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Dipartimento di Fisica e Astronomia "G. Galilei"

Scuola di Dottorato di Ricerca in Fisica
XXVIII Ciclo

Exploring Inflationary Perturbations with an Effective Field Theory Approach

Direttore della Scuola : *Prof. A. Vitturi*

Supervisore : *Prof. Nicola Bartolo*

Cosupervisore: *Prof. Sabino Matarrese*

Dottorando : *Dario Cannone*

Ecco che cosa ho pensato: affinché l'avvenimento più comune divenga un'avventura è necessario e sufficiente che ci si metta a raccontarlo. È questo che trae in inganno la gente: un uomo è sempre un narratore di storie, vive circondato delle sue storie e delle storie altrui, tutto quello che gli capita lo vede attraverso di esse, e cerca di vivere la sua vita come se la raccontasse. [...] Avrei voluto che i momenti della mia vita si susseguissero e s'ordinassero come quelli d'una vita che si rievoca. Sarebbe come tentar d'acchiappare il tempo per la coda.

J.P. Sartre

Riassunto

Il fondo cosmico di microonde (CMB) è per la cosmologia moderna quello che gli acceleratori sono per la Fisica delle Particelle. È stato un aiuto fondamentale nella costruzione di quello che oggi possiamo definire il Modello Standard della Cosmologia, dell'Inflazione e della formazione delle strutture cosmiche. Le sue attuali, precise misurazioni costituiscono la più forte conferma che l'Universo ha attraversato una fase di espansione esponenziale, in cui perturbazioni quantistiche sono evolute fino a formare la struttura su grande scala che oggi vediamo. In particolare, ogni modello di fisica delle alte energie che punti a spiegare i primi stadi di vita dell'Universo deve confrontarsi con i limiti che le osservazioni del CMB hanno posto, che sembrano favorire la più semplice realizzazione dell'Inflazione: un singolo campo scalare "in lento rotolamento" (slow-roll) che guida l'espansione dell'Universo e fa da sorgente alle perturbazioni adiabatiche. La nostra comprensione dell'Inflazione tuttavia è ben lontana dall'essere completa. Sia la mancanza di un'alternativa teorica completamente convincente che la totale esclusione di tutti i possibili altri effetti ammessi nelle perturbazioni primordiali continuano a spingere la ricerca teorica e sperimentale. Una delle possibilità più interessanti nello studio delle conseguenze osservative di modelli inflazionari è la non-Gaussianità primordiale, poiché permette un collegamento diretto con la fisica delle interazioni tra i campi attivi durante l'Inflazione.

In questa Tesi, analizzeremo parte dell'interessante fenomenologia di cui l'inflazione può essere responsabile, ponendo particolare enfasi alla questione della non-Gaussanità. In questo contesto, simmetrie e teorie di campo efficaci possono giocare ruoli decisivi e saranno uno degli argomenti principali di questo lavoro. L'elaborato si svilupperà come segue:

- Nel Capitolo 1 saranno introdotti i concetti base dell'Inflazione, con particolare attenzione alla dinamica delle perturbazioni primordiali.

- Nel Capitolo 2 rivedremo velocemente la fisica del CMB, gli osservabili legati alla fisica dell'Inflazione e le loro attuali misure. Qui verranno introdotti i concetti base della non-Gaussianità primordiale.

- Nel Capitolo 3 diamo un esempio di come la non-Gaussanità può essere prodotta andando oltre gli scenari inflazionari standard. Mostreremo come una modifica della gravità di Einstein durante l'Inflazione potrebbe aver lasciato impronte potenzialmente misurabili negli osservabili cosmologici sotto forma di non-Gaussianità. Queste modifiche infatti appaiono nella forma di un ulteriore campo, che potrebbe avere interazioni non bananli con l'inflatone. Mostreremo esplicitamente il caso $R+\alpha R^2$, in cui può esser prodotta una non-Gaussianità al livello $f_{\rm NL}\sim\mathcal{O}(1-10)$ in una configurazione detta quasi-locale.

- Il capitolo 4 contiene l'introduzione all'approccio della Teoria Effettiva dell'Inflazione (EFTI) alle perturbazioni cosmologiche e degli strumenti che saranno utilizzati nei capitoli successivi.

- I Capitoli 5 e 6 sono dedicati allo studio dei modelli inflazionari con "features" nel potenziale o velocità del suono dell'inflatone nel contesto della EFTI. Questo approccio permetto di studiare gli effetti delle features nello spettro di potenza e nel bispettro delle perturbazioni di curvatura da un punto di vista indipendente dal modello, parametrizzando le features direttamente in termini di parametri di "slow-roll" modificati. È così possibile ottenere un consistente spettro di potenza, insieme a non-Gaussianità che cresce con la quantità che parametrizza la larghezza della feature. Con questo trattamento sarà immediato generalizzare e includere features anche negli altri coefficienti dell'azione effettiva delle perturbazioni. La conclusione in questo caso è che, escludendo termini di curvatura estrinseca, effetti interessanti nel bispettro possono nascere solo da features nel primo parametro di slow-roll e nella velocità del suono. Infine, discuteremo la scala di energia a cui i contributi a loop alle interazioni sono dello stesso ordine dei contributi tree-level e l'espansione perturbativa smette di funzionare. Richiedendo che tutte le scale di energia rilevanti per il problema studiato siano sotto questo cutoff, deriveremo un forte limite sulla larghezza della feature, o, equivalentemente, sulla sua caratteristica scala temporale, indipendentemente dall'ampiezza della feature stessa. Faremo anche notare come una feature molto stretta, che sembra poter garantire un miglior fit ai dati dello spettro di potenza del CMB, potrebbe essere già oltre questo limit, mettendo in dubbio la consistenza del modello che la predice.

- Nei Capitoli 7 e 8 svilupperemo il concetto di rottura completa dei diffeomorfismi nella teoria effettiva delle perturbazioni primordiali. Durante l'inflazione con un singolo campo, l'invarianza per riparametrizzazioni temporali è rotta dal background cosmologico dipendente dal tempo. Qui vogliamo esplorare la situazione più generale in cui anche i diffeomorfismi spaziali sono rotti. Per prima cosa, considereremo la possibilità che questa rottura sia data da termini di massa o operatori derivativi per le perturbazioni della metrica nella cosiddetta Lagrangiana in gauge unitaria. Successivamente aggiungeremo anche operatori che rompono simmetrie discrete, come la parità e l'inversione temporale. Investigheremo le conseguenze cosmologiche di queste rotture, concentrandoci su operatori che hanno effetto sullo spettro delle fluttuazioni. Identificheremo gli operatori che possono produrre uno spettro blu per le perturbazioni tensoriali, senza la violazione della "null energy condition", e operatori che possono portare alla non conservazione delle perturbazioni comoventi di curvatura su scale oltre l'orizzonte anche in Inflazione "single-clock". Inoltre, troveremo che gli operatori che rompono simmetrie discrete producono nuove fasi, dipendenti dalla direzione, per le funzioni d'onda sia degli scalari che dei tensori.

- Nel Capitolo 9 continueremo a studiare la rottura dei diffeomorfismi. Usando i bosoni di Goldstone associati alla rottura di simmetria, esamineremo le conseguenze osservative sulla statistica dei modi scalari e tensoriali, con particolare enfasi alla struttura delle interazioni e delle funzioni a tre punti. Mostreremo che la rottura di queste simmetrie può portare ad un ampiezza aumentata per il bispettro nel limite "squeezed" e a una dipendenza angolare caratteristica tra i tre vettori d'onda.

- Il Capitolo 10 contiene considerazioni finali e possibili direzioni future. Le Appendici A e B rivisitano alcuni aspetti generali della quantizzazione delle pertubazioni primordiali e il formalismo in-in, usato per il calcolo dei bispettri presentati nel testo principale. Le Appendici C e D contengono alcuni dettagli tecnici sulla rottura dei diffeomorfismi temporali e spaziali. Nell'Appendice E discutiamo invece come i risultati del Capitolo 9 indicano prospettive per vincolare il livello della rottura di diffeomorfismi spaziali durante l'Inflazione.

Abstract

The Cosmic Microwave Background (CMB) is for nowadays cosmology what colliders are for Particle Physics. It has been an invaluable help to build what is now known as the Standard Model of Cosmology and shape our knowledge about Inflation and the formation of cosmic structures. More recently, the measurements of anisotropies in the temperature and polarization of the CMB are perfectly compatible with a Universe that has undergone an inflationary phase of exponential expansion, where quantum perturbations were stretched on cosmological scales and evolved into the Large Scale Structure (LSS) that we see today. In particular, any high-energy physics model which aims to explain the first stages of the evolution of the Universe, must face the bounds that CMB observations has put and that seem to favor the simplest realization of inflation, where a single slowly-rolling scalar field drives the expansion of the Universe and sources adiabatic perturbations. Our understanding of inflation however is far away to be complete. The lack of both a compelling theoretical UV mechanism and a definite exclusion of the many possible allowed effects in the primordial perturbations force us to push theoretical and observational research further on. One of the most intriguing possibility to study observational consequences of inflationary models is non-Gaussianity, as it provides a direct link to the interactions between the fields active during inflation.

In this thesis, we will review some of the interesting phenomenology that inflation could be responsible of, with particular attention to non-Gaussianity. In this context, symmetries and effective field theories can play a fundamental role and will be one of the main subjects of this work. The outline is as follows:

- In Chapter 1 the basic concepts of inflation will be introduced. The main focus will be on primordial quantum perturbations and their dynamics.

- In Chapter 2 we will briefly review the physics of the CMB, the observables related to the physics of inflation and their current measurements. Here basic concepts about primordial non-Gaussianity will be introduced.

- In Chapter 3 we give an example of how primordial non-Gaussianity can be pro-

duced when going beyond the simplest inflationary scenarios. We show that modification of Einstein gravity during inflation could leave potentially measurable imprints on cosmological observables in the form of non-Gaussian perturbations. This is due to the fact that these modifications appear in the form of an extra field that could have non-trivial interactions with the inflaton. We show it explicitly for the case $R + \alpha R^2$, where nearly scale-invariant non-Gaussianity at the level of $f_{\rm NL} \sim \mathcal{O}(1-10)$ can be obtained, in a "quasi-local" configuration.

- Chapter 4 contains a review of the approach of the Effective Field theory of Inflation (EFTI) to cosmological perturbations and of the tools that will be used in the following chapters.

- Chapters 5 and 6 are devoted to the study of inflationary models with features in the potential or speed of sound of the inflaton, in the context of the EFTI. This approach allows us to study the effects of features in the power-spectrum and in the bispectrum of curvature perturbations, from a model-independent point of view, by parametrizing the features directly with modified "slow-roll" parameters. We can obtain a self-consistent power-spectrum, together with enhanced non-Gaussianity, which grows with a quantity that parametrizes the sharpness of the step. With this treatment it will be straightforward to generalize and include features in other coefficients of the effective action of the inflaton field fluctuations. Our conclusion in this case is that, excluding extrinsic curvature terms, the most interesting effects at the level of the bispectrum could arise from features in the first slow-roll parameter or in the speed of sound. Finally, we find the energy-scale beyond which loop contributions have the same size of the tree-level ones and the perturbative expansion breaks down. Requiring that all the relevant energy scales of the problem are below this cutoff, we derive a strong upper bound on the sharpness of the feature, or equivalently on its characteristic time scale, which is independent on the amplitude of the feature itself. We point out that the sharp features which seem to provide better fits to the CMB power spectrum could already be outside this bound, questioning the consistency of the models that predict them.

- In Chapters 7 and 8 we will develop the concepts of full-diffeomorphism breaking in the effective theory of primordial perturbations. During single-field inflation, time-reparameterization invariance is broken by a time-dependent cosmological background. Here we want to explore more general setups where also spatial diffeomorphisms are broken. First, we will consider the possibility that this breaking is given by effective mass terms or by derivative operators for the metric fluctuations in the so-called unitary-gauge Lagrangian. Then we also add operators that break discrete symmetries like parity and time-reversal. We investigate the cosmological consequences of the breaking of spatial diffeomorphisms and discrete symmetries, focussing on operators that affect the power spectrum of fluctuations. We identify the operators for tensor fluctuations that can provide a blue spectrum without violating the null energy condition and operators for scalar fluctuations that can lead to non-conservation of the comoving curvature perturbation on superhorizon scales even in single-clock inflation. Moreover, we find that operators that break discrete symmetries lead to new direction-dependent phases for both scalar and tensor modes.

- In Chapter 9 we will investigate further the subject of diffeomorphism breaking. Using the Goldstone bosons associated to the symmetry breakings, we examine the observational consequences for the statistics of the scalar and tensor modes, paying particular attention to interactions and three-point functions. We show that this symmetry breaking pattern can lead to an enhanced amplitude for the squeezed bispectra and to a distinctive angle dependence between their three wavevectors.

- Chapter 10 contains final considerations and possible future directions. Appendices A and B review some general aspects related to the quantization of inflationary perturbations and the in-in formalism, used to compute the bispectra presented in the main text. Appendices C and D contain some technical details about time and spatial diffeomorphism breaking. In Appendix E we discuss how the results of Chapter 9 indicate prospects for constraining the level of spatial diffeomorphism breaking during inflation.

List of Papers

1. N. Bartolo, D. Cannone, A. Ricciardone and G. Tasinato, *"Distinctive signatures of space-time diffeomorphism breaking in EFT of inflation"*, arXiv:1511.07414 [astro-ph.CO].

2. D. Cannone, J. O. Gong and G. Tasinato, *"Breaking discrete symmetries in the effective field theory of inflation,"*, JCAP **1508** (2015) 08, 003 [arXiv:1505.05773 [hep-th]].

3. D. Cannone, G. Tasinato and D. Wands, *"Generalised tensor fluctuations and inflation"*, JCAP **1501** (2015) 01, 029 [arXiv:1409.6568 [astro-ph.CO]].

4. N. Bartolo, D. Cannone, R. Jimenez, S. Matarrese and L. Verde, *"Mild quasilocal non-Gaussianity as a signature of modified gravity during inflation"*, Phys. Rev. Lett. **113** (2014) 16, 161303 [arXiv:1407.6719 [astro-ph.CO]].

5. D. Cannone, N. Bartolo and S. Matarrese, *"Perturbative Unitarity of Inflationary Models with Features"*, Phys. Rev. D **89** (2014) 12, 127301 [arXiv:1402.2258 [astro-ph.CO]].

6. N. Bartolo, D. Cannone and S. Matarrese, *"The Effective Field Theory of Inflation Models with Sharp Features"*, JCAP **1310** (2013) 038 [arXiv:1307.3483 [astro-ph.CO]].

Contents

Part I

Introduction

CHAPTER 1

The Standard Model of Cosmology

1.1 Friedmann–Robertson–Walker Universe

The most important feature of our Universe is certainly its large scale homogeneity and isotropy. The observable patch of the Universe is of the order of 3000 Mpc and appears homogeneous and isotropic when coarse grained on 100 Mpc scales [7, 8]. This observation naturally suggests and supports the hypothesis that we do not occupy any special place in the Universe, hypothesis that is known as Cosmological Principle and is the starting point of Modern Cosmology. The Cosmological Principle can be formulated in a mathematical language as an assumpition on the geometry of spacetime:

- The hypersurfaces with constant time are maximally symmetric three-dimensional subspaces of the whole four-dimensional spacetime;
- All "cosmic" tensors (such as the metric $g_{\mu\nu}$ or the energy-momentum tensor $T_{\mu\nu}$) are form invariant with respect to the isometries of these subspaces.

The first point implies that it is always possible to cast the metric in the privileged form:

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)^2 \left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2) \right] , \tag{1.1}$$

where $a(t)$ is the scale factor, $k = 0, \pm 1$ is the spatial curvature and r, θ and ϕ are polar coordinates. This is the well-known Friedmann–Robertson–Walker (FRW) metric. The second point tells us about the behaviour of matter in a FRW Universe: the energy-momentum tensor $T_{\mu\nu}$ must be form invariant, $T'_{\mu\nu}(x) = T_{\mu\nu}(x)$, so that T_{00}, T_{0i}, and T_{ij} transform as three-scalars, three-vectors and three-tensor respectively, under purely spatial isometries. It can be shown then that the energy-momentum tensor of the Universe necessarily takes the same form as for a perfect fluid,

$$T_{\mu\nu} = (\rho(t) + p(t))u_\mu u_\nu + p(t)g_{\mu\nu} \; , \tag{1.2}$$

where u_μ is the 4-velocity, ρ and p the energy density and isotropic pressure respectively. We can insert this information into the Einstein's field equation,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu} \; , \tag{1.3}$$

(where $R_{\mu\nu}$ is the Ricci tensor build on the metric $g_{\mu\nu}$ and G the gravitational constant) and find the well-known Friedmann equations:

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} \; , \tag{1.4}$$

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 = -\frac{4\pi G}{3}(\rho + 3p) \; , \tag{1.5}$$

where $H = \dot{a}/a$ is the Hubble parameter. One of the Friedmann equations can be recovered from the other making use of the conservation law:

$$\mathrm{d}E + p\,\mathrm{d}V = 0 \qquad \Longrightarrow \qquad \dot{\rho} + 3H(\rho + p) = 0 \; . \tag{1.6}$$

Together with an equation of state $p = p(\rho)$, these equations form a complete system to determine the two unknown functions $a(t)$ and $\rho(t)$. Let us focus for a moment on the Friedmann equations (1.4) and (1.5). It is clear that as long as the quantity $\rho + 3p$ remains positive, the "acceleration" $\ddot{a}$ is negative, since $a > 0$ by definition. Moreover we know that at present the universe is expanding, thus $\dot{a}/a > 0$. It follows that the curve $a(t)$ versus t must be concave downward and must have reached $a(t) = 0$ at some finite time in the past. This is the singolarity universally known as Big Bang.

A homogeneous, isotropic universe whose evolution is governed by the Friedmann equations is the framework within which one can understand the formation of galaxies and cosmic structure. The standard cosmological model also predicts the existence of a background black-body radiation at the temperature $T \simeq 2.7K$, which is indeed observed and known as the Cosmic Microwave Background (CMB). One further outstanding success is the prediction of light-element abundances produced during cosmological nucleosyntesis, which agree with current observations (see [9] for a recent review). However it was soon realized that this picture suffers from (at least) two major unresolved problems and lacks the answer to a fundamental question. The question is the origin of the primordial inhomogeneities that will give rise to the cosmic structures that we observe today, whereas the problems regard initial conditions and "unlikeliness".

1.1.1 The Horizon Problem

The comoving particle horizon τ is defined as the maximum distance a light ray can travel from time 0 to time t. In a FRW Universe it can be written as

$$\tau = \int_0^t \frac{\mathrm{d}t'}{a(t')} = \int \mathrm{d}\ln a \left(\frac{1}{aH}\right) , \tag{1.7}$$

where we expressed the integral as a function of the *comoving Hubble radius* $(aH)^{-1}$. The physical size of the particle horizon is simply:

$$d(t) = a(t)\tau . \tag{1.8}$$

In a Universe filled with a fluid with equation of state

$$w = \frac{p}{\rho} , \tag{1.9}$$

it can be shown [10–12] that both the comoving Hubble radius and the particle horizon always increase in time,

$$\tau \sim (aH)^{-1} \sim a^{(1+3w)/2} . \tag{1.10}$$

This is true, for example, for a matter fluid $w = 0$ or radiation fluid $w = 1/3$. This means that the fraction of the Universe in causal contact grows in time, or, in other words, that the causally connected Universe was much smaller in the past. In particular, applying this to the Cosmic Microwave Background (CMB) and taking into account a finite-time singularity, one concludes that the last scattering surface is made of several independent patches that had never causally communicated in the past but incredibly share the same degree of isotropy. If no particles could have interacted, the situation of a photon bath with the same properties everywhere in the sky is extremely improbable. The lack of a microphysical explanation to this paradoxical fine tuning is known as the horizon problem.

1.1.2 The Flatness Problem

The first Friedmann equation (1.4) can be written as:

$$\Omega(a) - 1 = \frac{k}{(aH)^2} , \tag{1.11}$$

where:

$$\Omega(a) = \frac{\rho(a)}{\rho_{\mathrm{crit}}(a)} = \frac{\rho(a)}{3M_{\mathrm{Pl}}^2 H(a)^2} , \tag{1.12}$$

where $M_{\rm Pl} = (8\pi G)^{-1/2}$ is the Planck Mass. As we have seen, the comoving Hubble radius is growing (1.10), therefore the curvature parameter $\Omega_k = \Omega - 1$ decreases going backward in time. The measured value of Ω_k today is very close to zero, $|\Omega_k| < 0.005$ (95% CL) [13], implying that it should have been even smaller in the past. For example, one can show that $|\Omega_k| \sim \mathcal{O}(10^{-64})$ at the Planck scale. This means that the initial amount of energy density of matter and radiation in the universe had to be very tuned to the critical value, one part in 10^{64}! Like the horizon problem, this points again to an extreme fine tuning of initial conditions, which is known as the flatness problem.

1.2 The Inflationary Solution

In the previous section we saw how crucial is the role of the comoving Hubble radius in the formulation of the horizon and flatness problems: both of them appear since $(aH)^{-1}$ is strictly increasing. This however also suggests that the horizon and flatness problems can be solved by the same mechanism: make the comoving Hubble radius decrease in time in the very early Universe. In this way, the flatness problem is trivially solved as $\Omega - 1$ would naturally converge to zero at early times (1.11), before the standard FRW evolution begins. The horizon problem is also solved, as the region that will become the observable Universe today actually becomes smaller during this period, so that what appear now as causally disconnected regions in the sky were in causal contact in the past. This mechanism of shrinking the horizon requires:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{aH}\right) < 0 \quad \Leftrightarrow \quad \ddot{a} > 0 \, , \tag{1.13}$$

that is a period of accelerated expansion of the Universe. This period is called *Inflation*.

The search for the solution to the problems of horizon and flatness was the historical motivation for inflation. Its ability to motivate one or more of the initial conditions of the standard hot Big Bang model was noticed by several authors [14–17] and acquired widespread appreciation with the papers [18–20]. However it is mainly for another reason that inflation has now become a fundamental part of cosmology. Inflation provides us with a powerful mechanism to generate the perturbations in the energy density of the universe, necessary for the formation of large scale structure. Before the advent of inflationary solution, the initial fluctuations were postulated and taken as initial conditions designed to fit observational data. On the contrary, inflation explains the origin of primordial inhomogeneities as small quantum fluctuations of the inflation field that are stretched on very large scales by the enormous expansion. This leads to concrete predictions for the spectrum of these primordial perturbations, that are confirmed (for example) by the analysis of the CMB inhomogeneities (see Chapter 2).

As the evolution of the universe obeys Friedmann equations (1.4) and (1.5), it is clear that in order to have a period of accelerated expansion, $\ddot{a} > 0$, we need to satisfy the condition:

$$\rho + 3p < 0 \; . \tag{1.14}$$

As we can see, for an accelerated expansion it is necessary that the pressure of the Universe is negative $p < -\rho/3$. Neither a radiation-dominated phase or a matter-dominated phase (for which $p = \rho/3$ and $p = 0$) satisfies this condition. In this section we will see a simple field-theoretical model where this instead can be realized and discuss its consequences.

Consider the action of a scalar field ϕ, which we call the *inflaton*,

$$S = \int \mathrm{d}^4x\sqrt{-g}\mathcal{L} = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}\partial_\mu\phi\partial_\nu\,\phi g^{\mu\nu} + V(\phi)\right] \; , \tag{1.15}$$

where $g_{\mu\nu}$ is a FRW metric and $\sqrt{-g} = a^3$ is the square root of its determinant. Writing the energy-momentum tensor of the scalar field,

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\mathcal{L} \; , \tag{1.16}$$

we can define the corresponding density and pressure:

$$\rho \;=\; \frac{1}{2}\dot{\phi}^2 + V(\phi) + \frac{(\nabla\phi)^2}{2a^2} \; , \tag{1.17}$$

$$p \;=\; \frac{1}{2}\dot{\phi}^2 - V(\phi) - \frac{(\nabla\phi)^2}{2a^2} \; . \tag{1.18}$$

To follow the classical evolution, we separate the homogeneous and isotropic background vacuum expectation value of the field from the quantum perturbations:

$$\phi = \phi_0(t) + \delta\phi(t, \boldsymbol{x}) \; . \tag{1.19}$$

The homogeneous part behaves like a perfect fluid with

$$\rho_0 \;=\; \frac{1}{2}\dot{\phi_0}^2 + V(\phi_0) \; , \tag{1.20}$$

$$p_0 \;=\; \frac{1}{2}\dot{\phi_0}^2 - V(\phi_0) \; . \tag{1.21}$$

Under the hypothesis that the potential energy is larger than the kinetic energy $\dot{\phi}_0^2 \ll V(\phi_0)$, we now obtain what we were looking for,

$$w = \frac{p_0}{\rho_0} \simeq -1 < -\frac{1}{3} \; , \tag{1.22}$$

that gives an accelerated expansion of the Universe.

In order to be more accurate, let us write the equation of motion for the inflaton ϕ_0:

$$\Box\,\phi = V'(\phi_0) \qquad \Longrightarrow \qquad \ddot{\phi}_0 + 3H\dot{\phi}_0 + V'(\phi_0) = 0\ . \tag{1.23}$$

Physically, if $\dot{\phi}_0^2 \ll V(\phi_0)$, the field is slowly rolling down its potential, hence the name of *slow-roll inflation*. The Friedmann equation (1.4) becomes:

$$3M_{\rm Pl}^2 H^2 = \frac{1}{2}\dot{\phi}_0^2 + V(\phi_0) \simeq V(\phi_0)\ , \tag{1.24}$$

while, differentiating and using (1.23), one finds:

$$\dot{H} = -\frac{1}{2}\frac{\dot{\phi}_0^2}{M_{\rm Pl}^2} \simeq -\frac{1}{6}\frac{V'(\phi_0)^2}{3M_{\rm Pl}^2 H^2}\ . \tag{1.25}$$

This last equation tell us that for the potential energy to dominate the energy density of the Universe, the potential of the inflaton should be very flat:

$$\dot{\phi}_0^2 \ll V(\phi_0) \qquad \Longrightarrow \qquad \frac{V'(\phi_0)^2}{V(\phi_0)} \ll H^2\ . \tag{1.26}$$

This is the first slow-roll condition. Being the potential flat, we should also expect $\ddot{\phi}_0$ to be very small. Indeed if $\dot{\phi}_0^2 \ll V(\phi_0)$ has to be satisfied then also

$$\ddot{\phi}_0 \ll 3H\dot{\phi}_0 \qquad \Longrightarrow \qquad V''(\phi_0) \ll H^2\ , \tag{1.27}$$

must be satisfied, which is the second slow-roll condition.

Both conditions (1.26) and (1.27) can be expressed in more generality using only the Hubble parameter H. If H was constant, the expansion of the Universe would be almost exponential and inflation would be a quasi-de Sitter stage. To achieve this we should require that the fractional change of the Hubble parameter during one Hubble time H^{-1} is much less than unity:

$$\epsilon = -\frac{\dot{H}}{H^2} \ll 1\ . \tag{1.28}$$

This is the definition of the (first) *slow-roll parameter* and correspond to condition (1.26) if inflation is driven by a scalar field with action (1.15). At the same time, also the time variation of ϵ must be small during inflation,

$$\eta = \frac{\dot{\epsilon}}{\epsilon H} \ll 1\ , \tag{1.29}$$

that corresponds to condition (1.27). Notice that

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 = (1-\epsilon)\,H^2\ , \tag{1.30}$$

therefore the condition $\epsilon < 1$ is fundamental to achieve $\ddot{a} > 0$: as soon as it it violated, inflation comes to an end. In general, slow-roll inflation is a quasi-de Sitter stage of expansion of the Universe when $\epsilon \ll 1$ and $|\eta| \ll 1$.

The amount of inflation is measured by the number of e-folds of accelerated expansion:

$$N = \int_{a_i}^{a_f} \mathrm{d}\ln a = \int_{t_i}^{t_f} H(t)\mathrm{d}t \;, \tag{1.31}$$

where the subscripts i and f denotes respectively the beginning and the end of inflation. In the case of slow-roll inflation driven by a scalar field one can use

$$H\mathrm{d}t = \frac{H}{\dot{\phi}}\,\mathrm{d}\phi \simeq -\frac{3H}{V'}\,H\mathrm{d}\phi \simeq \frac{1}{\sqrt{2\epsilon}}\frac{\mathrm{d}\phi}{M_{\mathrm{Pl}}} \tag{1.32}$$

so that eq. (1.31) can be also written as an integral in field space:

$$N = \int_{\phi_i}^{\phi_f} \frac{1}{\sqrt{2\epsilon}}\frac{\mathrm{d}\phi}{M_{\mathrm{Pl}}} \;. \tag{1.33}$$

Inflation can succesfully solve the horizon and flatness problems if $N \gtrsim 50-60$ (see e.g. [21]).

1.3 Quantum Perturbations

If a perfectly homogeneous and isotropic background expansion was the end of the story, none of the cosmic structures we see today would have never formed. Indeed our current understanding of the large scale structures of the Universe is that they had their origin from tiny perturbations in the energy density of the early Universe. Afterwards, when the Universe becomes matter dominated, primeval density inhomogeneities ($\delta\rho/\rho \sim 10^{-5}$) were amplified by gravity[1] [10]. In order to do so, there should have been small (preexisting) fluctuations on physical lenght scales larger than the horizon during radiation

[1] The growth of small matter perturbations inside the horizon (i.e. with wavelenght $\lambda \lesssim H^{-1}$) is governed by the Newtonian equation:

$$\ddot{\delta}_k + 2H\dot{\delta}_k + v_s^2 k^2 \delta_k / a^2 = 4\pi G \rho_0 \delta_k \;,$$

where v_s^2 is their speed of sound. Only for fluctuations with wavenumber smaller than the Jeans wavenumber $k_J^2 = 4\pi G\rho_0 a^2/v_s^2$ gravity wins over pressure and matter perturbations can grow. Solving the previous equation in a matter-dominated Universe, where $a \sim t^{2/3}$, one can show that $\delta_k \sim t^{2/3}$, while in a radiation-dominated Universe, where $a \sim t^{1/2}$, the expansion is so rapid that matter pertubations grow too slowly $\delta \sim \log a$.

and matter eras, even though there is no causal mechanism in the Big Bang model to produce them. In absence of a better explanation, they must be put by hand as initial conditions. Fortunately, a better explanation is provided by the same mechanism that solves the horizon and flatness problems: during inflation, small quantum fluctuations are generated and, as the scale factor is growing exponentially while the Hubble radius remains almost constant, their wavelength soon exceeds the Hubble radius itself. At this point, microscopic physics does not affect their evolution any more and their amplitude "freezes" at some non-zero value, which remains almost unchanged until the end of inflation. Then, in the standard expansion of a radiation-dominated and matter-dominated eras, the Hubble radius increases faster than the scale factor and wavelenghts that had exited the horizon during inflation eventually reenter. The fluctuations that exited around 60 e-foldings or so before the end of inflation reenter with physical wavelengths in the range accessible to cosmological observations like the Cosmic Microwave Background (CMB) and provide us with distinctive signatures of the high-energy physics of the early Universe (see Chapter 2).

1.3.1 Scalar Fluctuations in a quasi-de Sitter Stage

Let us start studying the case of a scalar field ϕ (not necessarily the inflaton) during a de Sitter stage of the expansion of the Universe. After expanding the scalar field ϕ as in eq. (1.19), we can write the equation of motion of the perturbation $\delta\phi(t, \boldsymbol{x})$ as:

$$\ddot{\delta\phi} + 3H\dot{\delta\phi} - \frac{\partial_i^2 \delta\phi}{a^2} = -m^2(\phi)\delta\phi \; , \tag{1.34}$$

where dots are time derivatives while the mass $m^2(\phi) = V''(\phi)$ is the second derivative of the potential $V(\phi)$ with respect to ϕ. It is useful now to go to conformal time,

$$\mathrm{d}\tau = \frac{\mathrm{d}t}{a} \; , \qquad \Longrightarrow \qquad g_{\mu\nu} = a^2 \eta_{\mu\nu} \; , \tag{1.35}$$

where $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$ so that the equation of motion (1.34) becomes:

$$\delta\phi'' + 2aH\dot{\delta\phi} - \partial_i^2 \delta\phi + a^2 m^2 \delta\phi = 0 \; . \tag{1.36}$$

After Fourier expanding the scalar field,

$$\delta\phi = \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \, e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \delta\phi_{\boldsymbol{k}} \; , \tag{1.37}$$

and redefining

$$\delta\phi_{\boldsymbol{k}} = \frac{u_{\boldsymbol{k}}}{a} \; , \tag{1.38}$$

we find an equation of motion of a harmonic oscillator with a time-dependent frequency:

$$u_{\boldsymbol{k}}'' + \left(k^2 + a^2 m^2 - \frac{a''}{a}\right) u_{\boldsymbol{k}} = 0 \,. \tag{1.39}$$

To get physical insight, let us consider eq. (1.39) in different regimes:

- on subhorizon scales, for $k^2 \gg a^2 H^2$, one recovers a free oscillator in conformal time, whose solution is a plane wave:

  $$u_{\boldsymbol{k}}(\tau) = \frac{e^{-ik\tau}}{\sqrt{2k}} \,. \tag{1.40}$$

  Fluctuations with wavelenghts well within the horizon oscillates as they were in a flat spacetime.

- on superhorizon scales, for $k^2 \ll a^2 H^2$, (take for simplicity $m = 0$ for the moment):

  $$u_{\boldsymbol{k}}'' - \frac{a''}{a} u_{\boldsymbol{k}} = 0 \,, \tag{1.41}$$

  which is satisfied by:

  $$u_{\boldsymbol{k}} = a B(k) \,, \tag{1.42}$$

  where the constant of integration $B(k)$ can be fixed with a rough matching with the previous solution at the horizon $aH = k$, so that:

  $$u_{\boldsymbol{k}} = \frac{H}{\sqrt{2k^3}} \,. \tag{1.43}$$

  Fluctuations with wavelenghts much larger than the horizon freeze out and their amplitude remain constant.

In fact, if the mass term is constant, also an exact solution to eq. (1.39) can be found. During a de Sitter stage of inflationary expansion one can write:

$$a(\tau) = -\frac{1}{H\tau} + \mathcal{O}(\epsilon) \,, \tag{1.44}$$

where we neglect subleading terms proportional to the slow-roll parameter ϵ (1.28). Now the equation of motion (1.39) can be written as

$$u_{\boldsymbol{k}}'' + \left[k^2 - \frac{1}{\tau^2}\left(\nu^2 - \frac{1}{4}\right)\right] u_{\boldsymbol{k}} = 0 \,, \tag{1.45}$$

where

$$\nu^2 = \frac{9}{4} - \frac{m^2}{H^2} \,. \tag{1.46}$$

Eq. (1.45) is a Bessel equation and its solution can be written in terms of Hankel function of first and second kind,

$$u_{\boldsymbol{k}} = \sqrt{-\tau}\left[c_1(k)H_\nu^{(1)}(-k\tau) + c_s(k)H_\nu^{(2)}(-k\tau)\right] . \tag{1.47}$$

In the ultraviolet regime, i.e. well within the horizon $-k\tau \gg 1$ ($k \gg aH$), we expect this solution to match the plane-wave solution (1.40). This fixes the integration constants[2] and give us the exact solution:

$$u_{\boldsymbol{k}}(\tau) = \frac{\sqrt{\pi}}{2} e^{\frac{i\pi}{2}(\nu+1/2)}\sqrt{-\tau}H_\nu^{(1)}(-k\tau) . \tag{1.48}$$

Using the asymptotic behaviour of the Hankel function we can write a simple expression for the superhorizon limit:

$$u_{\boldsymbol{k}} = 2^{\nu-3/2}e^{\frac{i\pi}{2}(\nu-1/2)}\frac{\Gamma(\nu)}{\Gamma(3/2)}\left(\frac{(-k\tau)^{1/2-\nu}}{\sqrt{2k}}\right) \qquad (k \ll aH) , \tag{1.49}$$

where Γ is the Euler Gamma function. In the case of a light field, $m^2/H^2 \ll 1$, at lowest order in the small quantity $(3/2 - \nu)$ we can write:

$$|\delta\phi_{\boldsymbol{k}}| \simeq \frac{H}{\sqrt{2k^3}}\left(\frac{k}{aH}\right)^{3/2-\nu} , \tag{1.50}$$

which shows that the amplitude perturbation of a light field in a quasi-de Sitter Universe remains almost constant on super horizon scales, with a tiny time dependence proportional to its effective mass.

As an aside, it is interesting to see what happens if the field is heavier than the Hubble parameter, more precisely when $m^2/H^2 > 9/4$. In this case, the ν parameter (1.46) becomes imaginary, but one can define a new $\tilde{\nu}_k = i\nu$ and proceed in the same way. In the superhorizon limit now we have the asymptotic expression:

$$|\delta\phi_{\boldsymbol{k}}| \simeq \frac{\sqrt{\pi}}{2}e^{\frac{i\pi}{2}(1/2+i\tilde{\nu})}H(-\tau)^{3/2}\left[\frac{1}{\Gamma(i\tilde{\nu}+1)}\left(\frac{-k\tau}{2}\right)^{i\tilde{\nu}} - i\frac{\Gamma(i\nu)}{\pi}\left(\frac{-k\tau}{2}\right)^{-i\tilde{\nu}}\right] . \tag{1.51}$$

The evolution now contains an oscillating factor $\tau^{\pm i\tilde{\nu}}$ and a decaying factor $(-\tau)^{3/2}$, which is the signal that massive fields decay on super horizon scales and their amplitude eventually drops to zero.

[2] Because of quantization, actually these steps are not really straightforward. In Appendix A, we will briefly review the quantization process and the choice of the vacuum.

1.3.2 Metric Fluctuations

The important simplification hidden in the previous section is that, in studying perturbations, we completely neglected the dynamics of gravity, considering it as a fixed background which does not receive any back-reaction from the "spectator" scalar field. However, when we perturb the scalar field (1.19), at the same time we are perturbing its energy-momentum tensor $T_{\mu\nu}$ (1.16), which, in turn, will perturb the metric, since they are connected by the Einstein equations. The most generic first order metric perturbation of a spatially flat[3] FRW metric (1.1), can be written as:

$$\mathrm{d}s^2 = -(1+2\Phi)\mathrm{d}t^2 - 2a(t)B_i\mathrm{d}x^i\mathrm{d}t + a^2(t)\left[(1-2\Psi)\delta_{ij} + E_{ij}\right]\mathrm{d}x^i\mathrm{d}x^j \ , \tag{1.52}$$

where

$$B_i = \partial_i B - S_i \ , \qquad \partial_i S_i = 0 \ , \tag{1.53}$$

$$E_{ij} = 2\partial_i\partial_j E + 2\partial_{(i}F_{j)} + \gamma_{ij} \ , \qquad \partial_i F_i = 0 = \gamma^i{}_i = \partial_i \gamma^i{}_j \ . \tag{1.54}$$

Φ, B, Ψ and E are scalars, S_i and F_i are vectors and γ_{ij} is a tensor, as they trasform respectively as scalars, vectors and tensors under rotations on spatial hypersurfaces. Among these fields, many will not be dynamical when substituted into the action. In the the case we are going to consider here, only one dynamical scalar perturbation will survive (but notice that more general situations are possible, see Chapters 7, 8 and 9).

Let us consider the action of a scalar inflaton ϕ with potential $V(\phi)$ minimally coupled to gravity:

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}M_{\rm Pl}^2 R - \frac{1}{2}\partial_\mu\phi\partial_\nu\phi g^{\mu\nu} - V(\phi)\right] \ . \tag{1.55}$$

The easiest way to proceed here is not via the decompisition (1.52), but with the use of the ADM formalism [22, 23]. The metric is written as

$$\mathrm{d}s^2 = -N^2\mathrm{d}t^2 + h_{ij}\left(\mathrm{d}x^i + N^i\mathrm{d}t\right)\left(\mathrm{d}x^j + N^j\mathrm{d}t\right) \ , \tag{1.56}$$

where N is called the lapse function and N^i the shift function. Although using eq. (1.52) or eq. (1.56) is equivalent, the ADM metric is designed so that the N and N^i functions enter the action (1.55) as Lagrange multipliers and algebraic equations of motion can be solved and substituted back. Using eq. (1.56) into the action (1.55) one finds:

$$S = \frac{1}{2}\int \mathrm{d}^4x\sqrt{h}\left[N(R^{(3)} - h^{ij}\partial_i\phi\partial_j\phi - 2V) + N^{-1}(\dot{\phi} - N^i\partial_i\phi)^2 + N^{-1}(E_{ij}E^{ij} - E^2)\right] \ , \tag{1.57}$$

where

$$E_{ij} = \frac{1}{2}(\dot{h}_{ij} - D_iN_j - D_jN_i) \ , \qquad E = E^i{}_i \ , \tag{1.58}$$

[3]As we are interested in very early universe, where spatial curvature can be neglected, we will work out this case only, though results can be extended to non-zero spatial curvature.

$R^{(3)}$ is the 3d Ricci scalar and D_i is the three-dimensional covariant derivative. Now, the key issue is that General Relativity is a *gauge theory* where the gauge transformations are the generic coordinate transformations

$$x^\mu \;\to\; x^\mu + \xi^\mu(t, \boldsymbol{x}) \;, \tag{1.59}$$

so that any coordinate frame is equivalent. The redundancy of this description is solved by specifying a map that allows to univocally link the same spacetime point on the two different geometries of uniform FRW background and perturbed Universe. This "choice of coordinate" is what is called *gauge fixing*. The issue of gauge invariance in Cosmology is well-known in literature [24–29] and will not be discussed further in this Chapter. A convenient gauge choice here is the *comoving gauge*:

$$\delta\phi = 0 \;, \qquad\qquad h_{ij} = a(t)^2\big(1 + 2\mathcal{R}(t, \boldsymbol{x})\big)\delta_{ij} + \gamma_{ij}(t, \boldsymbol{x}) \;, \tag{1.60}$$

where $\mathcal{R}$ is called *comoving curvature perturbation* and γ_{ij} is the tensor perturbation. The variable $\mathcal{R}$ can be defined also through the pertubed energy-momentum tensor [29],

$$\delta T_{00} \;=\; -\rho_0 \delta g_{00} + \delta\rho \tag{1.61}$$

$$\delta T_{0i} \;=\; p_0 \delta g_{0i} + (\rho + p)(\partial_i \delta u + \delta u_i^T) \;, \tag{1.62}$$

$$\delta T_{ij} \;=\; p\delta g_{ij} + a^2(\delta p \delta_{ij} + \sigma_{ij}) \tag{1.63}$$

(where $\delta\rho$, δp, δu, $\delta u_i T$ are the perturbed density, pressure and 4-velocity, longitudinal and transverse components, and σ_{ij} the anisotropic stress) as:

$$\mathcal{R} = \Psi - H\delta u \;, \tag{1.64}$$

where Ψ was defined in eq. (1.52). Another useful variable to define is the curvature perturbation on uniform density slices, ζ,

$$\zeta = -\Psi - H\frac{\delta\rho}{\dot\rho} \;. \tag{1.65}$$

It can be shown that in single-field inflation these two variables coincide in the large scale limit [30–33]. For this reason, here we will work only with $\mathcal{R}$[4]. As it can be seen from the expression (1.60), in the comoving gauge $\mathcal{R}$ physically represents a spacetime-dependent rescaling of the scale factor a. The gauge choice (1.60) is very similar to the Coulomb gauge in elecrodynamics, where one sets $\partial_i A^i = 0$, solves the equations of motion for A^0

[4] There are different convention in the literature in the definitions of the $\mathcal{R}$ and ζ variables. For example in [34] and many other works, the comoving curvature perturbation is denoted with ζ. Here we adopt the convention of [29], where $\mathcal{R}$ is the comoving curvature perturbation, while ζ is the curvature on uniform density slices.

and puts its solution back in the action. In this case, the equations of motion for N and N^i are the momentum and hamiltonian constraints [34]:

$$D_j\left(N^{-1}(E_i^j - E\delta_i^j)\right) = 0 \tag{1.66}$$

$$R^{(3)} - 2V(\phi) - N^{-2}(E_{ij}E^{ij} - E^2) - N^{-2}\dot{\phi}^2 = 0\ . \tag{1.67}$$

These equations can be solved perturbatively setting

$$N = 1 + \delta N + \ldots\ , \qquad N^i = \partial^i\psi + N_T^i\ , \qquad (\partial_i N_T^i = 0)\ . \tag{1.68}$$

As we are interested in the action up to third order in the perturbed fields, solutions at first order are enough [34]: the reason is that the second order term in N will be multiplying the hamiltonian constraint evaluated to zeroth order, which vanishes since the zeroth order solution obeys the equations of motion. The third order terms multiply the constraints evaluated to first order, which vanish due to the first order expressions for N and N^i. The first order solutions are then:

$$\delta N = \frac{\dot{\mathcal{R}}}{H}\ , \tag{1.69}$$

$$\psi = -\frac{\mathcal{R}}{H} + \frac{a^2}{H}\partial^{-2}\dot{\mathcal{R}}\ . \tag{1.70}$$

Substituting them into the action (1.55), after performing some integrations by parts one can finally obtain the action for $\mathcal{R}$ at second order in pertubations:

$$S = \int \mathrm{d}^4x a^3 M_{\mathrm{Pl}}^2 \epsilon \left(\dot{\mathcal{R}}^2 - \frac{(\partial_i\mathcal{R})^2}{a^2}\right)\ . \tag{1.71}$$

This is the quadratic action of the comoving curvature pertubations during inflation, taking into account all pertubations, both from the inflaton and the metric sector. The form of this action is very simple and its evolution can be studied in the same way as we did in the previous section. Going to conformal time, the equation of motion in Fourier space of the normalized field,

$$a\sqrt{2\epsilon}M_{\mathrm{Pl}}\mathcal{R}_{\boldsymbol{k}} = u_{\boldsymbol{k}}\ , \tag{1.72}$$

takes the form (1.45) and has solution (1.48), with

$$\nu^2 = \frac{9}{4} + 3\epsilon + \frac{3}{2}\eta\ . \tag{1.73}$$

We know that during inflation the slow-roll parameters (1.28), (1.29) are small, by definition. Therefore we can expand the previous expression for ϵ, $|\eta| \ll 1$. In this case the Hankel function $H_{3/2}^{(1)}$, at lowest order in ϵ and η, has a simpler expression and we can read the wave function for $\mathcal{R}$:

$$\mathcal{R} = \frac{H}{M_{\mathrm{Pl}}\sqrt{4\epsilon k^3}}(1 + ik\tau)e^{-ik\tau}\ . \tag{1.74}$$

As one can see in eq. (1.60), also tensor perturbations γ_{ij} are generated during inflation. As they are traceless and transverse (1.52), they describe two degrees of freedom, that are the two helicities of the gravitational waves. More precisely, we can go in Fourier space and decompose γ as:

$$\gamma_{ij} = \int \frac{\mathrm{d}^3k}{(2\pi)^3} \sum_\lambda \boldsymbol{e}_{ij}^\lambda \gamma_{\boldsymbol{k}} e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \ , \tag{1.75}$$

where e_{ij}^λ with $\lambda = +, \times$ are the polarization tensors and satisfy:

$$\begin{array}{rcl} \boldsymbol{e}_{ij}^\lambda & = & \boldsymbol{e}_{ji}^\lambda \\ \boldsymbol{e}_{ii}^\lambda & = & 0 = k^i \boldsymbol{e}_{ij}^\lambda \\ \boldsymbol{e}_{ij}^\lambda(-\boldsymbol{k}) & = & \boldsymbol{e}_{ij}^\lambda(\boldsymbol{k})^* \\ \sum_\lambda \boldsymbol{e}_{ij}^{\lambda\,*} \boldsymbol{e}_\lambda^{ij} & = & 4 \end{array} \ . \tag{1.76}$$

If the action is the one of single-field slow-roll inflation (1.55), only the Ricci scalar contain tensors terms and their action is simply:

$$S = \frac{1}{8} M_{\rm Pl}^2 \int \mathrm{d}^4x \sqrt{-g} \left(\dot{\gamma}_{ij} \dot{\gamma}^{ij} - \frac{\partial_k \gamma_{ij} \partial^k \gamma^{ij}}{a^2} \right) \ . \tag{1.77}$$

After normalizing the field as $\gamma_{\boldsymbol{k}} = \sqrt{2} h_{\boldsymbol{k}} / a M_{\rm Pl}$, the equation of motion for the tensor mode functions in conformal time read:

$$h_{\boldsymbol{k}}'' + \left(k^2 - \frac{a''}{a} \right) h_{\boldsymbol{k}} = 0 \ , \tag{1.78}$$

which is formally equal to the equation of motion of a massless scalar field in de Sitter (1.39). We can therefore make use of the same machinery to conclude that the superhorizon tensor modes scale as:

$$|h_{\boldsymbol{k}}| = \left(\frac{H}{2\pi} \right) \left(\frac{k}{aH} \right)^{3/2 - \nu_T} \ , \tag{1.79}$$

where $\nu_T \simeq 3/2 - \epsilon$ at lowest order in the slow-roll parameters.

1.3.3 The Power Spectrum

Whereas perturbations have a well defined time dependence, viewed as function of position at fixed time, they have random distribution, whose statistical properties are exactly what we wish to uncover via observations. Within the standard single-field slow-roll models of inflation, the primordial density perturbations are (almost) Gaussian, that is, its Fourier components have no correlations except for the reality condition. In this situation the statistical information about the distribution is completely encoded in the two-point function, or its Fourier transform, the power spectrum:

$$\langle \mathcal{R}_{\boldsymbol{k}_1} \mathcal{R}_{\boldsymbol{k}_2} \rangle = (2\pi)^3 \delta(\boldsymbol{k}_1 + \boldsymbol{k}_2) P_{\mathcal{R}}(k_1) = (2\pi)^3 \delta(\boldsymbol{k}_1 + \boldsymbol{k}_2) \frac{2\pi^2}{k^3} \mathcal{P}_{\mathcal{R}} \ , \tag{1.80}$$

where $P(k)$ is the power spectrum and $\mathcal{P}$ is its dimensionless version. For Gaussian perturbations the statistical information about the distribution is completely encoded in the two-point function, as odd-n correlators all vanish and even-n correlators are products of two-point correlators and their permutations. The power spectrum is therefore the first observable we are interested in. Following the discussion of the previous section and using eqs. (1.80), (1.48), it is now very easy to find the expression of the power spectrum of the comoving curvature fluctuations at the end of inflation, that is $\tau = 0$:

$$\mathcal{P}_{\mathcal{R}} = \frac{H^2}{8\pi^2 M_{\rm Pl}^2 \epsilon} \left(\frac{k}{aH}\right)^{n_{\rm s}-1} = \mathcal{A}_{\rm s}^2 \left(\frac{k}{aH}\right)^{n_{\rm s}-1} \ , \tag{1.81}$$

where $\mathcal{A}_{\rm s}$ is its amplitude. The quantity $n_{\rm s}$ is the scalar spectral index and is defined as:

$$n_{\rm s} - 1 = \frac{{\rm d} \ln \mathcal{P}_{\mathcal{R}}}{{\rm d} \ln k} \ . \tag{1.82}$$

In this case, using (1.73) , it is equal to

$$n_{\rm s} - 1 = -\eta - 2\epsilon \ . \tag{1.83}$$

We then learn that the spectrum of curvature perturbations generated during inflation is "almost" scale-invariant on superhorizon scales, that is to say that the amplitude of a fluctuation at a scale k is almost independent on the scale itself. Current bounds on the amplitude and tilt of the power spectrum, together with constraints on departure from Gaussianity, will be the main subject of the next Chapter.

In exactly the same way, one can derive the spectrum of tensor perturbations γ_{ij} from eq. (1.79). Summing over the two polarizations, the power spectrum of inflationary gravitational waves is:

$$\mathcal{P}_T = \frac{k^2}{2\pi^2} \sum_{\lambda} |\gamma_{\boldsymbol{k}}|^2 = \frac{2H^2}{\pi^2 M_{\rm Pl}^2} \left(\frac{k}{aH}\right)^{n_{\rm T}} = \mathcal{A}_{\rm T}^2 \left(\frac{k}{aH}\right)^{n_{\rm T}} \ , \tag{1.84}$$

where the tensor tilt is given by:

$$n_{\rm T} = -2\epsilon \ . \tag{1.85}$$

Also the tensor power spectrum is almost scale invariant. Notice also that the amplitude depends only on the value of the Hubble parameter during inflation, which, if measured, would provide important information about the energy scale of inflation. Moreover, it is very interesting to compare the amplitudes of the tensor and scalar spectra, which is done defining the tensor-to-scalar ratio as:

$$r = \frac{\mathcal{A}_T^2}{\mathcal{A}_S^2} . \tag{1.86}$$

In the standard slow-roll single-field models of inflation that we have studied so far, the tensor to scalar ratio becomes:

$$r = 16\epsilon , \tag{1.87}$$

as one can see from eqs. (1.84) and (1.81). Thus, constraints on $n_{\rm s}$ and r are also contraints on the two slow-roll parameter ϵ and η. If we assume to be in single-field inflation, these are directly related to the the first two derivatives of the inflaton potential, so that a measurement of n_s and r can put stringent bounds on the shape of the scalar potential given by an inflationary model (see Figure 2.2 in the next Chapter). Notice also that

$$r = -8n_{\rm T} , \tag{1.88}$$

which is known as the "consistency relation" between the tensor-to-scalar ratio and tensor tilt [35–37]. This relation is valid for *any* single-field model fo inflation. If future measurments will falsify this relation (see e.g. [38, 39]), this would mean that inflation is not driven by the simple single-field dynamics we have seen and would point to non-trivial realizations of inflation.

CHAPTER 2

CMB Anisotropies

During the 50 years since its discovery [40], the Cosmic Microwave Background (CMB) has been one of the main probes of cosmological theories. Looking at the CMB, the first conclusion that observers were able to draw was that, in contrast to the very non-linear clustered structures of matter in the Universe, the CMB is extremely uniform. Later on, also the tiny 10^{-5} temperature fluctuations were discovered [41]. The CMB data we have today [13, 42–44] are by far the most precise and accurate way to test our knowledge of the Universe (see Table 2.3 for an up-to-date list of past and present CMB experiments).

