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JUMPS DIFFUSION AND JUMP RISK PRICING

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Introduction

Every day market operators exchange tens of thousand of stocks, creating an extremely rich information set to study price dynamics. Indeed, the pattern followed by asset returns have been a fundamental topic in finance literature for decades. Several studies provide evidence, Ball and Torous (1983), Jarrow and Rosenfeld (1984), and Jorion (1988) among others, that stock prices show sudden but infrequent movements of large magnitude, that are commonly known as jumps. Thus, it is a standard to design the dynamic of stock prices as a combination of a continuous diffusion component, plus discontinuous jumps.

Because of their relevance in economics, finance, and decision sciences, the present thesis focuses on jumps in stock returns. Note, Chapters 1 and 2 represent two different papers, respectively entitled *"Jump risk and pricing implications"* and *"The cross-sectional diffusion of jumps and the identification of collective sectorial movements"*, each of them develops the main theme in a different direction:

Chapter 1: construction of a jump risk factor. A central model in the description of market returns and risks is the Sharpe (1964), Lintner (1965), Mossin (1966) and Black (1972) CAPM. Subsequently, Fama and French (1993) and Carhart (1997) among others, proposed alternative asset-pricing models that add to the CAPM additional risk sources.

Chapter 1 contributes to the existing literature by proposing a factor which captures investors fear of future jumps. Moreover, we add it to the Carhart (1997) model, thus putting forward a 5-factor model, and show that not only our factor is able to capture common variation in stock returns, but also that its use improves the model performance. We additionally compute the risk premiums for the five risk sources of the model and find that they are always positive and not significantly different from their factor means. In doing this we employ all CRSP stocks over the 1925-2014 sample period, which leads to 89 years of assets prices and more than 24,000 stocks.

Chapter 2: cross-sectional jump diffusion. Even if there is evidence of price jumps in various markets, there is still little understanding about their cross-sectional diffusion.

Chapter 2 investigates the presence of contemporaneous jumps among a large number of stocks, the multivariate jumps (or MJs), using a high-frequency dataset of considerable dimension. The database includes 1-minute prices for all $3,509$ stocks belonging to the Russell 3000 index between January 2, 1998 and June 5, 2015 (4,344 days), data that we treat both as a whole as well by focusing on its 11 industries.

Using the information about MJs, we then propose two indexes which summarize data on cross-sectional jump diffusion: the daily diffusion index (or DID), and the intraday diffusion index (or DII). Results confirm the usefulness of both DID and DII, which trends and residuals show more and higher spikes in correspondence of important economic moments, as in 2008 and 2010. Moreover, we observe a positive and significant association of diffusion indexes with the market, and highlight that limiting the analysis to systemic events could be misleading and incomplete, while we suggest a combined use of systemic and non-systemic MJs. We additionally establish a relationship between detected MJs and market-level news.

Our results have important implication not only for asset allocation and hedging, but also in asset pricing. Regarding this last point, by including our diffusion indexes to the CAPM model, we prove that DID and DII capture common variation in stock returns that is missed by the market factor. This advocates to employ mulivariate jump information to build a factor capturing the cross-sectional jump risk, which could then be added, e.g., to the 5-factor model we propose in Chapter 1.

Conference presentations

The paper corresponding to Chapter 1, *"Jump risk and pricing implications"*, will be the object of a presentation during the 10^{th} International Conference on Computational and Financial Econometrics at the University of Seville (Spain, 9-11 December 2016), and at the 7^{th} Italian Congress of Econometrics and Empirical Economics, ICEEE 2017, in Messina (Italy, 25-27 January 2017).

Chapter 1

Jump risk and pricing implications

A joint work with Prof. Massimiliano Caporin[1], and Prof. Walter Distaso[2].

This paper identifies a new common risk factor in stock returns related to the fear of future jumps. The factor can be added to standard asset-pricing models leading to a five-factor model which is directed at capturing the size, value, profitability, momentum and fear in stock returns. The model outperforms the four-factor model of Carhart (1997).

1.1 Introduction

Finance literature has been focusing for decades on the patterns followed by asset returns. A cornerstone in the description of market returns and risks is the asset-pricing model of Sharpe (1964), Lintner (1965), Mossin (1966) and Black (1972). According to the CAPM the market portfolio is mean-variance efficient in the sense of Markowitz (1959). The efficiency implies that there is a linear relation between expected returns and their market betas. Another consequence of the efficiency is that market betas are sufficient to explain the cross-section of expected returns, in other words that only systematic market risk, measured by the beta of an asset, should be priced. Despite several studies, for example Reinganum (1981), Lakonishok and Shapiro (1986), and Fama and French (1992), challenge the ability of the CAPM to explain the cross-section of expected stock returns, the CAPM is still a common pricing scheme in finance. Fama and French (1993) propose an extension of the one-beta CAPM starting from the observation that average stock returns are not positively related to market

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betas. They suggest that stock risks are multidimensional and include other two sources of risk: one proxied by *size* (or market value, or market equity price, or ME), and one proxied by *value* (or BE/ME), defined as the ratio of book value (or BE) to market value (or ME). Carhart (1997) showed that there are patterns in average returns related to the momentum factor (or *MOM*) based on Jegadeesh and Titman (1993).

In this paper we examine if perceptions of price uncertainty, the fear of rough price movements, constitute a common risk factor in returns and how they impact on asset prices. Uncertainty plays a primary role in economics, finance, and decision sciences and may help explaining the observed empirical“fat tails" in stock returns. A possible explanation for the leptokurtosis might be found in the presence of discontinuous variations in the price process. There is evidence, Ball and Torous (1983), Jarrow and Rosenfeld (1984), and Jorion (1988) among others, that stock prices show sudden but infrequent movements of large magnitude, that are commonly known as jumps. The first models that incorporate jumps in the dynamic of stock prices are those of Press (1967) and Merton (1976), and several subsequent studies prove that such a structure is necessary to fit the observed prices. Evidence of this are provided for the option market by Ball and Torous (1985), Naik and Lee (1990), Bakshi et al. (1997), Duffie et al. (2000), Andersen et al. (2002), and Eraker et al. (2003) among others. In particular Pan (2002) shows the presence of a priced aggregate jump risk in option prices. The existence of the premium is analyzed using the Bates (2000) model who extends the Heston (1993) stochastic volatility model by incorporating jumps. More recently, Bollerslev et al. (2016) study the contribution of jump risk in explaining the cross-section of expected stock returns. They extend the CAPM by decomposing the market beta into three separate parts, continuous, discontinuous and overnight, and find that there are significant risk premiums, in the cross-section, for discontinuous and overnight movements and that their estimated betas are generally higher that the corresponding continuous betas.

Our work differs from these studies since it analyzes the presence of a jump risk factor and is related to Yan (2011) and Cremers et al. (2015). Both studies use option prices because they contain forward-looking information that helps matching the time-varying nature of the jump risk. Focusing on the behavior of put prices around the crash of October 1987, Bates (1991) finds that jump expectations in stock market returns change over time. Also Christoffersen et al. (2012) find that jump intensity is significantly time-varying and that discrete-time models have better performances when incorporating jumps. Yan (2011) proxies the average jump size using the slope of option implied volatility smile and finds that stocks with high positive (negative) slopes more probably will have large positive (negative) jumps in the future. He also proves the existence of a negative relation between average jumps sizes and expected stock returns. The latter, Cremers et al. (2015), study the effect of jump risk using factor-mimicking portfolios constructed by straddles. Their results show that stocks with high sensitivity to jump risk present lower expected returns and that the aggregate stock market jump risk is significantly priced in the cross-section of returns.

While the precedent researchers justify their use of the option market recalling the synchronized information content of stock and option markets, we make a step further by focusing directly on the former. In order to maintain a forward-looking perspective, we focus on the market expectation of future jumps encoded in stock prices. To this end we follow Chan and Maheu (2002) who propose a model for stock returns with time-varying conditional jump intensity. Once the model is estimated, using past stock returns, it is possible to compute the expected jump component as a function of the mean of the conditional jump size and the time-varying jump intensity. The factor-mimicking portfolio for the jump risk is then constructed in the same way as Fama and French (1993) build the portfolio mimicking the BE/ME risk source.

Our work is also related to the literature about rare disasters and tail risk. Relative to the former it is important to notice that, even if jumps and disasters show various similarities, jumps happen more frequently than disasters. About the latter, Bollerslev and Todorov (2011) and Gabaix (2012), among others, show that an important part of the aggregate equity risk premium and embedded temporal variation may be due to jump tail risk. According to Bollerslev and Todorov (2011) compensation for rare events accounts for a large fraction of the average equity and variance risk premia. By using high-frequency intraday data and short maturity out-of-the-money options they show that the market usually incorporates the possible occurrence of rare disasters in the way it prices risky payoffs. Furthermore, they discuss how the fear of that events account for a large part of the historically observed premia.

The main contribution of this paper is the construction of a jump factor (or JF) capturing investors fear of future sudden and sharp price movements, using a dataset of considerable dimension: 89 years of assets prices and more than 24,000 stocks. Our factor is modeled using all CRSP stocks over the 1925-2014 sample period and is the return differential between the high and low expected jump component quantile portfolios. The simple plot of JF time series, makes clear its ability of capturing jump forecasts changes over time. This is important since it proves that our factor is able to reflect the jump probability evolution over time. The factor average return is significantly different from zero and it is about 1.5% per year. To empirically test its relevance in explaining time-series and cross sectional return variations, we add the factor to the Carhart 4-factor model. Its importance is suggested by the high mean and variance values and the low correlations with the other factors. Time series regressions results, both on the full sample period and on sub-periods, confirm that the JF captures common variation in stock returns. We compute factor loadings and coefficients of determination using as dependent variables both individual stocks and portfolios. In all cases we observe that the estimated jump factor loadings are statistically different from 0, at conventional levels, in a relevant number of regressions and that their values are also large. For the former minimum and maximum fractions of slopes on JF that are more than 1.645 standard errors from 0 are both observed in the sub-periods and are respectively 12% and 60%. For all time-series regressions we also obtain important $R^2_{adjusted}$ increases when our jump factor is added to the asset-pricing model. Factor loadings and R^2 results suggest that the expected jump component proxy, in stock returns, for sensitivity to a common risk factor.

We also test the usefulness of adding the JF to the Carhart (1997) asset-pricing model. Our empirical investigation makes use of the one-month abnormal returns from the Fama-French model which become the dependent variables in time-series regressions that investigate if JF and MOM can be considered missing factors in the 3-factor model. Estimated factor loadings show, especially when focusing on short time-windows, similar outcomes for the two factors. The highly positive significance slope results, both for JF and MOM, justify the extension of the Carhart (1997) model with the inclusion of our new factor.

Lastly, we compute the risk premiums associated with the five factors of our 5-factor model using two approaches, Black et al. (1972) and Fama and MacBeth (1973), and applying the Hou and Kimmel (2006) extrapolation correction. The risk premiums are not always positive but also not significantly different from their factor means. JF risk premiums range from 0.07 to 0.17, while the factor mean monthly percentage excess return is 0.12. Differently from the premiums for the Carhart (1997) factors that are in most of the cases statistically different from 0, using standard confidence levels, JF premiums are more than 1.645 standard errors from 0 in just one case. *A possible explanation for the low JF significance may be found in the frequency we use to estimate the parameters of the Chan and Maheu (2002) model. To improve the JF capability to reflect the short time nature of the jumps, we are repeating our analysis using monthly estimated parameters instead of the yearly estimated parameters of this paper. An update of the paper is in progress.*

The paper proceeds as follows. Section 1.2 presents the models implied to study the presence of a priced risk factor in stock returns. It also describes the construction of our jump risk factor-mimicking portfolio. Section 1.3 presents and describes summary statistics for the jump factor and other factors returns. It also introduces and describes some portfolios that will be used for the subsequent analysis. Section 1.4 presents our main results on the ability of the jump factor to capture common variation in stock returns. It also describes how well a model including our jump factor explains average returns in the dependent portfolios. Section 1.5 presents the same results of section 1.4 but for sub-periods. Section 1.6 investigates if the jump factor is a missing factor in a standard asset-pricing model. Section 1.7 presents the estimated risk premiums. Section 1.8 concludes.

1.2 Jump factor and relative asset-pricing model

1.2.1 Modeling returns with jumps

Our goal is the construction of a risk factor reflecting the jump impact on returns in a forward-looking perspective. This requires a preliminary step: we must recover a measure of jump sensitiveness. To this end, we refer to Chan and Maheu (2002) who propose a model for stock returns with GARCH volatility and time-varying conditional jump intensity. In the vast literature focusing on jumps, only few works analyze the presence of a jump risk factor. They recover a Jump Factor from a specific option database and check if the jump component is priced by the market (see for example Yan (2011) and Cremers et al. (2015)). The model of Chan and Maheu (2002) differs from the previous ones by the possibility of recovering asset-specific jump expectations and, in turn, to use they for pricing, at the market level, the jump risk.

According to Chan and Maheu (2002) we model stock returns including a jump component, Z_t:

$$R_t = \mu + \sum_{i=1}^{l} \phi_i R_{t-i} + Z_t + \varepsilon_t, \tag{1.1}$$

where R_t is the daily stock log return, $\Phi_t = \{R_t, ..., R_1\}$ is the information set at time t, $Z_t = \sum_{k=1}^{N_t} Y_{t,k}$ is the sum of the conditional jump sizes $Y_{t,k}$, and ε_t follows a conditionally normal density with GARCH error. The conditional jump size, given the information set Φ_{t-1}, is normally and independently distributed: $Y_{t,k}|\Phi_{t-1} \sim N(\Theta, \Delta)$ with constant mean and variance. The jump component is obtained by summing up the sizes of the jumps arriving between $t-1$ and t, where the number of jumps, N_t, is distributed as a Poisson random variable with parameter $\lambda_t > 0$. Recalling that the mean and variance of a Poisson random variable both equal its parameter, it is easy to compute the conditional mean of the counting process

$$\lambda_t \equiv E[N_t|\Phi_{t-1}] \equiv Var[N_t|\Phi_{t-1}].$$

Moreover, λ_t, the conditional jump intensity, follows an approximate autoregressive moving average (ARMA) process

$$\lambda_t = \lambda_0 + \sum_{i=1}^{r} \rho_i \lambda_{t-i} + \sum_{i=1}^{s} \gamma_i \xi_{t-i} \tag{1.2}$$

where ξ_t is an innovation equal to the difference between

$$\xi_t \equiv E[N_t|\Phi_t] - \lambda_t \equiv E[N_t|\Phi_t] - E[N_t|\Phi_{t-1}] \tag{1.3}$$

that is, the expectation of the Poisson random variable conditional to time t and its expectation conditional to time $t-1$.

In this framework, it is possible to show that, conditional to the information set, the expected jump component equals the mean of the conditional jump size, Θ, times the time-varying jump intensity, λ_t:

$$E[Z_t|\Phi_{t-1}] = \Theta\lambda_t. \tag{1.4}$$

While the former is a constant value, the latter follows an endogenous autoregressive process. We can not directly recover from the market the elements of interest, Θ and λ_t, that are, consequently, estimated. Chan and Maheu (2002) model conditional jump intensity and size as function of observables and allow simple maximum likelihood estimation. We refer the reader to Chan and Maheu (2002) for further details about the model and the estimation approach. Differently from Chan and Maheu (2002) we set the parameters of $Y_{t,k}$ to be time invariant. The expected jump is a key element in our approach, it represents the basis for constructing the jump factor.

1.2.2 Jump factor construction

Our large dataset includes all the Center for Research in Security Prices (CRSP) assets with share code equal to 10 or 11, which covers NYSE and AMEX stocks until 1973 and adds NASDAQ stocks after that date. The sample includes a total of 24,122 equity over 89 years, from December 1925 until December 2014. We obtain the parameters of the model using non-overlapping rolling windows with a size of one year each. Estimations take place every year at the end of June, in order to stick with the timing that Fama and French (1993) use to construct the mimicking portfolios for the size and BE/ME risk sources. We run about 250,000 estimations where each of them uses previous year simple daily returns as defined by CRSP.[3]

In order to capture the temporary presence of serial correlation, we consider an AR(2) process in equation 1.1 by imposing $l = 2$. The resulting equation does not change in time and across assets,

$$R_t = \mu + \phi_1 R_{t-1} + \phi_2 R_{t-2} + Z_t + \varepsilon_t$$

We also use an ARMA(1,1) process for the jump intensity; this allows us to rewrite equation 1.2 as

$$\lambda_t = \lambda_0 + \lambda_{t-1}(\rho_1 - \gamma_1) + \gamma_1 E[N_{t-1}|\phi_{t-1}]$$

[3] CRSP daily return: $R_t = \frac{Price_t \times PriceAdjustmentFactor_t + CashAdjustment_t}{Price_{t-1}} - 1$

Lastly, it is important to point out that we focus on the one-step-ahead values of the expected jump component in equation 1.4. In this way we obtain the asset jump expectation for the following day.

Since each asset can show either positive or negative jumps, the range of values of the expected jump component (equation 1.4) spans from negative to positive values. This comes from the sign of Θ that can be either positive or negative while λ_t is always positive. Figure 1.1 shows, for each estimation date, the number of assets with positive and negative expected jump component. Even if the dimension of the two groups never differ too much, it is possible to observe that in 73% of the cases the number of negative expectations (expected jumps with negative sign) overcomes the number of positive expectations (expected jumps with positive sign).

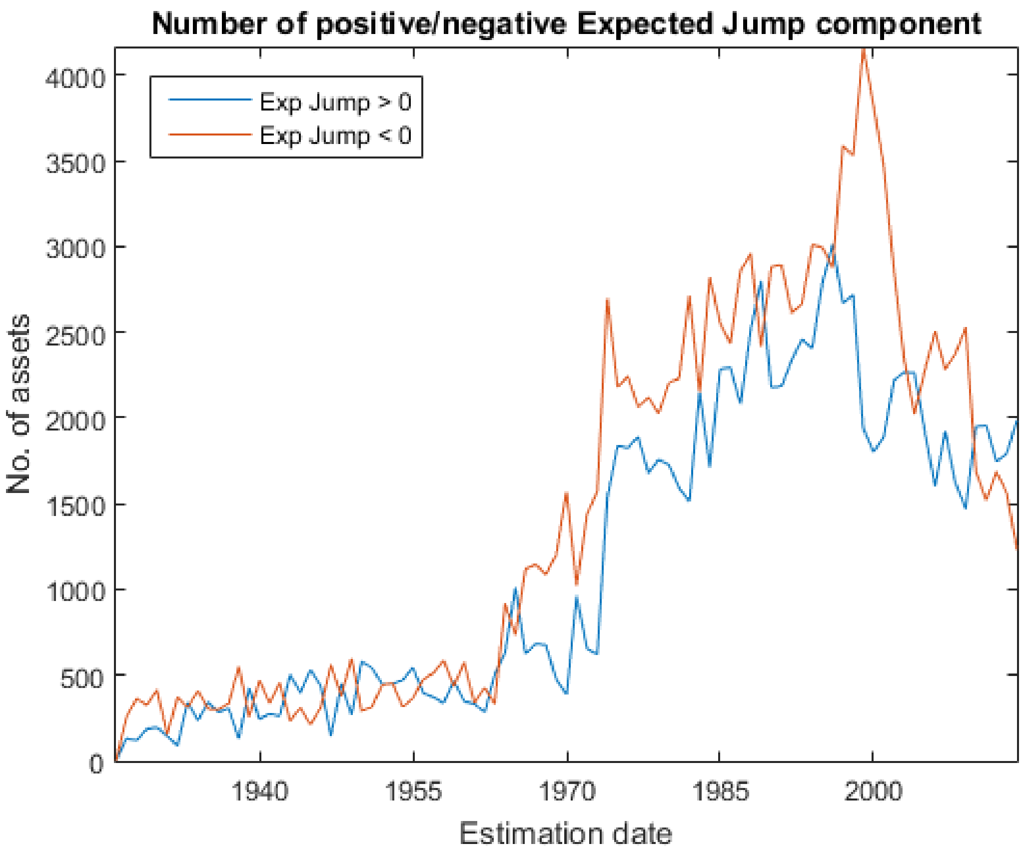

Figure 1.1 Expected Jump Factor sign. For each estimation date, from June 1926 till June 2014, it reports the number of assets for which the expected jump component ($\Theta\lambda_t$) is positive or negative. The estimation of the parameters uses non-overlapping rolling windows with a size of one year. Estimations take place every year at the end of June and use previous year simple daily returns as defined by CRSP.

We then propose to recover a Jump Factor (or JF) as a factor-mimicking portfolio for the jump risk. For its construction we follow the approach that Fama and French (1993) use to build the SMB (Small [market capitalization] Minus Big) and HML (High [book-to-market ratio] Minus Low) factors. In fact, both are portfolios mimicking risk sources, the size and BE/ME respectively. In June of each year t from 1926 to 2014, we sort all NYSE stocks on CRSP by *size* to determine the median breakpoint. The subsequent step is the allocation of all NYSE, Amex, and NASDAQ stocks to the two portfolios, Small and Big, according on the NYSE breakpoint. At the same dates we also split the NYSE, Amex, and NASDAQ stocks into three expected jump groups using $E[Z_t|\Phi_{t-1}]$: Low, Medium, and High. Using the ranked values of Expected jump component for NYSE stocks, we determine the breakpoints for the bottom 30%, the medium 40%, and the top 30%. The choice of forming three groups is arbitrary, but we have no reason to think that the tests are sensitive to this choice. From the intersection of the two size and the three expected jump groups we construct six portfolios: S/L, S/M, S/H, B/L, B/M, and B/H; adopting the same notation of Fama and French (1993). The re-balance of the portfolios takes place at the end of June of each year, and from July of year t to June of year $t+1$ we calculate the monthly value-weighted returns for four of the six portfolios - S/L, S/H, B/L, and B/H. The Jump factor (JF) is then constructed as the monthly difference between the average of the returns on the two low expected jump portfolios and the average of the returns on the two high expected jump portfolios:

$$JF = (1/2\, r_{Small\&High} + 1/2\, r_{Big\&High}) - (1/2\, r_{Small\&Low} + 1/2\, r_{Big\&Low})$$

Coherently with Fama and French (1993), we believe the influence of the size should be limited, therefore, we focus only on the different return behaviors of low and high expected jump stocks.

To understand which is the proportion of assets that present a negative (positive) expectation with respect to the total number of assets, we report in Figure 1.2 the composition of the Low and High expected jump portfolios at each estimation date. It is clear that assets with negative expected jump component almost exclusively belong to the Low portfolio. Similarly, almost all assets with positive expected jump component flow into the High portfolio. The two portfolios, consequently, represent two opposite strategies: stocks with negative expected jumps versus stocks with positive expected jumps. From a risk-premium perspective, the sign of the expectation is useful in forecasting and understanding the sign of the factor loading. Investors demand a positive risk premium, measured as the extra return relative to the risk-free rate, for investing in risky assets. To allow the JF premium to be positive the signs of factor and corresponding factor loading should coincide. Since the JF covers negative and positive values we expect also the factor loading to assume positive and negative values.

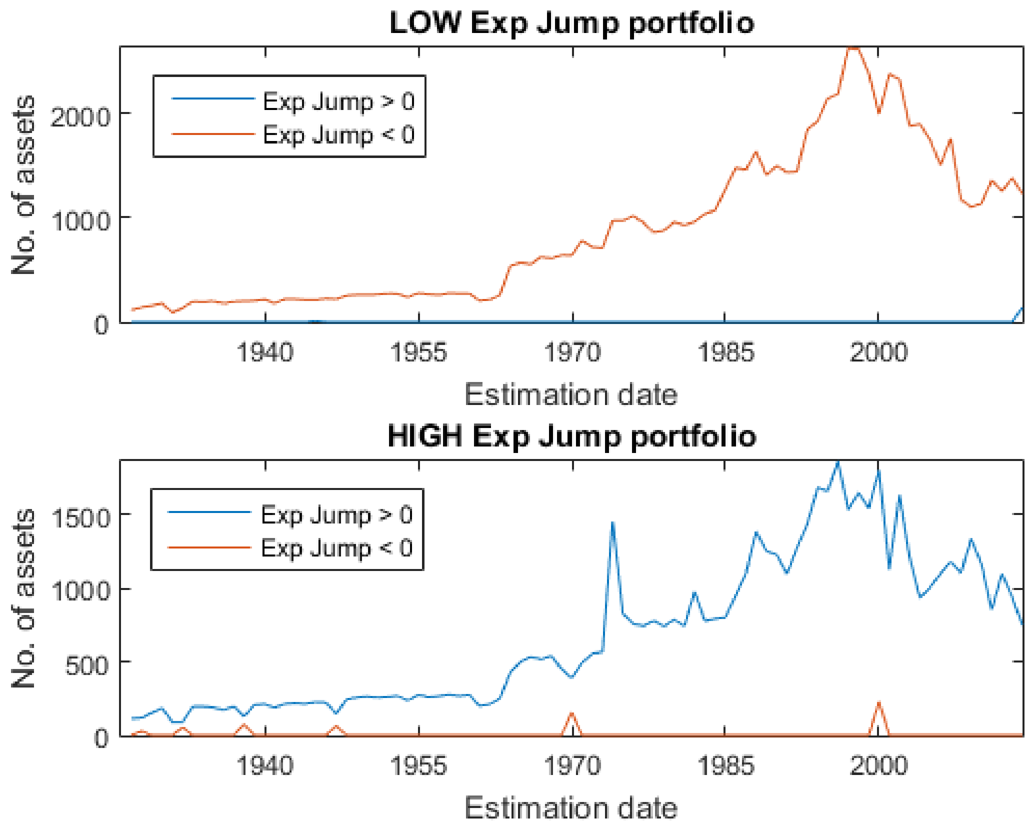

Figure 1.2 JF portfolios composition. For each estimation date, from June 1926 till June 2014, it shows the number of assets with positive and negative expected jump component ($\Theta\lambda_t$). The top panel plots the total number of assets with positive and negative expected jump component, considering only the stocks that are in the Low expected jump portfolio (expected jump component $<$ bottom 30% breakpoint). Similarly, the bottom panel plots the same indicators but taking into account only the stocks that are in the High expected jump portfolio (expected jump component $>$ top 30% breakpoint). The estimation of the parameters uses non-overlapping rolling windows with a size of one year. Estimations take place every year at the end of June and use previous year simple daily returns as defined by CRSP.

Finally, Figure 1.3 shows the time series of the JF. The top panel compares the JF and its rolling mean, where the latter is computed using the last twelve monthly values of the JF. The bottom panel, instead, reports JF values that are preceded by a Jump Factor value of the same sign, that is we focus on the JF runs. The goal is to obtain a clear image of the JF clusters.

The plots not only make clear the tendency of the jump factor values to be clustered by sign, but also show some peculiar behaviors associated with the JF levels. We identify the processes in charge of them by focusing on the behavior of the jump probability. Poisson jump model, introduced by Press (1967), assumes that a Poisson distribution leads the number of events that result in price movements. The subsequent extensions of the jump model shared the assumption of a constant Poisson distribution governing the jump probability.

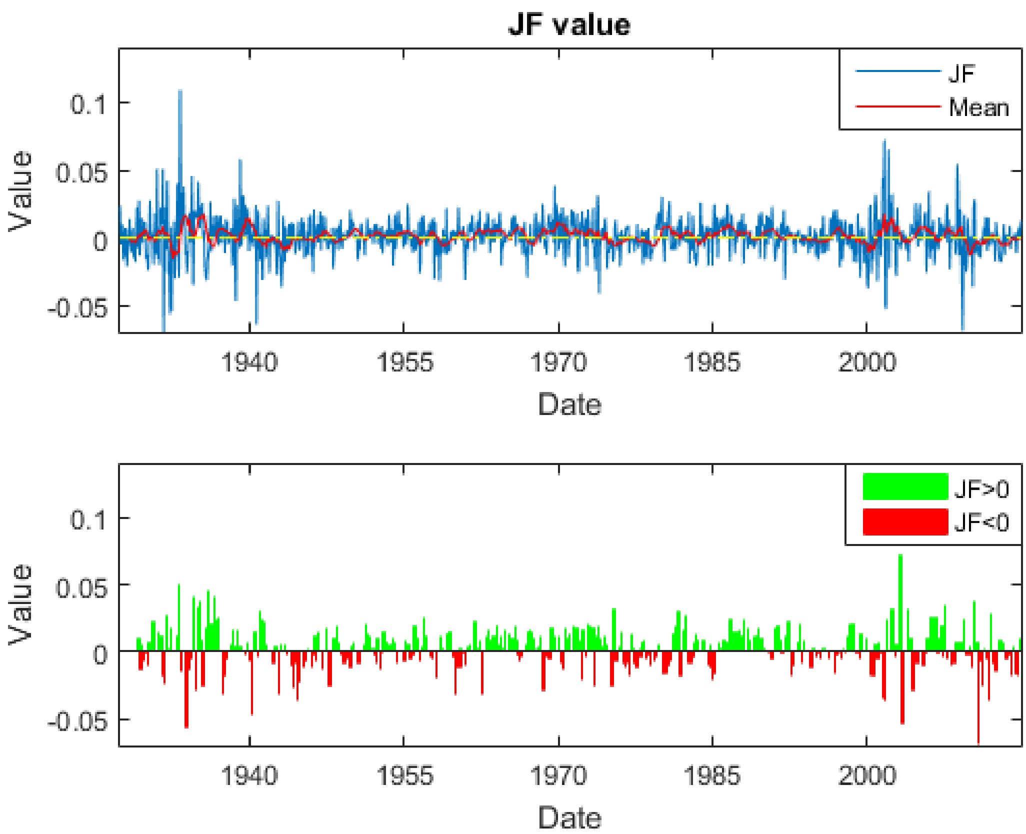

Figure 1.3 JF historical values. In June of each year t from 1926 to 2014, all NYSE, Amex, and NASDAQ stocks are allocated to the two size portfolios (Small and Big) and the three expected jump portfolios (Low, Medium, and High) according on the NYSE breakpoints. From their intersection we construct six portfolios: S/L, S/M, S/H, B/L, B/M, and B/H. The re-balance of the portfolios takes place at the end of June of each year, and from July of year t to June of year $t+1$ we calculate the monthly value-weighted returns for four of the six portfolios - S/L, S/H, B/L, and B/H. The Jump factor (JF) is then constructed as the monthly difference between the average of the returns on the two low expected jump portfolios and the average of the returns on the two high expected jump portfolios. Top panel plots the time series of the JF and its mean. For the JF mean we use non-overlapping rolling windows with a size of one year. The calculation of the mean takes place at the end of every month and uses previous year monthly JF values. The bottom panel plots the monthly Jump Factors that preceded by a Jump Factor of the same sign.

The acceptability of this hypothesis has been undermine by Bates (1991) who verified the presence of a systematic behavior in expected jumps before the crash in 1987. This proves what conventional wisdom would suggest: jump probability changes over time. Following this intuition Chan and Maheu (2002) assumes that the conditional jump intensity follows an approximate autoregressive moving average form. We expect the transmission of this property to the factor via the time-varying jump intensity, λ_t.

The top panel seems to corroborate our prevision since the factor and its mean assume values far from zero in periods of market turmoils as the 1929-1932 crises and the market crash in 2007-2009. It is reasonable to forecast that the fear of future jumps increases and is more relevant in periods of greater market uncertainty. The empirical findings of Chan and Maheu (2002) suggest an explanation to this behavior: autocorrelation in the conditional jump intensity is positive and persistent, which means that high probability of few (many) jumps today is generally followed by a high probability of few (many) jumps tomorrow. The JF historical behavior suggests that its use could be relevant especially in periods of market turmoils.

1.2.3 A model including Jump Factor

We now briefly describe and evaluate traditional asset pricing models with a double objective: first of all, we are interested in verifying our postulated impact of the JF on returns and the sign of its factor loading; secondly, we are interested in detecting potential changes on other more traditional factors, both in terms of their loadings as well as for their significance. We empirically assess this goal by starting from a model that can work as benchmark due to its recognized performances: the Carhart (1997) 4-factor model. We will then extend the Carhart (1997) model by adding the JF returns and thus putting forward a 5-factor model. The relative performance of the two models is studied by considering the performance estimates on portfolios formed by NYSE, Amex, and NASDAQ stocks. This allows us not only to follow the existing standard in literature but also to make it perfectly comparable with the reference works in this field of study (see for example Fama and French (1993) and Fama and French (2015)).
A following section will discuss a related topic, naming the estimation and relevance risk premiums for both the JF and the traditional factors.

The 5-factor model is designed as an extension of the Carhart (1997) 4-factor model where the innovation comes in as a factor capturing the market expectation of future price jumps. In line with the common interpretation of factor asset-pricing models, we can consider our 5-factor model as a performance-attribution model where coefficients and premia, on the factor-mimicking portfolios, represent the proportion of mean return due to five elementary strategies. For the 4-factor model these strategies cover stocks with high or low betas, stocks with large or small market capitalization, value or growth stocks, and one-year return momentum or contrarian stocks. Our new factor represents, instead, the elementary strategy of high versus low expected jump stocks.

The Carhart (1997) 4-factor model is, on its own, an evolution of the Fama and French (1993) 3-factor model. The 3-factor model is designed to capture the relation between average return and *Size* and average return and book-to-market equity. Its time-series regression representation is:

$$R_{i,t} - R_{F,t} = \alpha_i + \beta_i MKT_t + \beta_{SMB,i} SMB_t + \beta_{HML,i} HML_t + e_{i,t} \tag{1.5}$$

where $R_{i,t}$ is the return on a security or portfolio i, $R_{F,t}$ is the risk-free return, $MKT_t = (R_{M,t} - R_{F,t})$ is the excess return on a value weighted market portfolio, SMB_t is the return on a value-weighted factor-mimicking portfolio for *Size*, HML_t is the return on a value-weighted factor-mimicking portfolio for book-to-market equity, and e_{it} is a zero-mean residual.

Carhart (1997) adds to the above model a factor, MOM, aimed at describe the one-year momentum in stock returns. It enters equation (1.5) as a value-weighted factor-mimicking portfolio for the one-year momentum as it should capture the excess return of past winning over past losing stocks. Updating the model with this information leads to the 4-factor model:

$$\begin{aligned} R_{i,t} - R_{F,t} = {} & \alpha_i + \\ & \beta_i MKT_t + \beta_{SMB,i} SMB_t + \beta_{HML,i} HML_t + \beta_{MOM,i} MOM_t + e_{i,t} \end{aligned} \tag{1.6}$$

Our contribution to the existing models centers on the inclusion of a measure of jump sensitiveness, JF_t, that is the difference between the returns on portfolios of stocks with high and low jump expectations, the 5-factor model thus becomes:

$$\begin{aligned} R_{i,t} - R_{F,t} = {} & \alpha_i + \\ & \beta_i MKT_t + \beta_{SMB,i} SMB_t + \beta_{HML,i} HML_t + \beta_{MOM,i} MOM_t + \beta_{JF,i} JF_t + e_{i,t} \end{aligned} \tag{1.7}$$

If the exposure to the five factors, β_i, $\beta_{SMB,i}$, $\beta_{HML,i}$, $\beta_{MOM,i}$ and $\beta_{JF,i}$ capture all variation in expected returns, the intercept α_i in 1.7 is zero for all securities and portfolios i.

1.3 Jumps factors and other factors returns

In this section we present and discuss summary statistics for some factors of interest, with particular attention to the jump factor. The structure of our 5-factor model (described in the previous section) suggests the factors we are interested in: the market portfolio of stocks (MKT) and the mimicking portfolios for *size* (SMB), book-to-market equity (HML), momentum (MOM), and expected jump (JF). The following subsection illustrates the characteristics of these factors, focusing on the comparison between traditional factors and JF.

The subsequent subsection, instead, introduces some portfolios of interest for the analysis in the remainder of the paper. By combining different couples of risk sources, it is possible to construct sorted portfolios whose returns can be used to further investigate the importance of the JF. In particular we are interested in portfolios built by sorting assets on size and book-to-market equity and size and expected jump component.

1.3.1 JF and other risk factors

Tables 1.1 and 1.2 show the summary statistics for the monthly MKT, SMB, HML, MOM and JF returns. For each factor portfolio, of our 5-factor model, we report standard descriptive statistics.

Table 1.1 Factors summary statistics. Summary statistics for the monthly factor returns in percent. RF is the one-month Treasury bill return. MKT is the market proxy. SMB (small minus big) and HML (high minus low) are Fama and French's factor-mimicking portfolios for size and book-to-market equity. MOM and JF are the factor-mimicking portfolio respectively for one-year momentum and expected jump component.

Factor portfolio	Mean monthly return %	Standard deviation	t-statistic for Mean=0
RF	0.28	0.25	36.38
MKT	0.65	5.40	3.94
SMB	0.22	3.23	2.22
HML	0.40	3.54	3.65
MOM	0.67	4.74	4.62
JF	0.12	1.46	2.66

Volatilities and correlations give a perspective about the model capability of explaining time-series variations. For the former we can observe that the JF variance is the smaller among the factor-mimicking portfolio variances but it is still relatively high. For the latter, instead, factor-mimicking portfolio correlations, both among each other and with the market proxy, are low. In addition, all the correlations among factors and Risk-Free rate are not

Table 1.2 Factors correlations. RF is the one-month Treasury bill return. MKT is the market proxy. SMB (small minus big) and HML (high minus low) are Fama and French's factor-mimicking portfolios for size and book-to-market equity. MOM and JF are the factor-mimicking portfolio respectively for one-year momentum and expected jump component.

Factor	Cross correlations					
	RF	MKT	SMB	HML	MOM	JF
RF	1.00					
MKT	-0.07	1.00				
SMB	-0.05	0.33	1.00			
HML	0.02	0.23	0.11	1.00		
MOM	0.05	-0.34	-0.15	-0.40	1.00	
JF	0.03	-0.23	-0.20	-0.08	0.28	1.00
	P-value for corr=0					
	RF	MKT	SMB	HML	MOM	JF
RF	0.00					
MKT	0.03	0.00				
SMB	0.08	0.00	0.00			
HML	0.62	0.00	0.00	0.00		
MOM	0.08	0.00	0.00	0.00	0.00	
JF	0.32	0.00	0.00	0.01	0.00	0.00

significantly different from zero at the 99% confidence level. Focusing on the JF we have only three correlations for which we can not reject the null hypothesis (correlation=0) at 1% significance level, and their values run from a minimum (in absolute value) 0.2 with SMB to a maximum 0.28 with the momentum factor. The combined observation of high variances and low correlations suggests that the 5-factor model, and our new factor, can describe sizeable time-series variation. In addition, the low values of the cross-correlations indicate that multicollinearity should not affect the estimation of the factor loadings.

Turning now our attention to the first moment, we observe a range of values that goes from 0.67% per month for the momentum to a still considerably high 0.12% per month for the JF. In a time-series regression approach, they correspond to the average premiums per unit of risk (slope) and, from a statistical point of view, they are all significantly different from zero (1% significance level). The minimum value is reached by the JF but, from an investment perspective, it is still large (about 1.5% per year). The high value of the JF mean also suggests that it explains a considerable part of the mean return variation on stock portfolios, at a cross-sectional level.

Summing up, results suggest that the 5-factor model can explain much of the variation in returns both in time and cross-section.

1.3.2 JF and sorted portfolios

It is possible to use the five risk sources of our 5-factor model, alone or in combination, to form portfolios of stocks. When just two of the five characteristics are taken contemporaneously under consideration, we obtain ten different sets of portfolios. Of all the possible combinations, we focus on those that are more relevant for our case study: stock portfolios formed according to size and book-to-market equity, and stock portfolios formed on the basis of size and expected jump.

The size-BE/ME portfolios were first introduced by Fama and French in 1993 who also proposed their use as dependent variables in time-series regressions. They not only produce a wide range of average excess returns, but also allow to study if SMB and HML capture common factors in stock returns. Similar reasons drive the choice of the second group of portfolios.

