

Essays on information gathering and the use of
natural resources

Luca Di Corato

Submitted for the Degree of Doctor of Philosophy in Economics

University of York and Università di Padova

March 2008

Contents

Abstract	9
Abstract (in Italian)	11
Acknowledgements	15
Declaration	17
1 Introduction	19
2 Mechanism design for conservation contracts in developing countries	25
2.1 Introduction	25
2.2 The basic set-up	29
2.2.1 Landowner and government agency's preferences	29
2.2.2 Conservation in First Best	33
2.3 Mechanism under adverse selection	35
2.3.1 Analysis of the optimal Conservation Program	39
2.3.2 Transfers	43
2.3.3 Optimal CP vs general subsidy	44
2.4 Conservation program at work	45

2.5	Conclusions	47
3	Optimal conservation policy under imperfect intergenerational altruism	51
3.1	Introduction	51
3.2	The basic set-up	55
3.2.1	Harvesting or conserving	57
3.3	Optimal harvest timing: imperfect altruism and naiveté	60
3.4	Optimal harvest timing: imperfect altruism and sophistication	63
3.4.1	A three governments model	64
3.4.2	A <i>I</i> -governments model	68
3.5	Government targeting and instability	71
3.6	Conclusions	72
4	Option value of old-growth forest and Pigovian taxation under time inconsistency	73
4.1	Introduction	73
4.2	The basic set-up	78
4.2.1	Sketch of an agent with hyperbolic preferences	79
4.2.2	Conserving or harvesting: time-consistent case	81
4.3	Conserving or harvesting under time inconsistency	82
4.3.1	Strategies under sophistication	83
4.3.2	Conservation or harvest: a discussion on timing	87
4.4	Regulatory intervention	88
4.4.1	Passage time	88
4.4.2	Time for regulation	89
4.4.3	Numerical and graphical analysis	91

4.5	Conclusion	93
5	Optimal profit sharing under the risk of expropriation	95
5.1	Introduction	95
5.2	The basic set-up	99
5.2.1	The HC's and MNF's objective functions	100
5.2.2	The bargaining	101
5.3	Efficient bargaining set under uncertainty and irreversibility	103
5.3.1	The host country	103
5.3.2	The multinational firm	106
5.4	Nash bargaining and cooperative equilibrium	109
5.4.1	Cooperative equilibrium	109
5.4.2	Some analytical results	110
5.4.3	Final considerations on the cooperative agreement	114
5.5	Conclusions	115
6	Concluding remarks	117
A	Appendix to Chapter 2	121
A.1	Proposition 2.1	121
A.2	Proposition 2.2	123
A.3	Larger total rents for the higher type	125
A.4	Proposition 2.3	126
A.5	Proposition 2.4	127
A.6	Binding perverse incentive constraint	128
A.7	Feasibility of a GS program	129
A.8	Bunching types	131

B Appendix to Chapter 3	133
B.1 Strategies under naïve belief	133
B.2 Proposition 3.3	136
B.3 Strategies under sophisticated belief: three governments	137
B.3.1 Continuation value function	137
B.3.2 Value function	139
B.4 Proposition 3.6	142
B.5 Strategies under sophisticated belief: I governments	144
B.5.1 Continuation value function	144
B.5.2 Value function	146
C Appendix to Chapter 4	151
C.1 Strategies under sophisticated belief	151
C.2 Pigovian taxation	153
References	154

List of Figures

Figures 4.1-4.4	92
---------------------------	----

Abstract

The objective in this thesis is to pose and to answer to some questions concerning the role played by information in decisions on the economic allocation of natural resources.

In chapter 2 the design of a voluntary incentive scheme for the provision of ecosystem services is considered, having in mind the forested areas in developing countries where a governmental agency plans to introduce a set-aside policy. Payments are offered to the landowners to compensate the economic loss for not converting land to agriculture. The information asymmetry between the agency and the landowners on the opportunity cost of conservation gives incentive to the landowners to misreport their own "type". A principal - agent analysis is developed, adapted and extended to capture real issues concerning conservation programs in developing countries. I show that the information asymmetry may seriously impact on the optimal scheme performance and, under certain conditions, may lead to pay a compensation even if any additional conservation is induced with respect to that in absence of the scheme.

In chapter 3 an intergenerational dynamic game is solved under time- inconsistency. The optimal harvest timing for a natural forest is determined under uncertainty on the flow of amenity value derived from conservation and irreversibility. Due to time-varying declining discount rates intertemporal inconsistent harvest

strategies arise. The value of the option to harvest is eroded and earlier harvest occurs under both the assumptions of naïve and sophisticated belief on future generations time-preferences. This results in a bias toward the current generation gratification which affects the intergenerational allocation of benefits and costs from harvesting and conserving.

In chapter 4 a forest owner with hyperbolic time preferences is considered. At each period the irreversible decision to harvest an old-growth forest could be taken, while conservation is the alternative. Flows of future amenity value are uncertain while the net value of stumpage timber is known and constant. The decision problem is expressed as an optimal stopping problem and solved analytically in a time-inconsistent framework under the assumption of sophisticated belief on future trigger strategies. Premature harvesting occurs. To avoid socially undesirable harvesting the impact of hyperbolic discounting must be accounted and a modified optimal pigovian tax on the wood revenues is proposed.

Finally, in chapter 5 a government bargains a mutually convenient agreement with a foreign firm to extract a natural resource. The firm bears the initial investment in field research and infrastructures and earns as a return a share on the profits. The firm must cope with uncertainty due to market conditions and, as initial investment is totally sunk, also due to the risk of successive expropriation. In a real options framework where the government holds an American call option on expropriation I show under which conditions Nash bargaining is feasible and leads to attain a cooperative agreement maximizing the joint venture surplus keeping into account both the sources of uncertainty on profit realizations. I show that the investment trigger does not change under the threat of expropriation, while the set of feasible bargaining outcomes is restricted and the distributive parameter is adjusted to account for the additional risk of expropriation.

Abstract (in Italian)

L'obiettivo di questa tesi è quello di presentare e rispondere ad alcune domande riguardanti il ruolo svolto dall'informazione nelle decisioni riguardanti l'allocazione economica delle risorse naturali.

Nel capitolo 2, viene considerato uno schema volontario per l'incentivazione della fornitura di servizi di ecosistema. In particolare, si fa riferimento all'intervento da parte di un'agenzia governativa teso all'introduzione di un piano di *set-aside* nelle aree boschive dei Paesi in via di sviluppo. Il piano prevede di ricompensare tramite un trasferimento i proprietari terrieri per la perdita economica sofferta non convertendo l'area di proprietà ad agricoltura. L'asimmetria informativa esistente tra agenzia e proprietario terriero rispetto al costo opportunità della conservazione incentiva quest'ultimo a non rivelarne la corretta entità. Viene quindi sviluppata un'analisi principale - agente adattata ed estesa al fine di incorporare gli aspetti problematici che caratterizzano i programmi per la conservazione in Paesi in via di sviluppo nella realtà. Viene mostrato il drastico impatto che l'informazione asimmetrica può avere sulla performance dello schema ottimale. Si verifica che, sotto certe condizioni, paradossalmente si potrebbe dover compensare anche un proprietario terriero che ha conservato nell'ambito del programma la stessa area che avrebbe conservato in assenza del programma governativo.

Nel capitolo 3, si risolve un gioco dinamico intergenerazionale tra agenti incoerenti temporalmente. Il timing ottimale del taglio di una foresta naturale viene determinato tenendo in considerazione l'incertezza relativa al valore di cui si potrebbe beneficiare attraverso la conservazione e l'irreversibilità delle conseguenze del taglio una volta avvenuto. La strategia ottimale, a causa dei tassi di preferenza intertemporale varianti col tempo, può risultare incoerente. L'erosione del valore dell'opzione di taglio ne induce un esercizio più affrettato sia sotto l'ipotesi di aspettative rispetto alle preferenze temporali delle future generazioni di tipo *naïve* che di tipo sofisticato. Tutto ciò si riflette in una distorsione della ripartizione intergenerazionale dei benefici e dei costi derivanti dalla gestione della risorsa a vantaggio della generazione vivente.

Nel capitolo 4, si assume che il proprietario privato di una foresta abbia preferenze temporali iperboliche e possa decidere il taglio, con conseguenze irreversibili della foresta, oppure conservarla. Il flusso di valore di cui beneficia se conserva è incerto mentre il valore netto del legno tagliato è noto e costante nel tempo. Tale problema decisionale viene rappresentato nei termini di un problema di *optimal stopping time* e risolto analiticamente in un contesto caratterizzato da incoerenza temporale sotto l'ipotesi di aspettative di tipo sofisticato rispetto alle strategie preferite in futuro. Ne risulta che il taglio è realizzato prematuramente. Si mostra quindi come modificare la tassa Pigouviana sul legno per evitare effetti socialmente non desiderati dovuti alla particolare definizione delle personali preferenze temporali.

Infine, nel capitolo 5 il governo di un Paese ospitante negozia con un'impresa estera un accordo reciprocamente conveniente per lo sfruttamento di una risorsa naturale. L'impresa dovrebbe farsi carico dell'investimento iniziale necessario a sondare la consistenza del giacimento e a costruire le infrastrutture necessarie ot-

tenendo in cambio una quota sui profitti derivanti dall'estrazione. L'impresa oltre a far fronte all'incertezza sui profitti futuri dovuta alle variabili condizioni di mercato deve tener conto anche del rischio di una successiva espropriazione, dato che l'investimento è totalmente irrecuperabile. Utilizzando un modello teorico di opzioni reali in cui il governo può essere visto come detenere un'opzione di tipo *American call* sull'espropriazione, si mostra sotto quali condizioni, tenendo conto dell'incertezza di mercato e dell'addizionale rischio di espropriazione, un *Nash Bargaining* sia realizzabile e permetta di definire un accordo che massimizzi il valore complessivo dell'attività economica. Tra i risultati, si mostra che la soglia temporale alla quale sostenere in maniera ottimale l'investimento non varia in presenza di una minaccia di espropriazione rispetto al caso in cui tale rischio non esista, mentre l'insieme degli accordi potenzialmente realizzabili si riduce. Si mostra infine come le quote sui profitti vadano aggiustate per incorporare il rischio supplementare di espropriazione.

Acknowledgements

I am deeply indebted to my supervisors Bipasa Datta and Michele Moretto. I am also grateful to Paola Valbonesi for having supervised me in the beginning. My gratitude goes to them not only for their comments and suggestions but also for their advice, guidance and patience.

Finally, this thesis is dedicated to who in the darkest hours of the night was and will be there, close to me.

Declaration

An earlier version of chapter 2 has been presented at the European Summer School in Resource and Environmental Economics “Trade, Property Rights and Biodiversity”, San Servolo, Venice, July 4-10, 2007, at an Environmental and Resource Economics seminar at the LERNA, Université de Toulouse I, Toulouse, February 8, 2007, at the VIII annual BIOECON Conference “Economic Analysis of Ecology and Biodiversity”, Kings College, Cambridge, August 29-30, 2006 and at an internal seminar at the University of Padua. An earlier version has appeared as “Marco Fanno” Working Paper no. 34/2006, University of Padua.

Chapter 4 will be presented at the AFSE annual Thematic Meeting "Frontiers in Environmental economics and Natural Resources Management", Toulouse, June 9-11, 2008. An earlier version of Chapter 4 has been presented at the “Giornata Levi-Cases” doctoral seminar, Faculty of Economics, University of Padua, November 23, 2007, at the IX annual BIOECON Conference “Economics and Institutions for Biodiversity Conservation”, Kings College, Cambridge, September 19-21, 2007, and at an internal seminar at the University of Padua.

Chapter 1

Introduction

Natural resources play an important role for current and future societies since they represent an endowment whose use is crucial to support human welfare (Heal, 1998). The channels through which natural assets may impact on human felicity are diverse. For resources such as oil, natural gas and minerals, utility is derived mainly from their exploitation while for natural assets such as forests, wetlands, watersheds and related environmental goods and services, welfare could accrue not only from exploitation but also from conservation.

Decisions regarding the use of these assets must be taken in the light of current and future costs and benefits. Normally, in order to assess actual net benefits and to support strategies, information should be gathered. Several research questions may arise from this simple consideration and a number of them have been answered by social scientists. Nevertheless, some questions still remain. The objective of this thesis is to pose and to answer to some questions concerning the role that information may play for decisions about the economic allocation of natural resources.

In particular, chapter 2 investigates the problem related to the design of an

incentive-compatible conservation contract scheme which allowing for the collection of information needed to optimally allocate forested land to two alternative uses: agriculture or ecosystem services provision. The idea behind a conservation contract is relatively simple: an environmental agency proposes to landowners a contract scheme which specifying the extent of land that should be set aside for conservation, and the transfer compensating for the economic loss suffered for not converting such an extent to agriculture. The cost opportunity of conservation varies among landowners according to the quality of their land and it is often private information of landowners (Smith and Shogren, 2002). The information asymmetry between the landowner and the agency is an advantage for the former in that, by misreporting the land type, she may be over-compensated. This clearly represents a problem for the agency which must deal with limited and costly raised funds for conservation.

In this chapter I deal with such a problem by developing a standard principal - agent analysis, adapted and extended to capture real issues concerning conservation programs in developing countries. In these countries a substantial extent of land is still forested but "slash and burn" agriculture has become aggressive (Brocas and Carrillo, 1998). I assume first, that the private level of conservation may be positive and second, that agriculture is risky in that, due to primitive agricultural practices, the crop yield may be severely reduced by exogenous shocks such as pest and soil erosion (Arguedas et al., 2007). The second assumption represents a novelty in the conservation contracts literature but in my opinion is an important issue to be considered since it may have an impact on the actual extent of land conversion. Finally, imposing a restriction on the set of feasible incentive-compatible contracts, I address another important aspect concerning the perverse effects which may be induced through the conservation

program. In fact, inconsistently with the agency target, the program may relax credit constraints and give incentive to clear more land than that cleared without a program.

In all three chapters 3, 4 and 5, the perspective on the role played by information differs from that in chapter 2. Within different contexts, decision-making accounts for the value of information disclosing as time rolls on. In fact, if this is the case, it may be profitable to postpone a decision and to collect information in order to reduce uncertainty about future realizations of benefits and costs. This consideration becomes crucial, in particular, when the consequences of a decision are costly or impossible to reverse (Arrow and Fisher, 1974; Henry, 1974).

In particular, the model set-up of chapters 3 and 4 is quite similar from a technical point of view. In both chapters, I merge two different strands of literature: on the one hand the real option theory which emphasises the importance of waiting for collecting information, on the other hand the literature on hyperbolic time-preferences, where decision-makers affected by time-varying impatience are time-inconsistent and have incentive to rush because of future sub-optimal plan revisions. The results provided in these two chapters extend the real options tool box for the analysis of a wider class of economic problems entailing the exercise of options similar to an "American put" such as an option to exit or an option to shut down (Dixit and Pindyck, 1994).

Chapter 3 provides a rational for the observed tendency of governments to rush in undertaking projects which irreversibly impact the stock of natural resources available to future generations and for the time inconsistency of the conservation policies. An intergenerational dynamic game is considered to determine the optimal conservation policy set by the government. I assume that the government is truly democratic and at each time period perfectly represents the will

and the preferences of the politic body, namely the generation living at that time period (Phelps and Pollak, 1968). Each generation is imperfectly altruist and lives over a random lifespan benefiting from its own welfare and that of following generations. The value of the stand of forest is known and constant and accrues to society when irreversible harvest occurs, while the flow of amenity value from conservation randomly fluctuates according to a geometric Brownian motion and stops forever when the forest is harvested. Under these assumptions I show that the government is equivalent to an hyperbolic agent with a finite number of selves. Intertemporal inconsistent harvest strategies arise and due to time-varying declining discount rates, the value of keeping the option to harvest is lowered. Therefore, an earlier harvest is induced under both the assumptions of naïve and sophisticated belief on future generations time-preferences.

In chapter 4, the research question is how second best tools for government intervention must be adjusted to account for non standard time preferences (Shogren, 2007). Goods and services provided by a natural forest when conserved are public in nature and government intervention may be needed to guarantee the intertemporal socially desirable allocation of this natural asset. I show that a pigouvian tax on wood revenues should be modified to lead agents with hyperbolic time preferences toward the social optimum because otherwise the policy target could not be met. In this chapter, the optimal stopping problem in continuous time solved in the previous chapter for a finite number of government "incarnations", is now solved for the case of a private forest owner represented by an infinite sequence of selves with hyperbolic time preferences. The solution for this case is more tractable but is qualitatively equivalent to the one for the finite selves case.

In chapter 5, I analyse the problem of foreign direct investment for the

exploitation of a natural resource. In developing countries, due to limited budget often the governments cannot afford the initial investment for the exploitation of their natural resources and attempt to attain a mutually convenient agreement with foreign firms willing to bear the initial costs. According to these agreements, the firm bears the initial investment in field research and infrastructures and earns, as a return, a share of the profits derived from the resource extraction. In this context, when assessing the convenience of the investment, the firm must deal with profit uncertainty due to market conditions. Moreover, since the initial investment is totally sunk, the firm should also deal with the risk of successive expropriation. In high-profit states in fact the host country's government may have incentive to expropriate. I develop the analysis in a real options framework where the government is seen as holding an American call option on expropriation while the firm as holding a similar option on investment. Both parties wish to attain an agreement matching their different economic interests. I show under which conditions Nash Bargaining is feasible and leads to a cooperative agreement maximizing the joint venture value, keeping into account both sources of uncertainty on profits.

In chapter 6, I provide a summary of the main issues discussed in this thesis and suggestions for future research. All the proofs are available in the appendix.

Chapter 2

Mechanism design for conservation contracts in developing countries

2.1 Introduction

In the last decades the Payments for the provision of Ecosystem Services (hereafter, PES) have become an increasingly popular instrument to induce the provision of ecosystem services on private lands.¹ The target for most of the land managed under PES programs has usually been the conservation of biodiversity and the soil protection (Salzman, 2005; Ferraro, 2001; Ferraro and Kiss, 2002; Pagiola et al., 2002). Under a PES program a contract is usually proposed by a governmental agency to a landowner. The landowner sets aside a part of her own land and receives a compensation for the economic loss suffered. The contract is

¹A well known example is given by the PSA (Pagos por Servicios Ambientales) program in Costa Rica (FONAFIFO, 2000; Pagiola et al., 2002; Salzman, 2005). For other examples <http://www2.gsu.edu/~wwwcec/special/ci/index.html>.

designed to allow for the voluntary participation of the landowner to the program and specifies the extent of land that should be conserved and the compensation paid for the environmental service provided. To guarantee a voluntary participation the payment should be at least equal to the landowners' opportunity cost and no higher than the value of the benefit provided.

The landowners know their property and the opportunity cost of managing it for environmental services better than the governmental agency. Landowners could then have incentive to misreport their true type in order to be over compensated. This opportunistic behaviour produces an additional burden for the agency and impacts on the total level of conservation which may be induced through a program becoming a serious issue when funds for conservation are limited and/or are costly raised through distortionary taxation. This problem is common to a number of other situations where agents with different cost opportunity type may take advantage of their private information and the principal searches to differentiate them through a proper contract scheme. In these cases mechanism design theory can be used to design contract scheme which induces truth-telling (Mirrlees 1971; Groves, 1973; Dasgupta, Hammond and Maskin, 1979; Baron and Myerson, 1982; Guesnerie and Laffont, 1984). This is what has been also broadly done to deal with information failures impacting on the design of conservation contracts (Smith and Shogren, 2002; Wu and Babcock, 1996; Smith, 1995; Goeschl and Lin, 2004).

In the reality despite the fact that optimal incentive schemes could be designed, PES programs are usually general subsidy schemes.² A general subsidy scheme is surely easier to implement but it allocates sub-optimally the funds for

²This is the case for example for the PSA program in Costa Rica where each land unit conserved is paid the same amount and any landowner in the country is allowed to participate and choose the extent of land to be conserved (Pagiola et al., 2004).

conservation in that overpays³ landowners which misreport their cost opportunity⁴ type.

The aim of this chapter is to address such concern and design a voluntary incentive scheme for habitat conservation in developing countries where a substantial extent of land is still forested but "slash and burn" practices have become intense.

We investigate the adverse selection issue due to the information asymmetry between the governmental agency and the landowner on the environmental characteristics of each property. This set of characteristics affects the land agricultural productivity and determine the opportunity cost of each unit of land conserved. We are clearly aware that reality is even more complex for the presence of moral hazard in the contract compliance and for the asymmetry in gathering information about conservation costs but we prefer to abstract from these issues and work on a simpler model.⁵

We model the agricultural activity undertaken after land conversion as a risky activity suffering exogenous shocks which negatively affects the landowner's crop yield. This is an aspect which has not been considered in the previous contributions on this topic but that is in our opinion very relevant in that risk affects the landowner private allocation choice and consequently the actual cost opportunity of conservation.⁶ Moreover, this consideration can be even more important

³This has probably been the case in Costa Rica where the compensation paid has been quite attractive and a number of applications to the program were not considered because of funding limits (Pagiola et al., 2004).

⁴By principle also the different levels of benefit provided by the service should be taken into account. But as in the case of biodiversity conservation, such benefit is extremely difficult to assess. In contrast, to collect information on and estimate the landowner's opportunity cost may be easier and less costly.

⁵See White (2002) for the moral hazard problem and Goeschl and Lin (2004) for the asymmetry in the information gathering.

⁶There have been in fact no studies up to date assessing how much land managed under a conservation program would have been cleared in the absence of the program.

in developing countries where the agricultural activity is still primitive and the investment in technology is low.

The set-up of our model is completed by first, assuming that the level of conservation pursued by the governmental agency through the conservation program is not fixed ex-ante but results from the social welfare maximization, second, assuming that the private level of conservation is not necessarily zero but it is optimally determined by the landowner according to the expected profit associated to converting land and third, introducing as in the paper by Motte et al. (2004) a constraint on the surface conserved to control for the effectiveness of the policy.⁷ The purpose of this constraint is to control for a policy perverse effect which could induce landowners to clear more forest than they would have cleared without a contract.

In this frame a program consistent with the conservation target is designed to guarantee voluntary participation and truthful revelation of land opportunity cost. We show that the information asymmetry may seriously impact on the optimal second-best scheme leading under certain conditions to pooling types. First best conservation can only be attained if raising funds for the transfers comes at no cost. We also verify that even if any additional conservation is induced with respect to the extent privately undertaken a compensation must be paid in some cases to landowners. This is done only to induce them to reveal their private information and limit the information rent that must be paid to other types. We finally prove that the program designed is the optimal or best feasible contract scheme available and that social surplus under a general subsidy conservation program cannot be higher than under the optimal second best

⁷In Motte et al. (2004) the information asymmetry is on the individual cost of clearing effort. A "policy consistency" constraint is introduced in the standard principal-agent problem to restrict the set of incentive compatible contract schedules to the one where the conservation undertaken under the CP is at least equal to that without CP.

conservation program.

The structure of the chapter is the following: in section 2.2, the landowner and governmental agency's preferences are presented; the private allocation in the absence of a conservation program and the first best allocation with a conservation program in place are presented and discussed. In section 2.3, the second best outcome is derived and its properties are discussed. Section 2.4 proposes a parametric example of the optimal conservation program at work. Section 2.5 concludes.

2.2 The basic set-up

We assume that each landowner owns \bar{A} units of land and that each plot is in its pristine natural state. Each landowner's plot is of the same size but not necessarily has the environmental characteristics⁸ of the one owned by another landowner. On these private lands the governmental agency (hereafter, GA) plans to preserve some critical habitat for biodiversity conservation and to induce that proposes a voluntary contract scheme. According to the scheme, each landowner is paid to set aside a units of her plot for conservation. We further assume that the GA and the landowners are risk-neutral agents and that the funding of the transfers is raised as standard by taxation.

2.2.1 Landowner and government agency's preferences

Each landowner's plot is characterized by a set of characteristics, such as soil quality, soil erosion and water and distance to market. We use a scale index θ to represent these characteristics (Wu and Babcock, 1996). This parameter

⁸Hereafter, we would simply use "type".

varies among landowners and defines their type. We assume that the agricultural productivity of the plot is positively related to θ . The index θ is private information of the landowner. However, it is common knowledge that it is drawn from the interval $\Theta = [\underline{\theta}, \bar{\theta}]$ with a cumulative distribution function $F(\theta)$ and a density function $f(\theta)$. The density function is assumed to be strictly positive on the support Θ . Moreover, $f(\theta)$ satisfies the regularity conditions⁹ such that $\frac{\partial[F(\theta)/f(\theta)]}{\partial\theta} \geq 0$.

Crop yield to the landowner is represented by

$$(1 - v) Y(\bar{A} - a, \theta) \quad (2.1)$$

where $\bar{A} - a$ is the surface cultivated, θ is the land type and v is a random shock which may reduce the crop production and could be related to the technologically primitive "slash and burn" agricultural practice that is typical in developing countries still forested areas.¹⁰ We assume that v belongs to the set $V = \{\underline{v}, \bar{v}\}$ where $0 \leq \underline{v} < \bar{v} \leq 1$ and it is equal to \underline{v} or \bar{v} with probability q and $1 - q$ respectively. Therefore, the expected crop yield is

$$\begin{aligned} q(1 - \underline{v}) Y(\bar{A} - a, \theta) + (1 - q)(1 - \bar{v}) Y(\bar{A} - a, \theta) \\ = [1 - \bar{v} + q(\bar{v} - \underline{v})] Y(\bar{A} - a, \theta) \end{aligned} \quad (2.2)$$

Assume that the production is increasing and concave in the units of land converted, increasing in θ and that the marginal product with respect to land is increasing in the land type. This is equivalent to following set of

⁹Most parametric single-peak densities meet this sufficient condition (Bagnoli and Bergstrom, 1989).

¹⁰However, it could be assumed a constant yield and model in the same simple way a shock on the price of the crop due to changing market conditions. This could be done at no cost and keeping the model practically intact.

assumptions: $Y_1 > 0$, $Y_{11} < 0$, $Y_2 > 0$ and $Y_{12} > 0$ where $Y_1 = \partial Y / \partial (\bar{A} - a)$, $Y_2 = \partial Y / \partial \theta$, $Y_{11} = \partial^2 Y / \partial (\bar{A} - a)^2$, $Y_{12} = \partial^2 Y / \partial (\bar{A} - a) \partial \theta$.

In the absence of a conservation program (hereafter, CP), the expected profits to each landowner's $\bar{A} - a$ units of land are represented by

$$\pi(\bar{A} - a, \theta) = p[1 - \bar{v} + q(\bar{v} - \underline{v})]Y(\bar{A} - a, \theta) - c(\bar{A} - a) \quad (2.3)$$

where p is the price of the product and c is the private cost for converting a unit of land, i.e. the cost of clearing the new plot and settle it.

We assume as in Motte et al. (2004) that given the abundance of forested land convertible the constraint on land availability is non binding. Other factors like labour and other inputs, here represented by c , are scarcer and more costly for the landowner. This means that even in the absence of a CP the landowner do not convert all the available land ($a > 0$). This is often the case in developing countries, where landowners are often credit-constrained and can afford the conversion cost just up to a certain extent of land.

In this situation, each landowner maximizes her expected rents with respect to the converted surface $(\bar{A} - a)$

$$\max_{\bar{A}-a} \pi(\bar{A} - a, \theta) = p[1 - \bar{v} + q(\bar{v} - \underline{v})]Y(\bar{A} - a, \theta) - c(\bar{A} - a)$$

Rearranging the first order condition (hereafter, foc)

$$p[1 - \bar{v} + q(\bar{v} - \underline{v})]Y_1(\bar{A} - a, \theta) = c \quad (2.4)$$

it follows that

$$Y_1(\bar{A} - a, \theta) = \frac{c}{p[1 - \bar{v} + q(\bar{v} - \underline{v})]}$$

The surface to be cultivated is determined equalising the expected marginal land productivity with the private conversion cost. Note that being $Y_{11} < 0$ the surface converted increases as the private conversion cost, c/p , decreases. The crop yield depends on the magnitude of the exogenous shock and its likelihood and as one can easily check in (2.4) the landowner convert more land as the expected yield increases.

Define by $\bar{A} - \hat{a}(\theta)$ the private optimal level of conversion and substitute it into the expected profit function to derive the level of expected profit

$$\pi(\bar{A} - \hat{a}(\theta), \theta) = p[1 - \bar{v} + q(\bar{v} - \underline{v})]Y(\bar{A} - \hat{a}(\theta), \theta) - c(\bar{A} - \hat{a}(\theta)) \quad (2.5)$$

If the GA announces a CP then a voluntary contract schedule $\{[a(\theta), T(\theta)]; \underline{\theta} \leq \theta \leq \bar{\theta}\}$ is proposed to landowners. In the contract $a(\theta)$ represent the surface of land type θ to be conserved and $T(\theta)$ is the relative transfer. If the landowner accepts the contract then her expected program rents are given by

$$\begin{aligned} \Pi(\bar{A} - a(\theta), \theta) &= \pi(\bar{A} - a(\theta), \theta) + T(\theta) \\ &= p[1 - \bar{v} + q(\bar{v} - \underline{v})]Y(\bar{A} - a(\theta), \theta) - c(\bar{A} - a(\theta)) + T(\theta) \end{aligned} \quad (2.6)$$

The GA's objective¹¹ is the maximization of the social surplus, W , with respect to the pair $[a(\theta), T(\theta)]$. Social surplus is defined as

$$W = B(a(\theta)) - (1 + \lambda)T(\theta) + \Pi(\bar{A} - a(\theta), \theta) \quad (2.7)$$

where λ is the shadow cost of public funds.¹² The function $B(a(\theta))$ is the social

¹¹The multi-agent problem faced by the GA can be analysed as a single-agent problem repeated n times (Smith and Shogren, 2002).

¹²Funds have been raised by taxes and this parameter reflects the marginal deadweight loss

benefit deriving from setting aside $a(\theta)$ units of land. Social benefit may include the value of good and services such as flood control, carbon sequestration, erosion control, wildlife habitat, biodiversity conservation, recreation and tourism and option and existence value associated to the habitat conserved. We assume that $B(a(\theta))$ is increasing and strictly concave in its argument and that $\lambda \geq 0$.

2.2.2 Conservation in First Best

We set up the standard mechanism design problem to derive as solution the optimal CP. As standard we first solve the problem in a first best situation where there is perfect information and the GA knows each landowner's type. The definition of the properties of the first best solution will be useful later when we will refer to it as a benchmark. In this case the GA's problem is given by:

$$\max_{a(\theta), T(\theta)} W = B(a(\theta)) - (1 + \lambda)T(\theta) + \Pi(\bar{A} - a(\theta), \theta) \quad (2.8)$$

s.t.

$$\Pi(\bar{A} - a(\theta), \theta) \geq \pi(\bar{A} - \hat{a}(\theta), \theta)$$

$$a(\theta) \geq \hat{a}(\theta) \quad \text{for all } \theta \in [\underline{\theta}, \bar{\theta}]$$

The first constraint is the individual rationality constraint which ensures voluntary participation to the program. It guarantees that the landowners are at least not worse off accepting the contract than not accepting it. This constraint is type-dependent in that the return accruing to the landowner not participating to the CP is related to the productivity of her own plot. The second constraint is instead introduced to control that each landowner conserves at least the same surface of land that she would have conserved without contract. Not introducing

 from (distortionary) taxation (Wu and Babcock, 1996).

this constraint, the CP, could end up providing the perverse incentive to convert more land.

