}$ | $0.54{ }^{\text {b }}$ | $0.32^{\text {b }}$ | $0.19^{\text {a }}$ | $0.23{ }^{\text {a }}$ | $0.75{ }^{\text {b }}$ | $0.39{ }^{\text {b }}$ | $0.84{ }^{\text {b }}$ |
| III-Gj-NO-MS | $0.16^{\text {a }}$ | $0.54{ }^{\text {b }}$ | $0.58{ }^{\text {b }}$ | $0.19^{\text {a }}$ | 0.00 | $0.75{ }^{\text {b }}$ | $0.52^{\text {b }}$ | $0.84{ }^{\text {b }}$ |
| IV-Gj-NO-MS | 0.05 | $0.90{ }^{\text {b }}$ | $0.75{ }^{\text {b }}$ | $0.19^{\text {a }}$ | 0.00 | $0.75{ }^{\text {b }}$ | $0.39^{\text {b }}$ | $0.84{ }^{\text {b }}$ |
| V-Gj-NO-MS | 0.05 | $0.54{ }^{\text {b }}$ | $0.75{ }^{\text {b }}$ | $0.19^{\text {a }}$ | 0.00 | 0.02 | $0.52^{\text {b }}$ | $0.84{ }^{\text {b }}$ |
| VI-Gj-NO-MS | 0.00 | $0.54{ }^{\text {b }}$ | $0.75{ }^{\text {b }}$ | $0.19^{\text {a }}$ | 0.00 | 0.02 | $0.52^{\text {b }}$ | $0.84{ }^{\text {b }}$ |
| II-Co-NI-MS | $0.53{ }^{\text {b }}$ | $0.26{ }^{\text {b }}$ | 0.04 | $0.19^{\text {a }}$ | 0.00 | 0.02 | $0.52^{\text {b }}$ | $0.84{ }^{\text {b }}$ |
| III-Co-NI-MS | $0.16^{\text {a }}$ | $0.52^{\text {b }}$ | $0.58{ }^{\text {b }}$ | $0.36{ }^{\text {b }}$ | 0.00 | 0.02 | $0.79{ }^{\text {b }}$ | $0.86{ }^{\text {b }}$ |
| IV-Co-NI-MS | $0.16^{\text {a }}$ | $0.54{ }^{\text {b }}$ | $0.75{ }^{\text {b }}$ | $0.46{ }^{\text {b }}$ | 0.00 | 0.02 | $0.85{ }^{\text {b }}$ | $0.99{ }^{\text {b }}$ |
| V-Co-NI-MS | 0.05 | $0.23{ }^{\text {a }}$ | $0.75{ }^{\text {b }}$ | $0.19^{\text {a }}$ | 0.00 | 0.02 | $0.85{ }^{\text {b }}$ | $0.97{ }^{\text {b }}$ |
| VI-Co-NI-MS | 0.05 | $0.23{ }^{\text {a }}$ | $0.75{ }^{\text {b }}$ | $0.46{ }^{\text {b }}$ | 0.00 | 0.02 | $0.85{ }^{\text {b }}$ | $0.93{ }^{\text {b }}$ |
| II-Ga-NI-MS | $0.53{ }^{\text {b }}$ | $0.41^{\text {b }}$ | $0.12^{\text {a }}$ | $0.55{ }^{\text {b }}$ | $0.72{ }^{\text {b }}$ | $0.69{ }^{\text {b }}$ | $0.85{ }^{\text {b }}$ | $0.86{ }^{\text {b }}$ |
| III-Ga-NI-MS | 0.01 | $0.54{ }^{\text {b }}$ | $0.32^{\text {b }}$ | $0.94{ }^{\text {b }}$ | 0.00 | $1.00^{\text {b }}$ | $0.85{ }^{\text {b }}$ | $0.86{ }^{\text {b }}$ |
| IV-Ga-NI-MS | 0.01 | $0.54{ }^{\text {b }}$ | $0.58{ }^{\text {b }}$ | $0.94{ }^{\text {b }}$ | 0.00 | $0.75{ }^{\text {b }}$ | $0.96{ }^{\text {b }}$ | $1.00^{\text {b }}$ |
| V-Ga-NI-MS | 0.00 | $0.41^{\text {b }}$ | $0.58{ }^{\text {b }}$ | $0.55{ }^{\text {b }}$ | $0.23{ }^{\text {a }}$ | 0.02 | $1.00{ }^{\text {b }}$ | $0.97{ }^{\text {b }}$ |
| VI-Ga-NI-MS | 0.00 | $0.52^{\text {b }}$ | $0.58{ }^{\text {b }}$ | $0.81{ }^{\text {b }}$ | 0.00 | 0.02 | $0.96{ }^{\text {b }}$ | $0.93{ }^{\text {b }}$ |
| II-Gj-NI-MS | 0.05 | $0.52^{\text {b }}$ | 0.04 | $0.94{ }^{\text {b }}$ | $0.72^{\text {b }}$ | 0.02 | $0.85{ }^{\text {b }}$ | $0.86{ }^{\text {b }}$ |
| III-Gj-NI-MS | 0.05 | $0.54{ }^{\text {b }}$ | $0.12^{\text {a }}$ | $0.94{ }^{\text {b }}$ | 0.00 | $0.75{ }^{\text {b }}$ | $0.85{ }^{\text {b }}$ | $0.86{ }^{\text {b }}$ |
| IV-Gj-NI-MS | $0.16^{\text {a }}$ | $1.00^{\text {b }}$ | $0.58^{\text {b }}$ | $1.00^{\text {b }}$ | 0.00 | $0.75{ }^{\text {b }}$ | $0.52^{\text {b }}$ | $0.86{ }^{\text {b }}$ |
| V-Gj-NI-MS | 0.01 | $0.52^{\text {b }}$ | $0.32^{\text {b }}$ | $0.81{ }^{\text {b }}$ | $0.23{ }^{\text {a }}$ | 0.02 | $0.85{ }^{\text {b }}$ | $0.86{ }^{\text {b }}$ |
| VI-Gj-NI-MS | 0.00 | $0.54{ }^{\text {b }}$ | $0.32^{\text {b }}$ | $0.94{ }^{\text {b }}$ | 0.00 | 0.02 | $0.52^{\text {b }}$ | $0.86{ }^{\text {b }}$ |

Note: Forecast performance evaluation for the 16 series of stock volatility for the Crisis period from September 2008 to July 2009 ( 200 observation). See the note in table 2.7 for the description of the models. The MCS is a procedure to determine the "best" models from a collection of models based on the $M S E$ loss function. $p v_{R}$ are the p-values for the range deviation method. "a" and "b" denote that the model belongs to the $10 \%$ and $25 \%$ MCS.

Table 2.13: Out-of-sample forecast evaluation MCS - Crisis (cont.)

|  | JPM | KFT | PEP | PG | T | TWX | TXN | WFC |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| II-Co-NO | $0.19^{\text {a }}$ | 0.00 | $0.43{ }^{\text {b }}$ | $0.37{ }^{\text {b }}$ | $0.27^{\text {b }}$ | $0.38^{\text {b }}$ | $0.16^{\text {a }}$ | 0.02 |
| III-Co-NO | $0.27{ }^{\text {b }}$ | 0.00 | $0.17^{\text {a }}$ | $0.46{ }^{\text {b }}$ | $0.27^{\text {b }}$ | $0.33{ }^{\text {b }}$ | $0.16^{\text {a }}$ | $0.17^{\text {a }}$ |
| IV-Co-NO | $0.27^{\text {b }}$ | 0.00 | $0.17^{\text {a }}$ | $0.46{ }^{\text {b }}$ | $0.27^{\text {b }}$ | $0.33{ }^{\text {b }}$ | $0.29{ }^{\text {b }}$ | 0.02 |
| V-Co-NO | $0.27^{\text {b }}$ | 0.00 | $0.17^{\text {a }}$ | $0.46{ }^{\text {b }}$ | 0.01 | $0.38^{\text {b }}$ | $0.33^{\text {b }}$ | $0.20^{\text {a }}$ |
| VI-Co-NO | $0.28{ }^{\text {b }}$ | 0.00 | $0.17^{\text {a }}$ | $0.46{ }^{\text {b }}$ | 0.01 | $0.38^{\text {b }}$ | $0.16^{\text {a }}$ | $0.60{ }^{\text {b }}$ |
| II-Ga-NO | $0.19^{\text {a }}$ | 0.00 | $0.43{ }^{\text {b }}$ | $0.37{ }^{\text {b }}$ | $0.27^{\text {b }}$ | $0.38^{\text {b }}$ | $0.16^{\text {a }}$ | 0.02 |
| III-Ga-NO | $0.27{ }^{\text {b }}$ | 0.00 | $0.43{ }^{\text {b }}$ | $0.37{ }^{\text {b }}$ | $0.27^{\text {b }}$ | $0.38^{\text {b }}$ | $0.51{ }^{\text {b }}$ | 0.02 |
| IV-Ga-NO | $0.27^{\text {b }}$ | 0.00 | $0.17^{\text {a }}$ | $0.37{ }^{\text {b }}$ | $0.27^{\text {b }}$ | $0.38^{\text {b }}$ | $0.57{ }^{\text {b }}$ | 0.02 |
| V-Ga-NO | $0.29{ }^{\text {b }}$ | 0.00 | $0.17^{\text {a }}$ | $0.37{ }^{\text {b }}$ | $0.17^{\text {a }}$ | $0.38^{\text {b }}$ | $0.33{ }^{\text {b }}$ | $0.18^{\text {a }}$ |
| VI-Ga-NO | $0.75{ }^{\text {b }}$ | 0.00 | $0.17^{\text {a }}$ | $0.37{ }^{\text {b }}$ | $0.17^{\text {a }}$ | $0.38^{\text {b }}$ | $0.33{ }^{\text {b }}$ | $0.97{ }^{\text {b }}$ |
| II-Gj-NO | $0.19{ }^{\text {a }}$ | 0.00 | $0.43{ }^{\text {b }}$ | $0.37{ }^{\text {b }}$ | $0.27^{\text {b }}$ | $0.38^{\text {b }}$ | $0.16^{\text {a }}$ | 0.02 |
| III-Gj-NO | $0.27{ }^{\text {b }}$ | 0.00 | $0.43{ }^{\text {b }}$ | $0.37{ }^{\text {b }}$ | $0.27^{\text {b }}$ | $0.38^{\text {b }}$ | $0.50{ }^{\text {b }}$ | 0.02 |
| IV-Gj-NO | $0.28{ }^{\text {b }}$ | 0.00 | $0.43{ }^{\text {b }}$ | $0.37{ }^{\text {b }}$ | $0.27^{\text {b }}$ | $0.38^{\text {b }}$ | $0.51{ }^{\text {b }}$ | 0.02 |
| V-Gj-NO | $0.29{ }^{\text {b }}$ | 0.00 | $0.43{ }^{\text {b }}$ | $0.37{ }^{\text {b }}$ | $0.17^{\text {a }}$ | $0.38^{\text {b }}$ | $0.33{ }^{\text {b }}$ | $0.17^{\text {a }}$ |
| VI-Gj-NO | $0.75{ }^{\text {b }}$ | 0.00 | $0.43{ }^{\text {b }}$ | $0.37{ }^{\text {b }}$ | 0.01 | $0.38^{\text {b }}$ | $0.33{ }^{\text {b }}$ | $0.98{ }^{\text {b }}$ |
| II-Co-NI | $0.19^{\text {a }}$ | 0.00 | $0.43{ }^{\text {b }}$ | $0.37{ }^{\text {b }}$ | $0.17^{\text {a }}$ | $0.38^{\text {b }}$ | $0.16^{\text {a }}$ | 0.02 |
| III-Co-NI | $0.19^{\text {a }}$ | 0.00 | $0.17^{\text {a }}$ | $0.37{ }^{\text {b }}$ | $0.27^{\text {b }}$ | $0.33{ }^{\text {b }}$ | $0.33{ }^{\text {b }}$ | 0.02 |
| IV-Co-NI | $0.27{ }^{\text {b }}$ | 0.00 | $0.17^{\text {a }}$ | $0.37{ }^{\text {b }}$ | $0.27^{\text {b }}$ | $0.38^{\text {b }}$ | $0.50{ }^{\text {b }}$ | 0.02 |
| V-Co-NI | $0.27{ }^{\text {b }}$ | 0.00 | $0.17^{\text {a }}$ | $0.37{ }^{\text {b }}$ | $0.17^{\text {a }}$ | $0.38^{\text {b }}$ | $0.50{ }^{\text {b }}$ | 0.02 |
| VI-Co-NI | $0.27^{\text {b }}$ | 0.00 | $0.17^{\text {a }}$ | $0.37{ }^{\text {b }}$ | 0.01 | $0.38^{\text {b }}$ | $0.33{ }^{\text {b }}$ | $0.43{ }^{\text {b }}$ |
| II-Ga-NI | $0.19^{\text {a }}$ | 0.00 | $0.43{ }^{\text {b }}$ | $0.37{ }^{\text {b }}$ | $0.27^{\text {b }}$ | $0.38^{\text {b }}$ | $0.16^{\text {a }}$ | 0.02 |
| III-Ga-NI | $0.27^{\text {b }}$ | 0.00 | $0.43{ }^{\text {b }}$ | $0.37{ }^{\text {b }}$ | $0.27^{\text {b }}$ | $0.38^{\text {b }}$ | $0.94{ }^{\text {b }}$ | 0.02 |
| IV-Ga-NI | $0.27^{\text {b }}$ | 0.00 | $0.17^{\text {a }}$ | $0.37{ }^{\text {b }}$ | $0.27^{\text {b }}$ | $0.38^{\text {b }}$ | $0.94{ }^{\text {b }}$ | 0.02 |
| V-Ga-NI | $0.29{ }^{\text {b }}$ | 0.00 | $0.17^{\text {a }}$ | $0.37{ }^{\text {b }}$ | $0.17^{\text {a }}$ | $0.38^{\text {b }}$ | $0.82{ }^{\text {b }}$ | $0.17^{\text {a }}$ |
| VI-Ga-NI | $0.29{ }^{\text {b }}$ | 0.00 | $0.17^{\text {a }}$ | $0.37{ }^{\text {b }}$ | $0.17^{\text {a }}$ | $0.38^{\text {b }}$ | $0.33{ }^{\text {b }}$ | $0.60{ }^{\text {b }}$ |
| II-Gj-NI | $0.19^{\text {a }}$ | 0.00 | $0.43{ }^{\text {b }}$ | $0.37{ }^{\text {b }}$ | $0.27^{\text {b }}$ | $0.38^{\text {b }}$ | $0.16^{\text {a }}$ | 0.02 |
| III-Gj-NI | $0.27^{\text {b }}$ | 0.00 | $0.43{ }^{\text {b }}$ | $0.37{ }^{\text {b }}$ | $0.27^{\text {b }}$ | $0.38{ }^{\text {b }}$ | $0.94{ }^{\text {b }}$ | $0.20^{\text {a }}$ |
| IV-Gj-NI | $0.28{ }^{\text {b }}$ | 0.00 | $0.43{ }^{\text {b }}$ | $0.37{ }^{\text {b }}$ | $0.27^{\text {b }}$ | $0.38^{\text {b }}$ | $0.94{ }^{\text {b }}$ | $0.18^{\text {a }}$ |
| V-Gj-NI | $0.29{ }^{\text {b }}$ | 0.00 | $0.17^{\text {a }}$ | $0.37{ }^{\text {b }}$ | $0.27^{\text {b }}$ | $0.38{ }^{\text {b }}$ | $0.84{ }^{\text {b }}$ | $0.43{ }^{\text {b }}$ |
| VI-Gj-NI | $0.75{ }^{\text {b }}$ | 0.00 | $0.17^{\text {a }}$ | $0.37{ }^{\text {b }}$ | $0.17^{\text {a }}$ | $0.38{ }^{\text {b }}$ | $0.33{ }^{\text {b }}$ | $0.99{ }^{\text {b }}$ |
| II-Co-NO-MS | $0.29{ }^{\text {b }}$ | 0.01 | $0.44{ }^{\text {b }}$ | $0.37{ }^{\text {b }}$ | $0.27^{\text {b }}$ | $0.78{ }^{\text {b }}$ | $0.16^{\text {a }}$ | 0.02 |
| III-Co-NO-MS | $0.75{ }^{\text {b }}$ | 0.00 | $0.43{ }^{\text {b }}$ | $0.37{ }^{\text {b }}$ | $0.60{ }^{\text {b }}$ | $0.38{ }^{\text {b }}$ | $0.29{ }^{\text {b }}$ | $0.43{ }^{\text {b }}$ |
| IV-Co-NO-MS | $0.75{ }^{\text {b }}$ | 0.00 | $0.44{ }^{\text {b }}$ | $0.37{ }^{\text {b }}$ | $0.71{ }^{\text {b }}$ | $0.47^{\text {b }}$ | $0.16^{\text {a }}$ | $0.98{ }^{\text {b }}$ |
| V-Co-NO-MS | $0.92{ }^{\text {b }}$ | 0.00 | $0.43{ }^{\text {b }}$ | $0.37^{\text {b }}$ | $0.52^{\text {b }}$ | $0.58{ }^{\text {b }}$ | $0.16^{\text {a }}$ | $0.98{ }^{\text {b }}$ |
| VI-Co-NO-MS | $0.92{ }^{\text {b }}$ | 0.00 | $0.43{ }^{\text {b }}$ | $0.37{ }^{\text {b }}$ | $0.27^{\text {b }}$ | $0.59{ }^{\text {b }}$ | $0.16^{\text {a }}$ | $0.98{ }^{\text {b }}$ |
| II-Ga-NO-MS | $0.75{ }^{\text {b }}$ | 0.01 | $0.44{ }^{\text {b }}$ | $0.37{ }^{\text {b }}$ | $0.27^{\text {b }}$ | $1.00{ }^{\text {b }}$ | $0.94{ }^{\text {b }}$ | 0.02 |
| III-Ga-NO-MS | $0.98{ }^{\text {b }}$ | 0.00 | $0.44{ }^{\text {b }}$ | $0.37{ }^{\text {b }}$ | $0.88{ }^{\text {b }}$ | $0.47^{\text {b }}$ | $0.94{ }^{\text {b }}$ | $0.17^{\text {a }}$ |
| IV-Ga-NO-MS | $0.92{ }^{\text {b }}$ | 0.00 | $0.44{ }^{\text {b }}$ | $0.37{ }^{\text {b }}$ | $0.88{ }^{\text {b }}$ | $0.56{ }^{\text {b }}$ | $0.94{ }^{\text {b }}$ | $0.60{ }^{\text {b }}$ |
| V-Ga-NO-MS | $0.92{ }^{\text {b }}$ | 0.00 | $0.43{ }^{\text {b }}$ | $0.37{ }^{\text {b }}$ | $0.88{ }^{\text {b }}$ | $0.63{ }^{\text {b }}$ | $0.94{ }^{\text {b }}$ | $0.60{ }^{\text {b }}$ |
| VI-Ga-NO-MS | $0.92{ }^{\text {b }}$ | 0.00 | $0.43{ }^{\text {b }}$ | $0.37{ }^{\text {b }}$ | $0.88{ }^{\text {b }}$ | $0.59{ }^{\text {b }}$ | $0.94{ }^{\text {b }}$ | $0.60{ }^{\text {b }}$ |
| II-Gj-NO-MS | $0.75{ }^{\text {b }}$ | $0.12^{\text {a }}$ | $1.00{ }^{\text {b }}$ | $0.37{ }^{\text {b }}$ | $0.27^{\text {b }}$ | $0.86{ }^{\text {b }}$ | $0.94{ }^{\text {b }}$ | 0.02 |
| III-Gj-NO-MS | $1.00{ }^{\text {b }}$ | 0.00 | $0.43{ }^{\text {b }}$ | $0.37{ }^{\text {b }}$ | $0.88{ }^{\text {b }}$ | $0.47^{\text {b }}$ | $0.94{ }^{\text {b }}$ | $0.43{ }^{\text {b }}$ |
| IV-Gj-NO-MS | $0.92{ }^{\text {b }}$ | 0.01 | $0.44{ }^{\text {b }}$ | $0.37{ }^{\text {b }}$ | $0.88{ }^{\text {b }}$ | $0.58{ }^{\text {b }}$ | $1.00{ }^{\text {b }}$ | $0.98{ }^{\text {b }}$ |
| V-Gj-NO-MS | $0.92{ }^{\text {b }}$ | 0.00 | $0.44{ }^{\text {b }}$ | $0.37{ }^{\text {b }}$ | $0.88{ }^{\text {b }}$ | $0.78{ }^{\text {b }}$ | $0.94{ }^{\text {b }}$ | $1.00^{\text {b }}$ |
| VI-Gj-NO-MS | $0.92{ }^{\text {b }}$ | 0.00 | $0.43{ }^{\text {b }}$ | $0.37{ }^{\text {b }}$ | $0.79{ }^{\text {b }}$ | $0.75{ }^{\text {b }}$ | $0.94{ }^{\text {b }}$ | $0.99{ }^{\text {b }}$ |
| II-Co-NI-MS | $0.19{ }^{\text {a }}$ | $0.82{ }^{\text {b }}$ | $0.44{ }^{\text {b }}$ | $0.37{ }^{\text {b }}$ | $0.27^{\text {b }}$ | $0.56{ }^{\text {b }}$ | 0.03 | 0.02 |
| III-Co-NI-MS | $0.29{ }^{\text {b }}$ | $0.82^{\text {b }}$ | $0.43{ }^{\text {b }}$ | $0.86{ }^{\text {b }}$ | $0.88{ }^{\text {b }}$ | $0.38{ }^{\text {b }}$ | 0.03 | 0.02 |
| IV-Co-NI-MS | $0.29{ }^{\text {b }}$ | $1.00^{\text {b }}$ | $0.43{ }^{\text {b }}$ | $0.86{ }^{\text {b }}$ | $0.88{ }^{\text {b }}$ | $0.47^{\text {b }}$ | 0.03 | $0.17^{\text {a }}$ |
| V-Co-NI-MS | $0.29{ }^{\text {b }}$ | $0.82{ }^{\text {b }}$ | $0.43{ }^{\text {b }}$ | $0.46{ }^{\text {b }}$ | $0.71{ }^{\text {b }}$ | $0.47{ }^{\text {b }}$ | 0.03 | $0.18^{\text {a }}$ |
| VI-Co-NI-MS | $0.29{ }^{\text {b }}$ | $0.82^{\text {b }}$ | $0.43{ }^{\text {b }}$ | $0.56{ }^{\text {b }}$ | $0.71{ }^{\text {b }}$ | $0.47^{\text {b }}$ | 0.03 | $0.18^{\text {a }}$ |
| II-Ga-NI-MS | $0.27{ }^{\text {b }}$ | $0.82{ }^{\text {b }}$ | $0.44{ }^{\text {b }}$ | $0.37{ }^{\text {b }}$ | $0.27^{\text {b }}$ | $0.59{ }^{\text {b }}$ | $0.33{ }^{\text {b }}$ | 0.02 |
| III-Ga-NI-MS | $0.92{ }^{\text {b }}$ | $0.82^{\text {b }}$ | $0.44{ }^{\text {b }}$ | $0.56{ }^{\text {b }}$ | $1.00^{\text {b }}$ | $0.56{ }^{\text {b }}$ | $0.50{ }^{\text {b }}$ | 0.02 |
| IV-Ga-NI-MS | $0.92{ }^{\text {b }}$ | $0.82^{\text {b }}$ | $0.44{ }^{\text {b }}$ | $0.56{ }^{\text {b }}$ | $0.88{ }^{\text {b }}$ | $0.56{ }^{\text {b }}$ | $0.51{ }^{\text {b }}$ | $0.17^{\text {a }}$ |
| V-Ga-NI-MS | $0.75{ }^{\text {b }}$ | $0.82^{\text {b }}$ | $0.43{ }^{\text {b }}$ | $0.56{ }^{\text {b }}$ | $0.88{ }^{\text {b }}$ | $0.56{ }^{\text {b }}$ | $0.50{ }^{\text {b }}$ | $0.18^{\text {a }}$ |
| VI-Ga-NI-MS | $0.75{ }^{\text {b }}$ | $0.82^{\text {b }}$ | $0.43{ }^{\text {b }}$ | $0.56{ }^{\text {b }}$ | $0.88{ }^{\text {b }}$ | $0.56{ }^{\text {b }}$ | $0.50{ }^{\text {b }}$ | $0.18^{\text {a }}$ |
| II-Gj-NI-MS | $0.27{ }^{\text {b }}$ | $0.82^{\text {b }}$ | $0.44{ }^{\text {b }}$ | $0.37{ }^{\text {b }}$ | $0.27^{\text {b }}$ | $0.59{ }^{\text {b }}$ | $0.29{ }^{\text {b }}$ | 0.02 |
| III-Gj-NI-MS | $0.92{ }^{\text {b }}$ | $0.82^{\text {b }}$ | $0.44{ }^{\text {b }}$ | $0.86{ }^{\text {b }}$ | $0.88{ }^{\text {b }}$ | $0.47^{\text {b }}$ | $0.33{ }^{\text {b }}$ | $0.17^{\text {a }}$ |
| IV-Gj-NI-MS | $0.92{ }^{\text {b }}$ | $0.93{ }^{\text {b }}$ | $0.44{ }^{\text {b }}$ | $1.00^{\text {b }}$ | $0.88{ }^{\text {b }}$ | $0.56{ }^{\text {b }}$ | $0.50{ }^{\text {b }}$ | $0.43{ }^{\text {b }}$ |
| V-Gj-NI-MS | $0.75{ }^{\text {b }}$ | $0.82^{\text {b }}$ | $0.43{ }^{\text {b }}$ | $0.56{ }^{\text {b }}$ | $0.79{ }^{\text {b }}$ | $0.56{ }^{\text {b }}$ | $0.33{ }^{\text {b }}$ | $0.43{ }^{\text {b }}$ |
| VI-Gj-NI-MS | $0.75{ }^{\text {b }}$ | $0.82^{\text {b }}$ | $0.43{ }^{\text {b }}$ | $0.56{ }^{\text {b }}$ | $0.79{ }^{\text {b }}$ | $0.59{ }^{\text {b }}$ | $0.33{ }^{\text {b }}$ | $0.43{ }^{\text {b }}$ |

