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DOTTORATO DI RICERCA IN ECONOMIA E MANAGEMENT
CICLO XIX

Marital Fertility and Exogenous Constraints on Child Quality:

a Theoretical and Empirical Approach

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Sintesi

Lo scopo di lavoro consiste nello studio teorico ed empirico degli effetti di vincoli esogeni nella qualità dei figli sulla fertilità coniugale. La teoria neoclassica sulla fertilità, che vede come pionieri Becker ed altri autorevoli studiosi, tratta i figli alla stregua di un qualunque "bene durevole": i coniugi decidono di diventare genitori non solo perché traggono utilità dal numero di figli messi al mondo (quantità); ma pure da alcune loro caratteristiche "desiderabili" (qualità). L'interazione tra queste due dimensione dà origine al noto trade-off tra quantità e qualità nella "domanda" di figli. Ad ogni modo, è del tutto ovvio che i genitori non sono proprietari dei loro figli e che non possono disporre della loro prole a loro piacimento. A titolo d'esempio, si può pensare al *corpus* di leggi che regolamenta l'età minima in cui è consentito cominciare a lavorare e agli anni di istruzione obbligatoria. Inoltre, è possibile che pure le aspettative del gruppo di pari siano percepite dalla coppia vincolanti tanto quanto una legge vera e propria. Queste leggi ed aspettative rappresentano un vincolo stringente sulla qualità dei figli che i coniugi devono forzatamente considerare nelle loro decisioni di fertilità.

Il primo capitolo presenta un approccio teorico alla fertilità coniugale che generalizza il classico modello di trade-off tra quantità e qualità tramite l'introduzione dell'ipotesi che le coppie si trovino di fronte ad un vincolo stringente ed esogenamente determinato sulla qualità minima dei figli; viene ipotizzato, inoltre, che il non avere figli possa essere una soluzione ottimale. La decisione sul numero di figli è perciò un processo a due stadi: nel primo, i coniugi confrontano il loro benessere nelle due ipotesi (con e senza figli); nel secondo, qualora aver figli sia preferibile a non averne, la coppia ne decide il numero. Data la soglia esogena di qualità minima, la coppia massimizza una funzione di utilità *piecewise* sotto un vincolo di bilancio. Fin tanto che "qualità minima" (tutte quelle caratteristiche stabilite da entità estranee alla coppia) e "qualità discrezionale" (ovvero la qualità propriamente detta: le caratteristiche desiderate dai genitori) non sono perfette sostitute, l'esistenza di una soglia minima riduce tanto lo spazio in cui avere figli è ottimale quanto il numero di figli effettivamente messi al mondo. Inoltre, il trade-off tra qualità e quantità è viene esasperato rispetto al caso standard.

Il secondo capitolo propone una stima in forma ridotta dell'effetto prodotto da cambiamenti nella spesa marginale in qualità minima sulla fertilità completa. Infatti, la spesa in qualità minima è suscettibile di modifiche più frequenti rispetto al quelle del solo livello della soglia minima e tali cambiamenti sono più facilmente individuabili. Una conveniente controparte econometrica della funzione di utilità *piecewise* è rappresentata, nel caso in esame, da un modello ad ostacolo (*hurdle)* costituito da una regressione log-log complementare e da una Poisson. Il dataset (cross-sezionale) finale è stato costruito assemblando diversi strati del dataset longitudinale britannico NCDS mentre la spesa marginale in qualità minima è approssimata dal rapporto alunni/insegnanti nella scuola primaria osservato dalla coppia nell'anno delle nozze. Vi è una forte evidenza empirica che la spesa in qualità minima ha effetti fortemente depressivi tanto nella probabilità di "saltare l'ostacolo" e diventare perciò genitori quanto nel numero di figli effettivamente messo mondo dalla coppia. D'altro canto, il reddito osservato ha un effetto positivo sulla fertilità ma di entità alquanto ridotta: infatti, un incremento del reddito induce su nove coppie su dieci una riduzione della probabilità di diventare genitori: queste coppie, quindi, tendono a cedere alla suggestione di una maggiori consumi privati. La lettura congiunta di questi due risultati permette di sviluppare nuove interpretazioni a proposito della tendenza a bassi tassi di fertilità osservata in gran parte dei paesi sviluppati.

Abstract

This work aims to study the effect of exogenous constraints on child quality on marital completed fertility, both from a theoretical and empirical point of view. The neoclassical theory of fertility, pioneered by Becker et al., considers children in the same way as other "durable goods": the spouses decide become parents, not only because they get utility from the number of children they bear (quantity), but also from some "desirable" child characteristics (quality). The interaction between these two dimensions gives origin to the well-known quantity/quality trade-off in the demand for children. However, it is obvious that parents do not "own" their children and cannot dispose of their offspring at their will. For instance, legislations on compulsory education and regulating the minimum age to admission to work are almost universally widespread. Furthermore, the expectations of their group of peers may be perceived by the spouses as binding as the law itself. These laws and expectations represent a binding constraint on child quality that parent are compelled to take into account in their childbearing decisions.

The first chapter proposes a theoretical approach that generalizes the classic quantity/quality trade-off model by introducing the hypothesis that couples face an exogenously determined quality constraint and taking explicitly into account that remaining childless can be optimal. The childbearing decision consists therefore in a two-stages process: in the first step, the spouses evaluate whether they can be better-off with or without children; in the second one, provided that they decide to bear children, they decide their optimal number and quality. Given the exogenous minimum quality threshold, the couple maximizes then a piecewise utility function under a budget constraint. As long as "minimum quality" (i.e. those characteristics "decided" by an entity external to the couple) and "discretionary quality" (i.e. those child characteristics that parents desire) are no perfect substitutes, the existence of an exogenous minimum quality threshold reduces both the space where becoming parents is the optimal choice and the overall number of children born to the couple. Furthermore, the quantity/quality trade-off is significantly stronger than in the standard case.

The second chapter proposes a reduced form estimation of the effect of changes in the marginal expenditures on minimum quality on the total number of children born to the couple. In fact, changes in marginal expenditures take place more often than changes in the threshold only and are easier to locate. A suitable econometric counterpart of the piecewise utility function is, in the case under examination, the complementary log-log – Poisson hurdle model. The cross-sectional dataset used in this work is built from several sweeps of the British National Child Development Study (NCDS) , while the marginal expenditures on minimum quality are proxied by the pupil/teacher ratio in primary schools observed by the partners at the time they got married. There is strong evidence that high marginal expenditures have a strongly negative effect both on the probability of becoming parents and on the overall number of children born to the couple. In turn, the overall effect of the observed income is positive but of small magnitude: in fact, a higher income reduces the predicted probability of becoming parents for nine couples out of ten (i.e. they "surrender to the suggestion" of higher own-consumption). These two results suggests therefore new insights into the interpretation of the low fertility trends experienced in most developed countries.

Marital Fertility and Exogenous Constraints on Child Quality

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CHAPTER ONE
Marital Fertility and Exogenous Constraints on Child Quality

A THEORETICAL APPROACH

Abstract. The neoclassical "Beckerian" theory of fertility considers children as "durable goods" and parents get utility both from their quantity (their number) and their quality (their human capital). However, it cannot be neglected that parents cannot dispose of their children as they do with other durables: parents are compelled by law to provide their children with at least a basic care and also their peers may exert a strong influence in determining which child characteristics are desirable. This work generalizes the classic quality/quantity trade-off model by hypothesizing that parents have to take into account a binding exogenously determined quality ("mandatory quality") constraints in their child-bearing decisions. The couple maximizes a piecewise utility function under a budget constraint: the existence of a minimum quality threshold reduces both the space where becoming parents is the optimal choice and the overall number of children born to the couple. As long as mandatory and discretionary quality are less than perfect substitutes, the quantity/quality trade-off is significantly stronger than in the standard case. These results provide therefore an alternative (but not exclusive) interpretation of the low fertility rates experienced in most developed countries.

1.Introduction

During the 60s authors such as Becker, Lancaster, Mincer and others laid the foundations of a research branch that a decade later was baptized "New Economics of the Family". The basic idea was that the family is a sort of “transformation unit”: the couple uses its own time and market goods and services as inputs to produce more basic commodities according to its specific "production function" and to its preferences. In fact, the real arguments of the "family utility function " are not market goods and services: these are merely inputs used to produce a wide range of market and non market commodities such as social status, self confidence, the feeling of being part of a community, entertainment, nutrition and so on. In this framework, market goods are similar to health care expenditures: they do not yield satisfaction directly but are inputs used to produce "good health”. These authors claimed that the tools developed by the economic theory could be profitably used to gain a better understanding of the family behavior. In this context, they widened the use of the consumer theory to interpret phenomena such us marriage, fertility outcomes, and so on.

The neoclassic approach to the fertility behavior consists basically in the application of models of consumer demand. Parents constitute a single and indivisible decision unit which maximizes an utility function subject to a budget constraint and children are nothing but an argument of the utility function. Such an approach in intrinsically static, since the relevant unit of time coincide with parents’ lifetime: preferences, prices, and income are supposed to be stable over parents’ life cycle and there exist no uncertainty issues.

In a series of papers dating back to the 60s and 70s, Gary Becker et al. hypothesized that the demand for children would involve a qualitative dimension along the quantitative one. At that time, the main problem consisted in finding a plausible explanation for the observed negative correlation between income and fertility without having to rely on the assumption that children are inferior goods. The inspiration came from the hedonic price models pioneered in the early 50s by Theil [1952] and Houthakker [1952] . The main characteristic of this kind of problems is that quality and quantity enter multiplicatively in the budget constraint: the interaction between these two dimension, known as quantity/quality trade-off, is responsible of several particular features of the demand for children. The most peculiar feature of the quantity/quality trade-off model is that the shadow price of quantity is an increasing function of quality and vice versa: as long as the income elasticity of the demand for quality exceeds the one for quantity, whenever the income rises, a substitution effect toward per-child quality and against quantity takes place.

This branch of literature remains often voluntarily vague and "agnostic" about the concept of quality itself. Basically, child quality should be understood as a synthetic index (i.e. a scalar) of several characteristics of the offspring which are able to yield utility to the parents such as mental and physical health; intelligence; beauty; sex; capacity of high life-time earnings; and adhesion to specific religious, ethical or religious values[1]. Therefore, parents use their own time and market goods to endow their children with similar characteristics according to their own "recipe " or idea of quality. Many, if not all of these characteristics are themselves hardly measurable abstract constructs . On the other hand, from an operational point of view, the characteristic approach to child quality has not been widely used and child quality is usually interpreted at the light of the human capital theory (Hotz, Klerman & Willis [1993]). Furthermore, in most empirical application, quality is proxied with education related variables such as scores in standardized tests (see chapter two for references). In fact, all in all, the operational definition of child quality is much more an issue of empirical application than a theoretical hinder.

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[1] These characteristics could separately enter directly into the utility function but, as Willis [1987] points out, the gain in generality would be offset both by its overly complex mathematical and theoretical tractability and by the lack of data.

Aim of this work on the is to provide a generalization of Becker's quantity/quality trade-off model of childbearing decision through the introduction of an exogenous constraint on child quality.

It is unquestionable that parents do not "own" their children are not free to dispose of them at their will. Parents are rather compelled to provide their offspring with *at least* a basic care and to look after their education and health: in other words, parents face expenditures to endow each and every child with certain basic characteristics or *minimum quality*. By way of example, one may think of compulsory education. The foundation of this obligation is usually the law but also the expectations of the group of peers –which may be perceived as binding as the law itself- may play an important role in this context.

What really matters is that parents can chose neither the "composition" of this basic quality nor influence its level: if they want to have children, they have to commit to provide them with characteristics that *someone else* has decided (for instance, the mastery of the subjects constituting the compulsory education at a determined level). As no car lacking of some basic safety requirements is allowed to circulate, so there cannot be a child lacking of these basic characteristics or skills.

Furthermore, along Becker's line of argumentation, parents invest on quality in order to improve their children's "attractiveness": if they want to achieve an improvement from the basic level (which, in Beckerian models, is implicitly set to zero), they have to invest on further quality. We call such an edge over the minimum *discretionary quality.* This, and not the mere total quality (the sum of mandatory minimum quality and discretionary quality), is the most relevant concept of quality entering into the utility function. This does not necessarily mean that the minimum quality does not yield any utility to the parents, but simply that a marginal increase of discretionary quality yields to the parents more utility than a marginal increase of the mandatory quality: in fact, while it is sure that parents get utility from the discretionary quality (which is *chosen*), there is no *a priori* reason to assume that the same holds for the minimum quality (which is something that they put up with). Furthermore, we will see that under not particularly restrictive assumptions, parents' marginal utility of a further child is positively affected by the presence of a minimum quality threshold, even when the minimum quality does not enter in the utility function at all: that means that the utility of "quantity" takes already into account that some quality is already embedded in each and every child.

As a matter of fact, there exist also couples who decide not to bear children: the here-proposed model expands the basic structure of this family on models by taking explicitly into account -for the very first time in this kind of models- the possibility that renouncing to parenthood can be a rational and admissible choice.

Section 2 describes thoroughly the model: the standard hypotheses and the new ones; when having children is the optimal choice and the comparative statics.

Section 3 summarizes the main results and concludes.

2. The model

This section introduces a generalization of Becker's model on the interaction between the quantity and quality of children by hypothesizing that parents are compelled to provide their offspring with at least certain amount of quality. The nature and the level of this mandatory child care is exogenously determined: couples take this threshold as given and are not able to influence it by any means.