Proposition 2.1 *In first best the surface allocated to agriculture within the CP is less than without the CP for every $\theta \in [\underline{\theta}, \bar{\theta}]$.*

See appendix A.1 for the proof.

From the foc of the maximization problem it comes out that if $a(\theta) = a^{FB}(\theta)$ the following relation must hold if

$$p[1 - \bar{v} + q(\bar{v} - \underline{v})] Y_1 (\bar{A} - a(\theta), \theta) = c + \frac{B'(a(\theta))}{(1 + \lambda)} \quad (2.9)$$

The GA maximizes its objective function with respect to $a(\theta)$ when accepting the contract the landowner equalizes her expected land marginal productivity with her private cost of clearing land plus the negative externality generated by converting. The surface converted still depends on the private clearing cost and on the expectations in terms of crop yield. The risk in the production can have important consequences in landowner decisions and it has to be considered when a CP is designed. Internalizing the social cost of her action the landowner reduces the surface of land converted. Note in (2.9) that the marginal social benefit is adjusted by $(1 + \lambda)$ and this reflects the existence of a trade off between the cost of raising funds for the payments and the marginal benefit from conservation. In fact, as λ increases the surface cultivated is larger and less conservation is achieved.

The transfer is paid to each landowner accordingly to her type and is given by

$$T^{FB}(\theta) = \pi(\bar{A} - \hat{a}(\theta), \theta) - \pi(\bar{A} - a^{FB}(\theta), \theta) \quad (2.10)$$

2.3 Mechanism under adverse selection

The GA announces the voluntary contract scheme $\{[a(\theta), T(\theta)]; \underline{\theta} \leq \theta \leq \bar{\theta}\}$. Now, there is no perfect information and the landowners have more information about their type than the GA which only knows the types distribution, $F(\theta)$. In this context the first-best contract schedule may not be incentive compatible and there could be incentive for some landowners to mimic and earn a positive information rent. Hence, the contract schedule should be designed such that for each landowner it is optimal to report the land type truthfully.¹³ The participation must be voluntary and after observing the contract schedule proposed, each landowner chooses whether to enter or not into the CP.

To induce truth-telling an incentive compatibility constraint has to be added to the principal-agent problem. This will restrict the set of feasible contract schedules and the resulting optimal CP will be a second best solution.

If type- θ landowner chooses the contract designed for type- $\tilde{\theta}$ landowners, $[a(\tilde{\theta}), T(\tilde{\theta})]$, her expected program rents are

$$\Pi(\bar{A} - a(\tilde{\theta}), \theta) = p[1 - \bar{v} + q(\bar{v} - \underline{v})]Y(\bar{A} - a(\tilde{\theta}), \theta) - c(\bar{A} - a(\tilde{\theta})) + T(\tilde{\theta}) \quad (2.11)$$

Instead, if she chooses the schedule designed for her type, $[a(\theta), T(\theta)]$, her expected program rents are

$$\Pi(\bar{A} - a(\theta), \theta) = p[1 - \bar{v} + q(\bar{v} - \underline{v})]Y(\bar{A} - a(\theta), \theta) - c(\bar{A} - a(\theta)) + T(\theta) \quad (2.12)$$

A contract schedule $\{[a(\theta), T(\theta)]; \underline{\theta} \leq \theta \leq \bar{\theta}\}$ satisfies the incentive compatibility

¹³In addition to be voluntary the CP mechanism must satisfy a truth-telling condition (Dasgupta, Hammond and Maskin, 1979).

constraint if and only if

$$\Pi(\bar{A} - a(\theta), \theta) \geq \Pi(\bar{A} - a(\tilde{\theta}), \theta), \quad \text{for all } \theta \text{ and } \tilde{\theta} \in [\underline{\theta}, \bar{\theta}] \quad (2.13)$$

This means that type- θ landowners always prefer $[a(\theta), T(\theta)]$ to all other available contract schedules. Voluntary participation is instead guaranteed imposing as above in the first best case the incentive rationality constraint

$$\Pi(\bar{A} - a(\theta), \theta) \geq \pi(\bar{A} - \hat{a}(\theta), \theta) \quad (2.14)$$

Definition 2.1 *A CP is feasible if it satisfies both the incentive compatibility constraint and the individual rationality constraint.*

Under asymmetric information the GA's problem is then given by

$$\begin{aligned} \max_{a(\theta), T(\theta)} E_{\theta}[W] &= \int_{\underline{\theta}}^{\bar{\theta}} [B(a(\theta)) + \pi(\bar{A} - a(\theta), \theta) - \lambda T(\theta)] f(\theta) d\theta \\ &\text{s.t.} \\ d\Pi(\bar{A} - a(\theta), \theta) &\geq \pi(\bar{A} - \hat{a}(\theta), \theta) \\ \Pi(\bar{A} - a(\theta), \theta) &\geq \Pi(\bar{A} - a(\tilde{\theta}), \theta) \\ a(\theta) &\geq \hat{a}(\theta) \quad \text{for all } \theta \in [\underline{\theta}, \bar{\theta}] \end{aligned} \quad (2.15)$$

Now, we rearrange the incentive rationality and compatibility constraints and restate (2.15) in order to derive and describe the properties of the optimal second best contract schedule (see the appendix for the proofs).

Proposition 2.2 *A contract schedule $\{[a(\theta), T(\theta)]; \underline{\theta} \leq \theta \leq \bar{\theta}\}$ is incentive compatible if and only if*

(a) $a'(\theta) \leq 0$

(b) $T'(\theta) = \{p[1 - \bar{v} + q(\bar{v} - \underline{v})] Y_1(\bar{A} - a(\theta), \theta) - c\} a'(\theta)$

The differential equation stated by the first condition (a) and the monotonicity constraint (b) define the local incentive constraints set, which ensures local truth-telling and completely characterizes a truthful direct revelation mechanism¹⁴ (Laffont and Martimort, 2002).

Condition (a) simply states that an incentive compatible program requires to conserve more units of land where land productivity is low. The landowner's private land allocation is defined by $Y_1(\bar{A} - a, \theta) = \frac{c}{p[1 - \bar{v} + q(\bar{v} - \underline{v})]}$ while under CP in that there is more conservation $Y_1(\bar{A} - a(\theta), \theta) \geq \frac{c}{p[1 - \bar{v} + q(\bar{v} - \underline{v})]}$. Hence, from condition (b) it follows $T'(\theta) \leq 0$. This means that under an incentive compatible CP the GA must lower total transfers as land productivity increases. Otherwise, every landowner would have an incentive to mimic the highest land type in that for this type a larger compensation would be paid conserving less (condition a). Instead, the existence of this trade-off should reduce the incentive to misreport. However, even if the total transfer decreases with θ , the highest type landowner must end up earning larger total rents because otherwise she would mimic a lower type choosing the best combination between contract requirement and relative compensation (see appendix A.3).

¹⁴In the appendix we show that the landowner neither lie globally and that the local incentive constraints imply also global incentive constraints.

Proposition 2.3 *For any incentive compatible CP, the individual rationality constraint is satisfied for all θ when*

$$\Pi(\bar{A} - a(\bar{\theta}), \bar{\theta}) - \pi(\bar{A} - \hat{a}(\bar{\theta}), \bar{\theta}) \geq 0 \quad (2.16)$$

Provided that it holds, this is sufficient condition for all the land types. This means that if the highest type enters into the CP, all the other types may do the same in that their total rents are not reduced.

Proposition 2.4 *The GA's problem in equation (2.15) can be reformulated as follows:*

a)

$$\begin{aligned} & \max_{a(\theta)} \int_{\underline{\theta}}^{\bar{\theta}} \Phi[a(\theta), \theta] f(\theta) d\theta \\ & \text{s.t.} \\ & a'(\theta) \leq 0 \\ & a(\theta) \geq \hat{a}(\theta) \end{aligned} \quad (2.17)$$

where

$$\begin{aligned} \Phi[a(\theta), \theta] = & \frac{B(a(\theta))}{(1+\lambda)[1-\bar{v}+q(\bar{v}-\underline{v})]} + pY(\bar{A}-a(\theta), \theta) + \\ & - \frac{c(\bar{A}-a(\theta))}{[1-\bar{v}+q(\bar{v}-\underline{v})]} + \frac{\lambda}{(1+\lambda)} pY_2(\bar{A}-a(\theta), \theta) \frac{F(\theta)}{f(\theta)} \end{aligned}$$

b) Given the optimal conservation schedule, $a^{SB}(\theta)$, derived from (2.17), the optimal transfer schedule, $T^{SB}(\theta)$, is defined by

$$T^{SB}(\theta) = - \int_{\theta}^{\bar{\theta}} \{p[1 - \bar{v} + q(\bar{v} - \underline{v})] Y_1(\bar{A} - a^{SB}(\xi), \xi) - c\} a^{SB'}(\xi) d\xi + T^{SB}(\bar{\theta}) \quad (2.18)$$

where $T^{SB}(\bar{\theta})$ is the minimum transfer such that (2.16) holds.

The problem in (2.15) may be solved in three steps. At first, determine $a^{SB}(\theta)$ solving the problem in (2.17). Second, minimize $\Pi(\bar{A} - a^{SB}(\bar{\theta}), \bar{\theta})$ subject to (2.16) with respect to $T(\bar{\theta})$. Third, substitute $a^{SB}(\theta)$ and $T^{SB}(\bar{\theta})$ in (2.18) and compute the optimal transfer schedule.

2.3.1 Analysis of the optimal Conservation Program

We characterize some of the properties of the solution to (2.17) through the analysis of the constraints introduced into the problem. First, let start with the perverse incentive constraint taking apart for the moment the monotonicity constraint. The problem in (2.17) can be represented by the following Lagrangian:

$$L = \int_{\underline{\theta}}^{\bar{\theta}} \Phi[a(\theta), \theta] f(\theta) d\theta + \phi(\theta) (a(\theta) - \hat{a}(\theta))$$

Under imperfect information the necessary conditions for an optimum include:

$$\frac{\partial L}{\partial a(\theta)} = \frac{B'(a(\theta))}{(1 + \lambda)[1 - \bar{v} + q(\bar{v} - \underline{v})]} - pY_1(\bar{A} - a(\theta), \theta) + \frac{c}{[1 - \bar{v} + q(\bar{v} - \underline{v})]} + \frac{\lambda}{(1 + \lambda)} pY_{12}(\bar{A} - a(\theta), \theta) \frac{F(\theta)}{f(\theta)} + \phi(\theta) = 0 \quad (L.1)$$

$$\phi(\theta) (a(\theta) - \hat{a}(\theta)) = 0, \quad \phi(\theta) \geq 0 \quad (L.2)$$

Consider an interval $[\theta_1, \theta_2] \subseteq [\underline{\theta}, \bar{\theta}]$ with $\theta_1 < \theta_2$ and suppose $a(\theta) = \hat{a}(\theta)$ and $\phi(\theta) > 0$. Substituting (2.5) into (L.1)

$$\phi(\theta) = -\frac{B'(\hat{a}(\theta))}{(1+\lambda)[1-\bar{v}+q(\bar{v}-\underline{v})]} + \frac{\lambda}{(1+\lambda)} pY_{12}(\bar{A}-\hat{a}(\theta), \theta) \frac{F(\theta)}{f(\theta)} \quad (2.19)$$

Note that when $\theta = \underline{\theta}$, $F(\underline{\theta}) = 0$ and considering that $B'(a(\theta)) > 0$ by assumption

$$\phi(\theta) = -\frac{B'(\hat{a}(\underline{\theta}))}{(1+\lambda)[1-\bar{v}+q(\bar{v}-\underline{v})]} < 0$$

By contradiction we can then prove that at least for $\theta = \underline{\theta}$, $\phi(\theta)$ must be null and the constraint is not binding. This means that in lowest type land more conservation is undertaken under the CP than without it. It follows that $\underline{\theta} < \theta_1$. To analyze what happens in the rest of the interval one should study the derivative of $\phi(\theta)$

$$\begin{aligned} \phi'(\theta) = & -\frac{B''(\hat{a}(\theta))}{(1+\lambda)[1-\bar{v}+q(\bar{v}-\underline{v})]} \hat{a}'(\theta) + \quad (2.20) \\ & -\frac{\lambda}{(1+\lambda)} \left[pY_{112}(\bar{A}-\hat{a}(\theta), \theta) \frac{F(\theta)}{f(\theta)} \hat{a}'(\theta) - pY_{122}(\bar{A}-\hat{a}(\theta), \theta) \frac{F(\theta)}{f(\theta)} + \right. \\ & \left. -pY_{12}(\bar{A}-\hat{a}(\theta), \theta) \frac{\partial [F(\theta)/f(\theta)]}{\partial \theta} \right] \end{aligned}$$

At this point, given that any particular form has been assumed for the functions in the program $\phi'(\theta)$ can take both signs in $[\theta_1, \theta_2]$. This implies that the perverse incentive constraint may be binding somewhere.

From (2.19) $\phi(\theta) \geq 0$ when

$$\lambda p [1-\bar{v}+q(\bar{v}-\underline{v})] Y_{12}(\bar{A}-\hat{a}(\theta), \theta) \frac{F(\theta)}{f(\theta)} \geq B'(\hat{a}(\theta)) \quad (2.21)$$

The intuition behind (2.21) is straightforward, if the marginal cost of information

(LHS) is greater than the marginal social benefit from conservation (RHS) then the extent of conservation under CP is equivalent to that privately undertaken. If (2.21) does not hold then additional conservation can be induced implementing a CP. If this is the case then $a(\theta) > \hat{a}(\theta)$ and $\phi(\theta) = 0$. It follows that the optimal $a(\theta)$ must satisfy the following condition:

$$\begin{aligned} \frac{B'(a(\theta))}{(1+\lambda)[1-\bar{v}+q(\bar{v}-\underline{v})]} - pY_1(\bar{A}-a(\theta),\theta) + \frac{c}{[1-\bar{v}+q(\bar{v}-\underline{v})]} + \\ - \frac{\lambda}{(1+\lambda)} pY_{12}(\bar{A}-a(\theta),\theta) \frac{F(\theta)}{f(\theta)} = 0 \end{aligned} \quad (2.22)$$

Now, we focus on the monotonicity constraint. From condition (a) in Proposition 2.2 an optimal second best CP requires $a^{SB'}(\theta) \leq 0$. It can be proved that when $a^{SB}(\theta) = \hat{a}(\theta)$ the monotonicity constraint is always satisfied on the interval $[\theta_1, \theta_2]$ (see the appendix A.6).

Let consider then the case $a^{SB}(\theta) > \hat{a}(\theta)$. Differentiating (2.22) and solving for $a^{SB'}(\theta)$:

$$a^{SB'}(\theta) = \frac{pY_{12}(\bar{A}-a^{SB}(\theta),\theta) + v \frac{F(\theta)}{f(\theta)} pY_{122}(\bar{A}-a^{SB}(\theta),\theta) + v pY_{12}(\bar{A}-a^{SB}(\theta),\theta) \frac{\partial[F(\theta)/f(\theta)]}{\partial \theta}}{\omega B''(a^{SB}(\theta)) + pY_{11}(\bar{A}-a^{SB}(\theta),\theta) + v \frac{F(\theta)}{f(\theta)} pY_{112}(\bar{A}-a^{SB}(\theta),\theta)} \quad (2.23)$$

where $\omega = 1/(1+\lambda)[1-\bar{v}+q(\bar{v}-\underline{v})]$ and $v = \lambda/1+\lambda$.

Our model is general and given that no assumptions have been introduced for the sign of the third derivatives $Y_{122}(a(\theta),\theta)$, $Y_{112}(a(\theta),\theta)$ we can just say that the monotonicity constraint may or may not hold. Providing that it does then $\{[a^{SB}(\theta), T^{SB}(\theta)]; \underline{\theta} \leq \theta \leq \bar{\theta}\}$ is the optimal solution and it is separating in that all types choose the contract intended for them. In this case the optimal

extent of conservation in second best must satisfy the following relation

$$Y_1(\bar{A} - a(\theta), \theta) = \frac{1}{p[1 - \bar{v} + q(\bar{v} - \underline{v})]} \left[c + \frac{B'(a(\theta))}{(1 + \lambda)} \right] + \quad (2.24)$$

$$- \frac{\lambda}{(1 + \lambda)} Y_{12}(\bar{A} - a(\theta), \theta) \frac{F(\theta)}{f(\theta)}$$

Considering the restrictions imposed on $Y(\bar{A} - a(\theta), \theta)$ and comparing the first-best optimal allocation rule in (2.9) and the second best one in (2.24) it follows that

$$a^{FB}(\theta) \geq a^{SB}(\theta) \quad \forall \theta \in \Theta = [\underline{\theta}, \bar{\theta}] \quad (2.25)$$

Proposition 2.5 *Under symmetric information, the extent of conserved land is never less than that under asymmetric information.*

This distortion is due to the presence of the factor

$$\frac{\lambda}{(1 + \lambda)} Y_{12}(\bar{A} - a(\theta), \theta) \frac{F(\theta)}{f(\theta)}$$

This term represents the effect of the information rent that must be paid to landowners in order to give them appropriate incentives to truthfully report their type. Note that there is no distortion only for the landowners who own the lowest type land (since $F(\underline{\theta}) = 0$). Decreasing the surface of land conserved by higher land type holders ($a^{SB'}(\theta)$) and the compensation paid ($T'(\theta) \leq 0$) to higher land type holders the optimal scheme proposed reduces the information rents that must be paid to the lower land type holders.

Proposition 2.6 *If $\lambda = 0$ then the optimal CP is first best.*

First-best conservation can be attained under asymmetric information only in the case where the social cost for raising funds to pay ecosystem services is null.

Finally if the monotonicity constraint does not hold¹⁵ then $[a^{SB}(\theta), T^{SB}(\theta)]$ is not the solution to the GA problem. The solution (see appendix A.8), which involves bunching types on the whole support or on some intervals can be derived using the Pontryagin principle (Guesnerie and Laffont, 1984; Laffont and Martimort, 2002). When it is not possible to separate the types, the GA must consider that the CP may be costly in that higher type compensation may be paid to each landowner and less conservation than expected may finally be undertaken.

2.3.2 Transfers

When the perverse incentive constraint is not binding and the monotonicity constraint holds the transfers can be computed simply substituting $a^{SB}(\bar{\theta})$ and $a^{SB}(\theta)$ into (2.18). If the perverse incentive constraint is binding, the compensation structure changes. As proved in the appendix (A.6) the monotonicity constraint holds and the contract schedule is separating and all landowners who conserve $\hat{a}(\theta)$ within the contract receive the same transfer ($T'(\theta) = 0$). In particular, if $\bar{\theta}$ -type landowners conserve $\hat{a}(\bar{\theta})$ then all the landowners in the interval $[\theta_1, \bar{\theta}]$ where $a(\theta) = \hat{a}(\theta)$, will not receive any compensation. Instead if $a(\theta) = \hat{a}(\theta)$ is undertaken in $[\theta_1, \theta_2]$ and this interval is strictly included in $[\underline{\theta}, \bar{\theta}]$ then all the landowners in that interval will be paid the compensation computed for θ_2 for conserving the same extent of land they would have conserved privately. The GA is essentially paying them to correctly reveal their cost type.

However, without a constraint on the consistency of the policy, less conservation could have been induced for certain cost types and then controlling for this perverse effect of the CP at least avoids that payments are destined to convert more land (Motte et al. 2004). This could be actually the case in developing

¹⁵That is $a^{SB'}(\theta) > 0$ or $a^{SB'}(\theta)$ changes sign on the support Θ .

countries where landowners are normally credit constrained and can afford the conversion cost up to a certain extent of land. Under the program instead this constraint is relaxed in that conserving land is paying a certain return represented by the transfer and they may plan to convert more land.¹⁶

2.3.3 Optimal CP vs general subsidy

As said in the introduction the PES programs are implemented as general subsidy schemes (hereafter, GS). In practice any landowner may enter the program, choose the extent of land to conserve and earn a fixed compensation \bar{T} /ha/year. In principle, the GA should fix \bar{T} in order to attract cheapest land which cost opportunity is low. Now, suppose that the GA plans to develop a GS conservation program in areas where $\underline{\theta} \leq \theta \leq \bar{\theta}$. A GS scheme is equivalent to offer the contract schedule $\{[\bar{a}(\theta), \bar{T} \cdot \bar{a}(\theta)]; \underline{\theta} \leq \theta \leq \bar{\theta}\}$ where $\bar{a}(\theta)$ is the surface that the landowners voluntarily decides to conserve under the program. It can be proved (see appendix A.7).

Proposition 2.7 *Social surplus from agricultural production and habitat conservation is greater under the optimal conservation program (CP) than under a general subsidy conservation program (GS).*

The GS contract schedule $\{[\bar{a}(\theta), \bar{T} \cdot \bar{a}(\theta)]; \underline{\theta} \leq \theta \leq \bar{\theta}\}$ belongs to the feasible set in that it satisfies the incentive rationality and compatibility constraints. But, since $\{[a^{SB}(\theta), T^{SB}(\theta)]; \underline{\theta} \leq \theta \leq \bar{\theta}\}$ is the best feasible contract schedule and it is the unique solution to the GA's maximization problem, social surplus cannot be lower under the optimal CP than under the GS.

¹⁶In these countries land is surely cheaper than investing in technology to enhance the productivity of converted land.

A GA implementing the optimal CP designed needs to gather specific information regarding for example the structure of the landholder's profit function, the social benefit function, the cost of raising money, the distribution of types and with respect to the shock, the set of possible outcomes and their probability. The collection of this information could be costly and make less significant the gain in welfare that undoubtedly may be attained implementing this program. In fact, adding this cost to the information rent that must be paid to the landowners to reveal their type could more than balance this gain and justify the choice quite common in the reality of implementing general subsidy scheme.¹⁷

2.4 Conservation program at work

Let now illustrate the characteristics of the mechanism under incentive compatibility by using an example. Assume

(i) $B(a) = \beta a - \frac{a^2}{2}$ as social benefit function,

(ii) $Y(\bar{A} - a, \theta) = (\bar{A} - a) \theta - \frac{(\bar{A} - a)^2}{2}$ as agricultural production function,

(iii) the uniform distribution of θ with $F(\theta) = \frac{\theta - \underline{\theta}}{\bar{\theta} - \underline{\theta}}$, $f(\theta) = \frac{1}{\bar{\theta} - \underline{\theta}}$ and

(iv) $\beta > a$, $\underline{\theta} > \bar{A} - a$, $\bar{\theta} \leq \bar{A} + \frac{c}{p[1 - \bar{v} + q(\bar{v} - \underline{v})]}$, $k = [1 - \bar{v} + q(\bar{v} - \underline{v})]$.

Without any CP, the amount of land conserved is

$$\hat{a}(\theta) = \bar{A} - \theta + \frac{c}{pk}$$

¹⁷See Crepin (2005) and Arguedas et al. (2007).

With CP in place, first best allocations are given by

$$a^{FB}(\theta) = \frac{1}{1 + (1 + \lambda)pk} [((\bar{A} - \theta)pk - c)(1 + \lambda) + \beta]$$

$$T^{FB}(\theta) = (\hat{a}(\theta) - a^{FB}(\theta)) \left[pk(\bar{A} - \theta) - c \frac{(\hat{a}(\theta) + a^{FB}(\theta))}{2} \right]$$

Note that as proved the perverse incentive constraint does not bind in a first best scenario.

Now, assume that $a^{SB}(\theta) > \hat{a}(\theta)$. The monotonicity constraint holds given that

$$a^{SB'}(\theta) = -\frac{pk(1 + 2\lambda)}{1 + pk(1 + \lambda)} \leq 0$$

Second best allocations are then given by

$$a^{SB}(\theta) = \frac{1}{1 + (1 + \lambda)pk} [((\bar{A} - \theta)pk - c)(1 + \lambda) + \beta - (\theta - \underline{\theta})pk\lambda]$$

Comparing $a^{SB}(\theta)$ with $a^{FB}(\theta)$ one can see easily realize the impact of information asymmetry. The term representing the effect of the information rent is

$$-(\theta - \underline{\theta}) \frac{pk\lambda}{1 + (1 + \lambda)pk}$$

The land to be conserved decreases with θ and in this manner the optimal mechanism reduce the amount of information rent that should be paid to the low type landowners to correctly reveal their type. If $\theta = \underline{\theta}$ the surface conserved is as expected not distorted. To derive the transfer function $T^{SB}(\bar{\theta})$ must be determined.

Minimizing $\Pi(\bar{A} - a^{SB}(\bar{\theta}), \bar{\theta})$ subject to (2.16) with respect to $T(\bar{\theta})$, it follows

$$\begin{aligned} T^{SB}(\bar{\theta}) &= \pi(\bar{A} - \hat{a}(\bar{\theta}), \bar{\theta}) - \pi(\bar{A} - a^{SB}(\bar{\theta}), \bar{\theta}) \\ &= (\hat{a}(\bar{\theta}) - a^{SB}(\bar{\theta})) \left[pk(\bar{A} - \bar{\theta}) - c \frac{(\hat{a}(\bar{\theta}) + a^{SB}(\bar{\theta}))}{2} \right] \end{aligned}$$

The transfer function is then given by

$$\begin{aligned} T^{SB}(\theta) &= (\hat{a}(\bar{\theta}) - a^{SB}(\bar{\theta})) \left[pk(\bar{A} - \bar{\theta}) - c \frac{(\hat{a}(\bar{\theta}) + a^{SB}(\bar{\theta}))}{2} \right] + \\ &+ \frac{pk(1+2\lambda)}{1+pk(1+\lambda)} \int_{\theta}^{\bar{\theta}} \{pk[\xi - (\bar{A} - a^{SB}(\xi))] - c\} d\xi \end{aligned}$$

Note that $T^{SB}(\theta) \leq 0$ and that the contract proposed is separating. The value of the private information is higher for the low types and this types have no incentive to reveal their true cost if an informational rent is not paid.

2.5 Conclusions

Combining agriculture and habitat protection is an appealing but extremely challenging target. The debate over this issue in the past decades has highlighted the idea that ecosystem services are valuable and that conservation is an alternative land use. This is important in order to support the implementation of PES programs in developing countries not as the way richer countries subsidize the welfare of the poorer but as a tool for promoting their development paying them for the valuable contribution they can provide conserving the habitat.

However, some potential weaknesses in the PES programs implementation must be overcome. We refer in particular to the lack of proper targeting and the use of undifferentiated transfers (World Bank, 2000).

This chapter draws using the mechanism design theoretical framework a conservation program which allows for the differentiation of the payments with respect to the opportunity cost of providing ecosystem services. The contract schedule proposed in alternative to the more common general subsidy scheme keeps into account the risk of poor crop yield which characterizes the agricultural activity in developing countries and control for the likely perverse effect that a conservation program could have once a compensation is paid, namely less conversion than that which would be observed without a conservation policy. The recognition of the incentive for the rational landowner to select, even misreporting, the best combination of conservation and agriculture leads to impose in addition to the incentive rationality also the incentive compatibility of the contract schedule that should be announced. Transfers and contract requirements are then set to reduce information rents that must be paid for collecting private information on the conservation costs and maximize social welfare. We verify comparing the two alternatives that a gain in welfare can be attained implementing our incentive compatible program.

In the light of the debate on the opportunity of implementing incentive compatible programs for conservation we believe that our attempt to contribute to the broad literature on this topic is completely justified and that our framework allows for the analysis of several aspects characterizing this issue. Nevertheless, the analysis in this chapter may be weak in some respects and more research would be needed. In particular, we recognize the lack of an explicit modelling of the credit constraint for the landowners. Another aspect deserving more research is the relationship between the probability of unfavourable crop yields and the environmental characteristics of the land which may be converted to agriculture. This analysis could be developed in the standard principal - agent framework where

differently from the model here presented the private information on θ enters into the problem not only affecting the land productivity but also the probability of a scarce crop yield and as a direct consequence the actual probability that a certain land type will be cleared. Finally, in our view the uncertainty in the return from agriculture and the irreversibility of the conversion process matters and must be introduced into the mechanism design problem. Actually, the landowner can be seen as holding a portfolio including two assets, the land converted paying a risky return represented by the crop yield and the land conserved paying a certain return given by the transfer. It would be interesting to study in which proportions the two assets are held in the light of the uncertainty on the agricultural return and of the irreversibility characterizing the decision to convert. We think that extending the research presented in this chapter in the directions briefly sketched in these final lines may add insight to the analysis and significantly contribute to the literature on conservation contracts.

Chapter 3

Optimal conservation policy under imperfect intergenerational altruism

3.1 Introduction

In the debate on the reasons of natural resource depletion, an important role has been always given to the time preference or myopia resulting from discounted pay-offs attached to natural assets conservation. While this is surely a convincing argument, in my opinion however it is not sufficient to explain two characteristics of current conservation policies: excessive rush and time-inconsistency (Brocas and Carrillo, 1998; Hepburn, 2003). In this direction, one striking example is given by the management of publicly owned natural forests in Indonesia where despite a sustainable exploitation of this natural asset has been targeted by the government there is evidence of a faster depletion rate and of time-inconsistency in the application of the policy (Atje and Roesad, 2004).

Environmental issues such as forest conservation and species preservation are often characterized by the impact of uncertainty on the pay-offs and by the irreversibility of some decisions once taken. In such a context because of the attached option value waiting before taking an irreversible decision and collect information to reduce uncertainty may be a reasonable strategy (Arrow and Fisher, 1974; Henry, 1974; Dixit and Pindyck, 1994). As said above instead this seems not to be the case in the reality where often governments revise previous conservation policies and rush in undertaking projects which have irreversible impact on natural assets endowment and on the related provision of goods and services (Brocas and Carrillo, 1998).

The aim of this chapter is to give a rationale for haste and time-inconsistency developing the analysis of optimal conservation policies in an intergenerational framework where imperfect altruism is assumed.

In dynamic welfare economics the debate on the issue of intergenerational altruism and discounting is not a new one. It starts with a paper on optimal growth written by Ramsey (1928) where despite being termed "ethically indefensible" discounting at a constant rate of time-preference is allowed. In addition, Ramsey assumes perfect intergenerational altruism which implies that "each generation's preference for its own consumption relative to the next generation's consumption is no different from their preference for any future generation's consumption relative to the succeeding generation".¹ Phelps and Pollack (1968) instead discuss this assumption and extend the analysis introducing the possibility that a "truly democratic" government being representative of an "imperfectly altruistic" current generation defines its optimal policies according to its time preferences.

In a similar framework I intend to solve the classic problem of optimal timing

¹Phelps and Pollack (1968), p. 185.

of irreversible harvest with known and constant value of the wood harvested and uncertain flow of amenity value from conservation² (Reed, 1993; Conrad, 1997). Each generation is imperfectly altruistic and its welfare depends on its own and on future generations' consumption. Differently from Phelps and Pollack (1968) I allow for a finite number of succeeding generations living over a random period of time drawn by a birth/death Poisson process. Each generation compares the level of welfare deriving from harvesting the forest with the one attached to the conservation and sets a critical level for the amenity value that once met makes optimal to cut.