In this Chapter, we will briefly review the physics of the CMB, its observational consequences and their physical interpretations, in connection with the physics of the early Universe. For more details, we refer the reader to the many reviews in the literature, like for example [45–50]

2.1 Basics

The CMB is a very good black body with a temperature $T = 2.72548 \pm 0.00057$ K [51, 52]. The basic observable of the CMB is its intensity as a function of frequency and direction on the sky $\hat{\boldsymbol{n}}$, that can be described as temperature fluctuations,

$$\Theta(\hat{\boldsymbol{n}}) = \frac{\Delta T}{T} \, . \tag{2.1}$$

As we have already said, if the perturbations are Gaussian, all information about the multipole moments

$$a_{\ell m} = \int \mathrm{d}\hat{\boldsymbol{n}}\, Y^*_{\ell m}(\hat{\boldsymbol{n}})\Theta(\hat{\boldsymbol{n}}) \; , \tag{2.2}$$

where $Y^*_{\ell m}$ are the spherical harmonics, are encoded in the angular power spectrum:

$$\langle a_{\ell m} a_{\ell' m'} \rangle = \delta_{\ell\ell'}\delta_{mm'} C_\ell \; . \tag{2.3}$$

As $\theta = 2\pi/\ell$, small multipole moments correspond to large angular scales. The largest scales are the ones that were outside the horizon at the time of recombination, when the CMB was formed. As no physics could have affected them until they eventually re-enter the horizon, these modes carry almost unaltered information about early times and inflation. Smaller scale modes evolve in a more complicated way instead. Before the temperature of the Universe reached $T \sim 3000$ K and neutral hydrogen formed, photons and baryons were tightly coupled in a cosmological plasma (photon-baryon fluid). If any initial perturbations is present, the radiation pressure would act as a restoring force and the system oscillate at the speed of sound. Physically, this give rise to oscillations in the temperature fluctuations due to compression and rarefaction of a standing acoustic wave. The peaks that we observe in the CMB (see Figure 2.1) correspond to modes that have undergone these acoustic oscillations and are caught at their maxima or minima. On even smaller scales, one starts to see the effect due to shear viscosity and heat conduction in the fluid, since photons can travel only a finite distance before scattering again. This translates in the damping at high ℓ of the temperature power spectrum ("diffusion damping"). What an observer sees today is the projection of inhomogeneities produced at recombination onto anisotropies in the sky. In terms of spherical harmonics (2.2), the observed anisotropy today (i.e. at time τ_0) is:

$$\Theta(\hat{\boldsymbol{n}}, \tau_0) = \sum_{\ell m} Y_{\ell m}(\hat{\boldsymbol{n}}) \left[4\pi(-i)^\ell \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \Delta_\ell(k) \Phi(\boldsymbol{k}) Y^*_{\ell m}(\hat{\boldsymbol{k}}) \right] \; , \tag{2.4}$$

where $\Delta_\ell(k)$ is called radiation transfer function and encode all the typical effects observed in the CMB power spectrum at linear order. The C_ℓ's can then be written as:

$$C_\ell = \frac{2}{\pi} \int k^2 \mathrm{d}k\, P(k)\, |\Delta_\ell(k)|^2 \; . \tag{2.5}$$

where $P(k)$ is the power spectrum of primordial perturbations. It is clear then that the temperature fluctuations $\delta T/T$ are tightly bound to the initial gravitation potential perturbations, which are set by inflation, and are therefore an unvaluable probe of the physics of the early Universe. We are not going into the details of the complete expression of the transfer funcion $\Delta_\ell(k)$, which can be found and even solved analytically under some approximations (a nice derivation from Boltzmann equations to actual observables can be found for example in [11]). It is interesting however to write its expression for low multipoles, which describe the scales that were outside horizon at recombination and have been affected by no physical processes but inflation. In this case the transfer function is just the projection of inhomogeneities onto the spherical sky:

$$\Delta_\ell(k) \propto j_\ell \left[k(\tau_0 - \tau_*)\right] \ , \tag{2.6}$$

where j_ℓ is the spherical Bessel function and τ_* is the time at recombination, so $(\tau_0 - \tau_*)$ represents the (conformal) distance to recombination. Once put in the integral (2.5), one can find that on very large angular scales, the power spectrum is (almost) flat:

$$\ell(\ell+1)C_\ell \simeq \text{const.} \tag{2.7}$$

Figure 2.1 shows this behaviour of the temperature power spectrum $\mathcal{D}_\ell = \ell(\ell+1)C_\ell/2\pi$ in the *Planck* data [43], where one can appreciate all the processes we have seen in our flash review of the physics of the CMB, namely an almost flat spectrum at very large scales, acoustic peaks and diffusion damping. We can also notice that low multipoles have large errors. This is due to the fact that the predicted power spectrum is the average power in the multipole moment ℓ an observer would see in an ensemble of Universes. However a real observer can see only one Universe with its one set of $\Theta_{\ell m}$. This fundamental limitation, called "Cosmic Variance", is the fact that there are only $2\ell+1$ m-samples of power for each multipole, that leads to the unavoidable error:

$$\Delta C_l = \sqrt{\frac{2}{2\ell+1}} C_\ell \ . \tag{2.8}$$

This means that for the monopole $\ell = 0$ and the dipole $\ell = 1$ we actually have no information from the C_ℓ's. Physically we cannot say if the monopole is larger in our vicinity than its average value and cannot tell the difference between a true dipole and the peculiar motion of the Earth with respect to the CMB.

2.2 Constraints on the Primordial Power Spectrum

One of the most important results of the experimental studies of the CMB is the use of the temperature power spectrum to constrain the physics of inflation. The most interesting parameters are the scalar amplitude, the scalar tilt and the tensor-to-scalar ratio, which have been defined in eqs. (1.81), (1.82) and (1.86) and are summarized in Table 2.1.

The most recent and most accurate experimental results at our disposal are *Planck* 2015 data [13, 43, 44, 53]. Table 2.2 shows values and constraints for the primordial

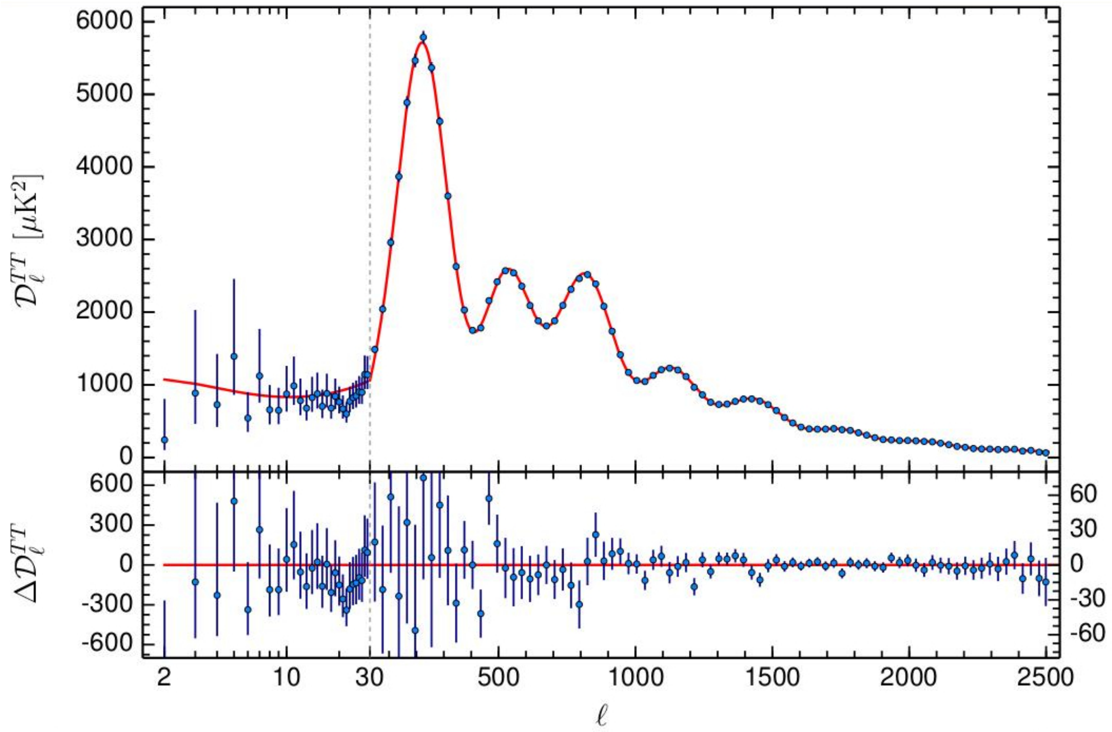


Figure 2.1: The *Planck* 2015 temperature power spectrum [13].

cosmological parameters $\mathcal{A}_\mathrm{s}$, n_s and r, while n_T has been fixed via the consistency relation (1.88) to $n_\mathrm{T} = -8r$. One of the most important result is the departure from exact scale invariance, $n_\mathrm{s} = 1$, at more than 5σ. Although in principle it does not prove that inflation is responsible for the generation of the primordial perturbations, it is a strong confirmation of the expectation of small deviations from scale invariance, in the red side, proportional to the slow-roll parameters (1.83). Another very important result is the constraint on the tensor-to-scalar ratio r. As we explained in the previous Chapter, the amplitude of the tensor spectrum is proportional only to the Hubble parameter, i.e. the energy scale of inflation. Therefore an upper bound on r gives us an upper bound on the quantity [43]

$$\frac{H}{M_\mathrm{Pl}} < 3.9 \times 10^{-5} \quad (95\%\ \mathrm{CL}) \tag{2.9}$$

during inflation. Equivalently, in terms of the potential $V(\phi)$ of a slowly rolling scalar field ϕ, a constraint on r translates into a constraint on $V(\phi)$ itself, since

$$V = \frac{3\pi^2}{2}\mathcal{A}_\mathrm{s} r M_\mathrm{Pl}^4 = (1.88 \times 10^{16}\ \mathrm{GeV})^4\ \frac{r}{0.10}\ . \tag{2.10}$$

The importance of measuring primordial gravitational waves is now clear: their ampli-

Table 2.1: Primordial cosmological parameters.

Parameters	Definition
$\mathcal{A}_\mathrm{s}$	Scalar power spectrum amplitude (at $k_* = 0.05$ Mpc^{-1})
n_s	Scalar spectral index (at $k_* = 0.05$ Mpc^{-1})
r	Tensor-to-scalar ratio (at $k_* = 0.002$ Mpc^{-1})
n_T	Tensor spectrum spectral index (at $k_* = 0.05$ Mpc^{-1})

Table 2.2: Constraints on primordial cosmological parameters [43].

Parameters	*Planck* results
$\ln(10^{10}\mathcal{A}_\mathrm{s})$	3.089 ± 0.036
n_s	0.9666 ± 0.0062 (68% CL)
r	< 0.10 (95% CL)

tude fixes the energy scale of inflation.

The couple of parameters (n_s, r) together can be used to exclude or put constraints on inflationary models, comparing observational results with theoretical predictions. Figure 2.2 shows the allowed region in the (n_s, r) plane together with the predictions of a selection of single-field inflationary models[1]. Here we briefly summarize the constraints for the considered models, refering the reader to the original papers and the *Planck* analysis for more details:

Chaotic Inflation Consider inflationary models with a monomial potential [56]

$$V(\phi) = \lambda M_{\mathrm{Pl}}^4 \left(\frac{\phi}{M_{\mathrm{Pl}}}\right)^n , \tag{2.11}$$

where inflation happens for $\phi > M_{\mathrm{Pl}}$. It can be seen that cubic potential is well outside the 95% CL region and is strongly disfavoured (quartic potential of the

[1]Notice that predictions of models move when changing the number of e-folds to the end of inflation. This reflect the uncertainty about the reheating process, which is the period connecting inflation and radiation era (see for example [54, 55] and references therein). The details of this mechanism go beyond the scope of this work.

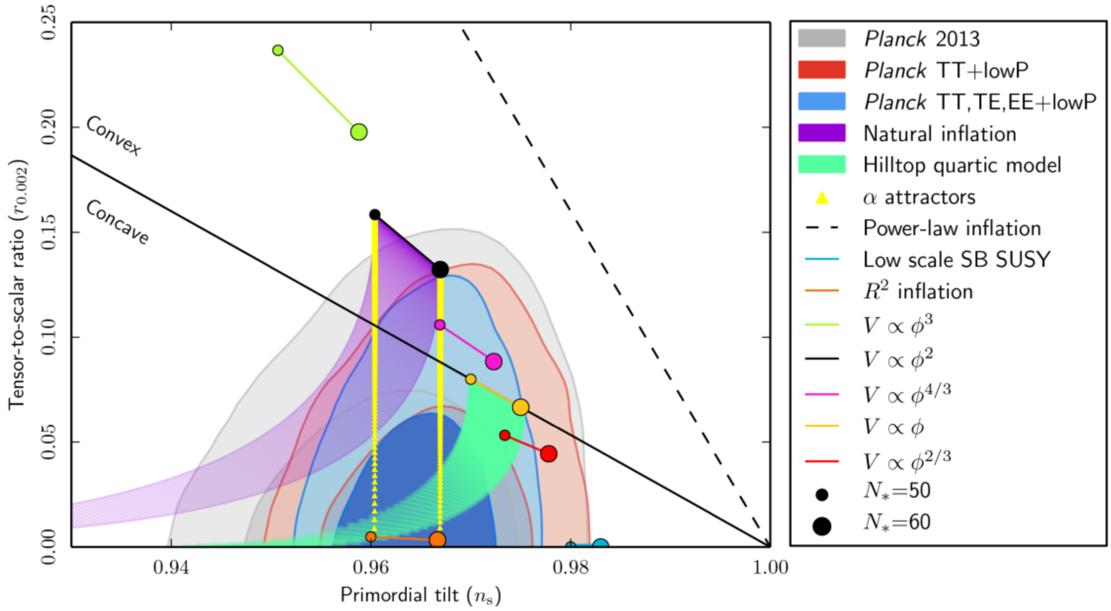


Figure 2.2: Marginalized joint 68% and 95% CL regions for n_s and r from *Planck* in combination with other data sets, compared to the theoretical predictions of selected inflationary models [43].

kind $\lambda\phi^4$ is not shown as it is even further away). Quadratic potentials lies at the margin of the 95% CL contour. Fractional values like $n = 4/3$ or $n = 2/3$ [57, 58] are instead compatible.

Hilltop models The potential has the form [59]:

$$V(\phi) = \Lambda^4 \left(1 - \frac{\phi^p}{\mu^p} + ...\right) . \tag{2.12}$$

where the ellipsis indicates higher order terms that are negligible during inflation but ensure positiveness of the potential later on. Figure 2.2 shows the results for $p = 4$, which is compatible with data.

Power-law Inflation Inflation with an exponential potential [60],

$$V(\phi) = \Lambda^4 e^{-\lambda\phi/M_{\rm Pl}} , \tag{2.13}$$

gives an exact analytical solution for the scale factor which grows in time as a power-law, $a \sim t^2/\lambda^2$, hence its name. This model now lies outside the joint 99.7% CL contour.

Natural Inflation The periodic potential

$$V(\phi) = \Lambda^4 \left[1 + \cos\left(\frac{\phi}{f}\right)\right] \tag{2.14}$$

characterizes what is called natural inflation [61, 62]. This model agrees with *Planck* data for $f/M_{\rm Pl} \gtrsim \mathcal{O}(1)$.

Spontaneously broken SUSY While Hybrid models [63, 64], predicting $n_{\rm s} > 1$, are generically disfavoured, an example of Hybrid model with $n_{\rm s} < 1$ is the spontaneously broken SUSY model [65] with potential:

$$V(\phi) = \Lambda^4 \left[1 + \alpha_h \log\left(\frac{\phi}{M_{\rm Pl}}\right)\right] . \tag{2.15}$$

Notice that for $\alpha_h \ll 1$ its prediction coincide with power-law potential with $p \ll 1$.

R^2 **inflation** This is the first inflationary model proposed in [66, 67], with action

$$S = \int {\rm d}^4x\sqrt{-g}\frac{1}{2}M_{\rm Pl}^2 \left[R + \frac{R^2}{6M}\right] , \tag{2.16}$$

which corresponds to a single-field slow-roll model with potential:

$$V(\phi) = \Lambda^4 \left[1 - \exp\left(\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm Pl}}\right)\right]^2 . \tag{2.17}$$

This model is at the center of the area favoured by *Planck* data.

α**-attractors** This class of models [68–70] have potentials of the form

$$V(\phi) = \Lambda^4 \tanh^{2m}\left(\frac{\phi}{\sqrt{6\alpha}M_{\rm Pl}}\right) , \tag{2.18}$$

Notice that it can interpolate between chaotic models $V \sim \phi^{2m}$, for $\alpha \gg 1$, and R^2 model, for $\alpha \ll 1$.

2.3 Primordial Non-Gaussianity

We have seen that the power spectrum of primordial perturbations provides important information about inflation. If perturbations were perfectly Gaussian, this would be the end of the story, as *all* information would have been encoded in the two-point function. Nonetheless, it is very difficult to discriminate between models, as even completely different scenarios can still give identical power spectra. In practice one can formulate a sort of "no-go theorem" [71], which states that every model

- of single-field inflation
- with canonical kinetic term
- which always slow rolls
- in Bunch–Davies vacuum[2]
- in Einstein gravity

deviate from Gaussianity in a neglibile way, as the amount of produced non-Gaussianity is proportional to the slow-roll parameters [31, 34]. However, though it would seem that this argument discourages the search for primordial non-Gaussianity, this is indeed one of the more interesting reasons to develop the subject. If, for example, non-Gaussianity was revealed by observation, it would make us discard the simplest models. Without these types of data, theoretical models with degenerate observational consequences in the power spectrum are very difficult to disentagle. On the other hand, as there are large differences in size and shape of non-Gaussianities between different models, the detection of such features would break the degeneracy of model building and shed light on the physics of inflation [72–74].

The lowest order additional correlator beyond the two-point function to take into account is the three-point function, or equivalently in Fourier space, the bispectrum:

$$\langle \mathcal{R}_{\boldsymbol{k}_1} \mathcal{R}_{\boldsymbol{k}_2} \mathcal{R}_{\boldsymbol{k}_3} \rangle = (2\pi)^3 \delta^3(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3)\, B(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)\ . \tag{2.19}$$

Under the assumption of statistical homogeneity and isotropy, the bispectrum $B(k_i)$ is a function of the magnitude of the momenta k_1, k_2 and k_3 forming a closed triangle configuration $\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3 = 0$ (condition enforced by the Dirac delta). Studies of the primordial bispectrum are usually characterized by constraints on a single amplitude parameter, denoted by $f_{\rm NL}$, once a specific model for $B(k_i)$ is assumed. The non-Gaussian parameter roughly quantifies the ratio

$$f_{\rm NL} \sim \frac{B(k,k,k)}{P(k)^2} \tag{2.20}$$

(where $P(k)$ is the primordial power spectrum), which measure the "strenght" of the bispectrum with respect to the power spectrum. More precisely, one can define the primordial shape function [75]

$$S(k_1,k_2,k_3) = \frac{1}{N}\,(k_1 k_2 k_3)^2\, B(k_1,k_2,k_3) \tag{2.21}$$

where the normalization factor N is often chosen such that $S(k,k,k) = 1$. The shape function contains a lot of information and, since different models can predict completely

[2] See Appendix A for more details.

different shapes, it can be useful to distinguish between inflationary scenarios. Among the several possible forms for the shape function S, the most studied ones are:

Local Shape This is a phenomenological model in which it is assumed that the observable quantity $\mathcal{R}$ acquires a non-linear correction in real space [76–80],

$$\mathcal{R}(x) = \mathcal{R}_g(x) - \frac{3}{5} f_{\rm NL}^{\rm local} \left(\mathcal{R}_g(x)^2 - \langle \mathcal{R}_g \rangle^2 \right) \ . \tag{2.22}$$

In this case the bispectrum becomes:

$$B^{\rm local}(k_1, k_2, k_3) = 2 f_{\rm NL}^{\rm local} \mathcal{A}_{\rm s}^2 \left(\frac{1}{k_1^{4-n_{\rm s}} k_2^{4-n_{\rm s}}} + \frac{1}{k_1^{4-n_{\rm s}} k_3^{4-n_{\rm s}}} + \frac{1}{k_2^{4-n_{\rm s}} k_3^{4-n_{\rm s}}} \right) \ . \tag{2.23}$$

where $\mathcal{A}_{\rm s}$ is the amplitude of the power spectrum (1.81) and $n_{\rm s}$ its tilt (1.82). This shape turns out to be physically relevant for all models where non-linearities develop outside the horizon, like for example in models of multifield inflation where additional light scalar fields besides the inflaton contribute to curvature perturbations (see e.g. [81]). This mechanism provides a correlation between large and small scale modes: indeed the bispectrum is larger in the *squeezed* configuration $k_1 \ll k_2 \simeq k_3$, where one of the momenta is much smaller than the others.

Equilateral and Orthogonal Shapes Inflationary models with non-canonical kinetic terms are able to generate large non-Gaussianity. An example is the effective Lagrangian:

$$\mathcal{L} = P(X, \phi) \ , \tag{2.24}$$

where $X = \partial_\mu \phi \partial^\mu \phi$. The inflaton fluctuations here propagate with an effective speed of sound $c_s^2 \neq 1$. The bispectrum produced by this class of models is generically well described by a superposition of the equilateral [82, 83],

$$\begin{aligned} B^{\rm equil}(k_1, k_2, k_3) &= 6 \mathcal{A}_{\rm s}^2 f_{\rm NL}^{\rm equil} \left\{ -\frac{1}{k_1^{4-n_{\rm s}} k_2^{4-n_{\rm s}}} - \frac{1}{k_1^{4-n_{\rm s}} k_3^{4-n_{\rm s}}} - \frac{1}{k_2^{4-n_{\rm s}} k_3^{4-n_{\rm s}}} \right. \\ &\left. - \frac{2}{(k_1 k_2 k_3)^{2(4-n_{\rm s})/3}} + \left[\frac{1}{k_1^{(4-n_{\rm s})/3} k_2^{(4-n_{\rm s})/3} k_3^{(4-n_{\rm s})/3}} + (5 \text{ perm.}) \right] \right\} \end{aligned} \tag{2.25}$$

and the orthogonal shapes [84],

$$\begin{aligned} B^{\rm ortho}(k_1, k_2, k_3) &= 6 \mathcal{A}_{\rm s}^2 f_{\rm NL}^{\rm ortho} \left\{ -\frac{3}{k_1^{4-n_{\rm s}} k_2^{4-n_{\rm s}}} - \frac{3}{k_1^{4-n_{\rm s}} k_3^{4-n_{\rm s}}} - \frac{3}{k_2^{4-n_{\rm s}} k_3^{4-n_{\rm s}}} \right. \\ &\left. - \frac{8}{(k_1 k_2 k_3)^{2(4-n_{\rm s})/3}} + \left[\frac{3}{k_1^{(4-n_{\rm s})/3} k_2^{(4-n_{\rm s})/3} k_3^{(4-n_{\rm s})/3}} + (5 \text{ perm.}) \right] \right\} \ . \end{aligned} \tag{2.26}$$

The underlying physics of these two shapes can be understood remembering that modes are frozen outside the horizon in single-field inflation. Then large interactions can occur only for modes with similar wavelenghts $k_1 \simeq k_2 \simeq k_3$, that are crossing the horizon at about the same time.

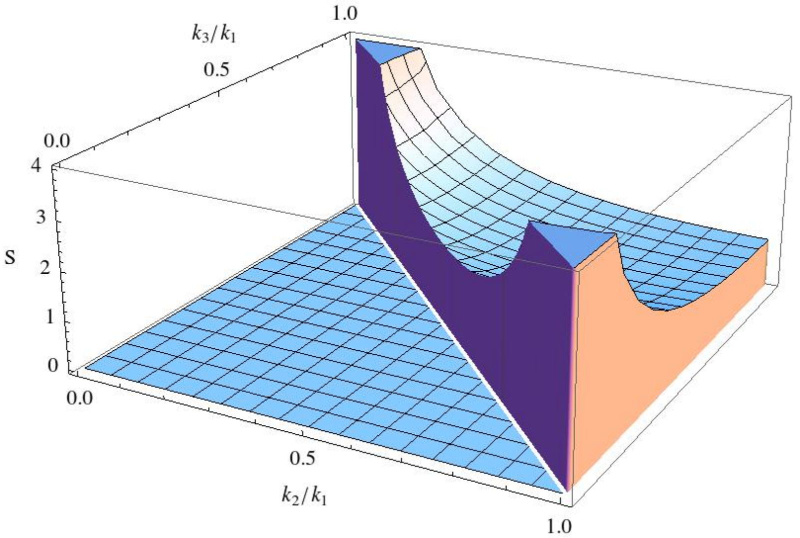


Figure 2.3: Example of local shape (2.23).

Observationally, the cosmological observable most directly related to the initial curvature bispectrum is given by the bispectrum of the CMB temperature fluctuations. We have seen that temperature anisotropies (2.1) are described with the spherical harmonics decomposition (2.2). The bispectrum is the three-point correlator of the $a_{\ell m}$ (2.4):

$$\begin{aligned} B^{m_1 m_2 m_3}_{\ell_1 \ell_2 \ell_3} &= \langle a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3} \rangle \\ &= \mathcal{G}^{m_1 m_2 m_3}_{\ell_1 \ell_2 \ell_3} \, b_{\ell_1 \ell_2 \ell_3} \\ &= \left(\frac{2}{\pi}\right)^3 \int x^2 \mathrm{d}x \int \mathrm{d}k_1 \mathrm{d}k_2 \mathrm{d}k_3 \, (k_1 k_2 k_3)^2 B(k_1, k_2 k_{,3}) \Delta_{\ell_1}(k_1) \Delta_{\ell_2}(k_2) \Delta_{\ell_3}(k_3) \\ &\quad \times j_{\ell_1}(k_1 x) j_{\ell_2}(k_2 x) j_{\ell_3}(k_3 x) \, \mathcal{G}^{m_1 m_2 m_3}_{\ell_1 \ell_2 \ell_3} \, , \end{aligned} \tag{2.27}$$

where the integral over the angular part of $\boldsymbol{x}$ is known as Gaunt integral $\mathcal{G}^{m_1 m_2 m_3}_{\ell_1 \ell_2 \ell_3}$ and can be written in terms of the Wigner-3j symbols as (for more details see e.g. [85, 86] and references therein):

$$\begin{aligned} \mathcal{G}^{m_1 m_2 m_3}_{\ell_1 \ell_2 \ell_3} &= \int \mathrm{d}\Omega_x Y_{\ell_1 m_1}(\boldsymbol{x}) Y_{\ell_2 m_2}(\boldsymbol{x}) Y_{\ell_3 m_3}(\boldsymbol{x}) \\ &= \sqrt{\frac{(2\ell_1+1)(2\ell_2+1)(2\ell_3+1)}{4\pi}} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \end{aligned} \tag{2.28}$$

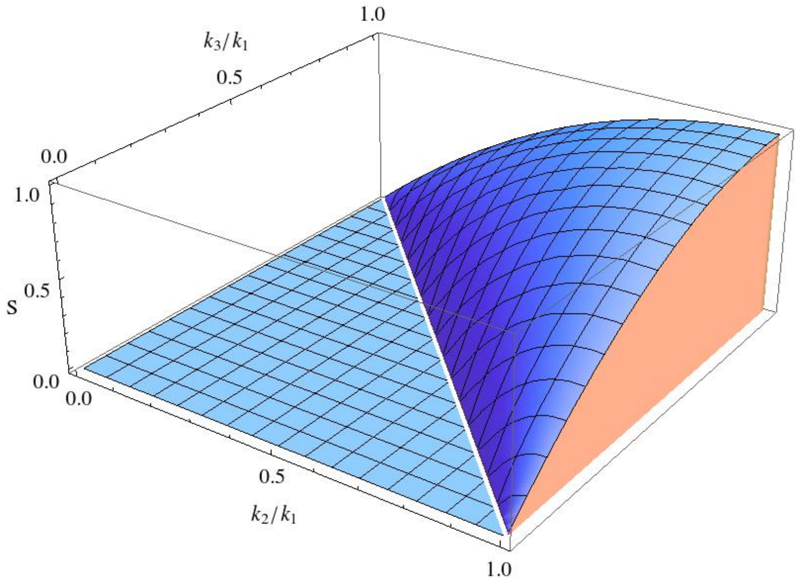


Figure 2.4: Example of equilateral shape (2.25).

The function $b_{\ell_1\ell_2\ell_3}$ in eq. (2.27) is called "reduced bispectrum". It is interesting to notice that the bispectrum $B_{\ell_1\ell_2\ell_3}$ is non-zero only if the sum of the ℓ's is even and that the triangle condition

$$|\ell_i - \ell_j| < \ell_k < \ell_i + \ell_j \tag{2.29}$$

is satisfied (exactly like the triangle condition on momenta $\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3 = 0$).

The goal of the analysis is to extract the non-Gaussian parameter $f_{\rm NL}$ (2.20) for different primordial shapes. Essentially this is achieved with a fit of a theoretical ansatz for the reduced bispectrum $b_{\ell_1\ell_2\ell_3}$ to the observed CMB bispectrum, finding an optimal statistical estimator for $f_{\rm NL}$ together with an efficient numerical implemetation[3]. For the latest analysis of *Planck*, three different techniques have been used to measure $f_{\rm NL}$ [44], as cross-validating and comparing different outputs improves the robustness of result. With the inclusion of polarization data, the constraints on local, equilateral and orthogonal non-Gaussianity are [44]:

$$\begin{aligned} f_{\rm NL}^{\rm local} &= 0.8 \pm 5.0 \quad (95\%{\rm CL}) \\ f_{\rm NL}^{\rm equil} &= -4 \pm 43 \quad (95\%{\rm CL}) \\ f_{\rm NL}^{\rm ortho} &= -26 \pm 21 \quad (95\%{\rm CL}) \end{aligned} \quad . \tag{2.30}$$

[3]This subject goes beyond the scope of this work and we are not going to develop it further. More details can be found, for example, in the review [72].

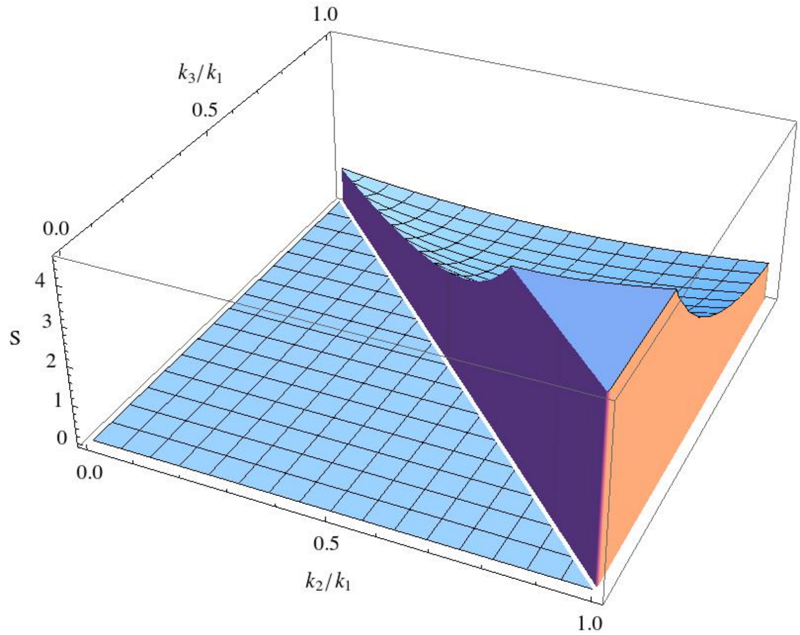


Figure 2.5: Example of orthogonal shape (2.26).

This means that perturbations deviate from Gaussianity for less than about one part in 10^5. These results, being compatible with zero, are telling us that primordial perturbations are Gaussian to a very high degree of accuracy and suggest that inflationary fluctuations were linear and weakly interacting. At the same time, they provide important constraints for model building, since they can put bounds on the allowed parameter space for theories. On the other hand, there is still space for physically-motivated models that go beyond simple Gaussian statistics and that are not well constrained yet. Being now so precise and accurate, observational constraints on non-Gaussianity are one of the main tests of inflationary scenarios. Then it becomes very important to develop theoretical tools which allow to translate observational results into physically meaningful quantities, at the same time allowing for more general possibilities which are not currently constrained by data.

Table 2.3: List of CMB experiments, with years and references.

Name	Years	Reference
Cobra	1982	Physical Review Letters, Vol. 65, pp. 537-540
Relkit	1983 – 1984	Soviet Astronomy Letters, Vol. 18, p. 153
Tenerife	1984 – 2000	The Astrophysical Journal, Vol. 529 (1) , pp. 47-55
BIMA	1986 – 2004	The Astrophysical Journal, Vol. 647(1) , pp. 13-24
ACME/HACME	1988 – 1996	The Astrophysical Journal, Vol. 541(2) , pp. 535-541
ARGO	1988, 1990, 1993	Astrophysical Journal Letters, Vol. 463, pp. L47-L50
FIRS	1989	Astrophysical Journal Letters, Vol. 432, pp. L15-L18
COBE	1989 – 1993	Astrophysical Journal Letters, Vol.464, pp. L17-L20
ATCA	1991 – 1997	MNRAS, Vol. 315(4) , pp. 808-822
MSAM	1992 – 1997	The Astrophysical Journal, Vol. 532(1) , pp. 57-64
Python	1992 – 1997	The Astrophysical Journal, Vol. 475(1) , pp. L1-L4
SK	1993 – 1995	The Astrophysical Journal, Vol. 474(1) , pp. 47-66
CAT	1994 – 1997	Astrophysical Journal Letters, Vol.461, pp.L1-L4
Tris	1994 – 2000	The Astrophysical Journal, Vol. 688(1) , pp. 24-31
APACHE	1995 – 1996	Astrophysics From Antarctica; ASP Conference Series; Vol. 141, p.81
BAM	1995	Astrophysical Journal Letters, Vol. 475, pp. L73-L76
MAXIMA	1995, 1998, 1999	Review of Scientific Instruments, Vol. 77(7) , pp. 071101-071101-25
QMAP	1996	The Astrophysical Journal, Vol. 509(2) , pp. L77-L80
BOOMERanG	1997 – 2003	The Astrophysical Journal, Vol. 647(2) , pp. 823-832
CG	1997 – *date*	Astrophysical Bulletin, Vol. 66(4) , pp.424-435
MAT	1997, 1998	The Astrophysical Journal, Vol. 524(1) , pp. L1-L4
COSMOSOMAS	1998 – *date*	MNRAS, Vol. 370(1) , pp. 15-24
Archeops	1999 – 2002	Astronomy and Astrophysics, Vol. 399, p.L19-L23
POLAR	2000	The Astrophysical Journal, Vol. 560(1) , pp. L1-L4
BEAST	2000 – *date*	MNRAS, Vol. 369(1) , pp. 441-448
ACBAR	2001 – 2008	The Astrophysical Journal, Vol. 694(2) , pp. 1200-1219
ARCADE	2001 – 2006	The Astrophysical Journal, Vol. 734(1) , id. 5, 11 pp.
DASI	2001 – 2003	The Astrophysical Journal, Vol. 568(1) , pp. 38-45
MINT	2001 – 2002	The Astrophysical Journal Supplement Series, Vol. 156(1) , pp. 1-11
WMAP	2001 – 2010	The Astrophysical Journal Supplement, Vol. 208(2) , id. 20, 54 pp.
CAPMAP	2002 – 2008	The Astrophysical Journal, Vol. 684(2) , pp. 771-789
CBI	2002 – 2008	The Astrophysical Journal, Vol. 549(1) , pp. L1-L5
PIQUE	2002	The Astrophysical Journal, Vol. 573(2) , pp. L73-L76
TopHat	2002 – 2004	The Astrophysical Journal, Vol. 532(1) , pp. 57-64
VSA	2002 – 2004	MNRAS, Vol. 341(4) , pp. 1076-1083
COMPASS	2003 – *date*	The Astrophysical Journal, Vol. 610(2) , pp. 625-634
KUPID	2003 – *date*	New Astronomy Reviews, Vol. 47(1) 1-12, p. 1097-1106
AMI	2005 – *date*	MNRAS, Vol. 391(4) , pp. 1545-1558
QUaD	2005 – 2010	The Astrophysical Journal, Vol. 705(1) , pp. 978-999
BICEP1	2006 – 2008	The Astrophysical Journal, Vol. 783(2) , id. 67, 18 pp.
AMiBA	2007 – *date*	Modern Physics Letters A, Vol. 23(1) 7-20, pp. 1675-1686
SPT	2007 – *date*	The Astrophysical Journal, Vol. 782(2) , id. 74, 24 pp.
ACT	2008 – *date*	The Astrophysical Journal, Vol. 749(1) , id. 90, 10 pp.
QUIET	2008 – 2010	The Astrophysical Journal, Vol. 760(2) , id. 145, 10 pp.
Planck	2009 – 2013	eprint arXiv:1502.01582
BICEP2	2009 – 2012	The Astrophysical Journal, Vol. 792(1) , id. 62, 29 pp.
KECKArray	2010 – *date*	eprint arXiv:1510.09217
ABS	2011 – *date*	Review of Scientific Instruments, Vol. 85(2) , id.024501
POLARBEAR	2012 – *date*	Physical Review Letters, Vol. 113(2) , id.021301
EBEX	2012 – 2013	Proceedings of the SPIE, Volume 7741, id. 77411C
QUIJOTE	2012 – *date*	MNRAS, Vol. 452(4) , p.4169-4182
SPTpol	2012 – *date*	The Astrophysical Journal, Vol. 807(2) , id. 151, 18 pp.
ACTpol	2013 – *date*	The Astrophysical Journal, Vol. 808(1) , id. 7, 9 pp.

CHAPTER 3

Quasi-local non-Gaussianity as a Signature of Modified Gravity

We have seen in Section 2.3 that standard models of single-field inflation do not generate large non-Gaussianity. To go beyond this, one must abandon one or more of the assumptions on which the simplest scenarios are based. In this Chapter we will make the example of a modification of Einstein gravity acting on the slowly-rolling scalar field that is driving inflation. Departures from Einstein gravity during inflation have been considered in the first inflationary model proposed [66] and in many following papers, for example in [87–94]. However, the non-Gaussianities that might be produced are generically below the sensitivity of future measurements and in fact well below the cosmic variance limit for the full sky. In [3], on which this Chapter is based, we have investigated whether deviations from General Relativity (GR) could be observable and measurable in the sky through the enhancement of non-Gaussianity (NG) of curvature perturbations. We found that this might be the case, in particular we show that modifications of Einstein gravity, if already relevant during the epoch of inflation, could lead to possibly measurable non-Gaussian signatures in the cosmological fluctuations. Also, we have also shown that, for a large part of the parameter space, the generated non-Gaussianities have a quasi-local shape. This is observationally promising given that future LSS surveys can be sensitive to values of local NG $f_{\rm NL} \sim \mathcal{O}(1)$ or even smaller (see, e.g., [95–98]). If supported by data, these findings would yield interesting insights into the physical mechanism behind inflation, pointing towards a non-trivial dynamics of the inflationary fields. Conversely, a null result would also be extremely useful, as it would place limits on possible departures from Einstein Gravity and the slow-roll paradigm.

3.1 Introduction

The scenario we are going to study is based on the concepts of *quasi-single field* inflationary dynamics, first introduced in [99] (to which we refer the reader for more details). In this setup, besides the usual light inflaton which is driving inflation, one more field is present. However this second field is neither too light to strongly modify the background flat slow-roll direction of the inflaton, or too heavy to be unimportant for the other light degree of freedom. The mass of the second field is in an intermediate range, namely $m \sim \mathcal{O}(H)$. If the mass was much larger, it would decouple from the inflaton and the standard predictions of single-field inflation would be recovered. Here however, with a mass of the order of the Hubble parameter, if large couplings exist, the massive field can still induce interesting effects on the inflaton perturbations.

Let us start from a Lagrangian that contains all generally covariant terms up to four derivatives built with the metric and one scalar field, that we will assume to drive inflation [100]:

$$\begin{aligned}
L \;=\; \sqrt{-g}\Bigg[& \frac{1}{2}M_{Pl}^2\Omega(\psi)^2 R - \frac{1}{2}h(\psi)g^{\mu\nu}\partial_\mu\psi\partial^\mu\psi - U(\psi) \\
& + f_1(\psi)\Big(g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi\Big)^2 + f_2(\psi)g^{\rho\sigma}\partial_\rho\psi\partial_\sigma\psi\Box\psi \\
& + f_3(\psi)\Big(\Box\psi\Big)^2 + f_4(\psi)R^{\mu\nu}\partial_\mu\psi\partial_\nu\psi \\
& + f_5(\psi)R\, g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi + f_6(\psi)R\,\Box\psi + f_7(\psi)R^2 \\
& + f_8(\psi)R^{\mu\nu}R_{\mu\nu} + f_9(\psi)C^{\mu\nu\rho\sigma}C_{\mu\nu\rho\sigma}\Bigg] \\
& + f_{10}(\psi)\epsilon^{\mu\nu\rho\sigma}C_{\mu\nu}{}^{\kappa\lambda}C_{\rho\sigma\kappa\lambda}\,. \qquad (3.1)
\end{aligned}$$

If the inflaton ψ is slowly-rolling, then the functions $\Omega(\psi)$, $h(\psi)$ and $f_i(\psi)$ are varying slowly and can be simply treated as constants up to slow-roll corrections, which we will neglect. In this case, the Weyl-squared term can be recast as a surface term (the Gauss-Bonnet term) plus R^2 and $R_{\mu\nu}R^{\mu\nu}$, which can then be reabsorbed. Moreover, in order to avoid ghosts, the terms proportional to f_2, f_3, f_6 and f_8 will be here set to zero, as well as f_{10} as we are not interested in parity violating signatures, which will be discussed in a following Chapter (see Chapter 8). We are interested only in the terms that could give rise to a possibly enhanced local (or quasi-local) NG in the squeezed limit, different from the well-known result $f_{\rm NL} \sim \mathcal{O}(\epsilon)$ that is valid in standard gravity [31, 34, 77]. Therefore we will not consider inflaton derivative self-interactions, which are known to generate NG mainly in the equilateral configuration[1]. This is valid also

[1]Fields self-interactions in this thesis will be considered mainly in the context of the Effective Field

for the ghost-free combination that can be built with the operators proportional to f_4 and f_5 [101], which would not generate significant NG in the local configuration. The only term left to consider is therefore the term R^2, which is nothing else than the first term in an expansion in powers of the Ricci scalar of a more general $f(R)$ theory:

$$\mathcal{L} = \sqrt{-g}\left[f(R) - \frac{1}{2}g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi - U(\psi)\right] . \tag{3.2}$$

This action describes one more degree of freedom associated to the $f(R)$ term. Through a standard procedure we use an auxiliary field $f'(\chi) = M_{\rm Pl}^2\phi/2$ to recast the action in the form

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}M_{\rm Pl}^2\phi R + \Lambda(\phi) - \frac{1}{2}g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi - U(\psi)\right] , \tag{3.3}$$

where $\Lambda(\phi) = f(\chi(\phi)) - M_{\rm Pl}^2\phi\chi/2$. By performing a Weyl transformation $g_{\mu\nu} \to {\rm e}^{-2\omega}g_{\mu\nu}$, with ${\rm e}^{2\omega} = \phi$, to go to the Einstein frame, the action appears as a two-field interacting model:

$$\begin{aligned}\tilde{\mathcal{L}} \;&= \sqrt{-g}\Bigg[\;\frac{1}{2}M_{\rm Pl}^2 R - \frac{1}{2}g^{\mu\nu}\gamma_{ab}\partial_\mu\varphi^a\partial_\nu\varphi^b \\ &\qquad -U_1(\varphi_1) - {\rm e}^{-4\varphi_1/\sqrt{6}M_{\rm Pl}}U(\varphi_2)\Bigg] ,\end{aligned} \tag{3.4}$$

where $a, b = 1, 2$ we have normalized the fields as

$$\sqrt{6}M_{\rm Pl}\omega = \varphi_1 , \qquad \psi = \varphi_2 , \tag{3.5}$$

defined U_1 as

$$U_1(\varphi_1) = -{\rm e}^{-4\varphi_1/\sqrt{6}M_{\rm Pl}}\Lambda\left(\phi\left(\omega\left(\varphi_1\right)\right)\right) , \tag{3.6}$$

and defined the field metric

$$\gamma_{ab} = \begin{pmatrix} 1 & 0 \\ 0 & {\rm e}^{-2\varphi_1/\sqrt{6}M_{\rm Pl}} \end{pmatrix} . \tag{3.7}$$

As expected, there is an equivalence between "$f(R)$+scalar" and a two-field model with a specific field metric, a generic potential for φ_1 and a "conformally-stretched" potential for φ_2. Then it is conceivable that the interactions between the two fields could induce some observable effects, possibly enhancing also local NG to an observable level. It is important to note here that if both fields contributed to the dynamics of the background, we should rigorously impose slow-roll conditions on both of them. However, if the field associated to the R^2 terms is subdominant, then this condition could be relaxed and its possible NG could be transferred to the inflaton field. In the Einstein frame this is equivalent to a transfer of non-Gaussian isocurvature perturbations to the adiabatic perturbation mode [73].

Theory of Inflation, as we will see in the following Chapters.

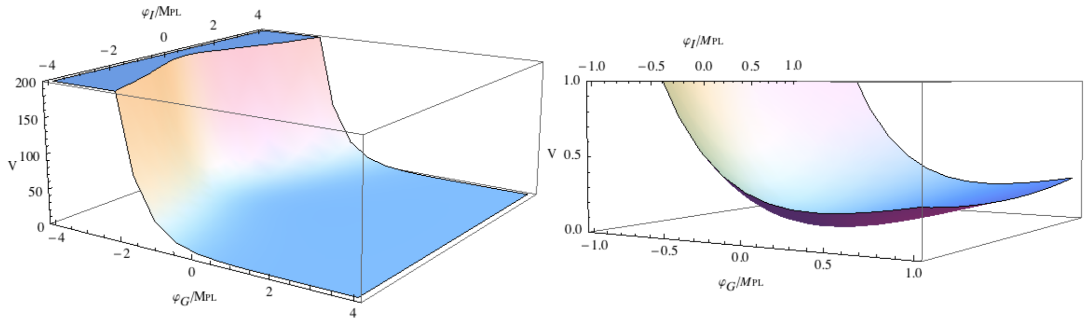


Figure 3.1: Potential as a function of the two scalar fields. φ_G describes the "scalaron" field that accounts for modifications of Einstein gravity while φ_I is the one driving inflation. Significant non-Gaussianities ($|f_{\rm NL}| \approx 1 - 30$) are generated for generic initial field values, provided $\varphi_G > -3$. Parameters are chosen for illustration purposes. In particular we chose a quadratic potential [102] for the inflaton field φ_I. The right panel shows the potential around the minimum.

3.2 The Size of non-Gaussianity

To study the possibly enchanced effect on non-Gaussianity, we will consider $f(R) = \frac{1}{2}M_{\rm Pl}^2 R + R^2/12M^2$. This choice is motivated by the fact that it corresponds to the leading order term in an expansion of a generic $f(R)$ in powers of R (or equivalently in derivatives of the metric). In this case, we obtain a complete potential $V(\varphi_1, \varphi_2)$ given by:

$$\begin{aligned} V(\varphi_1,\varphi_2) &= \frac{3}{4}M^2 M_{\rm Pl}^4 \left(1 - {\rm e}^{-2\varphi_1/\sqrt{6}M_{\rm Pl}}\right)^2 \\ &\quad + {\rm e}^{-4\varphi_1/\sqrt{6}M_{\rm Pl}} U(\varphi_2) \, . \end{aligned} \tag{3.8}$$

It is clear that if the field φ_1 is very heavy and the scale of the new physics induced by the R^2 term is much higher than the energy scale of the inflaton φ_2, then its effect should be vanishingly small. Indeed, if φ_1 is heavy enough, it could not be excited during inflation and its kinetic energy would be completely negligible. Therefore we could integrate it out of the action (3.4), coming back to a standard effective single-field scenario. This would correspond to a value of $M \sim 1$ or higher, which implies that the new physics simply enters at the Planck scale or beyond. On the other hand, lowering the scale $M \lesssim 1$, the first regime we encounter is the quasi-single field regime [99]. Progressively reducing the value of M, other regimes are possible: first the multi-field inflation where both scalar fields are actively at play and then, when the field φ_1 dominates the dynamics, single-field Starobinsky inflation [66]. Hereafter, we adopt a monomial potential $U(\varphi_2) = m^{4-\beta}\varphi_2^\beta$, with $\beta < 2$ (motivated by current Planck-satellite constraints [43]). Our results are however fairly insensitive to the choice of β.

We are interested in the quasi single-field regime, as observables do not depend on the particular choice of the initial conditions. In this sense we look for generic predictions. In this case, assuming that the adiabatic direction is given by $\varphi_2 \equiv \varphi_I$, we obtain non-trivial effects from the coupling with the isocurvature field $\varphi_1 \equiv \varphi_G$. (Here by using the subscripts I and G we have made explicit that the field φ_I is the inflation and φ_G describes the modifications of gravity). To make an estimate of the magnitude of the effect, we can expand the action Eq. (3.4) in the flat gauge and ignore metric perturbations for simplicity [99]. At second order, we find the leading transfer vertex:

$$\delta\mathcal{L}_2 = \frac{2}{\sqrt{6}M_{\rm Pl}} \mathrm{e}^{\frac{-2\bar{\varphi}_G}{\sqrt{6}M_{\rm Pl}}} \dot{\bar{\varphi}}_I \delta\varphi_G \delta\dot{\varphi}_I \; , \tag{3.9}$$

where the bar refers to homogeneous quantities computed on the background. At third order, as the isocurvature potential U_1''' is not subject to slow-roll conditions, the leading vertex is

$$\delta\mathcal{L}_3 = -\frac{1}{6} U_1'''(\bar{\varphi}_I)\delta\varphi_G^3. \tag{3.10}$$

Therefore we expect a contribution to the bispectrum of size [73]

$$\begin{aligned} f_{\rm NL} &\simeq \alpha(\nu) \left(\widehat{\delta\mathcal{L}_2}\right)^3 \widehat{\delta\mathcal{L}_3}\, \mathcal{P}_\zeta^{-1/2} \qquad (3.11)\\ &= -\frac{4}{9\pi}\alpha(\nu)\, \frac{\mathcal{P}_\zeta^{-1}}{\sqrt{\epsilon}}\, M^2 \left[\epsilon - 3\left(\frac{\dot{M}_{\rm Pl,eff}}{H M_{\rm Pl,eff}}\right)^2\right]^{3/2} \\ &\times \left[\left(\frac{M_{\rm Pl,eff}}{M_{\rm Pl}}\right)^2 - 4\right] \left(\frac{M_{\rm Pl,eff}}{M_{\rm Pl}}\right)^{-7} \end{aligned}$$

where $\widehat{\delta\mathcal{L}_2}$ and $\widehat{\delta\mathcal{L}_3}$ are the vertices of the interaction terms, eqs. (3.9-3.10), $\nu = \sqrt{9/4 - (M_{\rm eff}/H)^2}$, $M_{\rm eff}$ is the effective mass of the isocurvature mode and ϵ the total slow-roll parameter. In eq. (3.11) $M_{\rm Pl,eff} = M_{\rm Pl}\; \mathrm{e}^{\varphi_G/\sqrt{6}M_{\rm Pl}}$ is the effective (reduced) Planck mass during inflation in the Jordan frame. The numerical factor $\alpha(\nu)$ can range from 0.2, for heavier isocurvatons, to approximately 300; however, in the perturbative regime, NG can gain at most an effective enhancement factor proportional to the number of e-foldings, see [99]. The shape of the potential as a function of the two fields φ_I and φ_G is shown in Figure 3.1. On the left panel one can appreciate that the φ_I direction is flat but there are values of φ_G where the potential is steep. On the right panel we show the region around the global minimum. Figure (3.2) shows the NG parameter $f_{\rm NL}$ as a function of e-folds adopting $U(\varphi_I) = m^3\varphi$; our results are not sensitive to the specific value adopted for β. As an example, for $M = 10^{-3}$ and $m = 10^{-8/3}$, in Planck units, we obtain $f_{\rm NL} \sim \mathcal{O}(-3)$, for initial values of the field $\varphi_G = 3, \varphi_I = 12$. Note the nearly scale invariant dependence. For this particular example at 60 e-folds the field abandons slow-roll and re-heating starts. The characteristic shape of this kind of NG is intermediate between an equilateral shape, which is reached for small values of ν i.e., towards a single-field regime, and a local shape, for $\nu \geq 1/2$ i.e., closer to a multi-field scenario.

In this set up $f_{\rm NL}$ is generically negative. A quasi-local shape with $f_{\rm NL} \approx -1$ to -30 can thus be achieved without necessity of much fine tuning. The value of $f_{\rm NL}$ scales as a function of the masses of the two potentials,

$$f_{\rm NL} \propto -(M M_{\rm Pl}/m)^2 \alpha(\nu). \tag{3.12}$$

This makes it possible to test deviations from GR a couple of orders of magnitude above the mass scale of the inflaton. Note that eq. (3.11) gives a "consistency relation" between the amplitude of NG and its shape. In fact, $f_{\rm NL}$ measures departures from the effective gravitational constant $G_{\rm eff}$ during inflation as $G_{\rm eff}/G_{\rm GR} = {\rm e}^{-\varphi_G/\sqrt{6}M_{\rm Pl}}$.