Note: Forecast performance evaluation for the 16 series of stock volatility for the Crisis period from September 2008 to July 2009 ( 200 observation). See the note in table 2.7 for the description of the models. The $M C S$ is a procedure to determine the "best" models from a collection of models based on the $M S E$ loss function. $p v_{R}$ are the p-values for the range deviation method. "a" and "b" denote that the model belongs to the $10 \%$ and $25 \%$ MCS.

## Chapter 3

## Realized Volatility: Estimation, Forecasting and Option trading

### 3.1 Introduction

During the last years, with the increasing availability of high frequency financial data, the development and modeling of realized estimators of volatility have grown considerably ${ }^{1}$. The main goal is to obtain the most accurate estimation of the variation of the price process and the most precise forecasting that is needed in many financial applications. The realized volatility ( $R V$ ), introduced by Andersen, Bollerslev, Diebold, and Labys (2001) and Barndorff-Nielsen and Shephard (2002), is defined as the sum of squared intra-day returns and in theory, it is an unbiased and highly efficient estimator of the integrated variances and it converges to it when the length of the intra-day interval goes to zero. Moreover, the realized range volatility ( $R R V$ ), introduced by Martens and van Dijk (2007) and Christensen and Podolskij (2007), is another realized estimator consistent for the quadratic variation and it is based on the difference between the minimum and maximum prices observed during a certain intra-day interval. In theory, it is more efficient than the realized volatility.

Large part of the literature has concentrated and dealt with the Microstructure Noise that affects high frequency market data and as a consequence, the properties of the realized estimators that become biased and inconsistent. For example, some of the corrections for the realized variance are Zhang, Mykland, and Ait-Sahalia (2005) and Zhang (2006), Bandi and Russell (2008), Hansen and Lunde (2005) and Barndorff-Nielsen, Hansen, Lunde, and Shephard (2008), and Martens and van Dijk (2007) and Christensen, Podolskij, and Vetter (2009) for the realized range. In addition, the literature has also focused on modeling and forecasting the observed volatility with time series technique and analyzed the predictive accuracy from a statistical approach. The long memory behavior of realized volatility series has been modeled through different models. Among others, Andersen, Bollerslev, Diebold, and Labys (2003) considered an Autoregressive Fractionally Integrated Moving Average (ARFIMA) model, where forecasts for the

[^15]$R V$ generally dominate GARCH models. Corsi (2009) presented the Heterogenous autoregressive (HAR) model, a high order autoregressive model that reproduces the hyperbolic decay of the autocorrelation function observed in the data and that shows remarkably good forecasting performance. Moreover, other stylized facts of volatility (asymmetry, leverage effects, and fat tails) have been modeled to improve the forecasts. Martens, van Dijk, and de Pooter (2009) compared the performance of the ARFIMA models and HAR model with a flexible unrestricted high-order autoregressive model including leverage effects, days of the week effects and macroeconomics announcement. Corsi, Mittnik, Pigorsch, and Pigorsch (2008) introduced two important extensions to the standard models, a GARCH component modeling the volatility of volatility and non-Gaussian innovations, resulting in an improvement in the accuracy of the point forecasting and a better density forecast. Finally, Bollerslev, Kretschmer, Pigorsch, and Tauchen (2009) developed a multivariate discrete-time volatility model that jointly incorporates the returns, the realized continuous sample path and jump variation measures.

An alternative approach to determine the best estimator and model to obtain the most accurate forecast of volatility is to adopt an economic criteria. In the univariate context, Bandi, Russell, and Yang (2008) evaluated and compared the quality of several realized estimators based on the profits from option pricing and trading. In their work, agents price short-term options with alternative volatility estimates on the S\&P 100 index before trading with each other at average prices. In the multivariate case and considering the utility that an investor derives from alternative variance forecasts in a portfolio allocation problem, Fleming, Kirby, and Ostdiek (2003) measured the economic value of using intradaily returns to construct estimates of daily return volatility and they found substantial gains when switching from daily to intradaily returns to estimate the conditional covariance matrix. More recently, de Pooter, Martens, and van Dijk (2008) examined the economic significance of determining the optimal sampling frequency of realized covariance matrix and they found that choosing the optimal sampling frequency is relevant for the out-of-sample performance of portfolios constructed using realized covariances with the optimum near to the hour, while Bandi, Russell, and Zhu (2008) compared the utility of a representative investor for optimal portfolio weights constructed from forecast for daily variance/ covariance based on optimally-sampled procedures as well as 5 and 15 minute intervals.

The aim of this work is to analyze the performance of alternative volatility estimators and forecasting models in an economic set up, based on the profit that derives from a buy-and-hold option trading strategy that speculates on the future level of the volatility. We consider an agent that invests in a straddle (a combination of a call and a put option, with same exercise price and expiration date) depending on his expected level of volatility during the trading period. At the beginning of the period, the agent estimates and forecasts volatility with different estimators and models. Then, if high (low) volatility is expected he buys (sells) the straddle. Different estimators and models should result in different results. To evaluate the different strategies we consider the mean returns and the cumulative returns from a dynamic investment exercise. The trading strategy is implemented
with weekly options based on the S\&P 500 and traded at the Chicago Board Options Exchange (CBOE) for the period between October 2005 and October 2009.

More in details, we compute volatility series from a high frequency series on S\&P 500 Futures from January 1996 to October 2009 with estimators related to the realized volatility, which are based on the intradaily returns, as well as with estimators related to the realized range volatility, which are based on the intradaily range. In particular, the selected estimators are robust to the presence of microstructure noise and jumps in the price process. In order to obtain out-ofsample forecasts to drive our trading strategies, we fit these series with ARFIMA models. The long memory behavior of volatility series has been widely documented in the literature an ARFIMA models are able to capture the long range component of volatility series. In addition, to deal with the possibility of confusing long memory with process with short memory and structural breaks, as pointed out by Lu and Perron (2010) and Perron and Qu (2010), among others, we consider the ARFIMA model with random level shift component of Grassi and Santucci de Magistris (2011) and Varneskov and Perron (2011).

Our results suggest that some models and estimators generate positive mean and cumulative returns with different performances before the 2008-2009 financial crisis. In our set up and based on the profits obtained through the different strategies, the choice of the variance estimator seems to be more important than the specification of the time series model. In general, models tend to work better in prediction of low volatility than high volatility. The model and estimator that perform better during the period before the financial crisis resulted in annualized return higher than the $50 \%$.

The rest of the chapter is organized as follows. In section 3.2, we present the volatility estimators and the data. In section 3.3, we perform a statistical descriptive analysis of the volatility series. In section 3.4 , we present the models for the volatility and the forecasting framework. In section 3.5 , we introduce our trading strategies and timing. Section 3.6, introduces the hedging in our strategies. The payoffs are introduced in 3.7. In section 3.8 we describe the estimation results. Finally, section 3.9 and 3.10 discuss the trading results and exhibit the conclusions of the chapter.

### 3.2 The data, volatility estimators and corrections

### 3.2.1 Volatility estimation

Let us consider a price process that follows a continuous sample path semi martingale, at time $t$, the logarithmic price is:

$$
\begin{equation*}
p_{t}=p_{0}+\int_{0}^{t} \mu_{u} d u+\int_{0}^{t} \sigma_{u} d W u \tag{3.1}
\end{equation*}
$$

where $\mu=\left(\mu_{t}\right)_{t \geq 0}$ is the drift (locally bounded and predictable), $\sigma=\left(\sigma_{t}\right)_{t \geq 0}$ is the volatility $\left(\operatorname{cadlag}^{2}\right)$, and $W=\left(W_{t}\right)_{t \geq 0}$ is a standard Brownian motion. The objective is to define a measure of the return variation over a subinterval. This subinterval is assume to be a trading day and it is define between $[0,1]$. The quadratic variation $(Q V)$ is a natural measure of sample path variability for the semi martingales and it is defined by

$$
\begin{equation*}
\langle p\rangle=\operatorname{plim}_{n \rightarrow \infty} \sum_{i=1}^{n}\left(p_{t_{i}}-p_{t_{i-1}}\right)^{2} \tag{3.2}
\end{equation*}
$$

for any partition $0=t_{0}<t_{1}<\ldots<t_{n}=1$, such that $\max _{1 \leq i \leq n}\left\{t_{i}-t_{i-1}\right\} \rightarrow 0$ as $n \rightarrow \infty$.

In this framework, $Q V$ is induced only by the continuous path variation and it is equal to the integrated volatility (IV)

$$
\begin{equation*}
I V=\int_{0}^{1} \sigma_{u}^{2} d u \tag{3.3}
\end{equation*}
$$

$I V$ is our object of interest and it is central in financial economics.
With the increasing availability of high-frequency data, Andersen, Bollerslev, Diebold, and Labys (2001) and Barndorff-Nielsen and Shephard (2002) introduced the realized variance. Considering a series of high frequency prices $p$ recorded at the discrete points $i / n$ for $i=1, \ldots, n$ we define the intraday return

$$
\begin{equation*}
r_{i \Delta, \Delta}=p_{i / n}-p_{(i-1) / n} \tag{3.4}
\end{equation*}
$$

where $\Delta=1 / n$, and the realized variance at sampling frequency $n$ by

$$
\begin{equation*}
R V^{n}=\sum_{i=1}^{n} r_{i \Delta, \Delta}^{2} \tag{3.5}
\end{equation*}
$$

Moreover, they showed that

$$
\begin{equation*}
R V^{n} \rightarrow_{p} I V \tag{3.6}
\end{equation*}
$$

as $n \rightarrow \infty(\Delta \rightarrow 0)$. In addition, Barndorff-Nielsen and Shephard (2002) derived the distribution for $R V^{n}$ in relation to $I V$ as

$$
\begin{equation*}
\sqrt{n}\left(R V^{n}-I V\right) \rightarrow_{d} M N(0,2 I Q) \tag{3.7}
\end{equation*}
$$

where $M N$ is a mixed normal distribution and $I Q$ is the integrated quarticity, a latent variable that can be estimated through the realized quarticity.

The realized range volatility $(R R V)$, was introduced by Martens and van Dijk (2007) and Christensen and Podolskij (2007) and as they state the main idea of the $R R V$ is to reduce the information loss of $R V$ by replacing squared returns with squared ranges. Defining the range by:

$$
\begin{equation*}
s_{p_{i \Delta, \Delta, m}}=\max _{0 \leq s, t \leq m}\left(p_{\frac{i-1}{n}+\frac{t}{N}}-p_{\frac{i-1}{n}+\frac{s}{N}}\right) \tag{3.8}
\end{equation*}
$$

[^16]for $N=n m$ and $i=1, \ldots, n$, then we have
\[

$$
\begin{equation*}
R R V_{n, m}=\frac{1}{\lambda_{2, m}} \sum_{i=1}^{n} s_{p_{i \Delta, \Delta}, m}^{2} \tag{3.9}
\end{equation*}
$$

\]

where $N$ is the number of recorded prices per day, $n$ is the number of interval during the day, $m$ is the number of high frequency prices for each $n$ and $W$ is s standard Brownian Motion and

$$
\begin{equation*}
\lambda_{r, m}=E\left[\left\lvert\, \max _{0 \leq s, t \leq m}\left(W_{\frac{t}{m}}-\left.W_{\frac{s}{m}}^{m}\right|^{r}\right]\right.\right. \tag{3.10}
\end{equation*}
$$

Under certain conditions, Christensen, Podolskij, and Vetter (2009) showed that

$$
\begin{equation*}
R R V_{n, m} \rightarrow_{p} I V \tag{3.11}
\end{equation*}
$$

as $n \rightarrow \infty(\Delta \rightarrow 0)$ and derive the derived the distribution for $R R V_{n, m}$ as

$$
\begin{equation*}
\sqrt{n}\left(R R V_{n, m}-I V\right) \rightarrow_{d} M N(0, \Lambda I Q) \tag{3.12}
\end{equation*}
$$

where $M N$ is a mixed normal distribution and $I Q$ is the integrated quarticity, $\Lambda$ is a decreasing function in $m$ that takes values between 2 and 0.4 (for $m=1$ or $\infty)$. As a consequence, for any $m>1$, the $R R V_{n, m}$ is more efficient than $R V^{n}$.

### 3.2.2 Volatility estimation and the Microstructure noise

In presence of Microstructure noise in the high frequency data, the $R V$ becomes biased and inconsistent and several works have introduced different modifications to mitigate the impact or these errors. Zhang, Mykland, and Ait-Sahalia (2005) proposed a subsampler method and consistent estimator in the presence of noise for the $I V$, where the $R V$ is estimated across non-overlapping grids of returns and averaged. The Two Time Scale Estimator $(T S)$ is defined by

$$
\begin{equation*}
R V^{T S}=\frac{1}{K} \sum_{k=1}^{K} R V^{(k)}-\frac{\bar{n}}{n} R V^{n} \tag{3.13}
\end{equation*}
$$

where $k$ is the number of subgrids, $n$ is the number of returns or observations in the full grid and $n^{k}$ in each subgrid and $R V^{(k)}$ and $\bar{n}$ are

$$
\begin{equation*}
R V^{(k)}=\sum_{i=1}^{n^{k}} r_{i \Delta, \Delta}^{2} \tag{3.14}
\end{equation*}
$$

and

$$
\begin{equation*}
\bar{n}=\frac{1}{K} \sum_{k=1}^{K} n^{(k)}=\frac{n-K+1}{K} \tag{3.15}
\end{equation*}
$$

Moreover, Barndorff-Nielsen, Hansen, Lunde, and Shephard (2008) introduced a kernel-based estimator that incorporates the realized autocovariances and is defined by

$$
\begin{equation*}
R V^{K}=R V^{\Delta}+\sum_{h=1}^{H} k\left(\frac{h-1}{H}\right)\left(\hat{\gamma}_{h}+\hat{\gamma}_{-h}\right) \tag{3.16}
\end{equation*}
$$

where $k(x)$ is a non stochastic weight function, with $x \epsilon[0,1]$ and $\hat{\gamma}_{h}$ is the realized autocovariances.