I show that solving the intergenerational problem described above is equivalent to solving the standard optimal stopping problem in continuous time relaxing the assumption of exponential discounting and allowing for a decision-maker using an hyperbolic discount function³ which takes the functional form introduced by Harris and Laibson (2004). As noted by Strotz (1956) discount functions with time-varying declining discount rates implies inconsistent planning and belonging to that set this is also the case for the hyperbolic one.

I assume that the current generation is not able to impose any conservation plan to the following generations and I solve the problem by backward induction under the two standard assumptions of naïve and sophisticated belief on future generations time preferences (Strotz, 1956; Pollak; 1968). In the first case the current generation irrationally believes that future generations will act according to its own discount function as if they were committed.⁴ I find that under naïve

²Reed (1993) and Conrad (1997) determine the optimal harvest timing under a constant time preference rate. In an intergenerational framework this is equivalent to solve the problem under the assumption of perfect altruism.

³This is not a complete novelty in the real options literature. See Grenadier and Wang (2007) where the timing of investment is studied under the assumption of an hyperbolic discounting entrepreneur.

⁴One may think to a generation irrationally confident in an ineffective commitment device.

belief the critical amenity value level that must be met to harvest the forest is higher than that in the benchmark case represented by the solution of the same problem under perfect altruism. This implies that in expected terms the forest will be harvested earlier. The intuition behind this result is that the bias for current generation's gratification relative to the future generations' gratification due to imperfect altruism and the generational transition rate lowers the value attached to wait for collecting information and reduce uncertainty on benefit from conservation and induces haste in the exercise of the option to harvest. The conservation plan defined by current generation is defined on the basis of incorrect beliefs and is time-inconsistent in that the following generation will revise the previous policy setting a time trigger determined according to its own time preferences.

The solution of the problem under the assumption of sophisticated belief has even stronger implications for conservation and intergenerational forest value distribution. In fact, having perfect foresight with respect to future generations' strategies each generation internalizes the cost of sub-optimal (from its time perspective) future conservation plans and sets an higher critical threshold for harvesting relative to the "naïve generation". In this case the value of waiting is further eroded by an additional effect due to sophistication and I find that the critical thresholds set by each generation for harvesting the forest are increasing in the number of generations ahead. This makes sense considering that the less generations will succeed the less is the cost due their sub-optimal behaviour.

The chapter is structured as follows. In section 1, the set of assumptions on which I set up the model is presented. In this section I also briefly present the basic model by Conrad (1997) that will be used later as a benchmark. In section 3.2, the problem is solved under naïve belief and the solution is derived

and discussed. In section 3.3, I first solve under sophistication the problem for a three succeeding generations model. This allows to take things simpler at no cost in terms of insight. I finally provide the solution for the general case with a finite number of generations. In Section 3.4 I present and discuss an alternative application of the model based on political parties turnover at the government. Section 3.5 concludes. All the proofs and the details regarding the solving procedure are available in the appendix.

3.2 The basic set-up

Note that when harvest occurs the forest provides to the generation living at that time the value represented by wood revenue, the flow of amenity value stops forever and no value will accrue to succeeding generations.

Consider a government representing the will and the preferences of the generation currently belonging to the body politic. Assume that each generation is risk neutral and that its welfare depends on its own and on future generations' consumption but that the value of future generations' consumption relative to its own is lowered by a constant factor $0 < \delta \leq 1$. If $0 < \delta < 1$ the current generation is imperfectly altruistic while if $\delta = 1$ it is perfectly altruistic. Being risk-neutral to maximize the welfare objective function is equivalent to the maximization of the sum of current and discounted future generations' consumption of the value generated by harvesting or conserving the forest. Note that when harvest occurs the forests provides to the generation living at that time the value represented by wood revenue while the flow of amenity value stops forever and no value will accrue to succeeding generations. Assume also that each generation i discounts exponentially at a constant time preference rate ρ and lives over a random period

of time delimited by its birth at t_i and the birth of the next generation at t_{i+1} with births occurring according to a Poisson process with intensity⁵ $\lambda \in [0, \infty)$. It can be easily shown that given the assumptions above

Definition 3.1 *For any $\delta \in (0, 1]$ and $\lambda \in [0, \infty)$ the generation i discount function is given by*

$$D_i(t, s) = \begin{cases} e^{-\rho(s-t)} & \text{if } s \in [t_i, t_{i+1}] \\ \delta e^{-\rho(s-t)} & \text{if } s \in [t_{i+1}, \infty] \end{cases} \quad (3.1)$$

for $s > t$ and $t_i \leq t \leq t_{i+1}$

This stochastic function discounts at time t a \$1 pay-off accruing to generation i at time s . Generation i is discounting exponentially at rate ρ consumption occurring over its lifespan while consumption by future generations is additionally discounted by the factor δ . This functional form is equivalent to the one introduced by Harris and Laibson (2004) to model an hyperbolic discounting agent⁶ and as one can easily note consumption is discounted at a declining discount rate⁷ showing preference for the current relative to future generations consumption. Moreover as noted by Strotz (1956), being time-varying this preferences implies inconsistent planning. This means that each generation wish to revise according to its own time perspective and discount function the time trigger for harvesting determined by the previous generation.

In this frame I further and finally assume that the current generation is not able to commit future generations to any conservation strategy and that each generation defines its optimal conservation plan on the basis of its expectations

⁵This parameter represents the rate of generational transition.

⁶See Grenadier and Wang (2007) for an equivalent hyperbolic investor discount function.

⁷As $\frac{1-e^{-\rho}}{e^{-\rho}} < \frac{1-\delta e^{-\rho}}{\delta e^{-\rho}}$, the discount rate between two consecutive periods t and $t+1$ increases as date t comes close.

on the future generations optimal conservation plans. In this respect, as long as the absence of an effective commitment device may or may be not realized two different types of belief, respectively sophisticated and naïve, should be taken into consideration to model the decision-making process (Strotz, 1956; Pollak; 1968).

3.2.1 Harvesting or conserving

Let us focus on the management of a natural forest by each government⁸ in the light of the benefits and costs described above. The target is given by the maximization of the represented politic body welfare with respect to the two possible management policies, namely conservation and total and irreversible harvest.⁹

In the first case the net value of stumpage timber, M , is known and constant. Instead, if the forest is conserved at each time period t an uncertain flow of amenity value,¹⁰ $A = A(t)$, accrues to the society. Such flow randomly fluctuates according to the following geometric Brownian motion

$$dA(t) = \mu A(t)dt + \sigma A(t)dz \quad (3.2)$$

where $\mu > 0$ is the mean drift rate, $\sigma \geq 0$ is the standard deviation rate and $\{z(t)\}$ is a standard Wiener process.¹¹

Each government can be viewed as holding an option to harvest which pays

⁸From now on being totally representative of the current politic body preferences, each generation will be represented by the government in charge over its lifespan.

⁹This makes sense considering that recovering the forest in the initial state could take time. From a century up to several millennia, according to the cases (http://en.wikipedia.org/wiki/Old_growth_forest).

¹⁰Defined by the sum of option and existence values and of the value attached to the provision of services such as flood control, carbon sequestration, erosion control, wildlife habitat, biodiversity conservation, recreation and tourism (Reed, 1993; Conrad, 1997).

¹¹Where the usual conditions, $E[dz(t)] = 0$ and $E[dz(t)^2] = dt$ are satisfied. The upward drift draws the increasing consideration of society for the amenity services and the variance parameter captures the uncertainty about their actual and future value (Reed, 1993; Conrad, 1997).

a dividend represented by the flow $A(t)$ if unexercised. Harvesting being an irreversible action, an option value may be attached to the decision to conserve in that this strategy allows to the decision maker to collect information about the uncertain flow of amenity value. The question to be answered is then when is harvesting optimal with respect to conserving from the time perspective of the current generation. This can be decided solving the underlying stochastic optimal stopping problem.

Under the standard assumption of constant time-preference rate ($\delta = 1$) the solution to this problem has been provided by Conrad (1997). In the following I briefly describe how the problem has been solved and the characteristics of the solution.

Denote by $V(A)$ the value function that the government want to maximize. The Bellman equation of this problem is given by

$$V(A) = \max_A \{M, Adt + e^{-\rho dt} E [V(A + dA)]\} \quad (3.3)$$

where ρ is the time-preference rate.

Definition 3.2 *In the continuation region, $A \geq A^*$, the value function, $V(A)$, solves the following second-order non-homogenous differential equation¹²*

$$\frac{1}{2}\sigma^2 A^2 V''(A) + \mu A V'(A) - \rho V(A) = -A, \quad \text{for } A \geq A^* \quad (3.4)$$

where A^* represents the level of amenity value delimiting the continuation region where the option to harvest is kept alive. At A^* conserving or harvesting is indifferent and as soon as this level is hit the option is killed. Assuming¹³ $\rho > \mu$

¹²This equation is obtained using Ito's lemma to expand (3.3).

¹³Note that if $\rho \leq \mu$ conserving forever is the optimal plan.

(3.4) can be solved attaching the following value-matching and a smooth-pasting conditions to guarantee optimality:¹⁴

$$V(A^*) = M \quad (3.5)$$

$$V'(A^*) = 0 \quad (3.6)$$

Proposition 3.1 *Under constant time-preference the solution to the optimal stopping problem in (3.3) is given by*

$$A^* = \frac{\beta_1}{\beta_1 - 1} M(\rho - \mu) \quad (3.7)$$

$$V(A) = \begin{cases} \frac{M}{1-\beta_1} \left(\frac{A}{A^*}\right)^{\beta_1} + \frac{A}{\rho-\mu} & \text{for } A > A^* \\ M & \text{for } A \leq A^* \end{cases} \quad (3.8)$$

where β_1 is the negative root of the characteristic equation¹⁵ $\sigma^2\beta(\beta - 1)/2 + \mu\beta - \rho$.

The first term on the RHS of (3.8) represents the value of the option to harvest and it vanishes as $A \rightarrow \infty$. The second term is the expected present value of the randomly fluctuating flow A which accrues intertemporally to the society if the forest is never cut down. As soon as $A \leq A^*$ the option is exercised and the generation living at that time benefits from revenue M . Note that in this case, being $\delta = 1$, the discount function $D_i(t, s)$ reduces to the standard exponential form. This implies that the optimal harvest trigger strategy is time-consistent and that the conservation policy will not be revised by future governments.

¹⁴In the real options literature this is used as a no-arbitrage condition (Dixit, 1993).

¹⁵The solution is $\beta_1 = \left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right) - \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2\rho}{\sigma^2}} < 0$

3.3 Optimal harvest timing: imperfect altruism and naiveté

I relax now the assumption of perfect altruism and I assume that $0 < \delta < 1$. When in charge each government may exercise the option to harvest and earn the payoff M or keep it alive and let current generation benefiting from its and future generations' consumption of the goods and services provided by the forest. The option if not exercised is then left as a legacy to the succeeding government which in turn may or may not exercise it.

In this frame the solution to the optimal stopping problem that the current government solves to set its optimal harvest time trigger will be represented by the outcome of the game played over several periods by this government and the future ones and will internalize the effect of harvesting trigger strategies set by future governments according to their time perspective.

Let denote the current government by 0 and solve the problem under the assumption of naïve belief on future governments' harvesting strategies. Being "naïve" the current government believes that all succeeding governments will set their policies according to its discount function that is given by

$$D_0(t, s) = \begin{cases} e^{-\rho(s-t)} & \text{if } s \in [0, t_1] \\ \delta e^{-\rho(s-t)} & \text{if } s \in [t_1, \infty] \end{cases}$$

for $s > t$ and $0 \leq t \leq t_1$

This implies that all the succeeding governments are considered by the current practically as perfect altruistic and discounting exponentially at the same constant rate ρ .

According to $D_0(t, s)$ the current government discounts by $e^{-\rho(s-t)}$ the pay-offs from forest management occurring at $s < t_1$ and by $\delta e^{-\rho(s-t)}$ the pay-offs occurring at $s \geq t_1$. Also in this case, the optimal timing for the option exercise will be given by a critical threshold for the amenity value. If the next generation is born before such critical threshold is met the current generation enjoys the flow of amenity value, A , for the period $[0, t_1]$ and the continuation value $V_c^N(A)$ which is given by the expected present value of the pay-offs attached to future governments' strategies.

If as incorrectly believed all future governments are discounting at the constant rate ρ the optimal stopping problem they solve to define the harvest timing is equivalent to the one solved by Conrad (1997). Hence, their critical trigger and value function will be respectively given by A^* and $V(A)$. Given that the current government lowers by δ all pay-offs from future exercise it follows that $V_c^N(A) = \delta V(A)$. Now, let $V^N(A)$ and A_N be respectively the current government's value function and the optimal exercise threshold. In this case, the Bellman equation is given by

$$V^N(A) = \max_A \left\{ M, Adt + e^{-\lambda dt} E \left[e^{-\rho dt} V^N(A + dA) \right] + \right. \\ \left. + (1 - e^{-\lambda dt}) E \left[e^{-\rho dt} V_c^N(A + dA) \right] \right\} \quad (3.9)$$

Definition 3.3 *In the continuation region, $A \geq A_N$, the value function, $V^N(A)$, solves the following second-order non-homogenous differential equation¹⁶*

$$\frac{1}{2} \sigma^2 A^2 V^{N''}(A) + \mu A V^{N'}(A) - \rho V^N(A) \\ = - \left\{ A + \lambda \left[V_c^N(A) - V^N(A) \right] \right\}, \quad \text{for } A \geq A_N \quad (3.10)$$

¹⁶This equation is obtained using Ito's lemma to expand (3.9) (Dixit and Pindyck, 1994).

At the critical threshold A_N , where keeping the option open or exercising it is indifferent, the value-matching and smooth-pasting conditions respectively require

$$V^N(A_N) = M \quad (3.11)$$

$$V^{N'}(A_N) = 0 \quad (3.12)$$

Proposition 3.2 *Under declining time-preference rate and naïve belief the solution to the optimal stopping problem in (3.9) is given by*

$$A_N = \left[\frac{\beta_2}{\beta_2 - 1} - \delta \frac{\beta_2 - \beta_1}{(1 - \beta_1)(\beta_2 - 1)} \left(\frac{A_N}{A^*} \right)^{\beta_1} \right] M \left(\frac{\rho - \mu}{\eta} \right) \quad (3.13)$$

$$V^N(A) = \begin{cases} \left[M - \delta M \frac{1}{1 - \beta_1} \left(\frac{A_N}{A^*} \right)^{\beta_1} - A_N \left(\frac{\eta}{\rho - \mu} \right) \right] \left(\frac{A}{A_N} \right)^{\beta_2} + \\ + \frac{\delta M}{1 - \beta_1} \left(\frac{A}{A^*} \right)^{\beta_1} + A \left(\frac{\eta}{\rho - \mu} \right) & \text{for } A > A_N \\ M & \text{for } A \leq A_N \end{cases} \quad (3.14)$$

where $\eta = \frac{\rho + \lambda \delta - \mu}{\rho + \lambda - \mu} \leq 1$ and $\beta_2 \leq \beta_1$ is the negative root of the characteristic equation¹⁷ $\sigma^2 \beta(\beta - 1)/2 + \mu\beta - (\rho + \lambda)$. See B.1 for the solving procedure.

Proposition 3.3 *Under declining time-preference rate and naïve belief each government exercises the option to harvest at $A_N > A^*$.*

The time trigger for a naïve and imperfectly altruistic government is higher than the perfect altruistic one (see B.2 for the proof). The intuition behind this result is that the value of keeping alive the option has lower value in this case because due to its present-biased preferences the present value of the utility

¹⁷The solution is $\beta_2 = \left(\frac{1}{2} - \frac{\mu}{\sigma^2} \right) - \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2} \right)^2 + \frac{2(\rho + \lambda)}{\sigma^2}}$

resulting from the decisions of the future governments is lower than the one under perfect altruism ($0 < \delta < 1, \eta < 1$). There is then incentive for this generation to anticipate future ones in the exercise of the option and this incentive increases as less altruistic are the generations and the higher is the generational transition rate ($\frac{dA_N}{d\delta} < 0, \frac{dA_N}{d\lambda} > 0$). Note that the plan here defined is irrational in that is based on the false belief of being able to have the subsequent generations committed to the policy defined by the current one. Actually, as soon as the following generation will be born at $t = t_1$ the harvest trigger adopted will not be A^* as incorrectly believed by the current government but higher and fixed according to its discount function $D_1(s, t)$. This will happen also when succeeding generations will enter into the politic body at t_2, t_3 and so on. Note that if also the future governments are naïve then the problem they solve to fix the harvest time trigger is equivalent to one solved in this section and the time trigger is given by $A_N > A^*$.

3.4 Optimal harvest timing: imperfect altruism and sophistication

Assume now sophistication¹⁸ and imperfect altruism ($0 < \delta < 1$). In this case the current government has perfect foresight and anticipates that each future government is imperfect altruistic and will define its optimal harvesting strategy according to its own hyperbolic discount function $D_i(t, s)$. From the current government time-perspective all the future governments' harvest time triggers are sub-optimal and being this perfectly anticipated a cost attached to sub-optimality

¹⁸The agent decision making is based on rational expectations about future strategies (O'Donoghue and Rabin, 1999).

enters into its welfare maximization problem.¹⁹ This will produce an additional effect on the optimal harvest timing with respect to the naïve case where only a present-bias effect is present.

In the next paragraph the implications of perfect foresight will be shown through a three succeeding governments model where each government sets the optimal conservation policy fixing a critical threshold for the exercise of the option to harvest. Finally, I will present the solution to the same problem for the general case with a finite number of governments I .

3.4.1 A three governments model

Let G_0 be the current government. On each interval dt the subsequent government G_1 is in power with probability λdt . Once G_1 has replaced G_0 , according to the same process it will be replaced by G_2 which will be in charge forever. Given this structure the solution to the optimal stopping problem faced by G_0 will be derived using backward induction and will be represented by a subgame-perfect equilibrium sequence of critical thresholds.

Consider G_2 . Let $A_{S,2}$ and $V_2^S(A)$ denote its trigger value and her value function. Since it faces eternity, its maximization problem reduces to the time-consistent case and

$$A_{S,2} = A^* \tag{3.15}$$

$$V_2^S(A) = V(A) \tag{3.16}$$

¹⁹A sophisticated agent may follow two strategies: the "strategy of precommitment" which consists in committing to a certain plan of action and the "strategy of consistent planning" which leads the agent not to choose the plans that are going to be disobeyed in the future (Strotz, 1956).

Now, G_1 is in charge. Its plan is defined knowing that G_2 would exercise the option to harvest at $A_{S,2}$. Having present-biased preferences G_2 's value function is worth for G_1 only δ times its value. Note that the problem for G_1 is equivalent to the problem solved for a naive government ($A_N, V^N(A)$). The main difference is represented by the fact that now the underlying beliefs are rationally formed. It follows that

$$A_{S,1} = A_N \quad (3.17)$$

$$V_1^S(A) = V^N(A) \quad (3.18)$$

Finally, it is time for G_0 to formulate its optimal harvest plan. Denote respectively by $A_{S,0}$ and $V_0^S(A)$ its value function and its trigger strategy and let $V_{c,1}^S(A)$ represent its valuation of the exercise decisions that could be taken by G_1 and G_2 which strategies are perfectly anticipated ($A_{S,1}, A_{S,2}$). The continuation value, $V_{c,1}^S(A)$, is recursively determined. If G_1 is in charge when the trigger $A_{S,1}$ is hit then the option is exercised and the payoff for G_0 is δM . Instead if G_2 replaces G_1 before $A_{S,1}$ is met, then the G_0 continuation value is equal to G_1 's continuation value $V_{c,2}^S(A) = \delta V_2^S(A)$.

Definition 3.4 *In the continuation region, $A \geq A_{S,1}$, the continuation value function, $V_{c,1}^S(A)$, solves the following second-order non-homogenous differential equation*

$$\begin{aligned} & \frac{1}{2}\sigma^2 A^2 V_{c,1}^{S''}(A) + \mu A V_{c,1}^{S'}(A) - \rho V_{c,1}^S(A) \\ & = - \left\{ \delta A + \lambda [\delta V_2^S(A) - V_{c,1}^S(A)] \right\} \quad \text{for } A \geq A_{S,1} \end{aligned} \quad (3.19)$$

By the continuity of $V_{c,1}^S(A)$ it follows that

$$V_{c,1}^S(A_{S,1}) = \delta M \quad (3.20)$$

Solving (3.19) subject to (3.20) (see appendix B.3.1) one can derive

$$V_{c,1}^S(A) = \delta \left\{ [M - V_2^S(A_{S,1})] \left(\frac{A}{A_{S,1}} \right)^{\beta_2} + V_2^S(A) \right\} \quad \text{for } A \geq A_{S,1} \quad (3.21)$$

Having determined G_0 's continuation value I can now solve its optimal stopping problem represented by the following Bellman equation

$$\begin{aligned} V_0^S(A) = \max_A \{ & M, Adt + e^{-\lambda dt} E [e^{-\rho dt} V_0^S(A + dA)] + \\ & + (1 - e^{-\lambda dt}) E [e^{-\rho dt} V_{c,1}^S(A + dA)] \} \end{aligned} \quad (3.22)$$

Definition 3.5 *In the continuation region, $A \geq A_{S,1}$, the value function, $V_0^S(A)$, solves the following second-order non-homogenous differential equation*

$$\begin{aligned} \frac{1}{2} \sigma^2 A^2 V_0^{S''}(A) + \mu A V_0^{S'}(A) - \rho V_0^S(A) \\ = - \{ A + \lambda [V_{c,1}^S(A) - V_0^S(A)] \} \quad \text{for } A \geq A_{S,0} \end{aligned} \quad (3.23)$$

The solution can then be derived solving (3.23) subject to the value-matching and smooth-pasting conditions respectively requiring

$$V_0^S(A_{S,0}) = M \quad (3.24)$$

$$V_0^{S'}(A_{S,0}) = 0 \quad (3.25)$$

Proposition 3.4 *Under declining time-preference rate and sophisticated belief the solution to the optimal stopping problem in (3.22) is given by*

$$A_{S,0} = \left(\frac{\rho - \mu}{\eta} \right) \left\{ \frac{M}{(\beta_2 - 1)} \left[\beta_2 - \delta \frac{\beta_2 - \beta_1}{(1 - \beta_1)} \left(\frac{A_{S,0}}{A_{S,2}} \right)^{\beta_1} \right] + \frac{P_{0,1} A_{S,0}^{\beta_2}}{(\beta_2 - 1)} \right\} \quad (3.26)$$

$$V_0^S(A) = \begin{cases} (P_{0,0} - P_{0,1} \log A) A^{\beta_2} + \delta \frac{M}{1 - \beta_1} \left(\frac{A}{A_{S,2}} \right)^{\beta_1} + \\ + \left(\frac{\eta}{\rho - \mu} \right) A & \text{for } A > A_{S,0} \\ M & \text{for } A \leq A_{S,0} \end{cases} \quad (3.27)$$

where $P_{0,1} = \lambda \delta \frac{M - V_2^S(A_{S,1})}{\frac{1}{2} \sigma^2 (2\beta_2 - 1) + \mu} \left(\frac{1}{A_{S,1}} \right)^{\beta_2} > 0$ and $A_{S,0} > A_{S,1} > A_{S,2}$ (see B.3.2 and B.4).

Proposition 3.4 confirms the existence of an additional effect if sophistication is assumed. The exercise timing of the option to harvest and the value function are in fact affected by taking into account which threshold will be chosen by the following government and by how worth is for the current generation the value accruing to the following generations. For G_2 being the last generation there is any incentive to rush and anticipate. Instead, both G_1 and G_0 undervalue in that imperfect altruists the utility accruing to the following generations and this implies that there is a lower cost opportunity in taking the decision to harvest. This in turn lowers the value of the option to wait and induces earlier harvest. Having fixed a higher threshold, harvest by G_1 is more likely in expected terms and this lowers the value of following generations exercises for G_0 . It follows that G_0 has even less incentive to wait and this lead to fix an higher threshold with respect to G_1 .

3.4.2 A I -governments model

I generalize the previous model allowing for I governments randomly stepping in office. I describe in this section only the procedure that should be followed to solve the problem and the solutions. All the details are provided in the appendix (see B.5.1 and B.5.2).

As for the three-governments I use for the solution of the problem the backward induction concept. The solutions for governments G_I and G_{I-1} are known and are respectively given by $\{A_{S,I} = A^*, V_I^S(A) = V(A)\}$ and $\{A_{S,I-1} = A_N, V_{I-1}^S(A) = V^N(A)\}$. Instead, for $i \leq I - 2$, the solutions may be derived recursively keeping into account that at the tail of the program $A_{S,I-1} = A_N$ and $V_{c,I}^S(A) = \delta V(A)$. Let $V_{i+1}^S(A)$ and $V_{c,i+1}^S(A)$ be respectively the value function for G_{i+1} and the value for G_i of the pay-offs attached to the strategies of the following $I - i$ governments.

Definition 3.6 *In the continuation region, $A \geq A_{S,i+1}$, the continuation value function, $V_{c,i+1}^S(A)$, solves the following second-order non-homogenous differential equation*

$$\begin{aligned} & \frac{1}{2}\sigma^2 A^2 V_{c,i+1}^{S''}(A) + \mu A V_{c,i+1}^{S'}(A) - \rho V_{c,i+1}^S(A) \\ & = - \left\{ \delta A + \lambda [V_{c,i+2}^S(A) - V_{c,i+1}^S(A)] \right\} \quad \text{for } A \geq A_{S,i+1} \end{aligned} \quad (3.28)$$

This equation is then solved attaching the condition $V_{c,i+1}^S(A_{S,i+1}) = \delta M$ that as explained in the previous section holds by the continuity of $V_{c,i+1}^S(A)$.

The Bellman equation for G_{i+1} is given by

$$\begin{aligned} V_{i+1}^S(A) = & \max_A \left\{ M, A dt + e^{-\lambda dt} E \left[e^{-\rho dt} V_{i+1}^S(A + dA) \right] + \right. \\ & \left. + (1 - e^{-\lambda dt}) E \left[e^{-\rho dt} V_{c,i+2}^S(A + dA) \right] \right\} \end{aligned} \quad (3.29)$$

Definition 3.7 In the continuation region, $A \geq A_{S,i+1}$, the value function, $V_{i+1}^S(A)$, solves the following second-order non-homogenous differential equation

$$\begin{aligned} \frac{1}{2}\sigma^2 A^2 V_{i+1}^{S''}(A) + \mu A V_{i+1}^{S'}(A) - \rho V_{i+1}^S(A) \\ = - \{A + \lambda [V_{c,i+2}^S(A) - V_{i+1}^S(A)]\} \quad \text{for } A \geq A_{S,i+1} \end{aligned} \quad (3.30)$$

This equation can be solved subject to the value-matching and smooth-pasting conditions respectively requiring

$$V_{i+1}^S(A_{S,i+1}) = M \quad (3.31)$$

$$V_{i+1}^{S'}(A_{S,n+1}) = 0 \quad (3.32)$$

Proposition 3.5 Under declining time-preference rate and sophisticated belief the solution to the optimal stopping problem in (3.29) for $i + 1 \leq I - 2$ is given by

$$\begin{aligned} A_{S,i+1} = & \frac{M}{\beta_2 - 1} \left(\frac{\rho - \mu}{\eta} \right) \left[\beta_2 - \delta \frac{\beta_2 - \beta_1}{1 - \beta_1} \left(\frac{A_{S,i+1}}{A^*} \right)^{\beta_1} \right] + \\ & + \frac{\sum_{k=1}^{I-2-i} k P_{i+1,n} (\log A_{S,i+1})^{k-1} A_{S,i+1}^{\beta_2}}{\beta_2 - 1} \left(\frac{\rho - \mu}{\eta} \right) \end{aligned} \quad (3.33)$$

$$V_{i+1}^S(A) = \begin{cases} A \frac{\eta}{\rho - \mu} + \delta \frac{M}{1 - \beta_1} \left(\frac{A}{A^*} \right)^{\beta_1} + \\ \quad + \sum_{k=0}^{I-2-i} P_{i+1,k} (\log A)^k A^{\beta_2} \quad \text{for } A > A_{S,i+1} \\ M \quad \text{for } A \leq A_{S,i+1} \end{cases} \quad (3.34)$$

where $A_{S,I} = A^*$, $A_{S,I-1} = A_N$. See the appendix B.5.2 for the computation

of coefficients $P_{i+1,k}$.

Proposition 3.6 *Under declining time-preference rate and sophisticated belief and for $\delta \in (0, 1)$ the government exercises the option to harvest at $A_{S,0} > A_{S,1} > \dots > A_{S,i+1} > A_{S,i} > \dots > A_{S,I-1} > A_{S,I}$.*

See B.4 for the proof.

Proposition 3.6 implies that the continuation region enlarges as i increases and that the more governments the current government has ahead the higher is the critical threshold. This result generalizes the one provided in Proposition 3.4. Provided that $A > A_{S,I}$ it follows that in expected terms the more governments are ahead the less patient is the current government and the more likely is the harvesting. With respect to a naïve government, the current sophisticated government takes into account the burden represented by the sub-optimality (from its time perspective) of future policies. The more governments will succeed the more eroded will be the value attached to the option to wait. The anticipation of future generations exercises is needed in that their harvest plans negatively affect the current generation welfare and this could be done only fixing an higher threshold making so more likely the option exercise over the life of current generation. Finally the effect of changes in δ and λ on the thresholds is confirmed ($\frac{dA_S}{d\delta} < 0$, $\frac{dA_S}{d\lambda} > 0$).

3.5 Government targeting and instability

I suggest an alternative interpretation of the model.²⁰ Consider a political party,²¹ say X, assume that it is risk-neutral and currently in charge at the government. Suppose that it discounts exponentially at rate ρ the pay-offs occurring over all periods but that undervalues pay-offs occurring in the future periods by a factor $0 < \delta \leq 1$ to account for the probability of being in charge in the future periods. Assume $\delta = p + (1 - p)a \leq 1$ where p is the exogenous probability of winning an electoral round. Hence, each party gives weight 1 to social welfare when it is in charge and weight $a \leq 1$ when it is not. This could be due for example to the fact that political parties are aware that people when voting takes into account only their conduct when in charge. Differently from Brocas and Carrillo (1998) I suppose that due to populist or other parties pressure and/or unexpected events the current government may suddenly fall according to a Poisson process with intensity $\lambda \in [0, \infty)$ and that an electoral round follows. Hence, each government is in office for a period lasting from an electoral round (t_i) to the subsequent (t_{i+1}). The period in office, (t_i, t_{i+1}), and future periods (t_{i+1}, ∞) for each of these government are set randomly according to the occurrence of the political crisis. It is not difficult to see that on the basis of the assumptions made each government is an hyperbolic discounting decision maker and that its discount function is represented by (3.1). It follows that the analysis provided in the previous sections can be seen under a new light. In fact, it could allow for the investigation of the impact that the choice of different social objective functions by the political parties has on harvest timing and conservation

²⁰A similar frame is provided by Brocas and Carrillo (1998) to justify imperfect intergenerational altruism.