When used as dependent variables, our portfolios values define the range of returns that competing sets of risk factors must explain. In our case, these competing factors, are those forming the 4-factor and the 5-factor models since our interest is in understanding the importance and relevance of the JF.

Size-BE/ME portfolios

According to Fama and French (1993) portfolios formation takes place in June of each year, from 1926 until 2014, by the intersection of size and BE/ME quintiles. The allocation of NYSE, Amex, and NASDAQ stocks into five size quintiles and five book-to-market quintiles, makes use of the NYSE breakpoints. Value-weighted monthly excess returns are then calculated from July of year t to June year $t+1$, when portfolios are reformed.

Table 1.3 shows the average monthly excess returns for the 25 size-BE/ME portfolios. The 25 stock portfolios produce a wide range of average excess returns, from 0.58% to 1.38% per month. The patterns in average returns confirm the presence of size and value effects: controlling for book-to-market returns tend to decrease from small to big stocks, and controlling for *size* average returns tend to increase with BE/ME. The only exceptions for the size effect are in the first two columns: the first does not show a clear relation between *size* and average return, while in the second the only outlier is the low average return for the microcap. All but one average returns are more than two standard errors away from 0. The low-small portfolio shows a high standard deviation (12.36% per month) that makes its average return not significantly different from 0. This is a well-known problem already underlined by Merton (1980). Lastly, note that in each column volatility falls from small to

Table 1.3 Excess returns size-BE/ME portfolios. Value-weighted monthly percent excess returns of 25 stock portfolios formed on size and book-to-market. Portfolios are formed in June of year t, from June 1925 until June 2014, by the intersection of size and BE/ME quintiles. The allocation of NYSE, Amex, and NASDAQ stocks into five size quintiles and five book-to-market quintiles, makes use of the NYSE breakpoints.

	Book-to-market equity (BE/ME) quintiles									
Size quintile	Low	2	3	4	High	Low	2	3	4	High
	Means					Standard Deviations				
Small	0.58	0.71	1.02	1.18	1.38	12.36	9.91	9.08	8.40	9.40
2	0.63	0.91	1.02	1.08	1.26	8.03	7.55	7.32	7.51	8.76
3	0.71	0.90	0.96	1.01	1.17	7.47	6.52	6.57	6.96	8.53
4	0.72	0.75	0.87	0.96	1.05	6.29	6.15	6.51	6.87	8.72
Big	0.62	0.64	0.68	0.69	0.94	5.40	5.34	5.71	6.55	8.50
	t-statistic for Mean=0									
Small	1.53	2.33	3.68	4.59	4.79					
2	2.54	3.94	4.53	4.70	4.67					
3	3.10	4.50	4.74	4.75	4.45					
4	3.74	3.95	4.36	4.56	3.93					
Big	3.71	3.89	3.85	3.44	3.59					

big stocks, with the only exception of the High BE/ME portfolios (last column). For them we observe, in all but the small portfolio case, stable standard deviations with a value of about 8.6% per month.

Size-expected jump portfolios

The modeling of the size-expected jump portfolios is much like the six expected jump portfolios discussed in subsection 1.2.2 Using only NYSE stocks, we compute the breakpoints for *size* and *expected jump* that we then use to allocate NYSE, Amex, and NASDAQ stocks into five *size* groups and five *expected jump* groups in June of each year. From the interception of the groups we construct the 25 portfolios for which value weighted excess returns are computed monthly from July of year t until June of year $t+1$.

Table 1.4 reports average monthly excess returns for the 25 size-expected jump portfolios. The range of average excess returns covered by the stock portfolios goes from 0.58% to 2.10% per month. Similarly to Table 1.3, there is a negative relation between size and average return, when controlling for expected jump. There are only three exceptions to this general rule: the forth average return in column one is too high and microcap average return in columns two and four is too low. When controlling for size there is no evidence of a clear pattern between expected jump and average excess returns. All average excess returns are more than two standard errors away from 0, and correspondent standard deviations fall from small to big stocks when considering a single column. Focusing on the first three rows, we observe very high values for the standard deviations, and in particular for the microcaps. High volatility is a characteristic of the small stocks already observed in Table 1.3. In this case, however, standard deviations for small portfolios are particularly high: from 11.93 to 17.99. In all but the last row, we have standard deviations that are, with two exceptions, more than 22% higher in the size-expected jump case with respect to the $size - BE/ME$ case.

Table 1.4 Excess returns size-expected jump portfolios. Value-weighted monthly percent excess returns of 25 stock portfolios formed on size and expected jump component. Portfolios are formed in June of year t, from June 1925 until June 2014, by the intersection of size and expected jump quintiles. The allocation of NYSE, Amex, and NASDAQ stocks into five size quintiles and five expected jump quintiles, makes use of the NYSE breakpoints.

	expected jump (JF) quintiles									
Size quintile	Low	2	3	4	High	Low	2	3	4	High
	Means					Standard Deviations				
Small	1.51	1.24	1.88	1.22	2.10	13.43	11.93	17.99	13.25	17.77
2	1.38	1.41	1.41	1.25	1.46	12.16	12.44	13.47	11.29	11.33
3	0.89	1.35	1.15	1.20	1.25	10.17	11.25	9.99	9.05	10.68
4	0.94	0.90	0.99	0.99	1.06	9.16	8.33	7.98	8.41	8.45
Big	0.58	0.68	0.68	0.68	0.66	5.88	6.02	5.46	5.53	5.62
	t-statistic for Mean=0									
Small	3.45	2.89	3.10	2.62	3.53					
2	3.64	3.63	3.30	3.57	4.07					
3	2.83	3.88	3.70	4.30	3.80					
4	3.32	3.51	4.01	3.82	4.08					
Big	3.21	3.65	4.06	4.00	3.83					

1.4 Common variation in stock returns

We turn now to the asset-pricing tests: we use time series regressions to analyze if the JF captures common variation in stock returns. In a time-series regressions framework, variables related to average returns must proxy for sensitivity to common (shared and thus undiversifiable) risk factors in returns, when assets are priced rationally. Slopes and R^2_{adj} values give evidence if the JF captures shared variation in stock returns not explained by other factors. To judge the improvements provided by our new factor we employ three different sets of dependent variables: 25 size-BE/ME portfolios, 25 size-expected jump portfolios, and all CRSP single assets with share code 10 or 11.

1.4.1 25 size-BE/ME portfolios

The role of the JF is analyzed in two steps. We examine (a) regressions that use the four-factor model (equation 1.6) and (b) regressions that use the five-factor model (equation 1.7). Table 1.5 shows the results using model (a) and Table 1.6 summarizes the results obtained using model (b).

Tables make clear the importance of the standard three Fama and French factors: market, size and value. For β, β_{SMB}, and β_{HML} values we observe minor changes when moving from the 4-factor to the 5-factor model. Their statistical behavior is also very similar; market βs are always more than three standard errors from 0; and with few exceptions, the absolute t-statistics on the SMB and HML slopes are greater than 1.645 in both tables. Our results confirm previous Fama and French findings: the three factors capture strong common variation in stock returns.

When focusing, instead, on the momentum factor we still do not notice relevant variations in the values of β_{MOM}, but interesting changes in their t-statistics. The slopes on MOM that are significant (10% significance level), increase from 40% to 44% of the portfolios.

The most interesting results are those about the JF: β_{JF} assumes values from -0.16 to the maximum of 0.45, and in 28% of the cases its absolute t-statistic is greater than 1.645 (in three cases even greater than 2). Considering also Table 1.7, it is clear that the JF captures shared variation in stock returns that is missed by MKT, SMB, HML, and MOM. The values of the R^2_{adj}, increase 100% of the times when we include the JF in the regressions. The lower values of R^2_{adj} are, in the 4-factor case, in correspondence of the small portfolios (first row) where R^2_{adj} spans from 65.7% to 94.1%. This apparent lower ability of the model, that is a consequence of the high volatility of microcaps (see Table 1.3), seems partially captured by the JF. Adding the JF in the model increases its explanatory power, and the

Table 1.5 Size-BE/ME portfolios, 4-Factor regression results. Regressions of excess stock returns of 25 size-BE/ME portfolios on the excess market return and the mimicking returns for the size, book-to-market equity, and momentum factors: December 1925 to December 2014

	Book-to-market equity (BE/ME) quintiles									
Size quintile	Low	2	3	4	High	Low	2	3	4	High
			β					$t(\beta)$		
Small	1.25	1.07	1.02	0.94	0.98	14.66	36.79	26.30	46.43	36.41
2	1.07	1.00	0.99	0.97	1.06	41.68	61.15	41.60	52.96	53.58
3	1.11	1.02	0.99	0.99	1.11	63.3	67.46	38.96	53.23	49.63
4	1.08	1.02	1.02	1.02	1.19	82.81	45.73	48.98	51.5	56.21
Big	1.02	0.98	0.97	1.02	1.14	98.73	59.38	42.55	44.53	35.24
			β_{SMB}					$t(\beta_{SMB})$		
Small	1.44	1.53	1.25	1.21	1.31	7.59	9.07	24.40	12.12	17.31
2	1.12	0.97	0.84	0.83	0.90	14.93	13.18	12.63	12.17	17.93
3	0.81	0.50	0.44	0.46	0.58	20.45	9.88	8.76	8.11	8.86
4	0.32	0.24	0.20	0.21	0.28	8.48	4.87	3.96	6.25	6.18
Big	-0.15	-0.21	-0.24	-0.17	-0.14	-5.31	-7.95	-8.99	-5.08	-1.72
			β_{HML}					$t(\beta_{HML})$		
Small	0.36	0.23	0.46	0.57	0.89	1.72	3.15	12.26	14.04	18.73
2	-0.23	0.13	0.36	0.57	0.88	-6.17	2.47	7.15	12.72	24.14
3	-0.25	0.05	0.32	0.56	0.85	-5.55	1.21	7.08	11.23	19.11
4	-0.35	0.08	0.33	0.55	0.93	-14.12	1.80	7.39	11.47	20.97
Big	-0.27	0.02	0.31	0.63	0.96	-14.5	0.56	10.56	18.77	18.21
			β_{MOM}					$t(\beta_{MOM})$		
Small	-0.17	-0.01	-0.13	-0.03	-0.06	-1.77	-0.27	-3.26	-0.92	-1.84
2	-0.03	-0.05	0.01	0.02	-0.03	-1.05	-1.92	0.30	0.79	-1.41
3	-0.06	0.00	0.01	-0.01	-0.06	-2.45	-0.03	0.25	-0.17	-1.92
4	0.01	-0.01	-0.03	-0.05	-0.07	0.33	-0.48	-1.09	-1.81	-2.38
Big	-0.02	-0.01	-0.02	-0.04	-0.12	-1.25	-0.71	-0.96	-1.73	-2.32

Table 1.6 Size-BE/ME portfolios, 5-Factor regression results. Regressions of excess stock returns of 25 size-BE/ME portfolios on the excess market return and the mimicking returns for the size, book-to-market equity, momentum, and expected jump factors: December 1925 to December 2014

	Book-to-market equity (BE/ME) quintiles									
Size quintile	Low	2	3	4	High	Low	2	3	4	High
			β					$t(\beta)$		
Small	1.25	1.09	1.03	0.95	0.99	15.10	32.51	26.54	47.83	39.18
2	1.07	1.01	0.99	0.98	1.05	45.04	59.65	37.29	45.46	47.58
3	1.11	1.02	0.99	0.99	1.11	62.74	68.52	39.67	51.71	54.41
4	1.08	1.02	1.03	1.02	1.19	79.59	46.29	50.53	51.72	53.64
Big	1.03	0.98	0.98	1.02	1.13	101.54	60.16	41.10	42.71	32.95
			β_{SMB}					$t(\beta_{SMB})$		
Small	1.43	1.55	1.26	1.22	1.33	7.17	9.23	20.30	12.55	18.46
2	1.12	0.99	0.85	0.84	0.89	13.58	15.21	13.71	12.60	18.73
3	0.82	0.51	0.45	0.46	0.59	21.89	10.08	9.01	8.62	8.16
4	0.32	0.24	0.20	0.21	0.28	9.10	5.10	3.95	6.20	6.44
Big	-0.14	-0.20	-0.23	-0.17	-0.15	-5.60	-8.10	-9.66	-5.62	-1.70
			β_{HML}					$t(\beta_{HML})$		
Small	0.36	0.23	0.46	0.56	0.88	1.73	3.39	12.12	15.99	18.83
2	-0.23	0.12	0.35	0.57	0.88	-6.41	2.74	7.80	14.08	26.03
3	-0.26	0.05	0.32	0.55	0.84	-5.57	1.17	7.22	11.72	20.02
4	-0.35	0.08	0.33	0.55	0.93	-14.48	1.77	7.60	11.38	21.56
Big	-0.27	0.02	0.31	0.63	0.97	-15.36	0.52	10.87	19.29	18.46
			β_{MOM}					$t(\beta_{MOM})$		
Small	-0.16	-0.05	-0.15	-0.05	-0.08	-1.64	-0.99	-3.48	-2.28	-2.30
2	-0.03	-0.07	0.00	0.01	-0.02	-1.04	-2.82	-0.11	0.29	-0.99
3	-0.07	0.00	0.00	-0.01	-0.07	-2.62	-0.15	0.11	-0.40	-1.81
4	0.02	-0.02	-0.04	-0.05	-0.08	0.84	-0.72	-1.09	-1.89	-2.56
Big	-0.03	-0.02	-0.03	-0.04	-0.11	-1.81	-0.96	-1.13	-1.95	-1.78
			β_{JF}					$t(\beta_{JF})$		
Small	-0.13	0.45	0.28	0.29	0.22	-0.47	1.41	1.85	1.52	1.89
2	0.00	0.30	0.18	0.16	-0.16	-0.01	2.95	1.49	1.29	-1.77
3	0.13	0.05	0.07	0.12	0.06	1.59	0.81	1.09	1.48	0.48
4	-0.12	0.09	0.05	0.03	0.09	-2.48	1.71	0.67	0.38	0.95
Big	0.10	0.07	0.09	0.10	-0.09	2.23	1.42	1.38	1.37	-0.42

Table 1.7 Size-BE/ME portfolios, R^2_{adj}. Regressions of excess stock returns of 25 size-BE/ME portfolios on the returns of the 4-factor model (excess market return (MKT) and the mimicking returns for the size (SMB), book-to-market equity (HML), and momentum (MOM) factors), and on the returns of the 5-factor model (excess market return (MKT) and the mimicking returns for the size (SMB), book-to-market equity (HML), momentum (MOM), and expected jump (JF) factors). December 1925 to December 2014

	Book-to-market equity quintiles				
Size quintile	Low	2	3	4	High
	R^2_{adj} 4-factor model				
Small	0.657	0.820	0.892	0.927	0.941
2	0.909	0.931	0.938	0.951	0.951
3	0.932	0.926	0.926	0.932	0.928
4	0.934	0.922	0.914	0.921	0.915
Big	0.955	0.933	0.908	0.925	0.840
	R^2_{adj} 5-factor model				
Small	0.658	0.827	0.895	0.930	0.942
2	0.910	0.936	0.94	0.952	0.952
3	0.933	0.926	0.927	0.933	0.928
4	0.936	0.922	0.915	0.921	0.916
Big	0.956	0.933	0.909	0.926	0.840

larger improvements hit the small portfolios. Microcaps R^2_{adj} increase when moving from the 4-factor to the 5-factor model from 0.1 to 0.7 basis points, with an average increase of 0.3 basis points. Tables 1.8 and 1.9 compare residuals correlation and heteroskedasticity indicators for the 4-factor and the 5-factor models. The first and second panels of Table 1.8 show the P-values of the Breusch (1978)-Godfrey (1978) autocorrelation test, or AR, while the panels of Table 1.9 report the P-values of the Engle (1982) heteroskedasticity test, or ET, both with 3 lags. AR and ET results suggest that 5-factor model errors are affected by heteroskedasticity as much as 4-factor model errors. They also make clear that both models are slightly affected by autocorrelation of the errors. Moreover, the lower values of average residual correlations and average absolute residual correlations in Table 1.8, for the 5-factor model with respect to the 4-factor model, witness the superiority of the former.

1.4.2 25 size-expected jump portfolios

Similarly to the previous case, we consider (a) regressions that use the four-factor model, Table 1.10, and (b) regressions that use the five-factor model, Table 1.11.

The slopes of the standard three Fama and French factors show minor changes when moving from the 4-factor to the 5-factor model. We observe greater changes for β_{SMB} and

Table 1.8 Size-BE/ME portfolios, residual correlations. Regressions of excess stock returns of 25 size-BE/ME portfolios on the returns of the 4-factor model (excess market return (MKT) and the mimicking returns for the size (SMB), book-to-market equity (HML), and momentum (MOM) factors), and on the returns of the 5-factor model (excess market return (MKT) and the mimicking returns for the size (SMB), book-to-market equity (HML), momentum (MOM), and expected jump (JF) factors). December 1925 to December 2014. Panels report the P-values of the Breusch (1978)-Godfrey (1978) autocorrelation test with 3 lags. The table also shows statistics for residual correlations between portfolios: $\rho_{i,j}$ with $i \neq j$. In detail, $\bar{\rho} = \frac{1}{N(N-1)} \sum_{i=1}^{N} \sum_{j=1}^{N} \rho_{i,j}, i \neq j$ is the average residual correlation, $\ddot{\rho} = \frac{1}{N(N-1)} \sum_{i=1}^{N} \sum_{j=1}^{N} |\rho_{i,j}|, i \neq j$ is the average absolute residual correlation, while min(ρ) and max(ρ) are the minimum and maximum values of the residual correlations.

	Book-to-market equity quintiles				
Size quintile	Low	2	3	4	High
			AR 4-factor model		
Small	0.00	0.06	0.00	0.01	0.00
2	0.00	0.10	0.79	0.01	0.01
3	0.16	0.92	0.57	0.10	0.00
4	0.54	0.00	0.07	0.19	0.35
Big	0.22	0.75	0.00	0.03	0.15
			AR 5-factor model		
Small	0.00	0.02	0.00	0.00	0.01
2	0.00	0.14	0.84	0.02	0.03
3	0.12	0.91	0.57	0.07	0.00
4	0.73	0.00	0.10	0.19	0.52
Big	0.39	0.74	0.00	0.02	0.16
Model	$\bar{\rho}$	$\ddot{\rho}$	min(ρ)	max(ρ)	
4-factor	0.090	0.160	-0.431	0.473	
5-factor	0.087	0.158	-0.445	0.475	

β_{HML} when using as dependent variables size-expected jump portfolios with respect to the size-BE/ME portfolios. Similarly to the size-BE/ME case, the absolute t-statistics on the MKT slopes are always greater than 3, and the βs of SMB and HML are, with few exceptions, significant at the 10% significance level.

The values of β_{MOM} change sometimes considerably, the increase when the JF is added goes from -0.06 to 0.07. The slopes on MOM that are significant when considering a 90% confidence interval, decrease from 40% to 28% of the portfolios. Controlling for the size, we expect the value of β_{JF} to increase with the expected jump. As discussed in section 1.2.2, in order to get positive JF premiums the sign of the factor loadings must coincide with the sign of the factors. As shown in Figure 1.2, assets with negative (positive) expected jump principally belong to lower (higher) JF portfolios. This means that lower JF portfolios (left columns) present negative signs and higher JF portfolios (right columns) show positive signs. The associated slopes should, consequently, be < 0 for lower expected-jump portfolios and

Table 1.9 Size-BE/ME portfolios, residual heteroskedasticity. Regressions of excess stock returns of 25 size-BE/ME portfolios on the returns of the 4-factor model (excess market return (MKT) and the mimicking returns for the size (SMB), book-to-market equity (HML), and momentum (MOM) factors), and on the returns of the 5-factor model (excess market return (MKT) and the mimicking returns for the size (SMB), book-to-market equity (HML), momentum (MOM), and expected jump (JF) factors). December 1925 to December 2014. Tables report the P-values of the Engle (1982) heteroskedasticity test with 3 lags.

	Book-to-market equity quintiles				
Size quintile	Low	2	3	4	High
			ET 4-factor model		
Small	0.00	0.00	0.00	0.00	0.00
2	0.00	0.00	0.00	0.00	0.00
3	0.24	0.00	0.00	0.00	0.00
4	0.00	0.00	0.00	0.00	0.00
Big	0.00	0.00	0.00	0.00	0.00
			ET 5-factor model		
Small	0.00	0.00	0.00	0.00	0.00
2	0.00	0.00	0.00	0.00	0.00
3	0.25	0.00	0.00	0.00	0.00
4	0.00	0.00	0.00	0.00	0.00
Big	0.00	0.00	0.00	0.00	0.00

> 0 for higher expected-jump portfolios. When focusing on the β_{JF} with absolute t-statistics greater than 1.645 (in eleven cases even greater than 2), our expectations are confirmed with just one exception. Adding the information content of Table 1.12, it is possible to infer that the JF captures common variation missed by the other factors. When we add the JF in the regressions, the values of the R^2_{adj} increase 80% of the times.

In section 1.3 we observed how the 25 size-expected jump portfolios are characterized by high volatility. By comparing Table 1.4 and Table 1.12 it is evident a correspondence between higher volatilities and lower R^2_{adj}. The first three rows show volatilies from 9.05 to 17.99 and R^2_{adj} from 0.278 to 0.861 in the 5-factor model. For the microcaps R^2_{adj} assumes values from a minimum of 27.6% (4-factor model) to a maximum of 51% (5-factor model). The microcaps R^2_{adj} average increase when moving from the 4-factor to the 5-factor is of 0.26 basis points; it was 0.3 basis points in the $size - BE/ME$ case. Anyway, the table shows major improvements for the big portfolios. The R^2_{adj} performance is worse than in the $size - BE/ME$ case but we observe comparatively more significance for the β_{JF}. The bottom line is that, also in this case, the JF is able to capture the variation in the dependent variables left unexplained by the 4-factor model.

Table 1.10 Size-expected jump portfolios, 4-Factor regression results. Regressions of excess stock returns of 25 size-expected jump portfolios on the excess market return and the mimicking returns for the size, book-to-market equity, and momentum factors: December 1925 to December 2014

	Expected jump quintiles									
Size quintile	Low	2	3	4	High	Low	2	3	4	High
			β					$t(\beta)$		
Small	0.90	0.87	1.12	0.78	1.14	9.05	7.92	3.12	7.91	9.87
2	1.04	1.02	0.98	0.88	0.94	21.24	12.03	10.23	11.44	18.49
3	1.16	1.17	0.93	0.97	1.04	35.83	16.62	26.33	26.75	27.30
4	1.18	1.13	1.05	1.09	1.06	45.35	37.40	34.51	23.24	46.31
Big	1.02	1.04	0.97	1.01	1.04	69.32	42.69	85.58	84.69	69.46
			β_{SMB}					$t(\beta_{SMB})$		
Small	1.59	1.59	1.40	1.50	1.84	12.62	9.14	4.93	5.62	8.71
2	1.88	1.43	1.4	1.36	1.56	11.37	5.94	7.36	11.56	11.59
3	1.40	1.44	1.47	1.10	1.48	13.12	10.54	8.41	17.33	12.50
4	1.18	0.89	0.79	0.88	1.09	32.39	17.7	15.69	19.18	15.57
Big	0.00	0.02	-0.08	-0.07	-0.05	0.15	0.73	-1.95	-5.14	-1.92
			β_{HML}					$t(\beta_{HML})$		
Small	0.72	0.63	0.51	0.98	1.18	4.97	3.72	1.26	3.35	5.77
2	0.56	0.88	0.91	0.75	0.73	4.79	4.58	6.78	6.77	8.47
3	0.31	0.61	0.73	0.67	0.65	4.61	6.36	7.83	10.39	8.72
4	0.33	0.29	0.45	0.48	0.41	10.28	5.49	7.69	8.17	12.32
Big	0.05	0.10	0.09	0.00	0.00	1.38	3.07	3.48	0.09	0.06
			β_{MOM}					$t(\beta_{MOM})$		
Small	-0.09	-0.20	0.08	-0.33	-0.16	-0.82	-1.85	0.31	-2.05	-0.73
2	-0.20	-0.12	-0.18	-0.09	0.03	-2.33	-1.12	-2.25	-0.95	0.46
3	-0.15	-0.12	-0.02	-0.01	-0.05	-2.80	-1.67	-0.31	-0.28	-0.90
4	-0.03	-0.09	-0.09	-0.05	0.01	-1.35	-2.88	-2.62	-1.07	0.33
Big	-0.05	-0.04	-0.03	0.00	0.06	-2.26	-1.48	-1.49	0.14	3.00

Table 1.11 Size-expected jump portfolios, 5-Factor regression results. Regressions of excess stock returns of 25 size-expected jump portfolios on the excess market return and the mimicking returns for the size, book-to-market equity, momentum, and expected jump factors: December 1925 to December 2014

	Expected jump quintiles									
Size quintile	Low	2	3	4	High	Low	2	3	4	High
			β					$t(\beta)$		
Small	0.87	0.86	1.10	0.81	1.14	8.90	7.30	3.09	8.15	9.44
2	1.06	1.02	0.99	0.88	0.94	23.59	12.92	11.08	11.58	18.88
3	1.15	1.18	0.95	0.98	1.06	35.87	16.44	21.51	27.30	29.78
4	1.17	1.12	1.05	1.10	1.07	45.92	35.91	34.94	23.59	50.47
Big	1.01	1.03	0.97	1.02	1.05	68.85	44.18	85.97	89.47	86.38
			β_{SMB}					$t(\beta_{SMB})$		
Small	1.54	1.58	1.37	1.56	1.84	11.78	7.72	5.11	5.09	8.16
2	1.91	1.43	1.42	1.35	1.57	12.13	5.56	6.71	11.32	11.21
3	1.38	1.45	1.5	1.11	1.52	13.72	10.89	9.35	16.52	15.46
4	1.16	0.88	0.78	0.9	1.12	36.5	17.61	15.53	21.17	20.40
Big	-0.03	0	-0.08	-0.06	-0.02	-0.92	-0.10	-2.11	-3.22	-1.18
			β_{HML}					$t(\beta_{HML})$		
Small	0.74	0.64	0.53	0.98	1.18	5.27	3.76	1.32	3.43	5.97
2	0.54	0.88	0.9	0.75	0.72	4.97	4.5	6.64	6.77	8.59
3	0.32	0.60	0.71	0.66	0.63	5.02	6.46	8.69	10.46	9.30
4	0.34	0.30	0.45	0.47	0.4	9.83	5.60	7.69	8.76	12.56
Big	0.06	0.11	0.09	0.00	-0.01	2.18	3.16	3.64	-0.27	-0.58
			β_{MOM}					$t(\beta_{MOM})$		
Small	-0.02	-0.19	0.14	-0.37	-0.16	-0.13	-1.41	0.5	-2.00	-0.62
2	-0.25	-0.11	-0.21	-0.08	0.02	-2.58	-0.98	-2.60	-0.83	0.33
3	-0.13	-0.14	-0.06	-0.03	-0.11	-2.27	-1.71	-1.30	-0.62	-2.02
4	0.00	-0.08	-0.08	-0.08	-0.03	-0.10	-2.41	-2.29	-1.63	-1.01
Big	-0.01	-0.01	-0.03	-0.01	0.02	-0.42	-0.31	-1.07	-0.89	1.31
			β_{JF}					$t(\beta_{JF})$		
Small	-0.94	-0.12	-0.61	0.57	-0.08	-2.05	-0.24	-1.56	1.00	-0.16
2	0.60	-0.11	0.30	-0.04	0.08	1.87	-0.28	0.77	-0.19	0.29
3	-0.25	0.25	0.61	0.24	0.69	-1.23	1.22	2.03	1.88	3.63
4	-0.42	-0.19	-0.10	0.4	0.46	-3.85	-2.85	-1.47	2.67	5.14
Big	-0.55	-0.38	-0.08	0.21	0.46	-9.88	-6.39	-1.35	5.14	11.33

Table 1.12 Size-expected jump portfolios, R^2_{adj}. Regressions of excess stock returns of 25 size-expected jump portfolios on the returns of the 4-factor model (excess market return (MKT) and the mimicking returns for the size (SMB), book-to-market equity (HML), and momentum (MOM) factors), and on the returns of the 5-factor model (excess market return (MKT) and the mimicking returns for the size (SMB), book-to-market equity (HML), momentum (MOM), and expected jump (JF) factors). December 1925 to December 2014

	Expected jump quintiles				
Size quintile	Low	2	3	4	High
			R^2_{adj} 4-factor model		
Small	0.501	0.449	0.276	0.377	0.461
2	0.735	0.615	0.534	0.587	0.643
3	0.860	0.780	0.807	0.809	0.769
4	0.942	0.935	0.921	0.903	0.919
Big	0.937	0.934	0.946	0.959	0.952
			R^2_{adj} 5-factor model		
Small	0.510	0.448	0.278	0.380	0.461
2	0.739	0.614	0.535	0.587	0.642
3	0.861	0.781	0.814	0.810	0.776
4	0.946	0.936	0.921	0.907	0.924
Big	0.954	0.941	0.947	0.962	0.965

We report in Tables 1.13 and 1.14 residuals correlation and heteroskedasticity indicators for the 4-factor and the 5-factor models. Engle (1982) test results (Table 1.14) suggest that 5-factor model errors are slightly less affected by heteroskedasticity than 4-factor model errors. The P-values of the Breusch (1978)-Godfrey (1978) test (first and second panels of Table 1.13) show that both models errors are not affected by autocorrelation. We observe in Table 1.13 a small increase, when using the 5-factor model, of the average residual correlation, from 0.142 to 0.145, and of the average absolute residual correlation, from 0.188 to 0.196.

1.4.3 Single assets

These time-series regressions compare the results of 4-factor and 5-factor models using, differently from before, single asset excess returns as dependent variables. Our data-set is formed by all the CRSP stocks, with share code 10 or 11, quoted between December 1925 and December 2014. According to these characteristics, we consider 24,098 stocks and 89 years. Adding the JF in the regressions increases the R^2_{adj} in 39.84% of the times. Focusing on the results when using the 5-factor model, the absolute t-statistics are greater than 1.645 in 63%, 49%, 33%, and 22% of the cases respectively on the MKT, SMB, HML, and MOM

Table 1.13 Size-expected jump portfolios, residual correlations. Regressions of excess stock returns of 25 size-expected jump portfolios on the returns of the 4-factor model (excess market return (MKT) and the mimicking returns for the size (SMB), book-to-market equity (HML), and momentum (MOM) factors), and on the returns of the 5-factor model (excess market return (MKT) and the mimicking returns for the size (SMB), book-to-market equity (HML), momentum (MOM), and expected jump (JF) factors). December 1925 to December 2014. Panels report the P-values of the Breusch (1978)-Godfrey (1978) autocorrelation test with 3 lags. The table also shows statistics for residual correlations between portfolios: $\rho_{i,j}$ with $i \neq j$. In detail, $\bar{\rho} = \frac{1}{N(N-1)} \sum_{i=1}^{N} \sum_{j=1}^{N} \rho_{i,j}, i \neq j$ is the average residual correlation, $\ddot{\rho} = \frac{1}{N(N-1)} \sum_{i=1}^{N} \sum_{j=1}^{N} |\rho_{i,j}|, i \neq j$ is the average absolute residual correlation, while min(ρ) and max(ρ) are the minimum and maximum values of the residual correlations.

	Expected jump quintiles				
Size quintile	Low	2	3	4	High
			AR 4-factor model		
Small	0.64	0.06	0.83	0.61	0.93
2	0.73	0.17	0.02	0.70	0.60
3	0.59	0.44	0.48	0.10	0.19
4	0.20	0.81	0.13	0.01	0.28
Big	0.02	0.39	0.64	0.04	0.18
			AR 5-factor model		
Small	0.71	0.05	0.84	0.65	0.94
2	0.68	0.17	0.03	0.72	0.61
3	0.40	0.29	0.53	0.17	0.16
4	0.73	0.79	0.27	0.02	0.44
Big	0.28	0.19	0.64	0.07	0.90
Model	$\bar{\rho}$	$\ddot{\rho}$	min(ρ)	max(ρ)	
4-factor	0.142	0.188	-0.261	0.689	
5-factor	0.145	0.196	-0.374	0.682	

slopes. For the JF, instead, 17% of the times β_{JF} is significant at the 10% significance level. The results on single asset regressions, reinforce the conclusions obtained when considering portfolios as dependent variable: JF captures strong common variation in returns.

1.4.4 Model performance

The regression slopes and R^2_{adj} values in Tables 1.5 to 1.12 establish that the JF proxy for a common risk factors in stock returns. We now study how well the 4-factor and the 5-factor models explain average excess returns on the portfolios of Table 1.3 and Table 1.4. The focus is on their relative performances since they allow to judge the improvements provided by the JF. The time-series regressions in this section use excess returns, on portfolios and single assets, as dependent variables. The explanatory variables are, instead, either excess returns ($MKT = RM - RF$) or returns on zero-investment portfolios (SMB, HML, MOM, and JF).

Table 1.14 Size-expected jump portfolios, residual heteroskedasticity. Regressions of excess stock returns of 25 size-expected jump portfolios on the returns of the 4-factor model (excess market return (MKT) and the mimicking returns for the size (SMB), book-to-market equity (HML), and momentum (MOM) factors), and on the returns of the 5-factor model (excess market return (MKT) and the mimicking returns for the size (SMB), book-to-market equity (HML), momentum (MOM), and expected jump (JF) factors). December 1925 to December 2014. Tables report the P-values of the Engle (1982) heteroskedasticity test with 3 lags.

	Expected jump quintiles				
Size quintile	Low	2	3	4	High
			ET 4-factor model		
Small	0.02	0.45	0.00	0.00	0.00
2	0.00	0.00	0.00	0.00	0.01
3	0.00	0.00	0.00	0.00	0.05
4	0.06	0.00	0.00	0.00	0.00
Big	0.00	0.00	0.00	0.00	0.00
			ET 5-factor model		
Small	0.04	0.48	0.00	0.00	0.00
2	0.00	0.00	0.00	0.00	0.01
3	0.13	0.00	0.00	0.00	0.04
4	0.68	0.00	0.00	0.00	0.00
Big	0.00	0.00	0.00	0.00	0.00

In these regressions if the asset-pricing model completely captures expected returns, the intercept must be indistinguishable from 0 (Merton (1973)). We use the estimated intercepts to test whether the average premiums for the common risk factors in returns explain the cross-section of average returns. In addition to the simple comparison of the estimated values, we present three indicators, first introduced by Fama and French (2015), and one test on the intercepts. Detailed descriptions of the indicators and the test are reported in Table 1.15.

The first indicator (A1) considers the average absolute intercepts. The interpretation is straightforward: the model that better describes the cross-section of returns, has intercepts that, on average, are closer to zero. The other two ratios allow to compare the models in terms of proportion of cross-section of expected returns left unexplained. The numerators measure the dispersion of the estimated intercepts produced by a given model (4-factor or 5-factor model) for a set of dependent variables. The denominators, instead, measure the dispersion of the excess returns in the dependent portfolios (25 size-BE/ME portfolios or 25 size-expected jump portfolios).
The second ratio (A2) has as numerator the average absolute intercept and as denominator the average absolute deviation. $\bar{r}_i$, the deviation of portfolio i from the cross-sectional average, is obtained as the difference between its time-series average excess return, $\bar{R}_i$, and the cross-sectional average of all the 25 $\bar{R}_i$: $\bar{r}_i = \bar{R}_i - \bar{R}$. It is important to notice that we are

Table 1.15 Intercept indicators and test. a_i is the estimated intercept for portfolio i. $\bar{r}_i$ is the deviation of portfolio i from the cross-sectional average: $\bar{r}_i = \bar{R}_i - \bar{R}$ where $\bar{R}_i$ is the time-series mean for portfolio i and $\bar{R} = \frac{1}{n}\sum_{i=1}^{n}\bar{R}_i$. $\hat{\alpha}_i^2 = A(a_i^2) - SE_{a_i}^2$ and $\hat{\mu}_i^2 = A(\bar{r}_i^2) - SE_{\bar{r}_i}^2$ where SE are the standard errors. T is the length of the portfolios time series, N is the number of portfolios, and K is the number of factors. $\bar{f}$ is the vector of mean value of the factors. $\hat{\Sigma}_f$ is the estimated variance-covariance matrix of the factors. $\hat{\alpha}$ is the vector of estimated intercepts for the 25 portfolios. $\hat{\Omega}$ is the estimated variance-covariance matrix of the regression residuals.

A1	$\frac{1}{n}\sum_{i=1}^{n}\lvert a_i\rvert$
A2	$\frac{\frac{1}{n}\sum_{i=1}^{n}\lvert a_i\rvert}{\frac{1}{n}\sum_{i=1}^{n}\lvert\bar{r}_i\rvert}$
A3	$\frac{\frac{1}{n}\sum_{i=1}^{n}\lvert\hat{\alpha}_i^2\rvert}{\frac{1}{n}\sum_{i=1}^{n}\lvert\hat{\mu}_i^2\rvert}$
A4	$\frac{T-N-K}{N}(1+\bar{f}\hat{\Sigma}_f^{-1}\bar{f})^{-1}\hat{\alpha}'\hat{\Omega}^{-1}\hat{\alpha}$

using estimated values and not true values. As a consequence both the numerator and the denominator are inflated by estimation errors: the true intercept is just the difference between the estimated intercept and the estimation error, $\alpha_i = a_i - e_i$, and the expected deviation can be obtained by subtracting from the estimated deviation the estimation error, $\mu_i = \bar{r}_i - \varepsilon_i$.
The last ratio (A3), is designed to correct the measurement errors affecting (A2). Consider first the denominator where μ_i is the deviation of portfolio i from the mean: $\mu_i = x_i - \bar{x}$. Its average value is zero, $E(\mu_i) = E(x_i - \bar{x}) = E(x_i) - E(\bar{x}) = \bar{x} - \bar{x} = 0$, and its variance is $Var(\mu_i) = E[\mu_i^2] - [E(\mu_i)]^2 = E[\mu_i^2]$. It is now clear that the average value of μ_i^2, $A(\mu_i^2)$, is the cross-section variance of expected portfolio returns. Focus now on the numerator: α_i is a constant ($E(\alpha_i) = \alpha_i$ and $Var(\alpha_i) = 0$) and, consequently, $E(a_i^2) = E[(\alpha_i + e_i)^2] = E[\alpha_i^2 + e_i^2 + 2\alpha_i e_i] = \alpha_i^2 + E(e_i^2)$. The estimates of α_i^2 and μ_i^2 are respectively $\hat{\alpha}_i^2 = A(a_i^2) - SE_{a_i}^2$ and $\hat{\mu}_i^2 = A(\bar{r}_i^2) - SE_{\bar{r}_i}^2$ where SE are the standard errors. Summing up it is possible to rewrite (A2) in terms of squared intercepts and deviations, the proportion of portfolios variance left unexplained by a model.
Lastly, test (A4) is the Gibbons et al. (1989) or GRS test statistic that allows to investigate if the intercepts (α) are simultaneously equal to 0, $H_0 : \alpha = 0$. The GRS requires that the errors, u, are normal, uncorrelated, and homoskedastic. Under the hypothesis of normal Excess returns, the distribution of the estimated α, or $\hat{\alpha}$, conditional to the factors is: $\hat{\alpha}|F \sim \mathcal{N}\left[\alpha, \frac{1}{T}(1+\bar{f}'\hat{\Sigma}_f^{-1}\bar{f})\Omega\right]$. Where $\bar{f}$ is the vector of mean value of the factors, $\hat{\Sigma}_f$ is the estimated variance-covariance matrix of the factors $\hat{\Sigma}_f = \frac{1}{T}\sum_{t=1}^{T}[f_t - \bar{f}][f_t - \bar{f}]'$. $\hat{\alpha}$ is the vector of estimated intercepts for the 25 portfolios and $\hat{\Omega}$ is the estimated variance-

covariance matrix of the regression residuals $\hat{\Omega} = \frac{1}{T}\sum_{t=1}^{T}\hat{e}_t\hat{e}_t^{'}$. The test statistic, under H_0, has distribution $F_{N,T-N-K}$, where N equals the number of dependent portfolios, so in our case $N = 25$, T is the length of the portfolios time series and K is the number of factors, consequently $K = 4$ or $K = 5$.

