UNIVERSITÀ DEGLI STUDI DI PADOVA

Sede Amministrativa: Università degli Studi di Padova
Dipartimento: Psicologia dello Sviluppo e della Socializzazione

SCUOLA DI DOTTORATO DI RICERCA IN SCIENZE PSICOLOGICHE

INDIRIZZO: PSICOLOGIA DELLO SVILUPPO E DEI PROCESSI DI SOCIALIZZAZIONE

CICLO: XXIV

WHY MENTAL CALCULATION IS SO COMPLICATED? THE CONTRIBUTION OF WORKING MEMORY COMPONENTS IN CHILDREN WITH TYPICAL DEVELOPMENT AND LEARNING DISABILITIES.

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ENGLISH SUMMARY

Many studies support that working memory (WM Baddeley, 1986) is related to mental calculation in general but also confirm that this relationship is more complex than previously described. Although increasing numbers of recent studies have investigated these relationships, the involvement of various WM subcomponents in mental addition problems remains unclear, and empirical evidence of how one's WM tackles problems is sparse.

A theoretical framework that is particularly appropriate for studying WM involvement in mental addition is the multi-component WM model developed by Baddeley and Hitch (1974; Baddeley, 1986), which comprises three distinct components: a central executive and two slave systems, a verbal and a visuo-spatial WM components. Even though central executive involvement in mental calculations seems clear, the roles of the other two WM components in calculations are still not fully understood, especially in children. For this reason, the present study focuses only on the involvement of both verbal and visuo-spatial WM in children's mental calculations.

Thus, the main objective of this PhD dissertation is to increase the current understanding of the role of WM subcomponents in the execution of simple and complex mental addition problems in: *i)* typically developing children (Study I – Experiment 1-3), and in *ii)* children with a diagnosis of learning disabilities (Study II – Experiment 4-5).

A number of studies stated that the role of WM in multi-digit calculation seems to depend on several factors, such as the type of presentation format, the type of algorithm required (e.g.

additions or subtractions), the presence of carrying/borrowing or the nature of computation expected (Ashcraft & Kirk, 2001; DeStefano & LeFevre, 2004; Fürst & Hitch 2000; Imbo & LeFevre, 2010; Raghubar, Barnes, & Hecht, 2010; Trbovich & LeFevre, 2003).

Previous research analyzing WM involvement in mental calculation mainly used the dual task paradigm. In particular, participants perform a primary task (a mental calculation problem) in combination with a secondary task, which involves one WM component. The paradigm assumed that, if the primary and secondary tasks use overlapping cognitive resources, then performance on the primary task will worsen as the secondary task becomes more demanding. This approach has been widely used with typically achieving adults, but only rarely extended to children (McKenzie, Bull, & Gray, 2003; Imbo & Vandierendonck, 2007) – Study 1, and never used with children with learning disabilities – Study II.

Based on literature outlines, Study I has been carried out to examine as the presentation of a verbal or visuo-spatial WM task damages the execution of a mental calculation in children attending 3rd and 4th grade of primary school.

Specifically, three different experiments were conducted to analyze potential computation impairment (exact vs. approximate), the specific role of WM sub-components (i.e., letters recall vs. positions recall), manipulating both the presentation format of the operations (i.e., operations presented horizontally vs. operations presented vertically) and the complexity of operations themselves (presence vs. absence of carrying).

In Experiment 1, children were presented with exact and approximate addition problems with carrying, in Experiment 2 with exact and approximate addition problems without carrying, while in Experiment 3 approximate addition problems with and without carrying were directly compared.

The findings provide information about the specific involvement of WM components in children's mental addition operations, and suggest that WM components are deeply involved when calculation processes become more challenging and complex. In Experiment 1, analysis showed that horizontally presented problems were generally more impaired than vertically presented problems by verbal WM load, and, vice versa, vertical problems were more affected than horizontal one by visuo-spatial WM load. Moreover, this result was stronger for approximate than for exact calculation.

Differently from adult literature, results revealed that children generally found approximate calculation more difficult than exact calculation. In particular, in Experiment 2, where pupils had to compare exact and approximate calculations without carrying, children seemed to use the same strategy for both exact and approximate calculation in single tasks. Finally, in Experiment 3, in which children were asked to solve approximate addition problems only, the specific effect of verbal and visuo-spatial load emerged specifically in addition problems requiring a carrying procedure. Therefore, we assume that carrying is crucial to determine the specific involvement of different WM subcomponents in the solution process.

The association of learning disabilities with WM impairments has been demonstrated in a number of studies (Schuchardt, Maehler, & Hasselhorn, 2008; Swanson, 2006). The content of Study II was to extend the dual task paradigm to two different clinical samples: children diagnosed with developmental dyscalculia (DD – Experiment 4) and with non-verbal learning disability (NLD – Experiment 5).

Developmental Dyscalculia (DD), also called mathematical learning disability, is characterized by severe impairments in the acquisition of mathematical skills. Traditional classification systems (e.g. DSM-IV-TR; APA, 2000)state that the child must substantially underachieve on a standardized mathematical test relative to the level expected on the basis of his/her age, education, and intelligence and must experience disruption to academic achievement or daily living in order to receive a diagnosis of DD. In particular, there must be a considerable discrepancy between the child's general intellectual ability and the child's academic achievement.

Non-verbal learning disability (NLD) children are chiefly characterized by intact verbal abilities, but impaired visuospatial skills, showing a discrepancy between Verbal and Performance IQ, and major problems in areas of visuo-spatial working memory, psychomotor, visuo-constructive skills and mathematics, within a context of well-developed psycholinguistic skills. Although, this disorder is not included in any clinical classification systems, it is possible delineate some specific criteria for diagnosing children with NLD: a marked discrepancy between verbal and visuo-spatial intelligence associated with a specific pattern in academic achievement, characterized by major learning difficulties in arithmetic, geometry and science (Mammarella, Lucangeli & Cornoldi, 2010).

In both Experiment 4 and 5, it has been decide to present only exact addition problems with carrying. In fact, previous results of Study I showed that TD children do not spontaneously and easily use the most functional strategies employed by adults. In each Experiment, clinical sample performance has been compared to a control group, formed by typically developing children (TD) matched for age, schooling and socio-economic status, with no reported school difficulties.

Finding of Study II shown that the dual task paradigm applied to children with learning disabilities revealed that their performances did not completely overlap those observed in TD children. In particular, children with DD performed poorly when addition problems were presented in horizontal format and associated with verbal information, revealing that verbal weaknesses are critical in the majority of children with DD.

Conversely, NLD results are perfectly in line with those emerged in Study I with TD children for approximate calculation, revealing as the presence of carrying procedure makes the primary task sufficiently highly demanding on WM resources to produce a selective interference between presentation format and secondary task in NLD children

Taken as a whole, the results of the present study offer a general picture on how children meet with mental addition request and put forward important clinical and educational implications, further confirming that mathematical deficits could depend on poor WM resources. These findings are noteworthy not only in order to deepen the understanding of the relationship between memory processes and calculation, but also to provide scientific evidence to plan functional and specific intervention for learning disability treatments, based on the actual processes involved in the solution phase.

ITALIAN SUMMARY

Gli studi a sostegno del coinvolgimento della memoria di lavoro (ML, Baddeley, 1986) nell'esecuzione del calcolo a mente sono numerosi, ma allo stesso tempo confermano quanto complessa sia questa relazione. Nonostante l'interesse crescente per quest'ambito, il ruolo delle diverse componenti della ML nello svolgimento di addizioni a mente rimane ancora poco chiaro e le evidenze a riguardo sono spesso incoerenti.

Secondo il modello multi-componenziale di Baddeley (1986) sia l'esecutivo centrale, che il loop fonologico ed il taccuino visuo-spaziale sono coinvolti, a vari livelli, nell'esecuzione di calcoli a mente. Sulla base di tale modello sono stati effettuati diversi studi che, se da un lato permettono di confermare il coinvolgimento della ML, dall'altro non sempre hanno portato a risultati univoci e coerenti su come le diverse componenti entrino in gioco, ad esempio, nonostante il ruolo dell'esecutivo centrale sembri ampiamente riconosciuto, il ruolo delle altre sotto-componenti appare ancora molto confuso, soprattutto nei bambini. Per questa ragione, gli studi descritti in questa tesi si sono focalizzati nell'analisi del coinvolgimento specifico delle sole componenti verbale e visuo-spaziale della ML nell'esecuzione di calcoli a mente in bambini frequentanti la scuola primaria.

La presente tesi di Dottorato mira pertanto ad indagare il ruolo della ML, nello specifico le sue componenti verbale e visuo-spaziale, nella soluzione di operazioni di addizione, con o senza riporto, in *i)* bambini di età scolare a sviluppo tipico (Studio 1 – Esperimenti 1-3), e in *ii)* bambini con diagnosi di disturbo specifico dell'apprendimento (Studio 2 – Esperimenti 4-5).

Ricerche recenti hanno messo in luce come l'esecuzione del calcolo possa coinvolgere domini diversi e quindi diversi aspetti della ML, in relazione a quelli che sono le caratteristiche stesse del compito aritmetico o la complessità dell'algoritmo presentato (DeStefano & LeFevre, 2004; Raghubar, Barnes, & Hecht, 2010). Il ruolo della ML nella soluzione di calcoli a mente a più cifre sembra dunque dipendere da diversi fattori, come ad esempio il tipo di formato di presentazione (Trbovich & LeFevre, 2003), il tipo di algoritmo coinvolto (Imbo & LeFevre, 2010), oppure la complessità del calcolo (Fürst & Hitch 2000; Ashcraft & Kirk, 2001). Kalaman e LeFevre (2007) hanno inoltre analizzato come l'influenza della ML possa variare in relazione alla tipologia di stima richiesta: addizioni in condizione di calcolo esatto o approssimato.

La metodologia utilizzata con maggior frequenza per analizzare il ruolo della ML nel calcolo a mente è il paradigma del doppio compito. Tale paradigma richiede ai partecipanti di svolgere il compito principale (un calcolo a mente) in combinazione con uno specifico compito secondario, che coinvolge selettivamente una specifica componente della ML (ad es. dominio verbale vs. visuo-spaziale). Il paradigma assume che, se compito primario e secondario vanno ad attingere alle stesse risorse cognitive, la prestazione al compito primario sarà destinata a peggiorare. Questo approccio è stato ampiamente utilizzato con partecipanti adulti, ma solo raramente esteso a bambini a sviluppo tipico (McKenzie, Bull, & Gray, 2003; Imbo & Vandierendonck, 2007) – Studio I, e mai proposto a bambini con diagnosi di disturbo specifico dell'apprendimento – Studio II.

Sulla base dei risultati emersi dalla letteratura, attraverso il primo Studio si è voluto andare ad indagare come la presentazione di un compito di ML, verbale o visuo-spaziale, che precede l’esecuzione del calcolo a mente, possa compromettere o meno l’esecuzione di quest’ultimo in bambini frequentanti le classi 3^ e 4^ della scuola primaria. In modo specifico, tre diversi esperimenti hanno preso in esame l’eventuale compromissione del calcolo (esatto vs. approssimato) e la tipologia di compito di ML associato (ricordo di lettere vs. ricordo di posizioni) in relazione alla modalità di presentazione (in riga vs. in colonna) e alla complessità (riporto vs. no riporto) delle addizioni stesse.

Nell’Esperimento 1 è stato chiesto ai bambini di risolvere delle addizioni con riporto in condizione di calcolo esatto e approssimato, nell’Esperimento 2 di eseguire delle addizioni senza riporto sempre di calcolo esatto e approssimato, infine nell’Esperimento 3 è stata direttamente confrontata la prestazione di addizioni con e senza riporto nella sola condizione di calcolo approssimato.

I risultati forniscono inoltre informazioni importanti sullo specifico coinvolgimento del dominio verbale e visuo-spaziale nell’esecuzione delle addizioni, evidenziando come il ruolo di tali componenti diventi maggiormente esplicito a mano a mano che i processi di calcolo divengono sempre più impegnativi e complessi. Nell'esperimento 1, in cui è stato chiesto ai bambini di risolvere le operazioni di calcolo esatto e approssimato con il riporto, le analisi hanno mostrato come le addizioni presentate in riga fossero generalmente più compromesse di quelle presentate in colonna dalla presenza di un carico verbale, e, viceversa, come i problemi presentati in colonna fossero invece più danneggiati da un carico di natura visuo-spaziali rispetto a quelli presentati in riga. Questo effetto è emerso con maggior chiarezza nella condizione approssimata.

Diversamente da quanto emerge per gli adulti, le analisi rilevano che i bambini trovano più complessa l’esecuzione di calcoli approssimati piuttosto che di calcoli esatti. Nell'esperimento 2, in cui è stato chiesto ai bambini di risolvere i problemi senza il riporto, i bambini sembravano utilizzare la stessa strategia di soluzione sia per i calcoli esatti che per i calcoli approssimati.

Infine, nell’esperimento 3, in cui è stato chiesto ai bambini di risolvere addizioni con e senza riporto solamente in condizione di calcolo approssimato, l'effetto specifico di carico verbale e visuo-spaziale è emerso in particolare nelle addizioni che richiedevano l’esecuzione della procedura di riporto, confermando come la complessità del calcolo sia una caratteristica cruciale per discriminare il ruolo delle diverse componenti della ML all’interno del processo di soluzione del calcolo.

L’importante legame tra disturbi d’apprendimento e deficit a livello di ML è stato ampiamente dimostrato in un vasto numero di studi (Schuchardt, Maehler, & Hasselhorn, 2008; Swanson, 2006). Nel secondo Studio, il paradigma descritto in precedenza è stato applicato per analizzare il coinvolgimento della ML nell’esecuzione di addizioni in due differenti gruppi clinici: bambini con diagnosi di discalculia evolutiva (Developmental Dyscalculia, DD – Esperimento 4), e con diagnosi di disturbo dell’apprendimento non-verbale (Non-verbal Learning Disability, NLD – Esperimento 5).

La discalculia evolutiva (DD) è caratterizzata da disturbi nell'acquisizione di competenze matematiche. Sistemi di classificazione tradizionali (ad esempio, DSM-IV-TR; APA, 2000) affermano che un bambino per ricevere diagnosi di DD non deve raggiungere i normali livelli di competenza attesi in base a livello di età, di istruzione e di intelligenza ad un test standardizzato di matematica. In altre parole, ci deve essere una discrepanza notevole tra le capacità intellettive generali ed il rendimento scolastico in matematica.

Il disturbo dell’apprendimento non-verbale (NLD) si caratterizza invece per una difficoltà di carattere generale nell’elaborazione di informazioni visive e spaziali, all’interno di un profilo cognitivo in norma, in cui le abilità verbali sono preservate. Nonostante tale disturbo non trovi ancora spazio nei principali manuali diagnostici è possibile delineare alcuni chiari criteri per la diagnosi, quali ad esempio difficoltà cognitive specifiche di natura visuo-spaziale (ad esempio, discrepanza tra quoziente intellettivo verbale e di performance di almeno di 15 punti) associate ad un profilo di apprendimenti scolastici con cadute nell'area della matematica o in altre discipline che sottendono il coinvolgimento di abilità visuo-spaziali e grafo-motorie (Mammarella, Lucangeli & Cornoldi, 2010).

In entrambi gli esperimenti, è stato scelto di sottoporre ai bambini solo la condizione di calcolo esatto. Tale scelta è stata guidata dai risultati emersi dai precedenti esperimenti dello Studio I, che mostrano come bambini a sviluppo tipico non sappiano ancora spontaneamente usare strategie di arrotondamento che rendono, per gli adulti, i processi di stima veloci ed efficaci. In ciascuno dei due esperimenti, la prestazione dei gruppi clinici è stata inoltre confrontata con un gruppo di controllo formato da bambini a sviluppo tipico appaiati per genere età e livello socio-culturale.

I risultati di questo secondo Studio hanno portato in luce come le diverse componenti della ML siano diversamente coinvolte nell'esecuzione di calcoli a mente rispetto a quanto emerso analizzando le prestazioni dei bambini con sviluppo tipico (Studio I). In particolare, per quanto riguarda i bambini con DD il diverso coinvolgimento delle componenti della ML nell'esecuzione di operazioni di addizione sembra tradursi in una richiesta di risorse prevalentemente di natura verbale, indipendentemente dal formato di presentazione, diversamente da quanto emerso per il gruppo di controllo. Invece, per quanto riguarda il gruppo con NLD, il pattern che emerge sembra sostanzialmente coerente con i risultati dello Studio I riferiti al solo calcolo approssimato. Tali risultati evidenziano come la presenza della procedura di riporto renda l'esecuzione del calcolo altamente richiestiva per le risorse di ML possedute da questi bambini, tanto da determinare un'interferenza selettiva tra il formato di presentazione dell'operazione stessa e la natura del compito secondario.

In sintesi, i risultati dei presenti Studi offrono nel loro complesso un quadro generale piuttosto articolato di come i bambini affrontano e gestiscono le richieste cognitive derivate dall'esecuzione di addizioni a mente. Presi nel loro complesso, entrambi gli studi, offrono lo spunto per trarre delle importanti implicazioni, sia in ambito educativo che clinico, dimostrando come le difficoltà che i bambini incontrano nel risolvere calcoli a mente siano collegate a limitate risorse di ML. Tali evidenze risultano infatti significative non solo allo scopo di approfondire la comprensione della relazione che intercorre tra processi di memoria e di calcolo, ma anche per fornire evidenze scientifiche utili a impostare materiali per il trattamento delle difficoltà di apprendimento, capaci di tenere in considerazione gli effettivi processi attivati nella fase di soluzione.

CHAPTER 1

ARITHMETICAL LEARNING

Arithmetical learning is not only a fundamental part of children's education but it becomes also central expertise in several daily activities. The understanding of the simple four algorithms (i.e., being able to solve simple addition, multiplication, subtraction, and division problems) is a inescapable constraint of everyday life, providing essential means for dealing with a diverse variety of problem-solving situations. Actually in our daily lives most of our activities are related to and affected by numbers, from the important ability to comprehend the value of money when doing shopping, to the more entertaining ability to understand the score of our favourite sport team. Basic arithmetic also provides the foundation for more advanced mathematical skills that are central to more advanced scientific subjects. Consequently, understanding this fundamental intellectual skill is an important goal for cognitive science.

In this first chapter, the theoretical framework of numerical representation first, and of arithmetical knowledge after, will be delineated. The principal models and theories on mathematical cognition will be portrayed, analyzing different aspect of mathematical learning according to a normal development trajectory. The outline of the arithmetical capacities in young children is, by necessity, brief and illustrative, and a variety of research on individual differences, cultural factors, and early instruction that are not directly relevant to our immediate target will be omitted (to more detail see Baroody & Dowker, 2003; Bryant, 1995; Bryant & Nunes, 2002; Geary, 1994).

1. DEVELOPMENT OF NUMERICAL SKILLS: A BRIEF OVERVIEW FROM INFANTS TO PUPILS

A number is a mathematical object used to count and measure, could be outlined as a property of sets of elements. Mathematical algorithms are definite procedures that take one or more numbers as input and produce a number as output, and the study of numerical operations is called arithmetic. In addition to their use in counting and measuring, numbers are often used for labels (e.g., telephone numbers, postal code numbers), for ordering (serial numbers on banknotes), and for codes (e.g., the ISBN code for books or the ISO code for counties). In common use, the word number can mean the abstract object, the symbol, or the word for the number. To escape the ambiguity of the word “number”, the term *numerosity* is used to refer specifically to a measurable numerical quantity (Gelman & Gallistel, 1978). The term *numerosity* concerns the unique property of a set of elements that does not change when the characteristics of the elements vary. In other words, we recognize different sets of objects with the same number of items as equivalent, regardless the variation of perceptual variables as shape, color, spatial disposition or sensor modality of presentation (e.g., visual, auditory). In the next paragraph, the developmental perspective of numerical cognition will be shortly reviewed. First infants’ abilities with small and large numerosities will be discussed and followed by the description of preschoolers’ counting performance. Finally, the third section of this chapter will be dedicated to pupils' abilities both in solving simple and complex arithmetical tasks.

1.1 Number Sense: a Core ability

From birth, infants are sensitive to numerical information, in either the form of the exact number of objects in small sets (at least when set size is less than four) or in the form of the approximate number of objects in large sets. Both types of numerical sensitivity, for small and large quantities[1], are thought to be part of this *Number Sense* (Dehaene, 1997; Feigenson, Dehaene, & Spelke, 2004) a language independent system, shared with other animal species. This module, that can be define for humans as a pre-symbolic representation of number, is fundamental also in the animal world for survival and nurturing, such as, for example, discerning the number of approaching predators or choosing the largest herd of preys (Brannon & Terrace, 2000; Kilian, Yaman, von Fersen, & Gunturkun, 2003; McComb, Packer, & Pusey, 1994; Meck & Church, 1983; Uller, Jaeger, Guidry, & Martin, 2003).

Many studies suggest that since the first months of life, even in the first days of life, infants are able to discriminate small quantities of simple dots (Antell & Keating, 1983; Starkey & Cooper, 2002), moving objects and pool of objects (van Loosbroek & Smitsman, 1990; Wynn, Bloom & Chiang, 2002), with events presented sequentially such as doll jumps (Canfield & Smith, 1996; Wynn, 1996).

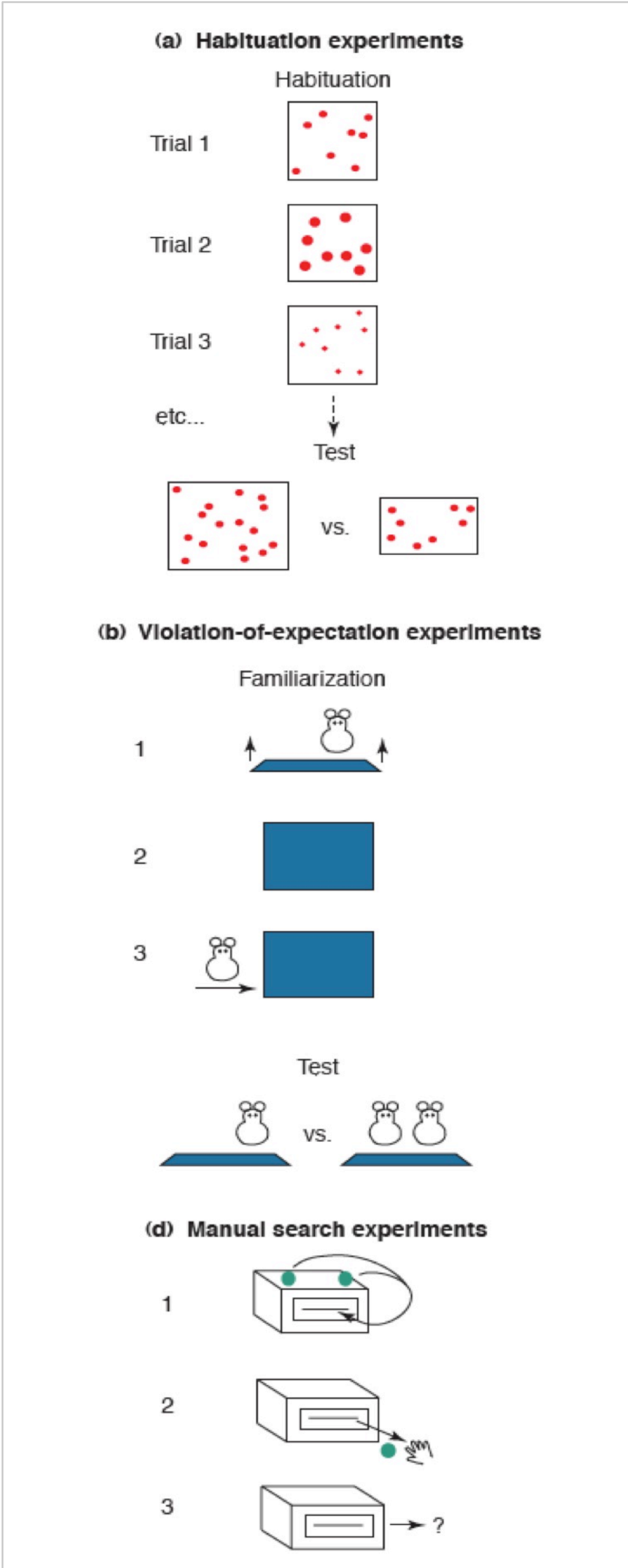
Antell and Keating (1983) conducted one of the first studies to assess numerical abilities in infants with *habituation-dishabituation paradigm*, demonstrating that neonates are able to detect numerical difference in arrays consisting of small numbers of discrete stimuli, but that they fail when the set becomes too large. Adopting the *violation-of-expectation paradigm*, Wynn (1992)

[1] These two processes are also known with the terms subitizing and estimation, rispectively.

showed that 5-month-olds are able to engage in numerical reasoning with small numerosities. In her experiments, infants are shown a small collection of objects, which then has an object added to or removed from it (i.e., 1+1; 2-1). The resulting number of objects shown to infants was either numerically consistent (i.e., respectively, 2; 1), or inconsistent (i.e., 1; 2) with the events. Since infants look longer at outcomes that violate their expectations, the author interpreted these finding as the proof that infants anticipate the number of objects that should result, and for this reason they look longer at the inconsistent outcomes than the consistent ones. Feigenson and Carey (2005) used the *manual search task* (Van de Walle, et al., 2000; Feigenson & Carey, 2003, 2005) to explore the limits of 12-month-old infants' quantification of small object arrays. Only under the condition with small numerosities all infants successfully retrieved the correct number of balls (see Figure 1).