The Microstructure noise in the price also impacts in the range introducing a bias. Christensen, Podolskij, and Vetter (2009) proposed a robust estimator based on the range for the $I V$ in the presence of errors. The realized range volatility bias corrected $\left(R R V_{n, m}^{B C}\right)$ is given as

$$
\begin{equation*}
R R V_{n, m}^{B C}=\frac{1}{\tilde{\lambda}_{2, m}} \sum_{i=1}^{n}\left(s_{p_{i \Delta, \Delta}, m}-2 \hat{\omega}_{N}\right)^{2} \tag{3.17}
\end{equation*}
$$

where $\omega^{2}$ is the variance of the noise process. Moreover, $\hat{\omega}_{N}^{2}$ is a consistent estimator of this measure, and it is defined as

$$
\begin{equation*}
\hat{\omega}_{N}=\sqrt{\hat{\omega}_{N}^{2}}=\sqrt{\frac{R V^{N}}{2 N}} \longrightarrow \omega \tag{3.18}
\end{equation*}
$$

Finally,

$$
\begin{equation*}
\tilde{\lambda}_{r, m}=E\left[\left.\left.\right|_{t: \eta \frac{t}{m}=\omega, s: \eta \frac{s}{m}=-\omega}\left(W_{\frac{t}{m}}-W_{\frac{s}{m}}\right)\right|^{r}\right] \tag{3.19}
\end{equation*}
$$

where $N$ is the number of recorded prices per day, $n$ is the number of interval during the day, $m$ is the number of high frequency prices for each $n$ and $W$ is a standard Brownian Motion. The authors show that as $m, n \rightarrow \infty$

$$
\begin{equation*}
R R V_{n, m}^{B C} \rightarrow_{p} I V \tag{3.20}
\end{equation*}
$$

and they derive its asymptotic distribution.

### 3.2.3 Volatility estimation and jumps

A generalization of the price process defined in equation 3.1 can include a jump component with the continuous semimartingale component. In this context, The $R V$ and the $R R V$ estimate the total $Q V$, being not informative about $I V$. Formally, if we consider a price process with a continuous sample path and a Jump component, the logarithmic price is:

$$
\begin{equation*}
p_{t}=p_{0}+\int_{0}^{t} \mu_{u} d u+\int_{0}^{t} \sigma_{u} d W u+\sum_{i=1}^{N_{t}} J_{i} \tag{3.21}
\end{equation*}
$$

where $N_{t}$ is the number of jumps and $J=\left(J_{i}\right)_{i=1 \ldots N_{t}}$ is the jump component and the $Q V$ is defined as

$$
\begin{equation*}
\langle p\rangle=\int_{0}^{1} \sigma_{u}^{2} d u+\sum_{i=1}^{N_{t}} J_{i} \tag{3.22}
\end{equation*}
$$

In order to disentangle the continuous from the discontinuous sample path movements in the asset prices, Barndorff-Nielsen and Shephard (2004) presented a robust estimator of the $I V$. The realized bipower variation was defined by

$$
\begin{equation*}
R V^{B V}=\sum_{i=1}^{n-1}\left|r_{i \Delta, \Delta}\right|\left|r_{(i+1) \Delta, \Delta}\right| \tag{3.23}
\end{equation*}
$$

Moreover, Christensen, Podolskij, and Vetter (2009) define the realized rangebased bias corrected bipower variation $\left(R R V^{B V B C}\right)$ as

$$
\begin{equation*}
R R V_{n, m}^{B V B C}=\frac{1}{\tilde{\lambda}_{1, m}^{2}} \sum_{i=1}^{n-1}\left|s_{p_{i \Delta, \Delta}, m}-2 \hat{\omega}_{N}\right|\left|s_{p_{i+1 \Delta, \Delta}, m}-2 \hat{\omega}_{N}\right| \tag{3.24}
\end{equation*}
$$

that is a robust estimator of the IV in the presence of noise and jumps.

### 3.2.4 Volatility data

Our volatility estimators are constructed from high frequency series on the S\&P 500 Futures from January 1996 to October 2009. In order to analyze the performances of estimators that are robust to microstructure noise and jumps, we consider seven realize measures based on the return and the range, previously presented. We consider four realized measures based on the return: the realized volatility $(R V)$, the realized bipower variation ( $R V^{B V}$ ), the Two Time Scale Estimator $\left(R V^{T S}\right)$ and the realized kernel $\left(R V^{K}\right)$. Moreover, we compute daily volatility series with the realized range volatility $\left(R R V_{n, m}\right)$, the corrected version and the realized range-based bias corrected $\left(R R V_{n, m}^{B C}\right)$ and the range-based bipower variation bias corrected ( $R R V_{n, m}^{B V B C}$ ) estimators of Christensen and Podolskij (2007) and Christensen, Podolskij, and Vetter (2009). The seven estimators are computed with two different sampling frequencies 1 and 5 minutes.

Finally, since our interest is to obtain the volatility forecasting for the time to expiration of the traded options or portfolio, we work on a weekly base. In particular, we aggregate volatility from Friday to Thursday because the options are listed each Friday morning and expire one week later. The point is to simplify our forecasting and work only one period ahead.

### 3.2.5 Options data

The Weeklys are options that are listed with only one week to expiration. The first weekly option was launched on October 28, 2005 by the Chicago Board Options Exchange (CBOE) and it was based on the Standard \& Poor's 500 Stock Index (SPX). They have an European exercise style, they may be exercised only on its expiration date, the Friday following the Friday of the listing of the options. The Short-term Standard \& Poor's 500 index options are AM-settled. Our database, obtained from OptionMetrics, consists in daily prices for the S\&P 500 Weekly options from November 2005 to October 2009. In total we have 208 trading periods, one per each week.

### 3.3 Preliminary data analysis

Descriptive statistics of the daily and weekly logarithmic realized series of volatility are displayed in table 3.1 and plots of the logarithmic volatility series and their sample autocorrelation functions (ACF) for some of the estimators are presented in figures 3.1. A statistical analysis of the different estimators for the daily and weekly volatility series confirms the characteristic of the different estimation methods. The mean annualized volatility ranges from $14.2147 \%$ to $16.2589 \%$ for the daily series. This difference is related to the presence of microstructure noise that biased the simple estimator, in particular at the highest sampling frequency or 1 minute. Moreover, the standard deviation is lower for the range estimators, which reflects the more efficiency of the range-based method. The skewness parameter ranges from 0.3967 to 0.8296 and the kurtosis ranges from 3.4161 to 3.9737.

As documented in the literature, the series display volatility clustering and show some periods of high level of volatility associated with different crisis (2000, 2009). The presence of these shifts in the volatility are important since they may cause an upward bias in the estimation of the long memory parameter (see for example, Lu and Perron (2010) and Perron and Qu (2010)). The ACF exhibits the typical hyperbolic decay of the long range series. In particular, the ACF decays to zero questioning the presence of shifts. According to Varneskov and Perron (2011), the autocorrelation function of a long memory process with shift has two components. The first one which is associated with the autocorrelation of the ARFIMA process and the second one which accumulates the shifts. The last component becomes more important when increasing the number of lags, making the convergence of the ACF to be different from zero. This pattern is not observed in the ACF of the series.

After the preliminary analysis of the statistical characteristic of the series that suggests the presence of a dominating long range process and the plots that display possible shifts in the series, we estimate the long memory parameter with alternative semiparametric models and two different bandwidths. Estimation results in table 3.1 show values in the non-stationary region with $d$ decreasing with the largest bandwidth, a result that suggests the presence of level shifts in the series that impose an upward bias.

Last, in order to formally test the presence of structural changes in the level and long memory, we perform the Perron and Qu (2010) test. This test is based on the difference in the estimates of $d$ from different number of frequencies included in the log-periodogram regression. T-statistic for the test are presented in the figure 3.2. In almost all the series, the null hypothesis of long memory cannot be rejected. We think that non-stationary of $d$ in the semiparametric estimation may be caused by the presence of a large shifts associated to the 2008-2009 financial crisis.

### 3.4 The models for the volatility

The volatility forecast plays a central role in our work. In particular, the implementation of the strategies (buy and sell) is based on the expected level of volatility and the estimated confidence interval associated with this prediction. Since we are modeling weekly volatility, we only need a one step ahead forecast. In total, we compute rolling forecasts for one step ahead with 6 different models and 14 volatility series. The resulting 42 forecast series and associated confidence intervals at $90 \%, 92.5 \%, 95 \%, 97.5 \%$ and $99 \%$, define the trading strategies.

### 3.4.1 Long memory process

Given $y_{t}$, a weekly measure of logarithmic volatility, we fit an Autoregressive Fractionally Integrated Moving Average ARFIMA(p,d,q) process introduced by Granger and Joyeux (1980) and Hosking (1981) defined by

$$
\begin{equation*}
\Phi(L)(1-L)^{d} y_{t}=\Theta(L) \epsilon_{t} \tag{3.25}
\end{equation*}
$$

where $(1-L)^{d}$ is a fractional differencing operator, $L$ is the lag operator and $\Phi(L)=1-\phi_{1} L-\ldots-\phi_{p} L^{p}$ and $\Theta(L)=1+\theta_{1} L+\ldots+\theta_{q} L^{q}$ are the autoregressive and moving average operator with no common roots. $\epsilon_{t}$ is a white noise sequence with zero mean and finite variance $\left(\sigma_{\epsilon}^{2}\right)$.

We estimate the ARFIMA(p,d,q) with an autoregressive approximation technique, proposed by Beran (1994). Following Hosking (1981), it is possible to recover a $\mathrm{MA}(\infty)$ and $\operatorname{AR}(\infty)$ expansion of an ARFIMA $(\mathrm{p}, \mathrm{d}, \mathrm{q})$ process as

$$
\begin{equation*}
y_{t}=\sum_{i=1}^{\infty} \psi_{i} \eta_{t-i} \quad \text { and } \quad y_{t}=\sum_{i=1}^{\infty} \pi_{i} y_{t-i}+\eta_{t} \tag{3.26}
\end{equation*}
$$

Given the series $\left\{y_{t}, t>0\right\}$, an approximate innovation sequence $\left\{u_{t}\right\}$ can be obtained as

$$
\begin{equation*}
u_{t}=y_{t}-\sum_{i=1}^{t-1} \pi_{i} y_{t-i} \tag{3.27}
\end{equation*}
$$

for $i=2 \ldots, n$ Finally, a Quasi maximum likelihood estimation for the vector of parameters $\theta$ is obtained by the minimization of

$$
\begin{equation*}
\log \operatorname{Like}\left(y_{t}, \theta\right)=2 n \log \left(\sigma_{\epsilon}\right)+\sum_{t=2}^{n} \frac{u_{t}^{2}}{\sigma_{\epsilon}^{2}} \tag{3.28}
\end{equation*}
$$

One of the advantages of this procedure is that since the $\operatorname{AR}(\infty)$ representation does not required the series to be stationary, the quasi maximum likelihood estimator is defined for any stationary and non stationary fractional processes.

### 3.4.2 Long memory and structural breaks

As early discussed, processes with structural breaks display similar stylized facts than long memory process. As a result, spurious long memory can be caused by the presence of these shifts. In order to jointly consider the presence of long memory and level shifts in the series we introduced the ARFIMA model with random level shift of Grassi and Santucci de Magistris (2011) and Varneskov and Perron (2011). The authors assume a data generating process (DGP) with random level shift component and an ARFIMA process. As Grassi and Santucci de Magistris (2011) point out this procedure provides estimation of the ARFIMA parameters and the probability and the magnitude of the shifts allowing to disentangle the long memory component from the shift process. The ARFIMA model with random level shift is an extension of the univariate model with short memory and jump or level shift component of Lu and Perron (2010). Assume the following GDP

$$
\begin{equation*}
x_{t}=v_{t}+y_{t} \tag{3.29}
\end{equation*}
$$

where $y_{t}$ is an $\operatorname{ARFIMA}(\mathrm{p}, \mathrm{d}, \mathrm{q})$ process with $\operatorname{AR}(\infty)$ representation, given by 3.26, and $v_{t}$ is a random shift component defined by

$$
\begin{equation*}
v_{t}=v_{t-1}+\gamma_{t} \kappa_{t} \tag{3.30}
\end{equation*}
$$

where $\kappa_{t} \sim N\left(0, \sigma_{\kappa}^{2}\right)$ and $\delta_{t}=\gamma_{t} \kappa_{t} . \gamma_{t}$ is a binomial variable equals to 1 when the shift occurs with probability $\alpha$.

In order to estimate the model, the authors recover the state space representation and implement the algorithm propose by Lu and Perron (2010). The model is specified in the first difference of the data

$$
\begin{equation*}
\Delta x_{t}=y_{t}-y_{t-1}+\delta_{t} \tag{3.31}
\end{equation*}
$$

Then, they have the following state space form

$$
\begin{gather*}
\Delta x_{t}=Z Y_{t}+\delta_{t}  \tag{3.32}\\
Y_{t}=T Y_{t-1}+H \epsilon_{t}
\end{gather*}
$$

where $Y_{t}=\left[y_{t}, y_{t-1}, \ldots, y_{t-M}\right]$ and $Z=[1,-1,0, \ldots, 0]$ are $M \times 1$ vectors, $T$ is an $M \times M$ matrix where the first row represents the parameters of the truncated $A R(\infty)$ in $M$ of the ARFIMA process, $H_{t}=[1,0, \ldots, 0]^{\prime}$ is a $M \times 1$ vector and $\epsilon_{t}$ is a vector of innovations with variance covariance matrix $Q$. The likelihood function ${ }^{3}$ is obtained with a Kalman filter on the state space model in 3.32.

[^17]
### 3.5 Option trading

### 3.5.1 The strategy

The aim of this chapter is to analyze the performance of different estimators and models when forecasting volatility adopting an economic and monetary approach. We compare the profits and losses from a buy-and-hold strategy that speculates on the future or expected level of volatility and that is driven by forecasts based on alternative realized estimators and models. We operate in an options exchange market and the trading consists in buying and selling straddles, the investor's portfolio. Bandi, Russell, and Yang (2008) also evaluate and compare the quality several volatility estimators in the context of an economic metric. However, they operate in an artificial option market and obtain the profits from option pricing and trading.

Our strategy takes place every week in two step. We denote by $t$ each week (from Friday to Thursday) and by $t(0)$ and $t(T)$ the beginning and the end of each week $t$. The trading strategy and the timing are the following:

- at $t(0)$ the volatility is estimated and the forecast is obtained for $\hat{V}_{t+1 \mid t}$ (one week ahead) with the associated confidence interval $\left[\widehat{C I}_{t+1 \mid t}^{-\alpha} ; \widehat{C I}_{t+1 \mid t}^{1-\alpha}\right]$ for different models,
- at $t(0)$ the portfolio is formed and the trading (buy or sell the straddle) is implemented based on the confidence interval associated to the volatility forecast,
- at $t(T)$ the options expire and the cost and benefit are realized.

A straddle is a strategy that involves taking a position in a call and a put with the same strike price and expiration date. If high volatility is expected, the investor will buy a straddle or go long, while if low volatility is expected, he will sell a straddle or go short. At $t(0)$, the investor trades contracts that are closest at-the-money. Finally, the investor decides between three different positions: he can go long or buy a straddle, go long or buy and go short or sell or only to go short or sell a straddle during the trading.

### 3.5.2 The determination of high-low expected volatility

A straddle is a combination of options that benefits from the movement of the price of the underlying stock and it does not depend on the direction of movement. In our strategy, we associate large changes in the price with periods of high volatility. Then, an investor who expects high volatility in a given period, which implies a change in the price in any direction, will buy a straddle. At the same time, an investor who expects low volatility will sell or go short in the same derivatives.

In this paper, we want to evaluate from an economic approach, the performances of different estimators and forecast models for volatility. Then, we analyze
the profits which results from the trading of a strategy that speculates on volatility and that is driven by competing models.

The main point is to decide whether a week of low or high volatility is expected. The investor compares his own forecast with the future level of volatility expected by the market (VEM). To obtain this measure, the investor extracts the volatility implied by the option prices. He computes the implied volatility with the Black and Scholes $\left(I V^{B S}\right)$ model ${ }^{4}$ Since the investor is not interested on volatilities which are higher or lower than the market expectation, but which are significantly different, he compares the market expectation with the estimated confidence interval.

Given the forecasted confidence interval $\left[\widehat{C I}_{t+1 \mid t}^{-\alpha} ; \widehat{C I}_{t+1 \mid t}^{1-\alpha}\right]$ for $\alpha=0.01,0.025$, $0.05,0.075$ and 0.10 and the $V E M$ (i.e. the $I V^{B S}$ ), we can define the investor's trading rule based as:

$$
\text { Strat }_{t}^{V E M}= \begin{cases}\text { if } \widehat{C I}_{t+1 \mid t}^{-\alpha}>V E M_{t}+\lambda_{t} & \text { he buys a straddle } \\ \text { if } \widehat{C I}_{t+1 \mid t}^{1-\alpha}<V E M_{t}+\lambda_{t} & \text { he sells a straddle }\end{cases}
$$

where $\lambda_{t}$ is the volatility risk premium.
In order to estimate the volatility risk premium, we exploit the information content of the realized volatility and the implied volatility, regressing the weekly realized estimation of volatility on the Black-Scholes implied volatility of the options.

The relation between realized and implied volatility has been studied in different works. For example in Christensen and Prabhala (1998), Bandi and Perron (2006), Christensen and Nielsen (2006) and Nielsen and Frederiksen (2011). Moreover, different approaches to estimate the volatility risk premium have been introduced (see for example Bollerslev, Gibson, and Zhou (2011) and Garcia, Lewis, Pastorello, and Renault (2011) and the references therein). Bandi and Perron (2006) and Christensen and Nielsen (2006) find that the realized and implied volatility are fractional cointegrated and as a consequence, the estimation of the relation between them with OLS is inconsistent. Following Christensen and Nielsen (2006), a time series p-vector $x_{t}$ is fractional cointegrated if the elements of $x_{t}$ are $I(d)$ and there exists a linear combination that is $I\left(d_{e}\right)$ with $d$ and $d_{e}$ positive and real number and $d>d_{e}$. Robinson (1994) presents a narrow-band least squares (NBLS), a semiparametric method, and proves its consistency in situations where the error term is correlated with the regressors as a result of fractional cointegration, for the $d>1 / 2$ and $d_{e} \geq 0$. Christensen and Nielsen (2006) introduce the limiting distribution for the stationary fractional cointegration case where ( $d>1 / 2, d_{e} \geq 0$ and $d-d_{e}<0$ ). Finally, Nielsen and Frederiksen (2011) consider the case where the regressors and the errors have long memory (including the non-stationary case of $d>1 / 2)$ but the errors have less memory than the regressors $d>d_{e}>0$ and

[^18]$d-d_{e}<1 / 2$. They extend the previous works and present a fully modified NBLS estimator.