The introduction of the minimum quality threshold allows to take explicitly into account situations where having no child is the best choice and makes the model more consistent with the reality, where parents cannot dispose of their children at their will. Moreover the minimum quality threshold allows brand new insights and results on the interaction between the demand of quantity and *total* quality of children.

2.1 Hypotheses

Both the standard set of hypotheses common to this family of models -which may be already known to the reader- and the new ones peculiar to the here proposed model are thoroughly discussed in the following two paragraphs.

2.1.1 Standard hypotheses

a) The couple acts as single agent: there is no bargaining game between partners since they share the same attitudes and preferences[1].

b) The couple is stable: couples do not divorce and each partner have children only within marriage[2].

c) Perfect information[3]. Parents behave as they had perfect knowledge of their economic environment. It is also assumed that there will be no major unexpected price or income shock during the marriage.

d) Allocation of resources. The couple can allocate its resources on "child services"[4] and goods not related to children. Goods not related to children (Z) can be thought as the level of consumption of the couple, i.e. their own standard of life. It is also assumed that both "child services" and parent's own consumption are normal goods.

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[1] This is logically equivalent to the hypothesis of a single sexless agent.

[2] This hypothesis is tightly related with the previous one: even if though it may seem to be quite restrictive, what really matters here is that partners make long-term plans and "act as one" for a fairly long period of time. Weiss & Willis [1985] relax the last two hypotheses and consider mother and father as two different agents who cooperate to "produce" a child and whose relationship may terminate.

[3] Becker, Landes & Michael [1977] stress the role of uncertainty and the lack of perfect information to explain the occurrence of divorce.

[4] Since children have a quality dimension, parents have to jointly decide their number *N and* quality *Q;* the resulting bundle NQ is called "child services" in this branch of literature.

e) Impartiality. Parents do not discriminate among children. All children are endowed with the same innate ability[5] and parents are committed to guarantee to each of them the same care: they will invest the same amount of resources on each child without taking into account factors like sex or birth order[6].

f) Fertility. Parents are able to bear the exact number of children they desire, i.e. no serious health problem compromises their fertility[7].

g) Timing and spacing. The model is static and timing and spacing of births are not taken into account[8]: parents decide jointly the number of children they want to have and their total quality at the beginning of their marriage *and do not modify their choice*[9]. Even with those simplifying

[5] Becker & Tomes [1984] consider the possibility that children of the same couple may be genetically differently endowed and develop a model where parents aim to compensate these differences.

[6] Ben-Porath & Welch [1976] take into consideration the sex composition of the offspring while several other empirical works try to estimate whenever sex and birth order matter or not (see next chapter for further references).

[7] On the other hand, fertility is considered a stochastic process in several dynamic frameworks: the idea behind is that parents cannot directly decide to have children or not at a given time; they can rather control their fertility indirectly through contraception, coital frequency, healthy life-style and so on. See Newman [1988] and Miller & Hotz [1984]

[8] It is necessary to point that reduced-form models of birth spacing are popular subjects of several empirical works. See Hotz & Miller [1984] and Heckman & Walker [1990].

[9] We explicitly exclude the possibility that parents may change their mind over time: arguably, most stable couples have a clear picture of the size of the family they want and usually get exactly the number of children they decided earlier in life. Having excluded health problems, they can achieve this result by controlling their fertility through contraceptive methods and, eventually, by resorting to abortion as well. Moreover, we do not consider that expenditures on children may change over time because of factors like the experience they gain with older children: while it is certainly true that under some circumstances experience may serve as cost-reducing and effectiveness-improving device in quality-producing activities, it must be also recognized that usually the core of these expenditures is not significantly affected by experience. This is particularly true if we take a look at the ongoing trend on all developed countries, where a growing share of quality-related services are directly bought in the market and not produced inside the household. Pre-school education is a good example: the percentage of 3 and 4 years old children

assumptions, the model takes into account all the salient facets of the problem related to compete fertility decisions while maintaining a tractable mathematics.

2.1.2 Specific hypotheses

New hypotheses specific to this model are separately introduced in this subsection. The total quality of children described by Becker is divided into two components: mandatory and discretionary quality. This operation represents a distinct departure from the standard approach indeed: it becomes therefore crucial to carefully examine these new hypotheses to avoid misreading that could occur because of similarity of the technical terminology involved.

h) Minimum quality threshold. There exists an *exogenous* minimum quality threshold that parents must take into account when they decide the optimal allocation of their resources. Parents are not completely free to decide how much to invest on quality: they have to consider legal constraints as well as the social expectations of the group of peers.

By no means children can be assimilated to other goods parents do not “own” their children and cannot freely dispose of them. Furthermore couples cannot neglect their children at least a *basic care*. The amount (and the nature) of this basic care goes far beyond the mere survival level (almost) everywhere. Nowadays, laws establishing the minimal age until which parents have to support their children, the mandatory years of full-time education, and minimum age to admission to any type of work or employment are luckily significantly widespread. Obviously, legislations sharply differ among countries, ranging from a minimal set of rights of the child in underdeveloped countries to much more pervasive disciplines in richer ones: notably, mandatory years of education and work legislation are two of the most representative

attending nursery schools in UK rose from 21% in 1970 to 65% in 2003. Whether pre-school education is positively or negatively related to child quality is still a debated issue.

features in this sense. Health-related measures like compulsory inoculations also belong to the set of requirements contributing to the minimum quality.

Even though governments normally subsidize at least a share of the costs related to the achievement of the required minimum quality, a variable portion of these costs still remains on the couple's back. We can think of countries where child labor is allowed or tolerated: raising the mandatory years of full-time education may represent an economic loss for the family, even if schooling is fully subsidized. Furthermore, this example makes clear that the mandatory quality does not necessarily coincide with parents' concept of quality, who may consider "quality" an earlier child's capability of producing an income.

We may think of all these legal constraints as an *objective threshold*: regardless their preferences in terms of number and quality of children, the couple must be ready to afford at least the *private* costs related to the achievement of this minimum quality (if they want to bear children at all) [10].

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[10] On the other hand, the couple may face a different set of *exogenous* parameters determining the minimum quality threshold: expectations of the social network to which the couple belongs may provide a sort of *perceived threshold*. To this extent, the couple is influenced –or better: it feels truly constrained- in its decisions by the predominant customs of its community: the Indian caste system provides a very good example of such a context. Although legally abolished, the caste system is still widely practiced: being member of a particular caste is by no means a free choice but it shapes one's customs and behaviors indeed; this holds in terms of education of children as well.

More generally, it may happen that a couple perceives as a duty to provide its children with *at least* a certain level of schooling and –broadly speaking- with a particular education. We may think, for instance, at the percentage of young people continuing their studies after the mandatory years of schooling: in most developed countries, children of middle and upper-classes parents stay at school longer than legally required. Not only schooling but also dressing codes, religious habits, etc. play a major role in this context. Nevertheless, if a couple can choose (at least to some extent) its reference community, it could be very hard in practice to disentangle investment in quality that are truly *voluntary* from expenditures that are *perceived* as *compulsory* but actually derive from the choice of a particular reference. That means that, whenever the affiliation to specific community (and the consequent set of "duties"

i) Discretionary quality. Parents are no longer *entirely* free to choose the quality of their children, since a third subject (the State and/or the peers) is able to impose a constraint and, therefore, the couple can exert its free will only after having fulfilled this duty. We label therefore *discretionary quality* the amount of quality that parents decide to demand on top of the mandatory level. The sum of mandatory and discretionary quality is the total quality per child.

Given the total demanded quality $\widetilde{Q}$, we assume that a couple draws more utility from a marginal increase due to the discretionary quality Q than from a rise of the mandatory quality $\bar{q}$[11].

According to Becker, reducing the number of children may be a good deal whether this allows parents to invest *sufficiently more* on quality, in order to make possible a *better* representation of their offspring in the following generation. In our context, parents who wish to improve the "attractiveness" of their children are compelled to invest on discretionary quality, since the mandatory quality represents now the new "level zero": therefore, if enhancing the

deriving from it) is not surely exogenous as in the case of a caste system, the exogenous minimum quality threshold is the one deriving from actual laws.

[11] This assumption is line with Becker's arguments on quality. In fact, in his "*Treatise on Family"*, Becker explains the role of quality in child-bearing decisions as follows: *"A reduction in the number of children born to a couple can increase the representation of their children in the next generation if this enables the couple to invest sufficiently more in the education, training, and 'attractiveness' of each child to increase markedly their probability of survival to reproductive age and the reproduction of each survivor".*

There is actually the possibility that the minimum quality may be no quality at all (or even harmful!) for the parents. For instance, if the teaching of a religion -in confessional terms - is compulsory, parents who are agnostic or not believer can find a "credolous" child somewhat less "attractive". Even the existence of a very basic compulsory education may be perceived as "damaging" whenever parents heavily count on child-labor. Nonetheless, cases of "negative utility" of the minimum quality will not be considered in this work, being the possibility that parents get utility only from the discretionary quality taken as the bottom limit.

quality of children from the minimum is *per se* valuable, an edge over the basic level generates a higher utility than an equal rise of the threshold.

Moreover, while minimum quality is by definition the same for each and every child, the *composition* of discretionary quality is not necessarily homogeneous among families. The existence of a compulsory minimum quality locks resources that parents could have employed otherwise (either for their own consumption or on different quality-related goods and services according to their *own* notion of quality) and, consequently, enforces homogeneity where parents wish heterogeneity. In short, mandatory quality and discretionary quality cannot be perfect substitutes[12].

2.2 The utility maximization problem

The couple's utility is represented by a piecewise function:

$$U = \begin{cases} v(Z) & \text{if } N = 0 \\ u(N, \breve{Q}, Z) & \text{if } N > 0 \end{cases} \tag{1}$$

where N[13] is the number of children and Z represents parents' own consumption respectively. The second branch of the utility function depends on quality as follows:

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[12] As a matter of fact, a case where mandatory and discretionary quality are perfect substitutes would be a very fortunate one, since the introduction of a minimum threshold would induce –as we will see- no distortion in the allocation of resources, as long as the threshold is lower or at most equal to the level of quality the couple would have chosen anyway.

[13] A common feature of this family of models –and the present one is no exception- is that the number of children *N* can take any positive real value: even though children are no divisible good in real life, this simplification is without a doubt sensible, since it allows the use of standard calculus techniques; moreover, the insights on the demand of children obtainable through this approach have proved to be very significant. It can be also assert that the costs in terms of complexity of the analysis and loss of generalization implied by more sophisticated maximization algorithms

$$\breve{Q} = Q + \sigma\overline{q} \quad (2)$$

where Q is the discretionary quality (the only choice variable), $\overline{q}$ is the mandatory quality and $\sigma \in [0,1)$ represents the level fungibility in terms of utility between mandatory and discretionary quality[14].

The total demand of quality *per child* $\tilde{Q}$ is therefore the sum of the discretionary and mandatory quality: $\tilde{Q} = Q + \overline{q}$

While a couple may decide not to have children, it is sensible to exclude that *Z* may be zero in either case:

$$\begin{cases} N \in \mathbb{R}_0^+ \\ Q \in \mathbb{R}_0^+ \\ Z \in \mathbb{R}^+ \end{cases}$$

It is also assumed that each branch of the utility function is "well behaved", i.e. both exhibit monotonicity, convexity and global non-satiation.

The main features of this utility function are:

1) it takes explicitly into account that the optimal consumption bundle may not include child-services at all, a brand new feature in this family of models;

2) given the total level $\tilde{Q}$, it allows for different marginal utility of mandatory and discretionary quality;

do not seem to pay off. In order to resolve problems of this nature, Mixed-Integer Nonlinear Programming (MINP) techniques are required. These problems are very hard to solve because they exhibit both the combinatorial nature of mixed integer programming and the difficulty proper to nonlinear programming. Furthermore, the utility function and *most* of the parameters have to be fully specified in order to apply the algorithms.

[14] This is an extremely flexible framework. In fact, if σ is zero, the minimum quality threshold is a sort of "subsistence level" *a la* Stone-Geary and the couple consider useful just the proportion of quality over the minimum; while if it approaches one, swapping discretionary quality for mandatory quality becomes increasingly less problematic.

In order to avoid situations where zero children is always the best option, we assume that the indirect utility of having children when the threshold is zero is greater than the indirect utility of not having any:

$$(3) \qquad W(\cdot)\Big|_{\overline{q}=0} < V(\cdot)\Big|_{\overline{q}=0}$$

where *W* and *V* are the indirect utility functions in the cases of zero and *N>0* children respectively.

The budget constraint is:

$$(4) \qquad p_n N + p_q \tilde{Q} + pN\tilde{Q} + p_z Z = Y$$ [15]

where:

- p_n is a component of the cost of children largely independent from the quality; it includes physical and psychological costs related to pregnancy and delivery (Becker & Lewis [1973]);
- p_q is a component of cost of quality largely independent from the number of children; it can be used to take into account possible "economies of scale" in the production of quality, since it includes factors like learning from parents[16], time spent preparing meals as well as second-hand stuff (clothes, books, toys, etc.);
- *p* represents costs that parents have to incur for each and every child (school and university fees, etc. and it is sometimes called "variable price of quality";
- p_z is the price-index of parents' own consumption;
- *Y* is the permanent income.