Only recently, a growing number of studies have investigated the ability to discriminate large number of elements in the first months of life. Using the visual habituation method, recent studies have replicated and extended the pioneering experiments of Xu and Spelke (2000), suggesting that from 6-months of age infants discriminate a wild range of quantities that differ by a 1: 2 ratio (e.g., Xu, Spelke &Goddard, 2005; Xu & Arriga, 2007). Also McCrink and Wynn (2004), using a procedure similar to Wynn (1992), extended the previous results also to large numerosities. All these data suggest that infants are capable of addition and subtraction, at least with very small set size and, consequently, that they can represent ordinal numerical relations.

Fugure 1. Three types of methodologies used to test infants' quantity representations [source: Feigenson, Dehaene, & Spelke, 2004].



1.2 From Preverbal Numerical Competences to Counting

Toddlers grow and progressively mature their numerical competencies both on non-symbolic and symbolic prospective. Thus, babies, little by little, learn to use number words and entering in education system they also learn the meaning of numerical symbols.

1.2.1 Counting

One of the main conquests reached by children in this phase is the counting ability. According to the theory proposed by Gelman, non-verbal numerical reasoning is the starting point to learn both number words to count and the rules for how to correctly use them in the counting process (Gelman, 2006; Zur & Gelman, 2004; Gelman & Butterworth, 2005).

Gelman (Gelman, 2006; Gelman & Meck, 1983) describes the different rules that a child needs to learn to make a proficient use of number words. Each number word has to be in a one-to-one correspondence with the item to be counted, words must have a stable order (i.e. relations of more and less among numbers) and finally, the last word represents the cardinal value of the set (i.e. the amount represented by a number).

According to the authors' model, the child has to learn the number words and learn to map them onto the internal numerical representation in order to create a memory for each numerosity (Gallistel & Gelman, 1992). By the age of 3 ½ children start understanding how the counting system determines numerosity and have acquired the cardinal meaning of all the number words within their counting range (Wynn, 1990). From this, children progressively learn the meaning of each following number until they generalize the principle that any following number in the number word sequence is one item more than the previous (Margolis & Laurence, 2007; Rips, Asmuth, & Bloomfield, 2006). By 5 years of age, many children know most of the essential features of counting described by Gelman and Gallistel but also believe that adjacency and start at an end are also essential features of counting. It takes about a year for the child to reach a complete understanding of the counting system.

Whereas Gelman and collaborators suggest an innate knowledge of the counting principles others argue that the count list is learned just as the letters of the alphabet, without attributing any significance to the order. The knowledge of counting principles would be constructed by attempting to make sense of the number words themselves (Le Corre & Carey, 2008; Le Corre & Carey, 2007). According to this hypothesis, an innate numerical representation would not play a central role. Fuson (1988) stated that children's performance is influenced by number size, with larger numbers being harder and mastery of the three principles is not completely synchronised: stable order being reliably earliest, one-one correspondence between counting words and objects following later, and the cardinal principle being the last one(also Butterworth, 2004). Theories on how they acquire the counting skills still diverge but it seems reasonable to state that an early numerical representation is present and guides future numerical learning.

Finally, recent studies demonstrated that a fair mastery of the counting principles in kindergarten was predictive for several mathematical abilities in the subsequent schooling years, especially for arithmetic achievement (Dowker, 2005; Stock, Desoete & Roeyers, 2009).

1.2.2 OTHER NUMERICAL SKILLS

During development, thanks to the increasing familiarity with the symbols, children acquire a more exact and discrete concept of both small and large quantities. Moreover, numerical discrimination improves with age and therefore also the numerical representation. Another important accomplishment is the ability to recognize as numerically equivalent different sets of objects, for example that four doggie is a set equivalent to four strawberry or four pencils.

Results indicated that the ability to match numerically equivalent sets improves with age (Mix, Huttenlocher & Levine, 1996; Mix, 1999) showing that conventional counting skills may play an important role when similarity is low. At age 3 only identical comparisons were successful, then progressively by 4½ children were able to match those sets that were composed of heterogeneous items. Moreover, children that had better knowledge of conventional count words had also better performance in recognizing numerical equality.

Regarding comparisons of large numerosities, different studies found that 3-year-old children failed to compare sets perceptually controlled using numerical cues but results demonstrated a relation between the development of numerically-based judgments and some cardinality knowledge (Rousselle, Palmers & Noël, 2004). Another aspect that influences the performance in numerosity comparison task in early childhood when large quantities are involved, is the value of the ratio applied to the sets of stimuli. Huntley-Fenner and Cannon (2000) investigated 3 to 5-years-old children's performances on pairs of arrays that varied by either ½ or ⅔ ratios. The authors analyzed the proportion of accurate responses and they observed that sets with a ⅔ ratio were harder to compare and that errors varied systematically with ratios. A same ratio effect was found in non-symbolic arithmetic tasks (Barth, La Mont, Lipton, Dehaene, Kanwisher, & Spelke, 2006). Barth and colleagues tested the ability of preschoolers to add and subtract large numerosities and proposed outcome sets varied according to four ratios from the correct outcome. Children, but adult also, were sensitive to the ratio: accuracy decreased with a ratio closer to one.

1.3 From Early Arithmetic To Complex Mathematic Tasks

From the previous paragraphs appears evident that the foundations of arithmetic emerge well before school begins, and preschool children often display striking knowledge of arithmetic facts, procedures, and concepts prior to entering school. Indeed, preschool children can show sensitivity to the directional effects of addition and subtractions problems in situations that do not require exact computation (Brush, 1978, Villette, 2002). Young children show also some skill in exact computations, but their levels of success vary greatly with age and depend on problem characteristics and how problems are presented (Hughes, 1981; Huttenlocher, Jordan & Levine, 1994; Starkey, 1992). Moreover, beyond this age and throughout school, children learn to write and read number words and they acquire formal computational procedures, as well as arithmetical facts and more proficient strategies.

In formal curricula, multiplication, division and fractions typically follow addition and subtraction, because they are explained in terms of repeated addition and repeated subtraction and partitions of sets, thus building on concepts of sets and numerosities. For example, the mathematics curriculum in Italy begins in kindergarten (4–5 yrs) with the pre-mathematical concepts, in grade 1 (5–6 yrs) children consolidate their counting skills and start learning adding and subtracting principles, In grade 2 the procedures for solving complex addition and subtraction are taught (Cornoldi & Lucangeli, 2004). The table facts and the multiplication concept are trained among grade 2-3. Fractions are introduced in grade 4, and division, as the complement of multiplication, in grade 5.

Another important aspect that is worth addressing is the discrepancies between mental and written calculation (Thompson, 1997). This may be reflected by different cognitive styles: mental calculation may be more dependent on cognitive resources (e.g., working memory), whereas written calculation on knowledge of procedures (e.g., accurate recognition of place value, processing of written symbols, correct application of the procedure). The difference in these two types of arithmetic may also reflect teaching methods and other aspects of life experience. Schools at different times and country give varying degrees of emphasis to mental and written calculation. For example, Italian educational policies have, in the last few years, placed a greatly increased emphasis on mental calculation in the primary school, whereas written calculation received much more emphasis in the past. It emerges that children are often more mindful of concepts, and more inclined to reason, when carrying out mental arithmetic; and more inclined to strictly execute procedures as recipes when carrying out written arithmetic. This is one reason why some countries have chosen to begin by teaching mental arithmetic before proceeding to written arithmetic.

1.3.1 ARITHMETICAL ESTIMATION

The discrepancy between mental and written arithmetic is not the only: another central divergence in arithmetic field emerged when computational estimation is considered. The ability to estimate an approximate answer to an arithmetic problem and the capacity to evaluate the goodness of estimation are both important aspects of arithmetical understanding. Dowker (1998) reported a number of children who did show marked discrepancies between calculation and estimation; and also found similar discrepancies among adults (Dowker, 1994; Macaruso & Sokol, 1998).

This discrepancy emerges clearer when the distinctions between arithmetical estimation and written calculation is considered (Rubenstein, 1985). Written calculation is seen as depending on standard, school-taught strategies and procedures, whereas estimation is more flexible, less dependent on any specific standard techniques and less likely to be taught in school. Moreover, in the case of multi-digit numbers, the two types of process differ with regard to the order in which units, tens, hundreds, etc. are calculated. Written calculations typically begin with the units, and continue from right to left, using carrying or borrowing procedures if necessary. By contrast, estimation typically begins, and sometimes ends, with the left-hand digits.

Nevertheless several research indicates considerable individual variation in estimation performance by children (Dowker, 1997; Lefevre, Greenham, & Waheed, 1993; Lemaire, 2000; Sowder, 1992), indicating a relationship between calculation ability and estimation. Results documented age-related differences in children's computational estimation performance and strategies (Baroody, 1989; Case & Sowder, 1990; Dowker, 1997; Dowker et al., 1996; LeFevre et al., 1993; Lemaire & Lecacheur, 2002; Lemaire & Lecacheur, 2011; Lemaire, Lecacheur, & Farioli, 2000; Levine, 1982; Reys et al., 1982; Sowder & Markovits, 1990).

Development of computational estimation begins surprisingly late and proceeds surprisingly slowly. At the youngest age, children overestimated small numbers and compressed large numbers to the end of the scale (logarithmic shape of the estimates). This was however modulated by numerical context. Indeed, when the context was familiar (i.e., the smaller interval) positions of estimates were linear.

Computational estimation does improve considerably, albeit gradually, after grade 3 and 4. Adults and 6th graders are more accurate than 4th graders in estimating both addition and multiplication (Lemaire & Lecacheur, 2002). Older children (10-11 yrs) positioned numbers linearly, showing a performance comparable to adults.

2. DEVELOPMENT AND SELECTION OF STRATEGIES

How can children solve simple (but also complex) arithmetic problems? Children typically resolve the problems by using a variety of overt and covert procedures. Four principal procedures have been acknowledged in numerous studies with young children (Bisanz, Morrison

& Dunn, 1995; Geary, 1994; Jordan, Huttenlocher & Levine 1992; Siegler & Jenkins, 1989; Siegler & Shrager, 1984):

a) *counting-based* procedure: children use their fingers to externally represent the addends and then counted their fingers. Children can count both addends (*counting all*) or only ones (*counting on from first*);
b) *finger recognition* procedure: children raise fingers to represent the addends but did not count their fingers;
c) *counting* procedure with no external representation;
d) *covert* procedure without overt signs of counting, presumably involved *retrieval* of answers from memory or other type of more complex strategies (i.e., *decomposition, regrouping, rounding on-off strategies*), without any sort of mental computation or counting.

Even in the earliest phases of the development of arithmetic abilities, children typically use more than one procedure and select their procedures flexibly so that, for example, they are more likely to use overt procedures on difficult than on easy problems. Preschool children, in contrast to their older counterparts, show the remarkable diversity and creativity in their solution processes that can often be expected in "immature" but capable learners (Bjorklund, 1997; Bransford & Heldmeyer, 1983).

Several research has found that children use varying strategies to accomplish cognitive tasks and select them on a trial-by-trial basis. They may thereby adapt flexibly in different contexts to inherent task characteristics, such as problem difficulty, and to situational demands, such as the need to answer quickly and/or accurately. With age, children use the most efficient and problem-appropriate strategies increasingly often, and they execute them more and more efficiently. (Barrouillet, Mignon, & Thevenot, 2008; Kuhn & Pease, 2009; Lemaire & Calliès, 2009;

Lemaire & Siegler, 1995; Lucangeli, Tressoldi, Bendotti, Bonanomi, & Siegel, 2003; Luwel, Lemaire, & Verschaffel, 2005).

2.1 ARITHMETICAL FACT RETRIEVAL

Increasing years of schooling leads children to use less frequently counting strategies and more often mature strategies, as the retrieval of arithmetical facts. The mechanisms according which arithmetical facts are retrieved from memory are described in a variety of models. One model postulated by Siegler and his colleagues (e.g., Shrager & Siegler, 1998; Siegler & Shipley, 1995; Siegler & Shrager, 1984) assumes the probability of retrieving a particular answer depends on the associative strength between the problem and that answer, relative to other answers. A *confidence criterion* establishes the activation threshold to retrieve a specific answer. How this confidence criterion is determined is not clear, but it may be related to individual differences in how children approach problems (Siegler, 1988).

Another model stated that facts are typically stored as specifically verbal associations, retrieval is likely to be a process that operates to a large extent on verbal representations of number (e.g., “eight”; Dehaene & Cohen, 1995). In both models, retrieval will depend on the education story of the individual. The only deepest reason against these prospective is that a strong problem-size effect emerges for the reaction times derived from retrieval of single-digit problems: larger is the sum or the product and longer the problem takes to solve (Ashcraft et al., 1992). This aspect is much more powerful than frequency of occurrence (see Butterworth, Girelli, Zorzi, & Jonckheere, 2001).

Regarding this latter aspect, LeFevre and colleagues (LeFevre, Sadesky et al., 1996a; LeFevre, Sadesky, & Bisanz, 1996b) showed that adults, just like children, use both retrieval and non-retrieval strategies to solve simple-arithmetic problems. Indeed, as the non-retrieval strategies (generally slower) are used more frequently on large problems than on small problems, LeFevre and colleagues maintained that the problem-size effect might, to a certain extent, be caused by strategy selection processes. Although the problem-size effect is significant in all four operations, a recent study demonstrated that the selection and the efficiency of strategies differ across operations (Campbell & Xue, 2001).

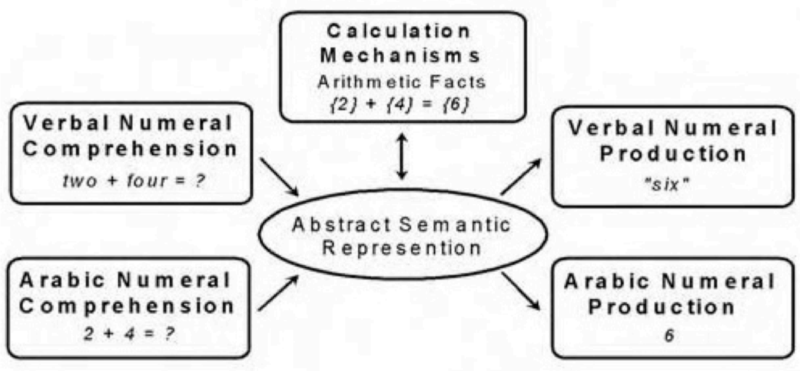
3. ARCHITECTURES FOR ARITHMETIC

During development children learn to familiarize with symbols, which denote a precise way to represents both small and large numerical information. Despite diverging ideas about the role and the relations among Arabic code and verbal representation, every model concerns the relation between arithmetic encoding and calculation process had to consider this dual nature of numerical representation. There are many models about how numerical cognition is represented and develop but in the following paragraphs, only some relevant models will be reviewed.

3.1 ABSTRACT CODE MODEL

One of the main models of number processing is the Abstract Code Model proposed by Michael McCloskey and his colleagues (e.g., McCloskey, 1992; McCloskey & Macaruso, 1994, 1995; Sokol, McCloskey, Cohen, & Aliminosa, 1991). The model recognises three types of systems in numerical processing: comprehension, calculation, and response production systems. Principal to this model is the statement that these subsystems are connected each other through the shared use of a single, abstract, semantic quantity code, as it is well portrays in Figure 2.

Figure 2. McCloskey's (1992) abstract code model (adapted from Cohen & Dehaene,1995)



The comprehension system encrypts different numerical input (e.g., written or spoken Arabic digits) into the abstract code, which converts the input upon which the other two systems operate.

The calculation system comprises memory for arithmetic facts and simple rules (e.g., $0 + N = N$) and is also involved in carrying out more complex arithmetic procedures (e.g., multi-digit problems). Arithmetic facts, as well as the numerical output of the calculation system, are in abstract code format.

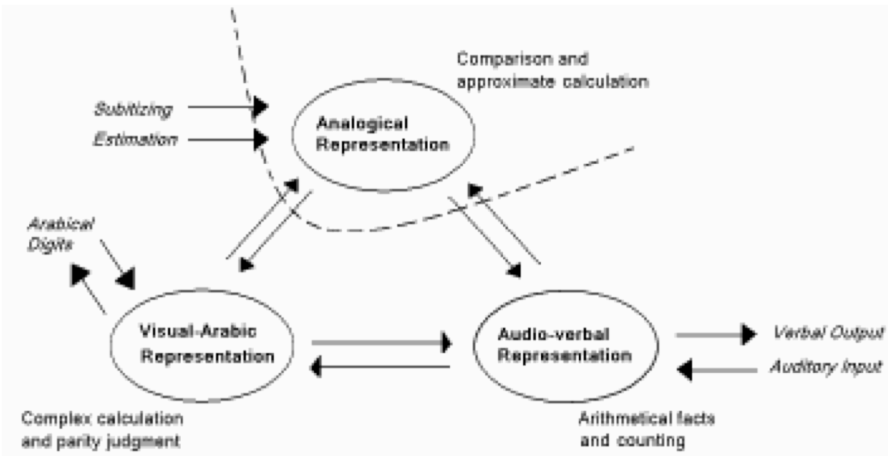
The production system converses the process of the comprehension system by translating the abstract output from the comprehension and calculation systems into Arabic numerical forms, written or spoken, as required.

The assumption of a unitary, abstract code as the core for numerical computation has repercussions for the independence of the processes involved. Since stimuli are recoded into the abstract code before calculation processes happen, input format necessarily has no influence upon calculation. Therefore, according to this model, calculation processes should show no differences as a result of the original format of the numerical input.

3.2 TRIPLE CODE MODEL

A substitute construct for number processing was proposed by Dehaene (1992; Dehaene & Cohen, 1995). The triple code model is shown in Figure 3. This theory postulates that there are three different codes upon which numerical processing is grounded: an analogue magnitude system, a visual-Arabic number form, and an auditory-verbal code system. Conversely to the abstract code model, it is expected that the three codes can directly trigger one another without the involvement of an abstract code. Similar to McCloskey's model, however, each subsystem is supposed to contribute to different number processing tasks. The analogue-magnitude code supports estimate processes and number size comparisons and, maybe, have a role in subitizing. Numerical input and output and multi-digit operations are intermediated by the Arabic form. The auditory-verbal system referees written and spoken input and output and provides the representative foundations for simple addition and multiplication facts.

Figure 3. Dehaene & Cohen's (1995) triple code model (adapted from Cohen & Dehaene,1995)



The triple-code model thus accepts language-based representations in memory for number facts, as opposed to the language-independent processes assumed by the abstract code model. There is a range of indication that confirms the postulation that memory for number facts involves verbal codes (Campbell, 1994, 1997, 1998; Cohen, Dehaene, Chochon, Lehericy, & Naccache, 2000; Dehaene, Spelke, Pinal, Stanescu & Tsivkin, 1999; Spelke & Tsivkin, 2001).

Nevertheless the lack of a central abstract code, Dehaene and Cohen's triple code model however predicts additive, encoding/retrieval processes. In this model, once input is codified into the appropriate code, processing happens in the same way regardless the nature of input. Thus, any format-based changes in performance for a given numerical operation (e.g., magnitude judgment, number dictation, arithmetic fact retrieval) should be imputable to differences in the competence of transcoding from the stimulus code to the type of internal code requisite for that specific task (Dehaene, 1996; Dehaene & Akhavein, 1995; Dehaene, Bossini, & Giraux, 1993).

3.3 ENCODING-COMPLEX HYPOTHESIS

Another important model was postulated by Campbell and Clark (1988, 1992; see also Campbell 1992, 1994; Clark & Campbell, 1991). The Authors pointed out the necessity to postulate an "encoding complex" model, also suggested by experimental evidence that demonstrated as resolution of interference among competing numerical responses and operations is central to capable numerical cognition. Numbers are toughly associated with a variety of numerical tasks (e.g., number reading or transcoding, number comparison, estimation, and arithmetic facts); accordingly, numbers mechanically activate a valuable network of connections that, in the context of a given task, includes both pertinent and irrelevant information.

Positive performance derives from the capability to overcome potential interference from irrelevant information. For example, when expert adults execute simple addition and multiplication problems with the instruction to be as fast as possible, the mistakes they produce show a overabundance of interferences. There are different variables that come into play and generate wrong performances. Errors' analysis usually reveals that individual engage associative or semantic neighbor in the same (4 x 7 = 32) or a related algorithm (4 x 7 = 11). Errors frequently involve interferences by one of the problem operands (e.g., 3 + 8 = 8) or by an answer given in a previous trial. Additionally, mistakes are most likely when such factors combine their influences on a specific trial. Factors that increment errors also incline to increase time for producing a correct response, which suggests that resolution of interference is a prevalent factor in performance. Moreover, the impact of these aspects can vary greatly depending upon the nature of format, such as Arabic digits versus written number words (Campbell, 1994; Campbell & Clark, 1992; Campbell, Kanz, & Xue, 1999).

The importance of the encoding-complex model is that potentially demonstrates that the modular systems that sub-serve number processing often communicate interactively rather than additively. As in the triple code model, the encoding-complex view is that number processing involves task-specific activation of information in one or more representational codes (e.g., visual, visuo-spatial, verbal, motoric). The encoding-complex model assumes furthermore that interaction between representational systems often comprises cooperative rather than strictly additive processes. Cooperative processes are products of task specific practice, which creates specific excitatory and inhibitory connections within and between systems to optimize resistance to interference (i.e., increase activation of pertinent information and reduce activation of inappropriate information). The development of such combined encoding-retrieval procedures is an elementary mechanism of acquisition of accomplished number processing. Hence, at the heart of the encoding complex hypothesis there is the concept of *skilled processing*.

4. CONCLUSION

Mathematical abilities involve a variety of complex mental activities such as identification of quantities, encoding and transcribing those quantities into an internal representation, application of procedural knowledge (e.g. borrowing in subtraction), keeping track of partial results while carrying out the next, etc. Some authors have proposed that cognitive capacities, not specific to number, are necessary to process and execute all the arithmetical tasks. These include working memory (e.g., Ashcraft, Donley, Halas & Vakali, 1992; Hulme & Mackenzie, 1992), spatial cognition (e.g., Rourke, 1993), and linguistic abilities (e.g., Bloom, 1994; Carey & Spelke, in press). Correlations between these cognitive abilities and standardised tests of arithmetic are well established.

In the next Chapter we will specifically focus on the relationship between working memory and arithmetic under a developmental prospective.

CHAPTER 2

COGNITIVE PROCESSES INVOLVED IN MATHEMATICAL COGNITION

Most individuals, as well children, can mentally compute mental calculation problems such as 13 + 5, 75 + 43, or even 159 + 25: trying to execute these examples provides a clear demonstration that several cognitive processes are involved in the solution of these mental arithmetic tasks. A great number of research confirmed the involvement of working memory as an essential aspect of numerical cognition (for a recent review see Raghubar, Barnes & Hecht, 2010; but see also Ashcraft, 1995; DeStefano & LeFevre, 2004; Fürst & Hitch, 2000; Heathcote, 1994; LeFevre, DeStefano, Coleman & Shanahan, 2005; Noel, Aubrun, Desert, Seron, 2001), thus making the relationship between numerical cognition and working memory a chief area of research for understanding the architecture of arithmetical processing. Therefore, the brief review of research described in the following Chapter are designed to examine the relation between working memory and multi-digit arithmetic problems.

The cognitive processes involved in mathematical cognition have been of interest both to developmental and experimental psychology. In fact, working memory plays a key role in supporting not only children's learning over school years, but also during every-day activities in adult populations.