To summarise, we have explored whether signatures of modified gravity during the period of inflation can produce observable effects. To be used to gain insight into the physics at play during inflation, these effects should be specific and not easily mimicked by standard gravity, yet arising under fairly generic conditions. For this reason we concentrated on local (or quasi-local) NG: we have found that it is possible, in a generic set-up, for modifications of gravity to generate deviations from Gaussian initial conditions where the NG is close to the local type and has values $f_{\rm NL} \approx -1$ to -30 [3]. It is interesting to note that in the same way that gravity, via its relativistic corrections, enhances the level of NG to $f_{\rm NL} \sim \mathcal{O}(-1)$ right after inflation (as pioneered by [96, 103]), a modification of GR *during* inflation will lead to an enhancement of similar magnitude. For quasi-local shapes NG is near maximal in the squeezed limit and the squeezed limit is made observationally accessible in the so-called large-scale halo bias. Thanks to the halo bias effect, a local NG of this amplitude is expected to be measurable in forthcoming and future LSS surveys (see, e.g., [95, 96, 96, 97, 104]) if systematic effects can be kept under control (e.g., [105]). On the other hand, the departures from exact single-field behaviour leave some imprint on the shape of NG, and in particular on the squeezed-limit dependence of the bispectrum on the (small) momentum. In fact, since the shape of the effective potential, eq. (3.8), is given, there is a "consistency relation" linking the amplitude of non-Gaussianity, $f_{\rm NL}$, to its shape (i.e., the parameter ν). For large enough values of $f_{\rm NL}$ it would be possible to constrain the scale-dependence of the bispectrum in the squeezed limit and hence ν, from forthcoming surveys [106, 107]. Thus, in case of a detection of NG, it may be possible to test the "consistency relation" between amplitude and shape. If such consistency relation were found to be satisfied to sufficient precision, it would require a fine tuning to be produced by any multi/quasi-single field inflation. Further, because the non-inflating field is related to gravity, the ratio between r (the tensor-to-scalar ratio) and its power law slope (n_T) will be modified from the standard single-field relation (1.88) with its counterpart in the two-field description in

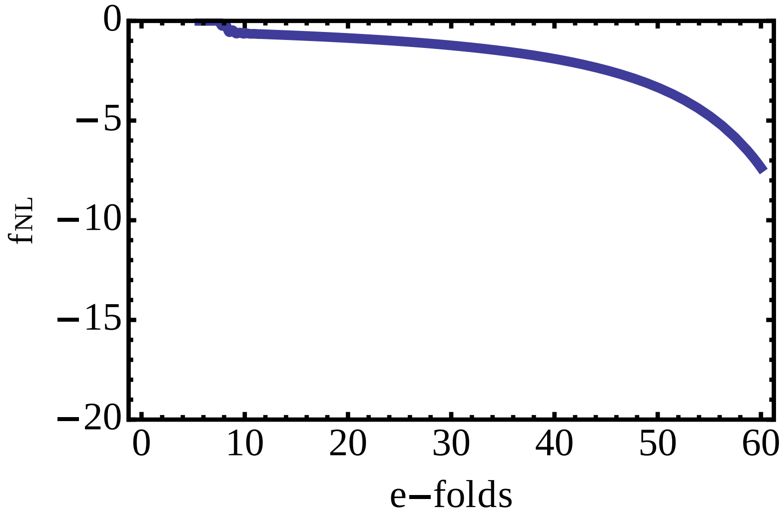


Figure 3.2: The NG parameter $f_{\rm NL}$ as a function of number of e-folds for $\alpha(\nu) = 1$, $M = 10^{-3}$ and $m = 10^{-8/3}$ in units of $M_{\rm Pl}$ to illustrate the scale dependence; $f_{\rm NL}$ can be smaller than -1 for fairly generic conditions.

the Einstein frame [81, 108]. A given form for $f(R)$ (corresponding to a given shape of $U_1(\varphi_G)$) would break the standard consistency relation in a specific way. Notice also that a specific running of the NG parameter $f_{\rm NL}$ in Eq. (3.11) can be left imprinted by the dynamics of the "scalaron" field φ_G, and interestingly the NG running will be correlated with the running of the scalar spectral index [99]. Specific signatures in the trispectrum of curvature perturbations, similar to those featured in eq. (3.11), are expected to arise as well.

CHAPTER 4

The Effective Field Theory of Inflation

The usual way to study inflation is to start from an action for the field (or fields), which is either postulated or derived from models of high-energy physics, supersymmetry, string stheory etc. After finding the background equations of an expanding inflationary Universe, one perturbs around this solution and study the dynamics of fluctuations. Observations then test the physics of these perturbations and hopefully put constraints on the model. In general, the conclusions one can draw depend on the model we choose to start with. Since is it possible to construct a huge quantity of very different inflationary models, it becomes very interesting to find an approach that allows the study of large classes of models at the same time and derive constraints that are less model-dependent. After all, what we really measure are fluctuations, therefore the most useful thing to do is to build an *effective* action for fluctuations, after fixing the background to the desidere FRW evolution. This approach, which directly studies inflationary perturbations with as less assumptions as possible about the model-dependent microphysics of the background, is known as the Effective Field Theory of Inflation (EFTI) [109][1].

4.1 The Action in Unitary Gauge

The effective field theory approach is the description of a system only in terms of the light degrees of fredoom with the systematic contruction of all the lowest dimension operators compatible with the underlying symmetries. In the absence of a fundamental theory of high-energy physics and gravity, applying this method to the theory of perturbations

[1]See also [100] for a slightly different approach.

during inflation can be very powerful as it does not require any assumptions on the field(s) driving inflation and study directly the action of perturbations. Now, following the original paper [109], we proceed to construct the most general effective action for the fluctuations around a given FRW inflationary background.

In constructing effective field theories, the first step is to identify the relevant degrees of freedom of interest. When studying early-universe perturbations, independently of what matter is actually driving the expansion, we are focusing attention on a scalar perturbation, corresponding to a common, local shift in time for the "matter field" ϕ[2]. Given a homogeneous FRW background $\phi_0(t)$ we consider the perturbation

$$\delta\phi(\boldsymbol{x}) = \phi(t + \pi(\boldsymbol{x})) - \phi_0(t) , \tag{4.1}$$

A time-dependent FRW background, such as the inflationary quasi-de Sitter spacetime, spontaneously breaks time-traslation invariance. Thus the scalar $\pi(x)$ represents the Goldstone boson associated with the spontaneous breakdown of this symmetry. Now, remember that General Relativity has a powerful gauge symmetry, that is diffeomorphism invariance:

$$x^\mu \to x^\mu + \xi^\mu(t, \boldsymbol{x}) . \tag{4.2}$$

Under a time-diffeomorphism the scalar perturbation $\delta\phi$ transforms as:

$$t \to t + \xi^0(t, \boldsymbol{x}) , \qquad \delta\phi \to \delta\phi + \dot{\phi}_0(t)\xi^0 . \tag{4.3}$$

We can now exploit the gauge freedom on ξ^0 to fix the so-called *unitary gauge*, which sets $\delta\phi = 0$. In this way, the scalar perturbation formally disappear from the action and the only dynamical field left is the metric, which now describes three degrees of freedom[3]: the two helicities of the gravitational waves and one scalar perturbation. Having fixed time diffeomorphisms, our theory will be invariant only under spatial diffeomorphisms:

$$x^i \to x^i + \xi^i(t, \boldsymbol{x}) . \tag{4.4}$$

If the symmetry was the full diffeomorphism invariance, the only 4-derivative operator built out of the metric that we could have written in the action would have been the Ricci scalar. Now, because of the reduced symmetry of the system, many more terms are allowed in the action:

1. Terms which are invariant under all diffeomorphisms: these are all the polynomials of the Riemann tensor $R_{\mu\nu\rho\sigma}$ and its covariant derivative, contracted to give scalars;

[2]Even though we are following the example of a scalar field, the EFTI is actually independent on what is actually driving inflation. It only requires that time diffeomorphisms are broken by only one "clock", which measures time during inflation and whose fluctuations can be described by a single scalar field.

[3]The scalar $\delta\phi$ is "eaten" by the metric, in exact analogy with the Higgs mechanism of the Standard Model of Particle Physics.

2. Generic functions of time $f(t)$ in front of any operators;

3. In unitary gauge, for a generic spatial diffeomorphism (4.4), the gradient $\partial_\mu \tilde{t}$ becomes δ^0_μ, so we can leave a free upper 0 index in every tensor. For example we can use g^{00} and functions of it;

4. It is useful to define a unit vector perpendicular to surfaceses of constant time:
$$n_\mu = \frac{\partial_\mu \tilde{t}}{\sqrt{-g^{\mu\nu}\partial_\mu \tilde{t}\partial_\nu \tilde{t}}} \ . \tag{4.5}$$
This allows to define the induced spatial metric on surfaces of constant time, $h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$, that we can use to project tensors on the surfaces (for example the Riemann tensor ${}^{(3)}R_{\mu\nu\rho\sigma}$ or 3d covariant derivative);

5. Covariant derivatives of n_μ, that we can decompose into a part projected on the surface of constant time and a part perpendicular to it. The first one is the extrinsic curvature of these surfaces[4]:
$$K_{\mu\nu} = h_\mu{}^\sigma \nabla_\sigma n_\nu \ . \tag{4.6}$$
The second one does not give rise to new terms because it can be rewritten as:
$$n^\sigma \nabla_\sigma n_\nu = -\frac{1}{2}\left(-g^{00}\right)^{-1} h^\mu{}_\nu \partial_\mu \left(-g^{00}\right) \ ; \tag{4.7}$$

6. Using at the same time the Riemann tensor of the induced spatial metric and the extrinsic curvature is redundant because one can be rewritten with the other and the 3d metric. We can also avoid to use $h_{\mu\nu}$ explicitely, writing it in terms of $g_{\mu\nu}$ and n_μ.

At this point, we can conclude that the most generic action in unitary gauge is given by [109, 110]:
$$S = \int \mathrm{d}^4x\sqrt{-g}\ F(R_{\mu\nu\rho\sigma}, g^{00}, K_{\mu\nu}, \nabla_\mu, t) \ , \tag{4.8}$$
where all the free indexes inside the function F are upper 0's. Expanding in perturbations around a FRW background, the action takes the form:
$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}M_{\mathrm{Pl}}^2 R - c(t)g^{00} - \Lambda(t) + \dots\right] \ , \tag{4.9}$$
where the dots stand for terms that start quadratic in the perturbations
$$\delta g^{00} = g^{00} + 1 \ , \qquad \delta K_{\mu\nu} = K_{\mu\nu} - K^{(0)}_{\mu\nu} \ , \qquad \delta R_{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma} - R^{(0)}_{\mu\nu\rho\sigma} \ . \tag{4.10}$$

[4]The index ν is already projected on the surface since $n^\nu \nabla_\sigma n_\nu = \frac{1}{2}\nabla_\sigma (n^\nu n_\nu) = 0$.

Notice that these terms start linearly in the perturbations, as we have explicitely removed their value on the given FRW solution, and they are well defined covariant operators. Notice also that every tensor evaluated on the background can be a function only of $g_{\mu\nu}$, n_μ and t (for example $K^{(0)}_{\mu\nu} = a^2 H h_{\mu\nu}$). The coefficient $c(t)$ and $\Lambda(t)$ in the action (4.9) are uniquely determined by the background evolution. In fact, the terms in eq. (4.9) are the only ones that produce a non-zero energy-momentum tensor at zero-order in fluctuations:

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S_{matter}}{\delta g^{\mu\nu}} = T^{(0)}_{\mu\nu} + \delta T_{\mu\nu}\ , \tag{4.11}$$

where we can recognise

$$T^{(0)}_{\mu\nu} = 2c(t)u^\mu u^\nu + (c(t) - \Lambda(t))g^{\mu\nu} \tag{4.12}$$

as the energy-momentum tensor of a perfect fluid with density $\rho = c(t) + \Lambda(t)$ and pressure $P = c(t) - \Lambda(t)$. Through the Einstein field equation for the background,

$$G^{(0)}_{\mu\nu} = 8\pi G\, T^{(0)}_{\mu\nu}\ , \tag{4.13}$$

we arrive at the Friedmann equations:

$$H^2 = \frac{1}{3M_{\rm Pl}^2}\left(c(t) + \Lambda(t)\right)\ , \tag{4.14}$$

$$\dot{H} + H^2 = -\frac{1}{3M_{\rm Pl}^2}\left(2c(t) - \Lambda(t)\right)\ . \tag{4.15}$$

Solving for $c(t)$ and $\Lambda(t)$, we can therefore write the most generic action with broken time diffeomorphisms in unitary gauge describing perturbations around a flat FRW background with a Hubble rate $H(t)$ [109]:

$$\begin{aligned} S \ = \ & \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{\rm Pl}^2 R + M_{\rm Pl}^2\dot{H}g^{00} - M_{\rm Pl}^2\left(3H^2(t) + \dot{H}(t)\right) + \frac{M_2(t)^4}{2!}(g^{00}+1)^2\right. \\ & \left. + \frac{M_3(t)^4}{2!}(g^{00}+1)^3 + \ldots - \frac{\bar{M}_2(t)^3}{2}(g^{00}+1)\delta K^\mu{}_\mu - \frac{\bar{M}_2(t)^2}{2}\delta K^\mu{}_\mu{}^2 + \ldots\right] \end{aligned} \tag{4.16}$$

where all the time-dependent coefficient $M_n(t)$ and $\bar{M}_m(t)$ are free and parametrize all the possible different effects on perturbations of any single-field models of inflation.

4.2 The Action for the Goldstone Boson

As we already said, the unitary gauge Lagrangian (4.16) describes three degrees of freedom: the two graviton helicities and a scalar mode, that represent the Goldstone boson associated with the breakdown of time diffeomorphisms. This mode will become explicit after one formally "restore" full diffeomorphism invariance through a broken time diffeomorphism (Stuekelberg trick). This is done in analogy with the gauge theory case and, as we will see, it will give us many advantages.

Les us review briefly what happens in a non-Abelian gauge theory and in our case, borrowing the examples in [109]. The unitary gauge action for a non-Abelian massive gauge field A_μ^a is:

$$S = \int \mathrm{d}^4x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} m^2 \,\mathrm{Tr} A_\mu A^\mu \right] \ , \tag{4.17}$$

where $A_\mu = A_\mu^a T^a$ and T^a are the generators of the Lie algebra. Under a gauge transformations,

$$A_\mu \quad \longrightarrow \quad U A_\mu U^\dagger + \frac{i}{g} U \partial_\mu U^\dagger = \frac{i}{g} U D_\mu U^\dagger \ , \tag{4.18}$$

the action is not invariant, because of the mass term for the gauge field, and becomes:

$$S = \int \mathrm{d}^4x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \frac{m^2}{g^2} \,\mathrm{Tr} D_\mu U D^\mu U^\dagger \right] \ . \tag{4.19}$$

The gauge invariance can be "restored" writing U in terms of the Goldstone boson π^a associated with the breakdown of the gauge simmetry:

$$U = \exp \left[i T^a \pi^a(t, \boldsymbol{x}) \right] . \tag{4.20}$$

The π's are scalars which transforms non-linearly under a gauge transformation as

$$e^{i T^a \tilde{\pi}^a(t,\boldsymbol{x})} = \Lambda(t, \boldsymbol{x})\, e^{i T^a \pi^a(t,\boldsymbol{x})} \ . \tag{4.21}$$

Going to canonical normalization $\pi_c = m/g \cdot \pi$, it can be shown that terms that mix Goldstones and gauge fields are of the form $m A_\mu^a \partial^\mu \pi_c^a$ and therefore are irrelevant with respect to the canonical kinetic term $\partial_\mu \pi_c^a \partial^\mu \pi_c^a$ for energies $E \gg m$. Thus in the window $m \ll E \ll 4\pi m/g$ (which is the scale at which boson self-interactions become strongly coupled) the physics of the Goldstone π is weakly coupled and it can be studied negleting the mixing with A_μ^a. Formally, in the *decoupling limit* $g \to 0$ and $m \to 0$ for $m/g =$const., the local gauge symmetry effectively becomes a global symmetry and there is no mixing between Goldstones and the gauge modes.

Now let us come back to inflationary fluctuations and consider the following action terms:

$$\int \mathrm{d}^4x \sqrt{-g} \left[A(t) + B(t) g^{00}(x) \right] \ . \tag{4.22}$$

Under a broken time diffeomorphism $t \to t + \xi^0(x)$, g^{00} transforms as:

$$g^{00}(x) \quad \longrightarrow \quad \tilde{g}^{00}(\tilde{x}(x)) = \frac{\partial \tilde{x}}{\partial x^\mu} \frac{\partial \tilde{x}}{\partial x^\nu} g^{\mu\nu}(x) \ . \tag{4.23}$$

Let us now write the action in terms of the transformed fields,

$$\int \mathrm{d}^4x \sqrt{-\tilde{g}(\tilde{x}(x))} \left|\frac{\partial \tilde{x}}{\partial x}\right| \left[A(t) + B(t)\frac{\partial x^0}{\partial \tilde{x}^\mu}\frac{\partial x^0}{\partial \tilde{x}^\nu}\tilde{g}^{\mu\nu}(\tilde{x(x)})\right] , \tag{4.24}$$

and then change integration variable to $\tilde{x}$:

$$\int \mathrm{d}^4\tilde{x}\sqrt{-\tilde{g}(\tilde{(x)})}\,\big[A(\tilde{t} - \xi^0(x(\tilde{x})))+ \\ +B(\tilde{t} - \xi^0(x(\tilde{x})))\frac{\partial(\tilde{t} - \xi(x(\tilde{x})))}{\partial \tilde{x}^\mu}\frac{\partial(\tilde{t} - \xi(x(\tilde{x})))}{\partial \tilde{x}^\nu}\tilde{g}^{\mu\nu}(\tilde{x})\Big] . \tag{4.25}$$

As in the gauge theory case, we promote the parameter $\xi^0(x)$ to a field (dropping the tildes):

$$\xi^0(x) = -\pi(x) . \tag{4.26}$$

This gives:

$$\int \mathrm{d}^4x\sqrt{-g(x)}\left[A(t+\pi(x)) + B(t+\pi(x))\frac{\partial(t+\pi(x))}{\partial x^\mu}\frac{\partial(t+\pi(x))}{\partial x^\nu}g^{\mu\nu}(x)\right] . \tag{4.27}$$

It is not difficult to show that this action is invariant under full spacetime diffeomorphism upon assigning to the field π the transformation rule:

$$\pi(x) \quad\longrightarrow\quad \tilde{\pi}(\tilde{x}(x)) = \pi(x) - \xi^0(x) . \tag{4.28}$$

We can then apply this procedure to the unitary gauge action (4.16). Under time reparametrization, the metric transforms as:

$$\begin{aligned} g^{ij} &\longrightarrow g^{ij} , \\ g^{0i} &\longrightarrow (1+\dot{\pi})g^{0i} + g^{ij}\partial_j\pi , \\ g^{00} &\longrightarrow (1+\dot{\pi})^2 g^{00} + 2(1+\dot{\pi})g^{0i}\partial_i\pi + g^{ij}\partial_i\pi\partial_j\pi , \end{aligned} \tag{4.29}$$

which allow us to rewrite the unitary gauge action (4.16) as:

$$\begin{aligned} S = & \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}M_{\mathrm{Pl}}^2 R - M_{\mathrm{Pl}}^2\left(3H^2(t+\pi) + \dot{H}(t+\pi)\right) + \right. \\ & + M_{\mathrm{Pl}}^2\dot{H}(t+\pi)\left((1+\dot{\pi})^2 g^{00} + 2(1+\dot{\pi})g^{0i}\partial_i\pi + g^{ij}\partial_i\pi\partial_j\pi\right) + \\ & \left. + \frac{M_2(t+\pi)^4}{2!}\left((1+\dot{\pi})^2 g^{00} + 2(1+\dot{\pi})g^{0i}\partial_i\pi + g^{ij}\partial_i\pi\partial_j\pi\right)^2 + \ldots\right] . \end{aligned} \tag{4.30}$$

Notice that the action (4.16) and (4.30) describe the same degrees of freedom, thus the same physics.

Just like in the gauge theory case, one can find a decoupling limit where all the dynamics is described by the Goldstone boson only. Following the gauge theory analogy and identifying $g \to M_{\rm Pl}^{-1}$, $m^2 \to \dot{H}$, the decoupling limit is reached when

$$M_{\rm Pl} \to \infty \ , \qquad \dot{H} \to 0 \ , \tag{4.31}$$

with $M_{\rm Pl}^2 \dot{H}$ =const. At energies much higher than the decoupling energy, $E \gg E_{mix}$, the mixing between gravity gauge modes and the Goldstones can be neglected. The scale E_{mix} generically depends on the terms that are present or not in the action. For example, let us assume $M_n = \bar{M}_m = 0$ for all n, m in (4.30). In this case, the term with the kinetic term for the Goldstone π is $M_{\rm Pl}^2 \dot{H}\, \dot{\pi}^2$. Then we choose the normalization:

$$\pi_c = M_{\rm Pl} \dot{H}^{\frac{1}{2}} \pi \ , \tag{4.32}$$

toegether with the standard $\delta g_c^{00} = M_{\rm Pl} \delta g^{00}$. The dominant mixing term between δg and π is:

$$M_{\rm Pl}^2 \dot{H} \dot{\pi}\, \delta g^{00} = \dot{H}^{\frac{1}{2}} \dot{\pi}_c\, \delta g_c^{00} \ll \dot{\pi}_c^2 \qquad \text{for } E \gg \dot{H}^{\frac{1}{2}} \ . \tag{4.33}$$

This case corresponds, as we will see, to the standard slow-roll inflation. Another useful example is the following: let the term M_2 in (4.30) get large. Then we should normalize:

$$\pi_c = M_2^2 \pi \ . \tag{4.34}$$

In this case the dominant mixing terms becomes:

$$M_2^4 \dot{\pi}\, \delta g^{00} = \frac{M_2^2}{M_{\rm Pl}} \dot{\pi}_c\, \delta g_c^{00} \ll \dot{\pi}_c^2 \qquad \text{for } E \gg \frac{M_2^2}{M_{\rm Pl}} \ . \tag{4.35}$$

At this point, a question could arise. As we are interested in computing predictions for present cosmological observations, it could seem that the decoupling limit (4.31) is completely irrelevant for these extremely infrared scales. However, as we said in the first introductory Chapter, it can be proved that there exist a quantity, the usual $\mathcal{R}$ or ζ variables, which is constant out of the horizon at any order in perturbation theory. Therefore the problem is reduced to calculating correlation functions just after horizon crossing, thus we are interested in studying our Lagrangian at energies of order H. If the decoupling scale is smaller than H, then the action in the decoupling limit will give us the correct answer up to terms suppressed by E_{mix}/H.

Moreover, a further simplification occurs when we look at the time dependence of the coefficients of any operator in (4.30). Althought they can depend generically on time, we are interested in solutions where they do not vary significantly in one Hubble time. If it was the case, the rapid time dependence of this coefficients could win against the friction created by the exponential expansion, so that inflation may cease to be a dynamical attractor, which is necessary to solve the problems of standard FRW cosmology. Thus we can conveniently neglect all the terms that, Taylor expanding the coefficients, would result from their time dependence:

$$f(t+\pi) = f(t) + \dot{f}(t)\pi + \frac{1}{2}\ddot{f}(t)\pi^2 + \ldots \simeq f(t) \; . \tag{4.36}$$

As we can see, this assumption allows us to neglect all terms in π without a derivative, that is to say we are assuming an approximate continuous shift simmetry for π.

Finally, using both these arguments together, in the regime where $E \gg E_{mix}$ and assuming an approximate shift symmetry for the Goldstone boson, the action (4.30) dramatically simplifies to [109]:

$$\begin{aligned} S \;\; = \;\; & \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}M_{\rm Pl}^2 R - M_{\rm Pl}^2\dot{H}\left(\dot{\pi}^2 - \frac{(\partial_i\pi)^2}{a^2}\right) + \right. \\ & +2M_2^4\left(\dot{\pi}^2 + \dot{\pi}^3 - \dot{\pi}\frac{(\partial_i\pi)^2}{a^2}\right) - \frac{4}{3}M_3^4\dot{\pi}^3 + \\ & \left. +\frac{1}{2}\bar{M}_1^3\left(2H\frac{(\partial_i\pi)^2}{a^2} - (\partial_j^2\pi)\,\frac{(\partial_i\pi)^2}{a^2}\right) + \ldots\right] \; . \end{aligned} \tag{4.37}$$

The relation between π and the variable $\mathcal{R}$ which we are interested in very simple [109]:

$$\mathcal{R} = -H\pi + \mathcal{O}(\epsilon) \; . \tag{4.38}$$

The advantages of this approach are now clear:

- The theories of perturbation of all the possible single-field inflationary models have a unified model-independent description, in terms only of fluctuations and symmetries. In the action (4.37) one can “switch on or off” some particular operators in order to recover various single field models of inflation (we will see some examples in the next sections). This is probably the most important point, because it allows us to generically study very different models with a unifying formalism (see Section 4.3).

- We have parametrized our ignorance about all the possible high energy effects in terms of the leading invariant operators. Experiments will put bounds on the size of the various operators (for example with measurements of non-Gaussianity of curvature perturbations), that generically describe deviations from the standard scenario. In some sense this is similar to what one does in particle physics, where one puts constraints on the size of the operators that describe deviations from the Standard Model and thus encode the effects of new physics.

- It is explicit what is forced by simmetries and what is not.

- As every effective theories, it is clear the regime of validity of the action and where an UV completion is required (see Section 4.3).

4.3 Speed of Sound and Non-Gaussianity

As we said, the EFTI approach encompasses all single-field models of inflation. The simplest of those models is the action for a slowly rolling scalar field (1.55):

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{\rm Pl}^2 R - \frac{1}{2}\partial_\mu\phi\partial_\nu\phi g^{\mu\nu} - V(\phi)\right] . \tag{4.39}$$

This action can be straightforwardly recast in the unitary gauge ($\delta\phi = 0$) form of eq. (4.16) ,

$$S = \int d^4x\sqrt{-g}\left[-\frac{\dot\phi_0(t)^2}{2}g^{00} - V(\phi_0)\right] , \tag{4.40}$$

as the Friedmann equations give $\dot\phi_0(t)^2 = -2M_{\rm Pl}^2\dot H$ and $V(\phi_0(t)) = M_{\rm Pl}^2(3H^2+\dot H)$. Then we can reintroduce the Goldstone π and finally write:

$$\int d^4x\sqrt{-g}\left[\frac{1}{2}M_{\rm Pl}^2 R - M_{\rm Pl}^2\dot H\left(\dot\pi^2 - \frac{(\partial_i\pi)^2}{a^2}\right)\right] . \tag{4.41}$$

This action is in the form of eq. (4.37) with $M_n = \bar M_n = 0$. The absence of interaction terms tell us that correlation function of order three or higher will be exactly zero in this case, then the fluctuations are perfectly Gaussian. Remember that we are working in the decoupling limit and with the assumption of an approximate shift symmetry for π, so every conclusion will be correct up to slow-roll corrections. Then:

$$f_{\rm NL} \sim \mathcal{O}(\epsilon) , \tag{4.42}$$

which is just a confirmation of what we have already discussed in Section 2.3, namely that standard single-field inflationary models produce slow-roll suppressed non-Gaussianity.

There are several models that allow us to go beyond simple slow-roll and generate non-neglibigle non-Gaussianity. One of the first and most studied possibility is the presence of non-canonical kinetic terms in the inflaton Langrangian. In the standard "ϕ language", the starting point is a general Lagrangian of the form [111–113]:

$$\mathcal{L} = P(X,\phi) \qquad \text{where} \qquad X = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi . \tag{4.43}$$

The matter energy-momentum tensor reads:

$$T_{\mu\nu} = \frac{\partial P}{\partial X}\partial_\mu\phi\partial_\nu\phi - P(X,\phi)g_{\mu\nu} . \tag{4.44}$$

Providing that $\partial_\mu\phi$ is time-like (i.e. $X > 0$), it has the same form of a perfect fluid with pressure $p = P(X,\phi)$ [111]. Together with the slow-roll parameters (1.28) and (1.29), it is useful here to define a speed of sound,

$$c_s^2 = \frac{P_{,X}}{P_{,X} + 2XP_{,XX}} , \tag{4.45}$$

where $_{,X}$ denotes derivative with respect to X, and a correspondent "slow-roll" parameter

$$s = \frac{\dot{c}_s}{c_s H} \,, \tag{4.46}$$

which is the relative time variation of c_s in one Hubble time and is usually kept small, $s \ll 1$, in order to naturally obtain a scale-invariant power spectrum. The action (4.43) obviously comprehends the standard slow-roll case (4.39) if one chooses:

$$P(X,\phi) = X - V(\phi) \,. \tag{4.47}$$

Despite the different physical mechanisms that could give rise to $P(X,\phi)$ action (a well-known example being DBI inflation [114, 115]), observable predictions for the curvature power spectrum are essentially degenerate with the standard scenario to leading order in the slow-roll parameters. Looking at the second order action for the curvature perturbation $\mathcal{R}$,

$$S_2 = \int \mathrm{d}t\mathrm{d}^3x\, \epsilon a^3 \left[\frac{\dot{\mathcal{R}}^2}{c_s^2} - \frac{(\partial_i \mathcal{R})^2}{a^2}\right] \,, \tag{4.48}$$

one can see that the only difference with respect to the slow-roll action (1.71) is the presence of the c_s^2 in the kinetic term. This gives an identical form of the power spectrum up to a simple rescaling proportional to the speed of sound:

$$\mathcal{P}_{\mathcal{R}} = \frac{H^2}{8\pi^2 M_{\rm Pl}^2 \epsilon c_s} \,. \tag{4.49}$$

On the other hand, distinct features are found at the level of the three-points functions. The cubic action contains terms like

$$\mathcal{L} \supset \left(1 - \frac{1}{c_s^2}\right) \dot{\mathcal{R}} \frac{(\partial_i \mathcal{R})^2}{a^2} \,, \tag{4.50}$$

which generate non-Gaussianity of the order[5]:

$$f_{NL} \sim \frac{1}{c_s^2} \,. \tag{4.51}$$

Differently from the slow-roll case with slow-roll suppressed bispectrum, the size of non-Gaussianity can now be large and, being proportional to c_s^{-2}, it can become important (and possibly detectable) in the limit of small speed of sound.

The entire class of models described by (4.43) can be easily recovered with the formalism of the EFTI, as in unitary gauge the action simply becomes:

$$S = \int \mathrm{d}^4x \sqrt{-g} P\left(\dot{\phi}_0(t)^2 g^{00}, \phi(t)\right) \,, \tag{4.52}$$

[5] The explicit and complete calculation together with the detailed profiles can be found, for example, in [83, 113].

where $\phi_0(t)$ is the unperturbed solution. This action takes the form (4.37) with

$$M_n^4(t) = \dot{\phi}_0(t)^{2n} \frac{\partial^n P}{\partial X^n} \,, \tag{4.53}$$

and can be written in the π language with the usual procedure. In particular, if we define the speed of sound of the π field as

$$\frac{1}{c_s^2} = 1 - \frac{2M_2^4}{M_{\rm Pl}^2 \dot{H}} \,, \tag{4.54}$$

the most generic action in the decoupling limit up to third order in perturbations is:

$$\begin{aligned} S &= \int d^4x \sqrt{-g} \Bigg[-\frac{M_{\rm Pl}^2 \dot{H}}{c_s^2} \left(\dot{\pi}^2 - c_s^2 \frac{(\partial_i \pi)^2}{a^2} \right) \\ &\quad + M_{\rm Pl}^2 \dot{H} \frac{1-c_s^2}{c_s^2} \dot{\pi} \frac{(\partial_i \pi)^2}{a^2} - M_{\rm Pl}^2 \dot{H} \frac{1-c_s^2}{c_s^2} \left(1 + \frac{2}{3} \frac{\tilde{c}_3}{c_s^4} \right) \dot{\pi}^3 \Bigg] \,, \end{aligned} \tag{4.55}$$

where

$$\frac{\tilde{c}_3}{c_s^2} = M_3^4 / M_2^4 \,. \tag{4.56}$$

In order to prevent pathological instabilities, the coefficient of the time kinetic term in the action must be positive. Comparing eqs. (4.37) and (4.55), one obtains the bound:

$$-M_{\rm Pl}^2 \dot{H} + 2M_2^4 > 0 \,. \tag{4.57}$$

Furthermore superluminal propagation, $c_s^2 > 1$, can be forbidden[6] imposing $M_2^4 > 0$. We have already discussed the mixing with gravity, which can be neglected at energies $E \gg E_{mix} \simeq M_2^2/M_{\rm Pl}$. This implies that action (4.55) can be consistently used to predict cosmological observables, which are done at energies of order H, if $H \gg M_2^2/M_{\rm Pl}$ or equivalently when $\epsilon/c_s^2 \ll 1$.

The two operators, $\dot{\pi}^3$ and $\dot{\pi}(\partial_i\pi)^2$, produce two kind of bispectra with amplitudes [83, 84]:

$$f_{\rm NL}^{\dot{\pi}^3} = \frac{10}{243}\left(1 - \frac{1}{c_s^2}\right)\left(\tilde{c}_3 + \frac{3}{2}c_s^2\right) \qquad f_{\rm NL}^{\dot{\pi}(\partial_i\pi)^2} = \frac{85}{324}\left(1 - \frac{1}{c_s^2}\right) \,. \tag{4.58}$$

The two shapes turns out to be a linear combination of the equilateral (2.25) and orthogonal (2.26) shapes [84] we have seen in the previous Chapter. The experimental constraints on $f_{\rm NL}$ can then be translated into constraints in the parameter space of the theory, as it can be seen in Figure 4.1. Marginalizing over $\tilde{c}_3$, one can also find a lower

[6]Superluminal propagation in effective field theories may not be a problem *per se* (see e.g. [116]), but implies that the theory can not have a Lorentz invariant UV completion [117].

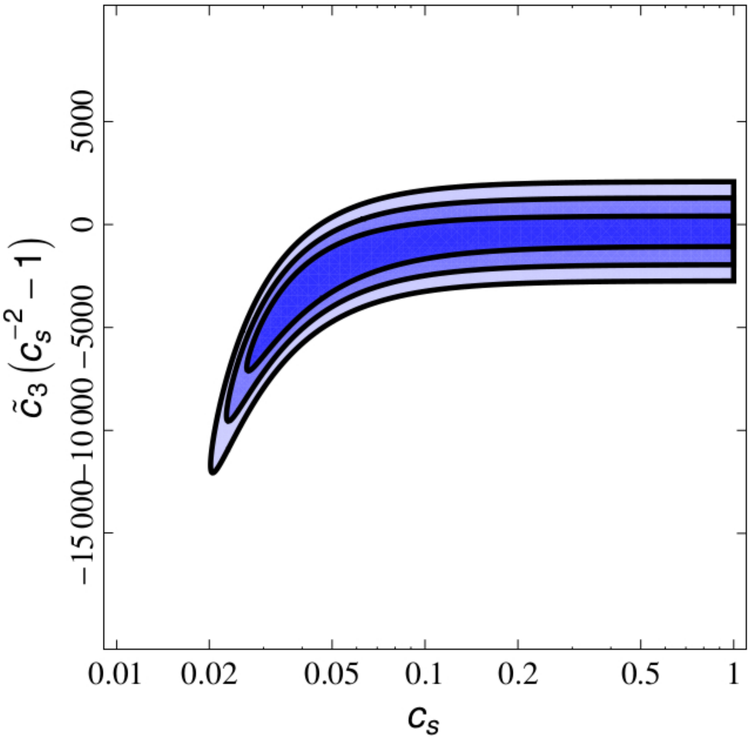


Figure 4.1: 68%, 95%, and 99.7% confidence regions in the EFTI parameter space $(c_s, \tilde{c}_3)$ [44].

bound on the speed of sound of fluctuations:

$$c_s^2 \geq 0.020 \qquad 95\%\text{CL} \,. \tag{4.59}$$

As the EFTI action (4.55) is describing the leading interaction terms of all single-field models of inflation, with the contraints in Figure 4.1 we are putting bounds on different specific models of inflation at the same time. For example, the DBI case [114, 115] corresponds to $\tilde{c}_3 = 3(1 - c_s^2)/2$ and can be mapped into the same constraints. The same happens for many other models, that we will not review here. The most recent oservational constraints, where the EFTI methods have been used, can be found in the *Planck* paper [44].

4.4 Strong Coupling

Since large interactions can be possible in the action we have seen, there could exist high-energy regimes where the theory is not consistent any more, as strong coupling would spoil perturbativity. In particular, the effective action (4.55) contains non-renormalizable self-interactions between the π's, which will become strongly coupled at a certain energy scale Λ. This scale sets the UV cutoff and therefore the regime of validity of the theory. To identify the cutoff of the non-Lorentz invariant action (4.55) [118], one first rescales spatial coordinates,

$$x^i \to \tilde{x}^i = \frac{x^i}{c_s} \ , \tag{4.60}$$

and then canonically normalize the Goldstone as:

$$\pi_c = (-2M_{\rm Pl}^2 \dot{H} c_s)^{1/2} \pi \ . \tag{4.61}$$

The action becomes:

$$S = \int {\rm d}t{\rm d}^3\tilde{x}\sqrt{-g}\left[\frac{1}{2}\left(\dot{\pi}_c^2 - \frac{(\tilde{\partial}_i\pi_c)^2}{a^2}\right) - \frac{1}{\sqrt{8M_{\rm Pl}^2|\dot{H}|c_s^5}}\dot{\pi}_c\left(\frac{(\tilde{\partial}_i\pi_c)^2}{a^2} - \frac{2}{3}\tilde{c_3}\dot{\pi}_c^2\right)\right] \ . \tag{4.62}$$

We can now read the strong coupling scale as the energy in the denominator of the cubic interactions:

$$\Lambda^4 \simeq M_{\rm Pl}^2|\dot{H}|c_s^5 \ . \tag{4.63}$$

As c_s approaches $c_s = 1$, the strong coupling scale becomes higher as in this limit the theory lose their possibly dangerous interaction and collapses back to the free action (4.41). On the contrary, as c_s becomes smaller, Λ decreases and the theory can become strongly coupled even at energies of our interest. As we are interested in making cosmological predictions at energies of the order of the Hubble scale H, we must ensure that Λ is bigger than H, which implies [109]:

$$H^4 \ll M_{\rm Pl}^2|\dot{H}|c_s^5 \qquad \Longrightarrow \qquad c_s \gg \mathcal{P}_{\mathcal{R}}^{1/4} \simeq 0.003 \ . \tag{4.64}$$

A more accurate result can be obtained looking at the energy scale where the scattering of π's lose perturbative unitarity, which signals the breakdown of the loop expansion, since higher order terms becomes equally important as the lower ones (the details of this calculation can be found for example in [119]). The final cutoff scale reads [109]:

$$\Lambda^4 \simeq 16\pi^2 M_{\rm Pl}^2|\dot{H}|\frac{c_s^5}{1-c_s^2} \tag{4.65}$$

Part II

Inflationary Models with Features

CHAPTER 5

Inflationary Models with Features

In this Chapter we focus on the problem of studying single-field models of inflation with sharp features in the inflaton potential in the context of the EFTI. The idea of allowing for features in the inflationary potential has a long history [79, 120–123], but it started to draw attention several years later as a possible explanation for the apparent "glitches" in the angular power-spectrum of the CMB [124–134]. Despite being not statistically significant from a Bayesian point of view, features seem to lead to marginal improvement in the likelihood of the primordial power spectrum also in the *Planck* data [43]. Beyond the power-spectrum features, it has been shown that these models generally predict enhanced non-Gaussianity [79, 135–139] and can be motivated by some high-energy physics mechanisms [140–150]. More generally, features can also be present in the speed of sound [151–159], giving also in this case characteristic effects on the power-spectrum, together with possible enhanced non-Gaussianity.

It becomes even more attractive then to find a common setup for this wide phenomenology. As we will show, in the case of very small and very sharp steps in the inflaton potential this is achievable in the context of the EFTI. This reformulation of feature models will allow us to provide a straightforward generalization to features in the speed of sound or in every coefficient of higher-dimension operators in the effective Lagrangian. One of the main advantages of our approach is model-independence and a better understanding of the regime of validity and energy scales involved. This Chapter, which is based on [1], shows how to describe models with features in the inflaton potential within EFTI and derive, using the in-in formalism [160], the predicted power-spectrum and bispectrum of curvature perturbations. We also generalize the same approach to features in other coefficients of the EFTI action and show that the most interesting case is the case of a feature in the speed of sound.

5.1 Features in the Hubble Parameter

The common characteristics of models with features are the breaking of the scale-invariance of the power-spectrum and an enhancement of higher-order correlators, that strongly depends on momenta. The traditional road followed to deal with models with step features is to specify a form for the inflaton potential $V(\phi)$ and then study the background evolution of the field, derive expressions for the modified slow-roll parameters and finally study their effects on the behavior of the correlation functions of curvature perturbations. In this section we want to show how to describe models with step features within the formalism of the EFTI, studying the effect of features in the Hubble parameter and its derivatives. Let us first restrict to the simplest scenario, where all the $M_n(t)$ and $\bar{M}_m(t)$ coefficients of higher-order operators in the effective action (4.30) are set to zero. Consider a potential for the inflaton field of the form [139]

$$V(\phi) = V_0(\phi)\left[1 + cF\left(\frac{\phi-\phi_f}{d}\right)\right] , \tag{5.1}$$

which describes a step of height c and width d centered at ϕ_f with a generic step function F. As the field crosses the feature, a potential energy $\Delta V \simeq cV$ is converted into kinetic energy $\dot{\phi}^2 = 2\dot{H}$. As long as the step is small, $c \ll 1$, it does not ruin the inflationary background evolution and its effect can be treated as a perturbation on a standard background. The idea is then simple: we can describe these models into the EFTI through a time-dependent Hubble parameter $\dot{H}$. This approach can be easily extended to features in the $M_n(t)$ and $\bar{M}_m(t)$ coefficients of higher-order operators, as we will show in the following Section.

We parametrize the derivative of the Hubble parameter as

$$\dot{H}(t) = \dot{H}_0(t)\left[1 + \epsilon_{step}(t)F\left(\frac{t-t_f}{b}\right)\right] , \tag{5.2}$$

which implies that the slow-roll parameter ϵ will be[1]

$$\epsilon = \epsilon_0(t)\left[1 + \epsilon_{step}(t)\,F\left(\frac{t-t_f}{b}\right)\right] . \tag{5.3}$$

The quantity ϵ_{step} represents the height of the step, while t_f is its position and b its characteristic width. The function $F(x)$ goes from -1 to $+1$ as its argument passes $x=0$ with a characteristic width $\Delta x = 1$. We do not give here any further requirement on the shape of the step and we shall remain as general as possible throughout the discussion. The background parameters $\dot{H}_0(t)$, $\epsilon_0(t)$ and even ϵ_{step} can in principle have a mild time

[1] In order to compare, notice that the parameters in Eqs. (5.3) and (5.1) are related by $\epsilon_{step} = 3c/\epsilon$, $1/b = H\sqrt{2\epsilon}/d$.

dependence, which is controlled by the zeroth-order slow-roll parameters ϵ_0, η_0, etc. However this variation should be very small in order not to spoil inflation. Moreover, we are interested here in the case in which the strongest time dependence comes from the step feature, therefore we shall take them to be constant in our calculations. It also is clear that, if we want an inflationary background, ϵ_{step} should be small, $|\epsilon_{step}| \ll 1$, otherwise we could have a violation of the necessary condition $\epsilon \ll 1$. Provided that, we can expand every quantity in ϵ_{step} around an unperturbed background, as, e.g.,

$$\epsilon = \epsilon_0 + \epsilon_1 + \dots , \tag{5.4}$$

where dots stand for terms which are higher than first order in ϵ_{step}. Although, as we said, ϵ is always small, this could not be the case for higher-order slow-roll parameters, which can temporarily become of order unity or larger. This happens, for example, for the parameter[2]:

$$\delta = \frac{1}{2}\frac{\mathrm{d}\ln\epsilon}{\mathrm{d}\ln\tau} = -\frac{\dot{\epsilon}}{2\epsilon H} , \tag{5.5}$$

where $\mathrm{d}\tau = \mathrm{d}t/a$ is the conformal time. We can expand δ in powers of ϵ_{step} as

$$\delta = \delta_0 + \delta_1 + \mathcal{O}(\epsilon_{step}^2) . \tag{5.6}$$

Notice that this parameter contains a derivative of ϵ (5.3) and hence is proportional to $1/b$, which in principle can be very large. The major contribution to δ_1 then comes from

$$\delta_1 \simeq -\frac{1}{2}\frac{\epsilon_{step}}{H}\dot{F}\left(\frac{t-t_f}{b}\right) , \tag{5.7}$$

This is the situation which we are interested in, as it corresponds to a sharp step feature. It is useful to rewrite quantities in conformal time. This can be easily done, as we are in a quasi-de Sitter space-time,

$$\tau \sim -e^{-Ht} \qquad \Longrightarrow \qquad \frac{t-t_f}{b} = -\beta\ln\frac{\tau}{\tau_f} , \tag{5.8}$$

where τ_f is the conformal time at which the step occurs and we defined

$$\beta = \frac{1}{bH} . \tag{5.9}$$

Then,

$$\delta_1 = -\frac{1}{2}\epsilon_{step}\beta\, F'\left(-\beta\ln\frac{\tau}{\tau_f}\right) , \tag{5.10}$$

where primes denote derivatives with respect to the argument of F.

[2]The choice of the second (and higher) order slow-roll parameters is somewhat arbitrary. Other conventions are possible, for example $\delta = \ddot{H}/2H\dot{H} = -\epsilon - \delta_{ours}$.

Now we come back to the effective action (4.30), with the Taylor expansion in eq. (4.36). Using eq. (5.2) for the time dependence of the Hubble parameter, we obtain an effective theory which can describe models with features in the inflaton potential. The advantage in using this approach is twofold: first, it becomes easier to identify the regime of validity of the theory and to assess the relative importance of operators. Second, from this point of view one could easily generalize feature models to the other couplings in the effective Lagrangian and study all the effects within the same formalism.

5.1.1 Power Spectrum

The first prediction we want to make is the power-spectrum of the curvature perturbations in the case of a sharp step in the inflaton potential ($\beta \gg 1$ i.e. $b \ll 1$). In order to obtain the equation of motion for the Goldstone boson π, we need the second-order action, in which the Hubble parameter is Taylor expanded around $\pi = 0$ [161]

$$S_2 = \int \mathrm{d}^4 x a^3 \left[-M_{Pl}^2 \dot{H} \left(\dot{\pi}^2 - \frac{(\nabla\pi)^2}{a^2} \right) + 3M_{Pl}^2 \dot{H}^2 \pi^2 \right] . \tag{5.11}$$

From the second-order action we derive the equation of motion for π:

$$\ddot{\pi} + \left(3H + \frac{\ddot{H}}{\dot{H}} \right) \dot{\pi} - \frac{\nabla^2 \pi}{a^2} = \ddot{\pi} + H\left(3 - 2\delta\right)\dot{\pi} - \frac{\nabla^2\pi}{a^2} = 0 \, , \tag{5.12}$$

where we have neglected a slow-roll suppressed term. It is easier to discuss the dynamics in conformal time $\mathrm{d}\tau = \mathrm{d}t/a$. We can rewrite the action (5.11) in the form

$$S_2 = \frac{1}{2} \int \mathrm{d}^3 x \mathrm{d}\tau \, z^2 \left[\pi'^2 - (\nabla\pi)^2 - 3a^2 \dot{H} \pi^2 \right] \, , \tag{5.13}$$

where primes denote differentiation with respect to τ and

$$z^2 = -2a^2 M_{Pl}^2 \dot{H} \, . \tag{5.14}$$

Making the redefinition $\pi = u/z$, we obtain

$$S_2 = \frac{1}{2} \int \mathrm{d}^3 x \mathrm{d}\tau \left[u'^2 - (\nabla u)^2 + \left(\frac{z''}{z} + 3a^2 H^2 \epsilon \right) u^2 \right] . \tag{5.15}$$

Notice that the second derivative of z (5.14) contains slow-roll parameters and their derivatives up to the second derivative of ϵ, which appears through the parameter

$$\frac{\dot{\delta}}{H} = -\frac{\mathrm{d}\delta}{\mathrm{d}\ln\tau} \, . \tag{5.16}$$

It is clear that $\dot{\delta}/H$ will give the largest contribution, being proportional to β^2. To study its effects on curvature perturbations, we look at the equation of motion for u in terms of the variable $x = -k\tau$,

$$\partial_x^2 u - \frac{2}{x^2}u + u = \frac{\dot{\delta}}{Hx^2} \ , \tag{5.17}$$

where we have neglected some other slow-roll terms, which are much smaller in the case of a small, $\epsilon_{step} \ll 1$, and sharp, $\beta \gg 1$, step.[3] This equation can be solved using the Green's function technique, treating the right-hand side of eq. (5.17) as a source function for the left-hand side. The machinery of the General Slow-Roll (GSR) approximation developed in [139, 162] helps us to accomplish this task and provides us with a useful formula for the resulting power-spectrum at late times, $\tau \to 0$,

$$\ln \mathcal{P}_{\mathcal{R}} = \ln \mathcal{P}_{\mathcal{R},0} + \frac{2}{3}\int_{-\infty}^{+\infty} \mathrm{d}\ln\tau \, W(k\tau)\, \frac{\mathrm{d}\delta}{\mathrm{d}\ln\tau} \ , \tag{5.18}$$

we have used the linear relation between π and the curvature perturbation (4.38), and $W(x)$ is the "window function":

$$W(x) = \frac{3\sin(2x)}{2x^3} - \frac{3\cos(2x)}{x^2} - \frac{3\sin(2x)}{2x} \ . \tag{5.19}$$

The zeroth-order power-spectrum is simply

$$\mathcal{P}_{\mathcal{R},0} = \frac{H^2}{8\pi^2\epsilon M_{Pl}^2} \ . \tag{5.20}$$

Now, from (5.18), integrating by parts and using eq. (5.10) we obtain

$$\ln \mathcal{P}_{\mathcal{R}} = \ln \mathcal{P}_{\mathcal{R},0} - \frac{1}{3}\epsilon_{step}\beta\int_{-\infty}^{+\infty} \mathrm{d}\ln\tau \, W'(k\tau)\, F'\left(-\beta\ln(\tau/\tau_f)\right) \ , \tag{5.21}$$

where

$$W'(x) = \left(-3 + \frac{9}{x^2}\right)\cos(2x) + \left(15 - \frac{9}{x^2}\right)\frac{\sin(2x)}{2x} \tag{5.22}$$

is the derivative of $W(x)$ with respect to $\ln x$. Notice that if we take the limit $\beta \to +\infty$, the derivative of the step, $F'(x)$, would become a Dirac delta function. Then the integration in the previous equation would give a power-spectrum which exhibits constant amplitude oscillations with frequency $2k\tau_f$ up to $k \to +\infty$. As we will see better in the next sections, the limit $\beta \to +\infty$ cannot be taken naively since it is not phyisical, and we must take into account the finite width of the step. The integral in eq. (5.21) can be analitically evaluated when $\beta \gg 1$ [139, 163], leading to

$$\ln \mathcal{P}_{\mathcal{R}} = \ln \mathcal{P}_{\mathcal{R},0} - \frac{2}{3}\epsilon_{step}\, W'(k\tau_f)\, \mathcal{D}\left(\frac{k\tau_f}{\beta}\right) \ , \tag{5.23}$$

[3] All other terms are suppressed at least by $1/\beta$, ϵ_0 or ϵ_{step}.

where $\mathcal{D}(y)$ is a damping function normalized to one.

Irrespective of the particular shape of the step, $\mathcal{D}$ corresponds to the Fourier transform of the step function F times $(-ik)$. The typical integrals that can be found when studying models with features. These integrals in conformal time are generally of the form:

$$I = \beta \int_{-\infty}^{+\infty} \mathrm{d}\ln\tau \;\; p(k\tau)\cos(2k\tau)F'\left(-\beta\ln(\tau/\tau_f)\right) \;, \tag{5.24}$$

or with sine instead of cosine and where $p(k\tau)$ is a sum of polynomials. Notice that, as $\beta \gg 1$, the derivative of the step $F'(x)$ is strongly peaked in its central value, namely $\tau = \tau_f$. The polynomials varies slowly in the small region where $F'(x)$ is non-zero and we can replace them by their value when $F'(x)$ is peaked, namely $(k\tau)^n \to (k\tau_f)^n$. Then, one can use the exponential form of sine and cosine and change variable to $y = -\beta\ln(\tau/\tau_f)$ to obtain

$$\frac{1}{2}p(k\tau_f)\left[\int_{-\infty}^{+\infty} \mathrm{d}y \; \exp\left\{2ik\tau_f e^{-y/\beta}\right\}F'(y) + \int_{-\infty}^{+\infty} \mathrm{d}y \; \exp\left\{-2ik\tau_f e^{-y/\beta}\right\}F'(y)\right] \;. \tag{5.25}$$

Now we linearize the exponential, $\exp(-y/\beta) \simeq 1 - y/\beta$, to give:

$$\frac{1}{2}p(k\tau_f)\left[e^{2ik\tau_f}\int_{-\infty}^{+\infty} \mathrm{d}y\, e^{-\frac{2ik\tau_f}{\beta}y}F'(y) + e^{-2ik\tau_f}\int_{-\infty}^{+\infty} \mathrm{d}y\, e^{\frac{2ik\tau_f}{\beta}y}F'(y)\right] \;. \tag{5.26}$$

We can make this substitution as long as $y \ll \beta$, that is to say that the validity of the approximation breaks down for $\tau \ll \tau_f$. However, since τ_f is the position of the step in conformal time, this corresponds to early times or much before the step, where we expect that the integral is already negligible. Notice now that the two integrals in the previous equation are actually the same integral: $F'(x)$ is even , being the derivative of the step $F(x)$, which is an odd function. As a consequence, we can reconstruct the cosine in front of the integral and write

$$I = p(k\tau_f)\cos(2k\tau_f)\int_{-\infty}^{+\infty} \mathrm{d}y\, e^{-\frac{2ik\tau_f}{\beta}y}F'(y) \;. \tag{5.27}$$

It is easy to recognize the Fourier transform of the derivative of the step with respect to the variables y and $2k\tau_f/\beta$, which is nothing else that the Fourier transform of the step itself

$$I = 2\,p(k\tau_f)\cos(2k\tau_f)\left(\frac{2ik\tau_f}{\beta}\hat{\mathcal{F}}\big[F(y)\big]\right) = 2\,p(k\tau_f)\cos(2k\tau_f)\mathcal{D}\left(\frac{k\tau_f}{\beta}\right) \;. \tag{5.28}$$

We have obtained an oscillating function (with sine or cosine), times a damping envelope $\mathcal{D}$ which is normalized to one. The further factor 2 is due to the fact that $F(x)$ goes from -1 to $+1$. This is a quite general result that depends only on the assumption of a very small and very sharp step. It is also reminiscent of the classical quantum mechanics

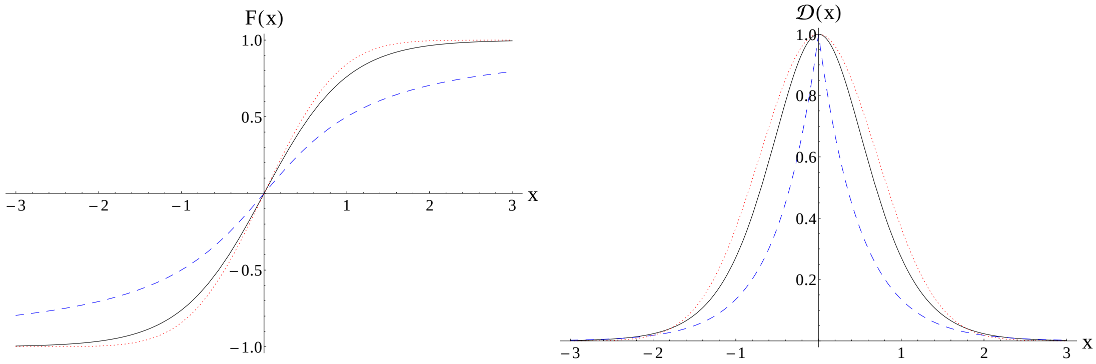


Figure 5.1: Step functions $F(x)$ and damping functions $\mathcal{D}(x)$ for different choice of the step shape, namely hyperbolic tangent (5.29) (black), arctangent (5.30) (dashed blue), gaussian integral (5.31) (dotted red) profiles.

problem of a potential barrier, where the reflection probability is proportional to the Fourier transform of the barrier itself. For different choice of the step shape, we obtain different damping effects (see Figure 5.1):

$$F(x) = \tanh(x) \qquad \Longrightarrow \qquad \mathcal{D}\left(\frac{k\tau_f}{\beta}\right) = \frac{\pi k\tau_f/\beta}{\sinh\left(\pi k\tau_f/\beta\right)}\,, \tag{5.29}$$

$$F(x) = \frac{2}{\pi}\arctan(x) \qquad \Longrightarrow \qquad \mathcal{D}\left(\frac{k\tau_f}{\beta}\right) = e^{-2\left|\frac{k\tau_f}{\beta}\right|}\,, \tag{5.30}$$

$$F(x) = \frac{2}{\sqrt{\pi}}\int_{-\infty}^{+\infty} e^{-x^2} \qquad \Longrightarrow \qquad \mathcal{D}\left(\frac{k\tau_f}{\beta}\right) = e^{-\left(\frac{k\tau_f}{\beta}\right)^2}\,. \tag{5.31}$$

Finally, consider the case $\beta \to \infty$, which is the case of an infinitely sharp step. This corresponds to a step function in the form of an Heaviside function, whose derivative is a Dirac delta function. In this case the integral (5.24) is straightforward and correspond to take $\tau = \tau_f$ everywhere. We can see explicitly that no damping envelope arises and oscillations persist in all k-space.