Tables 1.16 and 1.17 show the results using the 4-factor and the 5-factor models when considering 25 size-BE/ME portfolios. Results for the 25 size-expected jump portfolios are, instead, reported in Table 1.18 (using the 4-factor model) and Table 1.19 (using the 5-factor model).

The first thing to notice is that the number of intercepts significantly different from 0 ($|t$-statistic$| > 1.645$) always increase when the 5-factor model is applied. In the 25 $Size - B/M$ case they move from 8 to 9 while in the size-expected jump case from 3 to 4.

Tables 1.16 to 1.19 show that the average absolute intercept (A1) is not always smaller for the five-factor model. For the 25 size-expected jump portfolios A1 increases of 0.001 when moving from the 4-factor to the 5-factor model. We obtain better results when considering the 25 $Size - B/M$ portfolios where we observe an improvement of 0.15 basis points. These results suggest that apply the 4-factor model to portfolios with strong size and value inclinations may lead to poor results.

Also for A2 we observe a positive result only for the 25 $Size - B/M$ portfolios: it decreases from 0.5629 to 0.5546, which means that the 5-factor model reduces of 0.8% the unexplained dispersion of average excess returns. For the 25 size-expected jump portfolios we observe, instead an increase of 0.34 basis points.

A3 increases both when 25 $Size - B/M$ portfolios and 25 size-expected jump portfolios are used. It increases respectively of 3.85 and 0.94 basis points. The worse results observed for this ratio may be caused by the fact that it is in units of return squared. It is anyway interesting to notice that with the 5-factor model only 65.94% in the $Size - B/M$ and 4.64% in the size-expected jump case, of the cross-section variance of expected returns is left unexplained.

Lastly, the P-values of test A4, reject the null hypothesis ($H_0 : \alpha = 0$) for both the 4-factor and the 5-factor regressions when using size-BE/ME dependent portfolios. When using the size-expected jump portfolios, instead, the GRS tests say that the models are complete descriptions of expected returns at 1% significance level.

Despite the unclear results of the tests on the intercepts, when results are considered globally, it seems that the 5-factor model outperforms the 4-factor model. Major evidences of this are the increase in 100% ($Size - B/M$) and 80% (size-expected jump) of the portfolios R^2_{adj}, the grow of the average R^2_{adj} values from, respectively, 90.91% and 74.20% to 91.05% and 74.55%, and the positive results of the A4 tests.

Table 1.16 Size-BE/ME portfolios, 4-Factor regression intercepts. Regressions of excess stock returns of 25 size-BE/ME portfolios on the excess market return (MKT) and the mimicking returns for the size (SMB), book-to-market equity (HML), and momentum (MOM) factors: December 1925 to December 2014. A1, A2, A3, and A4 are intercept indicators and tests and their formulas are specified in Table 1.15. Test statistic A4, under H_0, has distribution $F_{N,T-N-K}$, where N equals the number of dependent portfolios, so in our case $N = 25$, T is the length of the portfolios time series and K is the number of factors, consequently $K = 4$. The table reports the P-value for test A4 (P(A4)).

	Book-to-market equity (BE/ME) quintiles									
Size quintile	Low	2	3	4	High	Low	2	3	4	High
			α					$t(\alpha)$		
Small	-0.59	-0.40	0.00	0.09	0.14	-3.48	-3.67	-0.05	1.43	2.05
2	-0.21	0.03	0.04	0.04	0.04	-3.11	0.47	0.84	0.75	0.60
3	-0.04	0.10	0.09	0.04	0.02	-0.87	1.84	1.60	0.8	0.31
4	0.08	0.00	0.05	0.07	-0.11	1.52	0.04	0.86	1.14	-1.37
Big	0.10	0.04	-0.01	-0.16	-0.07	2.62	0.89	-0.19	-2.99	-0.65
(A1)	0.1029									
(A2)	0.5629									
(A3)	0.6209									
P(A4)	1.09e-04									

Table 1.17 Size-BE/ME portfolios, 5-Factor regression intercepts. Regressions of excess stock returns of 25 size-BE/ME portfolios on the excess market return (MKT) and the mimicking returns for the size (SMB), book-to-market equity (HML), momentum (MOM), and expected jump (JF) factors: December 1925 to December 2014. A1, A2, A3, and A4 are intercept indicators and tests and their formulas are specified in Table 1.15. Test statistic A4, under H_0, has distribution $F_{N,T-N-K}$, where N equals the number of dependent portfolios, so in our case $N = 25$, T is the length of the portfolios time series and K is the number of factors, consequently $K = 5$. The table reports the P-value for test A4 (P(A4)).

	Book-to-market equity (BE/ME) quintiles									
Size quintile	Low	2	3	4	High	Low	2	3	4	High
			α					$t(\alpha)$		
Small	-0.58	-0.46	-0.02	0.06	0.12	-3.44	-3.88	-0.22	0.89	1.72
2	-0.22	0.00	0.02	0.02	0.04	-3.06	0.04	0.41	0.46	0.69
3	-0.06	0.10	0.08	0.03	0.01	-1.12	1.85	1.58	0.55	0.13
4	0.09	-0.01	0.06	0.07	-0.13	1.65	-0.11	0.96	1.12	-1.79
Big	0.09	0.03	-0.02	-0.17	-0.05	2.30	0.74	-0.25	-2.94	-0.46
(A1)	0.1014									
(A2)	0.5546									
(A3)	0.6594									
P(A4)	1.08e-04									

Table 1.18 Size-expected jump portfolios, 4-Factor regression intercepts. Regressions of excess stock returns of 25 size-BE/ME portfolios on the excess market return (MKT) and the mimicking returns for the size (SMB), book-to-market equity (HML), and momentum (MOM) factors: December 1925 to December 2014. A1, A2, A3, and A4 are intercept indicators and tests and their formulas are specified in Table 1.15. Test statistic A4, under H_0, has distribution $F_{N,T-N-K}$, where N equals the number of dependent portfolios, so in our case $N = 25$, T is the length of the portfolios time series and K is the number of factors, consequently $K = 4$. The table reports the P-value for test A4 (P(A4)).

	Expected jump (JF) quintiles									
Size quintile	Low	2	3	4	High	Low	2	3	4	High
			α					$t(\alpha)$		
Small	0.24	0.34	0.45	0.35	0.33	0.85	1.19	1.56	1.22	0.89
2	0.09	0.19	0.24	0.13	0.11	0.52	1.05	1.13	0.65	0.62
3	-0.20	0.11	-0.06	0.07	0.03	-1.86	0.83	-0.48	0.63	0.17
4	-0.20	-0.09	0.01	-0.07	-0.04	-3.26	-1.38	0.22	-0.88	-0.45
Big	-0.06	-0.01	0.06	0.04	-0.03	-1.32	-0.28	1.74	1.04	-0.91
(A1)	0.1422									
(A2)	0.4873									
(A3)	0.0370									
(A4)	0.0578									

Table 1.19 Size-expected jump portfolios, 5-Factor regression intercepts. Regressions of excess stock returns of 25 size-BE/ME portfolios on the excess market return (MKT) and the mimicking returns for the size (SMB), book-to-market equity (HML), momentum (MOM), and expected jump (JF) factors: December 1925 to December 2014. A1, A2, A3, and A4 are intercept indicators and tests and their formulas are specified in Table 1.15. Test statistic A4, under H_0, has distribution $F_{N,T-N-K}$, where N equals the number of dependent portfolios, so in our case $N = 25$, T is the length of the portfolios time series and K is the number of factors, consequently $K = 5$. The table reports the P-value for test A4 (P(A4)).

	Expected jump (JF) quintiles									
Size quintile	Low	2	3	4	High	Low	2	3	4	High
			α					$t(\alpha)$		
Small	0.33	0.36	0.49	0.29	0.34	1.12	1.18	1.70	1.01	0.93
2	0.02	0.20	0.21	0.14	0.10	0.15	1.12	0.96	0.67	0.58
3	-0.18	0.09	-0.12	0.05	-0.04	-1.54	0.66	-0.86	0.43	-0.24
4	-0.16	-0.07	0.02	-0.10	-0.08	-2.52	-1.09	0.38	-1.35	-0.98
Big	-0.01	0.02	0.07	0.02	-0.08	-0.31	0.42	1.92	0.57	-2.43
(A1)	0.1432									
(A2)	0.4907									
(A3)	0.0464									
(A4)	0.0466									

It is finally important to notice that the worse results for the size-expected jump case may be caused by high volatility affecting its dependent variables. The high returns volatility may mean that our asset-pricing tests lack power. The tests on the intercepts could be imprecise because the common factors in returns are not able to absorb most of the variation in stock returns.

1.5 Sub periods

From section 1.4, where we obtain the results using the full sample, we know that the JF proxies for common risk factors in stock returns, and that 5-factor model outperforms the 4-factor model in explaining average excess returns on the dependent portfolios. In this section we repeat the tests to judge how the relevance of the JF changes in different periods. To this end, we consider four equally spaced sub-periods of 267 months: from December 1925 till March 1948, from April 1948 till June 1970, from July 1971 till September 1992, and from October 1992 till December 2014.

We present the results separately for regressions that use as dependent portfolios: 25 size-BE/ME portfolios, and 25 size-expected jump portfolios. For each dependent variable we show and compare results for (a) the four-factor model (equation 1.6) and (b) the five-factor model (equation 1.7).

1.5.1 25 size-BE/ME portfolios

Table 1.20 and Table 1.21 show the estimated coefficient results when using model (a) and (b). The tables not only show the mean, the average standard error, the maximum and minimum values of the estimated parameter across the 25 portfolios, but also the percentage of portfolios with $|t$-statistic$| > 1.645$. Results for market, size, and value are in line with full-sample regressions: no important changes when moving from the 4-factor to the 5-factor model in all sub-periods. While market betas are always significant (90% confidence level), β_{SMB} and β_{HML} show little variation in time and the lowest significance level is registered, for both models, in the first sub-period. Focusing on the momentum, despite we observe an increase in the significance level for the first two sub-periods when adding the JF, it is in the last sub-period that the momentum factor seems to have greater importance (significance=36%). The most relevant results are for the JF. The range of values covered by β_{JF} is almost double in the first sub-period, 1.23, with respect to the others, about 0.5. Average values for β_{JF} move from $|0.006|$ to $|0.086|$. Finally, and most importantly, the JF slope shows high levels of significance in all the sub-periods and touches the minimum in the 1971-1992 window, 12%, and the maximum in the 1992-2014 window, 44%.

Considering also the R^2_{adj}s in Table 1.22, it is clear that the JF captures shared variation in stock returns that is missed by MKT, SMB, HML, and MOM. The R^2_{adj}s, in each sub-period, are higher when using the 5-factor model with respect to the 4-factor model for 92%, 40%, 36%, and 76% of the 25 dependent portfolios. The ability of the JF to capture common variation in stock returns, seems to be stronger in the first and last sub-periods for which we obtain greater β_{JF} and larger R^2_{adj} percentage increases with respect to the 4-factor case.

Table 1.20 Size-BE/ME portfolios, 4-Factor sub-periods coefficient results. Regressions of excess stock returns of 25 size-BE/ME portfolios on the excess market return and the mimicking returns for the size, book-to-market equity, and momentum factors. Period identifies the sub-period used for the regressions. Mean, A(SE), Max, and Min represents, respectively, the mean, the average Standard Error, the maximum value and the minimum value of the estimated parameter across the 25 portfolios. Significance reports the percentage of portfolios with $|t\text{-statistic}| > 1.645$.

		Book-to-market equity (BE/ME) portfolios			
Period	Mean	A(SE)	Max	Min	Significance
			β		
Dec1925-Mar1948	0.986	0.158	1.736	0.232	92%
Apr1948-Jun1970	1.006	0.101	1.247	0.706	100%
Jul1970-Sep1992	0.926	0.034	1.139	0.682	100%
Oct1992-Dec2014	0.894	0.048	1.146	0.624	100%
			β_{SMB}		
Dec1925-Mar1948	1.185	0.274	2.324	-0.088	84%
Apr1948-Jun1970	1.191	0.16	2.086	-0.095	88%
Jul1970-Sep1992	1.079	0.07	1.898	-0.096	92%
Oct1992-Dec2014	0.793	0.078	1.478	-0.197	92%
			β_{HML}		
Dec1925-Mar1948	0.675	0.216	2.203	-0.078	88%
Apr1948-Jun1970	0.549	0.154	1.366	-0.023	84%
Jul1970-Sep1992	0.216	0.07	0.518	-0.055	84%
Oct1992-Dec2014	0.185	0.073	0.596	-0.051	60%
			β_{MOM}		
Dec1925-Mar1948	-0.115	0.174	0.567	-0.825	24%
Apr1948-Jun1970	-0.047	0.141	0.15	-0.501	16%
Jul1970-Sep1992	-0.013	0.052	0.122	-0.165	28%
Oct1992-Dec2014	-0.036	0.052	0.149	-0.18	36%

Table 1.21 Size-BE/ME portfolios, 5-Factor sub-periods coefficient results. Regressions of excess stock returns of 25 size-BE/ME portfolios on the excess market return and the mimicking returns for the size, book-to-market equity, momentum, and expected jump factors. Period identifies the sub-period used for the regressions. Mean, A(SE), Max, and Min represents, respectively, the mean, the average Standard Error, the maximum value and the minimum value of the estimated parameter across the 25 portfolios. Significance reports the percentage of portfolios with $|t\text{-statistic}| > 1.645$.

Book-to-market equity (BE/ME) portfolios					
Period	Mean	A(SE)	Max	Min	Significance
		β			
Dec1925-Mar1948	1.051	0.051	1.225	0.901	100%
Apr1948-Jun1970	1.009	0.027	1.114	0.869	100%
Jul1970-Sep1992	1.015	0.026	1.168	0.876	100%
Oct1992-Dec2014	0.999	0.035	1.088	0.858	100%
		β_{SMB}			
Dec1925-Mar1948	0.631	0.104	1.873	-0.359	92%
Apr1948-Jun1970	0.606	0.048	1.632	-0.221	96%
Jul1970-Sep1992	0.548	0.04	1.364	-0.239	96%
Oct1992-Dec2014	0.524	0.049	1.295	-0.276	96%
		β_{HML}			
Dec1925-Mar1948	0.376	0.08	1.14	-0.275	76%
Apr1948-Jun1970	0.274	0.046	0.883	-0.379	84%
Jul1970-Sep1992	0.216	0.046	0.826	-0.48	80%
Oct1992-Dec2014	0.335	0.059	0.904	-0.446	96%
		β_{MOM}			
Dec1925-Mar1948	-0.041	0.058	0.069	-0.295	24%
Apr1948-Jun1970	-0.018	0.043	0.078	-0.2	20%
Jul1970-Sep1992	-0.018	0.037	0.09	-0.112	20%
Oct1992-Dec2014	-0.044	0.037	0.028	-0.1	36%
		β_{JF}			
Dec1925-Mar1948	0.052	0.160	0.786	-0.444	20%
Apr1948-Jun1970	-0.021	0.103	0.225	-0.306	20%
Jul1970-Sep1992	0.006	0.108	0.205	-0.246	12%
Oct1992-Dec2014	0.086	0.105	0.407	-0.197	44%

Table 1.22 Size-BE/ME portfolios, R^2_{adj} and intercept tests. Regressions of excess stock returns of 25 size-BE/ME portfolios on the returns of the 4-factor model, and on the returns of the 5-factor model: December 1925 to December 2014. R^2_{adj} increase reports the percentage of portfolios for which R^2_{adj} increases when moving from the 4-factor to the 5-factor model. R^2_{adj} max and R^2_{adj} min are respectively the maximum and minimum R^2_{adj} obtained using the 5-factor model. A1, A2, A3, and A4 are intercept tests and their formulas are specified in Table 1.15. Test statistic A4, under H_0, has distribution $F_{N,T-N-K}$, where N equals the number of dependent portfolios, so in our case $N = 25$, T is the length of the portfolios time series and K is the number of factors, consequently $K = 4$ or $K = 5$. The table reports the P-value for test A4 (P(A4)).

Book-to-market equity (BE/ME) portfolios				
Period	R^2_{adj} increase	R^2_{adj} max	R^2_{adj} min	
Dec1925-Mar1948	92%	0.980	0.605	
Apr1948-Jun1970	40%	0.957	0.727	
Jul1970-Sep1992	36%	0.966	0.802	
Oct1992-Dec2014	76%	0.952	0.786	
Period	A1 (4F)	A1 (5F)	A2 (4F)	A2 (5F)
Dec1925-Mar1948	0.19	0.21	0.59	0.65
Apr1948-Jun1970	0.12	0.12	0.82	0.80
Jul1970-Sep1992	0.11	0.11	0.61	0.59
Oct1992-Dec2014	0.13	0.12	0.81	0.80
Period	A3 (4F)	A3 (5F)	P(A4) (4F)	P(A4) (5F)
Dec1925-Mar1948	0.41	0.60	0.17	0.15
Apr1948-Jun1970	0.44	0.39	0.01	0.01
Jul1970-Sep1992	0.31	0.28	0.07	0.09
Oct1992-Dec2014	0.42	0.42	9.16e-06	9.42e-06

To study how well the 5-factor model explains average excess returns, we focus on αs in Table 1.23 and on intercept tests (A1, A2, A3 , and A4) in Table 1.22. The fraction of intercepts different from 0 ($|t$-statistic$| > 1.645$) decreases in the second sub-period, from 36% to 28%, and does not change in the other periods when the 5-factor model is applied. Intercept tests reinforce the idea of a superiority of the 5-factor model, as the values of the tests A1, A2, and A3 decrease when we add the JF to the model in all but the first sub-period. In addition, in three out of four sub-periods, the P-values of test A4 does not allow to reject the null hypothesis that $H_0 : \alpha = 0$, at the 1% significance level, when using the 5-factor model. This means that the model is a complete description of expected returns. The 5-factor model seems to better explain the cross-section of average returns in the last two sub-periods, where we observe lower α significance percentages and better values of the tests.

Table 1.23 Size-BE/ME portfolios, 4-Factor and 5-factor sub-periods intercept results. Regressions of excess stock returns of 25 size-BE/ME portfolios on the excess market return and the mimicking returns for the size, book-to-market equity, and momentum factors. Period identifies the sub-period used for the regressions. Mean, A(SE), Max, and Min represents, respectively, the mean, the average Standard Error, the maximum value and the minimum value of the estimated parameter across the 25 portfolios. Significance reports the percentage of portfolios with $|t\text{-statistic}| > 1.645$.

Book-to-market equity (BE/ME) portfolios					
Period	Mean	A(SE)	Max	Min	Significance
		$\alpha_{4-factor}$			
Dec1925-Mar1948	-0.011	0.189	0.336	-1.307	12%
Apr1948-Jun1970	-0.019	0.089	0.256	-0.349	36%
Jul1970-Sep1992	0.017	0.091	0.218	-0.544	20%
Oct1992-Dec2014	0.031	0.114	0.278	-0.555	24%
		$\alpha_{5-factor}$			
Dec1925-Mar1948	-0.021	0.192	0.438	-1.49	12%
Apr1948-Jun1970	-0.016	0.09	0.272	-0.317	28%
Jul1970-Sep1992	0.016	0.092	0.227	-0.52	20%
Oct1992-Dec2014	0.028	0.112	0.277	-0.549	24%

1.5.2 25 size-expected jump portfolios

Table 1.24 presents the estimated coefficient results for the 4-factor model while Table 1.25 shows the coefficient results for the 5-factor model. Similarly to the full-sample case, market, size, and value show minor changes when moving from the 4-factor to the 5-factor model. β_{SMB} shows some variation in time but its significance percentage is always above 80% and slightly decreases in two sub-periods when the JF is added to the model. β_{HML} presents lower significance levels, in particular for the last sub-period. Greater changes are observed in the first sub-period when we apply the 5-factor model: significance drops from 88% to 76%. Momentum is the factor that changes the most when the JF is introduced. In all but the first sub-periods, we observe important decreases in significance. With the 4-factor model we obtain significance percentages of 16% (1948-1970), 28% (1970-1992), and 36% (1992-2014) that become 12%, 20%, and 28% with the 5-factor model. Finally β_{JF} shows large average values, from -0.049 to -0.198, and high levels of significance. In the first and last sub-periods β_{JF} is more than 1.645 standard errors from 0 for 60% of the portfolios. Stunning significance percentages, suggest that the JF captures shared variation in stock returns that is missed by MKT, SMB, HML, and MOM. This finding is confirmed by the R^2_{adj} results in Table 1.27. In each sub-period, the R^2_{adj}s when using the 5-factor model are higher than those obtained using the 4-factor model, for 76%, 48%, 80%, and 84% of the 25 dependent portfolios.

Table 1.24 Size-expected jump portfolios, 4-Factor sub-periods coefficient results. Regressions of excess stock returns of 25 size-expected jump portfolios on the excess market return and the mimicking returns for the size, book-to-market equity, and momentum factors. Period identifies the sub-period used for the regressions. Mean, A(SE), Max, and Min represents, respectively, the mean, the average Standard Error, the maximum value and the minimum value of the estimated parameter across the 25 portfolios. Significance reports the percentage of portfolios with $|t\text{-statistic}| > 1.645$.

Expected jump (JF) portfolios					
Period	Mean	A(SE)	Max	Min	Significance
		β			
Dec1925-Mar1948	0.986	0.158	1.736	0.232	92%
Apr1948-Jun1970	1.006	0.101	1.247	0.706	100%
Jul1970-Sep1992	0.926	0.034	1.139	0.682	100%
Oct1992-Dec2014	0.894	0.048	1.146	0.624	100%
		β_{SMB}			
Dec1925-Mar1948	1.185	0.274	2.324	-0.088	84%
Apr1948-Jun1970	1.191	0.160	2.086	-0.095	88%
Jul1970-Sep1992	1.079	0.070	1.898	-0.096	92%
Oct1992-Dec2014	0.793	0.078	1.478	-0.197	92%
		β_{HML}			
Dec1925-Mar1948	0.675	0.216	2.203	-0.078	88%
Apr1948-Jun1970	0.549	0.154	1.366	-0.023	84%
Jul1970-Sep1992	0.216	0.070	0.518	-0.055	84%
Oct1992-Dec2014	0.185	0.073	0.596	-0.051	60%
		β_{MOM}			
Dec1925-Mar1948	-0.115	0.174	0.567	-0.825	24%
Apr1948-Jun1970	-0.047	0.141	0.150	-0.501	16%
Jul1970-Sep1992	-0.013	0.052	0.122	-0.165	28%
Oct1992-Dec2014	-0.036	0.052	0.149	-0.180	36%

Table 1.25 Size-expected jump portfolios, 5-Factor sub-periods coefficient results. Regressions of excess stock returns of 25 size-expected jump portfolios on the excess market return and the mimicking returns for the size, book-to-market equity, momentum, and expected jump factors. Period identifies the sub-period used for the regressions. Mean, A(SE), Max, and Min represents, respectively, the mean, the average Standard Error, the maximum value and the minimum value of the estimated parameter across the 25 portfolios. Significance reports the percentage of portfolios with $|t\text{-statistic}| > 1.645$.

		Expected jump (JF) portfolios			
Period	Mean	A(SE)	Max	Min	Significance
		β			
Dec1925-Mar1948	0.983	0.154	1.685	0.259	92%
Apr1948-Jun1970	1.001	0.101	1.242	0.703	100%
Jul1970-Sep1992	0.916	0.036	1.116	0.674	100%
Oct1992-Dec2014	0.885	0.047	1.124	0.605	100%
		β_{SMB}			
Dec1925-Mar1948	1.196	0.254	2.245	-0.098	88%
Apr1948-Jun1970	1.187	0.164	2.088	-0.053	84%
Jul1970-Sep1992	1.056	0.068	1.844	-0.100	92%
Oct1992-Dec2014	0.777	0.071	1.407	-0.18	88%
		β_{HML}			
Dec1925-Mar1948	0.666	0.212	2.310	-0.047	76%
Apr1948-Jun1970	0.549	0.152	1.376	-0.019	84%
Jul1970-Sep1992	0.206	0.068	0.495	-0.063	84%
Oct1992-Dec2014	0.188	0.07	0.589	-0.055	64%
		β_{MOM}			
Dec1925-Mar1948	-0.117	0.168	0.612	-0.772	24%
Apr1948-Jun1970	-0.04	0.154	0.157	-0.524	12%
Jul1970-Sep1992	0.014	0.054	0.193	-0.099	20%
Oct1992-Dec2014	-0.023	0.055	0.162	-0.153	28%
		β_{JF}			
Dec1925-Mar1948	-0.105	0.389	1.036	-1.972	60%
Apr1948-Jun1970	-0.049	0.374	0.537	-0.843	32%
Jul1970-Sep1992	-0.198	0.177	0.483	-1.062	44%
Oct1992-Dec2014	-0.114	0.158	0.579	-0.652	60%

As in the $size - BE/ME$ case, the ability of the JF to capture common variation in stock returns, seems to be stronger in the first and last sub-periods for which we obtain higher β_{JF} and larger R^2_{adj} percentage increases with respect to the 4-factor case.

To judge the ability of the model of explaining the cross-section of average returns we use the results for α in Table 1.26 and the tests on the intercepts in Table 1.27. The inclusion

Table 1.26 Size-expected jump portfolios, 4-Factor and 5-factor sub-periods intercept results. size-expected jump portfolios on the excess market return (MKT) and the mimicking returns for the size (SMB), book-to-market equity (HML), and momentum (MOM) factors. Period identifies the sub-period used for the regressions. Mean, A(SE), Max, and Min represents, respectively, the mean, the average Standard Error, the maximum value and the minimum value of the estimated parameter across the 25 portfolios. Significance reports the percentage of portfolios with $|t\text{-statistic}| > 1.645$.

Expected jump (JF) portfolios					
Period	Mean	A(SE)	Max	Min	Significance
		$\alpha_{4-factor}$			
Dec1925-Mar1948	0.548	0.555	2.612	-0.165	4%
Apr1948-Jun1970	0.187	0.293	1.347	-0.258	24%
Jul1970-Sep1992	-0.112	0.135	0.124	-0.528	32%
Oct1992-Dec2014	0.134	0.172	0.563	-0.138	20%
		$\alpha_{5-factor}$			
Dec1925-Mar1948	0.568	0.571	3.118	-0.113	4%
Apr1948-Jun1970	0.194	0.300	1.360	-0.212	20%
Jul1970-Sep1992	-0.092	0.135	0.143	-0.489	28%
Oct1992-Dec2014	0.137	0.168	0.577	-0.151	20%

of the JF in the model has a positive effect on α for which the fraction of intercepts different from 0 (90% confidence level) decreases in the second and third sub-periods respectively from 24% to 20% and from 32% to 28%. The minimum significance level is reached in the first sub-period, 4%. Tests on the intercept do not give clear evidence of a superiority of the 5-factor model. We observe lower A1-A3 test values, when the JF is included, only in the second and third sub-periods. The P-values of test A4 are, instead, clearly in favor of the 5-factor model: when the JF is added to the model the test never allows to reject the null hypothesis that $H_0 : \alpha = 0$.

Instead, when using the 4-factor model the test presents lower P-values that in the 5-factor case. It is finally interesting to notice that in the last two sub-periods, A2 assumes values greater than 1, that means that intercepts are more disperse than average dependent portfolio returns. The reason of this anomaly may be found in the inflation errors affecting test A2, that are instead corrected in test A3. As a consequence, for the same sub-periods we observe

Table 1.27 Size-expected jump portfolios, R^2_{adj} and intercept tests. Regressions of excess stock returns of 25 size-expected jump portfolios on the returns of the 4-factor model, and on the returns of the 5-factor model: December 1925 to December 2014. R^2_{adj} increase reports the percentage of portfolios for which R^2_{adj} increases when moving from the 4-factor to the 5-factor model. R^2_{adj} max and R^2_{adj} min are respectively the maximum and minimum R^2_{adj} obtained using the 5-factor model. A1, A2, A3, and A4 are intercept tests and their formulas are specified in Table 1.15. Test statistic A4, under H_0, has distribution $F_{N,T-N-K}$, where N equals the number of dependent portfolios, so in our case $N = 25$, T is the length of the portfolios time series and K is the number of factors, consequently $K = 4$ or $K = 5$. The table reports the P-value for test A4 (P(A4)).

Expected jump (JF) portfolios			
Period	R^2_{adj} increase	R^2_{adj} max	R^2_{adj} min
Dec1925-Mar1948	76%	0.972	0.268
Apr1948-Jun1970	48%	0.958	0.253
Jul1970-Sep1992	80%	0.977	0.703
Oct1992-Dec2014	84%	0.968	0.514

Period	A1 (4F)	A1 (5F)	A2 (4F)	A2 (5F)
Dec1925-Mar1948	0.59	0.60	0.67	0.68
Apr1948-Jun1970	0.29	0.29	0.79	0.78
Jul1970-Sep1992	0.15	0.14	1.25	1.18
Oct1992-Dec2014	0.17	0.18	1.04	1.06

Period	A3 (4F)	A3 (5F)	P(A4) (4F)	P(A4) (5F)
Dec1925-Mar1948	0.22	0.27	-	-
Apr1948-Jun1970	0.12	0.12	0.66	1.00
Jul1970-Sep1992	1.72	1.12	0.01	1.00
Oct1992-Dec2014	0.63	0.71	0.02	1.00

lower values of A3; in particular in the Oct1992-Dec2014 window, test values return below 1.

Unclear results of the tests on the intercepts were found also when using the full-sample. Similarly to that case, the high dependent portfolio return volatility may mean that our asset-pricing tests lack power. When results are considered globally, however, it seems that the 5-factor model outperforms the 4-factor model. Major evidences of this are the high percentages of increase of R^2_{adj} and the low fractions of α significance.

1.6 Missing factor

This section evaluates the appropriateness of adding the JF to the asset-pricing model by measuring its marginal effect on the abnormal performance.

Each month we estimate the 3-factor model loadings. Estimations cover the prior three years excess returns for all the 25 dependent portfolios, using a minimum of 30 observations. The corresponding time-series regression equation, already presented in section 1.2, is reported here for simplicity.

$$R_{it} - R_{Ft} = \alpha_i + \beta_i MKT_t + \beta_{SMB,i} SMB_t + \beta_{HML,i} HML_t + e_{it}$$

where R_{it} is the return on a security or portfolio i, R_{Ft} is the risk-free return, $MKT_t = (R_{Mt} - R_{Ft})$ is the excess return on a value weighted market portfolio, SMB_t is the return on a value-weighted factor-mimicking portfolio for *Size*, HML_t is the return on a value-weighted factor-mimicking portfolio for book-to-market equity, and e_{it} is a zero-mean residual.
We then use the results to compute the one-month abnormal return from the 3-factor model:

$$\alpha_{it} = (R_{it} - R_{Ft}) - \hat{\beta}_i MKT_t - \hat{\beta}_{SMB,i} SMB_t - \hat{\beta}_{HML,i} HML_t \tag{1.8}$$

Similarly to the previous sections, we consider two different sets of dependent portfolios: (i) 25 size-BE/ME portfolios, and (ii) 25 size-expected jump portfolios. For each kind of dependent variable, we estimate three different time series regressions:

$$\alpha_{it} = a_i + \beta_{Mi} MOM_t + \xi_{it} \tag{1.9}$$

$$\alpha_{it} = a_i + \beta_{Ji} JF_t + \xi_{it} \tag{1.10}$$

$$\alpha_{it} = a_i + \beta_{Mi} MOM_t + \beta_{Ji} JF_t + \xi_{it} \tag{1.11}$$

Equations 1.9, 1.10, and 1.11 use data from either the previous three years and the full sample.

Regression results allow to study if MOM and JF are missing factors in the 3-factor model. The significance of the corresponding betas signals that the factors explain shared variation in stock returns not explained by MKT, SMB, and HML.

1.6.1 25 size-BE/ME portfolios

Full sample results in Table 1.28, highlight the importance of including the momentum factor in the 3-factor model. The absolute t-statistics on β_M greater than 1.645 are six in the single factor regressions, and seven in the multi-factor regressions. The average value of the MOM slope, considering just those significantly different from 0, is in both cases about -0.04. Differently from the MOM, the relevance of the JF is not so clear: we observe just one portfolio for which β_J is more than two standard errors from 0 and with a value of about -0.1. The poor performance of the JF may be due to the long time windows used in the second regressions.

To further investigate this point it is useful to consider the three-years regressions. We present the results by focusing on portfolios, Table 1.29, and estimation dates, Figure 1.4.

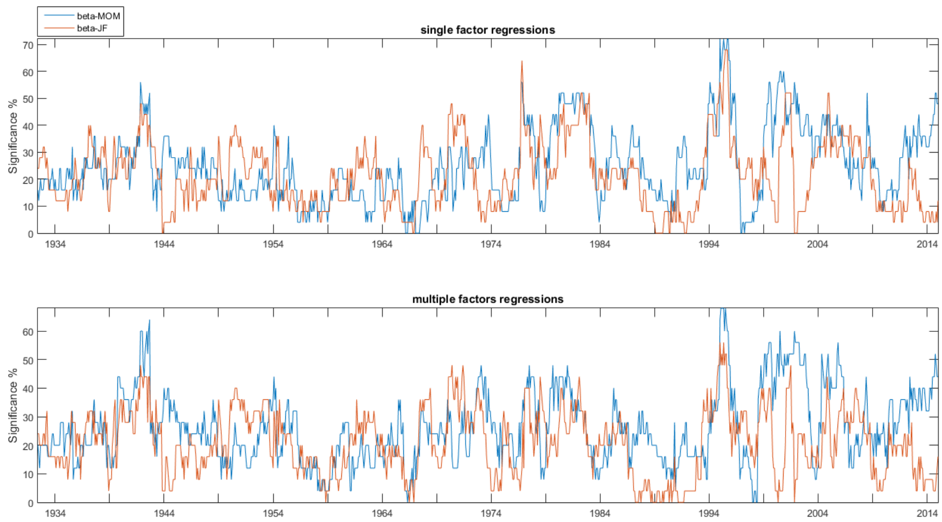

Figure 1.4 Size-BE/ME portfolios, three years window missing factors. Regressions of excess stock returns of 25 size-BE/ME portfolios on the excess market return (MKT) and the mimicking returns for the size (SMB), and book-to-market equity (HML) factors: December 1925 to December 2014. The correspondent abnormal returns, computed monthly using a rolling window of three years, are then used to test if MOM and JF are missing factors. The parameters of equations 1.9, 1.10, and 1.11 are estimated using the prior three years of monthly data. For each estimation date, the graph shows the fraction of portfolios with significant slopes (10% significance level). The top panel plots results for β_M of equation 1.9 and β_J of equation 1.10, while the bottom panel plots results for the slopes of equation 1.11.

Table 1.29 shows, for each portfolio, the fraction of significant β_M and β_J, when considering a 90% confidence level. Even if significance percentages are generally greater for momentum, results for MOM and JF are not too different and suggest that both factors are relevant. β_M, from equation 1.9, is on average significant in 25.18% of the regressions with a minimum of

Table 1.28 Size-BE/ME portfolios, full sample missing factors. Regressions of excess stock returns of 25 size-BE/ME portfolios on the excess market return and the mimicking returns for the size, and book-to-market equity factors: December 1925 to December 2014. The correspondent abnormal returns, computed monthly using a rolling window of three years, are then used to test if MOM and JF are missing factors. The parameters of equations 1.9, 1.10, and 1.11 are estimated using the full sample.

	Book-to-market equity (BE/ME) quintiles									
Size quintile	Low	2	3	4	High	Low	2	3	4	High
	$\alpha_{it} = a_i + \beta_{Mi} MOM_t + \xi_{it}$									
	β_M					$t(\beta_M)$				
Small	-0.07	-0.01	-0.06	0.00	-0.04	-1.10	-0.39	-1.61	-0.25	-1.65
2	-0.02	-0.03	-0.02	0.00	0.00	-0.88	-1.47	-1.35	-0.03	-0.30
3	-0.03	0	-0.01	-0.02	-0.04	-2.41	-0.22	-0.70	-1.15	-2.32
4	0.01	-0.02	-0.04	-0.02	-0.07	1.18	-1.35	-2.39	-1.00	-2.85
Big	-0.02	-0.01	-0.02	-0.03	0.03	-1.88	-1.43	-0.98	-1.23	0.39
	$\alpha_{it} = a_i + \beta_{Ji} JF_t + \xi_{it}$									
	β_J					$t(\beta_J)$				
Small	0.07	0.04	0.05	0.02	0.02	0.27	0.30	0.49	0.34	0.24
2	-0.08	0.05	0.00	0.00	-0.09	-1.12	0.77	-0.03	0.00	-2.16
3	0.03	0.04	-0.01	-0.05	-0.05	0.58	0.86	-0.23	-1.4	-0.66
4	0.01	0.01	-0.08	-0.02	-0.03	0.42	0.25	-1.51	-0.27	-0.37
Big	-0.01	-0.01	-0.01	-0.03	0.00	-0.21	-0.30	-0.12	-0.44	0.03
	$\alpha_{it} = a_i + \beta_{Mi} MOM_t + \beta_{Ji} JF_t + \xi_{it}$									
	β_M					$t(\beta_M)$				
Small	-0.08	-0.02	-0.07	-0.01	-0.04	-1.26	-0.49	-1.66	-0.35	-1.93
2	-0.01	-0.03	-0.02	0.00	0.00	-0.55	-1.74	-1.29	-0.03	0.30
3	-0.04	-0.01	-0.01	-0.02	-0.04	-2.65	-0.4	-0.67	-0.85	-2.31
4	0.01	-0.02	-0.04	-0.02	-0.07	1.07	-1.61	-1.96	-0.88	-2.84
Big	-0.02	-0.01	-0.02	-0.03	0.03	-2.00	-1.32	-0.89	-1.23	0.37
	β_J					$t(\beta_J)$				
Small	0.14	0.06	0.11	0.03	0.06	0.57	0.39	0.99	0.4	0.83
2	-0.07	0.08	0.02	0.00	-0.10	-0.91	1.23	0.41	0.01	-2.13
3	0.07	0.05	0.00	-0.04	0.00	1.15	0.94	-0.05	-0.85	-0.07
4	0.00	0.03	-0.05	0.00	0.04	0.00	0.63	-0.80	-0.06	0.48
Big	0.01	0.00	0.01	0.00	-0.02	0.38	0.02	0.16	-0.01	-0.17

Table 1.29 Size-BE/ME portfolios, three years window missing factors. Regressions of excess stock returns of 25 size-BE/ME portfolios on the excess market return (MKT) and the mimicking returns for the size (SMB), and book-to-market equity (HML) factors: December 1925 to December 2014. The correspondent abnormal returns, computed monthly using a rolling window of three years, are then used to test if MOM and JF are missing factors. The parameters of equations 1.9, 1.10, and 1.11 are estimated using the prior three years of monthly data. The table reports, for each portfolio, the fraction of regressions with absolute t-statistics on β greater than 1.645.