²¹We assume that also all the other parties currently not governing but competing with X at each electoral round have the same preferences.

policy consistency. Moreover, allowing for random electoral rounds additional insight may be provided incorporating into the analysis the role of political instability that is driven in this framework by the magnitude of λ . This frame is quite realistic and may represent another argument for explaining the rush in irreversible harvesting and the time inconsistency of conservation policies.

3.6 Conclusions

This chapter extends the model for the definition of optimal harvest timing under a real options approach by Conrad (1997) to a framework in which time-inconsistent preferences are considered. These preferences have been and are a research object which captures the interests of researchers in various fields of economics (Strotz, 1956; Phelps and Pollak, 1968; Harris and Laibson, 2004; Laibson, 1996, 1997; O'Donoghue and Rabin, 1999; Brocas and Carrillo, 2005; Dasgupta and Maskin, 2005).

I set up a model which gives a rationale to the governments' haste in undertaking irreversible projects leading to the commercial exploitation of natural resources such as forests and to the time-inconsistency of conservation policies.

As proved by the results provided in this chapter imperfect intergenerational altruism induces governments to rush the exercise of the option to harvest and leads through the inconsistent time-preferences to which it gives rise to inconsistent conservation policy.

Chapter 4

Option value of old-growth forest and Pigovian taxation under time inconsistency

4.1 Introduction

The use of option theory has become relevant in resource and environmental economics¹ (Brennan and Schwartz, 1985; Mcdonald and Siegal, 1986; Merton, 1998). This approach postulates that when decisions are characterized by irreversibility and uncertainty the option value of waiting for additional information² about future benefits and costs should be taken into account in the decision-making process (Arrow and Fisher, 1974; Henry, 1974; Dixit and Pindyck; 1994).

The standard real options approach is based on the assumption of agents

¹See also for example Clarke and Reed (1989), Reed (1993), Conrad (2000), Bulte et al. (2002), Kassari and Lasserre (2004), Insley and Rollins (2005).

²"When the purchase of an option or delay of an irreversible action allows an individual to ascertain the true state with certainty, option value is equivalent to the expected value of perfect information. When delay only affords the opportunity for probability revision (imperfect learning), option value will equal the expected value of information" (Conrad, 1980).

which exponentially discount future payoffs at a constant rate. Such assumption allows to characterize agent's decision-making as time-consistent.

A constant rate of time preference is a strong assumption that has been and is broadly discussed in economics since Strotz (1956) who proposed declining discount rates as an alternative³. The debate has become stronger when also experimental evidence in psychology has supported the idea of an individual taste for immediate gratification.⁴ Such evidence confirms Strotz's conjecture of individuals discounting at declining rates and recommends to model discount functions as hyperbolas rather than exponential functions.

However, as noted by Strotz (1956) declining discount rates almost always implies time-inconsistent planning.⁵ Time-inconsistency results from time preferences changing over time i.e. the discount rate between two succeeding periods t and $t+1$ increases as t comes close. This means that individuals could reconsider their plans at later dates and disobey the optimal plans originally defined.

In the last several years, the attention of economists on the implications of hyperbolic discounting in the economic analysis has increased and led to contributions in different fields.⁶ Also in resource economics there is increasing interest but just few contributions⁷ determining how hyperbolic discounting can impact the management of natural resources (Shogren, 2007).

This chapter aims is to contribute in two directions. First, I want to assess how the assumption of hyperbolic discounting affects the timing of the decision

³Referring to exponential discounting, Strotz (1955) argues that there is "no reason why an individual should have such a special discount function" (p.172).

⁴See Loewenstein and Prelec (1992) for a review.

⁵Heal (1998) shows that logarithmic discounting is an exception.

⁶See for example contributions by Laibson (1996, 1997), O'Donoghue and Rabin (1999), Brocas and Carrillo (2004, 2005), Harris and Laibson (2004), Dasgupta and Maskin (2005), Salanie and Treich (2006) and Frederick, Loewenstein and O'Donoghue (2002) for a review. See also Rubinstein (2003) for a more cautious position on the use of hyperbolic discounting.

⁷See for example Hepburn (2003) and Hepburn and Koundouri (2007).

to harvest a stand of old-growth forest.⁸ I combine two different strands of literature: the real option theory introduced above which remarks the importance of waiting for new information and the literature on hyperbolic time-preferences where individuals showing time-varying impatience may have incentive to rush because of future sub-optimal decisions. Up to my knowledge this chapter represents the second attempt of introducing hyperbolic discounting in the real options framework. The first is by Grenadier and Wang (2007) where the optimal timing of investment under uncertainty and time-inconsistent preferences is determined. Differently from them the problem solved in this chapter is instead equivalent to the optimal disinvestment timing of an activity when a flow of value is paid as a dividend if the shut-down option is kept alive.⁹

Second, I investigate the implications of hyperbolic discounting on second-best public policies used to guarantee the socially optimal allocation of forest resources when markets fail. As I show in this chapter second-best policy tools should be designed to account for the behavioural failure arising from time-inconsistent preferences in that otherwise they may miss the policy target (Shogren, 2007).

I start presenting the model by Conrad (1997) that will serve as benchmark. In this time-consistent framework, harvesting is irreversible, the value of the wood harvested is known while future flows of amenity value are uncertain and follow a geometric Brownian motion. Conrad (1997) solves the optimal stopping problem in continuous time and provide the analytical solutions for the value function and the critical level of amenity needed to sustain forest conservation. The main

⁸This is a standard problem that has been solved in a time-consistent framework using option-pricing theory (Reed, 1993; Conrad, 1997)

⁹Technically their problem resembles to optimal timing for exercising an American call option while in this paper the problem is equivalent to optimal timing for exercising an American put option. An important difference is given by the dividend earned, in fact, when holding the investment option the dividend paid is null while holding the harvesting/disinvestment option a "dividend" represented by the flow of amenity value is paid.

insight in this paper is represented by the impact of the option value on the definition of the harvest timing.

I solve then the same problem but in a time-inconsistent framework. Here, the decision-maker is represented by a sequence of infinite succeeding selves with time-inconsistent preferences. Each of them determines the optimal threshold for the exercise of the harvest option according to her discount function and to her belief about future selves behaviour. Each self is assumed sophisticated and perfectly foresee that her time-preferences are going to change over time (Strotz, 1956; Pollak; 1968). I solve the problem on the basis of a result contained in Di Corato (2007) where a similar problem is solved for a finite number of selves by backward induction. The solution is represented by the limit to which the critical threshold determined for a finite number of selves converges when the number of selves tends to infinity and still incorporates the insight behind the assumption of hyperbolic time preferences. It is in fact easy to show that the critical threshold for the exercise of the option to harvest is higher than the one for a time-consistent agent. The intuition behind this result is that the value of keeping open the option to harvest is less worth for a time-inconsistent agent in that at first impatience induces her to undervalue future payoffs and second because the sub-optimality (from her time perspective) of future selves' decisions is anticipated. This means that an effect other than the first due to a preference for current satisfaction is introduced by perfect foresight. Being sub-optimal in their decisions, the current self must anticipate future selves and so a higher threshold for the amenity value is fixed. A higher threshold in fact implies that in expected terms harvesting occurs earlier. This is done at a cost that is represented by giving up important amenity value flows but this cost is undervalued by our time-inconsistent agent.

The solution to the optimal stopping problem is derived as a steady state solution. Interestingly the critical threshold is represented by a convex combination of the thresholds that would be fixed by two time-consistent agents which differs in the discount rate. The higher discount rate term captures the taste for the present gratification and represents the short-run oriented view of the agent while the other partially correcting the first stands for the long-run view and is derived using a lower discount rate.

It follows that taking the Marshallian threshold of the time-consistent agent as a benchmark the hyperbolic discounting agent seems to exercise the option to harvest even if the expected net present value is negative but this is simply due to not having redefined the benchmark for the higher subjective discount rate.

Finally, I move the analysis to public intervention studying Pigovian taxation of wood revenues to correct market failures in the provision of ecosystem services. Having proved that hyperbolic discounting induce premature harvesting I show how the optimal taxation must be redesigned to internalize the behavioral failure due to time inconsistency and hit the environmental policy target.

The chapter is organized as follows. In section 4.2, I present the basic model basic and provide the solution to the standard time-consistent problem. In Section 4.3 the optimal timing of harvesting is studied under time-inconsistent preferences and a discussion of the results is provided. Section 4.4 proposes the analysis of regulatory intervention with the help of a numerical and graphical analysis. Section 4.5 concludes.

4.2 The basic set-up

Consider a privately owned stand of old-growth forest where the net value of stumpage timber, M , is known and constant. The old-growth forest generates at time t a flow of amenity value arising from services such as flood control, carbon sequestration, erosion control, wildlife habitat, biodiversity conservation, recreation and tourism, option and existence values (Reed, 1993; Conrad, 1997). Define by $A = A(t)$ the rent that society is willing to pay for the provision of such services.¹⁰ Assume that due to the uncertainty on the future evaluation of amenity services $A(t)$ is a stochastic process following geometric Brownian motion

$$dA(t) = \mu A(t)dt + \sigma A(t)dz \quad (4.1)$$

where μ is the mean drift rate, σ is the standard deviation rate and $\{z(t)\}$ is a standard Wiener process.¹¹

Consider now that the owner¹² can either conserve or totally harvest the forest. Assume the decision to harvest is irreversible.¹³ This is similar to having an option: at each time-period one can exercise the option, harvest and get the payoff M or keep it open, wait and get as a dividend the flow $A(t)$. Since harvesting is an irreversible action while preserving is not, there is an option value

¹⁰Considering that amenity services have generally public-good nature not all the value generated could accrue to the forest owner.

¹¹Where the usual conditions, $E[dz(t)] = 0$ and $E[dz(t)^2] = dt$ are satisfied. The upward drift draws the increasing consideration of society for the amenity services and the variance parameter captures the uncertainty about their actual and future value (Reed, 1993; Conrad, 1997). Nevertheless, the drift could be negative if for example scientific progress in chemistry makes less worth the genetic information provided through biodiversity conservation (Bulte et al., 2002).

¹²From now on, referring to the private owner of the stand of forest I will use the term agent and by using the term decision I will always refer to the decision to harvest or conserve.

¹³This seems plausible considering that depending on the location the regeneration process could last from a century up to several millennia (http://en.wikipedia.org/wiki/Old_growth_forest).

attached to the decision to preserve in that the latter action allows the decision maker for waiting and updating information about the flow of amenity value. In this framework, the question is when it would be optimal to exercise such option. The answer lies in the solution of an optimal stopping problem in continuous time.

4.2.1 Sketch of an agent with hyperbolic preferences

Assume that the agent is time-varying impatient, risk neutral and that she cannot commit to follow any plan. Having a taste for present gratification our agent overvalues current payoffs with respect to future ones. This present-biased (O'Donoghue and Rabin, 1999) preferences have been originally modelled by Laibson (1996, 1997) as quasi-hyperbolic using a discrete-time functional form introduced by Phelps and Pollak (1968) to study intergenerational time preferences.¹⁴

In this framework, such formulation cannot be applied and thus, to model hyperbolic preferences I will use the hyperbolic continuous-time discount function proposed by Harris and Laibson (2004).

In the version presented by Grenadier and Wang (2006), each self n 's present period lasts a random length of time and is equal to $L_n = l_{n+1} - l_n$ where l_n, l_{n+1} are respectively the birth date of self n and self $n + 1$. Each self takes decisions only in the present period and doing it she keeps into account what future selves may decide when they will be in charge. Future for self n lasts from l_{n+1} to ∞ . The birth of future selves is a Poisson process with intensity $\lambda \in [0, \infty)$. The self n 's life, L_n , is then stochastic and distributed exponentially with parameter λ .

Self n discounts exponentially with instantaneous discount rate ρ the present

¹⁴Generalizations in continuous-time are presented in Barro (1999) and Luttmer and Mariotti (2000).

and future payoffs but she values the future payoffs less because of the additional discount factor $0 < \delta \leq 1$. Her discount function, $D_n(l, t)$, is given by

$$D_n(l, t) = \begin{cases} e^{-\rho(t-l)} & \text{if } t \in [l_n, l_{n+1}] \\ \delta e^{-\rho(t-l)} & \text{if } t \in [l_{n+1}, \infty] \end{cases} \quad (4.2)$$

for $t > l$ and $l_n \leq l \leq l_{n+1}$

Each self n when in charge takes decisions discounting present and future according to her own discount function $D_n(l, t)$.

As noted by Strotz (1956), changing time-preferences are time-inconsistent. This can be easily verified comparing the per-period discount rate between the present and the future, $\frac{1-\delta e^{-\rho}}{\delta e^{-\rho}}$, with the per-period discount rate between any two future periods $\frac{1-e^{-\rho}}{e^{-\rho}}$. As $\frac{1-e^{-\rho}}{e^{-\rho}} < \frac{1-\delta e^{-\rho}}{\delta e^{-\rho}}$, the discount rate between two consecutive periods t and $t + 1$ increases as date t comes close.

Time-inconsistency may have serious implications on the individual planning because in the absence of any commitment device a decision taken by a previous self and entailing a future payoff may be considered not optimal by a future self and reconsidered.

In the continuous-time formulation of hyperbolic preferences, the degree of time-inconsistency is driven by δ and λ .¹⁵ It increases as $\delta \rightarrow 0$ and as $\lambda \rightarrow \infty$ (Harris and Laibson, 2004). Finally, note that this functional form allows also for the representation of standard time-consistent preferences ($\delta = 1$ or $\lambda = 0$).

¹⁵The hazard rate of transition from the present to the future (Harris and Laibson, 2004).

4.2.2 Conserving or harvesting: time-consistent case

Let $V(A)$ be the value function that the agent want to maximize¹⁶. The Bellman equation for her problem is

$$V(A) = \max \left\{ M, A dt + \frac{1}{1 + \rho dt} E [V(A + dA, t + dt)] \right\} \quad (4.3)$$

where ρ is the instantaneous discount rate. Expanding (4.3) by Ito's lemma, in the continuation region the value function, $V(A)$, solves the following second-order non-homogenous differential equation

$$\frac{1}{2} \sigma^2 A^2 V''(A) + \mu A V'(A) - \rho V(A) = -A, \quad \text{for } A \geq A^* \quad (4.4)$$

where A^* is the critical amenity value or the point at which the agent is indifferent between conserving and harvesting.

Assume¹⁷ $\rho > \mu$ and solve (4.4) attaching a value-matching condition (4.5) and a smooth-pasting condition (4.6) to guarantee optimality:¹⁸

$$V(A^*) = M \quad (4.5)$$

$$V'(A^*) = 0 \quad (4.6)$$

The critical amenity A^* is

$$A^* = \frac{\beta_1}{\beta_1 - 1} M(\rho - \mu) \quad (4.7)$$

¹⁶The optimal stopping problem for a time-consistent agent has been solved in Conrad (1997) and to save space the interested reader is referred to his paper for the details.

¹⁷Note that if $\rho \leq \mu$ it will be never optimal to harvest.

¹⁸It rules out arbitrary exercise of the option to harvest at a different point (Dixit, 1994).

where β_1 is the negative root of the characteristic equation¹⁹ $\sigma^2\beta(\beta-1)/2+\mu\beta-\rho$.

The value accruing to the agent is

$$V(A) = \frac{M}{1-\beta_1} \left(\frac{A}{A^*}\right)^{\beta_1} + \frac{A}{\rho-\mu} \quad \text{for } A > A^* \quad (4.8)$$

The first term on the RHS of (4.8) resembles the value of the option to harvest and it goes to zero as $A \rightarrow \infty$. The second term is the expected value of never harvesting and it is given by the discounted flow of amenity value A . As soon as $A \leq A^*$ the option is exercised and $V(A) = M$.

4.3 Conserving or harvesting under time inconsistency

Assume that the agent has sophisticated belief²⁰ (Strotz, 1956; Pollak; 1968). Having perfect foresight she knows in advance that her preferences will change as time rolls on and that she will wish to revise his original harvest plan according to her own $D_n(l, t)$. This leads a rational agent to take decisions over her lifespan which accounts for the sub-optimality (from her time perspective) of future selves' strategies. Actually, a sophisticated agent could choose a "strategy of pre-commitment" which consists in committing herself to a certain plan of action and never revise the critical threshold originally fixed for the exercise of the option to harvest (Strotz, 1956). This may be possible if an effective commitment device exists but this has been excluded by assumption in order to study the more interesting and real case of an agent not able to tie her hands. Let then proceed to

¹⁹The solution is $\beta_1 = (\frac{1}{2} - \frac{\mu}{\sigma^2}) - \sqrt{(\frac{1}{2} - \frac{\mu}{\sigma^2})^2 + \frac{2\rho}{\sigma^2}} < 0$

²⁰In a different framework, the solution to the optimal stopping problem under naïve belief is provided in Di Corato (2007).

the next paragraph and define the optimal strategy for this agent.

4.3.1 Strategies under sophistication

In Di Corato (2007), the optimal harvest time trigger is determined solving for a finite number of hyperbolic selves by backward induction. In this paper it is proved that the critical threshold fixed for harvesting by each self increases monotonically with the number of selves ahead. Hence, one may conjecture that the critical threshold for the first self in the sequence has a limit to which converges when the number of selves ahead tends to infinity. By assuming the existence of an infinite number of selves it follows that the optimization problem that each self must solve should be the same and that the position in the sequence, n , does not matter. Every self is going to play the same strategy in this intra-personal game and its outcome will be determined imposing stationarity to the solution of the optimization problem.

Denote by A_S the steady state solution. Now, consider the current self and let \tilde{A} represent her conjecture on the future selves' timing trigger. Assume that current self's optimal trigger, $H(\tilde{A})$, depends on her conjecture.²¹ Let $V^S(A; \tilde{A})$ and $V_c^S(A; \tilde{A})$ be respectively current self's value function and continuation value function. The continuation value function is the current self's valuation of the decisions that could be taken once the succeeding self is born. According to (4.2) the current self has present-biased preferences and the payoff from future selves' decisions accrues to her only for δ times its value.

Given that all future selves are supposed to exercise the option to harvest at

²¹Note that if $H(\tilde{A}) = \tilde{A}$ then $A_S = \tilde{A}$.

the same \tilde{A} then $V_c^S(A; \tilde{A})$ is given by

$$V_c^S(A; \tilde{A}) = \begin{cases} \delta \left[\frac{M}{1-\beta_1} \left(\frac{A}{\tilde{A}} \right)^{\beta_1} + \frac{A}{\rho-\mu} \right] & \text{for } A > \tilde{A} \\ \delta M & \text{for } A \leq \tilde{A} \end{cases} \quad (4.9)$$

The Bellman equation for the current self is given by

$$V^S(A; \tilde{A}) = \max_A \left\{ M, A dt + e^{-\lambda dt} E \left[e^{-\rho dt} V^S(A + dA; \tilde{A}) \right] + \right. \\ \left. + (1 - e^{-\lambda dt}) E \left[e^{-\rho dt} V_c^S(A + dA; \tilde{A}) \right] \right\} \quad (4.10)$$

The current self defines her optimal exercise trigger, A_S , maximizing her value function. $V^S(A; \tilde{A})$ solves in the continuation region ($A \geq \tilde{A}$) the following differential equation

$$\frac{1}{2} \sigma^2 A^2 \frac{\partial^2 V^S(A; \tilde{A})}{\partial A^2} + \mu A \frac{\partial V^S(A; \tilde{A})}{\partial A} - \rho V^S(A; \tilde{A}) \\ = - \left[A + \lambda \left(V_c^S(A; \tilde{A}) - V^S(A; \tilde{A}) \right) \right] \quad \text{for } A \geq \tilde{A} \quad (4.11)$$

where $V_c^S(A; \tilde{A})$ is defined by (4.9).

At the critical amenity value, A_S , the value-matching and smooth-pasting conditions require

$$V^S(H(\tilde{A}); \tilde{A}) = M \quad (4.12)$$

$$\frac{\partial V^S(H(\tilde{A}); \tilde{A})}{\partial A} = 0 \quad (4.13)$$

Plugging (4.9) into (4.11) and attaching the two boundary conditions one can solve the differential equation and find

$$H(\tilde{A}) = \left[\frac{\beta_2}{\beta_2 - 1} - \delta \frac{\beta_2 - \beta_1}{(1 - \beta_1)(\beta_2 - 1)} \left(\frac{H(\tilde{A})}{\tilde{A}} \right)^{\beta_1} \right] M \left(\frac{\rho - \mu}{\eta} \right) \quad (4.14)$$

$$\begin{aligned} V^S(H(\tilde{A}); \tilde{A}) &= \frac{M}{1 - \beta_2} \left[1 - \delta \left(\frac{H(\tilde{A})}{\tilde{A}} \right)^{\beta_1} \right] \left(\frac{A}{H(\tilde{A})} \right)^{\beta_2} \\ &+ \frac{\delta M}{1 - \beta_1} \left(\frac{A}{\tilde{A}} \right)^{\beta_1} + A \left(\frac{\eta}{\rho - \mu} \right) \quad \text{for } A > \tilde{A} \end{aligned} \quad (4.15)$$

Now, recalling that in the steady-state equilibrium $H(A_S) = A_S$ and substituting into $H(\tilde{A})$ and $V^S(A; \tilde{A})$ it follows that

$$\begin{aligned} A_S &= \frac{(1 - \delta)(\rho - \mu)}{(1 - \delta)(\rho - \mu) + \delta(\rho + \lambda - \mu)} \left[\frac{\beta_2}{\beta_2 - 1} M(\rho + \lambda - \mu) \right] + \\ &+ \frac{\delta(\rho + \lambda - \mu)}{(1 - \delta)(\rho - \mu) + \delta(\rho + \lambda - \mu)} \left[\frac{\beta_1}{\beta_1 - 1} M(\rho - \mu) \right] \\ &= (1 - \theta) A^{**} + \theta A^* \\ V^S(A) &= \begin{cases} (1 - \delta) \left[\frac{M}{1 - \beta_2} \left(\frac{A}{A_S} \right)^{\beta_2} + \frac{A}{\rho + \lambda - \mu} \right] + \\ + \delta \left[\frac{M}{1 - \beta_1} \left(\frac{A}{A_S} \right)^{\beta_1} + \frac{A}{(\rho - \mu)} \right] & \text{for } A > A_S \\ M & \text{for } A \leq A_S \end{cases} \end{aligned} \quad (4.17)$$

where $\theta = \frac{\delta(\rho + \lambda - \mu)}{(1 - \delta)(\rho - \mu) + \delta(\rho + \lambda - \mu)} \leq 1$, β_2 is the negative root of the equation²² $\sigma^2 \beta(\beta - 1)/2 + \mu\beta - (\rho + \lambda)$ and $A^{**} = \frac{\beta_2}{\beta_2 - 1} M(\rho + \lambda - \mu)$.

²²The solution is $\beta_2 = \left(\frac{1}{2} - \frac{\mu}{\sigma^2} \right) - \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2} \right)^2 + \frac{2(\rho + \lambda)}{\sigma^2}} < 0$

Proposition 4.1 *Under sophistication and for any δ such that $0 < \delta < 1$ the agent exercises the option to harvest at $A_S > A^*$.*

The critical threshold, A_S , is a convex combination of two time-consistent critical thresholds, A^* and A^{**} , respectively determined for the discount rates ρ and $\rho + \lambda$. As expected $\partial A_S / \partial \delta < 0$ and $\partial A_S / \partial \lambda > 0$. This means that the sophisticated critical threshold decreases with the degree of time-inconsistency. Both δ and λ are important to rule the intrapersonal conflict between different levels of patience in discounting and consequently fixing the critical threshold for the exercise of the cutting option. Note that A_S is always higher than A^* ($0 < \delta < 1, \eta < 1, A^{**} > A^*$).

The present value of the payoff resulting from future selves' decisions is lower than that one for a time-consistent agent and consequently there is a lower incentive for keeping open the option to harvest and waiting for more information.

The value function in (4.17) is a weighted sum of two time-consistent value functions respectively weighted by $1 - \delta$ and δ .²³ Both terms measures the value of the option to wait until A_S has been hit and the expected present value of the flow of amenity value if A_S is never touched but using two different discount rates, respectively $\rho + \lambda$ and ρ .

Note that if $\lambda > 0$ and $\delta \rightarrow 0$ then $A_S \rightarrow A^{**} = \frac{\beta_2}{(\beta_2 - 1)} M (\rho + \lambda - \mu) > A^*$. The extreme impatience leads the agent to practically burn value. In this extreme case the time trigger can be determined using the standard real option analysis with a time-consistent agent discounting the future at a rate adjusted by λ . The higher discount rate internalizes the fear of a "catastrophic" future self arrival that would result in this case as $\delta \rightarrow 0$ in the loss of any kind of revenue.

²³In Grenadier and Wang (2007), a similar result is derived solving an investment timing problem.

These results are in line with findings in Harris and Laibson²⁴ (2004) and in particular with the second interpretation they give to the discount function drawn by (4.2). They consider the birth of the succeeding self as an event likely to happen every instant and use for each self a deterministic discount function, $\bar{D}_n(t)$, simply equal to the expected value of the stochastic discount function in (4.2). For each self at the birth:

$$\begin{aligned}\bar{D}_n(t) &= e^{-\rho t}e^{-\lambda t} + (1 - e^{-\lambda t})\delta e^{-\rho t} \\ &= (1 - \delta)e^{-(\rho+\lambda)t} + \delta e^{-\rho t}\end{aligned}$$

The discount function, $\bar{D}_n(t)$, is a convex combination of two exponential discount functions with two different discount rates, respectively $\rho + \lambda$ and ρ , and it is straightforward to relate this result with (4.16) and (4.17).

4.3.2 Conservation or harvest: a discussion on timing

The trigger amenity value A^* is the level of benefits from conservation at which a time-consistent agent will find it profitable to exercise the option to harvest. Note that A^* is lower than $M(\rho - \mu)$ which represents a myopic flow-equivalent cost of preservation. The option value multiple $\beta_1/(\beta_1 - 1)$ lowers the critical trigger because the agent wants to take into account the irreversibility of harvesting and the uncertainty. This means that waiting and gathering more information on the randomly fluctuating amenity value A before harvesting could be a sensible strategy. Furthermore, as $\partial A^*/\partial \sigma < 0$, an increase in uncertainty over future level of A implies an increase in the wedge between A^* and $M(\rho - \mu)$ and as a

²⁴Harris and Laibson (2004) deals with intertemporal consumption and show that even if observationally not equivalent the dynamically-inconsistent optimization problem has the same value function of a related dynamically-consistent optimization problem.

consequence an additional increase in the waiting time before harvesting.

Proposition 4.1 states that time-inconsistent preferences lead to premature harvesting. Even if under sophisticated belief the effect is mitigated by the agent internalizing future sub-optimality, premature harvesting occurs under both the assumptions on self-awareness. Note that as $\partial A_S/\partial\delta < 0$, $\partial A_S/\partial\lambda > 0$, an increase in the strength of time-inconsistency induces an increase in the wedge between A_S and A^* with a further decrease in waiting time before harvesting.

Taking A^* as benchmark for decision making it can also be proved $A_S > M(\rho - \mu) > A^*$. In other words, the agent may seem to exercise the option to harvest even if the expected net present value is negative.²⁵ This is simply due to the definition of the Marshallian trigger using $(\rho - \mu)$ instead of the higher adjusted rate $(\rho - \mu)/\eta$.

Given that $\partial A_S/\partial\sigma < 0$ then also with time-inconsistent preferences an increase in uncertainty over future level of A implies lower critical thresholds and as a consequence an increase in the waiting time before harvesting.

4.4 Regulatory intervention

4.4.1 Passage time

The amenity value flow A randomly fluctuates following the process in (4.1). Above, different optimal stopping problems have been solved. The solutions provide timing thresholds at which it is optimal to exercise the option to harvest. Denote by $A(t) = \hat{A}$ a generic time trigger. The process (4.1) stops as soon as the absorbing barrier \hat{A} has been hit. The probability of ever reaching the barrier

²⁵For $\delta = \bar{\delta} = \frac{(\beta_1 - 1)[(\rho - \mu) + \lambda\beta_2]}{[\lambda(1 - \beta_1)(1 - \beta_2) - (\beta_2 - \beta_1)(\rho + \lambda - \mu)]}$ the option value is null.

\hat{A} starting from the current $A_0 > \hat{A}$ is given by:

$$P(A_0, \hat{A}) = \begin{cases} 1 & \text{if } \mu \leq \sigma^2/2 \\ \left(\frac{\hat{A}}{A_0}\right)^{(2\mu-\sigma^2)/\sigma^2} & \text{if } \mu > \sigma^2/2 \end{cases} \quad (4.18)$$

Note that when $\mu \leq \sigma^2/2$ there is a drift which does not bring away A from the barrier and the probability of attaining \hat{A} is unity. Instead, when $\mu > \sigma^2/2$ the upward drift moves A away from the barrier and reduces the probability of absorption to $P(A_0, \hat{A}) < 1$. In other words, there is a non-zero probability of never hitting the barrier.

Given that A follows a stochastic process then also the option exercise time $T = \inf(t > 0 \mid A(t) = \hat{A})$ is a stochastic variable. If the process (4.1) starts at A_0 then the expected time at which the barrier is reached is:

$$E(T) = \begin{cases} \infty & \text{if } \mu \geq \sigma^2/2 \\ \frac{2}{2\mu-\sigma^2} \ln\left(\frac{\hat{A}}{A_0}\right) & \text{if } \mu < \sigma^2/2 \end{cases} \quad (4.19)$$

Notice that as expected when the drift moves A away from the barrier²⁶ $E(T) = \infty$ (Dixit, 1993).

4.4.2 Time for regulation

In the management of forest resources, decision-making relies not only on market rules. Market institutions work efficiently when the goods and services provided by forests are private but they fail when these have public nature. Given that in some cases the expected private timing of harvesting an old-growth forest may not be socially acceptable and regulatory measures are needed to fix a nonmarket allocative rule correcting market failures.

²⁶This happens even when $\mu = \sigma^2/2$. See for a deeper analysis Cox and Miller (1965).

In forest economics, the design of policy intervention has not taken into account behavioural anomalies such as hyperbolic discounting. Above it has been shown that hyperbolic discounting can have serious implications on the option exercise timing and can induce premature harvesting. When policy measures are designed to cope with market failures in forestry, ignoring such behavioural failure could potentially lead to miss the policy target.

Suppose that the policy maker has identified the date \bar{T} as socially optimal for cutting the forest. Also assume that the policy tool used to drive the agents to the exercise of the cutting option at this date is a Pigovian tax on the wood revenues. A tax on the revenues makes less desirable to harvest and lowers the amenity critical threshold at which the forest owner exercises the cutting option.

Authorities must evaluate the opportunity of regulation and in order to make it the analysis provided in the previous subsection could be useful. Starting from the current $A_0 > A_S$ and if $\mu \geq \sigma^2/2$, according to (4.18) and (4.19) it is unlikely that the barrier A_s may ever be met. In this case, policy intervention may not be required. Instead, if $\mu < \sigma^2/2$ then $P(A_0, A_S) = 1$ and regulatory measures could be needed to move toward the socially optimal target.

