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FUTURE CONTINGENTS.
INDETERMINISM, TEMPORAL LOGIC AND SEMANTICS

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Contents

Abstarct

This essay is about the problem of future contingents, that is, statements predicting future events that are neither historically impossible, nor inevitable. The analysis intersects metaphysics, logic, philosophy of language and philosophy of science. In particular, the essay explores why indeterminism – viz., the doctrine that the present, the past, and the laws of nature do not necessitate the future – may be taken as a sensible thesis. Furthermore, several semantics for indeterministic, modal temporal languages will be considered. It is argued that the modal, temporal logic that best fits indeterminism is a version that mirrors the so-called TRL metaphysics. The advocates of the TRL metaphysics, indeed, can evoke a substantive notion of actuality to tell themselves apart from determinist, many worlds theorists. And the notion of substantive actuality assumed by TRL theorists may be easily reflected at the (post)semantic level, yielding a temporal logic which meets several *desiderata*.

Introduction

Intuitively, what is possible changes with time. At the present time, some things are still possible, others have become impossible. The future is contingent, it can turn out one way or another. The past is necessary, entirely settled.

To be sure, when I say that the past is necessary I do not mean necessary in the *logical* sense. Logic is consistent with any past one can conceive. Neither I mean that the past is *epistemically* necessary, that is, roughly, unquestionable or certain. One can doubt the past as well as nearly anything else.

When I say that the past is necessary I mean necessary in a *historical* sense. To be historically necessary is to be *inevitable*. What makes historical necessity special as compared to, say, logical or metaphysical necessity is its privileged relationship with time. The past is historically necessary *because* it is past: past things cannot go otherwise anymore.

But a *lot* of future things are now historically contingent: whether Brazil will win the next World Cup, who will be the next president of the USA, even the flavour of your next ice-cream. A *future contingent* is a statement that predicts some of these things. More generally:

Future Contingents A statement is a future contingent at a moment m if it predicts something (a state of affairs, or an event, or what have you) that is contingent at m.

Thus far, however, I have taken indeterminism – viz., the view that the future is contingent – to be an intuitive, true thesis. But there are two classical arguments – viz. the Aristotelian argument and the Master argument – which seem to deny that there are future contingents. According to these arguments, indeed, there are plausible premises that entail fatalism, that is, the view that if it will be the case that $\mathcal{A}$, it is inevitably so. And if fatalism holds, the future is settled, not contingent.

The first Chapter explores the Aristotelian and the Master arguments. As we shall see, these two arguments allow to isolate two separate questions. On the one hand, the Aristotelian argument entails that bivalence and indeterminism are incompatible. The Master argument, on the other hand, entails that indeterminism contradicts certain tense-logical principles. On the face of it, one may be tempted to drop indeterminism, and thus save both bivalence and those tense-logical principles that are employed in the Master argument. I shall argue that one should resist to make such a choice, for indeterminism seems to be required to make sense of some fundamental, physical theories.

Given that indeterminism is a sensible hypothesis, Chapter 2 compares two framework that are usually adopted to model future contingencies. On the one hand, the divergentist approach defines indeterminism over a set of non-overlapping, possible worlds. These worlds, in turn, may bear significant similarity relations to one another. On the other hand, the branching approach adopts a collection of overlapping possible worlds, branching towards the future. Both frameworks have rigorous mathematical translations, and they are apt to define two temporal logics which reject the fatalists arguments of Chapter 1. The two frameworks, however, are incompatible with one another, and they represent two alternative accounts of indeterminism. As I will try to argue, the branching conception has some virtues that divergence lacks. Thus the branching conception should be preferred to divergence when it comes to define indeterminism in terms of possible worlds.

Chapter 3 considers three semantics which interpret a temporal-modal language over a tree structure. Peirceanism, Supervaluationism and Relativism are very different, but they share the assumption that, given a branching conception of indeterminism, the truth of tensed statements should be relativised to moments only. These three strategies, even if promising, cannot account for several desiderata concerning the logic of a tensed language. The motivations for adopting supervaluationism, as well as those for subscribing to relativism, however, highlight an important, metaphysical problem. According to supervaluationists and relativists, indeterminism requires that actuality can only be a perspectival, relative property. The relational reading of actuality, in turn, is shared by those branching theorists who claim, against supervaluationism, that the truth of tensed statement should be relativised to moments and possible worlds (histories).

Chapter 4 presents a deterministic interpretation of the tree – viz., the many-worlds theory. This theory assumes that everything that is possible is bound to obtain, and thus it is perfectly compatible with the adoption of a relational account for actuality. Accordingly, it will be argued that, as far as indeterminism is concerned, branching theorists need a substantial, non-relational notion of actuality. This notion is required to tell apart branching indeterminists from branching, many-worlds determinists. Moreover, substantial actuality helps to define some principles that only an indeterminist may – and, in my view, should – accept. These principles, in turn, entail that there must exist a unique history (possible world) that is substantially actual.

Chapter 5 explores the so-called Thin red line semantics, which share the intuition that there exists a unique, privileged history. It will be argued that there is a semantics – the so-called post semantic thin red line –which perfectly mirrors the metaphysical principles that a branching indeterminism should adopt. The post semantic thin red line, furthermore, does not suffer from the logical and semantical flaws that affect other frameworks, and it overcomes several objections raised against the Thin red line approach. Hence, the post semantic thin red line is the most plausible semantics to interpret future contingents.

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Chapter 1

Fatalism and indeterminism

This chapter explores the Aristotelian and the Master arguments. As we shall see, these two arguments allow to isolate two separate questions. On the one hand, the Aristotelian argument entails that bivalence and indeterminism are incompatible. The Master argument, on the other hand, entails that indeterminism contradicts certain tense-logical principles.

On the face of it, one may be tempted to drop indeterminism, and thus save both bivalence and those tense-logical principles that are employed in the Master argument. I shall argue that one should resist to make such a choice, for indeterminism seems to be required to make sense of some fundamental, physical theories.

1.1 The Aristotelian argument

Que sera sera (*Whatever will be will be*) is a popular song from the 50's. The singer, Doris Day in the original, wonders about her future – Am I going be happy? What about my future child? etc. The answer these questions is always the same:

(1) Whatever will be will be.

It is natural to take (1) as a fatalist motto. Under this view, fatalism is an unhopeful, passive attitude toward the future. Clearly, we may *use* (1) to express this kind of attitude. However, both this attitude and (1) do not correspond to the philosophical *thesis* known as fatalism. Moreover, (1) is a logical truth that may be formulated as 'If it will be the case that $\mathcal{A}$, then it will be the case that $\mathcal{A}$'. Fatalism, however, is a more interesting claim: 'If it will be the case that $\mathcal{A}$, then, *necessarily*, it will be the case that $\mathcal{A}$'.

Aristotle elaborates a famous fatalist argument in his *De Interpretatione* (19a23-25):

> For if every affirmation or its negation is true or false it is necessary for everything either to be the case or not to be the case. [...] For if it is true to say that it is white or not white, it is necessary for it to be white or not white: and if it is white or not white, than it was true to say or deny this. If it is not the case it is false, if it is false it is not the case. So it is necessary for the affirmation or the negation to be true. It follows that nothing either is happening, or will be or will not be, by chance or as chance has it, but everything of necessity and not as chance has it [...].

Aristotle's fatalist argument can be summarised as follows.[1] Consider the statement:

(2) Tomorrow there will be a sea battle.

Even if now we do not know whether (2) is true or false, there is something we can be confident with, namely, that (2) is either true or false. Let us start by assuming that (2) is true. If it is now true that tomorrow there will be a sea battle, then the battle is fated, inevitable. (It is always too late to change the present.) What is inevitable is not contingent. Therefore:

(3) It is not contingent that tomorrow there will be a sea battle.

But the same conclusion is got if we assume that (2) is now false. For again, if it is now false that the sea battle will take place, then it is too late to make it happen. The battle is now historically impossible – and nothing impossible is contingent. As the argument does not depend on any particular feature of either the sea battle or (2), it generalises. Everything is either inevitable or impossible, nothing is contingent.

In general,

(P) (a) If $\mathcal{A}$ is true at a moment m, then 'Necessarily, $\mathcal{A}$' is true at m.
(b) If $\mathcal{A}$ is false at a moment m, then 'Possibly, $\mathcal{A}$' is false at m.

But given that bivalence holds relative to moments,

(B) Either $\mathcal{A}$ is true at a moment m, or $\mathcal{A}$ is false at m,

it follows that

[1] This is the standard reconstruction. For a discussion and alternative reconstructions, see Whitaker (2002).

(C) For any statement $\mathcal{A}$, either 'Necessarily $\mathcal{A}$' is true at m, or 'Possibly $\mathcal{A}$' is false at m.

Therefore, there are no future contingents and everything is necessary. The conclusion (C) entails *fatalism*, according to which the future is historically inevitable (see Rice 2015). Furthermore, (C) is a consequence of the so-called *causal determinism* (see Hoefer 2015), according to which, for any moment m, the state of the world at m and the laws of nature necessitate any later state. Whether fatalism is committed to the inevitability of the future, causal determinism grounds fatalism on the laws of nature.

It is worth noticing that a key role in the argument is played by the premise that truth satisfies *bivalence* relative to any moment: for every moment m, each statement (including future contingents) is either true at m or false at m. Accordingly, even if the argument is valid, it does not show that nothing is contingent. At best, it proves that contingency and bivalence (relative to moments) are incompatible, mutually exclusive. Indeed, Aristotle's solution – as standardly reconstructed – involves the rejection of bivalence.

There are three reasonable reactions to this fatalist argument: dropping contingency, giving up bivalence, or rejecting the argument (and so sticking to both bivalence and contingency). We can put this dialectical situation in the form of a question:

(Q1) Are we to drop indeterminism, bivalence, or neither?

As we shall see, each possible reply to (Q1) comes with its own difficulties. To offer a solution to the problem of future contingents is to choose an answer and solve, or put into perspective, the corresponding difficulties.

Today, to solve the problem of future contingents requires providing a semantics for modal and temporal languages, along with a rigorous interpretation of future contingents. Hence, it is impossible to give a picture of the contemporary debate on future contingents without taking into account formal languages and their semantics.

Be as it may, next section explores a further, fatalist argument.

1.2 The Master Argument

Indeterminists have to face another problem related to the logic of time. As Arthur Prior points out, there is an ancient argument – the so-called Master argument – that supports the fatalist view. Most likely, the Master Argument is due to the megaric Diodorus Cronus, even if it has been readapted by many medievals logicians. In what follows, I present the version of the argument that Arthur Prior presented in his famous *Time and Determinism*. The strength of the argument relies on the fact that (apparently) innocuous and intuitive premises about time logically entail the fatalist's thesis: if something will be the case, it is inevitably so.

In order to present the argument in a fully rigorous way, a modal temporal language is needed. For the present purposes, it suffices to enrich the standard propositional language with three sentential operators: $\mathsf{F}(n), \mathsf{P}(n), \Box$. The set of wffs of the new language thus obtained is specified by the following rule (p varies over atoms, $\mathcal{A}$ and $\mathcal{B}$ over wffs, and n over positive numbers).[2]

$$\mathcal{L}_T ::= p \mid \neg\mathcal{A} \mid \mathcal{A} \vee \mathcal{B} \mid \mathsf{F}(n)\mathcal{A} \mid \mathsf{P}(n)\mathcal{A} \mid \Box\mathcal{A}$$

Atoms represent present tensed statements (for instance, "There is a sea battle" is one of those). $\mathcal{L}_T$ has two temporal operators, $\mathsf{F}(n)$ and $\mathsf{P}(n)$. Their intuitive readings are "In n time units hence, it will be the case that" and "n time units ago, it was the case that" respectively. The primitive, modal operator of $\mathcal{L}_T$ is $\Box$, and it reads "It is now unpreventable that". For instance, if the atom p corresponds to "There is a sea battle" while time units are days, then

"$\mathsf{F}(1)p$" reads "Tomorrow, there will be a sea battle".
"$\mathsf{P}(1)p$" reads "Yesterday, there was a sea battle".
"$\Box p$" reads "It is now unpreventable that there is a sea battle".

With the language $\mathcal{L}_T$ at hand, let us run Prior's version of the Master argument:

1.	$\mathsf{P}(n)\mathcal{A} \to \Box\mathsf{P}(n)\mathcal{A}$	*PNP*
2.	$\mathsf{P}(n)\mathsf{F}(n+m)\mathcal{A} \to \Box\mathsf{P}(n)\mathsf{F}(n+m)\mathcal{A}$	By 1, substitutivity.
3.	$\mathsf{F}(m)\mathcal{A} \to \mathsf{P}(n)\mathsf{F}(n+m)\mathcal{A}$	*PR*
4.	$\mathsf{F}(m)\mathcal{A} \to \Box\mathsf{P}(n)\mathsf{F}(n+m)\mathcal{A}$	By 2, 3, transitivity.
5.	$\Box(\mathcal{A} \to \mathcal{B}) \to (\Box\mathcal{A} \to \Box\mathcal{B})$	*K*
6.	$\Box(\mathsf{P}(n)\mathsf{F}(n+m)\mathcal{A} \to \Box\mathsf{F}(m)\mathcal{A})$	*NCPR*
7.	$\Box\mathsf{P}(n)\mathsf{F}(n+m)\mathcal{A} \to \Box\mathsf{F}(m)\mathcal{A}$	By 5, 6, transitivity.
$\therefore$	$\mathsf{F}(m)\mathcal{A} \to \Box\mathsf{F}(m)\mathcal{A}$	By 4, 7, transitivity.

Notice that the inferential rules applied within the argument are pretty simple and intuitively sound. The argument involves the substitutivity of logically equivalent formulas (step 2), along with the transitivity of the

[2] The material conditional, classical conjunction and the historical possibility operator are defined in the usual way.

material conditional (steps 4, 7 and $\therefore$). Hence, from an indeterministic perspective, the Master argument could be rejected only by refuting (at least) one schema among *PNP*, *PR*, *K* and *NCPR*.

Principle *K* is the so-called Kripke's schema, used to define the set of normal modal logics. Moreover, it seems to capture a basic modal truth, that is, the distribution of necessity over the material conditional. Hence, it seems unreasonable to reject it.

The *NCPR* schema corresponds to the necessitation of the converse of the principle of retrogradation, *PR*. At a first sight, *NCPR* can strike as counterintuitive, but it is implied by some very basic principles, as the following inferential pattern shows.

1.	$\mathsf{P}(n)\mathsf{F}(n+m)\mathcal{A}$	Hypothesis.
2.	$\mathsf{P}(n)\mathsf{F}(n)\mathcal{A} \rightarrow \mathcal{A}$	First assumption.
3.	$\mathsf{F}(m)\mathsf{F}(n)\mathcal{A} \leftrightarrow \mathsf{F}(m+n)\mathcal{A}$	Second assumption.
4.	$\mathsf{P}(n)\mathsf{F}(n)\mathsf{F}(m)\mathcal{A}$	By 1, 3, substitutivity.
5.	$\mathsf{F}(m)\mathcal{A}$	By 2, 4, MP.
6.	$\mathsf{P}(n)\mathsf{F}(n+m)\mathcal{A} \rightarrow \mathsf{F}(m)\mathcal{A}$	By 1, 5, $\rightarrow$ intro.
$\therefore$	$\Box(\mathsf{P}(n)\mathsf{F}(n+m)\mathcal{A} \rightarrow \mathsf{F}(m)\mathcal{A})$	By 6, necessitation.

Thus, even rejecting *NCRP* seems a bad idea.

As for *PR*, the principle of retrogradation, it expresses the idea that if something is true today – namely, that tomorrow there will be a sea battle – then yesterday it was the case that "Tomorrow there will be a sea battle" would have been true a day later. One may reject *PR* by saying that, if a future event was neither inevitable, nor impossible (or, as Prior would say, if a future event was not 'present in its causes'), it would have been untrue to say that it was going to happen.[3] However, this criticism works only if the future operator $\mathsf{F}(n)$ does not only have a temporal meaning – it makes you look forward along the time line – but it also conveys a strong modal force – it expresses some sort of inevitability. Under a more natural understanding of $\mathsf{F}(n)$, *PR* seems to be fine. Be as it may, there is room to think that *PR* can fail to be valid, and an indeterminist may say a story in order to reject it.

A much more controversial principle is *PNP*, the principle of the necessity of the past. To see why, let us focus on one of its instances:

(PNP1) If yesterday it was the case that $\mathcal{A}$, then, inevitably, yesterday it was the case that $\mathcal{A}$.

[3] This is the reason why the Peircean semantics rejects *RP*, and, therefore, the Master argument.

$$\mathsf{P}(1)\mathcal{A} \to \Box\mathsf{P}(1)\mathcal{A}$$

Notice that the schematic letter $\mathcal{A}$ in (PNP1) can be replaced by any formula of $\mathcal{L}_T$. Accordingly, if *PNP* were valid, the following sentence – obtained by substituting $\mathcal{A}$ with "two days hence it would be rainy" in (PNP1)– would be valid as well.

(PNP2) If yesterday was the case that two days hence it would be rainy, then, inevitably, yesterday was the case that two days hence it would be rainy.

$$\mathsf{P}(1)\mathsf{F}(2)p \to \Box\mathsf{P}(1)\mathsf{F}(2)p$$

It is easy to see what is wrong with (PNP2); even though both its antecedent and consequent begin with the past operator $\mathsf{P}(1)$, both components are not about the past. What they really talk about is the future. As Prior (1967: 124) puts it, (PNP2) has "traces of futurity". Thus, in general, *PNP* allows to take as validities sentences with this kind of traces of futurity. Actually, any sentence of $\mathcal{L}_T$ that talks about the future is equivalent to a sentence that begins with $\mathsf{P}(n)$.[4] For instance, $\mathsf{F}(1)\mathcal{A}$ ("In one time unit hence, it will be the case that $\mathcal{A}$) is equivalent to $\mathsf{P}(1)\mathsf{F}(2)\mathcal{A}$ ("One time unit ago, it was the case that, two time units after, $\mathcal{A}$").

But one can notice that asking whether *PNP* should be considered as valid is asking whether one should endorse (or reject) fatalism (determinism). Thus, assuming this schema as a premise within an argument in favour of fatalism appears to beg the question that is at stake, that is, whether fatalism (or indeterminism) holds. The Master argument assumes fatalism to conclude that fatalism is true.[5]

Obviously, the objection just raised against *PNP* holds only $\mathsf{F}(n)$ does not convey a strong modal meaning. Indeed, if predicting that $\mathcal{A}$ will occur amounts to predict that it is inevitable that $\mathcal{A}$ will occur, *PNP* is perfectly right. Under this perspective, having traces of futurity just means having traces of future inevitability.[6] However, an indeterminist who takes this route would be much more recalcitrant to regard *PR*, instead of *PNP*, as a validity.

[4]See, for instance, step 6 in the derivation of *NCPR*.

[5]This is one motivation for adopting the so-called Ockhamist semantics.

[6]This is the way a Peircean would argue against an advocate of the Ockhamist semantics.

In conclusion, there are three possible reactions to the Master argument: dropping indeterminism, rejecting *PR*, or denying *PNP*'s validity. This dialectic situation is summarised in the following question:

(Q2) Are we to drop indeterminism, *PR*, or *PNP*?

As we shall see, there are several reasons suggesting that there are future contingencies. And if one can sensibly resist to the fatalistic motto that everything that will happen – or everything that it is possible that will happen – is inevitably going to happen, one can reasonably reject the fatalistic conclusions entailed by both the Aristotelian and the Master arguments. Next section highlights why one may doubt that fatalism is true.

1.3 Physical Indeterminism

1.3.1 The measurement problem

Thus far, both (Q1) and (Q2) leave open the option of rejecting indeterminism. As I'll try to argue, this option should be resisted, for there are good reasons to hold that future contingencies are required to make sense of physical reality.

In my view, the best way to argue in favour of indeterminism has to do with the stochastic behaviour that quantum physical systems may exhibit. The stochastic behaviour of quantum systems, in turn, can be appreciated by focusing on the classical quantum puzzle known as the measurement problem.

The measurement problem characterises the tension between two principles of Von Neumann (1955)'s standard formulation of quantum mechanics, namely, the *Dynamical* and the *Collapse* postulates. Let us start with the former.

Dynamical postulate: When no measurements are going on, the states of all physical systems invariably evolve in accordance with the dynamical equations of motion.[7]

The (standard) equation of motion for quantum systems is the Schrödinger's equation (or its relativistic generalisations), which allows to determine the undisturbed state of a system S at all times, given the state that S has at a time.

Experimental evidence tells us that any measurement of the component of the spin of an electron can have only two outcomes: the spin can

[7] See Albert (2009: 80).

either be measured "up" (relative to a given axis), or it can be measured "down" (relative to the same axis).[8] Suppose that, at t_0, it is known that the electron a has spin up along the z axis (that is, at t_0 it is known that a is in the state $\uparrow_z$). If one then measures the spin of a along the x axis at a time later than t_0, one has 1/2 chance to find a in the state $\uparrow_x$, and 1/2 chance to find a in the state $\downarrow_x$. And the same happens if, at t_0, it is known that a has spin down along the z axis: again, at a time later than t_0, one has 1/2 chance to find a in the state $\uparrow_x$, and 1/2 chance to find a in the state $\downarrow_x$. Apparently, the value of the spin along z at t_0 is not correlated with the value of the spin along x at t_1.

But given that the spin's state of a along z is known at t_0, one may apply the *Dynamical* postulate to see what the Schrödinger's equation predicts about the x-spin state that a should have at t_1. In general, the Schrödinger's equation is said to describe a at t_1 as being in a superposition of having spin up along x, and having spin down along x. Formally, one may express the state of a at t_1 – as it is predicted by the Schrödinger's equation – as follows.

(2) $| a, t_1 > = \sqrt{\frac{1}{2}}\,|\uparrow_x> + \sqrt{\frac{1}{2}}\,|\downarrow_x>$

It is pretty difficult to understand what kind of state a sentence such as (2) may represent. Even worst, the Schrödinger's equation predicts that macroscopical objects such as measuring devices can be in superpositions as well. Assume, for instance, that "|x-up>" denotes the state of a device measuring a being in state $\uparrow_x$, while "|x-down>" refers to the state of the same device as measuring a being in state $\downarrow_x$. And suppose that the device is set up right. And, once again, assume that the Schrödinger's equation predicts that, at t_1, a is in a superposition of being $\uparrow_x$ and $\downarrow_x$. Then, the very same equation predicts that, at the time of the measurement (t_1), the state of the system composed of both the electron a and the measurement device is in the following superposition:[9]

(3) $\sqrt{\frac{1}{2}}\,|x\text{-up}>\,|\uparrow_x> + \sqrt{\frac{1}{2}}\,|x\text{-down}>\,|\downarrow_x>$

But what does it mean for a measuring device, as well as for any other physical system, to be in a superposition? As Albert & Loewer (1988: 125) point out,

> When textbook writers attempt to explain what it is for an electron to be in, for example, the spin state $\uparrow_x + \downarrow_x$ they are reduced to saying things like "it neither has x-spin up nor x-spin

[8] The following elementary, quantum mechanical facts are taken from Weber (1993).

[9] See Albert (2009: 75).

> down but is in some sense in both states and in neither". [...] So one problem is to "interpret" superpositions. (Albert & Loewer 1988: 195)

Thus, one of the major problem of interpreting superpositions is that we never observe a physical system as "being in some sense in both states and in neither". It is an obvious characteristic of our experience, indeed, that we always observe physical objects as being in definite states, that is, as being either in a state or in another.

Recall that, by the *Dynamical* postulate, a statement such as (2) (as well as (3)) is meant to describe a, intended as an undisturbed, isolated physical system. And, obviously, any act of measurement perturbs the system thus measured. Hence, one may think that a measurement of a's spin along x at t_1 perturbs a itself, forcing the electron to "jump" to one definite state among $\uparrow_x$ and $\downarrow_x$. This is what the *Collapse* postulate guarantees.

Collapse postulate: When a measurement takes place at time t, the state of the measured system S collapses over one element of the superposition (if any) among those described by the equations of motion of S at t.[10]

According to the *Collapse* postulate, if one measures a's spin along x at t_1, the measurement induces a's state to collapse, and a's state is thus instantaneously reduced to one element of the superposition (i.e., either a is in $\uparrow_x$ at t_1, or it is in $\downarrow_x$ at t_1). Hence, the *Collapse* postulate seems to guarantee the accordance of standard quantum mechanics with our experience: indeed, it entails that whenever one observes a physical system, that system collapses over a definite state.

Furthermore, by applying the so-called Born rule, one can compute the probability of the outcomes, conditional on the fact that specific measurement procedures are actually performed. For instance, if the solution of the Schrödinger's equation for a at t_1 is (2), the Born rule tells us that, if one carries a measurement of the x-spin of a at that time, one has 1/2 chance to obtain $\uparrow_x$, and 1/2 chance to obtain $\downarrow_x$.

Accordingly, if standard quantum mechanics captures everything that there is to know about quantum systems, one can plausibly say that nature behaves stochastically. And thus, one may appeal to standard quantum mechanics to hold that fatalism is false. But this conclusion might be too hasty.

[10] See Albert (2009: 80).

As they are standardly understood, indeed, the *Dynamical* and the *Collapse* postulates are deeply problematic, for the notion of measurement plays a crucial role in both of them.

> What these laws actually *amount to* (that is: what they actually *say*) will depend on the precise meaning of the word *measurement* (because these two laws entail that which one of them is being *obeyed* at any given moment depends on whether or not a "measurement" is being *carried out* at that moment). And it happens that the word *measurement* simply doesn't have any absolutely precise meaning in ordinary language; and it happens (moreover) that von Neumann didn't make any attempt to cook up a meaning for it, either. (Albert 2009: 80-81)

Hence, the measurement problem: how to deal with the notion of measurement? And relatedly, how the *Dynamical* and the *Collapse* postulates have to be understood?

1.3.2 Collapse theories

Scientists and philosophers have developed several ways to overcome the measurement problem. The problem, however, has not been solved yet. More precisely, none of the attempts to solve the measurement problem has achieved the unanimous consensus of the scientific community.[11]

Be as it may, there is a family of candidate solutions according to which nature evolves stochastically. These solutions, in turn, are labeled as collapse theories, and their foundational program traces back to the work of Ghirardi et al. (1986).[12]

Collapse theorists hold that "measurement", whatever its meaning may be, does not have to occur within the postulates of any respectable account of nature.[13] This point, after all, is pretty sensible: measurements seem to be nothing over and above certain interactions taking place among physical systems, and the interactions occurring between physical systems are what physics is supposed to account for. What is controversial, still, is how to deal with those postulates that scientists apply in their daily work, and in which the notion of measurement plays a crucial role.

Recall that the *Collapse* postulate was invoked to establish a link between the *Dynamical* postulate and the world as we see it. If physical sys-

[11] For a review of these attempts, see Albert (2009).
[12] See Ghirardi (2016).
[13] See Albert & Loewer (1996: 83), Ghirardi et al. (1986: 471) and Ghirardi (1993: 186).

tems evolved in accordance with the Schrödinger's equation only, the device at t_1 should be in a superposition of measuring a in state $\uparrow_x$, and measuring a in state $\downarrow_x$. This latter conclusion obviously contradicts our experience, for we always see the device either as measuring a in state $\uparrow_x$, or as measuring a in state $\downarrow_x$.

Standard quantum theorists attempt to avoid the undesirable conclusion by adopting the *Collapse* postulate. Collapse theorists, on the contrary, claim that such a conclusion, being incompatible with our experience, forces to reject the Schrödinger's equation. In their view, the Schrödinger's equation has to be modified to obtain an equation that

> [...] represents a mathematically precise and successful attempt to achieve the two divergent aims [...] of leaving the physics of microscopic systems essentially unaltered [w.r.t standard quantum predictions] but at the same time of forbidding the occurrence of superpositions of macroscopically distinguishable states. (Ghirardi 1993: 189)

One may suspect that collapse theorists are introducing a dichotomy between microscopic and macroscopic objects. And one may object that this dichotomy is as arbitrary as that between measured and undisturbed systems. But this is not so. Let us see why.

According to collapse theorists, a system such as an electron can be in a superposition such as that described in (2). The equations of motion assumed by collapse theorists, however, predict that every so often the electron's state spontaneously collapses over one of the elements of the superposition (if any). The collapse, in turn, does not necessarily involve measurements, for it corresponds to a natural mechanism of nature. Collapse processes, indeed, are predicted by the very equations which govern a system's evolution, and which a collapse theorist adopts. For instance, whenever a is in a given z-spin state, collapse theories associate a to a given frequency λ. This frequency, in turn, measures the probability for a to collapse into $\downarrow_x$ or into $\uparrow_x$ within a given interval of time.[14] Thus, a collapse is taken to be a natural processes, predicted by specific equations. Furthermore, collapse processes occur at random. In other terms, the new equations, in general, do not establish with absolute certainty when a will collapse. They only allow to compute the probability for a to collapse over one definite state at a given time.

Furthermore, collapse theories also entail that, when a's collapse does occur, a has a certain probability of collapsing over $\uparrow_x$, and a certain prob-

[14]See Ghirardi (2015: 366).

ability of collapsing over $\downarrow_x$. In general, however, one cannot predict with absolute certainty which state will characterise a after the occurrence of a collapse. Hence, whenever a collapse takes place, a "jumps" stochastically into a unique element of the superposition. Accordingly, the time of the collapse of a system, as well as the state that characterises the system after the collapse, are governed by laws which are genuinely probabilistic.

> It is important to stress that both the specification of WHEN as well as the one of WHERE the localizations take place introduce stochastic elements, and that the rule about WHERE it occurs introduces nonlinear feature in the theory. (Ghirardi 1993: 190)

Clearly, one may ask whether collapse theories can avoid superpositions of macroscopic objects such as (3). As a matter of fact, collapse theories predict that the higher is the number of the components of a system, the higher is the probability for the system to collapse in a small period of time. For instance, the frequency λ is often chosen to ensure that the mean lifetime of a superposition of a single proton is 10^{16} seconds, a quite long period indeed. If a systems is composed by 10^{20} particles, however, the mean lifetime of a superposition for such a system is 10^{-5} seconds.[15] Superpositions of macroscopic objects are unstable, for macroscopic objects tend to collapse into definite states in a considerably short amount of time. This, in turn, is the solution that collapse theorists give to the measurement problem. As Albert & Loewer (1996) put it, any measurement-like interaction

> [h]as nothing whatever to do with the occurrence of [...] collapses: *all* superpositions of states in which macroscopic objects are in macroscopically different positions (no matter how those states may happen to have been gotten into) are clearly unstable on this theory. (Albert & Loewer 1996: 86)

To sum up, collapse theories are plausible solutions to the measurement problem, and they entail that nature behaves stochastically. The randomness that physical systems may exhibit, according to these theories, involves both the time of the occurrence of a collapse, and the state one system may assume after the occurrence of such a collapse. Collapse theories, moreover, assume that the statistical character of predictions is not an objective, non-epistemic matter. And since collapse theories are plausible physical accounts, there is room to resist to one of the options left open by (Q1)-(Q2) – viz., that of denying indeterminism.

[15]See Bell (2004: 203-204) and Ghirardi (2015: 370).

1.4 Let us look ahead

As we have seen, the Aristotelian argument – if sound – shows that bivalence is incompatible with indeterminism. The Master argument, moreover, points out that certain tense-logical principles entail the fatalists' thesis (i.e. if it will be the case that $\mathcal{A}$, it is inevitable that it will be the case that $\mathcal{A}$). Thus, the Aristotelian argument, along with the Master argument, highlight the following problems.

(Q1) Are we to drop indeterminism, bivalence, or neither?
(Q2) Are we to drop indeterminism, *PR*, or *PNP*?

Both questions leave open the possibility of rejecting indeterminism. However, it is reasonable to think that indeterminism may play a crucial role when it comes to account for nature's behaviour. Collapse theories, indeed, entail exactly that nature, at a fundamental level, evolves stochastically. Therefore, the option of rejecting indeterminism can be resisted.

But how to deal with bivalence? And how to deal with the premises occurring in the Master argument? In order to answer these questions, one needs a framework to interpret tensed languages. Such a framework, moreover, should reflect some features that an indeterministic, physical reality should have.

Next chapter compares two approavhes usually adopted by indeterminists, and explores their role in answering (Q1)-(Q2).

Chapter 2

Two frameworks for indeterminism

In what follows, I consider two frameworks which help to overcome the fatalist arguments we have encountered thus far. The two frameworks are very common in the philosophical literature, and they are known as the branching view and the divergence approach, respectively. As Placek (2012: 28) points out, each framework plays two roles.

On the one hand, they (partially) elucidate what may mean that the future is unsettled, or that the reality which we inhabit evolves indeterministically. In other terms, both framework are helpful to define objective indeterminism, intended as a metaphysical doctrine.[1]

On the other hand, the two frameworks are useful to interpret modal temporal languages, and to "offer a semantics for languages with historical modalities, tenses, and indexicals" (Placek 2012: 28). As we shall see, there are temporal logics – defined against branching (or divergentist) models – which invalidate the arguments for determinism we have seen in the first chapter.

But the two frameworks, however, are incompatible with one another. Indeed, there is a principle – what it will be called the *No-Overlap* condition – that is explicitly assumed by the divergentist approach, while it is

[1]For recent attempts aiming at rehabilitating objective indeterminism, see Williams 2008 and Barnes & Cameron 2009, 2011. In what follows, both the branching and the divergentist approach will be framed in purely B-theoretical terms. Thus, any possible world, as well as anything that exists at a moment atensionally exists. This is not to say that indeterminism requires a B-theoretic conception of reality. It is just that the debate concerning A- and B-theories *is not* the debate concerning indeterminism. And the B-theoretic approach, moreover, is particularly comfortable when it comes to account for indeterminism.

rejected on a branching view. Thus, even if both divergence and branching can reject the Aristotelian and the Master arguments, they represent two alternative accounts of indeterminism. In what follows, I argue that the branching conception of time has some virtues that divergence lacks. In particular, the branching view is better suited than the divergentist one when it comes to explain what does it mean that there are many ways things can turn out in the future. Furthermore, the two main arguments agains branching – those elaborated by David Lewis (1986a) – are far from being compelling. Thus, it is reasonable to reject the fatalistic arguments we encountered in the first chapter by adopting a branching conception of indeterminism.

2.1 Divergence

The divergentist approach is one way to refine of our intuitive, pre-analytic definition of objective indeterminism. Divergentist theorists analyse objective indeterminism in terms of physically possible worlds, where a physical possible world is a temporally complete course of events compatible with a given set of physical laws. Hence, whether a world is possible, it is so relative to a given set of laws. Divergentists assume – quite plausibly – that it is sensible to talk about the state that a physical world has at a time: thus, sentences of the form "World w is such-and-such at time t" are taken as perfectly meaningful. For the sake of simplicity, the state of a world at a time can be thought of as a spatially complete, temporally instantaneous slice of that world. Furthermore, the divergentist approach consists in two main thesis:

No-Overlap: Physically possible worlds do not overlap.

Duplicate: Physically possible worlds bear significant similarity relations to one another.

The *No-overlap* condition can be explained as follows: given a set W of physically possible worlds, if $w, w' \in W$ are numerically distinct at a time, they are numerically distinct at any time. Equivalently, if w and w' are identical at a time, they are identical at any time (that is, w and w' are the very same world). There are, as we shall see in the next sections, several arguments that philosophers have used to motivate the *No-overlap* condition. But for the time being, I shall confine myself to saying that, according to the *No-overlap* condition, distinct possible worlds are taken to be both spatiotemporally separated and causally inert to one another.

The second main feature of divergentism is the *Duplicate* principle, according to which worlds may bear interesting similarity relations to one another. But what kind of relation(s) do they have to bear to one another to be similar in the relevant way? John Earman answers as follows:

> [...] agreement [viz. similarity] of worlds at a time means agreement at that time of all relevant physical properties. (Earman 1986: 13)

According to Earman, to say that two worlds are similar at a time is to say that the two worlds are indiscernibile with respect to to the physical properties they instantiate at that time. This view seems to share with that of Lewis the idea that similarity means indistinguishability of *natural properties*.

According to Lewis, (perfect) natural properties are instinct properties that fundamental physical entities locally instantiate.

> Maybe [there are] points of spacetime itself, maybe point-sized bits of matter or aether or fields, maybe both. And at those points we have local qualities: perfectly natural intrinsic properties which need nothing bigger than a point at which to be instantiated. For short: we have an arrangement of qualities. And that is all. (Lewis 1986b: ix-x)

Physics – at least in its ideally, ultimate formulation – discovers (perfect) natural properties. These properties, in turn, help to specify when two physically possible worlds are indiscernible, or, as Lewis calls them, when two worlds are *duplicates*.

> Physics is relevant because it aspires to give an inventory of natural properties – not a complete inventory, perhaps, but a complete enough inventory to account for duplication among actual things. If physics succeeds in this, then duplication within our world amounts to sameness of physical description. But the natural properties themselves are what matter, not the theory that tells us what they are. (Lewis 1983a: 356-357)

Lewis then considers two entities, say x and y, as duplicates just in case (i) x and y have exactly the same perfect natural properties, and (ii) their parts can be put into correspondence in such a way that corresponding parts have exactly the same perfect natural properties, and stand in the same perfectly natural relations.[2]

[2]See Lewis (1983a: 360).

As long as the relevant similarity relations obtain between two worlds, they do not diverge. As soon as the similarity relations fail to hold between two worlds, each of them diverge relative to the other. As Lewis notices,

> I shall say that two possible worlds diverge iff they are not duplicates but they do have duplicate initial temporal segments. Thus our world and another might match perfectly up through the year 1945, and go their separate ways thereafter. (Lewis 1983a: 359)

For instance, imagine two worlds, say w and w', where any particle in w has a copy in w' and vice versa. Up to a time t, any particle in w has exactly the same mass, velocity and position of its copy (assuming that these properties are natural). Thus, up to t, the two worlds have two indiscernible, temporal evolutions, and they look as if they were the same world. But suppose that, at time t', a particle in w changes its velocity, where no particle in w' changes in the same way. Then, the two worlds cease to look perfectly alike, and they are said to diverge. The two worlds, however, are governed by the very same laws at any time.

2.1.1 Kamp Frames and Ockhamism

The *No-Overlap* and the *Duplicate* principles find a natural, formal representation in what are been called *Kamp frames*.[3] Kamp frames, as we shall see, have two roles: first, they allow to define objective indeterminism in terms of similarities among possible worlds. Secondly, they may be used to interpret a temporal language, and thus to define a temporal logic. In turn, the temporal logic based on a Kamp frame should give precise answers to the two questions of the previous chaprer, (Q1) and (Q2).

Definition 2.1.1 (*Kamp Frames*)**.** *A Kamp frame $\mathcal{K}$ is a triple $(W, T, \approx)$ such that*

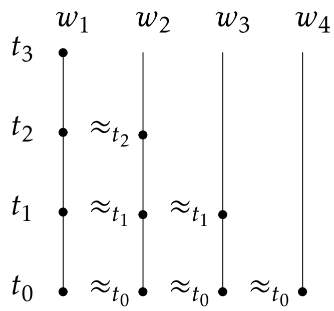
- *(a) W is a non empty set of worlds,*
- *(b) T a function assigning to any $w \in W$ an irreflexive, linear order $T(w) = (M_w, <_w)$,*
- *(c) $\approx$ is a relation on $\{(t, w, w') : w, w' \in W$ and $t \in M_w \cap M_{w'}\}$ such that*
  - *(c.1) for all t, $\approx_t$ is an equivalence relation and if $w \approx_t w'$, then $\{t' : t' \in M_w$ and $t' <_w t\} = \{t'; t' \in M_{w'}$ and $t' <_{w'} t\}$,*
  - *(c.2) if $w \approx_t w'$ and $t' <_w t$, then $w \approx_{t'} w'$.*

[3] See Thomason (1984: 147), Zanardo (2006: 386).

Condition (b) associates any world with its timeline: any world w has its own 'clock', where M_w is the set of the times of w, and $<_w$ is the later than relation. Hence, $t_0 <_w t_1$ means that time t_1 is later than t_0 relative to the timeline of world w. As clear from above, timelines have two roles. On the one hand, they track the'passage of time' taking place at each world. On the other hand, they signal for how long two worlds remain indistinguishable. From a metaphysical viewpoint, however, one is free to adopt a substantialist view – and, thus, conceive times as primitive, metaphysical entities – or a relationalist view – according to which times are reducible to the relations obtaining between worlds and the objects existing at them.

Clause (c) defines the similarity relation between worlds. (c.1) guarantees that, if w is similar to w' at time t ($w \approx_t w'$), the two worlds share the same timeline up to t. (c.2) says that, when w is similar to w' at time t, the two worlds continue to be similar at any earlier time. This latter clause reflects Lewis' suggestion, according to which two divergent worlds match together for an initial segment. The clause also entails that, whenever two worlds stop to match, they cannot match once again in the future. Thus, we can take $w \approx_t w'$ to mean that w and w' do not diverge, but are similar, at time t. Other equivalent readings are

- At time t, w and w' are indiscernible with respect to the natural properties they instantiate.
- At time t, w and w' are duplicates of one another.



Once we have *Kamp frames*, we can define Objective Indeterminism in the way suggested by Earman (1986: 13):

Definition 2.1.2 (*Divergence Objective Indeterminism* (DOI))**.** *Given a set of (physically) possible worlds W – that is, a collection of worlds which satisfy the same physical laws – a world $w \in W$ is objectively indeterministic iff there is a $w' \in W$ and two times $t, t' \in M_w$ such that $t <_w t'$, $w \approx_t w'$ but $w \not\approx_{t'} w'$.*

The figure above represents a Kamp frame with divergent worlds. $w_1,\ldots,$ w_4 are taken as physically possible worlds, where $t_0,\ldots,t_3$ form the timeline of w_1. World w_1 is $\approx$-related to w_2, w_3, and w_4 at t_0, but it diverges from w_4 after t_0, from w_3 after t_1, and from w_2 after t_2. Hence, there is a 'starting time', t_0, at which all the worlds are indiscernible from one another. As time goes by, w_1 evolves in such a way that it differentiates itself from the other possible worlds, and becomes qualitatively different from any other world at time t_3. Thus, by DOI, w_1 is indeterministic.[4]

Kamp frames allows to define what may be called the Kamp Ockhamist semantics, i.e. a semantics for a modal temporal language such as $\mathcal{L}_T$. As we shall see, the Kamp Ockhamist semantics is useful to answer the two questions raised in the previous section, that is:

(Q1) Are we to drop indeterminism, bivalence, or neither?

(Q2) Are we to drop indeterminism, *PR*, or *PNP*?

First, recall that $\mathcal{L}_T$ is a propositional, temporal modal language, whose set of wffs is obtained by this rule.

$$\mathcal{L}_T ::= p \mid \neg\mathcal{A} \mid \mathcal{A} \vee \mathcal{B} \mid \mathsf{F}(n)\mathcal{A} \mid \mathsf{P}(n)\mathcal{A} \mid \Box\mathcal{A}$$

Now, both $\mathsf{F}(n)$ and $\mathsf{P}(n)$ are *metric* tense operator, for they can convey how far in the future (past) an event takes place. Accordingly, we need a metric that tells us the temporal distance between, say, two events with different temporal locations. A *duration function* will do the job.[5]

Definition 2.1.3. *($\mathcal{K}$-duration function) Given a Kamp frame $\mathcal{K} = (W, T, \approx)$, for any $w \in W$, $d_{\mathcal{K}}$ maps couples of times in M_w to the set $Y - \{0\}$, where Y is a set of positive numbers whose cardinality is no more than continuous. In addition, $d_{\mathcal{K}}$ fulfils the following conditions.*

(a) $d_{\mathcal{K}}(t,t') =\uparrow$ just in case $t = t'$. This conditions avoids $n > 0$ as the value of the distance between an arbitrary time and itself, for any $n \in Y - \{0\}$. Moreover, it ensures that $d_{\mathcal{K}}$ is definite for any two times in the same timeline.

(b) If $t \neq t'$, then $d_{\mathcal{K}}(t,t') = d_{\mathcal{K}}(t',t)$. The value of the distance between t and t′ is the same as the value of the distance between t′ and t.

[4]One may attempt to extend the definition of divergence objective indeterminism to laws, and say that a set of laws is indeterministic iff – given the set of possible worlds compatible with them – there is (at least) an indeterministic world in that set.

[5]If $d_{\mathcal{K}}(t,t') =\uparrow$, function $d_{\mathcal{K}}$ is undefined for (t,t').

(c) *If $t \in M_w$, there exists at most one $t' \in M_w$ such that $d_{\mathcal{K}}(t,t') = n$ and $t <_w t'$ ($t' <_w t$). This condition ensures that there can be at most just one time t' in the timeline of world w which has n as the value of its distance from another time t in that timeline, and that is later (eralier) than t.*

(d) *If $d_{\mathcal{K}}(t,t') = n$, $d_{\mathcal{K}}(t,t'') = m$ and $t <_w t' <_w t''$, then $n < m$. This condition states that $d_{\mathcal{K}}$ preserves the order among times and the values of their distances.*

Intuitively, if $d_{\mathcal{K}}(t,t') = n$ and $t <_w t'(t' <_w t)$, n is the number of time units at which t' lies in the future (past) of t along the timeline associated to world w.

Now that we have a duration function, the Kamp Ockhamist semantics (KO) can be defined by evaluating formulas at time-world pairs (w,t) of $\mathcal{K}$, where the time t of such a point of evaluation is one of the timeline associated with world w (that is, $t \in M_w$). This strategy leads to the following definition.

Definition 2.1.4 (*KO-model*)**.** *A KO-model for $\mathcal{L}_T$ is a tuple $\mathcal{M}_{KO} = (\mathcal{K}, d_{\mathcal{K}}, I)$, where:*

(a) *$\mathcal{K} = (W, T, \approx)$ is a kamp frame,*

(b) *$d_{\mathcal{K}}$ is a $\mathcal{K}$-duration function,*

(c) *I is an interpretation function from the atoms of $\mathcal{L}_T$ to the set $\{(t,w) : w \in W \text{ and } t \in M_w\}$, and it satisfies*

   (c.1) *If $(t,w) \in I(p)$, then, for any w' s.t. $w \approx_t w'$, $(t,w') \in I(p)$.*

Intuitively, $(t,w) \in I(p)$ means that the atom p holds at time t in world w. Moreover, condition (c.1) captures the idea that, whenever two worlds are duplicates of one another at a time, the propositional language cannot distinguish them for what happens at them at that time. For instance, if there is a sea battle at time t in w, and w' is a duplicate of w at t, then there must be an indistinguishable sea battle at t in w' as well. This, obviously, does not prevent the propositional language to be able to tell apart w from w' in virtue of what will happen at each world at times later than t.

Then, it is easy to give recursive clauses for any wff of $\mathcal{L}_T$ (symbols "$\vDash_{KO}$" and "$\nvDash_{KO}$" denotes KO-truth and KO-falisty, respectively).

Definition 2.1.5 (*KO-semantics*)**.** *Given a KO-model $\mathcal{M}_{KO}$ for $\mathcal{L}_T$,*

(KO1) $(t,w) \vDash_{KO} p \Leftrightarrow (t,w) \in I(p)$

(KO2) $(t,w) \vDash_{KO} \neg\mathcal{A} \Leftrightarrow$ *not* $(t,w) \vDash_{KO} \mathcal{A} \Leftrightarrow (t,w) \nvDash_{KO} \mathcal{A}$

(KO3) $(t,w) \vDash_{KO} \mathcal{A} \vee \mathcal{B} \Leftrightarrow$ *either* $(t,w) \vDash_{KO} \mathcal{A}$ *or* $(t,w) \vDash_{KO} \mathcal{B}$

(KO4) $(t,w) \vDash_{KO} \mathsf{F}(n)\mathcal{A} \Leftrightarrow \exists t'(t' \in M_w \ \& \ t <_w t \ \& \ d_{\mathcal{K}}(t,t') = n \ \& \ (t',w) \vDash_{KO} \mathcal{A})$

(KO5) $(t,w) \vDash_{KO} \mathsf{P}(n)\mathcal{A} \Leftrightarrow \exists t'(t' \in M_w \ \& \ t' <_w t \ \& \ d(t,t')_{\mathcal{K}} = n \ \& \ (t',w) \vDash_{KO} \mathcal{A})$

(KO6) $(t,w) \vDash_{KO} \Box\mathcal{A} \Leftrightarrow \forall w'(w' \in W \ \& \ w \approx_t w' \Rightarrow (t,w') \vDash_{KO} \mathcal{A})$

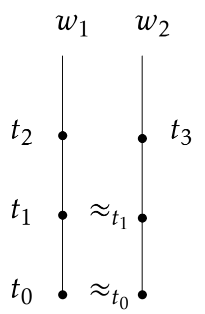
What is interesting about KO is the way in which it defines the truth conditions for modal and tensed statements. Intuitively, KO evaluates a sentence of the form $\mathsf{F}(n)\mathcal{A}$ as true at (t,w) iff $\mathcal{A}$ is true at the time that lies n time units in the future of t along world w (the truth conditions for statements of the form $\mathsf{P}(n)\mathcal{A}$ are symmetric, since they require to move n time units backward from t along w). Moreover, a sentence of the form $\Box\mathcal{A}$ is true at (t,w) iff $\mathcal{A}$ is true at any world which is $\approx_t$-related to w_t – that is, which is a duplicate of w a time t. Historical necessity at (t,w) is captured in terms of truth at any duplicate of w at time t.

KO allows to give precise answers to the questions (Q1)-(Q2). As for the answer the first question, (Q1), it is easy to see that KO is perfectly classical and bivalent, in the sense that:

KO-bivalence: For any wff $\mathcal{A}$ of $\mathcal{L}_T$ and any point (t,w), either $(t,w) \vDash_{KO} \mathcal{A}$ or $(t,w) \nvDash_{KO} \mathcal{A}$.

Notice, moreover, that KO-truth ($\vDash_{KO}$) and KO-falsity ($\nvDash_{KO}$) are mutually exclusive but jointly exhaustive. Thus, it is perfectly consistent to stick to the notion of indeterminism (as it is defined by DOI) and subscribe to the KO-bivalence.

As far as the second question is concerned, (Q2), it is easy to check that there are countermodels in which *PNP* ($\mathsf{P}(n)\mathcal{A} \to \Box\mathsf{P}(n)\mathcal{A}$) fails. See, for instance, the following partial representation of a $\mathcal{M}_{KO}$-model.



Assume that p is KO-true at (t_2,w_1) but KO-false at (t_3,w_2).

If p is KO-true at (t_2, w_1) but KO-false at (t_3, w_2), and $d_{\mathcal{K}}(t_0, t_1) = d_{\mathcal{K}}(t_1, t_2) = d_{\mathcal{K}}(t_1, t_3) = 1$, an instance of *PNP* such as $\mathsf{P}(1)\mathsf{F}(2)p \rightarrow \Box\mathsf{P}(1)\mathsf{F}(2)p$ is KO-false at (t_1, w_1). Thus, the Master argument can be contested within a divergentist approach, since the KO-semantics does not validate one of its premises.

In conclusion, divergence offers a way to define indeterminism in terms of duplicate possible worlds. Furthermore, it is possible to define a suitable semantics which is KO-bivalent and which refutes the Master argument. In the next section, the main indeterminist rival framework to divergence, that is, the branching approach, will be analysed.

2.2 Branching

The branching time approach is the main rival of the divergentist strategy in order to account for Objective Indeterminism. As the divergentists, branching theorists take possible worlds as temporally complete courses of events, but they stick to the following two main tenets.

Overlap: Any two physically possible worlds do overlap with one another.

Branching: Physically possible worlds branch towards the future only.

The *Overlap* condition can be explained by contrast to the divergentist approach. Recall one of the examples given above, according to which, up to a time t, any temporal slice of w has an exact copy in *another* world w'. According to the divergentist approach, w and w' are indiscernible up to t, even though they are numerically distinct. Branching theorists are happy to accept that, up to t, any slice of w has an exact copy in world w'. But contrary to divergentists, they hold that any slice of w up to t and its copy (at that time) cannot be conceived as two distinct entities: they are numerically the very same slice. This allows branching theorists to explain the divergentist claim that, up to t, w and w' look as if they were the same world by saying that, up to that moment, they are indeed the same world. Equivalently, there are no two initial segments of w and w' up to t, but there is just one initial segment of both w and w' up to that time. Suppose, for instance, that it is possible for Peter to turn left, but it is also possible for him to turn right. According to a branching theorist, there are two worlds (histories), and

> [...] each of the two histories literally contains Peter's indecision, and everything that led up to it. A mental or linguistic

> theory would make these pasts "similar" instead of straightforwardly identical. But that is wrong. We began by contemplating that there were for Peter two possibilities, left and right, just as I have two shoes, say L and R. And just as it is unhelpful to say that L's owner is only "similar" to R's owner, but not identical, so it is unhelpful to say that the indecision in the past of Peter's possible left turn is only "similar," but not identical, to the indecision lying in the past of Peter's possible right turn. (Belnap 2002: 6)

Thus, possible world are not spatiotemporally separated, for they share instantaneous but maximally extended slices.

The *Overlap* principle does not only say that possible worlds overlap, but that *any* possible world overlap with any other. This means that, for any stage that a possible world may achieve, not matter how remotely far in the future, that stage has an instantaneous slice which is its historical ancestor, and which is common to any possible world. Nuel Belnap calls this feature "Historical connection". In his view, it has a relevant metaphysical import:

> [...] every possible history, h, has a definite causal relation to the very moment in which we converse, since h must share with it a common past. Historical connection is the postulate that endows the theory with a sense of robust reality. (Belnap et al. 2001: 188)

The idea here is that anything that is possible in the future has a common root in the past. This should give to the branching approach a "sense of robust reality", for the common root that possible worlds share partially explains why they are possible future evolutions of the very same past. What has happened up to now can be viewed as a causal chain for the future possibilities which are now available. Obviously, this means that the notion of cause, given a branching setting, does not necessitate its effects.[6]

The branching view also states that physically possible worlds branch towards the future only (this is the *Branching* condition). Suppose that a divergentist says that there is a time later than t at which a particle a in w changes its velocity, but a's counterpart in w' does not change its velocity after t. According to the divergentist approach, the two worlds diverge after t. Branching theorists, on the other hand, would assess this description as inadequate. They would say that, up to t, a inhabits both

[6]See Belnap (2005) and Placek (2000: Ch. 1).

world w and w', but the two worlds split at t. Particle a has a possible future in w where it changes its velocity. But the very same particle has also another possible future in w', where its velocity does not change. As Lewis puts it,

> [In a branching setting] there is one initial spatiotemporal segment; it is continued by two different futures – different both numerically and qualitatively – and so there are two overlapping worlds. One world consists of the initial segment plus one of its future; the other world consists of the identical initial segment plus the other future. (Lewis 1986a: 206)

Thus, not only worlds do overlap, but an entity's having more than one possible future is explained in terms of forward branching.

Notice that the *Branching* condition prohibits worlds to branch towards the past. This prohibition reflects the idea that the past is settled or unique. If it was sunny yesterday, it is now inevitable that it was sunny. However, it may well be that it is possible that tomorrow it will be sunny, but it is also possible that tomorrow it will be rainy. Contingency is only available for future events. Hence, the *Branching* principle posits a modal asymmetry between the past and the future. In turn, this asymmetry is often justified by making reference to the so-called "arrow of time", which appears to be implicit within some aspects of modern physics.

> [...] the use of probabilities in statistical mechanics and quantum theory is necessarily always forward directed since the past is factual and the future open. If irreversibility is introduced on a fundamental level as proposed then the coincidence of the different "arrows of time" (psychological, thermodynamic, cosmological...) is immediate and in particular dissociated from any cosmological model. (Haag 1990: 250)

> It is not usually explicitly stated, but it is obvious that these transition chances which standard quantum mechanics prescribes are forwards transition chances, standard quantum mechanics does not prescribe any backwards transition chances. Thus quantum mechanics implies that there is an objective arrow of time. (Arntzenius 1995: 77)[7]

[7] According to McCall (1976, 1994), the branching structure that physical possibilities may display account for an objective notion of time flow.

Both the *Overlap* and the *Brancing* principles are useful to highlight some crucial differences between branching and divergence. In a branching setting, if an entity x exists at a world w and has more than one possible future after t, then x literally inhabits more than one world at t. Indeed, x is located at any world that overlaps at t and branches afterwards. Thus, the branching view is incompatible with the following, Lewisian thesis.

World-boundness: Nothing is in two worlds. (Lewis 1968: 114)

The *WorlWorldd-boundness* principle, on the other hand, is perfectly consistent with a divergentist framework. A divergentist can claim that the *World-boundness* principle is compatible with an entity's having more than one possible future. Indeed, one can say that x – existing *only* in world w – has more than one possible future after t iff it has a duplicate y in w' at t, but there is a time t' later than t such that y fails to be a duplicate of x at t'. As we shall see, the assessment of the *World-boundness* thesis will be relevant to compare divergence against branching.

2.2.1 Trees and Ockhamism

Now, let us see how to formalise the branching approach through the so-called branching time frames.

Definition 2.2.1. *(Branching time frames) A branching time frame is a couple* $\mathcal{T} = (M, <)$, *where* M *is a non-empty set of moments and* $<$ *is a strict partial order on* M *satisfying the following conditions.*

(a) $\forall m, m', m''(m' < m \wedge m'' < m \Rightarrow m' = m'' \vee m' < m'' \vee m'' < m')$,
(b) $\forall m, m'(m \neq m' \wedge m \not< m' \wedge m' \not< m \Rightarrow \exists m''(m'' < m \wedge m'' < m'))$.

The strict partial order $<$ mimics the later than relation between moments in time. Thus, if $m < m'$, m' is later than m or, alterntively, m' is a possible future moment for m. Notice that moments are generally taken to be instantaneous, temporally complete world-slices. Intuitively, each moment conveys an information analogous to that encoded in a world-time couple (w, t) of a Kamp frame. A relevant logical difference, however, is that branching frames take moments as primitive entities, and possible worlds (viz. histories) are constructed out of them. On the other hand, Kamp frames take both worlds and times as primitives, and instantaneous world-slices – such as (w, t) – are constructed out of those primitives.

Clause (a) is often called the *no backward branching* condition, and it reflects the idea that the past of any moment is settled or unique. In other terms, (a) guarantees that, given any moment m, there are no alternative

<-unrelated moments in the past of m. Condition (b), instead, says that if there are two moments that are not <-related, there is (at least) one moment in their past. This clause expresses that any two possible moments share at least a common past moment. Clause (b) corresponds to the "Historical connection" condition discussed above.

In a branching time setting, possible worlds are called histories, which are maximally <-chains of $\mathcal{T}$.

Definition 2.2.2 (*Histories*). *A history h is a subset of $\mathcal{T}$ such that*

(a) $\forall m, m'(m, m' \in h \Rightarrow m < m' \vee m' < m \vee m = m')$,
(b) No subset g of $\mathcal{T}$ which satisfies condition (a) is such that $h \subset g$.

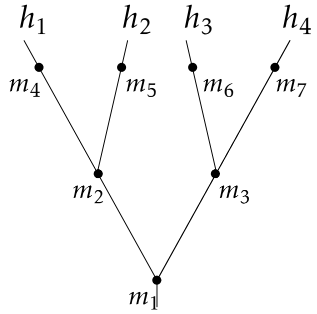
Any history, again, represents a possible, temporally complete courses of events specified by $\mathcal{T}$. Each history, thus, is a path reality might take in its evolution (if any) from the past towards the future. The set of histories obtainable from a branchin frame $\mathcal{T}$ will be denoted as $H(\mathcal{T})$. Histories h and h' overlap at m if $m \in h$ and $m \in h'$. Moreover, h and h' are undivided at m if there is a moment m' at which they overlap and which is later than m.

Branching time frames can be used to obtain the following definition of objective indeterminism:

Definition 2.2.3. *(Branching Objective Indeterminism* (BOI)) *Given a set of physically possible histories (worlds) $H(\mathcal{T})$, a history $h \in H(\mathcal{T})$ is objectively indeterministic iff there is a moment m and a history $h' \in H(\mathcal{T})$ such that $m \in h$, $m \in h'$ and $h \neq h'$.*

As Zanardo (2006: 380) notices, the second-order, left-hand side of the biconditional is equivalent to the following, first-oder clause:

$$\exists m, m', m''(m < m' \ \&\ \ m < m'' \ \&\ \ m' \neq m'' \ \&\ \ m' \not< m'' \ \&\ \ m'' \not< m').$$



The tree-like graph above is a representation of a branching time frame. History h_1 passes through moments m_1, m_2 and m_4. Histories h_1 and h_2 overlap at m_1 and m_2, are undivided at m_1, but they branch at m_2. It is worth noting that there are moments in the tree which do not bear the later than relation to one another. For instance, m_2 and m_3 are not $<$-related: they are two alternative moments belonging to histories that have already branched at an earlier moment. Thus, by BOI, histories h_1, h_2, h_3 and h_4 are indeterministic.

