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STRUCTURAL EQUATION MODELS WITH INTERACTING MEDIATORS: THEORY AND EMPIRICAL RESULTS

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Summary

In last decades researchers have focused on the analysis of SEM models with nonlinear effects involving exogenous variables, i.e. which are not linearly dependent on other variables. The main problems studied are the estimation process, the choice of the indicators for nonlinear terms, when the variables are unobserved and the possibility of distinguishing interaction models from curvilinear models, while the causal analysis is not taken into account.

In this thesis I introduce nonlinear terms involving endogenous variables in SEM model with 2 mediators. I focus my attention on the interaction and curvilinear effects as its particular case. This analysis is made both with observed variables and with unobserved or latent variables. To address causal analysis, I propose two different approaches and I compare them using simulated data with different sample sizes and different covariances between the 2 mediators.

I find that my model with Pearl's (2012) causal theory and exogenous interaction, i.e. which does not depend linearly on other variables, is preferable for its simplicity and because it requires a smaller sample size. Pearl's theory can be applied to very general models and for this reason it has problems when the mediators are correlated given the mediated variable. Then I propose a modified formula to apply this theory. This approach has good performances both for interaction models and for curvilinear models and I propose a procedure to recognize the true model.

Finally from a managerial perspective using the exogenous interaction model with Pearl's modified causal theory proposed by me, I show that, in a customer satisfaction context, positive emotions and negative emotions influence "jointly" future behavior. As emotions are in turn influenced by the design of the restaurant, managers can use it to enhance customers' loyalty both directly and indirectly by jointly inducing more positive emotions and less negative ones. This way a model with interacting mediators may help to better understand customers' behavior.

Riassunto

Negli ultimi decenni i ricercatori hanno focalizzato la loro attenzione sull'analisi di modelli SEM con effetti non lineari che coinvolgono variabili esogene, ossia che non sono linearmente dipendenti da altre variabili. I principali problemi studiati sono il processo di stima, la scelta degli indicatori per i termini non lineari quando le variabili sono non osservate e la possibilità di distinguere i modelli con interazione dai modelli curvilinei, non prendendo in considerazione l'analisi causale.

Introduco in questa tesi i termini non lineari che coinvolgono variabili endogene nel modello SEM con 2 mediatori. Focalizzo la mia attenzione sull'interazione e, come suo caso particolare, sugli effetti curvilinei. Questa analisi viene effettuata sia con le variabili osservate sia con le variabili non osservate o latenti. Per esaminare l'analisi causale, propongo due approcci diversi e li confronto utilizzando i dati simulati con differenti dimensioni del campione e con diverse covarianze tra i 2 mediatori.

Ho trovato che il modello con la teoria causale di Pearl (2012) e con l'interazione esogena, cioè che non dipende linearmente da altre variabili, è preferibile per la sua semplicità richiedendo un campione di dimensioni più piccole. La teoria di Pearl può essere applicata a modelli molto generali e quindi presenta problemi quando i mediatori sono correlati data la variabile mediata.Per applicare questa teoria propongo una formula da me modificata. Propongo una procedura per riconoscere il vero modello dando questo approcio buoni risultati sia per modelli con interazione sia per modelli curvilinei.

Infine dal punto di vista gestionale, utilizzando il modello con l'interazione esogena e con la teoria causale modificata di Pearl, dimostro che, in un contesto di soddisfazione del cliente, le emozioni positive e le emozioni negative influenzano "congiuntamente" il comportamento futuro. Essendo le emozioni a loro volta influenzate dal design del ristorante, i manager possono utilizzarlo per migliorare la fidelizzazione dei clienti sia direttamente che indirettamente e indurre congiuntamente più emozioni positive e meno quelle negative. In questo modo un modello con i mediatori che interagiscono può aiutare a comprendere meglio il comportamento dei clienti.

CONTENTS

1 Introduction **1**
1.1 Overview . . . 1
1.2 Main contributions of the thesis . . . 3

2 SEM with interaction **5**
2.1 Model with 2 mediators and uncorrelated errors . . . 5
2.1.1 Causal effects . . . 5
2.1.2 Estimation . . . 7
2.2 Model with 2 mediators and correlated errors . . . 8
2.2.1 Causal effects . . . 8
2.2.2 Estimation . . . 9
2.3 Model with 2 mediators with uncorrelated errors and interaction . . . 11
2.3.1 Causal effects . . . 11
2.3.2 Estimation . . . 13
2.4 Model with 2 mediators, correlated errors and interaction . . . 14
2.4.1 Causal effects . . . 14
2.4.2 Estimation . . . 15
2.5 Interaction: its interpretation . . . 16
2.6 Conclusions . . . 19

3 Nonlinear SEM: interaction vs curvilinear model **21**
3.1 Curvilinear models: a special case of the interaction model . . . 21
3.1.1 Causal effects . . . 21
3.1.2 Estimation . . . 22
3.2 The problem of spurious interaction . . . 22
3.2.1 A procedure to detect nonlinear effects . . . 26
3.3 Conclusions . . . 27

4 Nonlinear SEM with Latent variables **29**
4.1 Model with 2 mediators and interaction: estimation . . . 30
4.2 Curvilinear model: a special case of the interaction model . . . 32
4.3 Spurious interaction and latent variables . . . 34
4.3.1 What's in literature ? . . . 34
4.3.2 Multiple nonlinear effects model with endogenous variables . . . 37
4.3.3 Procedure to distinguish the nonlinear effects . . . 38
4.4 Conclusions . . . 40

5 Models with mediators and interaction: Measurement part and sample size 41
5.1 Measurement part and nonlinear effects . . . 41
5.1.1 Model with 2 mediators and interaction . . . 43
5.1.2 Curvilinear model . . . 45
5.1.3 Model with interaction and low reliability for the nonlinear terms . . . 45
5.2 Sample size . . . 48
5.3 Relation between sample size and reliability . . . 50
5.4 Conclusions . . . 56

6 Models with 2 mediators and interaction: practical applications in customer satisfaction 59
6.1 Interaction between positive and negative emotions . . . 59
6.2 Interaction between satisfaction and positive emotions . . . 62
6.3 Conclusions . . . 63

7 Conclusions 65

Appendix A Causality 67

Appendix B Model with 2 mediators vs model with 2 mediators and interaction 75

Appendix C Theorical standardized loadings and estimated standardized loadings 79

CHAPTER 1

INTRODUCTION

1.1 Overview

Economists, psychologists, and political scientists are often interested in interpreting relationships. the causation and the correlation, as it is clear correlation itself does not imply causation.

By using an example in the marketing field, I consider a customer which goes to the restaurant. The atmosphere which he finds inside influences the positive emotions perceived by the customer. These 2 variables are correlated but between them there is also a causal effect: the customer appreciates the friendly atmosphere and then feels a positive emotion. The atmosphere is the cause and the positive emotions are the effect. Now I consider the different emotions perceived by the same customer, which according to the marketing literature can be both positive and negative (Phillips & Baumgartner, 2002). I usually find that positive emotions are negatively correlated with negative ones, but between them there is not a causal effect: a customer who has high positive emotions has probably low negative emotions too, but I can not say that positive emotions cause negative emotions or vice versa.

With these 2 examples, I show that causation usually implies correlation while correlation does not imply causation. It is therefore very important to distinguish when the relationships among variables are causal or only due to correlation. For this reason I introduce two concepts: mediation and moderation. Mediator variables address how or why X causes Y while moderator variables address when or for whom X causes Y. To explain them better, I continue the previous example, naturally simplifying the relationships among the variables. If the positively emotioned customer will return more often to the restaurant, then atmosphere influences positive emotions which in turn influence customer future behavior. This is an example of mediation model, because atmosphere influences future behavior indirectly, i.e. through positive emotions, which are defined mediator variable. Then the effect of atmosphere on positive emotions is a direct effect as that of positive emotions on future behavior, while the effect of atmosphere on future behavior is called indirect effect. When atmosphere does not influence future behavior directly, this model is called complete mediation model. If the customer, instead, decides to return both for the atmosphere and for positive emotions perceived in the local, this model is called partial mediation model. Then the difference between the two models of mediation is the absence or presence of the direct effect of atmosphere on future behavior.

If I suppose that atmophere influences both positive emotions and negative emotions and both emotions influence future behavior, this is a model with 2 mediators. Now I suppose that the increase of positive emotions perceived by the customer with low negative emotions will produce a higher increase of loyalty than the same increase of positive emotions perceived by a customer with high negative emotions. This is a moderation model, because the causal effect of positive emotions on future behavior depends on the value of negative emotions, which are called moderator variable.

If I do not distinguish between the roles of the emotions, then I do not speak of moderation but of interaction, i.e. I consider both the causal effect of positive emotions on future behavior depending on the value of negative emotions and the causal effect of negative emotions on future behavior depending on the value of positive emotions. There are models which consider together the mediation and the moderation. Mediated moderation occurs when the interaction between two variables affects a mediator, which then affects a dependent variable, while moderated mediation occurs when the path from the intervention X to the mediator is constant, whereas the effect of the mediator on outcome Y depends on the level of a variable Z (Morgan-Lopez & MacKinnon 2006).

To analyze the causal effects, there are various causal theories, whose application may depend on the type of relation which exists between the variables. For this reason I introduce another important concept, which is linearity, in the parameters and/or in the variables. As an example, $Y = \beta F(X) + \alpha G(Z)$ is a "linear-in-parameters" model while $Y = \exp(\beta F(X) + \alpha G(Z))$ is "nonlinear-in-parameters" model. With nonlinear-in-parameters models the causal analysis for mediation has been studied mainly by Pearl (2009, 2012, 2014), who also analized the other type of non-linearity, i.e. in the variables, and nonparametric models. The linearity in the variables occurs when the causing variable itself affects the outcome (i.e. $Y = \beta X$), while in the case of nonlinearity the variable affects it through a function (i.e. $Y = \beta F(X)$). The most used among the nonlinear effects in the variables are interactions and curvilinear effects. The interaction occurs when the causal effect of a variable X on the variable Y depends on the value of another variable Z and similarly this happens for the causal effect of the variable Z on the variable Y. Mathematically, the interaction is the product of the variables Z and X. The curvilinear effect occurs when the variable X affects the variable Y through its square or successive powers. Mathematically, therefore, in the curvilinear model, $F(X)$ is for example $\beta X + \alpha X^2$. Two authors who study the causality in "linear-in-parameters" models are Preacher and Hayes (Preacher & Hayes 2008, Hayes & Preacher 2010). They focus on the determination of causal effects both in "linear-in-variables" models with mediation and in "nonlinear-in-variables" models with mediation.

The main difference between Hayes & Preacher's causal analysis and that proposed by Pearl is that: Hayes & Preacher calculate the effects using the linear equation which links the variables, while Pearl uses the moments and the overall distribution of variables. Consequently Pearl's causal analysis is used for any model ("linear-in-parameters", "nonlinear-in-parameters", "linear-in-variables", "nonlinear-in-variables", nonparametric model), while Hayes & Preacher's analysis is only suitable for "linear-in-parameters" models with any linearity in variables ("linear-in-parameters" and "linear-in-variables" models & "linear-in-parameters" and "nonlinear-in-variables" models). A special feature found by Pearl is that the total effect can not be always decomposed into the sum of the direct effect and the indirect effect.

A popular causal "linear-in-parameters" model is the structural equation model (SEM). In its literature mediation analysis has its roots (Pearl, 2014). In SEM 2 types of variables are analyzed: observed variables and latent ones, which are not directly observed but are rather obtained, through a mathematical model, from other variables that are observed, i.e directly measured. Then the SEM methodology estimates links between observed and latent variables and explains the causal relations among latent variables. In a general formulation SEM is composed of a measurement part, which uses the observed variables to measure latent variables, and of a structural part, which examines the causal relationships among latent variables determined in the measurement part. Factor analysis, which includes only the measurement part, is a particular case of SEM. SEM with observed variables, which considers only the structural part, is another particular case, having regression models as a special case. In SEM there is another distinction among exogenous and endogenous variables. The variables which causally affect other variables but are not affected by other variables are called exogenous, the others are called endogenous. SEM was initially specified as a "linear-in-parameters" and "linear-in-variables" model, but in recent decades many authors have focused in introducing nonlinearity even if only for exogenous variables. For example, therefore, two exogenous variables X and Z affect directly the endogenous variable Y through their interaction XZ. These models are defined in the literature Nonlinear SEM. In particular, the German school has focused on this analysis (for example Keleva et al. 2008, Moosbrugger et al. 2009, Brandt et. al 2014). Only Coenders et al. (2008) and Chen & Cheng (2014) consider the interaction between endogenous variables but in both papers the interaction variable is treated as exogenous although the 2 variables, which compose it, are endogenous. From the causal perspective, the 2 papers give few explications: Coenders makes a brief analysis using Hayes & Preacher, while Chen & Cheng omit the study of the topic.

1.2 Main contributions of the thesis

In my thesis, I analyses in details the problems in specifying models with nonlinear terms involving endogenous variables. I specify increasingly complex models and analyse their theoretical properties and their empirical performances.

In the second chapter I start considering a model in which all variables are observed and introducing a simple SEM, i.e. a linear-in-variables model, where an exogenous variable X affects an endogenous variable Y through two mediators Z and W, whose structural errors are not correlated. In this model both causal theories, i.e. that proposed by Pearl and that proposed by Hayes & Preacher, can be applied and they provide substantially the same effects.

I complicate the model by assuming that the structural errors of Z and W are correlated. Hayes & Preacher's theory can still be used because it does not consider the errors, while Pearl's analysis can no longer be used, because, being a theory for general models, the correlation between the errors affects the causal analysis. To overcome this problem I propose a modified formula which I call modified Pearl. The two theories continue substantially to provide the same result if I am mostly interested in the joint mediating effect of Z and W.

I then introduce the interaction term between endogenous variables in the model with two mediators and uncorrelated errors, moving from a linear-in-variables model to a nonlinear-in-variables model, i.e., from a SEM to a Nonlinear SEM. Initially I consider the interaction as exogenous following Coenders et al. and Chen & Cheng. In this model only Pearl's theory can be applied while that proposed by Hayes & Preacher can not be used, because there is not the causal relationship between the variable X and the interaction variable, which are only correlated. In order to obtain the causality, and then to endogenize the interaction variable, I introduce the term X^2, so the interaction is due to the variable X^2, and I call this "model with endogenous interaction". The causal analysis of the exogenous interaction model, made using Pearl's causal theory and the endogenous interaction model made using Hayes & Preacher provide substantially the same results.

I complicate this model introducing the correlation between the structural errors of Z and W, and so I get the model with two mediators with correlated errors and interaction. In this case I have still two options: exogenous interaction and causal analysis with modified Pearl or endogenous interaction and Hayes and Preacher's causal theory. As for the other models, the two theories provide substantially the same results. Because of its easier use my final suggestion is to use the model with exogenous interaction with my modified version of Pearl's causal analysis.

In the third chapter, I investigate the curvilinear model as a special case of the model with two mediators with correlated errors and interaction. The endogenous variables, which affect a variable in a curvilinear manner, have never been considered in SEM literature and in the section 3.1 I address this complication. In the second part of the chapter, starting from some considerations on the analysis of the exogenous interaction, I try to study if they are still valid for the interaction between endogenous variables and if there is the possibility to find some solutions for them. I start from Ganzach's concept of spurious interaction (1997), which is widely studied in the field of SEM by the German school. The spurious interaction consists in estimating a significant parameter for the interaction variable, even if it is not present in the true model. To overcome this problem Ganzach proposes to include also the quadratic variables in the model; in fact, he assumes that the significance of the parameter of the interaction is due to the fact that the true quadratic causal effects of the variables, which compose the interaction, are not considered. Using simulated data, I note that Ganzach's solution is not good if the two variables which form the interaction are highly correlated. This high correlation is easily found in the case of interaction between endogenous variables, as Z and W are correlated both through the exogenous variable X and through their structural errors. To overcome this problem I propose a method to detect the interaction models from the curvilinear ones.

In chapters 4 and 5 I introduce the measurement part to analyze causal models with unobserved variables. I find that the inclusion of the measurement part in nonlinear SEM models only affects the estimation methods, while the structural analysis and the causal theory remain the same. For this reason the problems of using the theory proposed by Hayes & Preacher or that proposed by Pearl remain the same as in Chapter 2. I investigate, therefore, how the measurement part affects the estimation of the model with two mediators, correlated structural errors and interaction. The measurement part consists of several items, i.e. observed variables, from which the unobserved or latent variables are derived. To define how much each item is linked to the latent variable the theory uses the concept of reliability. Moosbrugger et al. (2008) provide the formulas to derive the reliability of nonlinear variables from the reliability of linear variables. Unfortunately I note that good constructs for the linear variables are not necessarily good for their respective nonlinear variables, i.e. a good construct for Z may not be good for Z^2. This can be a problem in the estimation process. Another problem can be created by the choice of items of the nonlinear variables; when, in fact, I introduce the measurement part in nonlinear-in-variables models, I need also items for these nonlinear variables. Many authors have proposed various methods to obtain these items, I use those already used by Coenders et al (2008) and proposed by Marsh et al (2004). Estimating simulated data using Marsh' indicators I find that the reliability values are very close to those calculated by Moosbrugger et al (2008). I verify that Marsh' indicators for the nonlinear terms are good. When I have good constructs for linear variables and bad constructs for nonlinear ones, I recommend the use of newly defined constraints on the reliabilities, so that they lower the standard errors and improve the estimation. I find that when reliability decreases with the same sample size, it is increasingly difficult to distinguish between the two mediators if they are highly correlated and this causes problems in recognizing models with interaction from curvilinear models. I also note that the sample size affects the estimation process: the model with endogenous interaction requires a higher sample size than the model with exogenous interaction and when the goodness of fit of the measurement part decreases, then a higher sample size is required to have good estimates.

In chapter 6, I finally apply the proposed models to marketing data collected by a survey on McDonalds' italian fast food. I analyze various models in which I consider how much the atmosphere of the local affects the future behavior of the customer through positive emotions, negative emotions and satisfaction. In the different proposed models, the atmosphere positively affects future behavior both directly and indirectly. In a first case the effects of positive emotions and negative emotions are emphasized by their interaction. Positive emotions have a positive effect on future behavior, while negative emotions affect it negatively. Because of the interaction I find that for a customer with low negative emotions an increase of positive emotion leads to a greater positive change in future behavior than the same increase of positive emotion for a customer with average or high negative emotions. Using a different theoretical model as an example, also for positive emotions and satisfaction the causal effects can not be analyzed separately beacuse of the interaction term. Thus the manager who increases the positive emotions of the customer with high satisfaction increases the loyalty more than who increases the positive emotions of a customer with average or low satisfaction.

Chapter 7 sinthetizes the main conclusions from previous chapters both from a theorethical point of view and for practical implications of the newly proposed models.

CHAPTER 2
SEM WITH INTERACTION

In this chapter, I apply to SEM models with two mediators the causal theory proposed by Pearl (2012, 2014) and that proposed by Hayes & Preacher (2010); the problems in using this or that causal theory increase with model complexity, I start from the simple case of structural linear models with continuous variables and mediation, i.e. with at least a third variable which explains the relationship between a predictor and an outcome, and add complexity with different kinds of non-linearity. In this chapter I present, therefore, the following 4 structural models:

- model with 2 mediators and uncorrelated errors
- model with 2 mediators and correlated errors
- model with 2 mediators, uncorrelated errors and interaction
- model with 2 mediators, correlated errors and interaction

These models are studied both analytically and with the analyses of simulated datasets. For each model, therefore, I propose the model analytically and subsequently I control, using simulated data, if this model can be estimated and how the proposed method performs. In this chapter, I suppose that the variables are directly observed. More details on Pearl's theory are in Appendix A.

2.1 Model with 2 mediators and uncorrelated errors

2.1.1 Causal effects

I start analyzing a simple model with two mediators whose structural errors are uncorrelated. In this model, then, the effect of variable X on variable Y passes through other variables Z and W, called mediators. The mediated effect is also called indirect effect. As explained in Hayes (2013), when mediators are more than one, they can be in series (i.e. X influences Z, which influences W, which influences Y and X influences W), or parallel (i.e. X influences Z, which influences Y, and also X influences W, which influences Y). In this thesis I consider only the parallel case. The model is described mathematically so

$$Y = \beta_{41}X + \beta_{42}Z + \beta_{43}W + \zeta_4$$

$$Z = \beta_{21}X + \zeta_2$$

$$W = \beta_{31}X + \zeta_3$$

$$Cov(\zeta_3, \zeta_2) = 0$$

I suppose that the errors are indipendent of the regressors and that the error ζ_4 is indipendent of ζ_2 and ζ_3. The variables X,Z, W and Y and the error terms have zero means. Because the mediators are 2, i.e. Z and W, this is a classic example of a model with parallel multiple mediators and:

- if $\beta_{41} = 0$, then this model is defined complete mediation model
- if $\beta_{41} \neq 0$, then this model is defined partial mediation model

Now I calculate the effects according to 2 theories, that proposed by Pearl (2012, 2014) and that proposed by Hayes and Preacher (2010).

a) Pearl's theory considers the effects as function of $\Delta x = x_1 - x_0$ and requires to calculate the effects for each mediator

$$\text{effects with mediator Z: } \begin{cases} DE_{x_1,x_0}(Z) = (\beta_{41} + \beta_{43}\beta_{31})(x_1 - x_0) \\ IE_{x_1,x_0}(Z) = \beta_{42}\beta_{21}(x_1 - x_0) \\ TE_{x_1,x_0}(Z) = (\beta_{41} + \beta_{43}\beta_{31} + \beta_{42}\beta_{21})(x_1 - x_0) \end{cases}$$

$$\text{effects with mediator W: } \begin{cases} DE_{x_1,x_0}(W) = (\beta_{41} + \beta_{42}\beta_{21})(x_1 - x_0) \\ IE_{x_1,x_0}(W) = \beta_{43}\beta_{31}(x_1 - x_0) \\ TE_{x_1,x_0}(W) = (\beta_{41} + \beta_{43}\beta_{31} + \beta_{42}\beta_{21})(x_1 - x_0) \end{cases}$$

b) Hayes and Preacher's theory considers the effects as instantaneous change and, in Hayes (2013), the indirect effect, which is calculated for each mediator, is defined specific indirect effect, while the sum of the specific indirect effects is defined total indirect effect. In order to compare this theory with that proposed by Pearl, I calculate the specific indirect effect [1]

$$\text{effects with mediator Z: } \left\{ IE_{x_1,x_0}(Z) = \beta_{42}\beta_{21} \right.$$

$$\text{effects with mediator W: } \left\{ IE_{x_1,x_0}(W) = \beta_{43}\beta_{31} \right.$$

The direct effect proposed by Preacher & Hayes (2008) is always equal to β_{41} both when there is one mediator and when there are two mediators. This occurs because the direct effect is not influenced by the presence of more mediators as in the case of Pearl's effects , in which the indirect effect of the mediator, which is not considered, constitutes a part of the direct effect. The total effect is the sum of the direct effect and of the specific indirect effects:

$$TE_{x_1,x_0}(W, Z) = \beta_{41} + \beta_{43}\beta_{31} + \beta_{42}\beta_{21}$$

Now I compare the 2 theories. Pearl's effects are function of $\Delta x = \epsilon$ (i.e. I have as many effects as are the variations ϵ). To eliminate these many effects, Hayes and Preacher start considering the rate at which the value of Y changes with respect to the change of the variable X. Then

- Hayes and Preacher's effect is the ratio

$$=> \frac{\Delta Y}{\Delta X} = \text{Hayes and Preacher's effect}$$

[1] The indirect effect is calculated so

$$IE = \frac{\partial Y}{\partial Z}\frac{\partial Z}{\partial X}$$

- Pearl's effect is the variation

$$\Delta Y = \text{Pearl's effect}$$

$$=> \frac{\text{Pearl's effect}}{\Delta X} = \text{Hayes and Preacher's effect}$$

Because Hayes and Preacher don't consider a general variation of X but the istantaneous variation (very very small) $\Delta X \to 0$, then

$$lim_{\Delta X \to 0} \frac{\Delta Y}{\Delta X} = \text{Hayes and Preacher's effect}$$

and

$$lim_{\Delta X \to 0} \frac{\text{Pearl's effect}}{\Delta X} = \text{Hayes and Preacher's effect}$$

Then

$$lim_{x_1 \to x_0} \frac{IE(Z)_{\text{Pearl}}}{x_1 - x_0} = IE(Z)_{\text{Hayes \& Preacher}} \quad lim_{x_1 \to x_0} \frac{IE(W)_{\text{Pearl}}}{x_1 - x_0} = IE(W)_{\text{Hayes \& Preacher}}$$

A similar analysis can be made for Pearl's total effect and that proposed by Hayes & Preacher. Hayes & Preacher's direct effect is the istantaneous version of Pearl's direct effect considering simultaneously the two mediators. So the causal analysis according to Hayes & Preacher's theory and that according to Pearl's theory bring substantially to the same results.

2.1.2 Estimation

Now I consider the performance in estimating the previous model by simulating 1000 datasets of sample size 500 using the following theoretical model

$$Y = 0.27X + 0.45Z + 0.57W + \zeta_4$$

$$Z = 0.63X + \zeta_2$$

$$W = 0.77X + \zeta_3$$

$$Cov(\zeta_3, \zeta_2) = 0$$

and I estimate this model with the maximum likelihood (ML) method. The results are shown in Table 2.1 and are valued considering the following criteria:

- **95 % coverage** : it is the percentage of times in which the population value is included in the confidence interval of estimated parameters. According to Muthén and Muthén (2002) this index must be bigger than 0.91 and smaller than 0.98. According to Collins, Schafer and Kam (2001), this index must be bigger than 0.90.

- **Power of parameters**: it is measured as the ratio between the number of datasets in which the parameter is significant and the total number of datasets. According to Bradley (1978), a value between 0.025 and 0.075, is due to chance and it is coherent with a parameter restricted to 0. According to Thoemmes et al. (2010), the power must be bigger than 0.8 for nonzero parameters.

Cov=0								Bias: parameters		Bias: standard errors	
	ESTIMATES Population	Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff	beta_41	0.005185	beta_41	0.028097
Y ON								beta_42	-0.00111	beta_42	-0.01871
X	0.270	0.2714	0.0783	0.0805	0.0061	0.949	0.924	beta_43	0.000526	beta_43	0.016563
Z	0.450	0.4495	0.0481	0.0472	0.0023	0.946	1.000	beta_21	0.003175	beta_21	0.00659
W	0.570	0.5703	0.0483	0.0491	0.0023	0.964	1.000	beta_31	0.00013	beta_31	-0.02329
Z ON											
X	0.630	0.6320	0.0607	0.0611	0.0037	0.950	1.000				
W ON											
X	0.770	0.7701	0.0601	0.0587	0.0036	0.944	1.000				
Z WITH											
W	0.000	-0.0010	0.0232	0.0223	0.0005	0.944	0.056				

Table 2.1: Model with 2 mediators and uncorrelated errors

- **Relative bias of parameters** : it is measured as the difference between the estimated mean and the population value divided by the population value. According to Brandt et al. (2014), if it is included in the interval (0.05,0.1), it describes a "slightly bias", if it is bigger than 0.1, it describes a bias. According to Muthén et al.(1987), a value smaller than 0.1 or 0.15 can describe a negligible bias.
- **Relative bias of standard errors**: it is measured as the difference between the estimated standard error and the population value divided by the population value. According to Hoogland and Boomsma (1998), it must be smaller than 0.1.

From the results of Table 2.1, I can see that the estimate is good, in fact, the coverage index is always between 0.944 and 0.964, the biases of the parameters and of the standard errors are all less than 0.03, the powers of the parameters, which are present in the model, are greater than 0.924, and the power of the parameter, which is not present in the model, is 0.056.

2.2 Model with 2 mediators and correlated errors

2.2.1 Causal effects

I complicate the previous model adding the covariance between the structural errors of the mediators Z and W:

$$Y = \beta_{41}X + \beta_{42}Z + \beta_{43}W + \zeta_4$$

$$Z = \beta_{21}X + \zeta_2$$

$$W = \beta_{31}X + \zeta_3$$

$$Cov(\zeta_3, \zeta_2) \neq 0$$

This is still a classical example of a model with multiple parallel mediators as defined by Hayes (2013) and it is still true that:

- if $\beta_{41} = 0$, then this model is defined complete mediation model
- if $\beta_{41} \neq 0$, then this model is defined partial mediation model

Now I calculate the effects with the theories proposed respectively by Pearl and by Hayes & Preacher: Hayes and Preacher's effects are still calculable, while Pearl's effects are not calculable because of the correlation between the structural errors of the 2 mediators. Then I propose a modified formula (see Appendix A) in order to calculate these effects. In these new formulas, I consider jointly the two mediators.

a) Modified Pearl's theory:

$$\text{effects with mediators Z and W:} \begin{cases} DE_{x_1,x_0}(Z,W) = \beta_{41}(x_1 - x_0) \\ IE_{x_1,x_0}(Z,W) = (\beta_{43}\beta_{31} + \beta_{42}\beta_{21})(x_1 - x_0) \\ TE_{x_1,x_0}(Z,W) = (\beta_{41} + \beta_{43}\beta_{31} + \beta_{42}\beta_{21})(x_1 - x_0) \end{cases}$$

b) To compare Hayes and Preacher's theory with that proposed by Pearl, I calculate the total indirect effect:

$$\text{effects with mediators Z and W} \begin{cases} DE_{x_1,x_0}(Z,W) = \beta_{41} \\ IE_{x_1,x_0}(Z,W) = \beta_{42}\beta_{21} + \beta_{43}\beta_{31} \\ TE_{x_1,x_0}(Z,W) = \beta_{41} + \beta_{43}\beta_{31} + \beta_{42}\beta_{21} \end{cases}$$

I can therefore say that the value of covariance does not affect the causal effects. Recalling the concept of derivative (i.e. istantaneous variation), Modified Pearl's indirect effect is substantially equal to Hayes & Preacher's total indirect effect

$$lim_{x_1 \to x_0} \frac{IE_{\text{Modified Pearl}}(ZW)}{x_1 - x_0} = IE_{\text{Hayes \& Preacher}}(ZW)$$

Similarly, I obtain the equality between the direct effects and between the total effects in the two theories.