Denote by T^S the private harvesting timing. Given that A follows a stochastic process then also T^S becomes a stochastic variable and using (4.19) it is possible to calculate $E(T^S)$.

Suppose that the regulator use this rule to fix the tax:

$$E(T^S) - \bar{T} = m \tag{4.20}$$

where $m \geq 0$ is a constant parameter which represents a safety margin chosen by the regulator to define the interval into which private harvest should occur (Dosi

and Moretto, 1996, 1997). The parameter m represents a safety margin chosen by the regulator when designing the environmental policy. The tax, Γ , that should be levied on the wood revenue is then computed using (4.7), (4.16) and (4.19).

Proposition 4.2 *Under the policy rule $E(T^S) - \bar{T} = m$, the optimal tax is*

$$\Gamma^S = 1 - \frac{A_0}{\theta A^* + (1 - \theta) A^{**}} e^{-\left(\frac{\sigma^2}{2} - \mu\right)(m + \bar{T})} \quad (4.21)$$

If $E(T^S) > \bar{T} + m$, the regulator considers the expected private harvesting time socially acceptable and no taxes will be levied. Notice that, as $\partial\Gamma/\partial\delta < 0$, a decrease in δ leads to an increase in the tax rate, Γ^S , that should be charged on the wood revenues. Ignoring the behavioural failure due to people exhibiting changing time preferences means that the tax rate is calculated assuming $\delta = 1$. Actually, the tax rate turns out to be too low and being the critical threshold A_s too high, it leads the policy to miss the target, namely to $E(T^S) < \bar{T} + m$ (see C.2 for the procedure and the proof).

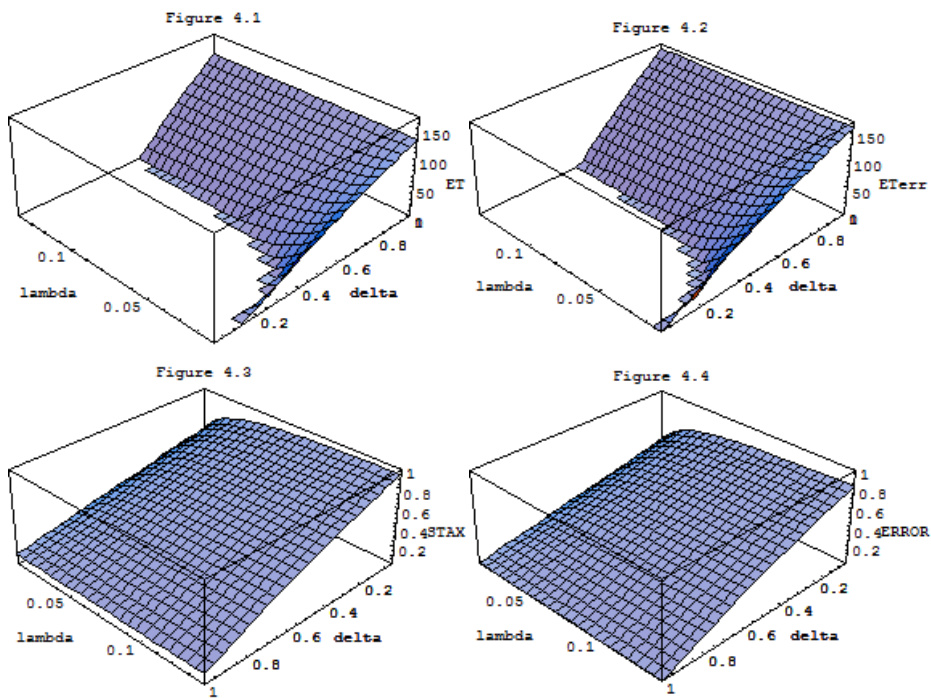
4.4.3 Numerical and graphical analysis

Some numerical solutions represented by graphs will help to illustrate the results provided in the previous sections. For the parameters we share I will use the values used in the numerical analysis provided in Conrad (1997), while for the others I will choose reasonable values. Let $M = \$550 \times 10^6$, $\rho = 0.06$, $\mu = 0.05$, $\sigma = 0.33$, $A_0 = 5$, $\bar{T} = 150$ and $m = 10$. Note that the value set for $\sigma = 0.33$ is chosen to analyse cases where it is actually likely that harvest occurs ($\mu < \sigma^2/2$). Finally, δ and λ are taken from the sets $0 < \delta \leq 1$ and $0 < \lambda \leq 0.14$.

Given the values hypothesized the expected private harvest timing is represented in Figure 4.1. As one can see in expected terms all private forest owner

hyperbolic types miss the policy target ($\bar{T} = 150$). Moreover, earlier harvest occurs as the magnitude of time-inconsistency increases. Note that for $A(t)$ starting at $A_0 = 5$ the expected harvest timing for a time consistent agent is about $\bar{T} = 150$ while it decreases rapidly and is dramatically equal to 0 for some time inconsistent forest owners.

Now, let consider the government intervention through pigovian taxation on wood revenues. Taxes are fixed according to the rule in (20). Figure 4.3 shows the level of taxation required to meet the policy target $E(T^S) = \bar{T} + m = 160$. Figure 4.2 and figure 4.4 shows instead respectively the impact of taxation on expected harvest timing when the hyperbolic nature of the agent is not accounted and the error made when setting the tax. Note that only the time consistent agent ($\delta = 1$) meets the policy target and that for certain hyperbolic types the impact of taxation may be even null.



4.5 Conclusion

This chapter has illustrated the implications of the assumption of hyperbolic time preferences in a specific context. According to previous contributions on optimal harvest timing, ongoing uncertainty induces agents to defer harvesting in order to keep open the option to harvest and to wait for collecting new information on the revenues from conservation. In this chapter I show that the effect due to the presence of option value may instead be significantly lowered if hyperbolic time preferences are assumed. Premature harvesting may occur in that the agent has incentive to rush in order to anticipate future selves' time-inconsistent and sub-optimal behaviour. In some extreme cases and if the long-run discount rate ρ is used as a benchmark, harvest with negative expected NPV may occur. The effects on the optimal rule of changes in uncertainty and in other parameters have been discussed. In this framework, I show and discuss how the regulator may intervene to correct market failures in presence of hyperbolic agents. As illustrated also with the help of a numerical and graphical analysis the regulator must adjust the optimal pigovian tax to account for the behavioural failure introduced by such time preferences. This is crucial to avoid that the environmental policy target is dramatically missed.

Chapter 5

Optimal profit sharing under the risk of expropriation

5.1 Introduction

Natural resources such as oil, natural gas and minerals represent a crucial endowment for many countries in that the profits deriving from the exploitation may fund their economic growth and welfare.¹ Developing countries in particular are often rich in natural resources but must often deal with the limited availability of funds to be destined to the exploration of resources fields and to the infrastructures required to extract such resources. Foreign direct investment (hereafter, FDI) may allow to overcome these difficulties in that multinational firms (hereafter, MNF) may be willing to bear the initial costs and extract the resource if an adequate return on their investment is paid.

Unfortunately, matching the economic interests of both parties is challenging and in particular once the investment in the project has been made undertaken.

¹See Brunnschweiler and Bulte (2008) for an empirical analysis and a critical discussion of the so-called "resource curse".

In fact, being the investment for the exploitation of natural resources high specific and totally sunk in nature, the HC may be tempted to exercise the option to expropriate the MNF investment and run the enterprise on its own (Guasch et al., 2003; Engel and Fischer, 2008). Expropriation is an extreme but still common event in developing and even developed countries.² When profits are high and the government is under populist pressures such opportunistic behaviour becomes particularly likely. Moreover, due to the weakness of the legal framework regulating the agreements between a sovereign country and a foreign firm and to the scarce weight of the threat of a fall of future FDI, the temptation is hard to resist in that benefits may largely cover the costs.

As long as there is a light penalty or no penalty at all for the violation of the agreement's terms it will be hard to have an HC credibly committed to their respect (Schnitzer, 1999). Hence, it follows that in addition to uncertainty about market conditions the MNF must account also for the possibility of expropriation as a source of uncertainty on the return on investment.

In order to meet the economic interests of both parties and reduce the risk of expropriation profit sharing agreements have been often proposed (Engel and Fischer, 2008). Through these arrangements a share of the profits from resources extraction is offered by the host country (hereafter, HC) to the MNF as a return on the investment made.

The aim of this chapter is to present a model of cooperative bargaining where uncertainty on profit level and risk of expropriation are considered and to investigate the impact they could have on the possibility of signing a mutually convenient agreement.

²Data on expropriations have been collected and presented in several studies. See Tomz and Wright (2007) for expropriations from 1900 to 1959, Kobrin (1984) from 1960 to 1979, Minor (1994) from 1980 to 1992 and finally Hajzler (2007) which updates available data to 2006.

This analysis is developed in a real options framework in that both the initial investment and the expropriation are economic decisions characterized by uncertainty in the pay-offs and irreversibility. In particular, as one can easily see both players, the MNF and the HC, can be viewed as respectively holding an American call option on investment and expropriation³ (Dixit and Pindyck, 1994; Mahajan, 1990).

Due to uncertainty about market conditions driven by a geometric Brownian motion, waiting before exercising both options is valuable in that additional information on profit future realizations can be collected.

Both parties have different economic targets but share the interest in reaching an agreement that makes them better off with respect to the alternative scenario where the extractive project is not undertaken. Before the extraction starts then mutual interest induces them to bargain on a sharing rule which maximizes the joint venture total rents. This situation resemble to a cooperative game which outcome can be determined applying the Nash Bargaining Solution concept.

The merger of the cooperative bargaining and real options frameworks proposed in this chapter is the first attempt to shed light on the use of profit sharing to shape agreements for the exploitation of natural resources under the risk of expropriation. Up to my knowledge only few contributions have approached the expropriation applying option pricing methods and differently from this chapter they were only focused on pricing its risk (Mahajan, 1990; Clark, 1997, 2003).

In this set-up, I present the conditions under which the bargaining may succeed and leads the two parties to attain a cooperative agreement which maximizes the joint venture value.

An interesting finding is given by the invariance of the investment time trigger.

³The only difference is the sort of dividend paid to the HC if the option to expropriate is not exercised and that is represented by the HC's share of profits fixed through the bargaining.

With or without expropriation risk the MNF invests at the same level of the state variable. The impact of the threat of expropriation is instead evident in the definition of the set of feasible levels of the distributive parameter. I show in fact that this set shrinks as expropriation risk increases leading at the extreme to the bargaining failure. Finally, under the risk of expropriation I prove that the share accruing to the MNF must be higher than without. This makes economic sense and two possible explanations can justify this result. On the one hand, this wedge can be simply seen as the way the MNF is compensated for facing this additional risk, while on the other hand the wedge may be viewed as balancing for the fact that the HC's participation to the venture is compensated not only through the share on profits but also indirectly through the option to expropriate that the HC gets as soon as the investment is undertaken.

The remainder of the chapter is organized as follows. In section 5.2 the basic ingredients to set up the model are presented. In section 5.3 I determine the efficient bargaining set on which the cooperative game is played. In section 5.4 the cooperative game outcome is derived and the agreement between the parties is characterized and deeply discussed. Section 5.5 finally concludes.

5.2 The basic set-up

Consider a project for the extraction of a natural resource in the HC. Assume that the extraction of such resource is lucrative and generates a flow of non-negative⁴ profits π_t which randomly fluctuates over time following a geometric Brownian motion with instantaneous growth rate $\alpha \geq 0$ and instantaneous volatility $\sigma \geq 0$:

$$d\pi_t = \alpha\pi_t dt + \sigma\pi_t dZ_t, \quad \pi_0 = \pi \quad (5.1)$$

where $\{Z_t\}$ is a standard Wiener process where the conditions, $E[dZ_t] = 0$ and $E[dZ_t^2] = dt$ are satisfied. The flow of profits is modelled in a simple way but at no cost in that one may interpret π_t as a reduced form of a more complex model $\pi_t = \pi(v_t)$ where v_t is a vector representing the several variables (market price, technology, taxes, market shocks, etc.) which may affect such flow in the reality (Moretto and Valbonesi, 2007).

Denote by I the sunk investment that the MNF is willing to make to explore the field and set up the required extractive infrastructure. As return on such investment the MNF is entitled to a share of the profits from resources extraction.

For simplicity assume that the venture that the two parties may agree to jointly run has a term sufficiently long that can be approximated by infinity. If the bargaining on profit sharing is feasible the two parties agree to divide each unit of profit in two parts, respectively θ to the MNF and $1 - \theta$ to the HC where $0 < \theta < 1$.

The MNF holds then an option to invest in a project paying if undertaken the flow of profits characterized above. The MNF faces uncertainty about market conditions and may gain by waiting for information relative to profit realization.

⁴Note that $\pi_t = 0$ is an absorbing barrier.

Market is not the only source of profit uncertainty for the MNF in that it has to take into account also the risk of being expropriated by HC. Once the specific investment I has been undertaken the HC has in fact the opportunity of expropriating MNF and run the venture on its own . But expropriation does not come at no cost and then let E represent the sunk cost attached to the expropriation and assume for simplicity that it is known and constant. This cost may include for instance the compensation that following a legal recourse by the MNF an international court may impose to the HC,⁵ the cost associated to the fall of future FDI due to the loss of reputation, the cost related to the lack of the technological and managerial competences to run the firm alone. Differently from previous contributions which applies option theory to evaluate the option to expropriate a dividend represented by the profit share $1 - \theta$ is paid if the option is not killed. Finally, given uncertainty about market conditions drawn by (5.1) also for the HC waiting to collect information on profit flow is valuable.

Expropriation may in fact results costly if it is very likely that after a legal recourse by the MNF an international arbitration is going to set an high compensation payment. Other considerations may include the lack of the necessary expertise to run the firm technology or the cost that the loss of reputation after an expropriation could have in terms of future foreign investments in the country.

5.2.1 The HC's and MNF's objective functions

Being the MNF a foreign firm the HC cares only for the rents accruing to it and has as unique objective their maximization (Engel and Fischer, 2008). Such rents are represented by the share of profits to which it is entitled as long as the

⁵Also the cost opportunity of the funds destined to pay the compensation should be taken into account.

venture is jointly run and by the entire profit once expropriation has occurred. The expected present value of such profits stream is represented by

$$\begin{aligned}
H(\pi, \theta) &= E[e^{-\rho T^F}] \cdot G(\pi, \theta) \\
&= \left(\frac{\pi}{\pi^F}\right)^\beta \left\{ E \left[\int_0^{T^H} e^{-\rho t} (1 - \theta) \pi_t dt + \int_{T^H}^{\infty} e^{-\rho t} \pi_t dt \mid \pi_t = \pi \right] \right\}
\end{aligned} \tag{5.2}$$

where $\rho (> \alpha)$ is the discount rate and $T^H = \inf(t > 0 \mid \pi_t = \pi_H)$ and $T^F = \inf(t > 0 \mid \pi_t = \pi_F)$ are respectively the stochastic expropriation time and the stochastic investment time. The HC's value is represented by the value function $G(\pi, \theta)$ discounted by the stochastic discount factor $\left(\frac{\pi}{\pi^F}\right)^\beta$ and it is a function of the distributive parameter θ .⁶

On the other side, the MNF maximizes instead the expected present value of the share of profits, θ , that is given by

$$F(\pi, \theta) = E \left[\int_0^{T^H} e^{-\rho t} \theta \pi_t dt \mid \pi_t = \pi \right] \tag{5.3}$$

The economic convenience of the investment I is assessed by the MNF consistently with the threat of expropriation which presence is represented by the upper limit of the integral in (5.3). So far I have implicitly assumed that $T^H > T^F$ ($\pi_F < \pi_H$) because as it will become clear later it is the only case which makes economic sense.

5.2.2 The bargaining

The MNF and the HC have different economic interest as shown by their objective function but share the interest of reaching an agreement on the distribution of

⁶See Dixit and Pindyck (1994, p. 315) for the computation of the expected values.

the rents deriving from the resource extraction. The two parties must sign a binding agreement before the venture starts and determine the sharing rule, θ^* , which maximizes the size of the "pie" they are going to share. This bargaining game can be solved by using the Nash Bargaining solution concept (Nash, 1950; Harsanyi, 1977).

The basic situation behind a Nash bargaining is very simple. Two agents may share a pie of size one and each of them simultaneously and without knowing the other agent's proposal presents to a referee her request. If the two requests are feasible, an agreement is reached and the pie is divided accordingly. Otherwise, the game ends and the two agents obtain the disagreement pay-off. Note that two requests are feasible if both parties have a positive share ($0 < \theta^* < 1$) and their sum is equal to 1. This implies that only internal solutions are considered.

The HC and MNF have the same information on the future dynamics of π_t and are averse to the risk of internal conflict. Hence, both parties can be represented by a concave Von Neumann-Morgenstern functions $W(H)$ and $U(F)$ respectively defined on the HC's and MNF's expected share of rents. If an agreement cannot be reached, the resource is not extracted and both parties earn the disagreement utility levels $\hat{w} = 0$ and $\hat{v} = 0$. The bargaining failure is the worst scenario that may occur in that both parties could get more cooperating. The Nash bargaining solution can be determined maximizing the following joint objective function⁷

$$\nabla = \log[W(H) - \hat{w}] + \log[U(F) - \hat{u}] \quad (5.4)$$

⁷See Breccia and Salgado-Banda (2005) and Moretto and Rossini (1995,1996) for bargaining games over a Nash product driven by a geometric Brownian motion.

5.3 Efficient bargaining set under uncertainty and irreversibility

I define in this section the set where the two parties play the efficient bargaining through which they will attempt to set the mutually agreed distribution of the rent from the resource extraction. As I will show in the next sections the definition of the set is affected by both the timing of investment and the timing of expropriation.

5.3.1 The host country

The HC's problem is given by the maximization of (5.2) with respect to T^H . This is a stochastic dynamic programming problem which solution can be determined applying the standard option pricing analysis⁸ (Dixit and Pindyck, 1994). Being T^F determined by the MNF it enters into the HC's problem as an exogenously given parameter. Hence, suppose for the moment that HC is assessing the proceedings just a while after the MNF has undertaken the investment.

Let $V_H(\pi, \theta)$ represent the expected present value of the stream of profits gained if the option to expropriate is never exercised. Such function is given by

$$\begin{aligned} V_H(\pi, \theta) &= E \left[\int_0^\infty e^{-\rho t} (1 - \theta) \pi_t dt \mid \pi_t = \pi \right] \\ &= E \left[\int_0^\infty e^{-(\rho - \alpha)t} (1 - \theta) \pi dt \right] \\ &= (1 - \theta) \frac{\pi}{(\rho - \alpha)} \end{aligned} \tag{5.5}$$

where $\rho > \alpha$ is the discount rate⁹ (Harrison, 1985).

⁸As discussed in the introduction the option to expropriate resembles to an American call option.

⁹Note that if $\rho \leq \alpha$ it would be never optimal for the MNF to invest and any profit would

Let $O_H(\pi, \theta)$ represent the value of such option and denote by π_H the critical threshold at which it is optimal to kill the option. In the region $\pi < \pi_H$ the option is unexercised and by applying Ito's lemma its expected capital gain is given by

$$E [dO_H(\pi, \theta)] = \left[\frac{1}{2} \sigma^2 \pi^2 O_H''(\pi, \theta) + \alpha \pi O_H'(\pi, \theta) \right] dt \quad (5.6)$$

In equilibrium¹⁰ the expected capital gain must be equal to the normal return, $\rho O_H(\pi, \theta) dt$, it follows then

$$\frac{1}{2} \sigma^2 \pi^2 O_H''(\pi, \theta) + \alpha \pi O_H'(\pi, \theta) - \rho O_H(\pi, \theta) = 0 \quad (5.7)$$

This differential equation has solution

$$O_H(\pi, \theta) = A_H \pi^\beta \quad (5.8)$$

where β is the positive root of the quadratic equation¹¹ $\sigma^2 \beta(\beta - 1)/2 + \alpha \beta - \rho$. As standard in the solution to (5.7) the term with the negative root is null to consider that when $\pi \rightarrow 0$ the option is valueless.

Now, one can jointly determine the constant A_H and π_H by solving for the value-matching and smooth-pasting¹² conditions

$$O_H(\pi_H, \theta) = \theta \frac{\pi_H}{(\rho - \alpha)} - E \quad (5.9)$$

$$O_H'(\pi_H, \theta) = \frac{\theta}{(\rho - \alpha)} \quad (5.10)$$

be shared.

¹⁰If a market for trading options to expropriate existed, in equilibrium the return from keeping the option must be equal to what the holder would receive selling the option and putting the proceeds in the bank at rate ρ .

¹¹The solution is $\beta = (\frac{1}{2} - \frac{\alpha}{\sigma^2}) + \sqrt{(\frac{1}{2} - \frac{\alpha}{\sigma^2})^2 + \frac{2\rho}{\sigma^2}}$

¹²It rules out the arbitrary exercise of the option to expropriate at a different point (Dixit, 1993).

The RHS of (5.9) represents the net benefit and cost of the expropriation. Note that (5.9) is equivalent to

$$O_H(\pi_H, \theta) = \frac{\pi_H}{(\rho - \alpha)} - (V_H(\pi_H, \theta) + E)$$

with the first term representing the expected present value of the entire flow of profits and the second term standing for the the cost associated to the expropriation. The cost is given by the sum of the expected present value at π_H of the share, $1 - \theta$, of the joint-venture future profits which are implicitly given up expropriating and of the expropriation E .

Attaching (5.9) and (5.10) to (5.8) and solving for π_H and A_H yields

$$\pi_H = \frac{\beta}{\beta - 1} \frac{(\rho - \alpha)}{\theta} E \quad (5.11)$$

$$A_H = \left[\theta \frac{\pi_H}{(\rho - \alpha)} - E \right] \pi_H^{-\beta} \quad (5.12)$$

Finally, plugging (5.12) into (5.8) and adding (5.5) gives

$$G(\pi, \theta) = \begin{cases} \left[\theta \frac{\pi_H}{(\rho - \alpha)} - E \right] \left(\frac{\pi}{\pi_H} \right)^\beta + \frac{(1 - \theta)}{(\rho - \alpha)} \pi & \text{for } \pi < \pi_H \\ \frac{\pi}{(\rho - \alpha)} - E & \text{for } \pi \geq \pi_H \end{cases} \quad (5.13)$$

In the first line equation, the first term represents the value of the option to expropriate while the second stands for the rents gained by the HC if the option is never exercised. On the second line instead the discounted net pay-off accruing to the HC once expropriation has occurred.

Note that π_H is decreasing in θ . This implies that as $\theta \rightarrow 1$, the expropriation becomes in expected terms more likely. This result simply confirms the reducing effect that profit sharing agreements should have on the risk of expropriation.

5.3.2 The multinational firm

The MNF maximizes (5.3) with respect to T^F and takes T^H as given. Also in this case the underlying stochastic dynamic programming problem can be solved applying the option pricing analysis.¹³

Let $V_F(\pi, \theta)$ represent the expected present value of the stream of profits gained by the HC

$$\begin{aligned} V_F(\pi, \theta) &= E \left[\int_0^{T^H} e^{-\rho t} \theta \pi_t dt \mid \pi_t = \pi \right] \\ &= E \left[\int_0^{T^H} e^{-(\rho-\alpha)t} \theta \pi dt \right] \\ &= \frac{\theta}{(\rho-\alpha)} \left[\pi - \pi_H \left(\frac{\pi}{\pi_H} \right)^\beta \right] \end{aligned} \quad (5.14)$$

From (5.14) follows that the MNF is accounting for the existence of the threshold π_H at which if reached the flow represented by its share of profits will stop.¹⁴

Now, let $F(\pi, \theta)$ represent the value of the option to invest and π_F be the critical threshold at which it is optimal to invest. Applying Ito's lemma to $F(\pi, \theta)$ such option has in the continuation region, $\pi < \pi_F$, an expected capital gain given by

$$E [dF(\pi, \theta)] = \left[\frac{1}{2} \sigma^2 \pi^2 F''(\pi, \theta) + \alpha \pi F'(\pi, \theta) \right] dt \quad (5.15)$$

By the asset market equilibrium condition the expected capital gain must be equal to the normal return $\rho F(\pi, \theta) dt$ and the following relationship must hold

$$\frac{1}{2} \sigma^2 \pi^2 F''(\pi, \theta) + \alpha \pi F'(\pi, \theta) - \rho F(\pi, \theta) = 0 \quad (5.16)$$

¹³Technically the option to invest and the option to expropriate are similar in that they both resemble to an American call option.

¹⁴This means that at least from the MNF perspective the threshold π_H is an absorbing barrier for (5.1). The other is given but for both parties by $\pi_t = 0$.

The solution to this differential equation is again given by

$$F(\pi, \theta) = A_F \pi^\beta \quad (5.17)$$

As above and for the same reasons the term with the negative root is dropped out.

Appending to (5.16) the value-matching and smooth-pasting conditions respectively requiring

$$\begin{aligned} F(\pi_F, \theta) &= V_F(\pi, \theta) - I \\ &= \frac{\theta}{(\rho - \alpha)} \left[\pi_F - \pi_H \left(\frac{\pi}{\pi_H} \right)^\beta \right] - I \end{aligned} \quad (5.18)$$

$$\begin{aligned} F'(\pi_F, \theta) &= V'_F(\pi, \theta) \\ &= \frac{\theta}{(\rho - \alpha)} \left[1 - \beta \left(\frac{\pi_F}{\pi_H} \right)^{\beta-1} \right] \end{aligned} \quad (5.19)$$

and solving the system

$$\begin{cases} A_F \pi_F^\beta = \frac{\theta}{(\rho - \alpha)} \left[\pi_F - \pi_H \left(\frac{\pi_F}{\pi_H} \right)^\beta \right] - I \\ A_F \beta \pi_F^{\beta-1} = \frac{\theta}{(\rho - \alpha)} \left[1 - \beta \left(\frac{\pi_F}{\pi_H} \right)^{\beta-1} \right] \end{cases}$$

yields

$$\pi_F = \frac{\beta}{\beta - 1} \frac{(\rho - \alpha)}{\theta} I \quad (5.20)$$

$$A_F = \left\{ \frac{\theta}{(\rho - \alpha)} \left[\pi_F - \pi_H \left(\frac{\pi_F}{\pi_H} \right)^\beta \right] - I \right\} \pi_F^{-\beta} \quad (5.21)$$

Note $\pi_F < \pi_H$ and that the threshold for the exercise of the option does not take into account the risk of expropriation. This can be easily seen by letting $\pi_H \rightarrow \infty$ and solving the MNF's problem. The threshold would be the same.

This represents an interesting result meaning that the timing of the investment is not affected by the presence of expropriation risk. It does not come as a surprise in that it follows from the dynamic programming principle of optimality applied to solve the problem. If at $t = 0$ the MNF fixes π_F as the optimal time trigger for the investment then the same trigger should be optimal for every $t > 0$, independently on any possible event occurring after π_F .¹⁵

Note also that π_F is decreasing in θ . The higher the share the earlier the investment occurs. This makes sense considering that the joint venture time horizon is restricted by π_H which is decreasing in θ as well. Being in fact the expropriation more likely for high θ , the MNF rushes to have sufficient time to benefit from the joint venture before being expropriated.

Substituting (5.21) into (5.17) gives

$$F(\pi, \theta) = \begin{cases} \left\{ \frac{\theta}{(\rho-\alpha)} \left[\pi_F - \pi_H \left(\frac{\pi_F}{\pi_H} \right)^\beta \right] - I \right\} \left(\frac{\pi}{\pi_F} \right)^\beta & \text{for } \pi < \pi_F \\ \frac{\theta}{(\rho-\alpha)} \left[\pi - \pi_H \left(\frac{\pi}{\pi_H} \right)^\beta \right] - I & \text{for } \pi_F \leq \pi < \pi_H \\ -I & \text{for } \pi \geq \pi_H \end{cases} \quad (5.22)$$

This function represents the expected present value of the net payoff accruing to the firm if the project is undertaken. The MNF is aware that investing is implicitly giving an option to expropriate to the HC and internalizes the risk of this event in the evaluation of the investment opportunity through the term $\pi_H \left(\frac{\pi_F}{\pi_H} \right)^\beta$. Note in fact that $\pi_H \left(\frac{\pi_F}{\pi_H} \right)^\beta = \pi_H \cdot E[e^{-\rho T^H} | \pi_t = \pi_F]$ which represents the amount of rents expropriated by the HC discounted for the random time period starting at T^F and ending at T^H .¹⁶

Finally, from (5.22) follows that $\pi_F < \pi_H$ is the only case to matter in our

¹⁵See Moretto and Valbonesi (2007) and chapters 8 and 9 in Dixit and Pindyck (1994) for similar results.

¹⁶See again Dixit and Pindyck (1994, p. 315) for further details.

analysis in that for $\pi_F \geq \pi_H$ the investment would be expropriated as soon as it is undertaken and would result in a loss equal to $-I$. This in turn implies that only the situations where $E > I$ should be considered.

5.4 Nash bargaining and cooperative equilibrium

The bargaining on the distributive parameter θ must occur before the resource extraction starts ($\pi < \pi_F < \pi_H$). In this region the MNF's and the HC's value function are respectively given by (5.22) and (5.2). Provided that $F(\pi, \theta) > 0$ must be positive for the bargaining to make economic sense,¹⁷ one can easily note that both derivatives, $\frac{dF(\pi, \theta)}{d\pi}$ and $\frac{dG(\pi, \theta)}{d\pi}$, are positive. This implies that the bargaining must occur just a "while" before the critical threshold for the investment (π_F) has been hit. It follows that the objective function (5.4) to be maximized should be evaluated at π_F .

5.4.1 Cooperative equilibrium

Now, denote respectively by $W(H) = H^{1-p}$ and $U(F) = F^q$ the HC's and MNF's utility functions where $0 \leq p < 1$ and $0 < q \leq 1$ represent the respective degree of relative risk aversion and let the two parties play the cooperative game at T^F . The equilibrium agreement will be represented by the level of θ^* which maximizes the objective function in (5.4).

Recalling that the proceedings are evaluated at π_F , that $\hat{w} = \hat{u} = 0$ and differentiating (5.4) with respect to θ the f.o.c. of the maximization problem is given by

$$\frac{1-p}{H(\pi_F, \theta^*)} \frac{dH(\pi_F, \theta^*)}{d\theta} + \frac{q}{F(\pi_F, \theta^*)} \frac{dF(\pi_F, \theta^*)}{d\theta} = 0 \quad (5.23)$$

¹⁷This holds if $\beta \left(\frac{E}{I}\right)^{1-\beta} < 1$.

Given that $\pi_F/\pi_H = E/I$ and rearranging (5.23) the relation that must hold in order to have a feasible agreement in equilibrium is given by

$$-\frac{1 - \left(\frac{E}{I}\right)^{1-\beta}}{1 - \beta \left(\frac{E}{I}\right)^{1-\beta}} \cdot \frac{\beta - \theta^* \left[\beta - \left(\frac{E}{I}\right)^{1-\beta}\right]}{\beta - 1 - \theta^* \left[\beta - \left(\frac{E}{I}\right)^{1-\beta}\right]} = \eta \quad (5.24)$$

where $\eta = \frac{1-p}{q}$.