Branching frames are usually coupled with a semantics – which it will be called Branching Ockhamist semantics, BO – which is analogous to KO, and which allows to give a straightforward interpretation to the modal temporal language $\mathcal{L}_T$. In turn, with the BO at hand, a branching indeterminist can attempt to answers to both (Q1) and (Q2).

Recall, once again, that $\mathcal{L}_T$ has two metric temporal operators, $\mathsf{P}(n)$ and $\mathsf{F}(n)$. Thus, one needs a duration function analogous to $d_{\mathcal{T}}$.

Definition 2.2.4 (*$\mathcal{T}$-duration function.*). *Given a tree $\mathcal{T}$, $d_{\mathcal{T}}$ is a $\mathcal{T}$-duration function which maps couples of $<$-related moments to elements of $Y - \{0\}$, where Y is a set of positive numbers whose cardinality is no more than continuous. In addition, $d_{\mathcal{T}}$ fulfils the following conditions.*

(a) $d_{\mathcal{T}}(m, m') =\uparrow$ just in case $m = m'$. This conditions avoids $n > 0$ as the value of the distance between an arbitrary time and itself, for any $n \in Y - \{0\}$. Moreover, it ensures that $d_{\mathcal{T}}$ is defined for any two $<$-related moments.

(b) If $m \neq m'$, then $d_{\mathcal{T}}(m, m') = d_{\mathcal{T}}(m', m)$. The value of the distance between m and m' is the same as the value of the distance between m' and m.

(c) For any $m \in h$, there exists at most one $m' \in h$ such that $d_{\mathcal{T}}(m, m') = n$ and $m < m'$ ($m' < m$). This condition ensures that, for each history h to which m belongs, there can be at most just one moment m' in the same h which has n as the value of its distance from m, and that is later (earlier) than m.

(d) If $d_{\mathcal{T}}(m, m') = n$, $d_{\mathcal{T}}(m, m'') = n'$ and $m < m' < m''$, then $n < n'$. This condition states that $d_{\mathcal{T}}$ preserves the order among moments and the values of their distances.

The Branching Ockhamist semantics, BO, can be defined by evaluating formulas at moment-history pairs (m, h) of $\mathcal{T}$, where the history of the pair passes trough the moment of the pair (i.e., $m \in h$; m/h abbreviates moment-history pairs that satisfy such a condition).

Definition 2.2.5 (*BO-model*). *A BO-model for $\mathcal{L}_T$ is a tuple $\mathcal{M}_{BO} = (\mathcal{T}, d_{\mathcal{T}}, I)$, where:*

(a) $\mathcal{T} = (M, <)$ *is a tree,*
(b) $d_{\mathcal{T}}$ *is a* $\mathcal{T}$*-duration function,*
(c) I *is an interpretation function from the atoms of* $\mathcal{L}_T$ *to the the set of moment history pairs* m/h*, and it satisfies*

(c.1) If $m/h \in I(p)$*, then, for any* h' *s.t.* $m \in h'$, $m/h' \in I(p)$.

If $m/h \in I(p)$, then p is BO-true at moment m, along history h. Furthermore, condition (c.1) ensures that the truth-values of present tensed sentences – i.e., the truth values of the atoms of $\mathcal{L}_T$ – are history-independent. This, in turn, mirrors that if it is true that there is a sea battle, it is (now) inevitably so. In a sense, the present cannot be modified, it is settled. Let us go on, and define the recurses clauses of BO as follows (again, "$\vDash_{BO}$" and "$\nvDash_{BO}$" denotes BO-truth and BO-falisty, respectively).

Definition 2.2.6 (*BO-semantics*)**.** *Given a BO-model* $\mathcal{M}_{BO}$ *for* $\mathcal{L}_T$*,*

(BO1) $m/h \vDash_{BO} p \Leftrightarrow m/h \in I(p)$

(BO2) $m/h \vDash_{BO} \neg\mathcal{A} \Leftrightarrow$ *not* $m/h \vDash_{BO} \mathcal{A} \Leftrightarrow m/h \nvDash_{BO} \mathcal{A}$

(BO3) $m/h \vDash_{BO} \mathcal{A} \vee \mathcal{B} \Leftrightarrow$ *either* $m/h \vDash_{BO} \mathcal{A}$ *or* $m/h \vDash_{BO} \mathcal{B}$

(BO4) $m/h \vDash_{BO} \mathsf{F}(n)\mathcal{A} \Leftrightarrow \exists m'(m' \in h \;\&\; m < m' \;\&\; d_{\mathcal{T}}(m, m') = n \;\&\; m'/h \vDash_{BO} \mathcal{A})$

(BO5) $m/h \vDash_{BO} \mathsf{P}(n)\mathcal{A} \Leftrightarrow \exists m'(m' \in h \;\&\; m' < m \;\&\; d_{\mathcal{T}}(m, m') = n \;\&\; m'/h \vDash_{BO} \mathcal{A})$

(KO6) $m/h \vDash_{BO} \Box\mathcal{A} \Leftrightarrow \forall h'(m \in h' \Rightarrow m/h' \vDash_{BO} \mathcal{A})$

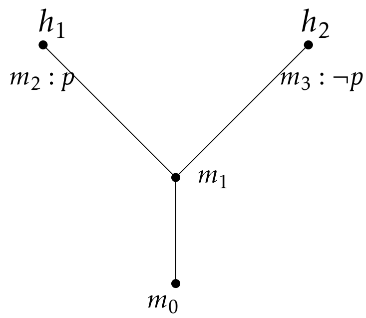
BO evaluates a sentence of the form $\mathsf{F}(n)\mathcal{A}$ as true at m/h iff $\mathcal{A}$ is true at the moment that lies n time units in the future of m, along history h (again, the truth conditions for statements of the form $\mathsf{P}(n)\mathcal{A}$ are symmetric). Historical necessity at a moment-history pair, on the other hand, means truth at any history passing through that moment.

It is easy to see that BO satisfy the following notion of bivalence,

BO-bivalence: For any wff $\mathcal{A}$ of $\mathcal{L}_T$ and any point m/h, either $m/h \vDash_{BO} \mathcal{A}$ or $m/h \nvDash_{BO} \mathcal{A}$,

where, as for the KO semantics, BO-truth ($\vDash_{BO}$) and BO-falsity ($\nvDash_{BO}$) are mutually exclusive but jointly exhaustive. Thus, branching indeterminism – as it is defined by BOI – is perfectly compatible with the BO-bivalence. And this seems to answer (Q1).

On the other hand, BO refutes the principle *PNP* (i.e., $\mathsf{P}(n)\mathcal{A} \to \Box\mathsf{P}(n)\mathcal{A}$), and the Master argument turns out to be unsound if its premises are interpreted with the BO-semantics. Here's a countermodel to *PNP*.



In this countermodel it is assumed that p is BO-true at m_2 but BO-false at m_3. Moreover, $d_{\mathcal{T}}(m_0, m_2) =_{\mathcal{T}} (m_0, m_3) = 2$, and $d_{\mathcal{T}}(m_0, m_1) = 1$. It is easy to check that $\mathsf{P}(1)\mathsf{F}(2)p \rightarrow \Box\mathsf{P}(1)\mathsf{F}(2)p$ is BO-false at m_1/h_1.

To sum up, the branching approach seems to be as good as the divergentist view in answering (Q1)-(Q2). Each strategy allows to define indeterminism in terms of possible worlds, and a semantics which satisfies (a specific kind of) bivalence but fails to validate one premise of the Master argument, namely, the principle of the necessitation of the past (*PNP*). As already noticed, however, the two frameworks are at odds with each other, for a branching theorist is happy to accept the *Overlap* condition, while an advocate of a divergentist view explicitly rejects it. This dialectical situation leads to a comparison between the two frameworks.

2.3 Divergence and Branching at work

In what follows, I consider two objections against divergence which may constitute some motivations for the adoption of the branching view. On the other hand, several objections that philosophers have raised against the branching conception will be explored. All of these objections point to several, alleged difficulties that branching frames should have in meeting some physical and metaphysical *desiderata*. It will be argued that these objections are far from being convincing, and they do not represent good reasons to reject the branching approach. Therefore, I conclude that the branching strategy is better suited than the divergentist one in accounting for objective indeterminism.

2.3.1 Perfect natural properties and collapsing towers.

Several philosophers assess divergentism as inadequate: as Belot (1995: 188) and Placek (2000: 31) have stressed, the notion of duplicate may fail to account for some specific indeterministic phenomena.

Let us start by assuming the relation of *being-a-duplicate-of*, as it is defined by Lewis:

> [T]wo things are duplicates iff (1) they have exactly the same perfectly natural properties, and (2) their parts can be put into correspondence in such a way that corresponding parts have exactly the same perfectly natural properties, and stand in the same perfectly natural relations. (Lewis 1986a: 61)

Belot (1995: 189) notices that, among divergentist theorists, part-whole relations and spatiotemporal relations are usually taken to be two kinds of natural properties.[8] Thus, sameness of natural properties entails sameness of part-whole and saptiotemporal relations among counterparts. Accordingly, if a is a duplicate of c, and b is a duplicate of d, then an object composed of a and b is a duplicate of an object composed of c and d just in case the spatiotemporal relations between a and b are the same as those between c and d. Notice that the notion of duplicate can be used to account for that of similarity between worlds at a time. Let f a one-to-one map from the entities existing at (t,w) to those existing at (t',w'), and let us denote with $D(t,w)$ the set of entities existing at (t,w). A function f is a duplication just in case, for any entity x in world w at time t, there is another possible world w' such that $t \in M_{w'}$ and $f(x)$ is a duplicate in w', at time t, of x (that is, f conserves spatiotemporal relations and perfectly natural properties of spacetime points and temporal stages of particles).

Thus, let us say that w and w' are duplicates at a time t ($w \approx_t w'$) just in case there is a duplication $f : D(t,w) \longrightarrow D(t,w')$.[9] Now, let us use the notion of duplication to refine the definition of indeterminism provided by divergentists.

Definition 2.3.1 (*Divergence Objective Indeterminism** (DOI*))**.** *Given a set of physically possible worlds W, world $w \in W$ is objectively indeterministic iff there is a $w' \in W$ and two times $t, t' \in M_w$ such that $t <_w t'$, f is a duplication $f : D(t,w) \longrightarrow D(t,w')$, but there is no duplication g from (t',w) to (t',w') whose restriction to $D(t,w)$ is f.*

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[8]See also Lewis (1986a: Ch.1.6).

[9]In order to conserve the intuition that, if two worlds are duplicates of one another at a time, they are duplicates at any earlier time, one should impose that

> If there is a duplication $f : D(t,w) \longrightarrow D(t,w')$, then, for any t' such that $t' <_w t$, there exists a duplication $g : D(t',w) \longrightarrow D(t',w')$, where g is the restriction of f to $D(t',w)$.

This latter definition is more fine-grained than that of DOI, since the similarity relation is now based on indistinguishability of perfect natural properties among counterparts. Nevertheless, DOI* seems to yield counterintuitive results. There are cases in which a world appears to be indeterministic, even if, according to DOI*, it should count as deterministic.[10]

Take a world w_1 such that only newtonian laws of motion hold in it. Furthermore, assume that w_1 has only three objects: (i) a large, homogeneous, perfectly spherical planet; (ii) a relatively small perfectly cylindrical column resting on the planet so that its axis of symmetry is normal to the planet at the point of contact; (iii) a cone which, at t_1, hits with its aphex the center of the column's top surface, causing the column to buckle.[11] Now, the (newtonian) laws of motion governing the three objects in w_1 determines the shape of the collapsing column, but they do not say anything about the direction of the buckling. As Wilson (1993) puts it,

> At a critical value of the weight [...] the column suddenly sags. But this collapse is not unique; although the *shape* of the buckling is the same, the column is free to bulge in any 360° direction. (Wilson 1993: 216)

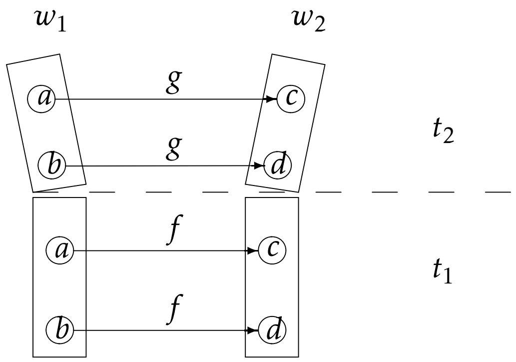
This latter feature suggests that the buckling should be classified as an indeterministic phenomena, and therefore w_1 should count as an indeterministic world.[12]

However, Belot (1995) has shown that, if there is there exists a world w_2 satisfying the newtonian laws, and if there is a duplication f such that $f : D(t_1, w_1) \longrightarrow D(t_1, w_1)$, then there must be an extension g of f from $D(t', w_1)$ to $D(t', w_2)$ that is itself a duplication, for any t' s.t. $t_1 <_{w_1} t'$. This kind of situation is pictured in the figure below.

[10]Obviously, if there is no w' that is physically possible with respect to w, then, by DOI*, w is indeterministic. However, this is nothing but a vacuous way to fulfil DOI*, and it will be ignored.

[11]The counterexample is due to Wilson (1993), and it is presented both in Belot 1995 and in Placek 2000.

[12]If the column argument is correct, it would show that, contrary to a common belief, even newtonian laws are indeterministic, given particular initial conditions. For a similar conclusion, see Earman 1986: 45, Norton 2008 and Echeverria et al. 1991.



The column in world w_1 – at the time of the buckling, t_1 – has two particles, a and b. The column which exists at t_1 in the other world w_2 has two particles as well, c and d. Moreover, at t_1, particle a is the duplicate of c (i.e., $f(a) = c$) and b is the duplicate of d ($f(b) = d$). At a time which immediately follows that of the buckling, t_2, the two towers start to collapse in different directions. However, there exists a duplication g which extends f at time t_2, and which ensures that both a and b continue to be duplicates of c ($g(a) = c$) and d ($g(b) = d$) even after the buckling. By DOI*, w_1 and w_2 are deterministic. But this contradicts the intuition that world w_1 (w_2) should be indeterministic, for the direction of the tower's collapse is not determined by the physical laws governing that world. As Wilson (1993: 216) concludes, in a universe consisting of such a column, the definition of indeterminism provided by DOI* would identify the differences among w_1 and w_2, "contrary to the conventional mathematical opinion that the governing theory dose *not* determine unique solutions". As a consequence, a statement such as

- It was possible that the tower would have collapsed in a different direction,

seems to be true at world w_1 (w_2) at t_2. But its truth cannot be traced back to a difference of natural properties between w_1 and w_2.

The counterexample, however, is far from being uncontroversial, given the key role that the spatial symmetries play in it. As Gordon Belot explains,

> [...] the laws have been chosen so that the only ambiguity in the evolution of the worlds is in the direction of collapse; but because of the symmetries of the worlds, it is always possible to give identical descriptions for them. (Belot 1995: 191)

Carolyn Brighouse (1997: 476) agrees with Belot in that the indiscernibility of the two worlds depends on the symmetries of the objects in w_1, but she holds that different spherically symmetrically tower collapses should be taken as physically equivalent phenomena. Indeed, according to Brighouse (1997), a divergentist may assess w_1 (w_2) as indeterministic only by individuating space-time points across possible worlds independently of their qualitative properties – that is, independently of whether they are occupied by matter, or fields, or what have you.

> [I]n the one world the outside of the elbow of collapse ends up lying at point p, and the inside of the elbow ends up lying at point q, while in the otherworld it is the other round inside lies on while the outside lies on way (the p' (p's counterpart), q'). We want to know the physical respect in which the futures differ. What is the property that p' has in the second world that guarantees that it's the counterpart of p in the first world? Since we have given up the hope of appealing to the qualitative properties of the points to determine counterparthood it is hard to see why we should favour p' as the counterpart of p over any other point in this second world, unless of course we think that p and p' have some non-qualitative property in common. (Brighouse 1997: 476)

The quotation suggests that, if a divergentist wants to assess w_1 as indeterministic, she has to say that spatial points exemplify haecceities, that is, non-qualitative properties that are responsible for their identity. Moreover, the duplicate link should be based on some relation among haecceities. This latter relation, however, cannot be that of sameness. Indeed, haecceity is the property in virtue of which a thing is what it is. Hence, if duplicate space points were to share the same haecceity, they would be the same identical point. This means that, if one takes Belot's argument as a genuine case for indeterminism, the identification of the counterparts of an object (at a time) requires a stronger principle that that of sameness of natural properties. And that's why an advocate of divergentism such as Brighouse (1997) wants to say that, at the end of the day, w_1 and w_2 are both deterministic, whatever one's intuitions may be.

A branching theorist would be happy to accept that the identification of the counterparts needs a stronger principle that that of sameness of natural properties. For in a branching perspective the only relation responsible for the identification of the counterparts of an object (at a time) is that of numerical identity. Moreover, a branching theorist can account for the intuition that w_1 and w_2 should count as indeterministic. A branching theorist may say that, at t_1, the very same tower inhabits both w_1 and

w_2. These two worlds branch at t_2, and the continuation of the tower at t_2 in world w_1 is thus numerically distinct from its continuation at time t_2 in world w_2. Then, classical possible worlds can be framed in the way suggested by John Earman (1986: 24), according to which each classical world is a triple $w = \langle Ma, G, P \rangle$, where Ma is the space-time manifold $\mathbb{R}^4$, G is a set of geometric object fields on Ma characterising the structure of space-time (including, for example, three dimensional planes of simultaneity), and P the set of geometric object fields characterising the physical contents of space-time. Then, a branching theorist may say that in w_1 the plane of simultaneity at t_2 has a distribution of space-time content that differs from that of w_2 at that time. In other terms, matter at w_1 (at t_2) has different locations than those occupied in w_2 (at t_2).

This does not show that the tower's collapse case is sufficient to favour branching over divergence. It only points out that, if one assumes the tower's collapse as a genuine indeterministic phenomena, a branching theorist would be advantaged over a divergentist one. Under a branching perspective, the tower's collapse can be viewed as indeterministic. But this characterisation doesn't force the branching theorist to change any of her fundamental tenets (such as the *No-overlap* condition or the *Branching* principle). A divergentist theorist, on the other hand, can characterise the tower's collapse as indeterministic, but she needs to identify counterparts with a principle that is stronger than that of sameness of natural properties.

2.3.2 The expressive adequacy argument and the duplicate talk

Thomason (1984) and Belnap et al. (2001) argue that Kamp frames have an alleged, expressive adequacy problem. It has to do with the satisfiability of two (apparently) contradictory formulas, whose conjunction can be satisfied in some Kamp frame. I will argue that the expressive adequacy problem is question-begging, for it argues against divergence by implicitly assuming a branching conception of future possibilities. This assumption, however, is only natural, and it helps to highlight an intuitive feature about modality and reference that makes the branching approach preferable to the divergentist one.

First, let us see how Kamp frames can be structurally compared to branching time frames. As Zanardo (2006: 387) notices, Kamp frames can be seen as hiding a tree-like representation of time, and (almost) any Kamp frame corresponds to a branching time frame. Recall that, in a branching time frame $\mathcal{T} = (M,<)$, the moments in M are instantaneous world-slices, where an instantaneous world-slice of a Kamp frame $\mathcal{K} = (W,T,\approx)$ is a couple (w,t). Accordingly, one might take (classes of) world-time pairs (w,t) as moments of a tree. Furthermore, any world w in $\mathcal{K}$ can be read as a history of a branching frame, for branching possible worlds are maximal $<$-chains of moments. Moreover, when a divergentist says that two worlds are indiscernible at a time (say $w \approx_t w'$), a branching theorist replies that the two worlds should be conceived as overlapping. Accordingly, $\approx$ can be interpreted as an overlapping relation.

Technically, these analogies can be formalised as follows. Given a Kamp frame $\mathcal{K}$, let us consider any element in

(i) $[(w,t)] = \{(w',t) : w \approx_t w'\}$

as a moment, and

(ii) $M_{\mathcal{K}} = \{[(w,t)] : w \in W \text{ and } t \in M_w\}$

as the set of moments obtained from $\mathcal{K}$. Let us define a binary relation $<_{\mathcal{K}}$ on $M_{\mathcal{K}}$ in the following way: given $(w'',t'') \in [(w,t)]$ and $(w''',t''') \in [(w',t')]$,

(iii) $[(w,t)] <_{\mathcal{K}} [(w',t')] \Leftrightarrow w'' \approx_{t'} w'''$ and $t <_{w''} t'$.

$<_{\mathcal{K}}$ is transitive, since both $\approx_w$ and $<_w$ are transitive; $<_{\mathcal{K}}$ is irreflexive, for $<_w$ is irreflexive. It is partial, given that whenever $w \not\approx_t w'$ and $w \not\approx_{t'} w'$, neither $[(w,t)] <_{\mathcal{K}} [(w',t')]$ nor $[(w',t')] <_{\mathcal{K}} [(w,t)]$. $<_{\mathcal{K}}$ is left-linear, given conditions (c.1)-(c.2) of Definition 1.2.1. Moreover, if it is the case that

(iv) For each world w specified by $\mathcal{K}$, there exists a w' and a time t in $\mathcal{K}$ such that $w \approx_t w'$,

whenever $[(w,t)]$ and $[(w',t')]$ are $<_{\mathcal{K}}$-unrelated, they have a common $<_{\mathcal{K}}$-ancestor. Hence, if (iv) is the case, $<_{\mathcal{K}}$ satisfies the properties of Definition 1.2.3, and $\mathcal{T}_{\mathcal{K}} = (M_{\mathcal{K}}, <_{\mathcal{K}})$ is a branching time frame.

One can go the other way round, and obtain a Kamp frame out of a tree. Given a tree $\mathcal{T} = (M,<)$, the correspondent Kamp frame $\mathcal{K}_{\mathcal{T}} = (W,T,\approx)$ is such that

(v) $W = H(\mathcal{T})$,
(vi) T is a map such that $T(h) = (\{m : m \in h\},<)$,
(vii) $h \approx_m h \Leftrightarrow m \in h \cap h'$.

As Zanardo (2006: 388) has shown, conditions (i)–(iv) and (v)–(vii) can be used to define two functions, $\mathcal{K} \mapsto \mathcal{T}_{\mathcal{K}}$ and $\mathcal{T} \mapsto \mathcal{K}_{\mathcal{T}}$. The function $\mathcal{K} \mapsto \mathcal{T}_{\mathcal{K}}$

yields the tree $\mathcal{T}_{\mathcal{K}}$ which corresponds to the Kamp frame $\mathcal{K}$, while $\mathcal{T} \mapsto \mathcal{K}_{\mathcal{T}}$ the Kamp frame $\mathcal{K}_{\mathcal{T}}$ corresponding to the tree $\mathcal{T}$.

One important feature of these functions is that $\mathcal{T}$ and $\mathcal{T}_{\mathcal{K}_{\mathcal{T}}}$ are always isomorphic structures, where it may be that $\mathcal{K}$ and $\mathcal{K}_{\mathcal{T}_{\mathcal{K}}}$ fail to be isomorphic. To see why, take a Kamp frame $\mathcal{K}_1 = (W, T, \approx)$ which has denumerably many worlds. One can enumerate these worlds simply by using naturals, w_0, w_1, etc. Suppose that T is such that, for any World $w_n \in W$, $T(w_n) = \{\mathbb{N}, <_{\mathbb{N}}\}$, where $<_{\mathbb{N}}$ is the usual order among natural numbers. Assume that $w_n \approx_t w_m$ just in case $t \leqslant_{\mathbb{N}} \min(n,m)$. Thus, for instance, $w_0 \approx_0 w_1$ but $w_0 \not\approx_1 w_1$. Analogously, $w_1 \approx_0 w_2$, $w_1 \approx_1 w_2$, but $w_1 \not\approx_2 w_2$. A partial representation of $\mathcal{K}_1$ is given in Figure A below.

Conditions (i)-(iv) allow to generate the correspondent tree $\mathcal{T}_{\mathcal{K}_1}$, and any world of $\mathcal{K}_1$ will correspond to a history in $\mathcal{T}_{\mathcal{K}_1}$. But there is a history in $\mathcal{T}_{\mathcal{K}_1}$ which does not correspond to any world of the original Kamp frame $\mathcal{K}_1$ (see Figure B below). This history is the chain

$$h_\omega = \{[(w_0, 0)], [(w_1, 1)], [(w_2, 2)], \ldots\}.$$

Hence, if one applies conditions (v)-(vii) and generates a Kamp frame $\mathcal{K}_{\mathcal{T}_{\mathcal{K}_1}}$ from $\mathcal{T}_{\mathcal{K}_1}$, it turns out that $\mathcal{K}_{\mathcal{T}_{\mathcal{K}_1}}$ has more worlds than the original frame $\mathcal{K}_1$. The structures thus obtained cannot be isomorphic.

In general, a tree $\mathcal{T}_{\mathcal{K}}$ has *emerging histories* that do not correspond to any world in $\mathcal{K}$, and this feature is responsible for the descriptive adequacy problem that Belnap et al. (2001) and Thomason (1984) attribute to Kamp frames. Let's run their expressive adequacy argument.

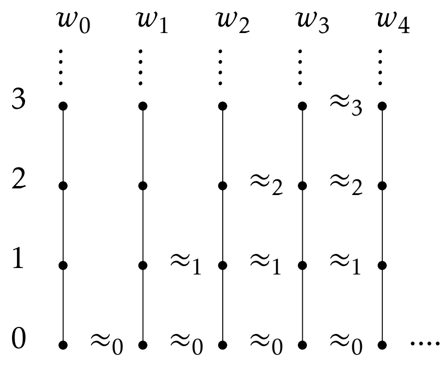


Figure A.

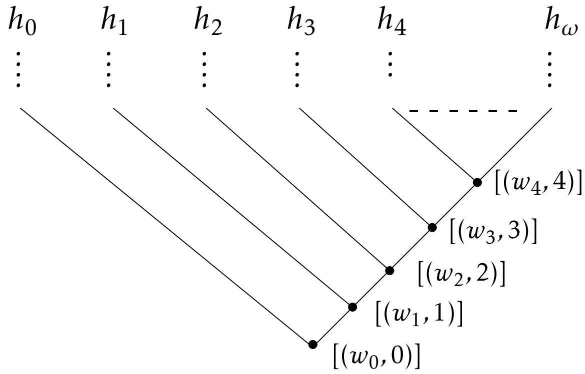


Figure B.

Suppose that any history in a tree is isomorphic to the reals, and has a clock that ticks at those moments which correspond to natural numbers. Assume that the following statement is true at the starting moment:

(1) As long as atom a has not yet decayed,

  (i) a might decay before the next tick, and
  (ii) a might not decay before the next tick.

The situation just described is pictured in Figure C, where p means "atom a has not yet decayed", and any node (viz. moment) in the three is a tick.

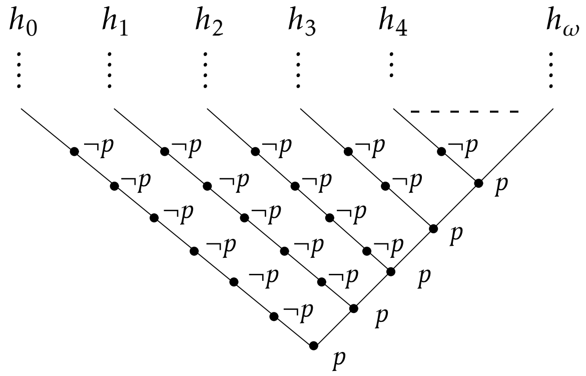


Figure C.

Statement (1), in turn, entails that

(2) Every no-decay chain of ticks of length n can be extended to a no-decay chain of length $n+1$.

Notice that the truth values of (1)-(2) do not depend on the existence of h_ω. On the contrary, the truth of

(3) At the starting moment, it is inevitable that a will decay after a finite number of ticks,

depends on whether h_ω is admitted or not. If it is admitted, (3) cannot be true, for h_ω is an infinite no-decay chain. If, on the other hand, history h_ω is excluded, (3) is true, since any possible course of events is such that a decays after a finite number of ticks on that course.

Belnap et al. (2001) and Thomason (1984) argue that (2)–(3) are contradictory claims, and that any adequate structure should falsify their conjunction. Nonetheless, the tree in Figure C can be seen as obtained by a Kamp frame whose missing history is h_ω. This Kamp frame would be analogous to that of Figure A. Just think of the $\approx$-related world-time pairs in Figure A as points at which p holds, where any pair which has no $\approx$-access to other pairs satisfies $\neg p$. It is easy to see that such a structure makes both (2) and (3) true. Indeed, any no-decay chain of thicks of length n can be extended to a no-decay chain of length $n+1$, even if at any chain, a decays after a finite number of ticks. According to Belnap et al. (2001) and Thomason (1984), this fact makes Kamp frames expressively inadequate.

This conclusion seems too hasty, and a particularly insightful reading of the expressive adequacy argument is given by Zanardo (2006):

> In private correspondence with Nuel Belnap, I suggested that the conviction that (2) and (3) contradict each other might depend on the implicit presupposition that time has a tree-like aspect (and not, for instance, a Kamp frame aspect). We feel that it is possible that atom a will never decay because the history h_ω is in some sense already there in our (tree-like) representation of time; we cannot pretend not to see h_ω. [...] This does not happen if, instead, we think of time in term of Kamp [...] frames. (Zanardo 2006: 394)

I think that Zanardo is right. The expressive adequacy argument is question-begging, for it presupposes a branching picture of time. There is no way to decide whether (2) and (3) are contradictory, unless a structure to interpret them is given.

There are further reasons that lead to reject the expressive adequacy argument. First, if (2) and (3) are jointly satisfiable – viz. they have a divergentist model – it is hard to see them as contradictory claims.

Secondly, the construction of the tree with the no-decay history, even if perfectly legitimate from a technical viewpoint, can be metaphysically suspicious. A divergentist may claim that the original Kamp frame is an exhaustive account of physically possible worlds. In other terms, that Kamp frame does not miss to specify any physically possible world. Thus, an advocate of the divergentist strategy may argue that h_w is physically impossible, and that the transition from a Kamp frame to a correspondent tree is wrongheaded. This reasoning allows the divergentist to say that, in a situation such as that of Figure A, (3) must be true, since it comes out false just in case h_ω is an admissible possible course of events. In turn, the apparent incompatibility between (2) and (3) can be explained away by saying that it rests on the wrong presupposition that the original Kamp frame is somehow incomplete. This supposition mistakenly suggests to implement the original Kamp frame with the emerging history h_ω, obtaining a tree similar to that of Figure C.

To sum up, the expressive adequacy argument is unsound, for it presupposes a branching representation of time. This illicit assumption, however, is only natural. When it comes to think about future possibilities, assuming a branching time structure is the most natural approach to adopt, for it is the one that best fits our intuitions about time, modality and reference. The intuitiveness that characterises the branching framework, in turn, makes it preferable to the divergentist approach. Let us see why.

The adequacy descriptive argument may hyde the different views that divergentists and branching theorists have about reference. Recall (1),

(1) As long as atom a has not yet decayed,
   (i) a might decay before the next tick, and
   (ii) a might not decay before the next tick.

Under a divergentist perspective, the truth of (1) at (w, t) entails that there is (at least) one duplicate of a, say b, which inhabits a world w' that spatiotemporally unrelated to w. Hence, if (1) is true at (w, t), it commits both to the existence of (at least) one duplicate of a, and to (at least) a possible world w' that is spatiotemporally unrelated to w. This means, furthermore, that there must be a possible world, other than w, at which "a" refers to b. On the other hand, a branching theorist doesn't want to say anything like that. The truth of (1) just means that the *very same temporal part*, say a–at–m, has access to two possible, future evolutions.In one of them, there is a decayed, temporal evolution of a–at–m, while in the other there is a stable, temporal evolution of a–at–m. But "a", even if refers to two distinct, temporal parts in the future of a–at–m, picks out two possible evolutions that the very same entity has after m. The latter interpretation is by far the closest one to what is intuitively conveyed by using modal statements. When someone says "It is possible that tomorrow it will be rainy", the information thus expressed is that there is a possible continuation of today in which it is rainy. In other terms, there is no evidence that one is talking about a spatiotemporal, causally unrelated world in which it will be rainy. Thus, the divergentist approach is recalcitrant to accept the most natural reading of the information conveyed by modal-tensed statements. As Kripke famously argued,

> [In a divergentist setting,] if we say 'Humphrey might have won the election (if only he had done such-and-such), we are not talking about something that might have happened to Humphrey but to someone else, a "counterpart".' Probably, however, Humphrey could not care less whether someone else, no matter how much resembling him, would have been victorious in another possible world. Thus, Lewis's view [viz. the divergentist approach] seems to me even more bizarre than the usual notions of transworld identification that it replaces. (Kripke 1980: 45, fn. 13)

Similarly, Thomas Placek argues that

> [...] about an hour ago I was nearly struck by a car. Hence I say, "I could have been run over, but fortunately I dodged out of the way". Am I really rejoicing because in another possible world my twin was run over, but in this one I was saved? Clearly not. (Placek 2012: 27)

If Kripke and Placek are right, the *World-boundness* thesis – i.e., nothing exists in two worlds – is at odds with a natural account of what we do when we talk about possible scenarios. If the *World-boundness* were true, "Humphrey" would refer to Humphrey in the actual world (assuming that Humphrey actually exists), but the same name would refer to one of his counterpart at a counterfactual world (assuming that Humphrey's counterpart exists at that counterfactual world). A divergentist is happy to accept this latter claim, and this makes divergence itself quite weird. But a branching theorist can obviously avoid this bizarreness, for in a tree temporal parts have several possible continuations, and any of them stem from a common source.

Surely, one can claim that the modal aspects related to indeterminism are theoretical notions, and pre-theoretical, linguistic intuitions are of no help in assessing the branching account against the divergentist one. But this reply is wrong, for it forgets that

> [...] the folk theory of time that is fossilized in the structure of our language does help to single out what the philosophy

> of time is talking about. We want an account of what *we* call time and we cannot reject too much of the folk theory without changing the topic. (Meyer 2013: 5)

When one does metaphysics, intuitions may be useful to isolate the very topic one is talking about. When one tries to figure out the subject matter of the future possibilities talk, the "folk theory" fossilized in our language clearly matters. Sure, intuitions might be dropped when there is sufficient evidence to do so. But when there is no such evidence, it is unreasonable to reject them. This methodological criterion appears to be shared even by David Lewis (1986a: Ch. 4), the leading advocate of divergence. Indeed, in adopting divergence, he feels the need of justifying his departure from intuitions by the overall theoretical role that counterparts play in a broad, metaphysical picture. As far as indeterminism is concerned, does one need to undertake the departure suggested by Lewis? More specifically, is there any reason to discharge branching in favour of divergence?

In the next section some arguments against branching will be explored. I will argue that these arguments are far from being convincing. Hence they do not collect any evidence to reject the most intuitive view among the two main rival frameworks that are relevant for indeterminism.

2.3.3 Lewis against the Overlap condition: demarcation of worlds.

David Lewis elaborates three arguments to motivate the *No-overlap* condition. Since this condition is assumed by divergentists but rejected by branching theorists, Lewis' arguments are meant to favour the former approach agains the latter.

According to his view, whoever is committed to overlapping worlds has to face a problem related to the demarcation of possible worlds. In Lewis' words,

> [...] overlap of worlds interferes with the most salient principle of demarcation for worlds, viz. that two possible individuals are part of the same world iff they are linked by some chain of external relations, e.g. of spatiotemporal relations. (Lewis 1983b: 360)

First, Lewis stresses that the external relations he's talking about are spatiotemporal. These relations have a pivotal metaphysical role, for they unify possible worlds. When it comes to assess whether two possible individuals, say x and y, exist at the same world w, one has to check whether there is some spatiotemporal relation obtaining between x and y. If x is a possible entity, but it doesn't bear any spatiotemporal relation with another possible entity y, x and y must exist at two different worlds, say w and w' respectively. Let "x_w" refer to an entity x, existing at a possible world w. Given two distinct entities $x_w \neq y_{w'}$, Lewis' criterion for the identification of worlds is

(4) $\forall x_w, y_{w'} \exists R(R(x_w, y_{w'}) \Rightarrow w = w'),$

where R is an external, spatiotemporal relation between possible individuals.

But why the *Overlap* condition should be in tension with (4)?

> What unifies a world, I suggested, is that its parts stand in suitable external relations, preferably spatiotemporal. But if we have overlap, we have spatiotemporal relations between the parts of different worlds. For instance, let P be the common part – say, a shared initial segment – of the two worlds W_1 and W_2, let R_1 be the remainder of W_1, and let R_2 be the remainder of W_2. Then the appropriate unifying relations obtain between parts of two different worlds: between P, which is *inter alia* a part of the world W_1, and R_2, which is part of the different world W_2. Of course, it is also true that P and R_2 are parts of a single world W_2. So at least, there is some world they are both parts of, even if they may be parts of other worlds besides. Or can we say even that? In a sense, even R_1 and R_2 are related, in a stepwise back-and-forth way, via P. For instance, R_1 and R_2 might stand to one another in the complex temporal relation: successor–of–a–predecessor–of. (Lewis 1986a: 208)

A crucial premise that Lewis endorses is that, if world w and world w' overlap, any two individuals, existing at w and w' respectively, bear some spatiotemporal link. This assumption can be written as

(5) If w and w' overlap, $\forall x_w, y_{w'} \exists R(R(x_w, y_{w'}))$

It is easy to see that (4) and (5) entail

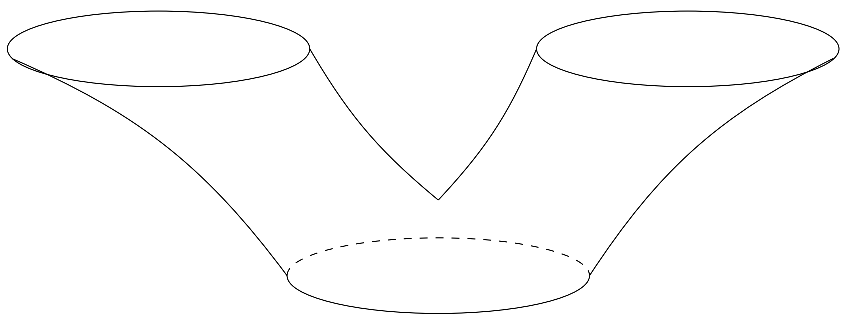
(6) If w and w' overlap, $w = w'$.

By (4)–(5), the plurality of possible courses of events that the branching approach predicts is a fake, for it collapses to one single, possible world. As Lewis comments,

> If we stay with the simple account of how worlds are defined [that is, if (4) holds], we will conclude that where there is this branching, there is one single world composed of all the branches. That would not be branching *of* worlds, but branching *within* worlds; and so the overlap of branches would not be the overlap of worlds. (Lewis 1986a: 209)

The collapse over one single possible world has several shortcomings. According to Lewis (1986a: 209), such a world cannot be the one in which we live, for otherwise we would be constantly deceived in thinking about 'the future'; in a single, branching possible world, its inhabitants would have many futures, not just one (more on this below).

If any branching frame represents just one single world, one could identify that world with an individual, relativistic spacetime manifold. This manifold, in turn, has an 'internal' branching structure, in the sense that it has a trunk that bifurcates in two legs, creating a "trousers universe" similar to that of the figure below.



According to Lewis' objection, there cannot be a branching universe that is not also a trousers universe. In a trousers universe, spacetime literally rips off, yielding two separate, spatiotemporal regions. As Earman (2008) has argued at length, this kind of structure involves a change in the spatial topology, and it contradicts several no-go results for topology change. Thus, such models are inconsistent with the laws that, as far as we know, govern the world we live in.

Furthermore, if Lewis' objection is on the right track, the branching approach cannot represent indeterminism. If any branching frame specifies a singular trousers universe, things may split in two copies in the future, but it would not be possible for them to do not split. If there were such a possibility, there would have been another possible world in which things do not split. Lewis' objection entails that any branching frame is deterministic, for it identifies just one possible way in which things can evolve.[13]

The force of Lewis' objection, if any, relies on the notion of spatiotemporal relation occurring in (5). The relation Lewis has in mind is *being–a–successor–of–a–predecessor–of*. For the sake of simplicity, call that relation $R*$, and assume that it has moments as its relata. Then, for any two moments m, m' in a tree $\mathcal{T}$,

$$R*(m, m') \equiv_{df} \exists m''(m'' < m \wedge m'' < m')$$

By condition (b) of Definition 2.2.1, any two moments that are not <-related must be $R*$-related. And if $\mathcal{T}$ is past-unbounded (i.e., there is no initial moment), any two moments of $\mathcal{T}$ are $R*$-related. Let us focus only on those trees which satisfy this latter condition.

Given that Lewis takes $R*$ to be spatiotemporal, one may ask whether that relation is indeed of the kind requested. At a first sight, it appears to be so. After all, $R*$ is defined on <, which has an obvious temporal meaning. Furthermore, < is often interpreted as a relation of causal connectedness.[14] If $m < m'$, what occurs at m' is a (possible) effect of what occurs at m. Given two spatially separated events e and e', e can have a causal influence on e' only if a signal can cover the spatial distance between the two. Hence, < can sometimes take a spatial meaning as well.[15] One may then claim something like: "Look, $R*$ is defined on <, which has both spatial and temporal characters. Thus, $R*$ must have a spatiotemporal character as well". But this is wrong.

The problem is that < encodes both spatiotemporal and *modal* information: if < does not hold between two moments, these two moments are *modally inconsistent*. Indeed, any two moments lying on different branches can be assimilated to alternative world states.[16] This means that two <-unrelated moments are modal alternatives of the very same past, precisely beacuse they are not *compossible*. They are two futures of the same past, but they are two modal alternatives of that past. Accordingly, if two moments are not compossible, they simply cannot be part of the same possible world. Recall that possible worlds are taken to be *physical* possible worlds. Hence, two compossible moments must identify two different

[13]For a similar conclusion, see Rosenkranz 2013.

[14]See, for instance, Belnap (1992), Belnap & Green (1994).

[15]This is particularly clear for the branching space-time structures elaborated in Belnap (1992). These structures take into account the frame-relativity of simultaneity required by the special theory of relativity. Mathematical details aside, in a branching space-time frame the < relation occurs between point events, and if $e < e'$, then e' is in one of the possible future light-cones of e. Thus, e''s spatial location can indeed be reached by a signal which starts moving form e's spatial location.

[16] See Bonomi & Del Prete (2008: 11).

spatiotemporal regions of the very same possible world. By parity of reasoning, two moments that fail to be compossible should be part of two different possible worlds. Thus, whenever $R*$ holds among two modally inconsistent moments, it cannot be taken to be a spatiotemporal relation between them, for otherwise the two moment would be compossible. If $R*$ holds among two modally inconsistent moments, it conveys the information that these moments are two future alternatives for a third, past moment: this latter moment is in the past of each of its future alternatives, but future alternatives do not bear any spatiotemporal relation to one another.

To sum up, the relation Lewis uses in (5) is $R*$, the relation of *being–a–successor–of–a–predecessor–of*. Lewis takes $R*$ to be spatiotemporal, and concludes via (4) that the *Overlap* condition entails the existence of a single, branching possible world. As we have seen, however, Lewis ignores that $R*$ may fail to be a relation of the kind required, since it can occur between modally inconsistent moments. Whenever $R*$ holds between two alternative possible moments, then $R*$ does not identify a spatiotemporal link between them. Hence, $R*$ cannot be used in (4) to conclude that any branching frame isolates a single possible world.

So far so good. Even Lewis recognises that his objection – based on the demarcation of possible worlds – cannot be taken as a knock down argument against branching. According to his view, however, branching yields a far more serious problem, involving the notion of individual taken as a structured entity and made out of temporal parts.

2.3.4 Lewis against the Overlap condition: contradictory individuals.

Branching theorists claim that the very same temporal part – such as the atom a at moment m – inhabits several, overlapping possible worlds (histories). These worlds branch towards the future, and so they specify different possible ways the future of a temporal part can turn out. For instance, a–at–m has both possible decay continuations and possible no-decay continuations. According to Lewis, this way of conceiving indeterminism leads to contradiction.

> [...] Humphrey, who is part of this world and here has five fingers on the left hand, is also part of some other world and there has six fingers on his left hand. *Qua* part of this world he has five fingers, *qua* part of that world he has six. He himself – one and the same altogether self-identical – has five fingers on the

> left hand, and he has not five but six. How can this be done? (Lewis 1986a: 200)

The objection is clear. Given a branching conception of indeterminism, individuals – which are taken to be complex entities, made up of several, temporal parts – have contradictory properties. In the example, Humphrey has several parts, distributed in distinct possible worlds. In some of them his left hand has six fingers, in others five. Thus, Humphrey's left hand has both five and six fingers. Contradiction.

This objection has an obvious rejoinder. In order to avoid the contradiction, one can simply relativize the identity of individuals (conceived as structured entities made of temporal parts) to possible worlds (histories). For instance, one may say that the individual Humphrey–on–h_1 has a left hand with six fingers, while another individual, Humphrey–on–h_2, has a left hand with just five fingers. Histories h_1 and h_2 overlap up to a moment, and thus Humphrey–on–h_1 shares some temporal parts with Humphrey–on–h_2. But h_1 and h_2 splits after a segment, representing two alternative, possible courses of events. Since any history is taken to be entirely consistent (that is, it is never the case that Humphrey–on–h_1 (Humphrey–on–h_2) has both exactly five and six fingers on his left hand), no contradiction arises. Thomas Placek seems to have flirted with this line of thought:

> The two histories contain different continuations of Humphrey in stage S, one continuation with five fingers on the left hand, and the other with six fingers on the left hand. [...] Thus, the two contradicting properties, of having five fingers and having six fingers on the left hand, refer to two different individuals that share some initial segment [...]. Hence, no contradiction ensues. (Placek 2001: 488-489)[17]

This solution, even if appealing, has one important shortcoming. One of the advantages that branching has over divergence is that the former, but not the latter, does justice to our tensed-modal talk. This means that, under a branching approach, it is plausible to take each of the following claims

(7) It is possible that Humphrey will have (exactly) five fingers on his left hand,

(8) It is possible that Humphrey will have six fingeres on his left hand,

as talking about two possible, but mutually incompatible future continuations of the very same individual. But the solution to Lewis' objection just

[17]Placek, however, seems to have changed his mind. See, for instance, Placek (2012).

outlined suggests that the two statements should talk about two numerically distinct individuals. Hence, one looses one reason to prefer branching over divergence.

However, this is not the whole story. There is another rejoinder to Lewis' objection which doesn't give up the idea that both (7)-(8) talk about the very same Humphrey. This solution has to provide a suitable interpretation for a modal, temporal language that mirrors that, when one uses (7) (or, for that matter, (8)), one is talking about just one possible, entirely consistent future continuation of Humphrey. Furthermore, the interpretation overcomes Lewis objection only if it avoids statements such as

(9) Humphrey has both (exactly) five and six finger on his left hand,
(10) Humphrey will have both (exactly) five and six finger on his left hand,
(11) It is possible that Humphrey will have both (exactly) five and six finger on his left hand,

to be true at any point of evaluation.

It is easy to yield the interpretation required by adopting adopting the so-called quantified Ockhamist semantics (QO). QO is definable by introducing a language $\mathcal{L}_{T-Pred}$ and a QO-model $\mathcal{M}_{QO}$. Language $\mathcal{L}_{T-Pred}$ is the standard language for predicate logic, equipped with a set of individual constants (c_1, c_2, etc.), a set of n-ary predicates (P_1, P_2, etc.), a set of individual variables (x, y, etc.), the boolean connectives ($\neg, \wedge$), the universal quantifier ($\forall$), and the sentential operators $\mathsf{P}(n), \mathsf{F}(n), \Box$. The formation rules for the wffs of $\mathcal{L}_{T-Pred}$ are the usual ones. A QO-model for $\mathcal{L}_{T-Pred}$ is defined as follows.

Definition 2.3.2 (*Q-model*)**.** *A QO-model for $\mathcal{L}$ is a tuple $\mathcal{M}_Q = (\mathcal{T}, d_{mt}, Domain, G, I)$, where*

(a) $\mathcal{T}$ is a tree,
(b) $d_{\mathcal{T}}$ is a $\mathcal{T}$-duration function,
(c) Domain is a non-empty set of individuals,
(d) G is the set of assignments, where each $g \in G$ is a function mapping each variable of $\mathcal{L}$ to an element of Domain,
(e) I is an interpretation function which maps each non-logical constant α of $\mathcal{L}$ to an intension, $\|\alpha\|$, where an intension is a map from moment-history pairs, m/h, to extensions, $\|\alpha\|_{m/h}$.[18] If α is an individual constant, $\|\alpha\|$ is constant and $\|\alpha\|_{m/h} \in Domain$. If α is a n-ary predicate, $\|\alpha\|_{m/h} \in Domain^n$.

[18] m/h means that $m \in h$, or that h passes through m.

Notice that, by condition (d), the existence of individuals is moment-history independent: if something exists at m/h, it exists at any m'/h'. However, one may want to say that individuals can fail to exist at some m/h. This can be done by introducing a null-object λ in $\mathcal{M}_Q$ s.t. $\lambda \notin Domain$. Furthermore, one could modify condition (d) as follows.

(d') If α is an individual constant, either $\|\alpha\|_{m/h} = \lambda$, or, for any m/h, m'/h' s.t. $\|\alpha\|_{m/h} \neq \lambda$ and $\|\alpha\|_{m'/h'} \neq \lambda$, $\|\alpha\|_{m/h} = \|\alpha\|_{m'/h'} \in Domain$. If α is a n-ary predicate, $\|\alpha\|_{m/h} \in Domain^n$.

Intuitively, whenever the extension of an individual constant c is λ at a point m/h, constant c fails to refer to any individual that exists at that point. In other terms, if $\|c\|_{m/h}$ = Humphrey and $\|c\|_{m'/h'} = \lambda$, Humphrey exists at m/h but not at m'/h'. However, if c denotes something that exists (say Humphrey) at a point m/h, whenever it denotes something that exists at another point m'/h', c refers to the same individual denoted at m/h (Humphrey). In order to answer Lewis' objection, however, one is free to adopt either condition (d) or (d').

Now let us define the quantified Ockhamist semantics.

Definition 2.3.3 (*Quantified Ockhamist Semantics* (QO)). *Given a model $\mathcal{M}_Q$ for $\mathcal{L}$,*

(QO1) $g, m/h \vDash_{QO} P(c_1, ..., c_n) \Leftrightarrow (\|c_1\|_{m/h}, ..., \|c_n\|_{m/h}) \in \|P\|_{m/h}$
(QO2) $g, m/h \vDash_{QO} \neg\mathcal{A} \Leftrightarrow$ *not* $g, m/h \vDash_{QO} \mathcal{A}$
(QO3) $g, m/h \vDash_{QO} \mathcal{A} \wedge \mathcal{B} \Leftrightarrow g, m/h \vDash_{QO} \mathcal{A}$ *and* $g, m/h \vDash_{QO} \mathcal{B}$
(QO4) $g, m/h \vDash_{QO} \forall\mathcal{A}(x) \Leftrightarrow$ *for any* g', $g', m/h \vDash_{QO} \mathcal{A}(g'(x))$, *where* g' *is such that, for any variable* y *in* $\mathcal{L}$ *other than* x, $g'(y) = g(x)$.
(QO5) $g, m/h \vDash_{QO} \mathsf{F}(n)\mathcal{A} \Leftrightarrow$ *there is a* m' *s.t.* $d_{\mathcal{T}}(m, m') = n$, $m < m', m' \in h$ *and* $g, m/h \vDash_Q \mathcal{A}$
(QO6) $g, m/h \vDash_{QO} \mathsf{P}(n)\mathcal{A} \Leftrightarrow$ *there is a* m' *s.t.* $d_{\mathcal{T}}(m, m') = n$, $m' < m, m' \in h$ *and* $g, m/h \vDash_Q \mathcal{A}$
(QO7) $g, m/h \vDash_{QO} \Diamond\mathcal{A} \Leftrightarrow$ *there is a* h' *s.t.* $m \in h'$ *and* $g, m/h' \vDash_{QO} \mathcal{A}$

It is straightforward to see that none of (9)-(11) – if correctly translated in $\mathcal{L}_{T-Pred}$ – comes out true at any point of evaluation admitted by QO. In other words, for claims such as

$Pc \wedge \neg Pc$ (the formal counterpart of (9))
$\mathsf{F}(n)(Pc \wedge \neg Pc)$ (the formal counterpart of (10))
$\Diamond\mathsf{F}(n)(Pc \wedge \neg Pc)$ (the formal counterpart of (11))[19]

[19]Obviously, $\Diamond$ is the dual of $\Box$.

there is no point of evaluation at which any of them is satisfied. The reason is pretty simple: the quantified Ockhamist semantics cannot satisfy contradictions. Moreover, if c denotes something at $g, m/h$, it denotes the very same individual at any point $g, m'/h'$ at which that individual exists, and the null object λ, at any $g, m''/h''$ at which that very same individual fails to exist. Thus, the QO-semantics is apt to avoid Lewis' objection, given that

(i) an individual can exist at several, modally inconsistent moments. Indeed, there is nothing in Definitions 1.2.7–1.2.8 that prevents $\exists x(x = c)$ to be satisfied at points $g, m/h$ and $g, m'/h'$, where $m \notin h'$ and $m' \notin h$,

(ii) point (i) does not entail that individuals are contradictory entities. Indeed, Definitions 1.2.7–1.2.8 prevents that there is a point $g, m/h$ s.t. $g, m/h \vDash_{QO} Pc$ and $g, m/h \vDash_{QO} \neg Pc$.

What point (i) clarifies is that the solution to Lewis' objection doesn't relativize the identity of individuals to histories. The proposal, however, relativizes the properties that individuals may have to moment-history pairs. Formally, this is reflected by the fact that – by definition 1.2.7 – if $i \in Domain$ and Q and Q' are two unary predicates denoting incompatible properties, no m/h is s.t. $i \in \|Q\|_{m/h}$ and $i \in \|Q'\|_{m/h}$ ($\|Q'\|_{m/h}$ can be viewed as the set-theoretic complement of $\|Q\|_{m/h}$). Hence, Lewis' objection – according to which the branching view entails the existence of contradictory individuals – doesn't cut much philosophical ice.

2.4 Some conclusions and a problem.

Both divergentism and branching enable to define indeterminism in terms of possible worlds. They have rigorous translations in mathematical terms (Kamp frames and trees), and are apt to interpret modal temporal languages in a precise, non-ambiguous way. The Ockhamist semantics defined on Kamp frames (KO), as well as its branching version (BO), are, respectively, KO- and BO-bivalent. These two semantics allow to answer the Aristotelian question (Q1) in a similar way. The divergentist variant of indeterminism, DOI, is compatible with KO-bivalence. The branching reading of indeterminism, BOI, is compatible with BO-bivalence. Moreover, both semantics enable to answer (Q2) by refuting the principle of the necessity of the past (*PNP*), and, therefore, they are apt to reject the fatalist conclusion of the Master argument.

Divergence and branching, however, are at odds with each other, since the former explicitly denies one main tenet of the latter. Under a divergentist view, possible worlds do not overlap. On a branching perspective, each history has at least a common part (i.e., it shares at least a moment) with any other history. Thus, an indeterminist must make a choice between the two theories. It has been argued that the branching conception of indeterminism has some advantages over its rival. An advocate of the branching perspective can characterise the tower's collapse cases as indeterministic, without giving up any fundamental tenets of her view. A divergentist theorist, on the other hand, can conceive the very same cases as indeterministic only by adopting a principle that is stronger than the one she is supposed to assume, that is, a principle for counterparts identification that is stronger than that of sameness of natural properties. Furthermore, the information that is conveyed by modal tensed statements – when they are interpreted within a branching approach – is much more intuitive than the one that is delivered by the divergentist view. Last but not least, two of the main objections that divergentists have raised against branching (those concerning the demarcation of worlds and the commitment to contradictory individuals) can be consistently overcome. Thus, the branching perspective appears to be preferable to divergence.

Is everything fine? Not really. One may rightly object that the Aristotelian question (Q1) wasn't properly answered by any semantics given so far. Recall that the branching Ockhamist semantics, BO, satisfies the following principle:

BO-bivalence: For any wff $\mathcal{A}$ of $\mathcal{L}_T$ and any point m/h, either $m/h \vDash_{BO} \mathcal{A}$ or $m/h \nvDash_{BO} \mathcal{A}$.

The *BO-bivalence* is relative to *both a moment and a history*, and it is *not* the kind of bivalence which can be used to answer (Q1). Indeed, the notion of bivalence which occurs in Aristotle's reasoning is much more simpler, being *relative to moments only*:

(B) Either $\mathcal{A}$ is true at a moment m, or $\mathcal{A}$ is false at m.

Accordingly, an indeterminist has to say something about the historical parameter occurring within the definition of BO-bivalence and, more generally, within any BO-point of evaluation.[20] There are different attitudes one may have towards that parameter.

Revisionary Strategy. A first reaction might be that the points of evaluation of BO are somewhat too complicated. This complexity, in turn,

[20] Adopting a KO semantics won't help, for the KO-bivalence is relative to a time and a world.

just mirrors that there is something fundamentally wrong in BO. Accordingly, one can either reject BO and make up a new semantics for tensed modal language (as the Peircean approach does), or can use BO as a technical tool useful to define a notion of truth at a moment (as the supervaluationists and the relativists).

Conservative Strategy. A second possible reaction is this: Aristotle's argument is ill conceived. It rises a question, (Q1), which is too much demanding. If we are to accept indeterminism, and if we accept to frame it in terms of possible worlds (histories), then we have to live with a notion of truth and bivalence that are bound to be relative to moment-history pairs. Under this view, the branching conception of indeterminism entails that any evaluation of tensed formulas makes sense only with respect to a couple of parameters (i.e., a moment of utterance and a history). This strategy, as we shall see, is the one adopted by Belnap et al. (2001).

Next chapters analyse both strategies.

Chapter 3

The revisionary strategy.

In general, a branching frame allows many histories to pass through a moment m. But BO-truth (BO-falsity) is defined relative to moment-history pairs, so that one can choose any h passing through m to compute the BO-truth value of a statement at m/h. From a formal perspective this is perfectly fine. However, these somewhat complicated points of evaluations yield a notion of bivalence (i.e., the *BO-bivalence*) which does not permit to answer the Aristotelian question, (Q1). According to what I have called the *Conservative Strategy*, this result is only natural once we accept to frame indeterminism in terms of branching possible worlds (histories). Under the *Revisionary Strategy*, however, the BO-semantics must be modified, since it does not reflect some fundamental features that an adequate indeterministic semantics should capture.

Next sections explore the reasons one may have to adopt the revisionary strategy. However, the three, revisionist semantic accounts – i.e., Peirceanism, supervaluationism and relativism – have several, significant flaws. Therefore, they will be assessed as inadequate.

3.1 Peirceanism

The approach known as Peirceanism was first introduced by Arthur Prior (1968a), and it is inspired by Charles Sander Peirce's philosophy of time.[1] Peirceanism sticks to the revisionary strategy, for it amounts to define a modal, temporal logic that is alternative to BO.[2] Peirceanism assesses the

[1]See Øhrstrøm & Hasle (1995: Ch. 2.2.) for a detailed analysis of Peirce's position about time and modality.

[2]Actually, Rumberg (2016) shows that there is a semantic machinery – what she calls the transition semantics – which generalises both Peirceanism and the BO semantics. This

branching Ockhamist semantics as inadequate, and the fact that the notion of BO-truth (as well as that of BO-bivalence) is moment-history relative reflects its inadequacy. The evaluations of future contingents cannot be relative to moment-history pairs, for the meaning conveyed by predictions is a necessary one. Under a Peircean perspective, saying that il will be the case that $\mathcal{A}$ amounts to say that, inevitably, it will be the case that $\mathcal{A}$.

We may introduce some motivations for Peirceanism by reporting Prior (1957: 85)'s "cliff paradox".