	Book-to-market equity (BE/ME) quintiles									
Size quintile	Low	2	3	4	High	Low	2	3	4	High
	$\alpha_{it} = a_i + \beta_{Mi} MOM_t + \xi_{it}$ β_M					$\alpha_{it} = a_i + \beta_{Ji} JF_t + \xi_{it}$ β_J				
Small	35.3	23.9	33.7	21.8	24.4	22.4	26.4	15.9	20.6	21.8
2	27.1	27.8	21.4	19.0	13.9	15.6	18.4	16.2	18.3	22.3
3	24.3	19.3	24.9	30.0	18.1	19.0	19.4	29.8	27.3	14.2
4	18.5	20.7	27.5	17.0	33.7	9.7	30.7	24.3	26.0	23.1
Big	22.7	36.3	24.4	32.9	31.0	24.8	20.4	25.1	26.3	22.5
	$\alpha_{it} = a_i + \beta_{Mi} MOM_t + \beta_{Ji} JF_t + \xi_{it}$									
	β_M					β_J				
Small	37.2	27.7	35	28.6	28.0	24.0	30.1	13.9	24.3	24.7
2	21.6	28.3	27.0	17.1	16.4	14.6	15.7	17.1	20.1	24.3
3	23.3	24.8	28.9	29.1	19.9	13.5	24.2	28.7	21.5	12.0
4	20.5	20.9	27.2	14.7	36.8	10.9	30.3	26.5	19.8	21.2
Big	28.8	33.9	23.0	28.5	28.7	21.7	21.8	21.0	22.6	18.3

13.9% and a maximum of 36.3%. Correspondent values for β_J, estimated using equation 1.10, are slightly lower: from 9.7% to 30.7%, with an average value of 21.62%. When both factors are included in the regression, equation 1.11, we observe a slight increase in average, 26.24%, minimum, 14.7%, and maximum, 37.2%, significance of β_M. For the JF, instead, only the minimum increases to 10.9% while the maximum and the average show a small decrease, respectively to 30.3% and 20.91%.

From Figure 1.4 we obtain, for each estimation date, information on the fraction of portfolios for which the slopes are more than 1.645 standard errors from 0. In the top panel there are the results for β_M of equation 1.9 and β_J of equation 1.10, while in the bottom panel the results for the slopes of equation 1.11. Not only the two factors show similar behaviors, but in 33.57% and 30.34% of the dates in, respectively, the single- and multi-factor regressions, the fraction of significant β_J is greater than the corresponding fraction of β_M.

A global interpretation of the results requires a preliminary consideration: since jumps are short-time phenomena, regressions that use short time windows can better capture and describe their behavior. In line with this statement, the use of the full time window leads to poor results for the JF. It is, however, more relevant to consider the results on shorter time-windows. In this last case performances for MOM an JF are similar and suggest that they are missing factors in the 3-factor model.

1.6.2 25 size-expected jump portfolios

Table 1.30 shows stronger results in favor of the inclusion in the 3-factor model for the JF with respect to the MOM. Considering a 90% confidence level, significant β_M are fourteen in the single factor regressions but just five in the multi-factor regressions. The average absolute values of the MOM slope, considering just those significantly different from 0, are respectively 0.1 and 0.21. For the JF, instead, the number of portfolios for which β_J is more than 1.645 standard errors from 0 decreases of just one unit, from thirteen to twelve. Average absolute slopes for significant portfolios are 0.26 in the single factor case and 0.24 in the multi-factor case.

As discussed before, since long time-windows impact on JF performance, we expect better results when using time-windows of three years. Results in Table 1.31 and Figure 1.5 confirm the importance of the JF.

In Table 1.31, the fraction of regressions with significant β_J is usually bigger than the corresponding fraction with significant β_M (10% significance level). Significance values on β_M, estimated using equation 1.9, show similar results to the 25 size-BE/ME case: average, minimum and maximum values are 26.16%, 16.5%, and 39%. β_J, from equation 1.10, shows a wider range of significance percentages, from 14% to 81.8%, and a higher average value,

Table 1.30 Size-expected jump portfolios, full sample missing factors. Regressions of excess stock returns of 25 size-expected jump portfolios on the excess market return and the mimicking returns for the size, and book-to-market equity factors: December 1925 to December 2014. The correspondent abnormal returns, computed monthly using a rolling window of three years, are then used to test if MOM and JF are missing factors. The parameters of equations 1.9, 1.10, and 1.11 are estimated using the full sample.

	Expected jump (JF) quintiles									
Size quintile	Low	2	3	4	High	Low	2	3	4	High
	$\alpha_{it} = a_i + \beta_{Mi} MOM_t + \xi_{it}$									
			β_M					$t(\beta_M)$		
Small	-0.07	-0.07	-0.28	-0.24	-0.13	-0.74	-1.50	-2.98	-2.20	-1.44
2	-0.15	0.02	-0.16	-0.12	0.04	-2.29	0.24	-1.95	-1.81	0.77
3	-0.05	-0.09	0.00	-0.07	0.04	-2.02	-1.81	-0.10	-2.28	0.94
4	-0.04	-0.02	-0.04	-0.05	0.02	-2.45	-1.21	-2.11	-1.31	1.36
Big	-0.04	-0.03	-0.02	0	0.02	-2.43	-1.66	-1.76	0.16	2.08
	$\alpha_{it} = a_i + \beta_{Ji} JF_t + \xi_{it}$									
			β_J					$t(\beta_J)$		
Small	-0.66	-0.15	-0.48	0.27	-0.45	-1.55	-0.90	-1.81	1.08	-1.54
2	-0.08	0.14	-0.01	-0.29	-0.04	-0.56	0.48	-0.03	-1.44	-0.23
3	-0.17	0.04	0.20	0.05	0.34	-1.65	0.28	1.55	0.43	3.09
4	-0.39	-0.13	-0.10	0.18	0.26	-4.48	-1.94	-2.18	1.86	4.47
Big	-0.44	-0.31	-0.09	0.15	0.34	-7.64	-6.07	-2.04	4.27	10.34
	$\alpha_{it} = a_i + \beta_{Mi} MOM_t + \beta_{Ji} JF_t + \xi_{it}$									
			β_M					$t(\beta_M)$		
Small	0.21	-0.07	0.17	0.17	0.16	0.60	-0.38	0.63	0.52	0.60
2	-0.13	0.35	0.15	-0.04	0.20	-0.86	1.99	0.41	-0.19	0.96
3	-0.34	0.14	0.02	0.11	-0.07	-3.37	1.08	0.18	1.10	-0.58
4	-0.19	-0.06	0.10	-0.03	-0.09	-3.24	-1.00	1.72	-0.48	-1.28
Big	-0.08	0.02	0.06	0.03	-0.03	-2.02	0.4	1.34	0.77	-0.93
			β_J					$t(\beta_J)$		
Small	-0.66	-0.15	-0.48	0.27	-0.45	-1.55	-0.90	-1.81	1.08	-1.54
2	-0.08	0.14	-0.01	-0.29	-0.04	-0.56	0.48	-0.03	-1.44	-0.23
3	-0.17	0.04	0.20	0.05	0.34	-1.65	0.28	1.55	0.43	3.09
4	-0.39	-0.13	-0.10	0.18	0.26	-4.48	-1.94	-2.18	1.86	4.47
Big	-0.44	-0.31	-0.09	0.15	0.34	-7.64	-6.07	-2.04	4.27	10.34

Table 1.31 Size-expected jump portfolios, three years window missing factors. Regressions of excess stock returns of 25 size-expected jump portfolios on the excess market return (MKT) and the mimicking returns for the size (SMB), and book-to-market equity (HML) factors: December 1925 to December 2014. The correspondent abnormal returns, computed monthly using a rolling window of three years, are then used to test if MOM and JF are missing factors. The parameters of equations 1.9, 1.10, and 1.11 are estimated using the prior three years of monthly data. The table reports, for each portfolio, the fraction of regressions with absolute t-statistics on β greater than 1.645.

	Expected jump (JF) quintiles									
Size quintile	Low	2	3	4	High	Low	2	3	4	High
	$\alpha_{it} = a_i + \beta_{Mi} MOM_t + \xi_{it}$					$\alpha_{it} = a_i + \beta_{Ji} JF_t + \xi_{it}$				
	β_M					β_J				
Small	21.8	17.9	29.0	19.5	32.0	29.2	19.8	28.5	16.4	29.7
2	30.2	22.2	22.1	29.9	20.6	28.5	24.0	25.0	19.1	28.6
3	21.7	28.7	17.7	16.5	30.0	24.4	26.4	26.7	14.0	39.4
4	29.6	19.7	33.1	33.4	16.6	45.2	29.5	19.7	26.5	28.9
Big	31.4	33.8	31.7	25.8	39.0	81.8	52.2	33.5	50.6	78.8
	$\alpha_{it} = a_i + \beta_{Mi} MOM_t + \beta_{Ji} JF_t + \xi_{it}$									
	β_M					β_J				
Small	27.3	19.5	20.7	12.5	20.9	29.2	19.8	28.5	16.4	29.7
2	28.5	25.6	24.9	17.1	16.9	28.5	24.0	25.0	19.1	28.6
3	33.6	23	16.2	18.1	22.5	24.4	26.4	26.7	14	39.4
4	38.9	18.5	22.6	26.1	22.1	45.2	29.5	19.7	26.5	28.9
Big	21.2	23.9	24.1	21.6	13.8	81.8	52.2	33.5	50.6	78.8

33.06%. Results for the JF do not change when both factors are included in the regressions, equation 1.11. For the MOM, instead, we observe a decrease in average, 22.40%, minimum, 12.5%, and maximum, 38.9%, significance with respect to the single-factor regressions.

Figure 1.5 shows, for each estimation date, the fraction of portfolios slopes that are more than 1.645 standard errors from 0. Both in the top panel, which considers the single-factor regressions (equation 1.9 and equation 1.10), and in the bottom panel, that focuses on the multi-factor regressions (equation 1.11) we observe a general higher level of significance with respect to the size-BE/ME case (Figure 1.4). We obtain similar results for MOM and JF when using single-factor regressions, and in 68.25% of the dates the fraction of significant β_J is greater than the corresponding fraction of β_M. Results are even more in favor of the JF when considering multi-factor regressions: levels of significance differs often substantially and in 71.07% of the dates the percentage of significant β_J is larger than the percentage of significant β_M.

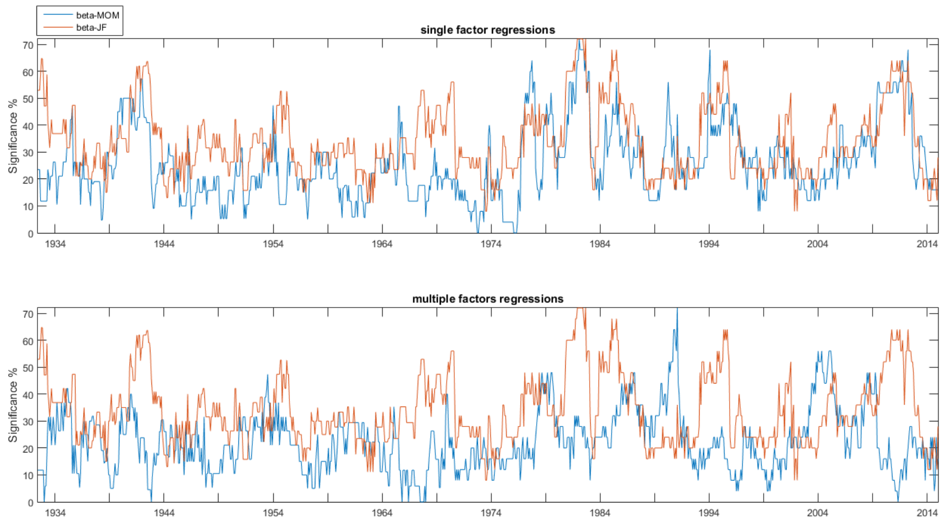

Figure 1.5 Size-expected jump portfolios, three years window missing factors. Regressions of excess stock returns of 25 size-expected jump portfolios on the excess market return (MKT) and the mimicking returns for the size (SMB), and book-to-market equity (HML) factors: December 1925 to December 2014. The correspondent abnormal returns, computed monthly using a rolling window of three years, are then used to test if MOM and JF are missing factors. The parameters of equations 1.9, 1.10, and 1.11 are estimated using the prior three years of monthly data. For each estimation date, the graph shows the fraction of portfolios with significant slopes (10% significance level). The top panel plots results for β_M of equation 1.9 and β_J of equation 1.10, while the bottom panel plots results for the slopes of equation 1.11.

Lastly, Table 1.32 reports the values, and corresponding P-values, of a Wald test statistics that verifies that the betas (βs) from full sample regressions are simultaneously equal to zero: $\beta = 0$. The correspondent test, $\beta' \Omega^{-1} \beta$ is distributed as a χ^2_N where N equals the number of dependent portfolios, so in our case $N = 25$. To empirically compute the test it is necessary

Table 1.32 3-Factor regression, missing factor slope test. Regressions of excess stock returns of 25 size-BE/ME portfolios and 25 size-expected jump portfolios on the excess market return (MKT) and the mimicking returns for the size (SMB), and book-to-market equity (HML) factors: December 1925 to December 2014. The correspondent abnormal returns are then used to test if MOM and JF are missing factors. The table reports the test values, $\beta'\Omega^{-1}\beta$, checking $H0: \beta = 0$, and the corresponding P-values. We substitute the unknown quantities β, and Ω with their estimated correspondents $\hat{\beta}$, and $\hat{\Omega} = \frac{1}{T}\sum_{t=1}^{T}\hat{\xi}_t\hat{\xi}_t'$. The test statistic, under H_0, has asymptotic distribution χ^2_N, where $N = 25$.

	P-value			
Portfolios	Single regression		Multiple regression	
	β_M	β_J	β_M	β_J
size-BE/ME	1.00	1.00	1.00	1.00
size-exp Jump	1.00	1.00	1.00	1.00

	Test-value			
Portfolios	Single regression		Multiple regression	
	β_M	β_J	β_M	β_J
size-BE/ME	0.0067	0.0157	0.0191	0.0075
size-exp Jump	0.0278	0.7819	0.1508	0.7857

to substitute the unknown quantities β, and Ω with their estimated correspondents $\hat{\beta}$, and $\hat{\Omega} = \frac{1}{T}\sum_{t=1}^{T}\hat{\xi}_t\hat{\xi}_t'$. $\hat{\beta}$ is the vector of estimated betas, $\hat{\Omega}$ is the variance-covariance matrix of the regression residuals, and T is is the length of the portfolios time series. Each row focus on a different set of dependent portfolios and reports, respectively, the P-values for β_M of equation 1.9, β_J of equation 1.10, and β_M and β_J of equation 1.11. It is clear from the results that it is not possible to reject the null hypothesis in all the cases in analysis. The negative results are motivated by the use of such a long time-window. The test assume greater values in the size-expected jump cases but β_M and β_J test values are closer in the size-BE/ME case.

The analysis in this section support the inclusion of the JF, as well the MOM, in the 3-factor model. Results for the JF are stronger when focusing on short time-windows, reflecting the short-term peculiarity of the jumps, and when using 25 size-expected jump portfolios. This last evidence, suggests that the inclusion of the JF is relevant, in particular, for portfolios with strong size and expected jump tilts.

1.7 Risk Premium

In section 1.2.3 we introduced our 5-factor model, designed such as the assets excess returns obey a linear relationship with their exposures to various sources of risk, equation 1.7. We are know interested in understanding what are the risk premia associated with those factors.

The risk premium, γ, measures the extra return demanded by an investor for investing in the asset relative to the risk-free rate. So, in our case, the total risk premium is the sum of different premiums:

$$\gamma_{tot} = \gamma_{MKT} + \gamma_{SMB} + \gamma_{HML} + \gamma_{MOM} + \gamma_{JF}.$$

To empirically compute the risk premiums we employ two similar approaches: Black et al. (1972) and Fama and MacBeth (1973). Both approaches use a two pass technique. In the first step, each asset's return is regressed on the time series of the factor realizations, to obtain the estimated beta coefficients of the portfolios on the factors:

$$R_{i,t} - R_{F,t} = \alpha_i + \beta_i MKT_t + \beta_{SMB,i} SMB_t + \beta_{HML,i} HML_t + \beta_{MOM,i} MOM_t + \beta_{JF,i} JF_t + e_{i,t}. \tag{1.12}$$

In the second pass, at each time t the cross-section of assets returns is regressed against their beta coefficients:

$$R_{i,t} - R_{F,t} = \hat{\beta}_i \gamma_{MKT,t} + \hat{\beta}_{SMB,i} \gamma_{SMB,t} + \hat{\beta}_{HML,i} \gamma_{HML,t} + \hat{\beta}_{MOM,i} \gamma_{MOM,t} + \hat{\beta}_{JF,i} \gamma_{JF,t} + \varepsilon_{i,t}. \tag{1.13}$$

In this way we obtain five time series of risk premia coefficients, $\hat{\gamma}$, each of length T.

It is important to notice that the regressors we use in the second step are not the real betas, which are unknown, but the estimated betas. This introduces the error-in-the-variables problem. As suggested among the others by Fama and MacBeth (1973), we solve this problem by using portfolios, 25 size-BE/ME portfolios and 25 size-expected jump portfolios, instead of assets as dependent variables. The $\hat{\beta}$'s of portfolios are more precise estimates of true β's than $\hat{\beta}$'s for single assets.

The difference between the Black et al. (1972) approach and the Fama and MacBeth (1973) approach lies in the choice of the explanatory variables in the second pass, $\hat{\beta}$s. According to the former we use full sample β estimates, while for the latter rolling β estimates.

The following subsections describe more in detail the two approaches and the variation to the standard two pass technique, the Hou and Kimmel (2006) correction, we use to ensure that the factors are spanned by assets.

1.7.1 Black, Jensen, and Scholes approach

Following the Black et al. (1972) approach, we run the first step (equation 1.12) using the full 1925-2014 sample of monthly percentage returns. From the T regressions of step two, we obtain the time series of risk premia coefficients, $\breve{\gamma}_t$, and residuals, $\breve{\varepsilon}_t$, that are then used to estimate γ and ε as the average of the cross-sectional regressions:

$$\breve{\gamma} = \frac{1}{T}\sum_{t=1}^{T} \breve{\gamma}_t \text{ and } \breve{\varepsilon} = \frac{1}{T}\sum_{t=1}^{T} \breve{\varepsilon}_t.$$

In order to test the statistical significance of the estimated risk premiums, $\breve{\gamma}$s, it is necessary to consider their asymptotic distribution:

$$\sqrt{T}(\breve{\gamma}-\gamma) \xrightarrow{d} \mathcal{N}[0,(1+\mu_f'\Sigma_f^{-1}\mu_f)(B'B)^{-1}B'\Omega B(B'B)^{-1}+\Sigma_f].$$

Where T is the length of the portfolios time series, B is the matrix of estimated coefficients for the 25 portfolios, μ_f is the vector of the expected value of the factors, Σ_f is the variance-covariance matrix of the factors, and Ω is the variance-covariance matrix of step one regression residuals. Moreover, the multiplicative term $(1+\mu_f'\Sigma_f^{-1}\mu_f)$ is due to the Shanken (1992) correction for the fact that $\hat{\beta}$ are generated regressors. To empirically test if a risk premium is equal to 0, $H_0 : \breve{\gamma} = 0$, we need to substitute the unknown quantities μ_f, Σ_f, and Ω with their estimated correspondents $\bar{f}$, $\hat{\Sigma}_f$, and $\hat{\Omega} = \frac{1}{T}\sum_{t=1}^{T} \hat{e}_t\hat{e}_t'$.

Table 1.33 shows the estimated monthly percentage risk premia and the correspondent t-statistics for $H_0 : \breve{\gamma} = 0$. In the size-BE/ME case all the estimates are positive and, with the exception of the momentum factor, fairly close to their factor portfolio mean monthly percentage excess return. Moreover, the estimated risk premiums for MKT, HML, and MOM are more than three standard errors from 0. For the size-expected jump case, instead, we observe a negative, even if not statistically significant, risk premium for the JF factor portfolio. In this case only the absolute t-statistics on the risk premiums for MKT and HML are greater than 1.645.

The time series of risk premia estimates, $\breve{\gamma}_t$, and errors, $\breve{\varepsilon}_t$, can also be used to approximate the variance-covariance matrix of the difference between estimated risk premiums and factor mean values: $\breve{\gamma}-\bar{f}$. Renaming the differences as $\breve{\phi}_t = \breve{\gamma}_t - f_t$ and $\breve{\phi} = \breve{\gamma}-\bar{f}$, we can define the variance-covariance matrix as:

$$V\breve{a}r(\breve{\gamma}-\bar{f}) \equiv V\breve{a}r(\breve{\phi}) = \frac{1}{T}V\breve{a}r(\breve{\phi}_t)$$

Table 1.33 5-factor risk premiums. Two-pass estimated risk premia of the 5-factor model, using monthly excess stock returns of 25 size-BE/ME portfolios and 25 size-expected jump portfolios: December 1925 to December 2014. The table reports the mean monthly percentage excess return of each factor portfolio, Mean return %, the Black et al. (1972) monthly percentage risk premia, $\breve{\gamma}$, and the corresponding t-statistic for $\breve{\gamma} = 0$, the Fama and MacBeth (1973) monthly percentage risk premia, $\hat{\gamma}$, and the corresponding t-statistic for $\hat{\gamma} = 0$.

Factor	Mean	25 size-BE/ME portfolios			
portfolio	return %	$\breve{\gamma}$	t-statistic	$\hat{\gamma}$	t-statistic
γ_{MKT}	0.65	0.69	4.04	0.78	4.82
γ_{SMB}	0.22	0.14	1.33	0.25	2.39
γ_{HML}	0.40	0.48	4.22	0.45	3.80
γ_{MOM}	0.67	2.47	3.19	0.36	1.62
γ_{JF}	0.12	0.28	1.13	0.02	0.18
		25 size-Expected jump portfolios			
		$\breve{\gamma}$	t-statistic	$\hat{\gamma}$	t-statistic
γ_{MKT}	0.65	0.40	2.16	0.47	2.38
γ_{SMB}	0.22	0.18	1.17	0.24	1.60
γ_{HML}	0.40	1.07	3.52	-0.13	-0.58
γ_{MOM}	0.67	0.17	0.34	-0.19	-0.64
γ_{JF}	0.12	-0.01	-0.17	0.01	0.13

where $V\breve{a}r(\breve{\phi}_t) = \frac{1}{T}\sum_{t=1}^{T}(\breve{\phi}_t - \breve{\phi})(\breve{\phi}_t - \breve{\phi})'$. This estimates are used to test if the differences between estimated parameters and factor means are statistically simultaneously different from 0: $H_0 : \breve{\gamma} - \bar{f} = 0$. The correspondent test statistic $\breve{\phi}'[V\breve{a}r(\breve{\phi})]^{-1}\breve{\phi}$, under H_0, has distribution χ^2_N, where N equals the number of dependent portfolios, so in our case $N = 25$.

The P-values of the tests for the 25 size-BE/ME portfolios and 25 size-expected jump portfolios reported in Table 1.34, show that the null hypothesis cannot be rejected, no matter the dependent portfolios in use.

Table 1.34 Risk premiums/factor means divergence tests. Two-pass regressions of the 5-factor model using monthly excess stock returns of 25 size-BE/ME portfolios and 25 size-expected jump portfolios: December 1925 to December 2014. The table reports the P-values of the tests that check if the differences between estimated parameters and factor means are statistically simultaneously equal to 0. The test statistics for the Black et al. (1972) and the Fama and MacBeth (1973) approaches are respectively $\breve{\phi}'[V\breve{a}r(\breve{\phi})]^{-1}\breve{\phi}$ and $\hat{\phi}'[V\hat{a}r(\hat{\phi})]^{-1}\hat{\phi}$, where $\breve{\phi} = \breve{\gamma} - \bar{f}$ and $\hat{\phi} = \hat{\gamma} - \bar{f}$. Both tests, under H_0, have distribution χ^2_N, where N equals the number of dependent portfolios, so in our case $N = 25$.

Dependent portfolio	P-value $H_0 : \breve{\gamma} - \bar{f} = 0$	P-value $H_0 : \hat{\gamma} - \bar{f} = 0$
25 size-BE/ME	0.980	1.000
25 size-Exp. jump	0.987	0.246

1.7.2 Fama and MacBeth approach

The Fama and MacBeth (1973) approach requires the use of five-years rolling windows of monthly percentage returns in the first step. We then estimate the risk premia, γs, and the residuals, ε, as the average of the T cross-sectional regressions of step two:

$$\hat{\gamma} = \frac{1}{T}\sum_{t=1}^{T}\hat{\gamma}_t \text{ and } \hat{\varepsilon} = \frac{1}{T}\sum_{t=1}^{T}\hat{\varepsilon}_t.$$

The sampling errors for the estimates are computing following Fama and MacBeth (1973) who suggest the use of the standard deviations of the cross-sectional regression estimates: $\sigma^2(\hat{\gamma}) = \frac{1}{T}\sum_{t=1}^{T}(\hat{\gamma}_t - \hat{\gamma})^2$. Since it does not correct for the fact that $\hat{\beta}$ are generated regressors, it is necessary to check that the Shanken (1992) correction factor, $(1 + \mu'_f \Sigma_f^{-1} \mu_f)$, is not too large. To empirically compute the correction factor we need to substitute the unknown quantities, μ_f and Σ_f, with their estimated correspondents, $\bar{f}$ and $\hat{\Sigma}_f$. In our case for a monthly interval $\bar{f}\hat{\Sigma}_f^{-1}\bar{f} \approx 0.08$. Since it is quite small, ignoring the multiplicative term does not make big difference.

We report in Table 1.33 the estimated monthly percentage risk premia and the correspondent t-statistics for $H_0 : \hat{\gamma} = 0$. For the size-BE/ME case we observe positive estimated values which are also fairly close to their factor portfolio mean monthly percentage excess return. The absolute t-statistics on risk premiums for MKT, SMB, and HML are greater than 1.645. In the size-expected jump case, instead, we observe negative but not statistically significant, risk premiums for the HML and the MOM factor portfolios. Only the estimated risk premium for MKT is more than two standard errors from 0.

Similarly to the Black, Jensen, and Scholes approach, we can use the time series of risk premia estimates, $\hat{\gamma}_t$, and errors, $\hat{\varepsilon}_t$ to approximate the variance-covariance matrix of the difference between estimated risk premiums and factor mean values: $\hat{\gamma} - \bar{f}$. Calling the differences $\hat{\phi}_t = \hat{\gamma}_t - f_t$ and $\hat{\phi} = \hat{\gamma} - \bar{f}$, we define the variance-covariance matrix as:

$$\hat{Var}(\hat{\gamma} - \bar{f}) \equiv \hat{Var}(\hat{\phi}) = \frac{1}{T}\hat{Var}(\hat{\phi}_t)$$

where $\hat{Var}(\hat{\phi}_t) = \frac{1}{T}\sum_{t=1}^{T}(\hat{\phi}_t - \hat{\phi})(\hat{\phi}_t - \hat{\phi})'$. The test $\hat{\phi}'[\hat{Var}(\hat{\phi})]^{-1}\hat{\phi}$ for $H_0 : \hat{\gamma} - \bar{f} = 0$, under H_0, has distribution χ^2_N, where N equals the number of dependent portfolios.

We show in Table 1.34 the P-values of the tests for the 25 size-BE/ME portfolios and 25 size-expected jump portfolios. In both cases, the null hypothesis that the differences between estimated risk premiums and factor means are simultaneously equal to 0 cannot be rejected.

1.7.3 Hou and Kimmel extrapolation correction

Hou and Kimmel (2006) define extrapolation as the phenomenon that arises when the factors of a linear factor model are not spanned by assets. In this case, in fact, the risk premium of a factor is formed by two components: the risk premium of the factor mimicking portfolio and an extrapolation of the risk premiums of the factors spanned components to the unspanned components. By purchasing the appropriate securities we can realize only the former. The latter, to be consider a real risk premium, requires the existence of additional assets that complete the market which are also correctly priced by the model.

Using 25 size-BE/ME portfolios, and 25 size-expected jump portfolios in the Black et al. (1972) and the Fama and MacBeth (1973) approaches, leads to compute risk premiums that are affected by extrapolation. In fact, we treat the factors as if they are unspanned even if they are traded assets that investors can buy. An investor who can only trade in the 25 portfolios (25 size-BE/ME portfolios or 25 size-expected jump portfolios) is not able to perfectly replicate the returns on the 5 factor portfolios. Hou and Kimmel (2006) suggest to augment the 25 dependent portfolios with the five factor portfolios, thus making the factors spanned.

To understand the change in the investment opportunities that we would introduce by the Hou and Kimmel (2006) correction, we regress the monthly returns of the factor portfolios on the monthly returns of the 25 other portfolios:

$$\begin{aligned}
MKT_t &= \alpha_{MKT} + \beta_{MKT,1}R_{1,t} + \cdots + \beta_{MKT,25}R_{25,t} + \varepsilon_{MKT,t} \\
SMB_t &= \alpha_{SMB} + \beta_{SMB,1}R_{1,t} + \cdots + \beta_{SMB,25}R_{25,t} + \varepsilon_{SMB,t} \\
HML_t &= \alpha_{HML} + \beta_{HML,1}R_{1,t} + \cdots + \beta_{HML,25}R_{25,t} + \varepsilon_{HML,t} \\
MOM_t &= \alpha_{MOM} + \beta_{MOM,1}R_{1,t} + \cdots + \beta_{MOM,25}R_{25,t} + \varepsilon_{MOM,t} \\
JF_t &= \alpha_{JF} + \beta_{JF,1}R_{1,t} + \cdots + \beta_{JF,25}R_{25,t} + \varepsilon_{JF,t}
\end{aligned}$$

The corresponding R^2 values, reported in Table 1.35, give us an idea of how well, using the 25 portfolios, we can replicate the returns of the factor portfolios. In the size-BE/ME case the high R^2 values for MKT, SMB, and HML suggest that the part not spanned by the 25

Table 1.35 Factor portfolios, R^2**.** Table reports the R^2 values resulting from regressions of excess returns of 5 factor portfolios (MKT, SMB, HML, MOM, and JF) on the returns of 25 size-BE/ME portfolios or 25 size-expected jump portfolios: December 1925 to December 2014.

	R^2 for the dependent portfolio				
Explanatory portfolios	MKT	SMB	HML	MOM	JF
25 size-BE/ME	0.99	0.97	0.96	0.30	0.14
25 size-Expected jump	1.00	0.92	0.28	0.35	0.88

portfolios of the three Fama and French factors is small. The same conclusion does not hold for MOM and JF. Their low R^2 values, respectively 0.30 and 0.14, tell us that if constrained to purchase only the 25 size-BE/ME portfolios, an investor would not be able to replicate the returns on MOM and JF. In the size-expected jump case, instead, we observe large R^2 values for MKT, SMB, and JF. The R^2 values for HML and MOM are both very low: 0.28 for the former and 0.35 for the latter.

The addition of the five factor portfolios significantly changes the investment opportunity set both in the size-BE/ME case and in the size-expected jump case. These results justify the inclusion of the factor portfolios in the set of dependent assets.

We repeat the regressions of the previous subsections but applying the Hou and Kimmel (2006) extrapolation correction. The results for both the Black et al. (1972) approach and the Fama and MacBeth (1973) approach are presented in Tables 1.36 and 1.37.

Table 1.36 5-factor corrected risk premiums. Two-pass estimated risk premia of the 5-factor model, using monthly excess stock returns of 25 size-BE/ME portfolios and 25 size-expected jump portfolios augmented by the 5 factor portfolios: December 1925 to December 2014. The table reports the mean monthly percentage excess return of each factor portfolio, Mean return %, the Black et al. (1972) monthly percentage risk premia, $\check{\gamma}$, and the corresponding t-statistic for $\check{\gamma} = 0$, the Fama and MacBeth (1973) monthly percentage risk premia, $\hat{\gamma}$, and the corresponding t-statistic for $\hat{\gamma} = 0$.

Factor portfolio	Mean return %	25 size-BE/ME portfolios + 5 factor portfolios			
		$\check{\gamma}$	t-statistic	$\hat{\gamma}$	t-statistic
γ_{MKT}	0.65	0.65	3.85	0.45	2.45
γ_{SMB}	0.22	0.13	1.21	0.30	2.11
γ_{HML}	0.40	0.42	3.72	0.47	3.71
γ_{MOM}	0.67	0.73	4.90	0.62	3.48
γ_{JF}	0.12	0.17	1.98	0.09	1.25
		25 size-Expected jump portfolios + 5 factor portfolios			
		$\check{\gamma}$	t-statistic	$\hat{\gamma}$	t-statistic
γ_{MKT}	0.65	0.79	4.86	0.49	2.47
γ_{SMB}	0.22	0.24	2.27	0.16	1.13
γ_{HML}	0.40	0.45	3.88	0.19	1.33
γ_{MOM}	0.67	0.51	3.43	0.49	2.49
γ_{JF}	0.12	0.07	0.97	0.09	1.22

Relative to the Black et al. (1972) approach, we observe that in the size-BE/ME case all the estimates are positive and close to their factor portfolio mean monthly percentage excess returns. In particular, the estimated risk premium for the MOM factor is much closer to its factor mean value, that is 0.67, with respect to the non-corrected case; it moves from 2.47 to 0.73. In addition, results show that not only the absolute t-statistics on the risk premiums for MKT, HML, and MOM are greater than 1.96, but also that the estimated JF risk premium is

Table 1.37 Corrected risk premiums/factor means divergence tests. Two-pass regressions of the 5-factor model using monthly excess stock returns of 25 size-BE/ME portfolios and 25 size-expected jump portfolios augmented by the 5 factor portfolios: December 1925 to December 2014. The table reports the P-values of the tests that check if the differences between estimated parameters and factor means are statistically simultaneously equal to 0. The test statistics for the Black et al. (1972) and the Fama and MacBeth (1973) approaches are respectively $\breve{\phi}'[\breve{Var}(\breve{\phi})]^{-1}\breve{\phi}$ and $\hat{\phi}'[\hat{Var}(\hat{\phi})]^{-1}\hat{\phi}$, where $\breve{\phi} = \breve{\gamma} - \bar{f}$ and $\hat{\phi} = \hat{\gamma} - \bar{f}$. Both tests, under H_0, have distribution χ^2_N, where N equals the number of dependent portfolios, so in our case $N = 30$.

Dependent portfolio	P-value $H_0 : \breve{\gamma} - \bar{f} = 0$	P-value $H_0 : \hat{\gamma} - \bar{f} = 0$
25 size-BE/ME + 5 factor	0.998	1.000
25 size-E. jump + 5 factor	0.999	0.794

significantly different from 0, at the 5% significance level. For the size-expected jump case, the correction resolves the anomaly we observed in the uncorrected case of a negative JF risk premium. Applying the correction we obtain all positive risk premiums that are also, with the exception of the JF, more than two standard errors from 0.

In the Fama and MacBeth (1973) case, we observe positive estimated values both using the size-BE/ME and the size-expected jump portfolios. Also in this case, the negative estimated risk premiums observed in the uncorrected case are no more present when we apply the Hou and Kimmel (2006) extrapolation correction. While in the size-BE/ME case we obtain risk premiums that, with the only exception of the JF, are more than two standard errors from 0, in the size-expected jump case only the absolute t-statistics on risk premiums for MKT and MOM are greater than 2.

Lastly, comparing Table 1.34 and Table 1.37 we do not observe large differences in the P-values. Therefore, also using Hou and Kimmel (2006) corrected regressions, the null hypothesis that the differences between estimated risk premiums and factor means are simultaneously equal to 0 cannot be rejected, no matter the dependent portfolios in use.

1.8 Conclusions

This paper investigates the presence of a new common jump risk factor in stock returns and tests whether it captures the cross-section of average returns. We construct our factor, the Jump Factor (or JF), starting from the observation that market fear of future jumps can be inferred from observed returns using a model for stock returns with time-varying conditional jump intensity: the model of Chan and Maheu (2002). The high values of JF mean (0.12% monthly return) and volatility (1.46) and its low correlations with the other factors (minimum and maximum correlations are respectively -0.23 and 0.28), show that the JF can explain much of the variation in returns both in time and cross-section.

Empirical evidence also supports the hypothesis that the expected jump component proxy, in stock returns, for sensitivity to a common risk factor. We empirically investigate an extended capm model, our 5-factor model, and find that the new factor captures shared variation in stock returns that is missed by the four factors of the Carhart (1997) model: MKT, SMB, HML, and MOM. The slopes on JF, resulting from 5-factor time series regressions, range respectively from -0.16 to 0.45 using 25 size-BE/ME dependent portfolios, and from -0.94 to 0.69 for 25 size-expected jump dependent portfolios. They are not only large, in absolute value, but also often statistically different from 0: t-statistic is greater than 1.645 respectively in 28% and 52% of the cases. The power of our factor in capturing common variation, is also supported by the values of the coefficients of determination. Indeed, we observe that the inclusion of the JF in the asset-pricing model, increases R^2_{adj} values at least 80% of the times.

Missing factor analysis provides further supporting results about the usefulness of adding the JF to the Carhart (1997) asset-pricing model and justifies its inclusion. JF and MOM slopes, resulting from correspondent regressions, show similar behaviors thus suggesting that both factors are relevant and can be considered missing factors in the Fama and French (1993) 3-factor model.

Lastly, we compute the risk premiums associated with the five factors of our new model (γ_{MKT}, γ_{SMB}, γ_{HML}, γ_{MOM}, and γ_{JF}), and find that they are positive and close to their factor portfolio mean monthly percentage excess returns. Their signs and values are in line with our expectations since the premiums reflect the extra return demanded by an investor for investing in the asset relative to the risk-free rate. Moreover, there is no statistical difference between estimated risk premiums and factor means. For all but the JF, we also observe risk premiums that are, in most of the cases, statistically different from 0 at standard confidence levels. The Jump Factor premium, instead, is more than 1.645 standard errors from 0 in just one case.

The low significance of our factor may be due to the frequency in use to estimate the parameters of the Chan and Maheu (2002) model. In this paper estimations take place at the end of June of each year using previous year daily returns. A yearly updating of the parameters could lead to build a factor that has reduced power in reflecting the short-time nature of the jumps. An update of the paper, using estimations that can better reflect jumps time variability, is in progress and employs monthly estimations using previous year daily returns.

Chapter 2

The cross-sectional diffusion of jumps and the identification of collective sectorial movements

A joint work with Giovanni Bonaccolto[1].

This paper investigates co-jumps which involve a relatively large number of stocks and propose two indexes informative of the cross-sectional diffusion of jumps. They have important implication not only for asset allocation and hedging, but also in asset pricing. Specifically, their inclusion in a standard CAPM model gives evidence that diffusion indexes capture common variation in stock returns that is missed by the market factor.

2.1 Introduction

Every day market operators exchange tens of thousand of stocks, creating an extremely rich information set to study price dynamics. In addition, the availability of high-frequency data, has recently stimulated the construction of non-parametric tests aim to detect if, in addition to a continuous diffusion component, asset prices are driven by discontinuous jumps. To take advantage of both these elements, we employ a very large dataset of high-frequency asset prices.

Even if a vast literature proves the presence of price jumps, Ball and Torous (1983), Jarrow and Rosenfeld (1984), Jorion (1988), Duffie et al. (2000), and Eraker et al. (2003)

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among others, their empirical identification is not easy since jumps are rare events and long samples of stock prices are often unavailable. The possibility to access high-frequency data, has created new techniques of jump detection. A cornerstone in this field are Barndorff-Nielsen and Shephard (2004b, 2006), who identify jumps using realized measures of volatility based on high-frequency data. Their method consists on separating the volatility into its continuous part, driven by continuous price variations, and another component that captures large and infrequent price changes, in other words the jumps. Following their approach, several subsequent studies proposed alternative tests to explicitly identify intraday jumps, see among others Andersen et al. (2007), Lee and Mykland (2008), Corsi et al. (2010), and Andersen et al. (2010).