I note that for the condition in (5.24) to hold

$$\theta^* > \frac{\beta - 1}{\beta - \left(\frac{E}{I}\right)^{1-\beta}} \quad (5.25)$$

Being $\beta > 1$, $E > I$ it follows that there are values of θ^* which can support a cooperative outcome in the feasible set $0 < \theta^* < 1$. Moreover, as $E \rightarrow \infty$, the feasible region enlarges and at the limit has the following lower bound

$$\theta^{**} > \frac{\beta - 1}{\beta} \quad (5.26)$$

where θ^{**} is the distributive parameter defined if there is no expropriation risk or expropriation is extremely unlikely ($\pi_H \rightarrow \infty$). This implies that as long as the expropriation is perceived as a sensible threat the region which sustains a cooperative outcome is smaller and this makes more difficult to attain a mutually convenient agreement.

5.4.2 Some analytical results

In this section I derive and discuss some results which characterizes the properties of the cooperative agreement. The magnitude of E it is not the only factor affecting the outcome of the bargaining in that the extent of the feasible region

is influenced through β also by α and σ .

For simplicity, in the following I assume

$$E = \gamma I \tag{5.27}$$

where $\gamma > 1$. This is an useful and reasonable assumption that relating the cost of expropriation to the scale of the investment expropriated allows to discuss the implications that the magnitude of the penalty¹⁸ may have on the HC's opportunistic behaviour. The parameter γ can also be interpreted as a measure of HC's respect of property and contract law. The higher is γ the higher is time trigger at which the HC exercises the option to expropriate.

Under no risk of expropriation

If $\gamma \rightarrow \infty$, $\pi_H \rightarrow \infty$ in that the expropriation is too costly for the HC. In this case the region where feasible θ^{**} can be set is given by

$$1 - \frac{1}{\beta} < \theta^{**} < 1 \tag{5.28}$$

Plugging (5.27) into (5.24) and solving for θ^{**}

$$\theta^{**} = 1 - \frac{1}{\beta} \frac{\eta}{\eta + 1} > 1 - \frac{1}{\beta} \tag{5.29}$$

Equation (5.29) subject to (5.28) can be used to draw and discuss the outcome of the cooperative game for different α and σ . In particular

(i) as $\sigma \rightarrow \infty$, $\beta \rightarrow 1$ and $\pi_F \rightarrow \infty$. In this case even if the threat of ex-

¹⁸Being out of the focus of this paper it is not important to directly relate such penalty to an international court or to the market for foreign direct investment.

appropriation is practically extinguished the joint extractive project is never undertaken because of uncertainty about market conditions which makes always optimal for the MNF to wait.

(ii) as $\sigma \rightarrow 0$, if $\alpha > 0$ then $\beta \rightarrow \rho/\alpha$ and $\pi_F \rightarrow \left(\frac{\rho}{\theta}\right) I$ and

$$\theta^{**} = 1 - \frac{\alpha}{\rho} \frac{\eta}{(\eta + 1)} < 1$$

This result implies that when the uncertainty about market conditions falls the cooperative sharing rule is shaped by the drift, α , and the discount rate, ρ , and adjusted for the respective relative risk aversions.

(iii) as $\sigma \rightarrow 0$, if $\alpha = 0$ then $\beta \rightarrow \infty$ and $\pi_F \rightarrow \left(\frac{\rho - \alpha}{\theta}\right) I$ and

$$\theta^{**} = 1$$

In this scenario the feasible region drawn by (5.28) collapses and the bargaining fails. Note that for this set of parameters both value functions become linear. This means that to be maximised extreme θ must be chosen (1 or 0). This makes the two parties' requests' not conciliable and leads to the bargaining failure.

Under the risk of expropriation

Let turn now to the situations where the risk of expropriation is sensible. The feasible region for θ^* is given by

$$1 - \frac{1 - \gamma^{1-\beta}}{\beta - \gamma^{1-\beta}} < \theta^* < 1 \quad (5.30)$$

Instead, plugging (5.27) into (5.24) and solving for θ^* yields

$$\theta^* = 1 - \frac{1 - \gamma^{1-\beta}}{\beta - \gamma^{1-\beta}} \frac{\eta (1 - \beta \gamma^{1-\beta}) - \gamma^{1-\beta}}{\eta (1 - \beta \gamma^{1-\beta}) + 1 - \gamma^{1-\beta}} > 1 - \frac{1 - \gamma^{1-\beta}}{\beta - \gamma^{1-\beta}} \quad (5.31)$$

Note that $\theta^* < 1$ if and only if $\eta > (\gamma^{\beta-1} - \beta)^{-1}$.

It follows that

(a) as $\sigma \rightarrow \infty$, $\beta \rightarrow 1$ and both $\pi_H \rightarrow \infty$ and $\pi_F \rightarrow \infty$. As in the previous section because of high uncertainty the joint extractive project never starts.

(b) as $\sigma \rightarrow 0$, if $\alpha > 0$ then $\beta \rightarrow \rho/\alpha$ and $\pi_F \rightarrow (\frac{\rho}{\theta}) I$, $\pi_H \rightarrow (\frac{\rho}{\theta}) E$ and

$$\theta^* = 1 - \frac{\alpha \left(1 - \gamma^{-\frac{\rho-\alpha}{\alpha}}\right)}{\rho - \alpha \gamma^{-\frac{\rho-\alpha}{\alpha}}} \frac{\eta \left(\alpha - \rho \gamma^{-\frac{\rho-\alpha}{\alpha}}\right) - \alpha \gamma^{-\frac{\rho-\alpha}{\alpha}}}{\eta \left(\alpha - \rho \gamma^{-\frac{\rho-\alpha}{\alpha}}\right) + \alpha \left(1 - \gamma^{-\frac{\rho-\alpha}{\alpha}}\right)}$$

In this case for the θ^* to be feasible it must be $\eta > \left(\gamma^{\frac{\rho-\alpha}{\alpha}} - \frac{\rho}{\alpha}\right)^{-1}$. With respect to (ii) it is evident that here the threat of expropriation plays a role in that

$$\theta^* > \theta^{**}$$

(c) as $\sigma \rightarrow 0$, if $\alpha = 0$ then $\beta \rightarrow \infty$, $\pi_F \rightarrow (\frac{\rho-\alpha}{\theta}) I$, $\pi_H \rightarrow (\frac{\rho-\alpha}{\theta}) E$ and $\theta^* = 1$.

Note that as $\beta \rightarrow \infty$, $\gamma^{1-\beta} \rightarrow 0$ and thus the same discussion provided in

(iii) applies.

5.4.3 Final considerations on the cooperative agreement

I propose now two alternative but related interpretations of a result that is obtained rearranging (5.31) as follows

$$\theta^* - \theta^{**} = \left[\frac{1}{\beta} \frac{\eta}{\eta + 1} - \left(\frac{1 - \gamma^{1-\beta}}{\beta - \gamma^{1-\beta}} \right) \frac{\eta (1 - \beta \gamma^{1-\beta}) - \gamma^{1-\beta}}{\eta (1 - \beta \gamma^{1-\beta}) + 1 - \gamma^{1-\beta}} \right] \geq 0 \quad (5.32)$$

Under the risk of expropriation the share of profits accruing to the MNF is higher than under no risk. This makes economic sense in that to induce the MNF to invest the HC must pay a premium for the risk of expropriation. The amount of this compensation is represented by the term into square brackets.

But changing perspective another interesting explanation could be given to this wedge. As discussed above as soon as the MNF invests the HC can exercise the option to expropriate. Being this anticipated by both parties one can see the option to expropriate and get the entire flow of profits as the way HC is compensated for taking part to the joint venture in addition to the share $1 - \theta$. This is taken into account when defining the distributive parameter, θ^* , which is consequently adjusted. It follows that then (5.32) express the way the two parties price the option to expropriate at π_F . This is an important aspect which deserves some comment. Note in fact that after the investment is undertaken the value of the option to expropriate will randomly fluctuates. This implies that according to the values taken the government may wish to reconsider the distribution of the profits. There could be then incentive for the so-called "creeping expropriation" (Schnitzer, 1999). This is the increasingly common practice by which governments subtly violate the agreements through a change in the fiscal treatment of MNF's earnings, or a change in the regulations regarding the firm's activity or simply imposing a new profit sharing rule.

5.5 Conclusions

Foreign investment may allow to developing countries to undertake the exploitation of their natural resources. The scarcity of this resources makes these projects a lucrative business for both parties and allow to developing countries to invest the proceeds in the provision of public goods and infrastructures needed for economic growth.

The room for these enterprises is unfortunately quite often limited by the presence of expropriation risk. Expropriation is a temptation hard to resist in particular when profits are high and governments have to deal with populist pressure for their redistribution. Moreover, being the punishment for such extreme act generally low with respect to benefits expropriation is definitely a sensible option in many cases.

The introduction of profit sharing agreements may reduce expropriation risk. In this chapter this cooperative situation is completely characterized by a model where the cooperative bargaining theoretical framework meets the real options approach.

The findings are interesting and are represented by the invariance of the investment time trigger with respect to the presence of expropriation risk, the restriction of the set of feasible bargains due to the threat of expropriation and the need to pay a premium to the MNF for the additional risk.

I believe that this framework may be extended at least in two respects. First, in this chapter I have considered only the case in which the governments takes all the "pie". It would be interesting to generalize the model and allow also for the risk of the so-called "creeping expropriation" (Schnitzer, 1999). Second, and in some respects related to the first point, the impact of shocks on the government

time preferences should be internalized in the model. I refer in particular to shocks caused by the random occurrence of political and economic events such as political crisis due to populist and political parties pressures and macroeconomic events which suddenly changes the economic scenario.

Chapter 6

Concluding remarks

In this thesis I have analysed several issues regarding the use of natural resources and their importance for social welfare. The analysis has been developed by looking at the role played by information in each context. In this final chapter, I intend to summarise the main issues discussed and I will identify lines for future research.

In chapter 2, I have applied the mechanism design theory to design a conservation program which differentiate payments with respect to the opportunity cost of providing ecosystem services. Poor targeting and sub-optimal use of the scarce funding available for conservation often characterizes the general subsidy schemes through which conservation programs are implemented in the reality (Salzman, 2005). The contract schedule proposed in this chapter can guarantee superior results in terms of targeting and efficient use of the funding. Nevertheless, when comparing the two schemes, one should also take into account the cost of the information required to implement the scheme and the rents that must be paid to induce revelation of true types.

These costs could be high and the actual welfare gain may be too little to justify the adoption of the scheme I propose (Crépin, 2005; Arguedas et al., 2007). In this respect, I have highlighted the impact that keeping into account the risk of poor crop yield and the related lower level of land converted by credit constrained landowners, could have when assessing possible welfare gains. Two aspects that deserve more future research are an explicit modelling of the credit constraint for the landowners in the model and exploring the relationship between the probability of unfavourable crop yields and the environmental characteristics of the land to be converted. Finally, an interesting extension for future research in this field will be the analysis of the mechanism design issues in a dynamic continuous time frame where uncertainty in the return from agriculture and the irreversibility of the conversion process once undertaken are considered.

In chapters 3 and 4, the model for an optimal harvest timing under a real options approach of Conrad (1997) is extended to incorporate hyperbolic time-inconsistent preferences. These preferences have been analysed in various fields of economics while have been hardly considered by resource economists (Shogren, 2007). I have attempted to start filling the gap first, by generalizing Conrad's basic model and second, by addressing in chapter 4 issues related to the impact of non-standard time preferences on the second best tools used to correct market failures in the provision of natural goods and services. The findings are interesting and, as shown chapter 3, provide a robust explanation about the haste of governments in undertaking projects irreversibly impacting on the intertemporal allocation of natural assets and to the time inconsistency of their conservation policies.

As highlighted in chapter 1, I contribute also to the real options literature presenting a more general framework for the evaluation of options such as the option to exit, to shut-down, or to abandon (Dixit and Pindyck, 1994). This framework can be easily applied to the analysis of several economic problems entailing the exercise of these options. One possible way one can enrich the model will be to extend the role of government targeting and political instability on environmental policies.

In chapter 5, the interesting problem of expropriation has been investigated. Foreign investment may allow to developing countries to undertake the exploitation of their natural resources. The derived revenue could then be used to fund their welfare improvement and their economic growth. Unfortunately, this opportunity can be discouraged by the risk of expropriation. Expropriation is a temptation hard to resist for the host country's government, in particular, when profits are high. The introduction of profit sharing agreements has been suggested to reduce such risk. In this chapter, I have modelled in an original way this situation merging the cooperative bargaining and the real options theoretical frameworks. The findings seem encouraging and although the model presented in this thesis is very simple, it provides significant insight for the analysis of the issue. I believe that this framework may be easily extended to capture other aspects of the more complex reality. This should be done at least in two respects. First, generalizing the model to allow also for the risk of the so-called "creeping expropriation" (Schnitzer, 1999). This extension requires in order to capture all the renegotiation aspects to develop a full contract game in a dynamic framework.

This would allow to investigate the increasingly common practice by governments to violate the initial agreement terms, for instance by changing the fiscal treatment of foreign firms' earnings, regulation on firms' activity or the profit sharing rule. Second, and somehow related to the previous point, I would suggest to internalize the impact of suddenly occurring shocks such as pressures on the government exerted by political parties, social and macroeconomic events (Engel and Fischer, 2008).

Appendix A

Appendix to Chapter 2

A.1 Proposition 2.1

The Lagrangian of the maximization problem in (2.8) is

$$L = B(a(\theta)) + (1 + \lambda) \pi(\bar{A} - a(\theta), \theta) - \lambda \Pi(\bar{A} - a(\theta), \theta) + \\ + \gamma(\theta) (\Pi(\bar{A} - a(\theta), \theta) - \pi(\bar{A} - \hat{a}(\theta), \theta)) + \phi(\theta) (a(\theta) - \hat{a}(\theta))$$

where $\gamma(\theta)$ and $\phi(\theta)$ are the lagrangian multipliers attached to the constraints.

Necessary conditions which must hold for an optimum are

$$\frac{\partial L}{\partial a(\theta)} = B'(a(\theta)) - (1 + \lambda) \{p[1 - \bar{v} + q(\bar{v} - \underline{v})] Y_1(\bar{A} - a(\theta), \theta) - c\} + \\ + (-\lambda + \gamma(\theta)) \frac{\partial \Pi(\bar{A} - a(\theta), \theta)}{\partial a(\theta)} + \phi(\theta) = 0 \tag{L.1}$$

$$\frac{\partial L}{\partial \Pi (\bar{A} - a(\theta), \theta)} = -\lambda + \gamma(\theta) = 0 \quad (\text{L.2})$$

$$\gamma(\theta) (\Pi (\bar{A} - a(\theta), \theta) - \pi (\bar{A} - \hat{a}(\theta), \theta)) = 0, \quad \gamma(\theta) \geq 0 \quad (\text{L.3})$$

$$\phi(\theta) (a(\theta) - \hat{a}(\theta)) = 0, \quad \phi(\theta) \geq 0 \quad (\text{L.4})$$

Under perfect information the payments are set to compensate the landowners for their actual economic loss. Hence, $\Pi (\bar{A} - a(\theta), \theta) = \pi (\bar{A} - \hat{a}(\theta), \theta)$. It is then easy to check that (L.3) holds being by (L.2), $\gamma(\theta) = \lambda \geq 0$.

Now, assume $a^{FB}(\theta) > \hat{a}(\theta)$ and $\phi(\theta) = 0$ and substitute (L.2) into (L.1).

Rearranging it follows that

$$\begin{aligned} Y_1 (\bar{A} - a^{FB}(\theta), \theta) &= \\ \frac{1}{p[1 - \bar{v} + q(\bar{v} - \underline{v})]} \left[c + \frac{B'(a^{FB}(\theta))}{(1 + \lambda)} \right] &> \frac{c}{p[1 - \bar{v} + q(\bar{v} - \underline{v})]} \\ &= Y_1 (\bar{A} - \hat{a}(\theta), \theta) \end{aligned}$$

and given the restrictions on the shape of $Y (\bar{A} - a(\theta), \theta)$

$$\begin{aligned} Y_1 (\bar{A} - a^{FB}(\theta), \theta) &> Y_1 (\bar{A} - \hat{a}(\theta), \theta) \\ \bar{A} - a^{FB}(\theta) &< \bar{A} - \hat{a}(\theta) \\ a^{FB}(\theta) &> \hat{a}(\theta) \end{aligned}$$

Our initial assumption is confirmed.

Checking instead the conjecture $a^{FB}(\theta) = \hat{a}(\theta)$ and $\phi(\theta) \geq 0$ it is not difficult to prove that falls by contradiction in that substituting (L.2) and (2.4) into (L.1) we get

$$\phi(\theta) = -B'(\hat{a}(\theta)) < 0$$

A.2 Proposition 2.2

If the contract schedule $\{[a(\theta), T(\theta)]; 0 \leq \theta \leq 1\}$ is incentive compatible the landowners maximize their program rents by revealing their true land type θ . Hence, θ must be the solution of the following maximization problem:

$$\begin{aligned} \max_{\tilde{\theta}} \left[\Pi \left(\bar{A} - a(\tilde{\theta}), \theta \right) \right] &= p [1 - \bar{v} + q(\bar{v} - \underline{v})] Y \left(\bar{A} - a(\tilde{\theta}), \theta \right) + \\ &\quad - c \left(\bar{A} - a(\tilde{\theta}) \right) + T(\tilde{\theta}) \end{aligned} \quad (\text{A.2.1})$$

If θ is the solution then the following first and second order conditions must hold:

$$\begin{aligned} \left. \frac{\partial \left[\Pi(\bar{A} - a(\tilde{\theta}), \theta) \right]}{\partial \tilde{\theta}} \right|_{\tilde{\theta}=\theta} &= - \left\{ p [1 - \bar{v} + q(\bar{v} - \underline{v})] Y_1 (\bar{A} - a(\theta), \theta) - c \right\} a'(\theta) + \\ &\quad + T'(\theta) = 0 \end{aligned} \quad (\text{A.2.2})$$

$$\begin{aligned} \left. \frac{\partial^2 \left[\Pi(\bar{A} - a(\tilde{\theta}), \theta) \right]}{\partial \tilde{\theta}^2} \right|_{\tilde{\theta}=\theta} &= p [1 - \bar{v} + q(\bar{v} - \underline{v})] Y_{11} (\bar{A} - a(\theta), \theta) a'(\theta)^2 + \\ &\quad - \left\{ p [1 - \bar{v} + q(\bar{v} - \underline{v})] Y_1 (\bar{A} - a(\theta), \theta) - c \right\} a''(\theta) + T''(\theta) \leq 0 \end{aligned} \quad (\text{A.2.3})$$

Condition (b) of Proposition 2.2 can be derived from (A.2.2).

Given that in the optimal contract schedule (A.2.2) must hold for every θ , it follows that its derivative with respect to θ must be zero:

$$p [1 - \bar{v} + q(\bar{v} - \underline{v})] [Y_{11} (\bar{A} - a(\theta), \theta) a'(\theta) - Y_{12} (\bar{A} - a(\theta), \theta)] a'(\theta) + \quad (\text{A.2.4}) \\ - \{p [1 - \bar{v} + q(\bar{v} - \underline{v})] Y_1 (\bar{A} - a(\theta), \theta) - c\} a''(\theta) + T''(\theta) = 0$$

Comparing (A.2.3) and (A.2.4):

$$p [1 - \bar{v} + q(\bar{v} - \underline{v})] Y_{12} (\bar{A} - a(\theta), \theta) a'(\theta) \leq 0 \quad (\text{A.2.5})$$

Condition (a) follows considering that by assumption $Y_{12} (\bar{A} - a(\theta), \theta) > 0$ and $p [1 - \bar{v} + q(\bar{v} - \underline{v})] \geq 0$.

Now, we prove that conditions (a) and (b) are met only if the contract schedule is incentive compatible. For every θ and $\tilde{\theta} \in [\underline{\theta}, \bar{\theta}]$,

$$\Pi (\bar{A} - a(\theta), \theta) - \Pi (\bar{A} - a(\tilde{\theta}), \theta) \geq \int_{\tilde{\theta}}^{\theta} \frac{\partial \Pi (\bar{A} - a(\xi), \theta)}{\partial \xi} d\xi \quad (\text{A.2.6})$$

where

$$\frac{\partial \Pi (\bar{A} - a(\xi), \theta)}{\partial \xi} = - \{p [1 - \bar{v} + q(\bar{v} - \underline{v})] Y_1 (\bar{A} - a(\xi), \theta) - c\} a'(\xi) + T'(\xi) \quad (\text{A.2.7})$$

By condition (b) $T'(\xi) = \{p [1 - \bar{v} + q(\bar{v} - \underline{v})] Y_1 (\bar{A} - a(\xi), \xi) - c\} a'(\xi)$.

Plugging it into (A.2.7)

$$\frac{\partial \Pi (\bar{A} - a(\xi), \theta)}{\partial \xi} = -p [1 - \bar{v} + q(\bar{v} - \underline{v})] [Y_1 (\bar{A} - a(\xi), \theta) + \\ - Y_1 (\bar{A} - a(\xi), \xi)] a'(\xi) \quad (\text{A.2.8})$$

If $\xi \in [\tilde{\theta}, \theta]$ with $\theta \geq \tilde{\theta}$ then $Y_1(\bar{A} - a(\xi), \theta) - Y_1(\bar{A} - a(\xi), \xi) \geq 0$ since $Y_{12}(\bar{A} - a(\theta), \theta) \geq 0$ by assumption. If condition (a) holds ($a'(\theta) \leq 0$) then the integrand in (A.2.6) is nonnegative and $\Pi(\bar{A} - a(\theta), \theta) - \Pi(\bar{A} - a(\tilde{\theta}), \theta) \geq 0$. By the same arguments, if $\theta \leq \tilde{\theta}$ then the integrand in (A.2.6) is nonpositive. But considering that we are integrating backwards then it still follows $\Pi(\bar{A} - a(\theta), \theta) - \Pi(\bar{A} - a(\tilde{\theta}), \theta) \geq 0$.

A.3 Larger total rents for the higher type

Total differentiating the program rent function in (2.12)

$$\begin{aligned} \frac{\partial [\Pi(\bar{A} - a(\theta), \theta)]}{\partial \theta} &= - [p[1 - \bar{v} + q(\bar{v} - \underline{v})] Y_1(\bar{A} - a(\theta), \theta) - c] a'(\theta) + \\ &+ p[1 - \bar{v} + q(\bar{v} - \underline{v})] Y_2(\bar{A} - a(\theta), \theta) + T'(\theta) \end{aligned} \quad (\text{A.3.1})$$

plugging condition (b) into (A.3.1), and considering that $Y_2(\bar{A} - a(\theta), \theta) > 0$ the following relation holds

$$\frac{\partial [\Pi(\bar{A} - a(\theta), \theta)]}{\partial \theta} = p[1 - \bar{v} + q(\bar{v} - \underline{v})] Y_2(\bar{A} - a(\theta), \theta) > 0 \quad (\text{A.3.2})$$

A.4 Proposition 2.3

By the envelope theorem and using (2.4)

$$\begin{aligned}
 \frac{\partial [\pi (\bar{A} - \hat{a}(\theta), \theta)]}{\partial \theta} &= - \{p[1 - \bar{v} + q(\bar{v} - \underline{v})] Y_1 (\bar{A} - \hat{a}(\theta), \theta) - c\} \hat{a}'(\theta) + \\
 &+ p[1 - \bar{v} + q(\bar{v} - \underline{v})] Y_2 (\bar{A} - \hat{a}(\theta), \theta) \quad (\text{A.4.1}) \\
 &= p[1 - \bar{v} + q(\bar{v} - \underline{v})] Y_2 (\bar{A} - \hat{a}(\theta), \theta) > 0
 \end{aligned}$$

Under the CP $a(\theta) \geq \hat{a}(\theta)$. Comparing (2.5) with (2.17) and being $Y_{12} > 0$ it follows that

$$\frac{\partial [\pi (\bar{A} - \hat{a}(\theta), \theta)]}{\partial \theta} \geq \frac{\partial [\pi (\bar{A} - a(\theta), \theta)]}{\partial \theta} \quad (\text{A.4.2})$$

That is, $\Pi (\bar{A} - a(\theta), \theta) - \pi (\bar{A} - \hat{a}(\theta), \theta)$ is non increasing in θ .

Hence, if $\Pi (\bar{A} - a(\bar{\theta}), \bar{\theta}) - \pi (\bar{A} - \hat{a}(\bar{\theta}), \bar{\theta}) \geq 0$ then $\Pi (\bar{A} - a(\theta), \theta) - \pi (\bar{A} - \hat{a}(\theta), \theta) \geq 0$ for every $\theta < \bar{\theta}$.

A.5 Proposition 2.4

Denote the term $[1 - \bar{v} + q(\bar{v} - \underline{v})]$ by k and use condition (b) in proposition 2.2 to rearrange $T(\theta)$ as follows

$$\begin{aligned}
T(\theta) &= T(\bar{\theta}) - \int_{\theta}^{\bar{\theta}} T'(\xi) d\xi \\
&= T(\bar{\theta}) - \int_{\theta}^{\bar{\theta}} \{pkY_1(\bar{A} - a(\xi), \xi) - c\} a'(\xi) d\xi \\
&= T(\bar{\theta}) + \int_{\theta}^{\bar{\theta}} \frac{d \{pkY(\bar{A} - a(\xi), \xi) - c(\bar{A} - a(\xi))\}}{d\xi} d\xi + \\
&\quad - \int_{\theta}^{\bar{\theta}} pkY_2(\bar{A} - a(\xi), \xi) d\xi \\
&= T(\bar{\theta}) + \{pkY(\bar{A} - a(\bar{\theta}), \bar{\theta}) - c(\bar{A} - a(\bar{\theta}))\} + \\
&\quad - \{pkY(\bar{A} - a(\theta), \theta) - c(\bar{A} - a(\theta))\} - k \int_{\theta}^{\bar{\theta}} pY_2(\bar{A} - a(\xi), \xi) d\xi \\
&= \Pi(\bar{A} - a(\bar{\theta}), \bar{\theta}) - \{pkY(\bar{A} - a(\theta), \theta) - c(\bar{A} - a(\theta))\} + \\
&\quad - k \int_{\theta}^{\bar{\theta}} pY_2(\bar{A} - a(\xi), \xi) d\xi \tag{A.5.1}
\end{aligned}$$

Substituting (A.5.1) into (2.15)

$$\begin{aligned}
E_{\theta}[W] &= \int_{\underline{\theta}}^{\bar{\theta}} \{B(a(\theta)) + (1 + \lambda)pkY(\bar{A} - a(\theta), \theta) - c(\bar{A} - a(\theta))\} f(\theta) d\theta + \\
&\quad + \lambda k \int_{\underline{\theta}}^{\bar{\theta}} \int_{\theta}^{\bar{\theta}} pY_2(\bar{A} - a(\xi), \xi) d\xi f(\theta) d\theta - \lambda \Pi(\bar{A} - a(\bar{\theta}), \bar{\theta})
\end{aligned}$$

Integrating by parts the last term of $E_{\theta}[W]$

$$\begin{aligned}
E_{\theta}[W] &= \int_{\underline{\theta}}^{\bar{\theta}} \{B(a(\theta)) + (1 + \lambda)pkY(\bar{A} - a(\theta), \theta) - c(\bar{A} - a(\theta))\} f(\theta) d\theta + \\
&\quad + \lambda k \int_{\underline{\theta}}^{\bar{\theta}} pY_2(\bar{A} - a(\theta), \theta) F(\theta) d\theta - \lambda \Pi(\bar{A} - a(\bar{\theta}), \bar{\theta})
\end{aligned}$$

$$\begin{aligned}
&= \int_{\underline{\theta}}^{\bar{\theta}} \{B(a(\theta)) + (1 + \lambda) [pkY(\bar{A} - a(\theta), \theta) - c(\bar{A} - a(\theta))] + \\
&+ \lambda kpY_2(\bar{A} - a(\theta), \theta) \frac{F(\theta)}{f(\theta)}\} f(\theta) d\theta - \lambda \Pi(\bar{A} - a(\bar{\theta}), \bar{\theta}) \\
&= (1 + \lambda) k \int_{\underline{\theta}}^{\bar{\theta}} \Phi[a(\theta), \theta] f(\theta) d\theta - \lambda \Pi(\bar{A} - a(\bar{\theta}), \bar{\theta}) \tag{A.5.2}
\end{aligned}$$

To maximize (A.5.2) or (2.17) is equivalent.

A.6 Binding perverse incentive constraint

By condition (a) in Proposition 2.2 $a^{SB'}(\theta) \leq 0$. Set $a^{SB}(\theta) = \hat{a}(\theta)$. Totally differentiate (2.4)

$$-p[1 - \bar{v} + q(\bar{v} - \underline{v})] [Y_{11}(\bar{A} - \hat{a}(\theta), \theta) \hat{a}'(\theta) - Y_{12}(\bar{A} - \hat{a}(\theta), \theta)] = 0$$

Solving for $\hat{a}'(\theta)$, it follows

$$\hat{a}'(\theta) = \frac{Y_{12}(\bar{A} - \hat{a}(\theta), \theta)}{Y_{11}(\bar{A} - \hat{a}(\theta), \theta)} < 0 \tag{A.6.1}$$

This means that the monotonicity constraint is always satisfied on the interval $[\theta_1, \theta_2]$.

Substituting $\hat{a}(\theta)$ into condition (b) of Proposition 2.2

$$T'(\theta) = \{p[1 - \bar{v} + q(\bar{v} - \underline{v})] Y_1(\bar{A} - \hat{a}(\theta), \theta) - c\} \hat{a}'(\theta) = 0$$

If type $\bar{\theta}$ landowners conserve $\hat{a}(\bar{\theta})$ then minimizing $T^{SB}(\bar{\theta})$ such that (2.16) holds involves

$$T^{SB}(\bar{\theta}) = 0 \tag{A.6.2}$$

Moreover, if $\theta_2 = \bar{\theta}$ being $T'(\theta) = 0$ it follows that all the landowners undertaking $a(\theta) = \hat{a}(\theta)$ in the interval $[\theta_1, \bar{\theta}]$ will not get any compensation.

A.7 Feasibility of a GS program

Under the GS program $T(\theta) = \bar{T} \cdot a(\theta)$ and the landowner chooses to conserve $\bar{a}(\theta)$. It follows that

$$\pi(\bar{A} - \bar{a}(\theta), \theta) + \bar{T} \cdot \bar{a}(\theta) \geq \pi(\bar{A} - \hat{a}(\theta), \theta) \quad (\text{A.7.1})$$

and this meet the incentive rationality requirement.