> Suppose A and B are being pushed towards the edge of a cliff, and there will be no stopping this process until there is only room for one of them. Then we may be able to say truly that it will definitely be the case that A or B will fall over, even though we cannot say truly either that A will definitely fall over or that B will definitely fall over. (Prior 1957: 85)

In the circumstances described by the example, we can say that (1) is definitely true, while we must say that (2) and (3) are *not*.

(1) It will be the case that either A or B fall over.
(2) It will be the case that A falls over.
(3) It will be the case that B falls over.

Then, the cliff paradox asks how one may define a semantics that evaluates (1)-(3) in the way just suggested.

The Peircean theorist answers that "It will be the case that $\mathcal{A}$ " is true at a moment m if and only if there are facts at m that make the truth of $\mathcal{A}$ at a future moment already settled at m. Since there are no facts that presently determine which one among A and B will fall over, but present facts determine that exactly one of them will fall over, then we are entitled to assess statement (1) – but not (2) and (3) – as presently true. (1) is presently true because now it is inevitably so: the future may take one way or another, but in any case one among A and B will fall over the cliff. On the other hand, (2) and (3) can't be presently true, for they predict contingent events. More generally, the Peircean view holds that one can truly predict something just in case the event predicted is inevitable, for "[...] nothing can be said truly 'going-to-happen' (*futurum*) until it is 'present in its causes' as to be beyond stopping" (Prior 1968a: 38).

result is surely interesting from a technical perspective. However, I doubt that it may have a philosophical relevance when one tries to solve the problem of indeterminism and future contingents.

It is easy to model these ideas into a rigorous semantics (PE). PE translates the future tense in terms of its primitive future operator $\mathsf{F}_p(n)$. Intuitively, $\mathsf{F}_p(n)$ is equivalent to the necessitation of the future operator provided by BO – that is, $\mathsf{F}_p(n)$ corresponds to $\Box\mathsf{F}(n)$. The Peircean (propositional) temporal language, $\mathcal{L}_{PE}$, differs from that adopted by BO, $\mathcal{L}_T$, since (i) its primitive future operator is $\mathsf{F}_p(n)$, and (ii) $\mathcal{L}_{PE}$ does not contain the necessity operator $\Box$. Thus, the wffs for a Peircean propositional temporal language are isolated by the following rule:

$$\mathcal{L}_{PE} ::= p \mid \neg\mathcal{A} \mid \mathcal{A} \vee \mathcal{B} \mid \mathsf{P}(n)\mathcal{A} \mid \mathsf{F}_p(n)\mathcal{A}$$

A model for $\mathcal{L}_{PE}$ is identical to a model for the propositional version of the branching Ockhamist semantics. In other terms, such a model is $\mathcal{M}_{PE} = (\mathcal{T}, d_{\mathcal{T}}, I)$, where $\mathcal{T}$ is a tree, $d_{\mathcal{T}}$ is a duration function, and I an interpretation from atoms of $\mathcal{L}_{PE}$ to set of moments of $\mathcal{T}$.The semantic clauses for the Peircean semantics are defined in the following manner.

Definition 3.1.1 (*PE-semantics*)**.** *Given a PE-model $\mathcal{M}_{PE}$ for $\mathcal{L}_{PE}$,*

(PE1) $m \vDash_{PE} p \Leftrightarrow m \in I(p)$

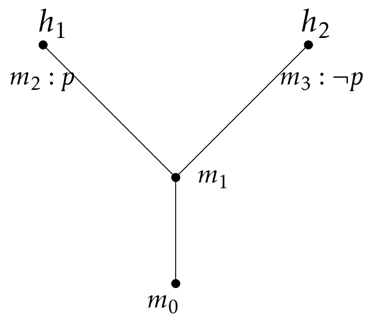
(PE2) $m \vDash_{PE} \neg\mathcal{A} \Leftrightarrow$ *not* $m \vDash_{PE} \mathcal{A} \Leftrightarrow m \nvDash_{PE} \mathcal{A}$

(PE3) $m \vDash_{PE} \mathcal{A} \vee \mathcal{B} \Leftrightarrow$ *either* $m \vDash_{PE} \mathcal{A}$ *or* $m \vDash_{PE} \mathcal{B}$

(PE4) $m \vDash_{PE} \mathsf{F}_p(n)\mathcal{A} \Leftrightarrow \forall h(m \in h \Rightarrow \exists m'(m' \in h \;\&\; m < m' \;\&\; d_{\mathcal{T}}(m, m') = n \;\&\; m' \vDash_{PE} \mathcal{A}))$

(PE5) $m \vDash_{PE} \mathsf{P}(n)\mathcal{A} \Leftrightarrow \exists m'(m' < m \;\&\; d_{\mathcal{T}}(m, m') = n \;\&\; m' \vDash_{PE} \mathcal{A})$

It is worth saying that the semantic clause for the (primitive) future operator, $(PE4)$, reflects the modal force that Peirceanism attributes to the future tense "Will". Indeed, $\mathsf{F}_p(n)$ acts as a universal quantification over the histories that pass through the moment of evaluation. In particular, $(PE4)$ asks to check whether the sentence embedded by $\mathsf{F}_p(n)$ is true at n time units in the future on *any* history which passes through the moment of evaluation. It follows that if one translates "Tomorrow it will be the case that $\mathcal{A}$" as $\mathsf{F}_p(1)\mathcal{A}$, such a statement is PE-true at m just in case it is inevitable at m. Therefore, any future contingent comes out PE-false. For instance, let us focus on the model pictured below, and let us assume that $d_{\mathcal{T}}(m_1, m_2) = d_{\mathcal{T}}(m_1, m_3) = 1$. Then, $\mathsf{F}_p(1)p$ is a future contingent at m_1. By $(PE4)$, it follows that $F_p(1)p$ is PE-false at m_1, since p is PE-false at m_3, that is, at one of the moments one time unit in the future of m_1.



Even if $\mathsf{F}_p(n)$ expresses future inevitability, the Peircean theorist has a way to talk about possible future events that may fail to be inevitable. First, one may define an operator which is the dual of $\mathsf{F}_p(n)$,

$$f_p(n)\mathcal{A} \equiv_{df} \neg\mathsf{F}_p(n)\neg\mathcal{A}$$

Given ($PE2$) and ($PE4$), the truth conditions for a sentence of the form $f_p(n)\mathcal{A}$ must be given by the following equivalences.

$$\begin{aligned}
&m \vDash_{PE} \neg\mathsf{F}_p(n)\neg\mathcal{A} \Leftrightarrow\\
&\neg\forall h(m \in h \Rightarrow \exists m'(m' \in h \wedge m < m' \wedge d_{\mathcal{T}}(m,m') = n \wedge m' \nvDash_{PE} \mathcal{A})) \Leftrightarrow\\
&\exists h\neg(m \in h \Rightarrow \exists m'(m' \in h \wedge m < m' \wedge d_{\mathcal{T}}(m,m') = n \wedge m' \nvDash_{PE} \mathcal{A})) \Leftrightarrow\\
&\exists h(m \in h \wedge \neg\exists m'(m' \in h \wedge m < m' \wedge d_{\mathcal{T}}(m,m') = n \wedge m' \nvDash_{PE} \mathcal{A})) \Leftrightarrow\\
&\exists h(m \in h \wedge \forall m'\neg(m' \in h \wedge m < m' \wedge d_{\mathcal{T}}(m,m') = n \wedge m' \nvDash_{PE} \mathcal{A})) \Leftrightarrow\\
&\exists h(m \in h \wedge \forall m'(m' \in h \wedge m < m' \wedge d_{\mathcal{T}}(m,m') = n \Rightarrow m' \vDash_{PE} \mathcal{A}))
\end{aligned}$$

Accordingly, $f_p(n)\mathcal{A}$ has the following truth conditions.

(PE6) $m \vDash_{PE} f_p(n)\mathcal{A} \Leftrightarrow \exists h(m \in h \wedge \forall m'(m' \in h \wedge m < m' \wedge d_{\mathcal{T}}(m,m') = n \Rightarrow m' \vDash_{PE} \mathcal{A}))$

Intuitively, $f_P(n)\mathcal{A}$ reads "It is possible that, in n time units, it will be the case that $\mathcal{A}$". As an example, both $f_p(1)p$ and $f_p(1)\neg p$ are true at m_1 in the previous model.

PE has another, relevant characteristic. The history parameter does not occur in its points of evaluation, and any statement is evaluated only relative to moments. Moreover, it is perfectly bivalent, in the sense that it satisfies the following principle:

PE-bivalence: For any wff $\mathcal{A}$ of $\mathcal{L}_{PE}$ and any moment m, either $m \vDash_{PE} \mathcal{A}$ or $m \nvDash_{PE} \mathcal{A}$.

As for BO, PE-truth ($\vDash_{PE}$) and PE-falsity ($\nvDash_{PE}$) are jointly exhaustive and mutually exclusive. Now that the main formal features of PE are clarified, let us see how a Peircean theorist may react to the Aristotelian and the Master argument.

3.1.1 Peirceanism, the Aristotelian argument and the Master argument

Notice that *PE-bivalence* is relative to moments only. Therefore, the Peircean semantics is apt to give a precise answer the Aristotelian question (Q1). The Peircean semantics accepts premise (Pa), but rejects (Pb).

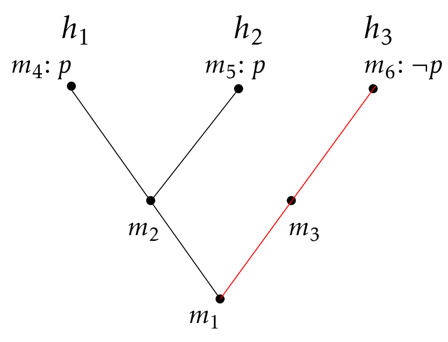
(P)(a) If $\mathcal{A}$ is true at a moment m, then 'Necessarily $\mathcal{A}$' is true at m.
(P)(b) If $\mathcal{A}$ is false at a moment m, then 'Possibly $\mathcal{A}$' is false at m.

Under a Peircean reading, if something is true at m, it is inevitably so at m. Thus, premise (Pa) is perfectly consistent with PE. But it may be the case that "In n time units, it will be the case that $\mathcal{A}$" (i.e., $\mathsf{F}_p(n)\mathcal{A}$) turns out to be PE-false at m, even though "Possibly, in n time units it will be the case that $\mathcal{A}$" – viz., $f_p(n)\mathcal{A}$ – is true at the very same moment m. Hence, premise (Pb) does not hold. This means that Peircean theorists can consistently stick to a branching conception of indeterminism (BOI) without giving up (PE-)bivalence. Indeed, they reject the Aristotelian argument by refuting premise (Pb).

As for the second question, (Q2), PE rejects the principle of retrogradation (*PR*): the Peircean perspective does not guarantee that what now is the case has always been going to be the case.

PR $\mathsf{F}(m)\mathcal{A} \rightarrow \mathsf{P}(n)\mathsf{F}(n+m)\mathcal{A}$

It is easy to yield a countermodel for *PR*. Take the graph pictured below, and assume that, if $m < m'$ and there is no m'' between them, then $d_{\mathcal{T}}(m, m') = 1$.



By *(PE4)-(PE5)*, it follows that $\mathsf{F}_p(1)p$ is PE-true at m_2, given that p holds at m_4 and m_5. But $\mathsf{P}(1)\mathsf{F}_p(2)p$ is PE-false at the very same moment, for p is false at m_6.

To sum up, the Peircean view conserves bivalence (relative to moments only), rejects the Aristotelian argument by refuting premise (Pb), and invalidates the Master argument by denying the validity of the principle of retrogradation. Even if PE behaves better than BO in answering (Q1), there are several features that make the Peircean approach questionable.

3.1.2 Against the Peircean view

Objection 1. *The "plain future" collapses over what is presently settled.*

Belnap et al. (2001: 160) point out that the expressive power of PE is actually very poor, and it does not allow to reflect some basic intuitions about what we mean when we utter predictions about future contingent events. Under a Peircean view,

(4) I bet that A will fall over,

really means

(5) I bet that present circumstances determine that A will fall over.

But these two latter statements surely express different contents. Indeed, whoever utters (4) wins the bet if and only if A will actually fall over the cliff. An agent who utters (5), however, can lose even when A will actually fall over. If, at the time of the utterance, it was possible for A to do not fall, then the bettor on (5) loses. This fact highlights an interesting feature of the "cliff's paradox", according to which (1) is definitely true, but (2) and (3) – being future contingents – cannot be said to be definitely true.

(1) It will be the case that either A or B fall over.
(2) It will be the case that A falls over.
(3) It will be the case that B falls over.

Now, it is natural to assume that whoever utters (2) (or (3)), as well as a bettor on (4), does commit herself to say something about what will actually be the case. But saying something about how things will actually turn out does not necessarily mean to say something about what is presently settled. Hence, the fact that (2) and (3) cannot be said to be *definitely true* does not suffice to assess them as *false*. PE cannot distinguish between predictions regarding what we may call the "plain" future, and predictions about what is determined by the present circumstances. Therefore, the Peircean view cannot capture one crucial aspect of our modal tensed talk.

Sure, a Peircean may frame indeterminism in such a way that any reference to what will actually be the case has to be assessed as misleading.[3] Thus, a Peircean may insist that, if indeterminism holds, then there is no such thing as the "plain" future: if we want to talk about the future, we must talk about what inevitably will be the case. As we shall see, this rejoinder is wrong. On the other hand, there are sensible ways to express, given indeterminism, what actually will be the case. On the other, it is a basic requirement on the notion of inevitable futurity that it entails (simple) futurity: if something inevitably will be the case, then it will be the case. And again, the Peircean theorist does not have the resources to say anything like this.

Objection 2. *The failure of the future excluded middle.*

PE does not validate a tensed logical schema that many authors, rightly in our mind, take as a tautology within an indeterministic setting.[4] This schema is known as the *future excluded middle*, and it says that

(FEM) Either it will be the case that $\mathcal{A}$, or it will be the case that not $\mathcal{A}$.

If one translates (FEM) in $\mathcal{L}_{PE}$, one obtains

(6) $\mathsf{F}_p(n)\mathcal{A} \vee \mathsf{F}_p(n)\neg\mathcal{A}$,

which may fail to be PE-true. Indeed, an instance of (6) such as $\mathsf{F}_p(2)p \vee \mathsf{F}_p(2)\neg p$ is PE-false if evaluated at moment m_1 in the model pictured just above. Thus, PE fails to validate what seems to be a valid schema.

Objection 3. *Negation and scope ambiguities.*

MacFarlane (2014: 216) rightly points out that there is no difference in asserting one of the two following statements.

(7) Tomorrow it won't be sunny,
(8) It is not the case that tomorrow it will be sunny.

Accordingly, an adequate semantics should make (7) and (8) equivalent. Notice, moreover, that the two statements differ only with respect to the scope of the negation: in (7), "will" takes wide scope over the negation, while in (8) it is the other way round. Now, if one translates both sentences in $\mathcal{L}_{PE}$, (7) becomes

[3] As we shall see in the next chapter, however, saying that actuality plays no legitimate role within an indeterministic setting is simply wrong.

[4] See, for instance, Thomason (1970: 267) and MacFarlane (2014: 217)

(9) $\mathsf{F}_p(n)\neg\mathcal{A}$,

while (8) is

(10) $\neg\mathsf{F}_p(n)\mathcal{A}$.

These latter two statements are not equivalent in PE, for an instance of (9) as $\mathsf{F}_p(1)\neg p$ is PE-false at m_1 in the model pictured above, but an instance of (10) as $\neg\mathsf{F}_p(1)p$ is PE-true at the very same point. And this is bad.

Objection 4. *Retrospective truth judgments.*

MacFarlane (2014) highlights that the Peircean approach is affected by another flaw. This time, Peirceanism doesn't capture some intuitive truth ascriptions which are relative to predictions made in the past.

Suppose that today is Tuesday, and it is actually sunny. Assume that someone, say Jake, asserted on Monday that it would be sunny the next day. On the basis of this data, it is only natural to conclude, on Tuesday, that what Jake has predicted on Monday was true. And this conclusion seems to be independent of whether Tuesday's weather is contingent from Monday's perspective. In a nutshell, if yesterday someone asserted that $\mathcal{A}$ would be the case the next day, and the next day it is the case that $\mathcal{A}$, what was predicted yesterday was true.

Jake's example generalises. From a wider perspective, an adequacy condition for a branching time semantics is to define a truth operator and a notion of logical consequence which make the following pattern sound.

(11) $\mathcal{A} \vdash \mathsf{P}(n) True \mathsf{F}(n)\mathcal{A}$
$\mathcal{A}$ logically entails that (n time units ago) it was true that (in n time units) $\mathcal{A}$ would be the case.[5]

Schema (11) is not sound within PE, at least if the future operator in the schema is treated as the primitive, Peircean future operator $\mathsf{F}_p(n)$. To see why, let us us enrich $\mathcal{L}_{PE}$ with a the sentential operator which expresses PE-truth.[6]

(12) $m \vDash_{PE} \mathsf{T}_p\mathcal{A} \Leftrightarrow m \vDash_{PE} \mathcal{A}$

Then, let us define the Peircean notion of logical consequence.

[5]Clearly, the pattern should be sound only at moments that are at least later than n time units from the first moment (if any).

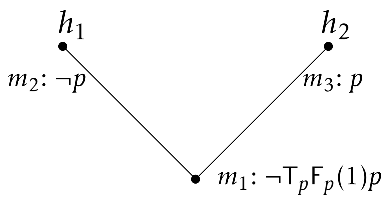
[6]For the sake of simplicity, we shall ignore the problems related to liar paradoxes and the like.

Definition 3.1.2 (Peircean logical consequence)**.** *Given two sets of wffs Γ, Δ of $\mathcal{L}_{PE} \cup \{\mathsf{T}_p\}$, Δ is a Peircean logical consequence of Γ (in symbols. $\Gamma \vdash_{PE} \Delta$) just in case, for any $\mathcal{M}_{PE}$ and any moment m in it, if $m \vDash_{PE} \mathcal{A}$ for any $\mathcal{A} \in \Gamma$, then there is a $\mathcal{B} \in \Delta$ such that $m \vDash_{PE} \mathcal{B}$.*

Accordingly, (11) finds its natural Peircean translation in

(13) $\mathcal{A} \vdash_{PE} \mathsf{P}(n)\mathsf{T}_p\mathsf{F}_p(n)\mathcal{A}$.

An inferential pattern such as $p \vdash_{PE} \mathsf{P}(1)\mathsf{T}_p\mathsf{F}_p(1)p$, which is an instance of (13), has a countermodel depicted in the following picture.



If $d_{\mathcal{T}}(m_1, m_2) = d_{\mathcal{T}}(m_1, m_3) = 1$, and if p is PE-true at m_3 but PE-false at m_2, then $\mathsf{T}_p\mathsf{F}_p(1)p$ is PE-false at m_1, and hence $\mathsf{P}(1)\mathsf{T}_p\mathsf{F}_p(1)p$ is PE-false at m_3. Accordingly, $p \vdash_{PE} \mathsf{P}(1)\mathsf{T}_p\mathsf{F}_p(1)p$ does not hold, for the PE-true truth of its premise does not guarantee the PE-truth of the consequent. Peirceanism does not capture retrospective truth judgments.

The reason why schema (13) is not sound is related to the way in which a Peircean theorist rejects the Master argument. First, let us notice that, by (12), the Peircean truth operator is redundant, and thus (13) is equivalent to

(14) $\mathcal{A} \vdash_{PE} \mathsf{P}(n)\mathsf{F}_p(n)\mathcal{A}$.

Now, notice that the following deductive rule,

Deduction Theorem $\Gamma, \mathcal{A} \vdash \mathcal{B} \Rightarrow \Gamma \vdash \mathcal{A} \to \mathcal{B}$

does hold within the Peircean semantics. Thus, if $\mathcal{A} \vdash_{PE} \mathsf{P}(n)\mathsf{F}_p(n)\mathcal{A}$ were sound, *Deduction Theorem* would guarantee the PE-validity of the following schema.

(15) $\mathcal{A} \to \mathsf{P}(n)\mathsf{F}_p(n)\mathcal{A}$

But recall that Peirceanism rejects the Master argument because the principle of retrogradation,

PR $\mathsf{F}_p(m)\mathcal{A} \to \mathsf{P}(n)\mathsf{F}_p(m+n)\mathcal{A}$,

is not PE-valid. The reason of the failure is that, if it is presently settled that $\mathcal{A}$ would occur, it does not follow that it was settled that $\mathcal{A}$ would occur. Thus, from a Peircean perspective, the Master argument doesn't go through in virtue of the strong modal reading that PE attributes to its primitive future operator. But *PR*, which is not PE-valid, is an instance of (15), which, in turn, is equivalent to $\mathcal{A} \to \mathsf{P}(n)\mathsf{T}_p\mathsf{F}_p(n)\mathcal{A}$. Thus, $\mathcal{A} \to \mathsf{P}(n)\mathsf{T}_p\mathsf{F}_p(n)\mathcal{A}$ cannot be PE-valid as well, and by the *Deciction Theorem*, it follows that (13) ($\mathcal{A} \vdash_{PE} \mathsf{P}(n)\mathsf{T}_p\mathsf{F}_p(n)\mathcal{A}$) cannot be PE-sound. The Peircean rejection of the principle of retrogradation entails that PE doesn't capture retrospective truth judgments. And, as we have seen, the principle of retrogradation is not a PE-validity given the strong modal reading of $\mathsf{F}_p(n)$. Hence, that reading allows PE to reject the Master argument, but it doesn't permit to account for retrospective truth judgments.

Objection 5. *A problem with propositions.*

The problems related with retrospective truth judgments help to highlight a tension between the Peircean semantics and some intuitive reading of what tensed statements are in general taken to mean. Thus, let us first clarify what a proposition may be taken to be. There are several, incompatible accounts which spell out the nature of propositions. In what follows, I will try to remain as neutral as possible about substantial issues on the nature of propositions. I would not say anything about whether propositions are structured entities, or if the fregean account fares better than the russellian one.

What I'll assume, however, is a Kaplanian-like approach.[7] As Kaplan (1989b: 502) suggests, let us call *circumstances of evaluation* those situations in which it is appropriate to ask for the extension of a given well-formed expression. For instance, it is sensible to ask whether "red" has an empty extension only with respect to a state of the world at a time. According to the Kaplanian view, "Something is red" expresses a content (i.e., a proposition) that is evaluable only with respect to a world and a time. And it is natural to hold that a sentence such as "Something is red" expresses a true proposition only relative to those circumstances in which there is at least one red thing. Under this view one may say that sentences

[7]A similar take on propositions is that of David Lewis (1980) and Robert Stalnaker (1999, 2014).

expresses propositions, and any proposition determines an intension. The intension of a proposition is a map from circumstances of evaluations to truth values.[8]

Accordingly, one may know if two sentences do not express the same proposition (at a context) by checking whether they have the same intension (at that context). If two sentences, as used at a given context, have different intensions, they express two distinct propositions.

But recall that Peirceanism evaluates sentences at moments of use. Thus, if one wants to know whether PE predicts that the intension of a sentence $\mathcal{A}$ differs from that of $\mathcal{B}$ at a context, one should be able to draw a link between moments in which $\mathcal{A}$ and $\mathcal{B}$ may be used – that is, moments at which $\mathcal{A}$ and $\mathcal{B}$ are associated with semantic values – and the intension associated to $\mathcal{A}$ and to $\mathcal{B}$ as used at those moments. Here's the link Kaplan brings into the scene.

> If c is a context, then an occurrence of $\mathcal{A}$ in c is true iff the content expressed by $\mathcal{A}$ in this context is true when evaluated with respect to the *circumstance of the context*. (Kaplan 1989b: 522)[9]

What is important is that a context of use – which in our case is just a moment – determines those circumstances of evaluation that, when combined with the intension of a sentence, yield a truth value. Accordingly, to identify the intension of the proposition expressed by $\mathcal{A}$, used at a given context, one has to apply a two-step strategy.

- First, the intension of a statement $\mathcal{A}$, used at m, should have as its value the truth status that a given semantics attributes to $\mathcal{A}$ used at m. For instance, if $\mathcal{A}$ is a future contingent at m, then the Peircean semantics predicts that, when one takes the circumstances determined by m as arguments of $\mathcal{A}$'s intension, $\mathcal{A}$'s (Peircean) intension yields the truth value false.
- Second, the kind of circumstances of evaluation which figure as the argument of the intension of $\mathcal{A}$ (used at m), and which are provided by a moment of use m, may reflect different philosophical views about the nature of propositions. In the case of tensed sentences, there are two main rivals.[10]

[8] Notice that we remain neutral as to whether propositions are intensions.

[9] Emphasis added.

[10] For a defence of the temporalist perspective, see Prior (1968b), Recanati (2007) and Brogaard (2012). For a defence of the eternalist view, see Evans (1985: 343-363) and Cappelen & Hawthorne (2009).

The temporalist perspective. According to temporalists, the proposition expressed by a tensed statement does not vary with the moment of utterance. For instance, "Tomorrow it will be rainy" says *that tomorrow it will be rainy* independently of the day in which it is uttered. And, intuitively, *that tomorrow it will be rainy* is true on Monday if it is rainy on Tuesday, but it is false on Tuesday if it is sunny on Wednesday. The point is that the proposition conveyed by a tensed statement, uttered at a moment, does not specify the moment relative to which the proposition has to be evaluated. Therefore, the circumstances determined by the context of use, being the parameters which determine the value of the intension (of a statement as used at that context) must specify a moment parameter.

The eternalist perspective. Eternalists, on the contrary, think that the proposition expressed by a statement used at a moment *does* vary relative to the moment of utterance. "Tomorrow it will be rainy", for instance, says different things if uttered in different days. It says *that Tuesday is rainy* if uttered on Monday, but the very same sentence means *that Wednesday is rainy* if uttered on Tuesday. Under an eternalists perspective, the proposition conveyed by a tensed sentence is not time-neutral, for it specifies the moment at which something was, is or will be the case. Accordingly, the circumstances of evaluation provided by the context of use, being the parameters which determine the value of the intension (of a statement as used at that context), do not specify a moment parameter.

As suggested by MacFarlane (2014: 207), one can implement these two perspectives on Peirceanism as follows.

Definition 3.1.3. (Temporalist propositions and temporalist intensions.) *$[\mathcal{A}]^T_m$ is the temporalist proposition expressed by $\mathcal{A}$ as used at m. The intension of $[\mathcal{A}]^T_m$ is the function $\|\mathcal{A}\|_T$ from moment-history pairs m/h to truth values* $\{1,0\}$, *such that*

- $\|\mathcal{A}\|_T(m/h) = 1 \Leftrightarrow m \vDash_{PE} \mathcal{A}$,
- $\|\mathcal{A}\|_T(m/h) = 0 \Leftrightarrow m \nvDash_{PE} \mathcal{A}$.

Definition 3.1.4. (Eternalist propositions and eternalist intensions.) *$[\mathcal{A}]^E_m$ is the eternalist proposition expressed by $\mathcal{A}$ as used at m. The intension of $[\mathcal{A}]^E_m$ is the function $\|\mathcal{A}\|_E$ from histories h to truth values* $\{1,0\}$, *such that*

- $\|\mathcal{A}\|_E(h) = 1 \Leftrightarrow m \vDash_{PE} \mathcal{A}$,
- $\|\mathcal{A}\|_E(h) = 0 \Leftrightarrow m \nvDash_{PE} \mathcal{A}$,

where h is one of the histories passing through the moment of utterance.

According to the temporalist account of Definition 3.1.3, the circumstances of evaluation that a context must provide are moment-history pairs, m/h, where m is the moment of use and h a history passing through it. One may object that, in this case, a context should privilege a single history, and this seems to be incompatible with the Peircean approach. This objection doesn't cut much ice, for any history h passing through the moment of use m would do the same job as any other $h' \ni m$. A sentence $\mathcal{A}$ is PE-true (PE-false) at m just in case $\mathcal{A}$ it is inevitable (impossible) at m, and thus the history parameter is redundant when it comes to determine the Peircean truth status of $\mathcal{A}$ at m. What is philosophically important in the temporalist position is that $\mathcal{A}$'s intension (at m) is function of the moment of use, and thus $\mathcal{A}$ must express at m a temporalist content. Analogously, what is important in the eternalist account of Definition 3.1.4 is that $\mathcal{A}$'s intension (at m) is function of the possible worlds (histories) passing through the moment of use. This trait, in turn, signals that $\mathcal{A}$ expresses an eternalist content at m.

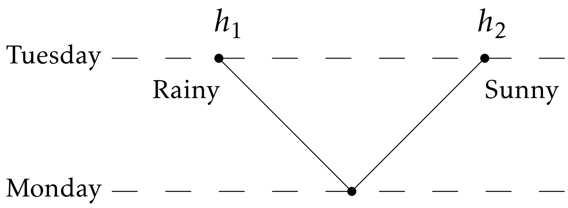
In my view, eternalism may be uncomfortable with Peirceanism. Suppose that today is Tuesday. It should then be natural to argue as follows.

a. A yesterday's utterance of "Tomorrow it will be sunny" means the same thing as a today's utterance of "Today it is sunny".
b. If $\mathcal{A}$ uttered at m means the same thing as $\mathcal{B}$ uttered at m', the value of the intension of $\mathcal{A}$ at m is the same as that of the intension of $\mathcal{B}$ at m'.

∴ The value of the intension of "Tomorrow it will be sunny", as uttered yesterday, is the same as the value of the intension of "Today it is sunny", as uttered today. [11]

[11] It may be objected that the occurrence of "means" in premise *a* is ambiguous, for it may either allude to the proposition expressed by a statement, uttered at a context (which varies from context to context) or, alternatively, to the intension associated with a sentence (which, being a function, does not vary with contexts). Under the former reading, premise *a* is intuitively true, and an eternalist, as well as a temporalist, have essentially no reason to reject it. Under the latter reading, premise *a* is clearly false, for the intuitive intensions of "Tomorrow" and "Today" are bluntly misunderstood. Thus, "means" in premise *a* has to be understood in accordance with the former reading: premise *a* says that the proposition expressed by a yesterday's utterance of "Tomorrow it will be the case" is the proposition expressed by a today's utterance of "Today it is sunny". Notice, moreover, that premises *a*-*b*, when rightly understood, entail what I take to be a quite plausible truth value link. Namely, that "Tomorrow there will be a sea battle" was true as uttered yesterday iff "Today there is a sea battle" is true as used today. Both Dummett (1978: 363) and Iacona (2013: 30-31) draw attention on it.

It can be shown that the eternalist view of Definition 3.1.4 cannot validate this argument. Indeed, suppose that we are dealing with the scenario pictured in the figure below.



First, suppose that we stick to a temporalist account for propositions, and assume that we are located at the sunny version of Tuesday. "Today is sunny" is PE-true on that Tuesday version, and its temporalist intension, say $||$Today it is sunny$||_T$(Tuesday/h_2), has value 1. According to the Peircean semantics, "Tomorrow it will be sunny", uttered on Monday, is PE-true just in case it is sunny on any possible version of tomorrow. But the graph pictured above tells us that "Tomorrow it will be sunny" is a future contingent on Monday, and thus it must be PE-false if uttered on that day. As a consequence, the temporalist intension of "Tomorrow it will be sunny" uttered yesterday, $||$Tomorrow it will be sunny$||_T$(Monday/h_2), must have value 0. Given that $||$Today it is sunny$||_T$ takes value 1 at (Tuesday/h_2), and $||$Tomorrow it will be sunny$||_T$ takes value 0 at (Monday/h_2), this doesn't suffice to show that the two intensions are different, for the arguments we just plugged in to the two maps differ.

But suppose that one adopts an eternalist view. Again, then the intension of the proposition expressed by "Today it is sunny" uttered on the sunny version of Tuesday, say $||$Today it is sunny$||_E(h_2)$, must take value 1. And the intension of the proposition expressed by "Tomorrow it will be sunny"uttered yesterday, $||$Tomorrow it will be sunny$||_E(h_2)$, must have value 0. An eternalist who sticks to Peirceanism cannot accept the conclusion of the above argument, according to which "Tomorrow it will be sunny", as uttered yesterday, has the same intension of "Today is sunny", as uttered today. Otherwise she would have to subscribe to a contradiction: $||$Tomorrow it will be sunny$||_E(h_2) = ||$Today it is sunny$||_E(h_2) = 0 = 1$.

A Peircean eternalist must reject premise *a* of the argument, and say that "Tomorrow it will be sunny", as uttered yesterday, means something different than "Today is sunny", as uttered today. I take for granted that any eternalist would hardly subscribe to something like that. The rejection of premise *a*, therefore, is a bad result, for it makes Peirceanism an attractive semantics only for the temporalist view.

It is time to recap the pros and cons of the Peircean semantics.

3.1.3 Assessing Peirceanism

To sum up, PE is apt to adequately answer (Q1)-(Q2). Indeed,

- ✓ PE refutes the Aristotelian argument and provides a bivalent notion of truth at a moment.
- ✓ PE refutes the Master argument by rejecting the retrogradation principle, *RP*.

The Peircean approach, however, has five significant flaws.

- ✗ PE equates "plain will" with "inevitably will".
- ✗ PE does not validate the future excluded middle.
- ✗ PE introduces a semantic distinction between $\mathsf{F}_p(n)\neg\mathcal{A}$ and $\neg\mathsf{F}_p(n)\mathcal{A}$.
- ✗ PE does not account for retrospective truth judgments.
- ✗ PE yields a notion of intension which is at odds with an eternalist understanding of the meaning of tensed statements.

I take these result as evidence that Peirceanism cannot be assessed as adequate. Can one do better?

3.2 Thomason's first interpretation

Recall that the main problem with the BO semantics was that it does not provide a notion of bivalence which is relative to moments only. This is due to the fact that BO-points of evaluations are moment history pairs, m/h. Prior (1967: 126) called the historical parameter in m/h a *prima facie assignment* of a future to m. However, it isn't clear what does it mean for a history to be a *prima facie assignment*. According to Thomason, prima facie assignments represent a serious, philosophical problem.

> [Ockhamism] is not above criticism. It says that more is needed to assign a truth-value to a formula at time m than a model structure and assignment to formulas not involving tense operators. Besides these a possible future for m must be specified, for on this view statements in the future tense do not in general

> take a truth-value at m unless a possible future is specified.[12] (Thomason 1970: 270)

Then, he goes on explaining why the historical parameter is problematic. He argues that the prima facie assignments can have two interpretations, each of which makes the Ockhamist semantics philosophically suspicious.

> According to the first interpretation we [evaluate predicitons] by *provisionally positing* a possible future for m, because a prediction made at m can only be true or false relative to such a future. [...] Since we may often be in situations in which we have made no suppositions concerning which of a variety of possible futures will come about, it should be often the case that certain statements in the future are neither true nor false. (Thomason 1970: 270–271)

Thomason's quotation suggests that uttering a future-tensed statement carries a supposition relative to which possible future (history) will come about. Thus, a *prima facie assigment h*, which occurs within a point of evaluation of a statement uttered at m, is the history the speaker takes to happen in the future. But since speakers fail to have these suppositions most of the time, we need to abandon the Ockhamist semantics. In particular, we need a semantic machinery which reflects these failures, making certain statements about the future neither true nor false.[13]

Notice that it is crucial for this interpretation that the two following thesis hold.

Supposition Uttering a future-tensed statement carries a supposition relative to which possible future (history) will come about,

Failure Agents fail to have these suppositions most of the time.

I shall argue that the *Supposition* principle has two readings. Under its *de re* interpretation, the principle is false. Under the second, *de dicto* reading, the *Supposition* principle is true but the *Failure* thesis is false. Thus, Thomason's first interpretation does not provide neither a rationale to abandon the Ockhamist semantics, nor a reason to drop bivalence.

[12] Notation changed.

[13] Thomason here has in mind some sort of semantic presupposition failure, probably the one Strawson 1952 talks about.

3.2.1 De re suppositions

The *Supposition* principle can be red in two ways. An agent's utterance of a future-tensed sentence may involve a *de re* supposition about a specific history h, that is, the specific h that an agent supposes to contain the future that will come about. On the other hand, an agent's utterance can carry a *de dicto* supposition, according to which the speaker supposes that there exists an h whose future will be actualised. Let us disentangle the *Supposition* thesis with respect to its two *de re*/*de dicto* readings.

De re Supposition If a utters $\mathcal{A}$ at m, $\mathcal{A}$ has to be evaluated w.r.t. m/h, where h is the history of which a, at m, supposes that will come about. If a utters $\mathcal{A}$ at m,$\exists h$(a supposes at m that h will come about $\wedge$ $m/h \vDash \mathcal{A}$)

Malpass & Wawer (2012: 122) noticed that the *de re* reading is also suggested by John P. Burgess. Indeed, he comments on the Ockhamist points of evaluations by saying that

> The [Ockhamist] truth-value of a future tense statement depends on which branch we think of as representing the course of events which is actually going to turn out to happen. (Burgess 1979: 575)

Be as it may, utterances of future-tended statements may carry another, the *de dicto* supposition as well.

De dicto Supposition If a utters $\mathcal{A}$ at m, a supposes at m that there is a unique h that will come about, and that $\mathcal{A}$ has to be evaluated w.r.t. m/h.If a utters $\mathcal{A}$ at m, a supposes at m that $\exists h$(h will come about $\wedge$ $m/h \vDash \mathcal{A}$)

One challenge these two readings have to face is whether they make sense of the way we ordinarily talk. Each of them should make sense of it, since both of them take agents – that is, ordinary speakers – as doing suppositions about what possible future will obtain.

It is easy to see that the *de re* reading yields counterintuitive results with respect to ordinary linguistic phenomena. For imagine that Alice takes h_1 as the history whose future will come about, where Bob supposes that another history, say h_2, is the one that will happen. Furthermore, assume that (1) is OC-true at m_1/h_1 but OC-false at m_1/h_2. Now, let us focus on the following dialogue (taking place at m_1):

Alice : Tomorrow there will be a sea battle.

Bob : That's false! There won't be any battle at all!

Intuitively, Bob is saying that Alice is wrong. But suppose that the *de re* account holds, and that the history a speaker takes to come about is part of the content expressed. Then, the *de re* account predicts that Bob is simply misunderstanding Alice. Indeed, Bob mistakenly takes Alice to refer to the same history he is referring to. This is not only intuitively wrong. It leaves open the absurd hypothesis that speakers may commit systematic errors when they try to understand each others.

On the other hand, if the history a speaker refers to is not part of the content expressed, Bob and Alice are contradicting each other. Each of them, however, is right from his/her own perspective. Alice is expressing a true content, relative to the history she takes to come, and the same holds for Bob. But again, this sort of faultless disagreement is misleading. As Stalnaker points out,

> [...] we don't get, from this particular kind of relativism, a feature that seems to be characteristic of some of the other applications: disagreement where neither party to the disagreement are making a mistake. If I predict (at m 1) the bombing the next day, and you predict (at that moment) that there will be no bombing the next day, [...] we will also agree that both of our statements should be assessed only tomorrow, and that at that time, one of us will be shown to have made a mistake. (Stalnaker 2014: 438)

Hence, the *de re* account is at odds with our ordinary way of speaking about the future.

One may object that, in these cases, either Alice or Bob fails to suppose what future will come about. But why one should say so? This objection is completely ad hoc, for it prevents speakers to fulfil what the *De re Supposition* principle requires in any situation similar to that of the dialogue. Therefore, the *De re Supposition* thesis cannot account for the way we use tensed statements, and it has to be rejected.

3.2.2 De dicto suppositions

On the other hand, the *de dicto* reading appears to be much more in accordance with the common talk. Take the first dialogue between Alice and Bob. Given that Alice utters "tomorrow there will be a sea battle" at m_1, Alice believes that there is an h that will come about, and that m_1/h satisfies "tomorrow there will be a sea battle". Similarly, Bob believes that there is an h that will come about, and that m_1/h satisfies "tomorrow there won't be any sea battle". If they also share – as plausibly any ordinary speaker do – the belief that there is a single history that will be actualised, and since no h is such that m_1/h satisfies both "tomorrow there will be a sea battle" and its negation, Alice and Bob must disagree with each other. In particular, they disagree whether the unique future that will be actualised has a sea battle in it. This result, in turn, is intuitively right.

Recall that I am assuming that both Alice and Bob suppose that only one future (history) will be actualised. This assumption is deeply rooted in the way we conceive how things will turn out. As a piece of evidence, there is a telling quotation by David Lewis.

> The trouble with branching exactly is that it conflicts with our ordinary presupposition that we have a single future. If two futures are equally mine, one with a sea fight tomorrow and one without, it is nonsense to wonder which way it will be [...] and yet I do wonder. (Lewis 1986a: 207-208)

Interestingly, Lewis argues against a branching conception of time, for he takes it to be incompatible with "our ordinary presupposition that we have a single future". It is easy to see that Lewis' worries vanish if we drop the hypothesis that "two futures are equally mine", while admitting that there is a single, privileged future. This move permits to make sense of the "presupposition that we have a single future" even in a branching time framework. Surely, several philosophers are not disposed to reject that all possible futures are on a par with one another. As a matter of fact, who endorses this view struggles with some version of the so-called "assertion problem": if there is no privileged future, one has to explain why it seems that we talk about it all the time.[14] The fact that those who reject a privileged future see the assertion problem *as* a problem is far from being an idle feature. Quite to the contrary, it shows that the "presupposition that we have a single future" is very intuitive, and can hardly be contested. But if it is so, it is contradictory to stick to the *Failure* thesis, according to which speakers fail to presuppose that they have a single future most of the time. This shows that Thomason's first interpretation does not motivate any departure neither from the Ockhamist semantics, nor from bivalence.

There is one point that has to be stressed. If an agent a believes at m that a history h will come about, it doesn't follow that there is a specific

[14]The two main versions of the assertion problem can be found in Belnap & Green (1994), Belnap et al. (2001) and MacFarlane (2014).

history of which *a* believes (at *m*) that it will come about. According to the *De dicto Supposition* principle, the reference to the history that will come about is, in an important sense, a generic one. A spatial analogy may be illuminating.

> In many action movies, there is a scene in which the good guy and the bad guy are in different rooms of the same house, each of them slowly walking without making noise in order to kill the other without being killed. [...] [Al] senses that a man is standing behind a door in front of him. He points his gun towards the centre of the door, fires off, and hits Bob. Since Al can't see Bob from his position, Al's intention is generically directed toward the man behind the door. It is not an intention to hit a specific man. (Iacona 2013: 38-39)

Andrea Iacona rightly points out that Al's intention to shoot the men behind the door is not an intention to shoot a specific man. Analogously, an agent's supposition that the history that will come about would satisfy a given formula is not a supposition towards a particular history. As for the spatial example, however,

> [...] once the gun fires off, the bullet is able to go through the door and hit Bob without further assistance. (Iacona 2013: 38-39)

Thus, even if Al's act is guided by a generic intention, its effects go beyond the generic nature of the intention. Indeed, the bullet hits Bob, the particular man standing behind the door. Similarly, when Al utters a future-tensed statement – leaded by a *de dicto* supposition – his speech act has semantic effects that go beyond the *de dicto* supposition which guides it. In particular, the speech act would be true just in case it predicts something that obtains at the particular future that will be actualised. Even if a *de dicto* supposition refers to the actual future generically, only a specific future, the one that will be actualised, is relevant for the truth value of a (future directed) utterance guided by that supposition.

As I said, it is pretty easy to make sense of the *de dicto* suppositions once that the possible futures are not on a par with one another. If there is one privileged possible future, any *de dicto* supposition would just reflect a metaphysical feature of reality. However, several philosophers reject the existence of such a privileged future, and their denial is related to the second interpretation that Thomason gives for the historical parameter.

3.3 Thomason's second interpretation

It used to be said of the British Empire that it was maintained by a thin red line of soldiers in service to the Queen. Analogously, some philosophers claim that, among the several branches of the tree, there is a history that is marked in red – that is, there is a distinguished, privileged history. As the thin red line of soldiers, the privileged history may be unknown. However, it is real as the Queen's soldiers were.

Under these assumptions, it is natural to hold that a BO-point of evaluation such as m/h is legitimate, and thus it can occur within a definition of truth for $\mathcal{L}_T$, only if the historical parameter refers to the actual history. This is what Thomason calls the second interpretation the historical parameter. According to his view, however, the second interpretation is incompatible with the branching conception of indeterminism defined as BOI.

> According to the second interpretation, just one of the futures of m is the right one – the one that will be actualised.This second view does not square very well with the whole project of indeterministic tense logic. For if a time m can only have one "real" future, times located in others alternative futures cannot really bear a temporal relation to m. They can bear an *epistemic* relation, being futures for a situation which for all we know is the actual one m, but strictly speaking this is not a temporal interpretation. Thus, indeterministic tense logic collapses to deterministic tense logic. (Thomason 1970: 270-271)

First, Thomason seems to endorse the following principle.

Second Interpretation: Moment-history pairs m/h can legitimately occur within a definition of truth for $\mathcal{L}_T$ only if h refers to the unique actual history.

But Thomason remarks that if there is only one actual future of m, any future moment of m which falls outside that privileged future can only represent an epistemic possibility. This means that only one history (passing through m) collects what it is objectively possible after m. Hence,

Unique Possible History: If there is a unique actual history, there is only one history that collects what is objectively possible.[15]

[15] This principle is shared by many branching theorists, such as Belnap & Green (1994), Belnap et al. (2001), MacFarlane (2003, 2008, 2014) and Todd (2015).

If the latter principle holds, there couldn't be two alternative, objective futures for m, and indeterminism would be contradicted. Thus, from the *Unique Possible History* principle it follows that

$\therefore$ If there is a unique actual history, branching indeterminism (BOI) is false.

Accordingly, by *Second Interpretation* one infers that

$\therefore$ If branching indeterminism (BOI) is true, moment-history pairs m/h can't legitimately occur within a definition of truth for $\mathcal{L}_T$.

In my view, the *Second Interpretation* principle is quite intuitive; it is sensible to evaluate a future contingent at m/h only if h contains what will actually happen after m. What is perplexing in Thomason's argument, however, is the *Unique Possible History* principle. It equates *the number* of actual histories with *the number* of objectively possible histories. And this equation has no intuitive appeal: if I say that there is a unique actual world, it hardly follows that no other objectively possible world exists.

The *Unique Possible History* principle is supported by the kind of actuality that many branching theorists have often adopted, and which Thomason summarises as follows.

> To the thoroughgoing indeterminist, the choice of a branch b through t has to be entirely *prima facie*; there is no special branch that deserves to be called the "actual" future through t. Consider two different branches, b_1 and b_2, through t, with $t < t_1 \in b_1$ and $t < t_2 \in b_2$. From the standpoint of t_1, b_1 is actual (at least, up to t_1). From the standpoint of t_2, b_2 is actual (at least up to t_2). And neither standpoint is correct in any absolute sense. (Thomason 1984: 145)

First, the quotation speaks about the actuality of a history up to a moment. In particular, it suggests that a set of moments b can be said to be actual relative to a moment m just in case, for any $m' \in b$, $m' \leq m$. Thus, actuality seems to be a property of sets of moments, and these sets may be proper subsets of histories. Secondly, the notion of actuality that Thomason has in mind is *relational* or *perspectival*: any set of moments can only be actual from the perspective of a given moment. No "standpoint is correct in any absolute sense." This reading of actuality is similar to that of Lewis:

> The inhabitants of other worlds may truly call their own world actual if they mean by 'actual' what we do; for the meaning we

> give to 'actual' is such that it refers at any world *i* to that world *i* itself. 'Actual' is indexical, like 'I' or 'here' or 'now': it depends for its reference on the circumstances of utterance, to wit, the world where the utterance is located. (Lewis 1973a: 85-86)

Both Lewis and Thomason subscribe to the semantical thesis that "actual" can only denote a relational property. Both Thomason and Lewis, however, holds that the relational analysis cannot be separated from the *metaphysical* thesis that the actuality of a branch (or, for that matter, of a world) is *nothing* over and above a relational property.[16]

> I suggest that "actual" and its cognates should be analyzed as indexical terms: terms whose reference varies, depending on relevant features of the context of utterance. The relevant feature of context, for the term "actual", is the world at which a given utterance occurs. According to the indexical analysis I propose, "actual" (in its primary sense) refers at any world *w* to the world *w*. "Actual" is analogous to "present", an indexical term whose reference varies depending on a different feature of context: "present" refers at any time *t* to the time *t*. (Lewis 1983b: 17)

> See D. Lewis (1983b) and substitute "the actual future" for "the actual world" in what he says. *That* is the view of the throughgoing indeterminist. (Thomason 1984: 160)

Notice, however, that in Lewis' favourite modal framework – that is, in a divergentist framework – worlds do not overlap. Hence, if one substitutes "the actual future" for "the actual world" in what he says, and takes "the actual future", as used at *t*, to mean the future of time *t* in the world of the utterance, the definite description "the actual future", evaluated at *t*, turns out to be proper. However, if one adopts a branching conception of reality, worlds overlap and branch. Hence, if one utters "the actual future" at a moment *m*, and takes "the actual future" to mean the future path of *m* in *the* history of the utterance, the description "the actual future" turns out to be improper most of the time. As Belnap & Green (1994) summarise,

> For a world to be actual is for it to be the world we inhabit. For a history to be actual would be for it to be the history to which the moment we inhabit belongs. [...] It is not [...] in general the

[16]For a similar diagnosis of David Lewis' position about actuality, see Stalnaker (1976: 69).

> case that the expression 'the history to which the moment we inhabit belongs' secures a referent, since uniqueness fails in the face of indeterminism. One does on the other hand always succeed in referring with the expression, 'the set of histories to which the moment we in- habit belongs', for which an alternative description might be, 'the actual situation'.(Belnap & Green 1994: 381)

What Belnap seems to suggest is that a history h is actual relative to a moment m just in case h is the only history passing through m. And if one takes "$@_1$" to denote this reading of actuality, both Belnap's quotation refer to the following principle:

Moment-History Actuality $@_1(h,m) \Leftrightarrow h$ is *the* history h' such that $(m \in h')$.

Notice, however, that $@_1(h,m)$ is false most of the time. I my viw, this is the reason that explains why Thomason holds that what what a branching indeterminism can say (at best) is that, whenever I claim "b is actual" at m, I am claiming that any moment in b is either earlier than or equal to the moment of my utterance. Hence, if we take a branch b to be a set of moments, we may define a notion of *Branch-Moment Actuality* as follows:

Branch-Moment Actuality $@_2(m,b) \Leftrightarrow \forall m' \in b(m' \leq m)$

Given the *Branch-Moment Actuality*, it is plausible to define a further notion of actuality which involves moments only:

Moment-Moment Actuality $@_3(m,m') \Leftrightarrow m' \leq m$

Notice, one again, that these notions of actuality are relational or perspectival: any set of moments (or any moment) can only be 'seen' as actual from the perspective of a given moment. Again, no "standpoint is correct in any absolute sense."

Once that the *Moment-History Actuality* principle is at its place, one can understand why a branching theorist may be tempted to accept the *Unique Possible History* principle.

Suppose that one wants to say that there is a history that is 'seen' as actual from any possible perspective – that is, there is a history which is actual from any possible moment. If one expresses this belief by using *Moment-History Actuality*, one has to be committed to the following principle.

Unique Actuality: $\forall m \exists ! h(@_1(h,m))$.

The *Unique Actuality* principle amounts to $\forall m \exists ! h(m \in h)$. It is immediate to see that this latter statement says that there exists exactly one possible history, and no other course of events can possibly exist.

On the one hand, if there were no histories at all, *Unique Actuality* would be contradicted. On the other hand, assume that there are two different histories, say h_1 and h_2. By condition (b) of Definition 2.2.1, there must be a moment, say m_1, that h_1 shares with h_2, (i.e., $m_1 \in h_1$ and $m_1 \in h_2$). But *Unique Actuality* says that there is only one history at which m_1 must belong. By reductio, *Unique Actuality* entails that exactly one history does exist.

Thus, *Unique Actuality* directly contradicts the branching conception of objective indeterminism, for BOI says there must exist at least two histories:

BOI: $\exists m, h, h'(m \in h \wedge m \in h' \wedge h \neq h')$.

Moreover, by assuming *Moment-History Actuality* and *Unique Actuality*, the *Unique Possible History* principle follows: if there is a unique actual history, there is only one objectively possible history.[17]

To sum up, Thomason's *Second Interpretation* of the historical parameter says that it would be legitimate to adopt moment-history couples m/h as truth-parameters only if the history (world) in the couple, h, were *the* actual history passing through the moment of the couple, m. And the *Unique Possible History* principle expresses an incompatibility between the branching conception of indeterminism and the commitment to a unique actual history. Therefore, if indeterminism is true, then moment-history couples cannot be taken as legitimate truth-parameters, for no history which passes trough a moment can be said to be *the* actual history, that is, the only history passing through that moment. And if one aims at a definition of truth-at-a-context, *no* history passing through the moment of the context can be said to be *the* history of the context. This conclusion is shared among many branching theorists. As MacFarlane puts it,

> [W]hat is it for such a proposition to be true at a context? [...] In frameworks without branching, these questions have a standard answer. A sentence S is true at a context c just in case $w_c, t_c \vDash S$, where w_c and t_c are the world and time of c. [...]

[17]Clearly, there might be several reactions to this argument. One may say, for instance, that the perspectival notions of actuality are misleading. An advocate of a single, actual history can invoke a substantial notion of actuality. Thus, one can refute the relational principles we have seen above, for they fail to express the non-indexical character that such a position may attribute to actuality.

> However, these simple answers aren't available on the branching picture. For they all assume that it makes sense to talk of "the world of the context of use." And
>
> **Nondetermination Thesis.** *In frameworks with branching worlds, a context of use does not, in general, determine a unique "world of the context of use," but at most a class of worlds that overlap at the context.* (MacFarlane 2014: 208)[18]

An analogous remark is made by Belnap et al. (2001),

> The history-of-evaluation parameter is *not* initialized by the context of use: Given indeterminism, there is no such thing as "the history of use". [...] The happening of an assertion that perchance uses "The coin will land heads" as its vehicle no more determines one of these histories than any other. A speech act determines "the moment of its occurrence", but not "the history of its occurrence". So since there is no "history of the context", the history-of-evaluation parameter cannot be initialized by context. (Belnap et al. 2001: 151)

Accordingly, whoever subscribes to both a branching conception of indeterminism (BOI) and to the *Unique Possible History* has to tell a story about how to handle the historical parameter. Such a branching theorist , indeed, should give an a account of truth that does not privilege any history among those passing through the moment of use. The supervaluationist semantics, as we shall see, is designed to satisfy such a requirement.

3.4 Supervaluationism

The supervaluationist view was originally proposed by Mehlberg (1958) for dealing with vagueness, and later was developed and formalised by Van Fraassen (1966). Thomason (1970) was the first logician who applied supervaluationism to the problem of future contingents.

As we have have just seen, Thomason aims at a notion of truth for wffs of $\mathcal{L}_T$ that does not attribute any privileged status to any history passing through a moment (of use). Moreover, he explicitly rejects the Peircean approach, for it preserves bivalence at a cost that appears to be too high: Peirceanism does not validate neither the principle of retrogradation, nor the future excluded middle.[19]

[18] Notation changed.
[19] See Thomason (1970: 268).

Recall that the BO semantics BO evaluates a statement $\mathcal{A}$ at moment-history pairs m/h. However, in several cases the historical parameter turns out to be redundant, since $\mathcal{A}$ may have the same BO-truth value in any history h passing through m. When that happens, supervaluationist assigns a truth value at $\mathcal{A}$ relative to m – naturally, he/she assigns to $\mathcal{A}$ the BO-truth value that the statement has at m/h, for any h passing through m. When it does not happen – viz., when $\mathcal{A}$'s BO-truth value is sensible to the historical parameter – the supervaluationist does not assign a truth value to $\mathcal{A}$ at m. This is so because it is not possible to identify the actual history among those passing through m: anyone of them is on a par with the others.[20] Accordingly, the evaluation of a statement at a moment m does not depend on a specific history passing through that moment, but is relative to *any* of that histories.

These ideas can be captured as follows.[21]

Definition 3.4.1 (Supervaluationism)**.** *Given a BO-model $\mathcal{M}_{BO} = (\mathcal{T}, d_{\mathcal{T}}, I)$, the correspondent supervaluationist model is $\mathcal{M}_{SU} = (\mathcal{T}, d_{\mathcal{T}}, ST, SF)$, where ST (supertruth) and SF (superfalsity) are supervaluationist evaluation functions from wffs of $\mathcal{L}_T$ to sets of moments, such that*

(a) $m \in ST(\mathcal{A}) \Leftrightarrow \forall h(m \in h \Rightarrow m/h \vDash_{BO} \mathcal{A})$
(b) $m \in SF(\mathcal{A}) \Leftrightarrow \forall h(m \in h \Rightarrow m/h \nvDash_{BO} \mathcal{A})$

We can denote supertruth (at a moment) and superfalsity (at a moment) by using $\vDash_{SU}$ and $\nvDash_{SU}$, respectively. These two relations are defined in an obvious manner: given a supervaluationist model $\mathcal{M}_{SU}$ and a wff $\mathcal{A}$ of $\mathcal{L}_T$,

(SU1) $m \vDash_{SU} \mathcal{A} \Leftrightarrow m \in ST(\mathcal{A})$
(SU2) $m \nvDash_{SU} \mathcal{A} \Leftrightarrow m \in SF(\mathcal{A})$

Notice that the supervaluationist semantics is perfectly compatible with MacFarlane (2014: 208)'s Nondetermination thesis, according to which a context of use does not, in general, determine a single world (history), but at most a class of worlds that overlap at the moment of the context. Indeed, a supervaluationist context of use can be identified with a moment, and a moment, in general, fails to be associate with a unique history which passes through it.

Furthermore, SU-truth and SU-falsity are relative to moments only, and thus an advocate of supervaluationism can answer the Aristotelian

[20] See Thomason 1970: 270 and Thomason 1984: 145.
[21] For an analogous definition, see Ciuni & Proietti 2013: 27-28.

question, (Q1), in an appropriate way. First, let us notice that, under a supervaluationist view, the premises (Pa)-(Pb) of the Aristotelian argument must hold.

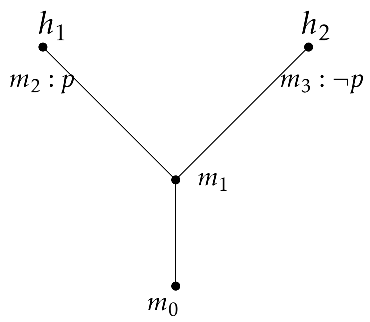
(P)(a) If $\mathcal{A}$ is true at a moment m, then 'Necessarily $\mathcal{A}$' is true at m.
(P)(b) If $\mathcal{A}$ is false at a moment m, then 'Possibly $\mathcal{A}$' is false at m.

As for (Pa), whenever $\mathcal{A}$ is SU-truth at m, then it is BO-true on any history passing through m. But if it is BO-true on any history passing through m, $\Box\mathcal{A}$ must be BO-true at m, for any $h \ni m$. By (SU1), it follows that $\Box\mathcal{A}$ is SU-true at m. Therefore, if $\mathcal{A}$ is true at a moment m, then 'Necessarily $\mathcal{A}$' is true at m, and thus premise (Pa) holds. As for the second premise, (Pb), whenever $\mathcal{A}$ is SU-false at m, there is no h such that $\mathcal{A}$ is BO-true at m/h. This means that $\Diamond\mathcal{A}$ is BO-false at m. Hence, $\Diamond\mathcal{A}$ must me SU-false at m. But then (Pb) follows: whenever $\mathcal{A}$ is SU-false at a moment m, then 'Possibly $\mathcal{A}$' is SU-false at m.

What supervaluationists do not accept is bivalence relative to moments, that is, premise (B) of the Aristotelian argument.

(B) Either $\mathcal{A}$ is true at m, or $\mathcal{A}$ is false at m.

As well known, SU-truth and SU-falsity are mutually exclusive but *not* jointly exhaustive. There is no statement of $\mathcal{L}_T$ that is both SU-true and SU-false at m. Nevertheless, there are statements that are neither SU-true nor SU-false at their moment of use – namely, future contingents. For example, take the model pictures below, and assume that $d_{\mathcal{T}}(m_1, m_2) = d_{\mathcal{T}}(m_1, m_3) = 1$. $\mathsf{F}(1)p$ is a future contingent at m_1, being BO-true at m_1/h_1 but BO-false at m_1/h_2. By (SU1)-(SU2), $\mathsf{F}(1)p$ is neither SU-true nor SU-false at m_1.