2.2.2 Estimation

Now I consider the estimation of the previous model. To make this, I simulate 1000 datasets of sample size 500 using the following theoretical model

$$Y = 0.27X + 0.45Z + 0.57W + \zeta_4$$

$$Z = 0.63X + \zeta_2$$

$$W = 0.77X + \zeta_3$$

$$Cov(\zeta_3, \zeta_2) = a \quad \text{with} \quad a = 0.4, 0.13, -0.4$$

Then I simulate three groups of 1,000 datasets which differ only in the value of covariance, in order to see if the covariance affects the estimation, having just seen that the causal effects are not influenced by it. The estimates are shown in Table 2.2. Using as criteria of goodness the coverage indices, the biases of the parameters and of the standard errors, I can say that the method estimates the parameters well in the three cases. The covariance between the structural errors affects the power of the direct effect of variable X on the variable Y, i.e. the power of β_{41}, which becomes below the minimum 0.8 in the case of covariance -0.4: this is due to an increase of the standard error. This could be explained by considering an analysis on multicollinearity in a SEM with latent variables made by Grewal et al. (2004). These authors observe that there is a relationship among a low power, a medium-high covariance between the variables, the sample size, a low explained variance in the endogenous constructs and a low goodness of the measurement part (see chapters 4-5), being the analysis done with latent variables in their case. In my case, the three groups of datasets differ only in the value of the covariance between ζ_3 and ζ_2 and therefore in the value of R^2 which measures the explained variance of the variable Y. When the covariance increases, the

Cov=0.4

	ESTIMATES Population	Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2713	0.0727	0.0748	0.0053	0.952	0.956
Z	0.450	0.4490	0.0782	0.0788	0.0061	0.949	1.000
W	0.570	0.5707	0.0805	0.0820	0.0065	0.960	1.000
Z ON							
X	0.630	0.6323	0.0602	0.0611	0.0036	0.954	1.000
W ON							
X	0.770	0.7712	0.0577	0.0586	0.0033	0.947	1.000
Z WITH							
W	0.400	0.3994	0.0284	0.0286	0.0008	0.958	1.000

Bias			
parameters		standard errors	
beta_41	0.004815	beta_41	0.028886
beta_42	-0.00222	beta_42	0.007673
beta_43	0.001228	beta_43	0.018634
beta_21	0.003651	beta_21	0.01495
beta_31	0.001558	beta_31	0.015598

Cov=0.13

	ESTIMATES Population	Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2713	0.0754	0.0776	0.0057	0.951	0.946
Z	0.450	0.4495	0.0496	0.0489	0.0025	0.947	1.000
W	0.570	0.5703	0.0500	0.0509	0.0025	0.963	1.000
Z ON							
X	0.630	0.6322	0.0604	0.0611	0.0037	0.957	1.000
W ON							
X	0.770	0.7702	0.0594	0.0587	0.0035	0.945	1.000
Z WITH							
W	0.130	0.1291	0.0236	0.0231	0.0006	0.941	1.000

Bias			
parameters		standard errors	
beta_41	0.004815	beta_41	0.029178
beta_42	-0.00111	beta_42	-0.01411
beta_43	0.000526	beta_43	0.018
beta_21	0.003492	beta_21	0.011589
beta_31	0.00026	beta_31	-0.01178

Cov=-0.4

	ESTIMATES Population	Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2716	0.1221	0.1252	0.0149	0.951	0.605
Z	0.450	0.4494	0.0791	0.0789	0.0062	0.947	1.000
W	0.570	0.5698	0.0808	0.0821	0.0065	0.954	1.000
Z ON							
X	0.630	0.6305	0.0630	0.0611	0.0040	0.940	1.000
W ON							
X	0.770	0.7709	0.0618	0.0587	0.0038	0.934	1.000
Z WITH							
W	-0.400	-0.4010	0.0298	0.0287	0.0009	0.944	1.000

Bias			
parameters		standard errors	
beta_41	0.005926	beta_41	0.025389
beta_42	-0.00133	beta_42	-0.00253
beta_43	-0.00035	beta_43	0.016089
beta_21	0.000794	beta_21	-0.03016
beta_31	0.001169	beta_31	-0.05016

Table 2.2: Models with 2 mediators and correlated errors

value of R^2 increases and then in datasets with covariance equal to -0.4 there is the lowest R^2.[2]

PROOF

> I start recalling the formula of R^2
>
> $$\begin{aligned} R^2 &= \frac{\text{explained variance}}{Var(Y)} = \frac{\text{explained variance}}{\text{explained variance} + Var(\zeta_4)} \\ &= \frac{\beta_{41}^2 Var(X) + \beta_{42}^2 Var(Z) + \beta_{43}^2 Var(W) + 2\beta_{41}\beta_{42}Cov(X,Z) + 2\beta_{41}\beta_{43}Cov(X,W) + 2\beta_{42}\beta_{43}Cov(Z,W)}{\text{explained variance} + Var(\zeta_4)} \end{aligned}$$
>
> When I increase the covariance between the structural errors, I increase also the covariance between Z and W, because:
>
> $$Cov(Z,W) = \beta_{21}\beta_{31}Var(X) + Cov(\zeta_2, \zeta_3)$$

[2] If I simulate 1000 datasets with sample size of 500 and with all values equal to those simulated here with cov = -0.4 and I change only the error variance of Y (from 0.58 to 0.2) in order to increase the R^2 and to hold fixed the covariance between the variables, the power of the direct effect becomes equal to 0.958 and the standard error drops to 0.0735. Then I can conclude that indeed the problem of power seems to be also linked to the explained variance as noted by Grewal et al. (2004)

Then I increase the explained variance because in my case β_{42} and β_{43} have the same sign. Because

$$\frac{\partial R^2}{\partial \text{explained variance}} = \frac{Var(\zeta_4)}{(\text{explained variance} + Var(\zeta_4))^2} > 0$$

if I increase the explained variance, I increase the R^2

2.3 Model with 2 mediators with uncorrelated errors and interaction

2.3.1 Causal effects

To introduce the non-linearity, I analyze a model with 2 mediators, uncorrelated structural errors and interaction, which consists in the product between the two mediators, $INT^{uc} = ZW$. [3] Most papers analyze the interaction between exogenous variables, i.e. between variables which don't depend linearly on other variables, with the exception of Coenders et al. (2008) and of Chen & Cheng (2014), which analyze the interaction between endogenous variables, i.e. between variables which depend linearly on other variables.[4] In both papers the interaction between endogenous variables is treated as exogenous, i.e. it is not linearly dependent on other variables, but is linked to them only through the covariance. The model is then

$$Y = \beta_{41}X + \beta_{42}Z + \beta_{43}W + \beta_{45}INT + \zeta_4$$

$$Z = \beta_{21}X + \zeta_2$$

$$W = \beta_{31}X + \zeta_3$$

$$Cov(\zeta_3, \zeta_2) = 0$$

While it is easy to calculate the effects according to the theory proposed by Pearl (2014), that proposed by Hayes and Preacher unfortunately can not be applied to this model where the relationship $INT^{uc} = ZW$ is not explicit in the model. To calculate Hayes and Preacher's indirect effect of the interaction, I use their theory proposed for the functions of one variable (for example, for $log(Z)$ or for Z^2) and not that proposed for moderation, which is mathematically equal to the interaction. The interaction, indeed, is a little more generic than moderation: the moderation distinguishes between the roles of the two variables involved in the product (for example, if Z is the predictor of the variable Y and W is the moderator, then W alters the direction or strength of the relationship between Z and Y), whereas the interaction gives the same role to the 2 variables which form the product.

Hayes and Preacher, indeed, consider only the effect of the moderation term (and not that of interaction term). The moderation gives 2 different roles to Z and W. Using my model with 2 mediators and interaction, if Z effect is moderated by W, the equation of Y can be written so :

$$Y = -\mu_{ZW} + \underbrace{\beta_{41}X + (\beta_{42} + \beta_{45}W)Z + \beta_{43}W + \zeta_4}_{Y^{uc}}$$

[3] $INT = INT^{uc} - E(INT^{uc}) = INT^{uc} - \mu_{ZW}$ then $Y = Y^{uc} - E(Y^{uc}) => Y^{uc} = Y + E(INT^{uc})$. Centering the variables does not affect the causal effects

[4] In Coenders the mediator Z causally affects the mediator W while in Chen & Cheng the 2 mediators are parallel with uncorrelated structural errors

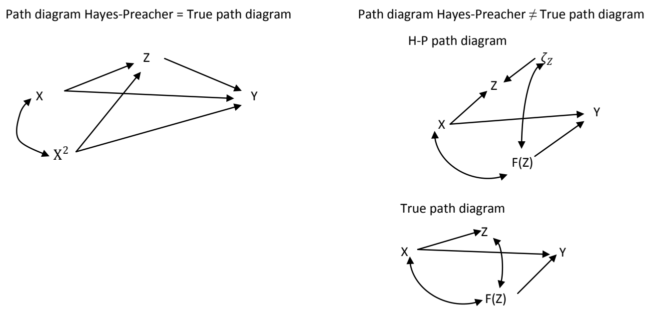

Figure 2.1: Hayes and Preacher's path diagram

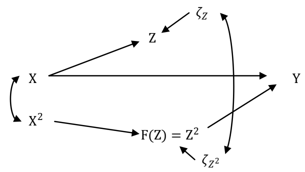

Figure 2.2: Path diagram of my solution with $F(Z) = Z^2$

In the traditional SEM, however, there is not this relation in the estimated model. An estimation method proposed by Muthén & Asparouhov (2003) inserts directly this relation in the estimated model and defines $(\beta_{42} + \beta_{45}W)$ random slope. In this situation, Hayes and Preacher calculate the conditional IE which is:

$$\text{Conditional}IE = (\beta_{42} + \beta_{45}W)\beta_{21}$$

If I want to consider INT only as an interaction, to calculate the indirect effect, I use the method for the nonlinear terms which now I explain and analyze. The nonlinear model, which Hayes and Preacher write mathematically, is not always equal to the SEM path diagram which they consider. Their SEM path diagram, also, cannot coincide with their estimated model (see Figure 2.1). Then if the nonlinear term is due to exogenous variables, all is correct, while if the nonlinear term is due to endogenous variables, it is not correct in the traditional SEM because it is not Z but its error term which correlates with the nonlinear term F(Z) and so the relation between F(Z) and X is not causal. Only the LMS method in SEM with latent variables (implemented only in the Mplus software and proposed by Moosbrugger and Klein, 2000) uses correctly the relation between Z and F(Z), where F(Z) can be only a power function and so the relation between F(Z) and X is causal. I propose a method to estimate the indirect effect using the traditional SEM when F(Z) is a power function. This is shown in Figure 2.2: I transform the exogenous variable F(Z) in an endogenous variable so:

$$F(Z) = \alpha + \beta G(X) + \epsilon => \text{if } Z = \gamma X + \zeta_Z \text{ and } F(Z) = Z^2 \text{ then } Z^2 = \alpha + \beta X^2 + \zeta_{Z^2}$$

.

In my SEM there is the same problem seen for F(Z). Then I propose transforming the exogenous variable INT in an endogenous variable:

$$INT^{uc} = \underbrace{+E(\zeta_2, \zeta_3)}_{\alpha} + \beta_{31}\beta_{21}X^2 + \underbrace{\beta_{31}X\zeta_2 + \beta_{21}X\zeta_3 + \zeta_3\zeta_2 - E(\zeta_2, \zeta_3)}_{error}$$
$$=> INT = \beta_{31}\beta_{21}\left(X^2 - E(X^2)\right) + \zeta_5 = \beta_{56}\left(X^2 - E(X^2)\right) + \zeta_5$$

which can be considered as a third mediator.

Now I can calculate the effects with both methodologies

a) Pearl's theory, which is applied to the model with exogenous interaction, requires to calculate the effects for each mediator

$$\text{effect with mediator Z:} \begin{cases} DE_{x_1,x_0}(Z) = (\beta_{41} + \beta_{43}\beta_{31})(x_1 - x_0) + \beta_{45}\beta_{31}\beta_{21}x_0(x_1 - x_0) \\ IE_{x_1,x_0}(Z) = (\beta_{42} + \beta_{45}\beta_{31}x_0)\beta_{21}(x_1 - x_0) \\ TE_{x_1,x_0}(Z) = (\beta_{41} + \beta_{43}\beta_{31} + \beta_{42}\beta_{21})(x_1 - x_0) + \\ \qquad + \beta_{45}\beta_{31}\beta_{21}(x_1^2 - x_0^2) \end{cases}$$

$$\text{effect with mediator W:} \begin{cases} DE_{x_1,x_0}(W) = (\beta_{41} + \beta_{42}\beta_{21})(x_1 - x_0) + \beta_{45}\beta_{31}\beta_{21}x_0(x_1 - x_0) \\ IE_{x_1,x_0}(W) = (\beta_{43} + \beta_{45}\beta_{21}x_0)\beta_{31}(x_1 - x_0) \\ TE_{x_1,x_0}(Z) = (\beta_{41} + \beta_{43}\beta_{31} + \beta_{42}\beta_{21})(x_1 - x_0) + \\ \qquad + \beta_{45}\beta_{31}\beta_{21}(x_1^2 - x_0^2) \end{cases}$$

b) Hayes and Preacher's theory can not consider separately the two mediators because of the nature of INT variable which is made endogenous by X^2:

$$\text{effects with mediators Z and W} \begin{cases} DE_{x_1,x_0}(Z,W) = \beta_{41} \\ IE_{x_1,x_0}(Z,W) = \beta_{42}\beta_{21} + \beta_{43}\beta_{31} + 2\beta_{45}\beta_{31}\beta_{21}x \\ TE_{x_1,x_0}(Z,W) = \beta_{41} + \beta_{43}\beta_{31} + \beta_{42}\beta_{21} + 2\beta_{45}\beta_{31}\beta_{21}x \end{cases}$$

Recalling the concept of derivative (i.e. instantaneous change), Pearl's indirect effects are equal to that proposed by Hayes & Preacher

$$lim_{x_1 \to x_0} \frac{IE_{\text{Pearl}}(Z) + IE_{\text{Pearl}}(W)}{x_1 - x_0} = IE_{\text{Hayes \& Preacher}}(Z,W)$$

The total effect proposed by Hayes & Preacher is the istantaneous version of the total effect proposed by Pearl and Hayes & Preacher's direct effect is the istantaneous version of Pearl's direct effect considering simultaneously the two mediators. The indirect effect of the two mediators can be decomposed into 2 parts, that due to the simple mediation and that due to the interaction:

$$\underbrace{\beta_{42}\beta_{21} + \beta_{43}\beta_{31}}_{\text{part due to the mediation}} \quad + \quad \underbrace{2\beta_{45}\beta_{31}\beta_{21}x}_{\text{part due to the interaction}}$$

It is important to note that the introduction of the interaction term leads the indirect effect to be a function of the variable X. The interpretative consequences will be seen in the following.

2.3.2 Estimation

Now I consider the estimation of the previous model. To make this, I simulate 1000 datasets of sample size 500 using the following theoretical model

$$Y = 0.27X + 0.45Z + 0.57W + 0.23INT + \zeta_4$$

$$Z = 0.63X + \zeta_2$$

$$W = 0.77X + \zeta_3$$

$$Cov(\zeta_2, \zeta_3) = 0$$

Exogenous interaction

Cov=0

	Population	Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2691	0.0785	0.0799	0.0062	0.952	0.923
INT	0.230	0.2276	0.0542	0.0516	0.0029	0.945	0.990
Z	0.450	0.4504	0.0479	0.0470	0.0023	0.937	1.000
W	0.570	0.5692	0.0491	0.0490	0.0024	0.937	1.000
Z ON							
X	0.630	0.6292	0.0606	0.0606	0.0037	0.963	1.000
W ON							
X	0.770	0.7702	0.0593	0.0581	0.0035	0.943	1.000
Z WITH							
W	0.000	0.0000	0.0223	0.0222	0.0005	0.949	0.051

Bias

parameters		standard errors	
beta_41	-0.003333	beta_41	0.0178344
beta_45	-0.010435	beta_45	-0.04797
beta_42	0.0008889	beta_42	-0.018789
beta_43	-0.001404	beta_43	-0.002037
beta_21	-0.00127	beta_21	0
beta_31	0.0002597	beta_31	-0.020236

Endogenous interaction

Cov=0

	Population	Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2690	0.0789	0.0799	0.0062	0.949	0.918
INT	0.230	0.2277	0.0568	0.0541	0.0032	0.942	0.977
Z	0.450	0.4504	0.0480	0.0470	0.0023	0.939	1.000
W	0.570	0.5692	0.0491	0.0490	0.0024	0.937	1.000
X2	0.000	0.0000	0.0919	0.0894	0.0084	0.933	0.067
INT ON							
X2	0.485	0.4853	0.1044	0.1027	0.0109	0.928	0.997
X	0.000	0.0003	0.0697	0.0671	0.0048	0.934	0.066
Z ON							
X	0.630	0.6291	0.0613	0.0606	0.0038	0.958	1.000
X2	0.000	-0.0004	0.0809	0.0807	0.0065	0.956	0.044
W ON							
X	0.770	0.7703	0.0597	0.0581	0.0036	0.941	1.000
X2	0.000	-0.0042	0.0789	0.0774	0.0062	0.942	0.058
Z WITH							
W	0.000	0.0000	0.0223	0.0222	0.0005	0.948	0.052

Bias

parameters		standard errors	
beta_41	-0.003704	beta_41	0.0126743
beta_45	-0.01	beta_45	-0.047535
beta_42	0.0008889	beta_42	-0.020833
beta_43	-0.001404	beta_43	-0.002037
beta_21	-0.001429	beta_46	-0.027203
beta_31	0.0003896	beta_21	-0.011419
beta_56	0.0004123	beta_26	-0.002472
		beta_31	-0.026801
		beta_36	-0.019011
		beta_56	-0.016284
		beta_51	-0.037303

Table 2.3: Model with 2 mediators with uncorrelated errors and interaction

I estimate these datasets using both the method of the endogenous interaction and that of the exogenous interaction. [5] In the endogenous interaction model, I do not constrain $\beta_{56} = \beta_{21}\beta_{31}$

In estimation, among the regressors of the mediators Z, W and among those of outcome Y I insert the variable X^2 with parameters β_{26}, β_{36} and β_{46}, which are equal to 0 in the true model. I also insert among the regressors of the variable INT X with the corresponding parameter β_{51}, which is equal to 0 in the true model. The estimated results are shown in Table 2.3. If I consider the coverage indices, the biases and the powers, the two methodologies (exogenous interaction and endogenous interaction) estimate the parameters equally well. If I compare the powers of the causal parameters which are present in both models, I realize that they are slightly lower in the endogenous interaction model than in the exogenous interaction model. I realize also that the powers of the parameters β_{26}, β_{36}, β_{46} and β_{51} are less than 0.075, i.e. their significance is due to chance.

2.4 Model with 2 mediators, correlated errors and interaction

2.4.1 Causal effects

I complicate the previous model (model with two mediators with uncorrelated errors and interaction) introducing the correlation between the structural errors of two mediators, generalizing Chen & Cheng's model in which the two structural errors have zero correlation.

[5] In the model with exogenous interaction, in addition to other variables, I center also the nonlinear variable INT. In the endogenous interaction model, in addition to what is already done for the exogenous interaction model, I center the variable X2.

$$Y = \beta_{41}X + \beta_{42}Z + \beta_{43}W + \beta_{45}INT + \zeta_4$$

$$Z = \beta_{21}X + \zeta_2$$

$$W = \beta_{31}X + \zeta_3$$

$$Cov(\zeta_3, \zeta_2) \neq 0$$

a) As seen in the case "model with two mediators and correlated errors", I must use my modified version of Pearl's theory considering jointly the two mediators. The effects become

$$\text{effects with mediators Z and W:} \begin{cases} DE_{x_1,x_0}(Z,W) = \beta_{41}(x_1 - x_0) \\ IE_{x_1,x_0}(Z,W) = (\beta_{42}\beta_{21} + \beta_{43}\beta_{31})(x_1 - x_0) + \\ \qquad + \beta_{45}\beta_{21}\beta_{31}(x_1^2 - x_0^2) \\ TE_{x_1,x_0}(Z,W) = (\beta_{41} + \beta_{43}\beta_{31} + \beta_{42}\beta_{21})(x_1 - x_0) + \\ \qquad + \beta_{45}\beta_{31}\beta_{21}(x_1^2 - x_0^2) \end{cases}$$

b) In Hayes and Preacher's theory the two mediators can not be considered separately because of the nature of INT variable which is made endogenous by X^2:

$$\text{effects with mediators Z,W and INT} \begin{cases} DE_{x_1,x_0}(Z,W) = \beta_{41} \\ IE_{x_1,x_0}(Z,W) = \beta_{42}\beta_{21} + \beta_{43}\beta_{31} + 2\beta_{45}\beta_{31}\beta_{21}x \\ TE_{x_1,x_0}(Z,W) = \beta_{41} + \beta_{43}\beta_{31} + \beta_{42}\beta_{21} + \\ \qquad + 2\beta_{45}\beta_{31}\beta_{21}x \end{cases}$$

In this case, as in 3 cases above, the 2 theories (Pearl or modified Pearl and Hayes & Preacher) lead substantially to the same causal effects

$$lim_{x_1 \to x_0} \frac{DE_{\text{Modified Pearl}}}{x_1 - x_0} = DE_{\text{Hayes \& Preacher}}$$

$$lim_{x_1 \to x_0} \frac{IE_{\text{Modified Pearl}}}{x_1 - x_0} = IE_{\text{Hayes \& Preacher}}$$

$$lim_{x_1 \to x_0} \frac{TE_{\text{Modified Pearl}}}{x_1 - x_0} = TE_{\text{Hayes \& Preacher}}$$

I can say then that the covariance does not affect the value of the indirect effects. The introduction of covariance leads to the impossibility of studying separately the effect of the two mediators in Pearl's theory and so it is not possible to determine which mediator contributes more to the indirect effect.

2.4.2 Estimation

Now I consider the estimation of the previous model. To make this, I simulate 1000 datasets of sample size 500 using the following theoretical model

$$Y = 0.27X + 0.45Z + 0.57W + 0.23INT + \zeta_4$$

$$Z = 0.63X + \zeta_2$$

$$W = 0.77X + \zeta_3$$

$$Cov(\zeta_3, \zeta_2) = a \quad \text{with} \quad a = 0.4, 0.13, -0.4$$

I hypothesize three values of covariance to see if it affects the estimate. Of course, for each group of datasets I estimate both the endogenous interaction model and the exogenous interaction model. The increase of the covariance between errors produces an increase of the covariance between Z and W. The results are presented in Tables 2.4 and 2.5. If I keep into account the coverage index and the biases, both methodologies (endogenous interaction and exogenous interaction) estimate the parameters equally well. The covariance affects mainly the power of the direct effect of X on Y (i.e., the power of the parameter β_{41} which becomes below the limit 0.8 in the case of negative covariance represented in Table 2.5). However, this low power occurs both in the exogenous and endogenous methodology and therefore it does not help to choose between the 2 methods.

2.5 Interaction: its interpretation

When I introduce the interaction, it is no longer possible to consider separately the effects of the two mediators Z and W on Y, indeed the variable INT is the combined effect of the variables Z and W. Now I analyze what changes causally by introducing the interaction in model with two mediators, following Hayes' theory for the moderation (2013). I start analyzing the model with two mediators. In Figure 2.3 (a) I show the effect of variation of Z on Y with X = 0 in a model with 2 mediators when I set the variable W equal to -st. error, 0 and st.error. Using the values of the simulated data, W is equal to 0 and ± 0.8037. In this case the 3 lines have the same slope and then the variation of Z leads to the same change of Y in the 3 cases. The distance between the parallel lines is equal to $\beta_{43}\Delta W$, i.e. to the direct effect of W on Y multiplied by the variation of W: in the case of the simulated data this distance is equal to $0.57\Delta W$. If I introduce the interaction, the effect of Z on Y is no longer independent of the values of W. This can be seen in Figure 2.3 (b). In this graph, I show the effect of the variation of Z on Y with X = 0 and W equal to -0.8037, 0 and 0.8037. In this case the slope of the lines changes in 3 cases and precisely as follows:

$$(\text{slope with} \quad W = 0) - (\text{slope with} \quad W = -0.8037) = \beta_{42} - (\beta_{42} - \beta_{45}0.8037) = \beta_{45}0.8037 = 0.1848$$

$$(\text{slope with} \quad W = 0.8037) - (\text{slope with} \quad W = 0) = (\beta_{42} + \beta_{45}0.8037) - \beta_{42} - = \beta_{45}0.8037 = 0.1848$$

The slope of the lines, indeed, measures the change in Y due to the variation of Z, i.e. $\Delta Y = (\beta_{42} + \beta_{45}W)\Delta Z$. The unitary variation of Z in the case with W = -0.8037, i.e. with a low level of W, causes a change of 0.2652, while it becomes equal to 0.45 in the case of E(W). The difference is exactly 0.1848. The same analysis can be made by comparing the average W and the high W: this difference is equal to 0.1848. The difference, then, between the low W and the high W is exactly 2 times 0.1848, i.e. 0.3696. Because the lines are not parallel, their distance changes with Z, when Z is equal to its average value, the distance between the lines is exactly equal to β_{43} 0.8037, i.e. to the parameter associated with the variable W multiplied by ΔW. This mathematically can be formulated as follows:

$$\begin{aligned}(\text{Y with} \quad W = X = Z = 0) - (\text{Y with} \quad W = -0.8037, X = Z = 0) &= -\mu_{ZW} - (-\mu_{ZW} - \beta_{43}0.8037) \\ &= \beta_{43}0.8037 = 0.4581\end{aligned}$$

$$\begin{aligned}(\text{Y with} \quad W = 0.8037, X = Z = 0) - (\text{Y with} \quad W = X = Z = 0) &= -\mu_{ZW} + \beta_{43}0.8037 - (-\mu_{ZW}) \\ &= \beta_{43}0.8037 = 0.4581\end{aligned}$$

Exogenous interaction								Bias			
Cov=0.4								parameters		standard errors	
	ESTIMATES Population	Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff	beta_41	-0.00111	beta_41	0.006757
Y ON								beta_45	0.001739	beta_45	-0.00732
X	0.270	0.2697	0.0740	0.0745	0.0055	0.945	0.943	beta_42	0.005111	beta_42	0.012903
INT	0.230	0.2304	0.0410	0.0407	0.0017	0.946	1.000	beta_43	-0.00281	beta_43	-0.00729
Z	0.450	0.4523	0.0775	0.0785	0.0060	0.953	1.000	beta_21	0.000794	beta_21	0
W	0.570	0.5684	0.0823	0.0817	0.0068	0.945	1.000	beta_31	-0.00091	beta_31	-0.0102
Z ON											
X	0.630	0.6305	0.0605	0.0605	0.0037	0.958	1.000				
W ON											
X	0.770	0.7693	0.0588	0.0582	0.0034	0.951	1.000				
Z WITH											
W	0.400	0.3991	0.0274	0.0285	0.0007	0.953	1.000				

Endogenous interaction								Bias			
Cov=0.4								parameters		standard errors	
	ESTIMATES Population	Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff	beta_41	-0.00185	beta_41	0.001344
Y ON								beta_45	0.003478	beta_45	-0.01182
X	0.270	0.2695	0.0744	0.0745	0.0055	0.943	0.949	beta_42	0.004889	beta_42	0.012903
INT	0.230	0.2308	0.0423	0.0418	0.0018	0.940	1.000	beta_43	-0.00246	beta_43	-0.00729
Z	0.450	0.4522	0.0775	0.0785	0.0060	0.954	1.000	beta_21	0.001111	beta_46	-0.03524
W	0.570	0.5686	0.0823	0.0817	0.0068	0.945	1.000	beta_31	-0.00091	beta_21	-0.00493
X2	0.000	-0.0026	0.0908	0.0876	0.0082	0.933	0.067	beta_56	-0.01031	beta_26	-0.01956
INT ON										beta_31	-0.01523
X2	0.485	0.4801	0.1403	0.1333	0.0197	0.911	0.976			beta_36	-0.01533
X	0.000	0.0006	0.0874	0.0878	0.0076	0.952	0.048			beta_56	-0.049893
Z ON										beta_51	0.0045767
X	0.630	0.6307	0.0609	0.0606	0.0037	0.955	1.000				
X2	0.000	0.0016	0.0818	0.0802	0.0067	0.947	0.053				
W ON											
X	0.770	0.7693	0.0591	0.0582	0.0035	0.947	1.000				
X2	0.000	0.0021	0.0783	0.0771	0.0061	0.938	0.062				
Z WITH											
W	0.400	0.3984	0.0273	0.0285	0.0007	0.953	1.000				

Exogenous interaction								Bias			
Cov=0.13								parameters		standard errors	
	ESTIMATES Population	Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff	beta_41	-0.00333	beta_41	0.029294
Y ON								beta_45	0.003913	beta_45	0.004098
X	0.270	0.2691	0.0751	0.0773	0.0056	0.960	0.940	beta_42	0.003111	beta_42	0.012474
INT	0.230	0.2309	0.0488	0.0490	0.0024	0.950	0.996	beta_43	0	beta_43	-0.01362
Z	0.450	0.4514	0.0481	0.0487	0.0023	0.953	1.000	beta_21	0.002381	beta_21	0.020202
W	0.570	0.5700	0.0514	0.0507	0.0026	0.941	1.000	beta_31	-0.00091	beta_31	-0.0102
Z ON											
X	0.630	0.6315	0.0594	0.0606	0.0035	0.947	1.000				
W ON											
X	0.770	0.7693	0.0588	0.0582	0.0034	0.951	1.000				
Z WITH											
W	0.130	0.1293	0.0228	0.0230	0.0005	0.948	0.999				

Endogenous interaction								Bias			
Cov=0.13								parameters		standard errors	
	ESTIMATES Population	Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff	beta_41	-0.004074	beta_41	0.0251989
Y ON								beta_45	0.0069565	beta_45	-0.001957
X	0.270	0.2689	0.0754	0.0773	0.0057	0.959	0.937	beta_42	0.0028889	beta_42	0.012474
INT	0.230	0.2316	0.0511	0.0510	0.0026	0.953	0.994	beta_43	0.0001754	beta_43	-0.013619
Z	0.450	0.4513	0.0481	0.0487	0.0023	0.953	1.000	beta_21	0.0030159	beta_46	-0.039003
W	0.570	0.5701	0.0514	0.0507	0.0026	0.942	1.000	beta_31	-0.000909	beta_21	0.0150502
X2	0.000	-0.0030	0.0923	0.0887	0.0085	0.935	0.065	beta_56	-0.002268	beta_26	-0.030157
INT ON										beta_31	-0.015228
X2	0.485	0.4840	0.1180	0.1120	0.0139	0.927	0.995			beta_36	-0.015326
X	0.000	0.0012	0.0718	0.0731	0.0051	0.951	0.049			beta_56	-0.050847
Z ON										beta_51	0.0181058
X	0.630	0.6319	0.0598	0.0607	0.0036	0.945	1.000				
X2	0.000	0.0004	0.0829	0.0804	0.0069	0.940	0.060				
W ON											
X	0.770	0.7693	0.0591	0.0582	0.0035	0.947	1.000				
X2	0.000	0.0021	0.0783	0.0771	0.0061	0.938	0.062				
Z WITH											
W	0.130	0.1291	0.0228	0.0230	0.0005	0.946	0.999				

Table 2.4: Models with 2 mediators, correlated errors and interaction

Exogenous interaction — Cov=-0.4

	ESTIMATES Population	Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2671	0.1189	0.1247	0.0141	0.960	0.575
INT	0.230	0.2311	0.0494	0.0489	0.0024	0.937	0.996
Z	0.450	0.4522	0.0775	0.0785	0.0060	0.951	1.000
W	0.570	0.5722	0.0800	0.0819	0.0064	0.954	1.000
Z ON							
X	0.630	0.6317	0.0605	0.0606	0.0037	0.942	1.000
W ON							
X	0.770	0.7693	0.0588	0.0582	0.0034	0.951	1.000
Z WITH							
W	-0.400	-0.3997	0.0276	0.0285	0.0008	0.954	1.000

Bias: parameters		Bias: standard errors	
beta_41	-0.01074	beta_41	0.04878
beta_45	0.004783	beta_45	-0.01012
beta_42	0.004889	beta_42	0.012903
beta_43	0.00386	beta_43	0.02375
beta_21	0.002698	beta_21	0.001653
beta_31	-0.00091	beta_31	-0.0102

Endogenous interaction — Cov=-0.4

	ESTIMATES Population	Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2669	0.1190	0.1247	0.0142	0.963	0.574
INT	0.230	0.2318	0.0515	0.0509	0.0027	0.941	0.996
Z	0.450	0.4522	0.0775	0.0785	0.0060	0.952	1.000
W	0.570	0.5723	0.0800	0.0819	0.0064	0.954	1.000
X2	0.000	-0.0033	0.0917	0.0888	0.0084	0.939	0.061
INT ON							
X2	0.485	0.4890	0.0832	0.0811	0.0069	0.940	0.994
X	0.000	0.0013	0.0576	0.0590	0.0033	0.951	0.049
Z ON							
X	0.630	0.6319	0.0608	0.0606	0.0037	0.944	1.000
X2	0.000	-0.0018	0.0824	0.0805	0.0068	0.939	0.061
W ON							
X	0.770	0.7693	0.0591	0.0582	0.0035	0.947	1.000
X2	0.000	0.0021	0.0783	0.0771	0.0061	0.938	0.062
Z WITH							
W	-0.400	-0.3989	0.0276	0.0284	0.0008	0.949	1.000

Bias: parameters		Bias: standard errors	
beta_41	-0.01148	beta_41	0.047899
beta_45	0.007826	beta_45	-0.01165
beta_42	0.004889	beta_42	0.012903
beta_43	0.004035	beta_43	0.02375
beta_21	0.003016	beta_46	-0.03162
beta_31	-0.00091	beta_21	-0.00329
beta_56	0.00804	beta_26	-0.02306
		beta_31	-0.01523
		beta_36	-0.01533
		beta_56	-0.02524
		beta_51	0.024306

Table 2.5: Models with 2 mediators, correlated errors and interaction

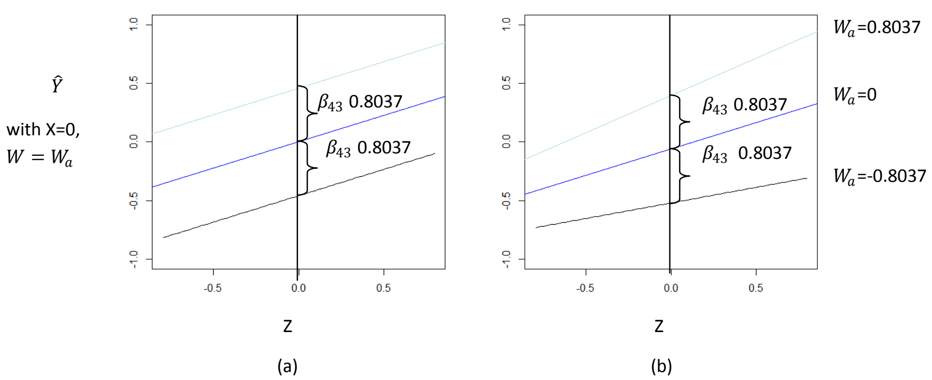

Figure 2.3: (a) model with 2 mediators (b) model with 2 mediators and interaction

When there is the interaction, there is not a single direct effect of Z on Y, but they are as many as the possible values of W. For this reason, if W is categorical, the direct effects of Z are as many as the number of categories of W, while, as in cases of this chapter, if W is continuous, then I have infinite direct effects of Z. In Pearl's theory these direct effects are defined controlled direct effect, while a direct effect of Z on Y equal for any single value of W is defined natural direct effect. Of course, in the linear case the natural direct effect coincides with the controlled direct effect. In the preceding paragraphs, for the variable X, the natural direct effect coincides with the controlled direct effect, because the variable X is not directly involved in the interaction, which affects, however, its indirect effect which becomes a function of the same X . I note, however, that the natural direct effect is simply the expected value of the natural direct effect (see Appendix A).