As solving arithmetic problems involve processing and storage of information, it was suggested that working memory processes are involved in children's arithmetic performance (Adams & Hitch, 1997). Actually, the main aim of this chapter is to deepen the role of working memory in children arithmetic's performance.

In the following paragraphs, an overview of the description of main working memory models will be provided. After the description of the working memory models, a non-exhaustive overview of studies, which investigated the role of working memory in simple and complex arithmetic problem solving will be given in order to provide a general idea about this wide area of research. Actually, the mental organization of simple arithmetic facts in memory has been documented fairly extensively (e.g., Ashcraft, l987; Campbell, 1987; Dehaene, 1992; Green & Parkman, 1972; Siegler, 1988), whereas the role of working memory in solving multi-digit arithmetic problems has received less attention (see LeFevre et al., 2005). Next, the principal methodological issues regarding this topic will be briefly illustrated. Finally, individual differences and their influences on cognitive processes will be dealt with.

Indeed, it is not within the aim of this Chapter to review all studies on the role of working memory in mental arithmetic. Actually, a detailed summary of these research will be also provided in the next Chapters according to the specific goals of each study. The implications for academic achievement of children either with poor working memory functions or with specific mathematical impairment, and in particular their characteristic failures in learning activities, will be described in detail in the Chapter 4.

1. WORKING MEMORY MODELS: AN OVERVIEW

Ulric Neisser defined cognition as "all the processes by which the sensory input is transformed, reduced, elaborated, stored, recovered, and used" (Neisser, 1967, p. 4). The concept of working memory refers to a set of processes or structures that are intimately associated with many of these processes, making it a cornerstone of cognitive psychology. Understanding how we temporarily store and process information is fundamental to understanding almost all other aspects of cognition.

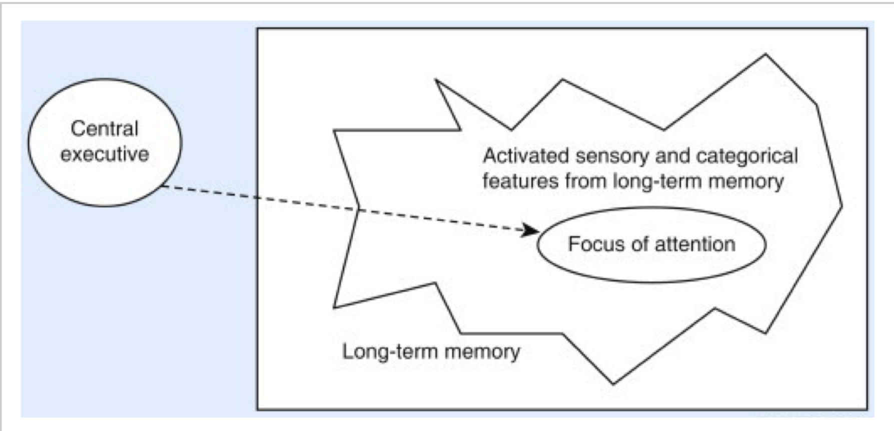
In the literature different theoretical perspectives on the construct of working memory have been provided and all these prominent models vary on a number of dimensions (Miyake & Shah, 1999). Nevertheless, the commonality among these models is that they describe how information is encoded into the working memory system, and how the system temporarily maintains this information. Working memory has been associated with a variety of everyday tasks that involve the temporary maintenance and processing of information (Baddeley, 1986, 1990, 1996; Baddeley & Hitch, 1974; Baddeley & Logie, 1999; Bull & Johnston, 1999; Hitch & McAuley, 1991; Logie, 1995; Logic & Pearson, 1997; Pearson, Logic, & Green, 1996; Pickering, Gathercole, Hall & Lloyd, 2001; Quinn, 1991; Robbins et al., 1996; Vecchi & Cornoldi, 1999; 2003).

A prominent aspect that characterized the conceptualization of working memory models refers to the presence of a central control unit that controls types and levels of processing, disposing commands executed by subordinate components. Conversely, the main difference states on the working memory architecture: unitary or multi-componential models (Baddeley, 1986, 1996; Baddeley & Hitch, 1974; Baddeley & Logie, 1999;Case, Kurland & Goldberg, 1982; Cowan, 2001; Daneman & Carpenter, 1980; Engle, Kane, & Tuholski, 1999; Just & Carpenter, 1992).

1.1 UNITARY SYSTEM

Single-resource frameworks have proposed that working memory is a system unitary in its nature. In this view, the modality-free storage and processing of information are switchable and compete for the same limited resource: controlled attention. A model that refers to working memory as a unitary system involved principally in attentional control was described by Cowan (1995, 1999, 2001; see Figure 2.1.). Cowan defines working memory as a limited-capacity attentional focus that operates across areas of activated long-term memory. According to this model, long-term memory can be seen in three ways: the larger portion that has relatively low activation at any particular point in time, a subset that is currently activated as a consequence of ongoing cognitive activities and perceptual experience, and a smaller subset of the activated portion that is the focus of attention and conscious awareness.

Figure 2.1. Cowan's Working memory model, (adapted from Baddeley, 2010).



The focus of attention is controlled primarily by the voluntary processes of the executive system, that are limited in capacity in chunks. Recent works indicated that typically between three and five chunks of information can be maintained in the focus of attention (Cowan, 2001; see also Chen & Cowan, 2005; Cowan et al., 2005).

A central issue of this model has been to define the capacity of the attentional resource and hence the capacity of working memory.

A similar view of working memory as control of attention in an on line mode including also storage processing has been also proposed by Engle and colleagues (Engle, 2001; Kane, Bleckley, Conway, & Engle, 2001; Engle & Kane, 2004; Engle, Tuholski, Laughlin, & Conway, 1999; Kane & Engle, 2002). Short-term memory consists of traces that have exceeded an activation threshold and represent indicators to specific regions of long-term memory. Controlled attention is a domain general resource that can accomplish activation through controlled retrieval, maintain activation, and block interference through the inhibition of distractors.

According to unitary-system theoretical prospective, a contrasting theoretical perspective on working memory was provided by Daneman and Carpenter (1980, 1983; Just and Carpenter, 1992). These researchers considered working memory as an undifferentiated resource that could be flexibly arrayed either to support temporary storage or processing activity. By this account, individuals with relatively low span scores on complex memory span tasks were relatively unskilled at the processing element of the activity (reading, in the case of reading span), thereby reducing the amount of resource available for storage of the memory items. This idea that working memory is a single adaptable system powered by a limited capacity resource that can be flexibly allocated to support processing and storage was applied by Case (Case, Kurland & Goldberg, 1982) to explain developmental increases in working memory performance across the childhood years. They proposed that the overall working memory resource remains constant as the child grows, but that the efficiency of processing increases, releasing additional resource to support temporary storage. Thus developmental increases in complex memory performance reflect improvements in processing speed and efficiency that release additional resources to support storage.

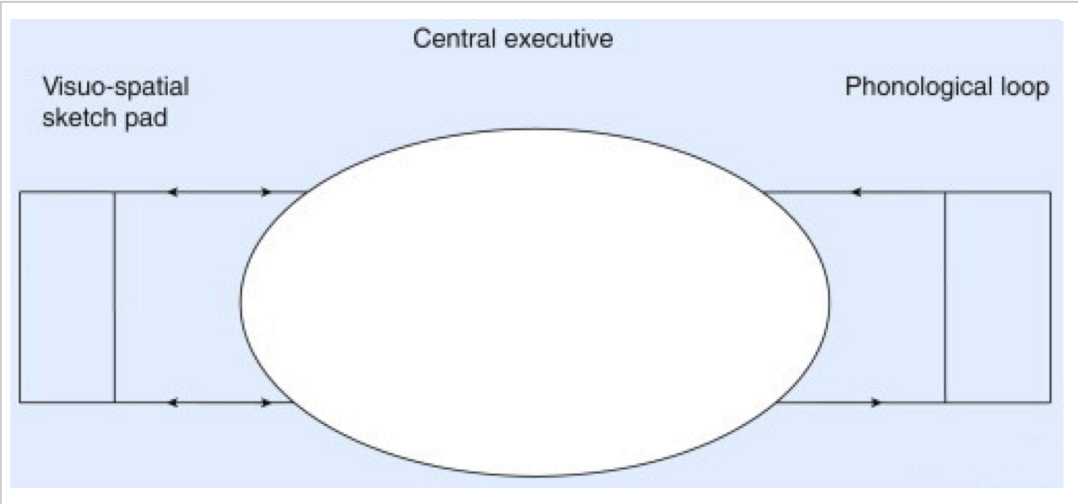
Another prospective, that can be define as time-based model, assumed that individuals executing for example a complex span tasks, do not process and store information at the same time, but strategically shift between the processing and storage components of the task (Towse and Hitch, 1995). Subsequent researches demonstrated that the period over which information was stored was a better predictor of complex memory span more than the difficulty of the processing activity, demonstrating as complex memory span is constrained by a time-based loss of activation of memory items (Towse, Hitch & Hutton, 1998; Hitch, Towse & Hutton, 2001).

Recently, Barrouillet and colleagues (Barrouillet & Camos, 2001; Barrouillet, Bernadin & Camos, 2004) integrated the temporal decay prospective with the source sharing account introducing the concept of cognitive cost. In this model, the cognitive cost is measured as the proportion of time that it requires limited-capacity attentional resources, for example, to support memory retrievals. When attention is deflected from remember items to handling information, memory representations cannot be refreshed and therefore traces decay with time.

1.2 THE MULTI-COMPONENTIAL SYSTEM

The most widely researched multiple resource framework is the multi-componential model initially proposed by Baddeley and Hitch (1974; Baddeley 1996; Baddeley & Logie, 1999; Baddeley, 2000). This multi-modal system of working memory states that storage is functionally independent from processing and consists of limited-capacity slave systems: phonological loop and visuo-spatial sketchpad responsible for a temporary storage and rehearsal of verbal and visuo- spatial material, respectively. A domain free processor, the central executive, coordinates the activity within the two slave systems (see Figure 2.2.).

Figure 2.2. The multi-componential model of working memory proposed by Baddeley and Hitch in 1974. The components are assumed to interact, and to be linked to both perception and long-term memory.



The central executive is responsible for a range of distinct processes including planning, switching, inhibition, monitoring, response selection, and the activation of representations within the long-term memory (Baddeley, 1996; Miyake et al., 2000). This executive component is a modality free, limited-capacity system that coordinates and integrates information incoming and outgoing to the two slave systems.

More recently, in a revised working memory model, Baddeley (2000) proposed a fourth component of working memory, the episodic buffer, which is capable of storing information in multiple codes and provides an interface between the slave systems and episodic long-term memory. The central executive is a higher level regulatory system, and the episodic buffer integrates and binds representations from different parts of the system, even if the empirical evidence in support of this component is limited (Baddeley, 2007).

The phonological working-memory component can be divided in two sub-components: an active sub-vocal rehearsal process and a passive phonologically based store (e.g., Baddeley, 1992; Baddeley & Logie, 1992; Logie & Baddeley, 1987). Phonological information is held in the phonological store. Because the contents of the phonological store are subject to decay, they have to be refreshed by the rehearsal process. This rehearsal process can be seen as some form of sub-vocal articulation, closely linked with the speech production system.

The visuo-spatial working-memory component functions as a mental blackboard or workspace for temporary storage of visual and spatial information. Also for this sub-component, Logie (1995) suggested the existence of two different module: a passive visual store (visual cache) and an active spatial store (inner scribe).

As noted by DeStefano & LeFevre (2004), the vast majority of empirical work on working memory and mental arithmetic has been conducted by applying the multi-componential model of working memory. According to a recent review (Raghubar et al., 2010), each component of the original working memory model is thought to play a role in mathematical cognition, thereby supporting a range of discrete steps in calculation such as encoding or manipulation of numerical information.

In the next paragraph, therefore Baddeley's model was used to scaffold the current doctoral dissertation as well, illustrating which is the role of each working memory component in the execution of mathematical task, according to both the specific methodology applied and the features of arithmetic problems.

2. WORKING MEMORY AND ARITHMETIC

Mathematical abilities involve a variety of complex mental activities such as identification of quantities, encoding and transcribing those quantities into an internal representation, application of procedural knowledge (e.g. borrowing in subtraction), keeping track of partial results while carrying out the next, etc. In recent years several research has been done on understanding the working memory system and its influence on mental arithmetic. Working memory is likely to be involved both in the early stages of mental arithmetic, where information is encoded verbally or visually and in the maintenance of information in further calculation processes.

In the next paragraphs, following a briefly description of the principal methodologies applied to unravel the problem of the relationship between working memory and arithmetic, an essential outline of the principal studies on the matter will be presented. As the issue of central executive involvement in execution of mental calculation seems clear and widely studied (Ashcraft, Donley, Halas, & Vakali, 1992; De Rammelaere, Stuyven, & Vandierendonck, 1999, 2001(a o b); De Rammelaere & Vandierendonck, 2001; Lemaire, Abdi, & Fayol, 1996), the next paragraphs will take into account specifically the role of the other two working-memory components that remains not fully understood (Heathcote, 1994; Logie, Gilhooly, & Wynn, 1994; Seitz & Schumann-Hengsteler, 2002; Trbovich & LeFevre, 2003).

2.1 METHODOLOGICAL ISSUES

An additional crucial element that resulted in the massive employment of this approach is its perfect fitting to two specific methodologies useful to investigate the modular structure of the working memory system. Actually, the model provided a large variety of tasks that levy specific working memory components thus giving the consequent possibility to make very specific predictions about the role of each component.

Two methods for testing the involvement of working memory are in general used: a) the selective interference paradigm, also called dual-task methodology and b) correlational studies.

The underlying philosophy of dual-task paradigm is to occupy particular components of the working memory system, which can then be used to investigate the extent to which particular activities engage one or another component. By the logic of dual-task methodology, any two activities that are unimpaired when conducted in combination do not tap common limited capacity systems. In contrast, performance decrements when two tasks are combined, indicate that they share a reliance on the same component. More specifically, performance on a primary task (e.g., mental arithmetic) is examined while participants perform a concurrent secondary task. This secondary task places demands on one specific working-memory component. When both tasks load the same working memory component, concurrent task execution should decrease performance on one of both tasks (or on both tasks).

Conversely, in the correlational approach, participants are given a working-memory assessment and are then tested on the task of interest (e.g., mental arithmetic). Performance differences on the task of interest may then be interpreted as due to differences in working-memory capacity assessed by specific tests. These studies address whether and how working memory is related to specific mathematical outcomes in children of different ages and abilities.

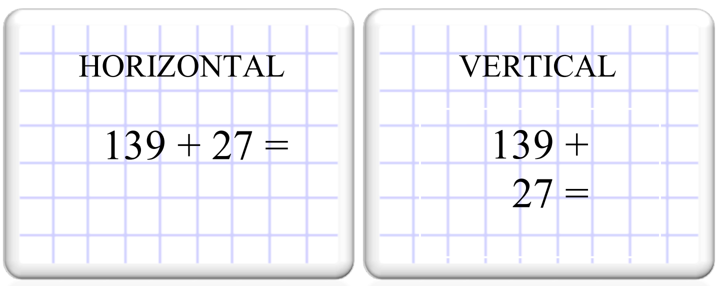
In the current doctoral dissertation, only multi-digit addition problems will be studied. We also decided to study children's arithmetic processing by means of a choosing task (e.g., 147 + 28 = 175 or 185; but see Chapter 3, in which the procedure is extensively described) rather than by means of a production task (e.g., 147 + 28 = ?). This decision was based on the fact that the production of arithmetic problems poses several problems in children. First, the production task is generally viewed as a four-stage process of encoding, retrieval and/or calculation and response execution. In contrast, even if the choosing task entails extra decision processes regarding which button to press, it generally considered easier than production task for young individual.

2.2 ENCODING

Research on the encoding processes of numbers has implicated the central executive when participants process both single-digit (Adams & Hitch, 1997; Logie & Baddeley, 1987) and multi-digit problems (Fürst & Hitch, 2000; Logic et al., 1994; Seitz & Schumann-Hengsteler, 2000, 2002). Phonological memory resources are assumed to be involved with retaining partial sums (Logie et al., 1994), and the verbal information representing the digits of the problem (Fürst & Hitch, 2000; Heathcote, 1994).

Also, both phonological and visual-spatial resources are believed to be necessary when encoding problems presented in different formats (vertical vs. horizontal; Heathcote, 1994; Trbovich & Lefevre, 2003). That is, participants usually make more errors and take longer to respond to multi-digit problems presented horizontally than problems presented vertically (see examples in Figure 3; Heathcote, 1994; Trbovich & LeFevre, 2003). For example, Heathcote (1994; Experiment 2) attempted to clarify the role of spatial working memory in maintaining the spatial codes of the arithmetic problem. Heathcote found that participants solved horizontal problems more slowly than vertical problems. From this evidence, he suggested that the visuo-spatial sketchpad was involved with solving arithmetic problems that required participants' to mentally re-arrange the operands into a more familiar format (i.e., vertical format; also see Hayes, l973; Hitch, 1978). This finding suggests that when people encode a horizontal problem they use a mental procedure (i.e., possibly re-aligning the operands into a vertical format) that uses visual-spatial resources. Thus Trbovich and LeFevre (2003) concluded that the extent to which phonological loop resources are utilised in arithmetic might depend on situational constraints such as presentation format of the arithmetic algorithm (see Figure 2.3. for an example).

Figure 2.3. Examples of presentation format of multi-digit numbers.



2.3 CALCULATION

After encoding the arithmetic problem, people must perform further mental calculations to produce the answer to the problem. Calculation of multi-digit problems may involve direct access to basic facts and stored procedural knowledge (Ashcraft, Donley, Halas, & Valcali, 1992; Hecht, 1999, 2002; LeFevre et al., 1996a, l996b). Research on mental calculation has concerned the involvement of the central executive in solving single-digit carry problems (De Rammelaere, Stuyven & Vandierendonck, 1996; 2001a, 2001b; Hecht, 2002; Lemaire, Abdi, & Fayol, 1996; Seitz & Schumann-Hengsteler, 2000), and in processing multi-digit carry problems (Ftirst & Hitch, 2000; Hitch, 1978; Logie et al., 1994; Noel et al., 2001; Seitz & Schumann-Hengsteler, 2002). Furthermore, calculation is thought to require phonological maintenance of the partial results (Ftirst & Hitch, 2000; Logic et al., 1994; Noel et al., 2001), and possibly visuo-spatial resources for the carry process (Hayes, 1973; Heathcote, 1994). This evidence suggests that the phonological loop and visuo-spatial sketchpad may have subsidiary roles in mentally calculating a multi-digit arithmetic problem.

Some researchers have found that verbal interference tasks do not disrupt processing of arithmetic problems (e.g., De Rammelaere, 1999; Noel et al., 2000), whereas others have found that verbal interference tasks do disrupt solution processes (Lernaire et al., 1996; Logie et al., 1994). For example, Logic et al. (1994) used a memory span task in which participants had to add a series of two-digit numbers that were presented verbally while concurrently participating in an articulatory suppression task or in an irrelevant picture task. Logie et al., found that concurrent performance of an arithmetic task was disrupted more by the articulatory suppression task than by the irrelevant picture task, therefore, suggesting that the role of the phonological loop was to keep track of running totals and maintaining accuracy in mental calculations.

Fürst and Hitch (2000) investigated the role of the phonological loop in solving multi-digit problems (e.g., 362 + 197). Their participants were required to solve either briefly presented or continuously presented problems while concurrently processing a secondary interference task. The secondary interference task involved a control condition and a verbal condition (i.e., articulatory suppression). Fürst and Hitch found that under dual-task conditions the articulatory suppression task had an effect when participants were solving briefly presented problems but it did not affect calculation of continuously presented problems. This evidence suggests that phonological resources are required in calculating multi-digit problems, possibly to temporarily maintain problem information. Therefore, the evidence from the studies mentioned above suggests that the phonological loop may have a subsidiary role in calculating arithmetic problems.

Few researchers have examined the role of the visuo-spatial sketchpad in mathematical processes (e.g., Hayes, 1973; Heathcote, 1994; Logie et al., 1994; Siegel & Linder, 1984; Trbovich & LeFevre, 2003). Heathcote (1994) assumed that the phonological loop and the visuo-spatial sketchpad would play different roles in solving carry and no-carry addition problems. Carry problems were defined by Heathcote to be problems where the sum of the units, or tens, or hundreds columns exceeded the value of nine. When the sum of the values in a column exceeded nine, then the carry (which is always a 1 in addition) was added to the first digit of the next column. Heathcote (1994) predicted that if carry problems involved visuo-spatial resources, then concurrent visual or spatial interference tasks would produce more interference on carry than on no-carry problems. Heathcote (l994) found that participants always solved carry problems more slowly than no-carry problems. He also found that viewing a visual pattern slowed participants' performance on carry problems but not on no-carry problems. However, spatial and verbal interference tasks did not differentially affect participants' performance on carry and no-carry problems. Thus, Heathcote suggested that the function of the visuo-spatial sketchpad was to maintain visual codes for carry problems. This finding suggests that the visuo-spatial sketchpad might be involved with maintaining the visual information of a carry problem (Hayes, 1973; Hitch, 1978) when mental calculations require producing an answer involving a multi-digit problem.

3. INDIVIDUAL DIFFERENCES AND DEVELOPMENTAL CHANGES

Arithmetic is one of the key mathematical skills in everyday life that is taught from early childhood. Although the research suggests that even in the first week of life humans are capable of basic discrimination based on numerosity (Butterworth, 2005), it is also clear that proficiency in arithmetic emerges developmentally as a result of education and interaction with the environment.

As regard population in growing, the investigation of the specific involvement of working memory on arithmetical skills is principally based on individual differences, correlational and longitudinal studies but only rarely based on experimental paradigm (Raghubar et al., 2010).

Associations between working memory and mathematical skills vary as a function of sample age as well as mathematical task. For example, older children perform arithmetic problems faster and more accurately than younger children do. This has been shown for simple addition problems (Adams & Hitch, 1997; Ashcraft & Fierman, 1982; Hamann & Ashcraft, 1985), simple multiplication problems (Butterworth, Marchesini, & Girelli, 1999; Campbell & Graham, 1985; De Brauwer, Verguts, & Fias, 2006; Kaye et al., 1989; Koshmider & Ashcraft, 1991), and mathematical word problems (Swanson, 2004; Swanson & Beebe-Frankenberger, 2004).

Accordingly, the problem-size effect also decreases with age. Based on these results, most developmental studies supposed a transition from counting-based performance (i.e., procedural knowledge) in young children to retrieval-based performance (i.e., declarative knowledge) in older children and adults (Ashcraft, 1982; Ashcraft and Fierman, 1982; Groen & Parkman, 1972; Hamann & Ashcraft, 1985; Koshmider & Ashcraft, 1991).

The great role of working memory in the development of arithmetic skill was also confirmed by Rasmussen and Bisanz (2005), who observed that measures of working memory accounted for a substantial proportion of the variability in arithmetic performance in both preschool children ($R^2 \geq .40$) and 1st grade children ($R^2 \geq .42$). These findings are consistent with Huttenlocher, Jordan, and Levine's (1994) proposal that preschoolers solve a variety of mathematical problems through the use of mental models. As language skills become stronger and verbal memory develops, children may begin to rely more on verbal memory codes to accomplish a variety of mathematical tasks including those that may have been solved using different cognitive resources at an earlier age.

Several recent studies of school age children that either contrast different age groups and/or take a wide variety of mathematical skills into account provide insight into the complexity of the relationships between working memory and math. In younger (7-8year olds) and older (9-10year olds) children, Holmes and Adams (2006) examined the contribution of the central executive, visuo-spatial sketchpad, and phonological loop to achievement in a variety of mathematical domains. In the younger children, the central executive, and to a minor degree the visuo-spatial sketchpad, contributed to performance on all the different areas of math. For the older children, the central executive predicted performance on both the easy and the hard items, but the phonological loop task predicted performance on the easy items, and the visuo-spatial sketchpad task predicted performance on the difficult items (also see Holmes, Adams, and Hamilton, 2008).

Other researches account the importance of the role of visuo-spatial memory on mathematical performance. It has been suggested that visuo-spatial memory functions as a mental blackboard, supporting number representation, such as place value and alignment in columns, in counting and arithmetic (Geary, 1990; McLean and Hitch, 1999; D'Amico and Gharnera, 2005). Visuo-spatial memory skills also uniquely predict variability in performance in non-verbal problems (operands presented with blocks) in pre-school children (Rasmussen and Bisanz, 2005). In contrast, the role of verbal short-term memory is restricted to temporary number storage during mental calculation (Fürst and Hitch, 2000; Hechet, 2002), rather than general mathematical skills (McLean and Hitch, 1999; Reuhkala, 2001).