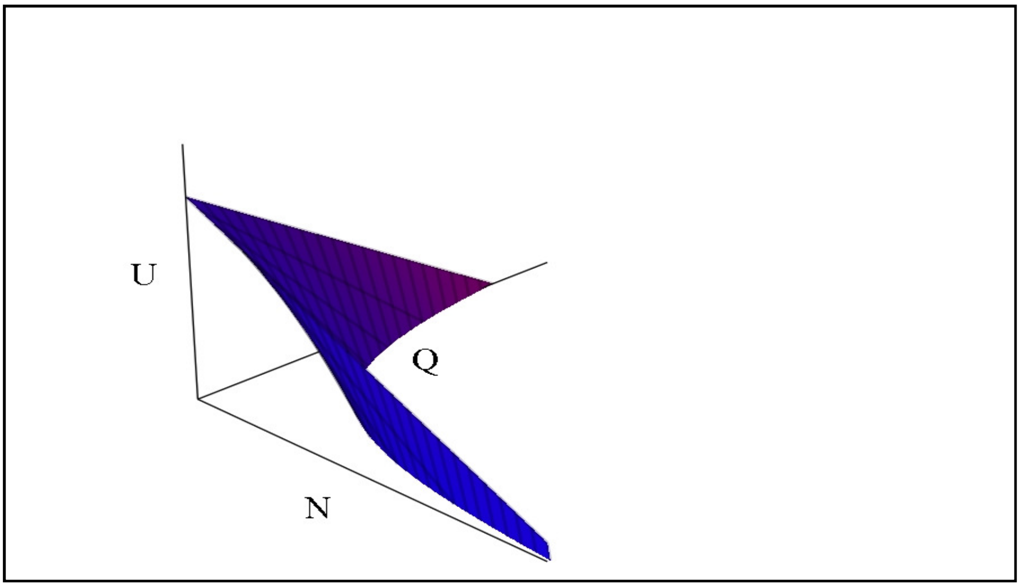
[15] It is obvious that the budget constraint collapses to $p_z Z = Y$ if $N = 0$, since quality does not enter in $v(\cdot)$.

[16] Here we are dealing with children learning from parents and not *vice versa*, so there is no contradiction with what we stated above about the neglectability of the parents' accumulation of experience in terms of reduction of rearing costs.

The most evident feature of this budget constraint is its non-linearity due to the component $pN\tilde{Q}$ which is responsible - as we will see soon - of many of the particular results we will obtain.

The couple then must maximize (1) subject to (4). The case where *N=0* brings the trivial solution $\left\{N^*, Q^*, Z^*\right\} = \left\{0, 0, Y/p_z\right\}$; we label the indirect utility in the case where the couple as no children as: $W(p_z, Y) = W^0$.

Now we have to establish under which conditions the solution is the one we have just seen and when it falls on the other branch of the utility function instead. The question we want to answer is: under which conditions will the couple bear children?



Plot 1: Budget Constraint for N>0 (Z kept constant)

2.2.1 Parenthood decision

Having supposed that equation (3) holds, we know that, whenever the minimum quality threshold is set to zero, the couple will bear children. In order to assess what the couple decides whenever $\overline{q} > 0$, we have to compare the indirect utility functions in the cases of *N=0* and *N>0*.

We already know that in the first case the indirect utility is W^0 and this quantity does not depend on $\overline{q}$. Let *V* be the indirect utility function when *N>0:*

$$(5) \qquad \mathrm{V} = \mathrm{V}\left(Y, p_n, p_q, p, p_z, \overline{q}\right)$$

We have that $\frac{\delta(V - W^0)}{\delta \overline{q}} = \frac{\delta V}{\delta \overline{q}}$. Applying the envelope theorem, we have:

$$(6) \qquad \frac{\delta V}{\delta \overline{q}} = \frac{\delta\Big(U(N^*, Q^*, Z^*) - \lambda^*(p_n N^* + p_q(Q^* + \overline{q}) + pN^*(Q^* + \overline{q}) + p_z Z^* - Y\Big)}{\delta \overline{q}}$$

Solving and simplifying (6) yields:

$$(7) \qquad \frac{\delta V}{\delta \overline{q}} = \sigma U_{Q^*} - \lambda^*\left(pN^* + p_q\right) = U_{Q^*}(\sigma - 1) < 0$$

where U_{Q^*} is the marginal utility of the discretionary quality at the optimum. Being the minimum quality threshold is a constraint, it is not surprising that couple's welfare decreases as $\overline{q}$ rises. The magnitude of this effect depends on the degree of substitutability between mandatory and discretionary quality, on the marginal utility of income λ^*, on the variable and fixed costs of quality p and p_q, and on the demanded number of children. That means that a couple willing to have a greater number of children and/or have a low income is more negatively affected by a rise of the threshold.

The second equality on the RHS shows that the loss of welfare is exactly the difference between the marginal utility of mandatory and discretionary quality: in fact, the resources needed to raise the minimum quality by the amount $d\overline{q}$ are detracted from the more appreciated discretionary quality[17].

[17] Even though the topic is outside the purposes of this study, it is interesting to notice that parents and State may have a different (and conflicting) focus regarding quality: while the latter is probably committed to increase the overall

Result 1: *A rise (fall) of the minimum quality threshold causes a decrease (increase) in couple's welfare. The welfare equals the difference between the marginal utility of mandatory and discretionary quality at the optimum.*

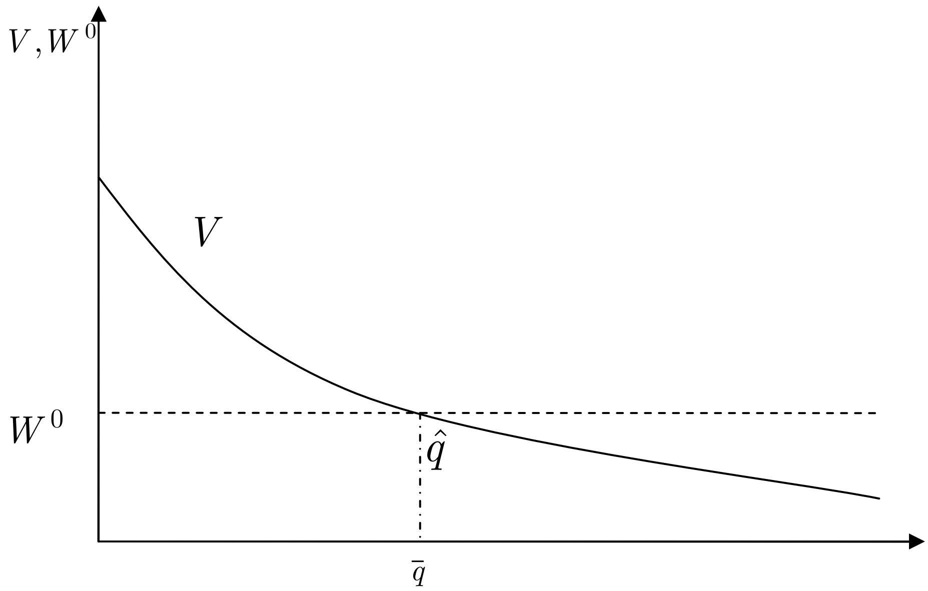


Figure 1: Indirect utility decreases with $\bar{q}$

We can summarize these findings:

level of quality $\tilde{Q}$ of each of its citizens, the formers are more interested in the edge that discretionary quality may provide to their own children (i.e. Q), a task that becomes more complicated as the minimum quality increases.

Result 2: T*here exists an* <u>*endogenously*</u> *determined level* $\hat{q}$[18] *representing the couple's maximum affordable (or critical) threshold. Everything else being equal, in* $\hat{q}$ *holds:* $V(\cdot)\big|_{\hat{q}} = W^0$ *: if* $\overline{q} < \hat{q}$*, the couple will be better off having children; if* $\overline{q} = \hat{q}$ *the couple is indifferent between having and not having children; if* $\overline{q} > \hat{q}$*, the couple won't have any.*

It is interesting to study how critical threshold $\hat{q}$ reacts to changes in income and prices. It is possible answer to these questions using the implicit function theorem. Let

$$(8) \qquad J\left(Y, p_n, p_q, p, p_z, \overline{q}\right)\big|_{\overline{q}=\hat{q}} = V\left(Y, p_n, p_q, p, p_z, \overline{q}\right)\big|_{\overline{q}=\hat{q}} - W(Y, p_z) = 0$$

which can be rewritten as:

$$(9) \qquad J\left(Y, p_n, p_q, p, p_z, \hat{q}(Y, p_n, p_q, p, p_z)\right) = 0$$

using the implicit function theorem we get:

$$(10) \qquad \frac{\delta \hat{q}}{\delta Y} = -\frac{\dfrac{\delta J}{\delta Y}}{\dfrac{\delta J}{\delta \hat{q}}} = -\frac{\dfrac{\delta V}{\delta Y} - \dfrac{\delta W}{\delta Y}}{\dfrac{\delta V}{\delta \hat{q}}} = \frac{\hat{\lambda} - \lambda_Z}{\hat{U}_{Q^*}(\sigma - 1)}$$

where $\hat{\lambda}$ and $\hat{U}_{Q^*}$ are the marginal utilities of income and discretionary quality respectively for *N>0* and $\overline{q} = \hat{q}$ and λ_Z is the marginal utility income for *N=0* (each measure is taken at the

[18] Figure 1 shows $V(\overline{q})$ such that $\dfrac{\delta V}{\delta^2 \overline{q}} > 0$: this result comes from the comparative statics analysis that will be performed in the next section.

optimum). Using Engel's aggregation and basic calculus[19], it can be shown that the sign of (10) depends exclusively on the relative magnitudes marginal utilities of *Z* for *N=0* and *N>0*. More precisely:

$$(11) \quad \begin{cases} \dfrac{\hat{\lambda} - \lambda_Z}{U_{Q^*}(\sigma - 1)} \leq 0 & \text{if} \quad u_Z(Z^*) \leq v_Z\left(\dfrac{Y}{p_z}\right) \\ \dfrac{\hat{\lambda} - \lambda_Z}{U_{Q^*}(\sigma - 1)} > 0 & \text{otherwise} \end{cases}$$

The actual sign of (10) can be empirically assessed but it is important to discuss right away why a rise of permanent income may lower the critical threshold. We have seen that if $\frac{\delta \hat{q}}{\delta Y} < 0$, then $u_Z(Z^*) \leq v_Z\left(Y/p_z\right)$: in order to grasp this result, we can think at particular goods in *Z,* the consume of which provides significantly more utility in absence of children (for instance a long cruise or time-consuming goods in general) and/or goods which cannot be afforded if the couple has children. Even if the negative signs holds, the difference between these marginal utilities will obviously quickly tend to zero as the consumption of *Z* rises so that the set of feasible thresholds will generally not collapse to $\bar{q} : \{\bar{q} = 0\}$ (i.e. $\hat{q} = 0$). This is the reason why we can expect very rich people to have children no matter the sign of (10).

Result 3: *The endogenous threshold $\hat{q}$ shifts in response to a change of income ; the direction of this shift depends on the difference between the marginal utilities of income for N>0 and N=0. In turn, that difference has the same sign of the one between the marginal utilities of Z in the two cases.*

Using analogous techniques, we obtain:

$$(12) \quad \frac{\delta \hat{q}}{\delta p} = -\frac{\frac{\delta J}{\delta p}}{\frac{\delta J}{\delta \hat{q}}} = -\frac{\frac{\delta V}{\delta p}}{\frac{\delta V}{\delta \hat{q}}} = \frac{\hat{\lambda}\hat{N}(\hat{Q} + \hat{q})}{\hat{U}_{Q^*}(\sigma - 1)} < 0$$

---

[19] Full derivation of this result can be found in appendix 1

$$(13) \qquad \frac{\delta \hat{q}}{\delta p_n} = -\frac{\dfrac{\delta J}{\delta p_n}}{\dfrac{\delta J}{\delta \hat{q}}} = \frac{\hat{\lambda}\hat{N}}{\hat{U}_{Q^*}(\sigma - 1)} < 0$$

$$(14) \qquad \frac{\delta \hat{q}}{\delta p_q} = -\frac{\dfrac{\delta J}{\delta p_q}}{\dfrac{\delta J}{\delta \hat{q}}} = \frac{\hat{\lambda}\left(\hat{Q} + \hat{q}\right)}{U_{Q^*}(\sigma - 1)} < 0$$

It is immediate to notice that:

Result 4: *A rise in any component of cost of children pushes the critical threshold to the left. The stronger effect is caused by a change in p, which measures the curvature of the budget constraint. Moreover, the closer substitutes are mandatory and discretionary quality, the greater is the magnitude of the effects.*

Equation (12) delivers an expected result: if variable costs of quality rise, parents' domain of choice where having children is the best choice reduces sharply. We will see that the demand of the child services $N\tilde{Q}$ declines with *p*. The overall magnitude of the effect depends then on the degree of substitutability between mandatory and discretionary quality and on the one between N and $\tilde{Q}$: if the inner allocation between N and $\tilde{Q}$ changes in favor of N, the effect is stronger.

Equation (13) says that whatever lowers p_n (for instance, whenever medical standards improve making therefore pregnancy and delivery less dangerous or the introduction of child allowances)also allows couples to afford higher thresholds.

Equation (14) can be also satisfactorily interpreted: recall that p_q refers to expenditures on goods and services where there is joint consumption by the children and learning from parents is a typical example of such an item. In order to obtain a given level of quality, less cultivated parents have to put more effort (i.e. spend more) than better educated ones: equation

(14) tells us that everything else being equal, more erudite parents can therefore afford higher thresholds.