Despite the evidence of jumps in various markets (see, e.g., Huang and Tauchen (2005); Andersen et al. (2007); Lee and Mykland (2008); Evans (2011) for jumps in equity indices and individual stocks), there is still little understanding about their cross-sectional diffusion. There are mainly two ways in which the literature has studied the tendency of jumps to arrive together: by detecting jumps in portfolios which include the stocks, such as stock indexes, or by detecting jumps in single stocks. Relative to the latter, the great majority of works propose tests suitable only to detect common jumps (or co-jumps) between $N = 2$ assets, and do not admit a trivial generalization to the case $N > 2$. A partial list of recent studies on this topic includes Barndorff-Nielsen and Shephard (2004a), and test specification of Jacod and Todorov (2009), Mancini and Gobbi (2012), Bibinger and Winkelmann (2015), and Bandi and Renò (2016).

In order to fill the gap in the literature on cross-sectional common jumps, this paper investigates the presence of contemporaneous jumps among several stocks. Our work differs from previous studies since it focuses on co-jumps which involve a relatively large number of stocks, the multivariate jumps (or MJs), using a high-frequency dataset of considerable dimension. The database includes 1-minute prices for all $N = 3,509$ stocks belonging to the Russell 3000 index between January 2, 1998 and June 5, 2015 (4,344 days). The potential advantage of having a large dataset of stocks, is limited by the market liquidity condition. Thus, to obtain accurate estimates, we focus on stocks with a sufficient number of non-null intraday returns (75%), considering 1, 5, and 11 minute observation intervals. It is then possible to observe that, in general, assets that do not match our liquidity condition are also low capitalized.

This paper relates to the strand of literature which investigates co-jumps with $N > 2$ assets. Relevant works are Bollerslev et al. (2008) who build a test to identify common jumps among multiple stocks, and Gilder et al. (2014) that propose a simple co-exceedance rule which identifies co-jumps by intersecting results from univariate jump tests. Both methods show similar power, but the latter is commonly used in empirical studies for its simplicity. We differ from Gilder et al. (2014) by employing a modified version of the co-exceedance rule to detect MJs, that are then used to build indexes informative of cross-sectional jump diffusion.

As a first step of the co-exceedance rule, we identify the jumps in the returns of the Russell 3000 constituents. Among all possible tests, we employ the *C*-Tz test introduced by (Corsi et al., 2010), because it has more power with respect to the tests based on multipower variation, and the *BNS* test (Barndorff-Nielsen and Shephard, 2004b, 2006) as a benchmark.

In order to allow accurate detection of the high-frequency returns affected by jumps, we preliminary correct them for their volatility periodicity (among others Wood et al. (1985); Harris (1986); Boudt et al. (2011)). According to Boudt et al. (2011), accounting for the U-shaped intraday volatility pattern of returns, enables us to improve the detection of small jumps during low volatility times and to reduce the spurious detection of jumps at high volatility times. Thus, after standardizing the returns, we apply the *C*-Tz test and the *BNS* test at both the daily and the intradaily levels using the sequential procedure suggested by Gilder et al. (2014).

We detect jumps using different observation intervals and find that 1-minute returns are affected by microstrucure noise while 11-minutes returns obscure jumps in the market index. Consequently, we move our focus prevalently on 5-minutes return results.

The subsequent step requires to combine jump results using Gilder et al. (2014) coexceedance method, which starting from univariate jump tests allow us to detect contemporaneous jumps in the cross-section. Results show that *C*-Tz co-jumps are more frequent and bigger, in terms of assets involved, than *BNS* ones. Moreover, the intraday distribution of jumps suggests that thanks to its greater efficiency, the *C*-Tz test corrects for the fact that many *BNS* single jumps ($N = 1$) are instead co-jumps. Interestingly, we also find a patter in jumps throughout the day. Around lunch it is possible to observe a large increase of common jumps and a correspondent decrease of single jumps, phenomenon that we name 'lunch effect'.

More importantly, results tell us that, even if co-jumps can involve a large number of assets, up to 956, they are usually small and negligible with respect to the whole stock market. Similarly to Bollerslev et al. (2008) and Gilder et al. (2014), we also analyze the relation of common jumps with the jumps in the market portfolio.

Common jumps which involve the market index are commonly known as systematic co-jumps, since they represent non-diversifiable events. In line with the results of Bollerslev et al. (2008) and Gilder et al. (2014) , we find an association between market jumps and co-jumps in the underlying stocks. However, since the majority of asset jumps and co-jumps are non-systematic, it is possible to assert that this association in weak. This conclusion is further confirmed by the correlations between RUA jumps and co-jumps, which are always positive and statistically significant, at conventional levels, but small (from 0.01 to 0.04).

We also find evidence that systematic co-jumps generally involve more stocks than non-systematic co-jumps, thus suggesting, similarly to Gilder et al. (2014), the existence of a positive relation between jumps in the market index and large co-jumps in the stocks. E.g., proportions of co-jumps which involve more than 10 stocks are 34.00%-65.10% for systematic jumps, and 0.72%-4.07% for non-systematic jumps.

As suggested among others by Bollerslev et al. (2008), it is more important to focus on multiple co-jumps because common jumps which involve only two assets have little impact on a huge portfolio. To this end, we modify Gilder et al. (2014) coexceedance method and detect multivariate jumps when at least 20 stocks jump together. Results show that these collective events are dramatically less frequent than co-jumps but that, for a relevant number of days, they appear multiple times. Similarly to co-jumps, MJs are not often reflected by a jump in the Russell 3000 index, suggesting that the comprised assets are small in size and, consequently, have no impact at the market level.

Using the information about common jumps, we propose two indexes which summarize data on cross-sectional jump diffusion: the daily diffusion index (or DID), and the intraday diffusion index (or DII). Each index tracks the distribution of MJs in time but focuses on a different time window. While the DID is informative of the maximum number of assets involved in a MJ per day, the DII reports the number of assets comprised in the MJs, if present, for each intraday interval. To make the information content more clear, we separate the trend from the noise component by filtering each index with the Local Level Model (LLM), see, e.g., Durbin and Koopman (2001). If the indexes are really informative, we expect trends and residuals to be more accentuated in periods of market turmoil. Results confirm the usefulness of both DID and DII. Trends and residuals show more and higher spikes in correspondence of important economic moments, as in 2008 and 2010.

Diffusion indexes are also positively and significantly correlated with the market, with correlations that are 9 to 15 times the correspondent jump and co-jump correlations. In addition, correlations between market jumps and indexes constructed using different MJ thresholds, tend to stabilize around $N = 20$.

Since some collective jumps may represent a relevant fraction of the market, we further investigate those MJs involving at least 20 stocks among the top 100 size stocks. We refer to this co-jumps as systemic jumps, since, in line with Das and Uppal (2004), they are infrequent events correlated across a large number of stocks. The systemic jumps we detect are rare but dramatic events. Indeed, we identify respectively 1 and 7 systemic jumps using *BNS* and

$C\text{-}Tz$ tests, which are just a tiny fraction of all detected MJs and confirm how multivariate jumps prevalently comprise small size stocks.

Interestingly, our results show that systemic co-jumps, which involve multiple large size stocks, are also the co-jumps comprising the greatest number of assets. Their relevance both in terms of size and population, suggest why they are almost always associated with jumps in the Russell 3000 Index. However, the presence of many non-systemic MJs correlated to a market jump, e.g. September 29, 2008, marks the importance of focusing on diffusion indexes together with systemic events.

We also examine if systematic common jumps are associated with the release of economic and financial news, since this may reveal their importance for portfolio selection and risk management activities. Among others Das and Uppal (2004) and Aït-Sahalia et al. (2009) study the impact of common jumps (systematic co-jumps) on portfolio choices, while Longin and Solnik (2001) discuss the reduction of portfolio diversification possibilities after a collective crash in the market.

We establish a relationship between detected MJs and market-level news, both focusing on large (residual) index values and on days with multiple MJs. Moreover, our systemic co-jumps can be easily associated with the release of macroeconomic news. Specifically, they are linked to Federal Reserve (or FED) announcements, Federal Open Market Committee (or FOMC) actions, and Associated Press news.

Our results are in line with the existing literature, among others Dungey and Hvozdyk (2012), Bollerslev et al. (2008), Lahaye et al. (2011), Gilder et al. (2014), and Caporin et al. (2016), which demonstrates an association of macroeconomic news with co-jumps. In particular, Gilder et al. (2014) suggest systematic co-jumps are produced by market-level news, which initially produce common jumps in the underlying stocks but that lastly generate jumps also in the market portfolio, and link co-jumps to Federal Funds Target Rate announcements. Caporin et al. (2016), instead, similarly to our analysis, find a relation between multiple jumps and FOMC/FED statements.

By splitting our database into its 11 industry portfolios, we discover that our sample is concentrated on financial, industrials, consumer services, technology, and health care stocks. To understand how jumps behave in each group and their impact on the full sample, we build diffusion indexes for each industry.

It is possible to observe that some sectors, namely technology, health care, industrials, financial, and consumer services, tend to be more affected both by jumps and collective jumps than others, and that this is not linked to the industry dimension. However, diffusion indexes show spikes with similar location for all industries, thus suggesting that MJs are market-wide and not industry specific. It is also important to notice that the great majority of industry collective jumps are non-systematic, but involve a considerable number of stocks. This reveal that some of the non-systematic common jumps, comprising a large number of stocks, we detect for the full sample, are initiated by industry co-jumps.

Also industry-level results are consistent in indicating the existence of a positive but weak association of diffusion indexes with index jumps. Specifically, the stronger associations, in terms of higher correlation, are for basic materials and consumer goods industries, while the tenuous relations are with oil & gas and health care sectors. Note, health care multivariate jumps are generally industry-specific, while basic materials and consumer goods collective jumps are often market multivariate jumps.

A deeper understanding of the cross-sectional diffusion of jumps, can be helpful not only for asset allocation and hedging, but also to gain a better comprehension of asset price dynamics. In this view, we analyze the impact of multivariate jumps on asset returns by including our diffusion indexes to the Sharpe (1964), Lintner (1965), Mossin (1966) and Black (1972) CAPM model, thus putting forward a 2-factor model. Coefficients of determination and estimated slope values give evidence that DID and DII capture common variation in stock returns that is missed by the market factor. Specifically, slopes on the diffusion indexes are significantly different from 0, using standard confidence levels, in a relevant number of cases both considering the full sample as well as sub-samples. Moreover, the coefficients of determination generally increase using the 2-factor model with respect to the CAPM, confirming the importance of our diffusion indexes. Results also suggest that we can successfully use the daily diffusion index especially in periods not importantly affected by market turmoils, while in more turbulent economic moments it is more appropriate to use the intraday diffusion index.

The remainder of the paper is organized as follows. Section 2.2 presents our database and the stocks we empirically use in the subsequent elaborations. Section 2.3 summarizes important realized measures of volatility and discusses the derived jump tests we employ in the paper. Section 2.4 applies the non-parametric jump detection techniques to the dataset. Section 2.5 investigates the presence of co-jumps and their link with the market. Section 2.6 presents our main results on the diffusion of jumps among a large number of stocks. It also introduces and describes two diffusion indexes, aimed to capture the presence of multivariate jumps, and analyzes systemic events. Section 2.7 applies jump analysis to eleven separate industries. Section 2.8 investigates if multivariate jumps help in explaining asset price dynamics. Section 2.9 concludes and outlines some implications for further research.

2.2 Data description

Our analysis focus on a large dataset including $N = 3,509$ stocks[2] belonging to the Russell 3000 index.[3] The stocks prices are sampled with a frequency of 1 minute from 09:30 am to 04:00 am, for all the days spanning the period between January 2, 1998 and June 5, 2015, for a total of 4,344 days. Then, for each stock, we collect, at the t-th day, $M+1 = 391$ 1-minute prices (390 closing prices + 9:30 opening price), denoted as $p_{t,i}$, for $t = 1,\ldots,T$ and $i = 1,\ldots,M+1$. For each stock, on intraday intervals in which no trade occurs we set the correspondent return equal to zero.

Following Gilder et al. (2014), we ignore the first 5 minutes of the day to avoid the potentially erratic price behavior induced by the market opening procedure, considering then

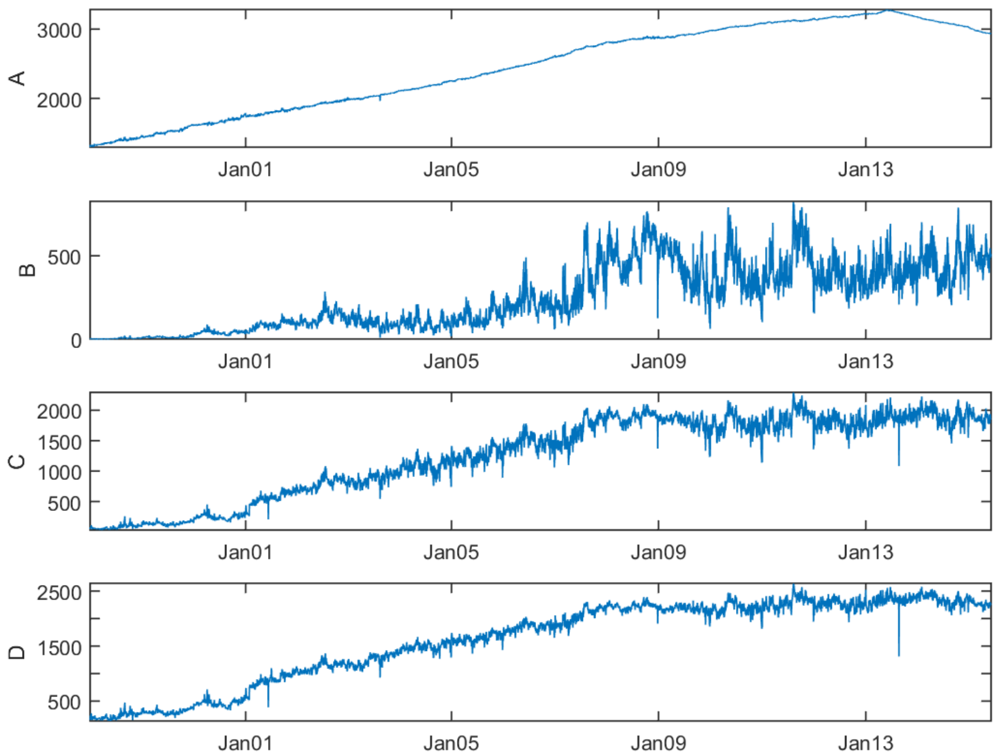

Figure 2.1 Quoted and liquid assets. The figure shows in the top panel, for each day between 01/02/1998 and 06/05/2015, the number of RUSSEL3000 quoted assets. The following three panels present, for each day, the number of assets with less than 25% null intraday returns, considering respectively intervals of 1 minute (panel B), 5 minutes (panel C), and 11 minutes (panel D).

[2]Our dataset comprise also dead stocks.

[3]The dataset is provided by Kibot; the details are available on http://www.kibot.com.

385 1-minute returns for each trading day. In general, the shorter the sampling interval the higher is the accuracy of the realized measures of volatility, see, e.g. Barndorff-Nielsen and Shephard (2004b), Andersen et al. (2010) and Corsi et al. (2010); nevertheless, this relationship could be altered by the presence of microstructure noise, arising from specific factors such as the bid-ask bounce, the price discreteness and irregular trading activities (Hasbrouck, 2006; Roll, 1984), leading to bias estimates. Overall there is not a simple solution about the optimal frequency to choose, due to the several pros and cons to take into account. The sampling frequencies we employ are the ones commonly used for the empirical jumps detection, see, e.g. Corsi et al. (2010) and Gilder et al. (2014) for the 5-minutes and the 11-minutes frequencies, respectively.

The potential advantage of having a large dataset of stocks, an important source of information for the jumps analysis, is limited by the market liquidity condition. The latter

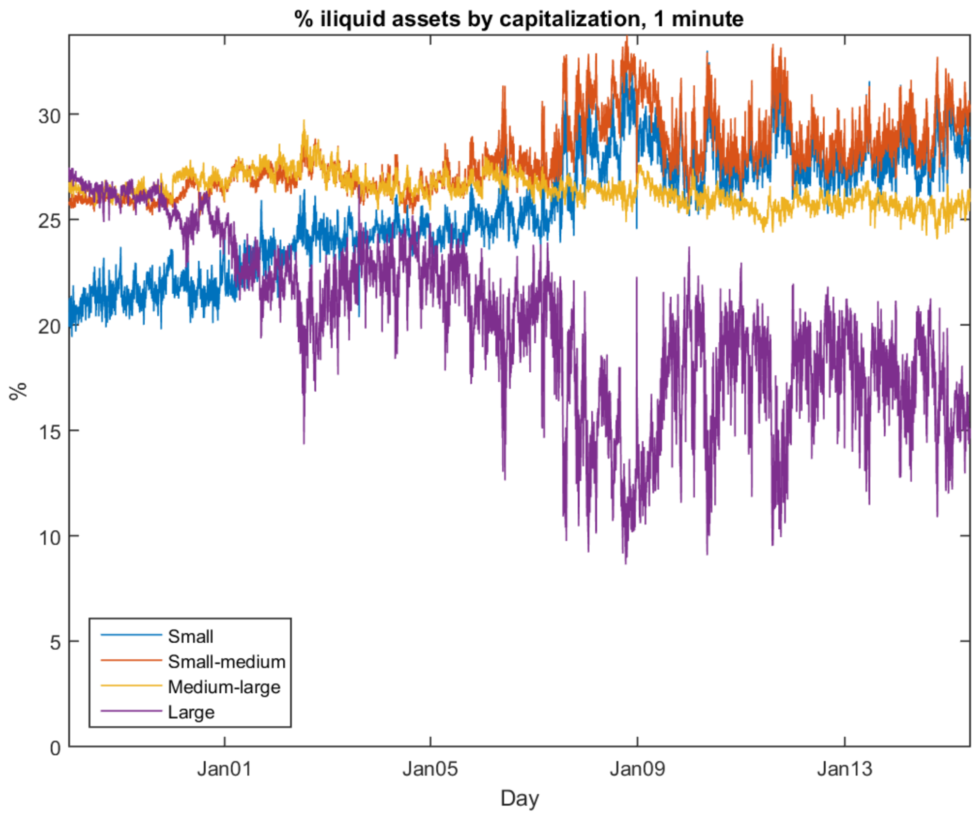

Figure 2.2 Illiquid assets, 1 minute. The figure shows the distribution, among four capitalization groups, of the quoted assets for which there are more than 25% daily null returns, using one minute intervals. The four capitalization groups are constructed at the beginning of January and at the beginning of July of each year, using the average capitalization in the previous semester: Small - Low capitalization, Small-medium - Low/Medium capitalization, group 3 - Medium-large capitalization, Large - High capitalization. For each day from 1998 to 2015 it reports the percentage of illiquid assets that belongs to each group.

states that, in order to guarantee accurate estimates, it is necessary to focus on stocks with a sufficient number of non-null intraday returns. Then, in studying the jump behaviour of a given stock, we implement the testing methodologies described in Section 2.3 under the condition that, in a given trading day, the percentage of the stock's non-zero intraday returns is greater or equal to the 75%; in contrast, we treat the days where the percentage of non-null returns is lower than the 75% as days where no jumps occur. With some abuse of wording, we label assets having more than 25% of intradaily returns equal to zero as illiquid. Clearly, the absence of price movements might be due to illiquidity but also to other different reasons. Panel A of Figure 2.1 shows, for each day of our dataset, the number of the quoted constituents of the Russell 3000 index.

In general, the small size stocks are the less traded; we check this phenomenon in Figures 2.2, 2.3 and 2.4, where we show, for the different sampling frequencies, the trend over time

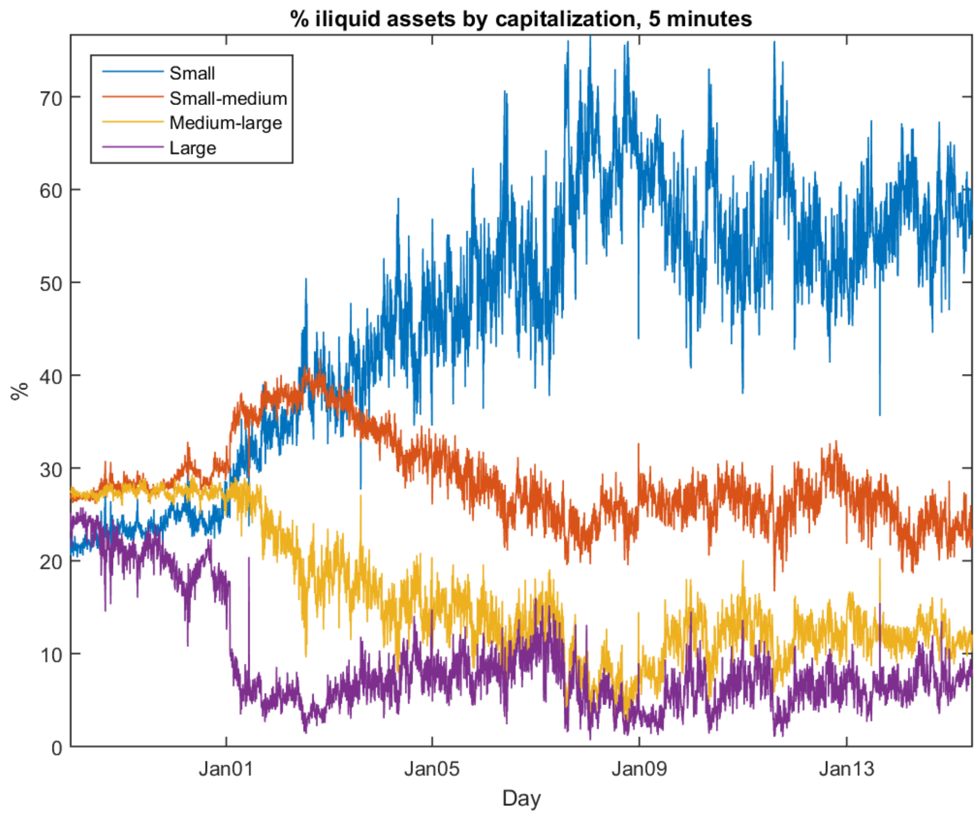

Figure 2.3 Illiquid assets, 5 minutes. The figure shows the distribution, among four capitalization groups, of the quoted assets for which there are more than 25% daily null returns, using five minutes intervals. The four capitalization groups are constructed at the beginning of January and at the beginning of July of each year, using the average capitalization in the previous semester: Small - Low capitalization, Small-medium - Low/Medium capitalization, group 3 - Medium-large capitalization, Large - High capitalization. For each day from 1998 to 2015 it reports the percentage of illiquid assets that belongs to each group.

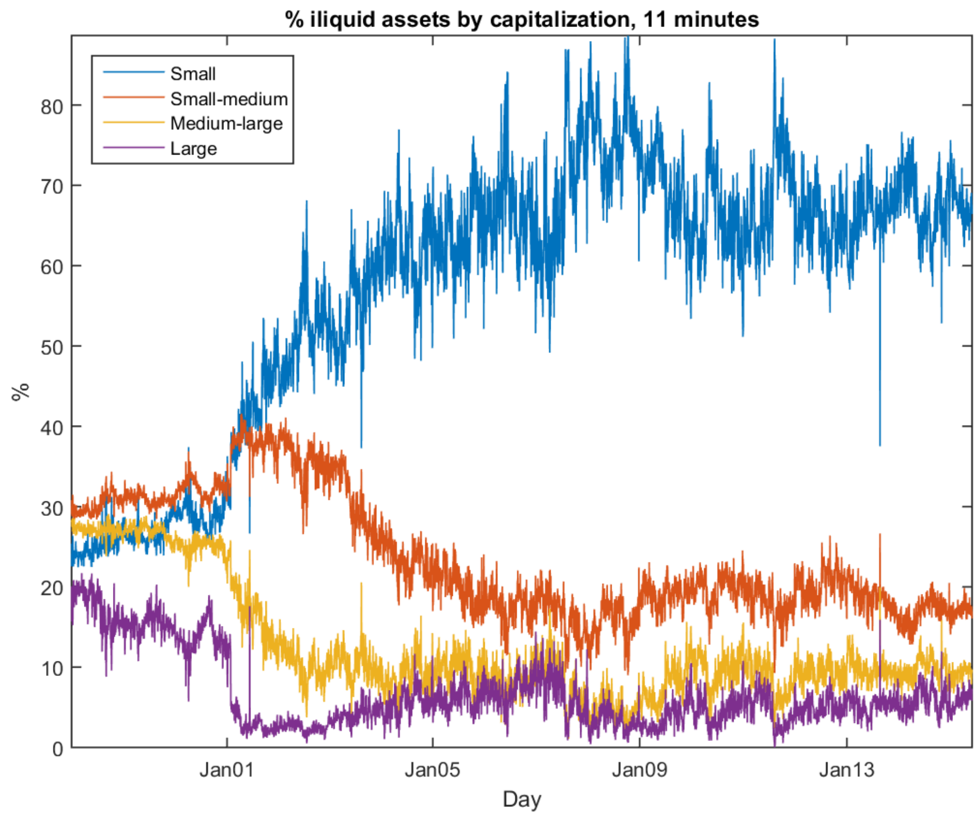

Figure 2.4 Illiquid assets, 11 minutes. The figure shows the distribution, among four capitalization groups, of the quoted assets for which there are more than 25% daily null returns, using eleven minutes intervals. The four capitalization groups are constructed at the beginning of January and at the beginning of July of each year, using the average capitalization in the previous semester: Small - Low capitalization, Small-medium - Low/Medium capitalization, group 3 - Medium-large capitalization, Large - High capitalization. For each day from 1998 to 2015 it reports the percentage of illiquid assets that belongs to each group.

of the percentages of the stocks that do not match our liquidity condition clustered according to their capitalization.

In particular, we compute the average capitalization of the Russell 3000 index constituents twice a year, at the beginning of January and at the beginning of July, for a total of 35 semesters. After excluding the ones that are not traded in the specific semester, we allocate the remaining stocks into four classes, determined according to the quartiles of their average capitalizations recorded during the six months of interest. Then, for each semester, we cluster the stocks into four classes of average capitalization: low, low-medium, medium-high and high. As expected, the Figures 2.2, 2.3 and 2.4 show that the great majority of assets that do not match our liquidity condition belongs to the low and the low-medium capitalization groups. This phenomenon is more evident with the 5-minutes and the 11-minutes frequencies, where we observe that more than the 50% of the illiquid assets are also low capitalized (see Figures 2.3 and 2.4).

Furthermore, we also analyze the behavior of the Russell 3000 index constituents according to the sector of their economic activity. We classify the assets constituents into the following 11 industries: oil & gas, basic materials, industrials, consumer goods, health care, consumer services, telecommunications, utilities, financials, technologies and others. From Figure 2.5 it is possible to see that the financial sector collects the highest number of the Russell 3000 index constituents (the 22%), followed by industrials (the 16%), consumer services and technology (the 13%), and health care (the 12%). Note, the total number of assets is greater than 3,000 since we include deads.

Sector	N
Oil & Gas	187
Basic Materials	176
Industrials	553
Consumer Goods	293
Health Care	438
Consumer Services	468
Telecommunications	39
Utilities	94
Financials	778
Technology	447
Others	36

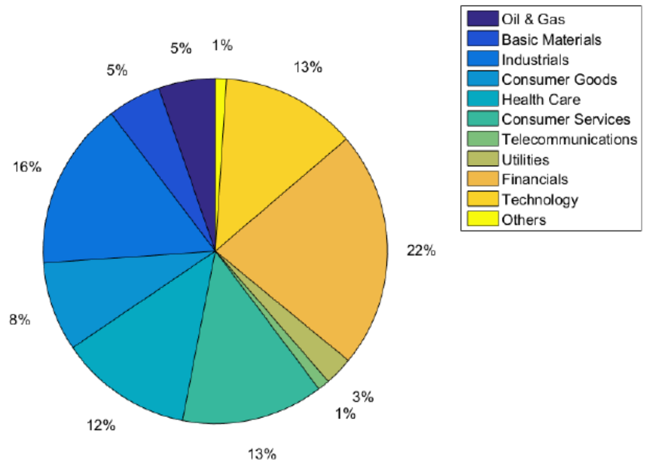

Figure 2.5 Russell 3000 industries. The figure shows the classification of the Russell 3000 index constituents among the following industries: oil & gas, basic materials, industrial goods, consumer goods, health care, consumer services, telecommunications, utilities, financials, technologies, others.

2.3 Realized measures of volatility and tests for jumps

Let p_t be the logarithmic price of a financial asset at time t. We can assume that p_t follows the Brownian semi-martingale process:

$$dp_t = \mu_t dt + \sigma_t dW_t, \tag{2.1}$$

with μ_t being the drift, locally bounded and predictable, whereas σ_t is a strictly positive process and càdlag, independent of the standard Brownian Motion W_t.

Let $r_{t,i}$ be the i-th intraday return of the t-th trading day, for $t = 1,...,T$ and $i = 1,...,M$ ($M = 385$, $M = 77$, and $M = 35$ using respectively observation intervals of 1, 5, and 11 minutes). Under the process in (2.1), the integrated volatility (IV), defined as

$$IV_t = \int_{t-1}^{t} \sigma_u^2 du, \tag{2.2}$$

can be estimated through the realized variance (RV):

$$RV_t = \sum_{i=1}^{M} r_{t,i}^2. \tag{2.3}$$

Notably, RV_t is a consistent estimator of IV as $M \to \infty$ (Barndorff-Nielsen and Shephard, 2002). Nevertheless, several studies in the literature, see, e.g., Barndorff-Nielsen and Shephard (2004b), highlight the presence of discontinuous jumps, in addition to the continuous diffusion component in Equation (2.1). Then, the process in (2.1) should be modified as follows:

$$dp_t = \mu_t dt + \sigma_t dW_t + k_t dN_t, \tag{2.4}$$

with N_t being a finite activity non-explosive Poisson counting process with intensity λ_t, whereas k_t are the random jump sizes.

RV is no longer a consistent estimator of IV in the presence of jumps,

$$\operatorname*{plim}_{M\to\infty} RV_t = \int_{t-1}^{t} \sigma^2(u)\mathrm{d}u + \sum_{(t-1)\le s\le t} (\Delta p(s))^2 = IV_t + JP_t \tag{2.5}$$

where JP_t is the sum of the instantaneous changes in the log price due to a jump at time s.

In contrast, Barndorff-Nielsen and Shephard (2004b, 2006) proposed the bipower variation (BV), that is robust to jumps. BV is defined as:

$$BV_t = \mu_1^{-2}\left(\frac{M}{M-1}\right)\sum_{i=2}^{M}|r_{t,i}||r_{t,i-1}|, \tag{2.6}$$

where $\mu_1^{-2} = (\mathbb{E}[|u|])^{-2} = \pi/2$.[4]

In estimating IV, we expect large differences between RV and BV in the presence of jumps. Building on such discrepancies, many non-parametric jump tests have been developed. Among them we focus on the $C\text{-}Tz$ test introduced by (Corsi et al., 2010), since it has more power with respect to the tests based on multipower variation. As benchmark, we also use the BNS test (Barndorff-Nielsen and Shephard, 2004b, 2006), that represents a reference in the literature.

As recommended in Huang and Tauchen (2005), we use the ratio form of the BNS test proposed by Barndorff-Nielsen and Shephard (2006), that is based on the difference between RV and BV:

$$Z_{BNS,t} = \frac{\frac{RV_t - BV_t}{RV_t}}{\sqrt{\left(\left(\frac{\pi}{2}\right)^2 + \pi - 5\right)\frac{1}{M}\max\left(1, \frac{TPV_t}{BV_t^2}\right)}} > \Phi_{1-\alpha}^{-1} \tag{2.7}$$

where

$$TPV_t = M\mu_{4/3}^{-3}\left(\frac{M}{M-2}\right)\sum_{i=3}^{M}|r_{t,i}|^{\frac{4}{3}}|r_{t,i-1}|^{\frac{4}{3}}|r_{t,i-2}|^{\frac{4}{3}}$$

and $\Phi_{1-\alpha}^{-1}$ is the inverse of the standard cumulative distribution function.

The $C\text{-}Tz$ test (Corsi et al., 2010) is a modified version of the BNS test, combining BV (Barndorff-Nielsen and Shephard, 2004b, 2006) and the threshold realized variance (TRV) discussed in Mancini (2009).[5] Notably, the small sample bias of the former, affecting in particular big jumps, is counterbalanced by the low effectiveness of TRV, with small jumps but also by its much more effectiveness with large ones. The test statistic proposed by Corsi et al. (2010) reads as:

$$Z_{C-Tz,t} = \frac{\frac{RV_t - C\text{-}TBV_t}{RV_t}}{\sqrt{\frac{1}{M}\left(\frac{\pi^2}{4} + \pi - 5\right)\max\left\{1, \frac{C\text{-}TTriPV_t}{C\text{-}TBV_t^2}\right\}}} > \Phi_{1-\alpha}^{-1}. \tag{2.8}$$

[4] $\mu_p = \mathbb{E}[|u|^p]$ and $u \sim \mathcal{N}(0,1)$.

[5] $TRV_t = \sum_{i=2}^{M}|r_{t,i}|^2\mathbb{I}_{\{|r_{t,i}|^2 \le \Theta(\delta)\}}$, where $\Theta(\delta)$ is a threshold function.

The *C-Tz* test substitutes the estimators based on the multipower variation in Equation (2.7), that is BV_t and TPV_t, with estimators based on the threshold multipower variation (TMV): $C\text{-}TBV_t$ and $C\text{-}TTriPV_t$. This corrects for the small sample bias in BV using indicator functions, which guarantee that returns with a jump larger than the threshold vanish asymptotically. In particular, $C\text{-}TBV_t$ is a modified version of TBV_t that accounts for returns variations larger than the threshold in absence of jumps:

$$TBV_t = \mu_1^{-2} \sum_{i=1}^{M} |r_{t,i}||r_{t,i-1}| \mathbb{I}_{\{|r_{t,i}|^2 \leq \vartheta_i\}} \mathbb{I}_{\{|r_{t,i-1}|^2 \leq \vartheta_{i-1}\}}, \tag{2.9}$$

where ϑ is the threshold function. Moreover, $\vartheta_t = c_\vartheta^2 \cdot \hat{V}_t$, with $\hat{V}$ being an auxiliary estimator of σ_t^2 and c_ϑ is a constant. In contrast, $C\text{-}TTriPV_t$ is a special case of threshold multipower variation:

$$C\text{-}TTriPV_t = \mu_{\frac{4}{3}}^{-3} C\text{-}TMV_t^{[\frac{4}{3},\frac{4}{3},\frac{4}{3}]}.$$

For further details about the *C-Tz* test and the threshold function specification we refer the reader to Corsi et al. (2010). We apply both the *BNS* (Barndorff-Nielsen and Shephard, 2006) and the *C-Tz* (Corsi et al., 2010) tests on standardized returns, to correct their volatility periodicity (among others Wood et al. (1985); Harris (1986); Boudt et al. (2011)). Indeed, Boudt et al. (2011) show that not accounting for the U-shaped intraday volatility pattern of returns, leads to non-parametric tests that overdetect (underdetect) jumps at periodically high (low) volatility intraday times. Assuming that the periodicity factor depends on the time of the day, the first step of the procedure proposed by Boudt et al. (2011) consists in computing $\bar{r}_{t,i}$:

$$\bar{r}_{t,i} = \frac{r_{t,i}}{\sqrt{\Delta \cdot BV_t}}, \tag{2.10}$$

where $\Delta = 1/M$.

Then, it is possible to compute the shortest half scale estimator ($ShortH_i$) of Rousseeuw and Leroy (1988):

$$ShortH_i = 0.741 \cdot \min\{\bar{r}_{(h_i),i} - \bar{r}_{(1),i}, \ldots, \bar{r}_{(T_i),i} - \bar{r}_{(T_i-h_i+1),i}\}, \tag{2.11}$$

where $\bar{r}_{(j),i}$ are the order statistics of $\bar{r}_{j,i}$ and $h_i = \lfloor T_i/2 \rfloor + 1$. For the latter $\lfloor T_i/2 \rfloor$ rounds $T_i/2$ to the lowest integer and T_i represents the total number of observations in intraday interval i. As third step, we define $\hat{s}^2_{ShortH,i}$:

$$\hat{s}^2_{ShortH,i} = \frac{M \cdot ShortH_i^2}{\sum_{i=1}^{M} ShortH_i^2} \tag{2.12}$$

The Weighted Standard Deviation (WSD) estimator is then computed as:

$$WSD_i^2 = 1.081 \frac{\sum_{t=1}^{T} w_{t,i} \bar{r}_{t,i}^2}{\sum_{t=1}^{T} w_{t,i}}, \tag{2.13}$$

where $w_{t,i} = w\left(\frac{\bar{r}_{t,i}}{\hat{s}_{ShortH,i}}\right)$ and $w(z) = 1$ if $z^2 \leq 6.635$, 0 otherwise.

Finally, we compute the Boudt et al. (2011)'s standard deviation robust estimator:

$$\hat{s}_{WSD,i}^2 = \frac{M \cdot WSD_i^2}{\sum_{i=1}^{M} WSD_i^2}. \tag{2.14}$$

It is now possible to compute Boudt et al. (2011) standardize returns:

$$\ddot{r}_{t,i} = \frac{r_{t,i}}{\sqrt{\hat{s}_{WSD,i}^2 \cdot \Delta \cdot BV_t}}. \tag{2.15}$$

The Boudt et al. (2011)'s correction enables us to improve the detection of small jumps during low volatility times and to reduce the spurious detection of jumps at high volatility times. After standardizing the returns, we apply the C-Tz test at both the daily and the intradaily levels. In particular, for a given stock, we compute $Z_{C-Tz,t}$, defined in Equation (2.8), using all the M intradaily returns, for $t = 1, \ldots, T$. Then, the t-th trading day has a daily jump if the null hypothesis of the C-Tz test is rejected at the significance level α. For all the days with daily jumps, we also test for intradaily jumps, by using the sequential procedure suggested by Gilder et al. (2014). This sequential procedure requires to firstly detect the jump days using the $Z_{C-Tz,t}$ test, as recommended by Huang and Tauchen (2005); notably, the t-th trading day is classified as jump day if the null hypothesis of the C-Tz test, applied on all the M intraday standardized returns, is rejected at the significance level α. Then, for each jump days, we select the maximum intraday standardized return to be the first intraday jump. The underlined assumption is that the jump size dominates the diffusion component. To detect all the further intraday jumps we repeat the procedure, after setting the previously identified intradaily jump-return equal to zero, until the null hypothesis of no jumps is not rejected at the significance level α. Likewise, we repeat the same procedure by using the *BNS* test, that in this case we call sequential *BNS* (or *s-BNS*).

2.4 Jump identification

In this section we summarize results from applying the *s-BNS* and the *C-Tz* jump tests to the Russell 3000 constituents. We set the significance level $\alpha = 0.01\%$ in all jump tests, and we follow Corsi et al. (2010) in imposing $\vartheta = 3$ for the *C-Tz* threshold function. As documented among others by Dumitru and Urga (2012) and Schwert (2011), different non-parametric jump tests lead to non-identical timing of jump arrivals. This can be due to different capabilities of jump identification as well as to spurious jump detection. In our case since the *C-Tz* test is an upgrade of the *s-BNS* test, we expect the differences to be principally driven by the the greater capability of the former with respect to the latter.

Table 2.1 presents the number of jump days detected by each detection method plus the percentage of positive jumps and the mean jump size. As expected, the *C-Tz* test detects more jump days than the *s-BNS* test for all frequencies.

Table 2.1 Jumps summary statistics. The table reports summary statistics for the market index, the RUA, and for the constituents of the Russell 3000. N_{RUA} is the number of days for which we observe at least one intraday jump in the RUA, % $J_{RUA} > 0$ is the percentage of RUA jump returns greater than 0, $Mean_{RUA}$ is the average RUA jump return, N is the number of days for which we observe at least one intraday jump in the constituents of the Russell 3000 index, % J$>$ 0 is the percentage of jump returns greater than 0 in constituents of the Russell 3000, and Mean is the average Russell 3000 jump return. Results are presented separately for each observation interval (1 minute, 5 minutes, and 11 minutes) and for the two tests (*s-BNS*, and *C-Tz*).