Very recent longitudinal studies further confirmed that working memory, central executive in particular, clearly predicts later mathematics achievement, but aged-related difference emerged in relation to the specific contribution of slave systems and the tasks used to assess both math achievement and working memory capacity. Indeed, research literature demonstrates that measures of the central executive are particularly strong predictors of children's mathematical ability (Fuchs et al., 2005; Gathercole & Pickering, 2000; Gathercole et al., 2004; Henry & MacLean, 2003; Holmes & Adams, 2006; Keeler & Swanson, 2001; Lee, Ng, Ng, & Lim, 2004; Lehto, 1995; Noel, Seron, & Trovarelli, 2004; Swanson, 1994; Swanson & Beebe-Frankenberger, 2004; Wilson & Swanson, 2001).

De Smedt et al., (2009) found that central executive was a unique predictor of both first- and second-grades mathematic achievements. According to the slave systems, the visuo-spatial sketchpad occurred as a unique predictor of first-grade, but not second-grade, mathematics achievement, whereas the phonological loop emerged as a unique predictor in 7-year-old children but not in 6-year-olds. Apparently contrary results emerged from Meyer and colleague study (Meyer, Salimpoor, Wu, Geary & Menon, 2010). They suggested that the central executive and phonological loop facilitate performance during second grade of mathematical learning whereas visuo-spatial representations play a progressively central role during third grade.

Finally, dual-tasks experiments have rarely been used with children to investigate the role of different working memory components (Raghubar et al., 2010). Nonetheless, McKenzie, Bull and Gray (2003) demonstrated that both a phonological and a visuo-spatial interference significantly impaired arithmetic performance in 8- and 9- year- olds. Conversely, 6-7 year olds children remained largely unaffected by phonological interference, yet their performance was severely impaired by visual-spatial interference (even though the problems were presented aurally). It was proposed that this pattern of data might reflect a developmental shift from the use of visual-spatial strategies principally used by younger children to a mix of strategies: older children seem primarily relay on verbal approach, such as retrieval of arithmetic facts, supplemented by visual-spatial resources. A second study conducted by Imbo and Vandierendonck (2007) integrated choice/no-choice paradigm with dual task methodology in which the secondary task loaded the executive component of working memory. Results confirmed that the effect of working memory load decreased across the grades (4^{th} to 6^{th} grades) because probably children become more efficient in the execution of calculation applying effectively the known strategies resulting in decrease of resources demand.

In Chapters 3 and 4, five experiments in which the role of working memory in multi-digit arithmetic addition will be presented employing an interference paradigm to both typically developing children (Chapter 3) and children with learning disabilities (Chapter 4). As previously mentioned, the theoretical framework will be the multi-componential model of Baddeley and Hitch (1974).

CHAPTER 3

STUDY I

THE ROLE OF WORKING MEMORY COMPONENTS WHEN PROBLEM COMPLEXITY IS MODULATED IN TYPICALLY ACHIEVEMENT CHILDREN

The present study aims to examined the involvement of working memory (WM) by means of two types of mental calculation tasks: exact and approximate calculation. Specifically, children attending grades 3 and 4 of primary school were involved in three experiments that examined the role of verbal and visuo-spatial WM in solving addition problems presented in vertical or horizontal format. For Experiment 1, the children were required to solve addition problems with carrying. For Experiment 2, they were required to solve addition problems without carrying. Then, for Experiment 3, the children had to solve approximate problems with and without carrying.

Results confirmed that different WM components are involved in solving mental addition problems. In Experiment 1 horizontally presented addition problems were more impaired than vertically presented ones, according to verbal WM load; vice versa, vertically presented addition problems were more affected by a visuo-spatial WM load, especially when the children were required to perform approximate calculations. In Experiment 2, this pattern emerged in neither exact nor approximate calculations. Finally, in Experiment 3, the specific involvement of WM components was only observed in problems with carrying. Overall, these results reveal that both approximate calculation and carrying procedures demand particularly high WM resources that vary according to the task's constraints.

1. INTRODUCTION

Learning different aspects of arithmetic is one of the main areas of academic achievement, and arithmetic skills have important implications for everyday life. In particular, mental calculation is required both at school and in many daily contexts (e.g., paying for purchases, games, decisions) that demand either accurate or approximate calculations. Thus, individuals must be proficient with both types of calculations to cope with ongoing situations. Unfortunately, although many children encounter difficulties with arithmetic, particularly with mental calculations, the underlying cognitive mechanisms and educational implications are still not fully understood. The present paper is a contribution to the development of knowledge in the area of mental addition when a working memory (WM) load is involved.

Many studies support the concept that WM is related to mental calculation in general (for reviews, see DeStefano & LeFevre, 2004, and Raghubar, Barnes, & Hecht, 2010) and to mental addition problems in particular (Logie, Gilhooly, & Wynn, 1994; Fürst, & Hitch, 2000; Hecht, 2002; Trbovich & LeFevre, 2003). Although increasing numbers of recent studies have investigated these relationships, the involvement of various WM subcomponents in mental addition problems remains unclear, and empirical evidence of how one's WM tackles problems is sparse (DeStefano & LeFevre, 2004). A theoretical framework that is particularly appropriate for studying WM involvement in mental addition is the WM model developed by Baddeley and Hitch (1974; Baddeley, 1986). In the original version, WM is described as a non-unitary function that comprises three distinct components: a central executive and two slave systems, a verbal WM component (phonological loop) and a visuo-spatial WM component (visuo-spatial sketchpad). In the most recent version of the model, Baddeley (2000) added a new component, called the episodic buffer, which is responsible for binding information across domains and memory subsystems into integrated chunks.

Although central executive involvement in mental calculations seems clear (De Rammelaere, Stuyven, & Vandierendonck, 1999, 2001; De Rammelaere & Vandierendonck, 2001; Lemaire, Abdi, & Fayol, 1996), the roles of the other two WM components in calculations are still not fully understood (Ashcraft & Kirk, 2001; Heathcote, 1994; Imbo &LeFevre, 2010; Logie, et al., 1994; Seitz & Schumann-Hengsteler, 2002; Trbovich & LeFevre, 2003), especially in children. For this reason, the present study focuses only on the involvement of both verbal and visuo-spatial WM in children's mental calculations.

Previous research analyzing WM involvement in mental calculation mainly applied the dual task paradigm. In this paradigm, the participants perform a primary task (a mental calculation problem) in combination with a secondary task that involves one WM component. The secondary task may require the concurrent processing of verbal information, such as the continuous articulation of a syllable, like "the the the" (see, for example, De Rammelaere et al., 1999; Hecht, 2002) or the maintenance of a WM load, such as remembering a series of letters or a pattern of locations (Imbo & LeFevre, 2010; Trbovich & LeFevre, 2003). The paradigm assumes that if both the primary and secondary tasks use overlapping cognitive resources, then performance on the primary task will worsen as the secondary task becomes more demanding. This approach has been widely used with typically achieving adults to study single-digit and multi-digit arithmetic, but it has only rarely been extended to children (McKenzie, Bull, & Gray, 2003; Imbo & Vandierendonck, 2007).

Whether verbal or visuo-spatial WM is involved in single and multi-digit calculation seems to depend on a series of factors, such as the type of operation required, the presentation format, and the problem's complexity (e.g. absence or presence of carrying) (DeStefano & LeFevre, 2004).

The type of mental calculation required has been considered in previous research with reference to two main aspects: (1) the specific arithmetical operation required (i.e., addition, subtraction, multiplication and division) and (2) the type of calculation task (i.e., exact vs. approximate). In the present study, we focused on this latter aspect. Exact and approximate calculations seem to engage different cognitive processes. In particular, adult populations typically find approximate calculation to be easier than exact arithmetic (Kalaman & LeFevre, 2007). This ease may offer fundamental support in everyday life when adults must make rapid estimates of an implied amount (price, distance, time of arrival, etc.). Unsurprisingly, exact mental arithmetic requires more calculations and more storage of interim results during the solution process than does approximate arithmetic (Lemaire & Lecacheur, 2002; Lemaire, Lecacheur, & Farioli, 2000). A study by Kalaman and LeFevre (2007) has thoroughly examined this aspect. In their study, young adults were presented with multi-digit addition problems with and without carrying, alone and in combination with a verbal WM load. Participants were required to perform either exact or approximate calculations. Results showed that verbal WM was implicated in both types, but WM played a greater role in exact calculation. Addition problems requiring carrying were also more demanding of verbal WM than were addition problems that did not require carrying. It is worth noting that the experiment of Kalaman and LeFevre (2007) only tested the involvement of verbal WM by requiring the concurrent recall of four consonant letter sequences.

In order to explore the effects of presentation format, Trbovich and LeFevre (2003) analyzed adult participants' performance on two-digits-plus-one-digit mental addition problems horizontally and vertically presented under either verbal or visuo-spatial WM loads. Verbal and visuo-spatial WM load conditions included two distinct no-load conditions in which participants were presented with the same material but were not required to perform the secondary task. Results showed that participants were quicker and more accurate in solving vertically presented addition (henceforth VA) problems; however, the verbal WM load impaired their performance on horizontally presented addition (henceforth HA) problems to a greater extent and, in contrast, the visuo-spatial WM load impaired their performance on VA problems to a greater extent. The authors concluded that participants used verbal WM resources while solving HA problems and visuo-spatial WM while solving VA problems.

Lastly, carrying requirements may interact with WM demands, but not consistently. The involvement of WM in mental calculations that require carrying was studied by Heathcote (1994), who showed that participants were slower at solving addition problems with carrying under a verbal WM load than they were at solving addition problems with carrying under either visual or spatial WM loads. Several other studies have found interactions between the type of WM load and carrying (e.g., Fürst & Hitch, 2000; Ashcraft & Kirk, 2001). Furthermore, it has been shown that the central executive component is involved when the number of carrying or borrowing operations increases and with higher carrying values (Imbo, Vandierendonck & De Rammelaere, 2007a; Imbo, Vandierendonck & Vergauwe, 2007b). In summary, experimental evidence offers relatively clear data on the involvement of the central executive component in mental calculation, whereas the roles of either verbal WM or visuo-spatial WM are still unclear.

1.1 OVERVIEW OF THE CURRENT STUDY

In this research, three experiments tested the involvement of both verbal and visuo-spatial WM loads in multi-digit mental addition problems with/without carrying in different groups of children attending grades 3 and 4 of primary school. Children were asked to solve either exact or approximate mental calculations, any of which was presented either horizontally or vertically. We focused on this particular age group, because children at these ages are at a sensitive moment in their development of mental calculation skills: they may have complete mastery of written calculation, but they usually do not possess completely automatized mental calculation skills and, in particular, they may be unfamiliar with approximate calculation.

In addition, the available evidence mainly concerns adults, because the role of WM in children's mental calculation has never been studied in depth (nevertheless, see McKenzie et al., 2003, and Imbo & Vandierendonck, 2007, for exceptions). For example, McKenzie et al. (2003) examined the importance of verbal and visuo-spatial WM in simple mental calculation problems at various ages. Younger (6-7-year-old) and older (8-9-year-old) children listened to mental calculation problems under three conditions: baseline, verbal interference and visuo-spatial interference. The younger children remained largely unaffected by verbal interference, yet their performance was severely impaired by visuo-spatial interference, whereas older children were equally affected by both types of interference, although not as much as the younger ones were. These results suggest that the younger children may have been relying almost solely on visuo-spatial strategies, whereas the older ones were probably using a mix of strategies, perhaps a primarily verbal approach supplemented by visuo-spatial resources. Thus, it appears that visuo-spatial WM may be implicated more in the mathematic performance of younger children who are in the process of acquiring basic arithmetic skills (see also Bull, Espy, & Wiebe, 2008; Holmes & Adams, 2006;

Holmes, Adams, & Hamilton, 2008; Kyttälä, Aunio, Lehto, Van Luit, & Hautamäki, 2003; Maybery & Do, 2003; Rasmussen & Bisanz, 2005). However, as children gain experience with arithmetic, verbal WM comes to support their arithmetic performance to a greater extent. We therefore decided to test children - first, because there are few dual task studies analyzing WM involvement in mental calculation and, second, because specific WM involvement could emerge. In three experiments, operation required (mental calculation task: exact vs. approximate), presentation format and presence of carrying were manipulated. The first variable manipulated in the present research was the type of calculation task. Following Kalaman and LeFevre's (2007) study, the exact versus approximate calculations with/without carrying were manipulated in a between-subjects condition during the first two experiments. Children had to solve mental addition problems by choosing the correct answer between two possible solutions. Previous results from adult populations (Kalaman & LeFevre, 2007) have shown that exact arithmetic requires more calculations and more storage of interim results during the solution process than does approximate arithmetic. However, no previous studies have compared children performing exact or approximate mental calculations. If children resemble adults in that they find approximate mental calculation easier than exact calculation, then the latter would make greater demands on children's WM (LeFevre, DeStefano, Coleman, & Shanahan, 2005). However, it is possible that children are not competent in approximate mental calculation and therefore do not spontaneously and easily use the most functional strategies employed by adults, particularly rounding off (i.e., 28 + 37 = 30 + 40) (Lemaire & Lecacheur, 2002; Lemaire et al., 2000). In this case, approximate mental calculation would be more demanding of WM than exact calculation. Concerning presentation format, participants were asked to solve horizontally and vertically presented mental addition problems in each experiment. Given our knowledge of previous studies with adults (see Trbovich & LeFevre, 2003, and Imbo & LeFevre, 2010), we expected that the presentation format would be differently affected according to type of WM load. Specifically, we hypothesized that the 3rd and 4th graders would be more impaired while solving HA problems with a verbal WM load than they would be while solving VA problems with a verbal WM load; in contrast, we expected that VA problems would be more affected than HA ones by a visuo-spatial WM load. Thus, we were particularly interested in observing how the presentation format interacts with the presence of verbal or visuo-spatial loads in exact or approximate addition problems with/without carrying.

Lastly, the effect of carrying was manipulated as recent studies indicate that operations such as carrying or borrowing may increase WM demands (Imbo et al., 2007a, 2007b; Venneri, Cornoldi & Garuti, 2003; Mammarella, Lucangeli, & Cornoldi, 2010). With this view in mind, we carried out a third experiment to test the hypothesis that problems which involve carrying from both the ones unit and the tens unit (e.g., 145+37) would demand more WM resources than similar problems which did not require any carrying at all (e.g., 153+21).

2. FIRST EXPERIMENT

Experiment 1 tested the ability of children to solve mental addition problems with carrying, letter recall, and location recall, alone and in combination. Letter recall was assumed to involve verbal WM, and location recall visuo-spatial WM. Half the children solved exact and half approximate calculations. Thus, we tested whether exact calculation is more demanding of WM than approximate calculation (Kalaman & LeFevre, 2007) in children. Finally, half of the problems were presented in horizontal format and half were presented in vertical format. In agreement with previous studies (Trbovich & LeFevre, 2003; Imbo & LeFevre, 2010), we expected that the presentation format would interact differently with either verbal or visuo-spatial WM load.

2.1 METHOD

2.1.1 PARTICIPANTS

A total of 91 children (44 males, 47 females) attending grade 3 (mean age=8.5 years; SD=3.9 months) and grade 4 (mean age=9.6 years; SD=3.1 months) of Italian primary schools participated in this experiment. For all children, parental consent was obtained prior to testing. Children who were reported as having a very low socio-economic level and/or special educational needs were not included in the study.

Children were first presented with a paper-and-pencil standardized arithmetic battery (Cornoldi, Lucangeli & Bellina, 2002) which included written calculations (addition, subtraction, multiplication and division problems), magnitude comparison tasks and number ordering tasks. The standardized arithmetic battery was collectively administered and used to select children with sufficient mathematical achievement. The numbers of correct responses in each area were summed to obtain a total score. Children who obtained low scores were excluded from analyses. In our sample, we found that two children did not understand the instructions and that three children scored 2 SD below the normative score. Therefore, our final sample number totaled 86 children (42 males, 44 females).

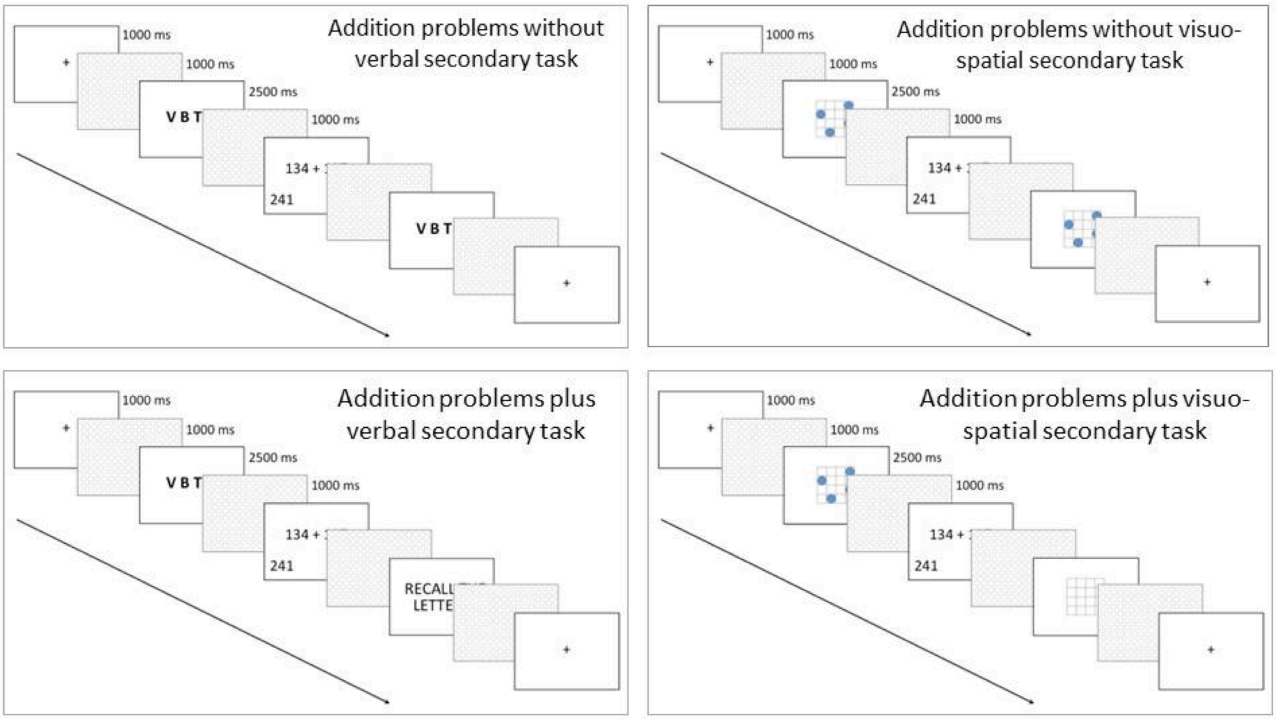
2.1.2 DESIGN

Forty-two (21 male, 21 female) children (mean age=9.1 years; SD=7.4 months) solved exact calculations and 44 (21 male, 23 female) (mean age=9 years; SD=6.9 months) approximate calculations.In one-third of the trials, addition problems were presented in association with either verbal or visuo-spatial information, but the children did not have to recall them (condition without secondary tasks). Another third of the trials required the children to solve addition problems as well as to recall either verbal or visuo-spatial information (condition of additions plus secondary tasks), and the final third required the children to recall either verbal or visuo-spatial information without solving addition problems (condition of secondary tasks only)[2]. The current study compared only four conditions (see Figure 1):

1) Addition problems without verbal secondary tasks
2) Addition problems without visuo-spatial secondary tasks
3) Addition problems plus verbal secondary tasks
4) Addition problems plus visuo-spatial secondary tasks

The first two sets were control conditions wherein the children were asked to perform only addition problems. In the remaining two sets, children carried out two tasks (addition problems plus a verbal task or addition problems plus a visuo-spatial secondary task).

Figure 1. Graphical representation of the experimental tasks used in the present study.



2.1.3 STIMULI

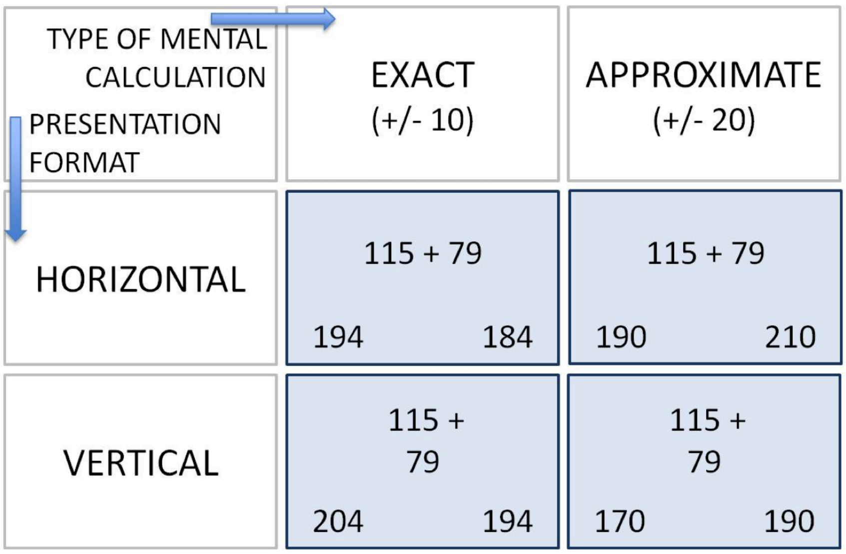
Experimental tasks. The material was programmed with E-Prime software (Psychology Software Tools, Inc., Pittsburgh, PA, USA) on a 15-inch computer screen. Children were seated about 60 cm in front of the screen.

Addition problems. There were four sets of 12 multi-digit addition problems (three or two-digit operands with the carrying operation only in the units) that were administered according to the procedure of Kalaman and LeFevre (2007). Each set of addition problems corresponded to sums of about 70, 80, 90, 140, 150, 160, 170, 180, 190, 240, 250 or 260. Half the problems were presented horizontally and half vertically, and the positions of both smaller and larger operands as well as the sums of the additions (i.e., odd or even) were controlled. The addition problems were the same in the two blocks (exact vs. approximate calculation) in that each problem presented one correct and one incorrect solution for both exact approximate task. For example, for the problem 115+79, the correct answer is 194 for the exact calculation task and 190 for the approximate calculation task. Following the criteria of Kalaman and LeFevre (2007), the exact calculation's incorrect response was created by adding or subtracting 10 units from the precise answer (i.e., either 204 or 184). In the approximate calculation, the incorrect solution was created by adding/subtracting 20 (i.e., either 210 or 170). For both exact and approximate calculations, almost half the incorrect solutions were greater than the correct ones (see Figure 2).

Verbal domain information. Participants were presented with four visual consonants that they had to recall. In half of the trials, the children were asked to recall the letters after each addition problem. Simply the identity of the letters (and not their correct order) was considered a correct response. Each sequence of consonants appeared once, and the consonants were not counterbalanced across the addition problem sets.

Visuo-spatial domain information. In a 4 x 4 matrix, four randomly filled squares were simultaneously presented. In half of the trials, the participants used the computer mouse to select the correct positions of the previously presented cells in an empty matrix after performing each addition problem. The filled squares were not counterbalanced across the addition problem sets. In other words, each matrix was paired with a particular addition problem.

Figure 2. Synthetic representations of the main features of addition problems implemented in the experimental tasks.



2.1.4 PROCEDURES

Each participant was given 16 practice trials and 48 experimental trials that were divided into four conditions (see the Design section). The task order was fully counterbalanced across the participants. Trial-by-trial feedback was given in the practice blocks. The time sequences of the procedure previously described for the addition problems were the same for all four conditions.