If conditions (a) and (b) of Proposition 2.2 are met then the GS program is incentive compatible. The landowner's rent is given by

$$\begin{aligned} \Pi(\bar{A} - \bar{a}(\theta), \theta) &= \pi(\bar{A} - \bar{a}(\theta), \theta) + \bar{T} \cdot \bar{a}(\theta) \\ &= p[1 - \bar{v} + q(\bar{v} - \underline{v})] Y(\bar{A} - \bar{a}(\theta), \theta) - c(\bar{A} - \bar{a}(\theta)) + \bar{T} \cdot \bar{a}(\theta) \end{aligned} \quad (\text{A.7.2})$$

Maximizing (A.7.2) with respect to $\bar{a}(\theta)$ the landowner defines the surface to be conserved. From the foc

$$Y_1(\bar{A} - \bar{a}(\theta), \theta) = \frac{c + \bar{T}}{p[1 - \bar{v} + q(\bar{v} - \underline{v})]} \quad (\text{A.7.3})$$

Differentiating totally (A.7.3) and solving for $\bar{a}'(\theta)$

$$\bar{a}'(\theta) = \frac{Y_{12}(\bar{A} - \bar{a}(\theta), \theta)}{Y_{11}(\bar{A} - \bar{a}(\theta), \theta)} < 0 \quad (\text{A.7.4})$$

and condition (a) is satisfied.

If $T(\theta) = \bar{T} \cdot \bar{a}(\theta)$ then $T'(\theta) = \bar{T} \cdot \bar{a}'(\theta)$. Substituting $T'(\theta)$ into condition (b)

$$\bar{T} \cdot \bar{a}'(\theta) = \{p[1 - \bar{v} + q(\bar{v} - \underline{v})] Y_1(\bar{A} - \bar{a}(\theta), \theta) - c\} \bar{a}'(\theta) \quad (\text{A.7.5})$$

The relation is satisfied considering that rearranging (A.7.3)

$$\bar{T} = p[1 - \bar{v} + q(\bar{v} - \underline{v})] Y_1(\bar{A} - \bar{a}(\theta), \theta) - c \quad (\text{A.7.6})$$

A.8 Bunching types

Bunching arises if the monotonicity constraint does not hold. We solve then (2.17) following Guesnerie and Laffont (1984). Restate the problem as follows

$$\max_{a(\theta), \gamma(\theta)} \int_{\underline{\theta}}^{\bar{\theta}} \Phi [a(\theta), \theta] f(\theta) d\theta$$

s.t.

$$\gamma(\theta) = a'(\theta) \tag{C1}$$

$$\gamma(\theta) \leq 0 \tag{C2}$$

where $a(\theta)$ and $\gamma(\theta)$ are respectively the state and the control variable. Attaching the multiplier $\mu(\theta)$ to (C2) the Hamiltonian for the problem is given by

$$H(a, \gamma, \mu, \theta) = \Phi [a(\theta), \theta] f(\theta) - \mu\gamma \tag{A.8.1}$$

From the Pontryagin principle:

$$\mu'(\theta) = -\frac{\partial H}{\partial a} = -\frac{\partial \Phi [a(\theta), \theta]}{\partial a(\theta)} f(\theta) \tag{A.8.2}$$

$$\mu(\theta) \gamma(\theta) = 0, \mu(\theta) \geq 0 \tag{A.8.3}$$

Suppose the existence of an interval where the monotonicity constraint (C2) is not binding. On this interval, $\mu(\theta) = 0$ everywhere and $\mu'(\theta) = 0$. In this case the optimal solution is $a^{SB}(\theta)$.

Consider now an interval $[\theta_m, \theta_M] \subseteq [\underline{\theta}, \bar{\theta}]$ where $a'(\theta) = 0$. It follows that $\gamma(\theta) = 0$ and $a(\theta)$ is constant and equal to a constant h . Observing that on the left and on the right of $[\theta_m, \theta_M]$ (C2) is not binding by continuity of $\mu(\theta)$ it follows that $\mu(\theta_m) = \mu(\theta_M) = 0$. Integrate (A.8.2) on $[\theta_m, \theta_M]$:

$$\int_{\theta_m}^{\theta_M} \frac{\partial \Phi[k, \theta]}{\partial a(\theta)} f(\theta) d\theta = 0$$

or

$$\begin{aligned} & \int_{\theta_m}^{\theta_M} \left\{ pY_1(h, \theta) f(\theta) + \frac{\lambda}{(1+\lambda)} pY_{12}(h, \theta) F(\theta) \right\} d\theta \quad (\text{A.8.4}) \\ &= \int_{\theta_m}^{\theta_M} \frac{1}{1 - \bar{v} + q(\bar{v} - \underline{v})} \left[\frac{B'(\bar{A} - h)}{(1+\lambda)} + c \right] f(\theta) d\theta \end{aligned}$$

One could compute the unknown θ_m, θ_M and h , setting the values which satisfies (A.8.4) and $h = a^{SB}(\theta_m) = a^{SB}(\theta_M)$.

To summarize if $a'(\theta) > 0$ on the whole support, Θ , then the agency will bunch types. All landowners will retire the same amount of land, $a(\theta) = h$, and receive the same transfer $T(\bar{\theta})$. Since landowner's profit is costly for the agency then the optimal transfer, $T^{SB}(\bar{\theta})$, is such that $\Pi(\bar{A} - h, \bar{\theta}) = \pi(\bar{A} - h, \bar{\theta})$. There is no alternative for the GA if she wants to keep feasible the program. If $a'(\theta) > 0$ on some intervals of Θ but $a'(\theta) \leq 0$ on others then it is not possible to separate some θ . The solution will pool some segments of the interval Θ with $a'(\theta) \leq 0$ and others with $a'(\theta) > 0$. On these segments the landowners retire the same amount of land and get the same transfer.

Appendix B

Appendix to Chapter 3

B.1 Strategies under naïve belief

Equation (3.10) can be rearranged as

$$\begin{aligned} & \frac{1}{2}\sigma^2 A^2 V^{N''}(A) + \mu A V^{N'}(A) - (\rho + \lambda) V^N(A) \\ & = - \left[A \left(1 + \frac{\lambda\delta}{\rho - \mu} \right) + \lambda\delta \frac{M}{1 - \beta_1} \left(\frac{A}{A^*} \right)^{\beta_1} \right] \quad \text{for } A \geq A_N \end{aligned} \quad (\text{B.1.1})$$

The solution to the homogenous part is¹

$$V_h^N(A) = k_2 A^{\beta_2}$$

where $\beta_2 = \left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right) - \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2(\rho+\lambda)}{\sigma^2}} < 0$.

Suppose that the particular solution takes the form $V_p^N(A) = c_1 A^{\beta_1} + c_2 A$.

Plug this candidate and its first two derivatives, $V_p^{N'}(A) = \beta_1 c_1 A^{\beta_1-1} + c_2$ and

¹The solution should have the form $V_h^N(A) = k_1 A^{\alpha_2} + k_2 A^{\beta_2}$ where k_1 and k_2 are coefficients to be specified and $\alpha_2 > 0$ and $\beta_2 < 0$ are the roots of the characteristic equation $\sigma^2\beta(\beta - 1)/2 + \mu\beta - (\rho + \lambda)$. As $A \rightarrow \infty$, the value of the option to harvest ($V_h^N(A)$) should go to zero. Since $\alpha_2 > 0$ then k_1 must be zero because if not $V_h^N(A) \rightarrow \infty$ as $A \rightarrow \infty$. The same argument holds when this functional form is used later.

$V_p^{N''}(A) = (\beta_1 - 1) \beta_1 c_1 A^{\beta_1 - 2}$ into (B.1.1)

$$\begin{aligned} & \frac{1}{2} \sigma^2 A^2 (\beta_1 - 1) \beta_1 c_1 A^{\beta_1 - 2} + \mu A (\beta_1 c_1 A^{\beta_1 - 1} + c_2) + \quad (\text{B.1.2}) \\ & - (\rho + \lambda) (c_1 A^{\beta_1} + c_2 A) = - \left[A \left(1 + \frac{\lambda \delta}{\rho - \mu} \right) + \lambda \delta \frac{M}{1 - \beta_1} \left(\frac{A}{A^*} \right)^{\beta_1} \right] \end{aligned}$$

In order to find the coefficients c_1 and c_2 (B.1.2) can be reduced to

$$\begin{aligned} \left[\frac{1}{2} \sigma^2 (\beta_1 - 1) \beta_1 c_1 + \mu \beta_1 c_1 - \rho \right] c_1 - \lambda c_1 &= -\lambda \delta \frac{M}{1 - \beta_1} \left(\frac{1}{A^*} \right)^{\beta_1} \\ [\mu - (\rho + \lambda)] c_2 &= - \left(1 + \frac{\lambda \delta}{\rho - \mu} \right) \end{aligned}$$

The candidate solution satisfies (B.1.1) if the following coefficients are set

$$\begin{aligned} c_1 &= \frac{\delta M}{1 - \beta_1} \left(\frac{1}{A^*} \right)^{\beta_1} \\ c_2 &= \left(1 + \frac{\lambda \delta}{\rho - \mu} \right) \frac{1}{(\rho + \lambda - \mu)} = \frac{\eta}{\rho - \mu} \end{aligned}$$

where $\eta = \frac{\rho + \lambda \delta - \mu}{\rho + \lambda - \mu} \leq 1$.

The general solution is given by the sum of $V_h^N(A)$ and $V_p^N(A)$. Substituting c_1 and c_2 into $V_p^N(A)$ it follows that

$$V^N(A) = k_2 A^{\beta_2} + \frac{\delta M}{1 - \beta_1} \left(\frac{A}{A^*} \right)^{\beta_1} + A \frac{\eta}{\rho - \mu} \quad (\text{B.1.3})$$

At the critical amenity value, A_N , the value-matching and smooth-pasting conditions respectively require $V^N(A_N) = M$ and $V^{N'}(A_N) = 0$.

Solving the system

$$\begin{cases} k_2 A_N^{\beta_2} + \frac{\delta M}{1-\beta_1} \left(\frac{A_N}{A^*}\right)^{\beta_1} + A_N \frac{\eta}{\rho-\mu} = M \\ k_2 \beta_2 A_N^{\beta_2-1} + \delta M \frac{\beta_1}{1-\beta_1} \left(\frac{A_N}{A^*}\right)^{\beta_1} \frac{1}{A_N} + \frac{\eta}{\rho-\mu} = 0 \end{cases}$$

one could find the optimal threshold (3.13) and

$k_2 = -\frac{1}{\beta_2} \left[\frac{\delta M \beta_1}{1-\beta_1} \left(\frac{A_N}{A^*}\right)^{\beta_1} + \frac{\eta A_N}{\rho-\mu} \right] (A_N)^{-\beta_2}$ and finally, plugging k_2 into (B.1.3) the value function (3.14)

$$\begin{aligned} V^N(A) &= -\frac{1}{\beta_2} \left[\delta M \frac{\beta_1}{1-\beta_1} \left(\frac{A_N}{A^*}\right)^{\beta_1} + A_N \left(\frac{\eta}{\rho-\mu}\right) \right] \left(\frac{A}{A_N}\right)^{\beta_2} + \\ &\quad + \frac{\delta M}{1-\beta_1} \left(\frac{A}{A^*}\right)^{\beta_1} + A \left(\frac{\eta}{\rho-\mu}\right) \\ &= \left[M - \delta M \frac{1}{1-\beta_1} \left(\frac{A_N}{A^*}\right)^{\beta_1} - A_N \left(\frac{\eta}{\rho-\mu}\right) \right] \left(\frac{A}{A_N}\right)^{\beta_2} + \\ &\quad + \frac{\delta M}{1-\beta_1} \left(\frac{A}{A^*}\right)^{\beta_1} + A \left(\frac{\eta}{\rho-\mu}\right) \quad \text{for } A > A_N \end{aligned}$$

B.2 Proposition 3.3

To prove this proposition one should look for the fixed point solution for $f(x) = x$, where

$$f(x) = M \frac{\rho - \mu}{\eta} \left(\frac{1}{\beta_2 - 1} \right) \left[\beta_2 - \delta \frac{\beta_2 - \beta_1}{1 - \beta_1} \left(\frac{x}{A^*} \right)^{\beta_1} \right] \quad (\text{B.2.1})$$

Note that $f'(x) > 0$ and $f''(x) \leq 0$. At A^* , $f(x)$ takes the following value

$$\begin{aligned} f(A^*) &= M \frac{\rho - \mu}{\eta} \left(\frac{1}{\beta_2 - 1} \right) \left[\beta_2 - \delta \frac{\beta_2 - \beta_1}{1 - \beta_1} \right] \\ &> M \frac{\rho - \mu}{\beta_2 - 1} \left[\beta_2 - \frac{\beta_2 - \beta_1}{1 - \beta_1} \right] = A^* \end{aligned} \quad (\text{B.2.2})$$

Given that $f'(x) > 0$, $f''(x) \leq 0$ and $f(A^*) > A^*$ it follows that there is a unique fixed point $A_N > A^*$ such that $f(A_N) = A_N$.

B.3 Strategies under sophisticated belief: three governments

B.3.1 Continuation value function

Equation (3.19) can be rearranged as

$$\begin{aligned} \frac{1}{2}\sigma^2 A^2 V_{c,1}^{S''}(A) + \mu A V_{c,1}^{S'}(A) - (\rho + \lambda) V_{c,1}^S(A) \\ = -\delta[A + \lambda V_2^S(A)] \quad \text{for } A \geq A_{S,1} \end{aligned} \quad (\text{B.3.1.1})$$

The solution for the homogenous part is given by

$$V_{c,1h}^S(A) = k_2 A^{\beta_2}$$

Suppose that the particular solution takes the form $V_{c,1p}^S(A) = w_1 A^{\beta_1} + w_2 A$.

Substitute the conjectured form and its first two derivatives, $\beta_1 w_1 A^{\beta_1-1} + w_2$ and $(\beta_1 - 1) \beta_1 w_1 A^{\beta_1-2}$ into (B.3.1.1)

$$\begin{aligned} \frac{1}{2}\sigma^2 A^2 (\beta_1 - 1) \beta_1 w_1 A^{\beta_1-2} + \mu A (\beta_1 w_1 A^{\beta_1-1} + w_2) + \\ - (\rho + \lambda) (w_1 A^{\beta_1} + w_2 A) = -\delta \left[A \left(1 + \frac{\lambda}{\rho - \mu} \right) + \lambda \frac{M}{1 - \beta_1} \left(\frac{A}{A_{S,2}} \right)^{\beta_1} \right] \end{aligned}$$

and solve for the undetermined coefficients w_1 and w_2

$$\begin{aligned} \left[\frac{1}{2}\sigma^2 (\beta_1 - 1) \beta_1 + \mu \beta_1 - \rho \right] w_1 - \lambda w_1 &= -\lambda \delta \frac{M}{1 - \beta_1} \left(\frac{1}{A_{S,2}} \right)^{\beta_1} \\ [\mu - (\rho + \lambda)] w_2 &= -\delta \left(1 + \frac{\lambda}{\rho - \mu} \right) \end{aligned}$$

The solution $V_{c,1p}^S(A)$ verifies (B.3.1.1) if the following coefficients are set

$$\begin{aligned} w_1 &= \frac{\delta M}{1 - \beta_1} \left(\frac{1}{A_{S,2}} \right)^{\beta_1} \\ w_2 &= \frac{\delta}{(\rho - \mu)} \end{aligned}$$

The continuation value function is then given by

$$\begin{aligned} V_{c,1}^S(A) &= kA^{\beta_2} + \frac{\delta M}{1 - \beta_1} \left(\frac{A}{A_{S,2}} \right)^{\beta_1} + \frac{\delta}{(\rho - \mu)} A = \quad (\text{B.3.1.2}) \\ &kA^{\beta_2} + \delta V_2^S(A) \end{aligned}$$

and solving (B.3.1.2) subject to the value-matching condition $V_{c,1}^S(A_{S,1}) = \delta M$ one can derive

$$\begin{aligned} k &= \delta \left\{ M \left[1 - \frac{1}{1 - \beta_1} \left(\frac{A_{S,1}}{A_{S,2}} \right)^{\beta_1} \right] - \frac{A_{S,1}}{(\rho - \mu)} \right\} A_{S,1}^{-\beta_2} \\ &= \delta \left\{ M - \left[\frac{M}{1 - \beta_1} \left(\frac{A_{S,1}}{A_{S,2}} \right)^{\beta_1} + \frac{A_{S,1}}{(\rho - \mu)} \right] \right\} A_{S,1}^{-\beta_2} \\ &= \delta [M - V_2^S(A_{S,1})] A_{S,1}^{-\beta_2} \end{aligned}$$

and

$$\begin{aligned} V_{c,1}^S(A) &= \left\{ \delta M \left[1 - \frac{1}{1 - \beta_1} \left(\frac{A_{S,1}}{A_{S,2}} \right)^{\beta_1} \right] - \frac{\delta}{(\rho - \mu)} A_{S,1} \right\} \left(\frac{A}{A_{S,1}} \right)^{\beta_2} + \quad (\text{B.3.1.3}) \\ &+ \frac{\delta M}{1 - \beta_1} \left(\frac{A}{A_{S,2}} \right)^{\beta_1} + \frac{\delta}{(\rho - \mu)} A \\ &= \delta \left\{ [M - V_2^S(A_{S,1})] \left(\frac{A}{A_{S,1}} \right)^{\beta_2} + V_2^S(A) \right\} \end{aligned}$$

Note that (B.3.1.1) is solved just appending a value matching condition. Here, I do not need to impose a smooth-matching condition to guarantee the optimality

of $A_{S,1}$ because I are taking it as given and optimally determined maximizing $V_1^S(A)$.

B.3.2 Value function

Equation (3.23) can be restated as

$$\begin{aligned} & \frac{1}{2}\sigma^2 A^2 V_0^{S''}(A) + \mu A V_0^{S'}(A) - (\rho + \lambda) V_0^S(A) \\ & = - \left\{ A + \lambda \delta \left[(M - V_2^S(A_{S,1})) \left(\frac{A}{A_{S,1}} \right)^{\beta_2} + V_2^S(A) \right] \right\} \quad \text{for } A \geq A_{S,0} \end{aligned} \quad (\text{B.3.2.1})$$

The solution to the homogenous part is standard

$$V_{0_h}^S(A) = k_2 A^{\beta_2}$$

Guessing for the particular solution to (B.3.2.1) one should be more careful and consider that $V_{c,1}^S(A)$ contains the A^{β_2} term. This means that there may be a potential problem with the conjectured functional form of the solution due to resonance. Suppose then that the particular solution takes the form

$$V_{0_p}^S(A) = q_1 A + q_2 A^{\beta_1} + q_3 A^{\beta_2} \log A + q_4 A^{\beta_2}$$

Substitute it and its first two derivatives into (B.3.2.1)

$$\begin{aligned} V_{0_p}^{S'}(A) &= q_1 + \beta_1 q_2 A^{\beta_1-1} + q_3 \beta_2 A^{\beta_2-1} \log A + q_3 A^{\beta_2-1} + q_4 \beta_2 A^{\beta_2-1} \\ V_{0_p}^{S''}(A) &= \beta_1 (\beta_1 - 1) q_2 A^{\beta_1-2} + q_3 \beta_2 (\beta_2 - 1) A^{\beta_2-2} \log A + \\ & \quad + q_3 \beta_2 A^{\beta_2-2} + q_3 (\beta_2 - 1) A^{\beta_2-2} + q_4 \beta_2 (\beta_2 - 1) A^{\beta_2-2} \end{aligned}$$

The guessed solution verifies (B.3.2.1) if the following parameter are set

$$\begin{aligned}
q_1 &= \left(\frac{\eta}{\rho - \mu} \right) \\
q_2 &= \delta \frac{M}{1 - \beta_1} A_{S,2}^{-\beta_1} \\
q_3 &= - \frac{\lambda \delta \left\{ M \left[1 - \frac{1}{1 - \beta_1} \left(\frac{A_{S,1}}{A_{S,2}} \right)^{\beta_1} \right] - \frac{A_{S,1}}{(\rho - \mu)} \right\} A_{S,1}^{-\beta_2}}{\frac{1}{2} \sigma^2 (2\beta_2 - 1) + \mu} \\
&= - \frac{\lambda \delta [M - V_2^S(A_{S,1})] A_{S,1}^{-\beta_2}}{\frac{1}{2} \sigma^2 (2\beta_2 - 1) + \mu} \\
q_4 &= 0
\end{aligned}$$

The general solution is then given by

$$\begin{aligned}
V_0^S(A) &= k_2 A^{\beta_2} + \left(\frac{\eta}{\rho - \mu} \right) A + \delta \frac{M}{1 - \beta_1} \left(\frac{A}{A_{S,2}} \right)^{\beta_1} + \quad (\text{B.3.2.2}) \\
&\quad - \lambda \delta \frac{M - V_2^S(A_{S,1})}{\frac{1}{2} \sigma^2 (2\beta_2 - 1) + \mu} \left(\frac{A}{A_{S,1}} \right)^{\beta_2} \log A
\end{aligned}$$

At the critical amenity value, $A_{S,0}$, the value-matching and smooth-pasting conditions respectively require $V_0^S(A_{S,0}) = M$ and $V_0^{S'}(A_N) = 0$. Solving the system

$$\left\{ \begin{array}{l}
k_2 A_{S,0}^{\beta_2} + \left(\frac{\eta}{\rho - \mu} \right) A_{S,0} + \delta \frac{M}{1 - \beta_1} \left(\frac{A_{S,0}}{A_{S,2}} \right)^{\beta_1} + \\
- \lambda \delta \frac{M - V_2^S(A_{S,1})}{\frac{1}{2} \sigma^2 (2\beta_2 - 1) + \mu} \left(\frac{A_{S,0}}{A_{S,1}} \right)^{\beta_2} \log A_{S,0} = M \\
k_2 \beta_2 A_{S,0}^{\beta_2 - 1} + \left(\frac{\eta}{\rho - \mu} \right) + \delta \frac{M \beta_1}{1 - \beta_1} \left(\frac{A_{S,0}}{A_{S,2}} \right)^{\beta_1} + \\
- \lambda \delta \frac{M - V_2^S(A_{S,1})}{\frac{1}{2} \sigma^2 (2\beta_2 - 1) + \mu} \left(\frac{A_{S,0}}{A_{S,1}} \right)^{\beta_2} \frac{(1 + \beta_2 \log A_{S,0})}{A_{S,0}} = 0
\end{array} \right.$$

yields

$$k_2 = A_{S,0}^{-\beta_2} \left\{ M - \left(\frac{\eta}{\rho - \mu} \right) A_{S,0} - \delta \frac{M}{1 - \beta_1} \left(\frac{A_{S,0}}{A_{S,2}} \right)^{\beta_1} + \right. \\ \left. + \lambda \delta \frac{M - V_2^S(A_{S,1})}{\frac{1}{2}\sigma^2(2\beta_2 - 1) + \mu} \left(\frac{A_{S,0}}{A_{S,1}} \right)^{\beta_2} \log A_{S,0} \right\}$$

Plugging $P_{0,1} = \lambda \delta \frac{M - V_2^S(A_{S,1})}{\frac{1}{2}\sigma^2(2\beta_2 - 1) + \mu} \left(\frac{1}{A_{S,1}} \right)^{\beta_2} > 0$ and $P_{0,0} = k_2$ into (B.3.2.2), (3.26)

and (3.27) are finally derived.

B.4 Proposition 3.6

Following Grenadier and Wang (2007) I prove these two propositions by induction logic. It can be easily proved using results provided in the three governments model that $A_{S,I-1} > A_{S,I}$, $V_{c,I-1}^S(A) < V_{c,I}^S(A)$ and $V_{I-1}^S(A) < V_I^S(A)$. Assume now, for a generic $1 \leq i \leq I-1$, that $A_{S,i} > A_{S,i+1}$, $V_{c,i}^S(A) < V_{c,i+1}^S(A)$ and $V_i^S(A) < V_{i+1}^S(A)$. If our conjecture is correct $A_{S,i-1} > A_{S,i}$, $V_{c,i-1}^S(A) < V_{c,i}^S(A)$ and $V_{i-1}^S(A) < V_i^S(A)$ must hold for the same i .

Equation (3.30) and the boundary conditions (3.31) and (3.32) can be used to characterize $V_i^S(A)$ as the function expressing the value of an asset paying a dividend equal to $\{A + \lambda V_{c,i+1}^S(A)\}$ and a strike price M when the time trigger $A_{S,i}$ has been hit. This asset resembles to a standard American put option. I can use the same arguments for $V_{i-1}^S(A)$. Comparing the two assets note that the only difference is in the dividend paid as $V_{c,i+1}^S(A) > V_{c,i}^S(A)$ by assumption. Provided that the first option is paying an higher dividend it should then be exercised later. This implies that $A_{S,i-1} > A_{S,i}$ and that $V_{i-1}^S(A) < V_i^S(A)$ being lower the option value for the second asset.

I characterize now by the same logic $V_{c,i}^S(A)$ that in fact can be seen as the function representing the value of an asset paying a dividend equal to $\{\delta A + \lambda V_{c,i+1}^S(A)\}$ and δM as strike price when the time trigger $A_{S,i}$ has been hit (use equation (3.28) and $V_{c,i}^S(A_{S,i}) = \delta M$). It is easy to see that the value of this asset is equivalent to $\delta V_i^S(A) + (1 - \delta) \lambda V_{c,i+1}^S(A)$. The same holds for $V_{c,i-1}^S(A)$ which is equivalent to $\delta V_{i-1}^S(A) + (1 - \delta) \lambda V_{c,i}^S(A)$. By the result proved above $\delta V_i^S(A) > \delta V_{i-1}^S(A)$. Being by assumption $V_{c,i}^S(A) < V_{c,i+1}^S(A)$ and $A_{S,i} > A_{S,i+1}$ it follows that $(1 - \delta) \lambda V_{c,i+1}^S(A) > (1 - \delta) \lambda V_{c,i}^S(A)$. Hence, comparing the two options $V_{c,i-1}^S(A) < V_{c,i}^S(A)$.

Finally, by proposition 3.3 $A_N = A_{S,1} > A_{S,2} = A^*$ and by proposition 3.6, being $A_{S,i}$ decreasing in i , $A_{S,0} > A_{S,I-1} = A_N = A_{S,1}$. It follows that $A_{S,0} > A_{S,1} > A_{S,2}$.

B.5 Strategies under sophisticated belief: I governments

B.5.1 Continuation value function

I solve for the continuation value function by the backward induction solution concept. Set $i = I - (j + 1)$ and suppose that for $j = 1, 2, \dots, I - 1$, is given by

$$V_{c,i+1}^S(A) = V_{c,I-j}^S(A) = \delta V(A) + \sum_{n=0}^{j-1} Q_{I-j,n} (\log A)^n A^{\beta_2} \quad (\text{B.5.1.1})$$

where $Q_{I-j,n}$ are parameters to be determined. To verify that (B.5.1.1) is the appropriate continuation value function I first check if it holds for the government $I - 2$. In this case

$$V_{c,I-1}^S(A) = \frac{\delta M}{1 - \beta_1} \left(\frac{A}{A^*} \right)^{\beta_1} + \frac{\delta}{\rho - \mu} A + Q_{I-1,0} A^{\beta_2} \quad (\text{B.5.1.2})$$

Solving $V_{c,I-1}^S(A)$ subject to $V_{c,I-1}^S(A_{S,I-1}) = \delta M$ for $Q_{I-1,0}$ yields

$$Q_{I-1,0} = \delta \left\{ M \left[1 - \frac{1}{1 - \beta_1} \left(\frac{A_{S,I-1}}{A^*} \right)^{\beta_1} \right] - \frac{A_{S,I-1}}{\rho - \mu} \right\} (A_{S,I-1})^{-\beta_2} \quad (\text{B.5.1.3})$$

Plugging (B.5.1.3) into (B.5.1.2)

$$\begin{aligned} V_{c,I-1}^S(A) &= \frac{\delta M}{1 - \beta_1} \left(\frac{A}{A^*} \right)^{\beta_1} + \frac{\delta}{\rho - \mu} A + \\ &+ \delta \left\{ M \left[1 - \frac{1}{1 - \beta_1} \left(\frac{A_{S,I-1}}{A^*} \right)^{\beta_1} \right] - \frac{A_{S,I-1}}{\rho - \mu} \right\} (A_{S,I-1})^{-\beta_2} = \\ &= \delta \left\{ [M - V(A_{S,I-1})] \left(\frac{A}{A_{S,I-1}} \right)^{\beta_2} + V(A) \right\} \end{aligned} \quad (\text{B.5.1.4})$$

knowing that $A_{S,I-1} = A_N = A_{S,1}$ and comparing $V_{c,I-1}^S(A)$ with (3.21) it follows that (B.5.1.1) is verified.

Second, if our conjecture is correct then (B.5.1.1) must hold also for $i+1 = I-j$. By induction then $V_{c,i+2}^S(A) = V_{c,I-j+1}^S(A)$. Plugging $V_{c,I-j}^S(A)$, its two first derivatives and $V_{c,I-j+1}^S(A)$ into (3.28)

$$\begin{aligned} & \left[\frac{1}{2} \sigma^2 \beta_1 (\beta_1 - 1) + \mu \beta_1 - \rho \right] \frac{\delta M}{1 - \beta_1} \left(\frac{A}{A^*} \right)^{\beta_1} + \delta A - \lambda \delta V(A) + \quad (\text{B.5.1.5}) \\ & + \left[\frac{1}{2} \sigma^2 \beta_2 (\beta_2 - 1) + \mu \beta_2 - (\rho + \lambda) \right] \sum_{n=0}^{j-1} Q_{I-j,n} (\log A)^n A^{\beta_2} + \end{aligned}$$

$$\begin{aligned} & + \frac{1}{2} \sigma^2 \sum_{n=0}^{j-1} n Q_{I-j,n} (\log A)^{n-1} A^{\beta_2} \left(2\beta_2 - 1 + \frac{n-1}{\log A} \right) + \\ & + \mu \left[\frac{\delta A}{\rho - \mu} + \sum_{n=0}^{j-1} n Q_{I-j,n} (\log A)^{n-1} A^{\beta_2} \right] - \rho \frac{\delta A}{\rho - \mu} + \\ & \quad + \lambda \left[\delta V(A) + \sum_{n=0}^{j-1} Q_{I-j+1,n} (\log A)^n A^{\beta_2} \right] \\ & = \frac{1}{2} \sigma^2 \sum_{n=0}^{j-1} n Q_{I-j,n} (\log A)^{n-1} A^{\beta_2} \left(2\beta_2 - 1 + \frac{n-1}{\log A} \right) + \\ & + \mu \sum_{n=0}^{j-1} n Q_{I-j,n} (\log A)^{n-1} A^{\beta_2} + \lambda \sum_{n=0}^{j-1} Q_{I-j+1,n} (\log A)^{n-1} A^{\beta_2} = 0 \end{aligned}$$

Now, group the terms by $(\log A)^k A^{\beta_2}$ for $k = 0, 1, \dots, j-1$. To satisfy (B.5.1.5) all the coefficients for each $(\log A)^k A^{\beta_2}$ must be null. It follows

$$\begin{aligned} & \frac{\sigma^2}{2} [(2\beta_2 - 1)(k+1)Q_{I-j,k+1} + (k+2)(k+1)Q_{I-j,k+2}] + \quad (\text{B.5.1.6}) \\ & + \mu(k+1)Q_{I-j,k+1} + \lambda Q_{I-j+1,k} = 0 \end{aligned}$$

Rearrange (B.5.1.6)

$$Q_{I-j,k+1} = \gamma \left[\frac{\sigma^2}{2}(k+2)Q_{I-j,k+2} + \frac{\lambda Q_{I-j+1,k}}{(k+1)} \right] \quad (\text{B.5.1.7})$$

where $\gamma = - \left[\frac{\sigma^2}{2}(2\beta_2 - 1) + \mu \right]^{-1}$.