To estimate the volatility risk premium $(\lambda)$, we regress RV on $I V^{B S}$ with the fully modified NBLS estimator

$$
\begin{equation*}
R V_{t}=\lambda+\beta I V_{t}^{B S}+\epsilon_{t} \tag{3.33}
\end{equation*}
$$

In particular, we compute a risk premium measure for each realized estimator and two different samples. One for the period before the crisis (October 2005 to September 2008) and another one for the full sample (October 2005 to October 2009). Then, each estimated series of risk premium is used in the trading decision in the previous equation. Estimated volatility risk premiums range from $1 \%$ to $10 \%$ for the alternative estimators of volatility.

### 3.5.3 The payoffs

The benefit and cost of a long position in a straddle, at the end of the week $t$ are defined as

$$
\begin{gather*}
\pi_{t}^{l}=\left|S_{t(T)}-K_{t}\right| \quad \text { and } \quad \operatorname{cost}_{t}^{l}=\left(c_{t}+p_{t}\right)  \tag{3.34}\\
\pi_{t}^{l}-\operatorname{cost}_{t}^{l}=\left|S_{t(T)}-K_{t}\right|-\left(c_{t}+p_{t}\right) \tag{3.35}
\end{gather*}
$$

and the benefit and the cost of a short position in a straddle are equal to

$$
\begin{gather*}
\pi_{t}^{s}=\left(c_{t}+p_{t}\right) \quad \text { and } \quad \operatorname{cost}_{t}^{s}=\left|S_{t(T)}-K_{t}\right|  \tag{3.36}\\
\pi_{t}^{s}-\cos _{t}^{s}=\left(c_{t}+p_{t}\right)-\left|S_{t(T)}-K_{t}\right| \tag{3.37}
\end{gather*}
$$

where, $S_{t(T)}$ is the level of the S\&P 500 at the end of the week or the settlement price, $K_{t}$ is the strike price of the put and the call that expires at the end of the week $t$, and $c_{t}$ and $p_{t}$ are the prices of the call and put options that expire at the end of the week $t$.

### 3.6 Hedging

The delta of a stock is the ratio of the change in the price of the stock option and the change in the price of the underlying stock. In other words, it is the number of units of the stock the investor should hold for each option shorted in order to create a riskless portfolio. The delta of a call is positive, whereas the delta of a put is negative. Based on preliminary results, we decide to introduce a static delta hedging to reduce the high volatility of the returns that result from the different trading decisions.

### 3.6.1 Delta hedging of straddles

Based on the Black and Scholes Model the delta ${ }^{5}$ of a call option can be defined by:

$$
\begin{equation*}
\Delta_{\text {call }}=N\left(d_{1}\right) \tag{3.38}
\end{equation*}
$$

Using a delta hedging for a long position in the call option involves maintaining a short position of $\Delta_{\text {call }}$ shares for each option purchased.

The delta of a put option is given by

$$
\begin{equation*}
\Delta_{p u t}=N\left(d_{1}\right)-1 \tag{3.39}
\end{equation*}
$$

since $\Delta_{p u t}$ is negative, a long position in the put option should be hedged with a long position in the stock.

Our strategy involves taking a position in a straddle, that is a combination of a call and a put option with the same strike and expiration date. In order to hedge our straddle, we need to hedge the call and the put at the same time. Basically, the delta of a portfolio is the sum of the delta of the individual options in the portfolio.

## Hedging a long position in a straddle

To hedge a long position in a straddle we have to hedge a long position in a call and a put. The hedging of a long position in the call, implies opening a short position in the underlying stock equal to $N\left(d_{1}\right)$, that has to be closed at the end of the contract. The hedging of the long position in the put implies opening a long position in the stock equal to $\left(N\left(d_{1}\right)-1\right)$, which also has to be closed.

- at $t(0)$, the investor has to:
- go short in $\Delta_{\text {call }}=N\left(d_{1}\right) \times S_{0}$ (for the call)
- go long in $\Delta_{p u t}=\left(N\left(d_{1}\right)-1\right) \times S_{0}$ (for the put)
- at $t(T)$, the investor has to:
- close the short position in $\Delta_{\text {call }}=N\left(d_{1}\right) \times S_{T}$ (for the call)
- close the long position in $\Delta_{p u t}=\left(N\left(d_{1}\right)-1\right) \times S_{T}$ (for the put)

The benefit and cost of a long position in a straddle, at the end of the week, are equal to

$$
\begin{align*}
& \pi_{t}^{l-h}=\left|S_{t(T)}-K_{t}\right|+N\left(d_{1}\right) \times S_{t(0)}-\left(N\left(d_{1}\right)-1\right) \times S_{t(T)}  \tag{3.40}\\
& \operatorname{cost}_{t}^{l-h}=\left(c_{t}+p_{t}\right)+N\left(d_{1}\right) \times S_{t(T)}-\left(N\left(d_{1}\right)-1\right) \times S_{t(0)} \tag{3.41}
\end{align*}
$$

[^19]\[

$$
\begin{equation*}
\pi_{t}^{l-h}-\operatorname{cost}_{t}^{l-h}=\left|S_{t(T)}-K_{t}\right|-\left(c_{t}+p_{t}\right)+\left(S_{t(T)}-S_{t(0)}\right) \times\left(1-2 \times N\left(d_{1}\right)\right) \tag{3.42}
\end{equation*}
$$

\]

where, $S_{t(0)}$ is the level of the S\&P 500 at the begging of the week used in the hedging, $S_{t(T)}$ is the level of the S\&P 500 at the end of the week or the settlement price, $K_{t}$ is the strike price of the put and the call that expires at the end of the week $t$, and $c_{t}$ and $p_{t}$ are the prices of the call and put options that expire at the end of the week $t$.

## Hedging a short position in a straddle

To hedge a short position in a straddle we have to hedge a short position in a call and a put. The hedging of a short position in the call implies opening a long position in the underlying stock equal to $N\left(d_{1}\right)$. The hedging of the short position in the put implies opening a short position in the stock equal to $\left(N\left(d_{1}\right)-1\right)$. Both positions have to be closed.

- at $t(0)$, the investor has to:
- go long in $\Delta_{\text {call }}=-N\left(d_{1}\right) \times S_{0}$ (for the call)
- go short in $\Delta_{p u t}=-\left(N\left(d_{1}\right)-1\right) \times S_{0}$ (for the put)
- at $t(T)$, the investor has to:
- close the long position in $\Delta_{\text {call }}=N\left(d_{1}\right) \times S_{T}$ (for the call)
- close the short position in $\Delta_{\text {put }}=\left(N\left(d_{1}\right)-1\right) \times S_{T}$ (for the put)

The benefit and the cost of a short position in a straddle are equal to

$$
\begin{gather*}
\pi_{t}^{s-h}=\left(c_{t}+p_{t}\right)+N\left(d_{1}\right) \times S_{t(T)}-\left(N\left(d_{1}\right)-1\right) \times S_{t(0)}  \tag{3.43}\\
\operatorname{cost}_{t}^{s-h}=\left|S_{t(T)}-K_{t}\right|+N\left(d_{1}\right) \times S_{t(0)}-\left(N\left(d_{1}\right)-1\right) \times S_{t(T)}  \tag{3.44}\\
\pi_{t}^{S-h}-\operatorname{cost}_{t}^{S-h}=\left(c_{t}+p_{t}\right)-\left|S_{t(T)}-K_{t}\right|-\left(S_{t(T)}-S_{t(0)}\right) \times\left(1-2 \times N\left(d_{1}\right)\right) \tag{3.45}
\end{gather*}
$$

where, $S_{t(0)}$ is the level of the S\&P 500 at the begging of the week used in the hedging, $S_{t(T)}$ is the level of the S\&P 500 at the end of the week or the settlement price, $K_{t}$ is the strike price of the put and the call that expires at the end of the week $t$, and $c_{t}$ and $p_{t}$ are the prices of the call and put options that expire at the end of the week $t$.

As we said, the investor can introduce a static hedging to cover his position. The hedge will be set up at the begging of each week $(t(0))$ and close at the end $(t(T))$.

### 3.7 The comparison of the profits

In order to compare the performance of the different strategies we compute two different metrics. $R T(\%)$ is the time series mean of the profits minus cost over the exposure of the strategy (expressed in \% terms).

$$
\begin{equation*}
R T(\%)=\frac{1}{t_{f}} \sum_{t=1}^{t f} \frac{\left(\pi_{t}-\cos _{t}\right)}{\operatorname{expo}_{t}} \tag{3.46}
\end{equation*}
$$

The exposure $\left(\right.$ expo $\left._{t}\right)$ can be considered as the initial investment that the investor incurs at the begging of the week (at time $t(0)$ ). In the case of the long position, it is equal to the cost of the portfolio. Whereas in the case of the short position, the exposure is equal to the call and put premiums. We also define the $R T(a b s)$ that is the time series average of the absolute profits minus cost of the strategy.

$$
\begin{equation*}
R T(a b s)=\frac{1}{t_{f}} \sum_{t=1}^{t f}\left(\pi_{t}-\cos _{t}\right) \tag{3.47}
\end{equation*}
$$

The cost was defined in subsection 3.5.3 and depends on the position. We compute the standard deviation (Std) of the mean and mean absolute returns and the Sharpe Ratio $(S R)$, which is define as the return over the standard deviation.

Moreover, in order to replicate a realistic situation, we suppose that the investor has an initial wealth of $V_{t}=\$ 100,000$ and he will be only able to invest up to $20 \%$ of the capital. At each trading week, the investor forms the portfolio. In the case the investor buys a straddle, the $20 \%$ of the capital is used to pay the cost of the strategy. While if the investor sells the straddle, the $20 \%$ represents the maximum exposure in case of a possible loss, that could be unlimited. Audrino and Colangelo (2009) implement the same exercise to analyze the performances of trading strategies with predicted option returns. As they say, the remaining capital is required as initial margin.

### 3.8 Estimation results

Consider the estimation of the ARFIMA models for the weekly series estimated with the $R V^{1 m}$ and $R V^{5 m}$ (see table 3.2). The memory parameter $d$ ranges from 0.6 to 0.7 indicating that the series is non-stationary. In these models we are not taking into consideration the presence of a level shift component and although the preliminary analysis of the series suggests a truly long memory component, a large shift is present at the end of the series related to the 2008-2009 financial crisis. As a consequence, we expect the estimated parameter to be upward biased. The AR and MA estimated coefficients are small and not significant.

Now consider the results of the ARFIMA with shift component models. The estimation of $d$ for the $(0, \mathrm{~d}, 0)$ specification is still in the non stationary region. The probability of the shift is equal to 0.17 , which represent a shift every six week, a very high probability. When introducing the AR and MA components, the estimation of the long memory parameters ranges from 0.32 to 0.46 displaying a
stationary long range component. The AR and MA coefficients are marginally significant. As in the ( $0, \mathrm{~d}, 0$ ) case, the probability of shift is very high and marginally significant. As Varneskov and Perron (2011) pointed out, this is an evidence of the spurious break phenomenon in structural models. When a long memory component dominates the series, the estimation of the probability of the shifts tries to capture the persistent process by overestimating the number of shifts.

The results described above are very similar across alternative realized estimators with non-stationary estimation of $d$ in the ARFIMA models, stationary long memory estimated parameters in the ARFIMA with shift component for $\mathrm{p}=1$ and $\mathrm{q}=0$ and $\mathrm{p}=0$ and $\mathrm{q}=1$. Moreover, the estimated probability of shifts is very high or non significant. These two patterns suggest the presence of a truly long memory process. Last, the non-stationary values of $d$ are not a problem for Beran's procedure and we think that these results are consequence of a large shift present at the end of the series.

After the in-sample analysis and in order to evaluate the performance of the different models, the out-of-sample forecasts are obtained using three different specifications ${ }^{6}$ for the ARFIMA and the ARFIMA with shift component models, they are the $(0, \mathrm{~d}, 0),(1, \mathrm{~d}, 0)$ and $(1, \mathrm{~d}, 0)$.

### 3.9 Trading results

We start analyzing the number of transactions for the three different possible strategies, the buying, selling and the buying and selling, for the different estimators and models and the length of the confidence intervals for the $I V^{B S}$ measure of market volatility expectation. Long positions are not taken by the investor except for two trades for some of the estimators in 208 weeks (less than $1 \%$ ). This result implies that the models are not able to capture periods of high volatility i.e. the lower limit of the forecasted confidence intervals is always smaller than the BlackScholes implied volatility. For the selling strategy positive profits are obtained for the alternative confidence interval. An alternative and natural measure for the market expectation would be the VIX, however, and as we said this model-free implied volatility index is obtained for options with a different time to expiration and it reflects the market beliefs over the next 30 day period.

The number of transactions for the short positions vary across the alternative estimators and between the two different forecasting models. Table 3.3 displays the mean return, standard deviation and the number of trades for the selling strategy for the full trading period and the ARFIMA $(0, \mathrm{~d}, 0)$ and $\operatorname{ARFIMA}(0, \mathrm{~d}, 0)$ with shift models and where the expected level of volatility is compared with the $I V^{B S}$. At the $99 \%$ and for the strategies based on the ARFIMA model the minimum number of transactions is four, which is the same for the ARFIMA with shift specification, in both cases for the $R V^{K, 1 m}$ estimator. The maximum number of trades is very different accross models and it is equal to 15 and 30 respectively. In general, the

[^20]strategies based on the realized range estimators produce more trade than the one associated to returns. Moreover, for the $95 \%$ and $90 \%$ confidence interval, the trades range from 16 to 68 and from 24 to 81 . In 208 weeks, this represents a maximum of one trade every two and a half weeks.

First, to evaluate the performance of the estimators and the time series models, we consider the mean return. At the $99 \%$ interval, almost all the estimators produce positive mean return with weekly means ranging from $-0.18 \%$ to $3.79 \%$ and different standard deviations, which implies an economic difference in their performance. When the confidence intervals become narrower the returns are positive for the ARFIMA models while they become negative for the ARFIMA with shift specification and the standard deviations increase. A thinner confidence interval produces more trades and eventually it becomes erratic, increasing the volatility of the profits. In addition, at the $90 \%$ interval, the standard deviation for the models with shift are higher. The maximum mean return is produced by the $R R V^{B V B C, 5 m}$ estimator and the ARFIMA( $0, \mathrm{~d}, 0$ ) with random level shift model. Table 3.4 shows the results for the ARFIMA (1,d,0) and $\operatorname{ARFIMA}(1, \mathrm{~d}$,$) with$ shifts. The number of trades is higher while the mean returns are similar to the previous models and to the ( $0, \mathrm{~d}, 1$ ) specification, not reported in the table. Since the performance of the different autoregressive and moving average specifications are very similar, we concentrate on the result for the ( $0, \mathrm{~d}, 0$ ) case.

Secondly, considered the high volatility of the weekly returns, we decided to introduce static delta-hedging. A reduction in the return and in the standard deviation is expected as a consequence of the cost of the implementation of the hedge and because of the diminution of the risk associated with price changes. Table 3.5 displays mean absolute returns for the selling strategies (with and without hedge). For the $99 \%$ interval, the reduction in the standard deviation is small, on the contrary, in some cases there is a significant increment in the variability of the profits with respect to the no hedged strategy and the returns become negative.

Thirdly, we present the results of the dynamic investment. As we said, the investor starts with an initial wealth of $\$ 100,000$ and at each trading week he invests up to $20 \%$ of the accumulated capital. The variability of the different strategies is displayed in figure 3.3 with the evolution of the capital. Table 3.6 presents the results for the no hedged strategies. The annualized returns for the different trading rules range from negative to positive values implying a very different performance across estimators and different model specifications. Similar to the mean returns, the estimator and model that produces the highest annualized return is the $R R V^{B V B C, 5 m}$, an estimator based on the range and robust to the presence of noise and jumps in the price process, and the ARFIMA( $0, \mathrm{~d}, 0$ ) with shift model and it is equal to $43,5 \%$. The associated Sharpe Ratio is 1.66 . The last one is also the highest. The performance of estimators based on the returns is also among the more profitable strategies. When we consider narrower confidence intervals, the annualized returns decline and become negative. At the $95 \%$ level, almost all the estimators and models result in negative accumulated returns. Table 3.7 displays the result of the introduction of the delta-hedging (ate the $99 \%$ level). As discussed before, hedged strategies show smaller returns and lowers than the
$10 \%$. Associated SR ratios are also smaller compared to the non hedged strategies. Then, the reduction in the standard deviation is proportionally higher than the reductions in the returns. For some estimators there is a considerable reduction in the number of weekly losses.

An interesting point is that the considered period includes the 2008-2009 financial crisis. During this period and in a few weeks most of the strategies suffer big losses (see figure 3.3). Table 3.8 presents mean returns for the trades carried out during the period before the financial crisis, until September 2008 and for 150 weeks. Mean returns are positive for the different confidence intervals and as we expected considerably higher than for the full sample period. At the $99 \%$ confidence interval, the returns range from $0.41 \%$ to $3.9 \%$ and the number of trades between 4 and 19. Interestingly, when the strategies are based on narrower confidence intervals, mean returns are still positive. At the $92 \%$ confidence interval, $R R V^{B V B C, 5 m}$ produces a $5.01 \%$ weekly return and at the $90 \%$ level, we have weekly returns equal to $4.43 \%, 4.91 \%$ and $4.94 \%$ for three different estimators, the $R V^{B V, 5 m}$, the $R R V^{B V B C, 5 m}$ and the $R V^{B V, 1 m}$ and the forecast obtained with the $\operatorname{ARFIMA}(0, \mathrm{~d}, 0)$ model. The maximum mean return is obtained with almost 1 trade every 3 weeks. The dynamic investment exercise reflects the previous result (see table 3.9). The maximum annualized return is obtained with the $92 \%$ confidence interval. The $R R V^{B V B C, 5 m}$ produces $65 \%$ annualized returns with a Sharpe ratio equal to 1.38 . At the $99 \%$ we obtain higher values for the SR but with smaller returns and equal to $49.8 \%$ for the same estimator and the ARFIMA( $0, \mathrm{~d}, 0$ ) with shift forecasts.

Finally, table 3.10 displays the performance of the strategies based on the $(1, \mathrm{~d}, 0)$ specification for the two models and the $90 \%$ interval. The annualized returns are similar to the previous specification. The introduction of the delta hedge reduces the annualized return but differently to the case of the full trading sample, the Sharpe ratios do not decrease suggesting a reduction in the volatility of the profits. Audrino and Colangelo (2009) construct trading strategies using predicted option returns with one month call and put options. They obtained returns of $56 \%$ with $50 \%$ standard annualized deviation for a period between 2002 and 2006. Similarly, the performance of our strategy produce high and positive annualized return. Despite the annualized returns for the trades carried out before the crisis are higher than the profits for the trades in the full sample period, the estimators that result in highest annualized profit are the same. These estimators are based on the range and corrected for the presence on noise and jumps. The performance of the different models for forecasting seems to be less important than the choice of the estimator.

### 3.10 Conclusions

In this Chapter, we have analyzed the economic performance of different realized estimators of volatility and of alternative time series forecasting models. Speculating on the future level of the volatility, we have defined a buy-and-hold option trading strategy that was implemented with weekly options based on the

S\&P 500 and traded at the Chicago Board Options Exchange (CBOE) for the period between October 2005 and October 2009.

We have constructed volatility series with estimators related to the realized volatility and to the realized range volatility. In particular, the estimators considered are robust to the presence of microstructure noise and jumps in the price process. We have obtained one-step-ahead rolling forecasts with ARFIMA models. Moreover, to account for the possibility of confusing long memory process and structural breaks, we have considered the ARFIMA model with random level shift component, recently introduced in the literature by Varneskov and Perron (2011) and Grassi and Santucci de Magistris (2011).

We find that some estimators for the volatility of the underlying asset and some time series models generate high positive mean and annualized profits. The estimator based on the range and corrected for noise and jumps produces the highest returns, when they are modeled by the ARFIMA model. Based on the returns obtained through the different strategies, the choice of the realized estimator for the volatility seems to be more important than the specification of the model that tends to work better in prediction of low volatility than high volatility, since only our strategy that speculates in lower future volatility is implemented by the investor.