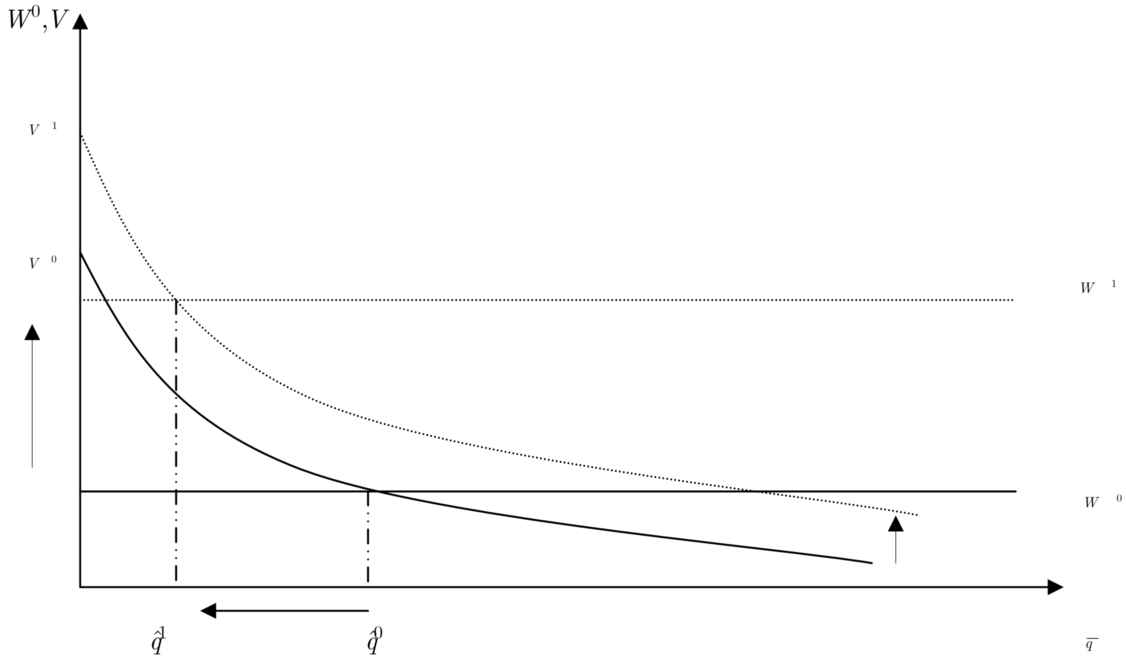


Figure 2: Shift of $\hat{q}$ due to a rise of Y (for $\hat{\lambda} - \lambda_Z < 0$)

A further result is given by:

$$(15) \qquad \frac{\delta \hat{q}}{\delta p_z} = -\frac{\frac{\delta J}{\delta p_z}}{\frac{\delta J}{\delta \hat{q}}} = -\frac{\frac{\delta V}{\delta p_z} - \frac{\delta W}{\delta p_z}}{\frac{\delta V}{\delta \hat{q}}} = \frac{\lambda_Z \frac{Y}{p_z} - \hat{\lambda}\hat{Z}}{\hat{U}_{Q^*}(\sigma - 1)}$$

The sign of {15} tells us that whether the welfare of a childless couple is more or less severely affected by a change in p_z than the one of a couple with children: it is negative in the first case and positive in the second one (see figure 4). Given that $Y/p_z > \hat{Z}$, a positive sign could arise only if the marginal utility of income for *N>0* was significantly greater than the one for *N=0*. Recalling the discussion on equation (10), this is what happens if the marginal utility of own consumption for the childless case is "small enough" with respect to the case where *N>0*. While this is certainly a possibility, if (10) is negative, we expect a positive sign of (15) instead.

We can summarize these findings:

Result 5: *The effect of a change in the price of parents' own consumption on the critical threshold cannot be signed in general but it can be positive only if the marginal utility of income is significantly greater for N>0 than for N=0.*

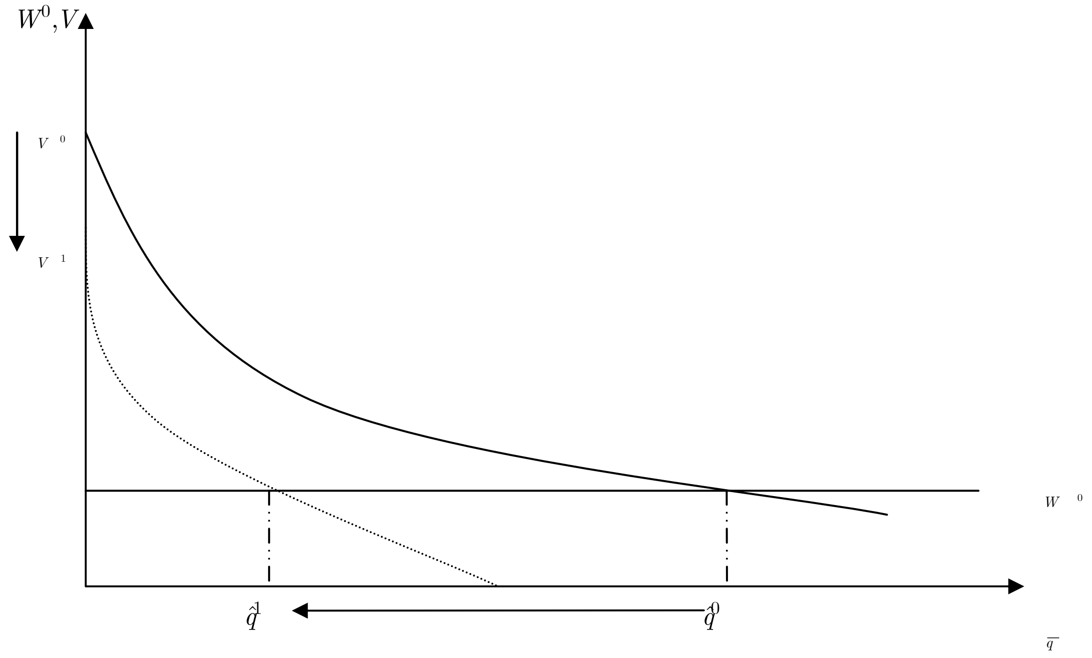


Figure 3: Effects of a rise in *p*

Results 3 and 5 show that theory does not provide unequivocal indications on the way the critical threshold moves as income and cost of parents' consumption change. Rather than being a weakness, this proves the flexibility of the model which can accommodate (and provide viable explanations to) many particular cases[20].

Now we are ready to proceed further with the utility maximization problem in the case *N>0*.

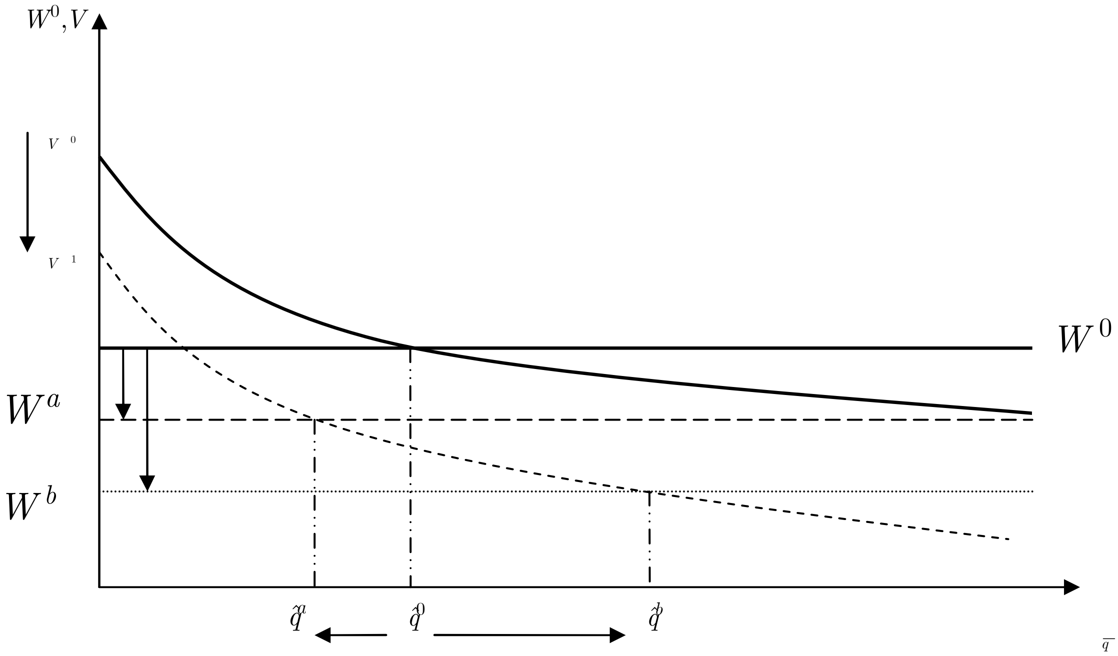


Figure 4: Effects of a change in p_z

[20] Nevertheless, it does not seem such a hazard to guess that {10} holds with negative sign: arguably childless couples may enjoy more certain goods and services in *Z,* the consumtpion of which may be hindered or even made impossible by the presence of children.

2.2.1 Utility maximization

The first order conditions are:

$$(16)\quad \begin{cases} u_n = \lambda(p_n + p\tilde{Q}) \\ u_q = \lambda(p_q + pN) \\ u_z = \lambda p_z \\ p_n N + p_q(Q + \overline{q}) + pN(Q + \overline{q}) + p_z Z = Y \end{cases}$$

The solution to the problem(supposed to be unique) is

$S = \left\{N^*, Q^*, Z^*\right\} = \left\{N(Y, p_n, p_q, p, p_z, \overline{q}), \tilde{Q}(Y, p_n, p_q, p, p_z, \overline{q}), Z(Y, p_n, p_q, p, p_z, \overline{q})\right\}$. It is immediate to notice that both the shadow prices of quantity and quality are endogenous: if these two goods are substitutes, a rise in the shadow price of *N* causes an increase on the demand of quality which, in turn, makes *N* ever more expansive and therefore induces a further substitution towards Q. Under these circumstances, the notion of shadow price itself loses much of its meaning and usefulness. Due to their variability, it seems more effective to label these prices "local prices":

$$(17)\quad \begin{cases} \rho_n = p_n + p\tilde{Q} \\ \rho_q = p_q + pN \\ R = Y + pN\tilde{Q} \end{cases}$$

w

here ρ_n and ρ_q are the local prices of quantity and quality respectively and *R* is the "virtual income"[21]. In order to have a better insight on this intuition, let's think of 3-goods maximization problem with a standard linear constraint:

$$(18)\quad \begin{cases} \max\ u(N, \breve{Q}, Z) \\ s.t.\ \rho_n N + \rho_q \tilde{Q} + p_z Z = R \end{cases}$$

[21] It is "virtual" because it represents the income the couple would need in order to achieve the same allocation if the budget constraint was linear. Being the local price of *Z* constant, in order to keep the notation as simple as possible, no special symbol is used.

Suppose that the solution to this problem is the same as before, i.e.

$$\left\{N^L, Q^L, Z^L\right\} = \left\{N(R,\rho_n,\rho_q,p_z), \breve{Q}(R,\rho_n,\rho_q,p_z), Z(R,\rho_n,\rho_q,p_z)\right\} = S = \left\{N^*, Q^*, Z^*\right\}$$

Figure 5 provides a graphical interpretation[22]: let a rise in p_q make the equilibrium shift from S to S^1; we can see that the change of just one parameter causes a change in *both* local prices. Local prices are the those prices providing the same optimal allocation under the following hypotheses: 1) linear constraint; 2) income is the virtual income. Luckily enough, it has been proved that most standard properties of the demand functions find their corresponding version in the non linear case (see Appendix 2).

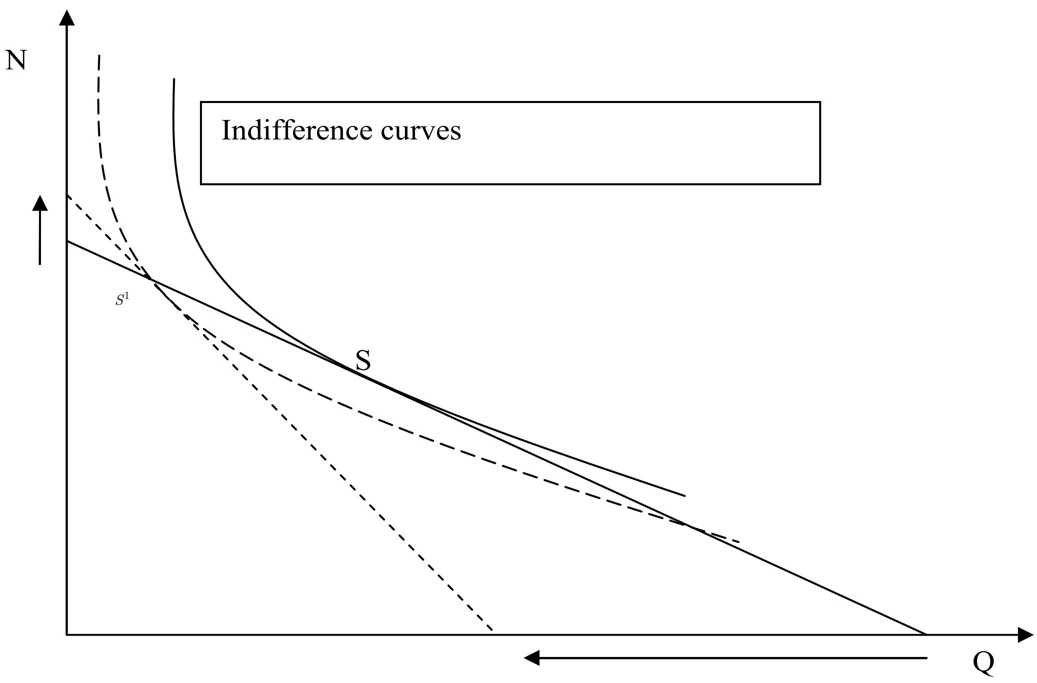


Figure 5: "Local prices" (non-linear budget constraint not shown)

[22] Clearly *Z* is kept constant; to avoid confusion, the non-linear budget constraint has not been drawn.