Accordingly, the supervaluationist answers (Q1) by saying that one can be a branching indeterminist only if one gives up bivalence (while conserving a notion of truth (falsity) that is relative to moments only).

Interestingly, the supervaluationist semantics is apt to reject the Master argument – an, thus, it can be used to answer (Q2) – given the particular relation it entertains with the BO semantics. SU-truth (SU-falsity) is relative to moments only, but it is defined in terms of the Ockhamist notion of truth (falsity) at a moment on a history. In particular, given the notion of SU-truth it is easy to see that $\mathcal{A}$ is BO-valid just in case $\mathcal{A}$ is SU-valid. This simple fact is proved below.

Definition 3.4.2 (BO-validity)**.** *For any wff $\mathcal{A}$ of $\mathcal{L}_T$,*
$\mathcal{A}$ is BO-valid (in symbols: $\vDash_{BO} \mathcal{A}$) $\Leftrightarrow$ $m/h \vDash_{BO} \mathcal{A}$, for any m/h in any BO-model $\mathcal{M}_{BO}$.

Definition 3.4.3 (SU-validity)**.** *For any wff $\mathcal{A}$ of $\mathcal{L}_T$,*
$\mathcal{A}$ is SU-valid (in symbols: $\vDash_{SU} \mathcal{A}$) $\Leftrightarrow$ $m \vDash_{SU} \mathcal{A}$, for any m in any SU-model $\mathcal{M}_{SU}$.

Fact 1. For any wff $\mathcal{A}$ of $\mathcal{L}_T$, $\vDash_{SU} \mathcal{A} \Leftrightarrow \vDash_{BO} \mathcal{A}$.
Proof. ($\Rightarrow$) If $\vDash_{SU} \mathcal{A}$, $m \vDash_{SU} \mathcal{A}$, for any m in any SU-model $\mathcal{M}_{SU}$. By *(SU1)*, it is the case that $\forall h(m \in h \Rightarrow m/h \vDash_{BO} \mathcal{A})$, for any $\mathcal{M}_{BO}$ corresponding to $\mathcal{M}_{SU}$. But then $\vDash_{BO} \mathcal{A}$ follows. ($\Leftarrow$) If $\vDash_{BO} \mathcal{A}$, then $m/h \vDash_{BO} \mathcal{A}$ for any m/h in any $\mathcal{M}_{BO}$. Hence, $m/h \vDash_{BO} \mathcal{A}$ for any $h \ni m$ in any $\mathcal{M}_{BO}$. By *(SU1)*, $m \vDash_{SU} \mathcal{A}$, for any $\mathcal{M}_{SU}$ correspondent to $\mathcal{M}_{BO}$. But then $\vDash_{SU} \mathcal{A}$ follows.

The equivalence between BO-validity and SU-validity explains why the advocates of supervaluationism reject the Master argument. Since one of the premise of the argument – the principle of the necessity of the past, $\mathsf{P}(n)\mathcal{A} \rightarrow \Box\mathsf{P}(n)\mathcal{A}$ – is *not* BO-valid, then it cannot be SU-valid. Therefore, a supervaluationist answers to (Q2) by saying that the Master argument is unsound.

Notice that if $\mathsf{F}(n)\neg\mathcal{A}$ is SU-true at m, $\neg\mathsf{F}(n)\mathcal{A}$ is SU-true at m. Supervaluationism does not introduce any ambiguities among the scope of the negation and that of the future operator. Under this respect, supervaluationism fares batter than Peirceanism.

An interesting feature of supervaluationism is that disjunctions (as well as conjunctions) are not truth-functional.[22]

To see why, let us evaluate evaluate an instance of the excluded middle, $F(1)p \vee \neg F(1)p$, by using (SU1)-(SU2) with respect to moment m_1 in the

[22]Jan Łukasiewicz (1968), as well as Craig Bourne (2004), deny bivalence without rejecting the truth-functionality of logical connectives. Their trivalent semantics, however, have been highly criticised by many authors (see Placek 2000, Prior 1953, MacFarlane 2003, 2014), and thus seen as a premature attempt to solve the semantical problems raised by future contingents.

model pictured just above. The disjunction is SU-true at m_1, since it is BO-true at both m_1/h_1 and m_1/h_2. However, each of its disjuncts is neither SU-true nor SU-false at m_1. This simple example shows that knowing the SU-truth value (if any) of $\mathcal{A}$ and $\mathcal{B}$ at m may be not sufficient to compute the SU-truth value (if any) of their disjunction $\mathcal{A} \vee \mathcal{B}$ at m (the same goes for conjunctions).

Another relevant characteristic is that future-tensed sentences of the form $\mathsf{F}(n)\mathcal{A}$ convey necessary contents, given that both (SU1) and (SU2) universally quantify over the histories passing through the moment of use. Under this respect, supervaluationism is similar to Peirceanism. Nevertheless, notice that the evaluations induced by clauses (SU1)-(SU2) involve *arbitray* wffs of $\mathcal{L}_T$, whatever their logical structure may be. This latter features is responsible for a crucial difference among supervaluationism, on the one hand, and the Peircean semantics, on the other. Recall that the Peircean does not validate the future excluded middle,

(FEM) Either it will be the case that $\mathcal{A}$, or it will be the case that $\neg\mathcal{A}$.

The Peircean semantics translates (FEM) as $\mathsf{F}_p(n)\mathcal{A} \vee \mathsf{F}_p(n)\neg\mathcal{A}$, which is PE-true at m just in case it is either inevitable that, in n time units in the future, it is the case that $\mathcal{A}$, or it is impossible that, in n time units in the future, it is the case that $\mathcal{A}$. Thus, the Peircean translation of (FEM) comes out PE-false when $\mathsf{F}_p(n)\mathcal{A}$ ($\mathsf{F}_p(n)\neg\mathcal{A}$) is a future contingent at m.

Under a supervaluationist semantics, things are radically different. SU translates (FEM) as $\mathsf{F}(n)\mathcal{A} \vee \mathsf{F}(n)\neg\mathcal{A}$, and since (SU1)-(SU2) evaluate any $\mathcal{A}$ at any history passing through a moment m, then $\mathsf{F}(n)\mathcal{A} \vee \mathsf{F}(n)\neg\mathcal{A}$ just needs to be BO-true (BO-false) at any singular $h \ni m$ in order to be SU-true (SU-false) at m. And it is easy to see that $\mathsf{F}(n)\mathcal{A} \vee \mathsf{F}(n)\neg\mathcal{A}$ is an BO-validity. Hence, supervaluationism validates (FEM), even if it is a semantics that ascribes necessary contents to stand-alone, future-tensed statements.

Supervaluationism has also another advantage over Peirceanism. Recall that this latter semantics introduces a distinction between “Tomorrow it won’t be rainy” and “It is not the case that Tomorrow it will be rainy”. Supervaluationism, on the contrary, does not predict any semantic distinction between $\mathsf{F}(n)\neg\mathcal{A}$ (“Tomorrow it won’t be rainy”) and $\neg\mathsf{F}(n)\mathcal{A}$ (“It is not the case that Tomorrow it will be rainy”). It is easy to check that $\mathsf{F}(n)\neg\mathcal{A} \leftrightarrow \neg\mathsf{F}(n)\mathcal{A}$ is a SU-validity, and $\mathsf{F}(n)\neg\mathcal{A}$ is SU-true just in case $\neg\mathsf{F}(n)\mathcal{A}$ is.

As we shall see, however, supervaluationism has several flaws.

3.4.1 Against the Supervaluationist view.

Objection 1. *Problems with Logical Consequence and Disquotational Truth*

Usually, logical consequence is defined as truth preservation from premises to conclusions. Since supervaluationism identifies truth with super-truth, one may be tempted to define the notion of logical consequence in terms of super-truth preservation.

Definition 3.4.4 (Global logical consequence)**.** *Given two sets of wffs* Γ, Δ *of* $\mathcal{L}_T$*,* Δ *is a global logical consequence of* Γ *(in symbols.* $\Gamma \vdash_G \Delta$*) just in case, for any* $\mathcal{M}_{SU}$ *and any moment* m *in it, if* $m \vDash_{SU} \mathcal{A}$ *for any* $\mathcal{A} \in \Gamma$*, then there is a* $\mathcal{B} \in \Delta$ *such that* $m \vDash_{SU} \mathcal{B}$*.*

As Williamson (2002: 151-152) has shown, $\vdash_G$ forces to reject some deductive rules which are classically sound. For instance, $\mathcal{A} \vdash_G \Box\mathcal{A}$ holds, for whenever $\mathcal{A}$ is SU-true at m, it is true at any $h \ni m$, and therefore $\Box\mathcal{A}$ must also be true at any $h \ni m$. Accordingly, $\Box\mathcal{A}$ must be SU-true at m as well. If $m \vDash_{SU} \neg\Box\mathcal{A}$, it follows that $m \vDash_{SU} \Diamond\neg\mathcal{A}$, which entails that, for any $h \ni m$, there is a $h' \ni m$ s.t. $m/h' \vDash_{BO} \neg\mathcal{A}$. But this does not suffice to rule out that there might be a $h'' \ni m$ s.t. $m/h'' \vDash_{BO} \mathcal{A}$. In which case, $\neg\mathcal{A}$ would be neither SU-true, nor SU-false at m. Hence, $\mathcal{A} \vdash_G \Box\mathcal{A}$ but $\neg\Box\mathcal{A} \nvdash_G \neg\mathcal{A}$. And this means that

Fact 2. *Contraposition fails: from* $\mathcal{A} \vdash_G \Box\mathcal{A}$ *it does not follow that* $\neg\Box\mathcal{A} \vdash_G \neg\mathcal{A}$*.*

There are other classical rules which do not preserve super-truth (in what follows, $\vdash_C$ is classical entailment, and Γ a set of auxiliary hypothesis).[23]

Deduction Theorem: $\Gamma, \mathcal{A} \vdash_C \mathcal{B} \Rightarrow \Gamma \vdash_C \mathcal{A} \rightarrow \mathcal{B}$

Fact 3. *The deduction theorem fails: from* $\mathcal{A} \vdash_G \Box\mathcal{A}$ *it does not follow that* $\vdash_G \mathcal{A} \rightarrow \Box\mathcal{A}$*.*

Proof. We already shown that $\mathcal{A} \vdash_G \Box\mathcal{A}$. It is easy to see that, if there is an m s.t $m \in h$ and $m \in h'$, and $m/h \vDash_{BO} \mathcal{A}$ but $m/h' \nvDash_{BO} \mathcal{A}$, then $m/h \nvDash_{BO} \mathcal{A} \rightarrow \Box\mathcal{A}$. Hence, $\mathcal{A} \rightarrow \Box\mathcal{A}$ would be neither SU-true nor SU-false at m, and $\mathcal{A} \vdash_G \Box\mathcal{A}$ but $\nvdash_G \mathcal{A} \rightarrow \Box\mathcal{A}$.

Argument by Cases: $\Gamma, \mathcal{A} \vdash_C \mathcal{C}$ *and* $\Gamma, \mathcal{B} \vdash_C \mathcal{C} \Rightarrow \Gamma, \mathcal{A} \vee \mathcal{B} \vdash_C \mathcal{C}$

[23] Notice that each rule fails to be globally sound only in the presence of the $\Box$ operator.

Fact 4. *Argument by cases fails: from* $\left[\mathcal{A} \vdash_G \Box\mathcal{A} \vee \Box\neg\mathcal{A} \text{ and } \neg\mathcal{A} \vdash_G \Box\mathcal{A} \vee \Box\neg\mathcal{A}\right]$ *it does not follow that* $\left[\mathcal{A} \vee \neg\mathcal{A} \vdash_G \Box\mathcal{A} \vee \Box\neg\mathcal{A}\right]$.
Proof. Given that $\mathcal{A} \vdash_G \Box\mathcal{A}$ holds, and given that the clause in BO for disjunction is classical (i.e., if a disjunct is true at m/h, then the disjunction is BO-true at m/h), then $\mathcal{A} \vdash_G \Box\mathcal{A} \vee \Box\neg\mathcal{A}$ and $\neg\mathcal{A} \vdash_G \Box\mathcal{A} \vee \Box\neg\mathcal{A}$ must hold as well. Now, take a moment m s.t. $\mathcal{A} \vee \neg\mathcal{A}$ is SU-true (clearly, the principle of the excluded middle is a BO-validiy, and, therefore, it is a SU-validity too). It is easy to see that it might be the case that both $\Box\mathcal{A}$ and $\Box\neg\mathcal{A}$ can fail to be BO-true at m/h; indeed, $\mathcal{A}$ might be a future contingent at m/h. In which case $\Box\mathcal{A} \vee \Box\neg\mathcal{A}$ would be SU-false at m, and the failure of the argument by cases simply follows.

Reductio ad Absurdum: $\Gamma, \mathcal{A} \vdash_C \mathcal{B}$ and $\Gamma, \mathcal{A} \vdash_C \neg\mathcal{B} \Rightarrow \Gamma \vdash_C \neg\mathcal{A}$

Fact 5. *Reduction ad absurdum fails: from* $\left[\mathcal{A} \wedge \neg\Box\mathcal{A} \vdash_G \Box\mathcal{A} \text{ and } \mathcal{A} \wedge \neg\Box\mathcal{A} \vdash_G \neg\Box\mathcal{A}\right]$ *it does not follow that* $\left[\vdash_G \neg(\mathcal{A} \wedge \neg\Box\mathcal{A})\right]$.
Proof. $\mathcal{A} \wedge \neg\Box\mathcal{A} \vdash_G \Box\mathcal{A}$ holds, for there is no m that makes $\mathcal{A} \wedge \neg\Box\mathcal{A}$ SU-true. Accordingly, the left-hand side of the biconditional in the definition of $\vdash_G$ is vacously satisfied. The same holds for $\mathcal{A} \wedge \neg\Box\mathcal{A} \vdash_G \neg\Box\mathcal{A}$. But $\neg(\mathcal{A} \wedge \neg\Box\mathcal{A})$ is not a SU-validity: if $m/h \vDash_{BO} \mathcal{A}$ but $m/h' \nvDash_{BO} \mathcal{A}$, then $\neg(\mathcal{A} \wedge \neg\Box\mathcal{A})$ is neither SU-true nor SU-false at m.

How one should react to these failures? According to Timothy Willamson,

> [c]onditional proof, argument by cases and reductio ad absurdum play a vital role in systems of natural deduction, the formal systems closest to our informal deductions. They are the rules by which premises are discharged, i.e. by which categorical conclusions can be drawn on the basis of hypothetical reasoning. Contraposition is another very natural deductive move. Thus supervaluations invalidate our natural mode of deductive thinking. (Williamson 2002: 152)

Williamson's reaction can be encoded in the following argument.

Uniformity: SU-logical consequence should be defined in terms of SU-truth preservation from premises to conclusions.

Naturalness: Contraposition, the Deduction theorem, Conditional proof, Argument by cases and Reductio ad absurdum are natural inferential rules, and they should be sound in any adequate logic.

$\therefore$ Given that global logical consequence does not make sound these rules, SU semantics yields an inadequate logic.

If Williamson's argument is sound, the failures we have just seen can lead to a rejection of supervaluationism. But such a conclusion is tenable only if both *Uniformity* and *Naturalness* are. As far as I know, supervaluationists try to overcome Williamson's objections in two ways. On the one hand, some supervaluationist endorses *Uniformity* and rejects *Naturalness*. Some other advocate of supervaluationism denies *Uniformity* but sticks to *Naturalness*. In what follows, I argue that supervaluationism has troubles either way.

What if one rejects Naturalness but conserves Uniformity?

If one rejects the *Naturalness* premise only, one may say that, at least in the case of future contingents, deference to classical logic is misguided. Indeed, supervaluationists embrace the Aristotelian argument, which appears to show that classical bivalence is at odds with indeterminism. And if one agrees that indeterminism requires the rejection of bivalence, some feature of classical logic must fail somewhere in the correspondent logic.

As we shall see in the next chapter, there is no reason to take seriously the Aristotelian argument, and thus there is no evidence at all that indeterminism and bivalence should be at odds with each other. But for the time being, let us notice that who drops *Naturalness* but retains *Unifromity* has to face a problem related to *Unifromity* itself. This principle yields a sound inferential pattern that licences to reason in a very counterintuitive way. As Tweedale (2004) suggests, *Uniformity* entails the following fact.

Fact 6. $\mathcal{A}, \Diamond\mathcal{B} \vdash_G \Diamond(\mathcal{A} \wedge \mathcal{B})$
Proof. If $\mathcal{A}$ and $\Diamond\mathcal{B}$ are both SU-true at m, then, for any $h \ni m$, $m/h \vDash_{BO} \mathcal{A}$ and $m/h \vDash_{BO} \Diamond\mathcal{B}$. Thus, there must be a history $h' \ni m$ such that $m/h' \vDash_{BO} \mathcal{A} \wedge \mathcal{B}$, which entails $m/h' \vDash_{BO} \Diamond(\mathcal{A} \wedge \mathcal{B})$. But then, for any history $h \ni m$, $m/h \vDash_{BO} \Diamond(\mathcal{A} \wedge \mathcal{B})$, which means that $\Diamond(\mathcal{A} \wedge \mathcal{B})$ is SU-true at m.

Now, if one substitutes $\mathcal{A}$ with $\mathsf{F}(n)\mathcal{A}$ and $\Diamond\mathcal{B}$ with $\Diamond\neg\mathsf{F}(n)\mathcal{A}$ in the pattern $\mathcal{A}, \Diamond\mathcal{B} \vdash_G \Diamond(\mathcal{A} \wedge \mathcal{B})$, one obtains that the formulas in $\{\mathsf{F}(n)\mathcal{A}, \Diamond\neg\mathsf{F}(n)\mathcal{A}\}$ globally entail $\Diamond(\mathsf{F}(n)\mathcal{A} \wedge \neg\mathsf{F}(n)\mathcal{A})$. But this seems to be wrong: intuitively, from the premise that "Tomorrow there will be a sea battle, even if it is not inevitable" one shouldn't be licensed to infer that "It is possible that tomorrow there will be a sea battle and there won't be any sea battle". Sure, a supervaluationist may say that the formulas in $\{\mathsf{F}(n)\mathcal{A}, \Diamond\neg\mathsf{F}(n)\mathcal{A}\}$ cannot be both SU-true at any moment m, and thus the inference from $\{\mathsf{F}(n)\mathcal{A}, \Diamond\neg\mathsf{F}(n)\mathcal{A}\}$ to $\Diamond(\mathsf{F}(n)\mathcal{A} \wedge \neg\mathsf{F}(n)\mathcal{A})$ is vacuously sound. But this fact doesn't avoid the flaw we have just noticed, i.e., that the inference pattern itself – $\mathcal{A}, \Diamond\mathcal{B} \vdash_G \Diamond(\mathcal{A} \wedge \mathcal{B})$ – has instances that seems to be suspect.

Uniformity yields other shortcomings. Let us enrich $\mathcal{L}_T$ with a sentential operator for expressing SU-truth.

(1) $m/h \vDash_{BO} \mathsf{T}\mathcal{A} \Leftrightarrow \forall h'(m \in h' \Rightarrow m/h' \vDash_{BO} \mathcal{A})$

Then the following pattern is globally sound.

(2) $\mathcal{A} \leftrightarrow \mathsf{T}\mathcal{A}, \neg\mathcal{A} \leftrightarrow \mathsf{T}\neg\mathcal{A} \vdash_G \mathsf{T}\mathcal{A} \vee \mathsf{T}\neg\mathcal{A}$

Thus, a supervaluationist should say that *if truth were disquotational, bivalence would hold*. And, as Williamson (2002: 162) has stressed, supervaluationists usually deny bivalence by rejecting the disquotational property of truth. Indeed, it is easy to see that

(3) $\vdash_G \mathsf{T}\mathcal{A} \rightarrow \mathcal{A}$

but

(4) $\nvdash_G \mathcal{A} \rightarrow \mathsf{T}\mathcal{A}$.

It is very odd to deny that if it is the case that $\mathcal{A}$, it is true that $\mathcal{A}$. And, in my view, saying that $\mathcal{A} \rightarrow \mathsf{T}\mathcal{A}$ fails when $\mathcal{A}$ is a future contingent is of no help. In assessing a material conditional, one takes what the antecedent says to be the case, and checks whether it is sufficient for the truth of the consequent. Thus, when one assesses a conditional such as "If tomorrow there will be a sea brattle, then it is true that tomorrow there will be a sea battle", it seems rather obvious that the truth of the antecedent – even if it is a future contingent – is indeed sufficient for the truth of the consequent. One can object that the antecedent's truth is not sufficient for its super-truth – i.e., it is not sufficient for its necessitation. Indeed, it it easy to see that T behaves as $\Box$, and thus there is no surprise that $\vdash_G \mathcal{A} \rightarrow \mathsf{T}\mathcal{A}$ fails, given that $\vdash_G \mathcal{A} \rightarrow \Box\mathcal{A}$ fails as well (see the failure of the Deduction theorem). But this reply just highlights that super-truth is not what we mean when we speak about truth: when we assess a statement as true, or entertain the hypothesis that it is true, we do not want necessarily to say (or hypothesise) that it is inevitable.

Notice that $\mathcal{A} \dashv\vdash \mathsf{T}\mathcal{A}$ holds when the supervaluationist logical consequence is global. Accordingly, one should accept that $\mathcal{A}$ logically entails $\mathsf{T}\mathcal{A}$ and vice versa, even if, via (4), one should hold that the material implication from $\mathcal{A}$ to $\mathsf{T}\mathcal{A}$ may fail. I take this as a bad result.[24]

[24]This diagnosis is, however, very controversial. For an opposite view on supervaluationism, disquotational truth and logical entailment, see Keefe (2000), McGee (1989) and Varzi (2007).

Accordingly, rejecting *Naturalness* and conserving *Uniformity* does not seem to be a promising strategy. Indeed, global logical consequence makes sound a bad inferential pattern, $\mathcal{A}, \Diamond\mathcal{B} \vdash_G \Diamond(\mathcal{A} \wedge \mathcal{B})$. Moreover, it prohibits the material inference from $\mathcal{A}$ to $\mathsf{T}\mathcal{A}$ to be valid, even if $\mathcal{A} \dashv\vdash \mathsf{T}\mathcal{A}$ does globally hold.

What if one rejects Uniformity but conserves Naturalness?

Many supervaluationists have contested Williamson's *Uniformity* principle, according to which the supervaluationist notion of logical consequence should be defined in terms of SU-truth preservation. For instance, Varzi (2007) adopts supervaluationism to deal with semantic vagueness, and comments on Williamson's *Uniformity* requirement as follows.

> For one thing, when we are dealing with a vague language, it seems perfectly reasonable to suppose that we may want to reason from premises that lack a definite truth-value, in which case super-truth cannot be our guidance. Indeed, one might suggest that it is precisely by reasoning according to [a notion of logical entailment alternative to the global one] that a supervaluationist finds it natural to accept so called principles of penumbral connection: 'Look, I'm not sure what 'small' exactly means, so I am not sure whether x is truly small. But I certainly know this: Assuming x is small, since y's height is less than x's, y must be small, too.'(Varzi 2007: 643-644)

Varzi suggests that, whenever a supervaluationist reasons about sentences which lack truth values, she usually applies inferential rules that wouldn't be sound if logical conseqeunce were global. Thus, super-truth preservation cannot be a reasonable candidate to isolate those patterns that are usually taken to be sound even by supervaluationists' own lights. Analogously, whenever a supervaluationist wants to reason about future contingencies, super-truth preservation cannot be apt to isolate sound inferential patterns.

Thus, one can reject the notion of global logical consequence, and adopt what Williamson (2002) calls 'local validity'. In the case of branching time structures, the supervaluationist form of 'local validity' simply holds that BO-logical entailment is the 'right' supervaluationist notion of logical consequence.

Definition 3.4.5. (Local logical consequence) *Given two sets of wffs* Γ, Δ *of* $\mathcal{L}_T$, Δ *is a local logical logical consequence of* Γ *(in symbols.* $\Gamma \vdash_L \Delta$*) just in case, for any* $\mathcal{M}_{SU}$ *and any moment-history pair* m/h *in it, if* $m/h \vDash_{BO} \mathcal{A}$ *for any* $\mathcal{A} \in \Gamma$*, then there is a* $\mathcal{B} \in \Delta$ *such that* $m/h \vDash_{BO} \mathcal{B}$.

It is easy to see that $\vdash_L$ violates the *Uniformity* principle, for SU-truth does not play any role in the definiens of local logical consequence. And once that local logical consequence is taken to be the right notion of logical consequence, a supervaluationist can easily restore the inferential rules that we have seen before. Contraposition, the Deduction theorem, Conditional proof, Argument by cases and Reductio ad absurdum turn out to be sound under the local reading of logical consequence. Furthermore, the problematic inferential pattern of Fact 6 – $\mathcal{A}, \Diamond\mathcal{B} \vdash \Diamond(\mathcal{A} \wedge \mathcal{B})$ – does not locally hold anymore (to see why, just suppose that $\{h : h \in m/h \wedge m/h \vDash_{BO} \mathcal{A}\} \cap \{h : h \in m/h \wedge m/h \vDash_{BO} \mathcal{B}\} = \emptyset$).

Varzi (2007: 665) highlights that settling on $\vdash_L$ does not amount to neglect its global variant, $\vdash_G$. The local logical consequence can recast the global one as a defined notion. One can define $\vdash_G$ in terms of $\vdash_L$ and T.

(5) $\Gamma \vdash_G \Delta \Leftrightarrow \{\mathsf{T}\mathcal{A} : \mathcal{A} \in \Gamma\} \vdash_L \mathsf{T}\mathcal{B}$, where $\mathcal{B} \in \Delta$.

Thus, even if an advocate of SU may deny *Uniformity*, she can always define the global logical consequence in terms of $\vdash_L$, properly combined with T.

In my view, even if this strategy may seem promising, it just highlights other difficulties that the supervaluationist can, in a sense, just swallow. First, notice that truth fails to be disquotational. Again, the conditional from $\mathcal{A}$ to $\mathsf{T}\mathcal{A}$ fails to be a local validity.

(6) $\nvdash_L \mathcal{A} \rightarrow \mathsf{T}\mathcal{A}$

On top of that, $\mathsf{T}\mathcal{A}$ is not a local logical conseqeunce of $\mathcal{A}$.

(7) $\mathcal{A} \nvdash_L \mathsf{T}\mathcal{A}$

A supervaluationist who assumes *Naturalness* but rejects *Uniformity* inhabits a strange world: she takes some classical inferential rules as "natural", but she is forced to attribute very counterintuitive properties to her favourite notion of truth. In particular, the tarskian intuitions appears to be incompatible with her approach.

A supervaluationist may object that there is no reason to require that the object language truth-operator must express super-truth. After all, once that the relevant notion of truth for logical entailment ceases to be that of super-truth, one seems to be entitled to give up an object language device that expresses super-truth as well. I think that this view is very unsatisfying, but let us see how far it can go.

Both Thomason (1970: 279), MacFarlane (2008: 96) and MacFarlane (2014: 223) define the supervaluationist truth operator as follows.

(8) $m/h \vDash_{BO} \mathsf{V}\mathcal{A} \Leftrightarrow m/h \vDash_{BO} \mathcal{A}$

If V is adopted, the following patterns hold both locally and globally, and the disquotational properties of truth are restored.

(9) $\mathcal{A} \dashv\vdash \mathsf{V}\mathcal{A}$

(10) $\vdash \mathcal{A} \leftrightarrow \mathsf{V}\mathcal{A}$

Hence, it seems that a supervaluationist who takes V seriously can recast the disquotational property of truth ((9)-(10)). Unfortunately, these virtues are only apparent.

First, recall that supervaluationists accept the Aristotelian premises (Pa)–(Pb), and thus they are forced to say that any indeterminist has to deny bivalence (relative to moments). But supervaluationists are also bound to accept the following validity, independently of their favourite notion of logical consequence.

(11) $\vdash (\mathsf{V}\mathcal{A} \vee \mathsf{V}\neg\mathcal{A}) \wedge \neg(\mathsf{V}\mathcal{A} \wedge \mathsf{V}\neg\mathcal{A})$

Thus, if a supervaluationist could be happy to use only V in her object language, she should subscribe to something that is really similar to bivalence. And, as we have already noticed, supervaluationism gives up bivalence at the metalanguage. Therefore, not only supervaluationists couldn't express one of their peculiar thesis within their object language, but they would be forced to subscribe to a principle, (11), which is *prima facie* incompatible with their fundamental beliefs. They should affirm bivalence in the object language, but they should also deny it at the metalanguage. I think that this is double-thinking.

To see how strange it would be for a supervaluationist to pick V as her favourite object language truth operator, one may form a formula that, intuitively, should be unsatisfiable, and which can indeed be satisfied within a supervaluationist framework.[25] Let us abbreviate the formula $\Diamond\mathcal{A} \wedge \Diamond\neg\mathcal{A}$ as follows.

(12) $Cont\mathcal{A} \equiv_{df} \Diamond\mathcal{A} \wedge \Diamond\neg\mathcal{A}$

The *Cont* operator signals that the formula it embeds is a future contingent, and if $Cont\mathcal{A}$ is BO-true at m/h, it must be SU-true at m. Notice moreover that $Cont\mathcal{A}$ is BO-true at m/h just in case $Cont\neg\mathcal{A}$ is BO-true at the very same point. Thus, whenever $Cont\mathcal{A}$ is SU-true at a moment, $Cont\neg\mathcal{A}$ is SU-true at that moment.

Now let us define a satisfiability operator, which isolates those object language formulas that can be SU-true.

[25] It will provided a formula that is similar to that of Graff Fara (2010) and Malpass (2013, 2016).

(13) $m/h \vDash_{BO} \Diamond_{Sat}\mathcal{A} \Leftrightarrow$ there is a model $\mathcal{M}_{SU}$ and a moment m' in it s.t. $m' \vDash_{SU} \mathcal{A}$.

According to (13), $\Diamond_{Sat}\mathcal{A}$ is BO-true at a moment history pair just in case there is a supervaluationist model $\mathcal{M}_{SU}$ and a moment m' in it, such that $\mathcal{A}$ comes out super-true at that moment of that model. Hence $\Diamond_{Sat}$ can identify those formulas that can be super true, since they can be SU-true in at least a moment of at least an SU-model.

With these two new operators at hand, one can form the following statement.

(14) $\bigvee(Cont\mathcal{A} \wedge \mathcal{A}) \vee \bigvee(Cont\mathcal{A} \wedge \neg\mathcal{A}) \wedge \neg\Diamond_{Sat}(Cont\mathcal{A} \wedge \mathcal{A}) \wedge \neg\Diamond_{Sat}(Cont\mathcal{A} \wedge \neg\mathcal{A})$

Statement (14) says that either it is true that $\mathcal{A}$ is contingent and $\mathcal{A}$ is the case, or it is true that $\mathcal{A}$ is contingent and $\neg\mathcal{A}$ is the case, but no model (and no moment) satisfies neither (a future contingent such as) $\mathcal{A}$, nor its negation.

Intuitively, (14) should be unsatisfiable, for whenever the disjunction occurring in it is satisfied – $\bigvee(Cont\mathcal{A} \wedge \mathcal{A}) \vee \bigvee(Cont\mathcal{A} \wedge \neg\mathcal{A})$ – it should follow that one among $\mathcal{A}$ and $\neg\mathcal{A}$ is satisfiable. But the conjunction $\neg\Diamond_{Sat}(Cont\mathcal{A} \wedge \mathcal{A}) \wedge \neg\Diamond_{Sat}(Cont\mathcal{A} \wedge \neg\mathcal{A})$ denies that. In other terms, if (14) were satisfiable, an obvious relation among truth (as it is expressed within the object language) and satisfiability would fail to obtain. Being true would be compatible with being unsatisfiable. Unfortunately, (14) is SU-satisfiable (i.e., if there is a model and a moment at which it is SU-true). Here's the proof.

Fact 7. *There is a moment m in a $\mathcal{M}_{SU}$ s.t. $m \vDash_{SU}$ (14).*
Proof. Take a point m/h s.t. $m/h \vDash_{BO} Cont\mathcal{A}$. By (12), it follows that $m/h' \vDash_{BO} \Diamond\mathcal{A} \wedge \Diamond\neg\mathcal{A}$ for any $h' \ni m$. Notice that $m/h' \vDash_{BO} Cont\mathcal{A} \wedge \mathcal{A} \vee \neg\mathcal{A}$ must hold for any $h' \ni m$, for $\mathcal{A} \vee \neg\mathcal{A}$ is a BO-validity. Thus $m/h' \vDash_{BO} (Cont\mathcal{A} \wedge \mathcal{A}) \vee (Cont\mathcal{A} \vee \neg\mathcal{A})$ for any $h' \ni m$, given that $Cont\mathcal{A}$ is true at any $h' \ni m$, and each history passing through m satisfies exactly one formula among $\mathcal{A}$ and $\neg\mathcal{A}$. By (8), $m/h' \vDash_{BO} \bigvee(Cont\mathcal{A} \wedge \mathcal{A}) \vee \bigvee(Cont\mathcal{A} \vee \neg\mathcal{A})$, for any $h' \ni m$. Now, it is easy to see that $\neg\Diamond_{Sat}(Cont\mathcal{A} \wedge \mathcal{A}) \wedge \neg\Diamond_{Sat}(Cont\mathcal{A} \wedge \neg\mathcal{A})$ is a BO-validity, for the two conjunctions $\Diamond\mathcal{A} \wedge \Diamond\neg\mathcal{A} \wedge \mathcal{A}$ and $\Diamond\mathcal{A} \wedge \Diamond\neg\mathcal{A} \wedge \neg\mathcal{A}$ can be either SU-false (when $\mathcal{A}$ is not a future contingent, and thus $\Diamond\mathcal{A} \wedge \Diamond\neg\mathcal{A}$ fails to hold at any history passing through a moment), or neither SU-true nor SU-false (when $\mathcal{A}$ is a future contingent). Accordingly, both $\Diamond\mathcal{A} \wedge \Diamond\neg\mathcal{A} \wedge \mathcal{A}$ and $\Diamond\mathcal{A} \wedge \Diamond\neg\mathcal{A} \wedge \neg\mathcal{A}$ cannot be SU-true. By (13) and the clause for negation, it follows that $\neg\Diamond_{Sat}(Cont\mathcal{A} \wedge \mathcal{A}) \wedge \neg\Diamond_{Sat}(Cont\mathcal{A} \wedge \neg\mathcal{A})$ is a BO-validity. But then the conjunction $m/h \vDash_{BO} \bigvee(Cont\mathcal{A} \wedge \mathcal{A}) \vee \bigvee(Cont\mathcal{A} \wedge$

$\neg\mathcal{A}) \wedge \neg\Diamond_{Sat}(Cont\mathcal{A} \wedge \mathcal{A}) \wedge \neg\Diamond_{Sat}(Cont\mathcal{A} \wedge \neg\mathcal{A})$ must be true at any $h' \ni m$, for $\mathsf{V}(Cont\mathcal{A} \wedge \mathcal{A}) \vee \mathsf{V}(Cont\mathcal{A} \wedge \neg\mathcal{A})$ is BO-true at each of these histories, and $\neg\Diamond_{Sat}(Cont\mathcal{A} \wedge \mathcal{A}) \wedge \neg\Diamond_{Sat}(Cont\mathcal{A} \wedge \neg\mathcal{A})$ is a BO-validity. Accordingly, Fact 7 follows.

In my view, the weirdness highlighted by Fact 7 is a further clue which suggests that taking V as the only object-language truth operator for supervaluationism is a bad idea.[26] And by looking at the semantic clause for V given in (8), it is easy to see that V expresses the Ockhamist truth at a point m/h, that is, that very notion of truth which a supervaluationist takes to be *wrong*.

Furthermore, nothing prevents to define the T operator – i.e., the super truth operator– in terms of $\Box$ and V.

(15) $m/h \vDash_{BO} \Box\mathsf{V}\mathcal{A} \Leftrightarrow m/h \vDash_{BO} \mathsf{T}\mathcal{A} \Leftrightarrow \forall h'(m \in h' \Rightarrow m/h' \vDash_{BO} \mathcal{A})$

And, by (15), it turns out that T has none of the virtues of V that we have noticed above. Again, $\mathcal{A} \dashv\vdash \mathsf{T}\mathcal{A}$ does not hold locally, for $\mathsf{T}\mathcal{A}$ is not a local logical consequence of $\mathcal{A}$. The conditional $\mathcal{A} \to \mathsf{T}\mathcal{A}$ is not a local validity, and therefore super-truth is not disquotational.

To sum up, rejecting *Uniformity* while conserving *Naturalness* doesn't solve some problems related to the disquotational properties of truth. In particular, the material implication from $\mathcal{A}$ to $\mathsf{T}\mathcal{A}$ is not locally valid, and the logical entailment from $\mathcal{A}$ to $\mathsf{T}\mathcal{A}$ is not locally sound. One may attempt to fix these shortcomings by using V instead of T. V seems to restore the tarskian character of truth. Nonetheless, whoever uses V to express the supervaluationist notion of truth is actually cheating.

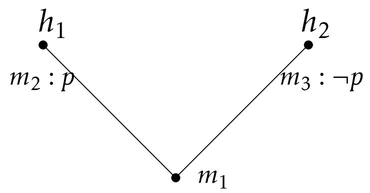
Objection 2. *Having been true, having been inevitable and retrospective truth judgments.*

According to Thomason (1970), supervaluationism allows to spell out the difference between *having been true* and *having been inevitable*. This difference is encoded in the following pattern, which fails to hold both locally and globally.

(16) $\mathsf{P}(n)\mathsf{V}\mathsf{F}(m)\mathcal{A} \vdash \mathsf{P}(n)\Box\mathsf{F}(m)\mathcal{A}$

The following tree falsifies (16).

[26]Notice, moreover, that if one substitutes in (14) the operator V with T – which indeed expresses super-truth – one obtains what is desirable, that is, an SU-unsatisfiable formula:
$\mathsf{T}(Cont\mathcal{A} \wedge \mathcal{A}) \vee \mathsf{T}(Cont\mathcal{A} \wedge \neg\mathcal{A}) \wedge \neg\Diamond_{Sat}(Cont\mathcal{A} \wedge \mathcal{A}) \wedge \neg\Diamond_{Sat}(Cont\mathcal{A} \wedge \neg\mathcal{A})$.



If $d_{\mathcal{T}}(m_1, m_2) = d_{\mathcal{T}}(m_1, m_3) = 1$, then $\mathsf{P}(1)\mathsf{V}\mathsf{F}(1)p$ is BO-true at m_2/h_1, and given that h_1 is the only history passing through m_2, $\mathsf{P}(1)\mathsf{V}\mathsf{F}(1)p$ is SU-true at m_2. But $\mathsf{P}(1)\Box\mathsf{F}(1)p$ is BO-false at m_2/h_1, for $\mathsf{F}(1)p$ fails to be inevitable at one unit in the past of m_2 (indeed, $m_1/h_2 \vDash_{BO} \neg\mathsf{F}(1)p$). And again, since h_1 is the unique history passing through m_2, $\mathsf{P}(1)\Box\mathsf{F}(1)p$ is SU-false at m_2.

Accordingly, (16) is neither locally nor globally sound. Thomason comments on this result by saying that

> [o]ur theory thus allows (indeed, forces) us to say that *having been true* is different from *having been inevitable*, as far as future-tense statements go. The latter is not a [local or global] consequence of the former, $\mathsf{P}(n)\mathsf{V}\mathsf{F}(m)\mathcal{A} \nvdash \mathsf{P}(n)\Box\mathsf{F}(m)\mathcal{A}$, because in an assertion that it was *true* that a thing would come about, truth is relative to events up to the present, whereas an assertion that it *was inevitable* that a thing would come about, inevitability is judged relative to some time in the past. (Thomason 1970: 279)

In my view, Thomason's diagnosis about *having-been-inevitable* statements is right. Asserting that it was inevitable that a thing would come about means that there is a moment m in the past at which the very same event happens in any possible future of m.

But I disagree with his interpretation of *having-been-true* statements. The truth of a prediction made in the past depends on whether reality has evolved or will evolve in such a way that it satisfies, sooner or later, what was predicted. Therefore, the truth of a past prediction does not necessarily depends on what is presently settled. It may depend on what will obtain in the future, and what will come about may well be contingent relative to the present moment. If I say that what I've predicted yesterday, namely that in two days it will be rainy, was true, I'm not committed to say that now it is presently settled that tomorrow it will be rainy. I am committed to hold that tomorrow's weather, being contingent or not, will be as I have predicted. As we shall see, it is pretty easy to make sense of this character of *having-been-true* statements once that a substantial notion of actuality enters the scene. But for the time being, let us focus on another flaw of Thomason's analysis, which highlights a more general flow of supervaluationism.

Is the difference between *having-been-true* statements and *having-been-inevitable* claims expressible within supervaluationism? The answer, as I shall argue, is negative.

Let us focus on (16) one again.

(16) $\mathsf{P}(n)\mathsf{VF}(m)\mathcal{A} \vdash \mathsf{P}(n)\Box\mathsf{F}(m)\mathcal{A}$

The conclusion in (16) – $\mathsf{P}(n)\Box\mathsf{F}(m)\mathcal{A}$ – is a schema for *having-been-inevitable* claims. $\mathsf{P}(n)\Box\mathsf{F}(m)\mathcal{A}$ is SU-true at m iff, for any $h \ni m$, there is a m' in n time units in the past of m, and $\mathsf{F}(m)\mathcal{A}$ is BO-true at any history h' passing through *that past moment* m'. On the other hand, the premise in (16), $\mathsf{P}(n)\mathsf{VF}(m)\mathcal{A}$, is a schema for *having-been-true* claims. Since $\mathsf{V}\mathcal{A}$ is BO-true at a point m/h iff $\mathcal{A}$ is BO-true at m/h, a statement such as $\mathsf{P}(n)\mathsf{VF}(m)\mathcal{A}$ would be SU-true at the present moment iff $\mathsf{F}(m)\mathcal{A}$ were BO-true at a moment lying at n time units in the past, at any history passing through *the present moment*. Having been true, under this reading, means being presently settled. But since the histories passing through the present moment are, in general, more than those passing through one of its past moments, what is presently settled may differ for what was settled. The desired result thus follows: *having-been-true* statements do not entail their *having-been-inevitable* counterparts.

The failure of (16) is surely desirable, but it doesn't show that the supervaluationist can tell apart *having-been-true* statements from *having-been-inevitable* sentences. The entailment in (16) fails because the notion of truth expressed by the *having-been-true* premise – $\mathsf{P}(n)\mathsf{VF}(m)\mathcal{A}$ – is encoded by the V operator. But it does *not* express super-truth, for V expresses BO-truth. And if one substitutes in (16) the V operator with the super truth operator T, then supervaluationism – *peace* Thomason – cannot tell the difference between *having been true* and *having been inevitable*. Indeed, a pattern such as

(17) $\mathsf{P}(n)\mathsf{TF}(m)\mathcal{A} \vdash \mathsf{P}(n)\Box\mathsf{F}(m)\mathcal{A}$

is equivalent to

(18) $\mathsf{P}(n)\Box\mathsf{VF}(m)\mathcal{A} \vdash \mathsf{P}(n)\Box\mathsf{F}(m)\mathcal{A}$.

Given that V is redundant, (18) amounts to

(19) $\mathsf{P}(n)\Box\mathsf{F}(m)\mathcal{A} \vdash \mathsf{P}(n)\Box\mathsf{F}(m)\mathcal{A}$,

which holds both locally and globally. Thus, whenever it *has been true* that $\mathcal{A}$ would come about, it logically follows, according to supervaluationism, that it *was inevitable* that $\mathcal{A}$ would come about.

Under a supervaluationist semantics, *have-been-true* statements cannot mean that what was predicted is presently settled. They must mean that what was predicted was super-true, and something was super-true only if it was inevitable. Hence, supervaluationist cannot tell apart what has been true form what has been inevitable. And this – as Thomason himself seems to suggest – is an unwelcome result.

The collapse of *having-been-true* statements over *having-been-inevitable* counterparts helps to explain why SU cannot account for retrospective truth judgments. A patterns such as

(20) $\mathcal{A} \vdash \mathsf{P}(n)True\mathsf{F}(n)\mathcal{A}$

says that $\mathcal{A}$ logically entails that, n time units ago, it was true that $\mathcal{A}$ would come about after n time units. But when one equips (20) with the supervaluationist truth operator T, one obtains

(21) $\mathcal{A} \vdash \mathsf{P}(n)\mathsf{T}\mathsf{F}(n)\mathcal{A}$,

which really means that $\mathcal{A}$ entails that, n time units ago it was inevitably the case that $\mathcal{A}$ would come about after n time units. (21) is neither locally nor globally sound, for what is presently the case could have been a future contingency. Indeed, whenever $\mathcal{A}$ is BO-true at m/h, but $\Diamond\mathsf{F}(n)\neg\mathcal{A}$ is BO-true at m'/h', where m' is n time units in the past of m, it follows that the BO-truth of the premise in (21) does not entail the BO-truth of the consequent. Hence, (21) is neither locally nor globally sound, and supervaluationism does not account for retrospective truth judgments.

Objection 3. *The plain future collapses over what inevitably will be the case.*

There is a weak spot that supervaluationism shares with Peircean semantics. They both identify the notion of truth at a moment m with truth at m on any history passing trough m. Thus, in both semantics truth at a moment (of use) means inevitability at that moment. But the way in which this identification is actually achieved differs between the two semantics.

Notice that the Peircean primitive future operator $\mathsf{F}_p(n)$ expresses future inevitability at any moment that happens to be that of its evaluation.

And if a sentence as $\mathsf{F}_p(n)\mathcal{A}$ is embedded by – for instance – a past operator $\mathsf{P}(m)$, the moment of evaluation of $\mathsf{F}_p(n)\mathcal{A}$ may differ from that of use of the whole sentence. For instance, if $\mathsf{P}(2n)\mathsf{F}_p(n)\mathcal{A}$ is uttered at m, $\mathsf{P}(2n)$ shifts the moment of use m to a moment of evaluation m' that is $2n$ time units in the past of m. Accordingly, $\mathsf{P}(2n)\mathsf{F}_p(n)\mathcal{A}$ is PE-true at m iff $\mathcal{A}$ is PE-true at n time units in the future of m', for any history passing through *the moment of evaluation* m'.

On the other hand, supervaluationism assesses a statement $\mathcal{A}$ as SU-true (SU-false) at m iff it is BO-true (BO-false) at m, for any $h \ni m$, no matter which logical form $\mathcal{A}$ may have. This means that the histories that are relevant for SU-truth (SU-falsity) at m may be only those passing through m – and not those passing trough the moment of evaluation. For instance, $\mathsf{P}(2n)\mathsf{F}(n)\mathcal{A}$ is SU-true as used at m iff it is BO-true at any history passing through that moment. As in the Peircean case, $\mathsf{P}(2n)$ shifts the moment of use m to a moment of evaluation m' that is $2n$ time units in the past. But $\mathsf{P}(2n)\mathsf{F}(n)\mathcal{A}$ is SU-true at m iff $\mathcal{A}$ is BO-true at n time units in the future of m', for any history passing through *the moment of use* m. In this case, the histories that are relevant for the evaluation of $\mathsf{P}(2n)\mathsf{F}(n)\mathcal{A}$ are only those that pass through the moment of use.

As a consequence, in SU a future tensed sentence, when embedded by other operators, may fail to express future inevitability w.r.t. the moment of its evaluation. On the other hand, in PE the primitive future operator, when embedded by other operators, does always express future inevitability w.r.t. the moment of its evaluation.

This difference is responsible of the fact that PE treats (22), (23) and (24) as three equivalent statements, while supervaluationism can reject the equivalence between (22) and (23).

(22) I bet that A will fall over.
(23) I bet that present circumstances determine that A will fall over.
(24) Present circumstances determine that I bet that A will fall over.

"I bet that" is a present tensed expression, and whatever its meaning may be, it must describe how reality is like at the moment of utterance, independently of which histories happen to pass through that moment.[27] In general, present tensed sentences receive their truth values at their moments of utterance independently of the histories passing trough those moments. And if settledness at m is thought as truth at any history passing trough m, a present tensed sentence that is true (false) at m must be

[27]One may object that whenever "I bet that" is embedded by a tense operator, it can fail to describe something that is happening at the moment of utterance. True. But now let us consider just (22)-(24), where the expression takes wide scope over tense operators.

settled true (false) at m. Accordingly, both PE and SU should take (22) – that is, a sentence which describes what is presently the case – to be equivalent with (24).

Moreover, since PE takes "will" to mean "inevitably will", (22) must be equivalent with (23). Thus, under a Peircean perspective (22), (23) and (24) are three equivalent sentences. As we have already argued, this result is bad, for (22) conveys an information which is intuitively different from what is expressed by (23).

An advocate of supervaluationism, on the other hand, has to acknowledge that if (22) is true at m, it is settled at m that I bet that A will fall over (which is the content expressed by (24)). But she is not committed to say that, if "I bet that A will fall over" is true at m, it is true at m that I bet that, *inevitably*, A will fall over. Indeed, in SU the future operator may not express future inevitability when embedded by other operators. Thus, a supervaluationist may tell a story about the meaning of "I bet that" to distinguish "I bet that: $\mathsf{F}(n)$(A falls over)" from that of "I bet that: $\Box\mathsf{F}(n)$(A falls over)". A supervaluationist, therefore, is not forced to say that (22) and (23) mean the same thing.

The advantages that a supervaluationist has over a Peircean theorist in telling apart (22) from (23) are, however, only apparent. Intuitively, (22) ("I bet that A will fall over") is equivalent to

(25) I bet that it is true that A will fall over.

But if one translates "it is true that" in (25) with the super truth operator T, then (25) expresses the same content of (23), i.e. that I bet that A will inevitably fall over. A supervaluationist has to say that (23) and (25) are equivalent, for in (25) the super truth operator T embeds the future-tensed sentence "A will fall over". Since T behaves as a necessity operator, then "it is true that A will fall over" means "inevitably, A will fall over".

In this context, a supervaluationist has two options. The first one is to say that (22) is not equivalent to (23). But if so, (22) is *not* equivalent to (25), for (25) is indeed equivalent to (23). And this is bad. At the end of the day, when I bet that A will fall over, I bet that it is true that A will fall over. This objection may appear question begging, for stating that $\mathcal{A}$ is the case, according to supervaluationism, is not equivalent to say that it is true that $\mathcal{A}$. In my view, I see no question begging here: at the end of the day, to say that $\mathcal{A}$ is the case *is* to say that it is true that $\mathcal{A}$, and any semantics which fails to capture this fact does not mirror the way we actually speak and reason.

Alternatively, a supervaluationist can hold that (22) is equivalent to (25). But then she would be forced to say that (22) is equivalent to (23).

This latter result restores a problem which affects Peirceanism, according to which "I bet that A will fall over" is (wrongly) taken to mean that I bet that it is inevitable that A will fall over. In both cases, supervaluationism yields counterintuitive results.

Moreover, if $\mathsf{F}(n)\mathcal{A}$ is a stand-alone sentence, and thus $\mathsf{F}(n)$ is the main operator of the formula we are considering, then $\mathsf{F}(n)\mathcal{A}$, uttered at m, must express inevitability at m. And this – as we have argued for the Peircean case – is wrong, for the "plain" future does not have to collapse over what is presently settled. This point is quite interesting, and deserves particular attention. First, notice that the collapse of the plain future over present settledness clearly depends on SU-truth (SU-falsity), which is the metalinguistic conjunction of two converse conditionals.[28]

(26) $\forall h(m \in h \Rightarrow m/h \vDash_{BO} \mathcal{A}) \Rightarrow m \vDash_{SU} \mathcal{A}$
(27) $m \vDash_{SU} \mathcal{A} \Rightarrow \forall h(m \in h \Rightarrow m/h \vDash_{BO} \mathcal{A})$

The conditional in (26) says that if something is inevitable at m, it is true at m. I take it as hardly contestable, for if $\mathcal{A}$ is true on any possible future, $\mathcal{A}$ is going to be the case *wathever* possible future would come about. Inevitability at m entails truth at m because what is inevitable takes place at any possible history accessible from m, and thus it must take place also on what is (or will become) the actual way things turn out to be. What is inevitable must happen.

What is controversial about SU-truth, however, is (27), i.e. the conditional from truth at a moment m to its inevitability at m. Intuitively, if I predict that $\mathcal{A}$ would occur, I do not commit myself to hold that present circumstances determine that $\mathcal{A}$ would occur. I commit myself to hold that $\mathcal{A}$ will occur in the future that would come about. And if it turns out that a future contingent does rightly predict how things evolve, why should I say that it is (or was) untrue at its moment of utterance? The idea that future contingents should be considered untrue *just because* they are contingent seems to be too strong.

Another objection to the conditional from truth to settledness is this. Again, (27) says that the present truth of $\mathcal{A}$ requires its present inevitability. Thus, one has to look to the present stage, and check whether it necessitates something to come about. But the present moment is the wrong place to look at when it comes to evaluate a future tensed sentence. A prediction such as "Tomorrow it will be sunny" says that *the future* stage of reality would be sunny, but it does not say that the *present stage* has the properties of necessitating what's going to happen next. The present

[28] An analogous observation obviously holds for SU-falsity.

stage can fail to be relevant in assessing the truth status of a prediction. This point can be explained by saying that the truth of propositions which happen to be expressed today can depend on future events.

> We shall say that the truth of an empirical proposition *supervenes upon* events in the sense of being wholly dependent upon them, while at the same time events in no way supervene upon truth. Thus the truth of the proposition that X is in Warsaw town square at noon next Friday depends upon what happens next Friday [...]. *What is true today depends upon what happens tomorrow, not the other way round. The set of true proposition in no way determines what the future is like. Instead, what the future is like determines the set of true propositions.* (McCall 1994: 14)[29]

In a nutshell, the truth status of a future contingent $\mathcal{A}$ depends on whether the actual future (i.e. the future that would come about) features what $\mathcal{A}$ predicts.

Sure, a supervaluationist like Thomason can object that any history is actual from the perspective of its moments. According to the *Relational Actuality* principle, a history is actual for a moment just in case that history passes through that moment. And given that, in general, a moment has more than one history passing through it, there would be more than a single history that is actual relative to a moment. Thus, the definite description "the actual future of moment m" fails to pick out a unique referent, and thus it has to be assessed as improper.[30] As we shall see in the next section, this rejoinder is right only if the notion encoded in the *Relational Actuality* principle is the only kind of actuality available to indeterminists. And, as we shall see, not only branching indeterminism is consistent with a notion of actuality that differs from the relational one, but a non-relational notion of actuality is indeed *needed* to tell apart an indeterminist reading of a tree from its deterministic interpretations. Be as it may, supervaluationists have problems with intuitive connections one may establish among propositions of tensed statements.

Objection 4. *Problems with propositions.*

As for the Peircean case, let us define the accounts for temporalist and eternalist intensions that one may impose on the supervaluationist semantics.

[29]Emphasis added.

[30]Notice that this diagnosis is quite shared in the literature. MacFarlane (2003, 2008, 2014), Belnap & Green (1994), Belnap et al. (2001), Todd (2015), among others, agree with Thomason's treatment of actuality in the context of branching indeterminism.

Definition 3.4.6. (Temporalist propositions and temporalist intensions.) $[\mathcal{A}]^T_m$ *is the temporalist proposition expressed by* $\mathcal{A}$ *as used at* m*. The intension of* $[\mathcal{A}]^T_m$ *is the function* $\|\mathcal{A}\|_T$ *from moment-history pairs* m/h *to truth values* $\{1, 0, i\}$*, such that*

- $\|\mathcal{A}\|_T(m/h) = 1 \Leftrightarrow m \vDash_{SU} \mathcal{A}$,
- $\|\mathcal{A}\|_T(m/h) = 0 \Leftrightarrow m \nvDash_{SU} \mathcal{A}$.
- $\|\mathcal{A}\|_T(m/h) = i \Leftrightarrow$ *neither* $m \vDash_{SU} \mathcal{A}$*, nor* $m \nvDash_{SU} \mathcal{A}$.

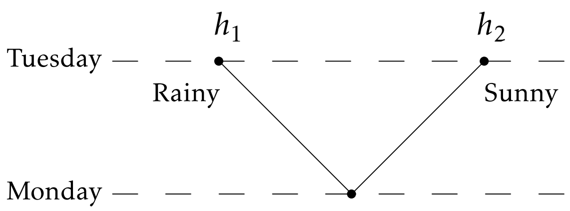
Definition 3.4.7. (Eternalist propositions and eternalist intensions.) $[\mathcal{A}]^E_m$ *is the eternalist proposition expressed by* $\mathcal{A}$ *as used at* m*. The intension of* $[\mathcal{A}]^E_m$ *is the function* $\|\mathcal{A}\|_E$ *from histories* h *to truth values* $\{1, 0, i\}$*, such that*

- $\|\mathcal{A}\|_E(h) = 1 \Leftrightarrow m \vDash_{SU} \mathcal{A}$,
- $\|\mathcal{A}\|_E(h) = 0 \Leftrightarrow m \nvDash_{SU} \mathcal{A}$,
- $\|\mathcal{A}\|_E(h) = i \Leftrightarrow$ *neither* $m \vDash_{SU} \mathcal{A}$*, nor* $m \nvDash_{SU} \mathcal{A}$,

where h *is one of the histories passing through the moment of utterance.*

Again, an eternalist may be uncomfortable with supervaluationism. If today it is Tuesday and it is sunny, but on Monday it was possible that it would have been rainy on Tuesday, then an eternalist supervaluationist couldn't accept the following argument as valid.

a. A yesterday's utterance of "Tomorrow it will be sunny" means the same thing as a today's utterance of "Today is sunny".
b. If $\mathcal{A}$ uttered at m means the same thing as $\mathcal{B}$ uttered at m', $\mathcal{A}$ at m and $\mathcal{B}$ at m' have the same intension.
∴ "Tomorrow it will be sunny", as uttered yesterday, has the same intension of "Today is sunny", as uttered today.



Take the graph represented above, and suppose that we are located at the sunny version of Tuesday. "Today it is sunny" is SU-true on that Tuesday version. Thus, the eternalist intension of the proposition expressed by "Today it is sunny", $||\text{Today it is sunny}||_E$, takes value 1 at h_2. But "Tomorrow it will be sunny", uttered on Monday, is neither SU-true nor SU-false. Accordingly, the intension of the correspondent eternalist proposition, $||\text{Today it is sunny}||_E$, must have value i at h_2. An eternalist supervaluationist must therefore deny that "Tomorrow it will be sunny", as uttered yesterday, has the same intension of "Today it is sunny", as uttered today. As Peirceanism, supervaluationism entails that premise a of the argument is not valid. In the example just given, "Tomorrow it will be sunny", as uttered yesterday, means something different than "Today it is sunny", as uttered today. And this, once again, is a bad result.

Now it is time to recap the pros and cons of supervaluationism.

3.4.2 Assessing Supervaluationism

The application of supervaluations to future contingents aims at a notion of truth at a moment which doesn't privilege any particular history of the moment of use. In turn, the democratic stance towards the histories of the moment of use is closely related to the idea that, in a branching context, actuality can only be a relational property. The relational character of actuality is encoded in the *Relational Actuality* principle. Moreover, supervaluationists want to validate the excluded middle and the principle of retrogradation, and thus they reject the definition of truth at a moment provided by the Peircean semantics.

Supervaluationism answers to (Q1) and (Q2) in an adequate way, given that

- ✓ SU refutes the Aristotelian argument and provides a non-bivalent notion of truth at a moment.
- ✓ SU refutes the Master argument by rejecting the principle of the necessity of the pasr, PNP.

SU has some advantages over the Peircean semantics. In particular,

- ✓ SU validates the future excluded middle.
- ✓ SU does not introduce a semantic distinction between $\mathsf{F}_p(n)\neg\mathcal{A}$ and $\neg\mathsf{F}_p(n)\mathcal{A}$.

But supervaluationism shares with Peirceanism three significant flaws.

- ✗ SU equates "plain will" with "inevitably will".

- ✗ SU does not account for retrospective truth judgments.
- ✗ SU yields a notion of intension which is at odds with an eternalist understanding of the propositions expressed by tensed statements.