Frequency	N_{RUA}	% $J_{RUA} > 0$	$Mean_{RUA}$	N	% J$>$ 0	Mean
			s-BNS			
1 min	847.00	51.19	0.15	4,057.00	50.19	-1.99
5 min	97.00	51.56	0.21	4,280.00	50.92	1.61
11 min	19.00	45.45	-0.23	4,313.00	52.37	1.42
			C-Tz			
1 min	1,512.00	51.05	0.09	4,119.00	50.23	-1.24
5 min	176.00	51.09	0.06	4,333.00	51.20	0.69
11 min	57.00	51.43	0.35	4,344.00	52.25	0.78

It is also important to notice that the number of jump days is negative related to the interval length for the index, and positive related to the observation interval for the underlying assets. In particular, the extreme low values obtained for the Russell 3000 index (or RUA) using an observation interval of 11 minutes, suggest that this observation interval in not suitable to detect jumps in the index.

The table also provides information about jump distribution: it shows that the percentages of positive and negative jumps are similar and that mean jump sizes are close to zero. This suggests that detected jumps are symmetrically distributed, a results in line with the findings of Lee and Mykland (2008) and Gilder et al. (2014).

A graphical comparison of the two methods in Figure 2.6, confirmes the idea of a

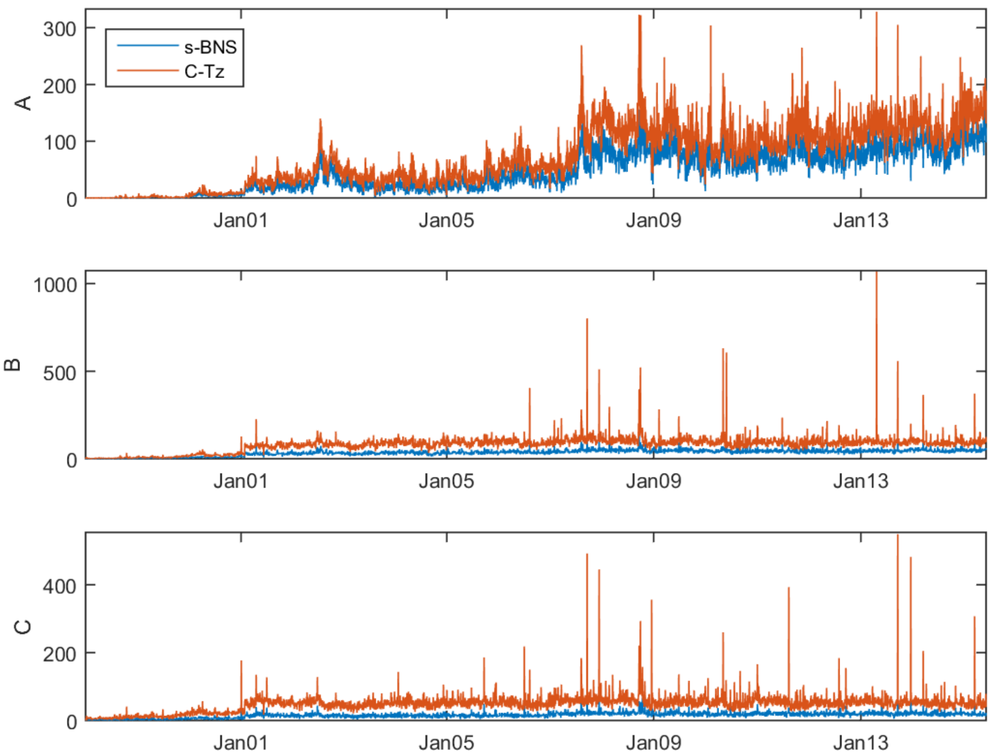

Figure 2.6 Assets that jump per day. The figure shows, for each day and for three frequencies (1 minute - panel A, 5 minutes - panel B, and 11 minutes - panel C), the number of assets for which we report at least one intraday jump using *s-BNS* and *C-Tz* tests.

supremacy of the *C-Tz* test. The figure reports for each of the 4,344 days, the number of assets for which we register at least one intraday jump using the two tests. It is evident that the *C-Tz* test has more power in detecting assets jumps for all frequencies. Note, while using high frequencies we observe spikes in correspondence of days where a large number of assets jump, this phenomenon is less clear using 1 minute intervals. A possible explanation is that jump detection is affected by the presence of microstructure noise.

Results suggest that the best observation interval is 5 minutes, since 1 minute observations are affected by microstrucure noise and 11 minute observations fail to detect index jumps. For this reason the remain of the paper will focus on 5 minute results while results for other frequencies will be available on request.

2.5 Common jumps detection

In this section we investigate the presence of common jumps, or co-jumps, among 5 minutes stock returns. The tendency of jumps to be contemporaneous has been little researched. Among the others Bollerslev et al. (2008) develop an intraday co-jump test that can be applied to a large panel of N securities (BLT test), and Gilder et al. (2014) propose a coexceedance rule that detects intraday co-jumps from the intersection of univariate jump tests. In our work we employ the latter for its simplicity and because it allows not only to choose among the univariate jump tests those more effective, but also to use returns corrected to account for the U-shaped intraday volatility pattern.

The coexceedance rule of Gilder et al. (2014) first requires to detect intraday jumps using a non-parametric univariate jump test, in our case either the *s-BNS* or the *C-Tz*, and then to verify if two or more assets present a jump in the same interval of interest,

$$\sum_{j=1}^{N} \mathbb{I}\{\text{Jump}_{t,i,j} > 0\} \begin{cases} \geq 2 & \text{Co-jump} \\ \leq 1 & \text{Single jump} \end{cases} \tag{2.16}$$

where the indicator function, $\mathbb{I}$, assumes a value equal to 1 when a jump is detected in asset j ($j = 1,\ldots,3,509$) at the intraday interval i ($i = 1,\ldots,77$, using an observation interval of 5 minutes) on day t ($t = 1,\ldots,4,344$).

Table 2.2 reports the results using the coexceedance rule in combinations with either the *s-BNS* test or the *C-Tz* test. It shows the number of jumps, single jumps, and co-jumps days and the maximum and mean numbers of stocks found to be involved in the co-jumps. It also

Table 2.2 Jumps and co-jumps summary statistics. The table reports jumps and co-jumps summary statistics for the constituents of the Russell 3000 and for an observation interval of 5 minutes. N is the number of days for which we observe at least one intraday jump, N_{sj} is the number of days with at least one intraday single jumps (if in the same interval there are no other assets that jump), N_{cj} is the number of days with at least one intraday co-jump, and Max_{cj} and $Mean_{cj}$ are respectively the maximum and average number of assets involved in a co-jump. Results are presented separately for the two jump tests: *s-BNS*, and *C-Tz*. The table also splits the number of days for which we register at least a jump (N), a single jump (N_{sj}), and a co-jump (N_{cj}) between those detected using only the *s-BNS* test (only *s-BNS*), using only the *C-Tz* test (only *C-Tz*), or using both methods (*s-BNS* ∩ *C-Tz*).

Jump test	N	N_{sj}	N_{cj}	Max_{cj}	$Mean_{cj}$
s-BNS	4,280.00	4,279.00	3,882.00	311.00	3.11
C-Tz	4,333.00	4,333.00	4,109.00	956.00	4.38
only *s-BNS*	0.00	0.00	0.00		
only *C-Tz*	53.00	54.00	227.00		
s-BNS ∩ *C-Tz*	4,280.00	4,279.00	3,882.00		

separately presents the number of days for which we register at least one event using only the *s-BNS* test or the *C-Tz* test, and the number of event days detected using both methods.

Table 2.2 shows that jumps and co-jumps days are usually detected using both jump tests. However, the *C-Tz* test identifies jumps and co-jumps days that are not detected using the *s-BNS* test. It also leads to higher mean, from 3.11 to 4.38, and especially maximum number of assets involved in co-jumps, 956 compared to 311 with the *s-BNS* test. Note, despite there are co-jumps that involve a large number of assets, the low mean values suggest that they are usually small and negligible with respect to the whole stock market. Moreover, small co-jumps may be due to spurious detection. In fact, Gilder et al. (2014) show that the coexceedance criterion produces small spurious co-jumps, with a median number of stock involved equal to 2.

Figure 2.7 shows the time distribution of jumps. For each day between January 2, 1998 through June 5, 2015 it reports the percentage of daily intervals (77 daily intervals of 5 minutes) with at least a jump, a co-jump, and a single jump. The figure reinforces the idea of

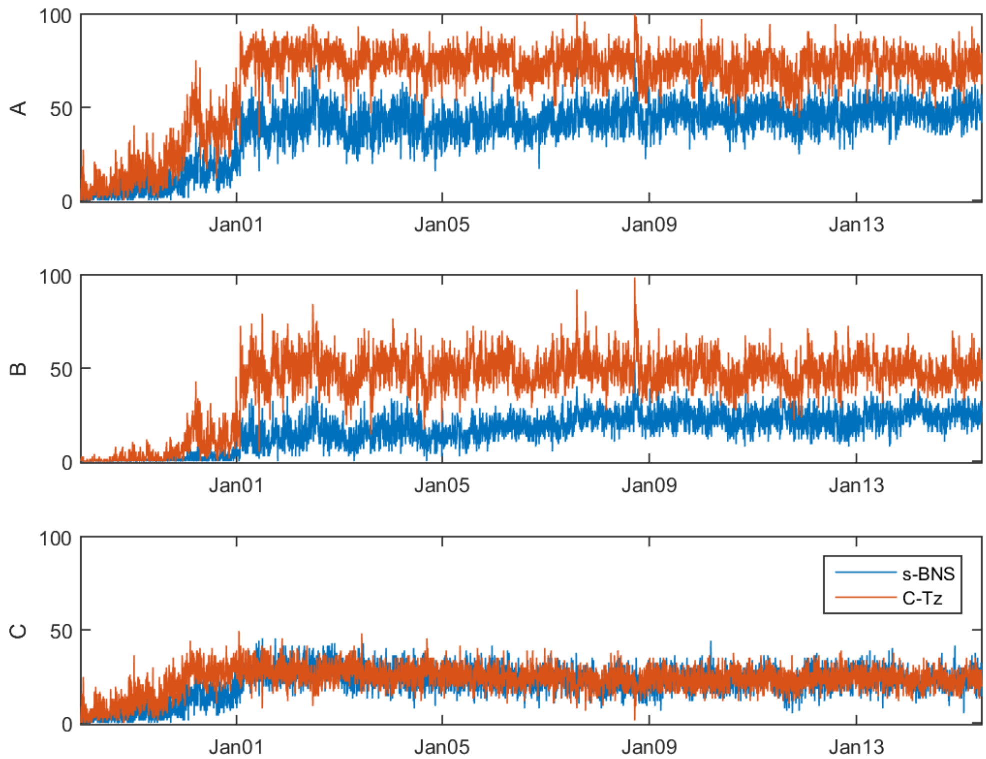

Figure 2.7 Intervals with jumps per day. The figure shows, for each of the 4,344 days, the percentages of daily intervals (77 daily intervals of 5 minutes) with at least a jump (panel A), a co-jump (panel B), and a single jump (panel C) using the *s-BNS* and the *C-Tz* tests.

a greater capability of the *C-Tz* test to detect common jumps. The figure also suggests that up to 2001 the dataset behaves in a rather different way.

Figure 2.8 investigates the distribution of jumps and co-jumps during the day. For each intraday interval of 5 minutes, it reports the number of days with at least a jump, a co-jump, and a single jump in that observation interval. The *C-Tz* test detects more jumps and co-jumps days in all intraday intervals, but the same does not hold for single jumps. A possible explanation is that the greater efficiency of the *C-Tz* test makes it possible to correct for the fact that many *s-BNS* single jumps are instead co-jumps. It is also interesting to notice that around lunch time we observe a great increase in co-jumps and a correspondent decrease of single jumps. The phenomenon is particularly evident using the *C-Tz* test and can be explained considering that during those hours there are more US active traders.

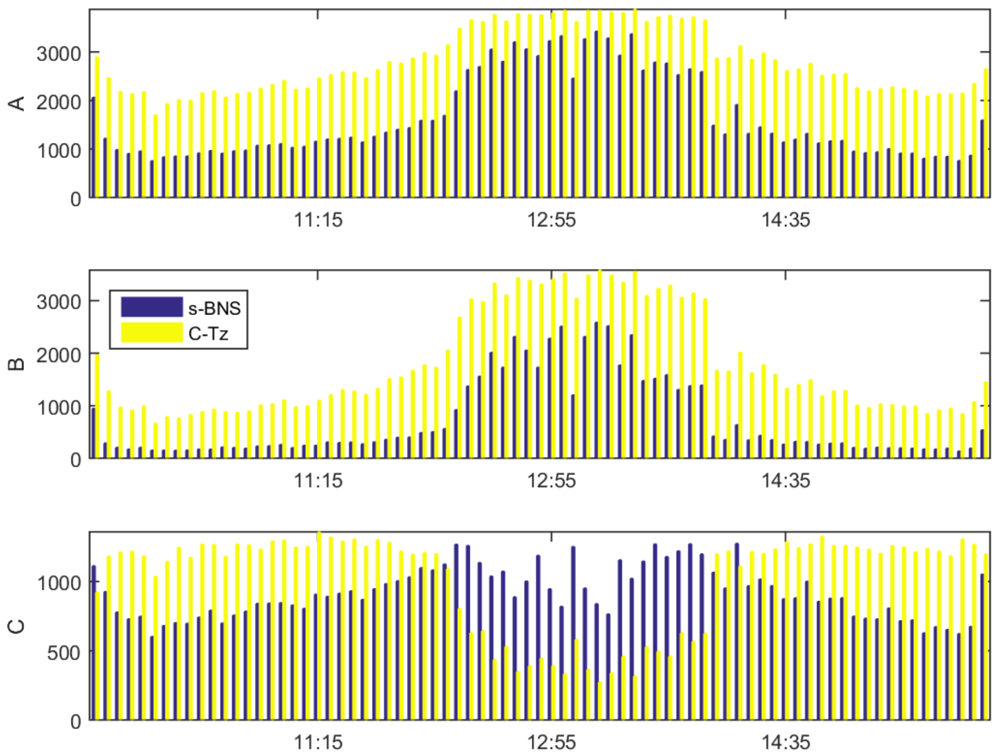

Figure 2.8 Days with jumps per intraday interval. The figure shows, for each of the 77 5-minutes daily intervals, the number of days for which we observe at least a jump (panel A), a co-jump (panel B), and a single jump (panel C) using the *s-BNS* and the *C-Tz* tests.

2.5.1 Co-jumps and RUA jumps

Market-level news that cause a co-jump in the stocks, may eventually be reflected also as a jump in the market portfolio. Co-jumps that involve the market index can be seen as non-diversifiable events, thus having important implications for portfolio selection and hedging. For this reason in the remainder of the paper we refer to them as systematic co-jumps.

Table 2.3 shows jumps days for the market index, jumps and co-jumps days in the underlying assets, and the amount of jumps and co-jumps days that are also RUA jump days. For completeness, we report results for all observation intervals.

Table 2.3 Asset jumps and market jumps. The table reports RUA jump days (N_{RUA}), the amount of days for which we observe at least one intraday jump (N) or co-jump (N_{cj}) in the constituents of the Russell 3000, and the days with both a jump in the index and a jump ($N_{RUA} \cap N$) or a co-jump ($N_{RUA} \cap N_{cj}$) in the underlying assets. Results are presented separately for three observation intervals, 1 minute, 5 minutes, and 11 minutes, and for the two jump tests (*s-BNS*, and *C-Tz*).

Frequency	N_{RUA}	N	$N_{RUA} \cap N$	N_{cj}	$N_{RUA} \cap N_{cj}$
			s-BNS		
1 min	847.00	4,057.00	771.00	3,739.00	696.00
5 min	97.00	4,280.00	96.00	3,882.00	84.00
11 min	19.00	4,313.00	19.00	3,818.00	17.00
			C-Tz		
1 min	1,512.00	4,119.00	1,418.00	3,858.00	1,345.00
5 min	176.00	4,333.00	175.00	4,109.00	172.00
11 min	57.00	4,344.00	57.00	4,269.00	57.00

We already observed in section 2.4 that the *C-Tz* test detects more jump days than the *s-BNS* test and that there is a negative relation between index jump days and interval length, and a positive relation between observation interval and Russell 3000 jump days. Now we can add that the *C-Tz* test detects more co-jump days, that are also positive related to the interval length. Even if the amount of days with at least an intraday co-jump is always lower than the correspondent number of days with at least an intraday jump, they are still very high: from a minimum of 3,739 days (85% of the sample days) to a maximum of 4,270 days (97% of the sample days). Thus, we observe a co-jump for almost each day of the sample.

Moreover, columns four and six present the intersections between jump days in the market index and jumps, or co-jumps, days detected in the underlying assets. These provides a first image on the RUA capability to reflect cross-sectional jump events. For both detection methods, a majority of jumps and co-jumps do not happen in correspondence of jumps in the index. This suggests that jumps in the index are not really informative on the presence

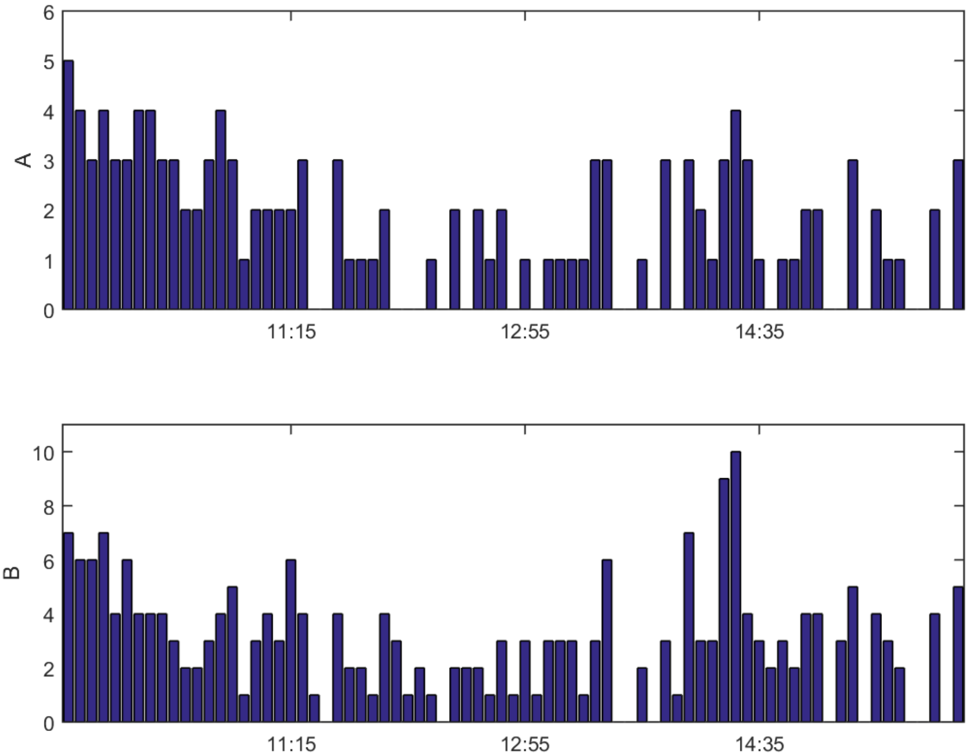

Figure 2.9 Days with jumps per intraday interval, RUA. The figure shows, for each of the 77 5-minutes daily intervals, the number of days with at least one RUA intraday jump using respectively the *s-BNS* test in panel A, and the *C-Tz* test in panel B.

of jumps and co-jumps in the cross-section. Since we are aggregating results to a daily frequency, outcomes when focusing on intraday intervals would show even less intersections.

Lastly, we observe that intersection days are more than double using the $C\text{-}Tz$ test, and this is due to its greater capability to detect jumps in the index.

Figure 2.9 investigates where index jumps are located during the day. It shows for each daily interval of 5 minutes, the amount of days with at least a RUA jump in the corresponding interval. Differently from Figure 2.8 where we observe a pattern in the distribution of Russell 3000 jumps and co-jumps during the day, RUA jumps do not present a clear behavior. We can only learn that jumps tend to be concentrated in the morning and around 2 p.m.. This reinforces the idea that index jumps are not informative of market co-jumps: the market lunch effect in not reflected in RUA jumps.

To deeper understand the relation between jumps in the index and jumps in the underlying assets, Table 2.4 shows correlations (or ρ), and correspondent P-values, of RUA jumps and Russell 3000 jumps, co-jumps, and single jumps. In addition to 5 minutes values, the table

Table 2.4 Correlations assets jumps and index jumps. The table reports the correlations (ρ), and the correspondent p-values (P-val(ρ)), between jumps (j), co-jumps (cj), and single jumps (sj) in the assets and jumps in the market index. Results are presented separately for two jump tests, *s-BNS* and *C-Tz*, and for six observation intervals: 1 minute, 5 minutes, 11 minutes, daily 1 minute - a jump/co-jump is detected in day t if there is at least one intraday 1 minute interval with a jump/co-jump, and daily 5 and 11 minutes that work exactly as the daily 1 minute but considering 5 and 11 minutes intervals.

Jump test	j	cj	sj	j	cj	sj
			s-BNS			
	ρ			P-val(ρ)		
1 min	0.02	0.02	-0.00	0.00	0.00	0.00
5 min	0.00	0.01	-0.01	0.04	0.00	0.00
11 min	0.01	0.01	-0.00	0.00	0.00	0.97
Daily 1 min	-0.05	-0.06	-0.05	0.00	0.00	0.00
Daily 5 min	0.01	-0.01	0.01	0.71	0.37	0.70
Daily 11 min	0.01	0.00	0.01	0.71	0.83	0.70
			C-Tz			
	ρ			P-val(ρ)		
1 min	0.03	0.04	-0.01	0.00	0.00	0.00
5 min	0.01	0.01	-0.01	0.00	0.00	0.00
11 min	0.01	0.01	-0.01	0.00	0.00	0.03
Daily 1 min	-0.03	0.00	-0.03	0.02	0.83	0.02
Daily 5 min	-0.01	0.03	-0.01	0.40	0.06	0.40
Daily 11 min		0.02	0.00		0.31	0.87

presents also results using 1 and 11 minutes observation intervals, together with aggregations to a daily level of intraday observations. In these last cases we observe a jump (co-jump or single jump) in day t, if in at least one intraday interval we detect a jump (co-jump or single jump).

For all but one intraday correlations, we observe P-values smaller than 0.1 (10% significance level) which lead to the not acceptance of the null hypothesis of $\rho = 0$. Moreover, intraday correlations are positive for jumps and co-jumps but negative for single jumps, thus indicating the existence of a positive relation between jumps in the index and jumps and co-jumps in the constituents of the Russell 3000 index. However, in line with the findings of Bollerslev et al. (2008), these associations are very weak and range respectively from 0 to 0.03 for jumps, from 0.01 to 0.04 for co-jumps, and from -0.01 to 0 for single jumps.

Moreover, when aggregating results to a daily level, correlations are often not statistically significant (at conventional levels) and, in the significant cases, they are small and, with just one exception, negative. Consequently, using daily information we are unable to identify a clear relation between stocks and market index about jumps. Recalling that when aggregating we lose the information on jump location within the day, previous results highlight the importance of using intraday detection and intersection methods.

Lastly, Figure 2.10 shows the time evolution of the daily correlation between index jumps and assets co-jumps using an interval frequency of 5 minutes. It is possible to observe that the correlation tends to be positive and constant around 0.2, thus confirming a weak positive association between market and co-jumps. We should however handle the information from this graph with prudence since correlations are computed using a small dataset that also presents few observations with jumps and co-jumps.

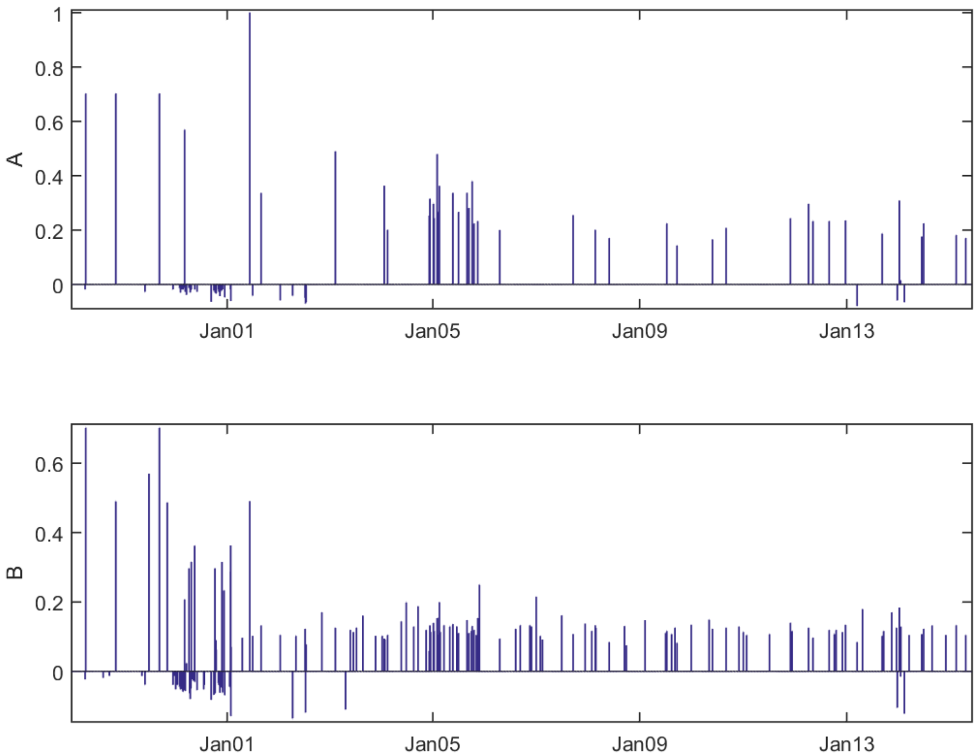

Figure 2.10 Daily correlation co-jumps. The figure shows, for each of the 4,344 days, the correlation between jumps in the index and co-jumps in the underlying assets, using 5 minutes returns. Results are presented separately for the *s-BNS* test, panel A, and the *C-Tz* test, panel B.

2.5.2 Dimension of systematic and non-systematic co-jumps

Following Gilder et al. (2014) we call systematic those co-jumps amongst the individual stocks that involve also the market index, while we define non-systematic those co-jumps not linked to the market proxy. Table 2.5 and Figure 2.11 provide information about the number

Table 2.5 Co-jumps distribution. The table reports the numbers of detected co-jumps involving different number of stocks, that are systematic (panel A: $\mathrm{cj} \cap \mathrm{j}_{RUA}$) or non-systematic (panel B: $\mathrm{cj} \not\cap \mathrm{j}_{RUA}$). The last three rows of each panel list the maximum, average, and median number of stocks detected to participate in the correspondent systematic or non-systematic co-jumps.

No. Stocks	*s-BNS*	*C-Tz*
	$\mathrm{cj} \cap \mathrm{j}_{RUA}$	
2	9.00	14.00
3	3.00	6.00
4	4.00	4.00
5	4.00	4.00
6-10	13.00	24.00
11-15	4.00	21.00
16-20	3.00	10.00
>20	10.00	66.00
Max	311.00	956.00
Mean	23.88	65.30
Median	8.00	16.00
	$\mathrm{cj} \not\cap \mathrm{j}_{RUA}$	
2	28,209.00	46,921.00
3	13,309.00	28,352.00
4	6,702.00	18,589.00
5	3,375.00	12,704.00
6-10	3,603.00	26,015.00
11-15	267.00	4,087.00
16-20	64.00	840.00
>20	69.00	698.00
Max	157.00	290.00
Mean	3.09	4.32
Median	2.00	3.00

and proportions of systematic ($\mathrm{cj} \cap \mathrm{j}_{RUA}$) and non-systematic ($\mathrm{cj} \not\cap \mathrm{j}_{RUA}$) co-jumps involving different number of stocks.

As expected, we detect more systematic and non-systematic co-jumps, involving different numbers of stocks, using the $C\text{-}Tz$ test. We also observe a clear difference between systematic and non-systematic co-jump distributions using both detection methods.

Table 2.5 shows that, for both detection methods, almost all co-jumps are not associated with a market jump. However, the mean and median number of stocks involved in systematic co-jumps are significantly higher than the mean and median number of assets involved in non-systematic co-jumps for both detection methods. Moreover, Figure 2.11 makes it evident, particularly using the $C\text{-}Tz$ test, that systematic co-jumps involve more stocks than non-systematic co-jumps. In particular panel B of Figure 2.11 confirms that non-systematic jumps are simply co-jumps among a small number of stocks that, due to the aggregation effect,

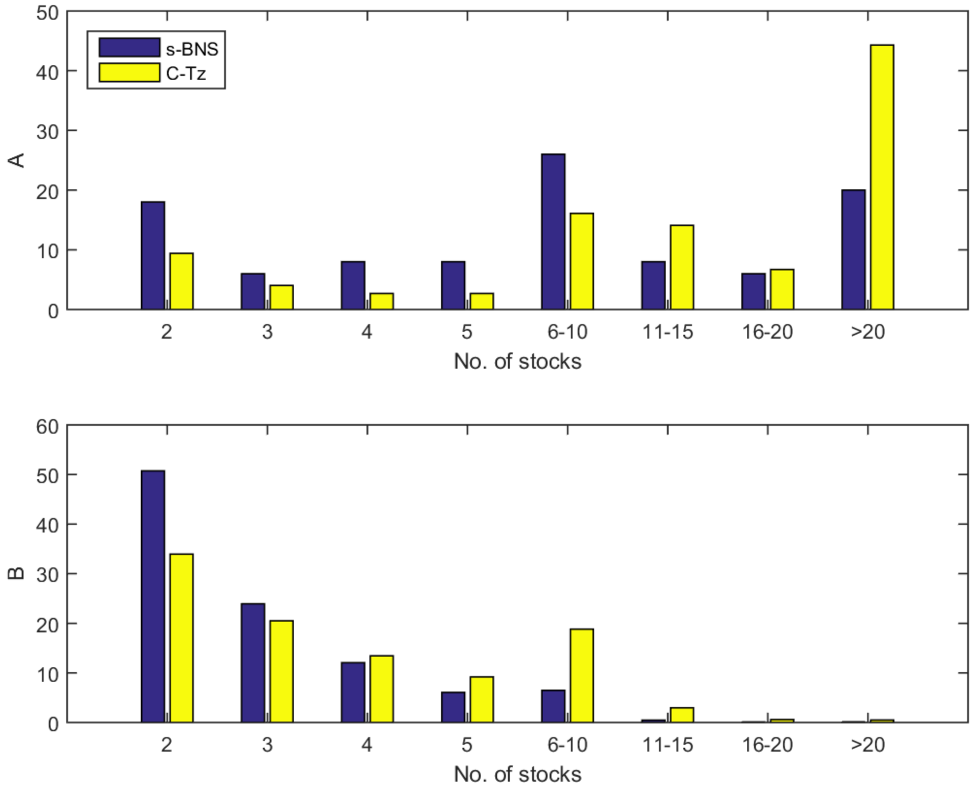

Figure 2.11 Co-jump distribution. The figure shows co-jump distributions for each detection method, focusing on jumps that involve the market index, or systematic co-jumps, (panel A) and stock jumps dissociated from RUA jumps, or non-systematic co-jumps, (panel B). Each panel presents the proportion of co-jumps that involve different numbers of stocks.

do not show up and do not give rise to a jump in the index. All the previous observations allow us to support the hypothesis of a positive association between jumps in the index and co-jumps in the underlying stocks.

Even if the number of stocks involved in systematic co-jumps is usually moderate relative to the dimension of our sample, it is important to notice that a high proportion of systematic co-jumps involve more than 10 stocks: 34.00% using the *s-BNS* test and 65.10% using the $C\text{-}Tz$ test. Comparatively, correspondent proportions for non-systematic co-jumps are 0.72% and 4.07%. Consequently, similarly to Gilder et al. (2014), we consider it an evidence for a positive relation between jumps in the market index and large co-jumps in the stocks. In addition, Gilder et al. (2014) and Bollerslev et al. (2008) propose an explanation for the presence of non-systematic co-jumps involving a large number of stocks. They suggest that these non-systematic co-jumps are missclassified due to a failure of jump tests to detect jumps in the market index. This reasoning does not seem sufficient. There is for sure an issue due to the power of the tests but also effects due to the aggregation of single stocks into the equity market index, and the different stock market values and liquidity.

2.5.3 Co-jumps and capitalization

In section 2.2 we discussed the importance of stock liquidity to guarantee accurate estimates for the univariate jump tests, and found that illiquid assets are usually also small capitalized. In this section we further investigates the relation between capitalization and detected jumps. At the beginning of January and July of each year y from 1998 to 2015, we sort all stocks according to the average capitalization in the previous semester, after excluding the ones that are not traded in the specific semester. We then determine the quartiles breakpoints (25%, 50%, and 75%) and allocate the assets into the four classes according to their average capitalization: small, small-medium, medium-large, and large capitalization. Table 2.6 shows the number of jump and co-jump days detected in each capitalization group for different observation frequencies and jump detection tests.

Table 2.6 Jumps and co-jumps by capitalization. The table reports the number of jumps and co-jumps days for four capitalization groups: S - small, S/M - small-medium, M/L - medium-large, L - large. N is the number of days for which we observe at least one intraday jump, and N_{cj} is the number of days with at least one intraday co-jump. Results are presented separately for the two jump tests: *s-BNS*, and *C-Tz*.

Frequency	g1	g2	g3	g4
		N, *s-BNS*		
1 min	453.00	1,705.00	3,346.00	4,042.00
5 min	3,151.00	3,759.00	4,096.00	4,188.00
11 min	3,308.00	3,952.00	4,066.00	3,592.00
		N_{cj}, *s-BNS*		
1 min	2.00	569.00	2,591.00	3,719.00
5 min	1,456.00	3,176.00	3,460.00	2,116.00
11 min	1,881.00	2,325.00	1,488.00	831.00
		N, *C-Tz*		
1 min	673.00	1,911.00	3,484.00	4,101.00
5 min	3,528.00	3,926.00	4,205.00	4,320.00
11 min	3,702.00	4,152.00	4,316.00	4,320.00
		N_{cj}, *C-Tz*		
1 min	6.00	776.00	2,800.00	3,849.00
5 min	2,442.00	3,515.00	3,861.00	3,796.00
11 min	2,950.00	3,708.00	3,801.00	3,229.00

Note, jump days, with only one exception, are positive related to capitalization. This means that we expect to observe a larger number of jumps as the stock capitalization increases:

large capitalized stocks jump more than small capitalized stocks. Instead, for co-jumps we observe an almost uniform distribution among the top three capitalization groups using the *C-Tz* test and a concentration in the two middle capitalization groups using the *s-BNS* test. Consequently, we expect to observe few co-jumps among small capitalization stocks and a similar number of co-jumps among the other groups. Moreover, this means that co-jumps are mainly formed by jumps of small-medium, medium, and medium-large stocks.

2.6 Multiple co-jumps

Similarly to the co-jumps, we identify a multivariate jump (or MJ) from the intersection of non-parametric univariate jump tests,

$$\sum_{j=1}^{N} \mathbb{I}\{\text{Jump}_{t,i,j} > 0\} \begin{cases} \geq 20 & \text{Multivariate Jump} \\ \leq 19 & \text{No Multivariate Jump} \end{cases} \tag{2.17}$$

where the indicator function, $\mathbb{I}$, equals 1 when a jump is detected in asset j, during the intraday interval i, on day t, and 0 otherwise. In the choice of the threshold we follow Caporin et al. (2016) who use $N = 20$ stocks to detect multi-jumps.

Table 2.7 presents the number of days with at least a MJ, and the days with both a RUA jump and a MJ, the systematic MJ days. The first thing to notice it is that the days with at least a multivariate jump are dramatically lower than the correspondent days with at least a co-jump or a jump. Moreover, the differences between the two jump tests, *s-BNS* test and *C-Tz* test, are larger when considering multivariate jump days than when focusing on jump and co-jump days.

While in the previous cases the two methods led to identify similar number of event days, when focusing on MJ days using the *C-Tz* test we detect more than two times the days we detect using the *s-BNS* test. Differently from the jump and co-jump cases we not only observe a negative relation between observation interval and Russell 3000 MJ days, but also that RUA jump days, that are almost always also jump and co-jump days, are usually not associated with a multivariate jump in the underlying assets.

Table 2.7 Asset jumps and market jumps. The table reports RUA jump days (N_{RUA}), the amount of days for which we observe at least one multivariate jump (N_{mj}) in the constituents of the Russell 3000, and the days with both a jump in the index and a multivariate jump ($N_{RUA} \cap N_{mj}$) in the underlying assets. Results are presented separately for three observation intervals, 1 minute, 5 minutes, and 11 minutes, and for the two jump tests (*s-BNS*, and *C-Tz*).

Frequency	N_{RUA}	N_{mj}	$N_{RUA} \cap N_{mj}$
		s-BNS	
1 min	847.00	571.00	113.00
5 min	97.00	84.00	10.00
11 min	19.00	25.00	2.00
		C-Tz	
1 min	1,512.00	1,291.00	422.00
5 min	176.00	643.00	65.00
11 min	57.00	275.00	21.00

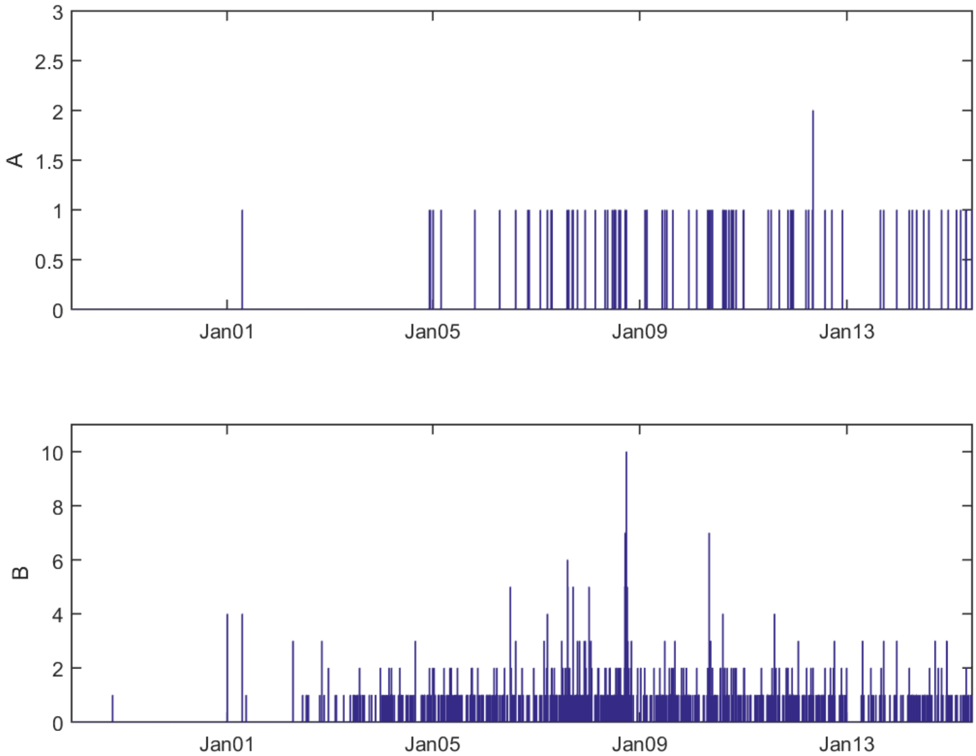

Figure 2.12 Intervals with multivariate jumps per day. The figure shows, for each of the 4,344 days, the number of daily intervals (77 daily intervals of 5 minutes) with at least a multivariate jump. All jumps are detected using the *s-BNS*, panel A, or the *C-Tz* test, panel B.