By conjecture (B.5.1.1) and $Q_{I-1,1} = 0$ it follows that $Q_{I-j,k} = 0$ for $k \geq j$.

Solving the recursive (B.5.1.7) yields

$$Q_{I-j,k} = \gamma \frac{\lambda}{k} \left[Q_{I-j+1,k-1} + \sum_{s=0}^{j-k-2} \left(\gamma \frac{\sigma^2}{2} \right)^s Q_{I-j+1,k+s} \prod_{t=0}^s (k+t) \right] \quad (\text{B.5.1.8})$$

for $k = 1, 2, \dots, j-1$. Note that by continuity of $V_{c,i+1}^S(A)$ I may append $V_{c,i+1}^S(A_{S,i+1}) = \delta M$ to (B.5.1.1) and solve for $Q_{I-j,0}$

$$\begin{aligned} Q_{I-j,0} &= \delta \left\{ M \left[1 - \frac{1}{1-\beta_1} \left(\frac{A_{S,I-j}}{A^*} \right)^{\beta_1} \right] - \frac{A_{S,I-j}}{\rho - \mu} \right\} (A_{S,I-j})^{-\beta_2} \quad (\text{B.5.1.8}) \\ &\quad - \sum_{n=1}^{j-1} Q_{I-j,n} (\log A_{S,I-j})^n \\ &= \delta [M - V(A_{S,I-j})] (A_{S,I-j})^{-\beta_2} - \sum_{n=1}^{j-1} Q_{I-j,n} (\log A_{S,I-j})^n \end{aligned}$$

where $A_{S,I-j} = A_{S,i+1}$ is the optimal time trigger for $i+1 = I-j$ that can be determined maximizing the value function $S_{i+1}(A) = S_{I-j}(A)$.

B.5.2 Value function

I proceed in this section as above. First suppose that for $j = 1, 2, \dots, I$, $V_{i+1}^S(A) = V_{I-j}^S(A)$ takes the following functional form

$$V_{I-j}^S(A) = A \frac{\eta}{\rho - \mu} + \delta \frac{M}{1 - \beta_1} \left(\frac{A}{A^*} \right)^{\beta_1} + \sum_{n=0}^{j-1} P_{I-j,n} (\log A)^n A^{\beta_2} \quad (\text{B.5.2.1})$$

where $P_{I-j,n}$ are parameters to be determined. To verify that (B.5.2.1) is the correct conjecture I check if it holds for the government $I - 1$. Solving $V_{I-1}^S(A)$ subject to $V_{I-1}^S(A_{S,I-1}) = M$ for $P_{I-1,0}$ yields

$$P_{I-1,0} = \left\{ M \left[1 - \frac{\delta}{1 - \beta_1} \left(\frac{A_{S,I-1}}{A^*} \right)^{\beta_1} \right] - A_{S,I-1} \frac{\eta}{\rho - \mu} \right\} (A_{S,I-1})^{-\beta_2} \quad (\text{B.5.2.2})$$

Substituting (B.5.2.2) into $V_{I-1}^S(A)$ it turns out that $V_{I-1}^S(A) = V^N(A) = V_1^S(A)$ where as I know $A_{S,I-1} = A_N = A_{S,1}$. Now, I must check if (B.5.2.1) is the appropriate form also for the generic government $i + 1 = I - j$.

I plug $V_{i+1}^S(A) = V_{I-j}^S(A)$, $V_{I-j}^{S'}(A)$, $V_{I-j}^{S''}(A)$ and $V_{c,i+2}^S(A) = V_{c,I-j+1}^S(A)$ into (3.30)

$$\begin{aligned} & \left[\frac{1}{2} \sigma^2 \beta_1 (\beta_1 - 1) + \mu \beta_1 - \rho \right] \frac{\delta M}{1 - \beta_1} \left(\frac{A}{A^*} \right)^{\beta_1} + A - A \eta \frac{\rho + \lambda}{\rho - \mu} \quad (\text{B.5.2.3}) \\ & + \left[\frac{1}{2} \sigma^2 \beta_2 (\beta_2 - 1) + \mu \beta_2 - (\rho + \lambda) \right] \sum_{n=0}^{j-1} P_{I-j,n} (\log A)^n A^{\beta_2} + \\ & + \frac{1}{2} \sigma^2 \sum_{n=0}^{j-1} n P_{I-j,n} (\log A)^{n-1} A^{\beta_2} \left(2\beta_2 - 1 + \frac{n-1}{\log A} \right) + \\ & + \mu \left[A \frac{\eta}{\rho - \mu} + \sum_{n=0}^{j-1} n P_{I-j,n} (\log A)^{n-1} A^{\beta_2} \right] - \lambda \frac{\delta M}{1 - \beta_1} \left(\frac{A}{A^*} \right)^{\beta_1} + \\ & + \lambda \left[\delta V(A) + \sum_{n=0}^{j-1} P_{I-j+1,n} (\log A)^n A^{\beta_2} \right] = \\ & \frac{1}{2} \sigma^2 \sum_{n=0}^{j-1} n P_{I-j,n} (\log A)^{n-1} A^{\beta_2} \left(2\beta_2 - 1 + \frac{n-1}{\log A} \right) + \\ & + \mu \sum_{n=0}^{j-1} n P_{I-j,n} (\log A)^{n-1} A^{\beta_2} + \lambda \sum_{n=0}^{j-1} P_{I-j+1,n} (\log A)^n A^{\beta_2} = 0 \end{aligned}$$

Grouping terms again by $(\log A)^k A^{\beta_2}$ for $k = 0, 1, \dots, j$ and imposing to all the coefficients to be null it follows

$$\begin{aligned} \frac{\sigma^2}{2} [(2\beta_2 - 1)(k+1)P_{I-j,k+1} + (k+2)(k+1)P_{I-j,k+2}] + \\ \mu(k+1)P_{I-j,k+1} + \lambda P_{I-j+1,k} = 0 \end{aligned} \quad (\text{B.5.2.4})$$

Comparing (B.5.2.4) with (B.5.1.6) yields

$$P_{I-j,k} = Q_{I-j,k} \quad (\text{B.5.2.5})$$

for $k = 1, \dots, j-1$. Last, let determine $P_{I-j,0}$. Rearrange (B.5.1.8) as follows

$$\sum_{n=1}^{j-1} Q_{I-j,n} (\log A_{S,I-j})^n (A_{S,I-j})^{\beta_2} = \delta [M - V(A_{S,I-j})] - Q_{I-j,0} (A_{S,I-j})^{\beta_2} \quad (\text{B.5.2.6})$$

I know that in $A_{S,I-j}$ the following relationship holds

$$\begin{aligned} V_{I-j}^S(A_{S,I-j}) = A_{S,I-j} \frac{\eta}{\rho - \mu} + \delta \frac{M}{1 - \beta_1} \left(\frac{A_{S,I-j}}{A^*} \right)^{\beta_1} + \\ + \sum_{n=0}^{j-1} P_{I-j,n} (\log A_{S,I-j})^n (A_{S,I-j})^{\beta_2} = M \end{aligned} \quad (\text{B.5.2.7})$$

Given that $P_{I-j,k} = Q_{I-j,k}$ for $k = 1, \dots, j-1$, I can substitute (B.5.2.6) into (B.5.2.7) and rearrange as follows

$$\begin{aligned} A_{S,I-j} \frac{\eta}{\rho - \mu} + \delta \frac{M}{1 - \beta_1} \left(\frac{A_{S,I-j}}{A^*} \right)^{\beta_1} + P_{I-j,0} (A_{S,I-j})^{\beta_2} + \\ + \delta [M - V(A_{S,I-j})] - Q_{I-j,0} (A_{S,I-j})^{\beta_2} = M \end{aligned}$$

and after a bit of algebra I determine

$$P_{I-j,0} = Q_{I-j,0} + (1 - \delta) \left(M - \frac{A_{S,I-j}}{\rho + \lambda - \mu} \right) (A_{S,I-j})^{-\beta_2} \quad (\text{B.5.2.8})$$

Appending the standard boundary conditions to $V_{I-j}^S(A)$ one can derive finally

$$A_{S,I-j} = A_{S,i+1}.$$

Appendix C

Appendix to Chapter 4

C.1 Strategies under sophisticated belief

Rearrange equation (4.11) as

$$\begin{aligned} \frac{1}{2}\sigma^2 A^2 \frac{\partial^2 V^S(A; \tilde{A})}{\partial A^2} + \mu A \frac{\partial V^S(A; \tilde{A})}{\partial A} - (\rho + \lambda) V^S(A; \tilde{A}) = & \quad (\text{C.1.1}) \\ - \left[A \left(1 + \frac{\lambda\delta}{\rho - \mu} \right) + \lambda\delta \frac{M}{1 - \beta_1} \left(\frac{A}{\tilde{A}} \right)^{\beta_1} \right] & \quad \text{for } A \geq \tilde{A} \end{aligned}$$

The solution to the homogenous part is¹

$$V^S(A; \tilde{A}) = k_2 A^{\beta_2}$$

Suppose that the particular solution takes the form $V_p^S(A; \tilde{A}) = c_1 A^{\beta_1} + c_2 A$.

Plug this candidate $\frac{\partial V_p^S(A; \tilde{A})}{\partial A} = \beta_1 c_1 A^{\beta_1 - 1} + c_2$ and $\frac{\partial^2 V_p^S(A; \tilde{A})}{\partial A^2} = (\beta_1 - 1) \beta_1 c_1 A^{\beta_1 - 2}$

¹The solution should have the form $V_h^S(A; \tilde{A}) = k_1 A^{\alpha_2} + k_2 A^{\beta_2}$ where k_1 and k_2 are coefficients to be specified and $\alpha_2 > 0$ and $\beta_2 < 0$ are the roots of the characteristic equation $\sigma^2 \beta(\beta - 1)/2 + \mu\beta - (\rho + \lambda)$. As $A \rightarrow \infty$, the value of the option to harvest ($V_h^S(A; \tilde{A})$) should go to zero. Since $\alpha_2 > 0$ then k_1 must be zero because if not $V_h^S(A; \tilde{A}) \rightarrow \infty$ as $A \rightarrow \infty$.

into (C.1.1)

$$\begin{aligned} & \frac{1}{2}\sigma^2 A^2 (\beta_1 - 1) \beta_1 c_1 A^{\beta_1 - 2} + \mu A (\beta_1 c_1 A^{\beta_1 - 1} + c_2) + \\ & - (\rho + \lambda) (c_1 A^{\beta_1} + c_2 A) = - \left[A \left(1 + \frac{\lambda \delta}{\rho - \mu} \right) + \lambda \delta \frac{M}{1 - \beta_1} \left(\frac{A}{\tilde{A}} \right)^{\beta_1} \right] \end{aligned}$$

and solve for the undetermined coefficients

$$\begin{aligned} \left[\frac{1}{2}\sigma^2 (\beta_1 - 1) \beta_1 + \mu \beta_1 - \rho \right] c_1 - \lambda c_1 &= -\lambda \delta \frac{M}{1 - \beta_1} \left(\frac{1}{\tilde{A}} \right)^{\beta_1} \\ [\mu - (\rho + \lambda)] c_2 &= - \left(1 + \frac{\lambda \delta}{\rho - \mu} \right) \end{aligned}$$

The candidate solution satisfies (C.1.1) if the following parameter are set

$$\begin{aligned} c_1 &= \frac{\delta M}{1 - \beta_1} \left(\frac{1}{\tilde{A}} \right)^{\beta_1} \\ c_2 &= \left(1 + \frac{\lambda \delta}{\rho - \mu} \right) \frac{1}{(\rho + \lambda - \mu)} = \frac{\eta}{\rho - \mu} \end{aligned}$$

The particular solution is then

$$V_p^N(A) = \frac{\delta M}{1 - \beta_1} \left(\frac{A}{\tilde{A}} \right)^{\beta_1} + A \frac{\eta}{\rho - \mu}$$

The general solution is given by the sum of $V_h^S(A; \tilde{A})$ and $V_p^S(A; \tilde{A})$

$$V^N(A) = k_2 A^{\beta_2} + \frac{\delta M}{1 - \beta_1} \left(\frac{A}{\tilde{A}} \right)^{\beta_1} + A \frac{\eta}{\rho - \mu} \quad (\text{C.1.2})$$

At the critical amenity value, A_N , the value-matching and smooth-pasting conditions respectively require $V^S(H(\tilde{A}); \tilde{A}) = M$ and $\frac{\partial V^S(H(\tilde{A}); \tilde{A})}{\partial A} = 0$. Solving the

system

$$\begin{cases} k_2(H(\tilde{A}))^{\beta_2} + \frac{\delta M}{1-\beta_1} \left(\frac{H(\tilde{A})}{\tilde{A}}\right)^{\beta_1} + H(\tilde{A})\frac{\eta}{\rho-\mu} = M \\ k_2\beta_2(H(\tilde{A}))^{\beta_2-1} + \delta M\frac{\beta_1}{1-\beta_1} \left(\frac{H(\tilde{A})}{\tilde{A}}\right)^{\beta_1} \frac{1}{H(\tilde{A})} + \frac{\eta}{\rho-\mu} = 0 \end{cases}$$

yields

$$\begin{aligned} H(\tilde{A}) &= \left[\frac{\beta_2}{\beta_2-1} - \delta \frac{\beta_2-\beta_1}{(1-\beta_1)(\beta_2-1)} \left(\frac{H(\tilde{A})}{\tilde{A}}\right)^{\beta_1} \right] M \left(\frac{\rho-\mu}{\eta}\right) \\ V^S(H(\tilde{A}); \tilde{A}) &= \frac{M}{1-\beta_2} \left[1 - \delta \left(\frac{H(\tilde{A})}{\tilde{A}}\right)^{\beta_1} \right] \left(\frac{A}{H(\tilde{A})}\right)^{\beta_2} \\ &\quad + \frac{\delta M}{1-\beta_1} \left(\frac{A}{\tilde{A}}\right)^{\beta_1} + A \left(\frac{\eta}{\rho-\mu}\right) \quad \text{for } A > \tilde{A} \end{aligned}$$

Finally imposing the intra-personal steady-state condition $H(A_S) = A_S$ and $V^S(A; A_S) = V^S(A)$ the solution follows

$$\begin{aligned} A_S &= \left[\frac{\beta_2}{\beta_2-1} - \delta \frac{\beta_2-\beta_1}{(1-\beta_1)(\beta_2-1)} \right] M \left(\frac{\rho-\mu}{\eta}\right) \\ V^S(A) &= M \left[\frac{1-\delta}{1-\beta_2} \left(\frac{A}{A_S}\right)^{\beta_2} + \frac{\delta}{1-\beta_1} \left(\frac{A}{A_S}\right)^{\beta_1} \right] + A \left(\frac{\eta}{\rho-\mu}\right) \end{aligned}$$

C.2 Pigovian taxation

The regulator's rule is given by

$$E(T^S) - \bar{T} = m \tag{C.2.1}$$

First, recall that $\mu < \sigma^2/2$ and that by (4.19) $E(T^S) = \frac{2}{2\mu-\sigma^2} \ln\left(\frac{A_S}{A_0}\right)$. Second,

If a pigovian tax is levied on M then

$$A_S^T = (1 - \Gamma^S) [(1 - \theta) A^{**} + \theta A^*]$$

Substitution into (C.2.1) yields

$$\begin{aligned}
\frac{2}{2\mu - \sigma^2} \ln \left(\frac{A_S^T}{A_0} \right) &= \bar{T} + m & (C.2.2) \\
\ln \left(\frac{A_S^T}{A_0} \right) &= \left(\mu - \frac{\sigma^2}{2} \right) (\bar{T} + m) \\
(1 - \Gamma^S) A_S &= A_0 e^{\left(\mu - \frac{\sigma^2}{2} \right) (\bar{T} + m)} \\
\Gamma^S &= 1 - \frac{A_0}{A_S} e^{-\left(\frac{\sigma^2}{2} - \mu \right) (\bar{T} + m)}
\end{aligned}$$

We prove now that if Γ^S is determined not considering the behavioural failure then $E(T^S) < (\bar{T} + m)$. Suppose is given by

$$\Gamma^S = 1 - \frac{A_0}{A^*} e^{\left(\mu - \frac{\sigma^2}{2} \right) (\bar{T} + m)}$$

If this is the case then

$$\begin{aligned}
E(T^S) &= \frac{2}{2\mu - \sigma^2} \ln \left(\frac{\frac{A_0}{A^*} e^{-\left(\frac{\sigma^2}{2} - \mu \right) (\bar{T} + m)} A_S}{A_0} \right) \\
&= \frac{2}{2\mu - \sigma^2} \ln \left(\frac{A_S}{A^*} e^{-\left(\frac{\sigma^2}{2} - \mu \right) (\bar{T} + m)} \right) \\
&= \frac{2}{2\mu - \sigma^2} \left[- \left(\frac{\sigma^2}{2} - \mu \right) (\bar{T} + m) + \ln \left(\frac{A_S}{A^*} \right) \right] \\
&= (\bar{T} + m) + \frac{2}{2\mu - \sigma^2} \ln \left(\frac{A_S}{A^*} \right) < (\bar{T} + m)
\end{aligned}$$

Bibliography

- [1] Arguedas, C., G. Meijerink and D.P. van Soest, 2007. Biodiversity conservation, asymmetric information and the role of fixed costs. Tilburg, Tilburg University.
- [2] Arrow, K.J. and A.C. Fisher, 1974. Environmental preservation, uncertainty and irreversibility. *Quarterly Journal of Economics*, 88, 312-319.
- [3] Atje, R. and K. Roesad, 2004. Who should own Indonesia's forests?: Exploring the links between economic incentives, property rights and sustainable forest management. WPE 076. Jakarta, Indonesia: Centre for Strategic and International Studies. Available at <http://www.csis.or.id/papers/wpeO76>.
- [4] Bagnoli, M. and T. Bergstrom, 1989. Log-concave probability and its applications. CREST WP # 89-23, University of Michigan.
- [5] Baron, D. and R. Myerson, 1982. Regulating a monopolist with unknown costs. *Econometrica*, 50, 911-930.
- [6] Barro, R., 1999. Laibson meets Ramsey in the neoclassical growth model. *Quarterly Journal of Economics*. 114, 4, 1125-52.
- [7] Breccia, A. and H. Salgado-Banda, 2005. Competing or colluding in a sto-

- chastic framework. Birkbeck Working Papers in Economics and Finance 0504, Birkbeck, School of Economics, Mathematics & Statistics.
- [8] Brennan, M.J. and E.S. Schwartz, 1985. Evaluating natural resource investments. *Journal of Business*, 58, 135-157.
- [9] Brocas, I. and J.D. Carrillo, 2005. A theory of haste. *Journal of Economic Behavior and Organization* , 56(1), 1-23.
- [10] Brocas, I. and J.D. Carrillo, 2004. Entrepreneurial Boldness and Excessive Investment. *Journal of Economics and Management Strategy*, 13 (2), 321–350.
- [11] Brocas, I. and J.D. Carrillo, 1998. A theory of haste with applications to impulse buying and destruction of the environment. CEPR DP 2027, London.
- [12] Brunnschweiler, C. N. and E. H. Bulte, 2008. The natural resource curse revisited and revised: a tale of paradoxes and red herrings. *Journal of Environmental Economics and Management*, forthcoming.
- [13] Bulte, E., D.P. van Soest, G.C. van Kooten and R.A. Schipper, 2002. Forest Conservation in Costa Rica When Nonuse Benefits Are Uncertain but Rising. *American Journal of Agricultural Economics*, 84, 150– 60.
- [14] Clark, E., 2003. Pricing the cost of expropriation risk. *Review of International Economics*, 11, 412-422.
- [15] Clark, E., 1997. Valuing political risk. *Journal of International Money and Finance*, 16, 477-490.
- [16] Clarke, H.R. and W.J. Reed, 1989. The tree-cutting problem in a stochastic environment. *Journal of Economic Dynamics and Control*, 13, 569-595.

- [17] Conrad, J.M., 2000. Wilderness: options to preserve, extract, or develop. *Resource and Energy Economics*, 22 (3), 205-219.
- [18] Conrad, J.M., 1997. On the option value of old-growth forest. *Ecological Economics*, 22, 97-102.
- [19] Conrad, J.M., 1980. Quasi-option value and the expected value of information. *Quarterly Journal of Economics*, 95, 813-820.
- [20] Cox, D.R. and H.D. Miller, 1965. *The theory of stochastic process*. London: Chapman and Hall.
- [21] Crépin, A., 2005. Incentives for wetland creation. *Journal of Environmental Economics and Management*, 50, 598–616.
- [22] Dasgupta, P.S., P.J. Hammond and E.S. Maskin, 1979. The implementation of social choice rules: some results on incentive compatibility. *Review of Economic Studies*, 46, 185-216.
- [23] Dasgupta, P.S. and E.S. Maskin, 2005. Uncertainty and hyperbolic discounting. *American Economic Review*, 95 (4), 1290-1299.
- [24] Dixit, A.K. and R.S. Pindyck, 1994. *Investment under uncertainty*. Princeton University Press, Princeton, NJ.
- [25] Dixit, A.K., 1993. *The art of smooth pasting*. Chur, CH: Harwood Academic Publisher.
- [26] Di Corato, L., 2007. Optimal harvesting under imperfect intergenerational altruism. Mimeo, University of Padua.

- [27] Dosi, C. and M. Moretto, 1997. Pollution accumulation and firm incentives to accelerate technological change under uncertain private benefits. *Environmental and Resource Economics*, 10, 285-300.
- [28] Dosi, C. and M. Moretto, 1996. Toward green technologies: switching rules and environmental policies. *Rivista Internazionale di Scienze Economiche e Commerciali*, 43, 13-30.
- [29] Engel, E. and R. Fischer, 2008. Optimal resource extraction contracts under threat of expropriation. NBER Working Paper No. W13742. Available at SSRN: <http://ssrn.com/abstract=1086993>.
- [30] Ferraro, P.J. and A. Kiss, 2002. Direct payments for biodiversity conservation. *Science*, 298, 1718-1719.
- [31] Ferraro, P.J., 2001. Global habitat protection: limitations of development interventions and a role for conservation performance payments. *Conservation Biology*, 15 (4), 1-12.
- [32] FONAFIFO, 2000. El desarrollo del sistema Pago de Servicios Ambientales en Costa Rica. San José, Costa Rica: Fondo Nacional de Financiamiento Forestal (FONAFIFO).
- [33] Frederick, S., G. Loewenstein and T. O'Donoghue, 2002. Time discounting and time preference: a critical review. *Journal of Economic Literature*, 40 (2), 351-401.
- [34] Goeschl, T. and T. Lin, 2004. Endogenous information structures in environmental contracting. Staff Paper No. 479, Department of Agricultural and Applied Economics, University of Wisconsin-Madison.

- [35] Grenadier, S.R. and N. Wang, 2007. Investment under uncertainty and time-inconsistent preferences. *Journal of Financial Economics*, 84, 2–39.
- [36] Groves, T., 1973. Incentive in teams. *Econometrica*, 41, 617-631
- [37] Guesnerie, R. and J.J. Laffont, 1984. A complete solution to a class of Principal-Agent problems with an application to the control of a self-managed firm. *Journal of Public Economics*, 25, 329-369.
- [38] Guasch, J.L., J.J. Laffont and S. Straub, 2003. Renegotiation of concession contracts in Latin America. Policy Research Working Paper Series 3011, The World Bank, revised.
- [39] Hajzler, C., 2007. Expropriation of foreign direct investments, 1990-2006. Unpublished Paper, University of Western Ontario.
- [40] Harsanyi, J., 1977. Rational behaviour and bargaining equilibrium in games and social situations. Cambridge, Cambridge University Press.
- [41] Harris, C. and D. Laibson, 2004. Instantaneous gratification. Working paper, Harvard University.
- [42] Heal, G.M., 1998. Valuing the Future: economic theory and sustainability. Columbia University Press.
- [43] Harrison, M. J., 1985. Brownian motion and stochastic flow systems. New York: Wiley.
- [44] Henry, C., 1974. Investment decisions under uncertainty: the irreversibility effect. *American Economic Review*, 64, 1006-1012.
- [45] Hepburn, C.J. and P. Koundouri, 2007. Recent advances in discounting: Implications for forest economics. *Journal of Forest Economics*, 13, 169-189.

- [46] Hepburn, C.J., 2003. Resource collapse and hyperbolic discounting. Discussion Paper 159, Oxford University, Department of Economics.
- [47] Insley, M. and K. Rollins, 2005. On solving the multirotational timber harvesting problem with stochastic prices: a linear complementarity formulation. *American Journal of Agricultural Economics*, 87 (3), 735–755.
- [48] Kassari, I. and P. Lasserre, 2004. Species preservation and biodiversity value: a real options approach. *Journal of Environmental Economics and Management*, 48(2), 857-879.
- [49] Kobrin, S. J., 1984. Expropriations as an attempt to control foreign firms in LDCs: trends from 1960 to 1979. *International Studies Quarterly*, vol. 28, 3, 329-348.
- [50] Laffont, J.J. and D. Martimort, 2002. *The theory of incentives: the Principal-Agent model*. Princeton University Press.
- [51] Laibson, D., 1996. Hyperbolic discounting, undersaving, and savings policy. NBER Working Paper No. 5635.
- [52] Laibson, D., 1997. Golden eggs and hyperbolic discounting. *Quarterly Journal of Economics*, 62 (2), 443-478.
- [53] Loewenstein, G. and D. Prelec, 1992. Anomalies in intertemporal choice: evidence and an interpretation. *Quarterly Journal of Economics*, 107, 573-598.
- [54] Luttmer, E. and T. Mariotti, 2003. Subjective discounting in an exchange economy. *Journal of Political Economy*, 111, 959–989.

- [55] Mirrlees, J.A., 1971. An exploration in the theory of optimum income taxation. *Review of Economic Studies*, 38, 175-208.
- [56] Motte, E., J.M. Salles and L. Thomas, 2004. Information asymmetry and incentive policies to farmers for conserving biodiversity in forested areas in developing countries. Montpellier, LAMETA, mimeo.
- [57] McDonald, R. and D. Siegal, 1986. The value of waiting to invest. *Quarterly Journal of Economics*, 101, 707-728.
- [58] Merton, R.C., 1998. Applications of option-pricing theory: twenty-five years later. *American Economic Review*, 88, 323-349.
- [59] Minor, M. S., 1994. The demise of expropriation as an Instrument of LDC Policy, 1980-1992. *Journal of International Business Studies*, vol. 25, 1, 177-188.
- [60] Mahajan, A., 1990. Pricing expropriation risk. *Financial Management*, 19, 77-86.
- [61] Moretto, M. and G. Rossini, 1996. Profit sharing regulation and repeated bargaining with a shut-down option. *Review of Economic Design*, 2, 1, 339-368.
- [62] Moretto, M. and G. Rossini, 1995. The shut-down option and profit sharing. *Journal of Comparative Economics* 21, 154-186.
- [63] Moretto, M. and P. Valbonesi, 2007. Firm regulation and profit sharing: a real option approach. *The B.E. Journal of Economic Analysis and Policy*, vol.7, 1 (Topics), Article 56.
- [64] Nash, J., 1950. The bargaining problem. *Econometrica*, vol. 18, 2, 155-162.

- [65] O'Donoghue, T. and M. Rabin, 1999. Doing it now or later. *American Economic Review*, 89, 103-124.
- [66] OECD, 1999. Handbook of incentive measures for biodiversity: design and implementation, OECD, Paris.
- [67] Pagiola, S., P. Agostini , J. Gobbi , C. de Haan, M. Ibrahim, E. Murgueitio, E. Ramírez, M. Rosales and J.P. Ruíz, 2004. Paying for biodiversity conservation services in agricultural landscapes. Environment Department Paper No. 96. Washington: World Bank.
- [68] Pagiola, S., N. Landell-Mills and J. Bishop, 2002. Making market-based mechanisms work for forests and people. In Pagiola, S., J. Bishop and N. Landell-Mills (eds.), *Selling forest environmental services: market-based mechanisms for conservation and development*. London: Earthscan.
- [69] Phelps, E. S. and R.A. Pollak, 1968. On second-best national saving and game equilibrium growth. *Review of Economic Studies*, 35, 185-199.
- [70] Pollak, R.A., 1968. Consistent Planning. *Review of Economic Studies*, 35 (2), 201-208.
- [71] Reed, W.J., 1993. The decision to conserve or harvest old-growth forest. *Ecological Economics*, 8, 45–69.
- [72] Rubinstein, A., 2003. Economics and psychology? The case of hyperbolic discounting. *International Economic Review* 44 (4), 1207-1216.
- [73] Salanie, F. and N. Treich, 2006. Over-savings and hyperbolic discounting. *European Economic Review*, 50 (6), 1557-1570.

- [74] Salzman, J.E., 2005. Creating markets for ecosystem services: notes from the field. *New York University Law Review*, vol. 80, no. 6. Available at SSRN: <http://ssrn.com/abstract=796772>.
- [75] Schnitzer, M., 1999. Expropriation and control rights: a dynamic model of foreign direct investment. *International Journal of Industrial Organization*, vol. 17, 8, 1119-1137.
- [76] Shogren, J.F., 2007. Behavior in forest economics. *Journal of Forest Economics*, 12, 233-235.
- [77] Smith, R. and J. Shogren, 2002. Voluntary incentive design for endangered species protection. *Journal of Environmental Economics and Management*, 43, 169-187.
- [78] Smith, R., 1995. The Conservation Reserve Program as a least-cost land retirement mechanism. *American Journal of Agricultural Economics*, 77, 93-105.
- [79] Strotz, R.H., 1956. Myopia and inconsistency in dynamic utility maximization. *Review of Economic Studies*, 23, 165-80.
- [80] Tomz, M. and M. Wright, 2007. Sovereign theft: theory and evidence about default and expropriation. Paper presented at the Conference on Populism in Natural Resources, held at the Kennedy School of Government, Harvard University, 1-2 November 2007. Available at http://ksghome.harvard.edu/~whogan/Populism_Nat_Res/Populism_Agenda_files/TomzWrightExpropriationDefault.pdf
- [81] White, B., 2002. Designing voluntary agri-environmental policy with hidden

information and hidden action: a note. *Journal of Agricultural Economics*, 53, 353-360.

[82] World Bank, 2000. Ecomarkets Project: project appraisal document. Report No. 20434-CR. Washington: World Bank.

[83] Wu, J. and B.A. Babcock, 1996. Contract design for the purchase of environmental goods from agriculture. *American Journal of Agricultural Economics*, 78, 935-945.

[84] http://en.wikipedia.org/wiki/Old_growth_forest