Moreover, SU has problems with logical consequence and disquotational truth. As we have seen, global logical consequence not only entails that several classical inferential rules are unsound, but it also implies that

- ✗ $\mathcal{A}, \Diamond\mathcal{B} \vdash_G \Diamond(\mathcal{A} \wedge \mathcal{B})$, and
- ✗ $\nvdash_G \mathcal{A} \rightarrow \mathsf{T}\mathcal{A}$.

If, on the other hand, one subscribes to the local reading of logical consequence, classical inferential rules are restored, and $\mathcal{A}, \Diamond\mathcal{B} \vdash_L \Diamond(\mathcal{A} \wedge \mathcal{B})$ fails to hold. Truth, however, continues to be non-disquotational. Indeed the local reading entails that:

- ✗ $\nvdash_L \mathcal{A} \rightarrow \mathsf{T}\mathcal{A}$, and
- ✗ $\mathcal{A} \nvdash_L \mathsf{T}\mathcal{A}$.

Can one do better? According to John MacFarlane (2014: 226), Thomason's analysis about actuality – according to which actuality can only be a relational property – is correct. Moreover, the supervaluationist semantics has many virtues, and we don't have to throw it out completely. In MacFarlane's view, supervaluationism just needs to be tweaked minimally. The modification MacFarlane has in mind is known as relativism.

3.5 Relativism

MacFarlane (2014: 222–223) disagrees with the analysis about supervaluationism just proposed. In his view, a supervaluationist can legitimately use an object-language truth operator which does not express super-truth. This asymmetry is justified as follows.

> [...] whichever view we take, the context-relativized truth predicate used in semantics is a technical term, which gets its meaning in part from an account of its pragmatic relevance (for example, in Lewis's theory, the view that speakers at c try to assert what is true at c, and trust others to be doing so). It is not the ordinary truth predicate used in everyday talk—a monadic predicate that applies to propositions, and is governed by the
>
> **Equivalence Schema.** *The proposition that* Φ *is true iff* Φ.

> The relativist (or nonindexical contextualist) can treat the monadic predicate "true" as just another predicate of the object language—the language for which she is giving a semantics. (MacFarlane 2014: 93)

The quotation says that the semantical notion of truth need not to be identical with that used at the object language level. Semantical truth is just a technical device whose role is to vindicate ordinary linguistic practices. Among these practices, there are also ordinary uses of the world "true". And there is no need that the way in which we ordinarily use "true" is the same as that in which a semanticist uses her technical notion of metalinguistic truth.

I think that, at least in the case of future contingents, there is room to resist MacFarlane's argument, and thus to pretend to express the semantical notion of truth within the object language. But for the sake of the argument, I'll assume that MacFarlane may be right, and concede that the object language may not be able to express the semantical truth.

If so, a supervaluationist may be happy to take the V operator to reproduce how we ordinarily characterise truth, and thus she can recast the disquotational properties of truth (both $\vdash \mathcal{A} \leftrightarrow \mathsf{V}\mathcal{A}$ and $\mathcal{A} \dashv\vdash \mathsf{V}\mathcal{A}$ are locally and globally sound.). Moreover, a supervaluationist can also capture retrospective truth judgments, for $\mathcal{A} \vdash \mathsf{P}(n)\mathsf{V}\mathsf{F}(n)\mathcal{A}$ holds both locally and globally. MacFarlane is not worried about the possible shortcomings of global logical consequence, and he doesn't see the collapse of the "plain" future over what inevitably would come about as a flaw.

What he seems to be concerned about are *retractions* involving future contingents. In his view, retractions are speech acts that have a significant pragmatic role. In general, speech acts involve commitments. For instance, an assertion commits the assertors to the truth of the content thus expressed, while a question is an obligation for the audience to answer. There are scenarios, however, where agents are obliged to withdraw some of the commitments undertaken by past speech acts, and to do so they perform retractions.

> So, for example, in retracting a question, one releases the audience from an obligation to answer it, and in retracting an offer, one withdraws a permission that one has extended. Similarly, in retracting an assertion, one disavows the assertoric commitment undertaken in the original assertion. (MacFarlane 2014: 108)

According to MacFarlane, retractions are speech acts governed by the following rule.

> **Retraction Rule.** An agent in context m_a is required to retract an (unretracted) assertion of $\mathcal{A}$ made at m_u if $\mathcal{A}$ is not true as used at m_u and assessed from m_a. (MacFarlane 2014: 108)[31]

Here m_u is the context (moment) in which a statement is asserted, while m_a is the context (moment) from which what is asserted at m_u is assessed. Intuitively, m_a represents a moment that *may* differ from that in which a sentence is uttered. A context of assessment can thus represent an improved viewpoint in order to evaluate a prediction made in the past. And given the retraction rule, it is easy to see that a context of assessment m_a is non-redundant only if the truth status of $\mathcal{A}$, as used and assessed at m_u, is different from its truth status as used at m_u but assessed at m_a. Indeed, if what was asserted at m_u conserved its truth status at any possible context of assessment, m_a would play no role in the application of the retraction rule. In other terms, m_a would be redundant and thus eliminable. An analogous remark applies to the characterisation of speech acts accuracy.

> **Accuracy.** *An attitude or speech act occurring at m_u* is accurate, as assessed from a moment m_a, *just in case its content is true as used at m_u and assessed from m_a.* (MacFarlane 2014: 127)

According to MacFarlane (2008, 2014), the retraction rule, along with an account for (non-redundant) contexts of assessment, are needed to capture two linguistic data involving retractions of future contingents. Here there are the two (alleged) data.

(1) Present assertions concerning the future can be shown to be inaccurate by a proof of present unsettledness. Thus, present utterance of future contingents should be retracted.
(2) Past claims concerning the present cannot be shown to have been inaccurate by a proof of past unsettledness. Hence, a proof of past unsettledness is not sufficient to compel to retract future contingents uttered in the past.[32]

In MacFarlane's view, (1)-(2) should hold for the following reason. Assume that yesterday I have asserted "Tomorrow will be sunny", and

> [s]uppose that the Director of the Bureau of Quantum Weather Prediction now offers me an irrefutable proof that, at the time of my utterance yesterday, it was still an open possibility that it would not be sunny today. Would such a proof compel me

[31] Notation changed. For a similar rule, see MacFarlane (2005: 320).
[32] See MacFarlane (2014: 206).

> to withdraw my assertion? Hardly. If I had asserted that it was settled that it would be sunny today, I would have to stand corrected. But I did not assert that. I just said that it would be sunny—and it is. My prediction was true, as we can demonstrate simply by looking outside. But suppose that the Director had visited me yesterday, just after I made my assertion, and confronted me with exactly the same facts. Wouldn't I have had to acknowledge that my claim was untrue? For it would have been arbitrary to understand what I said to concern any particular one of the many worlds I occupied. (It is useless, recall, to appeal to the "actual world" in this context.) By showing that some of those overlapping worlds contained a sunny tomorrow, while others did not, the Director would have shown that there was no objective basis for calling my utterance true rather than false. (MacFarlane 2008: 89)

A proof of the present unsettledness would show that present assertions of future contingents are inaccurate, for they would express an untrue content when assessed at the moment of their use (the present moment).[33] And the Retraction Rule obliges the assertor to withdraw her assertion of a future contingent. This is exactly what (1) says.

However, if what I have predicted – being contingent or not – turns out to be the case, a proof of the unsettledness of the moment in which I made the prediction (the moment of use) does not characterise my past speech act as inaccurate (this is data (2)). Now I am evaluating the accuracy of my past speech act from an improved viewpoint, that is, from a moment (of assessment) which is later than the moment of use. And in that moment (of assessment) what I had predicted is actually happening.

Let us assume that MacFarlane is right in claiming that (1)-(2) are linguistic data that need to be accounted for. It is easy to see that supervaluationism can only satisfy (1), and something different is needed to meet (2).[34] Let us introduce the moment of assessment as a parameter for supertruth, and let us do that without modifying the overall supervaluationist semantics: $\mathcal{A}$ is SU-true (SU-false) at a moment of use m_u and a moment of assessment m_a iff $\mathcal{A}$ is BO-true (BO-false) at any history passing through m_u.

[33]Notice, by the way, that future contingents, when assessed at their moments of use, are untrue. And this is so *because* there is no actual world. Here the truth which a future contingents may have,as assessed at its moment of use, is justified by assuming the relational view about actuality introduced by Thomason.

[34]The same of course holds for Peirceanism.

(3) $m_u, m_a \vDash_{SU} \mathcal{A} \Leftrightarrow \forall h(m_u \in h \Rightarrow m_u/h \vDash_{BO} \mathcal{A})$
(4) $m_u, m_a \nvDash_{SU} \mathcal{A} \Leftrightarrow \forall h(m_u \in h \Rightarrow m_u/h \nvDash_{BO} \mathcal{A})$

By (3), it is straightforward to see that any future contingent is untrue (and hence inaccurate) when used and assessed at its moment of use (that is, when one takes $m_u = m_a$). But then the Retraction Rule obliges an assertor to retract at that moment of use her utterance of a future contingent. And the retraction is mandatory at that moment of use because, by Accuracy, what is asserted is inaccurate as used and assessed at that moment. Thus, (1) is met: by (3)-(4), a present assertion of a future contingent – *qua* assertion of a future contingent – is inaccurate. And a proof of the present unsettledness – i.e., a proof that what is asserted is a future contingent – compel to retract a present utterance of a future contingent.

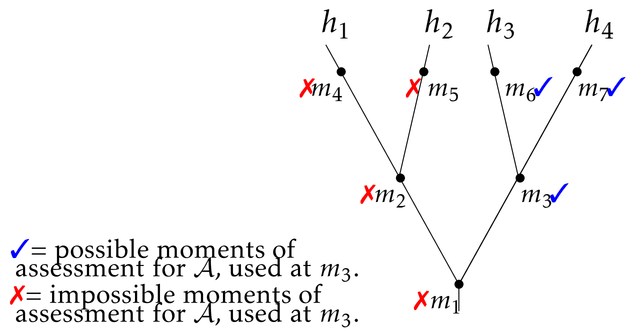
According to the supervaluationist clauses (3)-(4), if $\mathcal{A}$ is untrue as used at m_u, $\mathcal{A}$ remains untrue as used at m_u and assessed from m_a, for any value which m_a may take. The moment of assessment parameter in (3)-(4) is redundant. Therefore, if "Tomorrow it will be sunny", as used on Monday, is a future contingent, then it is untrue as used on Monday and assessed from Tuesday, even if Tuesday turns out to be sunny. Thus, if on Tuesday the whether is sunny, but someone comes along and proves that on Monday it was unsettled whether the next day would have been sunny, a supervaluationist should retract her assertion of "Tomorrow it will be sunny" made on Monday. Furthermore, she should take the proof as showing that "Tomorrow it will be sunny" is inaccurate even form Tuesday's perspective. But this counts against data (2): past claims concerning the present cannot be shown to have been inaccurate by a proof of past unsettledness.

In order to account for (2), MacFarlane suggests to modify (3)-(4) in such a way that the moment of assessment, m_a, ceases to be redundant. He adopts a technique originally developed by Belnap (2000), and formulates the following, relativistic semantics.

Definition 3.5.1. (Relativism) *Given a BO-model $\mathcal{M}_{BO} = (\mathcal{T}, d_{\mathcal{T}}, I)$, the correspondent relativist model is $\mathcal{M}_{RE} = (\mathcal{T}, d_{\mathcal{T}}, RT, RF)$, where RT (relative truth) and SF (relative falsity) are relativist evaluation functions from wffs of $\mathcal{L}_T$ to ordered pairs of moments (m_u, m_a), such that*

(a) $\{h : m_a \in h\} \subseteq \{h : m_u \in h\}$
(b) $(m_u, m_a) \in RT(\mathcal{A}) \Leftrightarrow \forall h(m_a \in h \Rightarrow m_u/h \vDash_{BO} \mathcal{A})$
(c) $(m_u, m_a) \in RF(\mathcal{A}) \Leftrightarrow \forall h(m_a \in h \Rightarrow m_u/h \nvDash_{BO} \mathcal{A})$

Condition (*a*) can be viewed as an accessibility relation between moments of use and moments of assessment. Indeed, a moment at which assess a statement uttered at m_u must be such that the histories passing through it form a subset of those passing through m_u. In general, if $m < m'$, then $\{h : m \in h\} \not\subseteq \{h : m' \in h\}$. This means that, in general, a statement used at a moment m_u cannot be assessed at moments that precede m_u itself.[35] It is possible to prove that, whenever $m \leqslant m'$, then $\{h : m' \in h\} \subseteq \{h : m \in h\}$. Thus any moment in the future of a moment of use m_u is a (possible) moment of assessment for a statement used at m_u.[36]In the picture below, m_3 is taken as a moment of use; the (possible) moments of assessment for $\mathcal{A}$ used at m_3 are ticked (✓), while those that cannot be moments of assessment for $\mathcal{A}$, as used at m_3, are crossed (✗).



Conditions (b)–(c) of Definition 3.5.1 attributes to m_a a non-redundant semantical role. (b) says that $\mathcal{A}$ is RE-true as used at m_u and assessed at m_a iff is BO-true at m_u/h, for any history *h passing through the moment of assessment* m_a. Analogously, (c) says that $\mathcal{A}$ is RE-false as used at m_u and assessed at m_a iff is BO-false at m_u/h, for any history *h passing through the*

[35] I said "in general" because there are trees which falsify the conditional from $m < m'$ to $\{h : m \in h\} \not\subseteq \{h : m' \in h\}$. Suppose, indeed, that $m < m'$, but there is no m'' s.t. $m < m''$, $m' \not\leqslant m''$ and $m'' \not< m'$. Then the set of histories passing through m must be the same as that of the histories passing through m', and thus $\{h : m \in h\} \not\subseteq \{h : m' \in h\}$ tourns out to be false.

[36] Here is the proof.

Fact 8. *If $m \leqslant m'$, then* $\{h : m' \in h\} \subseteq \{h : m \in h\}$

Proof. Assume that $m \leqslant m'$ but $\{h : m' \in h\} \not\subseteq \{h : m \in h\}$. By $\{h : m' \in h\} \not\subseteq \{h : m \in h\}$, there must be a history h_1 s.t. $m' \in h_1$ and $m \notin h_1$. By $m \leqslant m'$, either $m < m'$ or $m = m'$. If the latter, by $m' \in h_1$ it follows that $m \in h_1$, and thus one has the contradiction that $m \in h_1$ and $m \notin h_1$. If $m \leqslant m'$, the pairing axiom and the union axiom guarantee that there is a set g s.t. $g = h_1 \cup \{m\}$. Therefore, $m \in g$ and $h_1 \subseteq g$. Recall that h_1 is linearly ordered by $<$. But given that $m' \in h_1$, $m \in g$, and $m < m'$, then g must be linearly ordered by $<$ as well. Since h_1 is a $<$-maximal chain, $h_1 = g$. Thus, $m \in h_1$ and $m \notin h_1$. By reductio, Fact 8 follows.

moment of assessment m_a. Thus, we can rewrite (b) and (c) as follows (as usual, "$\vDash_{RE}$" and "$\nvDash_{RE}$" denote RE-truth and RE-falsity, respectively).

(RE1) $m_u, m_a \vDash_{RE} \mathcal{A} \Leftrightarrow \forall h(m_a \in h \Rightarrow m_u/h \vDash_{BO} \mathcal{A})$
(RE2) $m_u, m_a \nvDash_{RE} \mathcal{A} \Leftrightarrow \forall h(m_a \in h \Rightarrow m_u/h \nvDash_{BO} \mathcal{A})$

A remarkable feature of (RE1)-(RE2) is that the moment of use, as in the supervaluationism semantics, initialises the time at which evaluate a given sentence $\mathcal{A}$, though the history variable in the truth conditions of $\mathcal{A}$ ranges over a domain individuated by the moment of assessment, m_a. And given that, in general, m_a is either identical or later than m_u, it may well happen that the histories passing through m_a are less than those passing through m_u.

Notice, however, that RE-truth (RE-falsity) is relative not just to a moment (of use), but to couples of a moment of use and a moment of assessment. This trait may give raise to a qualm, according to which relativism cannot adequately answer the Aristotelian question (Q1). Indeed, question (Q1) asks for a definition of truth that is relative to a unique parameter only, i.e it asks to define a notion of truth which is relative to a moment of use only. In my view, this qualm can be easily overcome, for (RE1)-(RE2) can be used to define a notion of truth at a context of use. Such a context, in turn, is formed by a moment and a set of histories.[37]

Definition 3.5.2. (Relativistic context) *A relativistic context of use, c_{RE}, is a couple $(m_{c_{RE}}, H_{c_{RE}})$, where*

(a) $m_{c_{RE}}$ is the moment at which a statement $\mathcal{A}$ may be uttered, and
(b) $H_{c_{RE}}$ is a set of histories such that $H_{c_{RE}} \subseteq \{h : m_{c_{RE}} \in h\}$.

If $m_{c_{RE}}$ is a usual moment of use, $H_{c_{RE}}$ represents the set of histories at which one assesses a statement uttered at $m_{c_{RE}}$. Definition 3.5.2 entails that, given a subset H of the histories passing through m_1, any couple of the form (m_1, H) is a context for an utterance of $\mathcal{A}$ at m_1. As a consequence,

[37]Recall that MacFarlane agrees with Thomason's thesis, according to which, in an indeterministic setting, actuality can only be a relational notion. And thus relativism, if it has to be consistent with indeterminism, does not have to privilege a unique, actual history. As a result, MacFarlane elaborates the following criterion for contexts.

> **Nondetermination Thesis.** In frameworks with branching worlds, a context of use does not, in general, determine a unique "world of the context of use," but at most a class of worlds that overlap at the context. (MacFarlane 2014: 208)

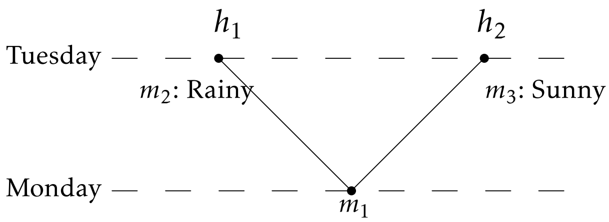
It is easy to see that Definition 3.5.2 satisfies the non-determination thesis, for a relativistic context specifies a set possible histories, overlapping at the moment of use.

it is compatible with the idea that the very same utterance can have many contexts of use.[38] And it is easy to check that the following clauses for a relativistic notion of truth at a context are equivalent with (RE1)-(RE2).

(RE1') $c_{RE} \vDash_{RE} \mathcal{A} \Leftrightarrow \forall h(h \in H_{c_{RE}} \Rightarrow m_{c_{RE}}/h \nvDash_{BO} \mathcal{A})$
(RE2') $c_{RE} \nvDash_{RE} \mathcal{A} \Leftrightarrow \forall h(h \in H_{c_{RE}} \Rightarrow m_{c_{RE}}/h \nvDash_{BO} \mathcal{A})$

Clauses (RE1')-(RE2') show that relativism can consistently yield a notion of truth (falsity) at a context of use, and that notion is perfectly adequate to answer the Aristotelian question (Q1) ("Are we to drop contingency, bivalence (relative to a context of use), or neither?"). But before we go on and explore some of the properties relativism has, let us use MacFarlane's notation, and speak of relative truth at a moment of use and a moment of assessment.

As noticed above, relativism is meant to improve supervaluationism. The relativistic semantics is designed to account for the (alleged) linguistic data described in (2), which is not capture by supervaluationism. Let's see how this can be done.



Given the picture above, if one takes m_1 as a monent of use and as a moment of assessment for "Tomorrow it will be sunny", then this sentence is RE-true (RE-false) just in case it is BO-true (BO-false) at any history passing through m_1. And since it is a future contingent as used and assessed at m_1, it is neither RE-true nor RE-false at that point. More generally, it is easy to see that whenever $m_u = m_a$, the relativist semantics boils down down to supervaluationism.

Accordingly, relativism inherits many of the features of supervaluationism, such as

- the acceptance of premises (Pa)-(Pb) of the Aristotelian argument,

[38]See Bonomi & Del Prete (2008) and Del Prete (2010).

- the rejection of bivalence,
- the rejection of the Master argument, given that the principle of retrogradation $\mathsf{P}(n)\mathcal{A} \to \Box\mathsf{P}(n)\mathcal{A}$ is not RE-valid,
- the validation of the future excluded middle, $\mathsf{F}(n)\mathcal{A} \vee \mathsf{F}(n)\neg\mathcal{A}$,
- the rejection of truth-functionality for logical connectives,
- the equivalence between $\mathsf{F}(n)\neg\mathcal{A}$ and $\neg\mathsf{F}(n)\mathcal{A}$,
- relativism, as well as supervaluationism, does account for (1): present assertions concerning the future can be shown to be inaccurate by a proof of present unsettledness. Therefore they should be retracted at the moments of their utterance.

But look at the picture above once again, and suppose that "Tomorrow it will be sunny", used on Monday (m_1), is assessed from the sunny version of Tuesday (m_3). Given that "Tomorrow it will be sunny", used on Monday (m_1), is BO-true at any history passing through the sunny version of Tuesday (m_3), then "Tomorrow it will be sunny", used on Monday (m_1), is RE-true as assessed at m_3. The retraction rule, therefore, cannot be used to infer that who asserted "Tomorrow it will be sunny" on Monday should retract her speech act at m_3, for that speech act is accurate as assessed from m_3. Contrary to supervaluationism, relativism vindicates (2): past claims concerning the present cannot be shown to have been inaccurate by a proof of past unsettledness. In MacFarlane's view, this latter feature would make relativism the adequate semantics for future contingents. Relativism accounts for (2), and thus it would fix the unique flaw affecting supervaluationism.

But is it true that the only flaw of supervaluationism has to do with (2)? And is the fix proposed by MacFarlane really needed?

3.5.1 Against Relativism

Objection 1. *Relative truth – which is not diquotational – should be expressible within the object language.*

According to MacFarlane, semantical truth may not be mirrored at the object language level. The role played by a object language truth operator is to mimic how we ordinarily use the word "true", while semantical truth must provide a distribution of truth values which fits linguistic data taken from ordinary discourse. Under this view, the disquotational properties of truth only involve the way in which we ordinarily use the word "true", whose behaviour may differ from that of the metalinguistic notion of semantical truth.

> [...] whichever view we take, the context-relativized truth predicate used in semantics is a technical term, which gets its meaning in part from an account of its pragmatic relevance (for example, in Lewis's theory, the view that speakers at c try to assert what is true at c, and trust others to be doing so). It is not the ordinary truth predicate used in everyday talk—a monadic predicate that applies to propositions, and is governed by the
>
> **Equivalence Schema.** *The proposition that* Φ *is true iff* Φ.
>
> The relativist (or nonindexical contextualist) can treat the monadic predicate "true" as just another predicate of the object language—the language for which she is giving a semantics. (MacFarlane 2014: 93)

This explains why MacFarlane holds that V is the only truth operator that both supervaluationists and relativists should contemplate in their object languages. Thus, both supervaluationism and relativism can fail to have an object language device that explicitly expresses semantical truth. This failure, as far as it does not preclude to account for ordinary talk, cannot be considered a bug.

I'll try to argue that, if (1) is correct, one should have an operator that explicitly expresses semantical truth. And given that MacFarlane considers (1) as correct, his object language should have a device which explicitly mirrors relative truth. If so, MacFarlane's relativism cannot restore the disquotational properties of truth.

Statement (1) says that present assertions concerning the future can be shown to be inaccurate by a proof of present unsettledness. Thus, there must be some kind of disagreement between who asserts a future contingent and someone who proves that the future is unsettled. Accordingly, the following dialogue should sound natural to whoever believes in (1).

Alice: Tomorrow I will take the train.
Bob: You're wrong, but it might be that you'll take it.

Intuitively, Bob uses "wrong" to signal that Alice has said something *untrue*. If one follows MacFarlane's advice, and use only V as our object language truth operator, Bob's answer would be something like

(5) $\neg\mathsf{V}(\mathsf{F}(1)p) \wedge \Diamond\mathsf{F}(1)p$.

Statement (5) cannot be RE-true when $\mathsf{F}(1)p$ is a future contingent at m_u, and $m_a = m_u$. But this seems strange. In MacFarlane's view, if Alice's statement is a future contingent, Bob's reply – formalised as (5) – should be RE-true as assessed at its moment of use.

A relativist who wants to adequately formalise Bob's objection should first equip her objet language with a truth operator that expresses relative truth. This can be done by following a proposal due to Cobreros (2016).

(6) $m/h \vDash_{BO} \mathsf{T}_{RE}\mathcal{A} \Leftrightarrow \forall h'(m_a \in h' \Rightarrow m/h' \vDash_{BO} \mathcal{A})$.

The T_{RE} operator expresses relative truth, and clause (6) says that $\mathsf{T}_{RE}\mathcal{A}$ is BO-true at a point m/h just in case $\mathcal{A}$ is BO-true at m, at any history passing trough the moment of assessment m_a. Notice that when one tries to evaluate a statement of the form $\mathsf{T}_{RE}\mathcal{A}$ by (RE1)-(RE2), the moment at which one has to evaluate $\mathcal{A}$ may be initialised by the moment of use. Nevertheless, it can also be a different one if $\mathsf{T}_{RE}\mathcal{A}$ is embedded by a tense operator. In each case, however, the set of histories at which $\mathcal{A}$ must be evaluated remain fixed by the moment of assessment. This trait is meant to reflect the semantical role that moments of assessment have within relativism: they isolate the relevant set of histories at which one has to look at for the evaluation.

With clause (6) at hand, Bob's answer may be formalised as

(7) $\neg\mathsf{T}_{RE}(\mathsf{F}(1)p) \wedge \Diamond\mathsf{F}(1)p$,

which is RE-true when used and assessed at m_u, and $\mathsf{F}(1)p$ is a future contingent at m_u. And this, in turn, is a desired result for a relativist, for it perfectly reflects what one may want to say once that (1) is accepted.

But T_{RE} has a non-disquotational nature. Indeed, when $m_u = m_a$, the T_{RE} operator expresses super-truth at m_u, and a conditional as $\mathcal{A} \to \mathsf{T}_{SU}\mathcal{A}$ is not SU-valid. Therefore, there are couples (m_u, m_a) at which $\mathcal{A} \to \mathsf{T}_{RE}\mathcal{A}$ is not RE-true. Thus, if a relativist finds (1) perfectly natural, she should be able to evaluate Bob's answer Alice in a way that is consistent with (1) itself. But then she has to express RE-truth within her object language. This, in turn, entails that a relativist cannot subscribe to a disquotational notion of truth.

Some advantages of Relativism. *Having been true, having been inevitable and retrospective truth judgments.*

It is fair to say that the relativist truth operator fares better than the supervaluationist. To see why, assume the following notion of relativist logical consequence, based on RE-truth preservation.[39]

[39]The notion of logical consequence of Definition 3.5.3 is the so-called absolute logical consequence presented in MacFarlane (2014: 69). It differs from the so-called diagonal logical consequence – see MacFarlane (2014: 70) – which boils down to the global logical consequence defined for supervaluationism.

Definition 3.5.3. (Relativist logical consequence) *Given two sets of wffs* Γ, Δ *of* $\mathcal{L}_T \cup \{\mathsf{T}_{RE}\}$*,* Δ *is a relativist logical consequence of* Γ *(in symbols.* $\Gamma \vdash_{RE} \Delta$*) just in case, for any* $\mathcal{M}_{RE}$ *and any couples* (m_u, m_a) *in it, if* $m_u, m_a \vDash_{RE} \mathcal{A}$ *for any* $\mathcal{A} \in \Gamma$*, then there is a* $\mathcal{B} \in \Delta$ *such that* $m_u, m_a \vDash_{RE} \mathcal{B}$*.*

The relativist logical consequence, as the global logical consequence, invalidates Contraposition, the Deduction theorem, Conditional proof, Argument by cases and Reductio ad absurdum.

However, it is easy to check that Definition 3.5.3, along with T_{RE}, enables to distinguish *having-been-true* statements form *having-been-necessary* sentences. Indeed,

(8) $\mathsf{P}(n)\mathsf{T}_{RE}\mathsf{F}(m)\mathcal{A} \vdash_{RE} \mathsf{P}(n)\Box\mathsf{F}(m)\mathcal{A}$

is not sound. If $\mathsf{P}(n)\mathsf{T}_{RE}\mathsf{F}(m)\mathcal{A}$ is RE-true at (m_u, m_u), but $\mathsf{F}(m)\mathcal{A}$ is a future contingent at n time units in the past of m_u, then $\mathsf{P}(n)\Box\mathsf{F}(m)\mathcal{A}$ is RE-false at (m_u, m_u). When one asserts that it was true that a thing would come about, truth is relative to events up to the moment of assessment. If something in the past was inevitable, it was settled relative to any history passing through a moment that may be in the past of that of assessment. It follows that what has been true does not collapse over what was inevitable.

Analogously, a relativist can easily capture the phenomenon of retrospective truth judgments, given that

(9) $\mathcal{A} \vdash_{RE} \mathsf{P}(n)\mathsf{T}_{RE}\mathsf{F}(n)\mathcal{A}$

is a sound pattern. If $\mathcal{A}$ is *settled* relative to any h passing through the moment of assessment m_a, then, in the past of the moment of use of $\mathcal{A}$, it was true to say that any $h \ni m_a$ would have featured $\mathcal{A}$.

Objection 2. *Intensions and the plain future.*

Relativism may appear to have a further advantage over both Peirceanism and supervaluationism. *Prima facie*, relativism yields two notions of intension which seems to be compatible with our natural understanding of tensed sentences. To see why, let us define both temporalist and eternalist intensions in a relativistic fashion. As a notational remark, "h_{m_a}" refers a history passing through the moment of assessment m_a.

Definition 3.5.4. (Temporalist propositions and temporalist intensions.) $[\mathcal{A}]^T_{(m_u,m_a)}$ *is the temporalist proposition expressed by* $\mathcal{A}$ *as used at* m_u *and assessed from* m_a*. The intension of* $[\mathcal{A}]^T_{(m_u,m_a)}$ *is the function* $\|\mathcal{A}\|_T$ *from moment-history pairs* m_u/h_{m_a} *to truth values* $\{1, 0, i\}$*, such that*

- $||\mathcal{A}||_T(m_u/h_{m_a}) = 1 \Leftrightarrow m_u, m_a \vDash_{RE} \mathcal{A}$
- $||\mathcal{A}||_T(m_u/h_{m_a}) = 0 \Leftrightarrow m_u, m_a \nvDash_{RE} \mathcal{A}$
- $||\mathcal{A}||_T(m_u/h_{m_a}) = i \Leftrightarrow$ *neither* $m_u, m_a \vDash_{RE} \mathcal{A}$, *nor* $m_u, m_a \nvDash_{RE} \mathcal{A}$

Definition 3.5.5. (Eternalist propositions and eternalist intensions.) $[\mathcal{A}]^E_{(m_u,m_a)}$ *is the eternalist proposition expressed by* $\mathcal{A}$ *as used at* m_u *and assessed at* m_a. *The intension of* $[\mathcal{A}]^E_{(m_u,m_a)}$ *is the function* $||\mathcal{A}||_E$ *from histories* h_{m_a} *to truth values* $\{1, 0, i\}$, *such that*

- $||\mathcal{A}||_E(h_{m_a}) = 1 \Leftrightarrow m_u, m_a \vDash_{RE} \mathcal{A}$
- $||\mathcal{A}||_E(h_{m_a}) = 0 \Leftrightarrow m_u, m_a \nvDash_{RE} \mathcal{A}$
- $||\mathcal{A}||_E(h_{m_a}) = i \Leftrightarrow$ *neither* $m_u, m_a \vDash_{RE} \mathcal{A}$, *nor* $m_u, m_a \nvDash_{RE} \mathcal{A}$

Given Definition 3.5.4 and 3.5.5, relativism can easily vindicate the following reasoning.

a. A yesterday's utterance of "Tomorrow it will be sunny", as assessed today, means the same thing as a today's utterance of "Today is sunny", as assessed today.

b. If $\mathcal{A}$, as used at m_u and assessed from m_a, means the same thing as $\mathcal{B}$, used at m'_u and assessed from m_a, $\mathcal{A}$ at (m_u, m_a) and $\mathcal{B}$ at (m'_u, m_a) have the same intension.

∴ "Tomorrow it will be sunny", as uttered yesterday but assessed from today, has the same intension of "Today is sunny", as uttered and assessed today.

Relativism is perfectly compatible with the conclusion just given. Suppose that days are expressed in time units, and formalise "Tomorrow it will be sunny" as $\mathsf{F}(1)p$ (p, obviously, means "Today it is sunny"). Now, suppose that today we are located at m_2, which is the moment we assume to be that of assessment. If $\mathsf{F}(1)p$ was used yesterday, it was used at a moment m_1 s.t. $m_1 < m_2$, where $d_{\mathcal{T}}(m_1, m_2) = 1$. By (RE1)-(RE2) and Definition 3.5.4, $||\mathsf{F}(1)p||_T(m_1/h_{m_2}) = 1$ just in case $\forall h(h \ni m_2 \Rightarrow m_2/h \vDash_{BO} p)$. In turn, $\forall h(h \ni m_2 \Rightarrow m_2/h \vDash_{BO} p)$ iff $||p||_T(m_2, h_{m_2}) = 1$. Thus,

$$||p||_E(h_{m_2}) = 1 \Leftrightarrow ||\mathsf{F}(1)p||_E(h_{m_2}) = 1.^{40}$$

It is easy to see that an analogous reasoning shows that:

$$||p||_E(h_{m_2}) = 0 \Leftrightarrow ||\mathsf{F}(1)p||_E(h_{m_2}) = 0,$$
$$||p||_E(h_{m_2}) = i \Leftrightarrow ||\mathsf{F}(1)p||_E(h_{m_2}) = i.$$

[40]Notice that an analogous conclusion follows for the temporalist view.

The value of the (eternalist) intension of "Tomorrow it will be sunny", as used yesterday and assessed today, is the same of the (eternalist) intension of "Today is sunny", used and assessed today. Therefore, relativism, unlike supervaluationism and Peirceanism, seems to be compatible with an eternalist understanding of what is said by tensed statement.

However, it is worth to notice that premise *a* says that a yesterday's utterance of "Tomorrow it will be sunny" means the same as a today's utterance of "Today it is sunny" when both utterances are assessed at the *same* moment (namely, today). And it is immediate to notice that, if both utterances are assessed at their respective moments of use, relativism yields the same results of supervaluationism. Hence, an eternalist relativist cannot subscribe to the validity of the following principle.

a'. A yesterday's utterance of "Tomorrow it will be sunny", as assessed yesterday, means the same thing as a today's utterance of "Today it is sunny", as assessed today.

But this seems strange. Relativism is compatible with the view that a yesterday's utterance of "Tomorrow it will be sunny" means the same thing as a today's utterance of "Today it is sunny" when both utterances are assessed today. But relativism also entails that, if one sticks to the eternalist view, then a yesterday's utterance of "Tomorrow it will be sunny" may fail to mean the same thing as a today's utterance of "Today it is sunny", when both utterances are assessed at their moments of use, respectively. I take it as a bad result.

A relativist can insist that the intension of a future tensed statement does not depend on the moment of use only, for it is sensible to the moment of assessment as well. Therefore, there is no surprise that a principle such as *a'* fails within a relativistic framework. Under this view, rejecting relativism *because* the very same utterance can express different intensions at different moments of assessment is to beg the question against relativism.

Maybe. But the rejection of *a'* – which an eternalist relativist is forces to subscribe to – helps to highlight a more general problem. "Tomorrow it will be sunny", as uttered and assessed yesterday, expresses that yesterday's future was inevitably sunny. And this strong modal reading of a future tensed sentence signals that, whenever the moment of use is the moment of assessment, the plain future collapses over what inevitably will come about. In my view, this result is wrong, and it is wrong independently of whether utterances' intensions are allowed to vary relative to moments of assessment. Again, if "Tomorrow it will be rainy" and "Inevitably, tomorrow it will be rainy" are uttered at the same moment, they mean different things.

Objection 3. *Retractions and the rationality of relativism.*

In my view, MacFarlane is right in subscribing to (2).

(2) Past claims concerning the present cannot be shown to have been inaccurate by a proof of past unsettledness. Hence, a proof of past unsettledness is not sufficient to compel to retract future contingents uttered in the past.

I also perfectly agree with his explanation about why (2) is a reasonable thing to believe. If yesterday I uttered that tomorrow it would be sunny, and if someone offers me a proof of past unsettledness, I can ask:

> [w]ould such a proof compel me to withdraw my assertion? Hardly. If I had asserted that it was settled that it would be sunny today, I would have to stand corrected. But I did not assert that. I just said that it would be sunny—and it is. My prediction was true, as we can demonstrate simply by looking outside. (MacFarlane 2008: 89)

Principle (2) holds because yesterday, when I asserted that the next day it would be sunny, *I didn't meant that it was settled that next day's weather would have been sunny (and, therefore, not contingent)*. This explains why a present proof that yesterday's future was unsettled does not contradict what I have said back then. But this explanation, in my view, is sufficient to reject (1),

(1) Present assertions concerning the future can be shown to be inaccurate by a proof of present unsettledness. Thus, present utterance of future contingents should be retracted.

If my yesterday's utterance of "Tomorrow it will be sunny" didn't mean that the next day the weather would have been inevitably sunny, why a yesterday's proof of yesterday's unsettledness should have been sufficient to compel me retract what I have predicted? Intuitively, that proof would be incompatible with the content of my assertion only if I had predicted that some future contingent event was inevitably going to happen. As already said, if plain future does not collapse over what inevitably is going to happen, then a yesterday's utterance of a future contingent does not express future inevitability. Again, "It will be the case that $\mathcal{A}$" simply does not mean that, inevitably, it will be the case that $\mathcal{A}$. This difference, furthermore, is independent of where I place the moment at which assess an utterance of a tensed sentences. Accordingly, a proof of present unsettledness cannot be sufficient to show that present assertions of future contingents are untrue. And it cannot be sufficient to compel to retract present assertions of future contingents.

Sure, a proof of present unsettledness may be epistemically significant. For instance, it may highlight that one cannot know how the future would be. Thus, a proof of the present unsettledness may show that one may lack a justification for asserting either that $\mathsf{F}(n)\mathcal{A}$ or that $\mathsf{F}(n)\neg\mathcal{A}$. But unless one adopts an epistemic notion of truth, lack of justification does not entail untruth. And if (2) holds, then a proof of past unsettledness is irrelevant for the truth of yesterday's prediction concerning the present. And this, in turn, suggests that yesterdays' lack of justification does not entail anything about the truth status of a future contingent uttered yesterday. And if retractions should only involve utterances of untrue contents, a proof of the present unsettledness does not entail that one should retract present utterances of future contingents.

Recall that MacFarlane presents a motivation for a transition from supervaluationism to relativism. Supervaluationism can only account for (1), but relativism meets both (1) and (2). Thus, one should tweak supervaluationism in order to obtain the relativistic semantics. As I tried to argue, what MacFarlane takes to be a reason to tweak supervaluationism is indeed a motivation to reject *both* supervaluationism and relativism. This because (2), which is a sensible principle, suggests to deny (1). And given that supervaluationism, as well as relativism, meet (1), both of them seems to be inadequate.

There is a sense, however, in which relativism is even more inadequate that supervaluationism in accounting for retractions. Suppose that on Monday I said that tomorrow it will be sunny, but assume that it is possible that Tuesday it will be rainy. According to supervaluationism, I should retract my assertion made on Monday because it is untrue. And, in general, if yesterday (i.e., on Monday) I predicted something untrue, and today (i.e., on Tuesday) I didn't retracted it yet, I should withdraw my past assertion. And this, in general, seems a reasonable thing to do. What I tried to argue against supervaluationism, among other things, is that future contingents cannot be taken to be untrue – and therefore, that they should not be retracted – *just because* they are contingent.

But relativism seems to entail something even more controversial. According to relativism, if it turns out that Tuesday it is sunny, on Tuesday I am not obliged to retract a yesterday's utterance of "Tomorrow it will be sunny", even if I should have retracted it on Monday. Thus, for a relativist is perfectly reasonable to hold that yesterday I should have retracted something that today I am not obliged to withdraw anymore.

Moreover, let us focus on a norm that, according to MacFarlane, regulates assertions.

Reflexive Truth Rule. An agent is permitted to assert that $\mathcal{A}$ at m_u only if $m_u, m_u \vDash_{RE} \mathcal{A}$.[41]

Now, by (RE1)-(RE2) a future contingent is RE-untrue when assessed at its moment of use and, by the Reflexive Truth Rule, one is never permitted to assert a future contingent. This consequence is quite weird, for competent speakers would be responsible of a massive infraction of the Reflexive Truth Rule. Competent speakers, indeed, assert future contingents all the time.[42]

And since a future contingent can turn out to be RE-true when assessed at a moment later than that of its use, the Retraction Rule cannot be used to entail that one is required to retract what predicted at that moment (of assessment). Thus, relativism is compatible with the idea that *I can keep myself from retracting a past assertion that I shouldn't have made*. There is room to think that these kinds of permissions and obligations are far from being fully rational.[43]

3.5.2 Assessing Relativism.

MacFarlane agrees with Thomason's analysis of actuality, according to which actuality itself can only be a relational property. However, MacFarlane wants to modify supervaluationism in order to account for two alleged linguistic data, (1)-(2). In doing that, MacFarlane elaborates a semantics which allows to answer (Q1)-(Q2) as follows:

- ✓ RE refutes the Aristotelian argument and provides a non-bivalent notion of truth at a moment, and
- ✓ RE refutes the Master argument by rejecting the principle of the necessity of the past, PNP.

Relativism is advantaged over the Peircean semantics, for

[41] See MacFarlane (2014: 103), notation slightly changed.

[42] Notice that, once that the Reflexive Truth Rule is assumed, supervaluationism and Peirceanism suffer of the same assertion problem.

[43] For a similar view about the rationality of relativism, see García-Carpintero (2013) and Moruzzi (2008).

✓ RE validates the future excluded middle.
✓ RE does not introduce a semantic distinction between $\mathsf{F}(n)\neg\mathcal{A}$ and $\neg\mathsf{F}(n)\mathcal{A}$.

Furthermore, contrary to supervaluationism,

✓ RE does account for retrospective truth judgments.

But relativism shares with supervaluationism several flaws. For instance, many classical inferential rules are not relativistically sound. Moreover,

✗ Relative truth is not disquotational, $\nvdash_{RE} \mathcal{A} \rightarrow \mathsf{T}_{RE}\mathcal{A}$.
✗ Anytime that a prediction is assessed at its moment of use, RE equates "will" with "inevitably will"..

It may be debatable whether relativism is compatible with an eternalist view about propositions. But apart from that, several qualms about the rationality of relativism still remain. Indeed

✗ RE accounts for (1), which I take to be a wrong principle,
✗ RE is consistent with the claim that yesterday I should have retracted something that today I am not obliged to withdraw anymore,
✗ RE entails that one is never permitted to assert a future contingent,
✗ RE is consistent with the claim that I can keep myself from retracting a past assertion that I shouldn't have performed.

Thus, even relativism has several, significant flaws. To sum up, the revisionist strategy seems to be at odds with several, natural requirements one may adopt in interpreting a tensed language.

According to the view that will explored in the next chapter, the flaws affecting the revisionist approach suggest that, given a branching conception of reality, truth must be relative to the historical parameter.

Chapter 4

The conservative strategy and actuality

As we have seen in the previous sections, the BO-semantics does not allow us to provide an adequate answer for the Aristotelian question:

(Q1) Are we to drop indeterminism, bivalence, or neither?

The reason of this failure should be familiar by now: (Q1) presupposes a notion of bivalence in which truth is relative to moments only. In the BO-semantics, however, truth is relative to moment-history pairs.

As we have seen, in the revisionary strategy, this discrepancy is conceived as a flaw of the BO-semantics. A revisionary philosopher, in turn, can take two different routes. On the one hand, the advocates of Peirceanism define a notion of truth at a moment which allows them to answer (Q1) by conserving both contingency and bivalence. On the other hand, a revisionary philosopher may adopt supervaluationism, or relativism, and define a notion of truth at a moment (or, in the case of relativism, truth at a relativist context) that allows him to answer (Q1) by conserving indeterminism but rejecting bivalence.

The revisionary strategy, nevertheless, is not the only reaction one may have to the discrepancy between the BO-semantics and the Aristotelian question. Revisionists charge the BO-semantics of being somehow inadequate, and its inadequacy explains why BO cannot yield an answer (Q1). Nonetheless, one may go the other way round, and claim that there's nothing wrong with BO, for the Aristotelian question itself is misleading. Indeed, it is natural to take the three alternatives that (Q1) evokes (viz. (i) dropping contingency, (ii) rejecting bivalence, (iii) neither of the two) as jointly exhaustive. But they need not be such.

The conservative strategy says that, if we accept indeterminism, and adopt a branching conception of reality, we should take truth (and bivalence) as relative to moment-history pairs. The Aristotelian question (Q1) presupposes that truth (falsity) is relative to moments only. But such presupposition is at odds with branching indeterminism, properly understood. Or at least, this is so according to the view we explore in the next section.

4.1 Belnap's viewpoint

One of the most influential branching theorists is Nuel Belnap. According to his view, the conservative strategy is perfectly right.

> [T]he indispensable idea from Prior-Thomason is that *truth shall be relativized to moment-history pairs*, where the moment belongs to the history. So, in addition to the immobile tree structure itself, there are two new mobile parameters to which truth (as well as denotation, etc.) is relativized: *the moment of evaluation*, and *the history of evaluation*. [...] The most difficult part of this Prior-Thomason idea, the part that goes beyond linear tense logic, is negative in character:
>
> > *Given indeterminism, it does not suffice to think of truth (or denotation, etc.) as relative only to moments.* (Belnap et al. 2001: 224-225)

As we have seen, Peirceanism defines a notion of truth at a moment by quantifying over the histories passing through the moment of evaluation of future tensed statements.[1] Supervaluationism, on the other hand, defines a notion of truth at a moment by quantifying over the histories passing through the moment of use of an arbitrary statement. And when the moment of assessment is identical with the moment of use, relativism boils down to supervaluationism.

Belnap resolutely rejects Peirceanism, for it cannot make sense

> of someone who purports to assert that the coin will land heads even though it might not, that is, who sincerely asserts both that *Will:*$\mathcal{A}$ and that *Poss:*$\neg$*Will:*$\mathcal{A}$. (Belnap et al. 2001: 159)

The reason is that, under the Peircean semantics, any statement of the form

[1] Again, recall that PE predicts that the set of histories passing through the moment of evaluation may differ from those passing through the moment of use.

(1) $\mathsf{F}_p(n)\mathcal{A} \wedge f_p(n)\neg\mathcal{A}$

is false at any moment.

Belnap seems to suggest that there is a motivation to reject supervaluationism (and hence, relativism) too. It has to do with plain-will statements of the form "It will be the case that $\mathcal{A}$". In Belnap's view, the truth value of "It will be the case that $\mathcal{A}$" cannot be only relative to the moment of use, and

> [o]ne must relativize truth to the history parameter as well. The reason is that only thus can we make sense, in branching time, out of plain (linear) future-tense sentences such as
>
> (2) There will be a sea battle tomorrow.
>
> Think of (2) as uttered before the admirals have made their decisions. Then the truth of that sentence (given indeterminism) depends not only on the moment at which the sentence is uttered. It depends in addition on which future course of events—which history—is being considered. [...] To put the matter in easily understood words: Given indeterminism, what will happen (for example, whether or not there will be a sea battle tomorrow) depends on what will happen (i.e., on which history is being considered). (Belnap et al. 2001: 225)

If the truth of "there will be a sea battle tomorrow" is sensitive to the historical parameter, it cannot only depend on the moment of use. Hence, it seems that Peirceanism, supervaluationism and relativism are essentially misguided.

These considerations lead Belnap to adopt the following principle:

Openess by constancy: The truth-values of future contingents are not constant as the history of evaluation varies. (Belnap & Green 1994: 377)

To sum up, Belnap rejects the revisionary strategy. Peirceanism is wrong, for it entails that $\mathsf{F}(n)\mathcal{A} \wedge \Diamond\neg\mathsf{F}(n)\mathcal{A}$ is always false. Supervaluationism (and hence, relativism) does not account for plain future-tense sentences such as "It will be the case that $\mathcal{A}$". According to Belnap, such sentences should be always assessed at moment-history pairs. Furthermore, Belnap endorses the *Openess by constancy* principle, which suggests that the relativisation of truth to moment-history pairs is the best we can get in an indeterministic context.

However, Belnap's approach shares with revisionism several assumptions. In his view, contexts of use should satisfy the following property.

Openness by the context of use: The context in which a future contingent is uttered cannot supply a privileged history for its evaluation. (Belnap & Green 1994: 377)

In turn, a context cannot provide a privileged, actual history, for there is no such a thing as the actual history. And the rejection of the existence of a unique, privileged course of event is justified by the adoption of the *Relational actuality* principle developed by Lewis and Thomason.

> As Lewis (1970) has argued, this world's being the actual world does not favor it over any others, but is just a reflection of the fact that this is the world at which we are conversing. Thinking now about *Our World*, suppose we have arrived at an indeterministic moment m. To suppose that there is one from among the histories flowing out of m that is the actual history is rather like purporting to stand outside Lewis' realm of concrete possibilia and pointing to the one that is actual. But this is wrong. *For a world to be actual is for it to be the world we inhabit. For a history to be actual would be for it to be the history to which the moment we inhabit belongs.* (Belnap & Green 1994: 381)[2]

This quotation is pretty important. Indeed, it clearly establish that Belnap endorses the relational actuality we encountered above.

Moment-History Actuality $@_1(h,m) \Leftrightarrow h$ is *the* history h' such that $(m \in h')$.

Belnap shares with Thomason the intuition that actuality can only be an indexical, relative matter: "$@_1$" is a relational term, and the truth (falsity) of $@_1(m,h)$ depends on whether h happens to be the unique history passing through m. And, as we have seen before, the *Moment-History Actuality*, along with the Definition 2.2.3 of objective branching indeterminism, are incompatible with the commitment to a unique history that is 'seen' as actual from any moment. And given that, in general, a moment of use has more than one history passing through it, a context of use cannot provide *the* history of the moment it specifies. Be as it may, nothing in Belnap's view appears to be incompatible with the other relational readings of actuality that we encountered above.

Branch-Moment Actuality: $@_2(m,b) \Leftrightarrow \forall m' \in b\ (m' \leq m)$
Moment-Moment Actuality: $@_3(m,m') \Leftrightarrow m' \leq m$

[2]Emphasis added.

The *Openness by the context of use* principle, along with the *Openess by constancy* principle, help to understand why Belnap's view is incompatible with Thomason's (and MacFarlane's) motivations to reform the BO-semantics. One main motivation for supervaluationism is the following assumption:

Second Interpretation: Moment-history pairs m/h can legitimately occur within a definition of truth for $\mathcal{L}_T$ only if h refers to the unique actual history.

Now, the *Openness by constancy* principle says that the truth values of future contingents are not constant as the history of evaluation varies. Thus, it seems that moment-history pairs should be taken as legitimate candidate for a definition of truth for a tensed language. However, the *Openess by the context of use* principle says that a context cannot yield any privileged history among those passing through the moment of the context. And this principe is justified on the basis of the *Moment-History Actuality* principle and Definition 2.2.3. They jointly entail that a unique actual history (viz. a history that is 'seen' as actual from any moment) cannot exist. Hence, moment-history pairs m/h can legitimately occur within a definition of truth for $\mathcal{L}_T$, even if there is no absolute, actual history. Therefore, the *Second Interpretation* principle, according to Belnap, must be assessed as false.

Even if Belnap's conservative strategy rejects one principle that motivates supervaluationism, it has to face a difficult task. This task has to do with the elaboration of a notion of truth-at-a-context which is compatible with both *Openness by the context of use* and the *Openness by constancy* principle, but which does not collapse over supervaluationism.

4.2 Belanp's view and Supervaluationism

Recall two principles of Belnap's view, that is

Openness by the context of use: The context in which a future contingent is uttered cannot supply a privileged history for its evaluation.
Openness by constancy: The truth-values of future contingents are not constant as the history of evaluation varies.

In his mind, these two principles should allow to draw an analogy between future contingents and open formulas. Belnap takes contexts of use to be moments.[3] Now, let us see how Belnap defines the truth conditions for future tensed statements and quantified sentences, *as evaluated at a point initialised by a context of use*.

> **Quantifiers.** $m_c, g, m/h \vDash \forall x_i \mathcal{A}$ iff $m_c, g_1, m/h \vDash \mathcal{A}$ for every assignment g_1 that does not differ from g except perhaps at the variable x_i.
>
> **Tense connectives.** $m_c, g, m/h \vDash \mathsf{P}\mathcal{A}$ iff there is a moment m_1 in the past of m such that $m_c, g, m_1/h \vDash \mathcal{A}$. Also $m_c, g, m/h \vDash \mathsf{F}\mathcal{A}$ iff there is a moment m_1 in the future of m along history h such that $m_c, g, m_1/h \vDash \mathcal{A}$. (Tenses never move off h.) (Belnap et al. 2001: 152)

The clauses in **Quantifiers** and in **Tense connectives** are metalinguistic conditions of truth-at-a-context-at-a-point (truth-at-an-index for short). Given the standard treatment of quantifiers, an open formula such as "x_1 is brindle" is open relative to the assignment parameter: indeed, there are points (as, say, $m_c, g_1, m/h$) where it comes out true (assuming $g_1(x_1)$ is indeed brindle), but others points (such as $m_c, g_2, m/h$) where it comes out untrue (assuming $g_2(x_1)$ is not brindle).[4] Analogously, if $\mathcal{A}$ is a future contingent at m_c, there are points (as, say, $m_c, g, m_c/h_1$) where it comes out true, but other points (such as $m_c, g, m_c/h_2$) where it comes out untrue.

According to Belnap, the analogy between future contingents and open formulas has significant consequences for the evaluation of predictions relative to a context. Consider the following, metalinguistic sentences, and assume that "F(The coin lands tails)" is a future contingent at m_c:

(2) $m_c \vDash x_1$ is brindle
(3) $m_c \vDash \mathsf{F}$(The coin lands tails)

Belnap et al. (2001) claim that both (2) and (3) have no meaning, for both "x_1 is brindle" and "F (The coin lands tails)" can only be evaluated if a further parameter, which is not settled by the context, is specified (an assignment and a history, respectively). Thus, open formulas, as well as future contingents, have no truth value in their contexts of use. As Belnap and Green put it,

[3]Clearly, one may enrich contexts with parameters that may be useful to interpret indexical expression or non-historical modalities. However, as far as $\mathcal{L}_T$ is concerned, Belnap's notion of a context of use coincides with a moment of use only.

[4]Recall the definition of open sentences: a sentence, $\mathcal{A}$, considered as stand-alone, is open in a parameter, Z, iff $\mathcal{A}$'s evaluation depends on Z, and Z is not initialized by context.

> In the absence of a specific convention, there is no sense to saying that an open stand-alone sentence has a truth value independently of the evaluation parameter with respect to which it is open. If a sentence is neither closed by context nor closed by constancy, it has no truth value save relative to the parameter in question. (Belnap & Green 1994: 377)

Notice that (2)-(3) use a notion of truth-at-a-context, while **Quantifiers** and **Tense connectives** use the notion of truth-at-an-index. Given a Kaplanian, Lewisian framework, it is usual to define truth-at-a-context as a function of truth-at-an-index. Belnap et al. (2001) do not provide such a definition. However, Belnap himself seems to recognise the need of defining truth-at-a-context in terms of truth-at-an-index.[5]

> 'Occamist' semantics, also mentioned by (Prior 1967: 121-127), employs an auxiliary history parameter, the 'history of evaluation', and takes truth to be relative to a triple (m_c, m, h) (moment of utterance, moment of evaluation, and history of evaluation), where h must contain m; [...] *Being auxiliary, the history of evaluation parameter must eventually be canceled.* (Belnap & Müller 2010: 690)[6]

Thus one may wonder how the auxiliary history may be cancelled, and what kind of connection between the notions of truth-at-a-context and truth-at-an-index best fits what Belnap wants to say.

Future contingents lack truth values at contexts because (a) their truth-at-an-index is function of the historical parameter, but (b) contexts do not initialise the historical parameter. A uniform semantical treatment, however, requires that any sentence whose truth-at-an-index is function of the historical parameter has to lack truth values at contexts. But the semantic value that *any* sentence takes at an index such as $m_c, g, m/h$ is function of the historical parameter. Thus, a uniform semantical treatment requires that any sentence lacks truth value at a context. This conclusion, however, is absurd.

Belnap et al. (2001: 225) do not face this difficulty directly, but they highlight a way to overcome it. Quantified statements, as well as sentences

[5]Belnap & Müller (2010: 694) adopts a semantical notion of truth-at-a-context which boils down to relativism. However, I take this choice to be incompatible with several remarks of Belnap et al. (2001), according to which truth (and plausibly, truth-at-a-context) should be relativised to the historical parameter. Moreover, relativism clearly contradicts the conservative approach.

[6]Emphasis added.

about the present and the past, are "closed" in both the historical and the assignment parameters. Their truth value at an index does not vary as the history or the assignment varies. Thus, these kinds of statements should turn out to be either true or false at a context.

> "$m_c \vDash$ Meg is hungry" makes sense since "Meg is hungry" is closed in each of the mobile parameters. We could indeed define "truth" given model and context as follows: For every (or some, or most, or your favourite) g and h, $m_c, g, m_c/h \vDash$ Meg is hungry. (Belnap et al. 2001: 155)[7]

If one tries to generalise this suggestion, a quantification over assignments and histories is needed. This quantification, in turn, allows to attribute truth values to closed sentences at a context. But an existential quantification won't work.

(4) $m_c \vDash \mathcal{A} \Leftrightarrow \exists g \exists h(h \ni m_c \Rightarrow m_c, g, m_c/h \vDash \mathcal{A})$

According to (4), future contingents are true at their contexts of use, as well as their negations.[8] This is not only weird, but it is incompatible with Belnap's requirement, according to which future contingents should not have a truth value at their moments of use. Assuming we do not want to substitute the existential quantification in (4) with "most" or "your favourite", only a universal quantification can do the job:

(5) $m_c \vDash \mathcal{A} \Leftrightarrow \forall g \forall h(h \ni m_c \Rightarrow m_c, g, m_c/h \vDash \mathcal{A})$

$m_c \nvDash \mathcal{A} \Leftrightarrow \forall g \forall h(h \ni m_c \Rightarrow m_c, g, m_c/h \nvDash \mathcal{A})$

It is easy to see that (5) captures several things Belnap wants to say:

- only future contingents, along with some open formulas, lack truth values at their contexts of use.
- the context of use does not provide any value for the historical parameter. Hence, the *Openness by the context of use* principle holds.
- Sentences about the present and the past, as well as quantified statements, are either true or false at their contexts.

However, (5) entails that the truth-at-a-context of a future contingent is *not* sensitive to the historical parameter. Indeed, (5) defines truth-at-a-context by a universal quantification over the histories passing through m_c. Accordingly, the *Openness by constancy* principle does not hold at the

[7] Notation slightly changed.

[8] (4) is actually a form of subvaluationism, see Ciuni & Proietti (2013).

truth-at-a-context level, but it holds at the truth-at-an-index level only. Moreover, (5) boils down to supervaluationism.

These are bad results, for they undermine several reasons that lead Belnap to adopt the conservative strategy. The conservative strategy (i) requires a notion of truth that is relative to moment-history pairs, (ii) it demands to distinguish plain predictions from predictions about what inevitably will be the case, and (iii) it calls for a semantics which makes $\mathsf{F}(n)\mathcal{A} \wedge \Diamond\neg\mathsf{F}(n)\mathcal{A}$ satisfiable when $\mathsf{F}(n)\mathcal{A}$ is a future contingent. Supervaluationism, however, cannot account for (i)-(iii). Moreover, truth-at-a-context is not sensitive to the historical parameter.

In my view, these results underline something relevant. If one wants to adopt a uniform semantic machinery, which, as usual, defines truth-at-a-context in terms of truth-at-an-index, one has to make a choice.

(i) Either one adds an historical parameter to the context and allows the *Openness by the constancy* principle to hold also at the truth-at-a-context level,

(ii) or one assumes the *Openness by the context* principle and (a) either one evaluates any statement as neither true nor false at its context of use, or (b) one adopts the supervaluationist semantics, which entails that future contingents do not have truth values at their contexts of use.