Addition problems. Each trial began with a fixation point on the computer screen for 1000 ms, followed by a 1000-ms blank interval. Thereafter, the associated information (verbal or visuo-spatial, as described below) appeared. It was displayed for 2500 ms, and then the screen went blank for 1000 ms in order to allow participants time to rehearse the load. Next, the addition problem appeared and remained on the screen until the participant responded by keying in one of the two possible responses. Each operation problem appeared in the center of the computer screen, simultaneously with the two possible solutions at the bottom of the display that were to the left and right sides of the screen. Half the addition problems showed the correct solution on the right side and half on the left. After the child keyed in a response, the screen went blank for another 1000 ms, and then, for the conditions in which the secondary task had to be recalled, the answer screen of the secondary task appeared and remained visible until the child responded. After the children had responded to the WM load, the screen went blank for another 1000 ms after which the fixation point appeared, indicating the beginning of another trial. The children were asked to respond as quickly and accurately as possible. No feedback was given to them during the experimental trials. Particular attention was devoted to the instructions given to children regarding approximate calculations. Specifically, children were presented with several examples of approximate calculations followed by practice trials, and only after they had performed correctly the majority of the cases and demonstrated understanding of the task could they start the experiment.

Verbal condition. Participants saw four consonants arranged horizontally in the centre of the screen and were instructed to read the letters aloud. When the secondary task did not require a response, the same letters appeared on the screen after each addition problem and participants simply pressed a key to continue. Conversely, when the secondary task required a response, the children were asked to recall the letters.

Visuo-spatial condition. On each trial, participants saw four randomly selected filled squares in a 4x4 matrix. When the secondary task did not require a response, after each addition problem, the same filled squares in the 4x4 matrix appeared and participants simply pressed a key to continue. Conversely, when the secondary task required a response, the children were asked to recall the filled squares previously presented, selecting them with the mouse.

2.2 RESULTS

2.2.1 STANDARDIZED ARITHMETIC BATTERY

Participants' mean scores on the standardized arithmetic battery were used to determine whether the groups of children performing exact and approximate calculations were equivalent in overall computational skill. Scores were analyzed with a one-way ANOVA. The results did not reveal any difference $F(1,85)=.003$, $p=.957$, $\eta^2<.001$.

2.2.2 EXPERIMENTAL TASKS

Before proceeding with the statistical analyses, the accuracy of the secondary tasks was controlled. All pupils who had obtained a score of at least 60% accuracy both for verbal and visuo-spatial secondary tasks were included in the analyses. All pupils were able to successfully complete the secondary task with a high rate of accuracy. Two variables were taken into account in the following analyses: (1) the percentage of correct responses to the addition problems, derived from different blocks and divided on the basis of presentation format; and (2) mean correct latencies to addition problems, calculated by considering only the trials in which children selected the correct answers for addition problems. Table 1 reports the descriptive statistics. Preliminary analyses were performed in order to examine whether there were relevant developmental changes. No difference between the 3rd and 4th graders were observed in the percentage of correct responses, whereas a main effect of grade level was found on mean correct latencies in approximate calculation only. Since the grade effect emerged for only one type of mental calculation, the grade effect was not included in the following analyses[3].

Separate mixed ANOVAs were performed. Interactions were decomposed by means of post-hoc pair-wise comparisons with Bonferroni's correction at $p < .05$, adjusted for multiple comparisons.

[3] The analysis revealed that children attending 4th grade solved approximate additions problems faster than pupils attending 3rd grade $F(1,42)=6.948$ $p=.012$, $\eta^2=.142$ (7626 ms vs. 11890 ms). Since the effect was only found in the mean correct latency of approximate calculations, it was supposed to reflect a greater expertise of fourth graders in solving approximate mental additions.

Table 1. Exp. 1. Percentage of correct responses, mean correct latencies (in seconds) and standard errors (SE) as a function of task (exact vs. approximate), presentation format (HA vs. VA problems) domain (verbal vs. visuo-spatial) and load (single vs. dual task).

		Percentage of correct responses				Mean correct latencies			
		Verbal domain		Visuo-spatial domain		Verbal domain		Visuo-spatial domain	
		Single task	Dual task	Single task	Dual task	Single task	Dual task	Single task	Dual task
Task	Presentation Format	M (SE)	M (SE)	M (SE)	M (SE)	M (SE)	M (SE)	M (SE)	M (SE)
Exact	HA problems	73.02 (3.81)	70.63 (3.76)	79.76 (3.57)	71.03 (4.37)	9.62 (.64)	8.10 (.75)	8.83 (.66)	8.38 (.63)
	VA problems	77.38 (3.45)	73.81 (3.94)	75.79 (4.44)	73.81 (4.48)	8.52 (.50)	7.40 (.67)	6.94 (.47)	7.39 (.43)
Approximate	HA problems	73.11 (2.88)	60.98 (2.92)	74.24 (2.62)	78.03 (2.91)	10.36 (1.07)	9.67 (.94)	9.84 (.83)	10.51 (1.29)
	VA problems	80.30 (2.19)	77.65 (2.54)	78.41 (3.19)	64.77 (2.83)	9.04 (.86)	9.25 (1.09)	8.41 (.71)	9.43 (1.14)

2.2.3 PERCENTAGE OF CORRECT RESPONSES

A 2 (task: exact vs. approximate) x 2 (presentation format: HA vs. VA problems) x 2 (domain: verbal vs. visuo-spatial) x 2 (load: dual tasks vs. single task) mixed ANOVA was performed. Neither the main effect of the task (exact vs. approximate) $F(1,84)$=.089 p=.767, η^2=.001 nor of the domain (verbal vs. visuo-spatial) $F(1,84)$=.537 p=.466, η^2=.006 were significant. However, the load effect (dual vs. single task) $F(1,84)$=11.021 p=.001, η^2=.118 (71.3% vs. 76.5%, respectively) was significant: children performed worse on addition problems that were associated with the dual task. The results also revealed a main effect of presentation format (HA vs. VA problems) $F(1,84)$=4.629 p=.034, η^2=.052 (72.6% vs. 75.2%, respectively), showing that the VA problems facilitated the children's performance.

The interaction presentation format by domain was significant $F(1,84)$=15.871 p<.001, η_p^2=.159, and revealed that the children were more impaired by visuo-spatial information than by verbal information while attempting to solve VA problems (M*diff.* = -4.09, p < .05). Inversely, they were more impaired by verbal information while attempting to solve HA problems (M*diff.* = -6.33, p < .01). In addition, the interaction task by presentation format by domain was significant $F(1,84)$=5.360 p=.023, η_p^2=.060, showing that children's performance on HA problems was more damaged by verbal than by visuo-spatial information for approximate calculations (M*diff.* = -9.09, p < .01); in contrast, their performance on VA problems was more affected by visuo-spatial information than by verbal associated information (M*diff.* = -7.39, p < .05). A similar trend was observed for exact calculation, although not statistically significant (p >.21).

The interaction presentation format by domain by load was also significant $F(1,84)$=4.333 p=.040, η_p^2=.049. In particular, the verbal load impaired performance on the HA problems more than it did on VA ones during the dual task (M*diff.* = -8.72, p < .01). In contrast, the visuo-spatial load impaired performance on the VA problems more than it did on the HA ones during the dual task (M*diff.* = -6.44, $p < .01$). There were no significant differences in the single task ($p > .16$). Lastly, the interaction task by presentation format by domain by load was significant $F(1,84)=14.628$ $p<.001$, $\eta_p^2=.148$. Confirming previous results, performance on the approximate HA problems was more impaired by verbal load than by visuo-spatial load only during the dual tasks (M*diff.* = -17.05, $p < .001$). In contrast, performance on the approximate VA problems was more impaired by visuo-spatial load than by verbal load (M*diff.* = -12.88, $p < .001$). No differences were found in the dual tasks for exact addition problems, nor did the single tasks show any differences in approximate or in exact calculations.

2.2.4 MEAN CORRECT LATENCIES

A 2 (task: exact vs. approximate) x 2 (presentation format: HA vs. VA problems) x 2 (domain: verbal vs. visuo-spatial) x 2 (load: dual tasks vs. single task) mixed ANOVA was run. The main effects of task $F(1,84)=1.966$ $p=.165$, $\eta^2=.023$, domain $F(1,84)<1$, and load $F(1,84)<1$ were not significant.Similar to Trbovich and LeFevre's results (2003), children performed HA problems more slowly than they performed VA ones $F(1,84)=57.733$ $p<.001$, $\eta^2=.407$ (9413 ms vs. 8297 ms). The interaction presentation format by domain was not significant $F(1,84)=2.704$ $p=.104$ $\eta_p^2=.031$, whereas the interaction presentation format by load was significant $F(1,84)=4{,}515$ $p=.037$ $\eta_p^2=.051$, showing that during both single and dual tasks, the children were faster at performing VA problems than HA problems (M*diff.* = -1435.83, $p < .001$ and M*diff.* = -795.66, $p < .001$, respectively).

2.3 DISCUSSION

Experiment 1 offered the possibility of examining the generalizability of two main results obtained with adults, i.e.: (1) whether – as observed by LeFevre et al., 2005 and Kalaman, and LeFevre, 2007 – approximate mental calculation makes lesser demands on WM than does exact mental calculation; and (2) whether the verbal WM load specifically impairs the performance of HA problems and, in contrast, whether the visuo-spatial WM load specifically impairs the performance of VA problems (Trbovich & LeFevre, 2003).

Regarding the first point, our results revealed that, unlike in research on adults (LeFevre, et al., 2005; Kalaman, & LeFevre, 2007), approximate calculation is more demanding of WM resources than exact calculation in children attending the 3rd and 4th grades of primary school. In fact, no differences were found between exact and approximate calculations, either for the percentages of correct responses or for the mean correct latencies, which indicates that both approximate and exact calculations present the same degree of difficulty to children. However, the influence of calculation tasks emerged as interacting with presentation format, domain and load in our analysis of percentages of correct responses. Our findings indicated that the presentation of verbal and visuo-spatial information in relation to presentation format was stronger than in exact calculation for approximate calculations, and was only significant when a dual task was required. In other words, only when children were required to recall either letters or positions while solving approximate addition problems did the other effects reach significance. Therefore, unlike the case of adult populations (Kalaman & LeFevre, 2007), approximate calculation involves children's WM resources to a greater extent than does exact calculation. It is possible that children do not spontaneously use the rounding strategy employed by adults for performing approximate mental calculations (Lemaire & Lecacheur, 2002; Lemaire et al., 2000).

On the contrary, our results confirmed that the children's performance is affected by the presentation format of the operation (vertical vs. horizontal), similarly to what happens in adults. Results revealed that, in general, HA problems were more impaired than VA ones when the children had to recall verbal information, and, conversely, VA problems were more impaired than HA ones when the children had to recall visuo-spatial information. Moreover, analysis of correct responses emphasized that the interaction between presentation format and domain was significant only under the dual task condition, and – more interestingly – only when children solved approximate calculations. In summary, when children attending grades 3 and 4 carried out approximate calculation with carrying, their performance was impaired by the verbal WM load when addition problems were horizontally presented and by the visuo-spatial WM load when the problems were vertically presented.

In conclusion, Experiment 1 showed that WM is deeply involved in approximate calculation, but its involvement depends upon the specific involvement of either verbal or visuo-spatial information as well as upon the presentation format. Nevertheless, some aspects remained unclear: mental additions with carrying involve a series of specific features that add complexity to calculations without carrying, thus increasing WM demands (Imbo et al., 2007a; 2007b). In order to clarify our findings, we decided to replicate the first experiment by using mental addition problems without carrying.

3. SECOND EXPERIMENT

Our second experiment tested the ability of children to solve mental addition problems without carrying. As in Experiment 1, children performed three different tasks: mental addition problems, letter recall and location recall, alone and in combination. Half the problems were presented in horizontal format and half in vertical format, and half the children solved exact calculations and the other half solved approximate calculations. If the results of Experiment 1 could be explained simply by considering the type of mental calculation required (exact vs. Approximate), then we should find similar results. However, if the presence of carrying is crucial, then we should find different results.

3.1 METHOD

3.1.1 PARTICIPANTS

A total of 88 children (47 male, 41 female) attending grades 3 (mean age=8.3 years; SD=3.6 months) and 4 (mean age=9.3 years; SD=3.9 months) of primary school participated in this experiment. As in Experiment 1, for all children, parental consent was obtained prior to testing, and children from a very low socio-economic background and/or children with special educational needs were not included in the study.

Again, as in Experiment 1, children were presented with a standardized paper-and-pencil arithmetic battery (Cornoldi et al., 2002). Children who obtained low scores were excluded from the analysis. In particular, three children scored below 2 SD of the mean sample and did not take part in the experiment. Our final sample therefore included 85 children (44 male, 41 female).

3.1.2 Procedure and Stimuli

The design and procedure exactly replicated those of Experiment 1, with the sole exception that addition problems did not involve carrying. The sample was randomly split between two types of mental calculations: 42 (21M, 21F) children (mean age=8.79 years; SD=7.2 months) were asked to solve exact calculation problems, and 43 (23M, 20F) (mean age=8.77 years; SD=7.3 months) devoted themselves to solving approximate calculations.

Addition problems. We created 48 simple addition problems (i.e., without carrying) by following identical manipulations for the addition problems with carrying. The same range of sums was used, and half the problems were presented horizontally and half vertically. The same problems were employed in both exact and approximate conditions.

3.2 Results

3.2.1 Standardized arithmetic battery

The means of the participants' overall mathematic scores were analyzed with one-way ANOVA. Any differences between children performing exact and approximate calculations, $F(1,83)=1.441$ $p=.233$, $\eta^2=.017$, were observed.

3.2.2 Experimental tasks

As in Experiment 1, we controlled for accuracy in the secondary tasks. Every pupil gained at least 60% accuracy both for the verbal and visuo-spatial secondary tasks. Preliminary analyses were run in order to control for a possible influence of grade level. Regarding the percentage of correct responses, no difference between 3rd and 4th graders was observed. However, a main effect of grade on mean correct latencies was found in both exact and approximate calculations[4]. The grade effect was not considered in the following analyses. Table 2 reports the descriptive statistics.

Both percentages of correct responses and correct mean latencies were examined with separate mixed ANOVAs, and interactions were decomposed by using Bonferroni's correction at $p < .05$, adjusted for multiple comparisons.

3.2.3 PERCENTAGE OF CORRECT RESPONSES

A 2 (task: exact vs. approximate) x 2 (presentation format: HA vs. VA problems) x 2 (domain: verbal vs. visuo-spatial) x 2 (load: dual tasks vs. single task) mixed ANOVA was performed. A main effect of task $F(1,83)=49.197$ $p<.001$, $\eta^2=.372$ was found, showing that exact calculation was easier than approximate calculation (95,8% vs. 84,9%). The main effects of the type of presentation format $F(1,83)=2.599$ $p=.113$, $\eta^2=.030$, domain $F(1,83)=.599$ $p=.441$, $\eta^2=.007$, and load $F(1,83)=1.591$ $p=.211$, $\eta^2=.019$ were not significant.

Unlike the previous experiment, neither the interaction presentation format by domain $F(1,83)<1$ nor the third-level interaction task by presentation format by domain $F(1,83)=2.211$ $p=.141$ $\eta_p^2=.026$ were significant. However, the interaction task by presentation format was significant $F(1,83)=8.206$ $p=.005$ $\eta_p^2=.090$, showing that children performed better on VA problems than they did on HA ones while making exact calculations (M*diff.* = 3.37, $p < .05$), whereas they performed no differently across the different presentation formats while making approximate calculations ($p>.14$). The interaction presentation format by load $F(1,83)=25.739$ $p<.001$ $\eta_p^2=.237$ was also significant, showing that children solved single tasks better than dual tasks in VA

[4] Statistical analyses revealed that 4th grade children solved faster than 3rd graders both exact additions problems $F(1,40)=12.715$ $p=.001$, $\eta^2=.241$ (7946 ms vs. 13065 ms) and approximate additions problems $F(1,41)=7.687$ $p=.008$, $\eta^2=.158$ (12499 ms vs. 17324 ms).

problems (M*diff.* = 4.95, $p < .001$), whereas they exhibited no differences while solving single or dual tasks in HA problems ($p>.05$).

Lastly, the interaction task by presentation format by load $F(1,83)$=8.394 p=.005 η_p^2 =.092 was significant. In particular, in the single task, the children performed exact VA problems better than they performed HA ones (M*diff.* = 4.96, $p < .01$); no differences emerged in the dual tasks for exact addition problems ($p>. 32$). Regarding approximate calculation, we found that children performed VA problems better than HA ones while solving single tasks, as previously described (M*diff.* = 3.88, $p < .05$); in contrast, they obtained higher scores for HA problems than for VA ones while solving dual tasks (M*diff.* = 7.75, $p < .001$).

3.2.4 MEAN CORRECT LATENCIES

A 2 (task: exact vs. approximate) x 2 (presentation format: HA vs. VA problems) x 2 (domain: verbal vs. visuo-spatial) x 2 (load: dual tasks vs. single task) mixed ANOVA was carried out. Children solved the exact problems more quickly than they did the approximate addition problems $F(1,83)$=13.577 p<.001 η^2=.141 (10506 ms vs. 15080 ms), and they performed the HA problems more slowly than they did the VA ones $F(1,83)$=35.426 p<.001 η^2 =.299 (13587 ms vs. 11999 ms). There was no significant main effect of either domain (F<1) or load $F(1,83)$=2.067 p=.154 η^2 =.024.

The interaction task by presentation format $F(1,83)$=4.249 p=.042 η_p^2 =.049 was significant, showing that children performed HA problems more slowly than they performed VA ones for both exact and approximate calculations (M*diff.* = 2137.39, $p < .001$ and M*diff.* = 1037.77, $p < .01$, respectively).

Lastly, the interaction presentation format by domain was significant $F(1,83)$=8.901 p=.004 η_p^2 =.097, indicating that children performed addition problems associated with verbal information more slowly when these problems were presented in the horizontal format than in the vertical format (M*diff.* = 2327.84, $p < .001$); vice versa, they performed addition problems associated with visuo-spatial information more slowly when these problems were presented in the vertical format than in the horizontal format (M*diff.* = 847.32, $p < .05$). The three-way interaction task by presentation format by domain was not significant $F(1,83)<1$.

Table 2. Exp. 2. Percentage of correct responses, mean correct latencies (in seconds) and standard errors (SE) as a function of task (exact vs. approximate), presentation format (HA vs. VA problems) domain (verbal vs. visuo-spatial) and load (single vs. dual task).

		Percentage of correct responses				Mean correct latencies			
		Verbal domain		Visuo-spatial domain		Verbal domain		Visuo-spatial domain	
		Single task	Dual task	Single task	Dual task	Single task	Dual task	Single task	Dual task
Task	Presentation Format	M (SE)	M (SE)	M (SE)	M (SE)	M (SE)	M (SE)	M (SE)	M (SE)
Exact	HA problems	91.27 (1.42)	93.25 (1.71)	95.24 (1.53)	96.83 (1.02)	12.81 (1.25)	11.47 (1.19)	12.81 (1.25)	11.27 (.98)
	VA problems	99.21 (.55)	95.63 (1.61)	97.22 (1.12)	98.02 (.84)	9.53 (.79)	8.55 (.67)	9.53 (.79)	10.02 (.76)
Approximate	HA problems	84.88 (2.41)	86.43 (2.24)	83.72 (2.51)	88.37 (2.69)	16.28 (1.16)	15.83 (1.47)	15.77 (1.31)	14.51 (1.11)
	VA problems	86.82 (2.19)	79.46 (2.93)	89.53 (2.08)	79.85 (3.07)	14.40 (.89)	14.60 (1.37)	15.66 (1.20)	13.58 (1.11)

3.3 DISCUSSION

Experiment 2 examined whether the results obtained in Experiment 1, which tested mental addition without carrying, are generalizable to problems wherein carrying is required.

Concerning one of the two main points that emerged from Experiment 1 we examined whether for children a verbal WM load impairs the performance of HA problems whereas a visuo-spatial WM load impairs the performance of VA problems, especially when approximate calculation is required. Results on both the percentage of the correct responses and the mean correct latencies did not show these effects, indirectly revealing the crucial importance of the carrying request.

Nevertheless, findings observed in Experiment 2, should suggest which strategies are used by children in solving exact and approximate calculation without carrying.

Furthermore, our results showed that unlike findings in adults (Kalaman & LeFevre, 2007) – children find approximate calculations without carrying to be more difficult to solve than exact calculations. In fact, both percentages of correct responses and mean correct latencies revealed that children performed approximate calculations more slowly and less correctly than exact ones. In addition, the task interacted with the type of presentation format in both percentages of correct responses and mean correct latencies. The percentages of correct responses showed that children performed better on exact VA problems than they did on exact HA problems, whereas no differences in presentation format were found in approximate calculation. Differently, mean correct latencies revealed that children were faster at responding to VA problems of both exact and approximate calculation. These outcomes stress that children feel comfortable with solving exact calculations that are vertically presented. Lastly, in the percentages of correct responses, children performed better when they had to solve VA operations in the single task of exact calculations, whereas no differences emerged on the dual task of exact calculations. Instead, in approximate calculations, their performance was better when they had to carry out VA problems in the single task, but during the dual task, they performed better when the addition problems were horizontally presented (as suggested by the interaction among task, presentation format and load).

These results suggest that, in exact calculation, children prefer to use the typical procedure of written calculation when WM resources are still available (in single task); similarly, while performing a single task of approximate calculation, they are still aided to perform the task in the same way, thus solving approximate operations as they would solve exact ones: by using a strategy that is not suited to the task. Yet, during the dual task, the children may have tried to use the rounding off strategy in approximate calculation whenever the WM demands increased (Lemaire & Lecacheur, 2002; Lemaire et al., 2000), and they performed better when addition problems were horizontally presented. Unfortunately, we did not collect information about the strategies employed by children; hence, this hypothesis should be tested in further studies.

In conclusion, the results of Experiments 1 and 2 show that the carrying implies a different involvement of WM, especially when approximate calculation is required. However, like Experiment 1, Experiment 2 did not compare directly the approximate calculations with/without carrying. This was the goal of the third experiment.

4. THIRD EXPERIMENT

Our third experiment tested the ability of children to solve approximate mental addition problems with/without carrying. Half the problems were presented in horizontal format and half in vertical format. Half the children solved addition problems with carrying, and the other half solved addition problems without carrying.

If the results of Experiment 1 and 2 – i.e., that verbal load impairs performance on HA problems and visuo-spatial load impairs performance on VA ones – could be explained only by considering the calculation task (exact vs. approximate), then we should find similar results in solving problems with different complexity (carrying vs. no carrying). However, if problem complexity is crucial, then we should find interactions between load and presentation format, especially in the presence of carrying.

The experiment also offered the opportunity to examine whether problems with carrying demand more WM resources than those without carrying (Fürst & Hitch, 2000; Seitz & Schumann-Hengsteler, 2002; Imbo et al., 2007a; Imbo et al., 2007b), also in children.

4.1 METHOD

4.1.1 PARTICIPANTS

A total of 84 children (44 male, 40 female) attending grades 3 (mean age= 8.4 years; SD=3.6 months) and 4 (mean age=9.5 years; SD=3.2 months) of primary school were tested. As in Experiments 1 and 2, parental consent was obtained prior to testing, and children from a very low socio-economic background and/or with special educational needs were not included in the study.

As in the previous experiments, children who obtained low scores in the standardized paper-and-pencil arithmetic battery (Cornoldi et al., 2002) were excluded from analysis. In particular, three children scored below 2 SD of the mean sample, whereas one was excluded, because that child had received a diagnosis of learning disability. Our final sample therefore involved 80 children (42 male, 38 female).

4.1.2 Procedure and Stimuli

Design and procedure exactly replicated those of our previous experiments, with the only exception that addition problems required the children to perform only approximate calculations. In this way, the sample was randomly split up according to problem complexity: 40 (22 male, 18 female) children (mean age=8.9 years; SD=7.3 months) solved calculation problems without carrying and 40 (20M, 20F) children (mean age=9.0 years; SD=7 months) solved problems with carrying.

Addition problems. Exactly the same approximate calculations of the previous experiments were used (i.e., the approximate addition problems with carrying from Experiment 1 and the approximate addition problems without carrying from Experiment 2).

4.2 Results

4.2.1 Standardized arithmetic battery

The results did not reveal any difference in the overall mathematic scores between children performing problems with carrying and children performing problems without carrying $F(1,79)=3.134$ $p=.081$, $\eta^2=.03$.

4.2.2 EXPERIMENTAL TASKS

The accuracy of the secondary tasks and possible developmental change were controlled. Regarding the former, all children earned a score of at least 60% of accuracy on both the verbal and visuo-spatial secondary tasks. Regarding the latter, preliminary analyses did not show any difference between grades in the percentage of correct responses. Differently, a main effect of grade emerged in the mean correct latencies for the addition problems with and without carrying, revealing that, as expected, 4th graders solved problems quicker than did 3rd graders. The grade effect was not considered in the following analyses[5]. Table 3 reports the descriptive statistics.