2.6 Conclusions

Starting from a model with 2 mediators, the fact that the errors associated to them are correlated does not affect the causal effects but only the process to calculate them and it causes the impossibility of comparing the various indirect effects obtained by the single mediators. It affects the estimate and this can be seen in the case of negative covariance where the power of the parameter of the direct effect of X on Y decreases.

The introduction of the interaction between the 2 mediators leads to a modification both of the values of causal effects and of the procedure to calculate them. A part which depends on the value of X must be added to the indirect effect calculated only with the "traditional" mediators. Of course, as shown in the Appendix, the ratio IE / TE is invariant to changes of scale of the variable x. For each value of covariance (Cov = 0, cov = 0.4, cov = 0.13 and cov = -0.4), Pearl's causal effects can be turned into those of Hayes & Preacher and then I recommend the use of the exogenous interaction model with Pearl's causal analysis.

CHAPTER 3

NONLINEAR SEM: INTERACTION VS CURVILINEAR MODEL

3.1 Curvilinear models: a special case of the interaction model

The curvilinear model, in the literature, has been studied separately from the interaction model, while it could be considered as its special case, interaction being the product of the same variable. I consider the following model

$$Y = \beta_{41}X + \beta_{42}Z + \beta_{43}W + \beta_{47}ZQ + \beta_{48}WQ + \zeta_4$$

$$Z = \beta_{21}X + \zeta_2$$

$$W = \beta_{31}X + \zeta_3$$

$$Cov(\zeta_2, \zeta_3) = 0 \quad \text{or} \neq 0$$

where ZQ and WQ are the centered quadratic effects of the variables Z and W.[1]

3.1.1 Causal effects

As the curvilinear model is a special case of the model with 2 mediators and interaction, the analysis made for the interaction model is still valid for this model. To calculate the causal effects there are two possibilities: Z2 and W2 can be considered exogenous and analyzed with Pearl's theory or endogenous and studied with Hayes and Preacher's theory. If I consider ZQ and WQ endogenous, I introduce the following equations

$$ZQ = \beta_{76}\left(X^2 - E(X^2)\right) + \zeta_7 = \beta_{21}^2\left(X^2 - E(X^2)\right) + \zeta_7$$

$$WQ = \beta_{86}\left(X^2 - E(X^2)\right) + \zeta_8 = \beta_{31}^2\left(X^2 - E(X^2)\right) + \zeta_8$$

As seen for the general case (interaction model), even in the particular case (curvilinear model)

[1] I recall that the relationships $Z^2 = ZQ^{uc}$ and $W^2 = WQ^{uc}$ are not explicit in the model and that $ZQ = ZQ^{uc} - E(ZQ^{uc})$ and $WQ = WQ^{uc} - E(WQ^{uc})$

the 2 causal theories give substantially the same results. For this reason I recommend the use of exogenous curvilinear effects with Pearl's causal theory or its modified version if the structural errors ζ_2 and ζ_3 are correlated. When the errors are correlated, the causal effects are

$$DE_{x_0,x_1}(Z,W) = \beta_{41}(x_1 - x_0)$$

$$IE_{x_0,x_1}(Z,W) = (\beta_{42}\beta_{21} + \beta_{43}\beta_{31})(x_1 - x_0) + \left(\beta_{47}\beta_{21}^2 + \beta_{48}\beta_{31}^2\right)(x_1^2 - x_0^2)$$

$$TE_{x_0,x_1}(Z,W) = DE_{x_0,x_1}(Z,W) + IE_{x_0,x_1}(Z,W)$$

The indirect effect consists of 2 parts: the first part $(\beta_{42}\beta_{21} + \beta_{43}\beta_{31})(x_1 - x_0)$ is due to the mediation and the second part $\left(\beta_{47}\beta_{21}^2 + \beta_{48}\beta_{31}^2\right)(x_1^2 - x_0^2)$ is due to the quadratic elements. If $\left(\beta_{47}\beta_{21}^2 + \beta_{48}\beta_{31}^2\right) > 0$, the indirect relationship between X and Y is convex, while if $(\beta_{47}\beta_{21}^2 + \beta_{48}\beta_{31}^2) < 0$, the indirect relationship between X and Y is concave. According to Ganzach (1997) a concave relation is negatively accelerated, while a convex relation is positively accelerated, then the indirect relationship between X and Y can be both positively and negatively accelerated.

3.1.2 Estimation

Now I consider the estimation of the previous model. To make this, I simulate 1000 datasets of sample size 500 using the following theoretical model

$$Y = 0.27X + 0.45Z + 0.57W + 0.23ZQ + 0.28WQ + \zeta_4$$

$$Z = 0.63X + \zeta_2$$

$$W = 0.77X + \zeta_3$$

$$Cov(\zeta_2, \zeta_3) = a \quad \text{with} \quad a = 0.4, 0.13, -0.4$$

to see how the datasets are estimated considering the nonlinear effect exogenous. As previously mentioned, being the curvilinear model a special case of the interaction model, I expect that the method with exogenous quadratic elements has still good performances. Table 3.1 shows the results of the estimate. Considering the coverage index and the biases, the method estimates well all 3 groups of datasets. The powers are all consistent with the presence of parameters in the model, except the power of β_{41}, i.e. of the parameter of effect of X on Y, in the group of datasets with negative covariance. This is consistent with the fact that the curvilinear model is a special case of the model with interaction. This low power of the direct effect, in fact, is also found in the model with interaction in datasets with cov = -0.4, as seen in the previous chapter.

3.2 The problem of spurious interaction

The problem of spurious interaction, analyzed by Ganzach (1997), occurs if an interaction is estimated significant even when it is not present in the true model. To eliminate this problem, Ganzach recommends to introduce the quadratic terms of the two variables which create the interaction. The obtained model is called "multiple nonlinear effects" (Kelava et al. 2008). Ganzach's idea is useful when I find a significant interaction incorrectly when only curvilinear effects are present in the true model. This is the reason why in Kelava's papers (Dimitruk et al. 2007, Kelava et al. 2008, Brandt

Exogenous curvilinear effects — Cov=0.4

ESTIMATES	Population	Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2689	0.0755	0.0744	0.0057	0.953	0.948
Z	0.450	0.4479	0.0776	0.0785	0.0060	0.954	0.999
W	0.570	0.5725	0.0797	0.0817	0.0064	0.941	1.000
ZQ	0.230	0.2292	0.0552	0.0535	0.0031	0.936	0.988
WQ	0.280	0.2788	0.0526	0.0521	0.0028	0.946	1.000
Z ON							
X	0.630	0.6334	0.0629	0.0607	0.0040	0.939	1.000
W ON							
X	0.770	0.7754	0.0615	0.0582	0.0038	0.938	1.000
Z WITH							
W	0.400	0.4008	0.0280	0.0285	0.0008	0.949	1.000

Bias			
parameters		standard errors	
beta_41	-0.00407	beta_41	-0.01457
beta_42	-0.00467	beta_42	0.011598
beta_43	0.004386	beta_43	0.025094
beta_47	-0.00348	beta_47	-0.0308
beta_48	-0.00429	beta_48	-0.00951
beta_21	0.005397	beta_21	-0.03498
beta_31	0.007013	beta_31	-0.05366

Exogenous curvilinear effects — Cov=0.13

ESTIMATES	Population	Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2709	0.0762	0.0773	0.0058	0.950	0.938
Z	0.450	0.4474	0.0507	0.0487	0.0026	0.933	1.000
W	0.570	0.5702	0.0527	0.0508	0.0028	0.933	1.000
ZQ	0.230	0.2321	0.0389	0.0381	0.0015	0.942	0.999
WQ	0.280	0.2814	0.0386	0.0379	0.0015	0.938	1.000
Z ON							
X	0.630	0.6333	0.0624	0.0608	0.0039	0.940	1.000
W ON							
X	0.770	0.7693	0.0596	0.0580	0.0036	0.937	1.000
Z WITH							
W	0.130	0.1292	0.0228	0.0229	0.0005	0.959	1.000

Bias			
parameters		standard errors	
beta_41	0.0033333	beta_41	0.0144357
beta_42	-0.005778	beta_42	-0.039448
beta_43	0.0003509	beta_43	-0.036053
beta_47	0.0091304	beta_47	-0.020566
beta_48	0.005	beta_48	-0.018135
beta_21	0.0052381	beta_21	-0.025641
beta_31	-0.000909	beta_31	-0.026846

Exogenous curvilinear effects — Cov=-0.4

ESTIMATES	Population	Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2704	0.1270	0.1247	0.0161	0.946	0.573
Z	0.450	0.4452	0.0819	0.0786	0.0067	0.944	1.000
W	0.570	0.5703	0.0822	0.0818	0.0067	0.938	1.000
ZQ	0.230	0.2303	0.0393	0.0384	0.0015	0.946	1.000
WQ	0.280	0.2789	0.0376	0.0376	0.0014	0.939	1.000
Z ON							
X	0.630	0.6308	0.0613	0.0607	0.0038	0.949	1.000
W ON							
X	0.770	0.7690	0.0594	0.0584	0.0035	0.948	1.000
Z WITH							
W	-0.400	-0.3992	0.0286	0.0284	0.0008	0.936	1.000

Bias			
parameters		standard errors	
beta_41	0.0014815	beta_41	-0.01811
beta_42	-0.010667	beta_42	-0.040293
beta_43	0.0005263	beta_43	-0.004866
beta_47	0.0013043	beta_47	-0.022901
beta_48	-0.003929	beta_48	0
beta_21	0.0012698	beta_21	-0.009788
beta_31	-0.001299	beta_31	-0.016835

Table 3.1: Models with curvilinear effects

et al. 2014), which analyze the interaction between two exogenous variables, the quadratic terms of these 2 variables are introduced.

I try to look if the introduction of the quadratic terms of two mediators in the process of estimation of my model is a good idea. The model, which I consider, is the following

$$Y = \beta_{41}X + \beta_{42}Z + \beta_{43}W + \beta_{45}INT + \zeta_4$$

$$Z = \beta_{21}X + \zeta_2$$

$$W = \beta_{31}X + \zeta_3$$

$$Cov(\zeta_2, \zeta_3) = 0 \quad \text{or} \neq 0$$

but to see the effect of the introduction of the quadratic terms, I estimate the datasets created by this model using a "multiple nonlinear effects" model as the following:

$$Y = \beta_{41}X + \beta_{42}Z + \beta_{43}W + \beta_{45}INT + \beta_{47}ZQ + \beta_{48}WQ + \zeta_4$$

$$Z = \beta_{21}X + \zeta_2$$

$$W = \beta_{31}X + \zeta_3$$

The causal analysis remains the same made earlier. If I consider Pearl causal theory, the effects are

$$DE_{x_0,x_1}(Z,W) = \beta_{41}(x_1 - x_0)$$

$$IE_{x_0,x_1}(Z,W) = (\beta_{42}\beta_{21} + \beta_{43}\beta_{31})(x_1 - x_0) + \left(\beta_{47}\beta_{21}^2 + \beta_{48}\beta_{31}^2 + \beta_{45}\beta_{21}\beta_{31}\right)(x_1^2 - x_0^2)$$

$$TE_{x_0,x_1}(Z,W) = DE_{x_0,x_1}(Z,W) + IE_{x_0,x_1}(Z,W)$$

If the methodology proposed by Ganzach is valid, when I estimate a dataset simulated from an interaction model, the parameters of the curvilinear effects are not significant and the interaction parameter is significant.

I estimate the model proposed by Ganzach with all nonlinear effects considered exogenous as I concluded at the end of the previous chapter. The INT, ZQ and WQ variables are therefore linearly independent of any other variable of the model, but they are only linked through the covariance. The estimated results [2] are represented in Table 3.2. The 4 groups of datasets have the covariances between the structural errors equal to 0.4, 0.13, 0 and -0.4, which correspond to the correlation between Z and W respectively equal to 0.839, 0.416, 0.213 and -0.413. I suppose that when the correlation between Z and W increases (precisely if the correlation coefficient has negative sign, it decreases; if it has a positive sign, it increases[3]), it is more difficult to distinguish between INT and the variables ZQ and WQ because of the problems of multicollinearity. My theory is confirmed by the analysis of the simulated datasets. The estimates of the simulated data, indeed, confirm that in the datasets with the covariance between the structural errors equal to -0.4, 0 and 0.13 the power of the parameter of the interaction is greater than 0.8 while the powers of the curvilinear parameters, which are not present in the true model, are less than 0.075. In the datasets with the covariance between the structural errors equal to 0.4, the power of the interaction is too low and only in 17.3 % of the datasets the parameter β_{45} is significant. This is due to the fact that it is difficult to disentangle the effect of INT and the effects of ZQ and WQ: β_{47} and β_{48} increase and cannibalize β_{45}.

Then the interaction can be found not significant even if it is significant, if the researcher estimates an interaction model using a model with all nonlinear effects (multiple nonlinear model) . Alternatively a researcher, who analyzes a dataset where there are curvilinear effects, can find a significant interaction even if it is not significant, if he estimates it using a model with mediators and interaction. An example of the first problem is presented in the first table of Table 3.2. An example of the second problem is shown in Table 3.3: the datasets are simulated from a model in which only ZQ and WQ affect linearly Y and these datasets are estimated with a model in which only the interaction influences linearly Y.

For this reason, I believe that Ganzach' idea is excellent when the interaction is between exogenous variables, being hard, in this case, to find a very high correlation between the variables which create the interaction, while its use is difficult with the interaction between endogenous variables,

[2]by centering INT, ZQ and WQ

[3]Then when the covariance between the structural errors increases from 0.13 to 0.4, the correlation between Z and W increases and the power of INT decreases. If I simulate 1000 datasets of sample size equal to 500 with the same parameters but with $Cov(\zeta_3, \zeta_2) = -0.2$, then the correlation between Z and W is -0.1 and the power of INT is 0.987. Then when the covariance between the structural errors decreses from -0.2 to -0.4, the correlation between Z and W decreases and the power of INT decreases passing from 0.987 to 0.913

Exogenous multiple nonlinear effects	Bias			
Cov=0.4	parameters		standard errors	
	beta_41	-0.00148	beta_41	0.005398
	beta_45	-0.01478	beta_45	0.000858
	beta_42	0.004667	beta_42	0.007692
	beta_43	-0.00228	beta_43	-0.01329
	beta_21	0.000794	beta_47	-0.00234
	beta_31	-0.00091	beta_48	-0.01807
			beta_21	0
			beta_31	-0.0102

	ESTIMATES Population	Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2696	0.0741	0.0745	0.0055	0.949	0.943
INT	0.230	0.2266	0.2331	0.2333	0.0543	0.953	0.173
Z	0.450	0.4521	0.0780	0.0786	0.0061	0.946	1.000
W	0.570	0.5687	0.0828	0.0817	0.0068	0.942	1.000
ZQ	0.000	0.0027	0.1280	0.1277	0.0164	0.952	0.048
WQ	0.000	0.0012	0.1273	0.1250	0.0162	0.943	0.057
Z ON							
X	0.630	0.6305	0.0605	0.0605	0.0037	0.958	1.000
W ON							
X	0.770	0.7693	0.0588	0.0582	0.0034	0.951	1.000
Z WITH							
W	0.400	0.3991	0.0274	0.0285	0.0007	0.953	1.000

Exogenous multiple nonlinear effects	Bias			
Cov=0.13	parameters		standard errors	
	beta_41	-0.003333	beta_41	0.0265604
	beta_45	0.0004348	beta_45	-0.015603
	beta_42	0.0028889	beta_42	0.0082645
	beta_43	0.0001754	beta_43	-0.013619
	beta_21	0.002381	beta_47	0
	beta_31	-0.000909	beta_48	-0.038627
			beta_21	0.020202
			beta_31	-0.010204

	ESTIMATES Population	Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2691	0.0753	0.0773	0.0057	0.957	0.941
INT	0.230	0.2301	0.0705	0.0694	0.0050	0.935	0.893
Z	0.450	0.4513	0.0484	0.0488	0.0023	0.946	1.000
W	0.570	0.5701	0.0514	0.0507	0.0026	0.942	1.000
ZQ	0.000	0.0010	0.0458	0.0458	0.0021	0.949	0.051
WQ	0.000	0.0001	0.0466	0.0448	0.0022	0.938	0.062
Z ON							
X	0.630	0.6315	0.0594	0.0606	0.0035	0.947	1.000
W ON							
X	0.770	0.7693	0.0588	0.0582	0.0034	0.951	1.000
Z WITH							
W	0.130	0.1293	0.0228	0.0230	0.0005	0.948	0.999

Exogenous multiple nonlinear effects	Bias			
Cov=0	parameters		standard errors	
	beta_41	-0.00407	beta_41	0.016518
	beta_45	-0.01304	beta_45	-0.03565
	beta_42	0.001111	beta_42	-0.02287
	beta_43	-0.00105	beta_43	-0.0081
	beta_21	-0.00127	beta_47	-0.01489
	beta_31	0.00026	beta_48	-0.04433
			beta_21	0
			beta_31	-0.02024

	ESTIMATES Population	Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2689	0.0787	0.0800	0.0062	0.952	0.920
INT	0.230	0.2270	0.0589	0.0568	0.0035	0.937	0.972
Z	0.450	0.4505	0.0481	0.0470	0.0023	0.937	1.000
W	0.570	0.5694	0.0494	0.0490	0.0024	0.934	1.000
ZQ	0.000	-0.0002	0.0403	0.0397	0.0016	0.943	0.057
WQ	0.000	0.0022	0.0406	0.0388	0.0017	0.930	0.070
Z ON							
X	0.630	0.6292	0.0606	0.0606	0.0037	0.963	1.000
W ON							
X	0.770	0.7702	0.0593	0.0581	0.0035	0.943	1.000
Z WITH							
W	0.000	0.0000	0.0223	0.0222	0.0005	0.949	0.051

Exogenous multiple nonlinear effects	Bias			
Cov=-0.4	parameters		standard errors	
	beta_41	-0.01111	beta_41	0.047899
	beta_45	0.004348	beta_45	0.013158
	beta_42	0.005111	beta_42	0.011583
	beta_43	0.004035	beta_43	0.021197
	beta_21	0.002698	beta_47	-0.01299
	beta_31	-0.00091	beta_48	0
			beta_21	0.001653
			beta_31	-0.0102

	ESTIMATES Population	Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2670	0.1190	0.1247	0.0142	0.960	0.574
INT	0.230	0.2310	0.0684	0.0693	0.0047	0.945	0.913
Z	0.450	0.4523	0.0777	0.0786	0.0060	0.953	1.000
W	0.570	0.5723	0.0802	0.0819	0.0064	0.951	1.000
ZQ	0.000	-0.0006	0.0462	0.0456	0.0021	0.940	0.060
WQ	0.000	0.0008	0.0450	0.0450	0.0020	0.951	0.049
Z ON							
X	0.630	0.6317	0.0605	0.0606	0.0037	0.942	1.000
W ON							
X	0.770	0.7693	0.0588	0.0582	0.0034	0.951	1.000
Z WITH							
W	-0.400	-0.3997	0.0276	0.0285	0.0008	0.954	1.000

Table 3.2: Models with 2 mediators, correlated errors and interaction estimated according to Ganzach

Spurious interaction							
Cov=0.13							
	ESTIMATES Population	Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2721	0.0811	0.0813	0.0066	0.958	0.909
INT	0.000	0.3633	0.0581	0.0579	0.1354	0.000	1.000
Z	0.450	0.4449	0.0570	0.0559	0.0033	0.936	1.000
W	0.570	0.5719	0.0625	0.0593	0.0039	0.936	1.000
Z ON							
X	0.630	0.6333	0.0624	0.0608	0.0039	0.940	1.000
W ON							
X	0.770	0.7693	0.0596	0.0580	0.0036	0.937	1.000
Z WITH							
W	0.130	0.1292	0.0228	0.0229	0.0005	0.959	1.000

Bias			
parameters		standard errors	
beta_41	0.0077778	beta_41	0.0024661
beta_42	-0.011333	beta_45	-0.003442
beta_43	0.0033333	beta_42	-0.019298
beta_21	0.0052381	beta_43	-0.0512
beta_31	-0.000909	beta_21	-0.025641
		beta_31	-0.026846

Table 3.3: Curvilinear Datasets estimated with a model with 2 mediators and interaction

	Cov=0.4	Cov=0.13	Cov=-0.4
Curvilinear model	5373.201	7402.541	6934.441
Model with interaction (True model)	5370.256	7389.904	6921.653

	Cov=0.4	Cov=0.13	Cov=-0.4
Curvilinear model (True model)	5377.698	7378.658	6911.610
Model with interaction	5380.374	7427.958	6960.707

Table 3.4: Application of the procedure on simulated data

which have a high correlation because of the exogenous variable X and of the possible covariance between the structural errors. A possible solution of this problem is presented in the next section.

3.2.1 A procedure to detect nonlinear effects

To overcome the problem of spurious interaction I propose a procedure to understand which nonlinear effect is present in the model so the researcher can estimate the model with the true effect. The method consists of the following steps:

- estimation of the model with two mediators and interaction where the variables ZQ and WQ do not affect linearly any variable but are correlated with all variables
- estimation of the model with two mediators and curvilinear effects where INT does not affect linearly any variable but is correlated with all variables
- comparison between the SABIC (with datasets with high sample size the BIC can be used) of the previous models and choice of the model with the lowest value [4]

In Table 3.4, the application of the tests on the simulated data is shown. In the first table, the datasets are those simulated in the previous chapter, i.e. created by a model in which the variable INT affects linearly Y. I note that in the three groups of datasets (cov = 0.4, cov = 0.13 and cov = -0.4) the test always chooses the correct model. In the second table, the datasets are created by a model in which only the variables ZQ and WQ affect linearly Y (curvilinear model). I see that in the 3 groups of datasets (cov = 0.4, cov = 0.13 and cov = -0.4) the test always chooses the correct model again. Analyzing the simulated data, I can say that the procedure works well and then I can use it to avoid Ganzach's solution for the spurious interaction.

[4] In this thesis I consider only curvilinear models or interaction models and for this reason I compare only these 2 SABIC indices. In general case, because in the true model the interaction and the curvilinear effects can coexist, I advice to estimate also a model in which all nonlinear effects influence the variable Y and so I compare the SABIC of this model with that obtained from the others.

Now I check if the procedure works even in extreme situations. The limit situation is represented by the correlation between Z and W equal to 1. This situation has a problem of identification, i.e. an interaction model is no longer distinguishable from a curvilinear model.

$$
\begin{aligned}
Y^{int} = & \underbrace{\beta_{41}X + \beta_{42}Z + \beta_{43}W}_{c} + \underbrace{\beta_{45}\beta_{21}\beta_{31}}_{\gamma}(X^2 - E(X^2)) + \underbrace{\beta_{45}\beta_{31}}_{\alpha}X\zeta_2 + \underbrace{\beta_{45}\beta_{21}}_{\delta}X\zeta_3 + \\
& + \underbrace{\beta_{45}(\zeta_2\zeta_3 - E(\zeta_2\zeta_3)) + \zeta_4}_{errors}
\end{aligned}
$$

$$
\begin{aligned}
Y^{curv} = & \underbrace{\beta_{41}X + \beta_{42}Z + \beta_{43}W}_{c} + \underbrace{(\beta_{47}\beta_{21}^2 + \beta_{48}\beta_{31}^2)}_{\gamma}(X^2 - E(X^2)) + \underbrace{2\beta_{47}\beta_{21}}_{\alpha}X\zeta_2 + \underbrace{2\beta_{48}\beta_{31}}_{\delta}X\zeta_3 + \\
& + \underbrace{\beta_{47}(\zeta_2^2 - E(\zeta_2^2)) + \beta_{48}(\zeta_3^2 - E(\zeta_3^2))}_{errors}
\end{aligned}
$$

If $\beta_{31} = Cov(\zeta_3, \zeta_2)\,(Var(\zeta_2))^{-1}\,\beta_{21}$ and if $Var(\zeta_3) = Cov(\zeta_3, \zeta_2)^2\,(Var(\zeta_2))^{-1}$, then the variables Z and W have correlation equal to 1 (if $Corr(Z, W) = 1$, $INT = ZQ = WQ$) and the 2 models are not identified. Indeed Y^{curv} is equal to Y^{int} and Σ^{curv} is equal to Σ^{int}, where Σ is the variance-covariances matrix of the variables. In this limit case, the test chooses the interaction model for parsimony principle (same goodness of fit and fewer parameters). When the model approaches the limit case, the test chooses correctly the curvilinear model with greater difficulty and then a model with spurious interaction is estimated.

I control how much the test is able to distinguish the two models when the model approaches the limit case. To test this, I create a situation similar to the extreme case and I simulate 1000 curvilinear datasets of sample size 500 which meet the constraint on beta $\beta_{31} = Cov(\zeta_3, \zeta_2)\,(Var(\zeta_2))^{-1}\,\beta_{21}$ $= 0.4846$, but where the constraint on the variance is not true, i.e. $Var(\zeta_3) = Cov(\zeta_3, \zeta_2)^2$ $(Var(\zeta_2))^{-1} = 0.3077 \neq 0.48$. The test, even in this extreme case, chooses correctly the curvilinear model (SABIC (curv) = 5215.529 and SABIC (int) = 5217.962). Now I lower ζ_3's variance, which becomes equal to 0.35 and closer to the value 0.3077. The test still chooses the curvilinear model (SABIC (curv) = 5215.706 and SABIC (int) = 5218.148). With the observed variables, then, the test is able to distinguish between the 2 models also in the limit cases.

3.3 Conclusions

In the first part of the chapter, I consider the curvilinear model as a particular case of the model with interaction. For this reason, both the analyses of causal effects and the studies of estimation process can be made following those for the model with interaction. Causal effects, then, are substantially the same when I use Pearl, modified Pearl or Hayes & Preacher. Of course, Pearl's theory is applied to the model with exogenous curvilinear effects while Hayes & Preacher's theory is used in the model with endogenous curvilinear effects, so I recommend the use of the method with exogenous curvilinear effects and Pearl's causal theory, being it easier to use.

When I estimate a model with only the interaction, this can be found significant also if it is not present in the true model. This problem, showed by Ganzach (1997), occurs both for interaction between exogenous variables and for interaction between endogenous variables. This author shows that if I estimate a curvilinear dataset with an interaction model, the interaction turns out to be significant even if this effect is not the true effect. This interaction is called spurious interaction. To eliminate this problem, Ganzach proposes to estimate a model with interaction and quadratic effects ("multiple nonlinear effects model"). I try to estimate the datasets of the previous chapter, i.e. those created from an interaction model, with a "multiple nonlinear effects model". In the datasets with high covariance between Z and W (those created with $Cov(\zeta_2, \zeta_3) = 0.4$) the interaction is too often nonsignificant and therefore there is the risk that the estimate supports the theory of a model with two mediators rather than a model with two mediators and interaction.

To solve the problem of spurious interaction, I propose to use a procedure which determines the true effects of the model instead of estimating a "multiple nonlinear effects model" as recommended by Ganzach and used in Kelava's papers. My proposal is to estimate two models where there are all nonlinear effects, but in the first only INT influences linearly Y while in the second only the cuvilinear effects influence linearly Y. After comparing the SABIC indices of 2 models, one chooses the model with the lower SABIC and estimates a model in which there is only the true effect. In general then I recommend to test three models (curvilinear, multiple nonlinear effects, interaction) and to choose the model with the lowest SABIC.

CHAPTER 4

NONLINEAR SEM WITH LATENT VARIABLES

Latent variables are not directly observed but are rather obtained from other observable variables. This process is called measurement part in traditional SEM. To analize the interaction between latent variables, many authors recommend to create the product between the observed variables which measure the latent variables forming the interaction. In the previous chapters I consider the models with interaction between endogenous observed variables. In this chapter I introduce the latent variables performing the same analysis made in the previous chapters for the observed variables: then what is different from the literature in this thesis is the endogeneity of variables which create the interaction.

First of all, I analyze what changes in the previous analysis if I switch from observed to latent variables. The causal analysis is not affected by this transition, whereas it affects the estimation process because of the introduction of the measurement part. An example of the measurement part is the following:

$$X_1 = X + \epsilon_1$$

$$X_2 = \lambda_{21} X + \epsilon_2$$

$$X_3 = \lambda_{31} X + \epsilon_3$$

$$Z_1 = Z + \epsilon_4$$

$$Z_2 = \lambda_{52} Z + \epsilon_5$$

$$Z_3 = \lambda_{62} Z + \epsilon_6$$

$$W_1 = W + \epsilon_7$$

$$W_2 = \lambda_{83} W + \epsilon_8$$

$$W_3 = \lambda_{93} W + \epsilon_9$$

$$Y_1^c = \underbrace{Y^{uc} - E(Y_1)}_{Y} + \epsilon_{10}$$

$$Y_2^c = \underbrace{\lambda_{114}Y^{uc} - E(Y_2)}_{\lambda_{114}Y} + \epsilon_{11}$$

$$Y_3^c = \underbrace{\lambda_{124}Y^{uc} - E(Y_3)}_{\lambda_{124}Y} + \epsilon_{12}$$

$$(Z_1W_1)^c = \underbrace{ZW - E(Z_1W_1)}_{INT} + \underbrace{Z\epsilon_7 + W\epsilon_4 + \epsilon_7\epsilon_4}_{\epsilon_{13}}$$

$$(Z_2W_2)^c = \underbrace{\lambda_{52}\lambda_{83}ZW - E(Z_2W_2)}_{\lambda_{145}INT} + \underbrace{\lambda_{52}Z\epsilon_8 + \lambda_{83}W\epsilon_5 + \epsilon_8\epsilon_5}_{\epsilon_{14}}$$

$$(Z_3W_3)^c = \underbrace{\lambda_{62}\lambda_{93}ZW - E(Z_3W_3)}_{\lambda_{155}INT} + \underbrace{\lambda_{62}Z\epsilon_9 + \lambda_{93}W\epsilon_6 + \epsilon_9\epsilon_6}_{\epsilon_{15}}$$

where the variables X_i, Z_i, W_i and Y_i are observed variables used as indicators for the latent variables X, Z, W, Y[1], and INT. The parameters λ are called factor loadings and represent the effects of the factors (or latent variables) on the observed variables. The factor loadings of the observed variables with subscript 1 (i.e. those of X_1, Z_1, W_1, Y_1^c and therefore that of $(Z_1W_1)^c$) are restricted to 1 to assign the scale to latent variables. The introduction of the measurement part in the models with interaction has been widely studied and, indeed, many different types of indicators for the latent variable INT, which represents the interaction, have been proposed. I here use the nonoverlapping indicators proposed by Marsh et al. (2004): for example, the indicator Z_1 is multiplied only once with W_1.

Another analysis which has been developed in the literature is centering of indicators (Z_iW_i), i.e. of the interaction term. There are 2 theories: "the mean centered indicators" and "the double mean centered indicators". In the mean centered indicators method only Z_i and W_i are centered , while in the double mean centered indicators method besides Z_i and W_i also their product Z_iW_i is centered, so I obtain $(Z_iW_i)^c$. I use the double mean centered indicators proposed by Lin, Wen, Marsh and Lin (2010) and by Coenders et al. (2008).[2]

4.1 Model with 2 mediators and interaction: estimation

Now I simulate 1000 datasets of sample size 500 to see if in the estimation process, in the case of latent variables, it is better to use a model with exogenous interaction or with endogenous interaction. The structural parameters, with which I simulate, remain the same as those of the model with observed variables

$$Y = 0.27X + 0.45Z + 0.57W + 0.23INT + \zeta_4$$

$$Z = 0.63X + \zeta_2$$

$$W = 0.77X + \zeta_3$$

$$Cov(\zeta_3, \zeta_2) = a \quad \text{with} \quad a = 0.4, 0.13, -0.4$$

while the parameters of the measurement part are so chosen

$$X_i = X + \epsilon_i$$

[1]the latent variable Y^{uc} is not centered while the latent variable Y is centered

[2]Chen & Cheng (2014), unlike me, use overlapping and mean centered indicators

$$Z_i = Z + \epsilon_{3+i}$$

$$W_i = W + \epsilon_{6+i}$$

$$Y_i = Y + \epsilon_{9+i}$$

with i = 1,2,3. The measurement errors variance ϵ is equal to 0.05. The measurement part is selected such that the index CR ("reliability for the construct") is greater than 0.6, as it is correct for a good model. Reliability refers to the accuracy and precision of a measurement procedure (Thorndike, Cunningham, Thorndike & Hagen, 1991) and the CR is an index proposed by Fornell & Larcker (1981) to analyze the reliability of the measurement part of SEM.