Option (ii.a) entails something absurd, viz. that any statement whatsoever fails to have a truth value at a context. Option (ii.b) boils down to supervaluationism, and thus it undermines the reasons for adopting conservationism. In the next section, I'll argue that a conservationist can easily make sense of option (i).

4.3 Belnap's view reformed

In my mind, we can retain the pros of Belnap's conservative strategy by dropping the following principle:

Openness by the context of use: The context in which a future contingent is uttered cannot supply a privileged history for its evaluation.

Once that the *Openness by the context of use* principle is rejected, one is free to conceive a context of use as a tuple formed by a moment and a history, m_c/h_c.

One can suspect that this way of modelling contexts is in tension with something that Belnap assumes, viz. the *Moment-History Actuality* principle. This latter principle, indeed, entails that, in general, there is nothing as a unique actual history passing through a moment. But if one models contexts as moment-history pairs, one seems to contravene to what the *Moment-History Actuality* principle entails.

This tension can be dissolved by making a move that we have already encountered in connection with the Definition 3.5.2 of a relativist context of use. Recall that a relativistic context of use, c_{RE}, is a couple of a moment of use $m_{c_{RE}}$ and a set of histories, $H_{c_{RE}}$. Since $H_{c_{RE}}$ can be a subset of the histories passing through $m_{c_{RE}}$, for a relativist the very same utterance can have *many* contexts of use. Analogously, if a conservationist conceives a context as a moment-history couple, but she sticks to the *Moment-History Actuality* principle, she can say that an utterance made at m_c has, in general, *many* contexts of use: any (m_c, h) s.t. $m_c \in h$ would be a context for a statement used at m_c. If there is no privileged history passing through m_c, an utterance made at m_c can be conceived as something that happens in many contexts, none of which is privileged over the others.

And once that one can use such a notion of context, it is easy to see that one can identify BO-truth (falsity) with truth at-a-context:

(6) $m_c/h_c \vDash \mathcal{A} \Leftrightarrow m_c/h_c \vDash_{BO} \mathcal{A}$.

By (6), one can recover several desiderata that a conservationist wants to satisfy, that is:

- Truth-at-a-context is relative to moment-history pairs. Accordingly, the *Openness by constancy* principle holds at the truth-at-a-context level.
- BO does not equate "will" with "inevitably will".
- The formula $\mathsf{F}(n)\mathcal{A} \wedge \Diamond\neg\mathsf{F}(n)\mathcal{A}$ is satisfiable when $\mathsf{F}(n)\mathcal{A}$ is a future contingent at m_c.

Notice, moreover, that (6) solves several problems affecting the revisionist semantics, for

✓ BO validates the future excluded middle.

✓ BO does not introduce a semantic distinction between $\mathsf{F}(n)\neg\mathcal{A}$ and $\neg\mathsf{F}(n)\mathcal{A}$.

✓ BO does account for retrospective truth judgments. Indeed, $\mathcal{A} \vdash_L \mathsf{P}(n)\mathsf{V}\mathsf{F}(n)\mathcal{A}$ is sound (here $\vdash_L$ is the local logical consequence of Definition 3.4.5, which is the natural notion of logical consequence for the BO-semantics).

✓ BO-truth is disquotational; $\mathsf{V}\mathcal{A} \leftrightarrow \mathcal{A}$ is BO-valid, and $\mathsf{V}\mathcal{A} \dashv\vdash \mathcal{A}$ is sound w.r.t. the local notion of logical consequence.

Furthermore, by adopting the conservative approach and (6), one can reasonably say that

(7) $m_c \vDash \mathcal{A}$

does not make sense *independently* of the formula which one substitutes to $\mathcal{A}$. This because (7) employs a notion of truth-at-a-context which is relative to the moment of the context only, while a conservationist can hold that the *right* notion of truth-at-a-context must be relative to both the moment of the context and the history of the context. And this, in turn, vindicates the *Openness by constancy* principle, which holds at the truth-at-a-context level.

Nonetheless, there is an interesting analogy between a deterministic interpretation of a tree structure and (6). As we shall see, a semantics such as (6) is attractive especially for those that assume a deterministic reading of tree structures. Next section explores some connections between (6) and a deterministic interpretation of tree structures, i.e., the many-worlds view.

4.4 Many worlds and Ockhamism

The label "many-worlds theory" refers to an interpretation of quantum mechanics due to Hugh Everett III. The many-worlds theory is usually conceived as a proposal to solve the so-called measurement problem.[9] The measurement problem, as we have already seen, characterises the tension between two principles of Von Neumann (1955)'s standard formulation of quantum mechanics.

Dynamical postulate: When no measurements are going on, the states of all physical systems invariably evolve in accordance with the dynamical equations of motion.[10]

Collapse postulate: When a measurement takes place at time t, the state of the measured system S collapses over one element of the superposition (if any) among those described by the equations of motion of S at t.[11]

[9] See DeWitt & Graham (2015).

[10] See Albert (2009: 80).

[11] See Albert (2009: 80).

The *Dynamical* and the *Collapse* postulates are problematic, given that they use an undefined, ordinary notion such as that of measurement. Hence, the measurement problem: how to deal with the notion of measurement? And relatedly, how the *Dynamical* and the *Collapse* postulates have to be understood?

The many-worlds interpretation of quantum mechanics answers these questions by adopting a two-step strategy. First, many-worlds theorists claim that the notion of "measurement", as it is used in the *Dynamical* and the *Collapse* postulates, "[...] leads to an artificial dichotomy of the universe into ordinary phenomena, and measurements".[12]

As a second step, many-worlds theorists hold that there is a way to understand superpositions which does not require to attribute to measurements any distinguished role. This understanding, moreover, should be perfectly compatible with *both* our experience *and* with the fundamental equations of motion for quantum systems.

The general idea is this. At any instant, every physically possible outcome of any quantum interaction occurs. Suppose, for instance, that the Schrödinger's 's equation predicts that, at time t_1, the electron a will be in a superposition of having spin $\uparrow_x$ and $\downarrow_x$. The many-worlds view states that, at time t_1, reality splits into two sets of branches. One set contains the branches at which a is in $\uparrow_x$; the other is the set of those branches in which a is in $\downarrow_x$. In general, many-worlds theorists hold that

> [t]his universe is constantly splitting into a stupendous number of branches, all resulting from the measurement like interactions between its myriads of components. Moreover, every quantum transition taking place on every star, in every galaxy, in every remote corner of the universe is splitting our local world on earth into myriads of copies of itself. (DeWitt 1970: 161)

Any element of a superposition describes an outcome of a quantum interaction, obtaining in (at least) a branch of the universe. Thus, superpositions

> [...] are just states of the world in which more than one [...] definite thing is happening at once. [...] superpositions do not describe indefiniteness, they describe multiplicity. (Wallace 2012: 36-37)

[12]See Everett et al. (2012: 59).

A many-worlds theorist would tend to say that the definiteness of our experience is due to the fact that we are conscious of – we interact with – just a single collection of branches of reality at a time (say, the one with a in state $\uparrow_x$). But there are copies of us inhabiting other collections of overlapping branches: in particular, there are copies that measure those outcomes that were possible to obtain, but which we didn't measure (thus, there are copies of us located in other branches, and these copies measure a in state $\downarrow_x$).

If this is the case, the *Collapse* postulate is wrong, for it says precisely what a many-worlds theorist denies. In a many-worlds perspective, it is false that a measurement induces the measured system to collapse into a unique definite outcome. Sure, any measurement would measure just a single, definite outcome, but any possible outcome, as well as any possible measurement, actually takes place somewhere in the branching universe.

Furthermore, a new reading of the *Dynamical* postulate should be adopted:

*Dynamical** postulate: The states of all physical systems invariably evolve in accordance with the dynamical equations of motion.[13]

The new *Dynamical** postulate does not mention the notion of measurement, which thus does not have any privileged status over other physical interactions. Moreover, the dynamical equations of motion – plausibly, the Schrödinger's equation associated with the entire physical reality – is the only theoretical element that plays an explicatory role.

Given the *Dynamical** postulate, along with the interpretation of superpositions just highlighted, it appears quite natural for a many-worlds theorist to subscribe to a branching conception of reality.[14] And any many-worlds theorists who adopts a branching conception of reality is perfectly entitled to endorse the BO-semantics (or, equivalently, (6)). After all, in the BO-semantics all histories are 'equally real', in that none of them is privileged over the others. This is perfectly consistent with the idea that they are all bound to happen. Thus, relativising truth (and, in particular the truth of predictions) to moment-history pairs appears to be justified by the endorsed, background metaphysics. As a matter of fact, David Wallace assumes a version of the many-worlds view equipped with an Ockhamist-inspired semantics (what he calls the "second model").

> On the second model, certainly spin up will occur in some parts of pretty much any possible world (i.e. pretty much any

[13]See Wallace (2012: 38).

[14]There are several many-worlds theorists that accept a branching conception of reality: Bacciagaluppi (2002), Belnap & Müller (2010), Saunders & Wallace (2008), Wallace (2005) and Wallace (2012). For a critical perspective, see Wilson (2012).

> quantum-mechanical branching structure); so will spin down. Indeed, both spin up and spin down results will occur in some of the branches which branch off from any instance in which the measurement is correctly performed. So the truth conditions of future-directed sentences appear to tell us that 'X will occur', uttered at time t, is true in all those possible worlds which assign nonzero branch weight to X occurring conditional on the context of utterance (that is: to X occurring in some branch futurewards of the event of utterance).
> (Wallace 2012: 268)

Now, recall that the conservative view– properly reformed as (6) – amounts to adopting the BO-semantics. Thus, the conservative view adopts a semantical machinery that a many-worlds theorist should be happy to endorse.

The fact that many-worlds theorists have reasons to adopt the BO-semantics might be taken as an evidence for the conservative view. As far as indeterminism is concerned, however, this evidence is illusory.

4.5 The many-worlds view and determinism.

Given Definition 2.2.3 of Branching Objective Indeterminism (BOI), a many-worlds theorist who subscribes to a branching conception of reality would count as an indeterminist. Indeed, according to this definition, a history (possible world) is objectively indeterministic just in case it shares a moment with another, numerically distinct history. But this cannot be correct, though. And the reason why Definition 2.2.3 cannot be correct is that many-worlds theorists are willing to *restore* determinism at the very foundations of physics. After all, the fundamental law that rules the evolution of reality is the Schrödinger's equation, and there are good reasons to call such a rule "deterministic".

> [w]e have a causal and deterministic evolution prescribed by the Schrödinger's equation. It is causal in the sense that any later state of a system is functionally determined by its earlier states. It is deterministic since any two trajectories of a system in a relevant Hilbert space $\mathcal{H}$ agree on their earlier segment, they agree everywhere. (Placek 2000: 99)

> The Schrödinger's equation, governing the time evolution of the state, is deterministic in exactly the same way as the equations of classical mechanics, i.e. it allows us to determine the

> undisturbed state of the system at all times, given the state at $t = 0$. (Weber 1993: 201)

> [t]he basic dynamical assumption of quantum mechanics is that if you know the state at one time, then the quantum equations of motion tell you what it will be later. Without loss of generality, we can take the initial time to be zero and the later time to be t. The state at time t is given by some operation that we call $U(t)$, acting on the state at time zero. Without further specifying the properties of $U(t)$, this tells us very little except that $|\Psi(t)>$ is determined by $|\Psi(t_0)>$. [...] We are setting up $U(t)$ in such a way that the state-vector will evolve in a deterministic manner. Yes, you heard me correctly—the time evolution of the state-vector is *deterministic*. (Susskind & Friedman 2015: 95-96)

If the Schrödinger's equation governs nature's evolution, reality can hardly be said to evolve stochastically. And if many-worlds theorists are determinists, but they can perfectly adopt a branching conception of reality, having a branching conception of reality cannot be sufficient to be an indeterminist. Accordingly, if a history h shares a moment with another history, this does not suffice to say that the underlying metaphysical framework is indeterministic. Hence, it seems that there are good reasons to reform Definition 2.2.3 of Branching Objective Indeterminism.

Indeterminism, it is tempting to think, is the view that there are many possible futures. This is not quite right, though, for the existence of a variety of possible futures is, as such, consistent with determinism. The reason is that we can coherently hold that many futures are possible and, at the same time, maintain that all of them will happen. If so, we have a determinist scenario, for there is nothing contingent in the future evolution of reality: as of now, it is already determined what future possible moments are going to happen.

As John Bell puts it, the deterministic character of the many-worlds theory consists in the view that

> [i]t is just an illusion that the physical world makes a particular choice among many macroscopic possibilities [...]; they are *all* realized, and no reduction of the wave function occurs. (Bell 2004: 95)

It is interesting to compare this typical trait of the many-worlds theory with a feature shared by the stochastic versions of quantum mechanics. These latter theories fail to be deterministic, for

> [s]ystems prepared identically and with the maximum accuracy permitted by the theoretical scheme, when subjected to identical measurements, can in general give different results. This is the essence of quantum indeterminism. It compels us to give up the principle that different effects arise from different causes. (Weber 1993: 202)

Now, the many-worlds theory does not give up "the principle that different effects arise from different causes". *Anytime* that one measures the spin of an electron with z-spin down (up) along the x axis, two classes of outcomes would then obtain: the class of outcomes with the electron having x-spin down, and the class of outcomes with the electron having x-spin up. The failure of the collapse of superpositions guarantees that same effects arise from same causes.

If, on the other hand, different effects arise from same causes, different measures of the x-spin of an electron in $\uparrow_z$ would give different outcomes. But each time that such a measurement is performed, just one *single* outcome will obtain. In other terms, reality would evolve by making *one* choice among many possible routes.

> Of all the possible futures represented by space-time manifolds which branch off from the first branch point on the model, one and only one becomes 'actual', i.e. becomes part of the past. The other branches vanish. The universe model is a tree that 'grows' or ages by losing branches. (McCall 1994: 3)

> The emergence of actuality, and the progressive vanishing of all but one future branch, is one of the two principal differences between the present theory [i.e., the open future] and the many-worlds interpretation of quantum mechanics. (McCall 2009: 420-421)

> On the one hand, each continuation from a branch point is individually possible; on the other hand, it is impossible that more than one of these continuations should be realized. (Belnap et al. 2001: v)

> [T]here are alternative incompatible ways the future might be, which fit together not by "both ways happening," but precisely by having a branch point at which they both were possibilities. At the branch point, the future can be either way, but not both ways. (Belnap et al. 2001: 170fn)

> Current possibilities drop off (McCall (1994)) with passage into the future [...]. (Belnap et al. 2001: 207)

Branching indeterminism is definitely not a many-ways (or a many-worlds) view. What is characteristic of an indeterministic reading of a branching structure is that, at any fork, things evolve along just one way. Indeterminists, therefore, should be able to express the principle that only some possible outcomes (or, more plausibly, just one outcome) of an experiment will obtain. Expressing this principle would allow indeterminists to deny the deterministic claim that *none* of the future possibilities will obtain, and to deny the deterministic claim that *any* future possibility will obtain.

The moral is that genuine objective indeterminism requires that

Possible futures: There are many possible future alternatives, but
Choice: among the several, possible alternatives, only a single future alternative would obtain.[15]

BOI, as it is defined in Definition 2.2.3, guarantees the *Possible futures* condition only. And, as we have just seen, such a condition can be easily satisfied by those many-worlds theorists – which, again, are determinists – who adopt a branching conception of reality. What Definition 2.2.3 *alone* does not guarantee – and what many-worlds theorists firmly deny – is the *Choice* principle. Accordingly, one needs to enrich Definition 2.2.3 with some further requirement which captures the *Choice* condition.

To express the *Choice* condition, in turn, one has to have a notion that distinguishes merely possible moments (viz., possible states of the world that fail to obtain) from possible moments which fail to be merely possible (viz., possible states of the world that obtain). Traditionally, the philosophical notion that has been employed to draw such a distinction is precisely that of actuality.[16] Intuitively, the *Choice* condition says that, whenever there is a moment m that has more than one possible future, even though it is indeterminate which possible future will be actualised, one of them will.

Notice that expressing the *Choice* principle is not only relevant to get a clearer understanding of branching indeterminism. It is also useful to

[15] Sure, one may object that indeterminism requires that, among the several, possible futures, only a proper subset of them would obtain. In my view, this objection is misleading. It seems arbitrary, indeed, to say that more than one possible future would obtain, but deny that any possible future will. Why it should be so? If one agrees that more than one single future will obtain, the only plausible thing to say then is that it is so because any future possibility will.

[16] See Menzel (2016), Plantinga (1974), Plantinga (1976) and Stalnaker (1976).

assess the conservative view developed by Belnap and his collaborators. The conservative strategy assumes BOI, which, as I have tried to argue, is insufficient to characterise indeterminism. Moreover, conservationists adopt a semantic machinery which is perfectly compatible with what the many-worlds view. Thus, one may wonder whether a conservationist has enough resources to tell herself apart from a many-worlds theorist. After all, conservationists such as Belnap (and his collaborators) take themselves as indeterminists. Thus they should be able to say what kind of difference tells their position apart from that of an advocate of a determinist, many-worlds view.

It may be the case, for instance, that the analogies between the conservative view and the many-worlds view turn out to be only superficial, as it were. Sure, a many-worlds theorist can adopt the same semantics as a conservationist. And a many-worlds theorist can adopt the same branching conception of reality as a conservationist. But there may be some metaphysical difference that enables to tell apart the two positions from one other. This metaphysical difference, in turn, would make it possible to distinguish conservationists, *qua* indeterminists, from many-worlds theorists, *qua* determinists. And, as we have seen, such a metaphysical difference should plausibly concern the notion of actuality. At the end of the day, many-worlds theorists disagree with indeterminists about whether anything that's possible obtains, or just a proper subset of possibilities obtains. We have seen that conservationists have several readings for actuality, but each of these reading is relational. Thus, it is interesting to see what are the notions of actuality that a many-worlds theorist can adopt.

4.6 Many worlds and actuality

Here's a telling quotation from Hugh Everett III, the founding father of the many-worlds view.

> From the viewpoint of the [many-worlds] theory all elements of a superposition (all "branches") are "actual," none any more "real" than the rest. It is unnecessary to suppose that all but one are somehow destroyed, since all the separate elements of a superposition individually obey the wave equation with complete indifference to the presence or absence ("actuality" or not) of any other elements. (Everett et al. 2012: 189)

Here Everett does not use the relational notions of actuality we encountered above. When he claims that all "branches" are "actual", he is not claiming that there is a unique possible history, passing through any possible moment. Nor he is claiming that any possible moment is later than any other moment.

A clue which suggests that he's not assuming a relational notion of actuality is the fact that he is speaking from "the viewpoint of the [many-worlds] theory." He's abstracting from any location that a possible outcome may have, taking an absolute standpoint over the plethora of all physical possibilities. Once that one has taken such a viewpoint, it is misleading to use a relational notion of actuality. By taking such a view from nowhere, it is misleading to speak of what is 'seen' as actual from the particular viewpoint one is located at. Indeed, if one takes a view from nowhere, one is supposing not to be located at all.

More plausibly, when Everett uses "actual" from this God's eye standpoint, he's using a *substantial*, *non-relational* notion of actuality. The substantial notion of actuality divides the space of (physical) possibilities into two kinds: on the one hand, there are those (physical) possibilities that obtains, and are said to be substantially actual. On the other hand, if there is a physical possibility that does not obtain, it is said to be merely possible. What is important, though, is that the substantial actuality of a moment is not a perspectival matter. It does not depend on the perspective of a given moment. In other terms, the substantial actuality of a moment is a primitive, monadic property. Analogously, that a history is substantially actual means – plausibly – that any of its moment is substantially actual, and thus that any of its moments obtain. And the substantial actuality of a history h does not depend on the relations that h has with other histories, nor with the relations that its moments have with other histories. It only depends on whether any moment in h is substantially actual.

Once that it is acknowledged what kind of notion of actuality Everett is speaking about, it is easy to understand what he's saying. According to him, any possible outcome obtains, and thus any possible outcomes is substantially actual. Therefore, the many-worlds view assumes that to say that a possible moment (history) is substantially actual is *trivially true*. And to say that what is possible is merely possible is *trivially false*. According to Everett, the substantial actuality does not capture any difference among physical possibilities, and hence it is perfectly redundant.

Interestingly, David Lewis reasons in a way that is perfectly analogous to that of Everett.

> If we take a timeless point of view and ignore our own location in time, the big difference between the present time and other times vanishes. That is not because we regard all times

> as equally present, but rather because if we ignore our own location in time we cannot use temporally indexical terms like "present" at all. And similarly, I claim, if we take an a priori point of view and ignore our own location among the worlds, the big difference between the actual world and other worlds should vanish. (Lewis 1970: 19-20)

According to Lewis, if one takes a nowhere standpoint over possible worlds, and uses the notion of (substantial) actuality to draw a distinction from one possible world to another, that notion turns out to be perfectly useless. Any possible world, in Lewis' view, is (substantially) actual when 'seen' from nowhere.

Lewis also suggests an interesting link among the substantial and the relational readings of actuality. If one thinks that substantial actuality is redundant, one may reasonably apply the relational readings of actuality in order to convey non-trivial information.

> [possible worlds differ] not in kind, but only in what goes on at them. Our actual world is only one world among others. We call it alone actual not because it differs in kind from all the rest, but because it is the world we inhabit. (Lewis 1973a: 85)

If actuality receives one of its indexical, relational readings, it may convey non-trivial information. Recall the relational notions of actuality we have encountered thus far.

Moment-History Actuality $@_1(h, m) \Leftrightarrow h$ is *the* history h' such that $(m \in h')$
Branch-Moment Actuality: $@_2(m, b) \Leftrightarrow \forall m' \in b\ (m' \leq m)$
Moment-Moment Actuality: $@_3(m, m') \Leftrightarrow m' \leq m$

If $@_1(h, m)$ is true, m has a unique history passing through it. When $@_2(m, b)$ is true, any $m' \in b$ is earlier than or equal to m. And if $@_3(m, m')$ is true, m' is earlier than or it is equal to m.

On the contrary, under a many-worlds view, that a possible world is (substantially) actual is trivially true. Thus, the only non-trivial applications of the concept of actuality are the relational ones.

There is no surprise, therefore, that a many-worlds theorist such as Simon Sounders is happy to concede that

> [w]hat is "actual", just what is "now", is to be understood as facts as relations. There is nothing more to be put in; neither the "flow" of time, taking us from one "now" to the next, nor the reduction state, taking us from one "actuality" to another. (Saunders 1995: 244)

To sum up, if one thinks that anything that's possible happens, one naturally loses interest in the notion of substantial actuality: anything that's possible is substantially actual. Hence, the relational readings of actuality become the only interesting ones, for they are the only readings that allows to say non-trivial things. And a many-worlds theorist is thus perfectly justified in adopting a Lewisian view about actuality: if "actual" has to receive an informative reading, it can only be relational.

But this seems to undermine the conservationist view: a conservationist, indeed, shares with a many-worlds theorist too many things. The two share the same semantics, they have the same conception of branching reality, and they also agree that the interesting readings that actuality may have are the relational ones. This latter trait, as we shall see, prohibits conservationists to formulate the *Choice* principle in a way that tells conservationism itself apart from the family of branching determinists. This result, moreover, suggests that, if indeterminism is true, the substantial notion of actuality cannot be redundant.[17]

4.7 The Choice principle and relational actuality

As I have argued above, branching objective indeterminism – as it is defined in Definition 2.2.3 – consists in the following principle.

Possible futures: There are many possible future alternatives.

When one adopts a branching conception of reality, it is natural to translate the *Possible futures* postulate with these two, equivalent statements.

(8) $\exists m, m', m''(m < m' \;\&\; m < m'' \;\&\; m' \nleqslant m'' \& \; m' \nless m'')$
(9) $\exists m \, \exists h, h'(m \in h \;\&\; m \in h' \;\&\; h \neq h')$

However, Definition 2.2.3 does not say anything about a further condition, which is needed to distinguish an indeterministic reading of a tree from its determinist interpretations. This condition is:

Choice: Among the several, possible alternatives, only a single future alternative would obtain.

As we have seen, this is what many philosophers say when they claim that, according to indeterminism, just one single outcome, among those that are

[17]For a similar conclusion, see Borghini & Torrengo 2013.

possible at a given moment, will ‘be realised’. And the *Choice* principle is precisely what a many-worlds theorist would deny: according to the many-worlds perspective, any possible future is bound to ‘be realised’.

Intuitively, to say that only a single future obtains is to say that only a single future is actual. Thus, the *Choice* principle has to involve actuality. But recall that, in dealing with the many-worlds view, one may distinguish two main kinds of actuality. There are readings according to which actuality is a relation. The substantial reading of actuality, on the contrary, entails that *being actual* is a monadic property of moments, and for a history to be (substantially) actual is to contain only moments that are (substantially) actual. Thus, when one tries to formulate the *Choice* principle in an explicit way, one has to employ either one of the relational readings of actuality, or its substantial interpretation.

The *Choice* principle, moreover, is not mean to be a mere guess. On the contrary, it has to be intended as a law governing the way in which reality evolves. For instance, any time that a system S is in a superposition, it may evolve in different, possible ways. But anytime that a measurement-like interaction takes place on S, the system collapses into one among the elements of its superposition. Thus, anytime that things might go in several ways, they turn out to take just a singular path. The law-like character of the *Choice* principle allows to claim that, if the principle holds, it must hold anytime that there is a fork in the tree.

With these information at hand, let us try to formulate the principle in a branching friendly way. First, let us introduce the notion of instant.

Definition 4.7.1. (Instants) *The set* Instant *is a partition of a branching frame $\mathcal{T}$ into equivalence classes; that is,* Instant *is a set of nonempty sets of moments such that each moment in $\mathcal{T}$ belongs to exactly one member of* Instant. *Moreover,*

(i) Unique intersection. *Each $i \in$* Instant *intersects each history in a unique moment; that is, for each instant i and history h, $i \cap h$ has exactly one member.*

(ii) Order preservation. *Given two instants i_1 and i_2 and two histories h and h', if the moment at which i_1 intersects h precedes, or is the same as, or comes after the moment at which i_2 intersects h, then the same relation holds between the moment at which i_1 intersects h' and the moment at which i_2 intersects h'.*[18]

[18]The definition is taken from Belnap et al. (2001: 194-195). See also Di Maio & Zanardo (1994).

Intuitively, any instant i is a set of contemporaneous but alternative moments. Thus, if $i(m)$ is the instant at which m belongs, $i(m)$ contains m itself, plus any alternative moment which lies at the same chronological height of m. Instants can be pictured as horizontal lines (see Figure 4.1). Given Definition 4.7.1, one can impose a linear order over instants: let us

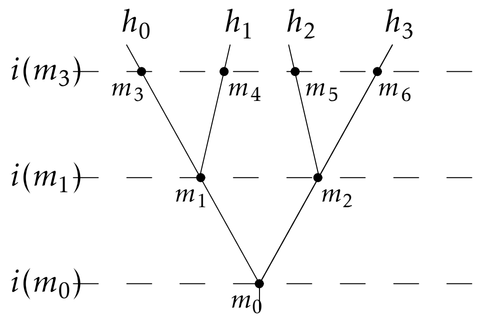


Figure 4.1

say that $i <_i i'$ just in case there exist $m \in i, m' \in i'$, and $m < m'$. Accordingly, in the model of Figure 4.1, it is the case that $i(m_0) <_i i(m_1) <_i i(m_3)$.

The notion of instant allows to express the intuition that, among several alternatives, just one of them would be realised as the physical world evolves. In particular, the *Choice* principle can be formulated as follows.

*Choice**: $\forall i, i'(i <_i i' \Rightarrow \exists! m''(m'' \in i' \ \&\ m'' \text{ is actual}))$

The *Choice** postulate says that, for any instant i' whose moments are later than those in i, there is a unique moment in i' that is actual. However, the notion of actuality occurring in the postulate is ambiguous. As we have seen, one may adopt either (one of) the relational notions of actuality, or a substantive reading.

If one adopts the relational notion, one has to relativise the actuality predicate occurring in the *Choice** principle. Let us see whether there is a relativisation that is plausible and, moreover, that enables to use the *Choice** principle to tell apart indeterminists from many-worlds theorists.

Recall the firts notion of relational actuality that we encountered above, namely:

Moment-History Actuality $@_1(h, m) \Leftrightarrow h$ is *the* history h' such that $(m \in h')$

Plausibly, if one relativises the actuality predicate occurring in the *Choice** principle with the help of $@_1$, one may obtain the two following versions.

(10) $\forall i, i'(i <_i i' \Rightarrow \exists h \exists ! m(m \in i' \ \& \ @_1(h, m)))$
(11) $\forall i, i'(i <_i i' \Rightarrow \forall h \exists ! m(m \in i' \ \& \ @_1(h, m)))$

Statement (10) says that, for any i' that is later than i, there is a unique moment in i' whose future is settled, for there is a unique history passing through it. Clearly, this principle seems to be perfectly arbitrary, and it should be denied by both indeterminists and many-worlds theorists.

Statement (11), furthermore, says that any history whatsoever is the unique history passing through m, where m is a moment in i', and i' is later than i. This statement directly contradicts the branching conception of reality encoded in (8)-(9), for it entails that any possible history is the unique history passing through m. Thus, (11) is denied by *any* branching theorist.

As a result, both many-worlds theorists and indeterminists should reject (10) and (11). Hence, it seems that $@_1$ cannot provide a version of the *Choice** principle which distinguishes the two parties. Let us try with another relational notion of actuality.[19]

Branch-Moment Actuality: $@_2(m, b) \Leftrightarrow \forall m' \in b \ (m' \leq m)$

If one relativises the *Choice** principle by using $@_2$, one should plausibly obtain the following two other versions of the principle.

(12) $\forall i, i'(i <_i i' \Rightarrow \exists b \exists ! m(m \in i' \ \& \ @_2(m, b)))$
(13) $\forall i, i'(i <_i i' \Rightarrow \forall b \exists ! m(m \in i' \ \& \ @_2(m, b)))$

Statement (12) says that, for any i' that is later than i, there is a set of moments b, and a unique moment m in i', such that any moment in b is either earler than or equal to m. This condition is easily satisfied by any branching frame, because for any moment m whatsoever there is a b whose moments are earlier than or equal to m: namely, $\{m\}$. Therefore, (12) is assumed by *any* branching theorist.

Statement (13) says that, for any i' that is later than i, there is a unique moment m in i' such that any moment in b is either earlier than or equal to m, for any set of moments b. If the antecedent in (13) holds, there are at leat two instants $i <_i i'$, and then two moments, m and m', such that $m' \in i$ and $m \in i'$. Since for any set of moments b, any moment in b is either earlier or equal to m, there cannot be a moment later than m. Furthermore, if there is a moment earlier than m', there would be a set that has a moment that is later than m', namely $\{m\}$. And this is something that (13) prohibits.

[19]Recall that b is a set of moments. Thus, b may fail to be a history, which is a maximal $<$-chain of moments.

And there cannot be a moment that is not <-related with m', for otherwise there would be a set with a moment that is neither earlier nor equal to m. Therefore, (13) entails that there exists a unique history with two moments only, $\{m', m\}$. This, in turn, is denied by *any* branching theorist.

To sum up, any branching theorist assumes (12) and rejects (13). Again, it seems that $@_2$ is not useful to provide a version of the *Choice** principle which tells apart branching determinists from branching indeterminists. Let us consider the third relational notion of actuality.

Moment-Moment Actuality: $@_3(m, m') \Leftrightarrow m' \leq m$

Let us try to relativise the actuality predicate in the *Choice** principle with the help of $@_3$. Let us see the two following relativised versions of the principle.

(14) $\forall i, i'(i <_i i' \Rightarrow \exists m' \exists ! m(m \in i' \,\&\, @_3(m, m')))$
(15) $\forall i, i'(i <_i i' \Rightarrow \forall m' \exists ! m(m \in i' \,\&\, @_3(m, m')))$

Statement (14) is trivial: any instant has at least a moment, and any moment is identical with itself. Statement (15) says that, for any i' that is later than i, there is a unique moment m in i' such that any moment whatsoever is either earlier than or equal to m. It is easy to see that (15), like (13), entails the existence of a unique history, and thus contradicts the branching conception of reality.

Once again, (14) and (15) cannot tell apart branching determinists form branching indeterminists: both parties assume (14) and reject (15).

These results suggest that the *Choice** principle can hardly involve a notion of relational actuality. And, in general, one cannot tell apart many-worlds and indeterminism *only* by using branching structures and the relational notions of actuality. Why? Because both views adopt a branching conception of reality – that is, both views adopt tree structures – and $@_1$, $@_2$ and $@_3$ just involve the relations that points (or sets of points) have within tree structures.

In general, the notions of relational actuality cannot – by themselves – be useful to tell apart indeterminists from those who adopt a deterministic reading of the tree. This, in turn, suggests that the relational notions of actuality are useless to convey the information that, at any fork, reality would take just one path. And if this is true, then the relational notions of actuality cannot express the intended meaning of the *Choice** principle. But then conservationists such as Belnap and his collaborators, as well as Thomason and MacFarlane, do not have enough resources to describe themselves as indeterminists.

4.8 The Choice principle and substantial actuality

In my view, indeterminists disagree with many-worlds theorists on the distribution that substantial actuality has over the tree. And since the *Choice** principle should describe the way reality evolves indeterministically, it is sensible to take it as talking about substantial actuality:

*Choice***: $\forall i, i'(i <_i i' \Rightarrow \exists! m''(m'' \in i'$ & m'' is substantially actual))

Once the principle is read this way, it is easy to see that a many-worlds theorist cannot accept it. For a many-worlds theorist, indeed, substantial actuality is redundant, in the sense that any possible moment is substantially actual. Hence, any moment of an instant is substantially actual. Now, given a tree $\mathcal{T}$ and (8)-(9), there must be at least two histories, say h and h'. This means, in turn, that there are at least two moments, m and m', s.t. $m < m'$, $m \in h \cap h'$, $m' \in h$ but $m' \notin h'$. By Definition 4.7.1, there are at least two instants, i and i', such that $i \cap h = i \cap h' = \{m\}$, but $i' \cap h = \{m'\} \neq i' \cap h'$. And since $m < m'$, then $i <_i i'$. Notice, moreover, that both $i' \cap h$ and $i' \cap h'$ are singlets. Hence, i' must have at least two elements. The *Choice*** principle entails that i' must have a unique (substantially) actual moment, and hence it has at least a merely possible moment.

This conclusion, as we have argued just above, cannot be accepted by a many-worlds theorist, for it entails that the notion of substantial actuality *is not* redundant. Furthermore, the *Choice*** principle, along with (8)-(9), entails something that any indeterminist should say: any time that the physical world can evolve in more than one way, it actually evolves by following just one alternative among those that are possible.

Furthermore, the *Choice*** principle and (8)-(9) express the idea that, according to indeterminism, same causes *can* have different effects. If one measures the x-spin of an electron being, say, in state $\uparrow_z$, the *Choice*** principle guarantees that the physical world would evolve in such a way that either $\uparrow_x$ or $\downarrow_x$ would obtain. Moreover, it guarantees that it is never the case that both outcomes will obtain. And the branching structure that physical possibilities take – and which is encoded in (8)-(9) – does not secure that, whenever one measures the x-spin of an electron in state $\uparrow_z$, it is settled that one would obtain $\uparrow_x$. It does not secure, moreover, that whenever one measures the x-spin of an electron in state $\uparrow_z$, it is settled that one would obtain $\downarrow_x$. Thus, the same cause (the measurement of the x-spin of an electron in state $\uparrow_z$) can have different effects (either $\uparrow_x$ or $\downarrow_x$ – but again, not both).

I have argued that a branching indeterminist needs the *Choice*** principle to be distinguished from a determinist such as a many-worlds theorist. Thus, one may reform the Definition 2.2.3 for branching indeterminism by adding to it the *Choice*** principle. This move, yet, would be too hasty, for there is another principle that one should reasonably invoke:

Past Actuality: $\forall m, m'$(m is substantially actual & $m' \leq m \Rightarrow m'$ is substantially actual)

The *Past Actuality* postulate says that the past of a substantially actual moment is substantially actual. I take this principle to be quite reasonable. First, notice that any moment in the past of a moment m is linearly ordered by $<$. Thus the past of m – that is, the set $\{m' : m' < m\}$ – can be taken to be as a singular course of event up to m. And if m happens to obtain – viz. if m happens to be substantially actual – it is so, plausibly, because how the physical world has evolved before m had m as one of its possible future outcomes. This means, in set theoretical terms, that any moment that did obtain in an instant i' such that $i' <_i i(m)$ is $<$-related with m. Hence, if m is substantially actual, the past of m must be substantially actual too.

It would be very odd to deny the *Past Actuality* principle. It would be really unreasonable, for instance, to stick to the *Choice** principle, but at the same time to say that there are two instants $i <_i i'$ whose (substantially) actual moments are not $<$-related. This, indeed, would amount to say that there are two moments that are substantially actual, but none of them is modally consistent with the other. The *Past Actuality* postulate, furthermore, prohibits that there is a merely possible moment in the past of a substantially actual moment. From this prohibition it follows that substantial actuality does not "jump": if $m < m' < m''$, and m and m'' are substantially actual, m' must be substantially actual as well. I take this as a reasonable thing to hold.

Notice that the *Past Actuality* postulate is independent of whether one accepts the *Choice*** principle. Take, once again, a many-worlds theorist. As we have seen before, such a theorist must deny the *Choice*** principle, for it entails that substantial actuality fails to be redundant. But she would be willing to hold the *Past Actuality* postulate. Indeed, if any moment is substantially actual, any past moment of a substantially actual moment must be substantially actual as well.

With the *Choice*** and the *Past Actuality* principles at hand, one can integrate Definition 2.2.3 of Branching Objective Indeterminism as follows.

Definition 4.8.1. (Branching Objective Indeterminism*(BOI*)) *Given a set of physically possible histories (worlds) $H(\mathcal{T})$, a history $h \in H(\mathcal{T})$ is objectively indeterministic iff*

Possible futures principle: *There is a moment m and a history $h' \in H(\mathcal{T})$ such that $m \in h$, $m \in h'$ and $h \neq h'$.*
Choice** principle: $\forall i, i'(i <_i i' \Rightarrow \exists! m''(m'' \in i'$ & m'' *is substantially actual*)).
Past Actuality principle: $\forall m, m'(m$ *is substantially actual* & $m' \leq m \Rightarrow m'$ *is substantially actual*).

The definition just given is pretty important. As we have seen, is something that a many-worlds theorist cannot accept. But Definition 4.8.1 has also another relevant feature. It entails that there must exist a unique (substantially) actual history. Before proving this fact, let us state some useful definitions.

Definition 4.8.2. (The set of substantially actual moments) *Given a tree $\mathcal{T} = (M, <)$, let $|\mathbf{A}|$ be a subset of the set of moments M in $\mathcal{T}$, such that $|\mathbf{A}| = \{m : m$ is substantially actual$\}$.*

Definition 4.8.3. (Trunk) *A set of moments p is a tunk iff (i) p is linearly ordered by $<$, (ii) p has a maximal element, that is, there is an m s.t., for any $m' \in p$, $m \leq m'$ entails $m = m'$ (iii) p contains any moment in the past of its maximal element. Let $p(m)$ be the trunk whose maximal element is m, that is $p(m) = \{m' : m' \leq m\}$. I write $m > p(m')$ $(p(m') > m)$ if $m > m'(m' > m)$. Analogously, $p(m) > p(m')'$ iff $m > m'$.*

Let us see two facts that will be useful later.

Fact 9. *Branching Objective Indeterminism* entails that, whenever $i <_i i'$, $m \in i$, $m' \in i'$, $m, m' \in |\mathbf{A}|$, then $p(m) < p(m')$.*
Proof. *Assume $i <_i i'$, $m \in i$, $m' \in i'$ and $m, m' \in |\mathbf{A}|$. If $p(m) \not< p(m')$, either $m' \leq m$, or m' and m are not $<$-related. $m' \leq m$ is not possible, for $m' \in i', m \in i$ and $i <_i i'$, and by the definition of instant, it follows that either m' and m are not $<$-related, or $m < m'$. If m and m' are not $<$-related, the definition of instant entails that $i \cap p(m')$ is non-empty and, by the* Past actuality principle, *$i \cap p(m') \subseteq |\mathbf{A}|$. Moreover, $i \cap p(m) \subseteq |\mathbf{A}|$, $i \cap p(m') \subseteq |\mathbf{A}|$, and $i \cap p(m) \neq i \cap p(m')$. Otherwise $m < m'$. But then i must have two actual moments, against the* Choice** principle. *By reductio, $p(m) < p(m')$.*

Fact 10. (Recursion Theorem) *For any set A, and any $a \in A$, and any function $g : A \times \mathbb{N} \longmapsto A$, there exists a unique infinite sequence $f : \mathbb{N} \longmapsto A$ such that*

(a) $f(0) = a$,
(b) $f(n+1) = g(f(n), n)$, *for all* $n \in \mathbb{N}$.[20]

[20] See Hrbacek & Jech (1999: 48) for a proof.

Facts 9 and 10 are useful to prove that:

Fact 11. *Branching Objective Indeterminism* entails the existence of a unique, substantially actual history.*
Proof. *Either (a) there is an i_f s.t. $i \leq_i i_f$ for any i, or (b) for any i, there is an i' s.t. $i <_i i'$.*

If (a), by the Choice** principle *and the* Past actuality principle*, there is an m_f in i_f such that $p(m_f) \subseteq |\mathbf{A}|$. If $p(m_f)$ is not a history then either $p(m_f)$ has two moments that are not $<$-related, or it fails to be maximal. $p(m_f)$ cannot contain two moments that are not $<$-related, for $p(m_f) = \{m : m \leq m_f\}$, and $<$ is linear towards the past. If $p(m_f)$ is not maximal, there must be a $m > p(m_f)$; but this cannot be, for i_f is the last instant that there is. Thus, $p(m_f)$ is a history. Now, either $p(m_f) = |\mathbf{A}|$ or $p(m_f) \neq |\mathbf{A}|$. If $p(m_f) \neq |\mathbf{A}|$, either $m \in |\mathbf{A}|$ but $m \notin p(m_f)$, or $m \notin |\mathbf{A}|$ but $m \in p(m_f)$, for some m. If $m \in |\mathbf{A}|$ but $m \notin p(m_f)$, then m belongs either to i_f or not. If $m \in i_f$, then i_f has two actual moments, m_f and m, against the* Choice** principle*. If $m \notin i_f$, it must belong to some i such that $i <_i i_f$. The* Past Actuality principle *entails that, for any $m' \in p(m_f)$, $m' \in |\mathbf{A}|$. According to the definition of instant, then, $i \cap p(m_f) \subseteq |\mathbf{A}|$. Hence, i has two actual moments, m and the element in $i \cap p(m_f)$, against the* Choice** principle*. Hence, there is no m s.t. $m \in |\mathbf{A}|$ but $m \notin p(m_f)$. If $m \notin |\mathbf{A}|$ but $m \in p(m_f)$, m must be either equal to or earlier than m_f. If $m = m_f$, then $m \in |\mathbf{A}|$. Thus, $m < m_f$. But $m_f \in |\mathbf{A}|$. By the* Past Actuality principle*, $m \in |\mathbf{A}|$. Accordingly, it must be the case that $p(m_f) = |\mathbf{A}|$, and $p(m_f)$ is a history.*

If (b), then there is no last instant i_f. And thus, by the Choice** principle *and* Fact 9*, it follows that, for any $p_0 \subseteq |\mathbf{A}|$, there exists a p_1 s.t. $p_0 < p_1$ and $p_1 \subseteq |\mathbf{A}|$. Accordingly, for any sequence of trunks $(p_0, ..., p_n)$ s.t. $p_0 < ... < p_n$ and $p_n \subseteq |\mathbf{A}|$, there is a p_{n+1} s.t. $p_n < p_{n+1}$ and $p_0 \subseteq ... \subseteq p_n \subseteq p_{n+1} \subseteq |\mathbf{A}|$. Let us define a set of functions F, such that any $f \in F$ maps sequences of trunks $s_n(p_0, ..., p_n)$, where $\bigcup_0^{n-1} s_n \subseteq p_n$ and $p_n \subseteq |\mathbf{A}|$, to sequences $s_{n+1} = (p_0, ..., p_{n+1})$ s.t. $\bigcup_0^n s_n \subseteq p_{n+1}$ and $p_{n+1} \subseteq |\mathbf{A}|$. Intuitively, any function f takes actuality 'one step further in the future', and yields a sequence s_{n+1} that has more actual trunks than s_n. The Recursion Theorem* (Fact 10) *guarantees that there is a unique infinite sequence $s_@ = \{p_n\}_{n=0}^{\infty}$ such that $p_{n+1} = f(p_0, ..., p_n)$, for any $n \in \mathbb{N}$. Then, $s_@$ is an infinite sequence of trunks: therefore, for any $m, m' \in \bigcup s_@$, either $m \leq m'$ or $m' < m$. Accordingly, $\bigcup s_@$ is linearly ordered by $<$. Moreover, for any $m \in \bigcup s_@$, $m \in |\mathbf{A}|$. If $\bigcup s_@$ is not a maximal $<$-chain, there must be some $m \in |\mathbf{A}|$ such that $m > p_n$, for any $p_n \in \bigcup s_@$. But given the construction of $s_@$, this is impossible. Therefore, $\bigcup s_@$ is a maximal $<$-chain of actual moments, and $|\mathbf{A}| = \bigcup s_@$. Hence, any actual moment is in history $\bigcup s_@$.*

The commitment to a unique, substantially actual history is directly entailed by Definition 4.8.1 of Branching Objective Indeterminism*. Indeterminism, therefore, is not only compatible with the existence of a unique, substantially actual history. Indeterminism requires such a commitment (with the proviso stated in refnote)

Notice, for clarity's sake, that Fact 11 does not contradict Thomason's and Belnap's claim, according to which a unique, *relationally* actual history cannot exist. If "there is a unique (relationally) actual history" means that there is a unique history which passes through any possible moment, whoever adopts Definition 4.8.1 (and hence, subscribes to Fact 11) denies the existence of such a history. What Fact 11 highlights, however, is that relational actuality cannot be everything that there is to know about actuality, at least within an indeterministic perspective. According to a many-worlds theorist, we have many futures. According to Fact 11, on the contrary, we have only one single future – i.e., the future that lies along the unique, substantially actual history.

4.9 "We have a single future"

Belnap, MacFarlane and Thomason claim that the *Choice* principle, viz.

Choice: among the several, possible alternatives, only a single future alternative would obtain,

can be expressed without positing a (substantially) actual history. Thus, the commitment to a unique a (substantially) actual history is unnecessary.[21] In particular, they suggest letthat the view that the physical world evolves by 'choosing' just one future alternative at a time is expressed by certain tense-logical validities.

On the one hand, certain validities encodes the view that *at least* one of tomorrow's alternatives will obtain.

> But 'It will or it won't' has the force of tautology. It is invariably true to say things such as 'Either it will rain tomorrow or it wont't', even in cases where there is no more justification for saying it will than for saying it won't rain. (Thomason 1970: 267)
>
> *One or another will happen*. On the Peircean view, it is always false to say

[21] See, for instance, Placek & Belnap (2012: 460-461).

(16) It is possible that it will be sunny tomorrow, and it is possible that it won't be, but either it will be or it won't be.

> [...] But this seems something that we can say truly [...]. It expresses the natural thought that, even if the future is open, one or the other of the alternatives will take place. (MacFarlane 2014: 217)

On the other hand, other validities guarantee that, among the several alternatives for tomorrow, *exactly one* will occur.

> "*We have a single future.*" [...] If it means that it is settled that incompatible events will never happen, it is true. (Belnap et al. 2001: 206)
>
> *It won't be both ways.* Perhaps it will be sunny here at this time tomorrow, perhaps it won't. But we know it won't be both ways—it will be one or the other. [...] All we conclude from the datum that it won't be both ways is that our semantic theory must avoid making

(17) Tomorrow it will be sunny here and won't be sunny here.
Tomorrow (Here is sunny ∧¬ Here is sunny)

> true at any context. (MacFarlane 2014: 211)

Apparently, according to belnap, MacFarlane and Thomason, the principle that, among future possibilities, just one of them will obtain is captured by these two schemas:

(18) $\mathsf{F}(n)(\mathcal{A}) \vee \mathsf{F}(n)(\neg\mathcal{A})$
(19) $\neg\mathsf{F}(n)(\mathcal{A} \wedge \neg\mathcal{A})$

Hence, the principle that we have a single future – that is, the *Choice* principle – can be expressed without positing any substantially actual history.

This cannot be true, however. Given the BO semantics, (18) is valid because any future moment (lying on a given history) satisfies either $\mathcal{A}$ or its negation (Clearly, I am assuming that histories have to be unbounded towards the past and the future). This, in turn, is due to nothing but the fact that, according to the BO semantics, negation and disjunction behave classically: for any $\mathcal{A}$ and every point m/h, either $\mathcal{A}$ is BO-true at m/h (and $\neg\mathcal{A}$ is BO-false at m/h) or $\mathcal{A}$ is BO-false at m/h (and $\neg\mathcal{A}$ is BO-true at m/h). Furthermore, the validity of (19) only ensures that any future moment is consistent.

Notice, moreover, that (18)-(19) are completely silent about (substantial) actuality. Therefore, the advocates of the many-worlds view can take (18)-(19) as valid. Many-worlds theorists are committed to the claim that any history is substantially actual, not to the (independent) views that (i) there are future moments that fail to satisfy both $\mathcal{A}$ and $\neg\mathcal{A}$, and (ii) there are future moments that are inconsistent. As a matter of fact, the many-worlds philosopher David Wallace (2012: 268) says explicitly that BO can be perfectly coherent with his favourite metaphysical picture. And notice, once again, that BO validates both (18) and (19) (if histories are taken to be unbounded both towards the past and the future). But surely Wallace wouldn't subscribe to the *Choice* principle: in his view, any future alternative would obtain.

Surely, there is a way to say that just a single future will obtain – which a many-worlds theorist cannot accept – and which is based on the validity of principles analogous to (18)-(19). This way forces the many-worlds theorist to reject a validity that an indeterminist may accept. It also entails, however, the existence of the actual history. Thus, if tense-logic validities analogous to (18)–(19) encode the view that we have a single future, they also entail that there is a unique, substantially actual history.

Let's see why. First, let's distribute the future operator over the conjuncts in (19):

(20) $\neg(\mathsf{F}(n)(\mathcal{A}) \land \mathsf{F}(n)(\neg\mathcal{A}))$

Notice that (20) is a BO validity: thus, as far as the BO semantics is concerned, (20) can be accepted both by branching indeterminists and many-worlds theorists. As a further step, let us introduce a bunch of definitions which will be useful later on.

Definition 4.9.1. (Metaphysical frames) *A metaphysical frame is a tuple* $\mathcal{F}_M = (\mathcal{T}, |\mathbf{A}|)$*, where* $\mathcal{T}$ *is a tree and* $|\mathbf{A}|$ *is a set of actual moments of* $\mathcal{T}$*, that is,* $|\mathbf{A}| \subseteq M \in \mathcal{T}$*.*

What is peculiar about metaphysical frames is that they specify the extension the actuality predicate may have over trees.

Definition 4.9.2. (Models based on metaphysical frames) *A model* $\mathcal{M}_{\mathcal{F}_M}$ *based on a metaphysical frame* $\mathcal{F}_M$ *is a sequence* $(\mathcal{F}_M, I)$*, where* I *is an interpretation of the atoms of* $\mathcal{L}_t$*.*

Definition 4.9.3. (Interpretation of 'Will') $\mathsf{F}(n)\mathcal{A}$ *is true at* m *in a model* $\mathcal{M}_{\mathcal{F}_M}$ *iff there is an actual moment* m' *which is* n *time units in the future of* m*,* *and $\mathcal{A}$ is true at m' in $\mathcal{M}_{\mathcal{F}_M}$.*[22] *In symbols,*

$$\mathcal{M}_{\mathcal{F}_M}, m \vDash \mathsf{F}(n)\mathcal{A} \Leftrightarrow \exists m'(d_{\mathcal{T}}(m,m') = n \ \&\ m < m' \ \&\ m' \in |\mathbf{A}| \ \&\ \mathcal{M}_{\mathcal{F}_M}, m' \vDash \mathcal{A})$$

Definition 4.9.4. (Validity on a model) *$\mathcal{A}$ is valid on a model $\mathcal{M}_{\mathcal{F}_M}$ just in case $\mathcal{A}$ is true at any moment $m \in |\mathbf{A}|$ in $\mathcal{M}_{\mathcal{F}_M}$. In symbols,*

$$\mathcal{M}_{\mathcal{F}_M} \vDash \mathcal{A} \Leftrightarrow \forall m(m \in |\mathbf{A}| \in \mathcal{M}_{\mathcal{F}_M} \Rightarrow \mathcal{M}_{\mathcal{F}_M}, m \vDash \mathcal{A})$$

Definition 4.9.5. (Non-pathological models) *$\mathcal{M}_{\mathcal{F}_M}$ is non-pathological only if, for any m, m' s.t. $m, m' \in i$ and $m \neq m$, then there is a formula $\mathcal{A}$ such that $\mathcal{M}_{\mathcal{F}_M}, m \vDash \mathcal{A}$ but $\mathcal{M}_{\mathcal{F}_M}, m' \vDash \neg\mathcal{A}$.*

The non-pathological requirement states that any two moments lying at the same instant satisfy different sets of formulas. This condition captures the intuition that two versions of tomorrow, say A and B, are alternative because something that would obtain if A were to happen wouldn't obtain if B were to happen. As we shall see, this requirement is crucial for the present purposes. However, it may appear problematic. For instance, one can satisfy the non-pathological condition by imposing that any actual moment has its own nominal (i.e, for any $m \in |\mathbf{A}|$, there is an atom p of $\mathcal{L}_t$ such that $I(p) = \{m\}$).[23] However, if $|\mathbf{A}|$'s cardinality is the continuum, one would need uncountable many nominals. It is possible to avoid these difficulties by following the method developed in Meyer (2009), and constructing moments as maximal consistent sets of sentences. In this case, the non-pathological requirement can be easily satisfied without assuming any problematic, infinitary language.

By Definitions 4.9.1-4.9.5 it follows that the conjunction of (18) and (20) encodes the principle that, among the many future alternatives, just one will obtain.

Fact 12. *If* (18) *and* (20) *are valid on a non-pathological model $\mathcal{M}_{\mathcal{F}_M}$, for any two instants i, i' s.t. $i <_i i'$ and $i, i' \subseteq M \in \mathcal{T} \in \mathcal{F}_M$, there is a unique m in i' s.t. $m \in |\mathbf{A}|$.*

If (18)-(20) are valid on a non-pathological model $\mathcal{M}_{\mathcal{F}_M}$, then $\mathcal{M}_{\mathcal{F}_M}$ is based on a metaphysical frame$\mathcal{F}_M = (\mathcal{T}, |\mathbf{A}|)$ that satisfies the *Choice** principle*. This is the sense in which (18)-(20) can express that we have a single future. However,

[22] In Definition 4.9.3, the requirement that the moment shifted by the future operator, m', must be actual is responsible for the failure of (18) at non-actual moments. However, bear in mind that the notion of validity that will be assumed in the proof of Fact 13 is defined as truth at every actual moment (see the following definition).

[23] See Blackburn (1994).

Fact 13. (18) *and* (20) *are valid on a non-pathological model $\mathcal{M}_{\mathcal{F}_M}$ just in case the metaphysical frame $\mathcal{F}_M = (\mathcal{F}_{BT}, |\mathbf{A}|)$ on which $\mathcal{M}_{\mathcal{F}_M}$ is such that $|\mathbf{A}|$ is a history.*

Proof. *As for* Fact 12, *if $\mathcal{M}_{\mathcal{F}_M} \vDash \mathsf{F}(n)(\mathcal{A}) \vee \mathsf{F}(n)(\neg\mathcal{A})$ for any $m \in |\mathbf{A}| \in \mathcal{M}_{\mathcal{F}_M}$, then*

$$\exists m'(d_{\mathcal{T}}(m,m') = n \;\&\; m < m' \;\&\; m' \in |\mathbf{A}| \;\&\; \mathcal{M}_{\mathcal{F}_M}, m' \vDash \mathcal{A}) \text{ or}$$
$$\exists m'(d_{\mathcal{T}}(m,m') = n \;\&\; m < m' \;\&\; m' \in |\mathbf{A}| \;\&\; \mathcal{M}_{\mathcal{F}_M}, m' \vDash \neg\mathcal{A})$$

Recall that histories are assumed to be unbounded both to the past and to the future. Moreover, any m is such that either $\mathcal{M}_{\mathcal{F}_M}, m \vDash \mathcal{A}$ or $\mathcal{M}_{\mathcal{F}_M}, m \vDash \neg\mathcal{A}$. Thus, (18)*'s validity means that, for any actual moment in $\mathcal{M}_{\mathcal{F}_M}$, there is at least one of its future moments that is actual. On the other hand, if $\mathcal{M}_{\mathcal{F}_M} \vDash \neg(\mathsf{F}(n)(\mathcal{A}) \;\&\; \mathsf{F}(n)(\neg\mathcal{A}))$ for any $m \in |\mathbf{A}| \in \mathcal{M}_{\mathcal{F}_M}$, then*

$$\neg(\exists m'(d_{\mathcal{T}}(m,m') = n \;\&\; m < m' \;\&\; m' \in |\mathbf{A}| \;\&\; \mathcal{M}_{\mathcal{F}_M}, m' \vDash \mathcal{A}) \;\&$$
$$\exists m'(d_{\mathcal{T}}(m,m') = n \;\&\; m < m' \;\&\; m' \in |\mathbf{A}| \;\&\; \mathcal{M}_{\mathcal{F}_M}, m' \vDash \neg\mathcal{A}))$$

which is equivalent to

$$\forall m'(d_{\mathcal{T}}(m,m') = n \;\&\; m < m' \;\&\; m' \in |\mathbf{A}| \Rightarrow \mathcal{M}_{\mathcal{F}_M}, m' \vDash \neg\mathcal{A}) \text{ or}$$
$$\forall m'(d_{\mathcal{T}}(m,m') = n \;\&\; m < m' \;\&\; m' \in |\mathbf{A}| \Rightarrow \mathcal{M}_{\mathcal{F}_M}, m' \vDash \mathcal{A}),$$

which says that any two actual moments, lying at n time units in the future of m, must satisfy the same set of formulas. Since we are dealing with non-pathological models, any two moments in the same instant must satisfy different sets of formulas. Hence, if (18) *and* (20) *are valid on a non-pathological model $\mathcal{M}_{\mathcal{F}_M}$, for any actual moment m in $\mathcal{M}_{\mathcal{F}_M}$, there is a unique actual moment at n time units in the future of m. This, obviously, entails the* Choice** principle*: for any two instants i, i' s.t. $i <_i i'$, there exists a unique $m \in i'$ s.t. $m \in |\mathbf{A}|$. This proves* Fact 12. *As for* Fact 13, *the right-to-left direction is quite trivial. Let us then proove its left-to-right direction only. Assume that* (18) *and* (20) *are valid in $\mathcal{M}_{\mathcal{F}_M}$. Thus, by* Fact 12, *it follows that $\mathcal{M}_{\mathcal{F}_M}$ must satisfy the* Choice** principle*. Suppose that $\mathcal{M}_{\mathcal{F}_M}$ does not satisfy the* Past Actuality principle, *for there are two moments in it, say m and m', s.t. $m, m' \in |\mathbf{A}|$, $m \in i$, $m' \in i'$, $i <_i i'$ but m and m' are not $<$-related. Assume that $d_{\mathcal{T}}(m,m') = n'$. Since* (18) *and* (20) *are valid in $\mathcal{M}_{\mathcal{F}_M}$, there must be a unique m'' s.t. $m' < m''$, $m'' \in |\mathbf{A}|$ and $d_{\mathcal{T}}(m',m'') = n$, for any n. And there must be a unique m''' s.t. $m < m'''$, $m''' \in |\mathbf{A}|$ and $d_{\mathcal{T}}(m,m''') = n$, for any n. Consider an instant i' s.t. $i(m) <_i i'$ and $i(m') <_i i'$. Then, there must be two moments in i', say m'' and m''', s.t. $m < m''$, $m' < m'''$, $d_{\mathcal{T}}(m,m'') \neq\uparrow$ and $d_{\mathcal{T}}(m',m''') \neq\uparrow$. Then, $m'', m''' \in |\mathbf{A}|$, and thus i' contains two actual moments. This contradicts the*

Choice** principle. *It follows that* $\mathcal{M}_{\mathcal{F}_M}$ *satisfies both the* Choice** principle *and the* Past actuality principle. *By* Fact 11, $\mathcal{M}_{\mathcal{F}_M}$ *must specify a unique, substantially actual history. Accordingly,* Fact 13 *follows.*

A many-worlds advocate must asses the non-pathological models in which (18) and (20) are both valid as metaphysically inadequate. In particular, the many-worlds theory entails that everything that is possible is substantially actual, and thus it is incompatible with the validity of (20) on non-pathological models. More generally, the class of metaphysical frames compatible with the many-worlds view are such that, for any frame $\mathcal{F}_M = (\mathcal{T}, |\mathbf{A}|)$ in that class, $\mathcal{T} = |\mathbf{A}|$. On the other hand, if (18) and (20) are valid according to Definitions 4.9.1-4.9.5, they express the *Choice*** principle. Hence, their validity can indeed convey the information that we have a single future. But (18) and (20) are valid in the class of models based on those metaphysical frames $\mathcal{F}_M = (\mathcal{T}, |\mathbf{A}|)$ such that $|\mathbf{A}| = h$. Accordingly, if (18) and (20) express that we have a single future, they forces to accept the existence of a unique, substantially actual history.