Some further comments about eq. (5.23) are in order. The function $W'(x)$ in (5.22) oscillates between -1 and $+1$ up to $k \to +\infty$ while the function $\mathcal{D}$ acts as a damping envelope. As $x \to 0$, $W'(x) \to 0$ and no spurious super-horizon contributions during inflation are generated. Moreover, the damping, decaying exponentially, "localizes" the oscillations in an effectively finite range in k-space. This was desirable and confirms our intuition that the feature should not affect modes either much before or much after the step. This is clearly visible from Figure 5.2: the largest contribution is in the range of frequencies $1 \lesssim k\tau_f \lesssim \beta$, which refers to the modes which are inside horizon at the time of the feature but whose momenta are not greater than the inverse of the time, $b = 1/\beta H$, characterizing the sharpness of the step. It is also clear that, as the parameter β becomes larger, the range in k-space in which there are oscillations also becomes larger. In the

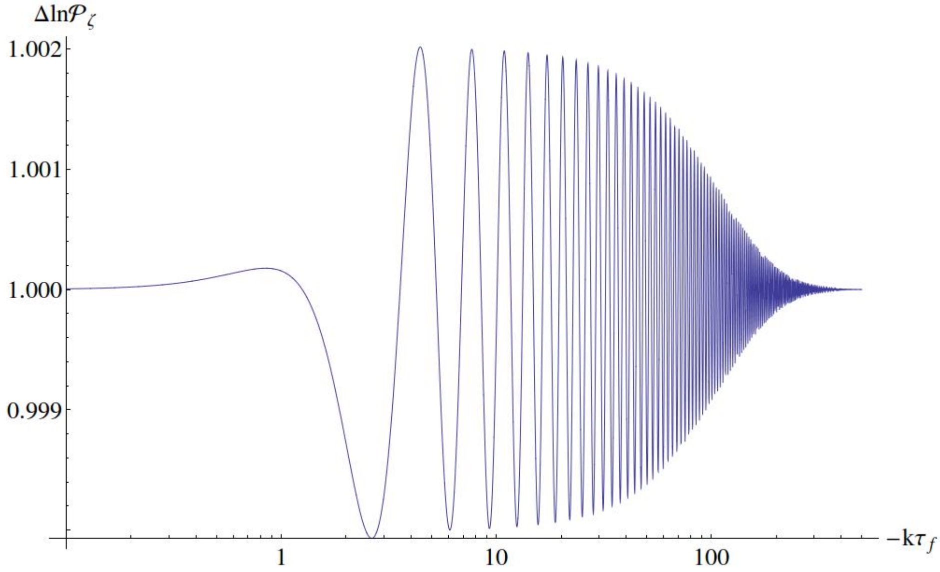


Figure 5.2: Non-scale invariant part of the power spectrum (5.21) for a hyperbolic tangent step (5.29), evaluated for $\epsilon_{step} = 0.001$ and $\beta = 43\,\pi$ for illustration purposes.

limit of an infinitely sharp step, $\beta \to +\infty$, as we already said, the power-spectrum would gain oscillations with constant amplitude up to $k \to +\infty$. Notice finally that the total height of the step, namely $2\epsilon_{step}$, does not affect neither the frequency of the oscillations nor the damping and appear in eq. (5.23) only as a multiplicative constant in front of the non-scale-invariant part of the spectrum.

5.1.2 Bispectrum

The starting point for computing the bispectrum is the third-order action, which can be derived from eq. (4.30) Taylor expanding around $\pi = 0$ as in (4.36):

$$S_3 = \int \mathrm{d}^4 x a^3 M_{Pl}^2 \left[-\ddot{H}\pi \left(\dot{\pi}^2 - \frac{(\nabla\pi)^2}{a^2} \right) - 3\dot{H}\ddot{H}\pi^3 \right] . \tag{5.32}$$

Notice that we could in principle work in the decoupling regime: after canonical normalization of the π field, $\pi_c = -M_{Pl}\dot{H}^{-1/2}\pi$, we see that we can neglect gravity-mixing interactions if we work at energies above $E_{mix} \sim \epsilon^{1/2} H$, which is surely below our infrared cutoff H, as long as $\epsilon \ll 1$.

For the study of non-Gaussianity, we will use the standard in-in formalism (see

Appendix B) and compute the expectation value

$$\begin{aligned}\langle\pi_{\boldsymbol{k}_1}\pi_{\boldsymbol{k}_2}\pi_{\boldsymbol{k}_3}\rangle &= -i\,(2\pi)^3\delta^3\left(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3\right)\int_{-\infty}^{0}\mathrm{d}\tau\, a\,\langle 0|\left[\pi^{(0)}_{\boldsymbol{k}_1}\pi^{(0)}_{\boldsymbol{k}_2}\pi^{(0)}_{\boldsymbol{k}_3}, H_I(\tau)\right]|0\rangle = \\ &= i\,(2\pi)^3\delta^3\left(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3\right)\pi^{(0)}_{\boldsymbol{k}_1}\pi^{(0)}_{\boldsymbol{k}_2}\pi^{(0)}_{\boldsymbol{k}_3}\int_{-\infty}^{0}\frac{\mathrm{d}\tau}{H^2\tau^2}\,\delta\left(-\beta\ln\tau/\tau_f\right)\times \\ &\times\ \pi_{\boldsymbol{k}_1}(\tau)^*\left[2\pi'_{\boldsymbol{k}_2}(\tau)^*\pi'_{\boldsymbol{k}_2}(\tau)^* - k_1^2\pi_{\boldsymbol{k}_2}(\tau)^*\pi_{\boldsymbol{k}_3}(\tau)^*\right] + \text{perm.} + \text{c.c.}\,, \end{aligned} \tag{5.33}$$

where the interaction Hamiltonian $H_I(\tau)$ can be easily read from the third-order action (5.32). Although also the operator π^3 should be present, it can be seen from the action (5.32) that it is proportional to one more factor $\dot{H}$. Therefore its contribution to the bispectrum will be suppressed by the ϵ slow-roll parameter. Notice also that for the computation of this three-point function at leading order we only need the unperturbed mode function

$$\pi^{(0)}_k(\tau) = \frac{i}{M_{Pl}\sqrt{4\epsilon k^3}}\left(1+ik\tau\right)e^{-ik\tau}\,. \tag{5.34}$$

As the deviation from the classic solution (5.34) is proportional to ϵ_{step}, its contribution inside the integral will be suppressed, being at least of order $\mathcal{O}\left(\epsilon^2_{step}\right)$[4]. The calculation simplifies using the dimensionless variable $y = z\sqrt{2k}\pi_k$, where z is given by (5.14), which has the form

$$y_0(-k\tau) = \left(1-\frac{i}{k\tau}\right)e^{-ik\tau} \tag{5.35}$$

in the unperturbed case. At leading order in the slow-roll parameters and ϵ_{step} we can evaluate the ϵ and H factors inside the integral at horizon crossing and use $\tau\sim -1/aH$. Then, using the linear relation between π and $\mathcal{R}$ (4.38) we can write

$$\begin{aligned}\langle\mathcal{R}_{k_1}\mathcal{R}_{k_2}\mathcal{R}_{k_3}\rangle &= (2\pi)^7\delta^3\left(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3\right)\frac{\mathcal{P}^2_{\mathcal{R},0}}{4}\int_{-\infty}^{0}\frac{\mathrm{d}\tau}{\tau^2}\,\tau y_0(k_1\tau)\,\delta\left(-\beta\ln\tau/\tau_f\right)\times \\ &\times\left[2\frac{\mathrm{d}}{\mathrm{d}\tau}\big(\tau y_0(k_2\tau)\big)\frac{\mathrm{d}}{\mathrm{d}\tau}\big(\tau y_0(k_3\tau)\big) - k_1^2\tau^2 y_0^*(-k_2\tau)y_0^*(-k_3\tau)\right] + \\ &+\text{perm} + \text{c.c.}\end{aligned} \tag{5.36}$$

where we have reconstructed the power-spectrum $\mathcal{P}_{\mathcal{R},0}$ (5.20) in front of the expression. This integral is very similar to the one in eq. (5.21) and can be treated in the same way: as we work with very sharp steps, $\beta \gg 1$, we can evaluate the polynomials at $\tau=\tau_f$ so that we are left with the Fourier transform of the step. At the end of the calculation we will obtain an oscillating function times a damping envelope. In order to focus on the particular scaling of this type of non-Gaussianity, it is often useful to consider the

[4]As we will see, this will be not true if also a speed of sound is taken into account.

dimensionless quantity [135]:

$$\frac{\mathcal{G}(k_1,k_2,k_3)}{k_1k_2k_3} = \frac{k_1^2k_2^2k_3^2}{(2\pi)^4\mathcal{P}^2_{\mathcal{R},0}}\mathcal{B}(k_1,k_2,k_3)\ , \tag{5.37}$$

where

$$\langle\mathcal{R}_{k_1}\mathcal{R}_{k_2}\mathcal{R}_{k_3}\rangle = (2\pi)^3\delta^3(k_1+k_2+k_3)\,\mathcal{B}(k_1,k_2,k_3)\,, \tag{5.38}$$

or the "effective" $\tilde{f}_{\rm NL}$,

$$\tilde{f}_{\rm NL}(k_1,k_2,k_3) = -\frac{10}{3}\frac{k_1k_2k_3}{k_1^3+k_2^3+k_3^3}\,\frac{\mathcal{G}(k_1,k_2,k_3)}{k_1k_2k_3}\ . \tag{5.39}$$

Since $10k_1k_2k_3/3\sum_i k_i^3$ is roughly of $\mathcal{O}(1)$, the two quantities are of the same order [135]. In our case, we find:

$$\begin{aligned}\frac{\mathcal{G}(k_1,k_2,k_3)}{k_1k_2k_3} \;&=\; \frac{1}{4}\,\epsilon_{step}\,\mathcal{D}\left(\frac{K\tau_f}{2\beta}\right)\left[\left(\frac{k_1^2+k_2^2+k_3^2}{k_1k_2k_3\,\tau_f}-K\tau_f\right)K\tau_f\cos(K\tau_f)-\right.\\ &\left.-\left(\frac{k_1^2+k_2^2+k_3^2}{k_1k_2k_3\,\tau_f}-\frac{\sum_{i\neq j}k_i^2k_j}{k_1k_2k_3}K\tau_f\right)\sin(K\tau_f)\right]\ ,\end{aligned} \tag{5.40}$$

where $K = k_1+k_2+k_3$.

Consistency Relation

It is well known that the bispectrum of curvature perturbations in single-field inflationary models satisfies a consistency relation which relates its squeezed limit to the slope of the power spectrum [34, 109, 164, 165] under very general assumptions: [5]

$$\lim_{k_L\to 0}\mathcal{B}(k_L,k_S,k_S) = -P_{\mathcal{R}}(k_L)P_{\mathcal{R}}(k_S)\left[(n_s-1)+\mathcal{O}\left(\frac{k_L^2}{k_S^2}\right)\right]\ . \tag{5.41}$$

In practice, eq. (5.41), which is an expansion in powers of k_L/k_S, tells us that the local physics is unaffected by long-wavelength modes that, being larger than the horizon, cannot be distinguished from a rescaling of the background. This provides us with a powerful check of our results. With our notation (5.37), eq. (5.41) becomes [139]

$$\lim_{k_L\to 0}\frac{\mathcal{G}(k_L,k_S,k_S)}{k_S^3} \simeq -\frac{1}{4}\frac{\mathrm{d}\ln\mathcal{P}_{\mathcal{R}}}{\mathrm{d}\ln k}\bigg|_{k_S} \simeq \epsilon_{step}\beta\left(\frac{x}{\beta}\right)\sin(2x)\,\mathcal{D}\left(\frac{x}{\beta}\right)\ . \tag{5.42}$$

The last equalities comes from the derivative of the power-spectrum (5.23), neglecting terms of order $\mathcal{O}(1/\beta)$, i.e. we ignore the variation of the envelope $\mathcal{D}$. For the squeezed

[5] See [166] and refs. therein for a detailed discussion of the conditions under which one can derive the consistency relation and for those cases where one can evade it. See also refs. [167, 168].

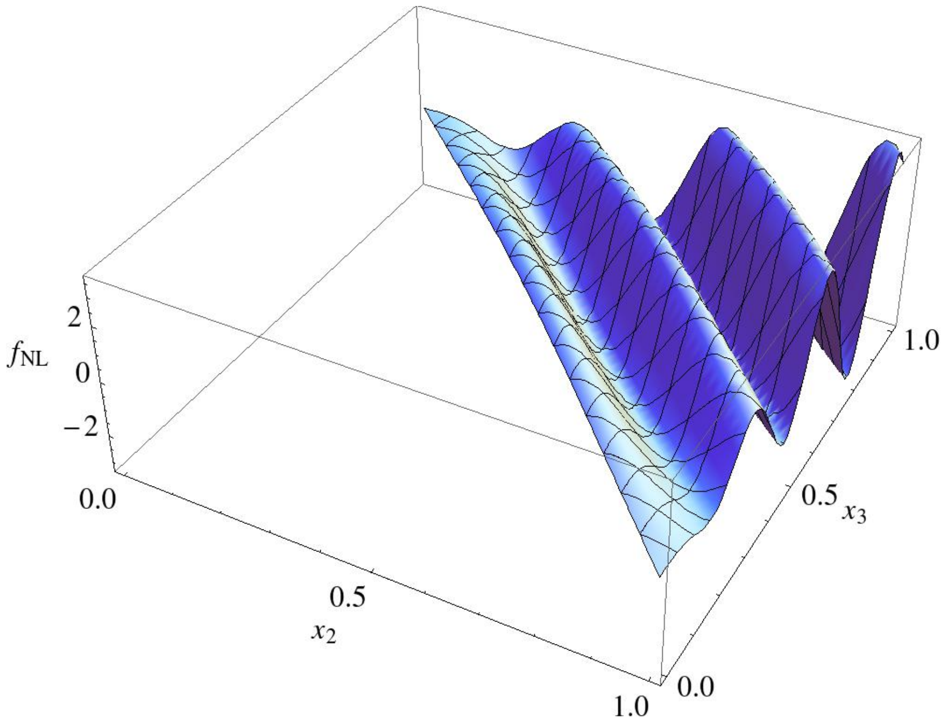


Figure 5.3: $\tilde{f}_{\rm NL}(k_1, k_2, k_3)$ for the bispectrum (5.40) for a hyperbolic tangent step (5.29) as function of $x_2 = k_2/k_1$ and $x_3 = k_3/k_1$. For illustration purposes we have fixed $k_1 = 1$ and chosen the values $\epsilon_{step} = 0.001$, $\beta = 43\,\pi$ and $\ln(-\tau_f) = 3$ for the parameters.

bispectrum, taking the limit $(\boldsymbol{k}_2 - \boldsymbol{k}_3)/2 = \boldsymbol{k}_S$, $\boldsymbol{k}_1 = \boldsymbol{k}_L \to 0$ of eq. (5.40) and focusing on the dominant term, we find

$$\lim_{k_L \to 0} \frac{\mathcal{G}(k_L, k_S, k_S)}{k_S^3} \simeq \epsilon_{step}\beta \left(\frac{k_S \tau_f}{\beta}\right) \sin(2k_S\tau_f)\, \mathcal{D}\left(\frac{k_S\tau_f}{\beta}\right) \,, \tag{5.43}$$

which therefore satisfies the consistency relation. It is important to notice that, for this kind of models, the consistency relation holds only for "very" squeezed triangles, that is to say, here it is not enough to require $k_L/k_S \ll 1$. The point is that when we assume that the only effect of the frozen super-horizon mode on the short wavelength one is a constant background rescaling, we are assuming that there are no interactions between modes when they are all within the horizon. This is not our case. Here the expansion in k_L/k_S will work only when k_L is sufficiently small that the mode is already frozen while the short ones are not yet perturbed by the occurrence of the step feature. To derive a bound on k_L/k_S, one can estimates the contribution of the non-Bunch-Davies state to the total energy density and require that it leaves the background evolution unaltered [169–171].

Equilateral Limit and Scaling

Notice that in the sharp-feature case, $\beta \gg 1$, the dominant contribution in eq. (5.40) comes from the terms with the steepest scaling with $K\tau_f$ and near the equilateral limit (as it can be seen for example in Figure 5.3), where all the momenta are of the same magnitude [139, 159]. Then we can approximate

$$\frac{\mathcal{G}(k_1,k_2,k_3)}{k_1k_2k_3} \simeq -\epsilon_{step}\beta^2\left(\frac{K\tau_f}{2\beta}\right)^2\cos(K\tau_f)\,\mathcal{D}\left(\frac{K\tau_f}{2\beta}\right)\,. \tag{5.44}$$

Focusing on the envelope only, we can clearly see that we have a maximum contribution for non-Gaussianity at a scale $K_{peak} \simeq 2\beta/\tau_f$, which implies: [6]

$$f_{\rm NL}\Big|_{peak} \sim \epsilon_{step}\beta^2\,. \tag{5.45}$$

It is clear however that the bispectrum for these models is strongly scale-dependent both for the oscillating behavior and the envelope. Then, for arbitrary choice of the parameters, the parameter $f_{\rm NL}(k)$ can change by several orders of magnitude from a scale to another. This means that an overall amplitude of the oscillations cannot be defined. We argue then that the ansatz proposed in the papers [135, 136],

$$f_{\rm NL}^{feat}\sin\left(\frac{K}{k_c}+\phi\right)\,, \tag{5.46}$$

does not capture the main characteristics of the bispectra of models with very sharp features (as noted also in [172]), if the right damping envelope is not considered. This approximation loses all the information about the sharpness of the feature, which actually sets the scale at which modes are most affected. The sharper the feature, the more inside the horizon non-Gaussianity is produced. Notice also that the ansatz (5.46) does not reproduce the correct physical behavior in the limits $K\tau_f \gg 1$ and $K\tau_f \ll 1$, since in both cases it does not vanish automatically. One could solve these problems by hand, multiplying the ansatz (5.46) by a suitable damping envelope [173], at the price of introducing new unknown parameters, where "suitable" means that it must reproduce the correct scaling of eq. (5.44), with a peak at $K\tau_f \simeq \beta$ and a maximum amplitude given by eq. (5.45). This however will not reproduce correctly the asymmetric behavior of the envelope (5.44), which first grows as K^2 and then decays exponentially fast.[7] Moreover, in the limit of an infinitely sharp step, which would correspond to have a very wide damping, we would obtain again oscillations with a constant amplitude, while, as it has already been noticed in [139], one should obtain a quadratic divergence in momenta

[6]Notice that the exact numerical factor in front of eq. (5.45) is model-dependent, as it depends on the normalization of the function $x^2\mathcal{D}(x)$, and hence on the form of the step.

[7]Different scaling with K are possible if the sharp feature is not a step but, e.g., a kink [174].

space. This behavior is easily understood: the parameter δ (5.10) in the limit $\beta \gg 1$ is a Dirac-delta function and its only effect in the integral (5.33) is to replace every τ with τ_f, without any damping coming into play. However, as we will see in a momentthe next Chapter, this limit can not be taken exactly, if we want to remain in a perturbative regime.

5.2 Generalizations

Beside features in the inflaton scalar potential, it could be interesting to study possible features, for example, in the speed of sound [151–159]. The effective field theory of inflation is the simplest setup for such a study, as it will be a generalization of features in the ϵ slow-roll parameter to other coefficients in the effective Lagrangian (4.30). This can be realized by simply "switching on" the coefficients of higher-order operators that we previously neglected. The coefficients of these new interactions could then be provided with time dependences in the form of step features, in the same way as we did for the Hubble parameter.

As we saw in the previous sections, if we allow for a time dependence of the ϵ parameter (5.3), we must also require that its deviation from a constant $\epsilon_0 \ll 1$ is small in order not to spoil inflation. This requirement is also necessary to obtain an approximate scale-invariant power-spectrum of curvature perturbations. In the spirit of the EFTI, a natural explanation is the presence of an approximate shift symmetry of the Goldstone boson π, that guarantees that the terms of the Taylor expansion (4.36) are all sub-leading with respect to the zeroth-order terms. This conclusion applies to *every* coefficient in the effective action and implies that every term in the Taylor expansion is completely negligible, including the ones coming from expanding $\dot{H}$. The results of the previous section, however, tell us that we can still have contributions from the expansion in π, if the time dependence of the Hubble parameter assumes a particular form like the one in (5.2). This effect will still give us an approximate scale-invariant power-spectrum as long as the shift symmetry still approximately holds, in the sense that it is explicitly broken in a small and controlled way. Then it is conceivable that also other coefficients in the effective action (4.30) could have the same form. This justifies the generalized study of possible step features in all the parameters appearing in the effective action. Therefore, we can parametrize the time dependence of the M_n coefficients as

$$M_n(t) = M_n^{(0)} \left[1 + m_n F_n \left(\frac{t - t_f}{b_n} \right) \right] . \tag{5.47}$$

The meaning of the function F_n and the parameters m_n, t_f and b_n are the same of eq.

(5.3) and we impose $m_n \ll 1$. Now, as we saw for the case of the ϵ parameter (5.3), interesting new effects arise with operators that are proportional to the derivative of the step, analogously to eq. (5.10), in spite of the step itself, as they are proportional to the factor $1/b_n$, which can be in principle very large. Looking at the Taylor expansion (4.36), we see that the n-th derivative of every coefficient M_n, only appears together with π^n. This means that if a coefficient M_n is present for the first time in the m-th order action, its derivative will appear in the $(m+1)$-order action. As an example, consider the coefficient M_3 in the effective action (4.30), which appears at third order in front of the operator $\dot{\pi}^3$. If it had the time dependence of eq. (5.47), at third order we would see one more term in the action, which is however proportional to $m_3 \ll 1$, so that its contribution would be suppressed with respect to the standard one given by $M_3^{(0)}$. The derivative $\dot{M}_3$, proportional to $1/b_3 \gg 1$, which therefore can be large, will appear however with the operator $\pi\dot{\pi}^3$ in the fourth-order action, that is, its effects must be searched for in the trispectrum. This leads us to argue that, *at any given order $n > 2$ in the effective action, features on a parameter M_n that can be parametrized by eq.* (5.47) *give non-negligible effects only if M_n itself has already appeared in the $(n-1)$-th order action*. Looking at the action of the EFTI and listing all the terms of the second-order action [175], one can see that only the coefficients $\dot{H}$, M_2, $\bar{M}_1$, $\bar{M}_2$, $\bar{M}_3$ are present: this means that only by adding a feature to these coefficients we could hope to see some feature-related effects at the level of the bispectrum. In practice, we obtain that, neglecting the extrinsic curvature terms, $\bar{M}_2$, $\bar{M}_3$, the only interesting effects in the bispectrum can come from features in the Hubble parameter or in the speed of sound.[8]

5.2.1 *Features in the Speed of Sound*

Focusing for simplicity only on the coefficient $M_2^4(t)$, we can easily see, from (4.30), that we get a coefficient in front of the time kinetic term which is different from the spatial kinetic one. In other words, we have a (time-dependent) speed of sound

$$c_s^2(t) = \frac{-M_{Pl}^2 \dot{H}(t)}{-M_{Pl}^2 \dot{H}(t) + 2M_2^4(t)} \,. \tag{5.48}$$

Then, it is clear that if we do not neglect the time evolution of the coefficients, we obtain also a time variation of the speed of sound.[9] Here we want to make the example of a

[8] Although also $\bar{M}_1$ is curvature-generated, the corresponding operator is a standard kinetic term and the parameter can be rewritten as an effective speed of sound for the perturbations [175, 176]. In this thesis, we shall not treat the case of non-vanishing $\bar{M}_2$, $\bar{M}_3$ and leave its study to a future work.

[9] Notice that even if we do not allow for the time evolution of the coefficient M_2^4, we still obtain a time-dependent speed of sound because of the time-varying Hubble parameter (5.2).

step feature in $M_2^4(t)$,

$$M_2^4(t) = M_{2,0}^4(t)\left[1 + \sigma_{step}(t) F\left(\frac{t - t_f}{b_s}\right)\right] , \tag{5.49}$$

where, as in the case of features in the Hubble slow-roll parameters, we could in principle allow for a mild time dependence of the zeroth-order parameters. Inserting (5.49) into (5.48), at first order in the parameter σ_{step} we find

$$c_{s,0}^2(t) = c_{s,0}^2\left[1 - \sigma_{step} F\left(\frac{t - t_f}{b_s}\right)\right] , \tag{5.50}$$

where $1/c_{s,0}^2 = 1 - 2M_{2,0}^2/M_{Pl}^2\dot{H}$. Notice that although the parameter σ_{step}, b_s, t_f and the step function $F(x)$ have similar physical interpretation as the ones in eq. (5.2), in principle they could be totally different. For the reasons we have already discussed, we must require that the time variation is small, namely $|\sigma_{step}| \ll 1$. This allows us to expand quantities in the parameter σ_{step} as in eq. (5.4). Now we can define a "slow-roll" parameter,

$$\sigma = \frac{\mathrm{d}\ln c_s}{\mathrm{d}\ln\tau} = -\frac{\dot{c}_s}{c_s H} , \tag{5.51}$$

which controls the time evolution of the sound speed. If we expand it in powers of σ_{step}, at first order we have

$$\sigma_1 \simeq \frac{1}{2}\sigma_{step}\beta_s F'\left(-\beta_s \ln\frac{\tau}{\tau_f}\right) , \tag{5.52}$$

where we have switched to conformal time. The important point here is that this expression is formally equal to the one found for the δ parameter in eq. (5.10).

Power Spectrum

Following the same steps of the previous sections, in order to study the effects of sharp features in the power-spectrum we start from the second-order action

$$S_2 = \int \mathrm{d}^4x a^3\left[-M_{Pl}^2\dot{H}\left(\frac{\dot{\pi}^2}{c_s^2} - \frac{(\nabla\pi)^2}{a^2}\right) + 3\frac{M_{Pl}^2\dot{H}^2}{c_s^2}\pi^2\right] . \tag{5.53}$$

The equation of motion for the Goldstone boson π reads

$$\ddot{\pi} + \left(3H + \frac{\ddot{H}}{\dot{H}} - \frac{2\dot{c}_s}{c_s}\right)\dot{\pi} - c_s^2\frac{\nabla^2\pi}{a^2} = \ddot{\pi} + H\left(3 - 2\delta + 2\sigma\right)\dot{\pi} - c_s^2\frac{\nabla^2\pi}{a^2} = 0 , \tag{5.54}$$

where we have neglected a slow-roll-suppressed term. Equation (5.54) is formally identical to eq. (5.12) and the parameter σ defined in (5.51) enters in the same place as δ (5.5). Both the parameters have also the same form ((5.10), (5.52)) at first order in the parameters σ_{step} and ϵ_{step}, therefore the main effect on the power-spectrum will be similar. Setting $\delta = 0$ to focus only on the effects of c_s, we can follow the same steps we have followed for the case of a feature in the ϵ parameter. Switching to conformal time and defining the variable

$$z^2 = -2a^2 M_{Pl}^2 \dot{H}/c_s^2 \,, \tag{5.55}$$

we find an action in the form of eq. (5.13). Now the second derivative of z contains the parameter

$$\frac{z''}{z} \supset \frac{\dot{\sigma}}{H} = -\frac{\mathrm{d}\sigma}{\mathrm{d}\ln\tau} \,, \tag{5.56}$$

which gives the dominant contribution in the case of a sharp step, being proportional to β_s^2. As it can be easily understood, at this point it is straightforward to write the expression of the power-spectrum at leading order in σ_{step}:

$$\mathcal{P}_{\mathcal{R}} = \mathcal{P}_{\mathcal{R},0}\left[1 - \frac{2}{3}\sigma_{step}\,\mathcal{D}\left(\frac{ks_f}{\beta_s}\right) W'(ks_f)\right] \,, \tag{5.57}$$

where we have used the variable [177]

$$s = \int \frac{c_s \mathrm{d}t}{a} \,, \tag{5.58}$$

so that s_f correspond to the time of the feature, $W'(ks_f)$ is the same oscillating function of eq. (5.22), $\mathcal{P}_{\mathcal{R},0}$ is the standard power-spectrum in the presence of a constant $c_s^2 \neq 1$:

$$\mathcal{P}_{\mathcal{R},0} = \frac{H^2}{8\pi^2\,\epsilon\, c_{s,0} M_{Pl}^2} \,. \tag{5.59}$$

Again, the damping function $\mathcal{D}$ is nothing else than the Fourier transform of the step itself. We can see, as already noticed in a previous paper [158] for DBI models, that very small and very sharp steps in the scalar potential or in the speed of sound have strongly degenerate effects on the power-spectrum, as both produce damped oscillations. If we want to break this degeneracy between the two physically different situations, we have to consider the effects on the bispectrum.

Bispectrum

In order to find the effects of the step in the speed of sound, we should consider the action (4.30) up to third order in π, after Taylor-expanding the coefficients of the various operators. If we focus only on $c_s^2(t+\pi)$, we see that at first order in σ_{step}, we get two new operators, namely,

$$-\frac{M_{Pl}^2 \dot{H}}{c_{s,0}^2}\,\sigma_{step} F_s\left(-\beta_s \ln\frac{\tau}{\tau_f}\right)\dot{\pi}\left(\dot{\pi}^2 - \frac{(\nabla\pi)^2}{a^2}\right) \,, \tag{5.60}$$

$$-\frac{2M_{Pl}^2\dot{H}}{c_{s,0}^2}H\sigma\,\pi\dot{\pi}^2\,, \tag{5.61}$$

where σ is given by eq. (5.52). Notice that the first of them is just the standard operator present in the EFTI with speed of sound [109], times the step function F_s and the parameter σ_{step}. It is clear then that the non-Gaussianity produced by this operator will be suppressed by $\sigma_{step} \ll 1$ with respect to the standard one, which scales as $f_{\rm NL} \sim 1/c_s^2$. The other operator instead is proportional also to β_s, which is very large in the case of a sharp step. To find the corresponding bispectrum, we use the in-in formalism, as in eq. (5.33). For the leading order result we only need the zeroth-order mode function,

$$\pi_k^{(0)}(s) = \frac{i}{M_{Pl}\sqrt{4\epsilon c_{s,0}k^3}}\,(1+iks)\,e^{-iks}\,. \tag{5.62}$$

and the linear relation (4.38). The calculation proceeds along the same path we followed in the case of steps in the Hubble parameter and the result assumes a similar form

$$\begin{aligned}\frac{\mathcal{G}(k_1,k_2,k_3)}{k_1k_2k_3} \;=\; & \frac{1}{4}\sigma_{step}\mathcal{D}\left(\frac{(k_1+k_2+k_3)s_f}{2\beta}\right)\Bigg[-2\sum_{i\neq j}k_ik_js_f^2\cos\big((k_1+k_2+k_3)s_f\big)+\\ & +\frac{\sum_{i\neq j}k_i^2k_j}{k_1k_2k_3}\sin\big((k_1+k_2+k_3)s_f\big)\Bigg]\,.\end{aligned} \tag{5.63}$$

The damping function has the same meaning and properties as the damping that we have already seen, as it arises from the same kind of integrals. Comparing eqs. (5.40) and (5.63), we see that, although very similar, the two bispectra can be in principle distinguished both for the different frequency of the oscillations and for the different combination of momenta k_1, k_2 and k_3. As an example, we show in Figure 5.4 the behaviour of the two bispectra in the equilateral limit, $k_1 = k_2 = k_3 = k$, in the case of very small steps in the speed of sound, $\sigma_{step} \ll 1$, where the variable s can be approximated with $s \simeq c_{s,0}\tau$. Looking at the profiles of the oscillations, we see that both peak at a scale x_{peak}, corresponding to the value $y = 1$, where y is the argument of the damping function $\mathcal{D}(y)$. Then, due to the presence of a factor c_s, we have that the first profile peak at x_{peak}, while the second at $x_{peak}^{c_s} = x_{peak}/c_s$, which is bigger than x_{peak} as long as $c_s < 1$. Therefore we expect that the two physically different cases are well distinguishable as we move from $c_s = 1$ to smaller values. This conclusion is also reinforced by the non-negligible presence of the characteristic operators of the small speed of sound scenario, namely the operators proportional to $(1 - 1/c_s^2)$ in the effective action of eq. (4.30)[10]. Moreover, as we will see in the next subsection, we should also consider now the correction to the mode functions that we previously neglected.

[10] The bispectrum of the operators $\dot{\pi}^3$ and $\dot{\pi}(\nabla\pi)^2$ gives just the well-known results for an inflaton with a non-standard kinetic term (see for example [135]), since correcting the coefficient $(1 - 1/c_s^2)$ with its step-like evolution will give only the σ_{step}-suppressed operator (5.60), which is negligible at first order.

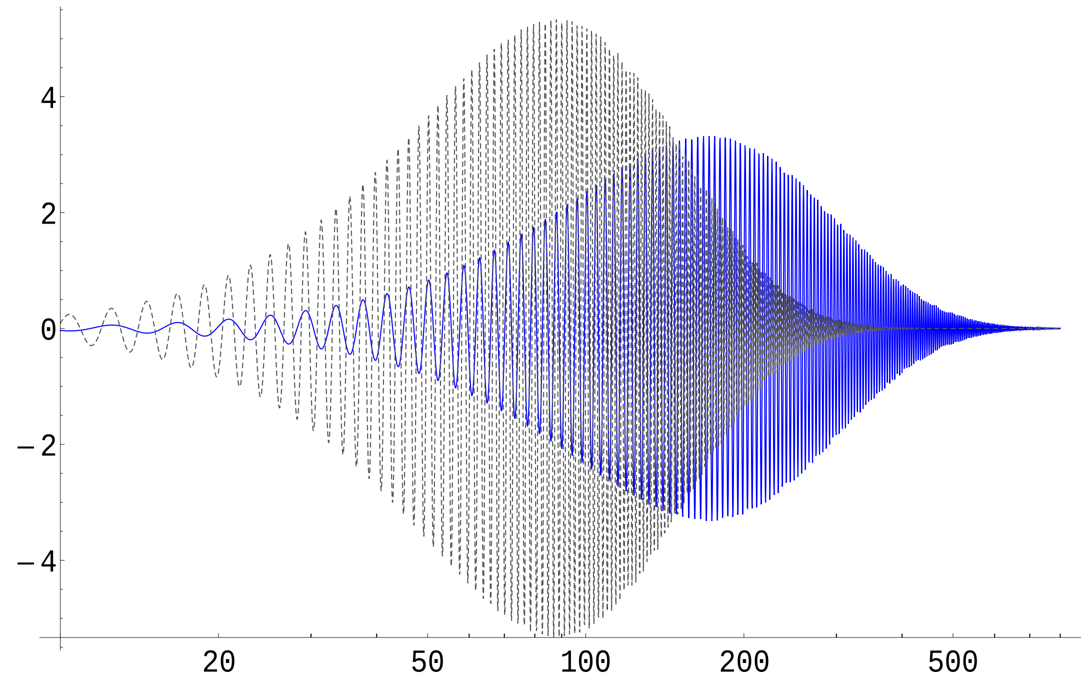


Figure 5.4: As an example, we plot the equilateral limit of the bispectra (5.40) (blue) and (5.63) (dashed black) as functions of $x = -k\tau_f$ in the case of a hyperbolic tangent step (5.29). The choice of the parameters are $\epsilon_{step} = \sigma_{step} = 0.001$, $\beta = \beta_s = 43\pi$, $c_s = 0.5$ for illustration purposes.

5.2.2 Accounting for a non-Bunch-Davies Wave Function: Folded Shape

Another interesting source of features in the bispectrum comes from the correction to the classical Bunch-Davies mode (see Appendix A). So far, we have considered only the standard Bunch-Davies mode (5.34) in the computation (5.33), as deviations enter with a factor ϵ_{step}. Thus, as the dominant cubic operators are already proportional to ϵ_{step}, the contribution would be suppressed. However, when considering for example speeds of sound different from one, we have also cubic operators which are zeroth-order in ϵ_{step}. As they are enhanced by c_s^{-2} in the case of small speed of sound, the effects of a non-Bunch-Davies wave function due to the presence of features can become relevant, as noted in [159]. This holds even more in general for every operator in the effective action which is zeroth-order in the parameter that controls the deviation from Bunch-Davies and happens both if we have features in the slow-roll parameters or in the speed of sound. In the presence of large interactions, these contributions to the non-Gaussianity can have a comparable size with the previously considered case. The main characteristic of this kind of non-Gaussianity is its enhancement in the folded triangle limit due to the presence of the negative-frequency mode.

In order to see how this mechanism works, we will compute the bispectrum arising from the operator $\dot{\pi}^3$ in the effective action (4.30) in the case of a sharp step in the slow-roll parameter ϵ and a constant speed of sound $c_s^2 < 1$. As we saw, the second order action gives us the equation of motion (5.54), where the non-negligible effect of the parameter δ results in a modification of the standard mode function (5.62). This is indeed the physical interpretation of the oscillations in the power-spectrum: the feature excites a non-Bunch-Davies component with negative frequency [135, 151, 173]. The contribution of this modification to the wave function in the calculation of the three-points functions at first order in ϵ_{step} is obtained substituting one of the three positive-frequency mode which enter the integration in the in-in formalism with a negative-frequency one, $u_- \sim e^{-ix}$ (and summing over the different possible choices of this negative-frequency mode):

$$\begin{aligned}\langle \pi_{k_1}\pi_{k_2}\pi_{k_3}\rangle &= (2\pi)^3\delta^3\left(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3\right)\left[\prod_{i=1}^{3}\frac{-i}{4M_{Pl}^2\epsilon c_s k_i^2}\right]\frac{1}{H}\times \\ &\quad\times\int_{-\infty}^{0}\frac{\mathrm{d}\tau}{\tau}\left[\frac{\mathrm{d}}{\mathrm{d}\tau}\left(\tau y^{NB}(-k_1\tau)^*\right)\frac{\mathrm{d}}{\mathrm{d}\tau}\left(\tau y(-k_2\tau)^*\right)\frac{\mathrm{d}}{\mathrm{d}\tau}\left(\tau y(-k_3\tau)^*\right)\right]+ \\ &\quad+\text{c.c}+\text{perm.}+\text{other choices of } y^{NB}\,, \qquad (5.64)\end{aligned}$$

where we used the dimensionless variable $y = \sqrt{2k}u_k = z\sqrt{2k}\,\pi_k$ and z is given by eq. (5.55). The superscript "NB" refers to the negative-frequency contribution. At first order in ϵ_{step} it can be computed solving the equation of motion (5.54) through the Green's Function technique [159, 162, 177]:

$$y_{NB}(-k\tau) = -iy_0^*(-k\tau)\int_{-\infty}^{\tau}\frac{\mathrm{d}\tau'}{\tau'}\left(1-\frac{i}{c_sk\tau'}\right)^2\frac{e^{-2ic_sk\tau'}}{2c_sk\tau'}\frac{\mathrm{d}\delta}{\mathrm{d}\ln\tau'}\,, \qquad (5.65)$$

where y_0 is given by eq. (5.35). After some lengthy algebra and an integration by parts, we are left in eq. (5.64) with the evaluation of integrals similar to those of the previous sections, where an oscillating exponential multiplies a polynomial in $k\tau$. Using again the same technique for the damping functions, we end up again with a bispectrum in the form of a oscillating function times a damping envelope. Instead of the full result, it is easier to focus only on the dominating factor, proportional to τ_f,

$$\begin{aligned}\frac{\mathcal{G}(k_1,k_2,k_3)}{k_1k_2k_3} &= \epsilon_{step}\left(1-\frac{1}{c_s^2}\right)\left[\frac{3}{k_1k_2k_3}\frac{\sum_{i\neq j}k_i^4k_j^2-2\sum_{i\neq j}k_i^3k_j^3-3k_1^2k_2^2k_3^2}{(k_1-k_2-k_3)(k_1-k_2+k_3)(k_1+k_2-k_3)}\right]\times \\ &\quad\times\mathcal{D}\left(\frac{Kc_s\tau_f}{2\beta}\right)Kc_s\tau_f\cos\left(Kc_s\tau_f\right)\,. \qquad (5.66)\end{aligned}$$

As it can be seen from Figure 5.5, the bispectrum (5.66) peaks in the folded limit $k_1 \to k_2+k_3$, as it should, given the negative-frequency correction to the mode functions,

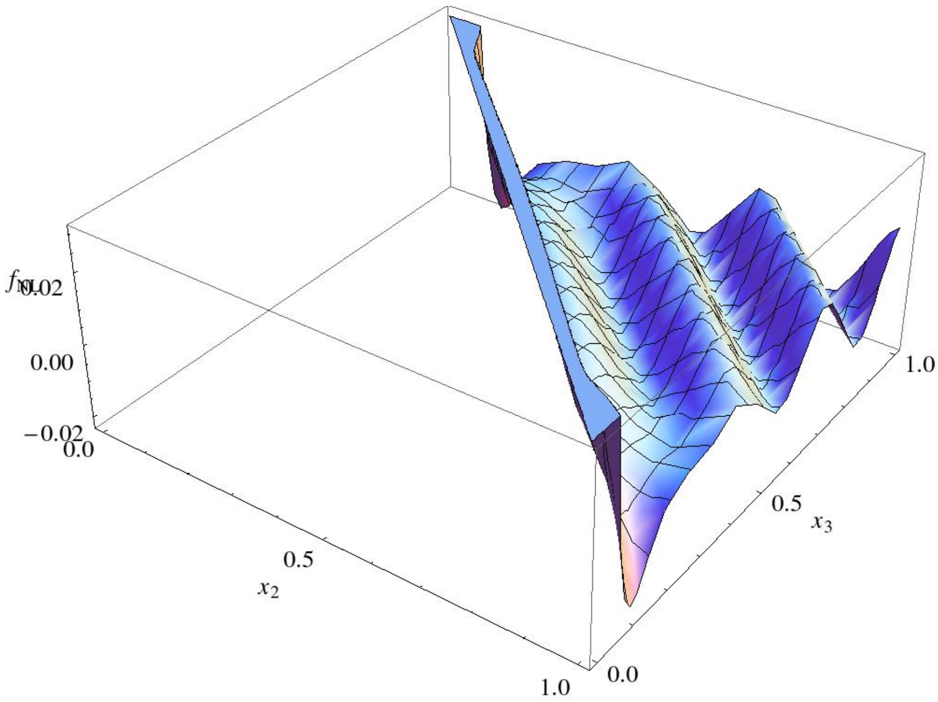


Figure 5.5: $\tilde{f}_{\rm NL}$ for the bispectrum (5.64) for a hyperbolic tangent step (5.29) as function of $x_2 = k_2/k_1$ and $x_3 = k_3/k_1$. For illustration purposes we have fixed $k_1 = 1$ and chosen the values $c_s = 0.4$, $\epsilon_{step} = 0.001$, $\beta = 43\,\pi$ and $\ln(-\tau_f) = 4$ for the parameters.

and has superimposed oscillations similar to those found for resonant models [161, 173]. To see the running of this bispectrum in the folded limit, one has to go back to eq. (5.64) and take $k_1 = k$, $k_2 \to k/2$, $k_3 \to k/2$. Focusing on the dominant contribution, that is with the steepest scaling with $x = -k\tau$, we find

$$\left.\frac{\mathcal{G}(k_1,k_2,k_3)}{k_1k_2k_3}\right|_{\rm folded} = -\frac{1}{2}\epsilon_{step}\left(1-\frac{1}{c_s^2}\right)\mathcal{D}\left(\frac{c_s x}{\beta_s}\right)c_s^2x^2\sin(2c_sx)\;. \tag{5.67}$$

Here, the maximum is at a scale $c_sk\tau_f \sim \beta$ and reach the value

$$\left.f_{\rm NL}^{folded}\right|_{peak} \sim \epsilon_{step}\beta^2\left(1-\frac{1}{c_s^2}\right)\;. \tag{5.68}$$

This differs significantly from the maximum non-Gaussianity estimated in eq. (5.45), where the speed of sound was exactly $c_s = 1$. Now, in the folded limit and at the scale k where the bispectrum peaks, we receive a further enhancement proportional to $1/c_s^2$. As it has been noted for resonant models [173], also for feature models the folded bispectrum can be less, equally or more important than the feature bispectrum depending on the values of the parameters.

CHAPTER 6

Perturbative Unitarity of Inflationary Models with Features

In the previous chapter we have studied models with features using the approach of the EFTI and we have derived predictions for the power-spectrum and bispectrum for models where a feature is present in the slow-roll parameter ϵ or in the speed of sound c_s. One of the advantages of this approach is that it makes the regime of validity of the theory more clear. Indeed large non-linear interactions can spoil the consistency of the theory, introducing strong couplings and losing perturbativity (see Section 4.4). Therefore it is very important to consider the energy scale of the modes most affected by interactions sourced by features and check that the strong coupling scale is higher than the relevant energy scale of the problem.

This Chapter, which is based on [2], addresses this problem. We considered the perturbative consistency of inflationary models with features by means of effective field theory methods. By estimating the size of loop contributions to the n-point functions and comparing them with the tree-level computation, one can identify the maximal energy scale at which the theory is unitary and perturbativity is safe. Then, by requiring that all the relevant energy scales of the physics we are interested in are below this UV cutoff, we can derive bounds on the parameters of the models. While in the standard slow-roll models of inflation, the only relevant energy scale is the Hubble parameter H, when features are present there is a new energy scale $E \simeq 1/\Delta t$ corresponding to the inverse of the characteristic time-scale of the interaction. In the case of feature models, that we have studied here, we estimated the size of one-loop contributions to the three-point functions and compared them to the tree-level expectation. Our main result is that there is a very strong upper bound on the sharpness $\beta = 1/\Delta t H$ of the feature beyond which the unitarity of the theory is lost. This constraint can be interesting even from the observational point of view: indeed, this bound can be used to compare the ratio of the signal-to-noise ratio for the three-point function to the one of the two-point function. Our result is that, within the range of validity of the effective approach, the two-point function has the highest signal-to-noise ratio, unless the height of the step is extremely small. However, as the amplitude of the bispectrum is proportional to the height of the step itself, in this case we expect at the same time a smaller value of non-Gaussianity. This suggests us that if a future experiment will show a statistically significant detection of feature effects in the bispectrum without an even more significant detection in the power-spectrum, the result would be difficult to explain only in the frame of models with features in the inflaton potential.

6.1 Energy Scales and Unitarity

The validity of the perturbative treatment one commonly uses relies on the assumption that higher-order contributions are small. This is what is done for example when one computes the equations of motion truncating the action at second order: it is implicitly assumed that the third-order contribution $\mathcal{L}_3$, for example, is small compared to the quadratic Lagrangian $\mathcal{L}_2$. To confirm that assumption, then one should check that $\mathcal{L}_3/\mathcal{L}_2 \ll 1$ in the relevant energy scales of the problem, so that the theory is perturbatively safe. In the standard cases, the only relevant energy scale is H, where fluctuations are crossing the horizon, so the bound is taken at $E \sim H$. However, for inflationary models with features (or resonances [178–181]), this should be required also for the scale where the largest interaction happens [1, 161], which corresponds to the inverse of the relevant time-scale b of the feature (or the resonance). In the case of inflationary models with features, we should make sure that $\mathcal{L}_3/\mathcal{L}_2 \ll 1$ is valid even in the worst possible case i.e. at the time of the feature t_f, when the interaction is maximized. As we have seen, the sharper the feature, the more inside the horizon large interactions among the modes are effective. The point is that the ratio $\mathcal{L}_3/\mathcal{L}_2$ depends on the energy scale [161]. Using the form of the third-order action (5.32), we find

$$\left.\frac{\mathcal{L}_3}{\mathcal{L}_2}\right|_E = \left.\frac{\ddot{H}}{\dot{H}}\pi\right|_E . \tag{6.1}$$

Now we use the fact that π_E at an energy scale E is related to π_H at Hubble, and hence to $\mathcal{R}$ by

$$\pi_E \sim \frac{E}{H}\pi_H \sim \frac{E}{H^2}\mathcal{R} . \tag{6.2}$$

Moreover, we know from eq. (5.2) the scaling for the time derivatives of the Hubble parameter H:

$$\dot{H} \sim \epsilon H^2 \,, \tag{6.3}$$

$$H^{(n)} \sim \epsilon\, \epsilon_{step} \beta^{n-1} H^{n+1} F\left(\frac{t-t_f}{b}\right) \,. \tag{6.4}$$

As the largest interactions happens when the inflaton is crossing the feature, we shall take $t = t_f$. In the case of a sharp feature, the modes which are most affected are inside horizon, $k\tau_f \sim k/a_f H \sim \beta$, and hence they have an energy proportional to the inverse of the characteristic time of the feature $1/b = \beta H$. Substituting it into eq. (6.1) and using eqs. (6.2), (6.3), (6.4), we find

$$\left.\frac{\mathcal{L}_3}{\mathcal{L}_2}\right|_{E\sim\beta H} \sim \epsilon_{step} \beta^2 \mathcal{R} \,, \tag{6.5}$$

which is indeed proportional to the $f_{NL}\big|_{peak}$ of eq. (5.45). Given that, one can find [1]

$$\left.\frac{\mathcal{L}_3}{\mathcal{L}_2}\right|_{E\sim\beta H} \ll 1 \qquad \Longrightarrow \qquad \beta^2 \lesssim \frac{1}{\epsilon_{step} \mathcal{P}_{\mathcal{R},0}^{1/2}} \,, \tag{6.6}$$

where $P_{\mathcal{R},0}$ is the standard power spectrum at zeroth order.

However we should check also that higher-order contributions from $\mathcal{L}_n$ satisfy a similar bound. In order to do this, notice that the most important interaction in the Lagrangian at nth-order (which comes from the Taylor expansion (4.36) of the term $\dot{H}(t+\pi)$ in the effective action [161]), parametrically scales as

$$\mathcal{L}_n \sim M_{Pl}^2 H^{(n-1)} \pi^{n-2} \dot{\pi}^2 \,, \tag{6.7}$$

while

$$\dot{H} \sim \epsilon H^2 \,, \tag{6.8}$$

$$H^{(n)} \sim \epsilon\, \epsilon_{step} \beta^{n-1} H^{n+1} \,. \tag{6.9}$$

Our perturbative expansion is then safe if:

$$\left.\frac{\mathcal{L}_n}{\mathcal{L}_2}\right|_{E\sim\beta H} \sim \epsilon_{step} \beta^{2n-4} \mathcal{R}^{n-2} \ll 1 \,, \tag{6.10}$$

which implies

$$\beta^2 \lesssim \frac{\mathcal{P}_{\mathcal{R},0}^{-1/2}}{\epsilon_{step}^{1/(n-2)}} \overset{n\gg 1}{\lesssim} \mathcal{P}_{\mathcal{R},0}^{-1/2} \,, \tag{6.11}$$

where in the last step we take the limit for $n \to \infty$. This simple argument then suggests that we should take $\beta^2 \lesssim \mathcal{P}_{\mathcal{R},0}^{-1/2}$ if we do not want higher-order corrections to threat perturbativity. An important thing to note here is that, being inside the horizon, our theory is a quantum theory, so the violation of (6.10) is signaling an actual quantum-mechanical strong coupling (in the sense that quantum loops are not suppressed), so that unitarity is lost and the model is not under control [109, 119, 182, 183]. In order to state the problem more rigorously, we will estimate the amplitudes of one-loop contributions to the three-point function and compare them to the tree-level amplitudes[1].

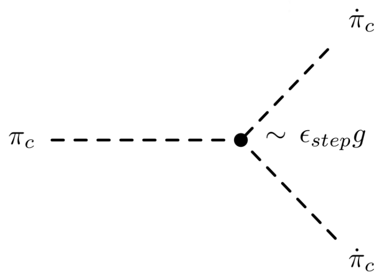
Consider the cubic operator,

$$\mathcal{L}_3 \ni M_{Pl}^2\, \ddot{H} \left(\frac{t-t_f}{b}\right) \pi\dot{\pi}^2 \,, \tag{6.12}$$

at the time of the feature, t_f, where the interaction is maximal. Upon canonical normalization, $(-2M_{Pl}^2\dot{H})^{-1/2}\pi = \pi_c$, and using (6.4), we have:

$$\frac{1}{2}\frac{\epsilon_{step}\beta}{M_{Pl}\sqrt{2\epsilon}}\pi_c\dot{\pi}_c^2 = \epsilon_{step}\, g\, \pi_c\dot{\pi}_c^2 \,. \tag{6.13}$$

Notice that, as the operator $\pi\dot{\pi}^2$ has mass-energy dimension E^5, the coupling g in front of it has dimension $1/E$. Diagrammatically, the correspongin vertex and amplitude (by dimensional analysis) are in Figure 6.1.