Now I estimate with these three groups of datasets an exogenous interaction model. Introducing the measurement part, some constraints are needed among the factor loadings

$$\lambda_{135} = \lambda_{42}\lambda_{73}$$

$$\lambda_{145} = \lambda_{52}\lambda_{83}$$

$$\lambda_{155} = \lambda_{62}\lambda_{93}$$

Table 4.1 shows the results of the estimation in the 3 groups of datasets (Cov = 0.4, 0.13 and -0.4). Considering the coverage index and the biases of the parameters and of the standard errors, the method with exogenous interaction estimates parameters well. The powers of the parameters are all greater than 0.8, except that of the direct effect of X on Y which is 0.435 in the datasets with negative covariance. This occurs also in the datasets created with the observed variables. Now I try to estimate the model with the endogenous interaction. Because there is the measurement part, I introduce the following equations

$$(X_1^2)^c = X_1^2 - E(X_1^2) = \underbrace{X^2 - E(X^2)}_{X2} + \underbrace{2X\epsilon_1 + \epsilon_1^2 - E(\epsilon_1^2)}_{\epsilon_{16}}$$

$$(X_2^2)^c = X_2^2 - E(X_2^2) = \underbrace{\lambda_{21}^2(X^2 - E(X^2))}_{\lambda_{176}X2} + \underbrace{2\lambda_{21}X\epsilon_2 + \epsilon_2^2 - E(\epsilon_2^2)}_{\epsilon_{17}}$$

$$(X_3^2)^c = X_3^2 - E(X_3^2) = \underbrace{\lambda_{31}^2(X^2 - E(X^2))}_{\lambda_{186}X2} + \underbrace{2\lambda_{31}X\epsilon_3 + \epsilon_3^2 - E(\epsilon_3^2)}_{\epsilon_{18}}$$

I constrain the factor loadings this way:

$$\lambda_{166} = \lambda_{11}^2$$

$$\lambda_{176} = \lambda_{21}^2$$

$$\lambda_{186} = \lambda_{31}^2$$

The measurement error of $(X_i^2)^c$ correlates with the measurement error of X_i and then their covariance is equal to the third moment of the structural error. This covariance can be constrained equal to 0 if the errors are supposed not only independent but also normal, indeed the third moment of a normal random variable with zero mean is equal to 0. In the normal datasets, the presence of these constraints has a very small impact on the estimates. The estimates of the model with the endogenous interaction are shown in Table 4.2. If I use the coverage index and the biases to evaluate the goodness of fit of the model, then the endogenous interaction model estimates the

Exogenous interaction — Cov=0.4

ESTIMATES	Population	Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2686	0.0801	0.0787	0.0064	0.946	0.920
INT	0.230	0.2293	0.0432	0.0417	0.0019	0.933	0.999
Z	0.450	0.4507	0.0881	0.0849	0.0078	0.939	1.000
W	0.570	0.5716	0.0923	0.0890	0.0085	0.938	1.000
Z ON							
X	0.630	0.6312	0.0633	0.0642	0.0040	0.952	1.000
W ON							
X	0.770	0.7715	0.0624	0.0628	0.0039	0.951	1.000
W WITH							
Z	0.400	0.3985	0.0300	0.0298	0.0009	0.943	1.000

Bias: parameters		Bias: standard errors	
beta_41	-0.005185	beta_41	-0.017478
beta_45	-0.003043	beta_45	-0.034722
beta_42	0.0015556	beta_42	-0.036322
beta_43	0.002807	beta_43	-0.035753
beta_21	0.0019048	beta_21	0.014218
beta_31	0.0019481	beta_31	0.0064103

Exogenous interaction — Cov=0.13

ESTIMATES	Population	Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2686	0.0829	0.0817	0.0069	0.951	0.901
INT	0.230	0.2289	0.0532	0.0505	0.0028	0.941	0.988
Z	0.450	0.4510	0.0529	0.0505	0.0028	0.940	1.000
W	0.570	0.5713	0.0548	0.0530	0.0030	0.940	1.000
Z ON							
X	0.630	0.6312	0.0631	0.0643	0.0040	0.956	1.000
W ON							
X	0.770	0.7708	0.0645	0.0628	0.0042	0.939	1.000
W WITH							
Z	0.130	0.1295	0.0240	0.0239	0.0006	0.943	1.000

Bias: parameters		Bias: standard errors	
beta_41	-0.005185	beta_41	-0.014475
beta_45	-0.004783	beta_45	-0.050752
beta_42	0.0022222	beta_42	-0.045369
beta_43	0.0022807	beta_43	-0.032847
beta_21	0.0019048	beta_21	0.0190174
beta_31	-0.899896	beta_31	-0.026357

Exogenous interaction — Cov=-0.4

ESTIMATES	Population	Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2668	0.1516	0.1465	0.0230	0.936	0.435
INT	0.230	0.2310	0.0533	0.0510	0.0028	0.938	0.995
Z	0.450	0.4528	0.0940	0.0908	0.0088	0.934	0.998
W	0.570	0.5725	0.0981	0.0949	0.0096	0.939	1.000
Z ON							
X	0.630	0.6312	0.0632	0.0643	0.0040	0.955	1.000
W ON							
X	0.770	0.7697	0.0644	0.0628	0.0041	0.941	1.000
W WITH							
Z	-0.400	-0.3986	0.0301	0.0292	0.0009	0.952	1.000

Bias: parameters		Bias: standard errors	
beta_41	-0.011852	beta_41	-0.039578
beta_45	0.0043478	beta_45	-0.043152
beta_42	0.0062222	beta_42	-0.034043
beta_43	0.004386	beta_43	-0.03262
beta_21	0.0019048	beta_21	0.0174051
beta_31	-0.00039	beta_31	-0.024845

Table 4.1: Model with 2 mediators with correlated errors and exogenous interaction

parameters well. The powers of the parameters , which are not present in the simulated model, i.e. $\beta_{46}, \beta_{51}, \beta_{26}$ and β_{36} are less than 0.075. The powers of the parameters which are present in the true model are greater than 0.8 except that of β_{41} which is 0.445 in the case of negative covariance. This low value occurs even when I estimate the model with the exogenous interaction. From this analysis, I can say that the method with the endogenous interaction and that with the exogenous interaction estimate equally well the datasets and then, as in the case with observed variables, I recommend the use of the exogenous method for simplicity.

4.2 Curvilinear model: a special case of the interaction model

The model with 2 mediators and interaction has as its particular case the curvilinear model, in which the interaction is the product of the same variable and, for this reason, the introduction of the measurement part does not change the causal effects. I can therefore prove that the causal effects are the same as the curvilinear model with the observed variables. The introduction of the measurement part, however, affects the estimation. The curvilinear model, being a particular case of the model with interaction, can have the quadratic effects treated both endogenously and exogenously. I prefer to study the exogenous quadratic terms because, from the analysis on the

Endogenous interaction	Bias	
Cov=0.4	parameters	standard errors

	Population	Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2688	0.0807	0.0783	0.0065	0.941	0.923
INT	0.230	0.2293	0.0447	0.0429	0.0020	0.931	0.999
Z	0.450	0.4507	0.0885	0.0847	0.0078	0.942	1.000
W	0.570	0.5716	0.0928	0.0888	0.0086	0.938	1.000
X2	0.000	0.0005	0.0987	0.0946	0.0097	0.942	0.058
INT ON							
X2	0.485	0.4946	0.1578	0.1437	0.0250	0.913	0.973
X	0.000	-0.0016	0.0937	0.0901	0.0088	0.939	0.061
Z ON							
X	0.630	0.6317	0.0643	0.0641	0.0041	0.951	1.000
X2	0.000	-0.0014	0.0891	0.0874	0.0079	0.943	0.057
W ON							
X	0.770	0.7720	0.0639	0.0629	0.0041	0.946	1.000
X2	0.000	-0.0001	0.0883	0.0847	0.0078	0.940	0.060
Z WITH							
W	0.400	0.3976	0.0300	0.0297	0.0009	0.940	1.000

Bias: parameters	
beta_41	-0.004444
beta_45	-0.003043
beta_42	0.0015556
beta_43	0.002807
beta_21	0.0026984
beta_31	0.0025974
beta_56	0.0195836

Bias: standard errors	
beta_41	-0.02974
beta_45	-0.040268
beta_42	-0.042938
beta_43	-0.043103
beta_46	-0.04154
beta_21	-0.00311
beta_26	-0.01908
beta_31	-0.015649
beta_36	-0.04077
beta_56	-0.089354
beta_51	-0.03842

Endogenous interaction	Bias	
Cov=0.13	parameters	standard errors

	Population	Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2687	0.0838	0.0814	0.0070	0.945	0.902
INT	0.230	0.2289	0.0560	0.0527	0.0031	0.940	0.982
Z	0.450	0.4510	0.0530	0.0503	0.0028	0.938	1.000
W	0.570	0.5713	0.0551	0.0528	0.0030	0.936	1.000
X2	0.000	0.0002	0.1001	0.0960	0.0100	0.937	0.063
INT ON							
X2	0.485	0.4925	0.1328	0.1206	0.0177	0.917	0.991
X	0.000	0.0006	0.0767	0.0748	0.0059	0.944	0.056
Z ON							
X	0.630	0.6317	0.0641	0.0642	0.0041	0.955	1.000
X2	0.000	-0.0014	0.0891	0.0871	0.0079	0.940	0.060
W ON							
X	0.770	0.7713	0.0659	0.0629	0.0043	0.939	1.000
X2	0.000	0.0013	0.0890	0.0841	0.0079	0.926	0.074
Z WITH							
W	0.130	0.1291	0.0240	0.0237	0.0006	0.946	1.000

Bias: parameters	
beta_41	-0.004815
beta_45	-0.004783
beta_42	0.0022222
beta_43	0.0022807
beta_21	0.0026984
beta_31	0.0016883
beta_56	0.0152546

Bias: standard errors	
beta_41	-0.02864
beta_45	-0.058929
beta_42	-0.050943
beta_43	-0.041742
beta_46	-0.040959
beta_21	0.0015601
beta_26	-0.056117
beta_31	-0.045524
beta_36	-0.055056
beta_56	-0.091867
beta_51	-0.024772

Endogenous interaction	Bias	
Cov=-0.4	parameters	standard errors

	Population	Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2668	0.1528	0.1461	0.0233	0.936	0.445
INT	0.230	0.2311	0.0566	0.0533	0.0032	0.933	0.988
Z	0.450	0.4529	0.0942	0.0905	0.0089	0.932	0.998
W	0.570	0.5726	0.0986	0.0946	0.0097	0.941	1.000
X2	0.000	-0.0002	0.1008	0.0960	0.0102	0.944	0.056
INT ON							
X2	0.485	0.4855	0.0990	0.0914	0.0098	0.930	0.989
X	0.000	0.0019	0.0635	0.0615	0.0040	0.944	0.056
Z ON							
X	0.630	0.6316	0.0642	0.0641	0.0041	0.953	1.000
X2	0.000	-0.0015	0.0891	0.0868	0.0079	0.942	0.058
W ON							
X	0.770	0.7704	0.0660	0.0628	0.0044	0.941	1.000
X2	0.000	0.0024	0.0856	0.0834	0.0073	0.938	0.062
Z WITH							
W	-0.400	-0.3977	0.0300	0.0290	0.0009	0.949	1.000

Bias: parameters	
beta_41	-0.011852
beta_45	0.0047826
beta_42	0.0064444
beta_43	0.0045614
beta_21	0.0025397
beta_31	0.0005195
beta_56	0.0008246

Bias: standard errors	
beta_41	-0.043848
beta_45	-0.058304
beta_42	-0.039278
beta_43	-0.040568
beta_46	-0.047619
beta_21	-0.001558
beta_26	-0.025814
beta_31	-0.048485
beta_36	-0.025701
beta_56	-0.076768
beta_51	-0.031496

Table 4.2: Model with 2 mediators with correlated errors and endogenous interaction

model with latent variables and interaction, I have observed that the exogeneity is preferred to the endogeneity for the simplicity of its formulation. To analyze the estimation procedure, I simulate 1000 datasets with sample size 500 using the following theoretical structural model

$$Y = 0.27X + 0.45Z + 0.57W + 0.23ZQ + 0.28WQ + \zeta_4$$

$$Z = 0.63X + \zeta_2$$

$$W = 0.77X + \zeta_3$$

$$Cov(\zeta_2, \zeta_3) = a \quad \text{with} \quad a = 0.4, 0.13, -0.4$$

and using the following measurement part

$$X_i = X + \epsilon_i$$

$$Z_i = Z + \epsilon_{3+i}$$

$$W_i = W + \epsilon_{6+i}$$

$$Y_i = Y + \epsilon_{9+i}$$

with i=1,2,3.

The Table 4.3 shows the results of the MLMV estimate [3] in the three groups of datasets (cov = 0.4, cov = 0.13 and cov = -0.4). The coverage index and the biases of the parameters and of the standard errors respect the limits and then lead me to say that the data are estimated well in 3 cases. The powers are all greater than 0.8, except that of the parameter of the direct effect of X on Y in the case of the datasets with negative covariance. This occurs in all datasets with covariance equal to -0.4:

- curvilinear model with observed variables
- model with 2 mediators and observed variables
- model with 2 mediators, interaction and observed variables
- model with 2 mediators, interaction and latent variables

4.3 Spurious interaction and latent variables

4.3.1 What's in literature ?

The problem of spurious interaction is introduced by Ganzach (1997), who advises the introduction of the quadratic terms of the variables which create the interaction. This model is called "multiple nonlinear effects model" and the German school in its papers (Dimitruk et al., 2007, Kelava et al., 2008, Moosgrugger et al. 2009) has always used it with latent variables. In Kelava et al. (2008), the authors show that the increase of covariance between the exogenous variables, which create the

[3] MLMV is a process of ML estimate proposed by Satorra and Bentler (1994) in which the chi-square and the standard errors are robust even if the variables are not normal. I choose to use this method because the nonlinear terms are definitely not normal even if derived from normal variables. In all thesis, when there is a nonlinear term, I use MLMV method.

Exogenous curvilinear effects — Cov=0.4

	Population	ESTIMATES Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2711	0.0805	0.0786	0.0065	0.942	0.928
Z	0.450	0.4548	0.0906	0.0855	0.0082	0.934	0.999
W	0.570	0.5669	0.0918	0.0895	0.0084	0.941	1.000
ZQ	0.230	0.2304	0.0595	0.0589	0.0035	0.945	0.964
WQ	0.280	0.2813	0.0610	0.0578	0.0037	0.927	0.999
Z ON							
X	0.630	0.6274	0.0656	0.0636	0.0043	0.940	1.000
W ON							
X	0.770	0.7674	0.0636	0.0625	0.0040	0.946	1.000
W WITH							
Z	0.400	0.3993	0.0305	0.0292	0.0009	0.931	1.000

Bias parameters		Bias standard errors	
beta_41	0.0040741	beta_41	-0.023602
beta_42	0.0106667	beta_42	-0.056291
beta_43	-0.005439	beta_43	-0.025054
beta_47	0.0017391	beta_47	-0.010084
beta_48	0.0046429	beta_48	-0.052459
beta_21	-0.004127	beta_21	-0.030488
beta_31	-0.003377	beta_31	-0.017296

Exogenous curvilinear effects — Cov=0.13

	Population	ESTIMATES Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2714	0.0835	0.0816	0.0070	0.940	0.908
Z	0.450	0.4520	0.0537	0.0507	0.0029	0.936	1.000
W	0.570	0.5687	0.0549	0.0532	0.0030	0.946	1.000
ZQ	0.230	0.2312	0.0411	0.0407	0.0017	0.952	1.000
WQ	0.280	0.2794	0.0411	0.0399	0.0017	0.942	1.000
Z ON							
X	0.630	0.6274	0.0656	0.0636	0.0043	0.939	1.000
W ON							
X	0.770	0.7687	0.0626	0.0626	0.0039	0.949	1.000
W WITH							
Z	0.130	0.1309	0.0244	0.0235	0.0006	0.942	1.000

Bias parameters		Bias standard errors	
beta_41	0.0051852	beta_41	-0.022754
beta_42	0.0044444	beta_42	-0.055866
beta_43	-0.002281	beta_43	-0.030965
beta_47	0.0052174	beta_47	-0.009732
beta_48	-0.002143	beta_48	-0.029197
beta_21	-0.004127	beta_21	-0.045732
beta_31	-0.001688	beta_31	0

Exogenous curvilinear effects — Cov=0.13

	Population	ESTIMATES Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2741	0.1501	0.1463	0.0225	0.941	0.487
Z	0.450	0.4490	0.0915	0.0908	0.0084	0.954	1.000
W	0.570	0.5667	0.0982	0.0949	0.0096	0.936	1.000
ZQ	0.230	0.2307	0.0416	0.0407	0.0017	0.944	1.000
WQ	0.280	0.2797	0.0426	0.0400	0.0018	0.927	1.000
Z ON							
X	0.630	0.6275	0.0655	0.0636	0.0043	0.940	1.000
W ON							
X	0.770	0.7715	0.0627	0.0622	0.0039	0.949	1.000
W WITH							
Z	-0.400	-0.3977	0.0309	0.0287	0.0010	0.924	1.000

Bias parameters		Bias standard errors	
beta_41	0.0151852	beta_41	-0.025316
beta_42	-0.002222	beta_42	-0.00765
beta_43	-0.023333	beta_43	-0.033605
beta_47	0.0030435	beta_47	-0.021635
beta_48	-0.001071	beta_48	-0.061033
beta_21	-0.003968	beta_21	-0.029008
beta_31	0.0019481	beta_31	-0.007974

Table 4.3: Curvilinear models

interaction in a dataset with only the interaction, leads to the increase of the problems of powers and to the consequent impossibility of distinguishing the interaction effect from quadratic effects in the various processes of estimation [4] of the "multiple nonlinear effects model". Using Kelava's parameters (except ζ_3's variance, which is not specified in the paper) [5], I simulate 1000 datasets of sample size 500 [6] with interaction between exogenous variables

$$Y = \beta_{31}Z + \beta_{32}W + \beta_{35}INT + \beta_{37}ZQ + \beta_{38}WQ + \zeta_3$$

[4] Jöreskong & Yang's method (1996), Ping's method (1995,1996) and Klein & Moosbrugger's method (LMS approach, 2000). The Jöreskong & Yang's approach uses the mean centered indicators and then introduces constraints on the means. They introduce also constraints on the variance of nonlinear variables and of the measurement errors of the nonlinear indicators, for example $Var(INT) = Var(Z)Var(W) + [Cov(Z, W)]^2$. Ping' approach proposes an estimation in two steps: in first step the loadings and the variances of the measurement errors are estimated, in second step these values are used as fixed and using similar constraints required for Joreskong & Yang' method, a complete nonlinear SEM is estimated.

[5] The reliabilities of Z and W are equal to 0.8; the CR of quadratic terms is 0.84. This is the CR with which I compare the measurement part of the models in the following sections

[6] I choose this sample size and the number of datasets to remain "consistent" with the other simulations, while Kelava uses 500 datasets with sample size 400

Kelava datasets								Bias			
Cov=0.8								parameters		standard errors	
	ESTIMATES Population	ESTIMATES Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff				
Y ON											
Z	0.316	0.3172	0.0533	0.0511	0.0028	0.947	1.000	beta_31	0.003797	beta_31	-0.04128
W	0.316	0.3165	0.0527	0.0512	0.0028	0.945	1.000	beta_32	0.001582	beta_32	-0.02846
INT	0.175	0.1831	0.1595	0.1524	0.0255	0.955	0.243	beta_35	0.046286	beta_35	-0.04451
ZQ	0.000	-0.0042	0.0849	0.0803	0.0072	0.946	0.054			beta_37	-0.05418
WQ	0.000	-0.0022	0.0851	0.0813	0.0072	0.950	0.050			beta_38	-0.04465
W WITH											
Z	0.800	0.7987	0.0694	0.0634	0.0048	0.917	1.000				

Kelava datasets								Bias			
Cov=0.5								parameters		standard errors	
	ESTIMATES Population	ESTIMATES Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff				
Y ON											
Z	0.316	0.3173	0.0340	0.0324	0.0012	0.945	1.000	beta_31	0.004114	beta_31	-0.04706
W	0.316	0.3165	0.0332	0.0324	0.0011	0.938	1.000	beta_32	0.001582	beta_32	-0.0241
INT	0.200	0.2033	0.0495	0.0453	0.0025	0.932	0.988	beta_35	0.0165	beta_35	-0.08485
ZQ	0.000	-0.0010	0.0299	0.0273	0.0009	0.933	0.067			beta_37	-0.08696
WQ	0.000	-0.0001	0.0291	0.0277	0.0008	0.939	0.061			beta_38	-0.04811
W WITH											
Z	0.500	0.4987	0.0588	0.0541	0.0035	0.922	1.000				

Table 4.4: Model with interaction between esogenous variables, Kelava's parameters (2008)

$$Cov(Z, W) \neq 0 \quad \text{and} \quad \beta_{38} = \beta_{37} = 0$$

and I analyze the behavior of the powers. The differences between the Jöreskog & Yang method used by Kelava and my method consists in the absence of the constraints due to the normality of the variables (i.e. for example covariances equal to 0 because the third moment is equal to 0), of the constraints on means and of constraints on the variances. The results are in Table 4.4. My method requires these constraints:

- constraints on factor loadings of Z_iW_i, Z_i and W_i
- Cov(measurement errors of $(Z_iW_i)^c$, measurement errors of $(W_i^2)^c) \neq 0$
- Cov(measurement errors of $(Z_iW_i)^c$, measurement errors of $(Z_i^2)^c) \neq 0$
- Cov(measurement errors of Z_i, measurement errors of $(Z_i^2)^c) \neq 0$
- Cov(measurement errors of W_i, measurement errors of $(W_i^2)^c) \neq 0$

and estimates the parameters well if I consider the coverage index and the biases, but there are problems if I considers the powers. In the datasets with cov = 0.5 (i.e. with correlation equal to 0.5 because Var(Z)=Var(W)=1) the powers of the parameters present in the model are all greater than 0.8, while the powers of β_{38} and β_{37} are less than 0.075. If I consider the model with cov = 0.8 (i.e. with correlation equal to 0.8 because Var(Z)=Var(W)=1), the coverage index and the biases are within their limits but the power of INT is too low and so it is difficult to distinguish between the interaction and the curvilinear effects, as noted by Kelava. So Ganzach's model is good for datasets with not very high covariance between the exogenous variables which form the interaction.[7] However in the case with exogenous variables it can be hard finding datasets with so high values. Unfortunately, these high covariances are easily found in the models with interactions between endogenous, which are analyzed in the next section.

[7]With Kelava's sample size, 400, the power of interaction with cov = 0.5 is 0.968, with cov = 0.8 is 0.188

4.3.2 Multiple nonlinear effects model with endogenous variables

Using the exogenous method, I estimate with the simulated datasets of section 4.1 the model proposed by Ganzach (Multiple nonlinear effects model). To do this, I introduce in the measurement

- the indicators of ZQ e WQ
- the constraints on loadings of ZQ and WQ
- the constraints on the covariance between the measurement errors of $(Z_i^2)^c$ and of $(Z_iW_i)^c$
- the constraints on the covariance between the measurement errors of $(W_i^2)^c$ and of $(Z_iW_i)^c$
- the constraints on the covariance between the measurement errors of $(Z_i^2)^c$ and of Z_i
- the constraints on the covariance between the measurement errors of $(W_i^2)^c$ and of W_i

These last two constraints are equal to 0 in the case of normal errors. The equations which must be added, are

$$(Z_i^2)^c = Z_i^2 - E(Z_i^2) = \underbrace{\lambda_{i+3\ 2}^2(Z^2 - E(Z^2))}_{\lambda_{i+18\ 7}ZQ} + \underbrace{2\lambda_{i+3\ 2}Z\epsilon_{i+3} + \epsilon_{i+3}^2 - E(\epsilon_{i+3}^2)}_{\epsilon_{i+18\ 7}}$$

$$(W_i^2)^c = W_i^2 - E(W_i^2) = \underbrace{\lambda_{i+6\ 3}^2(W^2 - E(W^2))}_{\lambda_{i+21\ 8}WQ} + \underbrace{2\lambda_{i+6\ 3}^2W\epsilon_{i+6} + \epsilon_{i+6}^2 - E(\epsilon_{i+6}^2)}_{\epsilon_{i+21}}$$

while the constraints, which must be added, are

- those on the covariance between the measurement errors of INT and the measurement errors of ZQ or of WQ

$$\begin{cases} Cov(\epsilon_{12+i}, \epsilon_{i+18}) = 2\lambda_{12+i}E(ZW)E(\epsilon_{i+3}^2) \\ Cov(\epsilon_{12+i}, \epsilon_{i+21}) = 2\lambda_{12+i}E(ZW)E(\epsilon_{i+6}^2) \end{cases}$$

- those on the covariance between the measurement errors of ZQ (or of WQ) and the measurement error of Z (or of W)

$$\begin{cases} Cov(\epsilon_{3+i}, \epsilon_{i+18}) \neq 0 \\ Cov(\epsilon_{6+i}, \epsilon_{i+21}) \neq 0 \end{cases}$$

with i = 1,2,3. The estimates are shown in Table 4.5. If I consider the coverage index and the biases, the "multiple nonlinear effects model" estimates well the parameters. There are some small problems when I consider the powers of the nonlinear effects. In the datasets with covariance -0.4, the power of ZQ slightly exceeds the upper limit of 0.075. A greater problem is in the case with cov = 0.4. The power of the interaction is much lower than 0.8, and then only in the 13.7 % of the cases I find β_{45} significant. As noted, therefore, by Kelava et al. (2008), in the models with interaction between exogenous variables, a high correlation between Z and W increases the difficulty of distinguishing between the interaction and the curvilinear effects. [8]Kelava et al. (2008) find that the transition from the observed variables to latent variables increases the multicollinearity.

Multicollinearity is manifested by lower powers and by higher standard errors. Naturally Kelava makes such analysis in a model with interaction between exogenous. Now in my datasets I control what happens with the introduction of measurement part in a model with interaction between

[8]Then when the covariance between the structural errors increases from 0.13 to 0.4, the correlation between Z and W increases and the power of INT decreases. If I simulate 1000 datasets of sample size equal to 500 with the same parameters but with $Cov(\zeta_3, \zeta_2) = -0.2$, then the correlation between Z and W is -0.1 and the power of INT is 0.977. Then when the covariance between the structural errors decreses from -0.2 to -0.4, the correlation between Z and W decreases and the power of INT decreases passing from 0.977 to 0.866

endogenous variables, comparing Tables 3.2 and 4.5. The power of the interaction term decreases when the measurement part is introduced, indeed, with cov = 0.4 the power passes from 0.173 to 0.137, with cov = 0.13 the power decreases from 0.893 to 0.868 and with cov = -0.4 it passes from 0.913 to 0.886. This lowering of power can lead to the problem of distinguishing between the various nonlinear effects. Then in the next section I check the performance of the procedure proposed as a solution in the case of the observed variables.

4.3.3 Procedure to distinguish the nonlinear effects

The introduction of the measurement part does not influence the procedure. The only thing, which is different, is that I introduce in the estimation process the constraints I proposed for Ganzach's model (multiple nonlinear effects model). I try my test on the simulated datasets of section 4.1 and of section 4.2, i.e. respectively datasets created with only the interaction and datasets created with only the curvilinear terms. Table 4.6 shows the results, which always chooses the true model, i.e. that with which the datasets are simulated.

As seen for the observed variables, the limit case is where the covariance between Z and W is equal to 1. This limit case happens under the same constraints found for the observed variables . These constraints are: $\beta_{31} = Cov(\zeta_3, \zeta_2)\,(Var(\zeta_2))^{-1}\,\beta_{21}$ and $Var(\zeta_3) = Cov(\zeta_3, \zeta_2)^2\,(Var(\zeta_2))^{-1}$. In this limit case it is impossible to distinguish between the interaction model and the curvilinear model.

When the models approach the extreme case, I want to see if the test chooses the true model. I simulate 1000 curvilinear datasets with sample size of 500 and cov = 0.4. In these datasets, I respect only the constraint $\beta_{31} = Cov(\zeta_3, \zeta_2)\ (Var(\zeta_2))^{-1}\ \beta_{21} = \beta_{31}^{cor1} = 0.4(0.52^{-1})0.63 = 0.4846$. The constraint on the variance is not respected, indeed $Var(\zeta_3) = 0.48 \neq Cov(\zeta_3, \zeta_2)^2\,(Var(\zeta_2))^{-1} = Var(\zeta_3)_{cor1} = 0.4^2(0.52)^{-1} = 0.3077$. The variance of measurement errors is changed to keep the CR index constant [9]. Then I try the test on these simulated data: SABIC (int) = 8919.756 and SABIC (curv) = 8920.386. The test chooses the interaction model. If I increase the variance of ζ_3 to 0.5, the test correctly chooses the curvilinear model (SABIC (int) = 9412.320 and SABIC (curv) = 9409.975). I can therefore say that unlike the datasets with observed variables, the test is less accurate in datasets with latent variables and indeed in a small neighborhood of $Var(\zeta_3)_{cor1}$ the test chooses the interaction model even if the dataset is constructed with a curvilinear model. This difference between the observed and the latent is intuitive and predictable, being the latent variables measured with less precision because of the error terms.

Now I simulate another group of 1000 curvilinear datasets where the two constraints on β_{31} and $Var(\zeta_3)$ are not respected, but the deviation from the value $\beta_{31}^{cor1} = 0.4846$ and from the value $Var(\zeta_3)_{cor1} = 0.31$ is small: $\beta_{31} = 0.4$ and $Var(\zeta_3) = 0.48$.

I still modify the measurement errors to keep the index CR constant. The test gives the following results: SABIC (int) = 8778.182, SABIC (curv) = 8778.012, then it chooses correctly the curvilinear model. I can therefore say that in a small neighborhood of β_{31}^{cov1}, if the proportion

$$P = \begin{cases} \text{if} \quad \beta_{31} = \beta_{31}^{cor1} => \frac{Var(\zeta_3) - Cov(\zeta_3,\zeta_2)^2 (Var(\zeta_2))^{-1}}{Cov(\zeta_3,\zeta_2)^2 (Var(\zeta_2))^{-1}} \\ \text{if} \quad \beta_{31} \neq \beta_{31}^{cor1} => \frac{Var(\zeta_3) - (\beta_{31})^2 (\beta_{21})^{-2} Var(\zeta_2)}{(\beta_{31})^2 (\beta_{21})^{-2} Var(\zeta_2)} \end{cases}$$

is small (for example 0.56, as in the case with $\beta_{31} = 0.4846$ and $Var(\zeta_3) = 0.48$), the SABIC method chooses the interaction model. I simulate 1000 datasets from a curvilinear model and from an interaction model, in these 2000 datasets the ratio P is the highest, i.e. 13.77. These 2 new datasets have the covariance equal to 0.13, $\beta_{31} = \beta_{31}^{cov1} = 0.1575$ and $Var(\zeta_3) = 0.48 \neq Var(\zeta_3)_{cor1} = 0.0325$ and the measurement errors are changed so that the CR index is constant. The results are presented in Table 4.7 in which I show that my test correctly chooses the true model. Keeping constant β_{31}, I simulate 1000 curvilinear datasets where $Var(\zeta_3)$ is lowered to 0.06. The proportion P becomes 0.85 and the test correctly chooses the curvilinear model (SABIC (int) = -

[9] I keep costantly equal to 0.95 the CR index of the quadratic terms, changing only the measurement errors of W_i. The reliabilities of Z and W are approximately 0.93, those of the curvilinear terms are about 0.86

Exogenous multiple nonlinear effects — Cov=0.4

	Population (ESTIMATES)	Average (ESTIMATES)	Std. Dev. (ESTIMATES)	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2695	0.0798	0.0782	0.0064	0.943	0.923
INT	0.230	0.2237	0.2969	0.2822	0.0881	0.940	0.137
Z	0.450	0.4506	0.0904	0.0847	0.0082	0.927	1.000
W	0.570	0.5718	0.0940	0.0888	0.0088	0.931	1.000
ZQ	0.000	0.0020	0.1593	0.1527	0.0254	0.940	0.060
WQ	0.000	0.0038	0.1576	0.1496	0.0248	0.940	0.060
Z ON							
X	0.630	0.6270	0.0658	0.0641	0.0043	0.943	1.000
W ON							
X	0.770	0.7674	0.0643	0.0627	0.0041	0.942	1.000
W WITH							
Z	0.400	0.3956	0.0320	0.0297	0.0010	0.919	1.000

Bias: parameters		Bias: standard errors	
beta_41	-0.00185	beta_41	-0.02005
beta_45	-0.02739	beta_45	-0.04951
beta_42	0.001333	beta_42	-0.06305
beta_43	0.003158	beta_43	-0.05532
beta_21	-0.00476	beta_47	-0.04143
beta_31	-0.00338	beta_48	-0.05076
		beta_21	-0.02584
		beta_31	-0.02488

Exogenous multiple nonlinear effects — Cov=0.13

	Population (ESTIMATES)	Average (ESTIMATES)	Std. Dev. (ESTIMATES)	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2691	0.0828	0.0812	0.0069	0.946	0.903
INT	0.230	0.2291	0.0765	0.0719	0.0058	0.925	0.868
Z	0.450	0.4513	0.0535	0.0499	0.0029	0.929	1.000
W	0.570	0.5717	0.0550	0.0525	0.0030	0.933	1.000
ZQ	0.000	-0.0003	0.0498	0.0475	0.0025	0.936	0.064
WQ	0.000	0.0008	0.0488	0.0463	0.0024	0.925	0.075
Z ON							
X	0.630	0.6314	0.0673	0.0652	0.0045	0.950	1.000
W ON							
X	0.770	0.7708	0.0668	0.0633	0.0045	0.935	1.000
W WITH							
Z	0.130	0.1295	0.0266	0.0251	0.0007	0.940	1.000

Bias: parameters		Bias: standard errors	
beta_41	-0.00333	beta_41	-0.01932
beta_45	-0.00391	beta_45	-0.06013
beta_42	0.002889	beta_42	-0.06729
beta_43	0.002982	beta_43	-0.04545
beta_21	0.002222	beta_47	-0.04618
beta_31	0.001039	beta_48	-0.05123
		beta_21	-0.0312
		beta_31	-0.0524

Exogenous multiple nonlinear effects — Cov=-0.4

	Population (ESTIMATES)	Average (ESTIMATES)	Std. Dev. (ESTIMATES)	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2683	0.1523	0.1456	0.0232	0.940	0.449
INT	0.230	0.2311	0.0765	0.0723	0.0058	0.927	0.886
Z	0.450	0.4528	0.0947	0.0901	0.0090	0.934	0.999
W	0.570	0.5725	0.0990	0.0943	0.0098	0.935	1.000
ZQ	0.000	-0.0006	0.0499	0.0472	0.0025	0.923	0.077
WQ	0.000	0.0001	0.0477	0.0465	0.0023	0.939	0.061
Z ON							
X	0.630	0.6312	0.0638	0.0633	0.0041	0.947	1.000
W ON							
X	0.770	0.7692	0.0643	0.0617	0.0041	0.940	1.000
W WITH							
Z	-0.400	-0.3974	0.0356	0.0315	0.0013	0.911	1.000

Bias: parameters		Bias: standard errors	
beta_41	-0.0063	beta_41	-0.04399
beta_45	0.004783	beta_45	-0.0549
beta_42	0.006222	beta_42	-0.04857
beta_43	0.004386	beta_43	-0.04747
beta_21	0.001905	beta_47	-0.05411
beta_31	-0.00104	beta_48	-0.02516
		beta_21	-0.00784
		beta_31	-0.04044

Table 4.5: Models with 2 mediators, correlated errors and interaction estimated according to Ganzach's method

	Cov 0.4	Cov 0.13	Cov -0.4
Model with interaction (True model)	9737.992	12419.757	12049.593
Curvilinear model	9741.020	12430.857	12060.988

	Cov 0.4	Cov 0.13	Cov -0.4
Model with interaction	9741.180	12467.523	12101.387
Curvilinear model (True model)	9738.574	12425.900	12058.609

Table 4.6: Application of my procedure

	True model: curvilinear model	True model: model with interaction
Curvilinear model	11196.861	11200.018
Model with interaction	11238.442	11188.794

Table 4.7: Models with β_{31} which respects the constraint

3011,668, SABIC (curv) = - 3029.710). I simulate another group of 1000 curvilinear datasets in which I lower the variance of ζ_3 to 0.041 so that the proportion P is 0.26. In these datasets the test chooses the interaction model (SABIC (int) = - 6969,317, SABIC (curv) = - 6855.532).