4.10 Metaphysical worries

Belnap et al. (2001) express a worry related to the commitment of a unique, substantially actual history.

> What in the structure of our world could determine a single possibility from among all the others to be "actual"? (Belnap et al. 2001: 162-163)

Suppose we name the unique, substantially actual history as **h**. Then – as Jacek Wawer (2014) rightly points out – the question «What in the structure of our world could determine a single possibility from among all the others to be "actual"?» has two readings.

First reading: What in the structure of our world makes it necessary that history **h** is substantially actual?
Second reading: What in the structure of our world makes it necessary that there is a unique history that is substantially actual?

The first reading asks whether something necessitates **h** to be actual. In my view, the notion of modality involved here cannot be historical. In other terms, the first reading does not use "necessity" in the same sense in which, given a moment m in a tree, one may say that at m it is inevitable that there will be a sea battle tomorrow. And this because the question uses the expression "the structure of our world", which, presumably, refers to those metaphysical principles – such as those in Definition 4.8.1 – which rule the physical possibilities of the reality we live in. In other terms, "the structure of our world" can be taken to refer to those principles that are satisfied by the metaphysical frame we are located at, but may be satisfied by other metaphysical frames as well. If this is so, I perfectly agree with Wawer:

> [T]he answer is: "Nothing!". The world is indeterministic and it can develop along any of the possible ways. It simply develops along one of them which we call TRL_h [that is, **h**]. (Wawer 2014: 392)

If one uses the notion of metaphysical frame, it is easy to see that nothing in Definition 4.8.1 of Branching Objective Indeterminism* necessitates **h** to be the actual history. Indeed, $\mathcal{F}_M = (\mathcal{T}, |\mathbf{A}| = \mathbf{h})$ is not the only metaphysical frame that satisfies that definition. There are, for instance, other metaphysical frames $\mathcal{F}'_M = (\mathcal{T}, |\mathbf{A}| \neq \mathbf{h})$ which are perfectly compatible with Branching Objective Indeterminism*, but they do not specify **h** as their substantially actual history. Definition 4.8.1 of Branching Objective Indeterminism*, indeed, does not isolate a unique metaphysical frame, but a class of frames. In this sense, nothing in the structure of our world necessitates **h** to be the actual history. However, any metaphysical frame compatible with Branching Objective Indeterminism* specifies a unique, actual history. Hence, in this sense it is necessary that there is a unique history that is substantially actual. This, in turn, directly answers the second reading of Belnap's question.

4.11 Towards the Thin red line semantics.

As I tried to argue, Belnap's conservative strategy should adopt the BO semantics to be distinguished from supervaluationism. The BO semantics, in turn, can only be attractive for those who subscribe to a many-worlds, deterministic view. And this because, on a many-worlds perspective, any possible history obtains.

Moreover, conservationists, as well as Thomason and MacFarlane, hold that actuality can only be a relational matter. This conception of actuality, moreover, cannot tell apart branching indeterminists from branching determinists. Thus, to express some basic tenets of indeterminism, one needs to use a notion of substantial actuality.

Once that the *Choice* and the *Past actuality* principles are interpreted with substantial actuality, one can easily define a notion of branching indeterminism that a many-worlds theorist cannot accept. However, this notion entails the commitment to a unique actual history.

Thus, it is tempting to think that, when it comes to interpret a tensed language against a tree structure, a branching indeterminist should attribute to the unique, substantially actual history a crucial role. Next chapter explores several semantics which employ the substantially actual history.

Chapter 5

Thin Red Line

The thin red line semantics model the intuition that histories are not on an equal footing with one another. One of them, the actual history, or, as it is called, the thin red line, is privileged. This approach was first suggested by Øhrstrøm (1981). It was defended by Borghini & Torrengo (2013), Braüner et al. (2000), Øhrstrøm (2009, 2014), Malpass & Wawer (2012) and Wawer (2014) from the criticism raised by Belnap & Green (1994), Belnap et al. (2001), MacFarlane (2014) and Iacona (2014).

The contemporary debate inherits the thin red line intuition from the solution William of Ockham (1978) gave to the puzzle involving free will and divine foreknowledge.[1] If today God knows what I will do tomorrow, then it is inevitable that tomorrow I will act as God's knowledge tells me to act. Hence, either God has foreknowledge but there is no free will, or I am free to will but God doesn't have foreknowledge. Ockham replies that not any truth known by God is inevitable. For instance, if today God knows that I will $\mathcal{A}$-ing tomorrow, today it is true that tomorrow I will $\mathcal{A}$-ing. But is not inevitably so. Future tensed statements that are true at a moment m are those that are true in the future of m along the actual history that that passes through m. But they might be true at m even if they are not inevitably so – even if they are not true at *some* history passing through m.

As we have seen, however, the commitment to a unique, substantially actual history is entailed by Definition 4.8.1 of Branching Objective Indeterminism*. This commitment, in turn, suggests that the substantially actual history should play a relevant semantical role in the interpretation of a tensed language such as $\mathcal{L}_T$. Recall, for instance, that according to David Lewis,

> [t]he trouble with branching exactly is that it conflicts with our

[1] See also Øhrstrøm (2009) and Malpass & Wawer (2012) on this matter.

> ordinary presupposition that we have a single future. If two futures are equally mine, one with a sea fight tomorrow and one without, it is nonsense to wonder which way it will be [...] and yet I do wonder. (Lewis 1986a: 207-208)

If the presupposition that we have a single future is ordinary, the way we talk should reflect it. And if indeterminism entails the existence of a unique actual future, it is natural to expect that the unique actual future helps to make sense of predictions about future contingent events – viz., it may help to make sense of how and why people "wonder which way it will be". Very roughly, one may say that

> [o]n the relevant semantics for 'will', something 'will' happen (as a first approximation) if and only if 'the unique actual future' features the thing happening. (Todd 2015: 2)

As we shall see, however, to yield a semantic framework in which the actual history plays a crucial role is not an easy task. Furthermore, authors such as Belnap and Placek to claim that

> [o]ur central argument against the TRL—the history containing the"actual future"— is this: Positing a TRL does no work in understanding statements involving a reference to future happenings, whether commonsensical or in the language of physics, and indeed tends to interfere with that understanding. (Placek & Belnap 2012: 463)

This chapter is devoted to defend the opposite claim: positing a TRL does work in understanding statements involving a reference to future happenings. And positing a TRL does not interfere with commonsensical language, nor with the language of physics. Moreover, I shall argue that (i) there is a suitable TRL semantics which helps to answer the Aristotelian question (Q1) by saving both contingency and bivalence, (ii) this semantics answers (Q2) by rejecting the principle of the necessity of the past, *PNP*, and (iii) it does not suffer of any of the flaws affecting the other semantics analysed thus far.

First, let us distinguish two semantic roles that the actual history may play.

Recursive Thin Red Line: The thin red line may occur in the recursive clauses of a given semantics. For instance, the actual history may occur within the truth conditions of future tensed statements. Under this view, one might say that $\mathsf{F}(n)\mathcal{A}$ is true at m iff $\mathcal{A}$ is true at the moment which is n time units in the future of m along the actual history.

Post-Semantic thin red line: The thin red line may occur within the definition of truth at a moment. For instance, one can state that a statement of $\mathcal{L}_T$ is true at m iff it is BO-true at m on the thin red line.

Among the semantics which attribute to the thin red line the recursive role, there are two main variants. The former assumes that the thin red line is unique. We call this theory the *absolute thin red line* (AT). The second kind subscribes to the idea that any moment of the tree has its own thin red line. The two semantics satisfying the latter condition are the *relative thin red line* (RT) and the *dynamic thin red line* (DT).

There are two semantics in which the thin red line plays the post-semantic role: these are the *post-semantic thin red line*, (PT), and the *supervaluationist thin red line* (ST). Next section analyses the recursive approaches. Next chapter analyses the recursive approaches.

5.1 Recursive thin red line

Let us start with the so-called Absolute thin red line semantics, AT.[2]

Definition 5.1.1. (AT-models) *An AT-model is a tuple* $\mathcal{M}_{AT} = (\mathcal{T}, d_{\mathcal{T}}, I, \mathbf{h})$, *such that*

- *(a)* $\mathcal{T}$ *is a tree,*
- *(b)* $d_{\mathcal{T}}$ *is a* $\mathcal{T}$*-duration function,*
- *(c)* I *is an interpretation function from atoms of* $\mathcal{L}_T$ *to the the set of moment history pairs* m/h*, such that, if* $m/h \in I(p)$*, then, for any* h' *s.t.* $m \in h'$, $m/h' \in I(p)$.
- *(d)* $\mathbf{h}$ *is the actual history of* $\mathcal{T}$.

The AT-semantics interprets $\mathcal{L}_T$ as follows.

Definition 5.1.2 (*AT-semantics*)**.** *Given a AT-model* $\mathcal{M}_{AT}$ *for* $\mathcal{L}_T$,

(AT1) $m \vDash_{AT} p \Leftrightarrow m \in I(p)$

(AT2) $m \vDash_{AT} \neg\mathcal{A} \Leftrightarrow \textit{not } m \vDash_{AT} \mathcal{A} \Leftrightarrow m \nvDash_{AT} \mathcal{A}$

(AT3) $m \vDash_{AT} \mathcal{A} \vee \mathcal{B} \Leftrightarrow \textit{either } m \vDash_{AT} \mathcal{A} \textit{ or } m \vDash_{AT} \mathcal{B}$

(AT4) $m \vDash_{AT} \mathsf{F}(n)\mathcal{A} \Leftrightarrow \exists m'(m' \in \mathbf{h} \;\&\; m < m' \;\&\; d_{\mathcal{T}}(m,m') = n \;\&\; m' \vDash_{AT} \mathcal{A}))$

(AT5) $m \vDash_{AT} \mathsf{P}(n)\mathcal{A} \Leftrightarrow \exists m'(m' < m \;\&\; d_{\mathcal{T}}(m,m') = n \;\&\; m' \vDash_{AT} \mathcal{A})$

(AT6) $m \vDash_{AT} \Diamond\mathcal{A} \Leftrightarrow \exists m'(m < m' \;\&\; m' \vDash_{AT} \mathcal{A})$

[2] See Wawer (2014: 370).

(AT7) $m \vDash_{AT} \Box\mathcal{A} \Leftrightarrow \forall h'(m \in h' \Rightarrow \exists m'(m' \in h' \,\&\, m < m' \,\&\, m' \vDash_{AT} \mathcal{A}))$

Notice that AT-truth (AT-falsity) is relative to moments only, and AT is perfectly bivalent. Furthermore, AT refelects what Branching Objective Indeterminism* entails: an $\mathcal{M}_{AT}$ model, indeed, specifies a unique actual history. Accordingly, AT provides an adequate answer the Aristotelian question (Q1), for it conserves both contingency and bivalence. Furthermore, it is easy to see that $\mathsf{P}(n)\mathcal{A} \to \Box\mathsf{P}(n)\mathcal{A}$ fails to be AT-valid. Hence, AT allows to answer (Q2) by rejecting the principle of the necessity of the past.

It is interesting to notice that $\Box$ and $\Diamond$ are not dual, and thus they have to be defined separately. In the AT-semantics, indeed, $\neg\Diamond\neg\mathcal{A}$, uttered at m, entails that any possible moment m' s.t. $m < m'$ satisfies $\mathcal{A}$. $m \vDash_{AT} \neg\Box\neg\mathcal{A}$, however, entails that there is a h passing through m s.t. any moment m' satisfies $\mathcal{A}$, where $m' \in h$ and $m < m'$. But this seems to be incorrect: saying at m that it is inevitable that $\mathcal{A}$ is to say that $\neg\mathcal{A}$ is historically impossible at m. But if there are two h, h' passing through m s.t. (i) for any moment m' that is in h s.t. $m < m'$, m' satisfies $\mathcal{A}$, and (ii) there is a moment $m'' \in h'$ that satisfies $\neg\mathcal{A}$, it follows that $m \vDash_{AT} \Box\mathcal{A} \wedge \Diamond\neg\mathcal{A}$. This blatantly violates the intended meaning of the historical operators $\Box$ and $\Diamond$.

Furthermore, according to clause (AT4), $\mathsf{F}(n)\mathcal{A}$ is true at m iff $\mathcal{A}$ is true at the moment m' that is n time units in the future of m along the actual history $\mathbf{h}$; otherwise $\mathsf{F}(n)\mathcal{A}$ is false at m. This entails that:

> We have no trouble with predictions that will be or have been made, but we have no way of understanding predictions that might have been made.(Belnap et al. 2001: 162)

What Belnap et al. (2001) are questioning is the way in which AT evaluates predictions that might have been made, viz., predictions made at non-actual moments. Whenever m is not actual ($m \notin \mathbf{h}$), AT predicts that any $\mathcal{A}$, uttered at m, must be assessed as false. This, in turn, seems wrong, for we should have a semantics that enables us to evaluate those predictions that one might have made. Furthermore, schemas such as the excluded middle, $\mathcal{A} \vee \neg\mathcal{A}$, as well as the principle of non-contradiction, $\neg(\mathcal{A} \wedge \neg\mathcal{A})$ fails at non-actual moments, and thus they cannot be AT-validities.[3]

The *relative thin red line* semantics, RT, is a natural strategy to overcome some of the difficulties which characterise AT. RT relativises the actual history to moments: the same history may be actual for a given moment, but may fail to be so for another moment. Formally, this move consists in

[3]Obviously, here I am assuming that $\mathcal{A}$ is AT-valid iff it is AT-true at any m.

defining a function TRL which maps any moment m to its actual history, $TRL(m)$. Once that the TRL function is introduced, it is possible to specify the truth conditions for future tensed statements as follows (the clauses for the other formulas of $\mathcal{L}_T$ are the same as those of AT):

(RT) $m \vDash_{RT} \mathsf{F}(n)\mathcal{A} \Leftrightarrow \exists m'(m < m' \; \& \; m' \in TRL(m) \; \& \; d_{\mathcal{T}}(m,m') = n \; \& \; m' \vDash_{RT} \mathcal{A})$

RT avoids some of the problems related to AT. For instance, RT does not entail that predictions that might have been made are false, for any m has its own thin red line $TRL(m)$. In a sense, RT predicts that there are no 'predictions that might have been made'. Furthermore, schemas such as the excluded middle and the principle of non-contradiction turn out to be RT-valid. However, Belnap & Green (1994) argue that RT contradicts the assumption that there are several possible futures accessible from a given arbitrary moment m. From their perspective, an adequate semantics for future contingents should validate the following, intuitive schema:

(1) $\mathsf{F}(n)\mathsf{F}(n')\mathcal{A} \rightarrow \mathsf{F}(n+n')\mathcal{A}$

In order to guarantee the validity of (1), one needs to specify some condition on the TRL function. One natural requirement is:

(A1) $\forall m(m \in TRL(m))$

Condition (A1) states that any moment is actual from its own 'perspective'. It is possible to prove that (1) is an RT-validity if the TRL function satisfies, in addition to (A1), also the following condition:

(A2) $\forall m,m'(m < m' \Rightarrow TRL(m) = TRL(m'))$

But the conjunction of (A1) and (A2) forbids forward branching, yielding a linear time structure. Here there is the proof.

Fact 14. (A1) *and* (A2) *entails the existence of a unique, possible history.*
Proof. *Assume that $m < m'$ and that $m < m''$. By* (A2), $TRL(m) = TRL(m')$ *and* $TRL(m) = TRL(m'')$. *Therefore,* $TRL(m') = TRL(m'')$. *By* (A1) *it follows that $m' \in TRL(m')$ and $m'' \in TRL(m')$, which entails that either $m' < m''$, or $m'' \leq m'$. Thus, there cannot be two moments that are not $<$-related. This directly entails that there is a unique possible history.*

Intuitively, if any moment has its own thin red line, which nonetheless is inherited by earlier moments, every moment must belong to the same thin red line. Braüner et al. (2000) and Øhrstrøm (2009) have objected that an advocate of RT is not forced to adopt (A2). Indeed, she can assume a weaker condition:

(A2') $\forall m, m'(m < m' \ \& \ m' \in TRL(m) \Rightarrow TRL(m) = TRL(m'))$

The new condition does not entail that, for a given moment m, any later moment m' has to belong to the thin red line $TRL(m)$ of m. (A2') simply requires that, if m' is in $TRL(m)$, then m' has $TRL(m)$ as its own thin red line. Hence, the conjunction of (A1) and (A2') is consistent with forward branching. Moreover, (A1) and (A2') guarantee that (1) is RT-valid.

Be as it may, recall that the clauses that RT gives for statements such as $\Box\mathcal{A}$ and $\Diamond\mathcal{A}$ are identical to those of AT. Hence, the RT-semantics, as well as AT, does not account for the intended meaning of $\Box$ and $\Diamond$. Indeed, statements of the form $\Box\mathcal{A} \wedge \Diamond\neg\mathcal{A}$ are still RT-satisfiable.

Furthermore, (A1) and (A2') are such that the principle of retrogradation is not an RT-validity. Consider Figure 5.1 and suppose that $TRL(m_0) = h_2$ and $d_{\mathcal{T}}(m_0, m_1) = d_{\mathcal{T}}(m_0, m_2) = 1$. Then the conditional $p \rightarrow \mathsf{P}(1)\mathsf{F}(1)p$ is RT-false at m_1, since p is RT-true at m_1, but $\mathsf{P}(1)\mathsf{F}(1)p$ is RT-false at m_1.

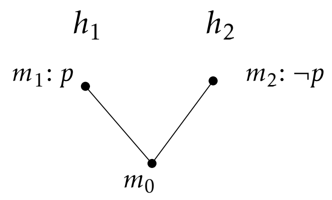


Figure 5.1

In order to overcome these difficulties, Braüner et al. (2000) and Øhrstrøm (2009, 2014) adopt a new semantics. Let us call it the dynamic thin red line, DT.

First of all, notice that the problem related to historical modalities may solved by interpreting $\Box$ and $\Diamond$ in the usual way, that is, as quantifications over the histories of evaluation. Moreover, the retrogradation principle, $\mathcal{A} \rightarrow \mathsf{P}(n)\mathsf{F}(n)\mathcal{A}$, fails to be a RT-validity since the TRL function may assign distinct actual histories to distinct but successive moments. Accordingly, one may restrict the number of histories that are relevant for the evaluation of a statement used at a given moment m. In particular, one may exclude those histories that pass through some moment m' that is later than m, but that are not the thin red line of m'. In other terms, the set $C(m)$ of those histories that can occur within a point of evaluation of a sentence used at m is:

(2) $C(m) = \{h : m \in h \;\&\; \forall m'(m' > m \;\&\; m' \in h \to TRL(m') = h)\}$

Once that (2) is introduced, let us define DT-models as follows.

Definition 5.1.3. (DT-models) *A DT-model is a tuple $\mathcal{M}_{DT} = (\mathcal{T}, d_{\mathcal{T}}, I, TRL)$, such that*

(a) $\mathcal{T}$ is a tree,
(b) $d_{\mathcal{T}}$ is a $\mathcal{T}$-duration function,
(c) I is an interpretation function from atoms of $\mathcal{L}_T$ to the the set of moment history pairs m/h, such that, if $m/h \in I(p)$, then, for any h' s.t. $m \in h'$, $m/h' \in I(p)$.
(d) TRL is a function from moments of $\mathcal{T}$ to histories of $\mathcal{T}$, and TRL satisfies (A1), (A2') .

DT interprets wffs of $\mathcal{L}_T$ as follows.

Definition 5.1.4 (*DT-semantics*). *Given a DT-model $\mathcal{M}_{DT}$ for $\mathcal{L}_T$, let h_c be a history in $C(m)$.*

(DT1) $m/h_c \vDash_{DT} p \Leftrightarrow m/h_c \in I(p)$

(DT2) $m/h_c \vDash_{DT} \neg\mathcal{A} \Leftrightarrow$ *not* $m/h_c \vDash_{DT} \mathcal{A} \Leftrightarrow m/h_c \nvDash_{DT} \mathcal{A}$

(DT3) $m/h_c \vDash_{DT} \mathcal{A} \vee \mathcal{B} \Leftrightarrow$ *either* $m/h_c \vDash_{DT} \mathcal{A}$ *or* $m/h_c \vDash_{DT} \mathcal{B}$

(DT4) $m/h_c \vDash_{DT} \mathsf{F}(n)\mathcal{A} \Leftrightarrow \exists m'(m' \in h_c \;\&\; m < m' \;\&\; d_{\mathcal{T}}(m,m') = n \;\&\; m'/h_c \vDash_{DT} \mathcal{A})$

(DT5) $m/h_c \vDash_{DT} \mathsf{P}(n)\mathcal{A} \Leftrightarrow \exists m'(m' \in h_c \;\&\; m' < m \;\&\; d_{\mathcal{T}}(m,m'') = n \;\&\; m'/h_c \vDash_{DT} \mathcal{A})$

(DT6) $m/h_c \vDash_{DT} \Box\mathcal{A} \Leftrightarrow \forall h'_c(m \in h'_c \Rightarrow m/h'_c \vDash_{DT} \mathcal{A})$

It is worth to noticing that the conjunction of (A1) and (A2') entails that the thin red line of m is one of the histories at which it is legitimate to evaluate a statement used at m. In general, DT validates both (1) and the retrogradation principle: $\mathcal{A} \to \mathsf{P}(n)\mathsf{F}(n)\mathcal{A}$. Moreover, intuitive schemas such as the excluded middle, as well as the principle of non-contradiction, turn out to be DT-valid. DT restores the intended meaning of historical modalities, in the sense that $\Box\mathcal{A} \wedge \Diamond\neg\mathcal{A}$ is unsatisfiable. These features save DT from the criticism raised against RT.

As far as (Q2) is concerned, DT, as well as BO, refutes the principle of the necessity of the past: $\mathsf{P}(n)\mathcal{A} \to \Box\mathsf{P}(n)\mathcal{A}$. Accordingly, the advocates of

DT reject the Master argument. Notice, however, that the recursive clauses provided by DT are very similar to that of BO. The only difference is that DT requires that the history occurring in a point of evaluation m/h_c must belong to $C(m)$. This, in turn, makes DT an inadequate semantic framework to answer the Aristotelian question (Q1), for DT – even if perfectly bivalent – defines truth (falsity) at moment-history pairs.

And there are other reasons to reject DT. First, notice that h_c in m/h_c can fail to be $TRL(m)$. This trait does not capture a basic tenet of the thin red line approaches, according to which a prediction, uttered at a given moment, is true since its 'real future' verifies what it is predicted. Even worst, DT doesn't capture one of the most basic principle purported by those indeterminists who – rightly in our mind – are committed to a unique, substantially actual history. The function TRL, indeed, attributes to any moment its actual history. But this is inadequate, at least from the indeterministic perspective defined in the previous chapter. If Branching Objective Indeterminism* is true, there are moments that are merely possible. And these moments, in turn, cannot be part of the unique substantially actual history (recall, indeed, that a history is substantially actual iff it contains substantially actual moment only). Thus, it is wrong to attribute to those moments a thin red line.

5.2 Post-semantic thin red line

If Branching Objective Indeterminism* entails the existence of a unique, substantially actual history, it is plausible to think that this history is significant for the truth of *arbitrary* statements. In particular, any statement $\mathcal{A}$ – whatever its logical form may be – is true at its moment of use just in case it is true at that moment on the unique actual history. To my mind, this is the most straightforward, natural way to attribute a semantical role to the actual history.

Accordingly, one may define a post-semantical, thin red line model as follows:

Definition 5.2.1. (PT models) *A PT-model is a tuple* $\mathcal{M}_{PT} = (\mathcal{T}, d_{\mathcal{T}}, \mathbf{h})$, *where* $\mathcal{T}$ *is a tree,* $d_{\mathcal{T}}$ *is a* $\mathcal{T}$*-duration function, and* $\mathbf{h}$ *the unique actual history.*

Definition 5.2.2. (PT semantics) *Given a PT model* $\mathcal{M}_{PT}$ *and an arbitrary statement* $\mathcal{A}$ *of* $\mathcal{L}_T$,

$m \vDash_{PT} \mathcal{A} \Leftrightarrow m/\mathbf{h} \vDash_{BO} \mathcal{A}$; $m \nvDash_{PT} \mathcal{A}$ *otherwise.*

The PT semantics has many virtues. It is perfectly bivalent and rejects the Aristotelian premises (Pa)-(Pb). The principle of the necessity of the past, $\mathsf{P}(n)\mathcal{A} \to \Box\mathsf{P}(n)\mathcal{A}$, cannot be PT-valid, and hence any advocate of PT rejects the Master argument. Moreover, PT-models are based on frames which specify a unique actual history, and this seems to be what branching indeterminism, properly understood, requires. This brief overview seems to save PT from many criticisms.

Nonetheless, Belnap et al. (2001: 162) and Iacona (2014: 2638) argue that PT has two serious logical flaws.

> **Objection 1**: PT falsifies any statement used at a moment which is not in **h**.

As we have seen before, also AT suffered of this feature. And **Objection 1** entails several unwelcome results. For instance, the excluded middle, the future excluded middle and the principle of retrogradation are all false at any non-actual moment. Hence, it seems that they cannot be PT-valid.

Malpass & Wawer (2012) take up the challenge raised by the objection, while maintaining their commitment to the unique actual history.[4] They observe that – by definition – a statement is inevitable (impossible) at m just in case it is BO-true (BO-false) at any history which passes through m. Hence, inevitable (impossible) statements have a determinate truth value, no matter whether their moment of use is either actual or merely possible.

On the other hand, future contingents have determinate truth values at their moment of use – so the argument goes – just in case there is an actual history passing through that moment, and against which an evaluation can be made. The existence of the actual history guarantees that any future contingent that is uttered at an actual moment has a determinate truth value. However, the uniqueness of the actual history entails that there is nothing as the actual history of a non-actual moment. Hence, future contingents, used at non-actual moments, must lack a determinate truth value.

In order to overcome the objection, Belnap et al. (2001) and Iacona (2014) raised against PT, one has to define a semantics which leaves contingent predictions, made at non-actual moments, devoid of any determinate truth value.

These considerations lead Malpass & Wawer (2012) to adopt the following disjunctive clauses, which define their *supervaluationist thin red line semantics* (ST). Given an arbitrary statement $\mathcal{A}$ of $\mathcal{L}_T$, and an $\mathcal{M}_{PT}$ model,

[4]See also Malpass (2013, 2016).

(ST1) $\mathcal{A}$ is true at m iff (for any history h which passes through m, $\mathcal{A}$ is BO-true at m/h) or ($\mathcal{A}$ is BO-true at $m/\mathbf{h}$)

(ST2) $\mathcal{A}$ is false at m iff (for any history h which passes through m, $\mathcal{A}$ is BO-false at m/h) or ($\mathcal{A}$ is BO-false at $m/\mathbf{h}$)

It is easy to see that (ST1)-(ST2) are apt to reject the two premises of the Aristotelian argument, (Pa) and (Pb). Moreover, the new semantics conserves any BO-validity and defines truth (falsity) only relative to moments.

It is worth noting that (ST1)-(ST2) assign determinate truth values only to two kinds of statements: inevitable (impossible) sentences and future contingents uttered at actual moments. For instance, suppose that $\mathcal{A}$ is a future contingent uttered at an actual moment m. By (ST1)-(ST2), it must be either true or false at m. Its determinate truth value depends on what happens both on the actual future and on the possible futures accessible from m. However, if $\mathcal{A}$ is a future contingent used at a non-actual moment, it is neither true nor false. This example shows that ST does not satisfy bivalence: any future contingent, used at a merely possible moment, comes out neither true nor false.

ST has a hybrid nature. It combines the intuition lying behind the post-semantics approach with the supervaluationist technique. As we shall see, supervaluationism assumes that a statement has a determinate truth value at its moment of use iff either it is settled true or settled false at that moment. Contingent sentences lack determinate truth values. In the ST semantics, the supervaluationist strategy is only applied to future contingents uttered at non-actual moment. In turn, this formal feature reflects the conclusion Malpass & Wawer (2012) inferred from the argument we have just seen above.

I think that one can reasonably reject the assumption that future contingents have truth values just in case they are used at actual moments. According to Iacona (2014: 2640), for instance, when one evaluates a prediction at a non-actual moment m, the fact that none of the possible futures of m is actual is irrelevant. What matter is which possibility would be actual if m were actual. Just as the truth value of a prediction made at an actual moment depends on the actual future, the truth value of a counterfactual prediction depends on what would happen if certain conditions were to obtain. Accordingly, there is theoretical room for the idea that future contingents satisfy bivalence, even when their moment of used is supposed to be non-actual. This kind of criticism against (ST1)–(ST2) is on the right track, and it is related to the arguments given in the following sections.

5.2.1 Actual history and contexts of use

The dialectical scenario which give raise to ST is ill-conceived. Recall that ST was designed to overcome PT, which entails that any sentence used at a non-actual moment is false. According to Wawer (2014), this very objection is false. Moreover, the notion on which the objection is based – namely, that of a statement used at a non-actual moment – presupposes a metaphysical picture that a thin red line theorist has to reject.

Wawer (2014) underlines that a moment occurring in an instance of

(PT) $m \vDash_{PT} \mathcal{A} \Leftrightarrow m/\mathbf{h} \vDash_{BO} \mathcal{A}$; $m \nvDash_{PT} \mathcal{A}$ otherwise,

is standardly assumed to be the moment of a context of use. In turn, the context of use is ordinarily thought as a sequence of semantic relevant parameters, describing the scenario in which a speech act may take place. Any parameter specified by a context of use represents a 'piece of reality' on which the semantic value of an expression, uttered at that context, may depend.

Several authors highlight that contexts of use have 'fact-of-the-matter' parameters. For instance, Kaplan (1989a: 597) claims that we have a-priori knowledge about what *facts* ought to obtain at a world for that world to contain a context.[5] In turn, these facts – i.e., language-independent features of reality – are the ingredients of contexts of use. Kaplan (1989a: 593) himself argues that assignments to variables have a language-dependent nature. Hence, they cannot occur as parameters of contexts. On the contrary, denotations of indexicals such as "I" and "here" – namely, the speaker and the utterance place – are not language-dependent. Thus, speakers and places are legitimate context parameters.[6]

Belnap et al. (2001: 145) share the same assumption:

> The context of use provides what is in fact determined by an idealized speech act using the given sentence as vehicle. The context provides only what Kaplan calls "fact-of-the-matter parameters" (Kaplan 1989a: 593). They are what they are, and the logician is not entitled to make them up.

[5] Part of this a-priori knowledge (see Kaplan 1989b: 512, footnote 37) should entail that a context of use must specify the agent of the context, existing during the time of the context at the world of the context. Furthermore, the agent should be located at the place of the context. However, this conception of a context of use has been recently criticised by Predelli (2005: ch. 2).

[6] Certainly, the fact that "I" has the character that it has is based on the way we use it. Hence, the character of "I" is, in a sense, conventional. But given the character that "I" actually has, its denotation at a context (the speaker) is language-independent.

Moreover, Belnap et al. (2001: 148) insist that context parameters cannot be stipulated:

> You cannot, however, make up features of the context as you go along. You can reasonably decide to treat "now" or "here" as context-dependent, but that is only because there is *in fact* a time of use and a place of use.

According to Wawer (2014: 380), the fact-of-the-matter character of contexts leads to the following conclusion:

> After all, a use of a sentence (an idealisation of a speech act) is a concrete event in our world, and a context consists of concrete circumstances that accompany this event. Therefore, I assume that postulating and using certain contexts in the semantic theory is tantamount to the metaphysical commitment to their *concrete* existence.[7]

Here 'concrete' means 'spatiotemporal': something is concrete if it has spatiotemporal properties. According to Wawer, any parameter of a context describes something that exists in space and in time. Wawer agrees with Belnap and Kaplan in that context parameters must have a fact-of-the-matter character. Moreover, this requirement leads him to stick to the following principle:

Concreteness: Using certain parameters as context parameters commits to their concrete existence.

If *Concreteness* is on the right track, there is room to reject that any possible moment is a moment of (a context of) use. If there is only one actual history, plausibly, any moment in that history is a concrete piece of reality. After all, an actual moment is nothing but an instantaneous slice of the actual world, the world we inhabit. By *Concreteness*, using actual moments as moments of use entails their concreteness. In turn, their concreteness is perfectly compatible with the commitment to a unique actual history.

But what about merely possible moments? Kripke (1980) famously argued that speaking about possibilities does not force to conceive each of them as concrete. Quite the contrary, he holds that just one possibility – the actual one – is concrete. The other possibilities are abstract entities, in the sense that no spatiotemporal entity, or state, or fact, corresponds to them.[8] Analogously, one may reject that merely possible moments are

[7] Emphasis added.

[8] For the opposite metaphysical view, see Lewis (1986a).

concrete, and, by *Concreteness*, merely possible moments are not moments of use. Wawer's argument may be summarised as follows:

Concreteness: Using certain parameters as context parameters commits to their concrete existence.

Actuality: Only moments lying on the actual history **h** are concrete.

$\therefore$ Moments of use must belong to the actual history **h**.

The conclusion allows to restrict the values the moment parameter can assume within any instance of (PT). Moreover, this latter restriction cannot be charged of being ad hoc, since it is motivated by the particular metaphysical view Wawer is assuming. Under this metaphysical reading, the objection that (PT) falsifies any statement used at merely possible moments is false, for there is no such thing as a merely possible moment of use. And if PT-validity is defined as PT-truth at any concrete moment – viz. truth at any moment in **h** – it is easy to check that $\mathcal{A}$ is PT-valid just in case it is a BO-validity.

5.2.2 On considering a history as actual

Wawer (2014)'s account of contexts overstates things. *Concreteness* is unnecessarily loaded from a metaphysical viewpoint, and it yields unwelcome results when applied to more general semantic approaches. To see why, let us focus on the so-called two dimensional semantics.

After the work of Kripke (1980), it is widely accepted that proper names are rigid designators – viz., a proper name denotes the same individual in all possible worlds. Thus, identity statements of coreferring proper names are necessary truths. For instance, "Hesperus is Phosphorus" is necessarily true, since both "Hesperus" and "Phosphorus" are rigid designators referring to Venus. Given that proper names are rigid designators, it is natural to interpret them with a *kripkean intension*: a constant function from possible worlds to individuals.

On the other hand, it is highly intuitive to hold that two rigid designators might have failed to corefer: "Hesperus" and "Phosphorus" might have been used differently than the way they are actually used. Hence, one may ask what the truth value of "Hesperus is Phosphorus" would be if "Hesperus" were used as it is actually used, but "Phosphorus" were used to name Jupiter. If the two expressions were used in this latter, 'deviant' way, "Hesperus is Phosphorus" would be necessarily false. In other terms, the particular kripkean intension associated with a name may be contingent, depending on how whatever fixes the references of names may turn out to be.

This little piece of reasoning shows that, as Stalnaker (2004: 295) puts it, the semantic values of expressions will depend on extra-linguistic features (let us call them facts) in two different ways. On the one hand, facts determine *what is said*. On the other hand, facts determine whether *what is said* is true. In order to capture these two different roles that facts may play in semantics, several philosophers have developed the so-called two dimensional semantics.[9] In the two dimensional approach, statements are evaluated at pairs of possible worlds, say (x,y). The first member, x, is the world at which a sentence may be used. Its role is to represent the facts determining *what is said* by a given sentence used at world x. The second member of the pair, y, is the circumstance against which *what is said* in x has to be evaluated. The role of y is to represent the facts that enable to state whether *what is said* is true.

Suppose that w_1 is the actual world, i.e., the world in which "Hesperus" and "Phosphorus" are used in such a way that they both refer to Venus. Assume also that w_2 is a (merely) possible world where people use "Hesperus" to refer to Venus, but use "Phosphorus" to refer to Jupiter.

If "Hesperus is Phosphorus" is used at w_1, it comes out true relative to both w_1 and w_2. In other terms, "Hesperus is Phosphorus" is true both at (w_1,w_1) and at (w_1,w_2). Indeed, if the two rigid designators "Hesperus" and "Phosphorus", as used in w_1, refer to Venus, "Hesperus is Phosphorus" states an identity which holds both at w_1 and at w_2. However, "Hesperus is Phosphorus", used at w_2, is false at (w_2,w_1) and at (w_2,w_2). In this latter case, "Hesperus" and "Phosphorus", as used at w_2, refer to Venus and Jupiter respectively, and Venus and Jupiter are distinct individuals both in w_1 and in w_2. These evaluations may be represented by the following matrix.

Matrix for "Hesperus is Phosphorus".

	w_1	w_2
w_1	True	True
w_2	False	False

Intuitively, worlds on the left column determine *what is said* by "Hesperus is Phosphorus". Worlds on the top row determine whether *what is*

[9] See Baldwin & Stalnaker 2001, Chalmers et al. 2006 and Stalnaker 1999, 2004: 78-95. In general, the two dimensional semantics is similar to the treatment Kaplan 1989b applies to indexicals. However, its applications go far beyond Kaplan's logic of indexicals. For an overview, see Schroeter 2012.

said by "Hesperus is Phosphorus" is true.[10] The matrix is the *propositional concept* expressed by the statement. The *diagonal* proposition is the diagonal row in the matrix (the one that goes from the top left to the bottom right), and identifies the proposition expressed by "Hesperus is Phosphorus" when the world which determines *what is said* is the same of the world against which *what is said* has to be evaluated.[11]

To wonder whether "Hesperus is Phosphorus" were true (or false) if uttered at w_2 amounts to consider w_2 both as the world of the context of utterance and as the world-parameter occurring in the circumstance of evaluation. In doing that, one is *considering a merely possible world*, w_2, *as actual*. What one does is wondering what the identity statement would say, and what its truth value would be, if w_2 turned out to be actual.

This latter feature helps to explain why Wawer (2014)'s interpretation of contexts is unnecessarily cumbersome from a metaphysical viewpoint. If *Conreteness* were correct – if it were true that "using certain contexts in the semantic theory is tantamount to the metaphysical commitment to their *concrete* existence" – any two dimensional semanticist would be committed to a pletora of concrete possible worlds. In turn, this conclusion is absurd. One thing is the metaphysical claim that there is just one world (history) which can be referred to as the 'true' or actual one. Quite another is to use certain sets of parameters as contexts of use. A given set of parameters, considered as a context of use, describes a situation as if it were actual. But from considering a scenario as actual, no commitment to its concreteness should plausibly follow.

But then a problem arises. Recall that *Concreteness* justifies Wawer's conclusion, according to which a moment of use must lie in **h**. In turn, this restriction on moments of use is needed to reject Objection 1. It seems that if one refutes *Concreteness*, one is forced to reject PT on the basis that it falsifies any statement used at merely possible moments. However, there is something wrong with this very objection.

In what follows, an argument for the restriction on moments of use inferred by Wawer is provided. However, that argument does not rely on any (implausible or unnecessary) metaphysical assumption. The idea that a moment of use must belong to the actual history, **h**, is nothing but a natural principle, deeply rooted in the way we understand and model contexts. The argument runs as follows.

[10]The first logicians who used this jargon were Davies & Humberstone (1980).

[11]The propositional concept expressed by a sentence $\mathcal{A}$ may be identified with the function $f_{\mathcal{A}} : W \times W \longmapsto \{1,0\}$, mapping pairs of possible worlds to truth values. The diagonal proposition d of $\mathcal{A}$, expressed at w, is the function $d_{\mathcal{A}}(w) = f_{\mathcal{A}}(w,w)$.

World of use : Given a PT-model $\mathcal{M}_{PT} = (\mathcal{T}, d_{\mathcal{T}}, \mathbf{h})$ and clause (PT), $\mathbf{h}$ is the world of use.

Admissible moments: If m is an admissible moment of use, it is located the world of use, $\mathbf{h}$ (that is, $m \in \mathbf{h}$).

$\therefore$ If m is not located at $\mathbf{h}$ (viz., $m \notin \mathbf{h}$), m is not an (admissible) moment of use.

The *World of use* premise can be motivated by looking once again at the two-dimensional semantics. To say that $\mathcal{A}$ has truth value n when used at world x amounts to consider x as actual. But when x is considered actual, it plays a special semantical role: its value initialises that of the world-parameter occurring within the circumstance at which $\mathcal{A}$ has to be evaluated. When one wonders whether "Hesperus is Phosphorus" is true (false) used at w_2, one considers w_2 *both* as the world of the context of utterance *and* as the world parameter occurring in the circumstance of evaluation. In the two-dimensional jargon, this is equivalent to check whether "Hesperus is Phosphorus" is true (false) at (w_2, w_2). This is quite natural: if one wants to know the truth value of "Hesperus is Phosphorus" used at w_2, one has to check whether *what is said* by the statement at w_2 fits with the facts obtaining at that world. In general, when it comes to define the notion of utterance-truth (that is, the notion of the truth of a statement used at an utterance scenario), *the world of use – the world considered actual – initialises the circumstances of evaluation*.

The most natural model for (PT) is a tuple $\mathcal{M}_{PT} = (\mathcal{T}, d_{\mathcal{T}}, \mathbf{h})$, where $\mathbf{h}$ is the actual history of the tree $\mathcal{T}$.[12] By (PT), the designated history in $\mathcal{M}_{PT} = (\mathcal{T}, d_{\mathcal{T}}, \mathbf{h})$ initialises the circumstances at which a sentence must be evaluated. In other terms, given a model $\mathcal{M}_{PT} = (\mathcal{T}, d_{\mathcal{T}}, \mathbf{h})$ and (PT), $\mathbf{h}$ is the initial value of the world (history) parameter occurring in the circumstance $m/\mathbf{h}$. But since the history which initialises the circumstances of evaluation *is* the history of use, the *World of use* principle follows: given a model as $\mathcal{M}_{PT} = (\mathcal{T}, d_{\mathcal{T}}, \mathbf{h})$ and clause (PT), $\mathbf{h}$ is the history of use. When one applies (PT) to evaluate a formula uttered at m, the model $\mathcal{M}_{PT} = (\mathcal{T}, d_{\mathcal{T}}, \mathbf{h})$ yields the world (history) of utterance by 'default'.

The *Admissible moments* premise may be justified as follows. Not any set of parameters is apt to describe a scenario in which an utterance may take place. For, as Lewis (1998: 29) suggests, context parameters do not vary independently. The 'interdependence' that contextual parameters bear with one another is a very deep intuition, and it relates to what can be con-

[12] Notice that the introduction of a designated history (world) in a model is a standard move in modal logic, tracing back to the work of Kripke (1963: 3).

sidered as an actual scenario of utterance. Kaplan (1989b: 512) assumes that a sequence (w,a,l,m) describes a possible utterance scenario only if agent a exists at the possible world w during moment m at the spatial location l. The interdependence introduced by Kaplan among contexts parameters is clear: suppose that a is born at moment m at a spatial location l in world w. As MacFarlane (2014: 56-57) points out, to take an utterance of a as made in world w at a time m' preceding the one of a's birth is preposterous. Agent a does not exist before m in w, so (w,a,l,m') cannot be taken as actual – hence, it cannot be considered as a context of use.

Predelli (2003, 2005) criticises the principle that agents must be located at the time and at the place in which their utterances are performed. Indeed, he notes that utterances of "I am not here now" can be true *because* they take place at spatiotemporal locations others of those of their agents. However, he continues to stick to the Kaplanian idea that a sentence used at a context is true just in case it is true at the time in which is uttered, relative to the world of the context. We think that this 'conservatism' is supported by a very deep and intuitive principle. What is basically conserved is that the moment of use signals *when* an utterance is supposed to take place *at* the world of use. If moments of use have this role, once that $\mathbf{h}$ is considered the world of use, it is unreasonable to take counterfactual moments as moments of use. *If one assumes* $\mathbf{h}$ *to be the world of use*, it is natural to hold that *what can be uttered at the world of use must be uttered at one of its moments*. This latter principle naturally correlates the moment of use with the world of use, and justifies the *Admissible moments* premise.

Hence, the value of the moment (of use) parameter must be located at the history (world) of use $\mathbf{h}$. In turn, this entails that any moment that is not located at the actual history $\mathbf{h}$ cannot be taken as a moment of use. Thus, Objection 1 is false.

The reasoning just provided has several virtues when compared with Wawer's argument. First, unlike Wawer's argument, the one just proposed does not assume *Concreteness*, avoiding its shortcomings. Furthermore, denying the *Concreteness* principle does not amount to give up the fact-of-the-matter character usually associated with contexts. Given an account of what does it mean to have a fact-of-the-matter character, one may still differentiate sets of parameters that can play the role of contexts from those that cannot. For instance, one may still require, as Kaplan (1989a: 596) does, that contexts must specify non-empty worlds. Alternatively, one may still agree with Wawer (2014: 380) and expect contexts to represent certain portions of space at a time. We remain as neutral as possible about what does it mean for a set of parameters to have a fact-of-the-matter character. We just impose that any pair (m,h) has a fact-of-the-matter character, and may represent an utterance scenario, only if $m \in h$. This minimal sense of fact-of-the-matter character is captured by the *admissible moments* premise.

If the values the moment of use are restricted in the way suggested, it is easy to check that:

- ✓ (PT) refutes the Aristotelian argument and provides a bivalent notion of truth at a moment.
- ✓ (PT) refutes the Master argument by rejecting the principle of the necessity of the past, PNP.
- ✓ (PT) validates the future excluded middle.
- ✓ (PT) does not introduce a semantic distinction between $\mathsf{F}(n)\neg\mathcal{A}$ and $\neg\mathsf{F}(n)\mathcal{A}$.
- ✓ (PT) does account for retrospective truth judgments.
- ✓ (PT) truth is disquotational.
- ✓ (PT) does not equates "plain" predictions with what inevitably will be the case.
- ✓ (PT) yields a notion of intension which is perfectly compatible with an eternalist understanding of the propositions.
- ✓ (PT) conserves any classical inferential rule.

Accordingly, (PT) seems to be advantaged over any other semantics analysed thus far. On the one hand, (PT) requires the specification of a unique actual history, in accordance with Branching Objective Indeterminism*. On the other hand, (PT) is compatible with what we do when we talk about time and possibility. As we shall see in the next section, however, many philosophers have further reasons for complaining (PT).

5.3 Counterfactuals and the thin red line

Even when the values the moment of use are restricted in the way suggested, several philosophers complains (PT) on the ground of another, alleged logical defect. This time, (PT) is charged of yielding wrong results for counterfactual reasoning. For instance, Belnap et al. (2001: 164) claim that

> We have no way of getting a grip on "Had things gone otherwise, Jack would have asserted the following: 'It will (eventually) rain'". Given the context of Jack's assertion, the TRL is no longer able to guide us in understanding his reference to his future.

The quotation suggests that the actual history fails to play an explanatory role when it comes to interpret counterfactual predictions.

Similarly, Iacona (2014: 2638) holds that (PT) does not yield correct evaluations for intuitively true counterfactuals. Suppose I do not toss the coin in my hand. However, it seems true to say that if I flipped the coin, It would have landed either heads or tails. According to Iacona, the requirement that moments of use must be actual prohibits to 'look' at what happens at (merely possible) histories that could be actual. But since this kind of shift is needed to evaluate counterfactuals, one cannot use (PT) to assign a truth-value to counterfactuals which are intuitively true.

These worries may be summarised as follows:

> **Objection 2**: (PT) does not account for counterfactual statements such as "Had things gone otherwise, it would be the case that $\mathcal{A}$".

Objection 2 has two readings, depending on what counterfactuals are taken to be. Counterfactuals may be thought as subjunctive conditionals of the object language. If so, Objection 2 says that (PT) cannot yield correct truth conditions for sentences of the form $\mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{B}$ (where $\mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{B}$ obviously reads "If $\mathcal{A}$ were the case, $\mathcal{B}$ would be the case"). On the other hand, counterfactuals may be taken as sentences formulated at the metalinguistic level. They express metalinguistic intuitions about what object language formulas would be true (false) if the actual history were different from the one that is marked as such. Under this latter reading, Objection 2 says that the truth value distributions provided by (PT) (for object language formulas) conflict which certain thoughts that we can express in the metalanguage by means of constructions in subjunctive form. To sum up, Objection 2 has the two following readings:

> **Objection 2.1**: (PT) does not provide correct truth conditions for (object language) sentences of the form $\mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{B}$.

> **Objection 2.2**: The truth value distributions provided by (PT) (for object language formulas) conflict with metalinguistic subjunctive conditionals.

Another objection against (PT) which involves counterfactual scenarios is the following. Suppose that there are just three moments, m_0, m_1 and m_2, such that $m_0 < m_1$, $m_0 < m_2$, $m_0, m_1 \in \mathbf{h}$ but $m_2 \notin \mathbf{h}$. If p is true at m_1 but false at m_2, by (PT) it follows that "It will be the case that p" is true at m_0. According to MacFarlane (2014), an agent at m_2 assesses the prediction uttered at m_0 as accurate (since it is true at m_0), even if $\neg p$ is true at the moment of the assessment, m_2. This, he argues, is a bad result. Hence,

Objection 3.(PT) provides counterintuitive results for the counterfactual retrospective assessments of future contingents.

In the next sections it will be argued that these three objections – based on counterfactual scenarios – are innocuous for (PT).

5.3.1 Objection 2.1

If Objection 2.1 is true, (PT) yields incorrect truth conditions for the object-language formulas such as $\mathcal{A} \boxright \mathcal{B}$. But what determines whether a biconditional expresses the correct truth conditions for counterfactuals? A tentative answer consists in selecting three kinds of inferential patterns involving $\boxright$: (i) those that are (taken to be) sound, (ii) those that are (taken to be) unsound, and (iii) those whose soundness is controversial. It is reasonable to assume that a semantics for $\boxright$ is incorrect if either it makes unsound one pattern among those of kind (i), or it makes sound one pattern among those of kind (ii). Thus, according to Objection 2.1, (PT) is incorrect, for it makes unsound some pattern of kind (i), or it makes sound some pattern of kind (ii).

Now, let us look at the patterns of the three different kinds ($\vdash$ is logical entailment).

Patterns that are (taken to be) uncontroversially sound.

A.	$\mathcal{A} \boxright \mathcal{B}, \mathcal{A} \vdash \mathcal{B}$	*Modus Ponens*
B.	$\mathcal{A} \boxright \mathcal{B}, \neg\mathcal{B} \vdash \neg\mathcal{A}$	*Modus Tollens*

Patterns that are (taken to be) uncontroversially unsound.

C.	$\mathcal{A} \boxright \mathcal{B}, \mathcal{B} \boxright \mathcal{C} \vdash \mathcal{A} \boxright \mathcal{C}$	*Transitivity*
D.	$\mathcal{A} \boxright \mathcal{B} \vdash (\mathcal{A} \wedge \mathcal{C}) \boxright \mathcal{B}$	*Antecedent Strenghtening*
E.	$\neg\mathcal{B} \boxright \neg\mathcal{A} \vdash \mathcal{A} \boxright \mathcal{B}$	*Contraposition*

Patterns whose soundness is controversial.

F.	$\vdash \neg(\mathcal{A} \boxright \mathcal{B}) \leftrightarrow (\mathcal{A} \boxright \neg\mathcal{B})$	*Equivalence*
G.	$\vdash (\mathcal{A} \boxright \mathcal{B}) \vee (\mathcal{A} \boxright \neg\mathcal{B})$	*Conditional Ex.Midd.*
H.	$\Box(\mathcal{A} \boxright \mathcal{B}) \vdash \mathcal{A} \boxright \Box(\mathcal{A} \rightarrow \mathcal{B})$	*Edelberg*
I.	$\mathcal{A} \boxright \Box\mathcal{A}, \Box(\mathcal{A} \boxright \mathcal{B}) \vdash \mathcal{A} \boxright \Box\mathcal{B}$	*Weak Edelberg*

Modus Ponens and *Modus Tollens* capture the conditional nature of $\boxright$. On the other hand, *Antecedent Strengthening* and *Transitivity* are related to each another, and both patterns seem to be inappropriate for counterfactual reasoning. First, *Transitivity* entails *Antecedent Strengthening*, as shown as follows.

1. $\mathcal{A} \boxright \mathcal{B}, \mathcal{B} \boxright \mathcal{C} \vdash \mathcal{A} \boxright \mathcal{C}$ *Transitivity*
2. $(\mathcal{A} \wedge \mathcal{B}) \boxright \mathcal{B}, \mathcal{B} \boxright \mathcal{C} \vdash (\mathcal{A} \wedge \mathcal{B}) \boxright \mathcal{C}$ by setting $\mathcal{A}$ as $(\mathcal{A} \wedge \mathcal{B})$
3. $\mathcal{B} \boxright \mathcal{C} \vdash (\mathcal{A} \wedge \mathcal{B}) \boxright \mathcal{C}$ for $(\mathcal{A} \wedge \mathcal{B}) \boxright \mathcal{B}$ is a logical truth

Step 3 is exactly the *Antecedent Strengthening* pattern, so whoever refutes the latter must reject *Transitivity* as well. And there are very good reasons to reject *Antecedent Strengthening*. As argued by Goodman (1947: 116), Stalnaker (1968: 106) and Bennett & Bennett (2003: 160), whenever

> If the match were struck, it would light,

is true, its strengthening

> If this match had been soaked in water overnight and it were struck, it would have light,

is false. *Contraposition* is another principle which fails in counterfactual reasoning.[13] It may be the case that

> If Bob apologised to Alice, she would not forgive him,

since Alice is very, very angry. But its contrapositive,

> If Alice did forgive Bob, then he would not apologised to her,

can be false as well, for Bob's apologies may be necessary for Alice to forgive him.

Moving to those patterns that are controversial, let us see some relationships obtaining between *Equivalence* and *Conditional Excluded Middle*. The left-to-right side of *Equivalence*, $\neg(\mathcal{A} \boxright \mathcal{B}) \rightarrow (\mathcal{A} \boxright \neg\mathcal{B})$, is just the *Conditional Excluded Middle*, $(\mathcal{A} \boxright \mathcal{B}) \vee (\mathcal{A} \boxright \neg\mathcal{B})$. Its right-to-left side,

L. $(\mathcal{A} \boxright \neg\mathcal{B}) \rightarrow \neg(\mathcal{A} \boxright \mathcal{B})$ *Weak Boethius Thesis*

fails if $\mathcal{A} \boxright \neg\mathcal{B}$ is vacuously true when $\mathcal{A}$ is impossible. But given that a counterfactual with impossible antecedent is usually taken to be true, *Equivalence*'s validity is in general restricted to counterfactuals with possible antecedents.

With this restriction in mind, Williamson (1988) noticed that, when we are focused on possibly true antecedents, *Equivalence* entails *Conditional Excluded Middle* once that two other patterns hold, namely

M. $\mathcal{B}_1, ..., \mathcal{B}_n \vdash \mathcal{C} \Rightarrow \mathcal{A} \boxright \mathcal{B}_1, ..., \mathcal{A} \boxright \mathcal{B}_n \vdash \mathcal{A} \boxright \mathcal{C}$.
N. $\vdash \mathcal{A} \boxright \mathcal{A}$

[13] The counterexample is readapted from Stalnaker (1968: 107).

It can be proved that

Fact 15. *M, N and Equivalence entail Conditional Excluded Middle (recall that we are supposing that the antecedents of counterfactuals are possibly true).*
Proof.

1. $\neg\Big[(\mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{B}) \vee (\mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{C})\Big]$ Assumption
2. $\neg(\mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{B}) \wedge \neg(\mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{C})$ By 1 and Class.Log.
3. $(\mathcal{A} \mathrel{\Box\!\!\rightarrow} \neg\mathcal{B}) \wedge (\mathcal{A} \mathrel{\Box\!\!\rightarrow} \neg\mathcal{C})$ By 2 and *Equivalence*
4. $\mathcal{A} \mathrel{\Box\!\!\rightarrow} \neg(\mathcal{B} \vee \mathcal{C})$ By 3, *M* and Class.Log.
5. $\neg\Big[(\mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{B}) \vee (\mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{C})\Big] \rightarrow \mathcal{A} \mathrel{\Box\!\!\rightarrow} \neg(\mathcal{B} \vee \mathcal{C})$ By 1–4 and I$\rightarrow$
6. $\neg\Big[(\mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{B}) \vee (\mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{C})\Big] \rightarrow \neg\Big[\mathcal{A} \mathrel{\Box\!\!\rightarrow} (\mathcal{B} \vee \mathcal{C})\Big]$ By 5 and *Equivalence*
7. $\mathcal{A} \mathrel{\Box\!\!\rightarrow} (\mathcal{B} \vee \mathcal{C}) \rightarrow (\mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{B}) \vee (\mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{C})$ By 6 and Class.Log.

Given that $\vdash \mathcal{A} \rightarrow \mathcal{B}$ holds only if $\mathcal{A} \vdash \mathcal{B}$, by statement 7 it follows that

8. $\mathcal{A} \mathrel{\Box\!\!\rightarrow} (\mathcal{B} \vee \mathcal{C}) \vdash (\mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{B}) \vee (\mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{C})$

In turn, 8 guarantees *Conditional Excluded Middle*.

9. $\mathcal{A} \vdash \mathcal{B} \vee \neg\mathcal{B}$ Class.Log.
10. $\mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{A} \vdash \mathcal{A} \mathrel{\Box\!\!\rightarrow} (\mathcal{B} \vee \neg\mathcal{B})$ By 9 and *M*
11. $\vdash \mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{A}$ By *N*
12. $\vdash \mathcal{A} \mathrel{\Box\!\!\rightarrow} (\mathcal{B} \vee \neg\mathcal{B})$ By 10–11
13. $\vdash (\mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{B}) \vee (\mathcal{A} \mathrel{\Box\!\!\rightarrow} \neg\mathcal{B})$ By 12 and 8

Now let us see why *Equivalence*, *Conditional Excluded Middle*, *Edelberg* and *Weak Edelberg* are controversial principles.

Equivalence

According to Stalnaker (1968: 107), *Equivalence* reflects the actual use of counterfactuals, for to deny a counterfactual (with a possibly true antecedent) is to use the same counterfactual, but with opposite consequent.

> This explains the fact, noted by Goodman and Chisholm in their early papers on counterfactuals, that the normal way to contradict a counterfactual is to contradict the consequent, keeping the same antecedent. To deny "If Kennedy were alive today, we wouldn't be in this Vietnam mess", we say, "If Kennedy were alive today, we would so be in this Vietnam mess". (Stalnaker 1968: 107)

However, if "If Kennedy were alive today, we would so be in this Vietnam mess" (i.e., $\mathcal{A} \boxright \mathcal{B}$) leads to deny "If Kennedy were alive today, we wouldn't be in this Vietnam mess" (i.e., $\neg(\mathcal{A} \boxright \neg\mathcal{B})$), then $(\mathcal{A} \boxright \mathcal{B}) \rightarrow \neg(\mathcal{A} \boxright \neg\mathcal{B})$ and, by contraposition, $(\mathcal{A} \boxright \neg\mathcal{B}) \rightarrow \neg(\mathcal{A} \boxright \mathcal{B})$ should be two validities. But these data only support *Weak Boethius Thesis*, and something stronger is needed to conclude that "to contradict a counterfactual is to contradict the consequent, keeping the same antecedent".