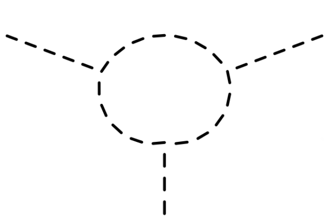
$$\mathcal{M}^{(0)} \sim \epsilon_{step}\, g\, E \tag{6.14}$$

Figure 6.1: Tree-level diagram for the interaction (6.13).

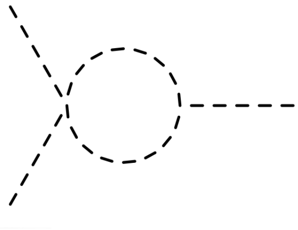
With the same simple arguments, one can see that the vertex with four πs is proportional to $\epsilon_{step}g^2$, with five πs to $\epsilon_{step}g^3$ and so on. Then we can list all the possible diagrams with three free legs and only one loop, that we show in Figure 6.2.

The list has only three diagrams, as there are no more ways to connect three free legs with only one loop. Notice also that the largest effect comes from the last diagram, where one has the lower power of ϵ_{step} and the higher power of β (as $\epsilon_{step} \lesssim 1$ and

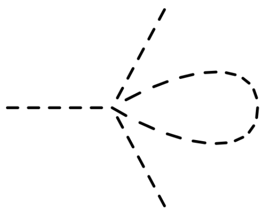
[1] Notice that one can obtain the same result considering, for example, one-loop contributions the two-point instead of the three-point function.



$$\mathcal{M}^{(1)} \sim 4\pi \, (\epsilon_{step} \, g \, E)^3 \tag{6.15}$$



$$\mathcal{M}^{(1)} \sim 4\pi \, \epsilon_{step}^2 \, (g \, E)^3 \tag{6.16}$$



$$\mathcal{M}^{(1)} \sim 4\pi \, \epsilon_{step} \, (g \, E)^3 \tag{6.17}$$

Figure 6.2: Loop diagrams with three external legs.

$\beta \gg 1$). Now, we can compare the tree-level amplitude with the loop contributions: the energy scale where the first one is comparable to the second, i.e.

$$\mathcal{M}^{(0)} \sim \mathcal{M}^{(1)} \, , \tag{6.18}$$

is to be considered as the maximum energy Λ, at which the loop expansion is under control. Beyond that, interactions become strongly coupled and the effective theory becomes non-unitary. It is easy to obtain Λ from the previous equation, using eqs. (6.14) and (6.17):

$$\Lambda^2 \simeq 16\pi \left(\frac{M_{Pl}\sqrt{2\epsilon}}{\beta} \right)^2 \, . \tag{6.19}$$

If we want to trust our predictions, we should then make sure that the energy scales we study are all below this cut-off[2]. In particular,

$$\beta H \ll \Lambda \qquad \Longrightarrow \qquad \beta^2 \ll \frac{2}{\sqrt{\pi}} \mathcal{P}_{\mathcal{R},0}^{-1/2} \, . \tag{6.20}$$

[2]The same happens for resonant models, where one requires that the frequency of the resonance is smaller than the UV cut-off of the effective theory [161].

Some comments are in order. The bound (6.20) is very strict and should be taken with care, even from an observational point of view. Indeed, from *Planck* 2013 data analysis, the best fit of the power spectrum seems to prefer very sharp features [133, 184], with $\beta \simeq 300$. However this is already out of the allowed region, as from (6.20) we have $\beta \lesssim 160$. This put serious questions on the consistency of these models for those values of β, as we have shown that problems with the unitarity of the theory then arise.

Beyond the simplest case, with no other coefficients in the action but $\dot{H}(t)$, the EFTI naturally contains higher order operators, which induce a speed of sound $c_s < 1$ and are source of non-Gaussianity. These interactions will have a new UV cutoff [109],

$$\Lambda_{c_s}^4 \simeq 16\pi^2 M_{Pl}^2 \dot{H} c_s^5 \, , \tag{6.21}$$

Then, it can be seen that there is an even stronger upper bound on β requiring βH be below this cutoff:

$$\beta^2 \lesssim c_s^2 \, \mathcal{P}_{\mathcal{R}}^{-1/2} \, . \tag{6.22}$$

This conclusion is very general and applies to every models where the slow-time dependence of the slow-roll parameters, the speed of sound or any coefficient in the effective action is broken by some temporary effects with a characteristic time scale $\Delta t = b = 1/\beta H$. Physically, this bound is just telling us that we cannot "effectively" consider features on arbitrary small time scales, as the theory of fluctuations is no more weakly coupled and perturbative unitarity is lost.

6.2 Signal to Noise Ratio

As a last step, let us make some considerations on the observability of features either in the power spectrum or in the bispectrum. As we saw, one of the most interesting characteristic of models with features is the fact that their effects in both these observables depend on the same set of parameters, which gives, in principle, the possibility to constrain them at the same time with two independent analyses. It could be interesting then to ask in which observable we should expect to see a stronger signal. To answer this question, let us estimate the signal-to-noise ratio as function of the parameters of the model. Following [139, 185], we write:

$$\left(\frac{S}{N}(\delta\langle\mathcal{R}^2\rangle)\right)^2 \simeq 2\pi \int \frac{\mathrm{d}^2 l}{(2\pi)^2} \left(\frac{\delta C_l}{C_l}\right)^2 , \tag{6.23}$$

$$\left(\frac{S}{N}(\langle\mathcal{R}^3\rangle)\right)^2 \simeq 4\pi \int \frac{\mathrm{d}^2 l_1}{(2\pi)^2} \int \frac{\mathrm{d}^2 l_2}{(2\pi)^2} \frac{B^2_{(l_1,l_2,l_3)}}{6 C_{l_1} C_{l_2} C_{l_3}} \, , \tag{6.24}$$

where

$$C_l = \frac{1}{5^2 D^2} \int \frac{\mathrm{d}k_{1||}}{2\pi} \mathcal{P}_{\mathcal{R}}(\boldsymbol{k_1}) \,, \tag{6.25}$$

$$B_{(l_1,l_2,l_3)} = -\frac{2}{5^3 D^4} \int \frac{\mathrm{d}k_{1||}}{2\pi} \int \frac{\mathrm{d}k_{2||}}{2\pi} \mathcal{B}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) \tag{6.26}$$

and $\boldsymbol{k}_{1,2} = (\boldsymbol{l}_{1,2}/D, k_{1,2||})$, $\boldsymbol{k}_3 = -\boldsymbol{k}_1 - \boldsymbol{k}_2$, D is the distance to recombination and $||$ is the direction along the line of sight. For the explicit calculation, let us focus for simplicity on the case of a feature in the ϵ slow-roll parameter in the form of hyperbolic tangent (see Section 5.1). From eqs. (5.23), (5.44) we can roughly approximate the maximal signal to noise accessible to CMB experiments in terms of ϵ_{step}, β, and τ_f [139]:

$$\left(\frac{S}{N}(\delta\langle\mathcal{R}^2\rangle)\right)^2 \simeq 2\pi\epsilon_{step}^2 \left(\frac{D}{|\tau_f|}\right) l_{max} \,, \tag{6.27}$$

$$\left(\frac{S}{N}(\langle\mathcal{R}^3\rangle)\right)^2 \simeq 480\,\epsilon_{step}^2 \left(\frac{\tau_f}{\mathrm{Gpc}}\right)^2 \left(\frac{l_{max}}{2000}\right)^4 . \tag{6.28}$$

Here l_{max} is the maximum multipole beyond which the signal-to-noise ratio saturates. This is set either by the resolution of the experiment or by the damped behaviour of our predicted observables. In fact we have seen that the amplitude of the spectrum and bispectrum is exponentially damped away for high k, which means that there is an effective maximum multipole beyond which the signal is strongly suppressed,

$$l_d \simeq \frac{2D\beta}{\pi|\tau_f|} \,. \tag{6.29}$$

Therefore we chose l_{max} to be the smallest values between the damping scale l_d and the resolution limit l_{res} that we fix at $l_{res} \simeq 2000$. Now we make the ratio between eqs. (6.27), (6.28) to compare the signals from the modifications of the two-point function and the three-point function. In the case $l_d < 2000$ we find:

$$\frac{\left(\frac{S}{N}(\langle\mathcal{R}^3\rangle)\right)}{\left(\frac{S}{N}(\delta\langle\mathcal{R}^2\rangle)\right)} \simeq 10^{-5}\,\beta^{3/2} \,. \tag{6.30}$$

Using the bound (6.6) for consistency of the perturbation expansion, we obtain the interesting result that:

$$\frac{\left(\frac{S}{N}(\langle\mathcal{R}^3\rangle)\right)}{\left(\frac{S}{N}(\delta\langle\mathcal{R}^2\rangle)\right)} \lesssim 1 \qquad \text{unless} \quad \epsilon_{step} \lesssim 10^{-3} \,. \tag{6.31}$$

On the other hand, if we take $l_{max} = 2000$, we obtain:

$$\frac{\left(\frac{S}{N}(\langle\mathcal{R}^3\rangle)\right)}{\left(\frac{S}{N}(\delta\langle\mathcal{R}^2\rangle)\right)} \simeq 10^{-3}\left(\frac{|\tau_f|}{\mathrm{Gpc}}\right)^3 \lesssim 1 \qquad \text{unless} \quad |\tau_f| \gtrsim 10\,\mathrm{Gpc}\,. \tag{6.32}$$

However, as we chose $l_{max} = 2000$, we have

$$l_d \simeq \frac{2D\beta}{\pi|\tau_f|} > 2000\,. \tag{6.33}$$

Then, violating the inequality (6.32) requires at the same time

$$10\,\mathrm{Gpc} \lesssim |\tau_f| \lesssim 10^{-3}\frac{D\beta}{\pi} \tag{6.34}$$

which requires in turn that

$$\beta \gtrsim 10^4\pi\left(\frac{D}{\mathrm{Gpc}}\right)^{-1}\,. \tag{6.35}$$

One more time, looking at the bound of eq. (6.6), we obtain that this can happen only for very small values of the ϵ_{step} parameter. However, as ϵ_{step} becomes smaller, we also expect that the signal-to-noise ratio of the bispectrum itself will become smaller. This can be seen from eq. (6.28):

$$\left(\frac{S}{N}(\langle\mathcal{R}^3\rangle)\right)^2 \lesssim 480\,\epsilon_{step}^2\left(\frac{\tau_f}{\mathrm{Gpc}}\right)^2 \lesssim 480\,\epsilon_{step}^2\left(\frac{D}{\mathrm{Gpc}}\right)^2\,. \tag{6.36}$$

The last inequality comes from the cutoff $|\tau_f| \lesssim D$ imposed by the projection onto the spherical sky [139]. It is clear then that we would not have a signal-to-noise ratio larger than one if we have $\epsilon_{step} \lesssim 10^{-3}$.

This means that the most sensitive test for these models is the feature part of the power spectrum, unless the height of the step is extremely small, so that one can increase the value of the sharpness of the feature without violating the bound (6.6). However, in this case, it would be too hard to detect any feature effect, as the signal-to-noise would be very small. This conclusion remains valid if we generalize to features in the speed of sound, where we have an even stronger bound (6.22) as we move away from $c_s = 1$. The only case that could in principle escape this conclusion would be the case of folded non-Gaussianity. Those configurations can potentially make the three-point function the leading observable for feature models since, for particular choices of the parameters, the folded bispectrum can become dominant and enhance the signal-to-noise ratio. This can be understood also focusing on the parametric scaling of $f_{NL}\big|_{peak}$ of eq. (5.68), which is proportional to β^2 but also to $1/c_s^2$, receiving then a further enhancement.

Part III

Breaking of Spatial Diffeomorphism

CHAPTER 7

Generalised Tensor Fluctuations

The recent results from the BICEP2 and *Planck* collaborations [186] suggest that CMB polarization measurements are reaching sufficient sensitivity to start detecting primordial B-modes, if foregrounds can be understood and the gravity wave amplitude is sufficiently large. In this optimistic situation, recent theoretical studies [38, 39, 187] suggest that if a sufficient delensing of the B-mode signal can be performed, then both the tensor-to-scalar ratio r, and the tilt of the tensor spectrum n_T might be measured with an accuracy sufficient to test the consistency relation (1.88)

$$n_T = -r/8\,, \tag{7.1}$$

that holds for standard single clock inflation in Einstein gravity. This motivates a general theoretical investigation of possible mechanisms for producing primordial tensor fluctuations during inflation, including scenarios that are more general than the ones studied so far. A generic prediction of standard single-field, slow-roll inflation is the production of a nearly scale invariant spectrum of tensor modes with an amplitude proportional to the Hubble parameter during inflation, a ratio $r < 1$ between the tensor and scalar power spectra, and a tilt $n_T < 0$ of the tensor spectrum related to r by eq. (7.1) (see e.g. [37] for a review). The single clock consistency relation (7.1) can be violated in multiple field models (see for example [81, 108, 188]); however, in inflationary scenarios based on a slow-roll expansion, that do not violate the Null Energy Condition, n_T is generically negative. On the other hand, various specific examples have been proposed in the literature that are able to obtain a positive n_T in a controllable way. One can include to eq. (7.1) contributions that are higher order in slow-roll [189], or violate the Null Energy Condition in Galileon or Hordenski constructions [190]. Alternatively, one can consider particle production during inflation [191], or investigate specific non-standard scenarios as solid/elastic inflation [192, 193].

This Chapter is based on the work [4] where we take a more general perspective to the problem of characterizing tensor fluctuations. By implementing an effective field theory approach to inflation, we examine novel properties of the spectrum of inflationary tensor fluctuations, that arise when breaking some of the symmetries or requirements usually imposed on the dynamics of perturbations. During single-clock inflation, time-diffeomorphism invariance is normally broken by the time-dependent cosmological background configuration: the construction of the most general theory for fluctuations that preserves spatial diffeomorphisms, but breaks the time reparametrization invariance, leads to the effective theory of single-field inflation initiated in [109], and developed by many groups over the past few years (see [194, 195] for recent reviews on this topic). On the other hand, it might very well be possible that during inflation also the *spatial* diffeomorphism invariance is broken in the lagrangian for fluctuations. This possibility has not been explored much in the literature, apart from interesting specific set-ups as solid inflation [192]. Alternatively, operators with more than two spatial derivatives acting on the tensor perturbations – preserving or not spatial diffeomorphism invariance – could become important in situations where the leading order Einstein-Hilbert contributions to the tensor sector can be neglected and provide interesting contributions to inflationary observables. In this Chapter, we explore these possibilities using an effective field theory approach. We consider the dynamics of metric fluctuations for single-clock inflation in a unitary gauge in which the clock perturbations are set to zero, and for simplicity we concentrate on operators that are at most quadratic in fluctuations, since our main aim is to try to understand how they can affect observables such as r and n_T, that are directly associated with the tensor power spectrum. The main conclusions of [4] is the identification of the single operator that contributes at leading order to the tensor spectral tilt n_T and that can change its sign, leading to a positive n_T without necessarily violating the null energy condition. We have then shown that this operator has important consequences also in the scalar sector. It generically leads to superhorizon non-conservation of the curvature perturbation ζ on uniform energy density slices, even in single clock inflation – since ζ acquires an effective mass – although additional allowed operators can render the mass of ζ (and its non-conservation after horizon exit) arbitrary small. Including also operators with more spatial derivatives, we have shown that non-trivial tensor sound speed can be generated and the formula for n_T receives new contributions that depend on the coefficients of these higher derivative operators. We also discussed a special case in which such operators can mimic the effect of a mass term in the tensor sector.

We do not wish to systematically investigate *all* possible operators with the properties we are interested in, but to study representative and promising examples that can be of some use to connect inflationary model building with observations, especially when focussing on the tensor sector. On the other hand, the tools that we develop can be further applied and generalized to study more general situations, for example in set-ups with broken isotropy in the effective action for fluctuations. Since we implement an effective field theory approach to the study of perturbations from inflation, we do not attempt to find actual theories or models whose cosmological fluctuations have the properties we investigate, although we will also comment on possible realizations for the operators we study. We limit our attention to operators that are quadratic in fluctuations. Given the fact that we break some of the symmetries such as spatial diffeomorphism invariance, many operators cubic or higher in fluctuations exist; this considerably complicates a systematic analysis of their effects, that we explore in Chapter 9.

7.1 Breaking Spatial Diffeomorphisms in Unitary Gauge

In this section we investigate an effective field theory for cosmological perturbations around quasi-de Sitter space, with broken spatial and time diffeomorphism invariance. We take a conformal (FRW) ansatz for the background metric,

$$ds^2 \,=\, \bar{g}_{\mu\nu}\, dx^\mu dx^\nu \,=\, a^2(\eta)\, \left(-\eta_{\mu\nu}\, dx^\mu dx^\nu\right) \tag{7.2}$$

with $a^2(\eta)$ the conformal scale factor and $a(\eta) \,=\, 1/(-H\eta)$ for de Sitter space. We denote the metric fluctuations by $h_{\mu\nu} = g_{\mu\nu} - \bar{g}_{\mu\nu}$. The time-reparameterization invariance for fluctuations is broken by the time dependence of the homogeneous background. In addition, we would like to study the effects of breaking spatial diffeomorphism invariance. The breaking of diffeomorphism invariance in the spatial sections is most easily achieved by mass terms, although derivative operators involving metric pertubations are also able to do so. First we consider the effects of mass terms, before including diffeomorphism-breaking derivative operators in the next subsections. These operators corresponding to mass terms do not necessarily originate by a theory of massive gravity holding during inflation; they simply correspond to the most general way to express quadratic non-derivative operators in the fluctuations that break diffeomorphism invariance.

We consider the Einstein-Hilbert action expanded to second order and add generic operators with no derivatives, that are quadratic in the metric fluctuations $h_{\mu\nu}$

$$\begin{aligned} S \;&=\; \int \mathrm{d}^4x\sqrt{-g}M_{\rm Pl}^2\left[R - 2\Lambda - 2\,c\,g^{00}\right] + \\ &\frac{1}{4}M_{\rm Pl}^2\int \mathrm{d}^4x\sqrt{-g}\left[m_0^2\,h_{00}^2 + 2\,m_1^2\,h_{0i}^2 - m_2^2\,h_{ij}^2 + m_3^2\,h_{ii}^2 - 2m_4^2\,h_{00}\,h_{ii}\right]. \end{aligned} \tag{7.3}$$

The terms in the first line are the ones that will give the homogeneous and isotropic background which we assume for inflation. They give a non-zero stress-energy tensor at the background level,

$$T^{(0)}_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}}\bigg|_{background} \tag{7.4}$$

and, using Friedmann equations, the parameters c and Λ can be expressed as functions of the Hubble parameter H and its time derivative $\dot{H}$ (that defines the slow-roll parameter $\epsilon = -\dot{H}/H^2$). The quadratic terms in the second line of eq. (7.3) break diffeomorphism invariance, yet they preserve a spatial $SO(3)$ invariance in order not to break spatial isotropy. The term proportional to m_0^2 breaks time reparameterization invariance, and is present also in the quadratic Lagrangian (4.16) of the standard EFTI [109]: the remaining terms in the second line of eq. (7.3), instead, are absent in [109] and break spatial diffeomorphism invariance. They have the same structure of the Lorentz violating mass terms of [196], this time applied to the case of an expanding (quasi-)de Sitter universe. They were dubbed "Lorentz violating" in [196, 197] since in the flat limit ($H \to 0$) they do break 4d Lorentz symmetry $SO(1,3)$ down to spatial rotational symmetry, $SO(3)$ [1]. Since the choice of operators we consider preserves isotropy at each point in space, they also preserve homogeneity in space. In the limit $m_i \to 0$ with $i \neq 0$, spatial diffeomorphisms are restored and, up to second order in perturbations, we recover the standard EFTI in [109] without extrinsic curvature terms, where only time diffeomorphisms are broken by powers of h_{00}. We can consider the "mass terms" in the second line of eq. (7.3) as arising from couplings between the metric and fields acquiring a nontrivial time-dependent profile during inflation. We assume that their coefficients (as well as the ones that we will meet in the following) are effectively constant in space and time during inflation, while these coefficients go to zero after inflation and hence are not constrained by present day observational limits. The constancy in space is not a strict requirement since effects of gradient terms are usually negligible at large scales during inflation. A (small) time dependence for these operators would instead be expected, proportional to slow-roll parameters quantifying the departure from an exact de Sitter phase during inflation: for simplicity we will neglect it. We will not consider interactions in this paper, but we will limit our attention to terms quadratic in perturbations. Nevertheless, for the class of mass terms contained in action (7.3), general considerations show that the maximal cut-off is of order $\Lambda_c \simeq \sqrt{m\, M_{Pl}}$ [199], assuming that all the non-vanishing mass parameters are of the same magnitude m. In

[1] For certain choices of the parameters, these mass terms (although breaking diffeomorphism invariance) can recover 4d Lorentz invariance in the flat limit $H \to 0$. The parameter choice one has to make is

$$m_0^2 = \alpha + \beta\,, \qquad m_1^2 = m_2^2 = -\alpha\,, \qquad m_3^2 = m_4^2 = \beta\,. \tag{7.5}$$

and the Fierz-Pauli theory corresponds to $\alpha + \beta = 0$. These arguments are reviewed in [198].

order to have a reliable theory, we must ensure that $\Lambda_c \geq H$, where H is the Hubble scale during inflation, so that

$$\frac{m}{H} \geq \frac{H}{M_{Pl}}\,. \tag{7.6}$$

Hence for inflation happening at high energy scales, the mass of the graviton must be quite large during the inflationary process (although it can be well below the Hubble scale). After inflation ends, we assume that the effective graviton mass becomes negligible, as we mentioned above.

Let us stress that in the spirit of our effective approach to cosmological fluctuations, only based on symmetry arguments, it is not necessary to specify the nature of the UV model that leads to the fluctuation Lagrangian we are examining. Our theory appears as a version of (Lorentz violating) massive gravity because we are selecting a specific gauge – the unitary gauge – in which fluctuations of the field(s) driving inflation are set to zero: the dynamics of perturbations is entirely described by the sector of metric fluctuations. The UV completion of our scenario might be some specific version of massive gravity coupled to an inflaton field (for reviews of massive gravity, see e.g. [198, 200]), or some model of inflation making use of vectors (see [201] for a review), or sets of scalars obeying specific symmetries. For example, solid inflation [192] is a set-up with broken spatial diffeomorphisms (but preserving time-reparameterization); the dynamics of its fluctuations might be considered as a subclass of our general discussion.

7.1.1 Tensor-vector-scalar Decomposition

It is helpful to rewrite the action (7.3) in terms of tensor, vector and scalar perturbations on spatial hypersurfaces, which evolve independently at linear order:

$$\begin{aligned}
h_{00} &= \psi\,, \\
h_{0i} &= u_i + \partial_i v\,, && \text{with} \quad \partial_i u_i = 0\,, \\
h_{ij} &= \chi_{ij} + \partial_{(i}s_{j)} + \partial_i\partial_j\sigma + \delta_{ij}\tau\,, && \text{with} \quad \partial_i s_i = \partial_j\chi_{ij} = \delta_{ij}\chi_{ij} = 0\,.
\end{aligned} \tag{7.7}$$

Under a diffeomorphism, $\eta \to \eta + \xi_0$, $x^i \to x^i + \xi^i$, these perturbations transform as

$$\begin{aligned}
\chi_{ij} &\to \chi_{ij} \\
u_i &\to u_i + \partial_0\xi_i^T \\
s_i &\to s_i + \xi_i^T \\
\psi &\to \psi + 2\partial_0\xi_0 + 2aH\xi_0 \\
v &\to v + \partial_0\xi^L + \xi_0 \\
\sigma &\to \sigma + 2\xi^L \\
\tau &\to \tau + 2aH\xi_0
\end{aligned} \tag{7.8}$$

where $\xi_i = \xi_i^T + \partial_i \xi^L$. Expanding (7.3) up to second order in these fluctuations, we find the following tensor-vector-scalar actions including the mass terms:

- Tensor action

$$S_m^{(T)} = \frac{1}{4} M_{\rm Pl}^2 \int {\rm d}^4 x a^2 \Big[-\eta^{\mu\nu}\partial_\mu \chi_{ij} \partial_\nu \chi_{ij} - a^2 m_2^2 \chi_{ij}^2 \Big] \,, \tag{7.9}$$

- Vector action

$$S_m^{(V)} = \frac{1}{2} M_{\rm Pl}^2 \int {\rm d}^4 x a^2 \Big[-(u_i - s_i')\nabla^2 (u_i - s_i') + a^2 (m_1^2 u_i^2 + m_2^2 s_i \nabla^2 s_i) \Big] \,, \tag{7.10}$$

- Scalar action

$$\begin{aligned} S_m^{(S)} &= \frac{1}{4} M_{\rm Pl}^2 \int {\rm d}^4 x a^2 \Big\{ -6(\tau' + aH\psi)^2 + 2(2\psi - \tau)\nabla^2 \tau \\ &+4(\tau' + aH\psi)\nabla^2(2v - \sigma') + a^2 \Big[(m_0^2 + 2\epsilon H^2)\psi^2 - 2m_1^2 v \nabla^2 v \\ &-m_2^2(\sigma \nabla^4 \sigma + 2\tau \nabla^2 \sigma + 3\tau^2) + m_3^2 (\nabla^2 \sigma + 3\tau)^2 - 2m_4^2 \psi (\nabla \sigma + 3\tau) \Big] \Big\} \end{aligned} \tag{7.11}$$

Since diffeomorphisms are broken, one would expect to find six propagating degrees of freedom, and one of these should generically be a ghost. Nevertheless, it has been shown that in a FRW background the theory can be ghost-free and potential instabilities avoided, if the masses m_i satisfy certain conditions [202]. In the next subsections, we will generalize this analysis including also the effect of a selection of derivative operators that break diffeomorphism invariance, studying each sector of the theory and also discussing possible phenomenological consequences. To the operators considered so far we will add new quadratic operators that contain at most two space-time derivatives in $h_{\mu\nu}$. They potentially break spatial diffeomorphism invariance, although they preserve Euclidean invariance in the spatial sections. See Appendix C.1 for a list of such operators. To conclude this section, let us point out that our analysis includes operators with higher spatial derivatives acting on the fields obtained after the tensor-vector-scalar decomposition of $h_{\mu\nu}$ (see for example the m_2^2 coefficient in eq. (7.11)) that have been removed by a parameter choice in [203]. See however [204] for a recent analysis including operators that are higher order in spatial derivatives.

7.1.2 Tensor Fluctuations

Let us start by discussing the tensor fluctuations, since this is the sector we are most interested in. We see from the action $S_m^{(T)}$ in eq. (7.9) that tensors acquire a mass only in the case $m_2^2 \neq 0$ and no instabilities arise if $m_2^2 \geq 0$. Hence only the operator proportional to m_2^2 in eq. (7.3) influences the tensor spectrum by giving an effective mass to the tensors. On the other hand, we can add to the mass term additional operators that contain up to two space-time derivatives and preserve isotropy: they can change speed of sound for tensor perturbations in eq. (7.9). In particular, the only allowed operators that can contribute to the tensor sound speed are the ones in eqs. (C.15), (C.17) in appendix C.1. We may add to the action (7.9) two derivative operators[2], with dimensionless coefficients b_1 and d_1:

$$S_d^{(T)} \equiv \frac{1}{4} M_{\rm Pl}^2 \left[b_1 (\partial_0 h_{ij})^2 + d_1 (\partial_i h_{jk})^2 \right] \, . \tag{7.12}$$

It is important to notice that these two derivative operators do not necessarily originate from contributions that break the 3-dimensional diffeomorphism invariance *per se*. In particular these terms can arise from the diffeomorphism invariant combination $b_1 \delta K_{ij} \delta K^{ij} - d_1 {}^{(3)}R$, where $\delta K_{ij} \delta K^{ij}$ is the perturbed extrinsic curvature and ${}^{(3)}\delta R$ is the three-dimensional Ricci scalar [109, 205]. These specific combinations, on the other hand, contain specific additional vector and scalar contributions that have to be taken into account. We will consider them in the next subsections, but for the moment we do not need to restrict to any special case; we can focus on (7.12) regardless of its origin.

The complete action for tensor fluctuations becomes

$$S^{(T)} = S_m^{(T)} + S_d^{(T)} = \frac{1}{4} M_{\rm Pl}^2 \int {\rm d}^4 x a^2 \left\{ (1+b_1) \left[(\dot{\chi}_{ij})^2 - c_T^2 (\partial_i \chi_{jk})^2 \right] - a^2 m_2^2 \chi_{ij}^2 \right\} \, , \tag{7.13}$$

where the speed of sound for tensors is

$$c_T^2 = \frac{1+d_1}{1+b_1} \, . \tag{7.14}$$

In this case, in order to avoid ghosts one should also require $b_1 > -1$, $d_1 \geq -1$; moreover we could also demand $d_1 \leq b_1$ not to have superluminal propagation. Taking the action (7.13), it is easy to derive the expression for the tensor power spectrum, quantizing the tensor fluctuations starting from the usual Bunch-Davies vacuum (see Section 1.3 and Appendix A). Upon canonical normalization and neglecting for simplicity time dependencies of c_T and m_2, the equation of motion for tensors has the usual Mukhanov–Sasaki form. It can be solved to give the following expression for the power spectrum and its scale dependence:

$$\mathcal{P}_T = \frac{2H^2}{\pi^2 M_{\rm Pl}^2 c_T} \left(\frac{k}{k_*} \right)^{n_T} \, , \qquad n_T = -2\epsilon + \frac{2}{3} \frac{m_2^2}{(1+b_1)^2 H^2} \left(1 + \frac{4}{3}\epsilon \right) \, , \tag{7.15}$$

[2] Notice that also a parity violating, one derivative operator could be included, $\epsilon^{ijk} \left(\partial_i \, h_{jm}\right) h_{km}$, with ϵ^{ijk} the totally antisymmetric operator in three spatial dimensions. On the other hand, in this work we concentrate on operators that preserve parity, so we do not consider its effects.

at leading order in slow-roll and in an expansion in $m_2/H \ll 1$. Notice that the mass term can render the tensor spectrum blue if m_2/H is sufficiently large and positive so that the second term in n_T wins out over the negative contribution from the first term. In this sense, a blue spectrum for tensors can be obtained without violating the Null Energy Condition or exploiting the time-dependence of parameters: it is the effect of the mass term proportional to m_2^2 and is not depending on the sign of $\dot{H}$. The amplitude of the tensor power spectrum is enhanced by the inverse of the sound speed c_T. On the other hand, it has been recently shown in [206] that, when focussing on operators containing at most two derivatives – as we do in this section – there exists a disformal redefinition of the metric which converts the theory with a speed of sound $c_T \neq 1$ into a theory (in the Einstein frame) with unit speed of sound. Thus, in the Einstein frame, during inflation the sound speed is equal to one. Hence – neglecting the scale dependence of $\mathcal{P}_T$ – the amplitude of the tensor power spectrum is directly linked to the scale of inflation. Notice that in our scenario we do have an additional source of scale-dependence though, associated with the mass term m_2 that breaks the spatial diffeomorphism invariance. The disformal transformation of [206] does not involve spatial coordinates hence does not modify our predictions for the scale dependence of the tensor spectrum, whose sign is still controlled by m_2^2/H^2 versus ϵ. It has been discussed in [206] that terms involving higher derivatives can actually change the situation and induce a non-trivial sound speed. While in [206] three-derivative terms were included, we will extend this possibility and study healthy four derivative terms (with at most two time derivatives) in a following section.

7.1.3 Vector Fluctuations

We now discuss the propagation of vector fluctuations in our set-up. In this and in the next subsection (where we will discuss the dynamics of scalars) we do not pretend to be exhaustive in our analysis, but only to investigate simple and interesting cases among the many possibilities allowed within our large parameter space. In particular, aiming for simplicity, our purpose is to reduce as much as we can the number of propagating degrees of freedom in our scenario, and choose parameters which can eliminate the vector degrees of freedom.

In principle we have two vector degrees of freedom, u_i and s_i, from the decomposition in eq (7.7). Examining the action (7.10) for vector perturbations including mass terms and in absence of additional derivative operators, it is straightforward to show that the field u_i is not dynamical, since we obtain

$$\nabla^2(u_i - s_i') - a^2 m_1^2 u_i = 0\,. \tag{7.16}$$

Hence u_i can be integrated out to give the effective action

$$S_m^{(V)} = \frac{1}{2}M_{\rm Pl}^2 \int \mathrm{d}^4x\, a^4 \left[m_1^2 s_i' \frac{\nabla^2}{\nabla^2 - a^2 m_1^2} s_i' + m_2^2 s_i \nabla^2 s_i \right] . \tag{7.17}$$

The action is free of instabilities for $m_1^2 \geq 0$ and $m_2^2 \geq 0$. The case $m_1^2 = 0$ is particularly interesting as there are no propagating vector modes, since the coefficient of the s_i kinetic term in (7.17) vanishes. Hence in order to eliminate vector degrees of freedom, we make the choice $m_1 = 0$.

On the other hand, the situation can drastically change if also other possible derivative contributions are included in the action, choosing from the list of allowed operators in Appendix C.1. There are six possible terms with up to two derivatives that contribute to the vector sector, that contribute to an effective Lagrangian that we dub $\mathcal{L}_d^{(V)}$:

$$\begin{aligned} \mathcal{L}_d^{(V)} \;\; = \;\; & \frac{1}{4}M_{\rm Pl}^2 \Big[b_1(\partial_0 h_{ij})^2 + b_2(\partial_i h_{0j})^2 + b_3(\partial_j h_{0i}\partial_0 h_{ij}) \\ & + d_1(\partial_i h_{jk})^2 + d_2(\partial_i h_{ij})^2 \Big] \\ & + \frac{1}{4}M_{\rm Pl}^3\, \alpha_4\, (h_{ij}\partial_i h_{0j}) \;, \end{aligned} \tag{7.18}$$

where b_i, d_i and α_4 are arbitrary constant coefficients. Notice that also a single derivative term is allowed in the last line of eq. (7.18). These derivative contributions in $S_d^{(V)}$ in general switch on a non-trivial dynamics for s_i even if $m_1^2 = 0$. On the other hand, it can be shown (c.f. Appendix C.1) that if one chooses the particular values

$$b_1 = \frac{1}{2} b_2 = -\frac{1}{4} b_3 \;, \tag{7.19}$$

then the structure of the action (7.10) would be unaltered and the vector s_i, when $m_1^2 = 0$, would still be non-dynamical. This corresponds to a combination of the operators forming the spatial diffeomorphism invariant quantity $(\delta K_{ij})^2$. Provided this condition (7.19) is satisfied, adding the operators proportional to d_1, d_2 and α_4 in eq. (7.18) does not change the conclusion such that s_i not dynamical. Hence, the condition $m_1^2 = 0$ is appealing since we can still ensure that no vectors propagate. As we will see, this condition also gives only one propagating mode in the scalar sector, since extrinsic curvature terms do not render a second scalar mode dynamical. Fine-tuning relations on mass parameters, such as $m_1^2 = 0$ can be motivated and protected by residual gauge symmetries [196]. Indeed, this is the case for $m_1^2 = 0$; if we require invariance under time-dependent diffeomorphisms,

$$x^i \to x^i + \xi^i(t) \;, \tag{7.20}$$

then the operator h_{0i}, associated with m_1^2, is forbidden in the action.

7.1.4 Scalar Fluctuations

Not surprisingly, the scalar sector is the most tricky to analyze due to the number of fields involved and their mixings. We separate the discussion in two parts. First we study the case in which only scalar masses are included, and no derivative operators are added to eq. (7.11). We show that an important physical consequence of our construction is that the curvature perturbation ζ is generally not conserved on super-horizon scales. We then proceed, including derivative operators in the second part of this section. The main aim is to find the conditions required to propagate at most one (healthy) scalar degree of freedom in our system.

Only masses are included

When only scalar masses are switched on, the action we are working with is eq. (7.11). This action potentially propagates two degrees of freedom, σ and τ. It can be shown that even in the case where all the masses are different from zero, the theory has no ghosts nor other instabilities provided that $m_1^2 > 0$, $6H^2 \geq m_0^2 - 2\dot{H} > 0$ and $\dot{H} < 0$ [202]. Here we focus instead on the case $m_1^2 = 0$ that, besides having no vectors, it also has only one propagating scalar, as we are going to discuss. From eq. (7.11) with $m_1^2 = 0$ one can obtain the equations of motion for the auxiliary fields ψ and v,

$$\begin{aligned} \psi &= -\frac{\tau'}{\mathcal{H}}\,, \\ \nabla^2 v &= \frac{a^2}{4\mathcal{H}}\left[(m_0^2 - 2\dot{H})\tau' - \frac{2}{a^2}\nabla^2\tau + \frac{2\mathcal{H}}{a^2}\nabla^2\sigma' + m_4^2(\nabla^2\sigma + 3\tau)\right]\,, \end{aligned} \tag{7.21}$$

and substitute them back into the action obtaining (where we write $\mathcal{H} = aH$ and $\dot{H} = -\epsilon H^2$)

$$\begin{aligned} S = \;& \frac{1}{4}M_{\rm Pl}^2 \int \mathrm{d}^4x\; a^2\Bigg[-2\left(\frac{\tau'}{aH} + \tau\right)\nabla^2\tau + a^2(m_0^2 + 2\epsilon H^2)\left(\frac{\tau'}{aH}\right)^2 \\ & -a^2 m_2^2(\sigma\nabla^4\sigma + 2\tau\nabla^2\sigma + 3\tau^2) \\ & +m_3^2(\nabla^2\sigma + 3\tau)^2 + \frac{2m_4^2 a^2}{aH}\tau'(\nabla^2\sigma + 3\tau)\Bigg]\,. \end{aligned} \tag{7.22}$$

This shows that σ is also an auxiliary field:

$$aH(m_2^2 - m_3^2)\nabla^2\sigma = m_4^2\tau' - aH(m_2^2 - 3m_3^2)\tau\,. \tag{7.23}$$

The action becomes

$$S = M_{\rm Pl}^2 \int d^4x \quad \frac{a^2}{H^2}\left[\frac{(m_0^2+2\epsilon H^2)(m_2^2-m_3^2)+m_4^2}{2(m_2^2-m_3^2)}\tau'^2 + \epsilon H^2 \tau\nabla^2\tau \right. \\ \left. -\frac{m_2^2 a^2 H^2 (m_2^2-3m_3^2+(3+\epsilon)m_4^2)}{m_2^2-m_3^2}\tau^2\right] \tag{7.24}$$

.

After canonical normalization of τ, the action finally is given by

$$S = \int d^4x a^2 \left[\hat{\tau}'^2 + c_s^2(\hat{\tau}\nabla^2\hat{\tau}) + a^2 M^2 \hat{\tau}^2\right] , \tag{7.25}$$

where effective mass and speed of sound are

$$c_s^2 = \frac{2\epsilon H^2 (m_3^2 - m_2^2)}{m_0^2(m_2^2-m_3^2)+m_4^2} , \tag{7.26}$$

$$M^2 = -\frac{2H^2 m_2^2\left(m_2^2 - 3m_3^2 + 3m_4^2\right)}{m_0^2(m_2^2-m_3^2)+m_4^4} , \tag{7.27}$$

at leading order in slow-roll.

An exhaustive analysys of all the possibilities for the scalar action is beyond the scope of this work. Other cases besides the one considered here could be interesting. For example, when $m_1^2 = 0$ and $m_2^2 = m_3^2$, case that is not included in (7.24), it can be shown that no scalar degrees of freedom propagate [202]. However this is true only if no derivative operators for h_{ij} are considered. When all the other combinations of h and derivatives are considered, they can provide kinetic terms for scalars, changing the previous conclusions. We will return to this later.

Non-conservation of $\mathcal{R}$ and ζ at super-horizon scales

Reconsidering the action (7.24), some interesting points can be made. There is only one scalar perturbation, τ, which is related to the comoving curvature perturbation $\mathcal{R}$. In an arbitrary gauge we define

$$\mathcal{R} = \tau - \frac{\mathcal{H}(\tau' - \mathcal{H}\psi)}{\mathcal{H}' - \mathcal{H}^2} . \tag{7.28}$$

However in the unitary gauge the equation of motion of the auxiliary field ψ, eq. (7.21), requires $\tau' = \mathcal{H}\psi$ and we have $\mathcal{R} = \tau$, even when diffeomorphisms are broken by the masses. In the limit where all masses go to zero, the scalar action (7.24) reduces to the standard slow-roll action for $\mathcal{R}$. Since $\mathcal{R}$ coincides with the (massive) scalar fluctuation τ, $\mathcal{R}$ (before canonical normalization) has a non-vanishing mass given by

$$M_{\mathcal{R}}^2 = \frac{m_2^2(m_2^2 - 3m_3^2 + (3+\epsilon)m_4^2)}{m_2^2 - m_3^2} . \tag{7.29}$$

Notice that this mass is present only if $m_2^2 \neq 0$, exactly as for tensor perturbations. A profound implication of this result is that $\mathcal{R}$ is in general *not* constant after horizon exit, as it is in standard single-field models of inflation. For $M_{\mathcal{R}}^2 > 0$ the solution of the Mukhanov-Sasaki equation for $\mathcal{R}$ will decay after horizon exit. The standard picture of different super-horizon patches of the universe evolving as separate universes with constant $\mathcal{R}$ [30] is not valid anymore. A simple physical interpretation is that, given that diffeomorphism invariance is broken in our set-up, very long wavelength fluctuations can no longer be considered as a gauge mode in the zero momentum limit, and there is actually a preferred frame (the unperturbed background, $\mathcal{R} = 0$) towards which the fluctuation dynamics is attracted for $M_{\mathcal{R}}^2 > 0$. This is analogous to what happens in the specific set-up of solid inflation [192], whose consequences can be considered as a special case of our general discussion. Notice that, phenomenologically, in order for the perturbations to remain over-damped on super-horizon scales (not to oscillate and decay rapidly), we require $M_{\mathcal{R}}^2 \ll H^2$, which gives a constraint on $M_{\mathcal{R}}^2$. On the other hand, given that the mass of the tensor depends only on m_2^2 while the mass of the scalar also on m_3^2 and m_4^2, there is still enough freedom to have a blue tilt for the tensor spectrum and a nearly constant $\mathcal{R}$ outside the horizon. Actually, making the particular choice $m_2^2 = 3m_3^2 - (3+\epsilon)m_4^2$ one finds that $\mathcal{R}$ is massless and conserved outside the horizon.

In our framework, analogously to solid inflation, the comoving curvature perturbations $\mathcal{R}$ and the curvature perturbations on uniform density slices ζ *do not coincide* in the large scale limit, as they do in standard single-field inflation. Indeed, taking the definition of the function ζ,

$$\zeta = \tau - H\frac{\delta\rho}{\dot{\rho}} \,, \tag{7.30}$$

and computing the density ρ and its perturbation from the energy-momentum tensor, one finds at leading order in gradients a contribution that does not vanish at large scales:

$$\zeta = \tau + \frac{(1-\epsilon)m_4^2}{m_0^2 + 2\epsilon H^2}\tau + \mathcal{O}(\nabla^2) \neq \mathcal{R} \,. \tag{7.31}$$

Also ζ is not conserved and evolves after horizon exit. Following [30],

$$\dot{\zeta} = -\frac{H}{\rho + p}\delta p_{\rm nad} + \mathcal{O}(\nabla^2)\,, \tag{7.32}$$

it can be understood that the reason for this non-conservation is the existence of a non-adiabatic stress induced by the presence of the masses. While in the standard case one finds that $\delta p_{\rm nad}$ is proportional only to gradient terms, here there is a non-trivial contribution in the perturbed (spatial) energy-momentum tensor even on super-horizon scales, given by

$$\mathrm{Tr}\,[\delta T_{ij}] = (m_2^2 - 3m_3^2)\,\mathrm{Tr}[h_{ij}] + 3(\epsilon H^2 + \frac{1}{2}m_4^2)h_{00} \,. \tag{7.33}$$

When diffeomorphisms are preserved, this trace is proportional only to $h_{00} = \psi$, which can then be substituted using the constraints (7.21) to see that indeed only gradients remain. When diffeomorphisms are broken by the masses, the use of the equation of motion (7.21) and (7.23) does not allow us to get rid of all the terms and we are left with

$$\mathrm{Tr}\,[\delta T_{ij}] = m_2^2 f(m_i)\tau + \mathcal{O}(\nabla^2) \; . \tag{7.34}$$

where $f(m_i)$ is a (complicated) function of all the mass parameters. This term will not vanish on large scales, making ζ evolve also after the horizon exit. The cause of the non conservation of ζ and $\mathcal{R}$ has to be understood in terms of the contribution m_2^2. Indeed if $m_2^2 = 0$ curvature perturbations are constant beyond the horizon. The operator proportional to m_2^2 is the only one that gives a non-trivial off-diagonal contribution to the energy-momentum tensor,

$$T_{ij} \sim m_2^2 h_{ij} \; , \tag{7.35}$$

and hence an anisotropic stress, that is sourced by the very same operator that gives an effective mass to the graviton (although we will see next that diffeomorphism breaking derivative operators can also play a role). This is coherent and very similar with what was found in [192], where it is shown that a non-vanishing anisotropic stress with certain characteristic on large scale violate some technical assumptions of Weinberg's theorem on the conservation of curvature perturbations [207].

Adding derivative operators

Let us now add derivative operators. We by adding the combination $(\delta K_{ij})^2$ corresponding to the first line of eq (7.18) with the condition (7.19) for the operators $(\partial_0 h_{ij})^2$, $(\partial_i h_{0j})^2$ and $(\partial_j h_{0i}\partial_0 h_{ij})]$, that as we have seen has the nice feature of avoiding the propagation of vectors. We then subtract $(\delta K_{ii})^2$, including the operators $(\partial_0 h_{ii})^2$, $(\partial_i h_{0i})^2$ and $(\partial_i h_{0i}\partial_0 h_{jj})$), in order to avoid the propagation of a second (ghostly) scalar mode. After this choice is made, we are free to add other derivative operators and write the Lagrangian density as

$$\begin{aligned}
\mathcal{L}_d^{(s)} \;\; = \;\; & M_{\rm Pl}^2\, b\, \left[(\delta K_{ij})^2 - (\delta K_{ii})^2\right] \\
& + \frac{1}{4} M_{\rm Pl}^2 \Big[d_1 (\partial_i h_{jk})^2 + d_2 (\partial_i h_{ij}) + d_3 (\partial_i h_{jj})^2 + d_4 (\partial_i h_{jj}\partial_k h_{ik}) \\
& \qquad + c_1(\partial_i h_{00}\partial_j h_{ij}) + c_2(\partial_i h_{0i}\partial_0 h_{jj}) + c_3(\partial_i h_{00})^2 \Big] + \\
& + \frac{1}{4} a M_{\rm Pl}^3 \left[\alpha_1 (h_{00}\partial_0 h_{ii}) + \alpha_2 (h_{00}\partial_i h_{0i}) + \alpha_3 (h_{ii}\partial_j h_{0j}) + \alpha_4 (h_{ij}\partial_i h_{0j})\right]
\end{aligned} \tag{7.36}$$

Interestingly, also first derivative terms can be added, however the condition $\alpha_1 = 2\alpha_2$ in the single derivative sector has to be imposed, in order to avoid the propagation of a second (ghostly) scalar mode. Collecting these pieces together, the new action for the scalars will then be

$$
\begin{aligned}
S^{(S)} \;=\; & \frac{1}{4}M_{\rm Pl}^2\int {\rm d}^4x a^2\Big\{ -6\left(\tau'+aH\psi\right)^2+2\left(2\psi-\tau\right)\nabla^2\tau \\
& +4\left(\tau'+aH\psi\right)\nabla^2\left(2v-\sigma'\right)+a^2\Big[\left(m_0^2+2\epsilon H^2\right)\psi^2-2m_1^2 v\nabla^2 v \\
& -m_2^2\left(\sigma\nabla^4\sigma+2\tau\nabla^2\sigma+3\tau^2\right)+m_3^2\left(\nabla^2\sigma+3\tau\right)^2-2m_4^2\psi\left(\nabla\sigma+3\tau\right)\Big] \\
& +b\left(8\tau'\nabla^2 v-4\tau'\nabla^2\sigma'-6\tau'^2\right)-c_1\nabla^2\psi\left(\nabla^2\sigma+\tau\right) \\
& -c_2\nabla^2\psi\left(\nabla^2\sigma+3\tau\right)-c_3\psi\nabla^2\psi-\left(d_1+d_2+d_3+d_4\right)\nabla^2\sigma\nabla^4\sigma \\
& -2\left(d_1+d_2+3d_3+2d_4\right)\tau\nabla^4\sigma-\left(3d_1+d_2+9d_3+3d_4\right)\tau\nabla^2\tau \\
& +aM_{\rm Pl}^3\Big[\alpha_1\psi(\nabla^2\sigma'+3\tau')+2\alpha_1\psi\nabla^2 v \\
& +\alpha_3\nabla^2 v(\nabla^2\sigma+3\tau)+\alpha_4\nabla^2 v(\nabla^2\sigma+\tau)\Big]\Big\}
\end{aligned} \tag{7.37}
$$

where the parameter b is associated to the combination $(\delta K_{ij})^2-(\delta K_{ii})^2$ expanded at quadratic order in fluctuations. As we said, the fields v and ψ are again auxiliary and their equations of motion can be solved algebraically. The main point is that the action resulting from their substitution does not contain any time derivative term σ', which means that the dangerous "sixth-mode" σ is not dynamical and can be integrated away. The action for the only remaining dynamical scalar has the following simple structure:

$$
S=M_{\rm Pl}^2\int {\rm d}^4x a^2\left[A_1\tau'^2+A_2\tau\tau'+A_3\tau^2+A_4\sigma^2+A_5\sigma\tau+A_6\sigma\tau'\right]\,, \tag{7.38}
$$

where the A_i are functions of all the parameters and the gradient ∇^2 (see Appendix C.2). The field σ can then be integrated out to give (after some integrations by parts)

$$
S=M_{\rm Pl}^2\int {\rm d}^4x a^2\left[B_1\tau'^2+B_2\tau^2\right]\,, \tag{7.39}
$$

At this point, one can canonically normalize $\hat\tau=\sqrt{B_1}\tau$ and symbolically expand in ∇^2 (which can be understood in Fourier space as an expansion in the momentum k), so that one can read the mass and the speed of sound of the scalar mode:

$$
S=\int {\rm d}^4x\, a^2\left[\hat\tau'^2+\hat c_s^2\hat\tau\nabla^2\hat\tau+a^2\hat M^2\hat\tau^2+\mathcal{O}(\nabla^4)\right]\,. \tag{7.40}
$$

The expression of $\hat c_s^2$ and $\hat M^2$ are complicated functions of all the parameters. It can be checked that in the limit where all the parameters of the modified kinetic terms b, c_i, d_i, δ_i, α_i vanish, we recover the expressions of the previous section where c_s is given by eq. (7.26) and mass is given by eq. (7.27), while higher-order derivative terms correctly drop to zero. As an example, we write here the effective mass and speed of sound at leading order in slow roll in the case where all the parameters are zero except for masses and α_1:

$$\hat{c}_s^2 = \frac{\alpha_1\Lambda(m_2^2-m_3^2)(\alpha_1\Lambda-4H)}{(m_2^2-m_3^2)\left(3\alpha_1\Lambda(\alpha_1\Lambda-8H)+8m_0^2\right)+8m_4^4}\,, \tag{7.41}$$

$$\hat{M}^2 = -\frac{m_2^2(4H-\alpha_1\Lambda)\left(4H\left(m_2^2-3m_3^2+3m_4^2\right)-\alpha_1\Lambda\left(m_2^2-3m_3^2\right)\right)}{(m_2^2-m_3^2)\left(3\alpha_1\Lambda(\alpha_1\Lambda-8H)+8m_0^2\right)+8m_4^4}\,. \tag{7.42}$$

One can see that "kinetic operators" like the one proportional to α_1 can also affect the effective mass. A natural question to ask is whether, by exploiting this fact, effective mass contributions can be generated even in the absence of explicit non-derivative terms in the action. This will be the subject of the next section.

Also after adding derivative contributions, the curvature perturbation is again not conserved and decays after horizon exit. As previously, this can be seen also from the trace of the spatial part of the energy-momentum tensor, which, in the simple example we do, now reads

$$\mathrm{Tr}\,[\delta T_{ij}] = m_2^2\tau + \frac{1}{2}\alpha_1 M_{\mathrm{Pl}}(a\psi)' + \mathcal{O}(\nabla^2)\,, \tag{7.43}$$

hence it does not vanish at superhorizon scales, due to the contributions proportional to m_2^2 and α_1. One might use the constraint equation (7.21) to express ψ' in terms of τ, the only propagating scalar degree of freedom in the system. It would be interesting to analyze how the curvature perturbation ζ evolves at superhorizon scales when α_1 or other diffeomorphism-breaking kinetic terms are included.