Comparative statics

Comparative statics analysis can be performed by taking the total derivatives of the first order conditions (16) at the optimum (the asterisks denoting the optima are omitted to keep notation as tidy as possible). The resulting equation (19) represents the generalized Slutsky equation as in the framework suggested by Edlefsen [1981].

$$\begin{pmatrix} \frac{\delta N}{\delta Y} & \frac{\delta N}{\delta \overline{q}} & \frac{\delta N}{\delta p} & \frac{\delta N}{\delta p_n} & \frac{\delta N}{\delta p_q} & \frac{\delta N}{\delta p_z} \\ \frac{\delta \breve{Q}}{\delta Y} & \frac{\delta \breve{Q}}{\delta \overline{q}} & \frac{\delta \breve{Q}}{\delta p} & \frac{\delta \breve{Q}}{\delta p_n} & \frac{\delta \breve{Q}}{\delta p_q} & \frac{\delta \breve{Q}}{\delta p_z} \\ \frac{\delta Z}{\delta Y} & \frac{\delta Z}{\delta \overline{q}} & \frac{\delta Z}{\delta p} & \frac{\delta Z}{\delta p_n} & \frac{\delta Z}{\delta p_q} & \frac{\delta Z}{\delta p_z} \\ \frac{\delta \lambda}{\delta Y} & \frac{\delta \lambda}{\delta \overline{q}} & \frac{\delta \lambda}{\delta p} & \frac{\delta \lambda}{\delta p_n} & \frac{\delta \lambda}{\delta p_q} & \frac{\delta \lambda}{\delta p_z} \end{pmatrix} = \begin{pmatrix} u_{nn} & u_{nq} - \lambda p & u_{nz} & -\rho_n \\ u_{nq} - \lambda p & u_{qq} & u_{qz} & -\rho_q \\ u_{nz} & u_{qz} & u_{zz} & -p_z \\ -\rho_n & -\rho_q & -p_z & 0 \end{pmatrix}^{-1} \begin{pmatrix} 0 & \lambda(1-\sigma) & \lambda\tilde{Q} & \lambda & 0 & 0 \\ 0 & 0 & \lambda N & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda \\ -1 & \rho_q(1-\sigma) & N\tilde{Q} & N & \tilde{Q} & Z \end{pmatrix}$$

(19)

Notice that the borders of the Hessian are local prices, which are the equivalent of prices in the non-linear case. Let's use the following notation to indicate the inverse of the 4×4 matrix on the RHS:

$$\begin{pmatrix} A_{nn} & A_{nq} & A_{nz} & -\alpha_n \\ A_{qn} & A_{qq} & A_{qz} & -\alpha_q \\ A_{zn} & A_{zq} & A_{zz} & -\alpha_z \\ -\alpha_n & -\alpha_q & -\alpha_z & \gamma \end{pmatrix}$$

By the second order conditions, the 3×3 sub-matrix $A = \{A_{i,j}\}$ is negative semidefinite. A change in just one parameter (excluding p_z) can affect *both* local prices: the hyperplane tangent to the indifference surface at the optimum rotates then in both direction; furthermore the constraint itself shifts. In order to make it clear, let's consider the following example: $\frac{\delta \breve{Q}}{\delta p} = \frac{\delta Q}{\delta p} = [\tilde{Q}(\lambda A_{nq}) + N(\lambda A_{qq})] - \alpha_q N\tilde{Q}$ Here $\tilde{Q}$ and N represent the changes in the local prices of quantity and quality respectively, while (λA_{nq}) and (λA_{qq}) are the per unit substitution effects; α_q is the per unit income effect and $(N\tilde{Q})$ measure its amount.

Defining $\lambda A_{i,j} = H_{i,j}$ makes possible to rewrite equation (19) in a much more compact way:

$$(20) \quad \begin{pmatrix} dN \\ d\breve{Q} \\ dZ \\ d\lambda \end{pmatrix} = \begin{pmatrix} H_{nn} & H_{nq} & H_{nz} & -\alpha_n \\ H_{qn} & H_{qq} & H_{qz} & -\alpha_q \\ H_{zn} & H_{zq} & H_{zz} & -\alpha_z \\ -\alpha_n & -\alpha_q & -\alpha_z & \gamma \end{pmatrix} \begin{pmatrix} d\rho_n \\ d\rho_q \\ dp_z \\ dC \end{pmatrix}$$

where $C \equiv \rho_n N + \rho_q \tilde{Q} + p_z Z - R = 0$ The virtue of this notation is to immediately show that the change in the demand of one good in response to a change in one parameter is mediated by changes in *all* local prices and in virtual income as well: in fact, the second matrix on the RHS is just the matrix of changes in local prices and local income. Being λ positive, the 3×3 sub-matrix $H = \{H_{i,j}\}$ remains negative semidefinite. Furthermore Theil and Edlefsen show that H is symmetric.

It turns out then that the diagonal elements of H have to be strictly negative. Due to Theil [1965][23] is also the following result:

$$(21) \quad \begin{cases} \rho_n H_{nn} + \rho_q H_{qn} + p_z H_{zn} = 0 \\ \rho_n H_{nq} + \rho_q H_{qq} + p_z H_{zq} = 0 \\ \rho_n H_{nz} + \rho_q H_{qz} + p_z H_{zz} = 0 \end{cases}$$

i.e. the sum of the substitution effects weighted by the local prices is zero(substitution effects are linearly dependent with respect to local prices).

If we had a linear budget constraint, we knew that the Hessian matrix arising from the dual problem of expenditure minimization would be equivalent to the one obtainable from the primal one. It is possible to show that a similar property carries on to the non-linear case: resolving the dual of (18) (i.e. performing a linearization of the original budget constraint), it can be shown that the Hessian of this problem with respect to the *local prices* is equivalent to the

[23] Blomquist [1989] comes to the same result following a different path.

matrix of substitution effects arising from the maximization of $u(N, \breve{Q}, Z)$ under (4)[24]. That means that H provides the compensated substitution effects. As we have already stressed, a change in one observable parameter may cause changes in several local prices: if we are willing to use H to study the compensated income effects of a change in any observable parameter, we have to take explicitly into account *all* those alterations. The Hicksian demands arising from the solution of dual of (18) are homogeneous of degree zero in local prices[25].

Compensated substitution effects – changes in $\overline{q}$

$$(22) \quad \left(\frac{\delta N}{\delta \overline{q}}\right)^{C} = p(1-\sigma)H_{nn} < 0$$

H_{nn} is strictly negative by the second order conditions and therefore the compensated demand for children (indicated by the subscript *C*) drops as the minimum quality threshold rises. Furthermore, the higher is the "variable" price of quality p (i.e.: the more curved the budget constraint is), the stronger is the magnitude of this effect. Clearly, the more interchangeable are mandatory and discretionary quality in the utility function, the less accentuated is this effect; in turn, if the couple gets utility only from the discretionary share of quality, then the effect is maximum.

It is important to notice that this result does not depend on particular assumptions on the relationship between quantity and discretionary quality in the utility function: the compensated

[24] Details can be found both in Edlefsen [1981] and Blomquist [1989].

[25] Using this result and (21), we can write:

$$\begin{cases} \text{a) } \varphi_{nn} + \varphi_{nq} + \varphi_{nz} = 0 \\ \text{b) } \varphi_{qn} + \varphi_{qq} + \varphi_{qz} = 0 \\ \text{c) } \varphi_{zn} + \varphi_{zq} + \varphi_{zz} = 0 \end{cases}$$

where the $\varphi_{x,j}$'s are the elasticities of Hicksian demands for good x with respect to the (local) price of j.

demand of children falls as the minimum quality threshold increases, no matter whether quality and quantity are substitutes or complements.

Equation (22) can provide an alternative explanation of the well-known negative link between mothers' education and their fertility from a brand new point of view. The fact that an increase in women's education reverberates in a lower fertility has been explained basically on the basis of two lines of arguments: first of all, the longer time needed to accumulate a higher human capital is subtracted from the fertile period; on the other hand, a better education increases the women's value of time, making child-care activities more expensive. The first argument may be not so strong: first of all, it is valid only if the schooling period significantly overlaps woman's fertile years; furthermore, nothing would prevent her from reducing the spacing of births in order to have the same number of children she would have had without (a better) education. The second argument is theoretically solid and several empirical works corroborate the hypothesis; nevertheless it does not exclude a further possibility: suppose that the government unexpectedly increases the minimum quality threshold by raising the mandatory years of school. The number of existing children is not affect by this decision (in fact, their parents decided the optimal number of children under a different constraint) but each of them will be better educated; once adult, they know that the threshold is higher than the one faced by their parents and react accordingly by reducing their fertility. Moreover, also all those couples who take their decision *just after* the raise of the threshold will reduce their fertility: these women, whose education is clearly *not* affected by the reform, *do* reduce their demand of children anyway[26]. This argument can peacefully coexist with the previous one on the opportunity cost of women's time and

[26] This leads to an empirically testable hypothesis: if a schooling reform which raises the mandatory years of school takes place, one should record an "immediate" (or rather fast) decline of fertility; that happens because all new potential mothers react to the rise of the minimum quality and not only the ones who get a better education because of the reform.

suggests that the effects of the minimum quality can be very pervasive and therefore able to explain much more than one could initially guess.

Let's turn on the effect of a change in the minimum quality threshold in the demand of discretionary quality. We have:

$$(23)\qquad \left(\frac{\delta\tilde{Q}}{\delta\overline{q}}\right)^{C} = \left(\frac{\delta\breve{Q}}{\delta\overline{q}}\right)^{C} + (1-\sigma) = (1-\sigma)(pH_{nq}+1)$$

and

$$(24)\qquad \left(\frac{\delta Q}{\delta\overline{q}}\right)^{C} = pH_{nq} - \sigma(pH_{nq}+1)$$

Given the assumption that $0 \leq \sigma < 1$, the sign of (23) depends exclusively on the relationship between quality and quantity[27]: assuming, as Becker does, that they are substitutes, then the (compensated) total demand for quality rises as the minimum threshold rises, i.e. total and discretionary quality are net complements. The magnitude of this effect depends positively on p, the variable price of quality; and negatively on σ, the degree of fungibility between mandatory and discretionary quality in the utility function.

Similar arguments hold for equation (24) but it is not possible to assess its sign without further assumptions on the form of the utility function. If we assume that quality and quantity are net substitutes, necessary and sufficient and condition for (24) to be positive is that $H_{nq} > {}^{\sigma}/_{p(\sigma-1)}$. The last expression makes clear that a positive sign becomes more and more likely as the variable price of quality increases. At the same time, the smaller is σ, the more probable is that mandatory and discretionary quality are net complements. In general, all effects from (22) to (24) are of higher magnitude when the degree of fungibility between mandatory and discretionary quality is low and reach their maximum when the utility function depends

[27] Recall that H_{nq} is the compensated cross-substitution effect under a "linear" constraint, i.e. the effect we had with constant local prices.

exclusively on the discretionary share of quality. Given these results, it turns out that assuming $\sigma = 0$ would not be particularly hazardous: by the first order conditions and given the ancillary assumption of substitutability between quantity and quality, we know by equation (23) that the marginal utility of quantity rises whenever the minimum quality threshold rises. Those basic characteristics are now "embedded" in each and every child and it is rather nonsense trying to conceptually split the number of children from the minimum quality; therefore the utility parents get from each child (i.e. from the quantity) increases along with the minimum quality, i.e. N "discounts" the fact that each child is "better" than before.

All in all, assuming that quality and quantity are substitutes, a rise of the threshold induces a couple to divert resources from N towards $\widetilde{Q}$: if the variable costs of quality p are high enough, even a small increase in $\bar{q}$ can severely depress fertility and encourage strong investments on quality[28], whenever the degree of fungibility between mandatory and discretionary quality is low (or zero). Casual observations seems to confirm this trend in most developed countries which have experienced sharply declining fertility rates: births per woman are lower and lower while children's education (in a broader sense)is far above the mandatory level.

The change in the overall demand of child services is given by:

$$(25) \quad \left(\frac{\delta N\tilde{Q}}{\delta \overline{q}}\right)^{C} = \tilde{Q}\left(\frac{\delta N}{\delta \overline{q}}\right)^{C} + N\left(\frac{\delta \tilde{Q}}{\delta \overline{q}}\right)^{C} + N = p(1-\sigma)\left[\tilde{Q}H_{nn} + N\left(NH_{nq} + \frac{1}{p}\right)\right]$$

which cannot be signed without further assumption on the actual form of the utility function. By equation (23), the sign of (25) is certainly negative if $\frac{\delta N}{\delta \rho_q} < \frac{1}{p}$. However, it is time to admit that it is rather uneasy to think at circumstances where N and Q should can be complements: stressing once again that we are dealing with *per child* quality, it is hard to picture a situation where parents wish either "lots of brilliant children" or just "very few mediocre ones"! Then, if

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[28] Recall that the level of the threshold affects the local price of quantity

quantity and quality are substitutes and the demand for quality significantly overcomes the one for quantity, the overall effect on the demand of child services will be probably negative. Nevertheless, the actual sign can be empirically assessed.