All these simulations show that only in a very small neighborhood of $(\beta_{31}^{cor1}, Var(\zeta_3)_{cor1})$ the test can not distinguish between the interaction model and the curvilinear model. When this occurs, for principle of parsimony my test chooses the interaction model.

4.4 Conclusions

The shift of the analysis from observed variables to latent ones and then the introduction of the measurement part do not affect the causal analysis but only the estimates. I estimate the simulated data both with the exogenous interaction model and with the endogenous interaction model: both methods estimates well the parameters. For this reason and because the causal analysis under the two methods provides the same results, I recommend the use of the exogenous method for simplicity. I consider, then, the curvilinear model as a special case of the model with interaction. For this reason, the introduction of the measurement part does not affect the causal analysis but only the estimates. Having observed in the model with latent variables and interaction that the exogenous method is preferable, I use it to estimate the three groups of curvilinear datasets. The method estimates the parameters well so I can prove that the datasets with interactions and/or curvilinear effects can always be estimated by the exogenous method.

In the estimation process the interaction can be found significant even if it is not present in the true model and this phenomenon is defined spurious interaction. To eliminate this problem, Ganzach (1997) suggests to introduce in the interaction model the quadratic effects of the variables which create the interaction. Kelava et al. (2008) note that when the correlation between the exogenous variables forming the interaction increases, it is increasingly difficult to distinguish between the curvilinear effects and the interaction effects. This problem is due to the multicollinearity. I control if it is true also for the endogenous variables forming the interaction. I estimate the model proposed by Ganzach with latent variables: with high correlation between the two mediators which create the interaction, it is difficult to distinguish between the interaction effect and the quadratic effects. The power of the interaction decreases when the correlation between the two mediators increases. If I compare the powers of the estimates made on the datasets with observed variables and these of the estimates made on datasets with the latent variables, I note that the power decreases in the latent case, because the transistion from the observed variables to latent variables increases the multicollinearity (Kelava et al. 2008).

To overcome the problem of the spurious interaction with high correlations, I propose a procedure used for both observed and latent variables. The test on the simulated datasets chooses correctly the true models. As for the observed variables, I analyze the behavior of the test when the dataset approaches the limit case in which the two mediators have a correlation equal to 1. In contrast to what happens in the datasets with observed variables, in the datasets with latent variables the test is less accurate in this limit case because of the increasing problem of multicollinearity, however I recommend using the test even in the case of latent variables.

CHAPTER 5

MODELS WITH MEDIATORS AND INTERACTION: MEASUREMENT PART AND SAMPLE SIZE

In previous chapters, the analysis is performed by fixing the sample size and the measurement part and by varying the covariance between structural errors ζ_3 and ζ_2. In this chapter I consider how the analysis changes if I change the measurement part and the sample size.

5.1 Measurement part and nonlinear effects

The measurement part introduced in chapter 4 consists in obtaining a variable, which can not be observed, from another variable, which can be observed. In mathematical form this can be formulated so

$$X_1 = \lambda_{11} X + \epsilon_1$$

where X_1 is the observed variable and X is the latent variable. The reliability is calculated so

$$Rel(X_1) = \frac{\lambda_{11}^2 Var(X)}{\lambda_{11}^2 Var(X) + Var(\epsilon_1)}$$

In general, the variable X is derived from many observed variables X_i

$$X_i = \lambda_{i1} X + \epsilon_i$$

with i = 1, ..., p. To know the reliability of the entire construct (CR) of the latent variable X, I use the index proposed by Fornell & Lacker (1981):

$$CR(X) = \frac{\left(\sum_{i=1}^{p} \lambda_{i1}^s\right)^2}{\left(\sum_{i=1}^{p} \lambda_{i1}^s\right)^2 + \sum_{i=1}^{p} Var(\epsilon_i)}$$

where λ_{i1}^s are the standardized loadings. This index must be bigger than 0.6 or, according to Garver & Mentzer (1999), bigger than 0.7. The standardized loadings are obtained in this way

$$\lambda_{i1}^s = \frac{\lambda_{i1}\sqrt{Var(X)}}{\sqrt{Var(X_i)}}$$

Variables	Reliability		St. loading		CR		AVE	
X	0.848485		0.921132		0.94382		0.848485	
Z	0.926593		0.962597		0.974272		0.926593	
W	0.928162		0.963412		0.97485		0.928162	
ZQ	0.858574		0.926593		0.947951		0.858574	
WQ	0.861485		0.928162		0.949131		0.861485	
INT		Cov		Cov		Cov		Cov
	0.917866	0.4	0.958053	0.4	0.971036	0.4	0.917866	0.4
	0.880704	0.13	0.938458	0.13	0.956799	0.13	0.880704	0.13
	0.880485	-0.4	0.938342	-0.4	0.956713	-0.4	0.880485	-0.4

Table 5.1: goodness of fit indices of simulated datasets of previous chapter

Standardized loadings should be bigger than 0.7 while Hair et al.(2010) affirm that the minimum λ_{i1}^s is equal to 0.5. A complementary measure of construct reliability is AVE index which must be bigger than 0.5. Recalling that the variance of the measurement error is $1-(\lambda_{i1}^s)^2$, then:

$$AVE(X) = \frac{\sum_{i=1}^{p}(\lambda_{i1}^s)^2}{\sum_{i=1}^{p}(\lambda_{i1}^s)^2 + \sum_{i=1}^{p} Var(\epsilon_i)} = \frac{\left(\sum_{i=1}^{p}(\lambda_{i1}^s)^2\right)}{p}$$

Using the formulas of reliability found by Dimitruck et al (2007) and by Moosbrugger et al. (2009) for the nonlinear terms, I find the relation between the reliability and the standardized loadings for the nonlinear effects:

$$Rel(X_i^2) = [Rel(X_i)]^2 = (\lambda_{i1}^s)^4 => \quad Rel(X2_i) = (\lambda_{i1}^s)^4 = (\lambda_{i6}^s)^2$$

$$Rel(Z_iW_i) = \frac{Rel(Z_i)Rel(W_i) + [Corr(W,Z)]^2}{1+[Corr(W,Z)]^2} = (\lambda_{i5}^s)^2$$

The reliability is always less than 1 and therefore the reliability of X_i^2 is always less than the reliability of X_i. while the reliability of the interaction term is related to the absolute value of the correlation of the two factors.

In the previous chapter, I used three indicators for each latent variable, i.e. p equal to 3. The indices above, applied to the simulated datasets of the previous sections, are shown in Table 5.1.[1] This table shows what has been demonstrated mathematically: the reliability of Z^2 is always less than that of Z, the reliability of WQ is always less than that of W and the reliability of the interaction depends on the covariance between Z and W. Because the covariance between the structural errors equal to 0.13 and that equal to -0.4 have the correlations between Z and W respectively equal to 0.416 and to -0.413, then their absolute values are very close and their reliabilities are very close. If I change β_{31} or $Var(\zeta_3)$ while keeping all other parameters constant, also the reliabilities, the standardized loadings, the AVE index and the CR index of the variable W, of its quadratic term WQ and of the interaction term change. In section 4.3.3 I modified the variance of the measurement error of W so that the CR of WQ is always close to 0.95. The choice of keeping the goodness of the measurement part constant is made because it affects the estimation. When the measurement part decreases its accuracy, the indices above decrease and it becomes difficult to distinguish Z from W if they are highly correlated as noticed in the simulations. In the following section I make some additional analyzes with other values of the indices of the measurement part.

[1] Because in the simulated datasets $\lambda_{1j} = \lambda_{2j} = \lambda_{3j}$ with j=1,2,3,4 and $Var(\epsilon_{l+1}) = Var(\epsilon_{l+2}) = Var(\epsilon_{l+3})$ with l=0,3,6,9, then

$$AVE(\text{latent variable}) = \frac{3\left(\lambda_{1j}^s\right)^2}{3} = Rel(\text{latent variable})$$

5.1.1 Model with 2 mediators and interaction

As discussed in chapter 4, the measurement part does not affect the causal analysis but only the estimates. For this reason I simulate 3 groups of 1000 datasets with sample size equal to 500 in which the structural part remains that of the previous chapters, i.e.

$$Y = 0.27X + 0.45Z + 0.57W + 0.23INT + \zeta_4$$

$$Z = 0.63X + \zeta_2$$

$$W = 0.77X + \zeta_3$$

$$Cov(\zeta_2, \zeta_3) = a \quad \text{with} \quad a = 0.4, 0.13, -0.4,$$

and the loadings of the measurement part remain unchanged

$$X_i = X + \epsilon_i \quad \text{with} \quad i = 1, 2, 3$$

$$Z_i = Z + \epsilon_{3+i} \quad \text{with} \quad i = 1, 2, 3$$

$$W_i = W + \epsilon_{6+i} \quad \text{with} \quad i = 1, 2, 3$$

$$Y_i = Y + \epsilon_{9+i} \quad \text{with} \quad i = 1, 2, 3$$

while the variances of measurement errors of Z_i and W_i change, i.e. they pass respectively from 0.05 to 0.125 and to 0.126: the choice is made to lower the reliabilities of ZQ and of WQ to 0.7. The goodness of fit indices of the measurement part are presented in Table 5.2. I compare the values of Table 5.2 with those of Table 5.1 and I observe that the increase of variances leads to a decrease of the reliabilities. The reliability of WQ reaches the value of 0.7. The CR indices of the quadratic terms pass from about 0.95 to about 0.87. The estimated standard loadings and those proposed by Moosbrugger et al. (2009) are very similar in both datasets, i.e in that with CR=0.95 and in that with CR=0.87 (see Appendix C).

As seen for the datasets with CR = 0.95, it is preferable to use the exogenous method for simplicity. Table 5.3 shows the results of the estimation of the 3 groups of datasets. If I consider the coverage index, the biases of the parameters and of the standard errors, the exogenous method estimates the parameters well. The powers of the parameters are all bigger than 0.8, except that of the direct effect of X on Y in the case with cov = -0.4. This low power is also found in all other simulated datasets with negative covariance. Finding that with a lowered CR, the exogenous method continues to estimate well the data, I compare these results with those of Table 4.1, which correspond to the datatsets of the interaction model with CR equal to 0.95. Many powers decrease slightly and the average standard errors increase and this leads more often to accept the null hypothesis. Considering, indeed, the statistic T to test the null hypothesis $\beta_{hr} = 0$ versus the alternative hypothesis $\beta_{hr} \neq 0$,

$$T = \frac{\beta_{hr}}{Var(\beta_{hr})} \quad h = 2, 3, 4 \quad r = 1, 2, 3, 5$$

an increase of the variance produces a decrease of T being the variation of β_{hr} very small. Consequently I accept more often the null hypothesis that the parameter β is equal to 0. This explains why the powers are diminished. The lowering of the CR, therefore, can lead to problems of significance of the parameters.

If I apply my procedure for comparing nonlinear models to the 3 groups of datasets to find the true effect, the test always chooses the correct model with interaction as Table 5.3 shows.

Variables	Reliability		St. loading		CR		AVE	
X	0.848485		0.921132		0.94382		0.848485	
Z	0.834685		0.913611		0.93807		0.834685	
W	0.83679		0.914762		0.938955		0.83679	
ZQ	0.696699		0.834685		0.873276		0.696699	
WQ	0.700218		0.83679		0.875113		0.700218	
INT		Cov		Cov		Cov		Cov
	0.823057	0.4	0.907225	0.4	0.933131	0.4	0.823057	0.4
	0.742999	0.13	0.861974	0.13	0.89662	0.13	0.742999	0.13
	0.742526	-0.4	0.8617	-0.4	0.896391	-0.4	0.742526	-0.4

Table 5.2: goodness of fit indices of the simulated datasets with $CR(Z^2) = CR(W^2) \simeq 0.87$

Exogenous interaction — Cov=0.4

		ESTIMATES Population	Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON								
	X	0.270	0.2683	0.0823	0.0803	0.0068	0.948	0.913
	INT	0.230	0.2294	0.0455	0.0440	0.0021	0.939	0.996
	Z	0.450	0.4496	0.0984	0.0950	0.0097	0.947	0.997
	W	0.570	0.5730	0.1032	0.0998	0.0106	0.943	1.000
Z ON								
	X	0.630	0.6312	0.0653	0.0663	0.0043	0.952	1.000
W ON								
	X	0.770	0.7717	0.0655	0.0653	0.0043	0.950	1.000
W WITH								
	X	0.400	0.3987	0.0325	0.0320	0.0011	0.942	1.000

Bias: parameters		standard errors	
beta_41	-0.0063	beta_41	-0.0243
beta_45	-0.00261	beta_45	-0.03297
beta_42	-0.00089	beta_42	-0.03455
beta_43	0.005263	beta_43	-0.03295
beta_21	0.001905	beta_21	0.015314
beta_31	0.002208	beta_31	-0.00305

Exogenous interaction — Cov=0.13

		ESTIMATES Population	Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON								
	X	0.270	0.2683	0.0849	0.0833	0.0072	0.952	0.885
	INT	0.230	0.2289	0.0565	0.0536	0.0032	0.940	0.982
	Z	0.450	0.4508	0.0553	0.0530	0.0031	0.944	1.000
	W	0.570	0.5721	0.0580	0.0559	0.0034	0.930	1.000
Z ON								
	X	0.630	0.6313	0.0650	0.0664	0.0042	0.956	1.000
W ON								
	X	0.770	0.7710	0.0675	0.0653	0.0045	0.941	1.000
W WITH								
	Z	0.130	0.1295	0.0254	0.0251	0.0006	0.945	1.000

Bias: parameters		standard errors	
beta_41	-0.0063	beta_41	-0.01885
beta_45	-0.00478	beta_45	-0.05133
beta_42	0.001778	beta_42	-0.04159
beta_43	0.003684	beta_43	-0.03621
beta_21	0.002063	beta_21	0.021538
beta_31	0.001299	beta_31	-0.03259

Exogenous interaction — Cov=-0.4

		ESTIMATES Population	Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON								
	X	0.270	0.2641	0.1651	0.1596	0.0273	0.938	0.381
	INT	0.230	0.2311	0.0565	0.0542	0.0032	0.931	0.982
	Z	0.450	0.4543	0.1037	0.1009	0.0108	0.944	0.994
	W	0.570	0.5750	0.1091	0.1058	0.0119	0.945	0.999
Z ON								
	X	0.630	0.6311	0.0652	0.0664	0.0042	0.957	1.000
W ON								
	X	0.770	0.7698	0.0672	0.0652	0.0045	0.940	1.000
W WITH								
	X	-0.400	-0.3986	0.0321	0.0313	0.0010	0.950	1.000

Bias: parameters		standard errors	
beta_41	-0.02185	beta_41	-0.03331
beta_45	0.004783	beta_45	-0.04071
beta_42	0.009556	beta_42	-0.027
beta_43	0.008772	beta_43	-0.03025
beta_21	0.001746	beta_21	0.018405
beta_31	-0.00026	beta_31	-0.02976

	Cov=0.4	Cov=0.13	Cov=-0.4
Curvilinear model	15195.037	17426.813	17104.699
Model with interaction (True model)	15192.871	17417.131	17094.717

Table 5.3: model with interaction (simulated datasets with $CR(ZQ) = CR(WQ) \simeq 0.87$)

5.1.2 Curvilinear model

As tested for the model with mediators, correlated errors and interaction, I analyze the decrease of CR in a curvilinear model. Recalling that it is a particular case of the model with interaction and that the causal analysis is not influenced by the measurement part, I analyze the estimation process. To do this, I simulate 3 groups of 1000 datastes with sample size of 500 using the following model

$$Y = 0.27X + 0.45Z + 0.57W + 0.23ZQ + 0.27WQ + \zeta_4$$

$$Z = 0.63X + \zeta_2$$

$$W = 0.77X + \zeta_3$$

$$Cov(\zeta_2, \zeta_3) = a \quad \text{with} \quad a = 0.4, 0.13, -0.4,$$

$$X_i = X + \epsilon_i \quad \text{with} \quad i = 1, 2, 3$$

$$Z_i = Z + \epsilon_{3+i} \quad \text{with} \quad i = 1, 2, 3$$

$$W_i = W + \epsilon_{6+i} \quad \text{with} \quad i = 1, 2, 3$$

$$Y_i = Y + \epsilon_{9+i} \quad \text{with} \quad i = 1, 2, 3$$

$$Var(\epsilon_{3+i}) = 0.125 \quad Var(\epsilon_{6+i}) = 0.126 \quad \text{with} \quad i = 1, 2, 3$$

Unlike the simulated curvilinear datasets of section 4.2, the variances of the measurement errors of the 2 mediators Z and W are increased to 0.125 and 0.126, i.e. the same change which occurs in datasets with interaction of the previous section 5.1.1. For this reason goodness of fit indices of Table of 5.2 are still valid.

Now I analyze the estimates with the exogenous method. The estimation results are shown in Table 5.4. Considering the coverage index and the biases, the method estimates well the parameters. The powers of the parameters are all greater than 0.8, except that of β_{41} in datasets with cov = -0.4, but this problem is encountered also in all the datasets created with negative covariance. Observing that the exogenous method continues to estimate well the data even if the CR decreases, I compare these results with those of Table 4.3, i.e. with the estimated values obtained from the three groups of datasets with CR = 0.95. I see that many powers decrease, as happens when I lower the CR in the interaction datasets.

If I apply my procedure to the three groups of datasets, this chooses the model correctly as seen in Table 5.4.

5.1.3 Model with interaction and low reliability for the nonlinear terms

As noted in the previous sections, the indices of the quadratic variables ZQ and WQ do not decrease proportionally to the decrease of their respective linear indices. To explain better, I take the example of the CR index of variable Z : in the dataset of Table 5.1 these indices are CR (Z) = 0.9743 and CR (ZQ) = 0.9479 while in the dataset of Table 5.2 the indices are CR (Z) = 0.9381 and CR (ZQ) = 0.8733. The variation of the CR index of the linear variable Z is 0.0362, while that of nonlinear variable Z2 is 0.0746. Then in situations where the indices are still good for the linear variables Z and W, they can not be good for the nonlinear variables ZQ and WQ. One question is whether the measurement part is good for the linear variables, I must also consider the measurement part of the

Exogenous curvilinear effects	Bias	
Cov=0.4	parameters	standard errors

	ESTIMATES Population	Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2714	0.0827	0.0808	0.0068	0.944	0.911
Z	0.450	0.4550	0.1040	0.0971	0.0108	0.934	0.992
W	0.570	0.5677	0.1052	0.1018	0.0111	0.943	1.000
ZQ	0.230	0.2307	0.0701	0.0691	0.0049	0.950	0.908
WQ	0.280	0.2817	0.0726	0.0679	0.0053	0.933	0.973
Z ON							
X	0.630	0.6267	0.0679	0.0657	0.0046	0.938	1.000
W ON							
X	0.770	0.7667	0.0664	0.0651	0.0044	0.946	1.000
Z WITH							
W	0.400	0.3992	0.0330	0.0314	0.0011	0.932	1.000

parameters		standard errors	
beta_41	0.005185	beta_41	-0.02297
beta_42	0.011111	beta_42	-0.06635
beta_43	-0.00404	beta_43	-0.03232
beta_47	0.003043	beta_47	-0.01427
beta_48	0.006071	beta_48	-0.06474
beta_21	-0.00524	beta_21	-0.0324
beta_31	-0.00429	beta_31	-0.01958

Exogenous curvilinear effects	Bias	
Cov=0.13	parameters	standard errors

	ESTIMATES Population	Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2716	0.0855	0.0837	0.0073	0.937	0.891
Z	0.450	0.4521	0.0575	0.0539	0.0033	0.934	1.000
W	0.570	0.5693	0.0590	0.0570	0.0035	0.945	1.000
ZQ	0.230	0.2314	0.0453	0.0445	0.0020	0.940	1.000
WQ	0.280	0.2795	0.0466	0.0442	0.0022	0.942	1.000
Z ON							
X	0.630	0.6267	0.0678	0.0658	0.0046	0.938	1.000
W ON							
X	0.770	0.7683	0.0651	0.0652	0.0042	0.950	1.000
Z WITH							
W	0.130	0.1312	0.0257	0.0247	0.0007	0.937	1.000

parameters		standard errors	
beta_41	0.005926	beta_41	-0.02105
beta_42	0.004667	beta_42	-0.06261
beta_43	-0.00123	beta_43	-0.0339
beta_47	0.006087	beta_47	-0.01766
beta_48	-0.00179	beta_48	-0.0515
beta_21	-0.00524	beta_21	-0.0295
beta_31	-0.00221	beta_31	0.001536

Exogenous curvilinear effects	Bias	
Cov=-0.4	parameters	standard errors

	ESTIMATES Population	Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2723	0.1646	0.1603	0.0271	0.941	0.421
Z	0.450	0.4505	0.1023	0.1018	0.0105	0.952	0.998
W	0.570	0.5686	0.1107	0.1067	0.0122	0.938	1.000
ZQ	0.230	0.2308	0.0459	0.0445	0.0021	0.949	1.000
WQ	0.280	0.2802	0.0471	0.0443	0.0022	0.931	1.000
Z ON							
X	0.630	0.6269	0.0678	0.0657	0.0046	0.936	1.000
W ON							
X	0.770	0.7716	0.0651	0.0648	0.0042	0.947	1.000
Z WITH							
W	-0.400	-0.3973	0.0333	0.0309	0.0011	0.922	1.000

parameters		standard errors	
beta_41	0.008519	beta_41	-0.02612
beta_42	0.001111	beta_42	-0.00489
beta_43	-0.00246	beta_43	-0.03613
beta_47	0.003478	beta_47	-0.0305
beta_48	0.000714	beta_48	-0.05945
beta_21	-0.00492	beta_21	-0.03097
beta_31	0.002078	beta_31	-0.00461

	Cov=0.4	Cov=0.13	Cov=-0.4
Curvilinear model (True model)	15201.158	17426.523	17106.614
Model with interaction	15202.976	17461.227	17142.526

Table 5.4: curvilinear model (simulated datasets with $CR(ZQ) = CR(WQ) \simeq 0.87$)

Variables	Reliability		St. loading		CR		AVE	
X	0.848485		0.921132		0.94382		0.848485	
Z	0.578419		0.760539		0.804538		0.578419	
W	0.578858		0.760827		0.80482		0.578858	
ZQ	0.334569		0.578419		0.601333		0.334569	
WQ	0.335076		0.578858		0.601878		0.335076	
INT		Cov		Cov		Cov		Cov
	0.609681	0.4	0.780821	0.4	0.82413	0.4	0.609681	0.4
	0.433079	0.13	0.658088	0.13	0.69621	0.13	0.433079	0.13
	0.432037	-0.4	0.657295	-0.4	0.695311	-0.4	0.432037	-0.4

Table 5.5: indices of goodness of the simulated datasets with $CR(ZQ) = CR(WQ) \simeq 0.6$

Exogenous curvilinear effects

CR=0.6 and Cov=0.4

	ESTIMATES Population	Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2692	0.0974	0.0972	0.0095	0.939	0.788
Z	0.450	0.4548	0.1822	0.1754	0.0332	0.954	0.781
W	0.570	0.5769	0.1880	0.1856	0.0354	0.964	0.902
Z2	0.230	0.2317	0.1687	0.1590	0.0284	0.970	0.461
W2	0.280	0.2900	0.1709	0.1572	0.0293	0.964	0.604
Z ON							
X	0.630	0.6258	0.0780	0.0749	0.0061	0.935	1.000
W ON							
X	0.770	0.7651	0.0786	0.0760	0.0062	0.942	1.000
W WITH							
Z	0.400	0.3995	0.0423	0.0404	0.0018	0.934	1.000

Bias: parameters		Bias: standard errors	
beta_41	-0.002963	beta_41	-0.002053
beta_42	0.0106667	beta_42	-0.037322
beta_43	0.0121053	beta_43	-0.012766
beta_47	0.0073913	beta_47	-0.057499
beta_48	0.0357143	beta_48	-0.080164
beta_21	-0.006667	beta_21	-0.039744
beta_31	-0.006364	beta_31	-0.033079

Exogenous interaction

CR=0.6 and Cov=0.4

	ESTIMATES Population	Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2668	0.0924	0.0888	0.0085	0.939	0.830
INT	0.230	0.2311	0.0590	0.0572	0.0035	0.941	0.977
Z	0.450	0.4466	0.1437	0.1405	0.0206	0.947	0.880
W	0.570	0.5791	0.1523	0.1487	0.0233	0.938	0.972
Z ON							
X	0.630	0.6316	0.0742	0.0753	0.0055	0.951	1.000
W ON							
X	0.770	0.7722	0.0777	0.0758	0.0060	0.940	1.000
W WITH							
Z	0.400	0.3994	0.0422	0.0410	0.0018	0.946	1.000

Bias: parameters		Bias: standard errors	
beta_41	-0.01185	beta_41	-0.03896
beta_45	0.004783	beta_45	-0.03051
beta_42	-0.00756	beta_42	-0.02227
beta_43	0.015965	beta_43	-0.02364
beta_21	0.00254	beta_21	0.014825
beta_31	0.002857	beta_31	-0.02445

Table 5.6: model with interaction and curvilinear model (simulated datasets with $CR(ZQ) = CR(WQ) \simeq 0.6$)

nonlinear variables. In literature this problem has never been analyzed concretely. To try an answer for this question, I simulate 1000 curvilinear datasets and 1000 datasets with interaction, both with sample size of 500. The structural parameters are respectively those of Table 5.3 and of Table 5.4 and I choose that the covariance between ζ_3 and ζ_2 is equal to 0.4 because it is the case with more problems of multicollinearity. The measurement part remains the same as in Tables 5.3 and 5.4, except for the variances of error terms ϵ_{3+i} and ϵ_{9+i} with i = 1,2,3 which become respectively equal to 0.46 and 0.47. The indices of the measurement part can be seen in Table 5.5. All the indexes of the nonlinear parameters are good except the AVE which is below the minimum value 0.5. The estimates are shown in Table 5.6 and I see that the parameters are estimated well, the standard errors are not too high (the maximum value is 0.1880) and the biases of the standard errors are less than 0.1. Then I affirm that the method still estimates the parameters well. Of course, the powers decrease if I compare them with those of the datasets with CR=0.95 and CR=0.87.

I propose then to increase the variances of the measurement errors to 0.62 and to 0.63 and with these values I simulate 1000 datasets with sample size of 500. The indices of the "linear" variables X, Z and W in Table 5.7 are all greater than the limits required in the literature, whereas

Variables	Reliability		St. loading		CR		AVE	
X	0.848485		0.921132		0.94382		0.848485	
Z	0.504449		0.710246		0.753322		0.504449	
W	0.506274		0.710246		0.754676		0.506274	
ZQ	0.254469		0.504449		0.505923		0.254469	
WQ	0.256314		0.506274		0.508348		0.256314	
INT		Cov		Cov		Cov		Cov
	0.563071	0.4	0.75038	0.4	0.794497	0.4	0.563071	0.4
	0.36538	0.13	0.604467	0.13	0.633328	0.13	0.36538	0.13
	0.364213	-0.4	0.603501	-0.4	0.632159	-0.4	0.364213	-0.4

Table 5.7: goodness of fit indices of the simulated datasets with $CR(ZQ) = CR(WQ) \simeq 0.5$

this does not occur for the indices of the nonlinear variables ZQ and WQ. This is an example where the indices of the linear variables X, Z and W are good while those of nonlinear variables ZQ and WQ are not good. The AVE index of the interaction INT is not good in the cases with the covariances equal to -0.4 and 0.13. For this reason I analyze the curvilinear models, being the most problematic case. The estimate of curvilinear datasets is shown in the first table of Table 5.8 (curvilinear effects without constraints). The parameters are estimated well, the standard errors are quite high (maximum value 0.4725) and many are biased (several biases exceed 2 %). To improve the estimation, I propose to add the following constraints which take into account the relationship between the reliability of the linear variable Z (or W) and that of the nonlinear variable ZQ (or WQ):

$$\frac{Va(ZQ)}{Var(Z2_i)} = \left(\frac{Var(Z)}{Var(Z_i)}\right)^2$$

which can be written also

$$\frac{Var(ZQ)}{\lambda_{12+i}^2 Var(ZQ) + Var(\epsilon_{12+i})} = \left[\frac{\beta_{21}^2 Var(X) + Var(\zeta_2)}{\lambda_{3+i}^2 \left(\beta_{21}^2 Var(X) + Var(\zeta_2)\right) + Var(\epsilon_{3+i})}\right]^2$$

In the left side of the formula, I place $Var(ZQ)$ and not its mathematical formula, which depends on the variance of Z or on the variance of X. This choice is made because in traditional SEM the variables Z and ZQ are linked only by the covariance between the structural error of Z and that of Z2 and because in the exogenous method the variables X and ZQ are linked only by the covariance and not by a linear function. Then Z2 is exogenous and consequently also its variance is not a function of the variance of variable X. The estimate with this constraint is shown in the second part of Table 5.8 (curvilinear effects with constraints). The estimates of the parameters are as good as those of the standard errors, which decrease (the maximum value is 0.3264). The powers improve although they do not reach the value of 0.8. When I apply this constraint to the datasets with the optimal measurement part (for example to the curvilinear datasets with CR = 0.95), it does not lead to a significant improvement and for this reason I recommend the use of this constraint only in the problematic cases.