Finally, in order to simulate a possible real situation we implement a dynamic investment exercise, where the investor is allowed to invest only the $20 \%$ of his wealth. Despite the limited profit of our short straddle strategy, the models that better perform produce a mean annual return of $65 \%$, associated to a Sharpe ratio of 1.38 .

Future steps are the introduction of other corrected estimators of the volatility for the presence of microstructure noise and jumps in the price process. Moreover the introduction of non-Normal distribution for the innovation and a GARCH term in the models may result in an improvement of the confidence interval estimation as suggested by Corsi, Mittnik, Pigorsch, and Pigorsch (2008). Last, in order to optimize the trading decision, an maximization of the profit based on the confidence interval and alternative estimators for the volatility risk premium should be consider.

### 3.11 Tables and figures



Figure 3.1: Plot and ACF for $R V^{1 m}$ and $R V^{K, 5 m} R R V^{1 m}$ and $R R V^{B V B C, 5 m}$

Table 3.1: Descriptive statistic and Semiparametric estimation of $d$

|  | Daily |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  | $\mathrm{m}=0.5$ |  | $\mathrm{m}=0.7$ |  |
|  | Mean | St.Dev | Skew | Kurt | $\sigma^{\text {annual }}$ | dgph | $d w$ | dgph | $d w$ |
| $R V^{1 m}$ | -9.398 | 0.918 | 0.667 | 3.772 | 16.258\% | 0.66 | 0.66 | 0.69 | 0.63 |
| $R V^{5 m}$ | -9.519 | 0.961 | 0.520 | 3.621 | 15.436\% | 0.65 | 0.64 | 0.65 | 0.59 |
| $R V^{B V, 1 m}$ | -9.595 | 0.888 | 0.829 | 4.383 | 14.670\% | 0.58 | 0.60 | 0.65 | 0.62 |
| $R V^{B V, 5 m}$ | -9.611 | 0.969 | 0.536 | 3.635 | 14.782\% | 0.65 | 0.62 | 0.66 | 0.59 |
| $R V^{K, 1 m}$ | -9.573 | 0.987 | 0.517 | 3.675 | 15.138\% | 0.63 | 0.63 | 0.64 | 0.57 |
| $R V^{K, 5 m}$ | -9.675 | 1.022 | 0.396 | 3.642 | 14.495\% | 0.64 | 0.64 | 0.60 | 0.55 |
| $R V^{T S, 1 m}$ | -9.484 | 0.952 | 0.528 | 3.681 | 15.674\% | 0.64 | 0.64 | 0.66 | 0.60 |
| $R V^{T S, 5 m}$ | -9.704 | 1.023 | 0.449 | 3.612 | 14.306\% | 0.60 | 0.63 | 0.59 | 0.54 |
| $R R V^{1 m}$ | -9.489 | 0.906 | 0.493 | 3.470 | 15.435\% | 0.66 | 0.65 | 0.69 | 0.63 |
| $R R V^{5 m}$ | -9.439 | 0.907 | 0.600 | 3.759 | 15.861\% | 0.65 | 0.64 | 0.68 | 0.62 |
| $R R V^{B C, 1 m}$ | -9.471 | 0.903 | 0.622 | 3.416 | 15.584\% | 0.68 | 0.68 | 0.65 | 0.63 |
| $R R V^{B C, 5 m}$ | -9.445 | 0.884 | 0.713 | 3.891 | 15.747\% | 0.65 | 0.65 | 0.68 | 0.63 |
| $R R V^{B V B C, 1 m}$ | -9.649 | 0.887 | 0.660 | 3.720 | 14.214\% | 0.62 | 0.63 | 0.62 | 0.61 |
| $R R V^{B V B C, 5 m}$ | -9.529 | 0.880 | 0.740 | 3.973 | 15.093\% | 0.65 | 0.64 | 0.69 | 0.63 |
|  | Weekly |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  | $\mathrm{m}=0.5$ |  | $\mathrm{m}=0.7$ |  |
|  | Mean | St.Dev | Skew | Kurt | $\sigma^{a}$ | dgph | $d w$ | dgph | $d w$ |
| $R V^{1 m}$ | -7.787 | 0.888 | 0.666 | 3.658 | 16.226\% | 0.67 | 0.64 | 0.65 | 0.61 |
| $R V^{5 m}$ | -7.889 | 0.910 | 0.550 | 3.559 | 15.477\% | 0.67 | 0.62 | 0.61 | 0.59 |
| $R V^{B V, 1 m}$ | -7.987 | 0.857 | 0.860 | 4.405 | 14.625\% | 0.67 | 0.61 | 0.57 | 0.59 |
| $R V^{B V, 5 m}$ | -7.980 | 0.915 | 0.570 | 3.617 | 14.818\% | 0.69 | 0.62 | 0.63 | 0.59 |
| $R V^{K, 1 m}$ | -7.927 | 0.919 | 0.596 | 3.644 | $15.236 \%$ | 0.65 | 0.61 | 0.59 | 0.58 |
| $R V^{K, 5 m}$ | -8.003 | 0.919 | 0.564 | 3.733 | 14.666\% | 0.67 | 0.62 | 0.59 | 0.59 |
| $R V^{T S, 1 m}$ | -7.862 | 0.906 | 0.580 | 3.609 | 15.686\% | 0.67 | 0.62 | 0.63 | 0.61 |
| $R V^{T S, 5 m}$ | -8.033 | 0.924 | 0.597 | 3.676 | 14.471\% | 0.64 | 0.61 | 0.57 | 0.57 |
| $R R V^{1 m}$ | -7.888 | 0.879 | 0.497 | 3.399 | 15.350\% | 0.64 | 0.60 | 0.60 | 0.61 |
| $R R V^{5 m}$ | -7.831 | 0.873 | 0.618 | 3.711 | 15.806\% | 0.65 | 0.61 | 0.61 | 0.60 |
| $R R V^{B C, 1 m}$ | -7.858 | 0.860 | 0.654 | 3.430 | 15.538\% | 0.60 | 0.59 | 0.59 | 0.61 |
| $R R V^{B C, 5 m}$ | -7.839 | 0.851 | 0.731 | 3.863 | 15.683\% | 0.64 | 0.61 | 0.61 | 0.61 |
| $R R V^{B V B C, 1 m}$ | -8.030 | 0.837 | 0.710 | 3.790 | 14.199\% | 0.57 | 0.55 | 0.58 | 0.57 |
| $R R V^{B V B C, 5 m}$ | -7.925 | 0.848 | 0.767 | 4.021 | 15.016\% | 0.68 | 0.62 | 0.63 | 0.61 |

Note: Descriptive statistics for the logarithmic daily and weekly volatility series and estimation of the $d$ parameter with alternative semiparametric methods. $\sigma^{\text {annual }}$ is the mean annualized volatility in percentage terms. $d g p h$ is the GPH estimator and $d w$ is the LW estimator. Total number of observations is 721 for the weekly series.



Figure 3.2: Perron and Qu (2010) test
Note: t-statistic for the Perron and Qu (2010) test for spurious long memory as function of $m=T^{a}$ with $1 / 2<a<1 / 3$ and $b=4 / 5$. Total number of observations is 721 for the weekly series.

Table 3.2: Estimation results for the $R V^{1 m}$ and $R V^{5 m}$ weekly series

|  | $R V^{1 m}$ |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | a | b | c | d | e | f | g | h |
| $d$ | $\begin{aligned} & 0.566^{\mathbf{a}} \\ & (0.056) \end{aligned}$ | $\begin{aligned} & 0.443^{\mathrm{a}} \\ & (0.137) \end{aligned}$ | $\begin{aligned} & 0.465^{\mathrm{a}} \\ & (0.101) \end{aligned}$ | $\begin{gathered} 0.324 \\ (0.316) \end{gathered}$ | $\begin{aligned} & 0.656^{\mathrm{a}} \\ & (0.031) \end{aligned}$ | $\begin{aligned} & 0.600^{\mathrm{a}} \\ & (0.052) \end{aligned}$ | $\begin{aligned} & 0.605^{\mathrm{a}} \\ & (0.046) \end{aligned}$ | $\begin{aligned} & 0.710^{\mathbf{a}} \\ & (0.070) \end{aligned}$ |
| $\phi$ | - | $\begin{gathered} 0.138 \\ (0.128) \end{gathered}$ | - | $\begin{gathered} 0.487 \\ (0.415) \end{gathered}$ | - | $\begin{gathered} 0.085 \\ (0.066) \end{gathered}$ | - | $\begin{aligned} & 0.887^{\mathrm{a}} \\ & (0.117) \end{aligned}$ |
| $\theta$ | ${ }^{-}$ | - | $\begin{gathered} 0.118 \\ (0.089) \end{gathered}$ | $\begin{aligned} & -0.239 \\ & (0.246) \end{aligned}$ | ${ }^{-}$ | - | $\begin{gathered} 0.080 \\ (0.057) \end{gathered}$ | $\begin{gathered} -0.921^{\mathbf{a}} \\ (0.082) \end{gathered}$ |
| $\sigma_{\epsilon}^{2}$ | $\begin{aligned} & 0.133^{\mathrm{a}} \\ & (0.013) \end{aligned}$ | $\begin{aligned} & 0.129^{\mathbf{a}} \\ & (0.014) \end{aligned}$ | $\begin{aligned} & 0.130^{a} \\ & (0.014) \end{aligned}$ | $\begin{aligned} & 0.128^{a} \\ & (0.015) \end{aligned}$ | $\begin{aligned} & 0.191^{\mathrm{a}} \\ & (0.010) \end{aligned}$ | $\begin{aligned} & 0.190^{\mathrm{a}} \\ & (0.010) \end{aligned}$ | $\begin{aligned} & 0.190^{\mathbf{a}} \\ & (0.010) \end{aligned}$ | $\begin{aligned} & 0.190^{a} \\ & (0.010) \end{aligned}$ |
| $\alpha$ | $\begin{aligned} & 0.176^{\mathbf{b}} \\ & (0.070) \end{aligned}$ | $\begin{aligned} & 0.210^{\text {b }} \\ & (0.087) \end{aligned}$ | $\begin{aligned} & 0.207^{\mathbf{b}} \\ & (0.084) \end{aligned}$ | $\begin{aligned} & 0.219^{\text {b }} \\ & (0.093) \end{aligned}$ | - | - | - | - |
| $\sigma_{\eta}^{2}$ | $\begin{aligned} & 0.305^{a} \\ & (0.110) \end{aligned}$ | $\begin{aligned} & 0.266^{\text {a }} \\ & (0.097) \end{aligned}$ | $\begin{aligned} & 0.267^{\mathrm{a}} \\ & (0.097) \end{aligned}$ | $\begin{aligned} & 0.259^{\mathrm{a}} \\ & (0.094) \end{aligned}$ | - | ${ }^{-}$ | - | - |
| loglik | -405.02 | -404.22 | -404.20 | -404.15 | -426.63 | -425.74 | -425.73 | -426.03 |
|  |  |  |  |  |  |  |  |  |
|  | a | b | c | d | e | f | g | h |
| $d$ | $\begin{aligned} & 0.563^{\mathrm{a}} \\ & (0.057) \end{aligned}$ | $\begin{gathered} 0.420^{\mathbf{a}} \\ (0.124) \end{gathered}$ | $\begin{aligned} & 0.439^{a} \\ & (0.094) \end{aligned}$ | $\begin{aligned} & 0.664^{\mathrm{a}} \\ & (0.100) \end{aligned}$ | $\begin{aligned} & 0.632^{\mathrm{a}} \\ & (0.031) \end{aligned}$ | $\begin{aligned} & 0.571^{\mathrm{a}} \\ & (0.052) \end{aligned}$ | $\begin{aligned} & 0.575^{\mathrm{a}} \\ & (0.045) \end{aligned}$ | $\begin{aligned} & 0.580^{\mathrm{a}} \\ & (0.065) \end{aligned}$ |
| $\phi$ | - | $\begin{gathered} 0.164 \\ (0.117) \end{gathered}$ | - | $\begin{aligned} & 0.874^{\mathrm{a}} \\ & (0.112) \end{aligned}$ | - | $\begin{gathered} 0.093 \\ (0.065) \end{gathered}$ | - | $\begin{aligned} & -0.064 \\ & (0.883) \end{aligned}$ |
| $\theta$ | ${ }^{-}$ | - | $\begin{aligned} & 0.148^{\text {c }} \\ & (0.082) \end{aligned}$ | $\begin{gathered} -0.963^{a} \\ (0.044) \end{gathered}$ | ${ }^{-}$ | - | $\begin{gathered} 0.090 \\ (0.057) \end{gathered}$ | $\begin{gathered} 0.150 \\ (0.837) \end{gathered}$ |
| $\sigma_{\epsilon}^{2}$ | $\begin{aligned} & 0.172^{\mathbf{a}} \\ & (0.018) \end{aligned}$ | $\begin{aligned} & 0.169^{\text {a }} \\ & (0.020) \end{aligned}$ | $\begin{aligned} & 0.169^{\mathrm{a}} \\ & (0.020) \end{aligned}$ | $\begin{aligned} & 0.168^{\mathrm{a}} \\ & (0.021) \end{aligned}$ | $\begin{aligned} & 0.222^{\mathrm{a}} \\ & (0.011) \end{aligned}$ | $\begin{aligned} & 0.221^{\mathrm{a}} \\ & (0.011) \end{aligned}$ | $\begin{aligned} & 0.221^{\mathrm{a}} \\ & (0.011) \end{aligned}$ | $\begin{aligned} & 0.221^{\mathrm{a}} \\ & (0.011) \end{aligned}$ |
| $\alpha$ | $\begin{aligned} & 0.176^{\mathrm{c}} \\ & (0.099) \end{aligned}$ | $\begin{aligned} & 0.231^{\mathrm{c}} \\ & (0.119) \end{aligned}$ | $\begin{aligned} & 0.231^{\text {c }} \\ & (0.118) \end{aligned}$ | $\begin{aligned} & 0.242^{\mathrm{b}} \\ & (0.119) \end{aligned}$ | ( |  | (0.010 | - |
| $\sigma_{\eta}^{2}$ | $\begin{aligned} & 0.248^{\mathrm{c}} \\ & (0.128) \end{aligned}$ | $\begin{aligned} & 0.197^{\mathrm{c}} \\ & (0.100) \end{aligned}$ | $\begin{aligned} & 0.197^{\mathrm{b}} \\ & (0.099) \end{aligned}$ | $\begin{aligned} & 0.191^{\text {c }} \\ & (0.097) \end{aligned}$ | ${ }^{-}$ | - | - | - |
| loglik | -459.22 | -457.90 | -457.77 | -457.58 | -480.91 | -479.84 | -479.79 | -479.76 |

Note: Estimation results for the ARFIMA(p,d,q) with random level shift model (column a-d) and ARFIMA(p,d,q) model (column e-h) for the weekly logarithmic series of volatility. "a", "b" and "c" indicate significance at the $1 \%, 5 \%$ and $10 \%$. 721 observations

Table 3.3: Mean returns and standard deviation I - Selling strat. - Full sample No hedge

| Est. | Model | RT(\%) | $\begin{gathered} 99 \\ \text { Std } \end{gathered}$ | Sell | RT(\%) | $\begin{gathered} 95 \\ \text { Std } \end{gathered}$ | Sell | RT(\%) | $\begin{gathered} 90 \\ \text { Std } \end{gathered}$ | Sell |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $R V^{1 m}$ | (0,d,0) | 2.78\% | 0.155 | 11 | 0.14\% | 0.296 | 34 | -1.43\% | 0.347 | 51 |
| $R V^{5 m}$ | (0,d, 0 ) | 2.26\% | 0.154 | 11 | -0.79\% | 0.330 | 46 | 0.91\% | 0.375 | 64 |
| $R V^{B V, 1 m}$ | (0,d, 0 ) | 2.60\% | 0.166 | 16 | 0.56\% | 0.318 | 46 | 3.06\% | 0.393 | 74 |
| $R V^{B V, 5 m}$ | (0,d,0) | 2.81\% | 0.271 | 24 | 0.40\% | 0.372 | 60 | 2.78\% | 0.417 | 87 |
| $R V^{K, 1 m}$ | (0,d,0) | 0.72\% | 0.218 | 9 | 0.45\% | 0.288 | 35 | 0.84\% | 0.372 | 63 |
| $R V^{K, 5 m}$ | (0,d,0) | 0.58\% | 0.214 | 9 | -0.56\% | 0.331 | 41 | 1.68\% | 0.384 | 70 |
| $R V^{T S, 1 m}$ | (0,d,0) | 1.38\% | 0.233 | 13 | 0.38\% | 0.303 | 44 | 0.66\% | 0.378 | 64 |
| $R V^{T S, 5 m}$ | (0,d,0) | 0.29\% | 0.214 | 10 | -0.78\% | 0.327 | 40 | 1.94\% | 0.382 | 69 |
| $R R V^{1 m}$ | (0,d,0) | -0.38\% | 0.306 | 35 | 1.97\% | 0.384 | 74 | 2.51\% | 0.401 | 85 |
| $R R V^{5 m}$ | (0,d,0) | 2.66\% | 0.167 | 14 | -0.68\% | 0.330 | 45 | 2.53\% | 0.373 | 66 |
| $R R V^{B C, 1 m}$ | (0,d,0) | -0.60\% | 0.267 | 19 | 0.18\% | 0.354 | 54 | 0.89\% | 0.382 | 73 |
| $R R V^{B C, 5 m}$ | (0,d, 0 ) | 2.19\% | 0.151 | 11 | 0.37\% | 0.313 | 42 | 1.17\% | 0.361 | 57 |
| $R R V^{B V B C, 1 m}$ | (0,d,0) | -0.87\% | 0.288 | 35 | 1.47\% | 0.389 | 78 | 0.80\% | 0.403 | 85 |
| $R R V^{B V B C, 5 m}$ | (0,d,0) | 2.41\% | 0.261 | 22 | -0.18\% | 0.363 | 56 | $3.37 \%$ | 0.402 | 83 |
| $R V^{1 m}$ | (0,d, 0 ) s | 1.48\% | 0.126 | 7 | 2.20\% | 0.204 | 28 | -1.41\% | 0.343 | 47 |
| $R V^{5 m}$ | (0,d, 0 ) s | 1.92\% | 0.147 | 10 | 1.51\% | 0.233 | 34 | -1.10\% | 0.354 | 55 |
| $R V^{B V, 1 m}$ | (0,d, 0 ) s | 2.39\% | 0.154 | 12 | 0.02\% | 0.310 | 40 | 1.56\% | 0.379 | 61 |
| $R V^{B V, 5 m}$ | (0,d, 0 ) s | 2.62\% | 0.180 | 17 | -0.12\% | 0.364 | 55 | 0.30\% | 0.395 | 73 |
| $R V^{K, 1 m}$ | (0,d, 0 ) s | 0.35\% | 0.083 | 4 | 0.79\% | 0.273 | 26 | -1.39\% | 0.337 | 47 |
| $R V^{K, 5 m}$ | $(0, d, 0) \mathrm{s}$ | -0.18\% | 0.198 | 7 | -0.34\% | 0.300 | 30 | -0.00\% | 0.352 | 53 |
| $R V^{T S, 1 m}$ | ( $0, \mathrm{~d}, 0$ ) s | 1.92\% | 0.147 | 10 | 0.72\% | 0.284 | 37 | -1.76\% | 0.346 | 51 |
| $R V^{T S, 5 m}$ | (0,d, 0 ) s | 1.18\% | 0.113 | 7 | -0.29\% | 0.299 | 30 | -0.18\% | 0.353 | 54 |
| $R R V^{1 m}$ | ( $0, \mathrm{~d}, 0$ ) s | 1.40\% | 0.230 | 30 | 0.45\% | 0.354 | 61 | 2.14\% | 0.391 | 79 |
| $R R V^{5 m}$ | $(0, d, 0) \mathrm{s}$ | 1.86\% | 0.144 | 10 | 1.20\% | 0.252 | 36 | -0.42\% | 0.346 | 51 |
| $R R V^{B C, 1 m}$ | (0,d, 0 ) s | 1.45\% | 0.184 | 17 | -0.29\% | 0.316 | 47 | -0.16\% | 0.374 | 68 |
| $R R V^{B C, 5 m}$ | (0,d, 0 ) s | 1.12\% | 0.123 | 8 | -0.25\% | 0.309 | 38 | -0.71\% | 0.344 | 49 |
| $R R V^{B V B C, 1 m}$ | (0,d, 0 ) s | 0.56\% | 0.226 | 30 | 0.89\% | 0.372 | 68 | 1.02\% | 0.399 | 81 |
| $R R V^{B V B C, 5 m}$ | (0,d, 0 ) s | 3.79\% | 0.181 | 19 | -0.96\% | 0.354 | 51 | 1.82\% | 0.381 | 72 |

Note: Mean returns (weekly) and standard deviation for the $I V^{B S}$ and different confidence intervals. sell is the number of weeks with short positions. We have in total 208 weeks of trading. First part of the table corresponds to the ARFIMA(p,d,q) model while the second part to the ARFIMA( $\mathrm{p}, \mathrm{d}, \mathrm{q}$ ) with level shift component.