The effect on Z is given by:

$$(26) \quad \left(\frac{\delta Z}{\delta \bar{q}}\right)^{C} = p(1-\sigma)H_{zn}$$

and cannot be signed in general. Clearly the negative sign holds only if parents' own consumption and the quantity of children are net complements, a hypothesis which may be appealing: this is the case of a couple who reduces its own consumption as the local price of quantity rises; such a couple has therefore very strong preferences towards the discretionary quality and is ready to "make sacrifices" in order to have (fewer but) better children.

We can then summarize these results:

Result 6: *The compensated demand for children decreases as the minimum quality threshold rises. The higher is the variable price of quality* p*, the sharper is the effect; the higher is the fungibility between mandatory and discretionary quality, the milder it becomes. This result does not rely on specific assumptions on the relationship between quantity and discretionary quality in the utility function.*

Result 7: *The compensated demand of quality* $\tilde{Q}$ *increases with the minimum quality threshold* $\bar{q}$ *(i.e. quality and mandatory quality are net complements) as long as N and Q are net substitutes. The magnitude of this effect depends positively on the variable price of quality* p *and negatively on the degree of fungibility between mandatory and discretionary quality* σ*.*

Result 7b: *The demand of discretionary quality may either rise or fall as the minimum quality threshold rises. If quantity and quality of children are net substitutes, then mandatory and discretionary quality are net complements as long as the variable price of quality* (p) *is high enough and the degree of fungibility between mandatory and discretionary quality in the utility function* (σ) *low.*

Result 7c: *Even in the case when the mandatory quality does not enter in the utility function (i.e. $\sigma = 0$), the marginal utility of quantity is positively affected by the level of the minimum quality threshold as long as quantity and quality are substitutes.*

Result 8: *The effect of a rise in $\bar{q}$ on the overall demand of child-services cannot be determined in general.*

Result 9: *The magnitude of the response of the compensated demands of Z to a rise in $\bar{q}$ depends positively on p and negatively on σ, while its sign is positive if Z and N are substitutes and negative otherwise.*

Compensated substitution effects – changes in p, p_n, p_q and p_z

Derivation of the following result are left to Appendix 3.

Result 10: *If the variable price of quality rises, the demand for child-services decreases.*

Result 11: *If N and $\widetilde{Q}$ are net substitutes, the compensated demands of N and $\widetilde{Q}$ change in opposite directions decrease as p rises.*

Result 12: *The effect of a change in p on the compensated demand of Z cannot be determined in general but it is surely positive if both N and $\widetilde{Q}$ are substitutes of Z.*

Result 13: *An increase in the fixed price of quality(quantity) induces a decrease in the compensated demand of quality(quantity). The effect on the compensated demand of quantity(quality) is positive if N and $\widetilde{Q}$ are net substitutes and negative otherwise.*

Result 14: *(Reciprocity conditions) The change in the compensated demand of child services in response to a change in p_n equals the one of the compensated demand for $\widetilde{Q}$ with respect to$p$; the change in the compensated demand of child services in response to a change in p_q equals the one of the compensated demand for N with respect to p.*

Having discussed the effects of changes in *N* caused by changes in either the level of the minimum quality threshold or in the variable price of quality, it is useful to define a parameter which will be important for the empirical application of the next chapter: the marginal expenditure on minimum quality. Once parents have decided to bear children, they have to spend for each (further) child the following amount on mandatory quality:

$$(27) \qquad \pi_q = \frac{\delta\left(\rho_q \overline{q}\right)}{\delta N} = p\overline{q}$$

The marginal expenditure on minimum quality can be useful to test the theory when the mandatory level of quality does not decrease but its price does: for instance, if the competences that a child has to develop during the compulsory years of school do not change but for any reason the level of the teaching deteriorates, a couple has to invest more of its own resources (private lessons or more parental time used to help with homework) in order to let their children achieve the required proficiency. It can be easily verified that: $\left(\partial N / \partial \pi_q\right)^C < 0$.

Income effects

Engel aggregation still holds even under a nonlinear budget constraint but the relevant prices are, as usual, the local ones. From this standpoint, our knowledge of the income effects is, in general, the same we have in standard problems. Nevertheless, it is important to stress that the assumption that the bundle $N\widetilde{Q}$ is a normal good is not enough to assure that when the permanent income rises, the demand for quantity grows along; nor sufficient is the hypothesis that quantity is normal under a linear constraint to assure that the same holds under a nonlinear one. The response of the demand of children to a rise in permanent income represented one of the most important topics in this branch of literature and the introduction of the minimum quality threshold does not induce any relevant change in this context. In fact, we have that the two relevant income effects are given by: 1) $\partial N/\partial Y$; and 2) $\partial \breve{Q}/\partial Y = \partial Q/\partial Y$ and none of them is affected by the existence of the minimum quality threshold.

Nevertheless, for the sake of completeness, following Edlefsen, it is possible to show the relationship between the income effects of our model with the ones we had under a linear constraint.

It is possible to split the bordered Hessian in order to isolate the one we had under a linear constraint; equation (20) becomes:

$$(28)\qquad \begin{pmatrix} \frac{\partial N}{\partial Y} \\ \frac{\partial Q}{\partial Y} \\ \frac{\partial Z}{\partial Y} \\ \frac{\partial \lambda}{\partial Y} \end{pmatrix} = \left[\begin{pmatrix} A^L & -\alpha^L \\ -\alpha^L & \gamma^L \end{pmatrix} - \begin{pmatrix} 0 & \lambda p & 0 & 0 \\ \lambda p & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right]^{-1} \begin{pmatrix} 0 \\ 0 \\ 0 \\ -1 \end{pmatrix}$$

where the subscript L indicates the linear case. Using the matrix property $(A-B)^{-1} = (I-A^{-1}B)^{-1}A^{-1}$ and defining $\lambda A^L = H^L$, we can write:

$$\begin{pmatrix} \alpha_n \\ \alpha_q \\ \alpha_z \\ \alpha_\lambda \end{pmatrix} = \begin{pmatrix} 1-pH^L_{nq} & -pH_{nn} & 0 & 0 \\ -pH_{qq} & 1-pH^L_{nq} & 0 & 0 \\ -pH_{zq} & -pH_{zn} & 1 & 0 \\ p\alpha^L_q & p\alpha^L_n & 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} \alpha^L_n \\ \alpha^L_q \\ \alpha^L_z \\ \gamma^L \end{pmatrix}$$

Inverting the matrix on the RHS and we obtain[29]:

$$(29)\qquad \begin{cases} \alpha_n = D\left[\alpha^L_n + p(H^L_{nn}\alpha^L_q - H^L_{nq}\alpha^L_n)\right] \\ \alpha_n = D\left[\alpha^L_q + p(H^L_{qq}\alpha^L_n - H^L_{nq}\alpha^L_q)\right] \end{cases}$$

where $D = \frac{1}{1-2pH^L_{nq}+p^2({H^L_{nq}}^2 - H^L_{nn}H^L_{qq})} > 1$ is the determinant of $(I-A^{-1}B)^{-1}$ matrix above, a positive quantity by the second order conditions[30]. In Appendix 4 the same technique will be used to compare the compensated substitution effects in the linear and nonlinear case.

[29] We show only the first two income effects for the sake of easier reading.

[30] By the second order conditions, the determinant of the bordered Hessian of the original problem (i.e. the determinant of $(A-B)^{-1}$) is positive and the same holds for the determinant of the bordered Hessian of the "linear" problem A^{-1}; it turns out that the determinant of $(I-A^{-1}B)^{-1}$ has to be positive as well. Furthermore, $({H^L_{nq}}^2 - H^L_{nn}H^L_{qq})$ is the opposit of the determinant of the second leading principal minor of the bordered Hessian relative to

It is clear then that each nonlinear income effect depends on *both* "linear" income effects as well as on "linear" compensated own- and cross-substitution effects. Mathematics cannot provide any other information and in order to say when nonlinear effects are smaller or greater than the linear ones other assumption on the utility function are therefore needed. In particular, we have that:

$$(30) \qquad \alpha_n = \alpha_n^L \left[1 + pD \left(\frac{\alpha_q^L}{\alpha_n^L} H_{nn}^L + H_{nq}^L - p\chi \right) \right]$$

where $\chi = {H_{nq}^L}^2 + H_{nn}^L H_{qq}^L$, which is positive by the second order conditions. So, the linear income effect is greater than the nonlinear one if:

$$\frac{\alpha_q^L}{\alpha_n^L} \geq \frac{p\chi - H_{nq}^L}{H_{nn}^L}$$

This is a rather complicated and "uncommunicative" expression but it allows, at least, to affirm that if in the linear case the quality response to income overcomes significantly the one of quantity, then in the nonlinear income effect of quantity will be smaller than the linear one: the relationship between linear and nonlinear effects describes the same argument that Becker and Lewis [1973] developed as difference between "true" and "observed" income elasticities of demand.

the linear constraint and is therefore positive. The denominator in D is therefore smaller than one and, consequantly, $D > 1$.

3. Conclusions

This work proposes a model of complete fertility which generalizes the famous quantity/quality trade-off theory originally due to Becker by acknowledging that parents are required to provide their children with at least some basic characteristics. The origin of this obligation may consist in both the *corpus* of laws in force in the country where the couple lives (*objective threshold*) and in the set of informal (but binding) requirements which are in force among its peers *(perceived threshold)*. In both cases, the couple faces an exogenous quality constraint in its child-bearing decisions.

This model shares a set of hypotheses with other ones belonging to this branch of literature, i.e. the couple acts like a single agent and is stable; allocates its resources in child services and not children-related goods; is impartial toward its children, who, in turn, are all equally genetically endowed and can have the exact number of children it wants. The new hypothesis are the above-mentioned existence of a binding *exogenous* minimum quality threshold and the higher value that parents attach to the share of quality above the minimum (*discretionary quality*). In fact, while mandatory quality is by definition the same for all children, it is not necessary true that different families share the same preferences on the nature of discretionary quality: the minimum threshold enforces somewhat homogeneity where parents desire heterogeneity. Furthermore, in such a context, only investing on discretionary quality is possible to increase children's "attractiveness".

The first and more evident departure from the standard model is the fact that zero children is an admissible solution: the utility function is piecewise, with each branch corresponding to the two different possible states, i.e. with or without children. This allows to model preferences about not children-related goods in a very flexible way, explicitly considering the possibility that a couple may enjoy the same goods (for instance, a pleasure cruise) at different extents in the two possible states.

The existence of a mandatory minimum quality reduces the couple's welfare and a rise of the threshold lowers its wellbeing exactly by the difference between the marginal utility of discretionary and mandatory quality. This happens because it becomes more expensive to improve children's "attractiveness" and the resources needed to finance a higher threshold are subtracted from other possible uses.

Being the couple's welfare a declining function of the minimum quality threshold, there exists an endogenously determined upper limit level of that threshold at which a couple becomes indifferent between having children or not; above that level, the couple is better off childless. Whatever makes child services more expensive also lowers that ceiling: this effect is particularly noticeable whenever the mark-up relates to those expenses on quality, which parents have to afford for each and every child (for instance, school fees).

The effect of a change in the permanent income on the critical endogenous threshold depends on the difference between the marginal utilities of parents own-consumption in the two possible states: if the consumption of non-children related goods is "more enjoyable" when childless, the effect of a rise of income will be negative; otherwise the effect will be positive. Even if the effect of income is negative, it quickly approaches to zero as income increases, i.e. very wealthy people are always better-off having children. Anyway, the actual sign can be empirically assessed.

Similar arguments hold for changes in the cost of parents' own consumption: the sign of this effect cannot be assessed in general but it can be positive only if the marginal utility of income is significantly greater in the case parenthood.

Given that the actual minimum quality threshold lies on the left of the critical values, it is possible to assess the effect of changes of parameters on the optimal demands of children, discretionary quality and non children-related goods.

It turns out that -regardless the relationship between quantity and discretionary quality- a rise of the minimum threshold depresses fertility and the higher is the price of child services (i.e. the more curved is the budget constraint), the stronger is this effect. On the other hand, if number and quality of children are net substitutes, it is possible to verify that mandatory and total quality are net complements: more specifically, if the degree of fungibility between mandatory and discretionary quality is low, the a rise of the threshold triggers an increase on investments in discretionary quality as well. Once again, these effects are proportional to the curvature of the budget constraint. If total quality and quantity are substitutes, the marginal utility of the quantity of children rises along the minimum quality threshold even if the minimum quality does not yield any direct utility to the couple: that means parents already take implicitly into account that each and every child is of a "higher standard" and therefore it would be possible to consider "quality" just its discretionary share. On the whole, it is not possible to predict the effect of a rise of the minimum quality threshold on the demand of child services without further assumptions on the utility function.

Not surprisingly, if quantity of children and own-consumption are net substitutes, the effect of a rise of the threshold on the demand for non child-related goods is positive and increases with the price of child-services.

Assuming that child-services considered as a bundle is a normal good, does not guarantee that quantity is also a normal good; neither does the assumption of normality under a linear constraint. Nevertheless, it is possible to say that whether the income response of quality is higher than the one of quantity under a linear constraint, the quantity response under a nonlinear income is lower than in the linear case. All in all, income effects are not affected by the presence of the minimum quality threshold.

To our knowledge, all the results concerning the effects of the introduction of a minimum quality threshold are new. The hypothesis of the existence a threshold that couples have to take into account has proved its capability to provide new insights on child-bearing decisions and moreover offers the possibility of several extensions. The first possible development is to build up a equilibrium model where parents choose for their offspring a specific rank on the distribution of quality among the children belonging to the same cohort. This can be a preparatory step to develop an overlapping generation model where the level of mandatory quality can be "endogenized", i.e. the choices of the government and of the parents of generation n jointly determine the level of the threshold for the generation $n+1$. Such a model could also be the basis for the study of a possible trade-off between per-capita productivity and fertility rate in a country closed to immigration. This appears to be a promising branch of research indeed.