5.2 Sample size

Using models with latent variables, there are many criteria to choose the right sample size for empirical datasets (Westland, 2010). In the previous chapters, I used a sample size of 500, which respects the various requirements for the 3 models considered (exogenous, endogenous, exogenous multiple nonlinear effects). The first criterion is called the "rule of thumb", which requires 10 observations for each indicator, then:

- Model with exogenous interaction: 5x3x10=150
- Model with endogenous interaction:6x3x10=180

Curvilinear effects without constraints

Cov=0.4

	ESTIMATES Population	Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2655	0.1236	0.1468	0.0153	0.949	0.679
Z	0.450	0.4669	0.3517	0.4381	0.1239	0.975	0.614
W	0.570	0.5727	0.3469	0.4348	0.1202	0.977	0.710
ZQ	0.230	0.2306	0.3828	0.4725	0.1464	0.988	0.302
WQ	0.280	0.3024	0.3681	0.4430	0.1359	0.981	0.390
Z ON							
X	0.630	0.6260	0.0826	0.0790	0.0068	0.936	1.000
W ON							
X	0.770	0.7651	0.0837	0.0806	0.0070	0.938	1.000
W WITH							
Z	0.400	0.3998	0.0461	0.0443	0.0021	0.933	1.000

Bias			
parameters		standard errors	
beta_41	-0.01667	beta_41	0.187702
beta_42	0.037556	beta_42	0.245664
beta_43	0.004737	beta_43	0.253387
beta_47	0.002609	beta_47	0.234326
beta_48	0.08	beta_48	0.203477
beta_21	-0.00635	beta_21	-0.04358
beta_31	-0.00636	beta_31	-0.03704

Curvilinear effects with constraints

Cov=0.4

	ESTIMATES Population	Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2672	0.1246	0.1337	0.0155	0.947	0.686
Z	0.450	0.4668	0.3135	0.3125	0.0985	0.968	0.618
W	0.570	0.5717	0.3145	0.3231	0.0988	0.968	0.730
ZQ	0.230	0.2336	0.3264	0.3130	0.1065	0.982	0.330
WQ	0.280	0.2905	0.3022	0.2970	0.0913	0.976	0.431
Z ON							
X	0.630	0.6282	0.0861	0.0813	0.0074	0.944	1.000
W ON							
X	0.770	0.7677	0.0876	0.0830	0.0077	0.931	1.000
Z WITH							
W	0.400	0.4020	0.0491	0.0464	0.0024	0.932	1.000

Bias			
parameters		standard errors	
beta_41	-0.01037	beta_41	0.0730337
beta_42	0.0373333	beta_42	-0.00319
beta_43	0.0029825	beta_43	0.027345
beta_47	0.0156522	beta_47	-0.041054
beta_48	0.0375	beta_48	-0.017207
beta_21	-0.002857	beta_21	-0.055749
beta_31	-0.002987	beta_31	-0.052511

Table 5.8: curvilinear model (simulated datasets with $CR(ZQ) = CR(WQ) \simeq 0.5$)

- Model with exogenous multiple nonlinear effects: 7x3x10=210

According to this rule, I must choose a sample size of at least 210. Another criterion considers the ratio between the sample size and the number of "free parameters", which must be 5: 1. This criterion is proposed by Bentler (1989) and requires the following sample sizes:

- Model with exogenous interaction: 44x5=220
- Model with endogenous interaction:59x5=295 [2]
- Model with exogenous multiple nonlinear effects:71x5=355

The third criterion requires that the sample size is a function of the ratio of indicators to latent variables (m=g/k where g=total number of the indicators and k=total number of the latent variables)

$$n \geq 50m^2 - 450m + 110$$

and so in this case being m = 3, n is 200. Muthén & Muthén (2002) affirm that from 5 to 10 observations for parameter are required

- Model with exogenous interaction: 44x10=440
- Model with endogenous interaction:59x10=590
- Model with exogenous multiple nonlinear effects:71x10=710

[2] This is the number of free parameters in the case without constraints on the covariance between the measurement error of X_i^2 and that of X_i. If I consider these constraints, the number of free parameters becomes 56.

Following, however, the statements made by Schermelleh-Engel, Moosbrugger & Müller (2003), for a robust ML estimate a sample size of 400 is required.

There is not a rule to apply in all situations, because the sample size depends on many factors such as the reliability and the strength of the relationships among the variables.

Given these rules, I propose to reduce the sample size of datasets to 300 although the model proposed by Ganzach (1997), i.e. the multiple nonlinear effects model, should not be estimated when I consider the Bentler's rule which requires at least a sample size of 355. I simulate 1000 datasets from an interaction model with sample size equal to 300 and with the same parameters of the datasets used in chapter 3. I estimate both the model in which the interaction is considered exogenous and that in which the interaction is considered endogenous. Table 5.9 shows the results of the estimation on these datasets. In both datasets (cov = 0.4 and cov = 0.13), both methods (exogenous method and endogenous method) cause a bias of the standard error of the interaction parameter. If I consider the endogenous method, there is also the bias of the standard error of the parameter β_{56}, which I introduce to consider the interaction endogenous. These results are coherent with Muthén's statement: although the bias of the parameters are small, the standard errors are more sensitive to sample size. In conclusion I can say that with the sample size of 300, both methods do not estimate well the standard errors.

I increase the sample size to 350, a value very close to that for which, according to Bentler, it is also possible to estimate the model proposed by Ganzach, i.e. the multiple nonlinear effects model. With the same parameters of the datasets of Table 5.9, I simulate 4 groups of 1000 datasets of sample size equal to 350, which only differ in the value of the covariance between the structural errors ζ_2 and ζ_3 (cov=0.4, 0.13, 0, -0.4). This way I may control if the covariance influences the choice of the minimum sample size. The estimate of these datasets is shown in Tables 5.10 and 5.11. The exogenous method estimates well the parameters, while the endogenous method has problems with the biases of the standard errors of β_{56}. A special feature, also noted in datasets with Kelava's parameters with sample size of 400, is that sometimes by lowering the covariance between Z and W (which in my case is due to lowering the covariance between structural errors) [3], the absolute value of the bias of the standard error of the interaction exceeds the limit 0.1. I note that when the covariance between Z and W decreases, the increase of the absolute value of the bias happens sometimes even in datasets with latent variables and sample size 500, but it does not lead to problems of biased standard errors. In the four groups of datasets (cov = 0.4, cov = 0.13, cov = 0 and cov = -0.4) the exogenous method estimates well the parameters, while the endogenous method does not estimate well because the free parameters increase from 44 to 59 [4] and this increase requires a greater sample size. For this reason, in addition to the motivations presented in the previous chapters, the exogenous method is preferable to the endogenous one. Now I consider a sample size equal to 400. To do this, I simulate 1000 datasets with interaction and with the same parameters of the datasets of Tables 5.10 and 5.11. The estimates are shown in Tables 5.12 and 5.13. The exogenous method estimates well the data and the bias of the standard errors of the interaction is no longer so close to the limit 0.1 as in some datasets with sample size equal to 350. Unfortunately, the endogenous method still does not estimate well the standard error of the direct effect of X^2 on interaction which is biased except that of the datasets with cov=0.4.

5.3 Relation between sample size and reliability

As seen in the previous sections, the sample size and the quality of the measurement part affect the estimates and the ability to identify the true model. In this section, I try to show the relationship between them. I consider the curvilinear model with the covariance between the structural errors equal to 0.4. In Table 4.3 I found the estimate of the datasets with CR=0.95 and sample size equal to 500 while in Table 5.8 I find the estimate of the datasets with CR=0.5 and sample size equal

[3] In the datasets where there is null covariance between the structural errors, Z and W are still related, but only through the variable X.

[4] If I constrain the covariance between the measurement error of X_i^2 and that of X_i, the free parameters become 56, but the problem remains. For example, in the datasets with covariance equal to 0.4, the bias of the st. error of β_{56} is -0.14226

Exogenous interaction — Cov=0.4

	ESTIMATES Population	Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2692	0.1035	0.1005	0.0107	0.942	0.750
INT	0.230	0.2295	0.0596	0.0531	0.0036	0.922	0.976
Z	0.450	0.4467	0.1137	0.1081	0.0129	0.934	0.978
W	0.570	0.5754	0.1189	0.1135	0.0141	0.928	1.000
Z ON							
X	0.630	0.6322	0.0850	0.0821	0.0072	0.934	1.000
W ON							
X	0.770	0.7734	0.0859	0.0803	0.0074	0.933	1.000
W WITH							
Z	0.400	0.3985	0.0392	0.0378	0.0015	0.941	1.000

Bias: parameters		Bias: standard errors	
beta_41	-0.002963	beta_41	-0.028986
beta_45	-0.002174	beta_45	-0.10906
beta_42	-0.007333	beta_42	-0.049252
beta_43	0.0094737	beta_43	-0.045416
beta_21	0.0034921	beta_21	-0.034118
beta_31	0.0044156	beta_31	-0.065192

Endogenous interaction — Cov=0.4

	ESTIMATES Population	Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2695	0.1049	0.0994	0.0110	0.940	0.756
INT	0.230	0.2292	0.0620	0.0545	0.0038	0.923	0.971
Z	0.450	0.4462	0.1139	0.1075	0.0130	0.936	0.977
W	0.570	0.5755	0.1194	0.1128	0.0143	0.930	1.000
X2	0.000	0.0013	0.1326	0.1211	0.0176	0.922	0.078
INT ON							
X2	0.485	0.4963	0.2023	0.1762	0.0410	0.889	0.832
X	0.000	-0.0037	0.1218	0.1126	0.0148	0.932	0.068
Z ON							
X	0.630	0.6336	0.0873	0.0814	0.0076	0.927	1.000
X2	0.000	-0.0047	0.1195	0.1127	0.0143	0.940	0.060
W ON							
X	0.770	0.7749	0.0882	0.0797	0.0078	0.922	1.000
X2	0.000	-0.0002	0.1143	0.1089	0.0131	0.937	0.063
W WITH							
Z	0.400	0.3970	0.0392	0.0375	0.0015	0.936	1.000

Bias: parameters		Bias: standard errors	
beta_41	-0.001852	beta_41	-0.052431
beta_45	-0.003478	beta_45	-0.120968
beta_42	-0.008444	beta_42	-0.05619
beta_43	0.0096491	beta_43	-0.055276
beta_21	0.0057143	beta_46	-0.086727
beta_31	0.0063636	beta_21	-0.067583
beta_56	0.023088	beta_26	-0.056904
		beta_31	-0.096372
		beta_36	-0.047244
		beta_56	-0.129016
		beta_51	-0.075534

Exogenous interaction — Cov=0.13

	ESTIMATES Population	Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2687	0.1073	0.1043	0.0115	0.942	0.728
INT	0.230	0.2292	0.0727	0.0644	0.0053	0.914	0.922
Z	0.450	0.4494	0.0673	0.0641	0.0045	0.945	1.000
W	0.570	0.5736	0.0711	0.0674	0.0051	0.930	1.000
Z ON							
X	0.630	0.6323	0.0850	0.0822	0.0072	0.935	1.000
W ON							
X	0.770	0.7725	0.0845	0.0802	0.0071	0.930	1.000
W WITH							
Z	0.130	0.1298	0.0307	0.0304	0.0009	0.932	0.998

Bias: parameters		Bias: standard errors	
beta_41	-0.00481	beta_41	-0.02796
beta_45	-0.00348	beta_45	-0.11417
beta_42	-0.00133	beta_42	-0.04755
beta_43	0.006316	beta_43	-0.05204
beta_21	0.003651	beta_21	-0.03294
beta_31	0.003247	beta_31	-0.05089

Endogenous interaction — Cov=0.13

	ESTIMATES Population	Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2690	0.1089	0.1033	0.0118	0.936	0.735
INT	0.230	0.2292	0.0764	0.0670	0.0058	0.916	0.896
Z	0.450	0.4491	0.0674	0.0637	0.0045	0.940	1.000
W	0.570	0.5735	0.0713	0.0670	0.0051	0.929	1.000
X2	0.000	0.0002	0.1334	0.1227	0.0178	0.917	0.083
INT ON							
X2	0.485	0.4944	0.1743	0.1477	0.0304	0.890	0.933
X	0.000	-0.0003	0.0997	0.0934	0.0099	0.945	0.055
Z ON							
X	0.630	0.6337	0.0873	0.0814	0.0076	0.928	1.000
X2	0.000	-0.0046	0.1195	0.1122	0.0143	0.938	0.062
W ON							
X	0.770	0.7739	0.0858	0.0797	0.0074	0.925	1.000
X2	0.000	0.0043	0.1127	0.1077	0.0127	0.935	0.065
W WITH							
Z	0.130	0.1293	0.0307	0.0301	0.0009	0.932	0.997

Bias: parameters		Bias: standard errors	
beta_41	-0.003704	beta_41	-0.051423
beta_45	-0.003478	beta_45	-0.123037
beta_42	-0.010889	beta_42	-0.054896
beta_43	0.0061404	beta_43	-0.060309
beta_21	0.005873	beta_46	-0.08021
beta_31	0.0050649	beta_21	-0.067583
beta_56	0.0191713	beta_26	-0.061088
		beta_31	-0.071096
		beta_36	-0.044366
		beta_56	-0.15261
		beta_51	-0.06319

Table 5.9: Model with interaction in datasets with sample size equal to 300

Exogenous interaction — **Cov=0.4**

	ESTIMATES Population	Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2684	0.0971	0.0934	0.0094	0.936	0.802
INT	0.230	0.2304	0.0539	0.0495	0.0029	0.926	0.987
Z	0.450	0.4477	0.1062	0.1007	0.0113	0.927	0.995
W	0.570	0.5738	0.1114	0.1058	0.0124	0.936	0.999
Z ON							
X	0.630	0.6317	0.0782	0.0763	0.0061	0.942	1.000
W ON							
X	0.770	0.7725	0.0785	0.0746	0.0062	0.928	1.000
W WITH							
Z	0.400	0.3989	0.0362	0.0353	0.0013	0.939	1.000

Bias: parameters		Bias: standard errors	
beta_41	-0.005926	beta_41	-0.038105
beta_45	0.0017391	beta_45	-0.081633
beta_42	-0.005111	beta_42	-0.051789
beta_43	0.0066667	beta_43	-0.050269
beta_21	0.0026984	beta_21	-0.024297
beta_31	0.0032468	beta_31	-0.049682

Endogenous interaction — **Cov=0.4**

	ESTIMATES Population	Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2688	0.0984	0.0929	0.0097	0.930	0.802
INT	0.230	0.2303	0.0555	0.0508	0.0031	0.930	0.984
Z	0.450	0.4479	0.1063	0.1003	0.0113	0.932	0.995
W	0.570	0.5737	0.1114	0.1052	0.0124	0.936	0.999
X2	0.000	0.0001	0.1219	0.1126	0.0148	0.929	0.071
INT ON							
X2	0.485	0.4922	0.1925	0.1651	0.0371	0.887	0.883
X	0.000	-0.0022	0.1128	0.1056	0.0127	0.930	0.070
Z ON							
X	0.630	0.6337	0.0802	0.0760	0.0064	0.933	1.000
X2	0.000	-0.0031	0.1075	0.1044	0.0116	0.945	0.055
W ON							
X	0.770	0.7745	0.0809	0.0745	0.0066	0.921	1.000
X2	0.000	0.0008	0.1056	0.1013	0.0111	0.950	0.050
W WITH							
X	0.400	0.3977	0.0363	0.0350	0.0013	0.938	1.000

Bias: parameters		Bias: standard errors	
beta_41	-0.004444	beta_41	-0.055894
beta_45	0.0013043	beta_45	-0.084685
beta_42	-0.005556	beta_42	-0.056444
beta_43	0.0064912	beta_43	-0.055655
beta_21	0.005873	beta_46	-0.076292
beta_31	0.0058442	beta_21	-0.052369
beta_56	-0.000206	beta_26	-0.028837
		beta_31	-0.07911
		beta_36	-0.024621
		beta_56	-0.142338
		beta_51	-0.06383

Exogenous interaction — **Cov=0.13**

	ESTIMATES Population	Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2682	0.1010	0.0970	0.0102	0.934	0.781
INT	0.230	0.2297	0.0663	0.0599	0.0044	0.916	0.944
Z	0.450	0.4496	0.0626	0.0597	0.0039	0.939	1.000
W	0.570	0.5727	0.0669	0.0627	0.0045	0.924	1.000
Z ON							
X	0.630	0.6318	0.0782	0.0763	0.0061	0.941	1.000
W ON							
X	0.770	0.7716	0.0786	0.0746	0.0062	0.926	1.000
W WITH							
Z	0.130	0.1298	0.0280	0.0283	0.0008	0.957	0.998

Bias: parameters		Bias: standard errors	
beta_41	-0.006667	beta_41	-0.039604
beta_45	-0.001304	beta_45	-0.096531
beta_42	-0.000889	beta_42	-0.046326
beta_43	0.0047368	beta_43	-0.06278
beta_21	0.0028571	beta_21	-0.024297
beta_31	0.0020779	beta_31	-0.050891

Endogenous interaction — **Cov=0.13**

	ESTIMATES Population	Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2685	0.1023	0.0965	0.0105	0.927	0.773
INT	0.230	0.2298	0.0698	0.0624	0.0049	0.924	0.927
Z	0.450	0.4496	0.0629	0.0594	0.0040	0.937	1.000
W	0.570	0.5726	0.0670	0.0624	0.0045	0.923	1.000
X2	0.000	-0.0008	0.1239	0.1143	0.0153	0.933	0.067
INT ON							
X2	0.485	0.4908	0.1628	0.1384	0.0265	0.895	0.961
X	0.000	0.0005	0.0926	0.0875	0.0086	0.939	0.061
Z ON							
X	0.630	0.6338	0.0801	0.0760	0.0064	0.931	1.000
X2	0.000	-0.0031	0.1075	0.1040	0.0115	0.945	0.055
W ON							
Z	0.770	0.7734	0.0809	0.0745	0.0066	0.929	1.000
X2	0.000	0.0043	0.1064	0.1005	0.0113	0.936	0.064
W WITH							
Z	0.400	0.1294	0.0280	0.0281	0.0740	0.000	0.998

Bias: parameters		Bias: standard errors	
beta_41	-0.005556	beta_41	-0.056696
beta_45	-0.00087	beta_45	-0.106017
beta_42	-0.000889	beta_42	-0.055644
beta_43	0.0045614	beta_43	-0.068657
beta_21	0.0060317	beta_46	-0.077482
beta_31	0.0044156	beta_21	-0.051186
beta_56	0.0117502	beta_26	-0.032558
		beta_31	-0.07911
		beta_36	-0.055451
		beta_56	-0.149877
		beta_51	-0.055076

Table 5.10: Model with interaction in datasets with sample size equal to 350

Exogenous interaction — Cov=-0.4

	ESTIMATES Population	ESTIMATES Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2649	0.1837	0.1745	0.0337	0.945	0.334
INT	0.230	0.2307	0.0661	0.0606	0.0044	0.913	0.959
Z	0.450	0.4533	0.1110	0.1079	0.0123	0.936	0.983
W	0.570	0.5747	0.1184	0.1128	0.0140	0.935	0.999
Z ON							
X	0.630	0.6316	0.0779	0.0763	0.0061	0.943	1.000
W ON							
X	0.770	0.7700	0.0767	0.0745	0.0059	0.934	1.000
W WITH							
Z	-0.400	-0.3985	0.0355	0.0344	0.0013	0.938	1.000

Bias: parameters		Bias: standard errors	
beta_41	-0.018889	beta_41	-0.050082
beta_45	0.0030435	beta_45	-0.083207
beta_42	0.0073333	beta_42	-0.027928
beta_43	0.0082456	beta_43	-0.047297
beta_21	0.0025397	beta_21	-0.020539
beta_31	0	beta_31	-0.028683

Endogenous interaction — Cov=-0.4

	ESTIMATES Population	ESTIMATES Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2647	0.1850	0.1739	0.0342	0.947	0.346
INT	0.230	0.2309	0.0704	0.0631	0.0049	0.912	0.932
Z	0.450	0.4537	0.1112	0.1074	0.0124	0.938	0.984
W	0.570	0.5751	0.1184	0.1123	0.0140	0.933	0.998
X2	0.000	-0.0005	0.1257	0.1144	0.0158	0.928	0.072
INT ON							
X2	0.485	0.4847	0.1237	0.1072	0.0153	0.917	0.956
X	0.000	0.0028	0.0752	0.0724	0.0057	0.939	0.061
Z ON							
X	0.630	0.6335	0.0798	0.0760	0.0064	0.932	1.000
X2	0.000	-0.0032	0.1075	0.1036	0.0116	0.944	0.056
W ON							
X	0.770	0.7712	0.0785	0.0744	0.0062	0.932	1.000
X2	0.000	0.0057	0.1037	0.0994	0.0108	0.932	0.068
W WITH							
Z	-0.400	-0.3972	0.0355	0.0341	0.0013	0.933	1.000

Bias: parameters		Bias: standard errors	
beta_41	-0.01963	beta_41	-0.06
beta_45	0.003913	beta_45	-0.103693
beta_42	0.0082222	beta_42	-0.034173
beta_43	0.0089474	beta_43	-0.05152
beta_21	0.0055556	beta_46	-0.089897
beta_31	0.0015584	beta_21	-0.047619
beta_56	-0.000825	beta_26	-0.036279
		beta_31	-0.052229
		beta_36	-0.041466
		beta_56	-0.133387
		beta_51	-0.037234

Exogenous interaction — Cov=0

	ESTIMATES Population	ESTIMATES Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2679	0.1055	0.1011	0.0111	0.931	0.751
INT	0.230	0.2293	0.0705	0.0638	0.0050	0.917	0.919
Z	0.450	0.4502	0.0600	0.0578	0.0036	0.950	1.000
W	0.570	0.5726	0.0648	0.0607	0.0042	0.929	1.000
Z ON							
X	0.630	0.6317	0.0781	0.0763	0.0061	0.941	1.000
W ON							
X	0.770	0.7712	0.0784	0.0746	0.0061	0.928	1.000
W WITH							
X	0.000	0.0002	0.0270	0.0272	0.0007	0.958	0.042

Bias: parameters		Bias: standard errors	
beta_41	-0.007778	beta_41	0.0436967
beta_45	-0.003043	beta_45	-0.095035
beta_42	0.0004444	beta_42	-0.036667
beta_43	0.0045614	beta_43	-0.063272
beta_21	0.0026984	beta_21	0.006402
beta_31	0.0015584	beta_31	0.002551

Endogenous interaction — Cov=0

	ESTIMATES Population	ESTIMATES Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2682	0.1069	0.1006	0.0114	0.927	0.748
INT	0.230	0.2295	0.0750	0.0668	0.0056	0.918	0.893
Z	0.450	0.4502	0.0603	0.0575	0.0036	0.944	1.000
W	0.570	0.5726	0.0649	0.0604	0.0042	0.924	1.000
X2	0.000	-0.0009	0.1252	0.1150	0.0157	0.931	0.069
INT ON							
X2	0.485	0.4897	0.1506	0.1276	0.0227	0.891	0.971
X	0.000	0.0014	0.0848	0.0808	0.0072	0.944	0.056
Z ON							
X	0.630	0.6338	0.0800	0.0760	0.0064	0.932	1.000
X2	0.000	-0.0031	0.1075	0.1038	0.0115	0.944	0.056
Z ON							
X	0.770	0.7728	0.0806	0.0745	0.0065	0.926	1.000
X2	0.000	0.0052	0.1061	0.1000	0.0113	0.928	0.072
W WITH							
Z	0.000	0.0002	0.0270	0.0270	0.0007	0.957	0.043

Bias: parameters		Bias: standard errors	
beta_41	-0.006667	beta_41	-0.058934
beta_45	-0.002174	beta_45	-0.109333
beta_42	0.0004444	beta_42	-0.046434
beta_43	0.0045614	beta_43	-0.069337
beta_21	0.0060317	beta_46	-0.08147
beta_31	0.0036364	beta_21	-0.05
beta_56	0.0094826	beta_26	-0.034419
		beta_31	-0.075682
		beta_36	-0.021678
		beta_56	-0.152722
		beta_51	-0.04717

Table 5.11: Model with interaction in datasets with sample size equal to 350

Exogenous interaction							
Cov=0.4							
	ESTIMATES			S. E.	M. S. E.	95%	% Sig
	Population	Average	Std. Dev.	Average		Cover	Coeff
Y ON							
X	0.270	0.2685	0.0896	0.0876	0.0080	0.944	0.863
INT	0.230	0.2306	0.0496	0.0464	0.0025	0.922	0.994
Z	0.450	0.4495	0.0989	0.0947	0.0098	0.936	0.997
W	0.570	0.5733	0.1029	0.0993	0.0106	0.939	1.000
Z ON							
X	0.630	0.6308	0.0729	0.0716	0.0053	0.945	1.000
W ON							
X	0.770	0.7718	0.0726	0.0699	0.0053	0.938	1.000
Z WITH							
W	0.400	0.3987	0.0348	0.0331	0.0012	0.933	1.000

Bias			
parameters		standard errors	
beta_41	-0.00556	beta_41	-0.02232
beta_45	0.002609	beta_45	-0.06452
beta_42	-0.00111	beta_42	-0.04247
beta_43	0.005789	beta_43	-0.03499
beta_21	0.00127	beta_21	-0.01783
beta_31	0.002338	beta_31	-0.03719

Endogenous interaction							
Cov=0.4							
	ESTIMATES			S. E.	M. S. E.	95%	% Sig
	Population	Average	Std. Dev.	Average		Cover	Coeff
Y ON							
X	0.270	0.2689	0.0902	0.0871	0.0081	0.943	0.862
INT	0.230	0.2307	0.0512	0.0476	0.0026	0.924	0.995
Z	0.450	0.4493	0.0991	0.0943	0.0098	0.937	0.997
W	0.570	0.5735	0.1033	0.0989	0.0107	0.935	1.000
X2	0.000	-0.0009	0.1091	0.1053	0.0119	0.937	0.063
INT ON							
X2	0.485	0.4917	0.1729	0.1563	0.0299	0.896	0.922
X	0.000	-0.0004	0.1043	0.0994	0.0109	0.932	0.068
Z ON							
X	0.630	0.6322	0.0742	0.0713	0.0055	0.942	1.000
X2	0.000	-0.0010	0.0995	0.0971	0.0099	0.938	0.062
W ON							
X	0.770	0.7730	0.0743	0.0698	0.0055	0.936	1.000
X2	0.000	0.0013	0.0980	0.0940	0.0096	0.941	0.059
Z WITH							
W	0.400	0.3976	0.0347	0.0329	0.0012	0.937	1.000

Bias			
parameters		standard errors	
beta_41	-0.004074	beta_41	-0.034368
beta_45	0.0030435	beta_45	-0.070313
beta_42	-0.001111	beta_42	-0.048436
beta_43	0.0061404	beta_43	-0.042594
beta_21	0.0034921	beta_46	-0.03483
beta_31	0.0038961	beta_21	-0.039084
beta_56	0.0136054	beta_26	-0.024121
		beta_31	-0.060565
		beta_36	-0.040816
		beta_56	-0.096009
		beta_52	-0.04698

Exogenous interaction							
Cov=0.13							
	ESTIMATES			S. E.	M. S. E.	95%	% Sig
	Population	Average	Std. Dev.	Average		Cover	Coeff
Y ON							
X	0.270	0.2680	0.0931	0.0910	0.0087	0.947	0.833
INT	0.230	0.2296	0.0615	0.0561	0.0038	0.927	0.973
Z	0.450	0.4509	0.0592	0.0561	0.0035	0.937	1.000
W	0.570	0.5726	0.0611	0.0590	0.0037	0.938	1.000
Z ON							
X	0.630	0.6309	0.0728	0.0716	0.0053	0.946	1.000
W ON							
X	0.770	0.7718	0.0733	0.0698	0.0054	0.931	1.000
Z WITH							
W	0.130	0.1298	0.0269	0.0265	0.0007	0.937	0.999

Bias			
parameters		standard errors	
beta_41	-0.007407	beta_41	-0.022556
beta_45	-0.001739	beta_45	-0.087805
beta_42	0.002	beta_42	-0.052365
beta_43	0.0045614	beta_43	-0.03437
beta_21	0.0014286	beta_21	-0.016484
beta_31	0.0023377	beta_31	-0.047749

Endogenous interaction							
Cov=0.13							
	ESTIMATES			S. E.	M. S. E.	95%	% Sig
	Population	Average	Std. Dev.	Average		Cover	Coeff
Y ON							
X	0.270	0.2684	0.0938	0.0906	0.0088	0.935	0.837
INT	0.230	0.2300	0.0645	0.0585	0.0042	0.925	0.958
Z	0.450	0.4509	0.0592	0.0559	0.0035	0.936	1.000
W	0.570	0.5726	0.0612	0.0587	0.0038	0.939	1.000
X2	0.000	-0.0016	0.1107	0.1070	0.0123	0.934	0.066
INT ON							
F6	0.485	0.4910	0.1504	0.1311	0.0226	0.891	0.973
F1	0.000	0.0016	0.0860	0.0825	0.0074	0.937	0.063
Z ON							
X	0.630	0.6323	0.0741	0.0713	0.0055	0.941	1.000
X2	0.000	-0.0011	0.0995	0.0968	0.0099	0.936	0.064
W ON							
X	0.770	0.7728	0.0750	0.0698	0.0056	0.924	1.000
X2	0.000	0.0032	0.0974	0.0933	0.0095	0.938	0.062
Z WITH							
W	0.130	0.1294	0.0268	0.0264	0.0007	0.937	1.000

Bias			
parameters		standard errors	
beta_41	-0.005926	beta_41	-0.034115
beta_45	0	beta_45	-0.093023
beta_42	0.002	beta_42	-0.055743
beta_43	0.0045614	beta_43	-0.04085
beta_21	0.0036508	beta_46	-0.033424
beta_31	0.0036364	beta_21	-0.037787
beta_56	0.0121624	beta_26	-0.027136
		beta_31	-0.069333
		beta_36	-0.042094
		beta_56	-0.128324
		beta_51	-0.040698

Table 5.12: Model with interaction in datasets with sample size equal to 400

Exogenous interaction — Cov=0

	ESTIMATES Population	ESTIMATES Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2678	0.0972	0.0949	0.0094	0.947	0.807
INT	0.230	0.2292	0.0654	0.0598	0.0043	0.923	0.963
Z	0.450	0.4513	0.0570	0.0543	0.0033	0.935	1.000
W	0.570	0.5726	0.0590	0.0570	0.0035	0.940	1.000
Z ON							
X	0.630	0.6310	0.0728	0.0717	0.0053	0.945	1.000
W ON							
X	0.770	0.7716	0.0732	0.0698	0.0054	0.926	1.000
W WITH							
X	0.000	0.0002	0.0257	0.0255	0.0007	0.938	0.062

Bias: parameters		Bias: standard errors	
beta_41	-0.008148	beta_41	-0.023663
beta_45	-0.003478	beta_45	-0.085627
beta_42	0.0028889	beta_42	-0.047368
beta_43	0.0045614	beta_43	-0.033898
beta_21	0.0015873	beta_21	-0.01511
beta_31	0.0020779	beta_31	-0.046448

Endogenous interaction — Cov=0

	ESTIMATES Population	ESTIMATES Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2681	0.0980	0.0944	0.0096	0.939	0.809
INT	0.230	0.2296	0.0691	0.0627	0.0048	0.919	0.938
Z	0.450	0.4514	0.0571	0.0541	0.0033	0.937	1.000
W	0.570	0.5726	0.0592	0.0568	0.0035	0.937	1.000
X2	0.000	-0.0017	0.1114	0.1077	0.0124	0.934	0.066
INT ON							
X2	0.485	0.4901	0.1410	0.1207	0.0199	0.899	0.977
X	0.000	0.0021	0.0791	0.0762	0.0063	0.946	0.054
Z ON							
X	0.630	0.6323	0.0741	0.0714	0.0055	0.938	1.000
X2	0.000	-0.0011	0.0995	0.0967	0.0099	0.935	0.065
W ON							
X	0.770	0.7725	0.0750	0.0698	0.0056	0.928	1.000
X2	0.000	0.0037	0.0962	0.0930	0.0093	0.933	0.067
W WITH							
Z	0.000	0.0001	0.0256	0.0254	0.0007	0.942	0.058

Bias: parameters		Bias: standard errors	
beta_41	-0.007037	beta_41	-0.036735
beta_45	-0.001739	beta_45	-0.092619
beta_42	0.0031111	beta_42	-0.052539
beta_43	0.0045614	beta_43	-0.040541
beta_21	0.0036508	beta_46	-0.033214
beta_31	0.0032468	beta_21	-0.036437
beta_56	0.0103072	beta_26	-0.028141
		beta_31	-0.069333
		beta_36	-0.033264
		beta_56	-0.143972
		beta_51	-0.036662

Exogenous interaction — Cov=-0.4

	ESTIMATES Population	ESTIMATES Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2652	0.1691	0.1637	0.0286	0.940	0.380
INT	0.230	0.2298	0.0611	0.0569	0.0037	0.926	0.973
Z	0.450	0.4540	0.1051	0.1013	0.0110	0.944	0.996
W	0.570	0.5742	0.1093	0.1060	0.0120	0.937	0.999
Z ON							
X	0.630	0.6309	0.0726	0.0717	0.0053	0.946	1.000
W ON							
X	0.770	0.7707	0.0714	0.0699	0.0051	0.935	1.000
W WITH							
Z	-0.400	-0.3986	0.0337	0.0324	0.0011	0.942	1.000