Furthermore, there are linguistic intuitions which pull towards the opposite direction of that of Stalnaker. Suppose Alice does not flip the coin in her hands, but says "If I flipped the coin, it would land heads" (i.e., $\mathcal{A} \boxright \mathcal{B}$). Bob can sensibly react to her by saying "Wrong Alice, it might land tails" (entailing $\neg(\mathcal{A} \boxright \mathcal{B})$). However, an equally reasonable reaction would be "Wrong Alice, it might land heads, but it also might land tails" (entailing $\neg(\mathcal{A} \boxright \mathcal{B}) \wedge \neg(\mathcal{A} \boxright \neg\mathcal{B})$). Notice that $\neg(\mathcal{A} \boxright \mathcal{B}) \wedge \neg(\mathcal{A} \boxright \neg\mathcal{B})$ amounts to the negation of the left-to-right side of *Equivalence*. Hence, Stalnaker's argument in favour of *Equivalence* can be resisted.

Equivalence has been defended by Von Fintel & Iatridou (2002), who claim that the pattern in question is crucial when it comes to interpret counterfactual statements embedded by quantifiers. They give the following argument. Intuitively, these two statements are equivalent:

(A) No student would have passed if they had goofed off.
(B) Every student would have failed to pass if they had goofed off.

In turn, (A) and (B) have to be formalized as follows:

(A') $\neg\exists x(x \text{ is a student } \wedge(x \text{ goofs off } \boxright x \text{ passess}))$
(B') $\forall x(x \text{ is a student} \rightarrow (x \text{ goofs off } \boxright \neg x \text{ passess}))$

But given the duality of $\forall$ and $\exists$ (i.e., $\neg\forall\neg = \exists$), it follows from (A') that

(A") $\forall x(x \text{ is a student} \rightarrow \neg(x \text{ goofs off } \boxright x \text{ passess}))$

Since (A") is equivalent to (A'), (A') to (A), (A) to (B), and (B) to (B'), by transitivity (A") must be equivalent to (B'). Von Fintel & Iatridou (2002) claims that their point generalises, to the extent that the two schemas

(A*) $\forall x(Fx \rightarrow \neg(Gx \boxright Hx))$
(B*) $\forall x(Fx \rightarrow (Gx \boxright \neg Hx))$

are equivalent. But then $\neg(Gx \boxright Hx)$ and $(Gx \boxright \neg Hx)$ must be coextensional, and a quantified version of *Equivalence* is thus obtained.

A crucial point in von Fintel's argument is that the equivalence between (A*) and (B*) concerns any interpretation of any triple of predicates.

But this assumption is pretty strong, and it can be contested: if no fair coin is such that, if tossed, it would land heads, it may well be that, for any such a coin, it is not the case that if it is tossed, it would land tails.

Interestingly, Williamson has another argument to reject *Equivalence* which is based on indeterminism. Suppose that a certain apparatus was not switched on at noon. Assume that nothing about the apparatus and its surroundings at noon determined whether, had it been switched on, it would have emitted a particle. Let us abbreviate "the apparatus is switched on" with p and "the apparatus will emit a particle" with Fq. According to Williamson, the only natural counterfactual form to express that "nothing about the apparatus and its surroundings at noon determined whether, had it been switched on, it would have emitted a particle"[14] is

$$(7) \quad \neg\Big[(p \mathrel{\Box\!\!\rightarrow} Fq) \vee (p \mathrel{\Box\!\!\rightarrow} \neg Fq)\Big]$$

However, if *Equivalence* were in place, (7) would be a contradiction. But indeterminism is not contradictory; thus, *Equivalence* has to be refuted.

The idea that (7) is not inconsistent appears to be convincing if we ask why asserting counterfactuals with contingent consequents may be a sensible practice. A natural answer is that counterfactuals aims to capture some regularities in the modal structure. In particular, both Fq and $\neg Fq$ in (7) are meant to be contingent at the counterfactual moments at which the antecedent holds. As argued in the previous section, counterfactual moments doesn't have any actual history which may guide the evaluation of a prediction. Thus, not only what (7) negates does not capture any genuine regularity in the counterfactual scenario evoked by the antecedent, but this fact explains why (7) is far from being inconsistent.

Conditional Excluded Middle

Stalnaker (1980: 92-93) thinks that principle 8,

8. $\mathcal{A} \mathrel{\Box\!\!\rightarrow} (\mathcal{B} \vee \mathcal{C}) \vdash (\mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{B}) \vee (\mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{C})$

reflects how people reason when they use counterfactuals. As a piece of evidence, he quotes the famous Verdi-Bizet example, where Quine uses something quite close to an instance of 8.

> It is clear that if Bizet and Verdi had been compatriots, then either Bizet would have been Italian, or Verdi French. But then one (and only one) of the two counterfactuals, *If Bizet and Verdi*

[14] Williamson (1988: 413).

> *had been compatriots, Verdi would have been French* or *If Bizet and Verdi had been compatriots, Bizet would have been Italian* must be true. (Quine 1950: 14)

Since Stalnaker accepts *Equivalence*, *M* and *N*, he sticks to *Conditional Excluded Middle*. Surely Stalnaker adduces evidence that we tend to think that 8 is a plausible pattern. But other examples show that it may fail to be so. For instance, Williamson argues that

> [Prinicple 8] takes us from the trivial premise that, had there been nothing but a gold or silver sphere, there would have been nothing but a gold or silver sphere [...] to the implausible conclusion that either, had there been nothing but a gold or silver sphere, it would have been gold or, had there been nothing but a gold or silver sphere, it would have been silver. (Williamson 1988: 412)[15]

Moreover, if asserting counterfactuals within an indeterminist context aims to express modal regularities, it seems strange to endorse *Conditional Excluded Middle*. Indeed, an instance such as $(p \mathrel{\Box\!\!\rightarrow} Fq) \vee (p \mathrel{\Box\!\!\rightarrow} \neg Fq)$ fails to capture any regularity when the consequents are (counterfactual) future contingents.

Lewis (1973a: 80-81) and Bennett & Bennett (2003: 189) claim that *Conditional Excluded Middle* gives wrong results for 'might' counterfactuals. First, Lewis (1973a) defines the 'might' counterfactual $\mathrel{\Diamond\!\!\rightarrow}$ as the dual of 'would':[16]

$$\mathcal{A} \mathrel{\Diamond\!\!\rightarrow} \mathcal{B} = \neg(\mathcal{A} \mathrel{\Box\!\!\rightarrow} \neg\mathcal{B}) \qquad \textit{Duality}$$

Then, he formulates the following argument:

1.	$(\mathcal{A} \mathrel{\Box\!\!\rightarrow} \neg\mathcal{B}) \vee (\mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{B})$	*Conditional Excluded Middle*
2.	$\neg(\mathcal{A} \mathrel{\Box\!\!\rightarrow} \neg\mathcal{B}) \rightarrow (\mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{B})$	By 1 and Class.Log.
3.	$(\mathcal{A} \mathrel{\Diamond\!\!\rightarrow} \mathcal{B}) \rightarrow (\mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{B})$	By 2 and *Duality*
4.	$(\mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{B}) \rightarrow (\mathcal{A} \mathrel{\Diamond\!\!\rightarrow} \mathcal{B})$	By *Duality*
5.	$(\mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{B}) \leftrightarrow (\mathcal{A} \mathrel{\Diamond\!\!\rightarrow} \mathcal{B})$	By 3 and 4

The conclusion is clearly wrong, for "If I tossed the coin, it might land heads" can be true and "If I tossed the coin, it would land heads" false.

[15] Here Williamson is arguing against the soundness of the following pattern: $(\mathcal{A} \vee \mathcal{B}) \mathrel{\Box\!\!\rightarrow} (\mathcal{A} \vee \mathcal{B}) \vdash ((\mathcal{A} \vee \mathcal{B}) \mathrel{\Box\!\!\rightarrow} \mathcal{A}) \vee ((\mathcal{A} \vee \mathcal{B}) \mathrel{\Box\!\!\rightarrow} \mathcal{B})$.

[16] $\mathcal{A} \mathrel{\Diamond\!\!\rightarrow} \mathcal{B}$ reads "If $\mathcal{A}$ were the case, $\mathcal{B}$ might be the case".

Furthermore, it is tempting to say that the latter statement is false (or not true) *because* the former 'might' conditional is true.

Clearly, one can reject Lewis' argument and deny *Duality*. For instance, Stalnaker (1980: 100) argues that there is nothing strange in saying that

> John might not have come to the party if he had been invited, but I believe he would have come,

even if, by *Duality*, it should sound as a moorean paradox. Be as it may, most philosophers who drop *Duality* read 'might' counterfactuals as epistemic (or quasi-epistemic) modals. Even if this is a strategy that is worth pursuing, the contemporary debate is far from having reached general consensus.

Furthermore, there are several philosophers – such as Lewis, Williamson and Bennett – who are happy to deny *Conditional Excluded Middle*, for they refute both *Equivalence* and principle 8 (see the previous sections).

Edelberg and Weak Edelberg

Edelberg and *Weak Edelberg* were introduced by Thomason & Gupta (1980: 74) in the context of branching time semantics, and are the only patterns which combine historical modalities with the 'would' connective. Each of them permits to infer a counterfactual with a necessitated consequent. In particular, *Edelberg* says that, from the fact that

1. It is inevitable that if $\mathcal{A}$ were the case, $\mathcal{B}$ would obtain,
   $\Box(\mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{B})$

one should infer that

$\therefore$ If $\mathcal{A}$ were the case, it would be inevitable that if $\mathcal{A}$, then $\mathcal{B}$.
$\mathcal{A} \mathrel{\Box\!\!\rightarrow} \Box(\mathcal{A} \rightarrow \mathcal{B})$

On the other hand, *Weak Edelberg* imposes that, whenever it is true that

1. If $\mathcal{A}$ were the case, it would be inevitably so,
   $\mathcal{A} \mathrel{\Box\!\!\rightarrow} \Box\mathcal{A}$
2. It is settled that, if $\mathcal{A}$ were the case, $\mathcal{B}$ would be the case,
   $\Box(\mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{B})$

it follows that

$\therefore$ If $\mathcal{A}$ were the case, $\mathcal{B}$ would inevitably be the case.
$\mathcal{A} \mathrel{\Box\!\!\rightarrow} \Box\mathcal{B}$

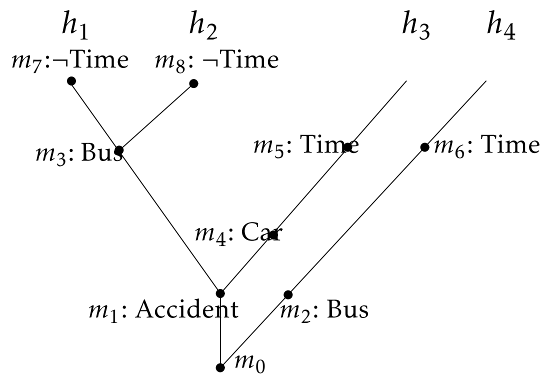


Figure 5.2

Thomason & Gupta (1980: 74) give an example to support the intuition that both patterns should hold (See figure 5.2).

Suppose that, at m_0, I have to go to work, and I am deciding whether to take the car or the bus. There is one possible future moment, m_2, where I take the bus which arrives at the scheduled time. At m_2 is then settled that I will arrive at work on time (m_6).

Thus, at m_0 the following statement is intuitively false.

> If I were take the bus, I would arrive late at work.
> $F(\text{Bus}) \mathrel{\Box\!\!\rightarrow} \neg F(\text{Time})$

However, imagine that, soon after m_0, the bus has an accident (m_1), so it won't arrive at the bus stop for the time scheduled (m_3), and it would be too late for me to get on work on time (m_7 and m_8). Thus, I decide to take the car (m_4) to get to work on time (m_5). Intuitively, the following statements should be true at m_4 (i.e., at the moment in which I take the car).

> It is inevitable that, If I were to take the bus, I would arrive late at work.
> $\Box(F(\text{Bus}) \mathrel{\Box\!\!\rightarrow} \neg F(\text{Time}))$
>
> If I were to take the bus, it would be inevitably so.
> $F(\text{Bus}) \mathrel{\Box\!\!\rightarrow} \Box F(\text{Bus})$

Given this amount of information, Thomason & Gupta (1980: 74) claim that it is perfectly natural to infer the following two statements (the informal reading of the second one is left unspecified):

If I were to take the bus, it would be settled that I would arrive late.
$F(\text{Bus}) \mathrel{\Box\!\!\rightarrow} \Box(\neg F(\text{Time}))$

$F(\text{Bus}) \mathrel{\Box\!\!\rightarrow} \Big[F(\text{Bus}) \rightarrow \neg F(\text{Time})\Big]$

Even if Thomason & Gupta (1980: 74) treat the example as perfectly intuitive, I find it rather artificial. That is why I consider *Edelber* and *Weak Edelberg* as controversial. However, these two patterns are interesting, for they challenge the semanticist: as they say, "the difficulty that needs to be solved is this: how to validate both conditional excluded middle and the Edelberg inference[s]". [17] In what follows, it will be shown that there's a semantic approach for counterfactuals which validates both kind of inferences, and which is consistent with (PT). Be as it may, it is time to recap.

Recapitulation

We have seen that, according to Objection 2.1, (PT) should yield incorrect truth conditions counterfatuals. In turn, I take that objection to mean that either (PT) makes sound some inferences that are (taken to be) uncontroversially unsound, or (PT) fails to validate some uncontroversial pattern. In the next sections, I show that Objection 2.1 is false. Recall that I tried to argue against *Equivalence* and *Conditional Exclude Middle*. I find also *Edelberg* and *Weak Edelberg* rather puzzling. However, since I didn't provide any conclusive argument against their adopotion, here there is my strategy:

- There is a semantic account (call it (C.1)) for $\mathrel{\Box\!\!\rightarrow}$ which validates *Conditional Excluded Middle*, *Equivalence*, *Edelberg*, and *Weak Edelberg*. However, it presupposes the so-called Limit Assumption.
- Another semantic account, (C.2), does not validate *Conditional Excluded Middle*,*Equivalence*, but it doesn't assume the Limit Assumption.

5.3.2 Counterfactual semantics with the limit assumption

Almost any proposal developed so far (see Thomason & Gupta (1980), Placek & Müller (2007), Wawer & Wroński (2015)) starts by assuming the 'similarity intuition' elaborated by Stalnaker (1968), Stalnaker & Thomason (1970), and Lewis (1973a).[18] Their idea is quite simple: take a world

[17] Thomason & Gupta (1980: 78).

[18] We won't take into account the counterfactual semantic offered in Øhrstrøm & Hasle (1995), since it didn't had much fortune in the philosophical literature.

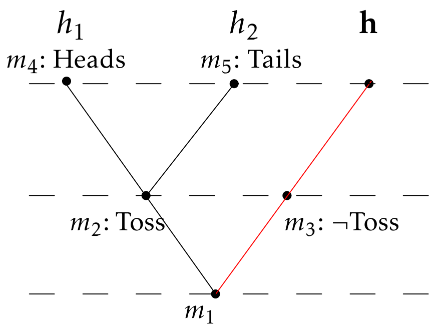


Figure 5.3

w_1 in which $\mathcal{A} \,\Box\!\!\rightarrow \mathcal{B}$ may be evaluated. If $\mathcal{A}$ is true at w_1, the truth of $\mathcal{B}$ at that very same world makes $\mathcal{A} \,\Box\!\!\rightarrow \mathcal{B}$ true at w_1. However, a counterfactual is interesting when its antecedent is false at the point of evaluation. Thus, suppose that $\mathcal{A} \,\Box\!\!\rightarrow \mathcal{B}$ is evaluated at w_1, but that very same world fails to satisfy $\mathcal{A}$. Then, the counterfactual takes you from w_1 to (at least) a different world, say w_2, at which the antecedent $\mathcal{A}$ is true. World w_2 is said to be minimally different from w_1, in the sense that there are no differences between the two worlds, except those required to make the antecedent $\mathcal{A}$ true at w_2. The truth of the consequent $\mathcal{B}$ at the most similar world w_2 makes the counterfactual $\mathcal{A} \,\Box\!\!\rightarrow \mathcal{B}$ true at the original world, w_1.

Let us use Figure 5.3 to see how this strategy may be applied to branching time frames. At m_3 I do not toss the coin in my hand, but I say

(7) If I tossed the coin, it would land tails.
Toss $\Box\!\!\rightarrow F(\text{Tails})$

The idea is to search those moments that are similar to m_3, but which satisfy the antecedent of (7).[19] It is standardly assumed that two moments m, m' are similar only if they belong to the very same instant (i.e., only if $m, m' \in \{m'' : m \not< m'' \wedge m'' \not< m\}$; if m, m' belong to the same instant, we will write $m \approx m'$). In the (oversimplified) case pictured in Figure 5.3, m_2 is the only moment which is similar to m_3, and satisfies the antecedent of (7). In other terms, m_2 is an alternative moment to m_3 at which it is is true that I toss the coin. Once we have reached this counterfactual moment, we need to check whether the consequent of (7) – "The coin will land tails" - comes out true (false) at m_2. If so, (7) is true at m_3; otherwise, it is false at that

[19]For the sake of simplicity, we won't consider counterfactuals whose antecedents are absurd.

moment. Clearly, if the components of a subjunctive conditional are future contingents, their truth value would depend on what history (or what set of histories) – among those passing through a similar, counterfactual moment – is considered relevant for the evaluation. In the toy-example of Figure 5.3, "The coin will land tails" is a future contingent at m_2: indeed it is BO-true at m_2/h_1, but it is BO-false at m_2/h_2. Thus, the truth of (7), used at m_3, depends on whether one takes either m_2/h_1 or m_2/h_2 to be relevant for the evaluation of the consequent.

In general, when one interprets counterfactuals within a branching conception of time, one has to tell a story about

- how to isolate counterfactual, similar moments, and
- what histories passing through those moments are relevant for the evaluation.

These two goals can be achieved by imposing suitable conditions to a *selection function s*, whose role is to tell which moment-history pairs are relevant for the evaluation of a counterfactual. s takes as argument pairs as $(\mathcal{A}, m/h)$, where $\mathcal{A}$ is a sentence that can occur as the antecedent of a counterfactual used at m. A selection s takes as values a moment-history pair m'/h' among those which are most similar to m/h, and it can be defined by the following four conditions.[20] Given a pair m/h at which $\mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{B}$ has to be evaluated,

Antecedent: If $s(\mathcal{A}, m/h) = m'/h'$, then $m'/h' \vDash_{BO} \mathcal{A}$

> The *Antecedent* condition says that s takes as value a moment-history pair which satisfies the antecedent. It is worth to notice that s is undefined (written $s(\mathcal{A}, m/h) = \uparrow$) whenever $\mathcal{A}$ is either logically impossible or counterfactually impossible – that is, when no m'/h' satisfies $\mathcal{A}$, where $m \approx m'$.

Similarity: If $m/h \vDash_{BO} \mathcal{A}$, then $s(\mathcal{A}, m/h) = m/h$

> The *Similarity* condition guarantees that s takes value m/h whenever m/h satisfies the antecedent. Intuitively, if m/h satisfies the antecedent in $\mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{B}$, the most similar couple to m/h at which the antecedent holds is m/h itself.

Contemporary: If $s(\mathcal{A}, m/h) = m'/h'$, then $m \approx m'$

[20] See Wawer & Wroński (2015: 297).

Intuitively, *Contemporary* says that m'/h' is similar to m/h only if m and m' are alternative but contemporary moments (i.e. they both lie on the same instant).

Past Predominance: If $s(\mathcal{A}, m/h) = m'/h'$, then $\forall m''/h''((m \approx m'' \wedge (h \cap h' \subset h \cap h'')) \Rightarrow m''/h'' \nvDash_{BO} \mathcal{A})$

This latter condition states that $s(\mathcal{A}, m/h)$ is the last moment-history pair which satisfies $\mathcal{A}$, since any other pair whose history branches off from h after that of $s(\mathcal{A}, m/h)$, and which specifies a moment lying at the same instant of m, fails to satisfy $\mathcal{A}$. The intuition behind *Past Predominance* is that a similar history to h must share with h as much past as possible.

With these four clauses at hand, it is tempting to define the truth conditions for counterfactuals as follows:

(C.1) $m/h, s \vDash_{BO} \mathcal{A} \,\Box\!\!\rightarrow \mathcal{B} \Leftrightarrow s(\mathcal{A}, m/h) = \uparrow$ or $s(\mathcal{A}, m/h), s \vDash_{BO} \mathcal{B}$

The first disjunct of the right-hand side in (C.1.) covers (a) the case in which an antecedent is logically impossible, and (b) the case in which, given an antecedent $\mathcal{A}$, no $m'/h', s$ is such that $m'/h', s \vDash_{BO} \mathcal{A}$, where $m' \approx m$. This means that $\mathcal{A}$ is counterfactually impossible. Hence, counterfactuals with impossible antecedents are true by default. The second disjunct shifts the evaluation to the most similar point to m/h, and checks whether the consequent is satisfied at that point.

It is worth to notice that *Antecedent* and (C.1) jointly entail that, given a point $m/h, s$, any counterfactual with a true antecedent at $m/h, s$ reduces to a material conditional. This captures *Modus Ponens* and *Modus Tollens*.[21]

Moreover, *Transitivity*, *Antecedent Strenghtening* and *Contraposition* aren't sound, and they have the countermodel pictured in Figure 5.4. Suppose that

1. $m_4/h_1 \vDash_{BO} \mathcal{A} \wedge \mathcal{B} \wedge \neg\mathcal{C}$
2. $m_5/h_2 \vDash_{BO} \neg\mathcal{A} \wedge \mathcal{B} \wedge \mathcal{C}$

[21] Here the following, natural notion of BO-entailment is presupposed:

BO-Entailment A set of wffs Δ BO-entails $\mathcal{A}$ just in case, for any point $m/h, s$, if $m/h, s \vDash_{BO} \Delta$, then $m/h, s \vDash_{BO} \mathcal{A}$. In symbols,

$$\Delta \vdash_{BO} \mathcal{A} \Leftrightarrow \forall m/h, s(m/h, s \vDash_{BO} \Delta \Rightarrow m/h, s \vDash_{BO} \mathcal{A})$$

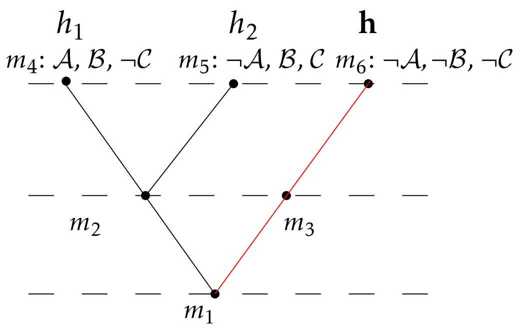


Figure 5.4

3. $m_6/\mathbf{h} \vDash_{BO} \neg\mathcal{A} \wedge \neg\mathcal{B} \wedge \neg\mathcal{C}$
4. $s(m_6/\mathbf{h}, \mathcal{A}) = m_4/h_1$
5. $s(m_6/\mathbf{h}, \mathcal{B}) = m_5/h_2$
6. $s(m_6/\mathbf{h}, \mathcal{A} \wedge \mathcal{B}) = m_4/h_1$

By 1-6 and (C.1), it follows that $\mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{B}, \mathcal{B} \mathrel{\Box\!\!\rightarrow} \mathcal{C}, \neg\mathcal{A} \mathrel{\Box\!\!\rightarrow} \neg\mathcal{C}$ are BO-true at $m_6/\mathbf{h}$, while both $\mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{C}$ and $\mathcal{A} \wedge \mathcal{B} \mathrel{\Box\!\!\rightarrow} \mathcal{C}$ are BO-false at the very same point. Hence,

$\mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{B}, \mathcal{B} \mathrel{\Box\!\!\rightarrow} \mathcal{C} \nvdash_{BO} \mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{C}$ Failure of *Transitivity*

$\mathcal{B} \mathrel{\Box\!\!\rightarrow} \mathcal{C} \nvdash_{BO} \mathcal{A} \wedge \mathcal{B} \mathrel{\Box\!\!\rightarrow} \mathcal{C}$ Failure of *Antecedent Strengthening*

$\neg\mathcal{C} \mathrel{\Box\!\!\rightarrow} \neg\mathcal{A} \nvdash_{BO} \mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{C}$ Failure of *Contraposition*

Thus, (C.1) validates the good patterns and invalidates the bad ones. As for the controversial inferences, the following results hold.

Fact 16. (C.1) *validates* Equivalence *and* Conditional Excluded Middle.
Proof. As for *Equivalence*: If $m/h, s \vDash_{BO} \neg(\mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{B})$, then $m/h, s \nvDash_{BO} \mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{B}$, which, by (C.1), entails that $s(\mathcal{A}, m/h), s \nvDash_{BO} \mathcal{B}$ (it is assumed that $s(\mathcal{A}, m/h) \neq \uparrow$). Thus, $s(\mathcal{A}, m/h), s \vDash_{BO} \neg\mathcal{B}$, and by (C.1) it follows that $m/h, s \vDash_{BO} (\mathcal{A} \mathrel{\Box\!\!\rightarrow} \neg\mathcal{B})$. Symmetrically, by assuming $m/h, s \vDash_{BO} (\mathcal{A} \mathrel{\Box\!\!\rightarrow} \neg\mathcal{B})$ it follows that $m/h, s \vDash_{BO} \neg(\mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{B})$. Hence, $\vdash_{BO} \neg(\mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{B}) \leftrightarrow (\mathcal{A} \mathrel{\Box\!\!\rightarrow} \neg\mathcal{B})$. As for *Conditional Excluded Middle*: since $\vdash_{BO} \neg(\mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{B}) \leftrightarrow (\mathcal{A} \mathrel{\Box\!\!\rightarrow} \neg\mathcal{B})$ holds, $\vdash_{BO} \neg(\mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{B}) \rightarrow (\mathcal{A} \mathrel{\Box\!\!\rightarrow} \neg\mathcal{B})$ holds as well, and it is equivalent to $\vdash_{BO} (\mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{B}) \vee (\mathcal{A} \mathrel{\Box\!\!\rightarrow} \neg\mathcal{B})$.

Fact 17. (C.1) *and the Ockhamist definition of* $\Box$ *invalidates both* Edelberg *and* Weak Edelberg.
Proof. Picture 5.5 shows a countermodel for both inferences. Suppose that:

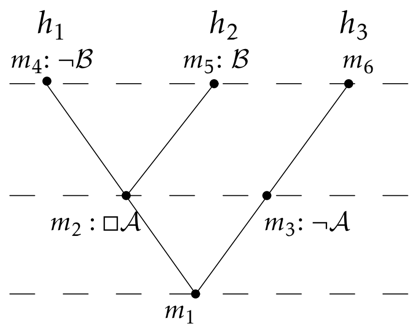


Figure 5.5

1. $m_3/h_3 \vDash_{BO} \neg\mathcal{A}$
2. $m_2/h_2 \vDash_{BO} \Box\mathcal{A}$
3. $m_4/h_1 \vDash_{BO} \neg\mathcal{B}$
4. $m_5/h_2 \vDash_{BO} \mathcal{B}$.
5. $s(m_3/h_3, \mathcal{A}) = m_2/h_2$

By 1–5 and (C.1), it follows that $\Box(\mathcal{A} \,\Box\!\!\rightarrow F(\mathcal{B})), \mathcal{A} \,\Box\!\!\rightarrow \Box\mathcal{A}$ are BO-true at m_3/h_3, but $\mathcal{A} \,\Box\!\!\rightarrow \Box(\mathcal{A} \rightarrow F(\mathcal{B}))$ and $\mathcal{A} \,\Box\!\!\rightarrow \Box F(\mathcal{B})$ are BO-false at the very same point. Thus,

$\Box(\mathcal{A} \,\Box\!\!\rightarrow F(\mathcal{B})) \nvdash_{BO} \mathcal{A} \,\Box\!\!\rightarrow \Box(\mathcal{A} \rightarrow F(\mathcal{B}))$ Failure of *Edelberg*

$\mathcal{A} \,\Box\!\!\rightarrow \Box\mathcal{A}, \Box(\mathcal{A} \,\Box\!\!\rightarrow F(\mathcal{B})) \nvdash_{BO} \mathcal{A} \,\Box\!\!\rightarrow \Box F(\mathcal{B})$ Failure of *Weak Edelberg*

Still, Wawer & Wroński (2015) suggest that the *Edelberg* inferences can be validated by modifying the Ockhamist definition of $\Box$ as follows.

(Def. $\Box_s$) $m/h, s \vDash_{BO} \Box\mathcal{A} \Leftrightarrow \forall s', h'(m/h', s' \vDash_{BO} \mathcal{A})$

Obviously, the duality between $\Box$ and $\Diamond$ is preserved.

Fact 18. *If the Ockhamist clause for* $\Box$ *is substituted with* (Def. $\Box_s$), (C.1) *validates both* Edelberg *and* Weak Edelberg.
Proof. As for *Edelberg*, suppose that $m/h, s \nvDash_{BO} \mathcal{A} \,\Box\!\!\rightarrow \Box(\mathcal{A} \rightarrow \mathcal{B})$. By (C.1) and (Def $\Box_s$), there must be an s'' such that $s''(\mathcal{A}, m/h), s'' \vDash_{BO} \mathcal{A} \wedge \neg\mathcal{B}$. By *Antecedent*, $s''(\mathcal{A}, m/h), s'' \vDash_{BO} \mathcal{A}$, thus $s''(\mathcal{A}, m/h), s'' \vDash_{BO} \neg\mathcal{B}$. It follows that s'' is such that $m/h, s'' \vDash_{BO} \mathcal{A} \,\Box\!\!\rightarrow \neg\mathcal{B}$. But this contradicts $m/h, s \vDash_{BO} \Box(\mathcal{A} \,\Box\!\!\rightarrow \mathcal{B})$, which, by (C.1) and (Def. $\Box_s$), is equivalent to $\forall h', s'(m \in h' \Rightarrow s'(\mathcal{A}, m/h'), s' \vDash_{BO} \mathcal{B})$. Hence, whenever $m/h, s \vDash_{BO} \Box(\mathcal{A} \,\Box\!\!\rightarrow \mathcal{B})$ holds, $m/h, s \vDash_{BO} \mathcal{A} \,\Box\!\!\rightarrow \Box(\mathcal{A} \rightarrow \mathcal{B})$ holds as well. As for *Weak Edelberg*, suppose that $\mathcal{A} \,\Box\!\!\rightarrow$

$\Box\mathcal{A}$, which, by (C.1) and (Def. $\Box_s$), is equivalent to $\forall h', s'(\mathcal{A}, m/h), s' \vDash_{BO} \mathcal{A}$ for any s' and any h' passing through the moment specified by $s'(\mathcal{A}, m/h)$. Assume also that $\Box(\mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{B})$ $m/h, s$. Now, if $m/h, s \nvDash_{BO} \mathcal{A} \mathrel{\Box\!\!\rightarrow} \Box\mathcal{B}$, by (C.1) and (Def $\Box_S$) it follows that $s(\mathcal{A}, m/h), s \vDash_{BO} \Diamond\neg\mathcal{B}$. If $s(\mathcal{A}, m/h) = m'/h'$, then there is an s'', h'', s.t. $m' \in h''$ and $m'/h'', s'' \nvDash_{BO} \mathcal{B}$. But this contradicts that $m/h, s \vDash_{BO} \Box(\mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{B})$. Indeed, this latter statement entails $m/h, s \vDash_{BO} \mathcal{A} \mathrel{\Box\!\!\rightarrow} \Box(\mathcal{A} \rightarrow \mathcal{B})$, which is equivalent, by (C.1), (Def $\Box_S$), and $s(\mathcal{A}, m/h) = m'/h'$, to $\forall h''', s'''(m' \in h''' \Rightarrow m'/h'', s''' \vDash \mathcal{B})$. Hence, whenever $m/h, s \vDash_{BO} \Box(\mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{B})$ and $m/h, s \vDash_{BO} \mathcal{A} \mathrel{\Box\!\!\rightarrow} \Box\mathcal{A}$ hold, it is the case that $m/h, s \vDash_{BO} \mathcal{A} \mathrel{\Box\!\!\rightarrow} \Box\mathcal{B}$.

Principle (C.1) and (Def. $\Box_s$) meet Thomason & Gupta (1980)'s challenge, since they yield a semantic account which validates *Conditional Excluded Middle*, *Edelberg* and *Weak Edelberg*. Given (C.1) and (Def. $\Box_s$), (PT) can be easily adapted to capture the truth conditions for counterfactuals, that is:[22]

(PT') $m \vDash_{PT} \mathcal{A} \Leftrightarrow m/\mathbf{h}, s \vDash_{BO} \mathcal{A}$.

Hence, Objection 2.1 appears to be false.

However, this might seem too hasty. To make (C.1) work, one has to enrich a TRL-model with a selection s. But there may be more functions satisfying the four conditions which define a selection. For instance, take the model pictured in Figure 5.4: both $s(\mathcal{A}, m_3/\mathbf{h}) = m_2/h_2$ and $s'(\mathcal{A}, m_3/\mathbf{h}) = m_2/h_1$ can be taken as legitimate values of the two, distinct selections s and s'. By (C.1), $\mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathsf{F}\mathcal{B}$ is true at $m_3/\mathbf{h}, s$, but it comes out false at $m_3/\mathbf{h}, s'$. Thus, the truth-values of counterfactuals vary with selections, but no selection is privileged over the others.

Moreover, our intuitions seems to be silent when it comes to decide whether – as in Figure 5.4 – m_2/h_1 is more similar to $m_3/\mathbf{h}$ than m_2/h_2. Furthermore, and most importantly, the point of asserting a counterfactual conditional is to capture certain regularities among different, possible scenarios. In turn, the tree represents physical possibilities. Thus, if a counterfactual whisks us to merely possible moments, with several, merely possible histories passing through them, selecting one of these history for the evaluation of the consequent forbids the counterfactual to express the regularities it is meant to capture.

However, it is not difficult to modify (PT') to overcome this difficulty. Indeed, one may state that:[23]

[22] Here, bivalence is assumed, that is, if a formula is not PT-true at a point $m/h, s$, it is PT-false at that point. Moreover, the moment in $m \vDash_{PT} \mathcal{A}$ is meant to be the moment of use of $\mathcal{A}$.

[23] Again, it is assumed that bivalence holds for formulas evaluated with (PT").

(PT”) $m \vDash_{PT} \mathcal{A} \Leftrightarrow \forall s(m/\mathbf{h}, s \vDash_{BO} \mathcal{A})$

(PT”), while conserves the validities we have just seen above, makes any counterfactual with future contingent consequent false. One can even drop bivalence for this latter kind of statements, by requiring that an untrue statement may not be false.

(PT”’) $m \vDash_{PT} \mathcal{A} \Leftrightarrow \forall s(m/\mathbf{h}, s \vDash_{BO} \mathcal{A})$

$m \nvDash_{PT} \mathcal{A} \Leftrightarrow \forall s(m/\mathbf{h}, s \nvDash_{BO} \mathcal{A})$

By (PT”’), counterfactuals with future contingent consequents are neither true nor false.

Be as it may, there can be qualms about (C.1). The clause depends on selection functions, and when both s and s' are defined for argument $(\mathcal{A}, m/h)$, *Past Predominance* entails that the histories specified by $s(\mathcal{A}, m/h)$ and $s'(\mathcal{A}, m/h)$ must branch off form h at the very same moment.[24] Put differently, if s is defined for argument $(\mathcal{A}, m/h)$, there is no infinite series of histories s.t., for any such history h', there is another history, say h'', and $h \cap h' \subset h \cap h''$. Thus, *Past Predominance* can be seen as a formulation of what Lewis (1973b: 63) calls the Limit Assumption: if “we proceed to closer and closer $\mathcal{A}$-worlds we eventually hit a limit and can go no farther”.

This restriction can be contested. There’s no *a priori* reason to prevent the $\mathcal{A}$-words to be infinitely closer to the world we are interested in, especially when the worlds represent objective modal features of physical reality. Thus, one may have good reasons to drop this principle. But what role does it play?

Past Predominance can be seen as a similarity criterion for histories. More precisely, given the antecedent $\mathcal{A}$ of a counterfactual evaluated at m/h, *Past Predominance* enables to choose – among the histories in $\{h' : m'/h' \vDash_{BO} \mathcal{A}$ and $m \approx m'\}$ – which of them shares with h as much past as possible. Hence, dropping *Past Predominance* amounts to identify a more general similarity criterion for histories.

Another reason why one may be discontent about (C.1) consists in that it permits to validate some controversial principles such as *Conditional Excluded Middle*, *Edelberg* and *Weak Edelberg*.

5.3.3 Counterfactual semantics without the limit assumption

As we have seen, *Past Predominance* can be dropped only if one introduces a more general relation of comparative similarity among histories. How-

[24]Wawer & Wroński (2015: 297).

ever, it is possible to conserve the idea that the more moments two histories share, the more they are similar. In particular, let us say that h' is as similar to h as h'' ($h' \preceq_h h'$) just in case h' branches off form h at the same moment of h'' or at a later moment. On the other hand, the most similar history to h is h itself. In symbols,

Global Similarity $h' \preceq_h h'' \Leftrightarrow (h \cap h'' \subseteq h \cap h')$

$\preceq_h$ is reflexive, transitive and connected (i.e., $\preceq_h$ is a preorder). Notice that history h is $\preceq_h$-strictly minimal; that is, for any $h' \neq h$, $h \preceq_h h'$. This features reflects the idea that there is no history which is more similar to h than h itself.[25]

Let us define a notion of local similarity, that is, a similarity relation obtaining among moment-history pairs $\preceq_{m/h}$. Let us conserve the intuition applied in the previous sections, according to which the counterfactual moments relevant for the evaluation of $\mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{B}$ at m/h – i.e., the moments similar to m – are those lying at the same instant of m.

Local Similarity $m'/h' \preceq_{m/h} m''/h'' \Leftrightarrow h' \preceq_h h''$ & $m \approx m' \approx m''$

$\preceq_{m/h}$ inherits the properties of $\preceq_h$. It is a preorder where m/h is $\preceq_{m/h}$-strictly- minimal.

Following Swanson (2011: 709), let $\|\mathcal{A}\|$ be the set of moment-history pairs at which $\mathcal{A}$ is BO-true, i.e. $\|\mathcal{A}\| = \{m/h : m/h \vDash_{BO} \mathcal{A}\}$. Hence, $\|\mathcal{A}\|^2$ is a set of couples whose intersection with $\preceq_{m/h}$ yields a preorder which agrees with $\preceq_{m/h}$ on the order of its couples. This technical detail is useful to define a new relation of similarity.

Antecedent Similarity $m'/h' \preceq^{\mathcal{A}}_{m/h} m''/h'' \Leftrightarrow (m'/h', m''/h'') \in (\|\mathcal{A}\|^2 \cap \preceq_{m/h})$

Notice that the similarity conditions defined thus far do not exclude that there may be an infinite series of $\mathcal{A}$-histories ever close to h. Hence, the Limit Assumption can fail.

Furthermore, let us define the truth conditions for counterfactuals in the following, Lewisian fashion:[26]

(C.2) $m/h \vDash_{BO} \mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{B} \Leftrightarrow \forall m'/h'(m'/h' \in \|\mathcal{A}\|$ & $m \approx m' \Rightarrow$
$\exists m''/h''(m''/h'' \preceq^{\mathcal{A}}_{m/h} m'/h'$ & $\forall m'''/h'''(m'''/h''' \preceq^{\mathcal{A}}_{m/h} m''/h'' \Rightarrow$
$m'''/h''' \vDash_{BO} \mathcal{B})))$

[25] For analogous notions of similarity between histories, moment-history pairs and tuples of moments, histories and formulas, see Placek & Müller (2007).

[26] See Lewis (1981: 230)

(C.2) says that $\mathcal{A} \boxright \mathcal{B}$ is BO-true at a point m/h iff, for any $\mathcal{A}$-point m'/h' that is comparable with m/h, there exists an $\mathcal{A}$-point m''/h'' that is at least as similar to m/h as m'/h', and for any $\mathcal{A}$-point that is at least as similar to m/h as m''/h'', that point satisfies $\mathcal{B}$.

When there is no m'/h' s.t. $m'/h' \in \|\mathcal{A}\|$ & $m \approx m'$, $\mathcal{A}$ is counterfactually impossible frome the perspective of m/h. Thus, (C.2) predicts conditionals with impossible antecedents to be true by 'default'. In these cases the antecedent in the right-hand side of (C.2) fails.

Whenever $\mathcal{A}$ is BO-true at m/h, it is easy to check that $\mathcal{A} \boxright \mathcal{B}$ comes out BO-true at that point just in case $\mathcal{B}$ holds at m/h (recall that m/h is $\preceq_{m/h}$-strictly-minimal). This feature validates *Modus Ponens* and *Modus Tollens*. Therefore, (C.2) validates the good patterns.

As for the bad ones, it is easy to check that the tree pictured in Figure 5.4 is a countermodel for *Transitivity*, *Antecedent Strenghtening* and *Contraposition*. Hence, (C.2) makes sound what should be sound, and invalidates what should be invalid.

(C.2) makes *Conditional Excluded Middle* and *Equivalence* invalid, as shown in the simple countermodel of Figure 5.5. If one evaluates $\mathcal{A} \boxright \mathsf{F}\mathcal{B} \vee \mathcal{A} \boxright \neg\mathsf{F}\mathcal{B}$ at m_3/h_3, by (C.2) it follows that neither of the two disjunct is satisfied. This entails that $\neg(\mathcal{A} \boxright \mathsf{F}\mathcal{B}) \to (\mathcal{A} \boxright \neg\mathsf{F}\mathcal{B})$ is false at the very same point, so that *Equivalence*, $\neg(\mathcal{A} \boxright \mathcal{B}) \leftrightarrow (\mathcal{A} \boxright \neg\mathcal{B})$, is not a validity according to (C.2). However, *Weak Boethius Thesis*, $(\mathcal{A} \boxright \neg\mathcal{B}) \to \neg(\mathcal{A} \boxright \mathcal{B})$, is validated.

Fact 19. (C.2) *validates* Weak Boethius Thesis.
Proof. The proof is straightforward by assuming (C.2) and $m/h \vDash_{BO} \mathcal{A} \boxright \neg\mathcal{B}$.

Notice that the failure of *Equivalence* and *Conditional Excluded Middle*, which is guaranteed by (C.2), enables to express indeterminism in a counterfactual form. Recall Williamson's example, according to which, if it is true that

> if nothing about an apparatus and its surroundings necessitate whether, had the apparatus been switched on, it would emit a particle,

then the two following statement should be false – and the negation of their disjunction true:

> If the apparatus were switched on, it would emit a particle,
> $p \boxright \mathsf{F}p$
>
> If the apparatus were switched on, it wouldn't emit a particle.
> $p \boxright \neg\mathsf{F}p$

According to (C.2), if $\mathsf{F}q$ is counterfactually contingent, the two counterfactuals are false, their conjunction false, and the negation of their conjunction true. Hence, (C.2) vindicates Williamson's way of expressing indeterminism. Furthermore, the following fact holds.

Fact 20. (C.2) *validates* Edelberg *and* Weak Edelberg.
Proof. *Recall that Edelberg is the pattern $\Box(\mathcal{A} \Box\!\!\rightarrow \mathcal{B}) \vdash_{BO} \mathcal{A} \Box\!\!\rightarrow \Box(\mathcal{A} \rightarrow \mathcal{B})$. Now assume that $\Box(\mathcal{A} \Box\!\!\rightarrow \mathcal{B})$ holds at m/h, which entails that $\mathcal{A} \Box\!\!\rightarrow \mathcal{B}$ is holds at m/h. By (C.2), it follows that there are no $m_1/h_1, m_2/h_2$ s.t. $m_1/h_1 \preceq^{\mathcal{A}}_{m/h} m_2/h_2$ and $m_1/h_1 \vDash_{BO} \neg\mathcal{B}$ but $m_2/h_2 \vDash_{BO} \mathcal{B}$. Now, suppose that $m/h \nvDash_{BO} \mathcal{A} \Box\!\!\rightarrow \Box(\mathcal{A} \rightarrow \mathcal{B})$. By (C.2) it follows that there are two points, say m_3/h_3 and m_4/h_4 s.t. $m_3/h_3 \preceq^{\mathcal{A}}_{m/h} m_4/h_4$, $m_3/h_3 \vDash_{BO} \Diamond(\mathcal{A} \wedge \neg\mathcal{B})$ and $m_4/h_4 \vDash_{BO} \Box(\mathcal{A} \rightarrow \mathcal{B})$. Now, $m_4/h_4 \vDash_{BO} \Box(\mathcal{A} \rightarrow \mathcal{B})$ entails $m_4/h_4 \vDash_{BO} \mathcal{A} \rightarrow \mathcal{B}$. Given that m_4/h_4 is a member of some couples in $\preceq^{\mathcal{A}}_{m/h}$, it must satisfies $\mathcal{A}$. Hence, from $m_4/h_4 \vDash_{BO} \mathcal{A} \rightarrow \mathcal{B}$, it follows that $m_4/h_4 \vDash_{BO} \mathcal{B}$. By $m_3/h_3 \vDash_{BO} \Diamond(\mathcal{A} \wedge \neg\mathcal{B})$, there must be a history, say h_5, s.t. $m_3/h_5 \vDash_{BO} \mathcal{A}$ but $m_3/h_5 \vDash_{BO} \neg\mathcal{B}$. Moreover, since $m_3/h_5 \vDash_{BO} \mathcal{A}$ and $m_3 \approx m$, it must be the case that m_3/h_5 is a member of at least a couple in $\preceq^{\mathcal{A}}_{m/h}$. Given that $\preceq^{\mathcal{A}}_{m/h}$ is connected, and since m_4/h_4 is a member of couples in $\preceq^{\mathcal{A}}_{m/h}$, either $m_3/h_5 \preceq^{\mathcal{A}}_{m/h} m_4/h_4$, or $m_4/h_4 \prec^{\mathcal{A}}_{m/h} m_3/h_5$. If $m_4/h_4 \prec^{\mathcal{A}}_{m/h} m_3/h_5$ then $h \cap h_5 \subset h \cap h_4$, and thus ther must be a moment, m_5, s.t. $m_5 \in h \cap h_4$ but $m_5 \notin h \cap h_5$. However, recall that $m_3/h_3 \preceq^{\mathcal{A}}_{m/h} m_4/h_4$, which entails that $h \cap h_4 \subseteq h \cap h_3$. But since $m_5 \in h \cap h_4$, then $m_5 \in h \cap h_3$, and thus $m_5 \in h_3$ and $m_5 \in h$. By $m_5 \in h_3$, $m_3 \in h_3$, either $m_3 = m_5$, or $m_3 < m_5$, or $m_5 < m_3$. If $m_5 < m_3$, by $m_5 \in h$ and $m_3 \in h_5$ it follows that $m_5 \in h \cap h_5$, against the assumption that $m_5 \notin h \cap h_5$. If $m_3 < m_5$, from $m_5 \in h_4$ it follows that $m_3 \in h_4$. But since $m_3 \approx m_4 \approx m$, h_4 would contain two $<$-unrelated moments, which is impossible. If $m_3 = m_5$, then, again, $m_5 \in h_5$, for $m_3 \in h_5$. But then $m_5 \in h \cap h_5$, against $m_5 \notin h \cap h_5$ Thus, to $m/h \nvDash_{BO} \mathcal{A} \Box\!\!\rightarrow \Box(\mathcal{A} \rightarrow \mathcal{B})$ to be true, it must be that $m_3/h_5 \preceq^{\mathcal{A}}_{m/h} m_4/h_4$. But if $m_3/h_5 \preceq^{\mathcal{A}}_{m/h} m_4/h_4$, then a $\neg\mathcal{B}$-point such as m_3/h_5 would be (at least) as similar to m/h as a $\mathcal{B}$-point m_4/h_4. Therefore, $m/h \vDash_{BO} \mathcal{A} \Box\!\!\rightarrow \mathcal{B}$ would be false, and $m/h \vDash_{BO} \Box(\mathcal{A} \Box\!\!\rightarrow \mathcal{B})$ could not hold, against our assumption. By reductio, $m/h \vDash_{BO} \Box(\mathcal{A} \Box\!\!\rightarrow \mathcal{B})$ entails $m/h \vDash_{BO} \mathcal{A} \Box\!\!\rightarrow \Box(\mathcal{A} \rightarrow \mathcal{B})$, and the Edelberg inference follows. It is easy to see that an analogous reasoning shows that Weak Edelberg, viz. $\mathcal{A} \Box\!\!\rightarrow \Box\mathcal{A}, \Box(\mathcal{A} \Box\!\!\rightarrow \mathcal{B}) \vdash_{BO} \mathcal{A} \Box\!\!\rightarrow \Box\mathcal{B}$, must be sound as well.*

Now, let us modify the truth conditions for counterfactuals in a (post-semantic) thin red line framework:

(PT'''') $m \vDash_{PT} \mathcal{A} \Leftrightarrow m/\mathbf{h}, \preceq_{m/h} \vDash_{BO} \mathcal{A}$

As we have seen, (PT'''') meets several desiderata, such as validating the good patterns (*Modus Ponens*, *Modus Tollens*) and invalidating the bad ones (*Transitivty, Antecedent Strenghtening* and *Contraposition*). Furthermore, (C.2) does not entail the limit assumption, and validates *Weak Boethius Thesis*, *Edelberg* and *Weak Edelberg*. However, if one assumes (C.2), *Conditional Excluded Middle* and *Equivalence* are not valid patterns. Last but not least, (C.2) allows to express indeterminism in a counterfactual construction. These features represents further reasons to consider Objection 2.1 false.

5.3.4 Objection 2.2

Once that Objection 2.1 is rejected, let's focus on another worry related to counterfactual claims:

> **Objection 2.2**: The truth value distributions provided by (PT) (for object language formulas) conflict with metalinguistic subjunctive conditionals.

As a motivation for the objection, consider the following example (the example is taken from Iacona 2014: 2638). Suppose that in one possible future of a merely possible moment, say, in the future of m_1 on history h_1, it rains. If one were to consider h_1 as actual, an adequate semantics should evaluate "It will rain", uttered at m_1, as true. But (PT) – so the objection goes – does not enable to assess this latter (metalinguistic) counterfactual as true. And this is so because (PT) is completely silent about the evaluations of statements uttered at non-actual moments.

Notice that the example just considered takes actual semantic facts for granted. In the example, indeed, "It will be rainy" is assessed as true as uttered at m_1 – under the assumption that h_1 is taken to be actual – because it rains in the future on m_1, along history h_1. This suggests that what "It will be rainy" says at m_1 – its (ordinary) intension – is what it says at the actual world. That is, the objection presupposes that "It will be rainy", when uttered at a merely possible moment, is intended to be a prediction about the weather (and not, for instance, a sentence saying *that the grass is green*). I shall notice that this very presupposition is not uncontroversial. In general, considering a merely possible history as actual does not only change the history which initialises the circumstances of evaluation, for it changes the relevant semantic facts as well. In other terms, to take a merely possible history as actual may change those facts that guarantee that expressions receive their ordinary, actual meanings. Nothing prevents, for instance, that "It will be rainy", as it is used at h_1, means *that the grass is green*.

But let us assume what it seems to be presupposed by the example which motivates Objection 2.2: for any expression $\mathcal{A}$ of $\mathcal{L}_T$ and any two histories h and h', what $\mathcal{A}$ says at h is the same as what $\mathcal{A}$ says at h'. From a two-dimensional viewpoint, this assumption amounts to assess secondary intensions as redundant, for what an expression says – which may be thought of as a function from circumstances of evaluation to extensions – does not depend on which world is taken to determine what it says.

If this is the case, Objection 2.2 has an obvious rejoinder. To take a merely possible history as actual, say h_1, is to change the model from $\mathcal{M}_{PT} = (\mathcal{T}, d_{\mathcal{T}}, \mathbf{h})$ to (MBT, h_1). In the latter model, $\mathbf{h}$ is considered merely possible, where h_1 is taken to be actual. Given $\mathcal{M}'_{PT} = (\mathcal{T}, d_{\mathcal{T}}, h_1)$, one can use (PT) to evaluate a statement used at a merely possible moment, which is now considered actual since it lies on h_1. And if one uses (PT) to evaluate "It will be rainy" at m_1 on h_1, one obtains the truth value distribution required by the example (the statement comes out true).

Iacona (2014) has an argument against this solution. He claims that

> Sentences can be evaluated at moments relative to different models, that is, models that differ only in the value of $\mathbf{h}$. The fact, however, is that relativity to models so understood is equivalent to relativity to histories. [...] This means that what can be expressed in a TRL semantics in terms of relativity to models can equally be expressed in [Ockhamist] semantics without postulating any actual history, hence that TRL semantics boils down to an unnecessarily convoluted variant of [Ockhamist] semantics.[27](Iacona 2014: 2643)

Now, the new objection charges (PT) of being unnecessarily complex, since it is expressively equivalent to the BO semantics. True, (PT) and the BO semantics are expressively equivalent. But this very fact does not entail that they are both well-suited for the same metaphysics.

For instance, the BO semantics is perfectly compatible with many-worlds view. And again, a many-worlds theorist such as Wallace (2012: 268) assumes a version of many-worlds equipped with an Ockhamist-inspired semantics. But Wallace would hardly accept (PT), since it defines the notion of truth at a moment as truth at a moment, at the actual history. The reason of the rejection is clear. According to many-worlds, "the actual history" is an improper definite description, for it doesn't meet the unicity

[27]Notation changed.

condition. Thus, granted that (PT) has the same expressive power of the BO semantics, endorsing the BO semantics does not force to stick to (PT).

Furthermore, the thin red line metaphysics – the claim that, among a plurality of possible courses of events, there exists a unique (substantially) actual history – motivates the post-semantic approach formulated in (PT). Indeed, if there is only one outcome that will actually obtain, it is natural to hold that a prediction such as "Outcome x will obtain" expresses a true content iff what is predicted obtains with respect to the actual course of events. From this latter perspective, the BO semantics fails to convey one crucial piece of metaphysical information.

5.3.5 Counterfactual retrospective assessments.

In his *Assessment Sensitivity*, John MacFarlane (2014: 209–213) develops several semantical objections against (PT). According to one of them, (PT) yields wrong results for counterfactual retrospective assessments of future contingents. Let us see why.

First, MacFarlane (2014: 27) introduces the following notion of accuracy for speech acts:

Accuracy. An attitude or speech act occurring at c_1 is accurate, as assessed from a context c_2, just in case its content is true as used at c_1 and assessed from c_2.

Let us identify a PT-context of use with a moment of utterance, and thus, with a moment lying on the thin red line. Notice, moreover, that (PT) doesn't invoke any notion of contexts of assessment, so they play no role in evaluating the accuracy of speech acts when their contents is evaluated against (PT).

Now, consider the figure below, and suppose that on Monday (at m_0) Jake says

(8) Tomorrow, it will be sunny.
$F(1)(\text{Sunny})$

Since on Tuesday, m_1 lies one day (one time unit) in the future of Monday (m_0) on the thin red line $\mathbf{h}$, (PT) predicts that Jake's claim is true on Monday (at m_0). At m_1, any assessor of Jake's claim should evaluate his utterance as accurate, for "the assessor has only to feel the sun on her skin to know that Jake's assertion was accurate".[28] And this result precisely follows from (PT) and *Accuracy*. So far so good.

[28] MacFarlane (2014: 210).

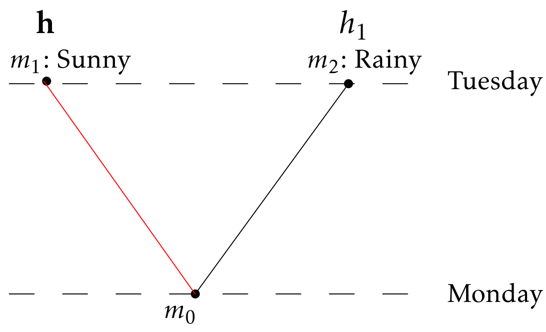


Figure 5.6

But imagine an assessor at m_2; by (PT) and *Accuracy*, she should evaluate Jake's speech act made at m_0 as accurate, even though it is rainy at m_2. And this, according to MacFarlane, is wrong. Therefore, MacFarlane concludes, (PT) cannot account for counterfactual retrospective assessments for future contingents.

I think that this objection is not fair. Indeed, if (PT) entails that a contingent prediction is true at a moment of use just in case it is true at that moment, relative to the thin red line, why one should care about (merely possible) assessments of actual speech acts? It is wrong to pretend that an utterance, whose truth value depends on what actually will be the case, can be assessed at a moment that is a sheer possibility. Having such a pretence seems like requiring to evaluate the accuracy of an utterance of "It is the case that p", made at a world w, relative to another world, say w', at at which $\neg p$ is the case. In other terms, if statement (8) is interpreted with (PT), then Jake's assertion aims at establishing what actually will be the case, but it doesn't state anything relative to what counterfactually will be the case. Therefore, whether Jake's assertion made at m_0 is either accurate or not depends on what is the case in the actual future of m_0.

To sum up, it seems that (PT) has no problem with counterfactual reasoning. More generally, positing a substantially actual history is not an obstacle for counterfactual reasoning.

Chapter 6

Conclusions

This essay aims to find a plausible semantics to interpret future tensed statements – and, in particular, future contingents – in an indeterministic context.

The problem is pressing, for indeterminism seems to be required to make sense of certain physical theories. Collapse theories are a plausible candidates to solve the measurement problem, and they clearly entail that physical systems, in general, evolve stochastically.

Moreover, the branching approach, when compared with the divergentist view, appears the most plausible framework in which one may model indeterminism. The branching view seems to be as good as the divergentist to define a semantics – i.e., the Ockhamist semantics – which refutes both the Aristotelian and the Master arguments. But a branching conception of reality is, surely, more intuitive than divergence. And the tower-collapse case seems to suggest that a branching structure can account for indeterministic phenomena in an easiest way than that of a divergentist.

However, interpreting future contingents against a branching structure is not an easy task. As we have seen, Peirceanism defines a notion of truth at a moment which violates the intended meaning of plain-will predictions. Moreover, Peirceanism fail to validate several tense-logical schemas (such as, for instance, the future excluded middle). And both Supervaluationism and Relativism suffer form analogous defects. Supervaluationism attributes to "plain" predictions a strong modal force, and it has several problems with logical consequence and with disquotational truth. Moreover, it cannot account for retrospective truth judgements, and it is at odds with an eternalist understanding of propositions. Relativism inherits several flaws that affect supervaluationism, such as those related to the notion of truth. Moreover, and most importantly, Relativism appears to have many problems when it comes to evaluate the rationality of retractions and assertions.

What is interesting about supervaluationism, though, is that one of its founding fathers, Richmond Thomason, explicitly adopts a relational notion of actuality. The analysis of the meanings that actuality may have within a branching structure, indeed, is pretty important to understand some basic, metaphysical commitments that a branching indeterminists should adopt.

As Belnap's viewpoint strongly suggests, the BO semantics clearly solves several problems that a supervaluationist can hardly overcome. But the BO-semantics may be appealing only for those who adopt a deterministic reading of the tree. Indeed, the metaphysical view of a many-worlds theorist is perfectly compatible with the relativisation of truth to moment-history pairs. Moreover, such a compatibility is grounded on the fact that, in a many-worlds perspective, substantial actuality is a redundant notion, for everything that is possible obtains. The dialectical scenario introduced by the many-world view is highly significant, for it allows to through light on the very notion of (in)determinism. As I try to argue, indeed, a branching theorist who claims that actuality can only be a relative, perspectival notion, cannot be distinguished from a branching, many-worlds determinist. And the relational meanings that actuality may assume, in turn, are insufficient to get a satisfactory definition for branching objective indeterminism. Once that the notion of substantial actuality is introduced, it is easy to distinguish many-worlds theorists form branching indeterminists. Moreover, if, among the possible alternatives, only one of them will obtain, it is easy to infer that a unique, substantially actual history must exist.

Hence, the Thin red line semantics appear to be the most sensible approaches to solve the problem of future contingents. Among these approaches, the post-semantic thin red line, (PT), defines a notion of truth that is relative to moments only. It is perfectly bivalent, and it does not suffer from the flaws which affect the other branching semantics. Moreover, I have argued that positing a Thin red line does not raise any problem related to counterfactual reasoning. Accordingly, I conclude that (PT) is the most plausible semantic framework for future tensed statement in an indeterministic context.

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