7.2 Generating a Mass without Mass: four Derivative Operators

We have learned in the previous section that by breaking spatial diffeomorphism invariance of the action for metric perturbations, by means of mass terms or derivative operators, we can change some of the properties of the tensor spectrum with respect to the standard inflationary predictions, in particular its tilt n_T and the value of the tensor sound speed c_T. It is natural to ask whether it is really necessary to explicitly break spatial diffeomorphism invariance to do so. The aim of this section is to show that the answer is no, provided that we allow for higher spatial derivative operators in the quadratic action for fluctuations. An effective field theory approach to inflation that takes into account of higher derivative operators has also been proposed in [100]. Adding such operators, one can avoid the argument [206] (based on operators with at most two space-time derivatives) and find genuine contributions to the tensor sound speed c_T, that cannot be eliminated by disformal transformations. This has interesting implications since the tensor sound speed enters in the amplitude of the tensor power spectrum (7.15) in a way that enhances the amplitude of $\mathcal{P}_T$ that scales as c_T^{-1}. It would be interesting to find explicit models able to avoid the Lyth bound using this fact, but would also need to consider the effect on the scalar modes and hence the observed tensor-to-scalar ratio r. In particular, we will explore the effect of 4-derivative contributions to the action for fluctuations, organized in such a way as not to break the spatial diffeomorphism invariance, and not to introduce instabilities. The starting point is to consider the quantities

$$\partial_0\partial_l\, h_{ij} = \partial_l\, \chi'_{ij} + \partial_l\partial_{(i}s'_{j)} + \partial_l\partial_i\partial_j\, \sigma' + \delta_{ij}\,\partial_l\tau'\,, \tag{7.44}$$

$$\partial_0\partial_i\, h_{ij} = \nabla^2 s'_j + \partial_j\,\nabla^2\,\sigma' + \partial_j\tau'\,, \tag{7.45}$$

$$\partial_0\partial_j\, h_{ii} = \partial_j\,\nabla^2\,\sigma' + 3\partial_j\tau'\,, \tag{7.46}$$

that we can use to build quadratic operators with four derivatives, that we can potentially add to the action for metric perturbations

$$L_1 = (\partial_l\,\partial_0\, h_{ij})^2 = \left(\partial_l\,\chi'_{ij}\right)^2 + 2\left(\nabla^2 s'_j\right)^2 - \nabla^2\,\sigma'\,\nabla^2\nabla^2\,\sigma' - 3\tau'\,\nabla^2\tau' - 2\,\nabla^2\,\sigma'\,\nabla^2\tau'\,, \tag{7.47}$$

$$L_2 = (\partial_0\partial_i\, h_{ij})^2 = \left(\nabla^2 s'_j\right)^2 - \nabla^2\,\sigma'\,\nabla^2\nabla^2\,\sigma' - \tau'\nabla^2\tau' - 2\,\nabla^2\,\sigma'\,\nabla^2\tau'\,, \tag{7.48}$$

$$L_3 = (\partial_0\partial_j\, h_{ii})^2 = -\nabla^2\,\sigma'\,\nabla^2\nabla^2\,\sigma' - 9\tau'\,\nabla^2\tau' - 6\,\nabla^2\,\sigma'\,\nabla^2\tau'\,, \tag{7.49}$$

$$L_4 = \partial_0\partial_i\, h_{ij}\partial_0\partial_j\, h_{ii} = -\nabla^2\,\sigma'\,\nabla^2\nabla^2\,\sigma' - 3\tau'\,\nabla^2\tau' - 4\,\nabla^2\,\sigma'\,\nabla^2\tau'\,, \tag{7.50}$$

where integrations by parts have been performed. We would like to build a combination of L_i such that only contributions associated with $\chi'_{ij}\,\nabla^2\,\chi'_{ij}$ and $\tau'\nabla^2\tau'$ are non-vanishing, while the vectors and the remaining scalars do not appear. If such combination can be found, it is invariant under spatial diffeomorphisms, since χ_{ij} and τ do not transform under this symmetry (see eq (7.8), noticing that τ transforms but only under time-reparameterization). The combination with the desired properties is

$$L_{\omega_1} = \omega_1(L_1 - 2L_2 - L_3 + 2L_4) \tag{7.51}$$

$$= -\omega_1\chi'_{ij}\nabla^2\chi'_{ij} + 2\omega_1\tau'\nabla^2\tau'\,. \tag{7.52}$$

In analogy to what happens for the two derivatives operators, see the comment after eq.(7.12)), this combination (7.51) corresponds to a particular combination of the extrinsic curvature perturbation,

$$(\partial_i\delta K_{jk})^2 - (\partial_i\delta K)^2 - 2(\partial_i\delta K_{ij})^2 - 2\partial_i\delta K\partial_j\delta K_{ij}\,, \tag{7.53}$$

expanded at quadratic order in perturbations. Analogously, one can consider four derivative operators that lead only to combinations involving four spatial derivatives acting on the tensors $\nabla^2\chi_{ij}\,\nabla^2\,\chi_{ij}$. The following Lagrangians arise from all possible contractions of two spatial derivatives and h_{ij} (once integrations by parts are taken into account):

$$\begin{aligned}
L_1 &= (\nabla^2 h_{ij})^2 = (\nabla^2\chi_{ij})^2 - 2s_i\nabla^4 s_i + (\nabla^4\sigma)^2 + 3(\nabla^2\tau)^2 + 2\nabla^2\tau\nabla^4\sigma\,, &(7.54)\\
L_2 &= (\partial_i\partial_j h_{ij}) = (\nabla^4\sigma + \nabla^2\tau)^2\,, &(7.55)\\
L_3 &= (\nabla^2 h_{ii})^2 = (\nabla^4\sigma + 3\nabla^2\tau)^2\,, &(7.56)\\
L_4 &= (\partial_k\partial_i h_{ij})^2 = -s_i\nabla^4 s_i + (\nabla^4\sigma + \nabla^2\tau)^2\,, &(7.57)\\
L_5 &= (\nabla^2 h_{kk}\partial_i\partial_j h_{ij}) = (\nabla^4\sigma + \nabla^2\tau)(\nabla^4\sigma + 3\nabla^2\tau)\,. &(7.58)
\end{aligned}$$

There exist combinations of these operators which allow us to avoid contributions from all vectors and scalars:

$$\begin{aligned}
L_{\omega_2} &= \omega_2(L_1 + \frac{1}{2}L_2 - \frac{1}{2}L_3 - 2L_4 + L_5) = &(7.59)\\
&= \omega_2(\nabla^2\chi_{ij})^2\,, &(7.60)
\end{aligned}$$

hence this combination preserves full four dimensional diffeomorphism invariance.

By adding the Lagrangians L_{ω_1} and L_{ω_2} to the quadratic EH Lagrangian plus the two derivative contribution (7.12) – that can preserve diffeomorphism invariance if it originates from a combination of δK_{ij}^2 and ${}^{(3)}R$ (see the comment after eq. (7.12)) – one obtains the effective Lagrangian for tensor modes[3]:

$$\begin{aligned}
\mathcal{L}^{(T)} &= \frac{M_{Pl}^2}{4}\,a^2\left[(1+b)(\chi'_{ij})^2 - \frac{\omega_1}{a^2\Lambda^2}\,\chi'_{ij}\nabla^2\chi'_{ij}\right.\\
&\qquad\left. + (1+d)\chi_{ij}\nabla^2\chi_{ij} + \frac{\omega_2}{a^2\,\Lambda^2}\chi_{ij}\nabla^2\nabla^2\chi_{ij}\right] \qquad (7.61)
\end{aligned}$$

with $\omega_{1,2}$ arbitrary parameters, and Λ some cut-off energy scale, that will depend on the UV completion, and that to be safe we take larger than the Hubble scale during inflation. Let us emphasize that we constructed the Lagrangians L_{ω_1} and L_{ω_2} as space diffeomorphism invariant combinations, with the specific aim to analyze the phenomenological consequences of higher order derivative operators in the tensor sector. These Lagrangians are characterized by a specific choice of parameters among their terms: it would be interesting to investigate whether such combinations can be enforced by some symmetry principle. To canonically normalize the tensor field appearing in the Lagragian $\mathcal{L}^{(T)}$ of eq. (7.61), we pass for simplicity to Fourier space, and define the quantity

$$\chi_{ij} = \frac{\sqrt{2}\,\tilde{\chi}_{ij}}{M_{Pl}\,a\,\sqrt{1+b+\omega_1\,k^2/(a^2\,\Lambda^2)}}\,. \qquad (7.62)$$

[3]The same operators will also modify the scalar sector. Considering for simplicity only the Einstein-Hilbert part plus these four-derivative operators, it can be easily seen that the action for the scalar has the same form of the action for the tensors (7.61) and that the arguments that can be developed for the scalar sector are very similar to the ones we are carring on for the tensors.

Using this tilde quantity $\tilde{\chi}_{ij}$, the Lagrangian, after an integration by parts, acquires a relatively simple form in a quasi-de Sitter universe

$$\mathcal{L}^{(T)} = \frac{1}{2}\left[(\tilde{\chi}_{ij}')^2 - F(k,\eta)\,\tilde{\chi}_{ij}^2\right] \tag{7.63}$$

with

$$\begin{aligned} F(k,\,\eta) \;=\; & \frac{1}{\left(1+b+\frac{\omega_1 k^2}{a^2\,\Lambda^2}\right)^2}\Bigg[-(1+b)^2\,(2-\epsilon)\,a^2\,H^2 \\ & +k^2\,(1+b)\left(1+d-(3-\epsilon)\,\frac{\omega_1 H^2}{\Lambda^2}\right) \\ & +\frac{k^4}{a^2\,\Lambda^2}\,(\omega_1+d\,\omega_1+\omega_2+b\,\omega_2)+\omega_1\omega_2\,\frac{k^6}{a^4\,\Lambda^4}\Bigg]\,. \end{aligned} \tag{7.64}$$

We can now work out some consequences of these results:

- By making the choice $b=-1$, the quadratic terms containing two time derivatives cancel from the action (7.61), and the dynamics is driven by the four derivative operator proportional to ω_1. In a certain sense, the situation can be seen as analogous to what happens in ghost inflation [208], where the leading terms in the gradients of the ghost field vanish, and the next-to-leading contributions in gradients become dominant. The expression for the function F above simplifies considerably:

$$F(k,\,\eta) = \frac{\omega_2}{\omega_1}\,k^2+\frac{(1+d)\,\Lambda^2}{\omega_1}\,a^2\,, \tag{7.65}$$

$$= \frac{\omega_2}{\omega_1}\,k^2-2\,H^2\,a^2+\frac{(1+d)\,\Lambda^2+2\,H^2\,\omega_1}{\omega_1}\,a^2\,. \tag{7.66}$$

  The first term in the right hand side of (7.66) can be recognized as the usual first contribution to the dispersion relation associated with $\tilde{\chi}_{ij}$, characterized by an effective sound speed $c_T^2 = \omega_1/\omega_2$. The second piece is the effective 'mass term' that usually arises in a quasi-de Sitter universe. Then, we have the third contribution, that mimics exactly a mass term with

$$m_{\tilde{\chi}}^2 = \frac{(1+d)\,\Lambda^2+2\,H^2\,\omega_1}{\omega_1}\,. \tag{7.67}$$

  Interestingly this effective mass arises only from the higher derivative terms, with no need to break diffeomorphism invariance! In this sense, 4-derivative contributions can be interpreted as being able to generate mass without an explicit mass parameter. On the other hand, notice that in this case the relation between the canonically normalized tensor field $\tilde{\chi}_{ij}$ and original one χ_{ij} scales as the inverse of the momentum: $\chi_{ij} \propto \tilde{\chi}_{ij}/k$: see eq. (7.62). This typically implies – by the arguments outlined around eq (7.6) – a low cut-off scale when focussing at large scales; on the other hand, this crucially depends on the tensor interactions during inflation, that might conspire in such a way to raise the cut-off. This is an interesting question that we intend to pursue in the future.

- Let us now consider the more general situation with $b \neq -1$, focusing on the large and small scale limits for the function F:

$$F(k,\,\eta) \overset{k\to 0}{\sim} (-2+\epsilon)a^2H^2 + \mathcal{O}(k^2)\,, \tag{7.68}$$

$$F(k,\,\eta) \overset{k\to +\infty}{\sim} \frac{\omega_2}{\omega_1}k^2 + \frac{a^2\Lambda^2}{\omega_1^2}\left[(1+d)\omega_1 - (1+b)\omega_2\right] + \mathcal{O}(k^{-2})\,. \tag{7.69}$$

  No major differences with respect to the standard case arise, apart from the presence of a non-trivial sound speed c_T: the system can be quantized selecting a Bunch-Davies vacuum at very small scales, while at large scales the tensors behave as in a standard quasi-de Sitter universe, with no mass.

This preliminary analysis of the role of operators with higher spatial derivatives shows their possible relevance for characterizing tensor modes, and can find some motivation for example (but not only) in the context of Horava-Lifshitz cosmology (see [209] for a review). It shows that in this set-up a non-unity tensor sound speed c_T can be generated, and that it cannot in general be set to one by a set of transformations of the metric [206].

CHAPTER 8

Breaking Discrete Symmetries

We have seen in the previous Chapter that the breaking of spatial diffeomorphisms together with time diffeomorphisms during inflation can lead to a very interesting phenomenology. Motivated by these considerations, we find interesting to add complexity and explore the subject further along the same line. In particular, we want to exploit the EFTI to find novel operators that can lead to possibly new effects associated with inflationary observables, as non-standard correlations among inflationary perturbations. In [5], on which this Chapter is based, we focus on the interesting class of operators that break discrete symmetries as parity and time-reversal during inflation. Unless discrete symmetries are imposed by hand on the theory under consideration, such operators will normally be generated, for example, renormalization effects: hence it is interesting to explore their consequences.

Parity-violating interactions have been studied in great detail for their consequences in the CMB, starting with [210]. These operators can be associated with the amplification of the amplitude of one of the circular polarization of tensor modes around horizon crossing, leading to distinctive effects associated with TB and EB cross correlations in the CMB [211–214]. Moreover, parity-violating operators can also affect the scalar sector, leading to statistical anisotropies in the bispectrum, or also explain anomalies in the CMB. The realization of models leading to parity violation during inflation and their observational consequences have been motivated by, for example, pseudoscalars coupled to gauge fields [215] or Chern-Simons modifications of gravity [216, 217] (see e.g. [93, 218–238] for a selection of papers discussing both theoretical and observational aspects of parity violation during inflation). To the best of our knowledge, there are no studies on the consequences of operators that contain a single derivative of time coordinate: in absence of better name, we say that these operators break time-reversal symmetry during inflation. In this Chapter, using the model-independent language of the EFTI, we study selected operators that break the aforementioned discrete symmetries and their effects for the dynamics of linearized perturbations around a homogeneous and isotropic Friedmann-Robertson-Walker (FRW) universe. Both in the scalar and tensor sectors, we show that such operators can lead to new direction-dependent phases for the fluctuations and also a small quadrupole contribution to the effective sound speed. A direction-dependent phase does not affect the power spectrum, but can have consequences for higher correlation functions.

8.1 Introduction

The starting point of the discussion is that if spatial diffeomorphisms are broken, in the absence of specific symmetries, one can expect also a small degree of background anisotropy during inflation. We consider a background metric that is decomposed as homogeneous FRW and anisotropic parts, denoted by $\bar{g}^{(0)}_{\mu\nu}$ and $\bar{g}^{(a)}_{\mu\nu}$ respectively, as

$$\bar{g}_{\mu\nu} = \bar{g}^{(0)}_{\mu\nu} + \bar{g}^{(a)}_{\mu\nu} = a^2(\eta) \begin{pmatrix} -1 & \\ & \delta_{ij} \end{pmatrix} + a^2(\eta) \begin{pmatrix} & \beta_i \\ \beta_i & \chi_{ij} \end{pmatrix} , \tag{8.1}$$

where β_i and χ_{ij} are transverse and traceless. We assume from now on that β_i and χ_{ij} are small: $|\beta_i| \ll 1$ and $|\chi_{ij}| \ll 1$. Hence we consider their contributions only at linearized order in our analysis. In other words, we develop a perturbative scheme in terms of the small quantities parameterizing the background anisotropy in the metric and (as we shall see in a moment) in the energy-momentum tensor. We stress that (8.1) is our background metric. On top of it, we will include inhomogeneous perturbations in the next sections. We now consider a background energy-momentum tensor that is able to support our small deformation (8.1) of a FRW background metric. For this aim, we start introducing the following anisotropy parameters that will enter in the energy-momentum tensor:

- A vector θ_i, selecting a preferred spatial direction.

- A shear σ_{ij}, a symmetric, traceless tensor.

To be consistent with the fact that the magnitudes of the anisotropic metric components β_i and χ_{ij} are small, we assume both these anisotropic parameters θ_i and σ_{ij} to be small, and treat them at linearized order in our discussion. We can think of these objects as *vevs* of some fields and, in realistic cosmological situations, at least a mild coordinate dependence is expected. Since we are implementing an EFT approach to describe inflationary fluctuations, we do not need to specify an underlying theory that provides such quantities and the equations of motion for the fields associated with them. This since by hypothesis the perturbations of energy-momentum tensor can be set zero by an unitary gauge choice and do not influence the dynamics of the metric perturbations on which we are focusing our attention. We only need to ensure that the energy-momentum tensor constructed using these quantities satisfies the Einstein equations, order by order in a perturbative expansion in the fluctuations.

In the spirit of the EFTI, the matter action that controls the background energy-momentum tensor breaks both time and space reparametrization invariance, and it is then written as

$$S_m = -\int d^4x\sqrt{-g}\left[\Lambda(\eta) + c_1(\eta)g^{00} + c_2(\eta)\delta_{ij}g^{ij} + d_1(\eta)\theta_i g^{0i} + d_2(\eta)\sigma_{ij}g^{ij}\right] . \quad (8.2)$$

Notice the presence of terms depending on g^{ij} and g^{0i}, that are absent in the standard EFTI (4.16), where spatial diffeomorphisms are preserved. Since the degree of anisotropy is assumed to be small, in what follows we only consider contributions at most linear in θ_i and σ_{ij}, and in the metric deformations β_i and χ_{ij}. Moreover, we neglect the possible spatial dependence of the coefficients in the previous action. The background energy-momentum tensor associated with the action (8.2) is

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S_m}{\delta g^{\mu\nu}} . \quad (8.3)$$

Combined with the Einstein tensor $G_{\mu\nu}$ – which can be constructed straightforwardly from (8.1) – the Einstein equations impose the following relations to be satisfied at the background level, in a linearized expansion for the anisotropy parameters (from now on we set the Planck mass $M_{\rm Pl} = 1$):

$$\begin{aligned} 3\mathcal{H}^2 &= c_1 + 3c_2 + a^2\Lambda , && (8.4)\\ \mathcal{H}^2 - \mathcal{H}' &= c_1 + c_2 , && (8.5)\\ d_1\theta_i &= c_2\beta_i , && (8.6)\\ 2d_2\sigma_{ij} &= \mathcal{H}\chi'_{ij} + \frac{1}{2}\chi''_{ij} + 2c_1\chi_{ij} . && (8.7) \end{aligned}$$

So we learn that in our linearized approximation the background quantity β_i in the metric is controlled by d_1 and the vector θ_i, while χ_{ij} is controlled by d_2 and the shear σ_{ij}. A configuration that solves these equations can lead to a solution with a small degree of anisotropy in the background during a quasi-de Sitter inflationary stage. By choosing appropriately the anisotropic parameters θ_i and σ_{ij} such a configuration can avoid Wald's no-hair theorem [239] and lead to anisotropic inflation.

Wald's theorem states that, under some hypotheses on the energy-momentum tensor, the inflationary expansion rapidly reduces the amplitude of background anisotropies to an unobservable level. The prerequisites behind the theorem are not necessarily satisfied in our case. We can write

$$\mathcal{H}' = \mathcal{H}^2(1-\epsilon)\,, \tag{8.8}$$

where $\epsilon \equiv -a^{-1}H'/H^2$ with $\mathcal{H} = aH$. Substituting this result into (8.5), we find that $c_1 + c_2 \,=\, \epsilon\,\mathcal{H}^2$. Using this information, the background energy-momentum tensor can be decomposed as

$$T_{\mu\nu} \;=\; -\Lambda(\eta)\,\bar{g}_{\mu\nu} + T^{(2)}_{\mu\nu} \tag{8.9}$$

with

$$\begin{aligned} T^{(2)}_{00} &= \epsilon\mathcal{H}^2 + 2c_2\,, \\ T^{(2)}_{0i} &= \left(\epsilon\mathcal{H}^2 - 2c_2\right)\beta_i\,, \\ T^{(2)}_{ij} &= \left(\epsilon\mathcal{H}^2 - 2c_2\right)\left(\delta_{ij} + 3\chi_{ij}\right) + \mathcal{H}\chi'_{ij} + \frac{1}{2}\chi''_{ij}\,. \end{aligned}$$

Wald's isotropization theorem states that anisotropies are rapidly suppressed during inflation if the strong and dominant energy conditions are satisfied:

$$\left(T^{(2)}_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}T^{(2)}\right)t^\mu t^\nu \quad\geq\quad 0 \quad \text{for all time-like vectors } t^\mu\,, \tag{8.10}$$

$$T^{(2)}_{\mu\nu}\hat{t}^\mu\hat{t}^\nu \;\;\geq 0 \qquad \text{for all future-directed, causal vectors } \hat{t}^\mu\,. \tag{8.11}$$

Time-like vectors t^μ satisfy the condition

$$\left(t^0\right)^2 \geq 2\beta_i t^0 t^i + \left(\delta_{ij} + \chi_{ij}\right)t^i t^j\,. \tag{8.12}$$

In our case, the dominant energy condition reads

$$\begin{aligned} &\left(\epsilon\mathcal{H}^2 - 2c_2\right)\left[\left(t^0\right)^2 + 2\beta_i t^0 t^i + \left(\delta_{ij} + \chi_{ij}\right)t^i t^j\right] \\ &+ 4c_2\left(t^0\right)^2 + 2\left(d_2\sigma_{ij} - c_2\chi_{ij}\right)t^i t^j \geq 0\,. \end{aligned} \tag{8.13}$$

In the EFTI scenarios with no breaking of spatial diffeomorphisms or isotropy, $c_2 = 0$, $\sigma_{ij} = 0$ and $\chi_{ij} = 0$. The second line in the above equation would vanish, while the first line would be positive definite, satisfying in this way the dominant energy condition (8.11). In our more general setup, instead, the second line is non-vanishing, and can render the previous quantity negative. Hence, in general the prerequisites underlying Wald's theorem can be expected to be violated in our context based on the EFT of inflation. Such situations can be realized in models of inflation with vector fields [240], or solid inflation, as discussed in the recent literature, see e.g. [241–243].

Hence, as a matter of principle, our approach based on the EFTI can accommodate a model-independent analysis of inflationary models with anisotropic backgrounds (see e.g. [244, 245] for specific models with these properties). On the other hand, the general analysis of such system can be very cumbersome, due to several new operators that can contribute. For the rest of this Chapter, we make some additional simplifying assumptions to remove the background anisotropies and facilitate as much as we can our analysis of fluctuations, yet covering some relevant features that are distinctive of our system with broken spatial diffeomorphism invariance. Our requirements are as follows:

1. We impose a residual symmetry [196],
$$x^i \to x^i + \xi^i(t) \tag{8.14}$$
for an arbitrary time-dependent function ξ^i. Notice that this symmetry invariance is *less restrictive* than spacetime-dependent spatial diffeomorphism. In our context, this residual symmetry is quite powerful. Since the $0i$ component of the metric perturbation transforms non-trivially under this symmetry (see next section), this symmetry eliminates it from our action, if there are no spatial derivatives acting on it. This requires to choose the parameter $d_1 = 0$ in the action (8.2), and consequently (8.6) tells us that the metric anisotropic parameter β^i vanishes:
$$\beta^i = 0\,. \tag{8.15}$$

2. In addition, from now on we set the shear equal to zero,
$$\sigma_{ij} = 0\,, \tag{8.16}$$
and focus on the effects of the vector θ_i only. Setting the shear to zero implies a vanishing source in (8.7) for the background anisotropic tensor χ_{ij}. For simplicity, in what follows we choose the solution corresponding to the configuration,
$$\chi_{ij} = 0\,. \tag{8.17}$$

After imposing these two requirements we obtain an *isotropic* and homogeneous FRW background metric. However, the anisotropic parameter θ_i contributing to the background energy-momentum tensor can be non-vanishing, and as we shall see next, it can play an important role to characterize quadratic operators that break discrete symmetries, in the quadratic action for perturbations.

8.2 Quadratic Action and New Operators

In this Section we discuss how to build a quadratic Lagrangian for the metric fluctuations in our setup. We mainly concentrate on operators that break discrete symmetries during inflation. The operators that we consider in this section are a selection chosen for the most notable phenomenological consequences. We stress that higher derivative symmetry breaking operators – even preserving spatial diffeomorphisms – can also be included, but ours are the leading ones in a derivative expansion given our symmetry choices. We make use of the background vector θ_i introduced in the previous section for constructing our quadratic operators and we work at linearized order on this small quantity.

The linearized perturbations around our isotropic background, $g_{\mu\nu} = \bar{g}_{\mu\nu} + a^2(\eta) h_{\mu\nu}$, can be decomposed into scalar, vector, and tensor sectors[1]:

$$h_{00} = 2A \,, \tag{8.18}$$
$$h_{i0} = S_i + \partial_i B \,, \tag{8.19}$$
$$h_{ij} = 2\varphi\delta_{ij} + 2\partial_i\partial_j E + \partial_i F_j + \partial_j F_i + \gamma_{ij} \,. \tag{8.20}$$

Under the most general diffeomorphism transformations $x^\mu \to x^\mu + \xi^\mu(t, \boldsymbol{x})$, the quantities that appear in the decomposition of $h_{\mu\nu}$ transform as, *with* $\beta^i = \sigma_{ij} = \chi_{ij} = 0$ [246],

$$A \to A - \partial_\eta \xi^0 - \mathcal{H}\xi^0 \,, \tag{8.21}$$
$$S_i \to S_i - \partial_\eta \xi_i^T \,, \tag{8.22}$$
$$B \to B - \partial_\eta \xi^L + \xi^0 \,, \tag{8.23}$$
$$\varphi \to \varphi - \mathcal{H}\xi^0 \,, \tag{8.24}$$
$$E \to E - \xi^L \,, \tag{8.25}$$
$$F_i \to F_i - \xi_i^T \,, \tag{8.26}$$
$$\gamma_{ij} \to \gamma_{ij} \,. \tag{8.27}$$

As we explained, our setup breaks both space and time diffeomorphism invariance, but we impose invariance under the residual symmetry transformation of (8.14) that ensures that the quadratic action for the metric perturbations does not contain contributions proportional to the metric components h_{0i}, if there are no spatial derivatives acting on them. In Appendix C.3 we list the new derivative operators that are allowed by the previous requirements. Here, after discussing the Einstein-Hilbert action and the leading operators that do not contain derivatives – the mass terms – we concentrate on derivative operators that break discrete symmetries. Our derivative operators can be considered as leading derivative corrections to the mass terms that break spatial diffeomorphisms and discrete symmetries in Lorentz violating theories of massive gravity [196–198].

We start with the Einstein-Hilbert action for quadratic fluctuations. Once decom-

[1]We adopt here the same notation of the original paper [5], which differs from the one in the previous Chapter only for variable names and some normalizations. In particular, the changes from (7.7) are: $\psi \to 2A$, $u_i \to S_i$, $v \to B$, $\tau \to 2\varphi$, $\sigma \to E$, $s_i \to F_i$, $\chi_{ij} \to \gamma_{ij}$.

posed into scalar, vector and tensor parts, they read respectively as follows [27]:

$$S^{(s)} = \int d^4x \frac{a^2}{2} \left[-6 \left(\varphi' - \mathcal{H}A \right)^2 - 2(2A + \varphi)\nabla^2\varphi + 4 \left(\varphi' - \mathcal{H}A \right) \nabla^2 \left(B - E' \right) \right] , \tag{8.28}$$

$$S^{(v)} = \int d^4x a^2 \left[-(S_i - F_i')\nabla^2(S_i - F_i') \right] , \tag{8.29}$$

$$S^{(t)} = \int d^4x \frac{a^2}{8} \left[{\gamma'_{ij}}^2 - (\nabla\gamma_{ij})^2 \right] . \tag{8.30}$$

Repeated spatial indices are contracted with δ_{ij}. To this action we can include the mass operators that are allowed by our symmetries:

$$\mathcal{O}_1^{(0)} = -m_1^2 a^4 h_{ij}^2 = -m_1^2 a^4 \left[12\varphi^2 + 2\left(\partial_i F_j\right)^2 + \gamma_{ij}^2 + 8\varphi\nabla^2 E + 4(\nabla^2 E)^2 \right] , \tag{8.31}$$

$$\mathcal{O}_2^{(0)} = -m_2^2 a^4 h_{ii}^2 = -m_2^2 a^4 \left(6\varphi + 2\nabla^2 E \right)^2 , \tag{8.32}$$

$$\mathcal{O}_3^{(0)} = -m_3^2 a^4 h_{00}^2 = -m_3^2 a^4 (4A^2) , \tag{8.33}$$

$$\mathcal{O}_4^{(0)} = -m_4^2 a^4 h_{00} h_{ii} = -m_4^2 a^4 \left(12A\varphi + 4A\nabla^2 E \right) . \tag{8.34}$$

These are the zero-derivative (hence the superscript (0)), leading operators that break diffeomorphism invariance. These operators, and the ones that we meet next, already contain the square root of the metric, and can be included as they stand into the action. For example the operator (8.31) can be included in the action as

$$\Delta S_1^{(0)} = \int d^4x \mathcal{O}_1^{(0)} . \tag{8.35}$$

These mass terms can lead to a non-vanishing anisotropic energy-momentum tensor, that among other things does not respect the adiabaticity condition and leads to non-conservation of the curvature perturbation on super-horizon scales. See [4] for a discussion on this point.

We now consider also some novel single-derivative operators, built with or without the anisotropic vector θ_i, that have the feature to break discrete symmetries in scalar and/or tensor sectors. As discussed in the introduction, there is a rich literature on possible interactions that violate the discrete parity symmetry, and their consequences for the CMB. The novelty of our model-independent approach is the use of EFT for inflation in a context where spatial diffeomorphism invariance can be explicitly broken (see also [100] for a discussion of parity violating operators in an EFT for inflation preserving spatial diffeomorphism invariance). As we discussed, spatial diffeomorphism invariance can be violated in inflationary systems where background fields acquire spatial-dependent background values, as in models with vectors or in solid inflation. If discrete symmetries are not imposed *a priori*, the operators that we consider can be expected to be generated by quantum effects in such inflationary scenarios. For this reason, it is interesting to explore them and their consequences. Here we introduce a couple of such operators, the ones with the most notable phenomenological consequences that will be studied in the next section.

The lowest dimensional, single derivative operator that breaks parity does not involve anisotropic parameters and reads

$$\mathcal{O}_1^{(1)} \,=\, \mu\, a^3 \epsilon_{ijk} \left(\partial_i h_{jm}\right) h_{km} \,=\, \mu\, a^3 \epsilon_{ijk} \left[\left(\partial_i \gamma_{jm}\right) \gamma_{km} - \partial_i F_j \nabla^2 F_k\right] . \tag{8.36}$$

It leads to *parity violation* in the tensor sector, since it is not invariant under the interchange $x^i \to -x^i$. μ is a mass scale we have included for dimensional reasons. In addition, there is another interesting single-derivative operator, built with the background vector θ_i, that contains a single derivative along time:

$$\begin{aligned}\mathcal{O}_2^{(1)} &= \mu\, a^3 \epsilon_{ijk} \theta_i h_{jm} h'_{km} \\ &= \mu\, a^3 \epsilon_{ijk} \theta_i \Big(\gamma_{jm} \gamma'_{km} - F_m \partial_j \gamma'_{km} - F'_m \partial_k \gamma_{jm} - F_j \nabla^2 F'_k \\ &\quad + 2\partial_j F'_k \nabla^2 E + 2\partial_k F_j \nabla^2 E'\Big) .\end{aligned} \tag{8.37}$$

We can say that such an operator breaks *time-reversal* in the tensor sector, since the contributions within the parenthesis are not invariant under a change of sign in the time direction. Notice that in order to build it we need to use the vector θ_i that selects a preferred direction. Recent papers discussed possible phenomenology of scenarios that contain together background anisotropies and parity violation: see e.g. [214, 247, 248]. We will see that such an operator can have interesting consequences for the dynamics of the tensor modes.

We will also include two-derivative operators. Among the many possibilities, we focus on two interesting operators that break discrete symmetries:

$$\begin{aligned}\mathcal{O}_1^{(2)} &= a^2 h'_{ij} \theta_j \partial_k h_{ik} \\ &= -a^2 \theta_j \Big(4\varphi' \partial_j \varphi + 2\varphi' \nabla^2 F_j + \gamma'_{ij} \nabla^2 F_i - 2\varphi \nabla^2 F'_j - 2F'_j \nabla^4 E \\ &\qquad + 4\varphi' \partial_j \nabla^2 E + 4\partial_j \varphi \nabla^2 E' + 4\nabla^2 E' \partial_j \nabla^2 E - F'_i \partial_j \nabla^2 F_i\Big) ,\end{aligned} \tag{8.38}$$

$$\begin{aligned}\mathcal{O}_2^{(2)} &= a^2 h'_{ij} \theta_k \partial_k h_{ij} \\ &= -a^2 \theta_k \Big(12\varphi' \partial_k \varphi + \gamma'_{ij} \partial_k \gamma_{ij} \\ &\qquad + 4\varphi' \partial_k \nabla^2 E + 4\partial_k \varphi \nabla^2 E' + 4\nabla^2 E' \partial_k \nabla^2 E - F'_i \partial_k \nabla^2 F_i\Big) .\end{aligned} \tag{8.39}$$

Notice that, considering their scalar and tensor parts, such operators are not invariant under an (independent) interchange of spatial and of time coordinates. Hence we can say that these operators break both parity and time-reversal, in the tensor as well as in the scalar sectors. In the next section we will discuss their consequences. Other single and two derivative operators that can break discrete symmetries are listed in Appendix C.3.

8.3 Dynamics of linearized fluctuations

We now discuss some consequences of the discrete symmetry breaking operators that we presented in the previous section. We concentrate our attention to the dynamics of linearized fluctuations. Within our approximation of small anisotropy parameter θ_i, we show that vector degrees of freedom do not propagate. Scalar degrees of freedom acquire a direction-dependent phase. Although this phase factor does not have consequences for the scalar power spectrum, nevertheless it might affect higher order correlators. We also show that small direction dependent contributions to the sound speed can arise. At the quadratic level, the most notable consequences occur in the tensor sector, where we find that some of our new operators lead to a chiral amplification of gravity waves. This is more effective than the one first pointed out in [210] discussing parity-breaking operators, because the modes can be continuously amplified during the whole inflationary epoch.

8.3.1 No propagating vector modes

At linear order in the anisotropy parameter θ_i, we can arrange our system such that there are no propagating vector degrees of freedom: the derivative operators of the previous section have been selected, among other things, to ensure this condition. To see this, we include for simplicity a single mass term, proportional to m_1^2, as given by (8.31), plus a combination of the discrete symmetry breaking operators proportional to θ_i that we have introduced in the previous section. The quadratic vector Lagrangian can be expressed as

$$\mathcal{L}^{(v)} = \frac{a^2}{2}\left[\partial_k\left(S_i - F_i'\right)\partial_k\left(S_i - F_i'\right)\right] - 2m_1^2 a^4\left(\partial_i F_j\right)^2 - \theta_i F_j\left(\cdots\right)\,, \tag{8.40}$$

where the dots contain contributions depending on F_k or on scalar and tensor fields, that we do not need to specify for our arguments. In the previous expression, the first part comes from the Einstein-Hilbert term, the second from a mass term, while the third part collects the contribution from the new derivative operators discussed in the previous section. In this context, the vector S_i appears only in the first term of (8.40). It can be readily integrated out, leaving a Lagrangian identical to (8.40) but with the first term missing. The equation of motion for F_i then reads

$$\nabla^2 F_i = \frac{\theta_i}{m_1^2}\left(\cdots\right)\,, \tag{8.41}$$

where again the dots contain contributions of the various fields involved, that we do not need to specify. Substituting (8.41) into (8.40), we find only terms of $\mathcal{O}(\theta_i^2)$ that are negligible within our approximation. Hence, although typically vector modes propagate in our context, at linearized order in θ_i, the vector degrees of freedom are not dynamical and will be set to zero.

8.3.2 Direction-dependent phase in the scalar sector

Let us examine the effects of parity breaking and time-reversal operators in the scalar sector. We consider a quadratic action built in terms of the Einstein-Hilbert contributions, mass terms, and a linear combination of the two-derivative operators $\mathcal{O}_i^{(2)}$ introduced in (8.38) and (8.39). We set the vector perturbations to zero as seen in the previous subsection. This scalar action contains four scalar degrees of freedom: A, B, E and φ. Among them, A and B are non-dynamical and can be integrated out, leaving a scalar Lagrangian for E and φ. We can proceed as done in [4], further solve the equation of motion for the non-dynamical field E, and plug it into the action. We find at linearized order in θ_i a two-derivative operator for φ (already present in our expressions for $\mathcal{O}_i^{(2)}$) that breaks the discrete parity and time-reversal symmetries:

$$\mathcal{L}^{(s)} \supset a^2 \varphi' \theta_i \partial_i \varphi \,. \tag{8.42}$$

Other contributions quadratic or higher in the parameter θ_i can be neglected, as done in the previous subsection for vector fluctuations. The scalar field φ in the unitary gauge is the curvature perturbation $\mathcal{R}$, hence its statistics can be directly connected with observable quantities. Here however we limit our attention to understand how the operator (8.42) modifies the mode function for φ, viz. $\mathcal{R}$. We consider then the action for the canonically normalized field $u = z\mathcal{R}$ with $z \propto a$ during quasi-de Sitter expansion:

$$S^{(s)} = \int d^4x \frac{1}{2} \left[u'^2 - (\nabla u)^2 + \frac{z''}{z} u^2 + 2b_1 \theta_i u' \partial_i u \right] , \tag{8.43}$$

where the last is our new term, weighted by a real coefficient b_1 that for simplicity we consider as constant. Here we do not explicitly discuss the consequence of the mass terms $\mathcal{O}_i^{(0)}$. Such contributions have been already studied for example in [4] and have been shown to lead to anisotropic stress and non-conservation of the curvature perturbation, generalizing the results first pointed out for solid inflation [192].

The equation of motion for the mode function u_k, that follows from (8.43) once converted to Fourier space, results

$$u''_{\boldsymbol{k}} + 2ib_1 \theta_i k_i u'_{\boldsymbol{k}} + \left(k^2 - \frac{z''}{z} \right) u_{\boldsymbol{k}} = 0 \,. \tag{8.44}$$

At early times the new operator proportional to b_1 is subdominant, so a standard Bunch–Davies vacuum can be unambiguously defined. It is convenient to express the mode function $u_{\boldsymbol{k}}$ as

$$u_{\boldsymbol{k}} = e^{-i\theta_i k_i b_1 \eta} u_k^{(0)} \tag{8.45}$$

so that (8.44) becomes

$$u_k^{(0)\prime\prime} + \left(k^2 - \frac{z''}{z}\right) u_k^{(0)} + k^2 \left(b_1 \theta_i \hat{k}_i\right)^2 u_k^{(0)} = 0 \,, \tag{8.46}$$

where $\hat{k}_i \equiv k_i/k$. The last term in the previous expression is quadratic in θ_i, so it can be neglected for consistency with our approximation (but see the comment at the end of this subsection). Doing so we end with the standard evolution equation in a FRW background, and the solution for $u_k^{(0)}$ can be expressed in terms of Hankel functions. On top of this, the complete solution for $u_{\boldsymbol{k}}$ gains a new direction-dependent contribution to the phase proportional to b_1 as in (8.45). Such a configuration is only reliable at linearized order in θ_i, hence on large scales, $k/(aH) \leq 1$. For smaller scales, contributions that are non-linear in θ_i can become large and change the solution: this fact is important when quantizing the system. Note that the power spectrum remains isotropic because (8.45) is different from the standard solution by a direction-dependent phase, which cancels when computing the power spectrum. It is also interesting to interpret the role of this phase in coordinate space, making a Fourier transform of (8.45). One finds that

$$u(\eta, x^i) = u^{(0)}(\eta, x^i + b_1\, \eta\, \theta^i) \,. \tag{8.47}$$

Hence its effect amounts to a time-dependent shift of the argument of the scalar mode function in coordinate space. Such shifts cancel when taking correlation functions among scalar fluctuations, due to the translational invariance of these quantities. On the other hand, they can have non-vanishing physical effects when taking higher order correlations functions between scalar and tensor modes, since the tensor perturbations do not necessarily share the same shifts. It would be interesting to study this topic further.

Let us end this subsection briefly commenting on the last term in (8.46): as we explained above, consistency of our approximations would require to neglect such terms, since at quadratic order in the anisotropy parameter θ_i other contributions of comparable size can arise – for example the coupled terms between scalar, vector and tensor fluctuations – that should be taken into account. Nevertheless, such a particular term would be present, and provide a quadrupole contribution to the scalar sound speed. It would be interesting to study its effects, noticing also that being of positive size it *increases* the amplitude of the sound speed rendering it larger than one. We leave the analysis of this topic to future work.

8.3.3 Chiral phase in the tensor sector

We now explore the consequences of our discrete symmetry breaking operators for the tensor sector. We do not consider the mass terms, which were studied e.g. in [4]. We consider here the effects of the single derivative operators $\mathcal{O}_1^{(1)}$ and $\mathcal{O}_2^{(1)}$ that break parity and time-reversal. The consequences of the two-derivative operator $\mathcal{O}_i^{(2)}$ are, as can be read from the derivative structure of the tensor perturbations in (8.39), identical to the ones discussed in the previous section on the scalar sector, so we do not analyze them here.

The action for tensor fluctuations is

$$S^{(t)} = \int d^4x \frac{a^2}{8} \left[{\gamma'_{ij}}^2 - (\nabla\gamma_{ij})^2 + 2q_1\,\mu\,a\,\epsilon_{ijk}\left(\partial_i\gamma_{jm}\right)\gamma_{km} + 2q_2\,\mu\,a\,\epsilon_{ijk}\theta_i\gamma_{jm}\gamma'_{km}\right], \tag{8.48}$$

where we have assumed the the coefficients q_1 and q_2 are constant dimensionless real parameters: the condition of being real is required by our conventions on the tensor polarizations. The equation of motion for the tensor degrees of freedom results

$$\gamma''_{ij} + 2\mathcal{H}\gamma'_{ij} - \nabla^2\gamma_{ij} - 2q_1\,\mu\,a\,\epsilon_{kmi}\partial_k\gamma_{mj} + 2q_2\,\mu\,a\,\epsilon_{kmi}\theta_k\gamma'_{mj} + 3q_2\,\mu\,a\,\mathcal{H}\,\epsilon_{kmi}\theta_k\gamma_{mj} = 0\,, \tag{8.49}$$

where all indices are contracted with the Kronecker symbol δ_{ij}. Now, we introduce the circular polarization tensor $\mathbf{e}_{ij}^{(\lambda)}(\hat{\boldsymbol{k}})$, with $\lambda = +\ (-)$ corresponding to the right (left) circular polarization, which satisfies the circular polarization conditions[2]:

$$\begin{aligned} \mathbf{e}_{ij}^{(\lambda)}k_j = \mathbf{e}_{ii}^{(\lambda)} &= 0\,, \\ \epsilon_{ilm}\mathbf{e}_{lj}^{(\lambda)}k_m &= i\lambda k\mathbf{e}_{ij}^{(\lambda)}\,, \\ \mathbf{e}_{ij}^{(\lambda)*}\mathbf{e}_{ij}^{(\lambda')} &= 2\delta_{\lambda\lambda'}\,. \end{aligned} \tag{8.56}$$

[2]Following[201, 249] The polarization vector $e_i^{(\lambda)}(\hat{\boldsymbol{k}})$ perpendicular to $\hat{\boldsymbol{k}}$ can be written as

$$e_i^{(\lambda)}(\hat{\boldsymbol{k}}) = \frac{\hat{\theta}_i(\hat{\boldsymbol{k}}) + i\lambda\hat{\phi}_i(\hat{\boldsymbol{k}})}{\sqrt{2}}\,, \tag{8.50}$$

with $\lambda = \pm$. This vector satisfies

$$k_i e_i^{(\lambda)} = 0\,, \tag{8.51}$$

$$e_i^{(\lambda)*}(\hat{\boldsymbol{k}}) = e_i^{(-\lambda)}(\hat{\boldsymbol{k}}) = e_i^{(\lambda)}(-\hat{\boldsymbol{k}})\,, \tag{8.52}$$

$$e_i^{(\lambda)*}e_i^{(\lambda')} = \delta_{\lambda\lambda'}\,, \tag{8.53}$$

$$\epsilon_{ijl}k_i e_j^{(\lambda)} = -i\lambda k e_l^{(\lambda)}\,. \tag{8.54}$$

By means of such a polarization vector we can construct the polarization tensor as

$$\mathbf{e}_{ij}^{(\lambda)} = \sqrt{2}e_i^{(\lambda)}e_j^{(\lambda)}\,. \tag{8.55}$$

It is straightforward to prove (8.56) using (8.51).

γ_{ij} can be Fourier expanded in terms of polarization mode functions as

$$\gamma_{ij}(\eta, \boldsymbol{x}) = \int \frac{d^3k}{(2\pi)^3} \sum_{\lambda=+,-} \left[\gamma_{(\lambda)}(\eta, \boldsymbol{k}) \mathbf{e}_{ij}^{(\lambda)}(\hat{\boldsymbol{k}}) e^{i\boldsymbol{k}\cdot\boldsymbol{x}} + h.c.\right] . \tag{8.57}$$

Then, we find the equation of motion for the mode function $\gamma_{(\lambda)}$, after contracting with $\mathbf{e}_{ij}^{(-\lambda)}$, as

$$\gamma''_{(\lambda)} + 2\mathcal{H}\gamma'_{(\lambda)} + k^2\gamma_{(\lambda)} - 2\lambda q_1\, \mu\, a\, k\gamma_{(\lambda)} - 2\lambda q_2\, \mu\, a\, \hat{\theta}\gamma'_{(\lambda)} - 3\lambda q_2\, \mu\, a\, \mathcal{H}\hat{\theta}\gamma_{(\lambda)} = 0\,, \tag{8.58}$$

where we have introduced

$$\hat{\theta} \equiv \frac{\lambda}{2}\mathbf{e}_{ij}^{(\lambda)}\epsilon_{lmi}\theta_l\mathbf{e}_{mj}^{(-\lambda)}\,. \tag{8.59}$$

Let us discuss the interpretation of $\hat{\theta}$. Using (8.55), we learn that $\hat{\theta} = \lambda e_i^{(\lambda)}\epsilon_{lmi}\theta_l e_m^{(-\lambda)}$. Here $e_i^{(\lambda)}$ and $e_m^{(-\lambda)}$ are two mutually orthogonal vectors, that are both orthogonal to the direction of the three-momentum $\boldsymbol{k}$. This implies that the cross product $e_i^{(\lambda)}e_m^{(-\lambda)}\,\epsilon_{mil}$ is a vector parallel to $\boldsymbol{k}$: contracting it with the vector θ_l and using (8.56) leads to the identity

$$\hat{\theta} = i\theta_i\hat{k}_i\,. \tag{8.60}$$

Notice at this stage the main difference between the operators proportional to q_1 and q_2. The operator proportional to q_2 is associated with time-derivatives of the mode function $\gamma_{(\lambda)}$ or the scale factor, while the operator q_1 with space-derivatives. The effect of the contribution of q_1 corresponds to the known parity-violating operators [210], and produces an enhancement/suppression of tensor mode polarization at horizon crossing only. Such effects are well studied in the literature (see as an example the review [201]) so we will not study them here. Let us instead concentrate on the consequences of the novel operator $\mathcal{O}_2^{(1)}$ proportional to q_2. We rescale the field $\gamma_{(\lambda)}$ in the standard manner as

$$v_{(\lambda)} \equiv \frac{a}{\sqrt{2}}\gamma_{(\lambda)}\,. \tag{8.61}$$

The equation of motion for $v_{(\lambda)}$ is then

$$v''_{(\lambda)} - 2i\lambda q_2\mu a\theta_i\hat{k}_i v'_{(\lambda)} + \left(k^2 - \frac{a''}{a} - i\lambda q_2\mu a\mathcal{H}\theta_i\hat{k}_i\right)v_{(\lambda)} = 0\,. \tag{8.62}$$

Similar to what we did for the scalar sector, it is convenient to rescale

$$v_{(\lambda)} \equiv e^{i\lambda q_2\mu\theta_i\hat{k}_i\int a d\eta}v_{(\lambda)}^{(0)}\,. \tag{8.63}$$

The equation for $v_{(\lambda)}^{(0)}$, at linear order in θ_i and so neglecting quadrupolar effects, reduces to the well-known form

$$v_{(\lambda)}^{(0)\,\prime\prime} + \left(k^2 - \frac{a''}{a}\right)v_{(\lambda)}^{(0)} = 0\,. \tag{8.64}$$

This equation is identical to the standard mode function equation for the tensor perturbations. Furthermore, we notice that, neglecting slow-roll corrections, we can write

$$\int a d\eta = \frac{N_e}{H}\,, \tag{8.65}$$

with N_e being the number of e-folds, and H the value of the Hubble parameter during inflation. So the solution for $\gamma_{(\lambda)}$ is given by

$$\gamma_{(\lambda)} \;=\; \exp\left(i\lambda q_2 \mu \theta_i \hat{k}_i \frac{N_e}{H}\right)\, \gamma_{(\lambda)}^{(0)}\,. \tag{8.66}$$

Therefore, we again find a phase modulation of the wavefunction – like in the scalar sector – but now the coefficient of this phase depends on the chirality of the specific gravity wave one is considering, and on the number of e-folds as well. Such a phase does not influence the power spectrum, since it can be read as a "chiral" translation of the modes when expressed in the coordinate space:

$$\gamma_{(\lambda)}(\eta, x^i) = \gamma_{(\lambda)}^{(0)}\left(\eta, x^i - \lambda q_2 \mu \frac{N_e}{H}\theta^i\right)\,. \tag{8.67}$$

A translation in the coordinates does not affect the power spectrum of the tensor modes, since the power spectrum is translationally invariant. On the other hand, since the translation depends on the chirality, it can affect the bispectra among tensor modes with different chirality (as studied for example in [91, 92]), as well as bispectra between tensor and scalar sectors. We hope to return to investigate these topics in the near future.

CHAPTER 9

Bispectrum Signatures of Diffeomorphism Breaking

We have seen that the EFTI is a powerful tool for obtaining model independent predictions for large classes of inflationary scenarios. It requires only information about the symmetries broken during the inflationary era and the study of general sets of operators that satisfies the symmetry requirements and connects the coefficients of such operators with observable quantities. We have seen in the previous chapters how EFTI methods can be succesfully applied to the exploration of scenarios where, besides time diffeomorphisms, also spatial diffeomorphisms are broken. This Chapter, which is based on [6], develop the subject further, paying particular attention to the specific signatures that this symmetry breaking pattern can leave on the three-point function of primordial fluctuations.

Spacetime diffeomorphisms correspond to the invariance of the theory under the gauge symmetry of General Relativity:

$$x^\mu \to x^\mu + \xi^\mu(t, \boldsymbol{x}) \ , \tag{9.1}$$

for arbitrary functions ξ^μ of the coordinates. During inflation, time-reparameterisation invariance

$$t \to t + \xi(x^\mu) \ , \tag{9.2}$$

is broken. This is due to the existence of an inflationary clock that breaks time diffeomorphisms and controls how much time is left before inflation ends. In models of single field inflation, there is a unique clock and only adiabatic modes can be generated on superhorizon scales. What is controlling the clock dynamics is what sources inflation, and at the same time causes the spontaneous breaking of de Sitter symmetry during the inflationary era. Studying the system at high energies, we can expect that gravity decouples. Time reparameterisation becomes a global symmetry and its breaking gives rise to a massless Goldstone boson π. Its high-energy dynamics faithfully describes the dynamics of the fluctuations of the inflationary clock. This thanks to Goldstone boson equivalence theorems, originally proven in quantum field theory for gauge symmetries [250] and more recently applied to the EFTI [109, 119]. While time reparameterization invariance is certainly spontaneously broken by the source of inflation, it is interesting to explore the possibility that *space diffeomorphisms* are also broken during the inflationary epoch. After all, we are ignorant about what was really happening at the high energy scales and early times characterising inflation. Here we will start from the case where also the symmetry

$$x^i \to x'^i(t,\, x^j)\ , \tag{9.3}$$

is violated during inflation. This can be realised if there are fields that acquire a *vev* depending on spatial coordinates, as scalars $\phi = \phi(x^i)$, or alternatively if there are fields that select a preferred direction, as vector configurations that break rotational invariance. Concrete realisations of both these possibilities can be found, for example, in models where inflationary fields acquire vacuum expectation values along space-like directions, motivated by Solid Inflation [192, 193, 251] or inflationary set-ups involving vector fields (see e.g. [201, 211, 240, 245, 252–257]). A general approach based on the EFTI allows us to study in a model independent way the consequences of this particular symmetry breaking. The phenomenology of these models can be quite different with respect to standard scenarios. They can lead to a blue spectrum of gravity waves, anisotropic features in non-Gaussianities and new couplings among different sectors (scalar-tensor-vector) of fluctuations (see e.g. [212, 244, 258–264]), as we have seen in Chapter 7. Also, at the level of the background, scenarios that break space diffeomorphisms can accommodate models that break isotropy, as we discussed in Chapter 8, possibly related to some of the anomalies in the CMB (see e.g. the recent review [265]). Here we further develop this subject, directly working with a Goldstone action for fluctuations. When working at sufficiently high energies, we can expect that gravity decouples and spatial diffeomorphisms reduce to global space translations and rotations: the breaking of these symmetries lead again to Goldstone bosons. In particular, a scalar "phonon" appears, that we call σ and that is associated with the broken translational invariance. This Goldstone field σ interacts with the Goldstone boson π associated with the breaking of time translations. Such couplings are constrained by non-linearly realised symmetries. They lead to interesting effects, that we analysed for the first time in [6] and that – as we will explain – are not obtained in the standard EFTI, where only time-reparameterisation invariance is broken, or Solid Inflation scenarios.

The main result are two broad physical effects, that are distinctive of our set-up and that we will review in detail:

- The first is specific of the scalar sector and exploits the new couplings between the two scalar Goldstone modes of broken symmetries. We find potentially large contributions to inflationary observables, that can give sizeable effects even in the limit of small breaking of space diffeomorphisms. Such contributions lead to a change in the amplitude of the power spectrum of scalar fluctuations and, more interestingly, direction dependent contributions to the squeezed limit of the scalar and tensor bispectra (in the sense that the bispectra non-trivially depend on the angle between the three wavevectors and can be parametrized with Legendre polynomials $P_{\rm L}$ and amplitude coefficients $c_{\rm L}$ as in [266]). We discuss the physical consequences of these findings, pointing our similarities and differences with previous results in the literature, as Solid Inflation [192], inflationary models involving vector [242, 243] or higher spin field components [267].

- The second effect is instead more specific of tensor sector, and exploits novel possibilities for tensors to couple with themselves and with scalars. Such possibilities are associated with operators that are allowed only if we break also space reparameterisation invariance during inflation. They can lead to a blue spectrum for gravitational waves and to a particular structure for the squeezed limit of tensor-scalar-scalar bispectra, that violates single field consistency relations.