Table 3.4: Mean returns and standard deviation II - Selling strat. - Full sample No hedge

| Est. | Model | RT(\%) | $\begin{gathered} 99 \\ \text { Std } \end{gathered}$ | Sell | RT(\%) | $\begin{gathered} 95 \\ \text { Std } \end{gathered}$ | Sell | RT(\%) | $90$ | Sell |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $R V^{1 m}$ | (1,d,0) | 2.78\% | 0.155 | 11 | 0.13\% | 0.300 | 36 | $-0.54 \%$ | 0.355 | 54 |
| $R V^{5 m}$ | (1,d,0) | 2.26\% | 0.154 | 11 | -0.18\% | 0.313 | 46 | 0.79\% | 0.376 | 66 |
| $R V^{B V, 1 m}$ | (1,d,0) | 2.35\% | 0.193 | 18 | $-0.24 \%$ | 0.337 | 48 | 2.67\% | 0.396 | 76 |
| $R V^{B V, 5 m}$ | (1,d,0) | 2.19\% | 0.278 | 24 | 1.31\% | 0.382 | 66 | 2.90\% | 0.417 | 88 |
| $R V^{K, 1 m}$ | (1,d,0) | 0.72\% | 0.218 | 9 | 1.06\% | 0.292 | 38 | 0.78\% | 0.383 | 67 |
| $R V^{K, 5 m}$ | (1,d,0) | 0.14\% | 0.204 | 8 | -0.20\% | 0.333 | 44 | 1.75\% | 0.385 | 70 |
| $R V^{T S, 1 m}$ | (1,d,0) | 1.38\% | 0.233 | 13 | 0.39\% | 0.307 | 45 | 0.47\% | 0.387 | 68 |
| $R V^{T S, 5 m}$ | (1,d,0) | 0.52\% | 0.211 | 9 | -0.27\% | 0.330 | 43 | 1.43\% | 0.390 | 72 |
| $R R V^{1 m}$ | (1,d,0) | -0.76\% | 0.308 | 36 | 1.82\% | 0.386 | 74 | 2.40\% | 0.400 | 85 |
| $R R V^{5 m}$ | (1,d,0) | 3.01\% | 0.168 | 14 | $-1.19 \%$ | 0.337 | 48 | 2.02\% | 0.373 | 67 |
| $R R V^{B C, 1 m}$ | (1,d,0) | -0.68\% | 0.267 | 20 | 0.18\% | 0.356 | 57 | 1.02\% | 0.386 | 75 |
| $R R V^{B C, 5 m}$ | (1,d,0) | 2.19\% | 0.151 | 11 | -0.11\% | 0.315 | 45 | 0.86\% | 0.364 | 61 |
| $R R V^{B V B C, 1 m}$ | (1,d,0) | -1.44\% | 0.299 | 36 | 0.74\% | 0.381 | 73 | 1.21\% | 0.412 | 88 |
| $R R V^{B V B C, 5 m}$ | (1,d,0) | 2.22\% | 0.278 | 26 | 0.46\% | 0.371 | 60 | $3.34 \%$ | 0.410 | 86 |
| $R V^{1 m}$ | (1,d,0)s | 1.48\% | 0.126 | 7 | 2.38\% | 0.202 | 27 | -0.73\% | 0.328 | 46 |
| $R V^{5 m}$ | (1,d,0)s | 1.92\% | 0.147 | 10 | 1.47\% | 0.233 | 33 | -1.16\% | 0.353 | 55 |
| $R V^{B V, 1 m}$ | (1,d,0)s | 2.39\% | 0.154 | 12 | -0.17\% | 0.314 | 40 | 1.30\% | 0.374 | 62 |
| $R V^{B V, 5 m}$ | (1,d,0)s | 2.38\% | 0.184 | 18 | 0.51\% | 0.348 | 55 | 0.42\% | 0.393 | 71 |
| $R V^{K, 1 m}$ | (1,d,0)s | 0.35\% | 0.083 | 4 | 0.34\% | 0.281 | 27 | -1.59\% | 0.335 | 46 |
| $R V^{K, 5 m}$ | (1,d,0)s | -0.18\% | 0.198 | 8 | -0.34\% | 0.300 | 30 | -0.00\% | 0.352 | 53 |
| $R V^{T S, 1 m}$ | (1,d,0)s | 1.83\% | 0.136 | 8 | 0.37\% | 0.291 | 36 | -1.99\% | 0.344 | 50 |
| $R V^{T S, 5 m}$ | (1,d,0)s | 0.02\% | 0.203 | 8 | $-0.47 \%$ | 0.298 | 29 | -0.54\% | 0.347 | 54 |
| $R R V^{1 m}$ | (1,d,0)s | 1.06\% | 0.227 | 28 | 0.51\% | 0.354 | 62 | 2.29\% | 0.392 | 80 |
| $R R V^{5 m}$ | (1,d,0)s | 1.86\% | 0.144 | 10 | -0.23\% | 0.311 | 39 | -0.51\% | 0.345 | 51 |
| $R R V^{B C, 1 m}$ | (1,d,0) s | 1.16\% | 0.179 | 15 | $-0.28 \%$ | 0.316 | 47 | -0.16\% | 0.374 | 68 |
| $R R V^{B C, 5 m}$ | (1,d,0)s | 1.12\% | 0.123 | 8 | -0.25\% | 0.309 | 37 | -0.94\% | 0.339 | 49 |
| $R R V^{B V B C, 1 m}$ | (1,d,0)s | -1.40\% | 0.295 | 32 | 0.86\% | 0.374 | 70 | 1.27\% | 0.400 | 83 |
| $R R V^{B V B C, 5 m}$ | (1,d,0)s | 3.08\% | 0.198 | 21 | -0.81\% | 0.355 | 53 | 2.44\% | 0.387 | 74 |

Note: Mean returns (weekly) and standard deviation for the $I V^{B S}$ and different confidence intervals. sell is the number of weeks with short positions. We have in total 208 weeks of trading. First part of the table corresponds to the ARFIMA(p,d,q) model while the second part to the ARFIMA( $\mathrm{p}, \mathrm{d}, \mathrm{q}$ ) with level shift component.

Table 3.5: Mean absolute returns and standard deviation - Selling strat. - Full sample - No hedge vs. Hedge

| Est. | Model | 99 |  |  |  | 95 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | No hedge |  | Hedge |  | No hedge |  | Hedge |  |
|  |  | RT(abs) | Std | RT(abs) | Std | RT(abs) | Std | RT(abs) | Std |
| $R V^{1 m}$ | (0,d,0) | 0.984 | 5.734 | 0.971 | 5.678 | -0.194 | 13.01 | -0.518 | 17.81 |
| $R V^{5 m}$ | (0,d,0) | 0.838 | 5.702 | 0.825 | 5.623 | -0.426 | 13.66 | -0.783 | 18.35 |
| $R V^{B V, 1 m}$ | (0,d,0) | 0.966 | 5.966 | 0.938 | 5.761 | -0.072 | 13.44 | -0.512 | 18.28 |
| $R V^{B V, 5 m}$ | (0,d,0) | 0.606 | 12.46 | -0.099 | 18.16 | -0.093 | 14.52 | -0.670 | 19.37 |
| $R V^{K, 1 m}$ | (0,d,0) | -0.005 | 11.46 | -0.473 | 17.59 | -0.166 | 13.14 | -0.443 | 18.30 |
| $R V^{K, 5 m}$ | (0,d,0) | -0.064 | 11.40 | -0.569 | 18.29 | -0.375 | 13.72 | -0.730 | 19.42 |
| $R V^{T S, 1 m}$ | (0,d,0) | 0.211 | 11.83 | -0.242 | 17.66 | -0.145 | 13.33 | -0.559 | 18.41 |
| $R V^{T S, 5 m}$ | (0,d,0) | -0.181 | 11.33 | -0.663 | 17.99 | -0.462 | 13.62 | -0.742 | 19.07 |
| $R R V^{1 m}$ | (0,d,0) | -0.539 | 13.11 | -0.663 | 18.09 | 0.375 | 15.35 | -0.023 | 19.93 |
| $R R V^{5 m}$ | (0,d,0) | 0.826 | 5.427 | 0.822 | 5.319 | -0.434 | 13.64 | -0.735 | 18.21 |
| $R R V^{B C, 1 m}$ | (0,d, 0 ) | -0.479 | 12.38 | -0.923 | 17.56 | -0.365 | 14.67 | -0.602 | 19.06 |
| $R R V^{B C, 5 m}$ | (0,d,0) | 0.750 | 5.292 | 0.743 | 5.229 | -0.190 | 13.43 | -0.500 | 18.02 |
| $R R V^{B V B C, 1 m}$ | (0,d,0) | -0.638 | 12.73 | -0.913 | 18.13 | 0.019 | 15.43 | -0.440 | 20.14 |
| $R R V^{B V B C, 5 m}$ | (0,d, 0 ) | 0.463 | 12.25 | -0.055 | 17.67 | -0.256 | 14.37 | -0.775 | 19.01 |
| $R V^{1 m}$ | (0,d, 0 ) s | 0.376 | 3.343 | 0.355 | 3.163 | 0.800 | 6.692 | 0.946 | 6.283 |
| $R V^{5 m}$ | (0,d, 0 ) s | 0.593 | 4.512 | 0.577 | 4.373 | 0.483 | 8.098 | 0.643 | 7.146 |
| $R V^{B V, 1 m}$ | (0,d, 0 ) s | 0.713 | 4.684 | 0.682 | 4.536 | -0.249 | 13.26 | -0.554 | 17.78 |
| $R V^{B V, 5 m}$ | $(0, d, 0) \mathrm{s}$ | 0.839 | 5.509 | 0.759 | 5.286 | -0.247 | 14.32 | -0.738 | 18.84 |
| $R V^{K, 1 m}$ | (0,d, 0 ) s | 0.140 | 2.483 | 0.162 | 2.586 | -0.044 | 12.53 | -0.264 | 17.37 |
| $R V^{K, 5 m}$ | (0,d, 0 ) s | -0.447 | 10.65 | -0.903 | 17.39 | -0.412 | 13.20 | -0.679 | 18.66 |
| $R V^{T S, 1 m}$ | (0,d, 0 ) s | 0.593 | 4.512 | 0.574 | 4.365 | 0.014 | 12.76 | -0.254 | 17.44 |
| $R V^{T S, 5 m}$ | (0, $\mathrm{d}, 0) \mathrm{s}$ | 0.466 | 4.495 | 0.491 | 4.568 | -0.381 | 13.18 | -0.594 | 18.40 |
| $R R V^{1 m}$ | (0,d, 0 ) s | 0.506 | 6.773 | 0.658 | 6.439 | -0.180 | 13.96 | -0.534 | 18.38 |
| $R R V^{5 m}$ | (0,d, 0 ) s | 0.505 | 3.975 | 0.493 | 3.837 | 0.483 | 7.866 | 0.651 | 7.306 |
| $R R V^{B C, 1 m}$ | (0,d, 0 ) s | 0.561 | 5.703 | 0.553 | 5.695 | -0.552 | 13.41 | -0.751 | 17.72 |
| $R R V^{B C, 5 m}$ | (0, d, 0 ) s | 0.283 | 3.241 | 0.289 | 3.228 | -0.420 | 13.32 | -0.578 | 17.50 |
| $R R V^{B V B C, 1 m}$ | (0,d, 0 ) s | 0.204 | 7.214 | 0.385 | 6.563 | -0.067 | 14.66 | -0.438 | 19.25 |
| $R R V^{B V B C, 5 m}$ | (0,d, 0 ) s | 1.254 | 6.184 | 1.148 | 5.923 | -0.505 | 14.15 | -0.930 | 18.55 |

Note: Mean absolute returns (weekly) and standard deviation for the $I V^{B S}$ and different confidence intervals. sell is the number of weeks with short positions. We have in total 208 weeks of trading. First part of the table corresponds to the ARFIMA(p,d,q) model while the second part to the ARFIMA( $\mathrm{p}, \mathrm{d}, \mathrm{q}$ ) with level shift component.


Figure 3.3: Dynamic investment - Selling strat. - Full sample

Table 3.6: Dynamic investment - Selling strat. - Full sample - No hedge

| Est. | Model | 99 |  | 97 |  | 95 |  | 92 |  | 90 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | An. ret. | SR | An. ret. | SR | An. ret. | SR | An. ret. | SR | An. ret. | SR |
| $R V^{1 m}$ | (0,d,0) | 30.3\% | 1.35 | -0.90\% | -0.02 | -8.90\% | -0.20 | -18.2\% | -0.39 | -25.5\% | $-0.51$ |
| $R V^{5 m}$ | (0,d,0) | 23.5\% | 1.05 | 4.61\% | 0.11 | -19.4\% | -0.40 | -14.6\% | -0.28 | -6.96\% | -0.12 |
| $R V^{B V, 1 m}$ | (0,d,0) | 27.4\% | 1.14 | -1.15\% | -0.02 | -6.18\% | -0.13 | -13.4\% | -0.25 | 14.7\% | 0.26 |
| $R V^{B V, 5 m}$ | (0,d,0) | 22.5\% | 0.57 | 2.76\% | 0.06 | -11.4\% | -0.21 | -7.90\% | -0.13 | 9.25\% | 0.15 |
| $R V^{K, 1 m}$ | (0,d,0) | 1.26\% | 0.04 | 7.22\% | 0.19 | -5.32\% | -0.12 | -11.7\% | -0.22 | -7.45\% | -0.13 |
| $R V^{K, 5 m}$ | (0,d,0) | -0.05\% | -0.00 | -5.03\% | -0.12 | -17.6\% | -0.36 | -5.50\% | -0.10 | 0.16\% | 0.00 |
| $R V^{T S, 1 m}$ | (0,d,0) | 7.74\% | 0.23 | 1.81\% | 0.04 | -6.91\% | -0.15 | -27.4\% | -0.55 | -9.49\% | -0.17 |
| $R V^{T S, 5 m}$ | (0,d,0) | -3.07\% | -0.09 | -4.79\% | -0.11 | -19.2\% | -0.40 | -12.5\% | -0.24 | 2.96\% | 0.05 |
| $R R V^{1 m}$ | (0,d,0) | -14.4\% | -0.32 | -19.9\% | -0.40 | 3.15\% | 0.05 | 7.18\% | 0.12 | 7.69\% | 0.13 |
| $R R V^{5 m}$ | (0,d,0) | 28.3\% | 1.17 | 1.22\% | 0.03 | -18.5\% | -0.38 | -12.4\% | -0.24 | 10.4\% | 0.19 |
| $R R V^{B C, 1 m}$ | (0,d,0) | -14.4\% | $-0.37$ | -20.6\% | -0.49 | -12.3\% | -0.24 | -14.6\% | -0.27 | -7.70\% | -0.13 |
| $R R V^{B C, 5 m}$ | (0,d,0) | 22.8\% | 1.04 | 4.11\% | 0.10 | -7.74\% | -0.17 | -17.1\% | -0.34 | -3.28\% | -0.06 |
| $R R V^{B V B C, 1 m}$ | (0,d,0) | -17.7\% | $-0.42$ | -10.4\% | -0.19 | $-2.45 \%$ | -0.04 | -1.97\% | -0.03 | -10.0\% | -0.17 |
| $R R V^{B V B C, 5 m}$ | (0,d,0) | 18.0\% | 0.47 | -4.58\% | -0.10 | -16.2\% | -0.31 | 18.7\% | 0.32 | 17.7\% | 0.30 |
| $R V^{1 m}$ | (0,d, 0 ) s | 14.8\% | 0.81 | 21.3\% | 0.93 | 20.5\% | 0.69 | -10.1\% | -0.23 | -25.2\% | -0.51 |
| $R V^{5 m}$ | (0,d, 0 ) s | 19.5\% | 0.92 | 40.2\% | 1.53 | 10.5\% | 0.31 | -23.9\% | -0.48 | -23.4\% | -0.45 |
| $R V^{B V, 1 m}$ | (0,d, 0 ) s | 25.2\% | 1.13 | 29.4\% | 1.01 | -10.9\% | -0.24 | -20.2\% | -0.39 | -0.66\% | -0.01 |
| $R V^{B V, 5 m}$ | (0,d, 0 ) s | 27.0\% | 1.03 | 1.58\% | 0.03 | -15.7\% | -0.30 | -13.1\% | -0.23 | -14.1\% | -0.24 |
| $R V^{K, 1 m}$ | (0,d, 0 ) s | 3.00\% | 0.25 | 29.2\% | 1.19 | -0.96\% | -0.02 | -16.4\% | -0.36 | -24.8\% | $-0.51$ |
| $R V^{K, 5 m}$ | (0,d, 0 ) s | -7.26\% | -0.25 | 14.1\% | 0.39 | -13.7\% | -0.31 | -20.4\% | -0.41 | -13.9\% | -0.27 |
| $R V^{T S, 1 m}$ | (0,d, 0 ) s | 19.5\% | 0.92 | 26.3\% | 1.04 | -2.31\% | -0.05 | -9.99\% | -0.22 | -28.0\% | $-0.56$ |
| $R V^{T S, 5 m}$ | (0,d, 0 ) s | 11.6\% | 0.70 | 2.82\% | 0.07 | -13.1\% | -0.30 | -24.2\% | -0.49 | -15.6\% | -0.30 |
| $R R V^{1 m}$ | (0,d, 0 ) s | 9.24\% | 0.27 | -25.0\% | -0.52 | -9.88\% | -0.19 | -2.61\% | -0.04 | 4.40\% | 0.07 |
| $R R V^{5 m}$ | (0,d, 0 ) s | 18.9\% | 0.90 | 18.1\% | 0.67 | 5.73\% | 0.15 | -21.1\% | -0.42 | -17.2\% | $-0.34$ |
| $R R V^{B C, 1 m}$ | (0,d, 0 ) s | 11.9\% | 0.45 | -0.26\% | -0.00 | -14.2\% | -0.31 | -5.67\% | -0.11 | -16.8\% | $-0.31$ |
| $R R V^{B C, 5 m}$ | (0,d, 0 ) s | 10.7\% | 0.60 | 24.3\% | 0.90 | -13.4\% | -0.30 | -21.6\% | -0.45 | -19.5\% | -0.39 |
| $R R V^{B V B C, 1 m}$ | (0,d, 0 ) s | 0.23\% | 0.00 | -17.7\% | -0.36 | -6.91\% | -0.12 | -3.91\% | -0.06 | -7.67\% | -0.13 |
| $R R V^{B V B C, 5 m}$ | (0,d, 0 ) s | 43.5\% | 1.66 | -9.16\% | -0.20 | -22.2\% | -0.43 | -9.18\% | -0.17 | 1.89\% | 0.03 |

Note: Annualized mean returns for the dynamic investment allocation for the $I V^{B S}$ and different confidence intervals. SR is the Sharpe Ratio. We have in total 208 weeks of trading. First part of the table corresponds to the $\operatorname{ARFIMA}(\mathrm{p}, \mathrm{d}, \mathrm{q})$ model while the second part to the $\operatorname{ARFIMA}(\mathrm{p}, \mathrm{d}, \mathrm{q})$ with level shift component.