The percentages of both correct responses and correct mean latencies were examined in separate mixed ANOVAs, and interactions were decomposed by using Bonferroni's corrections at $p < .05$, adjusted for multiple comparisons.

4.2.3 PERCENTAGE OF CORRECT RESPONSES

A 2 (complexity: carry vs. no carry) x 2 (presentation format: HA vs. VA problems) x 2 (domain: verbal vs. visuo-spatial) x 2 (load: dual tasks vs. single task) mixed ANOVA was performed. A main effect of problem complexity (carrying vs. no carrying) $F(1,78)=31.522$ $p<.001$, $\eta^2=.288$ was found, showing that children found the problems without carrying easier to execute than problems with carrying (85.6% vs. 73.6% respectively). The main effects of presentation format and domain were not significant ($F<1$). However, the effect of load $F(1,78)=10.285$ $p=.002$, $\eta^2=.116$ was significant, revealing that children performed worse on addition problems associated with a dual task (81.7% vs. 77.6%).

The interaction problem complexity by presentation format was significant $F(1,78)=6.679$ $p=.012$, $\eta^2=.079$. Pupils did not find any difference linked to the presentation format in solving

[5] Statistical analyses revealed that 4th graders solved faster than 3rd graders both additions with carrying $F(1,38)=9.013$ $p=.005$, $\eta^2=.192$ (12769 ms vs. 7626 ms) and without carrying $F(1,38)=7.076$ $p=.011$, $\eta^2=.157$ (17553 ms vs. 12623 ms).

additions without carrying ($p > .16$), whereas they obtained better performances in VA problems with carrying (M*diff.* = 3.646, $p < .05$). In addition, the interaction presentation format by load $F(1,78)$=15.791 p<.001 η_p^2 =.168 was significant, showing that children performed VA problems better than they performed HA ones during the single task (M*diff.* = 4.80, $p < .01$); conversely, during the dual task, children performed HA problems better than they performed VA problems (M*diff.* = 3.44, $p < .05$). Also the interaction presentation format by domain reached significance $F(1,78)$=11.280 p=.001 η_p^2 =.126, confirming that children performed more poorly on HA problems than on VA problems when the problems were associated with verbal information (M*diff.* = -3.438, $p < .05$), whereas they performed worse on VA problems than on HA problems, when the problems were associated with visuo-spatial information (M*diff.* = -4.792, $p < .01$).

The significant interaction among problem complexity, presentation format and domain $F(1,78)$=14.967 p<.001 η_p^2 =.161 revealed that children did not differ in addition problems without carrying, whatever presentation format and domain was associated (p >.24). Differently, in additions problems with carrying HA problems were more damaged by verbal information than VA one (M*diff.* = -12.500, $p < .01$) and VA problems were more impaired than HA problems by the presentation of visuo-spatial information (M*diff.* = -5.208, $p < .05$).

The interaction presentation format by domain by load $F(1,78)$=16.744 p<.001 η_p^2 =.177, revealed that - in dual task - children performed worse VA problems with a visuo-spatial load (M*diff.* = 10.625, $p < .01$) and they performed worse in HA problems with a verbal load (M*diff.* = 6.667, $p < .01$). No difference emerged in the single task condition ($p > .79$).

Lastly, the interaction complexity by presentation format by domain by load $F(1,78)$=8.780 p=.004 η_p^2 =.101 did not reveal any difference concerning additions without carrying (p>.42). Instead, in addition with carrying, when children performed the dual task, VA problems were specifically impaired by a visuo-spatial load (M*diff.* =19.167, $p < .01$), whereas, HA problems were damaged by a verbal load (M*diff.* =14.167, $p < .01$); no difference were found in the single task ($p > .49$).

4.2.4 MEAN OF CORRECT LATENCIES

A 2 (complexity: carry vs. no carry) x 2 (presentation format: HA vs. VA problems) x 2 (domain: verbal vs. visuo-spatial) x 2 (load: dual tasks vs. single task) mixed ANOVA with repeated measures on the last factor was carried out. Additions with carrying were solved more quickly than additions without carrying, $F(1,78)$=17.433 p<.001 η^2=.183 (9683 ms vs. 15334 ms); moreover, VA problems were solved more quickly than HA ones, $F(1,78)$=27.628 p<.001 η^2 =.262 (11959 ms vs. 13059 ms). There were no significant main effect of either domain or load (F<1). No other interactions turned out to be significant.

Table 3. Exp. 3. Percentage of correct responses, mean correct latencies (in seconds) and standard errors (SE) as a function of problem complexity (carry vs. no-carry) presentation format (HA vs. VA problems) domain (verbal vs. visuo-spatial) and load (single vs. dual task).

		Percentage of correct responses				Mean correct latencies			
		Verbal domain		Visuo-spatial domain		Verbal domain		Visuo-spatial domain	
		Single task	Dual task	Single task	Dual task	Single task	Dual task	Single task	Dual task
Problem complexity	Presentation Format	M (SE)	M (SE)	M (SE)	M (SE)	M (SE)	M (SE)	M (SE)	M (SE)
Without Carrying	HA problems	85.42 (2.40)	87.50 (2.29)	84.58 (2.63)	89.58 (2.51)	16.65 (1.23)	16.24 (1.55)	16.01 (1.40)	14.78 (1.18)
	VA problems	87.08 (2.27)	80.00 (3.00)	90.00 (2.13)	80.83 (3.07)	14.54 (.96)	14.99 (1.46)	15.74 (1.27)	13.74 (1.18)
With Carrying	HA problems	73.75 (2.98)	60.42 (3.02)	73.33 (2.79)	79.58 (2.70)	10.50 (1.16)	9.89 (1.01)	9.61 (.86)	10.81 (1.40)
	VA problems	80.83 (2.28)	78.33 (2.68)	78.33 (3.49)	64.17 (2.71)	9.28 (.92)	9.30 (1.17)	8.35 (.73)	9.73 (1.24)

4.3 DISCUSSION

In the third experiment, we directly compared approximate calculations with/without carrying in order to test the following hypotheses: (1) addition with carrying involves higher WM resources than addition without carrying, and (2) the presence of carrying is crucial for a specific involvement of both verbal and visuo-spatial WM on approximate calculation.

Our findings on percentages of correct responses revealed that children performed less carefully addition problems with carrying than without. These results are consistent with previous studies on multi-digit exact addition problems in adults (e.g., Fürst & Hitch, 2000; Heathcote, 1994; Seitz & Schumann-Hengsteler, 2002).

Moreover, our results showed that, as observed in adult populations for exact calculation (Imbo & LeFevre, 2010; Trbovich & LeFevre, 2003), the execution of approximate task in children involves different WM components according to the presentation format (as revealed the interaction among presentation format by domain by load). However, this pattern was significant only considering addition with carrying, as showed both the interactions problem complexity by presentation format by domain and the fourth-order interaction. In our view, this revealed that the presence of carrying in approximate calculation is crucial for the involvement of the two WM components in the function of the presentation format. In fact, our children performed worse when they were asked to estimate either HA problems with carrying associated with a verbal load, or to estimate VA problems with carrying associated with a visuo-spatial load.

5. GENERAL DISCUSSION AND CONCLUSION

The purpose of our study was to examine the involvement of both visuo-spatial and verbal WM in children asked to solve exact and approximate mental addition problems. Addition problems had to be solved alone or with a WM load and could be presented in either vertical or horizontal format. In Experiment 1, children were presented with exact and approximate addition problems with carrying, in Experiment 2 with exact and approximate addition problems without carrying, while in Experiment 3 approximate addition problems with and without carrying were directly compared.

Previous studies on adults' population suggested that the main difference between exact and approximate mental calculation is that exact calculation involves more computations and greater requirements for maintenance of intermediate sums than approximate calculation (Duverne, Lemaire, &Michel, 2003; Kalaman & LeFevre, 2007; LeFevre, Greenham & Waheed, 1993; Lemaire et al., 2000). Differently, our results revealed the opposite pattern: children generally found approximate calculation more difficult than exact calculation. Moreover, our findings seem to suggest that the strategies usually observed in adults performing approximate calculations are not completely developed in children attending 3rd and 4th grades of primary schools. In Experiment 2, comparing exact and approximate calculations without carrying, children seemed to use the same strategy for both exact and approximate calculation in single tasks (when no WM load was involved). Specifically, children performed better when they had to solve VA problems. Conversely, in dual task, when WM demands increased, they performed better when approximate calculations were horizontally presented, suggesting that they were trying to change their previous strategy. This result is consistent with the observations (LeFevre, et al., 1993) that the ability of computational estimation is poor in children and increases with age, and that the most important conquest in children's development regards the conceptual knowledge used to perform the task. Indeed, only from grade 6 do children seem to understand and use the different kinds of rounding-off strategies in estimation processes (see also Dowker, 2003).

The current findings also provide information about the specific involvement of WM components in children's mental addition operations, and suggest that WM components are deeply involved when calculation processes become more challenging and complex. In Experiment 1, in which children were asked to solve addition problems with carrying, analysis of percentages of correct responses showed that HA problems were generally more impaired than VA problems by verbal WM load, and, vice versa, VA problems were more affected than HA one by visuo-spatial WM load. This result was stronger for approximate than for exact calculation. In Experiment 2, in which children were asked to solve addition problems without carrying, considering both percentages of correct responses and mean correct latencies, the interaction among domain, load and presentation format did not reach significance. Finally, in Experiment 3, in which children were asked to solve approximate addition problems with/without carrying, the specific effect of verbal and visuo-spatial load emerged specifically in addition problems requiring a carrying procedure. Therefore, we assume that carrying is crucial to determine the specific involvement of different WM subcomponents in the solution process.

In sum, the results of the present study offer a general picture on how children meet with mental addition requests. Results offer a specification for mental addition of the general conclusions reached by recent studies (Andersson & Lyxell, 2007; Berch, 2008; D'Amico & Guarnera, 2005; Geary, Hoard, Byrd-Craven, Nugent & Numtee, 2007; Holmes et al., 2008; Passolunghi & Mammarella, 2010; Passolunghi, Mammarella, & Altoè, 2008) which showed that WM is related to and important for performance on mathematical tasks in children. Educational implications may be seen in our results, showing that children's difficulties in solving mental calculations are related with WM limitations and particular attention should be devoted to support children when mental additions involve great WM resources.

However, our study has a series of limitations which should be considered in future research. In particular, no strategy reports were collected, meaning then we could only infer the strategies employed by children in different conditions but could not know exactly whether they did in fact employ them. Further caution is needed in trying to generalize the present results to different populations. An increasing number of studies suggests that visuo-spatial WM is implicated to a greater extent in mathematic performance of younger children (see Bull, et al., 2008; Holmes & Adams, 2006; Holmes, et al., 2008; Kyttälä, et al., 2003; Maybery & Do, 2003; Rasmussen & Bisanz, 2005), and that verbal WM is involved in arithmetic performance to a greater extent in older children. Therefore, our results might be not replicated either in older children or adults' population.

In conclusion, the present study showed that WM is involved in children's mental addition problems, but the specific involvement respectively of its verbal vs. visuo-spatial components is related to the mental calculation task required, the presentation format and the presence of carrying. In agreement with Imbo et al. (2007a; 2007b), we found that children's mental calculation with carrying increases WM demands; however, differently to what has been observed in adult populations (Kalaman & LeFevre, 2007), approximate calculation involves to a greater extent WM resources than does exact calculation in children. Lastly, the specific involvement of visuo-spatial WM on VA problems and of verbal WM on HA ones (see Trbovich & LeFevre, 2003) only emerged when the task required high WM resources. In particular, only in approximate calculation with carrying the effect was clear and consistent.

CHAPTER 4

STUDY II

COMPLEX MENTAL ADDITION AND WORKING MEMORY IN CHILDREN WITH LEARNING DISABILITIES

As it has been widely pointed out in the first two Chapters, mathematical competence consists of multiple abilities (Aunola, Leskinen, Lerkkanen, & Nurmi, 2004; Campbell, 2005; Dowker, 2005; Geary, 2004;), revealing that not only basic skills, such as comparisons of quantities and counting, but also other cognitive structure (e.g. working memory) are fundamental prerequisites for solving arithmetic tasks, first by means of counting procedures and later by integrating direct retrieval of arithmetic facts from long-term memory with more complex procedural knowledge (Baroody & Wilkins, 1999; Dowker, 2003; Geary, Hamson, & Hoard, 2000).

The content of the current study was to extend the dual task paradigm to children diagnosed with developmental dyscalculia (DD) and with non-verbal learning disability (NLD), both matched to typically developing (TD) pupils. Thus, the main aim was to analyze the impact of verbal and visuo-spatial WM load in children with learning disabilities. It has been decided to present only exact addition problems with carrying because, the previous studies showed that children with typical development do not spontaneously and easily use the most functional rounding-off strategies employed by adults and older children in estimation processes (Study I).

1. DEVELOPMENTAL DYSCALCULIA (DD) AND NON-VERBAL LEARNING DISABILITY (NLD)

Learning disabilities are described as specific developmental disorders of scholastic achievement, which include reading, writing, and arithmetic skills. According to domain-specific contents, an estimated 4–7% of pupils are diagnosed as having specific learning disabilities (Geary, 2006; Hasselhorn & Schuchardt, 2006; Mercer & Pullen, 2005).

The rate of mathematic disabilities is similar to that of reading disabilities in school-aged children (Kosc, 1974; Shalev, Auerbach, Manor, & Gross-Tsur, 2000), and DD co-occurs in about 40% of individuals with reading disability (RD) or dyslexia (Ackerman & Dykman, 1995; Gathercole, Alloway, Willis, & Adams, 2006; Geary, 1993; Lewis, Hitch, & Walker, 1994). Reading skills seems also to have an impact on children's mathematical ability, since children with comorbid RD-DD display more severe and global functional difficulties than children with DD only (Andersson & Lyxell, 2007; Fuchs & Fuchs, 2002; Jordan & Hanich, 2000; Jordan & Montani, 1997).

Differently, NLD identification process is not enough strengthen in the clinical field as a general consensus about diagnostic criteria are not reached yet (Fine, Semrud-Clikeman, Bledsoe, & Musielak, in press; Spreen, 2011). Actually, in around 50 years, there is still no consensus definition about the diagnostic criteria of children with NLD and, for this reason the incidence rate for this disorder is not available.

The following paragraphs will illustrate the principal clinical features of the two samples by indicating the main inclusion/exclusion criteria useful for reach the correct diagnosis.

1.1 DD – Definition, Causes and Clinical features

Developmental Dyscalculia (DD), sometimes also called mathematical learning disability (for a discussion of the similarities between dyscalculia and mathematical learning disabilities literature, see Geary and Hoard, 2001), is characterized by severe impairments in the acquisition of mathematical skills. Traditional classification systems (e.g., DSM-IV-TR; American Psychiatric Association, 2000 and ICD-10; World Health Organization, 1992) state that the child must substantially underachieve on a standardized mathematical test relative to the level expected on the basis of his/her age, education, and intelligence and must experience disruption to academic achievement or daily living in order to receive a diagnosis of DD. In particular, there must be a considerable discrepancy between the child's general intellectual ability and the child's academic achievement (see Francis et al., 2005; Siegel, 1989; Stanovich, 2005 for alternative viewpoints).

Reviewing the literature, a distinction could be made on the basis of the performance cut-off established to identify children with difficulties. Accordingly, studies of severe DD should be distinguished from experimental studies on lighter mathematical difficulties, which tested children with scores above the 10th percentile in standardized math attainment tests (Murphy & Mazzocco, 2007; Mazzocco & Devlin, 2008). Despite the absence of consensus on terms and criteria, most researchers agree that children with DD fail to remember arithmetical facts, to use strategies and calculation procedures (Russell & Ginsburg, 1984; Kirby & Becker, 1988; Geary, 1993; Ginsburg, 1997; Jordan & Montani, 1997; Ostad, 1999; Geary & Hoard, 2001; Shalev &

Gross-Tsur, 2001; Landerl, Bevan, & Butterworth, 2004). Moreover, children with DD show difficulty when the task demands increase (Murphy & Mazzocco, 2009).

Although extensive studies have been conducted on the factors and cognitive deficits that might contribute to DD, the issue of its origins is still controversial. Specifically, the question of whether the difficulty in learning mathematics is due to a single impairment of a basic number specific core competence or a combination of impairments in a more general cognitive system is still open and several hypotheses have been put forward to account this matter (Butterworth, 2005; Geary, Hoard, Byrd-Craven, Nugent & Numtee 2007; Mix & Sandhofer, 2007; Rubinstein & Heink, 2009). In fact, children with DD are not only poor at school arithmetic and on standardized tests of arithmetic, they are also slower and less efficient at very basic numerical tasks, such as recognizing dots numerosities, and at comparing numerosities in a variety of number comparison tasks (Butterworth, 2005a; Landerl, et al., 2004). According to one of the main hypothesis, also called the core deficit hypothesis (Berch, 2005; Spelke & Kinzler, 2007; Wilson & Dehaene, 2007), mathematical difficulties lie on a specific impairment of the analog and approximate number system, which supports the ability to represent and manipulate numerical quantities. Butterworth (2005) proposed another outlook for a specific core deficit referring to the exact representation of magnitude.

Conversely, Rousselle and Noël (2007) proposed that DD could originate from impairments in accessing numerical meaning (i.e., their quantity) from symbols rather than from difficulties in processing numerosity *per se*. To date, findings are unclear and studies supporting both the defective number module hypotheses (Landerl, Fussenegger, Moll, & Willburger, 2009) and the access deficit hypothesis have been reported (Iuculano, Tang, Hall, & Butterworth, 2008; Rousselle & Noel, 2007).

1.1.1 WORKING MEMORY IMPAIRMENTS IN CHILDREN WITH DD

The association of learning disabilities with WM impairments has been demonstrated in a number of studies (Passolunghi & Siegel, 2001; Pickering, 2006; Schuchardt, Maehler, & Hasselhorn, 2008; Swanson, 2006). However, the nature of this relationship is still not well understood in children with DD (Passolunghi, 2006; Raghubar, et al., 2010). The central executive seems to be particularly impaired (Siegel & Ryan, 1989; Geary, Brown, & Samaranayake, 1991; Hitch & McAuley, 1991; Swanson, 1993; Geary, Hoard, & Hamson, 1999; McLean & Hitch, 1999; Geary, Hamson, & Hoard, 2000; Passolunghi & Siegel, 2001; Swanson & Sachse-Lee, 2001; Wilson & Swanson, 2001), whereas there is conflicting evidence for the role of the verbal and the visuo-spatial domain. Geary et al. (1991), Hitch and McAuley (1991), and Swanson and Sachse-Lee (2001) found that children with DD show deficits in verbal WM, but no evidence of such impairment was found in the studies by Bull, Johnston, and Roy (1999), Geary et al. (2000), Geary et al. (1999), McLean and Hitch (1999), and Landerl et al. (2004). Lastly, the role of the visuo-spatial domain in children with DD is less evident and seems associated with particular subgroups of children with DD (McLean and Hitch, 1999; Passolunghi, & Cornoldi, 2008; Passolunghi, & Mammarella, 2010; in press; van der Sluis, van der Leij, & de Jong, 2005). In fact, many children with DD do not seem to present problems in visuo-spatial WM (VSWM). For example, Bull et al. (1999) and Geary et al. (2000) reported that children with DD and their control peers have comparable outcomes on measures of VSWM.

The discrepancy concerning the role of either verbal or visuo-spatial domain in DD could be due to various reasons, such as developmental changes related to the participants' ages, different criteria for group selection and the different tasks employed.

1.2 NLD – Definition, Causes and Clinical features

Non-verbal learning disability (NLD) children are chiefly characterized by intact verbal abilities, but impaired visuospatial skills (Nichelli & Venneri, 1995; Rourke, 1989), showing a discrepancy between Verbal and Performance IQ, and major problems in areas of visuo-spatial working memory (Cornoldi, Rigoni, Tressoldi & Vio, 1999; Mammarella & Cornoldi, 2005a, 2005b), psychomotor, visuo-constructive skills and mathematics, within a context of well-developed psycholinguistic skills.

At the moment, this disorder is not included in any Clinical Classification Systems, such as DSM-IV-TR (2000) and ICD-10 (1992), and the majority of researchers and clinicians do not agree on which criteria used for the diagnosis have the greatest discriminative power. Hence, not only NLD do not have still received a clear definition, but also research and practice in the field are not developed as necessary.

However, in recent years, several researches tried to identify specific sub-types of NLD (Forrest, 2004; Grodzinsky, Forbes & Bernstein, 2010). For example, Forrest (2004) distinguished two main profile of NLD: *i)* a *visuo-spatial disability category* for children with severe visuo-spatial deficits affecting to academic achievement, in particular on math, and *ii)* a *social processing disorder category* for children whose social skills deficits are primary.

A recent meta-analysis individuated the following macro-criteria for diagnosing children with NLD (Mammarella & Cornoldi, in press):

Discrepancy between verbal and visuo-spatial intelligence, considering Verbal Intelligence Quotient (VIQ) greater than Performance Intelligence Quotient (PIQ);

Visuo-constructive, grapho-motor and motor-coordination impairments, detected using the Rey-Osterrieth Complex Figure (ROCF; Osterrieth, 1944), the Visual-Motor Integration (VMI) test (Beery, & Buktenica, 2006) and the Target test (Reitan, 1966). See Figure 1 for an exemple;

- Visuo- spatial WM (VSWM) deficit;
- Specific pattern in academic achievement, showing major academic learning difficulties in arithmetic, geometry and science;
- Poor social and emotional skills.

Different research teams (e.g. Cornoldi, et al., 2003) argued that, NLD must be considered such as a learning disability and the diagnosis must preliminarily examine whether the general criteria for a learning disability are present.

1.2.1 Mathematical impairments in children with NLD

Rourke and his colleagues (see Rourke, 1993 for a review) identified two subtypes of mathematically related deficits. The former subtype comprised children who manifest poor performance in mathematics and even poorer performance in reading and spelling, whereas the latter one, later called NLD, involved children who perform poorly in mathematics but perform sufficiently in reading and spelling. Therefore it appears evident as, from the beginning, a deficit in mathematical tasks has been played an important role in the correct identification of this disturb.

As previously mentioned, children with NLD usually showed good reading achievement and poor mathematics performances (Bloom & Heath, 2010; Galway & Metsala, 2011; Forbes & Bernstein, 2010; Forrest, 2004; Harnadek & Rourke, 1994; Worling, Humphries, & Tannock, 1999; Semrud-Clikeman, Walkowiak, Wilkinson, & Christopher, 2010). However, calculation skills have been studied in depth only on few research. Nichelli and Venneri (1995) reported a case study (AE) of a 22 year-old man with a developmental learning disorder consisting of visuospatial deficits and arithmetic difficulties. In particular, AE made errors in writing multi-digit numbers under dictation. In written calculation he gave incorrect answers arising through column confusion. Positron emission tomography scans revealed a marked hypo-metabolism of the right hemisphere. In a later study, Venneri, Cornoldi and Garuti (2003), comparing NLD and controls in arithmetic calculations, found that the disabled group had more severe difficulties with written calculation, especially where this involved borrowing/carrying. The authors hypothesized that children with NLD do not have a generalized problem with calculation *per se*; instead, their problems derive from dealing with specific processes, including visuo-spatial working memory (VSWM), which governs calculation. Thus, regarding the arithmetic achievement, NLD children made arithmetic errors typically associated with visuo-spatial processes, occurring carrying, partial calculation, and column confusion.

This hypothesis has been confirmed by a recent study of Mammarella, Lucangeli and Cornoldi (2010), in children displaying symptoms of NLD who were impaired in written calculations with carrying and failed in a number ordering tasks. However, covariance analyses showed that VSWM failures were primary with respect to calculation impairments.

2. FOURTH EXPERIMENT

The fourth experiment combines the dual task paradigm and the assessment of children with DD to examine whether horizontally presented addition (henceforth HA) problems and vertically presented addition (henceforth VA) problems involve different content domains, mainly verbal in the first case and visuo-spatial in the latter case. It also examines whether the result is emphasized in the case of children with DD. In particular, we examined whether these children were particularly affected by the HA problems associated with a verbal secondary task. In sum, the main aim of the current study was to analyze the impact of verbal and visuo-spatial secondary tasks on children with DD when solving HA and VA problems. We decided to present only exact addition problems with carrying. In fact, previous studies showed that TD children do not spontaneously and easily use the most functional strategies employed by adults (Kalaman & LeFevre, 2007; Lemaire & Lecacheur, 2002); in fact, only from the sixth grade onward do TD children seem to understand and use the different kinds of rounding-off strategies in estimation processes (see also Dowker, 2003) with the consequence that children with a DD should presumably acquire this kind of competence even later.