Appendix 1

Parenthood decision: relationship between changes in income and the marginal utility of parent's own consumption

Aim of this appendix is to prove equation (11). Let's start with the case *N=0.*

$$(1) \qquad \frac{\delta W}{\delta Y} = \frac{\delta v(Z(Y))}{\delta Y} = v_z Z_y$$

where v_z and Z_y are the marginal utility of *Z* and the derivative of *Z* with respect to income. Using Engel's aggregation $\left(p_z Z_y = 1\right)$, we obtain:

$$(2) \qquad \frac{\delta W}{\delta Y} = \lambda_z = \frac{v_z}{p_z}$$

Similarly, for *N>0*, we have:

$$(3) \qquad \frac{\delta V}{\delta Y} = \hat{\lambda} = u_n N_y + u_q Q_y + u_z Z_y$$

Where the terms all the terms can be interpreted as before. Recalling that Engel's aggregation is:

$$(4) \qquad \rho_n N_y + \rho_q Q_y + p_z Z_y = 1$$

Substituting *Z* and rearranging the equation, we obtain:

$$(5) \qquad \hat{\lambda} = \frac{u_z}{p_z} + N_y(u_n - \frac{u_z \rho_n}{p_z}) + Q_y(u_q - \frac{u_z \rho_q}{p_z})$$

We notice that the terms into the brackets derive from the first order conditions and therefore are zero;

$$(6) \qquad \hat{\lambda} = \frac{u_z}{p_z}$$

It becomes obvious that:

$$\frac{\hat{\lambda} - \lambda_Z}{\hat{U}_{Q^*}(\sigma - 1)} \leq 0 \qquad \text{if} \quad \hat{\lambda} < \lambda_Z \;\; \rightarrow \;\; u_Z(Z^*) \leq v_Z\left(\frac{Y}{p_z}\right)$$

Appendix 2

Properties of the demand functions derived under a non-linear constraint.

The following properties hold:

- **Marshallian demands are homogenous of degree zero in prices and income *given* $\overline{q}$.** For a given threshold, in fact, the budget constraint is homogenous of degree one.
- **Engel aggregation.** Differentiating the budget constraint at the optimum with respect to Y yields:

$$(7)\qquad (p_n + p\tilde{Q})\frac{\delta N}{\delta Y} + (p_q + pN)\frac{\delta \tilde{Q}}{\delta Y} + p_z\frac{\delta Z}{\delta Y} = \rho_n N_y + \rho_q \tilde{Q}_y + p_z Z_y = 1$$

Manipulating the expression further yields:

$$(8)\qquad \rho_n N_y + \rho_q \tilde{Q}_y + p_z Z_y = s_n \eta_{ny} + s_q \eta_{qy} + s_z \eta_{zy} = 1$$

- where the s's are the ratios of the local prices times the demand the good on actual income; the η's are the income elasticities of demand. The formula is very close to the standard one except for the fact that local prices are involved.
- **Cournot aggregation.** Differentiating the budget constraint with respect to $j = \{\overline{q}, p_n, p_q, p, p_z\}$ yields:

$$(9)\qquad \begin{cases} \rho_n N_{\overline{q}} + \rho_q Q_{\overline{q}} + p_z Z_{\overline{q}} = -\rho_q \\ \rho_n N_{p_n} + \rho_q Q_{p_n} + p_z Z_{p_n} = -N \\ \rho_n N_{p_q} + \rho_q Q_{p_q} + p_z Z_{p_q} = -\tilde{Q} \\ \rho_n N_p + \rho_q Q_p + p_z Z_p = -N\tilde{Q} \\ \rho_n N_{p_z} + \rho_q Q_{p_z} + p_z Z_{p_z} = -Z \end{cases}$$

These equations are just straightforward generalizations of the standard case. If we were to consider a standard two-goods case (child services are a single good) with no minimum quality threshold, we would obtain only the last two equations. Unfortunately this does not mean that the problem can be solved in two steps (first step: allocation between Z and (NQ); second step: inner distribution of resources between N and Q) as all decision are taken at once.

Appendix 3

Compensated substitution effects – changes in p, p_n, p_q and p_z

The effects of a change of the variable costs of quality p on the compensated demand for children N and on the demand of discretionary quality are:

$$(10) \quad \left(\frac{\delta N}{\delta p}\right)^C = \tilde{Q}H_{nn} + NH_{nq}$$

$$(11) \quad \left(\frac{\delta Q}{\delta p}\right)^C = \tilde{Q}H_{nq} + NH_{qq}$$

the signs of which cannot be determined without further assumptions on $u(\cdot)$: they depend, in fact, on the relative magnitudes of own- and cross-substitution effects and on the (Marshallian) demands for N and Q as well.

Something more can be said on the effect on the overall demand of child-services:

$$(12) \quad \left(\frac{\delta N\tilde{Q}}{\delta p}\right)^C = \tilde{Q}\left(\frac{\delta N}{\delta p}\right)^C + N\left(\frac{\delta Q}{\delta p}\right)^C = \begin{pmatrix}\tilde{Q} & N\end{pmatrix}\begin{pmatrix}H_{nn} & H_{nq}\\ H_{nq} & H_{nn}\end{pmatrix}\begin{pmatrix}\tilde{Q}\\ N\end{pmatrix} < 0$$

the sign of which comes from the fact that the quadratic form on the RHS is negative definite. Even if we cannot precisely sign (1) and (2), we know that the negative effect will prevail: an increase of the variable costs of quality induces a reduction of the demand of child-services.

Given the sign of (3), we can make a working hypothesis about the signs of the derivatives in (1) and (2): if quantity and discretionary quality of children are net substitutes, we know from (3) that either N or Q has to decrease with p while the other must increase. As long as the demand for quality overcomes significantly the one for N, we have: $\left(\frac{\delta N}{\delta p}\right)^C < 0$ and $\left(\frac{\delta Q}{\delta p}\right)^C > 0$. After all, p is the price of child-services considered as a whole: as the previous quotation from Becker makes clear, the entire logic behind this family of models takes (indirectly) as granted that couple's preferences go toward quality whenever the cost of child services p rises, i.e. quantity "succumbs" to quality. Therefore, it is reasonable to guess that in real life the demand of quantity N declines as the price of child-services p rises. Moreover, the time of the mother is an important component of the price p: hypothesising a negative effect on N is therefore in line with the *corpus* of literature predicting a decline of fertility as the opportunity cost of mother's time rises.

The effects on the demand of Z is just:

$$\text{(13)} \qquad \left(\frac{\delta Z}{\delta p}\right)^C = \tilde{Q}H_{zn} + NH_{zq}$$

Once again, the sign depends on the shape of the preferences. If both N and Q are substitutes of Z, then the sign is surely positive. Following results are self-explanatory and do not need any special discussion:

$$\text{(14)} \qquad \begin{array}{ll} \left(\dfrac{\delta N}{\delta p_n}\right)^C = H_{nn} < 0 & \left(\dfrac{\delta Q}{\delta p_n}\right)^C = H_{nq} \\ \left(\dfrac{\delta N}{\delta p_q}\right)^C = H_{nq} & \left(\dfrac{\delta Q}{\delta p_q}\right)^C = H_{qq} < 0 \end{array}$$

Furthermore we have:

$$\text{(15)} \qquad \begin{array}{l} \left(\dfrac{\delta N\tilde{Q}}{\delta p_n}\right)^C = N\left(\dfrac{\delta \tilde{Q}}{\delta p_n}\right)^C + \tilde{Q}\left(\dfrac{\delta N}{\delta p_n}\right)^C = \left(\dfrac{\delta Q}{\delta p}\right)^C \\ \left(\dfrac{\delta N\tilde{Q}}{\delta p_q}\right)^C = N\left(\dfrac{\delta \tilde{Q}}{\delta p_q}\right)^C + \tilde{Q}\left(\dfrac{\delta N}{\delta p_q}\right)^C = \left(\dfrac{\delta N}{\delta p}\right)^C \end{array}$$

which Edlefsen calls "reciprocity conditions" for obvious reasons. The effects of a change in p_z are standard and do not of particular interest.

Appendix 4

Magnitudes of the substitution effects in the linear and nonlinear cases: a comparison

Using the same technique we have seen for the income effects, it is possible to compare the magnitude of the substitution effects in the linear and nonlinear cases. We have:

$$(16)\qquad \begin{pmatrix} dN \\ d\breve{Q} \\ dZ \\ d\lambda \end{pmatrix} = \begin{pmatrix} 1-pH_{nq}^{L} & -pH_{nn} & 0 & 0 \\ -pH_{qq} & 1-pH_{nq}^{L} & 0 & 0 \\ -pH_{zq} & -pH_{zn} & 1 & 0 \\ p\alpha_{q}^{L} & p\alpha_{n}^{L} & 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} d\rho_n \\ d\rho_q \\ dp_z \\ dC \end{pmatrix}$$

Isolating the effects on quantity and quality, we get:

$$(17)\qquad \begin{pmatrix} dN \\ d\breve{Q} \end{pmatrix} = D \begin{pmatrix} 1-pH_{nq}^{L} & pH_{nn}^{L} & 0 & 0 \\ pH_{qq}^{L} & 1-pH_{nq}^{L} & 0 & 0 \end{pmatrix} \begin{pmatrix} d\rho_n \\ d\rho_q \\ dp_z \\ dC \end{pmatrix}$$

and therefore:

- $H_{nn} = DH_{nn}^{L} \rightarrow \left|H_{nn}\right| > \left|H_{nn}^{L}\right|$ and
- $H_{qq} = DH_{qq}^{L} \rightarrow \left|H_{qq}\right| > \left|H_{qq}^{L}\right|$, i.e. the own substitution effects are greater in the nonlinear case. Furthermore, being $\partial D / \partial p > 0$, the difference increases along with the variable cost of quality.
- $H_{nq} = D\left[H_{nq}^{L} + p\left(H_{nn}^{L}H_{qq}^{L} - \left(H_{nq}^{L}\right)^{2}\right)\right] \rightarrow H_{nq} > H_{nq}^{L}$,i.e. the compensated cross-substitution effect between quantity and quality is stronger in the nonlinear case and the degree of substitutability between them increases when the variable price of quality *p* rises, *regardless the presence of other goods*. Recall that $\left(H_{nn}^{L}H_{qq}^{L} - \left(H_{nq}^{L}\right)^{2}\right)$ is the opposite of the determinant of second leading principal minor of the bordered Hessian relative to the linear problem and is positive by the second order conditions.

Appendix 5

Utility maximization: Q=0

Given that our utility function depends on $\breve{Q} = Q + \sigma\bar{q}$, the case where the demand of discretionary quality is zero does not represent a problem in terms of the previous analysis and all the results we have seen still hold. Nevertheless, focusing on the effect of a rise of the minimum quality threshold when initial optimum is such a "corner solution" can be shed a further light on the inner mechanism of allocation of resources between quantity and (discretionary) quality.

The Kuhn-tucker conditions tell us that the following relationships hold:

$$\begin{cases} \dfrac{u_n}{u_q} > \dfrac{p_n + p\bar{q}}{\rho_q} \\ \dfrac{u_z}{u_q} > \dfrac{p_z}{\rho_q} \\ \dfrac{u_n}{u_z} = \dfrac{p_n + p\bar{q}}{p_z} \end{cases}$$

The local price of quantity is now a constant while the one of quality is still endogenous and depends on N through ρ_q. If the minimum quality threshold rises, we know that the compensated demand of quantity decreases and, as long as the degree of fungibility between mandatory and discretionary quality is low, the same happens to the one of discretionary quality. The previous discussion on income effects suggests that the quality response to income is likely to be positive and of greater magnitude than it would be in the linear case. If quality and quantity are net substitutes, then the demand of quantity declines while the one of discretionary quality becomes positive. The demand for discretionary quality would remain zero instead only if the degree of substitutability between mandatory and discretionary quality σ was very close to one[1]:

[1] We have: $\frac{\partial Q}{\partial \bar{q}} = (1-\sigma)\left(pH_{nq} + \rho_q \alpha_q\right) - \sigma$

in this case, all the resources "unlocked" from the child services contribute to increase the couple's own-consumption.

This example allows to stress that the overall effect of the introduction of the minimum quality threshold depends on the degree of fungibility between mandatory and discretionary quality in the utility function. As we have seen, the negative effect in terms of births is maximum if the couple does not get any utility from the mandatory quality while it totally disappears if parents care only about the total quality, regardless its nature. It is worthy to emphasize once again that the latter hypothesis is a very strong one: in fact, it implies that parents' preferences and the choices of the third subject who defines the minimum quality overlap perfectly, i.e. the couple would have spent the *same amount of resources in the very same items* even without the constraint. Moreover, investing in discretionary quality allows parents to have children above the basic standard and it is reasonable to guess that this edge over the minimum provides *per se* a higher (marginal) utility, in the very same way that a luxury car provides a higher utility than an economy vehicle barely allowed to circulate.