Bias: parameters		Bias: standard errors	
beta_41	-0.017778	beta_41	-0.031934
beta_45	-0.00087	beta_45	-0.06874
beta_42	0.0088889	beta_42	-0.036156
beta_43	0.0073684	beta_43	-0.030192
beta_21	0.0014286	beta_21	-0.012397
beta_31	0.0009091	beta_31	-0.021008

Endogenous interaction — Cov=-0.4

	ESTIMATES Population	ESTIMATES Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2652	0.1698	0.1632	0.0288	0.937	0.387
INT	0.230	0.2301	0.0644	0.0593	0.0041	0.918	0.957
Z	0.450	0.4543	0.1052	0.1009	0.0111	0.942	0.996
W	0.570	0.5744	0.1095	0.1055	0.0120	0.938	0.999
X2	0.000	-0.0012	0.1116	0.1071	0.0124	0.942	0.058
INT ON							
X2	0.485	0.4856	0.1171	0.1001	0.0137	0.917	0.968
X	0.000	0.0020	0.0708	0.0680	0.0050	0.934	0.066
Z ON							
F1	0.630	0.6322	0.0738	0.0713	0.0055	0.940	1.000
F6	0.000	-0.0011	0.0995	0.0965	0.0099	0.934	0.066
W ON							
X	0.770	0.7715	0.0733	0.0699	0.0054	0.934	1.000
X2	0.000	0.0033	0.0941	0.0927	0.0089	0.933	0.067
W WITH							
Z	-0.400	-0.3975	0.0335	0.0321	0.0011	0.943	1.000

Bias: parameters		Bias: standard errors	
beta_41	-0.017778	beta_41	-0.038869
beta_45	0.0004348	beta_45	-0.079193
beta_42	0.0095556	beta_42	-0.040875
beta_43	0.0077193	beta_43	-0.03653
beta_21	0.0034921	beta_46	-0.040323
beta_31	0.0019481	beta_21	-0.033875
beta_56	0.0010307	beta_26	-0.030151
		beta_31	-0.046385
		beta_36	-0.014878
		beta_56	-0.145175
		beta_51	-0.039548

Table 5.13: Model with interaction in datasets with sample size equal to 400

Exogenous curvilinear effect	Bias	
Cov=0.4, sample size =2000, CR=0.5	parameters	standard errors

	ESTIMATES		S. E.	M. S. E.	95%	% Sig
	Population	Average	Std. Dev.	Average	Cover	Coeff
Y ON						
X	0.270	0.2707	0.0506	0.0498	0.0026	0.942 1.000
Z	0.450	0.4491	0.0894	0.0881	0.0080	0.950 0.997
W	0.570	0.5743	0.0931	0.0937	0.0087	0.950 1.000
Z2	0.230	0.2322	0.0797	0.0774	0.0063	0.964 0.858
W2	0.280	0.2836	0.0809	0.0771	0.0066	0.957 0.956
Z ON						
X	0.630	0.6256	0.0410	0.0404	0.0017	0.944 1.000
W ON						
X	0.770	0.7660	0.0417	0.0412	0.0018	0.937 1.000
Z WITH						
W	0.400	0.3991	0.0238	0.0229	0.0006	0.936 1.000

parameters		standard errors	
beta_41	0.0025926	beta_41	-0.01581
beta_42	-0.002	beta_42	-0.014541
beta_43	0.0075439	beta_43	0.0064447
beta_47	0.0095652	beta_47	-0.028858
beta_48	0.0128571	beta_48	-0.046972
beta_21	-0.006984	beta_21	-0.014634
beta_31	-0.005195	beta_31	-0.01199

Exogenous curvilinear effect	Bias	
Cov=0.4, sample size =2000, CR=0.95	parameters	standard errors

	ESTIMATES		S. E.	M. S. E.	95%	% Sig
	Population	Average	Std. Dev.	Average	Cover	Coeff
Y ON						
X	0.270	0.2711	0.0397	0.0399	0.0016	0.946 1.000
Z	0.450	0.4505	0.0437	0.0434	0.0019	0.951 1.000
W	0.570	0.5693	0.0447	0.0455	0.0020	0.951 1.000
Z2	0.230	0.2304	0.0301	0.0300	0.0009	0.945 1.000
W2	0.280	0.2794	0.0300	0.0295	0.0009	0.949 1.000
Z ON						
X	0.630	0.6279	0.0327	0.0325	0.0011	0.945 1.000
W ON						
X	0.770	0.7675	0.0318	0.0318	0.0010	0.944 1.000
Z WITH						
W	0.400	0.3997	0.0155	0.0151	0.0002	0.947 1.000

parameters		standard errors	
beta_41	0.0040741	beta_41	0.0050378
beta_42	0.0011111	beta_42	-0.006865
beta_43	-0.001228	beta_43	0.0178971
beta_47	0.0017391	beta_47	-0.003322
beta_48	-0.002143	beta_48	-0.016667
beta_21	-0.003333	beta_21	-0.006116
beta_31	-0.003247	beta_31	0

	Sample size	
CR	500	2000
CR=0.95	No problem	No problem
CR=0.5	Biased st. errors, low powers	No problem

Table 5.14: Relation between reliability and sample size

to 500. If I compare these 2 estimates, I note that if the CR decreases with the same sample size, some standard errors become biased and some powers decrease. Table 5.14 shows respectively the estimate of the datasets with CR=0.5 and sample size 2000 and the estimate of the datasets with CR=0.95 and sample size 2000. If I compare these 2 estimates, I note that if the CR decreases with the same sample size, some powers decrease but, this time, no standard errors become biased. Now I compare the datasets with the same CR but with different sample size. I start from the case with CR=0.5, where an increase of the sample size leads to the absence of the problems of biased st. errors. Now I consider the case with CR=0.95, where both estimates are good: an increase of the sample size increases the powers, which become all equal to 1. Then increasing the sample size, the methods perform better even in situations where problems emerged with smaller samples.

5.4 Conclusions

In the first part of the chapter, I analyze how the goodness of estimation varies when there is a variation of the measurement part. I use the CR index which measures the reliability of the construct of each latent variable. As noted by Moosbrugger et al. (2008), the reliabilities of the non-linear elements depend on the reliabilities of the variables which compose them, for example, the reliability of Z2 depends on the reliability of Z. However, in general, the reliability of the nonlinear variables is lower than the reliability of the "linear" variables , i.e. $Rel(ZQ) < Rel(Z)$. This relationship means that good reliabilities for the linear terms becomes not good for the nonlinear terms. So two questions arise from mathematical analysis. The first is to understand if lowering the reliabilities but having all good indices (both those of linear variables, and those of nonlinear variables), the exogenous method still estimates the parameters well. The answer is that the exogenous method estimates well when the measurement part is good, both for linear and nonlinear variables. The second question is to analyze the estimates if the reliabilities of linear variables are good and those of nonlinear variables are not good. If only few indices, which measure the goodness of the nonlinear part, are not good, the exogenous method still estimates well the parameters. When almost all the indices of the nonlinear part are not good , it is advisable to use the exogenous method with constraints such that they significantly improve the standard errors of the structural parameters.

In the second part of the chapter, I deal with the problem of the minimum sample size required for good results of the estimation. Many authors recommend several rules to select the minimum sample size. I analyze three sample sizes: 300, 350 and 400. Both the exogenous method and the endogenous one do not estimate well the parameters with the sample size equal to 300. The exogenous method estimates well with sample size equal to 350, although often the bias of the standard error of the interaction parameter is very close to the upper limit 0.1. The endogenous method does not estimate well the parameters with sample size equal to 350. With sample size equal to 400, the exogenous method improves the estimation, while the endogenous method has still often problems of bias of the standard errors. This can be explained by the increased number of parameters in the endogenous method. Considering the analysis made in the previous chapters, where I showed that the causal analysis provides the same results both with exogenous method and with endogenous method, I prefer the exogenous method to estimate the data because it requires a lower sample size. If I have datasets with greater sample size than 350-400, I can estimate an exogenous model without problems of bias of the parameters and of the standard errors.

CHAPTER 6

MODELS WITH 2 MEDIATORS AND INTERACTION: PRACTICAL APPLICATIONS IN CUSTOMER SATISFACTION

6.1 Interaction between positive and negative emotions

A practical application of the models proposed in the previous chapters can be the customer satisfaction analysis, which serves the development of a truly customer-focused management and culture and offers a meaningful and objective feedback about client's preferences and expectations. In this specific case, I analyze how the positive and negative emotions affect the customer behavior and if the positive emotions effect is affected by that of negative emotions, and vice versa. This latter consideration is translated mathematically with the introduction of the interaction.

The analysis is performed on McDonalds' fast food chain in Italy. I use a dataset with sample size 465 in which the variables are measured by 7-points Likert scale. I want to analyze how the design (X), considered as a facility aestetics (Ryu & Han, 2010), affects the positive emotions (Z) and the absence of negative emotions (W) (i.e. measured by reversing the scale of values: 1 = high negative emotions, ..., 7 = low negative emotions) and how these variables influence the future behavior (Y). The items, which I use to calculate the latent variables X, Z, W and Y, are:

- X_1 = Externally the fast food looks attractive
- X_2 = The colors create a pleasant atmosphere
- X_3 = The overall design is eye-catching and attractive
- Z_1 = Happiness
- Z_2 = Interest
- Z_3 = Satisfaction
- W_1 = Tension
- W_2 = Irritation
- W_3 = Stress
- Y_1 = I will return in the future
- Y_2 = I will recommend this fast food to friends
- Y_3 = I will say good things about this fast food to others

	st. loadings	CR
X	0.7005	0.85
	0.9330	
	0.7723	
Z	0.6880	0.81
	0.7388	
	0.8515	
W	0.8276	0.93
	0.9551	
	0.9680	
Y	0.9363	0.85
	0.7624	
	0.9242	
ZQ	0.4733	0.61
	0.5458	
	0.7250	
WQ	0.6849	0.86
	0.9122	
	0.8542	
INT	0.7357	0.85
	0.8119	
	0.8845	

(a)

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Y ON				
X	0.266	0.076	3.503	0.000
INT	0.087	0.028	3.058	0.002
Z	0.341	0.091	3.740	0.000
W	0.207	0.083	2.493	0.013
Z ON				
X	0.720	0.074	9.770	0.000
W ON				
X	0.654	0.096	6.786	0.000
W WITH				
Z	0.437	0.054	8.143	0.000

(b)

Table 6.1: Customer satisfaction "positive-negative emotions" model on McDonalds

Table 6.1 (a) shows the CR indices and standardized loadings obtained from the CFA only using the "linear" variables (X_i, Z_i, W_i and Y_i) and those obtained by Moobrugger's formulas [1]. All the indices are within the bounds (standardized loadings > 0.5, CR > 0.6) with the exception of the standardized loading of ZQ_1, but the CR of variable ZQ is greater than 0.6. I estimate the dataset with the ML method and robust standard errors and robust chi-square according to Satorra & Bentler (1994). I apply this correction for robust estimation, which is necessary for non-normal data, because there is the interaction, i.e. a variable which is certainly nonnormal. In this dataset, however, all the variables are not normal. The indexes of goodness of the model are good according to Garver & Mentzer (1999): $chi_{square}/df = 2.13 < 3$, $RMSEA = 0.049 < 0.08$ and $CFI = 0.90$. The structural parameters are shown in Table 6.1 (b). The design (X) influences directly and positively the future behavior (parameter = 0.266), this exogenous variables affects positively the positive emotions (parameter = 0.72) and the absence of negative emotions (parameter = 0.654): a customer who appreciates the design has more positive emotions and less negative emotions and will be more inclined to return or have positive word of mouth. Positive emotions (Z) and absence of negative emotions (W) have a direct and positive effect on Y, respectively 0.341 and 0.207. The correlation between positive emotions and negative emotions is 0.683 while the covariance between the structural errors ζ_3 and ζ_2 is 0.437. In this analysis, however, to analyze the effect of positive emotions and negative emotions it is necessary also to take into account their interaction: the change of positive emotion produces a greater positive change in future behavior for a person who has a low negative emotion (i.e. high W) than for a subject with a medium or high negative emotion.

[1] As recommended by Coenders et al. (2008) the variables are associated according to the reliability, i.e.

$$\text{if} \quad Rel(Z_1) < Rel(Z_2) < Rel(Z_3) \quad \text{and} \quad Rel(W_1) < Rel(W_2) < Rel(W_3) \quad \text{then}$$

$$INT_1^{uc} = Z_1 W_1$$

$$INT_2^{uc} = Z_2 W_2$$

$$INT_3^{uc} = Z_3 W_3$$

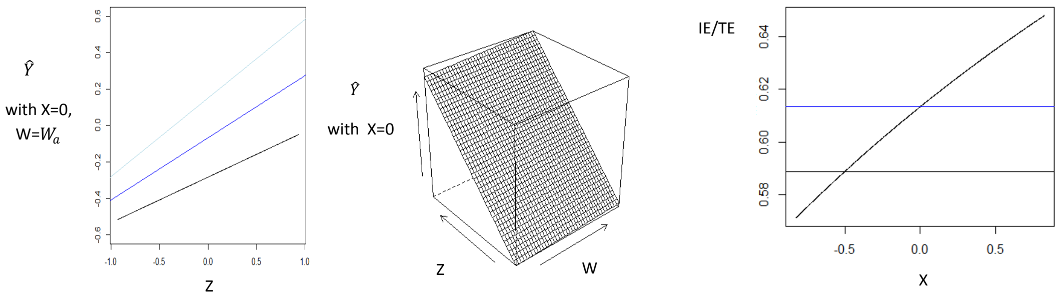

Figure 6.1: the effects of emotions on Y and ratio between IE and TE

The joint direct effect of positive emotions (Z) and of absence of negative emotions (W) on the future behavior (Y) can be seen by this relation (without considering the relations among X, Z and W)

$$\hat{Y}(X=0, Z, W) = -0.066 + 0.207W + 0.341Z + 0.087ZW$$

For example, the effect of Z given W can be studied by this relation (always without considering the relations among X, Z and W):

$$\hat{Y}(X=0, Z, W=W_a) = -0.066 + 0.207W_a + (0.341 + 0.087W_a)Z$$

and then when W increases, also the effect of Z increases and the trend is accentuated. When W decreases, the effect of Z decreases and the trend is attenuated. This behavior is represented in the first 2 graphs of Figure 6.1, in which the possible values of Z are in the range [-0.9333, 0.9333]. In the first graph, the black line represents the line with $W_a = -\sqrt{Var(W)} = -1$, the blue line is that with $W_a = 0$ and the light blue is that with $W_a = \sqrt{Var(W)} = 1$. When the negative emotion decreases (in order black, blue, light blue), the effect of positive emotion is accentuated: with high positive emotion (remembering that Z and W are variables with zero mean, $Z > 0$ corresponds to a high Z) the best future behavior occurs with low negative emotion; with low positive emotion low the worst future behavior occurs with high negative emotion. The line with the low negative emotion has a greater slope than the other two lines, and thus in this case the change of positive emotion influences most future behavior. All this analysis can be made for W, considering $\hat{Y}(X=0, W, Z=Z_b)$, because both the variables which form the interaction have the same role.

The final analysis studies the effect of the design mediated by positive and negative emotions. To calculate it I use my modified formulas for the indirect effect and for total effect

$$\begin{aligned} IE(Z,W) &= [0.341(0.72) + 0.207(0.654)]\Delta x + 0.087(0.72)(0.654)(x_1^2 - x_0^2) \\ &= 0.3809\Delta x + 0.041(x_1^2 - x_0^2) \end{aligned}$$

$$TE = DE + IE = 0.266\Delta x + 0.3809\Delta x + 0.041(x_1^2 - x_0^2) = 0.6469\Delta x + 0.041(x_1^2 - x_0^2)$$

Using these formulas, I find that the design (X) has a positive influence even indirectly[2] and therefore its effect on the future behavior is positive. The ratio IE / TE is equal to

$$\frac{IE}{TE} = \frac{0.3809 + 0.041(x_1 + x_0)}{0.6469 + 0.041(x_1 + x_0)}$$

The third graph in Figure 6.1 ($\epsilon = \Delta x = 1$) shows the development of this relationship. When the pleasure of the design (X) increases, the share of the indirect effect on the total effect increases

[2]the IE is positive if $x_0 + x_1 > -9.29$, which is always true in the range [-0.825,0.825]

too. The blue line represents the value of the ratio IE/TE when the appreciation of the design is at the average level ($E(X) = 0$) and is 0.6133, then in the midpoint the indirect effect is greater than half of total effect.

For subjects which give an average rating to the design, this exogenous variable affects more indirectly than directly future behavior. The black line represents the ratio IE/TE (0.5888) when I do not consider the interaction. This ratio is still higher than 50% and so even if I do not consider the interaction, the design effect is mainly due to its indirect part.

I apply my procedure proposed in the previous chapters to see if the interaction is spurious: SABIC (curv) = 27959.199, SABIC (int) = 27956.420, then the true model is with interaction.

6.2 Interaction between satisfaction and positive emotions

Another analysis, which is very important in customer satisfaction, is the relationship between satisfaction and positive emotions. To do this analysis, I still consider the dataset obtained from the survey on McDonalds's fast food. The variables used in this case are atmosphere (X), defined as ambient elements according to Ryu & Han (2010)[3], positive emotion (Z) (measured as wellness), satisfaction (W) and future behavior (Y). The items that I use to get these variables are:

- X_1 = The music creates a pleasant atmosphere
- X_2 =The music volume is appropriate
- X_3 = The temperature is adequate
- Z_1 = Fun
- Z_2 = Relax
- Z_3 = Comfort
- W_1 =I am satisfied with the experience which I have had in fast food
- W_2 = I had fun in this fast food
- W_3 = I'm glad to have had an experience in fast food
- Y_1 = I will return in the future
- Y_2 = I will recommend this fast food to friends
- Y_3 = I will say good things about this fast food to others

The estimated parameters of CFA analysis are represented in Table 6.2 (a). The indices of the measurement part are all greater than their lower limits. With the exogenous interaction model, the indices of goodness are good: $chi_{square}/df = 1.53 < 3$, $RMSEA = 0.034 < 0.08$ and $CFI = 0.96 > 0.9$. The structural parameters are shown in Table 6.2 (b). All parameters are positive. The atmosphere influences directly and positively positive emotion (0.622) and satisfaction (0.569), while future behavior is not significant, then this is a complete mediation model. I estimate the model without the direct effect of X on Y, the estimates are shown in Table 6.2. The indices of goodness are very close to those of the full model. The atmosphere influences future behavior only indirectly. Satisfaction and positive emotion affect directly and positively future behavior (respectively 0.517 and 0.386). The interaction is positive, then as in the previous dataset, increasing satisfaction, increases the influence of positive emotion on future behavior, even if this effect is lower than in the first example. The influence of atmosphere is mediated by positive emotions and satisfaction. Using the modified Pearl's formulas, I get:

[3]The ambient elements, such as music, scent and temperature, are not tangible and not visual sense

	st. loadings	CR
X	0.7160	0.7362
	0.7406	
	0.6235	
Z	0.8302	0.8260
	0.7053	
	0.8094	
W	0.9279	0.9521
	0.9374	
	0.7217	
Y	0.9337	0.8291
	0.7537	
	0.9309	
ZQ	0.6892	0.6473
	0.4974	
	0.6552	
WQ	0.8610	0.9023
	0.8787	
	0.8666	
INT	0.7862	0.7408
	0.6865	
	0.6175	

(a)

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Y ON				
X	0.083	0.074	1.125	0.261
INT	0.058	0.024	2.408	0.016
Z	0.484	0.096	5.038	0.000
W	0.371	0.072	5.145	0.000
Z ON				
X	0.622	0.100	6.235	0.000
W ON				
X	0.569	0.094	6.076	0.000
W WITH				
Z	0.394	0.071	5.554	0.000

(b)

Table 6.2: Customer satisfaction model, with "positive emotion-satisfaction" on McDonalds

$$\begin{aligned} IE &= [0.517(0.634) + 0.386(0.571)]\Delta x + 0.058(0.634)0.571(x_1^2 - x_0^2) \\ &= 0.5481\Delta x + 0.021(x_1^2 - x_0^2) \end{aligned}$$

$$TE = IE$$

From these formulas, I deduce that atmosphere has a positive influence indirectly on the future behavior. When the perception of the atmosphere increases, the positive future behavior increases indirectly [4]. The atmosphere influences future behavior only through positive emotions and satisfaction, i.e. influencing them, which influence future behavior.

Now I check if the interaction is spurious. To do this, I use the previous test: SABIC (int) = 31403.615 and SABIC (curv) = 31419.344. Even in this case, the test chooses the interaction model because its SABIC is minor, so the estimated interaction is not spurious.

6.3 Conclusions

In this chapter, I apply the models studied in the previous chapters to real marketing situations. Initially I focus on how the design of McDonalds' fast food affects customers positive and negative emotions and their suggesting the local and/or their returning to it. Customers, who appreciate the design, will have more positive emotions and less negative emotions than those which do not appreciate the design, this will positively influence future behavior. Of course, when positive emotions increase and negative emotions decrease, positive future behavior increases. However, the persons with high positive emotion and low negative emotion will be those who advise the local and return more often; while people with low positive emotion and high negative emotion will be those who advise or return less. This makes me believe that the positive emotion and the absence of negative emotion, if stimulated together, will increase their power and then the manager of a local, which considers both kinds of emotions, will be capable of increasing the customer loyalty.

[4] the IE is positive if $x_0 + x_1 > -26.1$, which is always true in the range [-0.826,0.826]

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Y ON				
INT	0.058	0.024	2.457	0.014
Z	0.517	0.083	6.224	0.000
W	0.386	0.072	5.347	0.000
Z ON				
X	0.634	0.098	6.463	0.000
W ON				
X	0.571	0.093	6.150	0.000
W WITH				
Z	0.388	0.070	5.531	0.000

Table 6.3: Customer satisfaction full mediation model, with "positive emotion-satisfaction" on McDonalds

In the second example, I try to understand how the atmosphere of McDonalds' fast food influences the future behavior through positive emotion and satisfaction. When the pleasure provided to the customer by the atmosphere of the local increases, it increases its positive emotion, his satisfaction and, only indirectly, his desire to advise the local and to return. Because of the presence of interaction, the manager which stimulates togheter positive emotion and satisfaction, will increase their power and so he will increase the customer loyalty.

CHAPTER 7

CONCLUSIONS

In recent decades, researchers have focused on introducing nonlinear terms in SEM model, obtaining what is called Nonlinear SEM. The analyses are mainly concentrated on estimation methods and the choice of indicators when unobserved variables are used. This analysis has a limit in considering only nonlinear terms involving exogenous variables. When nonlinear terms are obtained by endogenous variables, a causal analysis for mediation is required. For this reason I analyze the introduction of nonlinear terms involving endogenous variables considering both the estimation procedure and the causal analyis.

In SEM models with 2 mediators, I investigate the introduction of the interaction term between endogenous variables and I find that it influences both the causal analysis and the estimation process. I develop 2 statistical methods which differ both in the causal analysis and the estimation process. They give substantially the same results, but I recommend the use of the exogenous interaction models with Pearl's causal theory because of its simplicity. This model considers the interaction between endogenous variables as exogenous, i.e. this nonlinear term is linked to the other variables only through the correlation. Because of this artifice I recommend to use my modified Pearl's causal theory for models with two mediators and correlated errors.

If the correlation between the errors of the two mediators raises the correlation between the two mediators, this may cause problems in the estimation. I find that this increase can cause problems in recognizing a model with interaction from a curvilinear model. To overcome this problem I propose a procedure to detect the true model. Other factors which can cause problems in the estimate are the sample size and the quality of the measurement part when I work with latent variables. I find that measurement parts, which are good for linear variables, may not be good for the interaction variable and for the quadratic variables and this can lead to high and not well estimated standard errors. In this case I propose to include in the estimation process some constraints to improve the estimates.

Finally from a managerial perspective using the exogenous interaction model with Pearl modified causal theory I show that, in a customer satisfaction context, positive emotions and negative emotions influence "jointly" future behavior. The increase, in fact, of positive emotions of a customer with low negative emotions will be more profitable than the stimulus of positive emotions of a subject with high negative emotions. As emotions are in turn influenced by the design of the restaurant, managers can use it to enhance customers' loyalty both directly and indirectly by jointly inducing more positive emotions and less negative ones. This way a model with interacting mediators may help to better understand customers' behavior.

One weakness of the thesis can be considered to have only mediators with correlated errors and not mediators in series. Then a further development of the thesis could consist of models with any type of interacting mediators. Another development would be the introduction of another mediator K so that it is possible to introduce an interaction with 3 variables (ZWK) which influences Y. As an example, with this model it would be possible to analyze a model in which satisfaction, positive emotions and negative emotions together influence future behavior.

In general, some preliminary analyses show that the proposed methods should be easily adapted to specify and estimate more complex models as those typically found in empirical applications.

APPENDIX A

CAUSALITY

Pearl (2009,2012, 2014) proposes a theory to calculate the causal effects in the case of a model with mediation, whether it is linear or not. To explain his theory, I start from the simple model with mediation represented in Figure A.1 (figure from Pearl 2012): the pattern (a) is a model without confounders while the model (b) is a model with confounders, because of the variables W_1 and W_2. In models as those of Figure A1 (a) the effects are calculated as follows. First I describe the direct effect

$$DE_{x^0,x^1} = \sum_z \left[E(Y|x^1, z) - E(Y|x^0, z)\right] P(z|x^0)$$

where (x, z) are the variables which influence directly Y and X is the variable which influences Z. The variables Y and Z, then, are conditioned by the variables which directly affect them.

Now I consider the indirect effect calculated according to Pearl's theory

$$IE_{x^0,x^1} = \sum_z \left[E(Y|x^0, z)\right] \left[P(z|x^1) - P(z|x^0)\right]$$

where, as for the direct effect, (x, z) are the variables which influence directly Y and x is the variable which affects Z. [1]

The total effect is

$$TE_{x^0,x^1} = DE_{x^0,x^1} - IE_{x^1,x^0}$$

The total effect of the variation $x^1 - x^0$ is therefore a function of the indirect effect which consider the inverse variation $x^0 - x^1$. In linear models being $IE_{x^1,x^0} = -IE_{x^0,x^1}$ the classical decomposition of the total effect remains equal to the sum of the direct and the indirect effect. If I introduce the confounders, as in the graph of Figure A.1 (b), then the formulas of the direct effect and the indirect effect are so modified

$$DE_{x^0,x^1} = \sum_z \sum_{w_2} P(w_2) \left[E(Y|do(x^1, z), w_2) - E(Y|do(x^0, z), w_2)\right] \sum_{w_1} P(z|do(x^0), w_1, w_2)P(w_1)$$

$$IE_{x^0,x^1} = \sum_z \sum_{w_2} P(w_2) \left[E(Y|do(x^0, z), w_2)\right] \sum_{w_1} \left[P(z|do(x^1), w_1, w_2) - P(z|do(x^0), w_1, w_2)\right] P(w_1)$$

[1] In the formula (7) of Pearl's paper there is not x^0 in the conditional expected value of Y, but the calculations of the following pages and the subsequent Pearl's papers show that it is correct to use x^0

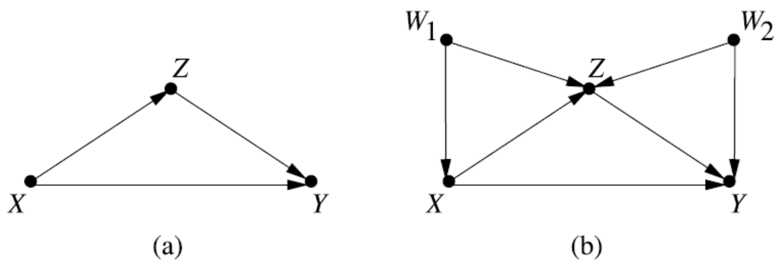

Figure A.1: (a) mediation model without confounders (b) mediation model with confounders

where the variables (x, z, w_2) are the variables which influence Y, while (x, w_1, w_2) are the variables that affect Z.

DO-OPERATOR

> The operator "do" is used to transform a variable into a "constant". To explain the do-operator, I use a modified version of an example taken from Pearl's paper (1998)
>
> $$X = U + \zeta_2$$
>
> $$Z = \beta_{32}X + \zeta_3$$
>
> $$Y = \beta_{43}Z + \zeta_4$$
>
> with $E(U) = E(\zeta_i) = 0$ i=2,3,4. If I consider the expected value with the "do" operator:
>
> $$\begin{aligned} E(Y|do(x^0)) &= E(\beta_{43}(\beta_{32}x^0 + \zeta_3) + \zeta_4) \\ &= \beta_{43}\beta_{32}x^0 \end{aligned}$$
>
> if instead I consider the traditional expected value
>
> $$E(Y|X = x^0) = \beta_{43}\beta_{32}x^0 + \beta_{43}E(\zeta_3|X = x^0) + E(\zeta_4|X = x^0)$$
>
> The conditional value with do-operator and the traditional conditional value, then, are equal if, for example, errors ζ are independent of the variable x, which happens in the traditional structural models.

In Pearl's theory two types of direct effects exist , that just shown also called natural direct effect, and the controlled direct effect. The latter is defined as the effect of X when I keep the mediator variable constant . Its formula is the same both for the model of Figure A.1 (a) and for that of Figure A.1 (b), then the presence of the confounders variables is not effective. Then the controlled direct effect is

$$CDE_{x^0,x^1} = E(Y|do(x^1), do(z)) - E(Y|do(x^0), do(z))$$

Pearl does not advise the use of the controlled direct effect because it focuses on one of the values of Z, or may focus on the average of all the values of Z, and this can not capture the underlying structure. Of course, in "linear-in-parameters and linear-in-variables models", the controlled direct effect coincides with the natural direct effect. In the case in which Z is independent of X, then the natural direct effect is nothing more than the expected value of the controlled direct effect

PROOF

$$\begin{aligned} DE_{x^0,x^1} &= \sum_z \left[E(Y|do(x^1,z)) - E(Y|do(x^0,z))\right] P(z|do(x^0)) \\ &= \sum_z \left[E(Y|do(x^1,z)) - E(Y|do(x^0,z))\right] P(z) \\ &= \sum_z CDE_{x^0,x^1} P(z) \end{aligned}$$

These formulas are the same used in the case with one mediator, however Pearl (2014) also studies the case with two mediators. Pearl affirms that in this case the effects are not always all identified. I take the two graphs of Figure A2 from Pearl's paper (2014) to explain his example in a linear-in-parameter model. The β_3 in figure serves to introduce the interaction.[2]. I start from the case without interaction, then $\beta_3 = 0$. If I consider M as a mediator, the direct effect is made by the effect $T \to Y$ and by the effect $T \to W \to Y$; the indirect effect is made by the effect $T \to M \to Y$ and by the effect $T \to W \to M \to Y$. If I consider W as a mediator, the direct effect is made by the effect $T \to Y$ and by the effect $T \to M \to Y$; the indirect effect is made by the effect $T \to W \to Y$ and by the effect $T \to W \to M \to Y$. Considering indifferently as mediators M or W, the total effect is the sum of the direct and the indirect effect. With the introduction of the interaction, i.e. $\beta_3 \neq 0$, the total effect is no longer equal to the sum of the direct and the indirect effect. Under a more general form of this model, i.e. with the parameters which vary across observations (for example $\alpha = \alpha_i$), Imai et al. (2014) show that the combined effect $T \to M \to Y$ and $T \to W \to M \to Y$ is not identified. If a non parametric model is applied to a model with 2 mediators W and M, where W influences causally M, Pearl (2014) shows that only the effects mediated by W are identifiable; if, instead, W and M are only correlated given T the effect mediated by either M or W can not be identified. Then I note that the effects are always calculable only if the two mediators are uncorrelated given the variable T.