2.1 METHOD

2.1.1 PARTICIPANTS

The total sample comprised 36 children aged 9 to 12 years. Eighteen of those children (13 boys and 5 girls, mean age 134.39 months, *SD*=16.72) had received a clinical diagnosis of DD at the University Center for Learning Disability, Padova (Italy), and the remaining eighteen (13 boys and 5 girls, mean age 131.72 months, *SD*=14.03) were typically developing (TD) children, attending fourth, fifth and sixth grades, as the DD group. In particular, the TD group was formed by children matched for age, schooling and socio-economic status, with no reported school difficulties.

Although children with DD were referred to a centre for learning disabilities and had received a clinical diagnosis, we also controlled to ensure that the clinical sample met specific criteria. The inclusion criteria were as follows: (1) diagnosis of DD; (2) age between 9 and 12 years; (3) total

IQ ≥ 80; (4) below-average arithmetic scores (i.e., ≤ 10^{th} percentile in at least two specific aspects of mathematical learning and a total mathematical score of ≤ 16^{th} percentile); and (5) a significant discrepancy between total IQ and overall performance on arithmetical academic achievement (see Schuchardt et al., 2008). Exclusion criteria were (1) being treated with psychoactive drugs; (2) fulfilling criteria for diagnosis of intellectual disability and attention deficit hyperactivity disorder; (3) history of seizures during the previous two years; (4) total IQ < 80; (5) poor socioeconomic situation; and (6) medical illness requiring immediate treatment.

The complete assessments included the most recent available standardized Italian version of the Wechsler Intelligence Scale for Children battery (WISC-III, Wechsler, 1991) and the MT battery (Cornoldi & Colpo, 1998), which measure children's reading skills. In particular, the MT battery obtains a measure of children's reading speed by computing the mean number of syllables read by the child while reading texts aloud, a measure considered the best index of a reading disability for transparent languages. Another measure collected by the MT battery concerns the number of errors made by a child (accuracy) while reading aloud. In line with recent literature (Fletcher, 2005; Raghubar et al., 2009), children diagnosed with DD are also often affected by reading difficulties (i.e., dyslexia). In our clinical sample, nine children revealed a severe impairment (< 2 SD) in reading speed and/or reading accuracy. Finally, another score refers to reading comprehension ability and is given by the total number of correct responses provided without time constraints in a multiple-choice questionnaire about the meaning of a passage. During the test, the child reads the passage silently, and then the child can refer to the passage at any time while answering the questions. Moreover, grapho-motor skills were measured using a subtest of the battery for the assessment of writing and orthographic skills (Tressoldi & Cornoldi, 1991). During this subtest, participants are given one minute to write either the word "uno" (one) or the word "le" (the), or the series of numbers in letters starting from "uno" (one) many times as possible without lifting the hand from the sheet of paper. Descriptive statistics are reported in Table 4.1.

Table 4.1. Demographic and clinical characteristics of children with DD

Characteristics	**M (SD)**	**Min**	**Max**
Age	134.39 (16.72)	113	163
General cognitive skills			
Full scale IQ	100.11 (11.33)	80	120
Verbal IQ	96.94 (11.61)	71	116
Performance IQ	103.67 (11.32)	88	124
Reading abilities			
Speed (z scores)	-1.41 (1.23)	-2.79	.49
Accuracy (percentiles)	49.17 (24.33)	5	80
Comprehension (percentiles)	43.89 (21.87)	20	75
Grapho-motor skills			
/le/(z scores)	-.33 (1.17)	-2.60	1.44
/one/(z scores)	.15 (1.66)	-2.81	2.71
/digit-letters/(z scores)	-.16 (1.50)	-2.98	2.79

A signed consent form was obtained from the parents. In the case of the TD group, the consent was given only for the arithmetical academic achievement and the experimental test. All the children spoke Italian as a first language, and none was primarily visually or hearing impaired or was identified as having a neurologically degenerative condition.

2.1.2 Materials and Procedure

Arithmetical academic achievement. In order to further confirm the groups' differences in arithmetic abilities, the children were presented with a paper-and-pencil standardized arithmetic battery (Cornoldi, Lucangeli & Bellina, 2002) that included the following subtests: *a)* written calculations that required children to perform as many correct answers as possible to a list of calculations (addition, subtraction, multiplication and division problems); *b)* mental calculations that required the children to find the solution to multi-digit calculation problems; *c)* number ordering tasks that required children to order digits from smallest to largest and vice-versa; *d)* number dictation that required them to write down in Arabic format a series of numbers spoken aloud by the experimenter; and *e)* arithmetical facts, which tested the number fact knowledge of children.

The standardized arithmetic battery was individually administered. The dependent variables from both performances of single subtests and of mean correct responses combined for the five subtests were considered.

Experimental tasks. The experimental tasks used are the same employed in the first experiment (Study I) and involved four sets of 12 multi-digit addition problems with carrying, half associated with visuo-spatial information and half with verbal information.

Each child was given 16 practice trials and 48 experimental trials divided into four sets and shaped from four different conditions. The presentation order was completely counterbalanced across participants. Trial-by-trial feedback on both arithmetic and secondary tasks was given in the practice blocks. The time sequence of the procedure described for addition problems was the same for all four conditions.

2.2 RESULTS

2.2.1 ARITHMETICAL ACADEMIC ACHIEVEMENT

As reported in Table 4.2., children with DD performed more poorly than TD children in all the subtests of the standardized battery. The results of one-way ANOVAs are summarized in Table 4.2.

Table 4.2. Performances in percentiles of children with DD and TD children in arithmetical academic achievement measures

	DD M (SD)	**TD** M (SD)	*df*	*F*	*p*	η^2
Written calculations	10.56 (8.38)	50.00 (17.15)	1,34	76.86	.001	.69
Mental calculation	8.89 (7.58)	57.50 (16.55)	1,34	127.05	.001	.79
Number ordering	11.67 (8.57)	59.72 (17.44)	1,34	110.00	.001	.76
Number dictation	9.17 (6.24)	56.11 (15.10)	1,34	148.53	.001	.81
Arithmetical facts	9.44 (10.83)	58.89 (16.05)	1,34	117.39	.001	.77
Total score	9.94 (3.72)	56.44 (10.06)	1,34	338.37	.001	.91

2.2.2 EXPERIMENTAL TASKS

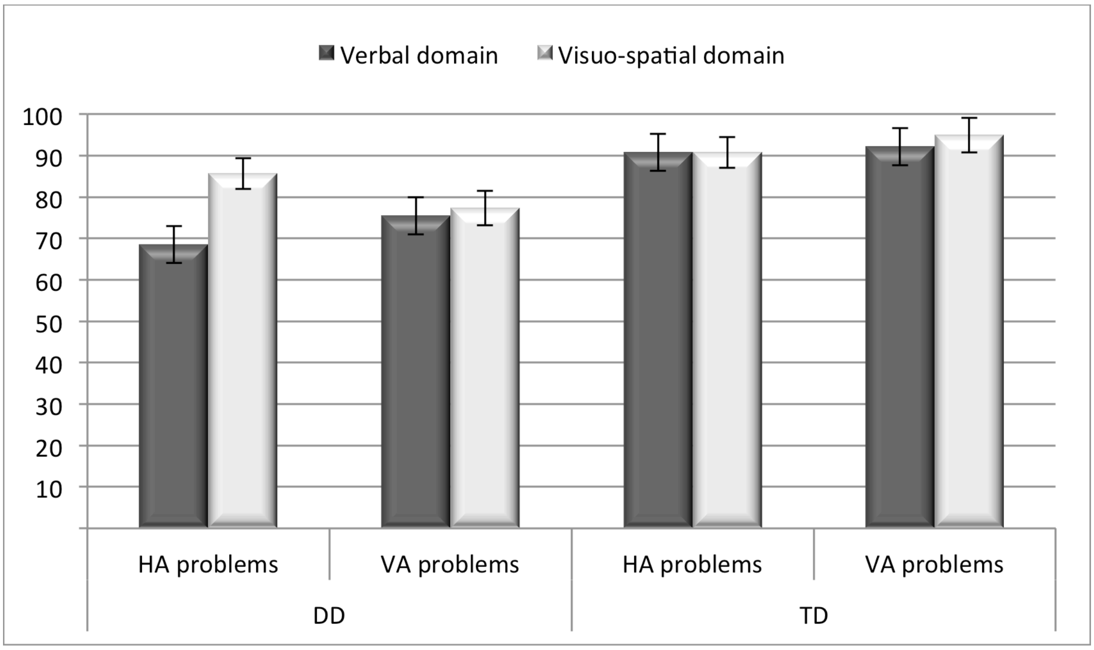
We computed the percentages of correct responses of addition problems for the four different conditions and the mean correct latencies to addition problems, calculated only by considering the trials in which children selected the correct answers for addition problems. Each of these variables was analysed in separate mixed ANOVAs. Interactions were decomposed by means of post-hoc, pair-wise comparisons with Bonferroni's correction at $p < .05$, adjusted for multiple comparisons.

A 2 (group: DD vs. TD children) x 2 (domain: verbal vs. visuo-spatial) x 2 (presentation format: HA vs. VA) x 2 (load: dual tasks vs. single task) mixed ANOVA was performed. A main effect of group $F(1,34)$=8.18 p=.007, η^2=.19 was found, showing that children with DD performed more poorly than TD children (76.7% vs. 92.1% correct responses). The main effect of the domain (verbal vs. visuo-spatial) $F(1,34)$=11.75 p=.002, η^2=.26 was significant, showing a poorer performance on addition problems with verbal information (81.71%) than on addition problems with visuo-spatial information (87.15%). Neither the main effect or the type of presentation format, $F(1,34)< 1$ or load $F(1,34)< 1$, were significant.

Moreover, the interaction between domain and presentation format, (HA vs. VA) $F(1,34)$=4.57 p=.04 η_p^2 =.12, was significant, showing that children performed worse on HA problems associated with verbal information than on HA problems with visuo-spatial information (M*diff.* = -8.56, p< .003), whereas there were no differences in VA problems between verbal and visuo-spatial domains (M*diff.* = -2.32, p>.11).

The group effect was better specified by the significant second-order interaction between domain and group and by the significant third-order interaction between domain, group and presentation format. The interaction domain x group, $F(1,34)$=6.51 p=.01 η_p^2 =.16, revealed that children with DD performed worse on addition problems associated with verbal information than on addition problems with visuo-spatial information (M*diff.* = -9.49, p< .001), whereas TD children exhibited no differences according to the domains (M*diff.* = -1.39, p>.54). Lastly, the third-order interaction domain x presentation format x group, $F(1,34)$=9.55 p=.004 η_p^2 =.22, showed that children with DD were more impaired while solving HA problems with verbal information than while solving HA problems with visuo-spatial information (M*diff.* = -17.13, p< .001); in contrast, no differences between visuo-spatial and verbal domains emerged in VA problems (M*diff.* = -1.85, p> .36) (see Figure 4.2.). In addition, no significant differences were found in TD children.

Figure 4.2. Interaction between domain (verbal vs. visuo-spatial) by presentation format (HA vs. VA problems) by group (children with DD vs. TD) for percentages of correct responses in addition problems. Error bars represents standard errors.



As a further control, we divided the sample of children with DD into two sub-groups: those also showing reading impairments (DD + dyslexia) and those revealing only mathematical impairments (DD). Children were included in the group with DD + dyslexia on the basis of a performance lower than 2 *SD* on reading speed and/or accuracy. Thus, nine children were identified as DD + dyslexia (3 F and 6M) and nine (1 F and 8M) were DD only.

A 3 (group: DD vs. DD + dyslexia vs. TD children) x 2 (domain: verbal vs. visuo-spatial) x 2 (presentation format: HA vs. VA) x 2 (load: dual tasks vs. single task) mixed ANOVA was performed, which generally replicated the outcomes of the previous ANOVA and showed a similar pattern for the two subgroups of children with DD. In particular, the main effect of group $F(2,33)=4.32$ $p=.022$, $\eta^2=.21$ was significant, showing that children with DD + dyslexia performed more poorly than did TD children (73.8% vs. 92.1% correct responses); however, no difference emerged between DD and TD children (M*diff.* = -12.50, $p>.20$) and between children with DD and DD + dyslexia (M*diff.* = -5.78, $p>1.00$). The main effect of domain $F(1,33)=16.11$

p=.002, η^2=.33 was significant, showing a poorer performance of addition problems associated with a verbal domain. Neither type of presentation format, $F(1,33)$< 1 or load $F(1,33)$< 1, were significant. Moreover, the interaction domain x group was significant, $F(2,33)$=3.32 p=.04 η_p^2 =.17, showing that both children with DD and DD + dyslexia performed worse on addition problems associated with verbal information than on addition problems with visuo-spatial information (M*diff.* = -8.33, p< .01; and M*diff.* = -10.65, p< .002, respectively), whereas TD children showed no performance differences according to the domain (M*diff.* = -1.39, p>.54). The interaction domain x presentation format, $F(1,33)$=8.85 p=.05 η_p^2 =.21, was also significant, showing that children performed HA problems worse when these addition problems were associated with verbal information (M*diff.* = -11.42, p< .001), whereas there were no differences in VA problems between the verbal and visuo-spatial domain (M*diff.* = -2.16, p>.16). Finally, the interaction domain x presentation format x group, $F(2,33)$=4.83 p=.01 η_p^2 =.23, was significant. In particular, in both children with DD and DD + dyslexia, HA problems were more impaired by verbal associated information (DD: 71.3%; DD+ dyslexia: 65.7%) than by visuo-spatial associated information (DD: 86.1%; DD+ dyslexia: 85.2%; DD: M*diff.* = -14.81, p< .01; DD + dyslexia: M*diff.* = -19.44, p< .001). Inversely, in VA problems, no differences between the visuo-spatial domain (DD: 81.5%; DD+ dyslexia: 73.1%) and the verbal domain (DD: 79.6%; DD+ dyslexia: 71.3%) had emerged (DD: M*diff.* = -1.85, p> .52; DD + dyslexia: M*diff.* = -1.85, p> .52). No differences were observed in performances on HA problems associated with the verbal domain between children with DD (71.3%) and DD + dyslexia (65.7%) (M*diff.* = -5.56, p> 1.00). No significant differences were found in TD children.

Finally, we computed the mean correct latencies to addition problems, calculated only considering the trials in which children selected the correct answers for addition problems. A 2 (group: DD vs. TD children) x 2 (domain: verbal vs. visuo-spatial) x 2 (presentation format: HA vs. VA) x 2 (load: dual tasks vs. single task) mixed ANOVA was carried out.

The main effect of group was significant, $F(1,34)$=7.70 p=.009 η^2=.19, showing that children with DD (M = 11299.09 ms, SE = 1156.86) solved addition problems more slowly than did TD children (M = 6758.18 ms, SE = 6758.18). The main effect of presentation format was also significant, $F(1,34)$=22.44 p<.001 η^2=.39, showing that children were faster at solving VA problems (M=8496.10 ms, SE=756.22) than HA problems (M=9561.17ms, SE=889.79). Additionally, the main effect of load was significant, $F(1,34)$=6.19 p<.019 η^2=.15, revealing that children solved addition problems faster when the recall task had to be performed after the arithmetic task (M= 8639.53 ms, SE=853.85) under the condition in which there was no memory request (M= 9417.74 ms, SE=811.28). The effect of the memory request was better clarified by the significant interaction group by load, $F(1,34)$=5.82 p<.021 η^2=.15, revealing that children with DD were particularly rapid at giving their responses under the condition in which the secondary task had to be performed (M*diff.* = -1532.93, p< .001), while no differences were observed in TD children (M*diff.* = -23.45, p>.96). There were no other significant effects. The same pattern of results emerged, distinguishing DD in two sub-groups (i.e., children with DD + dyslexia and DD).

2.3 DISCUSSION

The purpose of the fourth experiment was to examine the involvement of both verbal and visuo-spatial domains in children with DD and TD who were asked to solve vertically and horizontally presented mental addition problems with carrying. Hence, mental addition problems could be solved after the presentation either of verbal or visuo-spatial information and could be presented in either vertical or horizontal format. By combining the use of the dual task paradigm and the assessment of children with DD, we intended to examine whether HA and VA problems involve different processes, mainly verbal in the first case and visuo-spatial in the latter case. We also examined whether the result was emphasized in the case of children with DD.

Results showed that, as already observed in adults (Trbovich & LeFevre, 2003) and in TD children (Study I), different content domains (verbal and visuo-spatial) are involved in children's mental addition operations according to the presentation format. In particular, in the present experiment, children with DD were asked to solve addition problems with carrying, and an analysis of the percentages of correct responses showed that mental addition presented in horizontal format generally impaired the DD children more than those in the vertical format with verbal information. This result was previously observed by Trbovich and LeFevre (2003) in adult participants examining mean correct latencies, but never observed in children with learning disability. The inclusion of a group with a DD contributed evidence of the different characteristics of the two presentation formats in relationship with specific weaknesses of children with DD. In fact, children with DD were particularly disadvantaged when verbal information was presented with HA problems, revealing that verbal weaknesses are critical in the majority of children with DD, even in the absence of a comorbidity with dyslexia. Results were, in fact, replicated in a subsequent analysis, in which we split our sample into children with DD plus dyslexia and children with DD only, and are in agreement with previous studies showing that children with reading disability report phonological deficits (e.g., Siegel & Linder, 1984; Helland & Asbjørnsen, 2004; Kibby, Marks, Morgan, & Long, 2004; Gathercole, Alloway, Willis, & Adams, 2006; Schuchardt et al., 2008). Therefore, our findings showed that children with just DD outperformed children with combined math and reading disorders (Rubinstein, 2009), but both groups replicated the same pattern of results: horizontally presented addition problems were more impairing than vertically presented ones, according to the verbal domain.

However, it is worth noting that the present experiment differed from other studies with more competent calculators (e.g. Trbovich & LeFevre, 2003), because the memory load did not affect directly the pattern of results. This probably occurred because the presentation of information in the verbal domain was sufficient to impair both the conditions with and without a WM load. However, WM load affected mean correct latencies in some way, since children with DD gave a faster answer than did children with TD when the secondary task had to be performed, probably in order to avoid the decay of previously presented information. Moreover, mean correct latencies confirmed previous data, showing that children solved VA problems faster than HA problems, matching the results of both Studi I and Trbovich and LeFevre (2003).

Finally, the experiment confirm the slight tie between WM and DD as broad number of recent research have been showed, although the nature of this relationship is still not well-understood (Bull & Johnston, 1997; Bull, Johnston & Roy, 1999; McLean & Hitch, 1999; Passolunghi & Siegel, 2001, 2004; Passolunghi & Mammarella, 2010; in press; Raghubar, Barnes, & Hecht, 2010).

3. FIFTH EXPERIMENT

In the last experiment, the same methodology applied with DD children has been extended to children with diagnosis of NLD. The goal was deepen the understanding of the relationship between WM and mathematic from another prospective.

3.1 METHOD

3.1.1 PARTICIPANTS

The total sample comprised 30 children aged 8 to 11 years. Fifteen children (12 boys and 3 girls, mean age 118.07 months, *SD*=7.74) had received a clinical diagnosis of NLD at a Neuropsychiatric Developmental Centre for learning disorders in northern Italy, and fifteen (12 boys and 3 girls, mean age 113.67 months, *SD*=8.53) were typically developing (TD) children, attending from third to sixth grades, and were tested in local schools. In particular, the TD group was formed by children matched for age, schooling and socio-economic status, with no reported school difficulties.

All the children spoke Italian as first language, and none was primarily visually or hearing impaired, or identified as having a neurologically degenerative condition. A signed consent form was obtained from parents and an assent form from each child.

Although NLD children were referred to a specialized centre for learning disorders and had received a clinical diagnosis, we also controlled that the groups met specific criteria. The inclusion criteria for NLD group were as follows: (1) diagnosis of NLD; (2) age between 8 and 11 years; (3) Wechsler Intelligence Scale for Children Verbal Intelligence Quotient (VIQ) greater than Performance Intelligence Quotient (PIQ) by at least 15 points; (4) a significant discrepancy between verbal and perceptual/visuospatial intelligence: i.e., Verbal Comprehension index (VCI) greater than Perceptual Organization index (POI) of the WISC-III scale (5) visuo-constructive impairments; (6) low mathematical academic achievement and good reading decoding. Exclusion criteria were (1) being treated with psychoactive drugs; (2) fulfilling criteria for diagnosis of clinically significant autistic syndrome or Asperger's syndrome, developmental coordination disorder, traumatic brain injury; (3) history of seizures during the previous 2 years;

(4) total IQ<70; (5) poor socioeconomic situation; and (6) medical illness requiring immediate treatment. Descriptive statistics of clinical assessment are reported in Table 4.3.

Table 4.3. Demographic and clinical characteristics of children with NLD

Characteristics	M	SD	Min	Max
Age	118	7,74	106	**132**
General cognitive skills				
Full scale IQ	102,2	11	89	121
Verbal IQ	114,4	10,7	103	135
Performance IQ	88,9	10,6	75	103
Reading abilities				
Speed (z scores)	-0,7	0,8	-1,9	1
Accuracy (percentiles)	52	25,4	< 10	75
Grapho-motorskills				
/le/(z scores)	-1,5	0,9	< 3	0,1
/one/(z scores)	-2	1	< 3	-0,4
/digit-letters/(z scores)	-2	0,7	< 3	-0,7
Rey's Figure				
Copy (percentiles)	16,3	21	<10°	70°
Memory (percentiles)	18,8	21,3	<10°	60°

In the case of the TD group, the consent was given only for the arithmetical academic achievement and the experimental test (see below).

3.1.2 MATERIALS AND PROCEDURES

As concern the experimental tasks, this experiment exactly replicated those used in the fourth study. Regarding the arithmetic achievement battery, it has been used only a reduced form of the same battery (Cornoldi et al., 2002) for a collective and quick assessment of TD children.

Administrated subtests included written calculations, number comparison task, number ordering and number transcoding tasks.

3.2 Results

3.2.1 Arithmetical Academic Achievement

The results of one-way ANOVAs showed that, as expected, children with NLD performed poorly than TD children in the overall score of the standardized battery $F(1,28)=15.22$ $p=.001$, $\eta^2=.35$. In particular, their performance is poor than TD children in written calculations $F(1,28)=7.56$ $p=.01$, $\eta^2=.21$, and number ordering tasks $F(1,28)=8.95$ $p=.006$, $\eta^2=.24$. Both number comparison and transcoding tasks did not reveal any difference between the two groups ($F< 1$).

3.2.2 Experimental Tasks

Also for the arithmetical achievement score, the analyses for the experimental tasks followed the line of the previous experiment. Two variables were derived from addition problems: the percentages of correct responses for the four different conditions and the mean of correct latencies, computed only by considering the trials in which children answered correctly. Each of these variables was analysed in separate mixed ANOVAs. Interactions were decomposed by means of post-hoc, pair-wise comparisons with Bonferroni's correction at $p < .05$, adjusted for multiple comparisons.

A 2 (group: NLD vs. TD children) x 2 (domain: verbal vs. visuo-spatial) x 2 (presentation format: HA vs. VA) x 2 (load: dual tasks vs. single task) mixed ANOVA was performed. A main effect of group $F(1,28)=8.56$ $p=.007$, $\eta^2=.23$ was found, showing that children with NLD performed more poorly than TD children (70.9% vs. 87.5% correct responses). The main effect of the presentation format (HA vs. VA problems) $F(1,28)$=6.75 p=.015, η^2=.19 was significant, showing as children performed poorly HA (76,7%) than VA (81.8%) problems. Neither the main effect of domain, $F(1,28)$< 1 or WM load $F(1,28)$=1.62 p=.21, η^2=.05, were significant. Moreover, no other interaction resulted significant, neither second-order interactions nor third-order ones.