9.1 System under consideration

The study of this system that interests us, where all diffeomophisms (9.1) are broken, can be carried on following different approaches, that we now briefly discuss. The first approach consists on working in what is called the "unitary gauge". One makes the hypothesis that the system breaks diffeomorphism symmetries in such a way that a gauge can be selected, where the fluctuations of the fields sourcing inflation can be set to zero and perturbations are stored in the metric only [1]. This gauge choice makes the counting of the degrees of freedom particularly simple and provides a geometrical interpretation of the dynamical fluctuations. The possibility of making this gauge choice requires that we can work with at most four fields, that acquire vacuum expectation values spontaneously breaking the symmetry. We label them as ϕ^μ, $\mu = 0, .., 3$. We then assume that their own perturbations can be set to zero appropriately selecting the four

[1]Let us emphasise that this condition is not automatically satisfied in all models of inflation. Consider a system of two-fields inflation, ϕ_i with $i = 1, 2$, where both fields contribute to inflation acquiring a time-dependent *vev*. Their perturbations transform non-trivially under time reparameterization, $\delta\phi_i \to \delta\phi_i + \partial_t\phi_i\,\xi^0$. Having a unique function ξ^0 to play with, we don't have enough freedom for setting both $\delta\phi_i$ to zero.

functions ξ^μ in eq. (9.1). This is our definition of unitary gauge (a similar condition was studied in [192]). In this gauge, the dynamical degrees of freedom are stored in the metric: the usual transverse, plus all the *longitudinal* polarisations of the graviton. The resulting theory can be seen as an effective theory of (Lorentz violating) massive gravity in a cosmological spacetime [196, 197]. Besides the two transverse helicities, the longitudinal graviton polarisations can account for at most four more degrees of freedom: two form a transverse vector and two are scalars. Notice that the scalars can both have healthy dynamics around a cosmological spacetime (i.e. one of them does not necessarily correspond to a ghost, as in flat space [202]).

While the unitary gauge is well suited for geometrically understanding the dynamical degrees of freedom, this Chapter we adopt a second approach to study an inflationary system with broken spacetime diffeomorphisms. We interpret the new dynamical modes that arise as Goldstone bosons of broken spacetime symmetries. In order to do so, it is convenient to define our coordinates to be aligned with the background values of the fields that spontaneously break diffeomorphisms. The vacuum expectation values for the symmetry breaking fields are

$$\bar{\phi}^0 = t \, , \qquad \bar{\phi}^i = \alpha \, x^i \, , \tag{9.4}$$

$\bar{\phi}^0$ and $\bar{\phi}^i$ are respectively clock and rulers during inflation. The parameter α controls the breaking of spatial diffeomorphisms: we assume it to be small and we will use it as an expansion parameter. Using the Stückelberg trick, we can restore full diffeomorphism invariance by introducing a set of four fields, π and σ^i, and write the gauge invariant combinations ϕ^μ as

$$\phi^0 = t + \pi \, , \qquad \phi^i = \alpha \, x^i + \alpha \, \sigma^i \, . \tag{9.5}$$

The Stückelberg fields π and σ^i transform under diffeomorphisms such to render the previous combinations gauge invariant. For the system that we consider, σ_i can be decomposed into longitudinal σ_L and transverse components σ_i^T. The longitudinal component σ_L interacts with π, starting already at quadratic level: the interaction among these scalars is the main topic of our work. We make further assumptions: we would like to preserve homogeneity and isotropy, imposing extra internal symmetries on the field configuration [192],

$$\phi^i \to O^i_j \phi^j \, , \qquad \phi^i \to \phi^i + c^i \, , \tag{9.6}$$

where $O^i_j \in SO(3)$. We further assume an approximate shift symmetry $\phi^0 \to \phi^0 + c^0$, which is a technically natural assumption to protect the small time dependence of the coefficients that will appear in the action. Notice that these internal symmetries we impose act on field space. Diffeomorphism invariance of eq. (9.1) acts on coordinate space instead and is spontaneously broken in our system.

With this in mind, we can write – at lowest order in a derivative expansion – the diffeomorphisms invariant action describing our system

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}M_{\rm Pl}^2 R + F(X, Y^i, Z^{ij})\right] , \tag{9.7}$$

where F is an arbitrary function, respecting the internal group of spacetime shifts and rotations (9.6) and g is the determinant of the metric tensor. The building blocks that appear in the function F are the operators:

$$\begin{aligned} X &= \partial_\mu\phi^0\partial_\nu\phi^0 g^{\mu\nu} , \\ Y^i &= \partial_\mu\phi^0\partial_\nu\phi^i g^{\mu\nu} , \\ Z^{ij} &= \partial_\mu\phi^i\partial_\nu\phi^j g^{\mu\nu} , \end{aligned} \tag{9.8}$$

where $i = 1, 2, 3$. In what follows, we discuss the consequences of this form of the action for the dynamics of the Stückelberg fields.

9.2 Inflationary background and fluctuation dynamics

9.2.1 The equations for the background

Our first task is to determine the background evolution. We selected the background values for the fields that break diffeomorphisms to be aligned with the spacetime coordinates, as in eq. (9.4). Such background fields are expected to drive inflation. We now consider what conditions our function F have to satisfy, in order to generate a quasi-de Sitter period of inflationary expansion. We start assuming Friedmann-Robertson-Walker ansatz for the metric

$$g_{\mu\nu} = \mathrm{diag}(-1, a^2, a^2, a^2) , \tag{9.9}$$

where a is the scale factor of the universe. The energy-momentum tensor of our theory reads:

$$\begin{aligned} T_{\mu\nu} &= -\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}} = \\ &= g_{\mu\nu}F - 2\left(F_X\,\partial_\mu\phi^0\partial_\nu\phi^0 + F_{Y^i}\,\partial_\mu\phi^0\partial_\nu\phi^i + F_{Z^{ij}}\,\partial_\mu\phi^i\partial_\nu\phi^j\right) , \end{aligned} \tag{9.10}$$

where the subscripts of F stand for the partial derivatives with respect to the operators (9.8). When computed on the background values of the fields (9.4), the Einstein equations lead to the Friedmann equations (where $H = \dot{a}/a$ and overlines denote quantities evaluated on the background):

$$3M_{\rm Pl}^2 H^2 = \left(-\bar{F} - 2\bar{F}_X\right) , \tag{9.11}$$

$$-2M_{\rm Pl}^2 \dot{H} = -2\left(\bar{F}_X + \frac{\alpha^2}{a^2}\bar{F}_Z\right) . \tag{9.12}$$

On the background, the operators (9.8) read

$$\bar{X} = -1 , \qquad \bar{Y}^i = 0 , \qquad \bar{Z}^{ij} = \frac{\alpha^2 \delta^{ij}}{a^2(t)} . \tag{9.13}$$

Notice that Z^{ij} depends on α – being associated with the breaking of space diffeomorphisms – but it also explicitly depends on time, through the scale factor. The isotropy of the background requires

$$\bar{F}_{Z^{ij}} = \bar{F}_Z \delta_{ij} , \qquad \bar{F}_{Y^i} = 0 . \tag{9.14}$$

Our configuration solves all the background equations of motion (included the ones associated with the fields X, Y^i, Z^{ij}) if the following condition is satisfied [2]:

$$2\alpha^2 \bar{F}_{XZ} = a^2 F_X . \tag{9.15}$$

Using this information, we can express the slow-roll epsilon parameter $\epsilon = -\dot{H}/H^2$ as

$$\epsilon = \frac{3\bar{X}\bar{F}_X - \bar{Z}\bar{F}_Z}{-\bar{F} + 2\bar{X}\bar{F}_X} . \tag{9.16}$$

To obtain a phase of inflation we require $\epsilon \ll 1$, which, barring accidental cancellations, can be naturally obtained if the function F has only a weak dependence on both X and Z:

$$\left(\frac{\mathrm{d}\log F}{\mathrm{d}\log X}, \frac{\mathrm{d}\log F}{\mathrm{d}\log Z}\right) \ll 1 . \tag{9.17}$$

Physically, the slow-roll parameter ϵ is associated with the "ticks" of the inflationary clock. A small ϵ is associated with a configuration characterised by extremely slow ticks of the clock, corresponding to a quasi-de Sitter spacetime. The rhythm of the inflationary clock ticks also varies, and is controlled by a second, independent slow-roll parameter $\eta = \dot{\epsilon}/(\epsilon H)$. A small η ensures that changes in the rate of the inflationary clock occur slowly, so to provide a sufficiently long period of inflation. The condition $|\eta| \ll 1$ requires

$$\left|\eta = 2\epsilon + \frac{6\bar{F}_{XZ} + 2\bar{Z}\bar{F}_Z + 2\bar{Z}^2\bar{F}_{Z^2}}{-3\bar{F}_X - \bar{Z}\bar{F}_Z}\right| \ll 1 . \tag{9.18}$$

[2] Notice that this equation is equivalent to the continuity equation. In the limit $\alpha \to 0$, one consistently obtain a limit $F_X \to 0$, which is the limit of the continuity equation for a $F(X)$ theory when the symmetry $\phi_0 \to \phi_0 + c$ is taken as an exact symmetry.

Having a small value for ϵ implies, at leading order in slow-roll, that the quantities

$$\frac{F_X}{F} \qquad \text{and} \qquad \frac{F_Z}{a^2\,F}\,, \tag{9.19}$$

are constant that do not depend on spacetime coordinates (notice the explicit presence of the scale factor in the second one).

Our set-up does not correspond to a single clock model of inflation. We can identify two independent contributions that control the inflationary clock. The first is associated with the breaking of time reparameterisation, through an explicitly time-dependent background value for the field $\bar{\phi}^0$, as in eq. (9.4). The second is related with the time dependence of the quantity $\bar{Z}^{ij}$ introduced in eq. (9.13). $\bar{Z}^{ij}$ is associated with the breaking of space diffeomorphisms, and is defined in terms of the inflationary rulers $\bar{\phi}^i$ in eq. (9.4). $\bar{Z}^{ij}$ acquires a dependence on the scale factor $a(t)$ (due to the contraction with the spatial part of the metric). A similar fact is found also in Solid Inflation [192]. These two contributions to the energy momentum tensor both independently control the inflationary clock. Hence we are not dealing with a purely adiabatic system. And indeed, we will see next that we can identify two dynamical scalar fluctuations around our background configuration, each corresponding to a Goldstone boson of a different broken symmetry. The non-adiabatic properties of our set-up are quite distinctive though and are the topic of the remaining discussion.

9.2.2 Quadratic action for Stückelberg fields

We now discuss the structure of quadratic fluctuations of the transverse components of the metric and the Stückelberg fields π, σ^i introduced in eq. (9.5) as

$$\phi^0 = t + \pi\,, \qquad \phi^i = \alpha(x^i + \sigma^i)\,, \tag{9.20}$$

as fields restoring diffeomorphism invariance. In principle, besides the (self-)interactions of π and σ^i, also interactions of these fields with the metric components δg^{00}, δg^{0i}, δg^{ij} should be taken into account. However, we can consider the theory at very short distances – corresponding to energy scales $E = k/a \gg H$ – where the effects of gravity backreaction can be neglected. Gravitational modes decouple: the local diffeomorphisms of general relativity reduce to the global symmetries of Lorentz boosts and translations. In this decoupling limit the fields π and σ^i can be interpreted as Goldstone bosons of these broken global symmetries and these degrees of freedom interact only with themselves [3].

After these considerations, let us then focus on the system in a high energy decoupling limit, where the Stückelberg fields can be identified with Goldstone bosons of broken

[3] See Appendix D.1 for a technical discussion of decoupling limit in our set-up.

spacetime diffeomorphisms. We start writing the quadratic actions for these systems [4] at leading order in slow-roll parameters and the parameter α, neglecting gravitational corrections in our decoupling limit:

$$\begin{aligned} S^{(S)} &= \int \mathrm{d}^4x\, a^3\Bigg[\Bigg(-\bar{F}_X+2\bar{F}_{X^2}\Bigg)\dot{\pi}^2+\Bigg(\bar{F}_X+\frac{\alpha^2\bar{F}_{Y^2}}{2a^2}\Bigg)\frac{\partial_i\pi\partial^i\pi}{a^2} \\ + & \quad +\alpha^2\Bigg(\frac{\bar{F}_{Y^2}}{2}-\bar{F}_Z\Bigg)\dot{\sigma}_L^2+\alpha^2\Bigg(\bar{F}_Z+\alpha^2\frac{2\bar{F}_{ZZ}}{a^2}+\alpha^2\frac{2\bar{F}_{Z^2}}{a^2}\Bigg)\frac{\partial_i\sigma_L\partial^i\sigma_L}{a^2} \\ & \quad +\alpha^2\frac{4\bar{F}_{XZ}}{a^2}\sqrt{-\nabla^2}\dot{\pi}\sigma_L-\alpha^2\frac{\bar{F}_{Y^2}}{a^2}\sqrt{-\nabla^2}\pi\dot{\sigma}_L\Bigg]\,, \end{aligned} \tag{9.21}$$

$$S^{(V)} = \int \mathrm{d}^4x\, a^3\Bigg[\left(\frac{\bar{F}_{Y^2}}{2}-\bar{F}_Z\right)\dot{\sigma}_T^i\dot{\sigma}_{T,i}+\left(\bar{F}_Z+2\frac{\bar{F}_{ZZ}}{a^2}\right)\frac{\partial_j\sigma_T^i\partial^j\sigma_{T,i}}{a^2}\Bigg]\,, \tag{9.22}$$

$$S^{(T)} = \int \mathrm{d}^4x a^3\frac{1}{8}\Bigg[M_{\rm Pl}^2\left(\dot{\gamma}_{ij}\dot{\gamma}^{ij}-\frac{\partial_k\gamma_{ij}\partial^k\gamma^{ij}}{a^2}\right)+\alpha^2\left(\frac{\bar{F}_Z}{a^2}+\frac{\alpha^2\bar{F}_{ZZ}}{2a^4}\right)\gamma_{ij}\gamma^{ij}\Bigg] \tag{9.23}$$

where S, V, T represent the scalar, vector and tensor sectors respectively. The fields π has dimension of inverse of mass, and σ is dimensionless. The field σ^i has been decomposed in a (vector) transverse component and a (scalar) longitudinal one:

$$\sigma^i = \sigma_T^i+\frac{\partial^i\sigma_L}{\sqrt{-\nabla^2}}\,. \tag{9.24}$$

As explained above, when all diffeomorphisms are broken, in general six degrees of freedom are dynamical: two scalar fluctuations, the two components of a transverse vector, and the two helicities of a traceless transverse tensor.

The most evident consequence of our set-up is that we now have a system of two interacting scalars, π and σ, Goldstone bosons of two different symmetries. These scalars are coupled through distinctive derivative operators, controlled by the pattern of symmetry breaking in our system. Notice that masses, and non-derivative couplings among the fields, do not arise at our level of approximation, because of the symmetries (9.6) and since we are neglecting gravitational effects. We should also check that the actions (9.21), (9.22) and (9.23) do not lead to dangerous instabilities. For example, the coefficient of the time kinetic operators should have the right sign:

$$-\bar{F}_X+2\bar{F}_X^2 > 0\,, \tag{9.25}$$

$$\bar{F}_{Y^2}-2\bar{F}_Z > 0\,. \tag{9.26}$$

[4] The internal symmetries (9.6) limit the possible operators that can appear in the action. For example, deriving F twice with respect to Z^{ij} gives $\mathrm{d}F/\mathrm{d}Z^{ij}\mathrm{d}Z^{kl} = F_{ZZ}\delta_{ik}\delta_{jl}+F_{Z^2}\delta_{ij}\delta_{kl}$.

At the same time one should impose that the speeds of sound,

$$c_\pi^2 = \frac{\bar{F}_X + \bar{\alpha}^2 F_{Y^2}/2a^2}{\bar{F}_X - 2\bar{F}_{X^2}} \,, \tag{9.27}$$

$$c_\sigma^2 = \frac{\bar{F}_Z + 2\alpha^2 \bar{F}_{ZZ}/a^2 + 2\alpha^2 \bar{F}_{Z^2}/a^2}{F_Z - F_{Y^2}/2} \,, \tag{9.28}$$

$$c_T^2 = \frac{\bar{F}_Z + 2\alpha^2 \bar{F}_{ZZ}/a^2}{\bar{F}_Z + \bar{F}_{Y^2}/2} \,, \tag{9.29}$$

lie in the interval $0 < c_s^2 \leq 1$, where $s \equiv \pi, \sigma, T$. The complete list of relations between the coefficients that one can derive is not particularly illuminating, but we checked that there are regions of the parameter space where there are no dangerous instabilities. The allowed range of parameters will of course be important when trying to compare with cosmological observations, but this topic goes beyond the scope of the present work. Moreover, to avoid excessive time evolution for these quantities during inflation, we can impose that the "slow-roll" parameter associated with the speeds of sound should be small:

$$s_c = \frac{\dot{c}_s}{c_s H} \ll 1 \,. \tag{9.30}$$

This again can give constraints on combinations of parameters in the action, when comparing with observations. Moreover it suggests that, like $\bar{F}_X$ and $\bar{F}_Z/a^2$, also the other coefficients in the action, like for example $\bar{F}_{Y^2}/a^2$ are slowly varying and can be taken as constant. It would be interesting to see what are the consequences of relaxing this assumption and consider non-trivial time dependencies.

9.2.3 The expression for the curvature perturbation

There are two commonly used gauge invariant definitions of curvature perturbations, the curvature perturbation on uniform density hypersurfaces, ζ, and the comoving curvature perturbation $\mathcal{R}$. In single field inflation, on superhorizon scales these quantities are conserved, and coincide up to a sign (see e.g. [188] for a review). This implies that any result obtained in the aforementioned sub-horizon, decoupling limit remains valid also at superhorizon scales, since the curvature perturbation gets frozen there in single-field inflation. As we explained above, our system is, strictly speaking, not single field, and non-adiabatic contributions can arise. They are controlled by a small quantity though – the parameter α that characterises the breaking of space diffeomorphisms (9.4). The expression for the comoving curvature perturbation in the decoupling limit reads for our system[5]:

$$\mathcal{R} = \frac{H}{(-M_{\rm Pl}^2 \dot{H})} \left[\left(-\bar{F}_X + \frac{\alpha^2 \bar{F}_{Y^2}}{2a^2} \right) \pi + \alpha^2 \left(2\bar{F}_Z - \bar{F}_{Y^2} \right) \frac{\dot{\sigma}_L}{\sqrt{-\nabla^2}} \right] . \tag{9.31}$$

In the limit of α small, this expression reduces to the single-field expression

$$\mathcal{R} \, = \, -H \, \pi \, , \tag{9.32}$$

commonly used in the EFTI [109] (where $\mathcal{R}$ is dubbed ζ). In what follows, we will work in a small α limit, so that the definition (9.32) is sufficiently accurate and we can neglect its time-dependence at superhorizon scales. This is also justified because, nevertheless, we will find interesting potentially sizeable corrections to the n-point functions for $\mathcal{R}$ associated with the complete breaking of diffeomorphism invariance.

9.3 The two-point functions

9.3.1 The power spectrum for scalar fluctuations

In this section we consider the consequences of the new symmetry pattern in the second order action of the scalar perturbations. First, let us rewrite the action (9.21) in terms of the normalized fields $\hat{\pi}$ and $\hat{\sigma}$,

$$\hat{\pi} = \sqrt{2 \left(-\bar{F}_X + 2\bar{F}_{X^2} \right)} \pi \, , \qquad \hat{\sigma} = \alpha \sqrt{2 \left(\frac{\bar{F}_{Y^2}}{2a^2} - \frac{\bar{F}_Z}{a^2} \right)} \sigma_L \, , \tag{9.33}$$

$$\begin{aligned} S^{(S)} \;\; = \;\; & \int d^4x \, a^3 \left[\frac{1}{2} \left(\dot{\hat{\pi}}^2 - c_\pi^2 \frac{\partial_i \hat{\pi} \partial^i \hat{\pi}}{a^2} \right) + \frac{1}{2} a^2 \left(\dot{\hat{\sigma}}^2 - c_\sigma^2 \frac{\partial_i \hat{\sigma} \partial^i \hat{\sigma}}{a^2} \right) \right. \\ & \left. + \alpha \lambda_1 \sqrt{-\nabla^2} \dot{\hat{\pi}} \hat{\sigma} + \alpha \lambda_2 \sqrt{-\nabla^2} \hat{\pi} \dot{\hat{\sigma}} \right] , \end{aligned} \tag{9.34}$$

where the speeds of sound are written in eqs. (9.27), (9.28) and

$$\lambda_1 \;\; = \;\; \frac{2\bar{F}_{XZ}/a^2}{\sqrt{\left(-\bar{F}_X + 2\bar{F}_{X^2} \right) \left(\bar{F}_{Y^2}/2a^2 - \bar{F}_Z/a^2 \right)}} \, , \tag{9.35}$$

$$\lambda_2 \;\; = \;\; \frac{-\bar{F}_{Y^2}/a^2}{2\sqrt{\left(-\bar{F}_X + 2\bar{F}_{X^2} \right) \left(\bar{F}_{Y^2}/2a^2 - \bar{F}_Z/a^2 \right)}} \, . \tag{9.36}$$

[5]In the flat gauge the comoving curvature perturbation is defined as $\mathcal{R} = H\delta u$, where δu is the longitudinal component of the perturbed 4-velocity of the fluid [29].

The normalization of the fields have been defined so that the parameters λ_1, λ_2 are constant at leading order in slow-roll. The price to pay is that we leave an explicit factor of a^2 in front of the "kinetic action" for the Goldstone mode σ in eq. (9.34). We can see that, for small values of the parameter α, the interaction terms that mix the two fields can be treated as perturbations on top of a free Lagrangian for the two scalars involved. Thanks to this fact, we can perturbatively compute the spectrum for the fluctuation π by the following procedure. First, we evaluate it at zero order in the parameter α. Then, we compute perturbative corrections in α, using the in-in formalism. This calculation will provide a quantitative way to evaluate how the second Goldstone boson σ affects the properties of the two-point function of π and the curvature perturbation. Physically, we are interested to this question because we have learned that the contribution to curvature perturbation $\mathcal{R}$ is mostly due the field π, in the limit of small values for α (see Section (9.2.3)). On the other hand we will learn that contributions of σ to two and higher point functions of $\mathcal{R}$ can be sizeable even in the limit of small α.

Let us then proceed computing the power spectrum for π. The zeroth order power spectrum is straightforward to obtain:

$$\langle \hat{\pi}_{\vec{k}_1} \hat{\pi}_{\vec{k}_2} \rangle = (2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2) \, \frac{2\pi^2}{k_1^3} \hat{\mathcal{P}}_0 \, , \tag{9.37}$$

where

$$\hat{\mathcal{P}}_0 = \frac{H^2}{4\pi^2 c_\pi^3} \, . \tag{9.38}$$

Using the normalization coefficient (9.33), the power spectrum of the original fields π reads:

$$\hat{\mathcal{P}}_0 = \frac{H^2}{8\pi^2 c_\pi^3 (-\bar{F}_X + 2\bar{F}_{X^2})} = \frac{H^2}{8\pi^2 \, c_\pi \left(-\bar{F}_X - \alpha^2 \bar{F}_{Y^2}/2a^2\right)} \, , \tag{9.39}$$

where we used the definition of the speed of sound (9.27). Taking $\alpha \ll 1$ and using eq. (9.12), this result reduces to the standard result of single-field inflation with only time-diffeomorphism breaking, as in that case $\bar{F}_X = M_{\rm Pl}^2 \dot{H}$:

$$\hat{\mathcal{P}}_0 \, (\alpha \ll 1) \; = \; \frac{H^4}{8\pi^2 \, c_\pi \left(-\bar{F}_X\right)} = \frac{H^2}{8\pi^2 M_{\rm Pl}^2 \epsilon \, c_\pi} \, . \tag{9.40}$$

The effect of the interaction terms in the second order action (9.34) can be now computed using the in-in formalism [34, 160] (see Appendix B). The leading correction to the power spectrum is given by

$$\delta \langle \hat{\pi}_{\vec{k}_1} \hat{\pi}_{\vec{k}_2} \, (\tau) \rangle \;\; = \;\; - \int_{\tau_{\rm min}}^{\tau} d\tau_1 \int_{\tau_{\rm min}}^{\tau_1} d\tau_2 \left\langle \left[\left[\hat{\pi}^{(0)}_{\vec{k}_1} \hat{\pi}^{(0)}_{\vec{k}_2} \, (\tau) \, , \mathcal{H}^{(2)}_{\rm int} \, (\tau_1) \right] , \, \mathcal{H}^{(2)}_{\rm int} \, (\tau_2) \right] \right\rangle \tag{9.41}$$

where $\mathcal{H}^{(2)}_{\rm int}$ is the second order interaction hamiltonian and $\tau_{\rm min}$ corresponds to the time at which the contribution of the long mode starts to become dominant. This correction is represented as mass insertion diagram in Figure 9.1. In our case, from (9.34), we have:

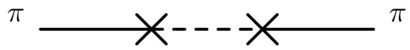


Figure 9.1: Leading diagram for computing the symmetry breaking contributions to $\langle\pi^2\rangle$.

$$\mathcal{H}^{(2)}_{\text{int}}(\tau) = \mathcal{H}^{(2)}_{\text{int},1}(\tau) + \mathcal{H}^{(2)}_{\text{int},2}(\tau) , \tag{9.42}$$

where

$$\mathcal{H}^{(2)}_{\text{int},1}(\tau) = \frac{\alpha\lambda_1}{(H\tau)^3}\int \frac{d^3k}{(2\pi)^3}\,|k|\hat{\sigma}^{(0)}_{\vec{k}}(\tau)\,\hat{\pi}'^{(0)}_{-\vec{k}}(\tau) , \tag{9.43}$$

$$\mathcal{H}^{(2)}_{\text{int},2}(\tau) = \frac{\alpha\lambda_2}{(H\tau)^3}\int \frac{d^3k}{(2\pi)^3}\,|k|\hat{\pi}^{(0)}_{\vec{k}}(\tau)\,\hat{\sigma}'^{(0)}_{-\vec{k}}(\tau) . \tag{9.44}$$

The field operators can be expanded in terms of their Fourier modes

$$\begin{aligned}\hat{\pi}_{\vec{k}} &= u_k\,a_{\vec{k}} + u^*_k\,a^\dagger_{-\vec{k}} ,\\ \hat{\sigma}_{\vec{k}} &= v_k\,b_{\vec{k}} + v^*_k\,b^\dagger_{-\vec{k}} ,\end{aligned} \tag{9.45}$$

where the creation and annihilation operators respect the commutation rules:

$$\left[a_{\vec{k}}, a^\dagger_{-\vec{k}'}\right] = (2\pi)^3\,\delta^{(3)}(\vec{k}+\vec{k}') , \qquad \left[b_{\vec{k}}, b^\dagger_{-\vec{k}'}\right] = (2\pi)^3\,\delta^{(3)}(\vec{k}+\vec{k}') , \quad \left[a_{\vec{k}}, b^\dagger_{-\vec{k}'}\right] = 0 . \tag{9.46}$$

This is because the eigenfunctions for the two scalar modes are the solution of the classical equations of motion, derived from the (free) action that can be read from (9.21). For the field π we have:

$$u''_k - \frac{2}{\tau}u'_k + c_\pi^2 k^2 u_k = 0 , \tag{9.47}$$

where we have used $aH = -1/\tau + \mathcal{O}(\epsilon)$. This has the standard solution (after choosing the Bunch–Davies vacuum and fixing the integration constants):

$$u_{\vec{k}}(\tau) = -\frac{H}{\sqrt{2c_\pi^3 k^3}}(1 + ikc_\pi\tau)e^{-ikc_\pi\tau} . \tag{9.48}$$

In the same way one can write the equation of motion for the field σ:

$$v''_k - \frac{4}{\tau}v'_k + c_\sigma^2 k^2 v_k = 0 , \tag{9.49}$$

whose solution is:

$$v_k(\tau) = -\frac{H^2}{\sqrt{2c_\sigma^5 k^5}}(-3 - 3ikc_\sigma\tau + c_\sigma^2 k^2\tau^2)e^{-ikc_\sigma\tau} . \tag{9.50}$$

Notice that the vacuum wave configuration for the field σ, (9.50), is different with respect to the vacuum configuration for π, eq. (9.48). The difference is due to the presence of the scale factor a^2 in front of the kinetic term for σ, in the quadratic action for fluctuations. Performing the commutators and plugging the interaction hamiltonian (9.43) into in (9.41), we arrive to integrals like:

$$\begin{aligned}\delta\langle\hat{\pi}_{\vec{k}_1}\hat{\pi}_{\vec{k}_2}(\tau)\rangle &= -\frac{4\alpha^2\lambda_1^2}{H^6}\mathrm{Re}\left[\int_{\tau_{\min}}^{\tau}\frac{\mathrm{d}\tau_1}{\tau_1^3}\int_{\tau_{\min}}^{\tau_1}\frac{\mathrm{d}\tau_2}{\tau_2^3}\ \mathrm{k}^2\times\right.\\ &\left.\left(v_{k_1}(\tau_2)v^*_{k_1}(\tau_1)u'_{k_1}(\tau_2)u^*_{k_1}(\tau)\left(u'_{k_1}(\tau_1)u^*_{k_1}(\tau)-c.c\right)\right)\right]\ . \qquad (9.51)\end{aligned}$$

Together with this, there are also the integrals coming from the substitution of $\mathcal{H}_{\mathrm{int},2}$ (9.44), proportional to λ_2^2 and the mixed contributions proportional to $\lambda_1\lambda_2$. These integrals are dominated by the contributions at the times when the modes are outside the horizon, as on sub-horizon scale the oscillatory phases in the mode functions suppress the result[6]. Setting $c_\sigma\simeq c_\pi$ for simplicity, we can take $\tau_{\min}=-1/c_\pi k$ and perform the integral analitically. We find the main contribution in the large scale limit:

$$\frac{\delta\mathcal{P}}{\mathcal{P}_0}=\frac{3\alpha^2\lambda_1 N_k(3\lambda_2-\lambda_1(3N_k+6\gamma_E+11-\log(64))}{c_\pi^2}\,, \qquad (9.52)$$

where γ_E is the Euler gamma and N_k is the number of efolds from the time when the mode k exits the horizon until the end of inflation. Notice that in eq. (9.51) the integrand contains a factor of k^2, associated with the derivative interactions among the modes π and σ. This factor compensates for the non-standard form of the vacuum solution (9.50) for σ, proportional to $k^{-5/2}$ at small scales (and not to $k^{-3/2}$ as it usually happens), leading to a scale invariant correction to the power spectrum.

One can see that even when the breaking of spatial diffeomorphisms is small, $\alpha\ll 1$, the effect of the interaction between the field σ and the field π can still be sizeable, as it is enhanced by the e-fold number N_k. In the limit of large N_k, the dominant correction to the power spectrum scales as

$$\frac{\delta\mathcal{P}}{\mathcal{P}_0}=-\frac{9\,\alpha^2\,\lambda_1^2\,N_k^2}{c_\pi^2}\,. \qquad (9.53)$$

This quantity can be non-negligible even if α is small, since the product $\alpha\,N_k$ can be sizeable (say of order one) being it enhanced by N_k. The logarithmic enhancement of the power spectrum has a similar behavior already met in other set-ups[7], see e.g. [270] or the review [271]. These are novel effects that we first point out in this paper, and are essentially due to the interplay between our two Goldstone bosons during inflation.

[6]The spurious divergences in the UV disappear when slightly deforming the countour of the time integration in the imaginary direction [34].

[7]Notice that, since we are *not* in single field inflation, these enhancement effects *cannot* be 'gauged away' by a rescaling of the curvature perturbation. See e.g. [268] or the reviews [269].

They are physical and arise even if the breaking of space diffeomorphisms is small, since they are enhanced by the e-fold number. Since for small α the curvature perturbation $\mathcal{R}$ is proportional to π through the simple relation (9.32), we can write the following modified expression to the power spectrum for $\mathcal{R}$, induced by a log-enhancement due to the Goldstone boson σ:

$$\mathcal{P}_{\mathcal{R}} \simeq \frac{H^2}{8\pi^2 M_{\rm Pl}^2 \epsilon\, c_\pi} \left(1 - \frac{9\,\alpha^2\,\lambda_1^2\,N_k^2}{c_\pi^2}\right) . \tag{9.54}$$

Although for the case of scalar power spectrum the correction induced by our pattern for breaking spatial diffeomorphisms amounts to a change in the amplitude, for higher point functions we can have more relevant, direction dependent effects, as we will discuss in Section 9.4.

9.3.2 The power spectrum for tensor fluctuations

The effect of breaking spatial diffeomorphisms can have interesting effects also in the tensor sector. This fact has been already explored in [4, 5]. At the level of two-point functions involving tensor modes, the main difference with the standard case is associated with the possibility of assigning a non-vanishing mass to the tensors, since a mass operator is allowed by the absence of diffeomorphism invariance. The resulting set-up can then be considered as an effective theory of (Lorentz-violating) massive gravity during inflation. It would be interesting to find a consistent UV complete theory of massive gravity that allows us to have a large graviton mass during cosmological inflation and a small graviton mass after inflation ends. Nevertheless, in our approach based on EFT we do not need to rely on the existence of any specific UV realisation and simply work with the most general set of operators, order by order in a field expansion.

Normalizing the tensor field as $\gamma_{ij} = \sqrt{2}\hat{\gamma}_{ij}/M_{\rm Pl}$ the quadratic Lagrangian for the two polarization modes of the gravitational fields has the form:

$$\mathcal{L} = \frac{1}{4}\sqrt{-g}\left[\partial_\mu \hat{\gamma}_{ij}\partial^\mu \hat{\gamma}^{ij} - m^2 \hat{\gamma}_{ij}\hat{\gamma}^{ij}\right] , \tag{9.55}$$

where $m^2 = \alpha^2(\bar{F}_Z + \alpha^2 \bar{F}_{ZZ}/2a^2)/M_{\rm Pl}^2 a^2$. To compute the power spectrum, one decomposes h_{ij} into helicity modes,

$$\hat{\gamma}_{ij} = \int \frac{{\rm d}^3 k}{(2\pi)^3} \sum_s \epsilon_{ij}^{(s)} \hat{\gamma}_{\vec{k}}^{(s)} {\rm e}^{{\rm i}\mathbf{k}\cdot\mathbf{x}} , \tag{9.56}$$

where $s = \pm$ is the helicity index and ϵ_{ij} is the polarization tensor. Making the redefinition $\hat{\gamma}_{\vec{k}} = h_{\vec{k}}/a$, the equation of motion (in conformal time $d\eta = dt/a(t)$ and neglecting slow-roll corrections) reads:

$$h''_{\vec{k}} + \left[k^2 - \frac{1}{\tau^2}\left(\nu^2 - \frac{1}{4}\right)\right] h_{\vec{k}} = 0 \ , \tag{9.57}$$

where $\nu = 9/4 - m^2/H^2$. The generic solution (for real ν) is [8]:

$$h_{\vec{k}} = \frac{\sqrt{\pi}}{2} \mathrm{e}^{\mathrm{i}(\nu+1/2)\frac{\pi}{2}} \sqrt{\eta}\, H^{(1)}_\nu(k\eta) \ , \tag{9.58}$$

where $H^{(1)}_\nu$ is the Hankel's function of the first kind. At this point one can easily find the tensor power spectrum

$$\mathcal{P}_T \simeq \frac{2H^2}{\pi^2 M_{\mathrm{Pl}}^2}\left(\frac{k}{k*}\right)^{3-2\nu} \ , \tag{9.59}$$

where:

$$n_T \simeq m^2/H^2 \ . \tag{9.60}$$

Together with the standard -2ϵ contribution to n_T, which can be easily found taking into account the slow-roll dependence in the equation of motion, this shows a non-trivial behaviour of the tensor tilt [4]:

$$n_T = -2\epsilon + \frac{2}{3}\frac{m^2}{H^2} \ . \tag{9.61}$$

As the m^2/H^2 contribution adds with a positive sign, if the mass of the tensor is large enough, then the spectrum could become blue [4, 272]. Moreover this would happen without inconsistencies, since it would not violate the Null Energy Condition, which is related to a change of sign of $\dot{H}$ [110]. The interplay between the negative contribution of ϵ given by the time-diffeomophism breaking part and the positive contribution given by the breaking of space diffeomorphisms is a non-standard feature of this particular symmetry pattern. Notice also that, even though massive tensors are not constant after horizon exit, their evolution is very small as it is controlled by the small parameter α. Indeed, if we take the limit $\alpha \ll 1$, tensor mass becomes completely negligible and we come back to the standard form of the tensor wave function:

$$\gamma_{\vec{k}} = \frac{H}{M_{\mathrm{Pl}} k^{3/2}}(1 + \mathrm{i}k\tau)\mathrm{e}^{-\mathrm{i}k\tau} \ . \tag{9.62}$$

So we find that breaking spatial diffeomorphisms provides qualitatively new effects in the power spectrum of tensor fluctuations. Other interesting effects arise when studying the bispectrum, as we are going to see in the next section.

[8]For imaginary ν one can define a new $\tilde{\nu} = \mathrm{i}\,\nu$ and solve the differential equations in the same way. However in this case the power spectrum would be suppressed by the ration H/m and fall rapidly on very large scales [99].

9.4 The three-point functions

In this section, we examine non-linear, cubic interactions among cosmological fluctuations. In particular, we will study bispectra involving scalar and tensor fluctuations. The study of bispectra is conceptually important since their squeezed limits are very informative for what concern general features of the physics driving inflation. For example, it is known that in models with adiabatic fluctuations only, appropriate squeezed limits of three-point functions involving scalars and tensors, e.g. $\langle \mathcal{R}^3 \rangle$ and $\langle \gamma \mathcal{R}^2 \rangle$, are related to the tilt of the scalar power spectrum [34, 164]. When non-adiabatic interactions are turned on, these consistency relations are violated in a way that depends on the model one considers. We are interested in understanding general features of how the breaking of spacetime diffeomorphisms affects the squeezed limits of three-point functions. We find that the breaking of such symmetry leads to (tunable) quadrupolar contributions to these quantities (corresponding to $c_{\mathrm{L}=2}$ contributions in the parameterisation of [266]) besides "pure" local (monopole $c_{\mathrm{L}=0}$) contributions in the squeezed limit. Similar results have been already found in specific models, as Solid Inflation or models with vector fields [192, 242, 251, 266], but our EFT approach allows to generalize these results and understand them as due to a specific pattern of symmetry breaking. As done in the case of the power spectrum, we are mostly interested on operators that are specifically associated with the simultaneous breaking of time *and* space reparameterization invariance, since these operators can lead to effects that have not been studied so far, when breaking separately time [109] and space [192] diffeomorphisms. Moreover, such effects can be sizeable, rendering them physically interesting even in a limit of small α, the parameter associated with the breaking of spatial diffeomorphisms.

Given these motivations, the operators that we consider are specific of our construction that simultaneously break space and time diffeomorphisms. Up to second order in the parameter α they are the following:

$$\frac{8\alpha^2 \bar{F}_{X^2 Z}}{3a^2} \dot{\pi}^2 \partial_i \sigma^i \, , \tag{9.63}$$

$$\alpha^2 \bar{F}_{Y^2} \left(\dot{\pi} \dot{\sigma}^i \dot{\sigma}_i - \frac{\dot{\pi} \dot{\sigma}^i \partial_i \pi}{a^2} + \frac{\gamma_{ij} \dot{\sigma}^i \partial^j \pi}{a^2} - \frac{\gamma_{ij} \partial^i \pi \partial^j \pi}{a^4} - \frac{\dot{\sigma}^i \partial_j \sigma_i \partial^j \pi}{a^2} + \frac{\partial_j \sigma_i \partial^i \pi \partial^j \pi}{a^4} \right) , \tag{9.64}$$

$$\frac{2}{3} \alpha^2 \bar{F}_{Y^2 X} \left(- \dot{\pi} \dot{\sigma}^i \dot{\sigma}_i + \frac{2 \dot{\pi} \dot{\sigma}^i \partial_i \pi}{a^2} \right) . \tag{9.65}$$

Here we are mostly interested in exploring interesting phenomenological consequences of our approach. On the other hand, the analysis of interactions can also be theoretically important to estimate the *strong coupling scale* at which unitarity bounds are violated in scattering experiments. We do not discuss this argument in the main text, but we develop it in Appendix D.2.

9.4.1 The bispectrum for scalar fluctuations

We start discussing how the operators breaking simultaneously space and time diffeomorphisms affect the squeezed limit of the curvature three-point function. The bispectrum of the curvature perturbation is defined as:

$$\langle \mathcal{R}(\vec{k}_1)\, \mathcal{R}(\vec{k}_2)\, \mathcal{R}(\vec{k}_3) \rangle = (2\pi)^3 \delta^3(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \mathcal{B}(\vec{k}_1, \vec{k}_2, \vec{k}_3)\ . \tag{9.66}$$

As for the two-point function, we compute the contributions of the symmetry breaking operators using a perturbative approach based on the in-in formalism. We take the quantity α controlling the breaking of space diffeomorphisms as a perturbation parameter. In the limit of small α, the curvature perturbation is proportional to the Goldstone mode π, up to small corrections, and is conserved on superhorizon scales (see the discussion in Section 9.2.3 and the expression (9.32) for the curvature perturbations).

We then start with computing the contribution to the tree level bispectrum of the Goldstone π, due to the mixing with the Goldstone σ. We consider the contribution associated with the diagram represented in Figure 9.2. In the limit of small α, the operators that we consider are associated with a mass insertion second order hamiltonian $\mathcal{H}^{(2)}$, given by (9.42), and by the third order operator

$$\frac{\alpha}{\sqrt{2\left(\bar{F}_X + 2\bar{F}_{X^2}\right)}} \left[(\lambda_2 + \lambda_3) \frac{\dot{\hat{\pi}} \partial^i \dot{\hat{\sigma}} \partial_i \hat{\pi}}{\sqrt{-\nabla^2}} - \lambda_2 \frac{\partial_j \partial_i \hat{\sigma} \partial^i \hat{\pi} \partial^j \hat{\pi}}{a^2 \sqrt{-\nabla^2}} - \lambda_4 \dot{\hat{\pi}}^2 \sqrt{-\nabla^2} \hat{\sigma} \right] , \tag{9.67}$$

that we express in terms of normalized fields (9.33). The new parameters λ_3 and λ_4 are defined as

$$\lambda_3 = \frac{4\bar{F}_{Y^2X}/a^2}{3\sqrt{\left(-\bar{F}_X + 2\bar{F}_X^2\right)\left(\bar{F}_{Y^2}/2a^2 - \bar{F}_Z/a^2\right)}}\ , \tag{9.68}$$

$$\lambda_4 = \frac{8\bar{F}_{X^2Z}/a^2}{3\sqrt{\left(-\bar{F}_X + 2\bar{F}_X^2\right)\left(\bar{F}_{Y^2}/2a^2 - \bar{F}_Z/a^2\right)}}\ , \tag{9.69}$$

while λ_1 and λ_2 are defined in (9.35), (9.36). Then, the integral that we need to compute (see Fig 9.2) is

$$\langle \hat{\pi}_{\vec{k}_1} \hat{\pi}_{\vec{k}_2} \hat{\pi}_{\vec{k}_3} \rangle = -\int_{\tau_{\min}}^{0} \mathrm{d}\tau_1 \int_{\tau_{\min}}^{\tau_1} \mathrm{d}\tau_2 \langle 0| \left[\mathcal{H}^{(3)}(\tau_1), \left[\mathcal{H}^{(2)}(\tau_2), \hat{\pi}_{k_1}(\tau) \hat{\pi}_{k_2}(\tau) \hat{\pi}_{k_3}(\tau) \right]\right] |0\rangle\ . \tag{9.70}$$

where the third-order interaction $\mathcal{H}^{(3)}$ can be extracted from (9.67). Let us make some example of the kind of integrals one has to study. Consider taking $\mathcal{H}^{(2)}$ as eq. (9.43) and $\mathcal{H}^{(3)}$ with only the operator $\partial_j \sigma_i \partial^i \pi \partial^j \pi$ from eq. (9.67). Then the form of the integral

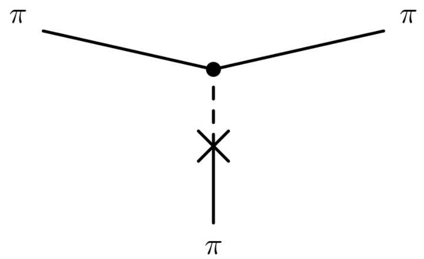


Figure 9.2: Leading diagram for computing the symmetry breaking contributions to $\langle \mathcal{R}^3 \rangle$.

is:

$$\langle \hat{\pi}^3 \rangle = 2\,\mathrm{Re}\Bigg\{ \alpha^2 \frac{-\lambda_1\lambda_2}{\sqrt{2\left(-\bar{F}_X + 2\bar{F}_{X^2}\right)}} \int_{-\infty}^{0} \frac{\mathrm{d}\tau_1}{(-H\tau_1)^2} \int_{-\infty}^{\tau_1} \frac{\mathrm{d}\tau_2}{(-H\tau_2)^3}\, \delta^3(\vec{k}_1+\vec{k}_2+\vec{k}_3)$$
$$(\vec{k}_1\cdot\vec{k}_2)(\vec{k}_1\cdot\vec{k}_3)\left[\lambda_1 \hat{\sigma}_{k_1}\hat{\sigma}^*_{k_1}\hat{\pi}'_{k_1}\hat{\pi}^*_{k_1}\left(\hat{\pi}_{k_2}\hat{\pi}^*_{k_2}\hat{\pi}_{k_3}\hat{\pi}^*_{k_3} - \mathrm{c.c}\right)\right.$$
$$\left. +\lambda_2 \hat{\sigma}'_{k_1}\hat{\sigma}^*_{k_1}\hat{\pi}_{k_1}\hat{\pi}^*_{k_1}\left(\hat{\pi}_{k_2}\hat{\pi}^*_{k_2}\hat{\pi}_{k_3}\hat{\pi}^*_{k_3} - \mathrm{c.c}\right)\right]\Bigg\}\,. \tag{9.71}$$

All the other integrals we compute have a similar structure. To keep computations simple and analytical, we assume that the sound speeds are equal, $c_\pi = c_\sigma$. We recall that, when computing the power spectrum, we were finding log-enhanced contributions. We can expect the same amplification effects to occur here for the case of a squeezed bispectrum when a long wavelength mode ($k \to 0$) is already outside the horizon. Then we can consider one of the momenta $k_1 = k_L \to 0$, while the other two are assumed with equal lenght $k_2 \sim k_3 = k_S$, and evaluate the integral from the time when the mode exits the horizon $\tau = -1/c_\pi k_1$ until the end of inflation. Summing all the terms arising from the operators (9.67) and focusing only on the leading contributions we obtain

$$\langle \hat{\pi}_{\vec{k}_1}\hat{\pi}_{\vec{k}_2}\hat{\pi}_{\vec{k}_3} \rangle = \frac{\alpha^2 H^5}{16 c_\pi^{10} k_L^3 k_S^3} \frac{1}{\sqrt{2\left(-\bar{F}_X + 2\bar{F}_{X^2}\right)}} \times \tag{9.72}$$
$$\left[9c_\pi^2\lambda_1\lambda_4 + (3\lambda_1+\lambda_2)(\lambda_2+\lambda_3)c_\pi^2\hat{S}_2 - 27\lambda_1\lambda_2\hat{S}_1\right] \log\left(\frac{k_L}{k_S}\right)\,,$$

where $\hat{S}_1$, $\hat{S}_2$ refer to scalar products between versors of momenta:

$$\hat{S}_1 = (\hat{k}_1\cdot\hat{k}_2)(\hat{k}_1\cdot\hat{k}_3) + (\hat{k}_2\cdot\hat{k}_3)(\hat{k}_1\cdot\hat{k}_2) + (\hat{k}_1\cdot\hat{k}_3)(\hat{k}_2\cdot\hat{k}_3)\,, \tag{9.73}$$
$$\hat{S}_2 = \hat{k}_1\cdot\hat{k}_2 + \hat{k}_2\cdot\hat{k}_3 + \hat{k}_1\cdot\hat{k}_3\,. \tag{9.74}$$

In the squeezed limit, they reduce to:

$$\hat{S}_1 = -\cos^2\theta \, , \qquad \hat{S}_2 = 1 - 2\cos^2\theta \, , \tag{9.75}$$

where θ is the angle between the long and the short wavelengths modes. From the three-point functions for π, as discussed above, we can extract the three-point function for the curvature perturbation $\mathcal{R}$. Using the normalization (9.33) together with the Friedmann eq. (9.12) and the definition of the speed of sound (9.27), we can write the squeezed limit of the three-point function for curvature pertubation, up to second order in α, as:

$$\begin{aligned} \langle \mathcal{R}^3 \rangle_{k_L \to 0} &\simeq (2\pi)^3 \delta^3(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \alpha^2 \frac{\mathcal{P}(k_L)\mathcal{P}(k_S)}{k_L^3 k_S^3} \frac{\pi^4}{c_\pi^4} \times \\ &\times \Big\{ \left[9c_\pi^2 \lambda_1 \lambda_4 + (3\lambda_1 + \lambda_2)(\lambda_2 + \lambda_3) c_\pi^2 \right] \\ &+ \left[27\lambda_1\lambda_2 - 2(3\lambda_1 + \lambda_2)(\lambda_2 + \lambda_3) c_\pi^2 \right] \cos^2\theta \Big\} \log\left(\frac{k_L}{k_S} \right) \, , \end{aligned} \tag{9.76}$$

where we only write the log-enhanced contributions to this quantity. Let us comment on the physical consequences of this result:

- Even if, in the squeezed limit, the curvature three-point function is suppressed by a factor of α^2 (a parameter that we consider small) it is nevertheless enhanced by a factor $\log(k_L/k_S)$, a quantity that can be of order of the number of e-folds of inflation:

$$\log\left(\frac{k_L}{k_S} \right) \simeq N_k \, . \tag{9.77}$$

  This means that, as for the case of the power spectrum, we find a log-enhanced contribution. The same considerations of Section 9.3.1 hold here: since we have non-adiabatic fluctuations only, these effects are physical and cannot be gauged away with a redefinition of coordinates. Notice that, moreover, the three-point function is enhanced by a large power of the sound speed $(1/c_\pi^4)$ that can also considerably increase its size, in the case that $c_\pi < 1$.

- Interestingly, we find a non-trivial angular dependence of the squeezed limit of the bispectrum. The squeezed bispectrum can be expressed as a sum of two contributions, a monopole plus a quadrupole, with tunable coefficients depending on the parameters λ_i. An angular dependent squeezed bispectrum has been also found in other works in the literature, as Solid Inflation [192], or inflation with vector fields [213, 214, 242, 266, 273, 274], or in models with higher spin fields [267]. In those realizations, the coefficients in front of each contributions (monopole and quadrupole) are fixed by the model. In our set-up based on an EFT approach to inflation, we have been able to identify classes of operators that allow to obtain more general squeezed limits for the bispectrum, with arbitrary coefficients in front of each angular-dependent contribution. We can then identify a possible origin of these effects as due to particular patterns of spacetime diffeomorphism breaking. It would be interesting to find concrete models that obtain our operators from a fundamental set-up.

9.4.2 Tensor-scalar-scalar bispectra and consistency relations

In this subsection we examine how breaking spacetime diffeomorphisms affects the bispectra involving tensor and scalar fluctuations. Observables associated with three-point functions involving tensor modes are becoming particularly interesting, since they are sensitive to the behavior of gravity at the high scales of inflation, and since the future promises advances in observational efforts to detect primordial tensor modes. We start with a brief review on the present theoretical and observational status of our knowledge of tensor-scalar-scalar bispectra; then we pass to discuss new results we obtain within the EFT of inflation with broken spacetime diffeomorphisms.

Motivations

In the next years we will see an increased dedicated effort in trying to detect gravitational waves [186, 275–279]. In light of the amount of precise measurements that are becoming available, it is important to select the best observables that will clarify the physics responsible for driving inflation. Among the predictions of inflation there is one that affects both the CMB and the LSS of the universe: it is the correlation between primordial scalar and tensor perturbations [34]. This tensor-scalar-scalar (TSS) correlation, that is present in all the inflationary models, generates a local power quadrupole in the power spectrum of the scalar perturbations when the wavelength of the tensor mode is much bigger than the scalar one, giving rise to an apparent local departure from statistical isotropy. This observable is a useful quantity to discriminate among the plethora of inflationary models. Moreover this long wavelength tensor mode leaves a precise imprints (dubbed *fossils*) on the observed mass distribution of the universe. The properties of the correlation functions are dictated by symmetries that have a crucial role in constraining the form of correlation functions, and the corresponding consistency relations and their violation [164, 280–286]. In [287] it has been shown that, in the case of single-clock models, that are space-diffeomorphism invariant, a quadrupole contribution to the TSS is cancelled. In particular, a quadrupole contribution arises, proportional to the number of efolds, that is exactly compensated by late-time projection effects that leave a negligible amplitude for the power quadrupole. However, when the conditions of single-clock [99], invariance under space diffeomophism [192, 251, 288], slow-roll evolution [166, 289] are evaded, then the consistency relation is violated, the cancellation is not perfect and we get a possibly detectable amplitude for the local power quadrupole. In [290] it has been shown how the violations of the slow-roll dynamics in non-attractor inflation and of space-diffeomorphism invariance in Solid Inflation bring to the violation of the consistency relation in the TSS correlation function with a consequent enhancement in the local quadrupole. In the case of non-attractor inflation, the limits from CMB on the statistical isotropy [291, 292] constrain the effects on non-observable scales since the transition from the non-attractor phase to the attractor one is found to happen before the time when the current observable universe left the horizon during the inflationary phase. In the Solid Inflation model, instead, the violation of the consistency relation is related to the violation of the diffeomophism invariance and, more interestingly, the observable anisotropic effects are spread on much smaller scales and so potentially detectable in the next future galaxies surveys. In a recent paper [293] the effect of the violation of the consistency relation has been computed in the Quasi-Single-Field model: a two fields model where one of the two has a mass near the Hubble scale H. From the non-trivial four-point function they estimate the size of the galaxy survey necessary to detect the effect of the tensor-scalar-scalar consistency violation.