Figures 2.12 and 2.13 show the distribution of MJ along our time window and during the 77 5-minutes observation intervals of a trading day. Relative to the former, we detect on the 3rd May, 2012 more than a MJ using both the *s-BNS* test and the *C-Tz* test. In addition to that day, using the latter test, we detect 140 additional multiple MJ days. In particular, we observe three days with more than six MJs: 09/19/2008, 09/29/2008, and 05/06/2010. Details on the days with more than 3 MJs are reported in Table 2.8. The first thing to notice it is that all multiple MJ days are associated with important economic and financial events.

Moreover, Table 2.8 tells us that the majority of these MJ days are not reflected by a jump in the RUA index. A possible explanation is that the stocks comprised in the co-jumps are small in size, thus their impact is negligible when considering the whole market and no jumps appear in the Russell 3000 index. We further analyze this point in section 2.6.2.

Regarding the distribution of MJs during the day, Figure 2.13 shows that using the *s-BNS* test we do not observe the clear pattern that is instead evident using the *C-Tz* test. In this

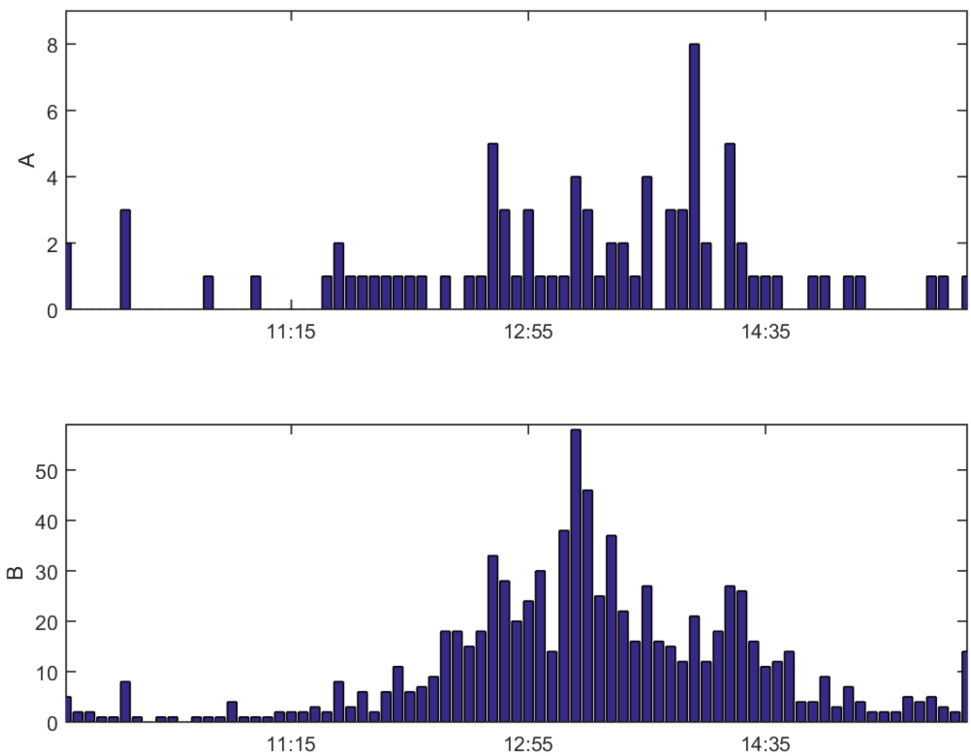

Figure 2.13 Days with multivariate jumps per intraday interval. The figure shows, for each of the 77 5-minutes daily intervals, the number of days for which we observe at least a multivariate jump. All jumps are detected using the *s-BNS*, panel A, or the *C-Tz* test, panel B.

last case, in fact, similarly to the co-jump case we observe that MJs are concentrated around lunch time.

Table 2.8 Multivariate jump days. The table shows, using a 5-minutes observation interval, the dates (Days) for which we report more than 3 multivariate jumps, if RUA jumps are present for the same days (RUA days), and the relevant Economic/Financial events during those days.

Days	RUA days	Economic/Financial event
03-Jan-01	no	FED cut fed funds rate
18-Apr-01	yes	FED cut short-term interest rates
29-Jun-06	no	FOMC statement
21-Mar-07	no	FOMC statement
09-Aug-07	no	BNP Paribas freeze three of their funds
10-Aug-07	no	FOMC statement
18-Sep-07	yes	FOMC lowers target for federal funds rate (50 bps)
10-Jan-08	no	FED chairman Ben Bernanke statement
18-Sep-08	no	FED measures against pressure in funding markets
19-Sep-08	no	FED announce liquidity programs
29-Sep-08	yes	FOMC meeting unscheduled, Emergency Economic Stabilization Act not approved
03-Oct-08	no	Sign of the Emergency Economic Stabilization Act
06-May-10	yes	The Flash Crash
10-Aug-10	no	FOMC statement
05-Aug-11	no	S&P downgrades US sovereign debt

2.6.1 Diffusion indexes

Using the information about MJs, we build two diffusion indexes: a daily diffusion index (or DID), and an intraday diffusion index (or DII). For the former, for each day from January 2, 1998 through June 5, 2015, we set, if present, the larger intraday MJ to be the t-element of DID. In this way we obtain a time series of 4,344 elements, where each one equals the daily maximum number of stocks involved in a multivariate jump, or 0 if there are no MJ in that day. The DII, instead, modifies the DID by considering for each day all 77 5-minutes intraday intervals. We thus obtain a time series of $4,344 \cdot 77 = 334,488$ elements, where each one reports the number of stocks involved in the multivariate jump, if present, and 0 otherwise.

Figures 2.14 and 2.15 show the DID and the DII we obtain employing 5-minutes observation intervals. There are no graphical evident differences between the diffusion indexes at

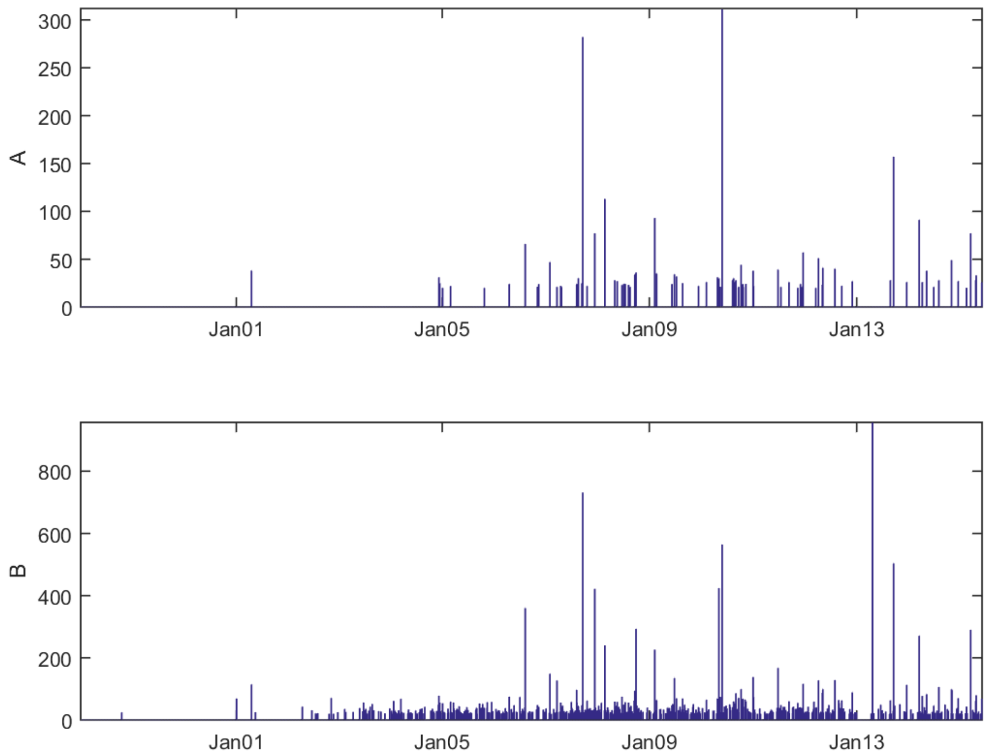

Figure 2.14 DID index. The figure shows, for each of the 4,344 days, the maximum number of stocks involved in a multivariate jump, if present, and 0 otherwise (the DID index). All jumps are detected focusing on 5-minutes observation intervals and using either the *s-BNS* test, panel A, or the *C-Tz* test, panel B.

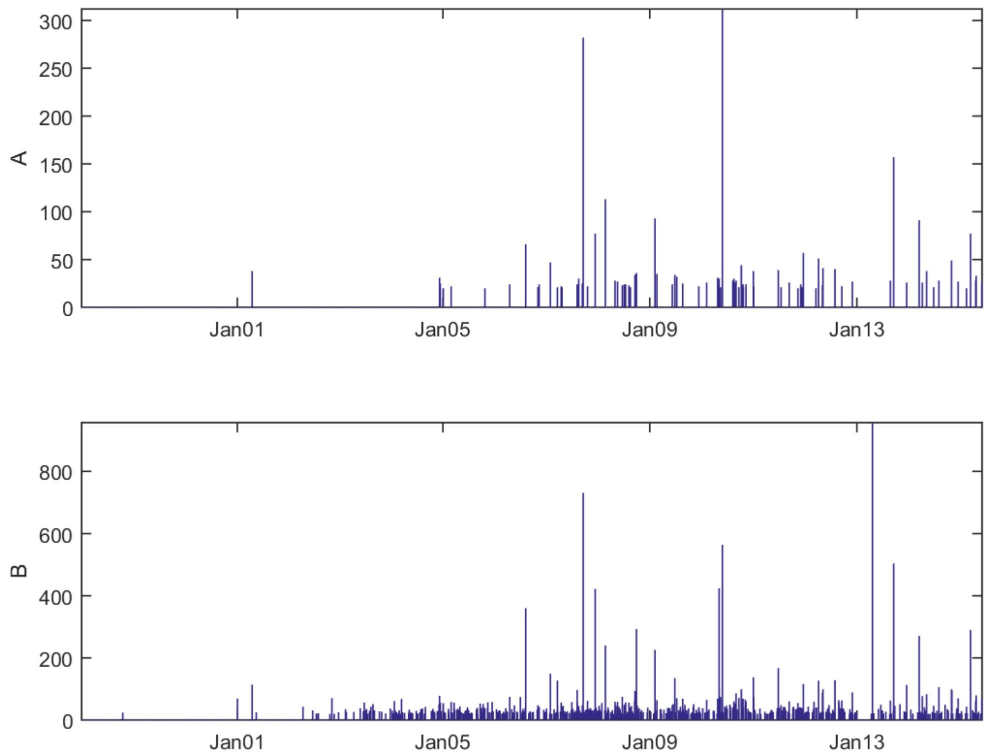

Figure 2.15 DII index. The figure shows, for each 5-minutes intraday interval between January 2, 1998 and June 5, 2015, the number of stocks involved in a multivariate jump, if present, and 0 otherwise (the DII index). All jumps are detected focusing on 5-minutes observation intervals and using either the *s-BNS* test, panel A, or the *C-Tz* test, panel B.

daily and intradaily levels, but using the *C-Tz* test we detect much more multivariate jumps and MJ days than using the *s-BNS* test.

It is important to notice that many MJs involve just a restricted number of assets and that the series exhibit relevant spikes. We then filter each diffusion index series to isolate the trend from the noise component. For this purpose, we use the Local Level Model (LLM), see, e.g., Durbin and Koopman (2001), that reads as follows:

$$\begin{aligned} x_t &= \mu_t + \varepsilon_t, \\ \mu_{t+1} &= \mu_t + z_t \end{aligned} \tag{2.18}$$

where x_t is the diffusion index series to filter with trend μ_t, whereas $\varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2)$ and $z_t \sim \mathcal{N}(0, \sigma_z^2)$ are the residuals. Smoothing the series could then be useful in identifying special events such as financial crises.

Figures 2.16 and 2.17, show the time series of μ (from equation 2.18) in addition to the index volatilities (or σ_{DID} and σ_{DII}).

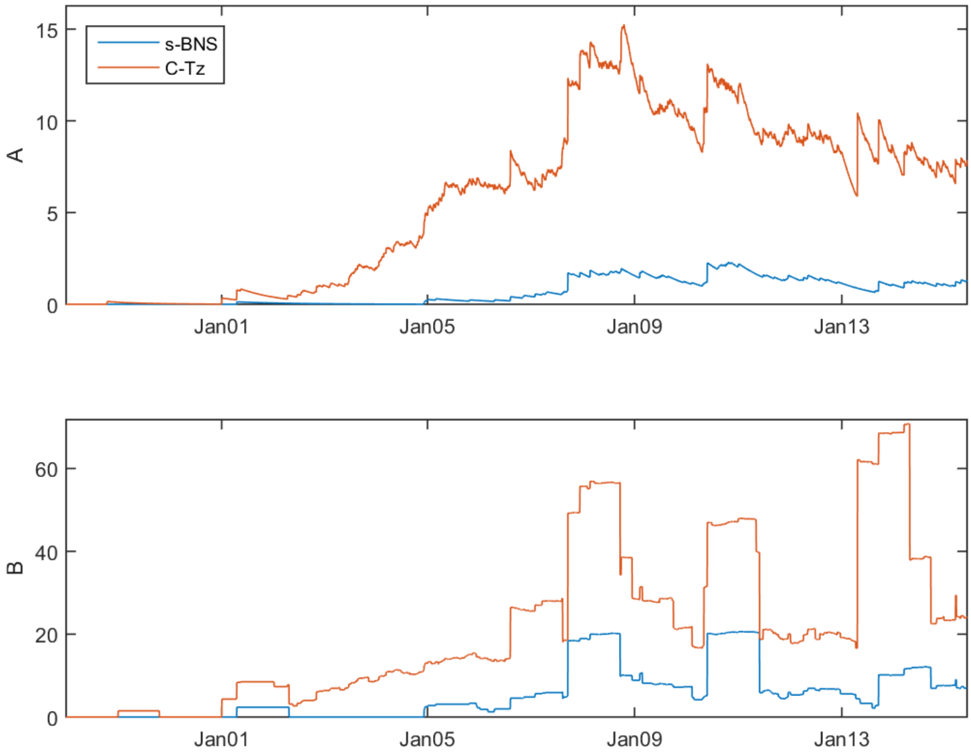

Figure 2.16 DID volatility and trend. The figure shows, for each of the 4,344 days, DID trend (μ_t), panel A, and volatility (σ_{DID}), panel B. μ_t is estimated daily as defined by equation 2.18, and, for each day t, σ_{DID} is computed using previous year daily DID observations. Each panel reports the results, using 5-minutes observation intervals, for both the DIDs defined using the *s-BNS* test and the *C-Tz* test.

For the daily index the volatility is computed daily using previous year daily DID observations, while for the intraday index we compute the volatility for each intraday 5-minutes interval using all previous year DII observations.[6]

Focusing on the daily index (see Figure 2.16), we observe that despite the series show similar behaviors, the *C-Tz* test leads to higher values of μ and σ than the *s-BNS* test. In fact, trend maximum and average values are respectively 2.30 and 0.69 using the former, and 15.26 and 5.80 for the latter. Correspondent volatility values are instead 20.61 and 5.43 for the *s-BNS* test, and 70.79 and 21.08 for the *C-Tz* test. Moreover, *C-Tz* results show larger time variations, thus making more evident trend and volatility increases during market turmoils, as in 2008 and 2010.

[6]We consider yearly rolling windows of: 252 days for the DID, and $252 \cdot 77 = 19,404$ intervals for the DII.

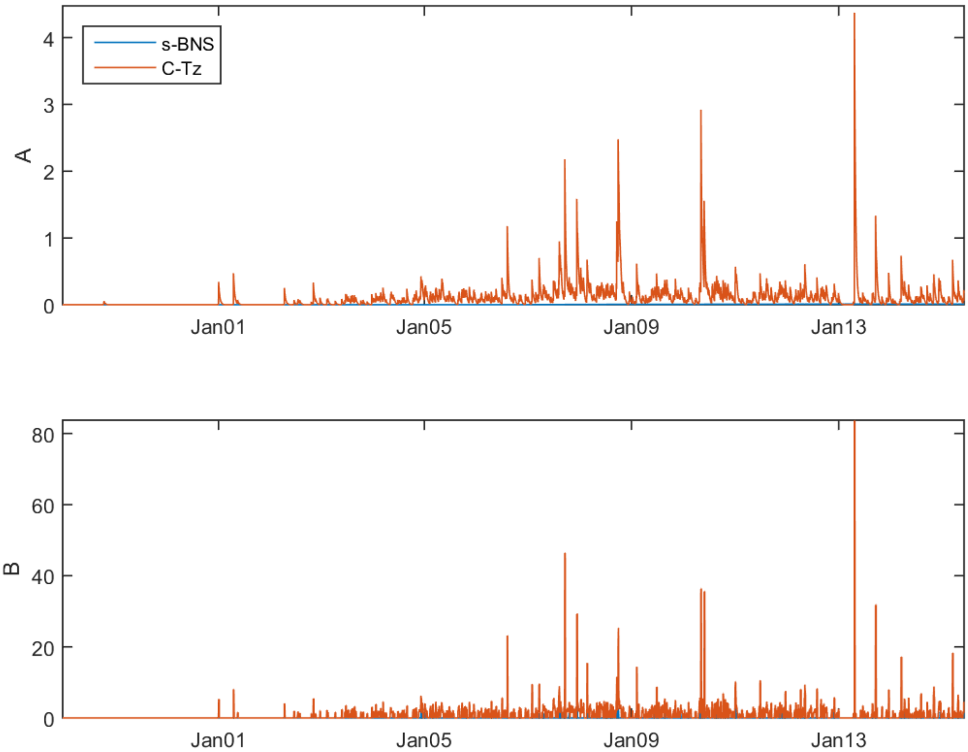

Figure 2.17 DII volatility and trend. The figure shows, for each 5-minutes intraday interval between January 2, 1998 and June 5, 2015, DII trend (μ_t), panel A, and volatility (σ_{DII}), panel B. μ_t is estimated for each intraday interval as defined by equation 2.18, and, for each daily interval, σ_{DII} is computed using previous year DII observations. Each panel reports the results, using 5-minutes observation intervals, for both the DIIs defined using the *s-BNS* test and the *C-Tz* test.

Moving our attention to the DII (see Figure 2.17), we observe important differences between the series computed using different tests. Trend maximum and average values are 0.01 and 0.004 using the *s-BNS* test, and 4.37 and 0.10 for the *C-Tz* test. Maximum and mean values of σ_{DII} are instead 19.59 and 0.16 for the former, and 83.63 and 1.22 for the latter. Similarly to the daily case, we observe higher spikes in correspondence of market turmoils.

It is however more interesting to focus on the irregular component of the filtering procedure, since it should help to identify special market events. Figures 2.18 and 2.19 show the time series of ε_t, from equation 2.18, using either x_t =DID and x_t =DII. In both figures we observe more and larger spikes in correspondence of special market moments, that are more evident using the *C-Tz* test. The days in correspondence of the ten larger spikes of Figure 2.18 are presented in Table 2.9. Note, in the great majority of the cases, days with larger spikes are

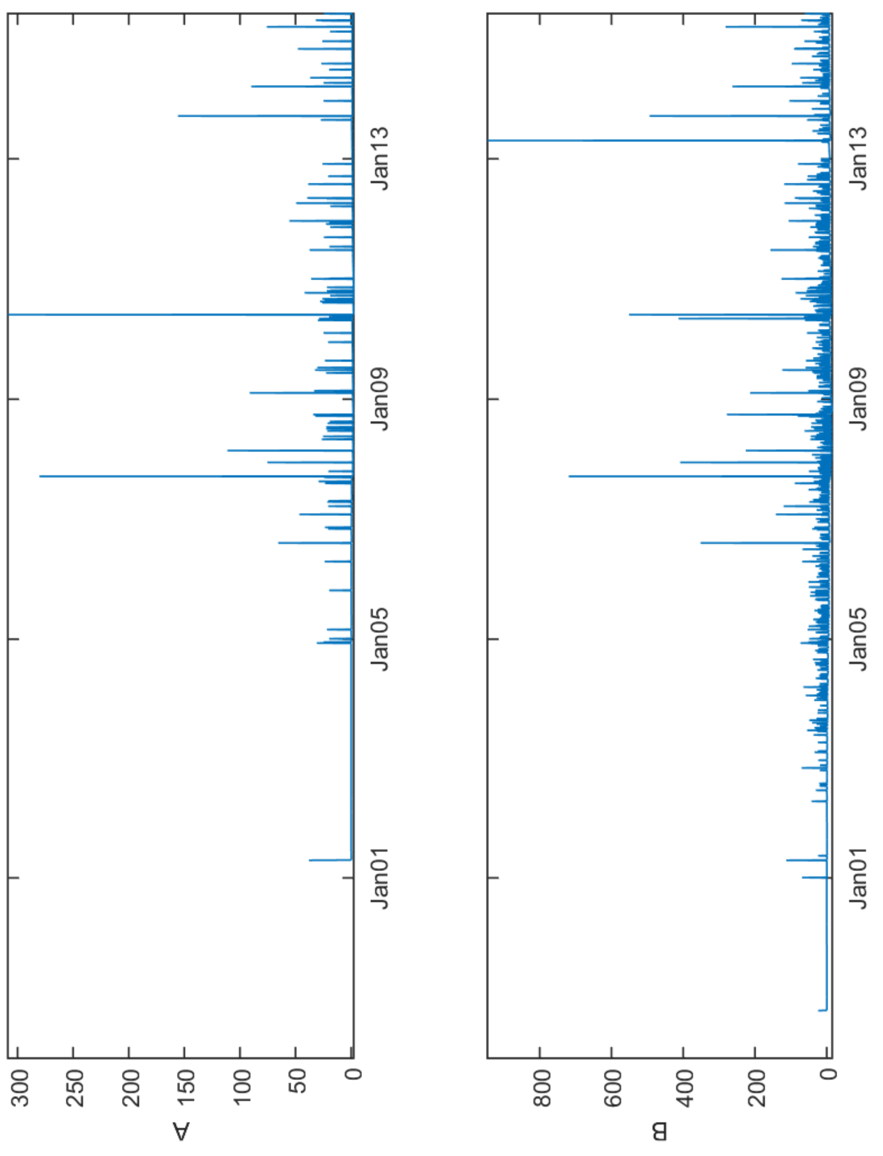

Figure 2.18 Filtered DID residuals. The figure shows, for each of the 4,344 days, DID residuals (ε_t) estimated as defined by equation 2.18. Panel A reports the results, for 5-minutes observation intervals, using the *s-BNS* test, while panel B shows the residuals we obtain using the *C-T$_Z$* test.

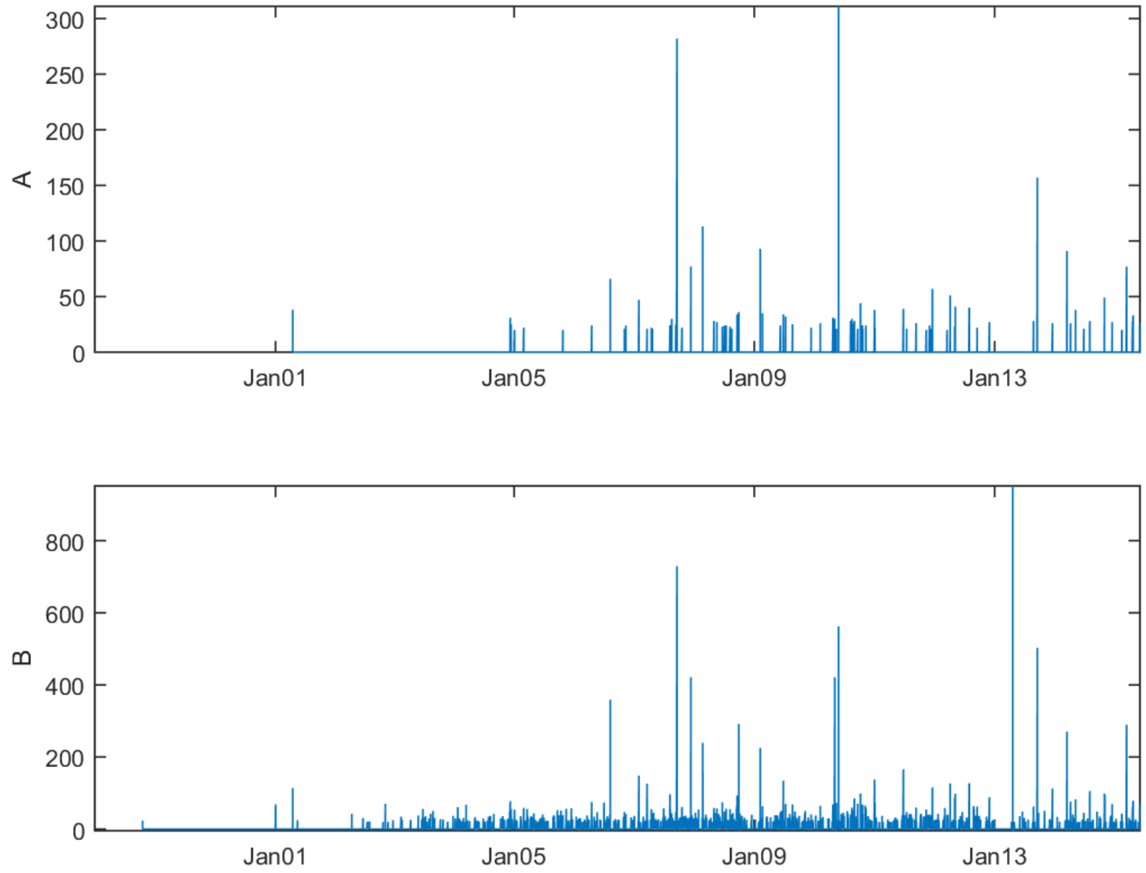

Figure 2.19 Filtered DII residuals. The figure shows, for each 5-minutes intraday interval between January 2, 1998 and June 5, 2015, DII residuals (ε_t) estimated as defined by equation 2.18. Panel A reports the results, for 5-minutes observation intervals, using the *s-BNS* test, while panel B shows the residuals we obtain using the *C-Tz* test.

Table 2.9 Residual spikes. The table shows, using a 5-minutes observation interval, the dates (Days) with the ten larger residual spikes, if RUA jumps are present for the same days (RUA days), and the relevant Economic/Financial events during those days.

	Days	RUA days	Economic/Financial event
			s-BNS
1	28-May-10	yes	FED announces three small auctions
2	18-Sep-07	yes	FOMC lowers target for federal funds rate (50 bps)
3	18-Sep-13	no	FOMC statement
4	25-Feb-08	yes	
5	10-Feb-09	no	
6	19-Mar-14	no	FOMC statement
7	18-Mar-15	no	FOMC statement
8	11-Dec-07	no	FOMC lowers target for federal funds rate (25 bps)
9	08-Aug-06	no	FOMC keeps its target for the federal funds rate
10	15-Dec-11	no	
			C-Tz
1	23-Apr-13	yes	AP fake tweet about explosions at the White House
2	18-Sep-07	yes	FOMC lowers target for federal funds rate (50 bps)
3	28-May-10	yes	FED announces three small auctions
4	18-Sep-13	yes	FOMC statement
5	06-May-10	yes	The Flash Crash
6	11-Dec-07	yes	FOMC lowers target for federal funds rate (25 bps)
7	08-Aug-06	yes	FOMC keeps its target for the federal funds rate
8	18-Mar-15	no	FOMC statement
9	29-Sep-08	yes	FOMC meeting unscheduled, Emergency Economic Stabilization Act not approved
10	19-Mar-14	yes	FOMC statement

Table 2.10 Correlations MJs and index jumps. The table reports the correlations (ρ), and the correspondent p-values (P-val(ρ)), between diffusion indexes and jumps in the market index. Results are presented separately for two jump tests, *s-BNS* and *C-Tz* using an observation interval of 5 minutes.

Index	DID	DII	DID	DII
	ρ		P-val(ρ)	
s-BNS	0.09	0.10	0.00	0.00
C-Tz	0.13	0.15	0.00	0.00

associated with relevant economic events, thus sustaining the importance of using diffusion indexes to detect important market moments. Results in the table confirm that this relation is more evident when using the *C-Tz* test. Moreover, days detected using the *s-BNS* test are usually not associated with a jump in the RUA index, while the opposite happens for

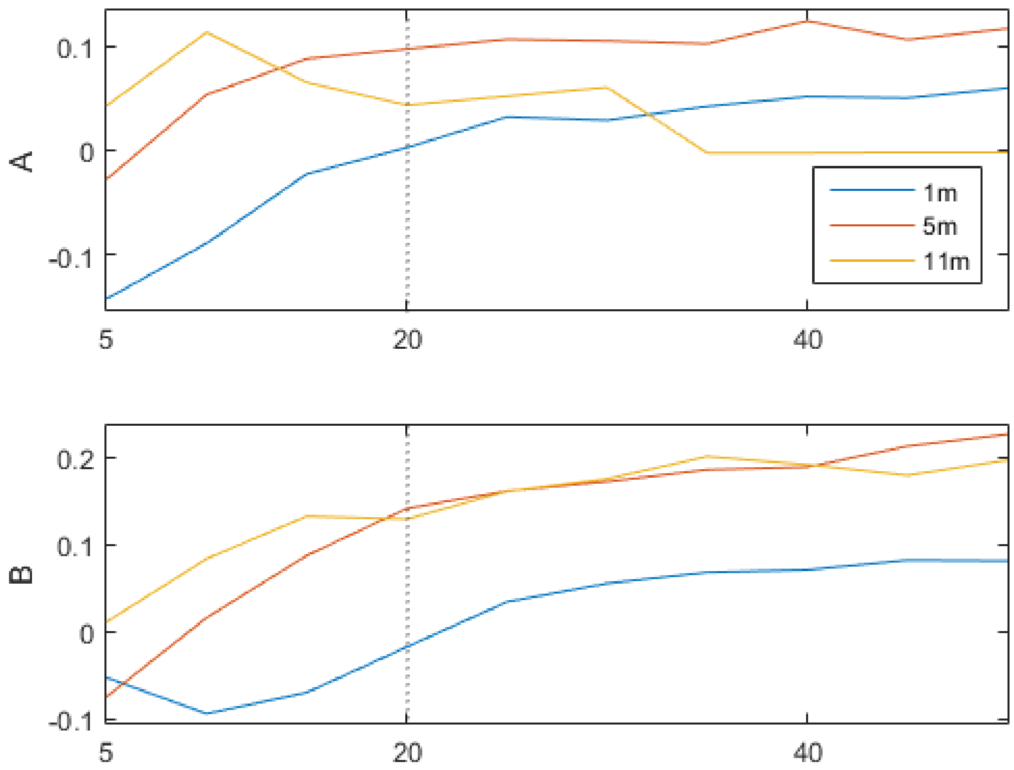

Figure 2.20 DID correlations. The figure shows, for different observation intervals, the correlation between RUA jump days and the DID index for different thresholds. The thresholds equal $N = 5, 10, 15, 20, 25, 30, 35, 40, 45, \text{and} 50$ stocks and all jumps are detected using either the *s-BNS* test, panel A, or the *C-Tz* test, panel B.

C-Tz days. Focusing on these last days, we also observe that 09/18/2007, 09/29/2008, and 05/06/2010 were also present in Table 2.8 since they are days with more than 3 MJs.

To study the relation between multivariate jumps and index jumps, Table 2.10 presents correlations (or ρ), and correspondent P-values, of RUA jumps and diffusion indexes, using an observation interval of 5 minutes. All P-values are smaller than 0.01 (1% significance level), thus allowing us to not accept the null hypothesis of $\rho = 0$. Correlations are positive and larger than in the jump and co-jump cases. Using 5-minutes observations, MJ correlations range from 0.09 to 0.15, while jump and co-jump correlations are about 0.01. Correlation results define the existence of a positive relation between jumps in the index and diffusion indexes in the constituents of the Russell 3000 index.

Figures 2.20 and 2.21 show how this relation depends on the choice of the threshold. They present the values of the correlation between RUA jumps and diffusion index, where a MJ is detected when at least 5, 10, 15, 20, 25, 30, 35, 40, 45, or 50 stocks jump together.

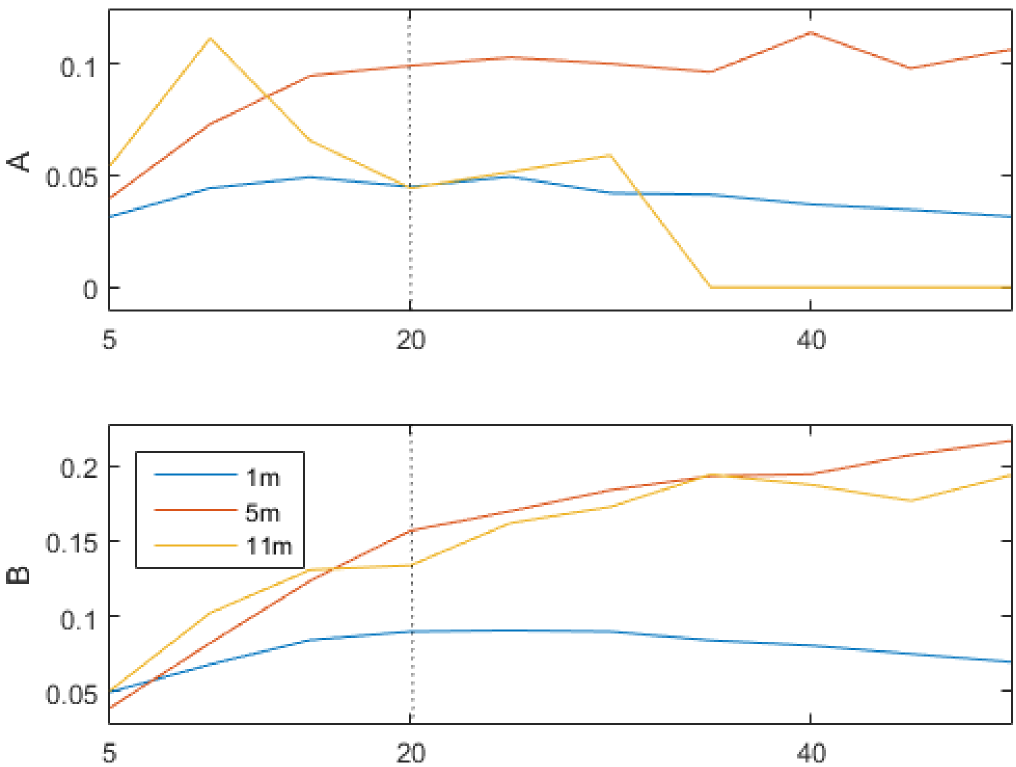

Figure 2.21 DII correlations. The figure shows, for different observation intervals, the correlation between RUA jumps and the DII index for different thresholds. The thresholds equal $N = 5, 10, 15, 20, 25, 30, 35, 40, 45,$ and 50 stocks and all jumps are detected using either the *s-BNS* test, panel A, or the *C-Tz* test, panel B.

The choice of the threshold seems to be relevant when using 11-minute observations, and especially for the DID. Instead, for the other two frequencies correlations tend to stabilize around $N = 20$, thus supporting our threshold choice (see equation 2.17).

2.6.2 Systemic events

In this section we focus on systemic co-jumps that, following Das and Uppal (2004), we define as infrequent events correlated across a large number of stocks. Among all co-jumps we are primarily interested in those rare and dramatic, hitting a large part of the market. To this end we identify a systemic jump (or Sy jump) when at least 20 stocks among the top 100 size stocks, jump together.

Figure 2.22 provides an image of where systemic jumps are located and their dimension, expressed as the number of assets involved in the co-jump. It is evident that systemic jumps are rare events: using the *s-BNS* test and the *C-Tz* test we identify respectively 1 and 6

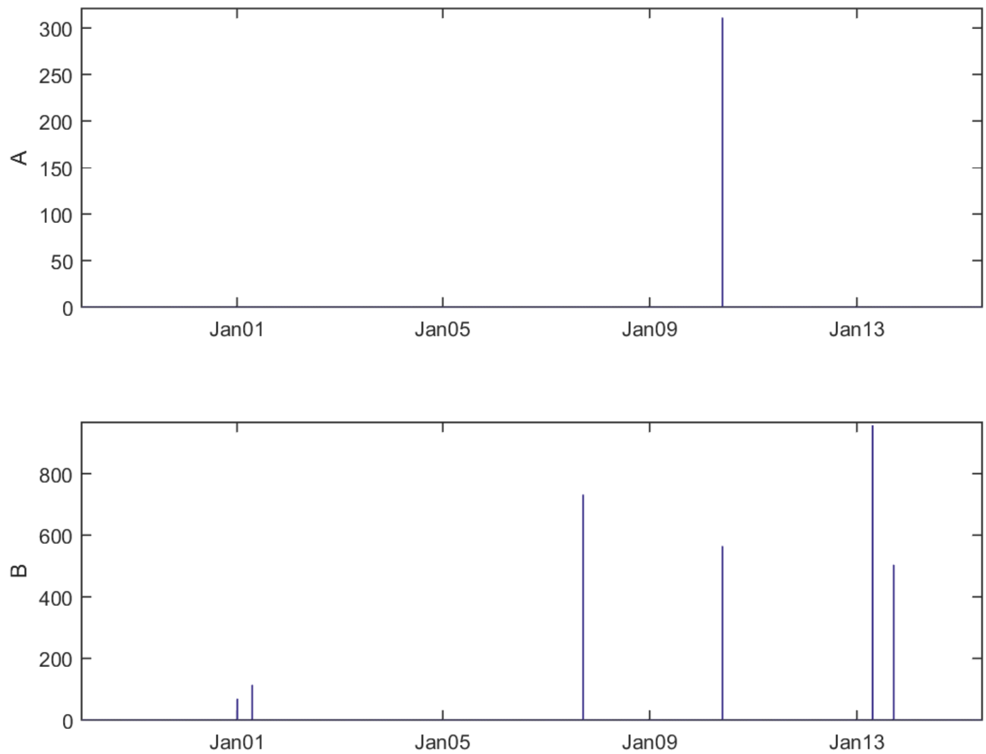

Figure 2.22 Systemic jumps. The figure shows, for each of the 4,344 days, the maximum number of stocks involved in a systemic jump, if present, and 0 otherwise. All jumps are detected focusing on 5-minutes observation intervals and using either the *s-BNS* test, panel A, or the *C-Tz* test, panel B.

systemic jump days. With respect to the total of 84 and 643 5-minutes MJ days in Table 2.7, systemic jump days are about 1%. This suggests that many MJs involve prevalently small size stocks and, despite they sometimes cover a large number of stocks, they might not be relevant when considering the full market. Instead, thanks to the size of the stocks involved, systemic jumps represent an important fraction of the market no matter the number of assets comprising in the co-jumps.

From Table 2.11, it is also clear that systemic jumps are economically significant events. The table reports the dates and times of the systemic jumps, and associates macroeconomic/financial information; it also reports, when present, the times of RUA index jumps for the same days.

Table 2.11 Systemic jump days. The table shows, using a 5-minutes observation interval, the dates (Days) and times (Time) for which we report a systemic jump, if RUA jumps are present for the same days (RUA days) and the corresponding times (RUA time), and the relevant Economic/Financial events during those days (Eco./Fin. event).