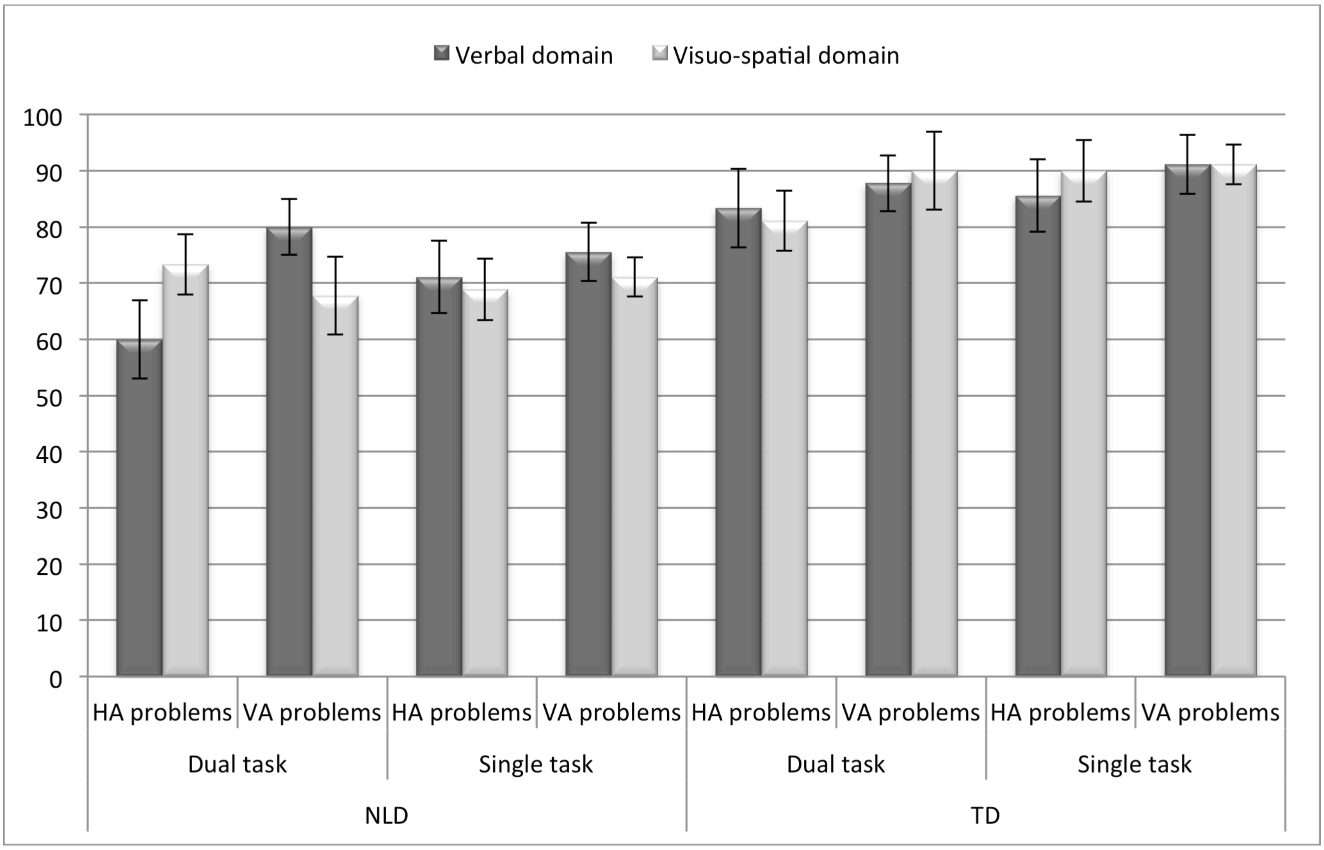
The group effect was better specified by the only significant fourth-order interaction among group, domain, presentation format and load $F(1,28)$=5.12 p=.03 η_p^2 =.16. As shown in Figure 4.3., the interaction revealed that only in the dual task conditions (i.e. when children have to perform both addition problem and secondary task), children with NLD found HA problems harder to solve when verbal information is associated (M*diff.* = -13.33, *p*= .034), whereas the performance on VA problems decreased when combined with visuo-spatial information (M*diff.* = -12.22, *p*= .048). Conversely, no difference emerged considering the percentages gained by NLD children in the single task conditions according to the interaction between domain and presentation format both considering HA problems (M*diff.* = -2.22, *p*= .77) and HA problems (M*diff.* = -4.45, *p*= .38). Similarly, TD children did not exhibit any significant differences (*p*> .56).

At length, the mean correct latencies to addition problems were analyzed by means a 2 (group: NLD vs. TD children) x 2 (domain: verbal vs. visuo-spatial) x 2 (presentation format: HA vs. VA) x 2 (load: dual tasks vs. single task) mixed ANOVA was carried out. Excepting for the main effect of group that was not significant, $F(1,28)$=1.64 p=.21 η^2=.05, other significant effects traced out what emerged for percentage of correct responses.

The main effect of presentation format was significant, $F(1,28)$=14.06 p=.001 η^2=.33, showing that children were faster at solving VA problems (M=9897.71 ms, SE=932.13) than HA problems (M=11767.46 ms, SE=1357.66). No other main effect emerged.

The effect of the domain was better clarified by the significant interaction group by domain by presentation format, $F(1,28)$=5.020 p<.033 η_p^2=.15, revealing that children with NLD were particularly rapid at giving their responses when have to solve VA problems, independently from the nature of the secondary task associated (verbal domain, M*diff.* = -3121.15, p=.003; visuo-spatial domain, M*diff.* = -1553.75, p=.031). Conversely, TD children answered faster VA problems than HA problems only when associated with visuo-spatial information (M*diff.* = -16.67.82, p=.008), while no difference was observed in the verbal domain (p> .38).

Figure 4.3. Interaction between domain (verbal vs. visuo-spatial) by presentation format (HA vs. VA problems) by load (dual tasks vs. single task) by group (children with DD vs. TD) for percentages of correct responses in addition problems. Error bars represents standard errors.



3.3 DISCUSSION

This last experiment was carried out in order to further deepen the relationship between WM and mathematical achievement in learning disabilities context. Actually, the experiment followed exactly the same methodology of the previous studies, combining the use of the interference paradigm and the assessment of children with NLD. The main goal was to examine whether a particular feature problem, such as the presentation format, involve different processes in relation to different task constraint. In agreement with the previous research both on adults (Trbovich & LeFevre, 2003) and on TD children (Study I), also NLD children' performance showed that different content domains (verbal and visuo-spatial) are specifically involved in the solution of complex mental addition problems in relation to the presentation format. However, compared to DD children' performance, some differences emerged.

Since the outcomes resulting from mean correct latencies analysis concurred to demonstrate only a marginal aspect of our hypothesis (i.e. by confirming that NLD children solved VA problems faster than HA ones), we mainly focused on the analysis of the percentages of correct responses. Considering the accuracy analysis, some aspects overlap between the two experiments, for example both clinical group gained lower scores than TD groups. Moreover the overall results in DD sample revealed that mental addition presented in horizontal format generally impaired the performance more than those in the vertical format with verbal information, without any influence of the secondary tasks. Conversely, NLD children found problems horizontally presented generally harder to solve than vertical one, independently from the associated domain. Moreover, the fourth-grade interaction highlighted that NLD children' performance was selectively impaired in relation to presentation format and type of domain associated, only in concomitance with the request of a secondary task execution.

Furthermore, NLD results are perfectly in line with those emerged in Study I with TD children for approximate calculation, revealing as the presence of carrying procedure makes the primary task sufficiently highly demanding on WM resources to produce a selective interference between presentation format and secondary task in NLD children.

4. GENERAL DISCUSSION AND CONCLUSION

Understanding the specific weakness of children with different profiles of mathematical difficulties is crucial to highlight the underlying processes. In the present study, children with DD and with NLD were administered with a dual-task paradigm in which they were asked to perform a primary task (i.e., addition problems with carrying) in combination with a secondary task (i.e.. recall either verbal or visuo-spatial material; Imbo & LeFevre, 2010; Trbovich & LeFevre, 2003) in order to analyse the impact of verbal and visuo-spatial secondary tasks according to the presentation format.

Regarding DD children, it is worth noting that in the fourth experiment we did not find that the vertical format was more affected than the horizontal one by the visuo-spatial domain (Trbovich & LeFevre, 2003). Children with DD showed only a specific impairment due to the verbal domain in exact addition problems with carrying emerged, supporting the hypothesis (Geary et al., 1991; Hitch & McAuley, 1991; Swanson & Sachse-Lee, 2001) that verbal processes are particularly critical, at least in the majority of children with a DD.

Differently, NLD children showed a response pattern more similar to TD children (Study I). Indeed, the results of the fifth experiment replicated the pattern emerged in approximate calculation both in the first and third experiments testing TD children: in particular NLD children showed a specific impairment in arithmetic performance in relation to presentation format, secondary task and WM request. As in the previous experiments (Imbo & LeFevre, 2010; Trbovich & LeFevre, 2003), results emphasized that the interaction between presentation format and domain was significant only under the dual task condition.

In conclusion, the current research found that the dual task paradigm applied to children with learning disabilities revealed that their performances did not completely overlap those observed in TD children. In particular, both children with DD and with DD + dyslexia performed poorly when addition problems were presented in horizontal format and associated with verbal information, showing that horizontally presented addition problems require a high quantity of verbal resources which are particularly compromised in children with DD. Moreover NLD children gained the same pattern of responses performing exact addition problems of those that TD children registered on approximate calculation in Study I. This indicates that an additive recall task is sufficient to disrupt the performance on arithmetic task in children with NLD. It should look surprising that children with NLD were specifically impaired by a verbal WM load when HA problems were presented. However, it is worth noting that verbal WM is usually the straight point for them. It is possible that, when a verbal WM load is required in association with HA problems, they could not use their preserved verbal WM, and this dramatically impaired their performance. Differently, the specific impairment due to a visuo-spatial WM load associated with the presentation of VA problems should be explained on the basis of their deficit on visuo-spatial WM (Cornoldi, et al., 1995; 1999; Cornoldi & Mammarella, 2005a; 2005b).

However, some limitation of the present study should be acknowledged. Actually, clinical samples involved in Study II include various different subtypes: both DD and NLD are associated to various neurological profiles (Geary, 2004; Forrest, 2004; Mammarella et al., 2006). Second, although confounding variables between the two clinical groups and respective TD group were carefully controlled for, the selection procedure may have influenced the pattern of results. Moreover, a manipulation better differentiating between the conditions with and without memory load could be introduced. Another limitation is that the current research involve a limited number of children with pure DD. It has been shown that children with comorbid DD + dyslexia display more severe and global functional difficulties than children with DD only (Andersson & Lyxell, 2007; Fuchs & Fuchs, 2002; Jordan & Hanich, 2000; Jordan & Montani, 1997).

Nevertheless, clinical and educational implications could be drawn on the basis of our study. In fact, our data support the suggestion that teachers should pay attention to the way in which addition problems are presented (i.e., vertically, vs horizontally), especially for children with learning disabilities. More specifically, one way to improve arithmetical skills of LD children, might be to reduce the demands on their WM system, both verbal and visuo-spatial WM for children with NLD, whereas reducing only verbal request for children with DD (see Gathercole & Alloway, 2004). This could be accomplished by providing external memory aids and giving simple instructions, as suggested by Gathercole and Alloway (2004) and Gathercole et al., (2006).

CHAPTER 5

GENERAL DISCUSSION

We spend our childhood growing complex cognitive skills, including reading, writing, comprehension, mathematics and problem solving. These competencies allow us to realize the greatest advantage from education and from everyday life in general. Especially in recent years, several psychological studies have been developed in order to delineate general cognitive mechanisms that underlie mathematics abilities. One among the most favourite candidates is working memory (WM), in charge of holding information in short-term memory; using this information to guide action; keeping track of the order of steps in solving a problem or carrying out an activity; keeping track of the results of one of these steps while carrying out the next. Actually, multi-digit arithmetic problems involve more than one single step and require several resources of WM, not only because of its demands on place value concepts, but because it necessary processes several steps while keeping track partial results.

The importance of the current thesis lays in the fact that when children solve mental calculation allocate WM resources differently according to both task complexity and features problem. How does it come? Which variables affect relationship between WM and arithmetic calculations? In the following paragraphs the most important findings are summarized and put in a broader context including practical and future implications.

1. RESEARCH OVERVIEW

The focus of this dissertation has been to analyze how children at different ages use limited mental resources to manage complex mental calculation from a developmental prospective. More specifically, the overall project aimed to increase the current understanding of cognitive processes involved in a specific algorithm, multi-digit additions, when a both verbal and visuo-spatial WM load is involved.

The theoretical framework widely applied in the literature for studying this matter is represented by the multi-component model developed by Baddeley and Hitch (1974, Baddeley, 1986) which described, in its first formulation, three components: a domain-general central executive, and two slave systems responsible for handling verbal and visuo-spatial information.

It is indubitably true that arithmetic tends to be impaired by (and to impair) the concurrent performance of any cognitive task that competes to achieve attention and planning resources; and that some features of arithmetic problems, such as the presence of carrying procedures, are more difficult than others to combine with such tasks (Fürst & Hitch, 2000).

Since in the analysis of the relation between arithmetic and WM, a great number of variables (and theoretical frameworks) come into paly the findings are even more conflicting than those concerning the relationships between other cognitive competencies. Although increasing numbers of recent studies have investigated these relationships, the involvement of various WM subcomponents in mental addition problems is still sparse and not clear (DeStefano & LeFevre, 2004; and Raghubar, Barnes, & Hecht, 2010, for reviews). In this dissertation a selective interference paradigm, also called dual task paradigm, has been used. It is widely employed to investigate dealings between WM and arithmetic performance. Thus, based on Kalaman and LeFevre's results (2007) and Trbovich and LeFevre's research (2003) on adults, three different experiments have been carried out to examine the performance on exact and approximate calculation of simple and complex addition problems in typically developing children attending grades 3 and 4 of primary school (Study I, Chapter 3). In Study II (Chapter 4) the same experimental methodology, but only considering exact complex addition problems, has been applied to children with specific profiles of learning disabilities.

1.1 TYPICAL DEVELOPMENT

Concerning Study I, results showed that, as already observed in adults, WM is also involved in children's mental addition problems and that different WM components are involved according to task constraints. As regards the specific feature problems, analysis of percentages of correct responses showed that mental additions presented in horizontal format were generally more impaired than those in vertical format by verbal WM load, and, vice versa, the vertical format was more affected than the horizontal one by visuo-spatial WM load. Nevertheless, these findings emerged only when additions with carrying are considered. Therefore, we assume that carrying procedure is crucial to determine the specific involvement of different WM components in the solution process. Moreover, this interaction was stronger for approximate than for exact calculation.

Thus, as regards of type of computation, the results showed that children generally found approximate calculation more difficult than exact calculation. While in adult population the main difference between the two forms of mental calculation is that exact calculation involves more computations and makes greater requirements for maintenance of intermediate sums than approximate calculation (Duverne, Lemaire & Michel, 2003; Lemaire, Lecacheur & Farioli, 2000), this is not the case in children. These results are consistent with the observations

(LeFevre, Greenham & Waheed, 1993) that the ability of computational estimation is poor in children and increases with age, and that the most important conquest regards the conceptual knowledge used to perform the task. Indeed, only from grade six do children seem to understand and use the different kinds of rounded strategies in estimation processes (see also Dowker, 2003).

1.2 ATYPICAL DEVELOPMENT

The content of Study II was to extend the dual task paradigm to children diagnosed with developmental dyscalculia (DD) and with non-verbal learning disability (NLD).

Findings revealed that the selective interference paradigm applied to children with learning disabilities leads to results not completely overlapping those observed in Study I. In particular, DD children performance was mostly damaged by horizontal problems presented in association with verbal information, revealing that verbal weaknesses are critical in the majority of children with DD. Moreover this outcome was replicated in a subsequent analysis, in which the sample was split into children with DD plus dyslexia and children with DD only. This further analysis simply confirmed that children with just DD outperformed children with combined math and reading disorders (Rubinstein, 2009).

Conversely, NLD children performance on exact calculation resulted more in line with those registered on approximate calculation in typically developing children. Actually, NLD children performance was selectively impaired in relation to presentation format and type of domain associated, revealing as the presence of carrying procedure makes calculation process sufficiently highly demanding on WM resources to produce a selective interference.

1.3 MERITS AND LIMITS OF THE CURRENT THESIS

Nevertheless both Studies address a potential interesting issue for both clinical and educational implications, and although the implementation of a dual task methodology with children is relatively novel and noteworthy, these researches present a series of limitations regarding both the methodology applied and the participant selection.

The main lack of these studies refers to the absence of any form of strategy investigation, which would have given a more complete prospective of the results. Actually, children often solve the same problem in different ways and the same child may solve the same problem in different ways at different times. In the literature, mainly referred to addition problems, strategies have been categorized into two main categories (Green, Lemaire & Dufau, 2007; Imbo & LeFevre, 2009). The first, called right-to-left, in which the addends are treated as concatenations of single digits and imply the right-to-left order for the purpose of calculation (Fuson, 1990). The prototypical exemplar of such strategy is the column-by column algorithm that is taught in school for the written calculation. In other words, a person might picture the problem in a column on a mental blackboard (transposing it to a vertical format) and solve the problem adding unit-to-unit and ten-to-ten. The second category, called left-to-right, comprises strategies in which the operands are represented and manipulated in a more holistic manner, and thus infer a left-to-right order of problem solving. For example, to solve the problem 64 + 12, a person might decompose the first operand in 60 + 4 and another in 10 + 2, storing this information in the phonological loop, then add 60 + 10 = 70 and 4 + 2, and then assemble the answer 'seventy-six' by combining all the information. On the basis of this short summary, the importance of include the analysis of strategies selection in the investigation of the role of WM sub-components appears evident, in order to have a more broad picture of cognitive processes underlie mental calculation, according to features problem.

Another methodological constraint refers to distinctive cognitive resources implied in the dual o single condition trials. Indeed, in the dual task paradigm, performance could suffer from the inevitable overlap between processing and retention of primary task and encoding or responding of secondary one. This implies that whichever task is presented first suffers from concurrent retention of the other task (which, in turn, suffers from response interference, Cowan & Morey, 2007). On this perspective, different task manipulations and presentation order could be better differentiating between the conditions with and without memory load.

It is important pointed up a further thoughtfulness concerning participants' selection. Indeed, as already turned out in Chapter 3, several research seems to indicate that different forms of WM are differently associated with arithmetic performance according different ages (e.g. verbal WM is involved in arithmetic performance to a greater extent in older children whereas visuo-spatial WM is more implicated in mathematic performance of younger children; Bull, et al., 2008; Holmes & Adams, 2006; Holmes, et al., 2008).

Lastly, also the high variability within each clinical samples (Chapter 4) might be reduced, by including more restrictive criteria both for age and for clinical features. Consequently, our results might be not replicated either in children at different ages or in adults' populations.

2. PRACTICAL IMPLICATIONS

There is an increasing body of evidence supporting the concept that mathematical deficits could depend on poor WM abilities. For example, poor computational abilities are often associated to low WM scores (Bull & Scerif, 2001; D'Amico & Gharnera, 2005; Gathercole & Pickering, 2001; Geary et al., 1999; McLean & Hitch, 1999; Passolunghi & Siegel, 2001; Swanson & Sachse-Lee, 2001; Wilson & Swanson, 2001). Thus the importance of adequate e specific educative training is evident not only for calculation, and above all, not only for children with learning disabilities, but also for typically developing children. Common failures in scholastic achievement might depend on WM weakness, which implies forgetting lengthy instructions, place-keeping errors (e.g., missing out numbers in a dictation tasks), and problem to cope with simultaneous processing and storage demands. Eventual other complex tasks, in turn, amplified the WM demands, leading to memory overload (see Gathercole & Alloway, 2005).

Educational and clinical implications may be easily seen in our results, showing as children's difficulties in solving mental calculations are related with WM limited resources. Thus, particular attention should be devoted to support children especially when mental computation increases complexity by involving great WM resources.

2.1 EDUCATIONAL AND CLINICAL IMPLICATIONS

In order to cope with WM loads effectively during the class activities, it is important that teacher is aware of the memory capacities typical for a specific age group. In the same way, the learning progress of grade contest can be improved dramatically by reducing working memory demands during the lesson. For example, on many circumstances, children simply forget what they are doing, leading to miss the mark of several learning activities. A worthwhile expedient to facilitate children's memory for instructions is to use concise and clear instructions, and, when possible, should be broken down into single steps. Another solution is to repeat frequently the crucial information contained in the original instructions, especially for tasks that take place over an extended period of time. Moreover, it is important to use a simple vocabulary and an easy syntax.

Further recommendations may refer to the complex activity itself that can be reduced by breaking down into simplest tasks, and can be integrated by providing external memory supports.

External memory aids such as useful squared notebooks are widely used in classrooms, even if not ever used with awareness from children. Lastly, but not less important, children with weak WM capacity often relied on less functional strategies (i.e. simple counting strategy) even when they have to execute complex tasks, resulting in a decrement of performance efficiency (Alloway, 2006; El-Naggar, 1996). Hence, teachers should help children "building" more adequate strategies, such as derived fact strategies, involving knowledge and reasoning about arithmetical properties (i.e. commutativity), which lead to work out arithmetical fact not store in the memory. In general, developing a metacognitive strategic behaviour for dealing with complex cognitive activities helps to improve learning successes. This strategic behaviour includes encouraging the pupil to ask for forgotten information where necessary, training in the correct use of memory aids, and encouragement to continue with complex tasks rather than abandoning them even if some of the steps are not completed due to memory failure.

By the same token, namely by reducing the demands on WM systems, several educational implications can be applied also to the clinical field. The outcomes of Study II may provide valuable feedbacks (obviously restricted to these particular clinical profiles only) that can be transformed in simple suggestions applicable both during a specific intervention program and during daily activity in the classroom. A comprehensive example could be addressed considering written calculation: a recommendation for NLD children is to present complex operation already displayed in column, in order to avoid possible error due to transcription process from horizontal presentation. Conversely, DD children would find helpful do not exceed in verbal requests, for example verbally summarize the procedure steps while they are executing a computation.

3. AVENUES FOR FURTHER STUDIES

Although the investigation of WM involvement in arithmetic abilities is noteworthy as well as highly motivating for the several implications in practical field, the issue leaves open several other aspects that may to be addressed in further research.

Since arithmetical content are composed by different aspects, it possible to find several dissociations among them. Only to provide some examples, multiplication and division tend to be more challenge than addition or subtraction; many children find particularly difficult to deal with fractions and decimals. Thus, future research my use the approaches adopted in the current doctoral dissertation (i.e. selective interference paradigm) to investigate the role of WM components in solving each different algorithm, as well as, processing of decimals numbers and symbolic fractions. Another important aspect of arithmetic abilities is written arithmetic. Written calculation may be dependent on WM for the accurate recognition and processing of written symbols. Actually, it requires that correct sequential organisation of material and procedures, the maintenance and elaboration of numerical information displayed in space, as well as a correct visual analysis of the graphic symbols. For these reasons, should be interesting start to deepen the tangled puzzle of WM and written calculations.

Finally, in order to broaden the picture referred to learning impairments, further research might involve the analysis of other specific clinical profiles, more or less strictly related with mathematical deficits, such as children with dyslexia or with diagnosis of Attention deficit hyperactivity disorder (ADHD).

3.1 DOES WORKING MEMORY PLAY A ROLE IN STRATEGY SELECTION?

As pointed out in the limitation paragraph, the conflicting findings with regard to the effects of different aspects of WM on arithmetic may reflect the fact that children, and people in general, use different strategies for arithmetic.

In order to better comprehend the cognitive processes involved in the solution of multi-digit additions (and subsequently extent to the other algorithms), future research may take into account what types of strategies are used by children. For this purpose, Siegler and Lemaire (1997) developed an interesting paradigm, the "choice/no-choice" paradigm, to examine the features of the strategies employed. In the *choice* condition, participants have to solve mental calculations choosing the solution method from two or three common strategies. In contrast, in the *no-choice* condition, all participants are instructed to use a particular strategy to re-solve the problems (Abbate & Di Nuovo, 1998; Lemaire & Lecacheeur, 2002). The only study that had integrated this paradigm with the dual task methodology has been conducted by Imbo and Vandierendonck, (2007). The Authors investigated the solution strategies of single-digit addition problem on children attending 4th and 6th grades. The findings related to WM and strategies use revealed that WM load did not seem to influence strategy selection.

Thus it is worth noting to investigate strategy chosen in children, not only at different school levels, but above all, in relation to different features problems, such as problem complexity (i.e. carrying or borrowing procedures), type of calculation (i.e. exact or approximate computation), and presentation format (i.e. horizontal or vertical presentation format).

To conclude, there is space for even more research in the combined domain of arithmetic content and WM resources. As becoming for scientific research, the current doctoral dissertation did not only answer to relevant (even if few) questions, but it also raised new and countless interrogations.

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