Table 3.7: Dynamic investment - Selling strat. - Full sample - No hedge vs. Hedge

| Est. | Model | 99 |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | No hedge |  |  |  | Hedge |  |  |  |
|  |  | Week g. | Week 1. | An. ret. | SR | Week g. | Week 1. | An. ret. | SR |
| $R V^{1 m}$ | (0,d,0) | 10 | 1 | 30.3\% | 1.35 | 10 | 1 | 8.83\% | 0.69 |
| $R V^{5 m}$ | (0,d,0) | 9 | 2 | 23.5\% | 1.05 | 9 | 2 | 7.01\% | 0.55 |
| $R V^{B V, 1 m}$ | (0,d,0) | 12 | 4 | 27.4\% | 1.14 | 14 | 2 | 7.62\% | 0.61 |
| $R V^{B V, 5 m}$ | (0,d,0) | 18 | 6 | 22.5\% | 0.57 | 19 | 5 | 5.53\% | 0.41 |
| $R V^{K, 1 m}$ | (0,d, 0 ) | 7 | 2 | 1.26\% | 0.04 | 7 | 2 | 2.29\% | 0.20 |
| $R V^{K, 5 m}$ | (0,d,0) | 7 | 2 | -0.05\% | -0.00 | 7 | 2 | 2.44\% | 0.21 |
| $R V^{T S, 1 m}$ | (0,d,0) | 10 | 3 | 7.74\% | 0.23 | 10 | 3 | 4.73\% | 0.35 |
| $R V^{T S, 5 m}$ | (0,d,0) | 7 | 3 | -3.07\% | -0.09 | 7 | 3 | 1.06\% | 0.09 |
| $R R V^{1 m}$ | (0,d,0) | 20 | 15 | -14.4\% | $-0.32$ | 25 | 10 | -0.15\% | -0.01 |
| $R R V^{5 m}$ | (0,d,0) | 11 | 3 | 28.3\% | 1.17 | 12 | 2 | 4.70\% | 0.45 |
| $R R V^{B C, 1 m}$ | (0,d,0) | 12 | 7 | -14.4\% | $-0.37$ | 13 | 6 | -2.51\% | -0.17 |
| $R R V^{B C, 5 m}$ | (0,d,0) | 9 | 2 | 22.8\% | 1.04 | 9 | 2 | 4.00\% | 0.38 |
| $R R V^{B V B C, 1 m}$ | (0,d,0) | 18 | 17 | -17.7\% | -0.42 | 27 | 8 | 1.81\% | 0.12 |
| $R R V^{B V B C, 5 m}$ | (0,d,0) | 17 | 5 | 18.0\% | 0.47 | 18 | 4 | 7.09\% | 0.52 |
| $R V^{1 m}$ | $(0, \mathrm{~d}, 0) \mathrm{s}$ | 6 | 1 | 14.8\% | 0.81 | 6 | 1 | -0.96\% | -0.18 |
| $R V^{5 m}$ | ( $0, \mathrm{~d}, 0) \mathrm{s}$ | 8 | 2 | 19.5\% | 0.92 | 8 | 2 | 3.31\% | 0.32 |
| $R V^{B V, 1 m}$ | ( $0, \mathrm{~d}, 0) \mathrm{s}$ | 10 | 2 | 25.2\% | 1.13 | 10 | 2 | 3.80\% | 0.38 |
| $R V^{B V, 5 m}$ | $(0, \mathrm{~d}, 0) \mathrm{s}$ | 13 | 4 | 27.0\% | 1.03 | 14 | 3 | 3.23\% | 0.32 |
| $R V^{K, 1 m}$ | $(0, \mathrm{~d}, 0) \mathrm{s}$ | 3 | 1 | 3.00\% | 0.25 | 3 | 1 | 0.11\% | 0.01 |
| $R V^{K, 5 m}$ | (0,d, $)$ s | 5 | 2 | -7.26\% | -0.25 | 5 | 2 | -1.63\% | $-0.20$ |
| $R V^{T S, 1 m}$ | $(0, \mathrm{~d}, 0) \mathrm{s}$ | 8 | 2 | 19.5\% | 0.92 | 8 | 2 | 3.25\% | 0.32 |
| $R V^{T S, 5 m}$ | $(0, \mathrm{~d}, 0) \mathrm{s}$ | 6 | 1 | 11.6\% | 0.70 | 6 | 1 | 4.08\% | 0.40 |
| $R R V^{1 m}$ | ( $0, \mathrm{~d}, 0$ ) s | 18 | 12 | 9.24\% | 0.27 | 23 | 7 | 2.28\% | 0.16 |
| $R R V^{5 m}$ | (0,d, 0 ) s | 8 | 2 | 18.9\% | 0.90 | 8 | 2 | 0.33\% | 0.04 |
| $R R V^{B C, 1 m}$ | $(0, \mathrm{~d}, 0) \mathrm{s}$ | 13 | 4 | 11.9\% | 0.45 | 14 | 3 | 1.33\% | 0.10 |
| $R R V^{B C, 5 m}$ | (0,d, 0 ) s | 6 | 2 | 10.7\% | 0.60 | 6 | 2 | -0.33\% | -0.04 |
| $R R V^{B V B C, 1 m}$ | $(0, \mathrm{~d}, 0) \mathrm{s}$ | 17 | 13 | 0.23\% | 0.00 | 25 | 5 | 4.10\% | 0.30 |
| $R R V^{B V B C, 5 m}$ | $(0, \mathrm{~d}, 0) \mathrm{s}$ | 16 | 3 | 43.5\% | 1.66 | 17 | 2 | 9.63\% | 0.77 |

Note: Annualized mean returns for the dynamic investment allocation for the $I V^{B S}$ and different confidence intervals. SR is the Sharpe Ratio. Week g. and Week l. are the number of weekly gains and losses. We have in total 208 weeks of trading. First part of the table corresponds to the $\operatorname{ARFIMA}(\mathrm{p}, \mathrm{d}, \mathrm{q})$ model while the second part to the ARFIMA( $\mathrm{p}, \mathrm{d}, \mathrm{q})$ with level shift component.

Table 3.8: Mean returns and standard deviation I - Selling strat. - Before crisis No hedge

| Est. | Model | RT(\%) | $\begin{gathered} 99 \\ \text { Std } \end{gathered}$ | Sell | RT(\%) | $\begin{gathered} 95 \\ \text { Std } \end{gathered}$ | Sell | RT(\%) | $\begin{gathered} 90 \\ \text { Std } \end{gathered}$ | Sell |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $R V^{1 m}$ | (0,d,0) | 2.42\% | 0.150 | 9 | 2.81\% | 0.193 | 18 | 1.17\% | 0.253 | 27 |
| $R V^{5 m}$ | (0,d,0) | 2.08\% | 0.143 | 8 | 2.19\% | 0.227 | 23 | 2.90\% | 0.282 | 35 |
| $R V^{B V, 1 m}$ | (0,d,0) | $2.64 \%$ | 0.152 | 10 | 2.72\% | 0.222 | 24 | 4.94\% | 0.301 | 43 |
| $R V^{B V, 5 m}$ | (0,d,0) | 3.90\% | 0.210 | 19 | 2.81\% | 0.283 | 35 | 4.43\% | 0.331 | 50 |
| $R V^{K, 1 m}$ | (0,d,0) | 1.38\% | 0.127 | 6 | $3.22 \%$ | 0.197 | 21 | 3.38\% | 0.287 | 37 |
| $R V^{K, 5 m}$ | (0,d,0) | 1.66\% | 0.132 | 7 | 2.65\% | 0.236 | 24 | 4.41\% | 0.300 | 41 |
| $R V^{T S, 1 m}$ | (0,d,0) | 2.42\% | 0.150 | 9 | 2.54\% | 0.213 | 23 | 3.20\% | 0.286 | 37 |
| $R V^{T S, 5 m}$ | (0,d,0) | 1.14\% | 0.113 | 6 | 2.17\% | 0.227 | 23 | 4.13\% | 0.284 | 38 |
| $R R V^{1 m}$ | (0,d,0) | 2.17\% | 0.183 | 15 | 3.53\% | 0.268 | 34 | 4.30\% | 0.287 | 42 |
| $R R V^{5 m}$ | (0,d,0) | 2.52\% | 0.155 | 9 | 1.92\% | 0.225 | 22 | 4.12\% | 0.270 | 34 |
| $R R V^{B C, 1 m}$ | (0,d,0) | 1.10\% | 0.145 | 9 | 2.82\% | 0.241 | 24 | 2.79\% | 0.267 | 33 |
| $R R V^{B C, 5 m}$ | (0,d,0) | 2.01\% | 0.140 | 8 | 2.97\% | 0.194 | 20 | 3.43\% | 0.262 | 31 |
| $R R V^{B V B C, 1 m}$ | (0,d,0) | 1.73\% | 0.167 | 14 | 3.00\% | 0.270 | 35 | 3.56\% | 0.276 | 38 |
| $R R V^{B V B C, 5 m}$ | (0,d,0) | 3.32\% | 0.190 | 15 | 2.11\% | 0.269 | 31 | 4.91\% | 0.310 | 47 |
| $R V^{1 m}$ | (0,d, 0 ) s | 1.29\% | 0.123 | 6 | 2.81\% | 0.193 | 18 | 1.16\% | 0.247 | 25 |
| $R V^{5 m}$ | (0,d, 0 ) s | 2.08\% | 0.143 | 8 | 2.75\% | 0.197 | 20 | 1.13\% | 0.256 | 31 |
| $R V^{B V, 1 m}$ | (0,d, 0 ) s | 2.42\% | 0.150 | 9 | 2.49\% | 0.211 | 22 | 3.39\% | 0.284 | 33 |
| $R V^{B V, 5 m}$ | (0,d, 0 ) s | 2.89\% | 0.182 | 15 | 2.93\% | 0.282 | 34 | 2.80\% | 0.309 | 44 |
| $R V^{K, 1 m}$ | (0,d, 0 ) s | 0.41\% | 0.090 | 4 | 2.81\% | 0.189 | 16 | 1.39\% | 0.241 | 26 |
| $R V^{K, 5 m}$ | (0,d, 0 ) s | 1.14\% | 0.113 | 6 | 2.00\% | 0.221 | 19 | 2.95\% | 0.262 | 32 |
| $R V^{T S, 1 m}$ | (0,d, 0 ) s | 2.08\% | 0.143 | 8 | 3.01\% | 0.199 | 21 | 1.01\% | 0.255 | 30 |
| $R V^{T S, 5 m}$ | (0,d, 0 ) s | 1.00\% | 0.112 | 6 | 2.22\% | 0.219 | 18 | 2.88\% | 0.263 | 32 |
| $R R V^{1 m}$ | (0,d, 0 ) s | 2.17\% | 0.183 | 15 | 1.05\% | 0.232 | 24 | 3.74\% | 0.275 | 38 |
| $R R V^{5 m}$ | (0,d, 0 ) s | 2.01\% | 0.140 | 8 | 2.59\% | 0.191 | 17 | 1.73\% | 0.240 | 26 |
| $R R V^{B C, 1 m}$ | (0,d, 0 ) s | 1.67\% | 0.124 | 9 | 2.34\% | 0.190 | 20 | 2.26\% | 0.260 | 31 |
| $R R V^{B C, 5 m}$ | (0,d, 0 ) s | 1.14\% | 0.113 | 6 | 2.58\% | 0.191 | 18 | 1.49\% | 0.237 | 24 |
| $R R V^{B V B C, 1 m}$ | (0,d, 0 ) s | 1.42\% | 0.162 | 13 | 2.65\% | 0.262 | 31 | 3.44\% | 0.276 | 37 |
| $R R V^{B V B C, 5 m}$ | (0,d, 0 ) s | 3.58\% | 0.170 | 13 | 1.82\% | 0.265 | 28 | 3.09\% | 0.285 | 40 |

Note: Mean returns (weekly) for the the $I V^{B S}$ and different confidence intervals. sell is the number of weeks with short positions. We have in total 150 weeks of trading. First part of the table corresponds to the $\operatorname{ARFIMA}(\mathrm{p}, \mathrm{d}, \mathrm{q})$ model while the second part to the ARFIMA $(\mathrm{p}, \mathrm{d}, \mathrm{q})$ with level shift component.

Table 3.9: Dynamic investment - Selling strat. - Before crisis - No hedge

| Est. | Model | 99 |  | 97 |  | 95 |  | 92 |  | 90 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | An. ret. | SR | An. ret. | SR | An. ret. | SR | An. ret. | SR | An. ret. | SR |
| $R V^{1 m}$ | (0,d,0) | 31.1\% | 1.32 | 31.0\% | 1.05 | 34.9\% | 1.15 | 23.0\% | 0.67 | 6.44\% | 0.16 |
| $R V^{5 m}$ | $(0, \mathrm{~d}, 0)$ | 26.0\% | 1.16 | 34.9\% | 1.15 | 22.5\% | 0.63 | 17.3\% | 0.43 | 29.1\% | 0.66 |
| $R V^{B V, 1 m}$ | $(0, \mathrm{~d}, 0)$ | 34.4\% | 1.44 | 34.1\% | 1.09 | 31.5\% | 0.90 | 18.4\% | 0.43 | 63.5\% | 1.34 |
| $R V^{B V, 5 m}$ | (0,d,0) | 53.0\% | 1.61 | 48.2\% | 1.33 | 27.7\% | 0.62 | 31.5\% | 0.64 | 50.1\% | 0.96 |
| $R V^{K, 1 m}$ | $(0, \mathrm{~d}, 0)$ | 16.2\% | 0.81 | 33.0\% | 1.12 | 41.6\% | 1.35 | 33.6\% | 0.80 | 36.5\% | 0.81 |
| $R V^{K, 5 m}$ | $(0, \mathrm{~d}, 0)$ | 20.1\% | 0.96 | 32.7\% | 1.08 | 29.0\% | 0.78 | 45.9\% | 1.06 | 53.5\% | 1.14 |
| $R V^{T S, 1 m}$ | $(0, \mathrm{~d}, 0)$ | 31.1\% | 1.32 | 34.8\% | 1.15 | 29.2\% | 0.87 | 4.16\% | 0.10 | 33.6\% | 0.75 |
| $R V^{T S, 5 m}$ | $(0, \mathrm{~d}, 0)$ | 13.3\% | 0.74 | 33.1\% | 1.12 | 22.2\% | 0.62 | 28.4\% | 0.69 | 50.0\% | 1.12 |
| $R R V^{1 m}$ | (0,d,0) | 25.3\% | 0.88 | 2.47\% | 0.06 | 40.9\% | 0.97 | 51.9\% | 1.20 | 52.8\% | 1.17 |
| $R R V^{5 m}$ | (0,d,0) | 32.4\% | 1.33 | 31.4\% | 1.04 | 18.6\% | 0.52 | 21.7\% | 0.55 | 51.1\% | 1.21 |
| $R R V^{B C, 1 m}$ | (0,d, 0 ) | 11.4\% | 0.50 | 16.1\% | 0.60 | 31.5\% | 0.83 | 23.0\% | 0.56 | 28.8\% | 0.68 |
| $R R V^{B C, 5 m}$ | (0,d, 0 ) | 25.0\% | 1.14 | 33.0\% | 1.12 | 37.4\% | 1.23 | 16.5\% | 0.42 | 39.6\% | 0.96 |
| $R R V^{B V B C, 1 m}$ | (0,d, 0 ) | 19.5\% | 0.74 | 33.4\% | 0.88 | 31.9\% | 0.75 | 38.9\% | 0.89 | 40.6\% | 0.93 |
| $R R V^{B V B C, 5 m}$ | (0,d, 0 ) | 43.7\% | 1.46 | 31.3\% | 0.89 | 18.2\% | 0.43 | 65.0\% | 1.38 | 61.9\% | 1.27 |
| $R V^{1 m}$ | $(0, \mathrm{~d}, 0) \mathrm{s}$ | 15.0\% | 0.77 | $31.2 \%$ | 1.33 | 34.9\% | 1.15 | 29.2\% | 0.91 | 6.55\% | 0.16 |
| $R V^{5 m}$ | $(0, \mathrm{~d}, 0) \mathrm{s}$ | 26.0\% | 1.16 | 49.8\% | 1.83 | 33.8\% | 1.09 | 11.0\% | 0.29 | 5.65\% | 0.14 |
| $R V^{B V, 1 m}$ | $(0, \mathrm{~d}, 0) \mathrm{s}$ | 31.1\% | 1.32 | 38.7\% | 1.29 | 28.5\% | 0.86 | 12.4\% | 0.30 | 37.0\% | 0.83 |
| $R V^{B V, 5 m}$ | $(0, \mathrm{~d}, 0) \mathrm{s}$ | 37.0\% | 1.29 | 47.1\% | 1.33 | 29.7\% | 0.67 | 21.6\% | 0.46 | 24.9\% | 0.51 |
| $R V^{K, 1 m}$ | $(0, \mathrm{~d}, 0) \mathrm{s}$ | 4.18\% | 0.29 | 34.2\% | 1.35 | 35.3\% | 1.19 | 33.3\% | 1.08 | 10.0\% | 0.26 |
| $R V^{K, 5 m}$ | $(0, \mathrm{~d}, 0) \mathrm{s}$ | 13.3\% | 0.74 | 43.8\% | 1.63 | 20.1\% | 0.58 | 17.5\% | 0.44 | 31.7\% | 0.77 |
| $R V^{T S, 1 m}$ | $(0, \mathrm{~d}, 0) \mathrm{s}$ | 26.0\% | 1.16 | 40.7\% | 1.52 | 37.8\% | 1.21 | 28.7\% | 0.83 | 4.16\% | 0.10 |
| $R V^{T S, 5 m}$ | $(0, d, 0) \mathrm{s}$ | 11.3\% | 0.64 | 20.0\% | 0.73 | 23.6\% | 0.68 | 10.0\% | 0.26 | 30.4\% | 0.73 |
| $R R V^{1 m}$ | $(0, \mathrm{~d}, 0) \mathrm{s}$ | 25.3\% | 0.88 | 10.3\% | 0.30 | 6.20\% | 0.17 | 21.7\% | 0.54 | 43.9\% | 1.01 |
| $R R V^{5 m}$ | $(0, \mathrm{~d}, 0) \mathrm{s}$ | 25.0\% | 1.14 | 26.8\% | 0.95 | 31.5\% | 1.05 | 10.2\% | 0.27 | 14.8\% | 0.39 |
| $R R V^{B C, 1 m}$ | $(0, \mathrm{~d}, 0) \mathrm{s}$ | 20.4\% | 1.05 | 19.5\% | 0.74 | 27.5\% | 0.92 | 33.8\% | 0.89 | 21.2\% | 0.51 |
| $R R V^{B C, 5 m}$ | (0,d,0)s | 13.3\% | 0.74 | 26.8\% | 0.95 | 31.3\% | 1.04 | 15.7\% | 0.44 | 11.7\% | 0.31 |
| $R R V^{B V B C, 1 m}$ | (0,d,0)s | 15.3\% | 0.60 | 18.5\% | 0.53 | 27.0\% | 0.65 | 31.9\% | 0.75 | 38.6\% | 0.89 |
| $R R V^{B V B C, 5 m}$ | $(0, \mathrm{~d}, 0) \mathrm{s}$ | 49.8\% | 1.87 | 32.5\% | 0.94 | 14.4\% | 0.34 | 26.3\% | 0.61 | 31.9\% | 0.71 |

Note: Annualized mean returns for the dynamic investment allocation for the $I V^{B S}$ and different confidence intervals. SR is the Sharpe Ratio. We have in total 150 weeks of trading. First part of the table corresponds to the ARFIMA( $\mathrm{p}, \mathrm{d}, \mathrm{q}$ ) model while the second part to the ARFIMA( $\mathrm{p}, \mathrm{d}, \mathrm{q}$ ) with level shift component.