Now I apply Pearl's formulas just explained to the models proposed in chapter 2. The first application is used in the model with two mediators with uncorrelated errors; then it is the simple model shown in Figure A2 with $\beta_3 = \gamma_2 = 0$. Assuming that the structural errors are independent of each other and independent of X and considering Z as mediator, I get the following effects (replacing W in Y, i.e. $\beta_{31}X + \zeta_3 => Y = (\beta_{41} + \beta_{43}\beta_{31})X + \beta_{42}Z + \zeta_4 + \beta_{43}\zeta_3)$.

$$\begin{aligned} DE_{x^0,x^1}(Z) &= \sum_z \left[E(Y|x^1,z) - E(Y|x^0,z)\right] P(z|x^0) \\ &= \sum_z \left[\beta_{41}(x^1 - x^0) + \beta_{43}\beta_{31}(x^1 - x^0)\right] P(z|x^0) \\ &= (\beta_{41} + \beta_{43}\beta_{31})(x^1 - x^0) \end{aligned}$$

[2] Hayes (2013) considers the model shown in the Figure A.2 without the interaction. He calls "parallel multiple mediator model" the model in which γ_2 is equal to 0, while he calls "serial multiple mediator model" the model with γ_2 different from 0. According to Hayes's theory, the parallel multiple mediators can be correlated. In the parallel multiple mediator model he defines specific IE the indirect effect which is obtained by controlling for all other mediators of the model, while the total IE is the sum of the specific IE. In this case the two specific IE are $\alpha\beta_4$ and $\gamma_1\beta_1$. If I consider the serial multiple mediator model, the total IE effect is equal to $\alpha\beta_4 + \gamma_1\beta_1 + \alpha\gamma_2\beta_1$

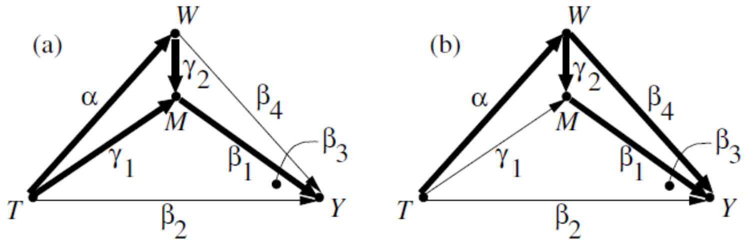

Figure A.2: (a) indirect effect with M mediator (b) indirect effect with W mediator

$$\begin{aligned}
IE_{x^0,x^1}(Z) &= \sum_z \left[E(Y|x^0,z)\right]\left[P(z|x^1)-P(z|x^0)\right]\\
&= \sum_z \left[(\beta_{41}+\beta_{43}\beta_{31})x^0+\beta_{42}z\right]\left[P(z|x^1)-P(z|x^0)\right]\\
&= \beta_{42}E(z|x^1)-\beta_{42}E(z|x^0)\\
&= \beta_{42}\beta_{21}(x^1-x^0)
\end{aligned}$$

$$TE_{x^0,x^1} = DE_{x^0,x^1}(Z)+IE_{x^0,x^1}$$

Symmetrically I calculate the effects with W as a mediator.

Now I calculate the model with two mediators and interaction. Unlike the model proposed by Pearl (Figure A2) in which the interaction is between the exogenous variable T and the endogenous variable M, my model considers the interaction between the two endogenous variables Z and W. Calculating the effects considering Z as mediator (replacing W in Y with $\beta_{31}X+\zeta_3 => Y = (\beta_{41}+\beta_{43}\beta_{31})X+(\beta_{42}+\beta_{45}\beta_{31}X)Z-\beta_{45}\mu_{ZW}+\zeta_4+\beta_{43}\zeta_3+\beta_{45}Z\zeta_3$)

$$\begin{aligned}
DE_{x^0,x^1}(Z) &= \sum_z \left[E(Y|x^1,z)-E(Y|x^0,z)\right]P(z|x^0)\\
&= \sum_z \left[\beta_{41}(x^1-x^0)+\beta_{43}\beta_{31}(x^1-x^0)+\beta_{45}\beta_{31}(x^1-x^0)z\right]P(z|x^0)\\
&= (\beta_{41}+\beta_{43}\beta_{31})(x^1-x^0)+\beta_{45}\beta_{31}(x^1-x^0)(\beta_{21}x^0)\\
&= (\beta_{41}+\beta_{43}\beta_{31}+\beta_{45}\beta_{31}\beta_{21}x^0)(x^1-x^0)
\end{aligned}$$

$$\begin{aligned}
IE_{x^0,x^1}(Z) &= \sum_z \left[E(Y|x^0,z)\right]\left[P(z|x^1)-P(z|x^0)\right]\\
&= \sum_z \left[-\beta_{45}\mu_{ZW}+(\beta_{41}+\beta_{43}\beta_{31})x^0+(\beta_{42}+\beta_{45}\beta_{31}x^0)z\right]\left[P(z|x^1)-P(z|x^0)\right]\\
&= \beta_{42}[E(z|x^1)-E(z|x^0)]+(\beta_{45}\beta_{31}x^0)[E(z|x^1)-E(z|x^0)]\\
&= (\beta_{42}\beta_{21}+\beta_{45}\beta_{31}\beta_{21}x^0)(x^1-x^0)
\end{aligned}$$

$$\begin{aligned}
TE_{x^0,x^1} &= DE_{x^0,x^1}(Z)-IE_{x^1,x^0}(Z)\\
&= (\beta_{41}+\beta_{43}\beta_{31}+\beta_{45}\beta_{31}\beta_{21}x^0)(x^1-x^0)-\left[(\beta_{42}\beta_{21}+\beta_{45}\beta_{31}\beta_{21}x^1)(x^0-x^1)\right]\\
&= (\beta_{41}+\beta_{42}\beta_{21}+\beta_{43}\beta_{31})(x^1-x^0)+\left(\beta_{45}\beta_{31}\beta_{21}x^0+\beta_{45}\beta_{31}\beta_{21}x^1\right)(x^1-x^0)\\
&= (\beta_{41}+\beta_{42}\beta_{21}+\beta_{43}\beta_{31})(x^1-x^0)+\beta_{45}\beta_{31}\beta_{21}(x^1+x^0)(x^1-x^0)\\
&= (\beta_{41}+\beta_{42}\beta_{21}+\beta_{43}\beta_{31})(x^1-x^0)+\beta_{45}\beta_{31}\beta_{21}\left[(x^1)^2-(x^0)^2\right]\\
&\neq DE_{x^0,x^1}(Z)+IE_{x^0,x^1}
\end{aligned}$$

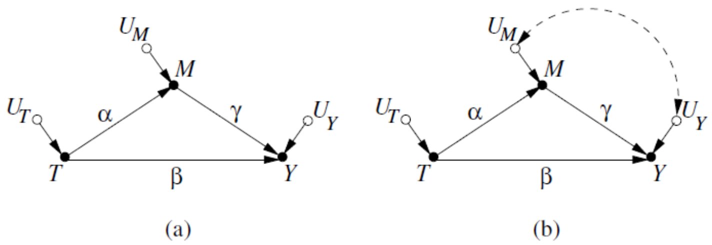

Figure A.3: (a) mediation model (b) mediation model with correlated errors

Symmetrically I calculate the effects with W mediator.

Now I consider the model with two mediators and correlated errors; before analyzing it, I consider an example made by Pearl (2014). The example can be simplified in Figure A3 (taken from Pearl 2014). Figure (a) represents a case of mediation without confounders in which the effects can be calculated. Figure (b) is a case where it is not possible to calculate the effects because there is the correlation between the errors: indeed "adjusting" for M, particular spurious correlations between T and Y are created. In the model I proposed in chapter 2, i.e. in the model with 2 mediators and correlated errors, unlike that of Figure A3 (b), it is possible to calculate the effects using my modification of Pearl's formulas. If I consider the 2 mediators Z and W together and I "adjust" for both, the spurious relationships are not created, indeed the correlation between the structural errors causes the variation of Z and of W together, but keeps costant the other variables (X) and the other errors (ζ_4). I consider the modified direct effect

$$DE^M_{x^0,x^1} = \sum_{\zeta_2,\zeta_3}\sum_{z,w}\left[E(Y|x^1,z,w) - E(Y|x^0,z,w)\right]P(z,w|x^0,\zeta_2,\zeta_3)P(\zeta_2,\zeta_3)$$

where (x, z and w) are the variables which affect directly Y and (x, ζ_2 and ζ_3) are the variables which influence directly Z and W. The indirect effect is

$$IE^M_{x^0,x^1} = \sum_{\zeta_2,\zeta_3}\sum_{z,w} E(Y|x^0,z,w)\left[P(z,w|x^1),\zeta_2,\zeta_3) - P(z,w|x^0,\zeta_2,\zeta_3)\right]P(\zeta_3,\zeta_2)$$

where, as for the direct effect, (x, z and w) are the variables which affect directly Y and (x, ζ_2 and ζ_3) are the variables which affect directly Z and W. The total effect is that of Pearl:

$$TE^M_{x^0,x^1} = DE_{x^0,x^1}(Z) - IE_{x^1,x^0}$$

Now I apply these formulas to the model with two mediators and correlated errors. Recalling that

$$E(X) = \sum_R xf(x,y)$$

I obtain

$$\begin{aligned}
DE^M_{x^0,x^1} &= \sum_{\zeta_2,\zeta_3}\sum_{z,w}\left[E(Y|x^1,z,w) - E(Y|x^0,z,w)\right]P(z,w|x^0,\zeta_2,\zeta_3)P(\zeta_2,\zeta_3)\\
&= \sum_{\zeta_2,\zeta_3}\sum_{z,w}(\beta_{41}x^1 - \beta_{41}x^0)P(z,w|x^0,\zeta_2,\zeta_3)\\
&= \beta_{41}(x^1 - x^0)
\end{aligned}$$

$$\begin{aligned} IE^M_{x^0,x^1} &= \sum_{\zeta_2,\zeta_3}\sum_{z,w} E(Y|x^0,z,w)\left[P(z,w|x^1,\zeta_2,\zeta_3) - P(z,w|x^0,\zeta_2,\zeta_3)\right]P(\zeta_3,\zeta_2) \\ &= \sum_{\zeta_2,\zeta_3}\sum_{z,w}(\beta_{41}x^0 + \beta_{42}z + \beta_{43}w)\left[P(z,w|x^1,\zeta_2,\zeta_3) - P(z,w|x^0,\zeta_2,\zeta_3)\right]P(\zeta_3,\zeta_2) \\ &= \sum_{\zeta_2,\zeta_3}\left[\beta_{42}\beta_{21}(x^1 - x^0) + \beta_{43}\beta_{31}(x^1 - x^0)\right]P(\zeta_3,\zeta_2) \\ &= (\beta_{42}\beta_{21} + \beta_{43}\beta_{31})(x^1 - x^0) \end{aligned}$$

$$TE_{x^0,x^1} = (\beta_{41} + \beta_{42}\beta_{21} + \beta_{43}\beta_{31})(x^1 - x^0)$$

Now I apply the modified formulas to the model with two mediators with correlated errors and interaction.

$$\begin{aligned} DE^M_{x^0,x^1} &= \sum_{\zeta_2,\zeta_3}\sum_{z,w}\left[E(Y|x^1,z,w) - E(Y|x^0,z,w)\right]P(z,w|x^0,\zeta_2,\zeta_3)P(\zeta_2,\zeta_3) \\ &= \sum_{\zeta_2,\zeta_3}\sum_{z,w}(\beta_{41}x^1 - \beta_{41}x^0)P(z,w|x^0,\zeta_2,\zeta_3) \\ &= \beta_{41}(x^1 - x^0) \end{aligned}$$

$$\begin{aligned} IE^M_{x^0,x^1} &= \sum_{\zeta_2,\zeta_3}\sum_{z,w} E(Y|x^0,z,w)\left[P(z,w|x^1,\zeta_2,\zeta_3) - P(z,w|do(x^0),\zeta_2,\zeta_3)\right]P(\zeta_3,\zeta_2) \\ &= \sum_{\zeta_2,\zeta_3}\sum_{z,w}(-\beta_{45}\mu_{ZW} + \beta_{41}x^0 + \beta_{42}z + \beta_{43}w + \beta_{45}zw)\left[P(z,w|x^1,\zeta_2,\zeta_3) - P(z,w|x^0,\zeta_2,\zeta_3)\right] \\ &\quad P(\zeta_3,\zeta_2) \\ &= \sum_{\zeta_2,\zeta_3}\{\beta_{42}\beta_{21}(x^1 - x^0) + \beta_{43}\beta_{31}(x^1 - x^0) + \beta_{45}\beta_{31}\beta_{21}[(x^1)^2 - (x^0)^2] + \beta_{45}\beta_{31}\zeta_2[x^1 - x^0] \\ &\quad + \beta_{45}\beta_{21}\zeta_3[x^1 - x^0]\}P(\zeta_3,\zeta_2) \\ &= (\beta_{42}\beta_{21} + \beta_{43}\beta_{31})(x^1 - x^0) + \beta_{45}\beta_{31}\beta_{21}[(x^1)^2 - (x^0)^2] \end{aligned}$$

$$TE_{x^0,x^1} = (\beta_{41} + \beta_{42}\beta_{21} + \beta_{43}\beta_{31})(x^1 - x^0) + \beta_{45}\beta_{21}\beta_{31}\left[(x^1)^2 - (x^0)^2\right]$$

The causal analysis is interested both in the direct, indirect and total effect and in the relationship between the indirect effect and the total effect. As noted by Pearl (2012), the ratio IE/TE is equal to the ratio $1 - (DE/TE)$ when $TE = DE + IE$. The ratio IE/TE represents the fraction for which mediation is sufficient, i.e. the fraction of Y which owes its value to one variable Z. The ratio $1 - (DE/TE)$ represents the fraction for which mediation is necessary, i.e. the fraction of Y which owes its value to the ability of X to influence the variable Z.

Now I calculate the ratio IE/TE for the model with two mediators with correlated errors. I hypothesize that $x^1 = x^0 + 1$, then in the case of a change of one unit the ratio is

$$\frac{IE}{TE} = \frac{\beta_{42}\beta_{21} + \beta_{43}\beta_{31}}{\beta_{41} + \beta_{42}\beta_{21} + \beta_{43}\beta_{31}}$$

and it is constant for each x^0. This does not happen if I calculate the same ratio for the model with two mediators with correlated errors and interaction

$$\frac{IE^M}{TE} = \frac{\beta_{42}\beta_{21} + \beta_{43}\beta_{31} + \beta_{45}\beta_{31}\beta_{21}(2x^0 + 1)}{\beta_{41} + \beta_{42}\beta_{21} + \beta_{43}\beta_{31} + \beta_{45}\beta_{31}\beta_{21}(2x^0 + 1)}$$

where IE^M/TE dipends on x^0, i.e. on initial value of the variation. However for any scale with which I measure x^0, the ratio remains the same.

PROOF

For simplicity I replace the parameters in this way

$$\begin{aligned} a &= \beta_{42}\beta_{21} + \beta_{43}\beta_{31} \\ b &= \beta_{45}\beta_{31}\beta_{21} \\ c &= \beta_{41} \end{aligned}$$

so

$$\frac{IE^M}{TE} = \frac{a + b(2x^0 + 1)}{c + a + b(2x^0 + 1)}$$

To proof the indipendence of scale, I start from Moosbrugger's analysis, made for interaction between exogenous variables (Moosbrugger et al , 1997). Now I change X scale:

$$\begin{aligned} z^0 &= \beta_{21}x^0 + \zeta_2 = \beta_{21}(x^s - \Delta x) + \zeta_2 = \beta_{21}x^s + m_z + \zeta_2 \\ w^0 &= \beta_{31}x^0 + \zeta_3 = \beta_{31}(x^s - \Delta x) + \zeta_3 = \beta_{31}x^s + m_w + \zeta_3 \end{aligned}$$

where $\Delta x = x^s - x^0$ As shown for the interaction between exogenous variables, the parameters β_{42} and β_{43} are not indipendent of the linear transformation because of presence of the interaction. For this reason, these parameters must be interpreted according to the scale of the variables Z and W (Moosbrugger et al., 1997) and in my case, considering the endogenous interaction, according to the scale of variable X. Now I consider the effect of the change of X scale on the interaction term:

$$\begin{aligned} \beta_{45}z^0w^0 &= \beta_{45}\left[\beta_{21}(x^s - \Delta x) + \zeta_2\right]\left[\beta_{31}(x^s - \Delta x) + \zeta_3\right] \\ &= \beta_{45}\left[\beta_{21}\beta_{31}(x^s - \Delta x)^2 + \underbrace{\beta_{21}(x^s - \Delta x)\zeta_3 + \beta_{31}(x^s - \Delta x)\zeta_2 + \zeta_2\zeta_3}_{error_{int}}\right] \\ &= \beta_{45}\beta_{21}\beta_{31}(x^s)^2 - 2\beta_{45}\beta_{21}\beta_{31}x^s\Delta x + \beta_{45}\beta_{21}\beta_{31}(\Delta x)^2 + \beta_{45}error_{int} \\ &= \beta_{45}\beta_{21}\beta_{31}(x^s)^2 - 2\beta_{45}\beta_{21}\beta_{31}\Delta x x^s + \beta_{45}m_{int} + \beta_{45}error_{int} \end{aligned}$$

=>

$$Y = [\beta_{41} + \beta_{42}\beta_{21} + \beta_{43}\beta_{31}]x^s + \beta_{45}\beta_{31}\beta_{21}(x^s)^2 - 2\beta_{45}\beta_{21}\beta_{31}\Delta x x^s + m_y + error_y$$

Then the parameters of IE^M/TE ratio becomes:

$$\begin{aligned} a^s &= \beta_{42}\beta_{21} + \beta_{43}\beta_{31} - 2\beta_{45}\beta_{21}\beta_{31}\Delta x = a - 2b\Delta x \\ b^s &= \beta_{45}\beta_{31}\beta_{21} = b \\ c^s &= \beta_{41} = c \end{aligned}$$

then I calculate the values for which the relation is true

$$\frac{a + b(2x^0 + 1)}{c + a + b(2x^0 + 1)} = \frac{a^s + b(2x^s + 1)}{c + a^s + b(2x^s + 1)}$$

=>

$$\left[c + a^s + b(2x^s + 1)\right]\left[a + b(2x^0 + 1)\right] = \left[c + a + b(2x^0 + 1)\right]\left[a^s + b(2x^s + 1)\right]$$

eliding $a^s a$ e $b(2x^s + 1)b(2x^0 + 1)$, I obtain

$$\begin{aligned} & c\left[a + b(2x^0 + 1)\right] + a^s b(2x^0 + 1) + ab(2x^s + 1) \\ & - c\left[a^s + b(2x^s + 1)\right] - ab(2x^s + 1) - a^s b(2x^0 + 1) = 0 \end{aligned}$$

=>

$$c\left[a + b(2x^0 + 1)\right] - c\left[a^s + b(2x^s + 1)\right] = 0$$

Recalling the relation between a and a^s:

$$c\left[a + b(2x^0 + 1)\right] - c\left[a - 2b\Delta x + b(2x^s + 1)\right] = 0$$

=>

$$cb(2x^0 + 1) + 2cb\Delta x - cb(2x^s + 1) = 2x^0 cb - 2x^s cb + 2cb\Delta x = 0$$

and I have that the equality holds for every value.

APPENDIX $\mathcal{B}$

MODEL WITH 2 MEDIATORS VS MODEL WITH 2 MEDIATORS AND INTERACTION

Now I analyze how the estimated parameters change if I estimate a model without interaction on a dataset where it is present, i.e.

$$Y = \beta_{41}X + \beta_{42}Z + \beta_{43}W + \underbrace{\beta_{45}INT + \zeta_4}_{\zeta_4^{new}}$$

$$Z = \beta_{21}X + \zeta_2$$

$$W = \beta_{31}X + \zeta_3$$

With normal datasets the rule that the regressors must be incorrelated with the errors term is respected, because the regressors X, Z e W are incorrelated with ζ_4^{new}. The first relation (Cov=expected value) is true because all variables are centered.

$$\begin{aligned} Cov(X, \zeta_4^{new}) &= E(X, \beta_{45}INT + \zeta_4) \\ &= \beta_{45}E\left(X, \beta_{21}\beta_{31}X^2 + \beta_{21}X\zeta_3 + \beta_{31}X\zeta_2 + \zeta_2\zeta_3 - \beta_{21}\beta_{31}Var(X) - E(\zeta_2, \zeta_3)\right) = 0 \end{aligned}$$

This can be so explained:

$$\begin{aligned} \beta_{21}\beta_{31}E(X^3) = 0 &\quad \text{because the third moment of a normal variable is equal to 0} \\ \beta_{31}E(X^2, \zeta_2) = 0 &\quad \text{because X and } \zeta_2 \text{ are indipendent} \\ \beta_{21}E(X^2, \zeta_3) = 0 &\quad \text{because X and } \zeta_3 \text{ are indipendent} \\ E(X, \zeta_2\zeta_3) = 0 &\quad \text{because X is indipendent from } \zeta_2 \text{ and } \zeta_3 \end{aligned}$$

Now I consider

$$\begin{aligned} Cov(Z, \zeta_4^{new}) &= E(Z, \beta_{45}INT + \zeta_4) \\ &= \beta_{45}E\left(\beta_{21}X + \zeta_2, \beta_{21}\beta_{31}X^2 + \beta_{21}X\zeta_3 + \beta_{31}X\zeta_2 + \zeta_2\zeta_3 - \beta_{21}\beta_{31}Var(X) - E(\zeta_2, \zeta_3)\right) \\ &= 0 \end{aligned}$$

$$
\begin{aligned}
\beta_{21}^2\beta_{31}E(X^3) = 0 &\quad \text{because the third moment of a normal variable is equal to 0} \\
\beta_{21}\beta_{31}E(X^2,\zeta_2) = 0 &\quad \text{because X and } \zeta_2 \text{ are indipendent and } E(\zeta_2) = 0 \\
\beta_{21}^2E(X^2,\zeta_3) = 0 &\quad \text{because X and } \zeta_3 \text{ are indipendent and } E(\zeta_3) = 0 \\
2\beta_{21}E(X,\zeta_2\zeta_3) = 0 &\quad \text{because X is indipendent from } \zeta_2 \text{ and } \zeta_3 \text{ and } E(X) = 0 \\
\beta_{21}\beta_{31}E(X^2,\zeta_2) = 0 &\quad \text{because X and } \zeta_2 \text{ are indipendent and } E(\zeta_2) = 0 \\
\beta_{31}E(X,\zeta_2^2) = 0 &\quad \text{because X and } \zeta_2 \text{ are indipendent and } E(X) = 0 \\
E(\zeta_2^2,\zeta_3) = 0 &\quad \text{because the distribution of } (\zeta_2,\zeta_3) \text{ is a multivariate normal} \\
&\quad \text{and the k-order moment is equal to 0 if k is odd}
\end{aligned}
$$

With the same analysis, I obtain that $Cov(W,\zeta_4^{new}) = 0$. Now I compare the estimatated parameters obtained using the model without interaction and those obtained using the model with interaction. This can be seen as a problem of omitted variables. Table B.1 shows the estimation for the dataset with observed variables and covariance equal to 0.4 and 0.13. I do not show the estimation of dataset with covariance -0.4, because the results are very similar to those obtained in these 2 groups of datasets. Of course the estimated parameters, which change, are those of regression on Y. The coverage index increases, slightly in the case with cov=0.4, when I introduce the interaction and it is correct because, being this the true model, the estimation accuracy increases. The residual variance of Y is different in the 2 models because:

- in the model without interaction it is equal to $Var(\zeta_4^{new}) = Var(\zeta_4) + \beta_{45}^2Var(INT) = 0.58 + 0.23^2Var(INT)$
- in the model with interaction it is equal to $Var(\zeta_4) = 0.58$

Then, I affirm that if the researcher does not insert the interaction term, a problem of omitted variables occurs. If the variables are normal, the estimation is still unbiased, if the variables are not normal, the dataset does not respect the rule of incorrelation between the regressors and the errros beacause the variables X, Z e W are correlated with error term ζ_4 (one of the principal rules of structural equation model) and then the estimates are biased. When I add the INT mediator, i.e. I add a regressor among the regressors of the variables Y, the r-squared increases.

Cov=0.4

MODEL without interaction

	Population	ESTIMATES Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2698	0.0764	0.0767	0.0058	0.944	0.932
Z	0.450	0.4525	0.0808	0.0818	0.0065	0.952	1.000
W	0.570	0.5687	0.0860	0.0849	0.0074	0.936	1.000
Z ON							
X	0.630	0.6305	0.0605	0.0605	0.0037	0.958	1.000
W ON							
X	0.770	0.7693	0.0588	0.0582	0.0034	0.951	1.000
Z WITH							
W	0.400	0.3991	0.0274	0.0285	0.0007	0.953	1.000
Variances							
X	0.280	0.2809	0.0172	0.0177	0.0003	0.946	1.000
Residual Variances							
Z	0.520	0.5186	0.0319	0.0328	0.0010	0.949	1.000
W	0.480	0.4793	0.0289	0.0302	0.0008	0.954	1.000
Y		0.6127	0.0389	0.0390			1.000

MODEL with interaction

	Population	ESTIMATES Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2697	0.0740	0.0745	0.0055	0.945	0.943
INT	0.230	0.2304	0.0410	0.0407	0.0017	0.946	1.000
Z	0.450	0.4523	0.0775	0.0785	0.0060	0.953	1.000
W	0.570	0.5684	0.0823	0.0817	0.0068	0.945	1.000
Z ON							
X	0.630	0.6305	0.0605	0.0605	0.0037	0.958	1.000
W ON							
X	0.770	0.7693	0.0588	0.0582	0.0034	0.951	1.000
Z WITH							
W	0.400	0.3991	0.0274	0.0285	0.0007	0.953	1.000
INT WITH							
W	0.000	0.0005	0.0483	0.0490	0.0023	0.963	0.037
Z	0.000	0.0005	0.0506	0.0513	0.0026	0.953	0.047
X	0.000	0.0004	0.0265	0.0268	0.0007	0.953	0.047
Variances							
X	0.280	0.2809	0.0172	0.0177	0.0003	0.946	1.000
INT		0.6946	0.1117	0.1095			1.000
Residual Variances							
Z	0.520	0.5186	0.0319	0.0328	0.0010	0.949	1.000
W	0.480	0.4793	0.0289	0.0302	0.0008	0.954	1.000
Y	0.580	0.5752	0.0363	0.0362	0.0013	0.934	1.000

R-SQUARE (MODEL without interaction)

Observed Variable	ESTIMATES Average	Std. Dev.	S. E. Average	% Sig Coeff
Z	0.178	0.031	0.031	1.000
W	0.258	0.034	0.033	1.000
Y	0.549	0.030	0.030	1.000

R-SQUARE (MODEL with interaction)

Observed Variable	ESTIMATES Average	Std. Dev.	S. E. Average	% Sig Coeff
Z	0.178	0.031	0.031	1.000
W	0.258	0.034	0.033	1.000
Y	0.576	0.031	0.031	1.000

Cov=0.13

MODEL without interaction

	Population	ESTIMATES Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2688	0.0775	0.0789	0.0060	0.955	0.931
Z	0.450	0.4512	0.0508	0.0511	0.0026	0.949	1.000
W	0.570	0.5710	0.0543	0.0531	0.0029	0.932	1.000
Z ON							
X	0.630	0.6315	0.0594	0.0606	0.0035	0.947	1.000
W ON							
X	0.770	0.7693	0.0588	0.0582	0.0034	0.951	1.000
Z WITH							
W	0.130	0.1293	0.0228	0.0230	0.0005	0.948	0.999
Variances							
X	0.280	0.2809	0.0172	0.0177	0.0003	0.946	1.000
Residual Variances							
Z	0.520	0.5183	0.0319	0.0327	0.0010	0.944	1.000
W	0.480	0.4793	0.0289	0.0302	0.0008	0.954	1.000
Y		0.6015	0.0382	0.0381			1.000

MODEL with interaction

	Population	ESTIMATES Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.270	0.2691	0.0751	0.0773	0.0056	0.960	0.940
INT	0.230	0.2309	0.0488	0.0490	0.0024	0.950	0.996
Z	0.450	0.4514	0.0481	0.0487	0.0023	0.953	1.000
W	0.570	0.5700	0.0514	0.0507	0.0026	0.941	1.000
Z ON							
X	0.630	0.6315	0.0594	0.0606	0.0035	0.947	1.000
W ON							
X	0.770	0.7693	0.0588	0.0582	0.0034	0.951	1.000
Z WITH							
W	0.130	0.1293	0.0228	0.0230	0.0005	0.948	0.999
INT WITH							
W	0.000	0.0018	0.0349	0.0345	0.0012	0.941	0.059
Z	0.000	0.0000	0.0360	0.0366	0.0013	0.956	0.044
X	0.000	0.0005	0.0222	0.0230	0.0005	0.952	0.048
Variances							
X	0.280	0.2809	0.0172	0.0177	0.0003	0.946	1.000
INT		0.4793	0.0687	0.0676			1.000
Residual Variances							
Z	0.520	0.5183	0.0319	0.0327	0.0010	0.944	1.000
W	0.480	0.4793	0.0289	0.0302	0.0008	0.954	1.000
Y	0.580	0.5752	0.0363	0.0362	0.0013	0.934	1.000

R-SQUARE (MODEL without interaction)

Observed Variable	ESTIMATES Average	Std. Dev.	S. E. Average	% Sig Coeff
Z	0.178	0.030	0.031	1.000
W	0.258	0.034	0.033	1.000
Y	0.502	0.032	0.032	1.000

R-SQUARE (MODEL with interaction)

Observed Variable	ESTIMATES Average	Std. Dev.	S. E. Average	% Sig Coeff
Z	0.178	0.030	0.031	1.000
W	0.258	0.034	0.033	1.000
Y	0.524	0.032	0.032	1.000

Table B.1: Models with 2 mediators and correlated errors

APPENDIX C

THEORICAL STANDARDIZED LOADINGS AND ESTIMATED STANDARDIZED LOADINGS

In this section I compare Moosbrugger's standardized loadings ant the estimated standardized loadings. To make this analysis, I use the datasets of Tables 4.1, 4.3, 5.4 and 5.5, i.e. respectively:

- interaction model with CR=0.95 and covariance between the structural errors equal to 0.4
- interaction model with CR=0.95 and covariance between the structural errors equal to 0.13
- interaction model with CR=0.95 and covariance between the structural errors equal to -0.4
- interaction model with CR=0.87 and covariance between the structural errors equal to 0.4
- interaction model with CR=0.87 and covariance between the structural errors equal to 0.13
- interaction model with CR=0.95 and covariance between the structural errors equal to -0.4

The values are shown in Table C1. I compare the bias of the standardized loadings, which is so calculated:

$$\text{BIAS} = \frac{\text{Estimated standardized loadings} - \text{ Moosbrugger'standardized loadings}}{\text{Moosbrugger'standardized loadings}}$$

I note that the estimated standardized loadings are very close to Moosbrugger's theorical loadings. For this reason, I affirm that Marsh's indicators produce good estimates. When the goodness of the measurement part decreases, i.e. when the CR decreases, the absolute value of the bias increses, for example

- the average of absolute value of bias of st. loadings of W2 with CR=0.95 and cov=0.4 is 0.000359, while in the same bias with CR=0.87 is 0.000796
- the average of absolute value bias of st. loadings of W2 with CR=0.95 and cov=0.13 is 0, while in the same bias with CR=0.87 is 0.000796
- the average of absolute value bias of st. loadings of W2 with CR=0.95 and cov=-0.4 is 0.00036, while in the same bias with CR=0.87 is 0.00119
- the average of absolute value bias of st. loadings of INT with CR=0.95 and cov=0.4 is 0.00278, while in the same bias with CR=0.87 is 0.0158
- the average of absolute value bias of st. loadings of INT with CR=0.95 and cov=0.13 is 0.00142, while in the same bias with CR=0.87 is 0.00928

		CR=0.95			CR=0.87		
		theorical loadings	Estimated loadings	bias	theorical loadings	Estimated loadings	bias
ZQ	0.4	0.927	0.926	-0.00108	0.835	0.834	-0.0012
		0.927	0.926	-0.00108	0.835	0.834	-0.0012
		0.927	0.926	-0.00108	0.835	0.834	-0.0012
	0.13	0.927	0.926	-0.00108	0.835	0.834	-0.0012
		0.927	0.926	-0.00108	0.835	0.834	-0.0012
		0.927	0.926	-0.00108	0.835	0.834	-0.0012
	-0.4	0.927	0.926	-0.00108	0.835	0.834	-0.0012
		0.927	0.926	-0.00108	0.835	0.834	-0.0012
		0.927	0.926	-0.00108	0.835	0.834	-0.0012
WQ	0.4	0.928	0.928	0	0.837	0.836	-0.00119
		0.928	0.928	0	0.837	0.837	0
		0.928	0.929	0.001078	0.837	0.838	0.001195
	0.13	0.928	0.928	0	0.837	0.837	0
		0.928	0.928	0	0.837	0.838	0.001195
		0.928	0.928	0	0.837	0.838	0.001195
	-0.4	0.928	0.928	0	0.837	0.836	-0.00119
		0.928	0.928	0	0.837	0.836	-0.00119
		0.928	0.927	-0.00108	0.837	0.836	-0.00119
INT	0.4	0.958	0.956	-0.00209	0.907	0.894	-0.01433
		0.958	0.955	-0.00313	0.907	0.892	-0.01654
		0.958	0.955	-0.00313	0.907	0.892	-0.01654
	0.13	0.938	0.937	-0.00107	0.862	0.855	-0.00812
		0.938	0.937	-0.00107	0.862	0.854	-0.00928
		0.938	0.936	-0.00213	0.862	0.853	-0.01044
	-0.4	0.938	0.937	-0.00107	0.862	0.854	-0.00928
		0.938	0.936	-0.00213	0.862	0.853	-0.01044
		0.938	0.936	-0.00213	0.862	0.852	-0.0116

Table C.1: Comparison between the Moosbrugger's st. loadings and estimated st. loadings

- the average of absolute value bias of st. loadings of INT with CR=0.95 and cov=0.4 is 0.00533, while in the same bias with CR=0.87 is 0.01044

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