Days	Time	RUA days	RUA time	Eco./Fin. event
		s-BNS		
28-May-10	12:35-12:40	yes	12:35-12:40	FED announces three small auctions
		C-Tz		
03-Jan-01	13:15-13:20	no		FED cut fed funds rate
18-Apr-01	10:55-11:00	yes	10:55-11:00	FED cut short-term interest rates
18-Sep-07	14:15-14:20	yes	14:15-14:20	FOMC lowers target for federal funds rate (50 bps)
28-May-10	12:35-12:40	yes	12:35-12:40	FED announces three small auctions
23-Apr-13	13:05-13:10	yes	13:05-13:10	AP fake tweet about explosions at the White House
23-Apr-13	13:10-13:15	yes	13:10-13:15	AP fake tweet about explosions at the White House
18-Sep-13	14:00-14:05	yes	14:00-14:05	FOMC statement

We can see that all systemic jumps are associated with important economic events as Federal Reserve (or FED) announcements, Federal Open Market Committee (or FOMC) actions, or Associated Press (or AP) news. Relative to the AP releases, the two systemic jumps on 04/23/2013 reflect the market reaction to a false claim of an attack on the White House. In fact, the AP announcement and following retraction caused a fall and rebound of the markets within a few minutes. Systemic events on 09/18/2007 and on 05/28/2010 are in line with the findings of Caporin et al. (2016) who investigate multi-jumps between 2 January 2003 and 29 June 2012.

Moreover, in Table 2.11 we observe that all but one systemic jumps, are also reflected by a jump in the Russell 3000 index. In Table 2.7 we noticed that systematic MJs are about 10-12% of all MJs (using an observation interval of 5 minutes). Among all systematic multivariate jumps, about 10% are also connected to systemic jumps.

Figure 2.23 shows all systematic MJs and highlights those that are also systemic events. It is clear that systemic jumps not only involve large size stocks, but they are also the larger

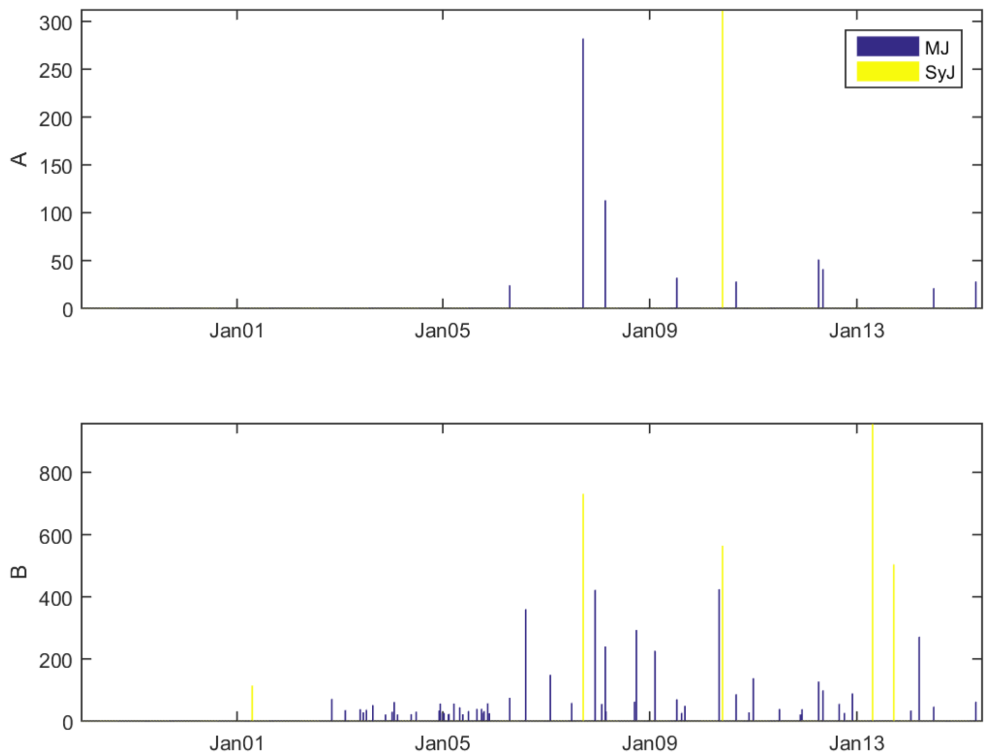

Figure 2.23 Systemic and non-systemic MJs. The figure presents, for each of the 4,344 days, the maximum number of stocks involved in a systematic multivariate jump, if present, and distinguishes between systemic (Mj) and non-systemic (SyJ) MJs. All jumps are detected focusing on 5-minutes observation intervals and using either the *s-BNS* test, panel A, or the *C-Tz* test, panel B.

co-jumps among all MJs. Because of their relevance in size and number of assets involved, we expect systemic jumps to be reflected also by a jump in the market index. Results in Table 2.11 and Figure 2.23 confirm our expectations. However, there are also many MJs that despite not being systemic events, are connected to a jump in the RUA. Note, e.g., that in correspondence to the Lehman crisis, on September 29, 2008, using the *C-Tz* test, we detect no systemic jumps but multiple multivariate jumps, Table 2.8, a large spike in the DID residuals, Table 2.9, and a at least a jump in the RUA. This example underlines the importance of focusing on diffusion indexes, in addiction to systemic events, to detect relevant market movements.

2.7 Jumps per industry

In this section we analyze the behavior of industry jumps, after sorting the constituents of the Russell 3000 index into 11 industry portfolios: oil & gas, basic materials, industrials, consumer goods, health care, consumer services, telecommunications, utilities, financials, technologies and others. The total number of assets belonging to each sector was presented in section 2.2, where we observed that our sample is concentrated on the financial, industrials, consumer services, technology, and health care industries.

To gain a deeper understanding on the industries characteristics, Table 2.12 reports the average and median number of quoted stocks (Mean_Q and Median_Q) belonging to each of the 11 industries, plus the correspondent mean and median number of liquid stocks (Mean_L and Median_L). As discussed in section 2.2, in order to guarantee accurate estimates, we focus on

Table 2.12 Industry population. The table reports, for each sector, the average and median number of assets quoted (Mean_Q and Median_Q) and that meet the liquidity condition (Mean_L and Median_L). The liquidity condition requires that, in a given trading day, the percentage of the stock's non-zero intraday returns is greater or equal to the 75%.

Sector	Mean_Q	Median_Q	Mean_L	Median_L
Oil & Gas	119.42	122.00	32.09	35.00
Basic Materials	121.12	120.00	16.04	13.00
Industrials	416.89	433.00	33.45	26.00
Consumer Goods	224.07	230.00	20.56	16.00
Health Care	288.78	300.00	26.72	24.00
Consumer Services	324.67	338.00	40.32	26.50
Telecommunications	25.20	26.00	1.92	1.00
Utilities	78.49	82.00	3.42	1.00
Financials	519.13	536.00	28.67	21.00
Technology	309.75	317.00	43.46	43.00
Others	22.58	27.00	2.20	2.00

liquid stocks: stocks that, in a given trading day, present a percentage of non-zero intraday returns greater or equal to the 75%.

In Table 2.12, the average and median number of quoted assets per industry, Mean_Q and Median_Q, confirm the results of Figure 2.5 about the distribution of the Russell 3000 constituents among the industries. It is however more interesting to observe the statistics on the liquid assets per industry, since they are the stocks on which we concretely run the jump tests. In line with the findings for the full sample, more than 50% of the assets in each sector do not meet the liquidity condition. When focusing on liquid instead of quoted assets, the composition of our sample slightly changes. The proportion of financial stocks decreases, since many financial assets are not much traded, while the fraction of oil & gas acquires importance because stocks of this industry are quite liquid. Moreover, we observe very low values of Mean_L and Median_L for telecommunications, and utilities industries and for the residual category, others. Due to the reduced number of assets effectively available to compute jump tests, we exclude this three categories in the subsequent elaborations.

We are now ready to apply the *s-BNS* and the *C-Tz* jump tests to the constituents of the remaining 8 industries, imposing the significance level $\alpha = 0.01\%$ in all jump tests, and $\vartheta = 3$ for the *C-Tz* threshold function. Table 2.13 and Figures 2.24 and 2.25 summarize jump results for each industry.

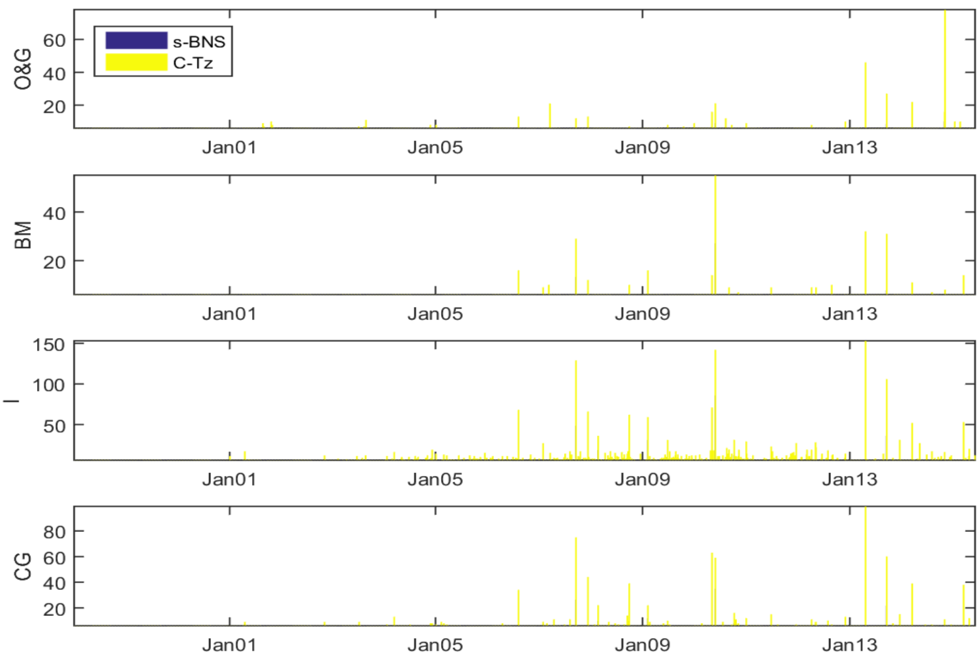

Figure 2.24 Industry DID/1. The figure shows, for each of the 4,344 days, the maximum number of stocks involved in a multivariate jump, if present, and 0 otherwise (the DID index). All jumps are detected focusing on 5-minutes observation intervals and using either the *s-BNS* test or the *C-Tz* test. Each panel present respectively the results for the industries: oil & gas, basic materials, industrials, and consumer goods.

Table 2.13 shows the number of jump, co-jump, and multivariate jump days we detect for each industry. While we identify co-jumps using the coexceedance rule of Gilder et al. (2014), equation 2.16, for multivariate jumps we modify equation 2.17 to account for the smaller dimension of the industries with respect to the full sample. In reducing the threshold we need to guarantee that co-jumps involve a number of assets sufficiently large, thus we identify a MJ if at least 6 stocks jump contemporaneously. Using detected MJs, we then build the diffusion indexes, DID and DID, for each industry (see Figures 2.24 and 2.25).

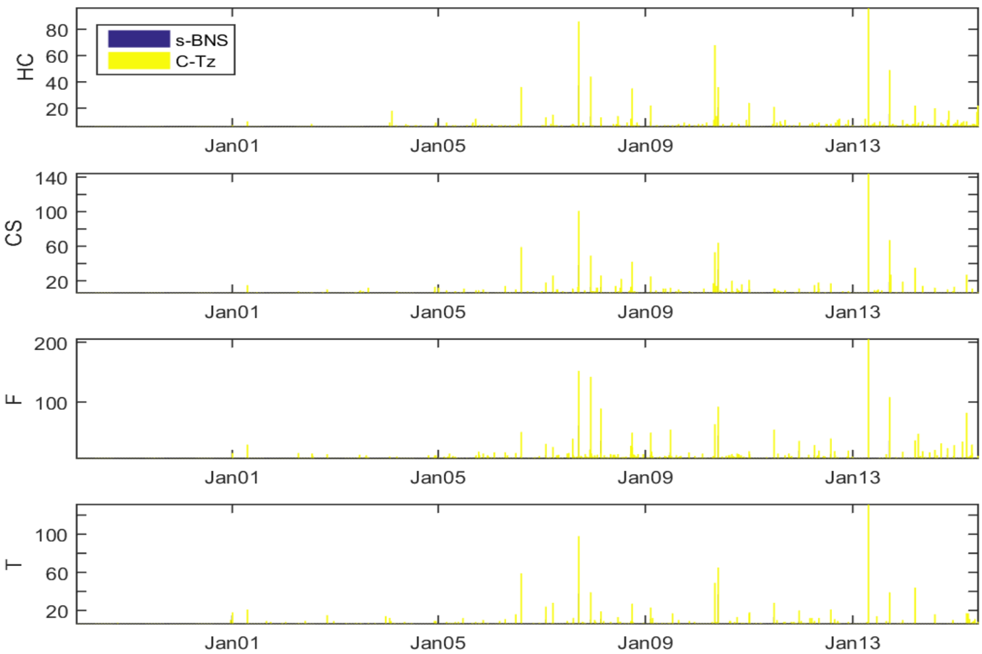

Figure 2.25 Industry DID/2. The figure shows, for each of the 4,344 days, the maximum number of stocks involved in a multivariate jump, if present, and 0 otherwise (the DID index). All jumps are detected focusing on 5-minutes observation intervals and using either the *s-BNS* test or the $C\text{-}Tz$ test. Each panel present respectively the results for the industries: health care, consumer services, financials, and technologies.

In line with previous results, Table 2.13 shows that using the $C\text{-}Tz$ test we detect more jump, co-jump, and multivariate jump days for each industry. Focusing on the differences among sectors it is important to notice that the dimension of the industry is not related to the number of detected jump (co-jump, or multivariate jump) days. For example, basic materials has the lowest number of liquid assets and the smallest amount of jump, cj, and MJ days. At the same time we observe that oil & gas presents few jump, co-jump and MJ days despite having a moderate number of liquid stocks.

Industries with more cj and MJ days are usually the same for which we detect more jump days. In particular, technology, health care, industrials, and consumer services are the sectors with more days with at least a jump. The same holds for co-jump and MJ days when using the $C\text{-}Tz$ test, while using the *s-BNS* the industries with more event days are health care, industrials, consumer services and financials. In particular, industrials has the larger number of multivariate jump days, 58 using the *s-BNS* test and 485 for the $C\text{-}Tz$ test. Lastly, it is interesting to notice that despite being a small industry, health care shows many contemporaneous jumps, thus indicating that its stocks tend to jump together.

Table 2.13 Industry jumps. The table reports jumps and co-jumps summary statistics for the constituents of the Russell 3000 index and for an observation interval of 5 minutes. N is the number of days for which we observe at least one intraday jump, N_{sj} is the number of days with at least one intraday single jumps (if in the same interval there are no other assets that jump), N_{cj} is the number of days with at least one intraday co-jump, and Max_{cj} and $Mean_{cj}$ are respectively the maximum and average number of assets involved in a co-jump. Results are presented separately for the two jump tests: *s-BNS*, and *C-Tz*. The table also splits the number of days for which we register at least a jump (N), a single jump (N_{sj}), and a co-jump (N_{cj}) between those detected using only the *s-BNS* test (only *s-BNS*), using only the *C-Tz* test (only *C-Tz*), or using both methods (*s-BNS* ∩ *C-Tz*).

Jump test	N	N_{cj}	N_{MJ}	$N_{RUA} \cap N_{cj}$	$N_{RUA} \cap N_{MJ}$
			s-BNS		
Oil & Gas	2,948.00	328.00	6.00	9.00	1.00
Basic Materials	2,903.00	324.00	3.00	12.00	2.00
Industrials	3,903.00	2,309.00	58.00	37.00	8.00
Consumer Goods	3,603.00	1,222.00	12.00	24.00	3.00
Health Care	3,928.00	2,531.00	24.00	40.00	3.00
Consumer Services	3,809.00	2,116.00	26.00	31.00	5.00
Financials	3,733.00	2,061.00	51.00	35.00	7.00
Technology	4,102.00	1,968.00	23.00	44.00	5.00
			C-Tz		
Oil & Gas	3,660.00	1,365.00	38.00	59.00	15.00
Basic Materials	3,644.00	1,334.00	30.00	65.00	16.00
Industrials	4,110.00	3,577.00	485.00	125.00	52.00
Consumer Goods	3,859.00	2,880.00	92.00	106.00	30.00
Health Care	4,087.00	3,622.00	482.00	133.00	41.00
Consumer Services	4,050.00	3,417.00	220.00	109.00	33.00
Financials	4,003.00	3,278.00	433.00	108.00	48.00
Technology	4,258.00	3,589.00	236.00	152.00	41.00

A view on the time evolution of MJs is possible using diffusion indexes. Figures 2.24 and 2.25 show, if present, the maximum number of assets involved in a multivariate jump per day, the DID, for each sector. The first thing to notice is that lager DID spikes tend to be concentrated in the same time periods for all industries. This suggests that when large multivariate jumps occur, they tend to be market-wide and not industry specific.

Moreover, many MJs involve a considerable number of stocks, and the phenomenon is more accentuated for industrials, consumer services, financial, and technology jumps. Consequently, some full sample systematic/non-systematic co-jumps which involve a relatively large number of stocks may be explained by co-jumps occurring exclusively amongst the stocks within these industries. To further study this point, Table 2.13 reports, for each type of industry, the number of systematic co-jump days, days for which we report at least a co-jump and a jump in the market index, and systematic multivariate jump days, days with both a MJ and a RUA jump.

It is clear that the great majority of contemporaneous jumps are not reflected by a jump in the Russell 3000 index, they are about 1 to 5% of all co-jump days, and 9 to 67% of the MJ days. Moreover, with the exception of basic materials, for all other industries systematic MJ

Table 2.14 Industry correlations MJs and index jumps. The table reports the correlations (ρ), and the correspondent p-values (P-val(ρ)), between diffusion indexes and jumps in the market index. Results are presented separately for two jump tests, *s-BNS* and *C-Tz*, using an observation interval of 5 minutes.

Index	ρ DID	ρ DII	P-val(ρ) DID	P-val(ρ) DII
		s-BNS		
Oil & Gas	0.04	0.04	0.02	0.00
Basic Materials	0.11	0.10	0.00	0.00
Industrials	0.09	0.09	0.00	0.00
Consumer Goods	0.08	0.08	0.00	0.00
Health Care	0.05	0.04	0.00	0.00
Consumer Services	0.09	0.09	0.00	0.00
Financials	0.08	0.08	0.00	0.00
Technology	0.10	0.09	0.00	0.00
		C-Tz		
Oil & Gas	0.17	0.17	0.00	0.00
Basic Materials	0.21	0.20	0.00	0.00
Industrials	0.12	0.15	0.00	0.00
Consumer Goods	0.21	0.21	0.00	0.00
Health Care	0.08	0.08	0.00	0.00
Consumer Services	0.13	0.15	0.00	0.00
Financials	0.12	0.13	0.00	0.00
Technology	0.16	0.16	0.00	0.00

days are less than 40%. This suggests that some of the non-systematic co-jumps involving a large number of stocks we detected in section 2.5.2, may be explained by industry co-jumps.

To further investigate the relation between industry contemporaneous jumps and jumps in the RUA, Table 2.14 shows the correlations of industry diffusion indexes and index jumps, and the correspondent P-values. All correlation are positive and statistically different from 0 (P-value < 0.05). They range from 0.04, for oil & gas and health care, to 0.11, for basic materials, using the *s-BNS* test. Similarly, adopting the $C\text{-}Tz$ test the minimum correlation is 0.08 for health care, while the maximum is 0.21 for basic materials and consumer goods. Results suggest that there exist a positive but weak relation between jumps in the index and industry MJs. Lastly, it is particularly interesting to notice that while the health care industry seems to be affected by many industry-specific multivariate jumps that are usually not reflected by a jump in the index, the few MJs in the basic materials and consumer goods industries are generally accompanied by a RUA jump, thus suggesting they belong to a market MJ.

2.8 Association between MJs and asset prices

Our previous empirical evidences suggest that the multivariate jumps we detect on stocks may affect the dynamics of prices in the whole market. In this section we assess the impact of multivariate jumps on asset returns by extending a traditional asset pricing model. This allows to verify our postulated impact of large cross-sectional jumps on returns. We empirically achieve this goal by adding to a simple asset-pricing model, the Sharpe (1964), Lintner (1965), Mossin (1966) and Black (1972) CAPM, our daily(intraday) diffusion index and thus putting forward a 2-factor model (see equations 2.20 and 2.21).

To analyze if DID and DII capture common variation in stock returns, we use time series regressions and check if slopes and R^2_{adj} values give evidence of a positive answer. Consequently, to judge the improvements provided by our new factors, we employ (a) regressions that use the CAPM and (b) regressions that use our 2-factor model, both focusing on the full sample (1998-2015) and on sub-samples.

For the DID, recalling that it is a daily index, we estimate the following regression models, where the latter is our 2-factor model for daily data:

$$R_{t,j} - R_{t,F} = \alpha_j + \beta_j MKT_t + e_{t,j} \tag{2.19}$$

$$R_{t,j} - R_{t,F} = \alpha_j + \beta_j MKT_t + \beta_{DID,j} DID_t + e_{t,j} \tag{2.20}$$

where $R_{t,j}$ is the daily return on a security j, computed excluding the first 5 minutes of the trading day, $R_{t,F}$ is the risk-free return available on the Kenneth R. French data library, DID_t is the daily diffusion index computed either using the *s-BNS* test or the *C-Tz* test, and $e_{t,j}$ is a zero-mean residual. Lastly, $MKT_t = (R_{t,M} - R_{t,F})$ is the excess return on a capitalization-weighted stock market portfolio, where $R_{t,M}$ is the daily Russell 3000 Index return. The proxy for the market portfolio seems adequate since we observe a correlation of 99.97% with the Kenneth French's data library correspondent portfolio, for the period 01/02/1998-06/05/2015.

Table 2.15 β_{DID} **significance**. Regressions of excess stock returns on the excess market return (MKT) and the daily diffusion index (DID). Regressions take place under the condition that, in the window of interest, stocks presents at least 251 days (about a year) of non-null returns. The table reports the fraction of β_{DID} with absolute t-statistic greater than 1.645, differentiating between the DID constructed using *s-BNS* or *C-Tz* test results.

Window	BNS	CPR
1998-2015	11%	10%
2002-2006	42%	35%
2007-2011	14%	11%
2012-2015	18%	20%

Table 2.15 and Figure 2.26 report information on the distribution of estimated β_{DID} (or $\hat{\beta}_{DID}$) and their statistical significance. Specifically, Table 2.15 shows the fraction of $\hat{\beta}_{DID}$ with absolute t-statistic greater than 1.645 (10% significance level), using different time windows. Considering the full sample, January 2, 1998 to June 5, 2015, we obtain about 10% of significant $\hat{\beta}_{DID}$s using both the *s-BNS* DID and the *C-Tz* DID. This suggests that the two diffusion indexes proxy for common risk factors in stock returns.

The table also shows results for different sub-periods, chose to reveal how the relevance of DID changes over time, and, specifically, with respect to economic crises. We also exclude the sample until 2001 because, as observed in section 2.5, up to that year our database behaves in a quite different way and presents an extremely low number of MJs. Note, DID slopes are significantly different from zero, using a 90% confidence interval, in a relevant number of cases for all sub-periods, with values larger with respect to the full-sample regressions.

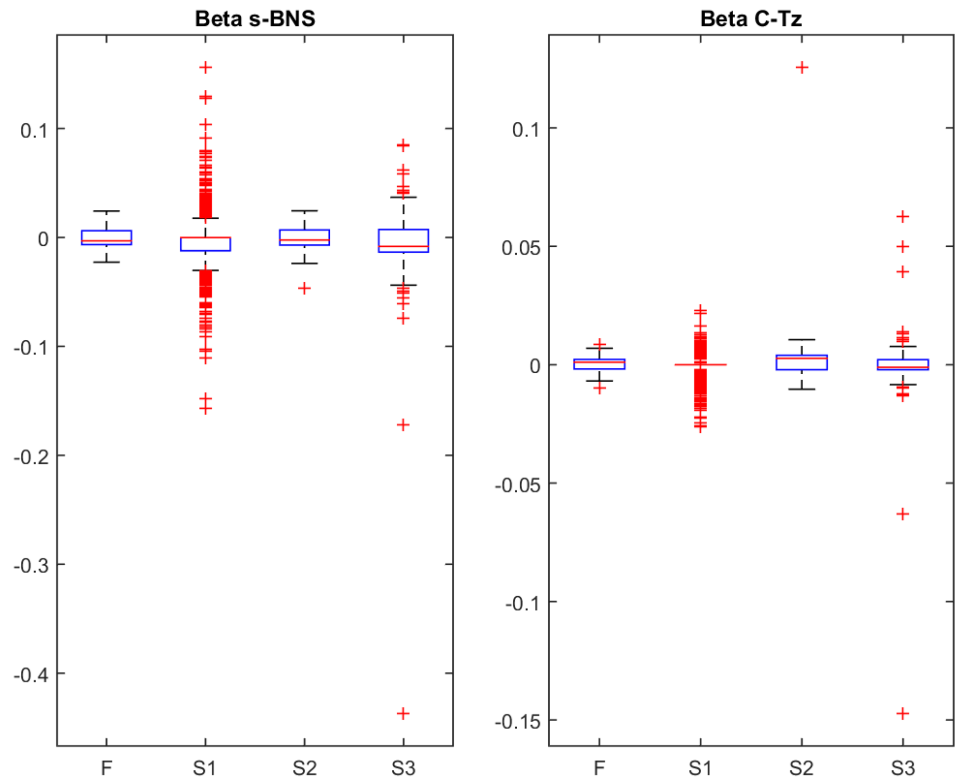

Figure 2.26 Significant β_{DID} distribution. Regressions of daily excess stock returns on the excess market return and the daily diffusion index. Regressions take place under the condition that, in the window of interest, stocks presents at least 251 days (about a year) of non-null returns. The figure reports the estimated β_{DID} with absolute t-statistic greater than 1.645, differentiating between the DID constructed using *s-BNS* or *C-Tz* test results. Each panel shows the distribution of the $\hat{\beta}$s resulting from regressions that use the full 1998-2015 sample (F), or 2002-2006 (S1), 2007-2011 (S2), and 2012-2015 (S3) data.

Moreover, it appears that DID slopes are more frequently significant during the pre-2008 economic crisis, namely from 2002 until 2006.

Figure 2.26 reports the distributions of significant $\hat{\beta}_{DID}$s for the same time windows of Table 2.15. Since DID values are sometimes very large, indeed they are equal to 0 or greater than 19, with respect to stock returns (average daily stock return = 0.0105%), we expect correspondent $\hat{\beta}$ to be small. In line with our previsions, the figure shows that $\hat{\beta}$ median, upper and lower quartiles (75^{th} and 25^{th} percentiles), and maximum and minimum values, excluding outliers, are close to 0, especially for *C-Tz* results. However, there are also many outliers, which are more(less) than 3/2 times the upper(lower) quartile values, in the sub-periods 2002-2006 and 2012-2015, for which we register the more extreme values, but almost none in the 2007-2011 sub-period and for the full sample.

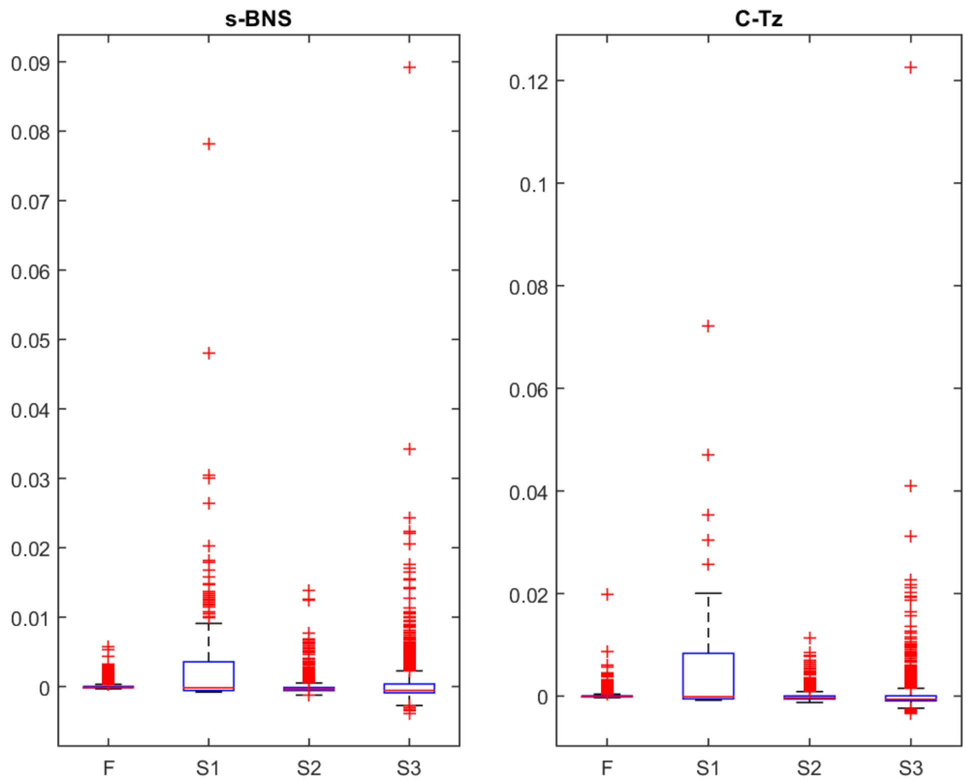

Figure 2.27 DID R^2_{adj} variation. Regressions of daily excess stock returns on the excess market return and the daily diffusion index (or 2-factor model), or exclusively on the excess market return (or CAPM). Regressions take place under the condition that, in the window of interest, stocks presents at least 251 days (about a year) of non-null returns. The figure reports the variation in the R^2_{adj} we observe using the 2-factor model with respect to the CAPM (R^2_{adj}(2-factor) $-$ R^2_{adj}(CAPM)), differentiating between the DID constructed using *s-BNS* or *C-Tz* test results. Each panel shows the distribution of R^2_{adj} differences resulting from regressions that use the full 1998-2015 sample (F), or 2002-2006 (S1), 2007-2011 (S2), and 2012-2015 (S3) data.

Note, while full sample $\hat{\beta}$s are approximately uniformly distributed around 0, they show different behaviors in the sub-periods. *s-BNS* results indicate that betas are more often negative in the first and last sub-periods, and positive in the second sub-period. Differently, *C-Tz* DID slopes tend to be symmetrically distributed around 0 for all regression windows.

Coefficients of determination confirm the ability of DID to capture shared variation in stock returns that is missed by the market factor. Figure 2.27, shows the variations in the values of R^2_{adj} we obtain including the diffusion index in the CAPM model. We observe few negative variations in correspondence of the second and third sub-periods, however, R^2_{adj} values increase for the vast majority of stocks, independently from the regression window in use. Increases are particularly pronounced for 2002-2006 and 2012-2015 data.

To gain a closer look on the impact of MJs on asset prices, we move our focus to intraday data. Similarly to the daily case, to study how DII helps in explaining stock returns, we run monthly regressions using 5-minutes data. Our 2-factor model for intraday data is:

$$R_{t,i,j} - R_{t,i,F} = \alpha_j + \beta_j MKT_{t,i} + \beta_{DII,j} DII_{t,i} + e_{t,i,j} \tag{2.21}$$

where $R_{t,i,j}$ is the return on a security j, on day t for the intraday interval i, $R_{t,i,F}$ is the risk-free return that we approximate equal to 0, $DII_{t,i}$ is the, *s-BNS* or *C-Tz*, intraday diffusion index, $MKT_{t,i} = (R_{t,i,M} - R_{t,i,F})$ is the excess return on the Russell 3000 market portfolio, and $e_{t,i,j}$ is a zero-mean residual.

The use high-frequency data allows to obtain long samples of stock returns, e.g., for our time window of 4,344 days we get 334,488 5-minutes observations. Consequently, it is possible to run regressions using data from a reduced number of days, and thus tracking how the significance of β_{DII} changes over time. We estimate the parameters of the model using non-overlapping rolling windows with a size of 22 days, 1,694 5-minutes observations, which corresponds to about one month of data. Figure 2.28 shows the time evolution of the significance of β_{DII} over the resulting 198 22-days intervals. As expected, the DII constructed using *C-Tz* results, appears to be more relevant in explaining asset returns. Indeed, we observe high fractions of significant betas for almost all intervals from 2004 until 2015. This confirms that multivariate jumps help to explain stock returns by capturing common variation that is missed by the market factor. Moreover, focusing on high-frequency data we do not observe higher levels of significance during the 2008 pre-crisis months but, instead, high picks clustered in 2007, 2008, 2010, and 2013.

A global reading of the results in this section, lead to multiple observations. First, the inclusion of the DID in the CAPM model is particularly effective, in terms of significant $\hat{\beta}$s and R^2_{adj} increases, for the sub-period with fewer and smaller MJs, the 2002-2006 window. For the second sub-period, which includes the years of the sub-prime crises, we, instead,

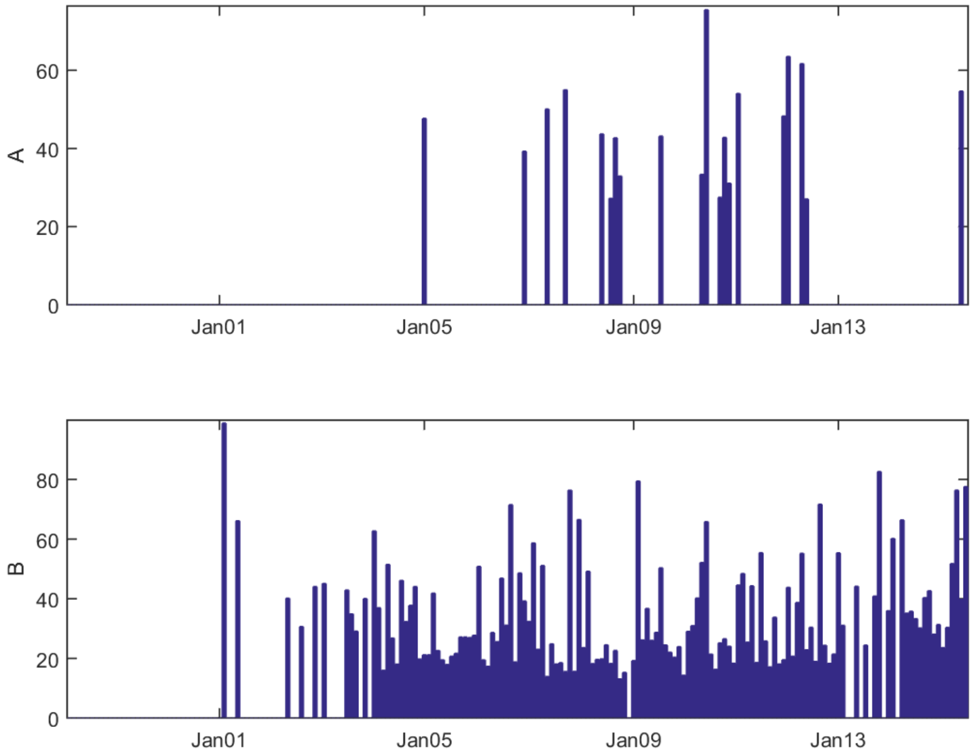

Figure 2.28 β_{DII} **significance**. Regressions of 5-minutes excess stock returns on the excess market return (MKT) and the intraday diffusion index (DII), using non-overlapping rolling windows with a size of 1,694 observations. Regressions take place under the condition that, in the window of interest, stocks presents at least 75% of non-null returns and the DII shows at least 2 non-null values. For each 22-days interval between 01/02/1998 and 06/05/2015, the figure reports the fraction of $\hat{\beta}_{DII}$ with absolute t-statistic greater than 1.645 (10% significance level). Panel A and B respectively show the results using *s-BNS* test and *C-Tz* test.

detect numerous and large MJs, and two systemic jumps. It seems however, that, for this window, the DID enhances only a little the CAPM. On the other hand, we observe relevant DII percentages of significance, especially in 2007, 2008, and 2010. Lastly, the 2013-2015 sub-period, represents the intermediate case. It presents several MJs, but less that in the previous window, some large collective jumps, and three systemic jumps. The usefulness of adding the DID in the asset-pricing model is stronger than for the second sub-period but weaker than in the first one. For the DII slopes, instead, we obtain good significance results.

We can conclude that, in periods less affected by market turmoils, the daily diffusion index makes a good job, while, in more turbulent economic moments, using the intraday index is more effective.

2.9 Conclusions

This paper identifies and analyzes common jumps which involve a relatively large number of stocks, the multivariate jumps (or MJs), that we then use to build indexes informative of the diffusion of jumps in the cross-section.

We start from the detection of jumps in the returns of the Russell 3000 constituents, employing the Corsi et al. (2010) $C\text{-}Tz$ test and the Barndorff-Nielsen and Shephard (2004b, 2006) *BNS* test. Results are then combined using the Gilder et al. (2014) coexceedance method, which makes it possible to detect contemporaneous jumps in the cross-section. The co-jumps we identify involve up to 956 assets, but are usually small and show a weak association with market jumps. The distribution of systematic and non-systematic co-jumps also suggest the existence of a positive relation between jumps in the market index and large co-jumps in the stocks. Indeed, systematic common jumps generally involve more stocks than non-systematic ones.

We then move to common jumps which involve a large number of assets, thus focusing on co-jumps which should have a relevant impact on a huge portfolio. Using a modified version of the Gilder et al. (2014) coexceedance method, we identify a multivariate jump if at least 20 stocks jump together. This information constitutes the starting point to derive our two diffusion indexes: the daily diffusion index (or DID), and the intraday diffusion index (or DII). Both indexes are informative of the cross-sectional diffusion of jumps but focus respectively on a daily and 5-minute intraday level. Results show how the indexes tend to be subject to more and higher spikes in correspondence of important economic moments, as in 2008 and 2010, and confirm their usefulness. Diffusion indexes are also positively associated with the market with correlations that are 9 to 15 times the correspondent jump and co-jump correlations.

A further analysis of MJs which represent a relevant fraction of the market, the systemic co-jumps, underline the importance of a joint evaluation of systemic events and diffusion indexes. Together with the systemic jumps which are rare events involving multiple large size stocks and the greatest number of assets among all co-jumps, we also detect many large non-systemic MJs correlated to a market jump, e.g. September 29, 2008. This suggests that limiting the analysis to systemic events could be misleading and incomplete.

We also examine the existence of a linkage between common jumps and market-level news, and establish a relationship between detected MJs and important economic and financial news. We also detect an association of systemic co-jumps with Federal Reserve (or FED), Federal Open Market Committee (or FOMC), and Associated Press announcements. The relevance of these relationships is particularly evident in portfolio selection and risk management activities (see among others Dungey and Hvozdyk (2012), Bollerslev et al. (2008), Lahaye et al. (2011), Gilder et al. (2014), and Caporin et al. (2016)).

We confirm previous results also focusing on the 11 industries of the Russell 3000 Index. Specifically, we notice a positive but weak association of diffusion indexes with market jumps, relation that is stronger for basic materials and consumer goods industries, and less important for oil & gas and health care. The similar behavior, across all industries, of the diffusion indexes suggests that common jumps are market-level jumps and not industry-specific co-jumps. Results also imply that some full-sample non-systematic common jumps, involving a large number of stocks, are due to industry co-jumps. Indeed, many industry MJs involve a considerable number of stocks despite being non-systematic.

The importance and usefulness of our indexes appears also clear when considering their contribution to asset-pricing models. By running regressions using a modified version of the CAPM model which includes our diffusion indexes, the 2-factor model, we register a relevant impact of multivariate jumps on asset returns. Both focusing on the 1998-2015 window as well as on some sub-periods, we observe that DID and DII slopes are significantly different from zero in a relevant number of cases, and that R^2_{adj} values usually increase when the DID is included in the CAPM. Lastly, results show that the DID is less effective in periods of market turmoils, in correspondence of which, instead, the DII appears more powerful.

Our results have important implication not only for asset allocation and hedging, but also in asset pricing. About this last point, mulivariate jump information can be used in the construction of a factor capturing the cross-sectional jump risk. This appears to us an interesting topic for future research.

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