New results using the EFTI for broken spacetime diffeomorphisms

Our model, violating the invariance under space diffeomorphism, leads to a violations of the consistency relation of the tensor-scalar-scalar correlator, as we are going to discuss. Following [282] the three-point function, in the case of a tensor-scalar-scalar interaction, can be re-defined as

$$\left\langle \gamma^s_{\vec{k}_1} \mathcal{R}_{\vec{k}_2} \mathcal{R}_{\vec{k}_3} \right\rangle \equiv (2\pi)^3 \delta^3(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \left\langle \gamma^s_{\vec{k}_1} \mathcal{R}_{\vec{k}_2} \mathcal{R}_{\vec{k}_3} \right\rangle' , \tag{9.78}$$

where the primed correlator is related to the bispectrum by

$$\left\langle \gamma^s_{\vec{k}_1} \mathcal{R}_{\vec{k}_2} \mathcal{R}_{\vec{k}_3} \right\rangle' \equiv \epsilon^s_{ij} \, \hat{k}^i_2 \, \hat{k}^j_3 \, \mathcal{B}(k_1, k_2, k_3) , \tag{9.79}$$

and ϵ^s_{ij} is the polarization tensor of the tensor mode. Considering the limit in which the momentum of the tensor (k_1) is identified with k_L (long wavelength) and the momenta of the scalars (k_2, k_3) are identified with k_S (short wavelengths), when the consistency relation for the tensor-scalar-scalar correlator is satisfied, the bispectrum can be expressed like

$$\mathcal{B}(k_L, k_S, k_S) \equiv -\frac{1}{2} P_\gamma(k_L) P_\mathcal{R}(k_S) \frac{\partial \ln P_\mathcal{R}(k_S)}{\partial \ln k_S} , \tag{9.80}$$

that, in single-field slow-roll models, translates in a quantity proportional to $(n_s - 4)$, where n_s is the scalar spectral index, as calculated by Maldacena in [34].

In our case from the third order action (9.64) we can read that the tensor-scalar-scalar bispectrum has two contributions, that we can schematically write as

$$\mathcal{B}(k_1, k_2, k_3) = \mathcal{B}_{[\gamma\partial\pi\partial\pi]}(k_1, k_2, k_3) + \mathcal{B}_{[\gamma\partial\dot{\sigma}_L\partial\pi]}(k_1, k_2, k_3) \, . \tag{9.81}$$

These two contributions are associated with our novel operators corresponding to the fourth and third terms in (9.64). They add to the other contributions already present in EFTI and Solid Inflation (that we do not consider here) and can be computed using the in-in formalism. The (normalized) scalar Fourier wavefunctions are defined in (9.45) while for the tensor perturbations we use

$$\gamma_{ij,\vec{k}} = \sum_{s=\pm} \epsilon^s_{ij}(\hat{k}) \left[c^s_{\vec{k}} \, \gamma_k + (c^s)^\dagger_{-\vec{k}} \, \gamma^*_k \right] \, , \tag{9.82}$$

where "s" represents the two polarizations of the tensor and the creation and annihilation operators respect the following commutation relation

$$\left[c_{\vec{k}}, c^\dagger_{-\vec{k}'} \right] = (2\pi)^3 \, \delta^{(3)}(\vec{k} + \vec{k}') \, \delta_{ss'} \, . \tag{9.83}$$

The scalar wave functions for the two scalar goldstones are given by (9.48) and (9.50) while for the tensor we can take the standard expression (9.62), since α^2-correction to the wave function would be subleading when considered in this interactions[9], that are already proportional to α^2.

The effect of the long wavelenght tensor mode on the two scalars is encoded in the squezeed limit ($k_L \ll k_S$) of the bispectrum $\langle \gamma_{k_L} \pi_{k_S} \pi_{k_S} \rangle$. The first contribution that we obtain can be computed at tree-level, following [34]

$$\langle \gamma_{k_L} \hat{\pi}_{k_S} \hat{\pi}_{k_S}(\tau_0) \rangle = -\, i \int_{\tau_{min}}^{\tau_0} d\tau \left\langle \left[\gamma_{\vec{k}_L}(\tau_0) \hat{\pi}_{\vec{k}_S}(\tau_0) \hat{\pi}_{\vec{k}_S}(\tau_0) \, , \mathcal{H}^{(3)}_{\gamma\partial\pi\partial\pi}(\tau) \right] \right\rangle \, , \tag{9.84}$$

and it gives

$$\mathcal{B}_{[\gamma\partial\mathcal{R}\partial\mathcal{R}]}(k_L, k_S, k_S) = \frac{H^2}{M^2_{Pl}} \frac{\alpha^2}{(F_X - 2F_{X^2})^2} \frac{F_{Y^2}}{a^2} \frac{3H^4}{4c^5_\pi} \left(\frac{1}{k^3_S \, k^3_L} \right) \, . \tag{9.85}$$

Rewriting this contribution in terms of the curvature and tensor power spectra, we find a violation of the consistency relation in the tensor-scalar-scalar bispectrum. This since our result is proportional to the quantity F^2_Y that is *not related* to the scalar spectral tilt; moreover it is not associated with the other observables met so far, so we do not have bounds on its size, although for naturalness reasons we do not expect it to be large. Let us also emphasize that such contribution to the TSS bispectrum is a distinctive feature of our set-up that simultaneously breaks time and space diffeomorphisms.

[9]The effects of modified wavefunctions could be interesting, in principle, when considered in the other bispectra which are not proportional to the small α^2, like e.g. the standard $1/c^2_s$ bispectrum.

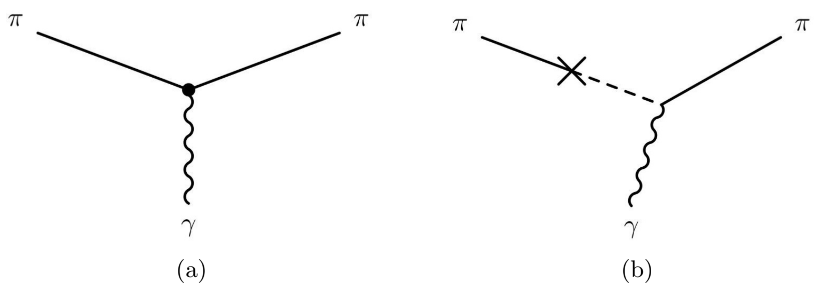


Figure 9.3: Leading consistency violating contributions to the TSS bispectrum $\langle\gamma\pi\pi\rangle$

Interestingly, this is *not* the dominant contribution to the bispectrum: from the second interaction term in eq. (9.81) we have

$$\langle\gamma_{k_L}\hat{\pi}_{k_2}\hat{\pi}_{k_3}\rangle = -\int_{\tau_{min}}^{0} \mathrm{d}\tau_1 \int_{\tau_{min}}^{\tau_1} \mathrm{d}\tau_2 \left\langle \left[\mathcal{H}^{(3)}(\tau_1), \left[\mathcal{H}^{(2)}(\tau_2), \gamma_{k_L}(\tau)\hat{\pi}_{k_2}(\tau)\hat{\pi}_{k_3}(\tau)\right]\right]\right\rangle , \tag{9.86}$$

where $\mathcal{H}^{(2)}$ is given by (9.42) and

$$\mathcal{H}^{(3)}_{[\gamma\partial\dot{\hat{\sigma}}\partial\hat{\pi}]} = -\frac{2\alpha}{M_{Pl}}\lambda_2 \int \mathrm{d}^3x\, a^3\, \frac{\gamma_{ij}\partial^i\hat{\sigma}'\partial^j\hat{\pi}}{\sqrt{-\nabla^2}} \,. \tag{9.87}$$

Performing the integral, considering the limit in which the tensor momentum $k_L \to 0$ is much smaller than the scalar momenta k_S, the result reads:

$$\mathcal{B}_{[\gamma\partial\dot{\sigma}\partial\pi]}(k_L, k_S, k_S) = \frac{\alpha^2\lambda_2\lambda_1}{8M_{Pl}}\frac{H^4}{c_\pi^5}\left(\frac{1}{k_S^3\,k_L^3}\right)\log\left(\frac{2c_\pi k_S}{k_L}\right) . \tag{9.88}$$

This contribution is the dominant violating contribution to the three-point tensor-scalar-scalar correlation function. Indeed, although it is suppressed by a small parameter α, it has a log-enhancement of the same kind we studied in the previous sections, that can be of the order of the number of e-folds. Going back to the original correlator $\langle\gamma\pi\pi\rangle$, and considering only the leading contribution, rewriting it in terms of the curvature perturbation we find

$$\mathcal{B}_{[\gamma\partial\mathcal{R}\partial\mathcal{R}]}(k_L, k_S, k_S) \supset \frac{\alpha^2}{M_{Pl}^2}\frac{\lambda_2\lambda_1}{(-F_X + 2F_{X^2})}\frac{H^6}{4c_\pi^5}\left(\frac{1}{k_S^3\,k_L^3}\right)\log\left(\frac{2c_\pi k_S}{k_L}\right) . \tag{9.89}$$

or in terms of the scalar and tensor power spectra

$$\mathcal{B}_{[\gamma\partial\mathcal{R}\partial\mathcal{R}]}(k_L, k_S, k_S) \supset P_\gamma(k_L)P_\mathcal{R}(k_S)\frac{\alpha^2\lambda_1\lambda_2}{4c_\pi^2}\log\left(\frac{2c_\pi k_S}{k_L}\right) , \tag{9.90}$$

and this our final result.

As anticipated, when a long wavelength tensor mode correlates with the density (scalar) fluctuations in the tensor-scalar-scalar squeezed bispectrum a local power quadrupole is generated. This contribution, that appears like a departure from statistical isotropy, shows an infrared-divergent behaviour, that becomes negligible $\mathcal{O}\left(\frac{{k_L}^2}{{k_S}^2}\right)$ when late time projection effects are taken into account [287] in the case when the consistency relation is satisfied, but *not* in our case. The local quadrupole Q enters in the power spectrum as anisotropic contribution

$$P_{\mathcal{R}}(\vec{k}_S)|_{\gamma(\vec{k}_L)} = P_{\mathcal{R}}(\vec{k}_S)\left[1 + \mathcal{Q}^p_{ij}(\vec{k}_L)\,\hat{k}^i_S\,\hat{k}^j_S\right] \,, \tag{9.91}$$

and it is defined as the ratio between the consistency-relation-violating contribution of the tensor-scalar-scalar bispectrum $\mathcal{B}_{\rm cv}(k_L, k_S, k_S)$ and the power spectra of the scalar and tensor modes. Estimating the variance of the quadrupole, that is the observable quantity, it is possibile to extract informations about the parameters of the theory. In our case its value is not so informative in putting constraints on the model with respect to the previous observables.

On the other hand a long wavelength tensor mode can leave "*fossil*" imprints also on the Large Scale Structure. In this case a tensor mode with wavelength smaller than our observable universe is considered and from an estimator for the tensor power spectrum and its variance it is possibile to extract informations on the minimum size of the galaxy survey on which the tensor can be detected. We report an estimate of the survey size in Appendix E deserving a careful parameter space analysis of the theory in future work. From the estimate we can see that in the next galaxy survey, like EUCLID or even better in 21-cm will be possibile to put bounds and test our theory. So we want to emphasize that even though some (null) searches for power asymmetry in the CMB [291, 294] and Large Scale Structure [295] have already been done, much effort is needed because we have seen how this signatures becomes important in order to rule out inflationary models and also to give informations on the pattern of symmetries in the early universe.

Part IV

Conclusions

CHAPTER 10

Final Considerations

Year after year, Cosmology enters more and more into the era of high-precision experiments. In particular, the past three years of early-Universe Cosmology have been marked by the analysis of the *Planck* CMB data [13, 43, 44]. The window on the inflationary physics opened by COBE [41] and WMAP [42] now shows results with unprecedented accuracy in the field of primordial perturbations. Among the many scientific achievements, one of the most fascinating one is the precision that the measurements of primordial non-Gaussianity have reached. They can now be successfully used together with other observables to disentangle inflationary models, put stringent bounds and hopefully, one day, choose the best candidate. At the moment, no primordial non-Gaussianity has been discovered: this means that the fields active during inflation were weakly coupled and non-linearities were small. Inflationary models beyond the simplest slow-roll scenario can generically predict non-Gaussianity with appreciable size, therefore this non-detection means selection in the space of the possible inflationary theories. Given the powerful experimental tests we can now use, the theoretical and observational study of the many possible effects that deviates from the simplest cases of single-field slow-roll inflation is of fundamental importance, in order to understand the physics of the Early Universe.

In this Thesis, we have taken this path: exploring inflationary perturbations with the aim of identifying and understanding possible departures from the simplest scenarios. A first example has been given in Chapter 3, where we investigated the role of a departure from Einstein gravity in the dynamics of fluctuations. We have shown that this could leave potentially measurable effects, in the form of non-Gaussianity in a quasi-local configuration.

In the following Chapters, we exploited the techniques of the effective field theory [109], as it can be used to derive general model-independent conclusions on the physics of inflationary perturbations, without relying on particular UV realizations of the inflationary models. First (Chapters 5 and 6), we have followed the hints given by the apparent "glitches" in the power spectrum of the CMB [43]. A possible explaination could be the presence of "features" in the potential of the inflaton, which can temporarily deviate from a simple slow-roll evolution and leave imprints on the dynamics of fluctuations [124–134]. We have reformulated the problem with the language of the EFT [1]: the starting consideration is that on very small time-scales the background evolution could be very different from de Sitter, as long as the deviation is small enough to preserve inflation and soon comes back to the attractor solution. As a step feature in the potential of the inflaton translates into a similar feature in the slow-roll parameter $\epsilon = -\dot{H}/H^2$, we can describe these models in the EFT giving a specific form to the time-variation of the Hubble parameter and its derivatives. This is valid in the case of a very small and very sharp step. Here, "small" means that the total deviation of the slow-roll parameter must be controlled by a parameter ϵ_{step}, which is indeed related to the height of the step, while "sharp" means that the characteristic time-scale of the variation should be much smaller than the characteristic time, $\Delta t = H^{-1}$, of inflationary evolution. Under these assumptions, it is possibile to analytically compute the effects of features in the power-spectrum and bispectrum. These effects are larger for modes still inside horizon at the time of the feature. Our technique also allows for a straightforward generalization to include possible features in other coefficients of the EFT Lagrangian. Very interestingly, we found that in this case, at the level of the three-point function, the most interesting scenario is the one of a feature in the speed of sound. Finally, the study of the energy-scale of the modes most affected by non-linear interactions has allowed us to put also a strong upper bound on the sharpness of the step, which comes from the requirement of validity of a perturbative treatment [2]. This severely restricts the space of parameters allowed for models with sharp features and suggests that the exact limit of an infinitely sharp step is theoretically inconsistent. Moreover, this bound can be used to compare the ratio of the signal-to-noise ratio for the three-point function to the one of the two-point function. Our result is that, within the range of validity of the effective approach, the two-point function has the highest signal-to-noise ratio.

One of the most interesting aspects of the EFT of inflation is its use of symmetry principles. Indeed, the theory of perturbations in standard single-field models of inflation is the theory of fluctuations around a FRW background, which spontaneously breaks time-diffeomorphisms invariance [109]. However, nothing forbids *a priori* that also spatial diffeomorphisms could be broken during inflation. This is the theoretical motivation behind Chapters 7, 8 and 9. If all diffeomorphisms are broken, in general more than one degree of freedom is dynamic and a plethora of effects becomes possible. We decided to focus mainly on the tensor and scalar sector. Our first result [4] is that the reduced symmetry allows the presence of masses for tensor perturbations, which can yield blue-tilted power spectra, without violating the null energy condition. The presence of a mass is also responsible for the non-conservation of super-horizon fluctuations, even in the presence of only one scalar perturbation. Along the same line, we have also studied the effect of a selection of operators that break discrete symmetries, such as parity and time reversal [5]. Both in the scalar and tensor sectors, we have shown that such operators can lead to a new direction-dependent phase for modes involved. Such a directional phase does not affect the power spectrum, but could have consequences for higher correlation functions. Moreover, a small quadrupole contribution to the sound speed can be generated.

The natural following step has been the study of non-linearities in the context of full-diffeomorphism breaking [6]. We built an action describing the physics of Goldstone bosons associated with our symmetry breaking pattern, where time and space diffeomorphisms are broken, though preserving homogeneity and isotropy of the background. In our scenario we find two scalar Goldstone bosons: one scalar π associated with the breaking of time reparameterisation, and one scalar σ – playing the role of a phonon – associated with the breaking of space translations. We discussed observables relative both to scalar and tensor sector, associated with two- and three-point functions among fluctuations. The scalar bispectrum receives new direction-dependent contributions in the squeezed limit, because of the interactions between the two Goldstone bosons. Scalars can also couple to tensor perturbations and generate a particular structure for the squeezed limit of tensor-scalar-scalar bispectra, that violate single field consistency relations and can lead to distinctive observable signatures.

Following the points highlighted in this work, one can clearly see that a long road has still to be walked until we will be finally sure of the physical mechanisms behind inflation and inflationary perturbations. In particular, we find that the observational power Cosmology has achieved suggests and requires a careful study of all the possible effects on inflationary perturbations, in order to hunt down departures from the simplest models and hints of new physics. We found the exploration of diffeomorphism breaking during inflation very interesting, full of new phenomenology and particularly interesting for the role that symmetries and broken symmetries play. This exploration is just at the beginning and many are the possible future directions. At the theoretical level, it would be interesting to find examples of inflationary models that break all spacetime diffeomorphisms and then can concretely realize the new observable consequences that we pointed out using an EFT approach. Finding explicit realizations of such set-ups would help also to understand what happens after inflation and possibly find a dynamical mechanism to recover space diffeomorphism invariance. At the observational level, more work is needed to fully characterize the properties of n-point functions in these scenarios – possibly not only three but also higher point functions. Given the present and future experimental effort aimed to measure gravitational waves, it is important to develop the subject further also in this direction, for example studying all the non-Gaussian effects that also the tensor sector could receive. Moreover it would be interesting to find distinctive consistency relations associated with this symmetry breaking pattern or new observables that specifically test particular features of breaking spatial diffeomorphism.

Part V

Appendix

APPENDIX A

Quantization

In this Appendix, we review some details of the quantization of a scalar fluctuation in de Sitter, that we have used in Section 1.3. For simplicity, let us follow the example we have already discussed in the main text: a spectator scalar field $\delta\phi$ in a de Sitter stage of expansion of the Universe. We will also take the field as massless. The action is:

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}\dot{\delta\phi}^2 - \frac{(\partial_i\delta\phi)^2}{a^2}\right] . \tag{A.1}$$

The system can be quantized with the creation and annihilation operators,

$$\delta\phi_{\boldsymbol{k}} = u_k(t)a_{\boldsymbol{k}} + u_k^*(t)a^\dagger_{-\boldsymbol{k}} , \tag{A.2}$$

which satisfy the usual commutation relations:

$$[a_{\boldsymbol{k}},\, a_{\boldsymbol{k}'}] = 0 \qquad \left[a_{\boldsymbol{k}},\, a^\dagger_{\boldsymbol{k}'}\right] = (2\pi)^3\delta(\boldsymbol{k}-\boldsymbol{k}') . \tag{A.3}$$

The mode function u_k in conformal time follows the classical equation of motion:

$$(au_k)'' + \left(k^2 - \frac{a''}{a}\right)(au_k) = 0 . \tag{A.4}$$

In de Sitter we have $a = -1/H\tau$ and the above equation has the exact solution:

$$u_k = c_1(1+ik\tau)^{-ik\tau} + c_2(1-ik\tau)e^{ik\tau} . \tag{A.5}$$

The integration constants c_1 and c_2 need to be fixed. First, from the uncertainty principle,

$$\pi_{\boldsymbol{k}} = \frac{\delta S}{\delta(\dot{\delta\phi}_{\boldsymbol{k}})} = a^3\dot{\delta\phi}_{-\boldsymbol{k}} , \qquad [\delta\phi_{\boldsymbol{k}},\, \pi_{\boldsymbol{k}'}] = i(2\pi)^3\delta(\boldsymbol{k}-\boldsymbol{k}') , \tag{A.6}$$

we obtain the normalization condition:

$$a^2(u_k u_k^{*\prime} - u_k' u_k^*) = i \ , \tag{A.7}$$

which translates into a condition on the integration constants:

$$c_1 c_1^* - c_2 c_2^* = \frac{H^2}{2k^3} \ . \tag{A.8}$$

As we are in a time-dependent background, this condition is not sufficient to determine the mode function and reflects the intrisic ambiguity in choosing the vacuum state of the system. To fix a vacuum state,

$$a_{\boldsymbol{k}}|0\rangle = 0 \ , \tag{A.9}$$

we aim to find the mode function that minimizes the Hamiltonian at $\tau \to -\infty$, so that the true physical vacuum corresponds to the lowest energy state at initial time. The Hamiltonian here is:

$$\hat{H} = \frac{1}{2}\left[a^4 \delta\dot{\phi}^2 + a^2(\partial_i \delta\phi)^2\right] \ . \tag{A.10}$$

Using the solution of the equation of motion (A.5) at time $\tau \to -\infty$ one finds:

$$\hat{H} \simeq \frac{1}{2H^2}\left(c_1 c_1^* + c_2 c_2^*\right) k^4 \ . \tag{A.11}$$

Together with the constraint (A.8), one finally concludes that:

$$c_1 = \frac{H^2}{\sqrt{2k^3}} \ , \qquad c_2 = 0 \ . \tag{A.12}$$

Physically, this confirms the expectation that the canonically normalized field au_k for $\tau \to -\infty$ should have the same solution as in Minkowski, since a mode far within the horizon effectively lives in flat spacetime.

APPENDIX B

The In-In Formalism

The in-in formalism [160, 296, 297] is the appropriate tool for the calculation of cosmological correlation functions. This problem is quite different from the more familiar one in quantum field theory as:

- we are not interested here in the calculation of S-matrix elements, but rather in evaluating expectation values of products of fields at a fixed time;
- conditions are not imposed on the fields at both very early and very late times, as in the calculation of S-matrix elements, but only at very early times, when the wavelength is deep inside the horizon and according to the Equivalence Principle the interaction picture fields should have the same form as in Minkowski spacetime;
- although the Hamiltonian H that generates the time dependence of the various quantum fields is constant in time, the time-dependence of the fluctuations in these fields are governed by a fluctuation Hamiltonian H with an explicit time dependence.

In this Appendix we will briefly review the formalism, following [160]. Consider a general Hamiltonian system, with canonical variable $\phi(\boldsymbol{x}, t)$ and conjugate $\pi(\boldsymbol{x}, t)$ satisfying the commutation relations

$$[\phi(\boldsymbol{x}, t), \pi(\boldsymbol{y}, t)] = i\delta^3(\boldsymbol{x} - \boldsymbol{y}) \ , \qquad [\phi(\boldsymbol{x}, t), \phi(\boldsymbol{y}, t)] = 0 = [\pi(\boldsymbol{x}, t), \pi(\boldsymbol{y}, t)] \ , \tag{B.1}$$

and the equation of motion:

$$\dot{\phi}(\boldsymbol{x}, t) = [H[\phi(t), \pi(t)], \phi(\boldsymbol{x}, t)] \ , \qquad \dot{\pi}(\boldsymbol{x}, t) = [H[\phi(t), \pi(t)], \pi(\boldsymbol{x}, t)] \ . \tag{B.2}$$

The Hamiltonian H does not explicitly depends on time. Now we expand around the solution of the classical equation of motion:

$$\phi(\boldsymbol{x}, t) = \phi_0(\boldsymbol{x}, t) + \delta\phi(\boldsymbol{x}, t) \ , \qquad \pi(\boldsymbol{x}, t) = \pi_0(\boldsymbol{x}, t) + \delta\pi(\boldsymbol{x}, t) \tag{B.3}$$

where

$$\dot{\phi}_0(\boldsymbol{x},t) = \frac{\delta H[\phi_0(t),\pi_0(t)]}{\delta \pi_0(\boldsymbol{x},t)} \qquad \dot{\pi}_0(\boldsymbol{x},t) = -\frac{\delta H[\phi_0(t),\pi_0(t)]}{\delta \phi_0(\boldsymbol{x},t)} \ . \tag{B.4}$$

The perturbations satisfy the same commutation relations as the total fields, as the classical solutions, being c-number, commutes with everything:

$$[\delta\phi(\boldsymbol{x},t),\delta\pi(\boldsymbol{y},t)] = i\delta^3(\boldsymbol{x}-\boldsymbol{y}) \ , \qquad [\delta\phi(\boldsymbol{x},t),\delta\phi(\boldsymbol{y},t)] = 0 = [\delta\pi(\boldsymbol{x},t),\delta\pi(\boldsymbol{y},t)] \ . \tag{B.5}$$

Expanding the Hamiltonian H in powers of the fluctuations,

$$\begin{aligned} H[\phi(t),\pi(t)] \;&=\; H[\phi_0(t),\pi_0(t)]+ \\ &+\left[\frac{\delta H[\phi_0(t),\pi_0(t)]}{\delta\phi_0(\boldsymbol{x},t)}\delta\phi_0(\boldsymbol{x},t)+\frac{\delta H[\phi_0(t),\pi_0(t)]}{\delta\pi_0(\boldsymbol{x},t)}\delta\pi(\boldsymbol{x},t)\right]+ \\ &+\tilde{H}[\delta\phi(t),\delta\pi(t);t] \ , \end{aligned} \tag{B.6}$$

we find terms of zeroth and first order in the perturbations, plus a term $\tilde{H}$ which is the sum of all higher-order contributions. Using (B.4), (B.5) and (B.6) it is easy to show that the time evolution of the perturbations $\delta\phi(\boldsymbol{x},t)$ and $\delta\pi(\boldsymbol{x},t)$ is generated by the time-dependent Hamiltonian $\tilde{H}$:

$$\delta\dot{\phi}(t) = \left[\tilde{H}[\delta\phi(t),\delta\pi(t);t],\delta\phi(t)\right] \ , \qquad \delta\dot{\pi}(t) = \left[\tilde{H}[\delta\phi(t),\delta\pi(t);t],\delta\pi(t)\right] \ . \tag{B.7}$$

The fluctuations at a generic time t can be expressed as

$$\delta\phi(t) = U^{-1}(t,t_0)\delta\phi(t_0)U(t,t_0) \qquad \delta\pi(t) = U^{-1}(t,t_0)\delta\pi(t_0)U(t,t_0) \ , \tag{B.8}$$

where t_0 is some very early time and the unitary operator $U(t,t_0)$ is defined by

$$\frac{\mathrm{d}}{\mathrm{d}t}U(t,t_0) = -i\tilde{H}[\delta\phi(t),\delta\pi(t);t]U(t,t_0) \tag{B.9}$$

with the initial conditions $U(t_0,t_0)=1$. Notice that in the application that concerns us in cosmology, the classical solution would describe the FRW background and we can take $t_0=-\infty$, by which we mean any time early enough so that the wavelengths of interest are deep inside the horizon. We now further decompose $\tilde{H}$ into a kinematic term H_0 that is quadratic in the fluctuations, and an interaction term H_I:

$$\tilde{H}[\delta\phi(t),\delta\pi(t);t] = H_0[\delta\phi(t),\delta\pi(t);t] + H_I[\delta\phi(t),\delta\pi(t);t] \ . \tag{B.10}$$

As in standard quantum field theory, the interaction picture is introduced defining fluctuations operators $\delta\phi_I(\boldsymbol{x},t)$ e $\delta\pi_I(\boldsymbol{x},t)$ whose time dependence is generated by the quadratic part of the Hamiltonian:

$$\delta\dot{\phi}_I(t) = [H_0[\delta\phi_I(t),\delta\pi_I(t);t],\delta\phi_I(t)] \ , \qquad \delta\dot{\pi}_I(t) = [H_0[\delta\phi_I(t),\delta\pi_I(t);t],\delta\pi_I(t)] \ , \tag{B.11}$$

together with the initial conditions

$$\delta\phi_I(t_0) = \delta\phi(t_0) \ , \qquad \delta\pi_I(t_0) = \delta\pi(t_0) \ . \tag{B.12}$$

The solutions can again be written as unitary transformations,

$$\delta\phi_I(t) = U_0^{-1}(t,t_0)\delta\phi(t_0)U_0(t,t_0) \qquad \delta\pi_I(t) = U_0^{-1}(t,t_0)\delta\pi(t_0)U_0(t,t_0) \ , \tag{B.13}$$

where $U_0(t,t_0)$ defined by the differential equation

$$\frac{\mathrm{d}}{\mathrm{d}t}U_0(t,t_0) = -iH_0[\delta\phi(t_0),\delta\pi(t_0);t]U_0(t,t_0) \tag{B.14}$$

with intial conditions $U_0(t_0,t_0) = 1$. It can be shown that if we write $U(t,t_0) = U_0(t,t_0)F(t,t_0)$, the operator $F(t,t_0)$ satisfies the equation

$$\frac{\mathrm{d}}{\mathrm{d}t}F(t,t_0) = -iH_I(t)F(t,t_0) \ , \qquad F(t_0,t_0) = 1 \ , \tag{B.15}$$

where $H_I(t)$ is the interaction Hamiltonian in the interaction picture,

$$H_I(t) = U_0^{-1}(t,t_0)H_I[\delta\phi_I(t),\delta\pi_I(t);t]U_0(t,t_0) \ . \tag{B.16}$$

The solution of equation (B.15) is:

$$F(t,t_0) = T\exp\left(-i\int_{t_0}^{t} H_I(t)\mathrm{d}t\right) \ , \tag{B.17}$$

where, as usual, "$T\exp$" indicates the time-ordered product of the operators in the series expansion of the exponential. It is now straighforward to show that, given any operator $Q(t)$ which is generally a product of $\delta\phi$'s and $\delta\pi$'s, its expectation value will be [160]:

$$\langle Q(t)\rangle = \langle 0|\left[\bar{T}\exp\left(i\int_{t_0}^{t} H_I(t)\mathrm{d}t\right)\right] Q_I(t)\left[T\exp\left(-i\int_{t_0}^{t} H_I(t)\mathrm{d}t\right)\right]|0\rangle \ , \tag{B.18}$$

where $\bar{T}$ denotes anti-time ordering. The terms of the series expansion are usually rearranged as:

$$i^n\int_{t_0}^{t}\mathrm{d}t_1\int_{t_0}^{t_1}\mathrm{d}t_2\cdots\int_{t_0}^{t_{n-1}}\mathrm{d}t_n\langle[H_I(t_n),[H_I(t_{n-1}),\,\ldots,[H_I(t_1),Q_I(t)]\cdots]]\rangle \ . \tag{B.19}$$

For the calculation, one can stop the expansion at the desidered order in the interaction Hamiltonian H_I. For example, the tree-level amplitude of the three-point functions for the scalar curvature perturbation ζ is given by:

$$\langle\zeta_{\boldsymbol{k}_1}\zeta_{\boldsymbol{k}_2}\zeta_{\boldsymbol{k}_3}\rangle = -i\int_{t_0}^{t}\mathrm{d}t\langle 0|\Big[\zeta_{\boldsymbol{k}_1}\zeta_{\boldsymbol{k}_2}\zeta_{\boldsymbol{k}_3}, H_I(t)\Big]|0\rangle \ . \tag{B.20}$$

APPENDIX C

Some Details on Breaking Diffeomorphisms in Unitary Gauge

C.1 Combinations of h and derivatives

Combinations up to second order in h and up to two derivatives, avoiding time derivatives on N or N^i (some integrations by parts have already been performed).

$$h_{00}\partial_0 h_{ii} = \psi(\nabla^2\sigma' + 3\tau') \tag{C.1}$$
$$h_{00}\partial_i h_{0i} = \psi\nabla^2 v \tag{C.2}$$
$$h_{ii}\partial_j h_{0j} = \nabla^2 v(\nabla^2\sigma + 3\tau) \tag{C.3}$$
$$h_{ij}\partial_i h_{0j} = \nabla^2 v(\nabla^2\sigma + \tau) - u_i\nabla^2 s_i \tag{C.4}$$
$$(\partial_i h_{00})^2 = -\psi\nabla^2\psi \tag{C.5}$$
$$(\partial_0 h_{ii})^2 = (\nabla^2\sigma' + 3\tau')^2 \tag{C.6}$$
$$(\partial_i h_{0i})^2 = (\nabla^2 v)^2 \tag{C.7}$$
$$\partial_i h_{0i}\partial_0 h_{jj} = \nabla^2 v(\nabla\sigma' + 3\tau') \tag{C.8}$$
$$(\partial_i h_{jj})^2 = -(\nabla^2\sigma + 3\tau)\nabla^2(\nabla^2\sigma + 3\tau) \tag{C.9}$$
$$(\partial_i h_{ij})^2 = -(\nabla^2\sigma + \tau)\nabla^2(\nabla^2\sigma + \tau) + (\nabla^2 s_j)^2 \tag{C.10}$$
$$\partial_i h_{jj}\partial_k h_{ik} = -(\nabla^2\sigma + 3\tau)\nabla^2(\nabla^2\sigma + \tau) \tag{C.11}$$
$$\partial_i h_{00}\partial_i h_{jj} = -\nabla^2\psi(\nabla^2\sigma + 3\tau) \tag{C.12}$$
$$\partial_i h_{00}\partial_j h_{ij} = -\nabla^2\psi(\nabla^2\sigma + \tau) \tag{C.13}$$

$$
\begin{aligned}
\partial_j h_{0i}\partial_0 h_{ij} &= \nabla^2 v(\nabla^2\sigma' + \tau') - u_i\nabla^2 s_i' && \text{(C.14)}\\
(\partial_0 h_{ij})^2 &= (\chi_{ij}')^2 + (\nabla^2\sigma')^2 + 2\tau'\nabla^2\sigma' + 3\tau'^2 - 2s_j'\nabla^2 s_j' && \text{(C.15)}\\
(\partial_i h_{0j})^2 &= (\partial_i u_j)^2 + (\nabla^2 v)^2 && \text{(C.16)}\\
(\partial_i h_{jk})^2 &= (\partial_i\chi_{jk})^2 + (\partial_i\partial_j\partial_k\sigma)^2 - 2\nabla^2\sigma\nabla^2\tau - 3\tau\nabla^2\tau + 2(\nabla^2 s_i)^2 && \text{(C.17)}
\end{aligned}
$$

C.2 Speed of sound and mass

Coefficients A_i for the scalar action (7.38)

$$
\begin{aligned}
A_1 &= -\frac{M_{\rm Pl}^2(1+b)}{2(\alpha_1\Lambda - 4H)^2}\Big[-8(1+b)\left(c_3k^2 + (m_0^2 + 2\epsilon H^2)\right) \\
&\quad +48baH - 3a^2\alpha_1\Lambda(\alpha_1\Lambda - 8H)\Big] && \text{(C.18)}\\
A_2 &= \frac{aM_{\rm Pl}^2(1+b)}{(\alpha_1\Lambda-4H)^2}\Bigg\{\Big[(3c_2+c_1-4)(\alpha_1\Lambda-4H) + c_3(3\alpha_3+\alpha_4)\Lambda\Big]k^2 + \\
&\quad +\left[\left(m_0^2 + 2\epsilon H^2 - \frac{6bH^2}{1+b}\right)(3\alpha_3+\alpha_4)\Lambda - 6m_4^2(\alpha_1\Lambda - 4H)\right]\Bigg\} + && \text{(C.19)}\\
&\quad +\frac{a^3M_{\rm Pl}^2\alpha_1(3\alpha_3+\alpha_4)(\alpha_1\Lambda-8H)\Lambda^2}{8(\alpha_1\Lambda-4H)}\\
A_3 &= \frac{a^2M_{\rm Pl}^2k^2}{(\alpha_1\Lambda-4H)^2}\Big[4(2+3d_1+d_2+9d_3+d_4)(\alpha_1\Lambda-4H)^2\\
&\quad +(3\alpha_3+\alpha_4)(2(3c_2+c_1-4)(\alpha_1\Lambda-4H) + c_3(3\alpha_3+\alpha_4))\Lambda\Big] + \\
&\quad +\frac{a^4M_{\rm Pl}^2}{(\alpha_1\Lambda-4H)^2}\left[6H(m_2^2-3m_3^2)(\alpha_1-2H) + 3(3\alpha+\alpha_4)m_4^2H\Lambda\right] + && \text{(C.20)}\\
&\quad -\frac{a^4M_{\rm Pl}^2\Lambda^2}{16(\alpha_1\Lambda-4H)^2}\Big[12\alpha_1^2(m_2^2-m_3^2) + (3\alpha_3+\alpha_4)\times\\
&\quad (12\alpha_1m_4^2 - (3\alpha_3+\alpha_4)(m_0^2+2\epsilon H^2 - 6H^2))\Big]\\
A_4 &= \frac{a^2M_{\rm Pl}^2k^6}{16(\alpha_1\Lambda-4H)^2}\Big[4(d_1+d_2+d_3+d_4)(\alpha_1\Lambda-4H)^2\\
&\quad +2(\alpha_3+\alpha_4)(c_1+c_2)(\alpha_1\Lambda-4H)\Lambda + c_3(\alpha_3+\alpha_4)^2\Lambda^2\Big] && \text{(C.21)}\\
&\quad -\frac{a^4M_{\rm Pl}^2k^4}{16(\alpha_1\Lambda-4H)^2}\Big[4(m_2^2-m_3^2)(\alpha_1\Lambda-4H)^2 + 4m_4^2(\alpha_1\Lambda-4H)(\alpha_3+\alpha_4)\Lambda + \\
&\quad +(m_0^2+2\epsilon H^2-6H^2)(\alpha_3+\alpha_4)^2\Lambda^2\Big]
\end{aligned}
$$

$$
\begin{aligned}
A_5 &= -\frac{a^2 M_{\rm Pl}^2 k^4}{8(\alpha_1\Lambda-4H)^2}\Big[4(d_1+d_2+5d_4)(\alpha_1\Lambda-4H)^2 - c_3(\alpha_3+\alpha_4)(3\alpha_3+2\alpha_4)\Lambda^2 \\
&\quad -2\alpha_3(\alpha_1\Lambda-4H)(3c_2+2c_1-2)\Lambda - 2\alpha_4(\alpha_1\Lambda-4H)(2c_2+c_1-1)\Lambda\Big] \\
&\quad +\frac{a^4 M_{\rm Pl}^2 k^2}{16(\alpha_1\Lambda-4H)^2}\Big[4(m_2^2-3m_3^2)(\alpha_1\Lambda-4H)^2 \qquad \text{(C.22)} \\
&\quad -(m_0^2+2\epsilon H^2-6H^2)(\alpha_3+\alpha_4)(3\alpha_3+2\alpha_4)\Lambda^2 + 4m_4^2(\alpha_1\Lambda-4H)(3\alpha_3+2\alpha_4)\Lambda\Big] \\
A_6 &= -\frac{a^2 M_{\rm Pl}^2 k^4}{8(\alpha_1\Lambda-4H)}\Big[(c_1+c_2)(\alpha_1\Lambda-4H) - c_3(\alpha_3+\alpha_4)\Lambda\Big] + \\
&\quad +\frac{a^3 M_{\rm Pl}^2 k^2}{8(\alpha_1\Lambda-4H)}\big[16m_4^2(1+b)(\alpha_1\Lambda-4H) - 8(1+b)(m_0^2+2\epsilon H^2)(\alpha_3+\alpha_4)\Lambda + \\
&\quad +3(16bH^2-\alpha_1\Lambda(\alpha_1\Lambda-8H))(\alpha_3+\alpha_4)\Lambda\big] \qquad \text{(C.23)}
\end{aligned}
$$

C.3 Discrete-Symmetry breaking operators

In this Appendix, we list new derivative operators that satisfy the requirements of Chapter 8, besides the ones already presented in the main text and in [4]. To avoid possible ghost pathologies, we do not consider operators that contain time derivatives on h_{00} and h_{0i}. Moreover, to satisfy the residual symmetry (8.14) we consider operators containing h_{0i} only when spatial derivatives act on it.

The new single-derivative operators are the following:

$$
\begin{aligned}
&h_{0i,i}h_{00}\,, \quad h_{0i,i}h_{jj}\,, \quad h_{0i,j}h_{ij}\,, \quad h'_{ii}h_{00}\,, \quad h'_{ii}h_{jj}\,, \quad h'_{ij}h_{ij}\,, \quad \epsilon_{ijk}h_{00,i}h_{jk}\,, \\
&\theta_i h_{ij,j}h_{00}\,, \quad \theta_i h_{jj,i}h_{00}\,, \quad \theta_i h_{ij,j}h_{kk}\,, \quad \theta_i h_{ij,k}h_{jk}\,, \qquad \text{(C.24)} \\
&\theta_i\epsilon_{jkl}h_{0j,k}h_{il}\,, \quad \theta_i\epsilon_{ijk}h_{0j,k}h_{ll}\,, \quad \theta_i\epsilon_{ijk}h_{0j,l}h_{kl}\,, \quad \theta_i\epsilon_{ijk}h_{0l,j}h_{kl}\,.
\end{aligned}
$$

Note that $\theta_i h_{jj,i}h_{kk}$ and $\theta_i h_{jk,i}h_{jk}$ are allowed but can be made as total derivatives, thus we have omitted these operators.

The new two-derivative operators are:

$$
\begin{aligned}
&h_{0i,i}h'_{jj}\,, \quad h_{0i,j}h'_{ij}\,, \quad \epsilon_{jkl}h_{ij,k}h'_{il}\,, \\
&\theta_i h_{00,i}h'_{jj}\,, \quad \theta_i h_{00,j}h'_{ij}\,, \quad \theta_i h_{ij,j}h'_{kk}\,, \quad \theta_i h_{ij,k}h'_{jk}\,, \quad \theta_i h_{jj,i}h'_{kk}\,, \quad \theta_i h_{jj,k}h'_{ik}\,, \qquad \text{(C.25)} \\
&\theta_i\epsilon_{jkl}h_{0j,k}h'_{il}\,, \quad \theta_i\epsilon_{ijk}h_{0j,k}h'_{ll}\,, \quad \theta_i\epsilon_{ijk}h_{0j,l}h'_{kl}\,, \quad \theta_i\epsilon_{ijk}h_{0l,j}h'_{jl}\,.
\end{aligned}
$$

APPENDIX D

Decoupling and Strong Coupling with Broken Diffeomorphisms

D.1 Mixing with gravity and decoupling Limit

In this Appendix we will show why taking the decoupling limit is a consistent approximation in the case under study in Chapter 9 [6]. Similarly to the equivalence theorem for massive gauge bosons, we expect that the physics of the Goldstone decouples from the transverse modes above a certain energy scale, E_{mix}. For example, in a non-Abelian gauge theory,

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2}(\partial_\mu \pi)^2 - \frac{1}{2}m^2 A_\mu^2 + im\partial_\mu \pi A^\mu \ , \tag{D.1}$$

where $m^2 = f_\pi^2 g^2$, the decoupling limit is reached taking the limit $g \to 0$, $m \to 0$ with f_π. Therefore, for energies $E > m$, the mixing between the Goldstone and the gauge modes becomes irrelevant and the two sectors effectively decouple.

Just like the gauge theory analogy, in our case we can find a decoupling limit which corresponds to the limits $M_{\rm Pl} \to \infty$, $\dot{H} \to 0$ with $M_{\rm Pl}^2 \dot{H}$ fixed[1]. To see that taking this limit in our case effectively lead to the decoupling of Goldstones and gravity, let us consider first a simplified case, where all the derivatives of F are zero but F_X. When expanding the the operator X (9.8) according to (9.20), one obtains

$$X = (1+\dot{\pi})^2 g^{00} + 2\partial_i \pi g^{0i} + \partial_i \pi \partial_j \pi g^{ij} \ . \tag{D.2}$$

Substituted back into the action (9.7), the leading mixing of the Goldstone π with gravity will be of the form:

$$\bar{F}_X \dot{\pi} \delta g^{00} \ . \tag{D.3}$$

[1] This is the same as the previous example with the indentifications $g \to M_{\rm Pl}^{-1}$ and $m \to \dot{H}$.

After canonical normalization $\pi_c \sim \sqrt{-\bar{F}_X}\pi$, $\hat{g}_c^{00} \sim M_{\rm Pl}\delta g^{00}$ (which gives to the fields the dimension of a mass), we can see that taking the decoupling limit $M_{\rm Pl} \to \infty$, $\dot{H} \to 0$ with $M_{\rm Pl}^2\dot{H}$ implies that mixing terms becomes irrelevant with respect to the standard kinetic term π_c^2 and can be neglected above a certain energy E_{mix}:

$$E_{mix}^2 \sim \frac{\bar{F}_X}{M_{\rm Pl}^2} \sim \frac{\bar{F}_X}{M_{\rm Pl}^2 H^2} H^2 \sim \epsilon H^2 \ll H^2 \ , \tag{D.4}$$

where we have used (9.11) and (9.16). Therefore as long as E_{mix} is smaller than H, we can safely neglect mixing terms, as they would appear in the action suppressed by powers of $(E_{mix}/H)^2 \sim \epsilon$, since H is our infrared cutoff. The same will happen for the other terms present in the action, but in general the answer depends on which operators are present and significant. For example, from Z^{ij}, after the canonical normalization $\sigma_c \sim \sqrt{-\bar{F}_Z + \bar{F}_{Y^2}/2}$, one has:

$$\bar{F}_Z \dot{\sigma}^i g^{0j}\delta_{ij} \qquad \Longrightarrow \qquad E_{mix} \propto \begin{cases} \dfrac{\alpha\sqrt{-\bar{F}_Z/a^2}}{M_{\rm Pl}} \ , & \text{for} \quad |\bar{F}_{Y^2}| \lesssim |\bar{F}_Z| \\ \dfrac{-\alpha \bar{F}_Z/a^2}{M_{\rm Pl}\sqrt{\bar{F}_{Y^2}/a^2}} \ , & \text{for} \quad |\bar{F}_Z| \ll |\bar{F}_{Y^2}| \end{cases} \tag{D.5}$$

In the first case, as $\alpha \lesssim 1$,

$$\frac{E_{mix}^2}{H^2} \sim \frac{-\alpha^2 \bar{F}_Z/a^2}{M_{\rm Pl}^2 H^2} \lesssim \epsilon \ , \tag{D.6}$$

where we have used (9.16). In the second case, $\bar{F}_Z \ll \bar{F}_{Y^2}$, one can find a similar expression too:

$$\frac{E_{mix}^2}{H^2} \sim \frac{-\alpha^2 \bar{F}_Z/a^2}{M_{\rm Pl}^2 H^2}\frac{\bar{F}_Z}{\bar{F}_{Y^2}} \lesssim \frac{-\alpha^2 \bar{F}_Z/a^2}{M_{\rm Pl}^2 H^2} \lesssim \epsilon \ . \tag{D.7}$$

Also in this case E_{mix} is smaller than H and the decopling limit can be safely taken. However, if for example one has $|\bar{F}_{Y^2}| \gg |\bar{F}_Z|$, then, looking at the expansion of the operator Y_iY^i one can see that working in the decoupling limit can restrict the range of the allowed parameters:

$$\bar{F}_{Y^2} \dot{\sigma}^i g^{0j}\delta_{ij} \qquad \Longrightarrow \qquad E_{mix}^2 \sim \frac{\alpha^2 \bar{F}_{Y^2}/a^2}{M_{\rm Pl}^2} \ , \tag{D.8}$$

which is lower than H only if $\bar{Z}\bar{F}_{Y^2}/M_{\rm Pl}^2 H^2 \ll 1$.

D.2 Strong coupling

As it is usual in effective field theories, the non-renormalizable self-interactions of the Goldstone fields will become strongly coupled at a certain energy scale, Λ_{st}, beyond which the theory ceases to make sense and new physics must enter. In our case, we have to make sure that $\Lambda_{st} \gg H$ so that the theory is weakly-coupled in the energy regime we are interested in.

Stronger interactions are related to smaller kinetic energy: indeed if the time-kinetic terms in (9.21) have prefactors of order $\sim \epsilon$, we would canonically normalize the fields (collectively denoted with π for simplicity) like $\epsilon F(\partial\pi)^2 \sim (\partial\hat{\pi})^2$ and inverse power of ϵ will appear in higher order terms, which would mean stronger interactions or, equivalently, a lower strong coupling scale. Of course, if the coefficients of the kinetic terms were bigger or the coefficients of higher-order terms were smaller, interactions would be accordingly weaker. As we have to impose a lower bound on Λ_{st}, from now on we will focus only on the "worst possible case", when prefactors of time-kinetic terms are as small as $\sim \epsilon F$, while interactions, which are proportional to higher derivatives of F with respect to the operators X, Y^i and Z^{ij}, are as big as F itself.

Let us first consider the case with speeds of sound very close to unity. In this case, after canonical normalization, we can directly read the strong coupling scale as the scale suppressing higher-order operators in the action, $(\partial\hat{\pi})^3/\Lambda^2$. The result is simply:

$$\Lambda_{st}^4 \simeq \epsilon^3 F \, . \tag{D.9}$$

If the speed of sound are non-relativistic, the cut-off can not be immediately read from the action as there is an hierarchy between time and spatial derivatives and the theory is not Lorentz invariant. Let's assume for simplicity that $c_\pi \simeq c_\sigma = c_s \ll 1$. We can rescale the time coordinate [118, 298] as $t \to t/c_s$, in order to remove this hierarchy. The quadratic action has now the form:

$$S_2 \simeq \int \mathrm{d}^4x\sqrt{-g}\,\epsilon F c_s (\partial_\mu \pi)^2 \, , \tag{D.10}$$

and the fields would be normalized as $\epsilon c_s F \pi = \hat{\pi}$. Schematically, after canonical normalization, the cubic interactions will have the form

$$S_3 \simeq \int \mathrm{d}^4x\sqrt{-g}\,\frac{(\partial\hat{\pi})^3}{c_s^{5/2}\epsilon^{3/2}\,F^{1/2}} \, . \tag{D.11}$$

where in the denominator the strong coupling *momentum* scale appears. We can obtain the energy scale Λ_{st} multiplying by an extra c_s. The result is:

$$\Lambda_{st}^4 \simeq \epsilon^3 c_s^9 F \, . \tag{D.12}$$

As we said, our theory is under control if $\Lambda_{st} \gg H$, which will give the constraint:

$$\epsilon\, c_s^3 \gg \left(\frac{H}{M_{\mathrm{Pl}}}\right)^{2/3} \, , \tag{D.13}$$

where we have used the Friedmann equation (9.11). This is only an order-of-magnitude estimate and, given the many possible combinations of free parameters that are allowed in our action, this constraint can also be not very restrictive. However it is still an important bound to respect for the consistency of the theory.

APPENDIX E

Tensor fossil estimation

In order to extract precise informations about the size of the galaxy surveys on which the long wavelength tensor mode can leave "*fossil*" imprints we need to use the optimal estimator for the tensor power spectrum constructed in [299]. In this case we consider a tensor mode which wavelength is smaller than the size of the observable universe and then we compute the variance of the optimal estimator

$$\sigma_\gamma^{-2} = \frac{1}{2}\sum_{\vec{k}_L,p}\left[k_L^3 P_p^n(k_L)\right]^{-2} \ , \tag{E.1}$$

where p refer to the two polarizations of the tensor, P_p^n is the noise power spectrum, defined as the ratio between the consistency violating contribution to the bispectrum and the total power spectrum

$$P_p^n(k_L) = \left[\sum_{\vec{k}_S}\frac{|\mathcal{B}_{\rm cv}(k_L, k_S, |\vec{k}_L - \vec{k}_S|)\epsilon_{ij}^p \hat{k}_S^i \hat{k}_{LS}^j|^2}{2V P_\gamma^2(k_L) P^{tot}(k_S) P^{tot}(|\vec{k}_L - \vec{k}_S|)}\right]^{-1} \ , \tag{E.2}$$

where $V \equiv (2\pi)^3/k_{min}^3$ is the total volume of the survey and ϵ_{ij}^p is the polarization tensor. The total power spectrum, that is the measured one, includes both the noise and the signal, $P^{tot}(k) = P(k) + P^n(k)$. The bispectrum can be written in terms of a function $f(\vec{k}_1, \vec{k}_2)$, that describes the coupling of the soft mode, and the "long" mode power spectrum $P(k_L)$

$$B(\vec{k}_L, \vec{k}_1, \vec{k}_2) = P(k_L) f(\vec{k}_1, \vec{k}_2)\epsilon_{ij}^p(\hat{k}_L)\hat{k}_1^i \hat{k}_2^j = \mathcal{B}(k_L, k_1, k_2)\epsilon_{ij}^p(\hat{k}_L)\hat{k}_1^i \hat{k}_2^j \,, \tag{E.3}$$

in such a way that the noise power spectrum becomes [299]

$$P_p^n(k_L) = \left[\sum_{\vec{k}_S}\frac{|f(\vec{k}_S, \vec{k}_L - \vec{k}_S)\epsilon_{ij}^p \, k_S^i \, (k_L - k_S)^j|^2}{2V P^{tot}(k_S) P^{tot}(|\vec{k}_L - \vec{k}_S|)}\right]^{-1} \ . \tag{E.4}$$

k_L and k_S are the wave number of the long wavelength mode and the short wavelength one. The function $f(\vec{k}_1, \vec{k}_2)$ can be easily read from the tensor-scalar-scalar bispectrum (9.89)

$$f(k_S, k_L) = \frac{C\ P(k_S)}{k_S^2} \log\left(\frac{2c_\pi k_S}{k_L}\right) \qquad , \qquad C = \frac{\alpha^2}{4c_\pi^2}\lambda_1\lambda_2\ , \tag{E.5}$$

where we see the novel dependence from the number of modes in the survey. Even if we know that in our case the tensor power spectrum is not exactly scale invariant, at lowest order in α we can assume a nearly scale invariant fiducial power spectrum with amplitude A_γ, $P_\gamma = A_\gamma k_L^{n_\gamma - 3}$ with $n_\gamma \simeq 0$. Assuming $P^{(0)}(k_S)/P^{tot}(k_S) \simeq 1$ ("correction" to the power spectrum much smaller than 1) if $k_S \leq k_{max}$ and equal to zero otherwise, where k_{max} is the largest wavenumber that allows for a large signal-to-noise measurement, we compute the noise power spectrum. Plugging this quantity in (E.1) and considering that a signal is detected if it has an amplitude larger than 3σ we obtain

$$3\sigma_\gamma \simeq \frac{18\sqrt{3}\ \pi^{3/2}}{C^2}\left(\frac{k_{\rm min}}{k_{\rm max}}\right)^3 \log\left(\frac{2c_\pi k_{\rm min}}{k_{\rm max}}\right)^{-2} . \tag{E.6}$$

Inverting this relation we can find the size of the galaxy survey necessary to detect at 3σ the imprints of primordial tensor mode with a given amplitude A_γ. The estimation, as we can see, would be model dependent and require an improved parameter space analysis of the model, but in order to have a rough estimation, if we assume $c_\pi \simeq 10^{-1}$, for the parameter C in the range $(0.1 - 1)$, one finds that a detectable primordial tensor mode with an amplitude $A_\gamma \simeq 2 \times 10^{-9}$, that is a value close to the current upper limits, requires a survey with size in the range $\frac{k_{max}}{k_{min}} \sim (4000 - 1000)$, a value that can be achievable with the next survey like 21-cm [300].

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