Table 3.10: Dynamic investment I - Selling strat. - Before crisis - No hedge vs. Hedge

| Est. | Model | 90 |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | No hedge |  |  |  | Hedge |  |  |  |
|  |  | Week g. | Week 1. | An. ret. | SR | Week g. | Week 1. | An. ret. | SR |
| $R V^{1 m}$ | (1,d,0) | 19 | 10 | 16.0\% | 0.38 | 21 | 8 | 6.97\% | 0.47 |
| $R V^{5 m}$ | (1,d,0) | 24 | 12 | 29.6\% | 0.67 | 26 | 10 | 13.3\% | 0.83 |
| $R V^{B V, 1 m}$ | (1,d,0) | 29 | 13 | 57.9\% | 1.24 | 32 | 10 | 17.4\% | 1.08 |
| $R V^{B V, 5 m}$ | (1,d,0) | 34 | 17 | 52.6\% | 1.01 | 36 | 15 | 21.8\% | 1.26 |
| $R V^{K, 1 m}$ | (1,d,0) | 26 | 12 | 43.8\% | 0.95 | 28 | 10 | 18.2\% | 1.03 |
| $R V^{K, 5 m}$ | (1,d,0) | 27 | 13 | 49.0\% | 1.05 | 29 | 11 | 18.2\% | 1.04 |
| $R V^{T S, 1 m}$ | (1,d,0) | 24 | 13 | 33.6\% | 0.75 | 26 | 11 | 15.8\% | 0.90 |
| $R V^{T S, 5 m}$ | (1,d,0) | 26 | 12 | 53.4\% | 1.18 | 28 | 10 | 21.0\% | 1.23 |
| $R R V^{1 m}$ | (1,d,0) | 28 | 14 | 50.5\% | 1.13 | 31 | 11 | 23.1\% | 1.32 |
| $R R V^{5 m}$ | (1,d,0) | 24 | 11 | 41.6\% | 0.98 | 27 | 8 | 21.3\% | 1.23 |
| $R R V^{B C, 1 m}$ | (1,d,0) | 22 | 11 | 28.8\% | 0.68 | 25 | 8 | 14.8\% | 1.01 |
| $R R V^{B C, 5 m}$ | (1,d,0) | 22 | 9 | 35.5\% | 0.87 | 25 | 6 | 16.8\% | 1.05 |
| $R R V^{B V B C, 1 m}$ | (1,d,0) | 27 | 14 | 47.7\% | 1.04 | 30 | 11 | 28.2\% | 1.43 |
| $R R V^{B V B C, 5 m}$ | (1,d,0) | 32 | 16 | 64.2\% | 1.32 | 35 | 13 | 24.7\% | 1.40 |
| $R V^{1 m}$ | (1,d,0)s | 16 | 8 | 19.6\% | 0.56 | 18 | 6 | 8.90\% | 0.64 |
| $R V^{5 m}$ | (1,d,0)s | 19 | 12 | 10.0\% | 0.24 | 21 | 10 | 5.71\% | 0.39 |
| $R V^{B V, 1 m}$ | (1, $\mathrm{d}, 0) \mathrm{s}$ | 24 | 11 | 38.7\% | 0.87 | 26 | 9 | 9.21\% | 0.63 |
| $R V^{B V, 5 m}$ | (1, $\mathrm{d}, 0) \mathrm{s}$ | 27 | 15 | 20.5\% | 0.43 | 29 | 13 | 8.95\% | 0.63 |
| $R V^{K, 1 m}$ | (1,d,0)s | 17 | 10 | 11.0\% | 0.29 | 19 | 8 | 7.81\% | 0.54 |
| $R V^{K, 5 m}$ | (1,d,0)s | 22 | 10 | $31.7 \%$ | 0.77 | 24 | 8 | 15.0\% | 0.95 |
| $R V^{T S, 1 m}$ | (1,d,0)s | 18 | 12 | 4.16\% | 0.10 | 20 | 10 | 4.37\% | 0.30 |
| $R V^{T S, 5 m}$ | (1,d,0)s | 20 | 11 | 22.9\% | 0.57 | 22 | 9 | 13.0\% | 0.83 |
| $R R V^{1 m}$ | (1,d,0)s | 25 | 13 | 43.9\% | 1.01 | 28 | 10 | 23.4\% | 1.30 |
| $R R V^{5 m}$ | (1,d,0)s | 17 | 9 | 14.8\% | 0.39 | 19 | 7 | 8.21\% | 0.57 |
| $R R V^{B C, 1 m}$ | (1, $\mathrm{d}, 0) \mathrm{s}$ | 20 | 11 | 21.2\% | 0.51 | 23 | 8 | 10.2\% | 0.79 |
| $R R V^{B C, 5 m}$ | (1,d,0)s | 17 | 8 | 15.0\% | 0.40 | 19 | 6 | 8.58\% | 0.58 |
| $R R V^{B V B C, 1 m}$ | (1,d,0)s | 25 | 13 | 40.6\% | 0.93 | 28 | 10 | 21.9\% | 1.29 |
| $R R V^{B V B C, 5 m}$ | (1,d,0)s | 28 | 13 | 46.2\% | 1.00 | 30 | 11 | 15.9\% | 0.99 |

Note: Annualized mean returns for the dynamic investment allocation for the $I V^{B S}$ and different confidence intervals. SR is the Sharpe Ratio. Week g. and Week l. are the number of weekly gains and losses. We have in total 150 weeks of trading. First part of the table corresponds to the ARFIMA ( $\mathrm{p}, \mathrm{d}, \mathrm{q}$ ) model while the second part to the ARFIMA( $\mathrm{p}, \mathrm{d}, \mathrm{q}$ ) with level shift component.

Table 3.11: Dynamic investment II - Selling strat. - Before crisis - No hedge vs. Hedge

| Est. | Model | 92 |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | No hedge |  |  |  | Hedge |  |  |  |
|  |  | Week g. | Week 1. | An. ret. | SR | Week g. | Week 1. | An. ret. | SR |
| $R V^{1 m}$ | (0,d,0) | 16 | 8 | 23.0\% | 0.67 | 18 | 6 | 10.8\% | 0.85 |
| $R V^{5 m}$ | (0,d,0) | 19 | 11 | 17.3\% | 0.43 | 21 | 9 | 11.2\% | 0.74 |
| $R V^{B V, 1 m}$ | (0,d,0) | 19 | 10 | 18.4\% | 0.43 | 21 | 8 | 5.94\% | 0.42 |
| $R V^{B V, 5 m}$ | (0,d,0) | 29 | 15 | 31.5\% | 0.64 | 30 | 14 | 13.8\% | 0.89 |
| $R V^{K, 1 m}$ | (0,d,0) | 20 | 10 | 33.6\% | 0.80 | 22 | 8 | 18.0\% | 1.05 |
| $R V^{K, 5 m}$ | (0,d,0) | 24 | 9 | 45.9\% | 1.06 | 26 | 7 | 21.3\% | 1.25 |
| $R V^{T S, 1 m}$ | (0,d,0) | 18 | 12 | 4.16\% | 0.10 | 20 | 10 | 4.25\% | 0.29 |
| $R V^{T S, 5 m}$ | (0,d,0) | 21 | 11 | 28.4\% | 0.69 | 23 | 9 | 14.6\% | 0.94 |
| $R R V^{1 m}$ | (0,d,0) | 26 | 12 | 51.9\% | 1.20 | 29 | 9 | 23.3\% | 1.33 |
| $R R V^{5 m}$ | (0,d,0) | 18 | 8 | 21.7\% | 0.55 | 20 | 6 | 12.2\% | 0.78 |
| $R R V^{B C, 1 m}$ | (0,d,0) | 20 | 10 | 23.0\% | 0.56 | 23 | 7 | 10.2\% | 0.79 |
| $R R V^{B C, 5 m}$ | (0,d,0) | 16 | 8 | 16.5\% | 0.42 | 18 | 6 | 11.9\% | 0.75 |
| $R R V^{B V B C, 1 m}$ | (0,d,0) | 24 | 12 | 38.9\% | 0.89 | 27 | 9 | 20.1\% | 1.21 |
| $R R V^{B V B C, 5 m}$ | (0,d,0) | 30 | 11 | 65.0\% | 1.38 | 32 | 9 | 18.3\% | 1.14 |
| $R V^{1 m}$ | (0,d, 0 ) s | 15 | 5 | 29.2\% | 0.91 | 16 | 4 | 8.75\% | 0.63 |
| $R V^{5 m}$ | (0,d, 0 ) s | 17 | 10 | 11.0\% | 0.29 | 19 | 8 | 7.37\% | 0.53 |
| $R V^{B V, 1 m}$ | (0,d, 0 ) s | 18 | 10 | 12.4\% | 0.30 | 20 | 8 | 5.93\% | 0.41 |
| $R V^{B V, 5 m}$ | (0,d, 0 ) s | 25 | 14 | 21.6\% | 0.46 | 27 | 12 | 9.52\% | 0.68 |
| $R V^{K, 1 m}$ | (0,d, $)^{\text {e }}$ | 15 | 7 | 33.3\% | 1.08 | 17 | 5 | 11.2\% | 0.86 |
| $R V^{K, 5 m}$ | (0,d, 0 ) s | 18 | 9 | 17.5\% | 0.44 | 20 | 7 | 9.07\% | 0.65 |
| $R V^{T S, 1 m}$ | (0,d, $)_{\text {s }}$ | 18 | 8 | 28.7\% | 0.83 | 19 | 7 | 10.5\% | 0.83 |
| $R V^{T S, 5 m}$ | (0,d, 0 ) s | 16 | 10 | 10.0\% | 0.26 | 18 | 8 | 7.55\% | 0.54 |
| $R R V^{1 m}$ | (0,d, 0 ) s | 20 | 12 | 21.7\% | 0.54 | 23 | 9 | 10.1\% | 0.78 |
| $R R V^{5 m}$ | (0,d, $)_{\text {s }}$ | 15 | 8 | 10.2\% | 0.27 | 17 | 6 | 7.72\% | 0.53 |
| $R R V^{B C, 1 m}$ | (0,d, 0 ) s | 19 | 7 | 33.8\% | 0.89 | 21 | 5 | 10.9\% | 0.86 |
| $R R V^{B C, 5 m}$ | (0,d, 0 ) s | 14 | 7 | 15.7\% | 0.44 | 16 | 5 | 8.54\% | 0.58 |
| $R R V^{B V B C, 1 m}$ | (0,d, 0 ) s | 23 | 12 | 31.9\% | 0.75 | 26 | 9 | 15.6\% | 1.02 |
| $R R V^{B V B C, 5 m}$ | (0,d, 0 ) s | 23 | 11 | 26.3\% | 0.61 | 25 | 9 | 8.91\% | 0.62 |

Note: Annualized mean returns for the dynamic investment allocation for the $I V^{B S}$ and different confidence intervals. SR is the Sharpe Ratio. Week $g$. and Week $l$. are the number of weekly gains and losses. We have in total 150 weeks of trading. First part of the table corresponds to the ARFIMA( $\mathrm{p}, \mathrm{d}, \mathrm{q}$ ) model while the second part to the ARFIMA( $\mathrm{p}, \mathrm{d}, \mathrm{q}$ ) with level shift component.

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[^0]:    ${ }^{1}$ Una versione ridotta di questo lavoro è stata publicata in Advances in Theoretical and Applied Statistics (SIS2010 Scientific Meeting), Springer Book.

[^1]:    ${ }^{2}$ A short version of this work has been published in Advances in Theoretical and Applied Statistics (SIS2010 Scientific Meeting), Springer Book.

[^2]:    ${ }^{1}$ The out-of-sample forecasting exercise is repeated using the bias corrected range-based bipower variation of Christensen, Podolskij, and Vetter (2009), a consistent and robust estimator of the integrated variance in the presence of jumps and microstructure noise. The results are very similar to the ones reported in the work and the conclusions are not affected.

[^3]:    ${ }^{2}$ In non reported table

[^4]:    ${ }^{3}$ In this version, we present the results based on the $M S E$ loss function.

[^5]:    Note: Estimation results for the 16 series of stock volatility for the period January 2003 to March 2010. $L L F$ is the Log-likelihood function, $A I C$ is the Akaike Information Criteria
    and $B I C$ is the Bayesian information criterion. $L J_{30}$ and $L J_{40}$ are the Ljung Box test for 30 and 40 lags. $J B$ is the Jarque-Bera test for Normality, $K S$ is the Kolmogorov-Smirnov and BIC is the Bayesian information criterion. $L J_{30}$ and $L J_{40}$ are the Ljung Box test for 30 and 40 lags. $J B$ is the Jarque-Bera test for Normality, $K S$ is the Kolmogorov-Smirnov

[^6]:    Table 1．4：HAR model with asymmetric effects with respect to the returns－Normal distribution and Constant variance

[^7]:    Note: Forecast performance evaluation for the HAR model with different variance specifications and distributions for the 16 series of stock volatility for the full out-of-sample period ( 1067 observation). Co is a constant variance specification, $G a$ is a $G A R C H$ and $G j$ is a $G J R-G A R C H$ variance specification. NO is Normal distribution and NI have the same performance. T-statistic in the table. "a", "b" and "c" indicate significance at the $1 \%, 5 \%$ and $10 \%$. Positive T-statistic favors the row model. The $M C S$ is a procedure to determine the "best" models from a collection of models based on the MSE loss function. $p v_{R}$ are the p -values for the range deviation method. "a" and "b" denote that the model belongs to the $10 \%$ and $25 \%$ MCS.

[^8]:    Note: Diebold and Mariano (1995) test for the row model vs. the HAR model with same variance equation and distribution assumption. Forecast performance evaluation for the 16 series of stock volatility for the Crisis period from September 2008 to July $2009(200$ observation). Model $I$ is a $A R(1)$ specification, $I I$ is an $H A R$, $I I I$ is an a constant variance specification, $G a$ is a $G A R C H$ and $G j$ is a $G J R$ variance specification. NO is Normal distribution and NI indicates Normal Inverse Gaussian distribution, The $D M$ is a test for equal predictive accuracy between two models based on the $M S E$ loss function. Under Ho, both models have the same performance. T-statistic in the table. "a", "b" and "c" indicate significance at the $1 \%, 5 \%$ and $10 \%$. Positive T-statistic favors the row model. Results for the model $I$ not reported to save space.

[^9]:    Note: Forecast performance evaluation for the the 16 series of stock volatility for the Crisis period from September 2008 to July 2009 ( 200 observation). Model $I$ is a $A R(1)$
     $H A R+I_{5}\left(h_{t-1}\right) h_{t-1}+I\left(R_{t-1}\right) R_{t-1}$. Co is a constant variance specification, Ga is a GARCH and Gj is a GJR variance specification. NO is Normal distribution and $N I$
    indicates Normal Inverse Gaussian distribution. The $M C S$ is a procedure to determine the "best" models from a collection of models based on the $M S E$ loss function. pv are the p-values for the range deviation method. "a" and "b" denote that the model belongs to the $10 \%$ and $25 \%$ MCS. Results for the model $I$ not reported to save space.

[^10]:    ${ }^{1}$ Some references for the topic include Zhang, Mykland, and Ait-Sahalia (2005), Bandi, Russell, Bandi, and Russell (2006) and Bandi and Russell (2008), Hansen and Lunde (2006) and Barndorff-Nielsen, Hansen, Lunde, and Shephard (2008) for the realized volatility estimator and Martens and van Dijk (2007) and Christensen, Podolskij, and Vetter (2009) for the realized range-based estimator. McAleer and Medeiros (2008) review the literature on the realized volatility.

[^11]:    ${ }^{2}$ See for example Welch and Goyal (2008).

[^12]:    ${ }^{3}$ The out-of-sample forecasting exercise is repeated using the bias corrected range-based bipower variation of Christensen, Podolskij, and Vetter (2009), a consistent and robust estimator of the integrated variance in the presence of jumps and microstructure noise, and the same set of macroeconomic and financial variables. The results are very similar to the ones reported in the work and the conclusions are not affected.

[^13]:    ${ }^{4}$ As in Caporin and Velo (2011) we sample every five minutes and to correct with the 66 previous days, where a more detailed description of the volatility series can be found.

[^14]:    Note：Diebold and Mariano（1995）test for the row model vs．the same specification without macroeconomic variables．Forecast performance evaluation for the 16 series of stock olatility for the full out－of－sample period（1067 observation）．Mod $I$ is an $A R(1)$ specification，$I F$ is an $H A R, I I I$ is an $H A R+I_{u m}\left(h_{t-1}\right) h_{t-1}+R_{t-1}+I\left(R_{t-1}\right) R_{t-1}, I V$ is an $H A R+R_{t-1}+I\left(R_{t-1}\right) R_{t-1}, V$ is an $H A R+I\left(R_{t-1}\right) R_{t-1}$ and $V I$ is an $H A R+I_{5}\left(h_{t-1}\right) h_{t-1}+I\left(R_{t-1}\right) R_{t-1}$ ．Co is a constant variance specification，Ga is a $G A R C H$
    and $G j$ is a $G J R$ variance specification．$N O$ is Normal distribution and $N I$ indicates Normal Inverse Gaussian distribution，MS indicates that the model includes the set of 5 macroeconomic variables（see section 2．4）．The DM is a test for equal predictive accuracy between two models based on the MSE loss function．Under Ho，both models have the same performance．T－statistic in the table．＂a＂，＂b＂and＂c＂indicate significance at the $1 \%, 5 \%$ and $10 \%$ ．Positive T－statistic favors the row model．Results for the model $I$ not reported to save space．Results for the model $I$ not reported to save space．

[^15]:    ${ }^{1}$ See for example McAleer and Medeiros (2008).

[^16]:    ${ }^{2}$ it is right continuous with left limits.

[^17]:    ${ }^{3}$ See Grassi and Santucci de Magistris (2011), Varneskov and Perron (2011) and Lu and Perron (2010) for a detailed explanation of the estimation methodology. Paolo Santucci de Magistris kindly provided the estimation code.

[^18]:    ${ }^{4}$ In a previous version, we also consider the $V I X$ as an alternative measure. The $V I X$ is a model-free implied volatility index that represents the expected future volatility of the $S \& P 500$ by the options over the next month. Although it is a natural quantity for the volatility in the market, it is constructed for a different period of time than the strategy implemented by the investor. The trading based on the $V I X$ results in negative results.

[^19]:    ${ }^{5} d_{1}=\frac{\ln \left(S_{t(0)} / K\right)+\left(r+\sigma^{2} / 2\right) T}{\sigma / \sqrt{T}}$, where $S_{t(0)}$ is the level of the stock, $K$ the strike price, $r$ is the continuously compounded risk-free interest rate, $\sigma$ the volatility of the stock and $T$ the time to expiration, see for example Hull (2008)

[^20]:    ${ }^{6}$ We do not consider the $(1, \mathrm{~d}, 0)$ specification since it seems to have a common root and it results in convergency problem during the mazimization.