Chapter Two
Marital Fertility and Exogenous Constraints on Child Quality

An Empirical Approach

Abstract. The aim of this work is to provide a reduced form estimation of the effect of changes in the marginal expenditures on minimum quality on the total number of children born to married couples. In fact, changes in marginal expenditures take place more often than changes in the threshold only and are easier to locate. In order to adhere as closely as possible to the piecewise formulation of the utility function proposed in the previous chapter, the couple's childbearing decision is modeled as a two-stages process through a complementary log-log - Poisson hurdle model. The cross-sectional dataset used in this work is built from several sweeps of the British National Child Development Study (NCDS), a huge longitudinal survey collecting data from every child born during a specific week in march 1958. In turn, the marginal expenditures on minimum quality are proxied by the pupil/teacher ratio in primary schools (at a State level) observed by the partners at the time they got married. There is strong evidence that increases in marginal expenditures on minimum quality have a strongly negative effect both on the probability of becoming parents and on the overall number of children born to the couple. In turn, the overall effect of the observed income is positive but of small magnitude: in fact, a higher income reduces the predicted probability of becoming parents for nine couples out of ten (i.e. they "surrender to the suggestion" of higher own-consumption). These two results suggests therefore new insights into the interpretation of trend of low fertility in most developed countries.

1.Introduction

Aim of this chapter is to propose an empirical application inspired by the theory introduced in the previous part. The theoretical discussion dealt with consequences of the introduction of a “compulsory” minimum standard of quality of children on couples’ fertility choices but remained voluntarily rather “agnostic” about the concept of quality itself. Such an agnosticism is instrumental to the purpose of making the theory as general as possible, in order to allow its implementation in the broadest possible set of real world situations.

As a matter of fact, the theory can represent an useful interpretation tool whenever an *entity* external to the couple has the *right* (and the *power*) to impose *certain behaviours* toward its children. Each word in italics can take different incarnations according to the concrete case under examination[1]: as long as the source of the obligation, i.e. the minimum quality that parents must guarantee to their offspring, is an actual law, the “entity external to the couple” is the State itself and the couple faces, therefore, a truly exogenous (or *objective*) constraint[2].

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[1] For instance, the “external entity” may be the group of peers in the neighbourhood: these persons may expect that children respect a certain dressing code, attend to religious rituals and so on. In this case, the above-mentioned “right” is just a social expectation that parent feel *compelled* to meet. To stress this concept further, we may think of the Indian caste system: even though this social structure is not legally recognised, it is able to enforce certain behaviours (also related to the education of the offspring) to the members of the different castes almost as if it was law. Both the expectations of the neighbourhood and the caste system represent examples of what we called “perceived thresholds” in the previous chapter. On the other hand, disregarding the example of the caste system, where the membership is clearly exogenous, one could except that people choose –at least to some extent- their reference community and that the perceived minimum quality threshold is, therefore, at least partly endogenous. Speculating on the extent at which such a choice is actually possible is surely an interesting digression, but it would take the discussion too far.

[2] Moreover, there exists no readily available dataset from which one can infer a workable measure of the perceived threshold: in fact, it would be necessary to plan an *ad hoc* constructed survey in order to assess both the *nature* and the

However, it is still necessary to translate a theoretical construct such as the child quality into an observable entity. Although several alternative concepts of child quality can be found in the theoretical literature, human capital has become almost a synonym for quality in the literature on fertility since the middle '70s (Leibowitz [1974]; Becker & Tomes [1976]). In turn, education-related variables such as years of schooling and school enrolments and attainments are almost invariably used as measure of children's human capital, being health-related variables a notable exception: for instance, Desai [1995] uses children's nutrition status while Schultz & Mwabu [2003] exploit height for age by sex and weight for height by age and sex. It is usually assumed that parents do not discriminate children on sex basis. Nevertheless, if a particular sex represents a valuable characteristic to the parents, i.e. is a component of child quality, (Ben-Porath & Welch [1976]), then both total fertility and investments in human capital may be affected by the sex of the first one or two children born, especially if they are female (Lee [2004], Quian [2006] and Jensen [2005]). However, it is important to stress that the role of sex is heavily dependent on the particular cultural environments under examination. In fact, the sex of the child may also capture some price effects: we can think, for instance, of the due to constitute a dowry for a daughter. The desire to have at least a boy and a girl can be a driver of higher fertility also in countries where there is no actual sex discrimination (Angrist & Evan [1998]).

From a methodological point of view, it is possible to distinguish between two main approaches in estimating fertility outcomes: reduced and structural models. The first approach

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level of such a minimum quality threshold. Even if the financial resources necessary to collect the data were available, planning such a survey would not be free from substantial complications, as it is often the case when one tries to record people's "perceptions" (Skrondal & Rabe-Hesketh [2004]). In particular, it could be rather complicated to elaborate questions able to call for answers from which one can easily and *effectively* disentangle the perceived minimum threshold (determined, by definition, outside the couple) from the overall desired level of quality (which is a choice instead).

consists in expressing the total fertility as function of all observed environmentally fixed household's constraints and characteristics. This choice is popular because it provides a description of how fertility changes in response to exogenous variations and, therefore, it may provide an approximation of the outcomes of specific policy measures which can influence parental choices at the margin. Examples of this approach are Rosenzweig & Wolpin [1982], Hussein [1989], and Benefo & Schultz [1996]. In order to deal with possible endogeneity of some of the covariates (especially the ones reflecting quality-related prices), several different methods have been used. The most popular (and standard) choice is the instrumental variables (IV) regression (Schultz [1985]); other alternatives are the fixed effects (difference-of-different) approach (Blank, George & London [1984]) and even social experiments (Burtless [1995]) and random assignments (Maynard & Rangarajan [1994]).

The second approach, i.e. the structural model, is preferred when the focus is on the cross-effect of fertility on other household's choices, such as the woman's participation to the labour market and the accumulation of human capital and these outcomes are simultaneously determined. Structural models are clearly more demanding because they have to rely on exclusion restrictions that may be quite controversial: the task here is to specify a variable that affects fertility but not the other outcome. The most famous example is due to Rosenzweig and Wolpin [1980] and consists in use of twin births as exogenous determinant of completed fertility to test whether a quantity-quality trade-off takes actually place or not. For an up-to-date review of these branch of models see Schultz [2007].

The here proposed application is a reduced form model which aims to test the effect of changes in the expenditures on compulsory minimum quality on married couples' complete fertility.

Following the tradition, the minimum quality will be modelled as an education-related variable: rather than just considering the mere number of compulsory years of schooling, it is more useful to rely on *levels of proficiency* or *skills* that students are required to meet and develop in certain matters at the end of any cycle of the compulsory education instead[3]. In general, the existence of standardized assessments allows to test the effect of the minimum quality threshold on the complete fertility in a variety of situations: for instance, the overall number of years of mandatory schooling may not change but further teachings may be introduced (for instance, foreign languages in primary schools) on top of the existing ones, with the consequence that the minimum quality threshold becomes higher. In fact, from the couple's perspective, what really matters are their (marginal) *expenditures* on minimum quality; if, for whatever reason, the quality of teaching declines but the skills that children have to develop do not drop accordingly, parents have to invest more of their own resources (time and money) in order to let their children reach the minimum standards, i.e. the minimum quality becomes more expensive. So, even if the number of years of compulsory education do not change but either the required competences and/or their cost do, it is possible to test the consequent effect on total fertility. According to this setting, the expenditures on minimum quality are proxied by the pupil/teacher ratio in primary schools at a State level in the year of marriage provided by the UK Department for Children, Schools and Family.

Leaving these issues on quality momentarily aside, it is necessary to stress that the theoretical model is equally quite demanding in terms of other required data: not only couples where the woman has already completed her fertile life; but also information on "attitudes" toward parenthood of both partners at the time they got married are needed; ideally, one should try to piece together the information set available to the spouses at the time of their wedding and all the relevant changes that took place afterwards . Finding a dataset providing all those information has revealed to be quite a challenge: surveys excelling in one aspect were lacking in

[3] The very fact that, in most countries, the level of proficiency are systematically assessed through standardized tests (or series of tests) proves that the Authorities' focus is on the competences that children should develop, rather than on the mere number of years of schooling.

indispensable information on others. The British NCDS (National Child Development Survey) has proved to be the best possible compromise.

The next section illustrates in detail the hurdle count model used in this work. This is an popular approach in health econometrics (Jones [2007]) which is gaining increasing popularity in fertility studies as well (Kalwij [2000] and Santos Silva & Covas [2000]). Section 3 introduces the NCDS while section 4 illustrates the actual estimation strategy. Section 5 illustrates and comments the estimation results. Section 6 concludes.

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2. Hurdle count data models

Aim of this study is to provide one of the possible empirical applications inspired from the theoretical model discussed in the previous chapter. It has been chosen to restrict the attention to the total number of children born to the couple (denoted by *N*) and, therefore, to single out a reduced-form econometric model resembling the structure of the theoretical model in the closest possible way.

The piecewise formulation of the utility function reflects a two-stage decision process which can find a straightforward econometric counterpart in two-part models (Cragg [1971]): in the first stage, the couple decides whether to have children or not; in the second one, provided that in the first stage the partners have decided to become parents, the couple decides the number of children it wants to bear. Being the number of children a discrete variable, it is reasonable to rely on that subset of two-part models known as hurdle count data models[4] (Mullahy [1986]).

According to the theory, a couple will have children as long as the level of the minimum quality threshold $\bar{q}$ is lower than the (unobservable) critical threshold $\hat{q}$, i.e. the "highest" minimum threshold the couple is willing to afford. We can think at that difference in terms of a latent variable D^*, such as:

$$(18) \qquad d_i^* = \bar{q} - \hat{q} = w_i'\delta + \mathrm{u}_i$$

where the d_i^* represents the realization of D^* for the couple i; w is a vector of relevant demographic and socio-economic and demographic characteristics of the couple determining its attitudes toward parenthood; δ is a vector of parameters and u is an error term. We observe a

[4] Given that the number of children is count variable, it could be tempting to rely on a simple Poisson regression. Unfortunately, data on complete fertility exhibit with virtually no exception underdispersion: being the variance smaller than the mean, the basic assumption of Poisson regression is violated and this leads to inefficient estimates.

positive number of children N only if $D^* > 0$; therefore, let's define the index variable D as follows:

$$D = 1[D^* > 0] \qquad (19)$$

Let d_i be the realization of D for the i-th couple; the "participation decision" (the decision whether or not to have children) can be described as follows:

$$\begin{aligned} &\Pr(d_i = 0 \mid w_i) = f_1(w_i'\delta) \\ &\Pr(d_i = 1 \mid w_i) = 1 - f_1(w_i'\delta) \end{aligned} \qquad (20)$$

Once the hurdle has been crossed, i.e. the couple has decided to bear children, it is assumed that a different process is responsible for the actual number of children born to the couple; we can write:

$$\begin{aligned} &\Pr(N = n_i \neq 0 \mid x_i, d_i = 0) = 0 \\ &\Pr(N = n_i > 0 \mid x_i, d_i = 1) = \hat{f}_2(x_i'\beta) \end{aligned} \qquad (21)$$

where $\hat{f}_2(\cdot)$ is a zero-truncated process (while the untruncated process $f_2(\cdot)$ is called *parent process* for reasons that will soon become clear); x is a vector of other socioeconomic and demographic characteristics of the couple which may partially or totally coincide with w; and β is a vector of parameters. We can rephrase these assumptions by saying that the reasons underlying the decision of becoming parents are not necessarily the same which lead the couple to give birth to a specific number of children.

The joint density of N and D can be written as follows:

$$g(n_i, d_i \mid x_i, w_i) = (1 - d_i)f_1(w_i'\delta) + d_i[1 - f_1(w_i'\delta)]\hat{f}_2(x_i'\beta) \qquad (22)$$

It is possible to choose $f_1(\cdot)$ and $\widehat{f_2}(\cdot)$ in several different ways according to problem under examination. Popular choices for $\widehat{f_2}(\cdot)$ are the truncated Poisson and the truncated negative binomial while logit and probit are widely used for $f_1(\cdot)$. As we will see, the overwhelming majority of the couples of our sample has given birth to at least one child; this fact suggests to model the participation decision using a complementary log-log link instead, being its greater skweness more suitable for the description of "rare" events:

$$(23) \quad f_1(w_i^{'}\delta) = \exp[-\exp(w_i^{'}\delta)]$$

If the parent process is Poisson, we have:

$$(24) \quad \hat{f}_2(x_i^{'}\beta) = \frac{\lambda_i^{n_i}}{[\exp(\lambda_i) - 1]n_i!}, \qquad \lambda_i = \exp(x_i^{'}\beta)$$

Therefore the log-likelihood of the hurdle model is:

$$\ln L = \ln\left\{ \prod_{d_i=0}\left(e^{-e^{w_i^{'}\delta}}\right) \prod_{d_i=1}\left(1 - e^{-e^{w_i^{'}\delta}}\right) \prod_{d_i=1} \frac{e^{n_i(x_i^{'}\beta)}}{[e^{(x_i^{'}\beta)} - 1]n_i\,!} \right\}$$

$$(25) \quad = \left\{ \sum_{d_i=0} -e^{\left(w_i^{'}\delta\right)} + \sum_{d_i=1} \ln\left[1 - e^{-\left(w_i^{'}\delta\right)}\right] \right\} + \left\{ \sum_{d_i=1} n_i(x_i^{'}\beta) - \sum_{d_i=1} \ln\left[e^{(x_i^{'}\beta)} - 1\right] - \sum_{d_i=1} \ln(n_i!) \right\}$$

$$= \ell_1(\delta) + \ell_2(\beta)$$

It turns out that the log-likelihood of the hurdle model can be decomposed in the sum of the log-likelihood of the binary outcome model, which depends just on δ, and the log-likelihood the truncated at zero model, which depends only on β. This holds for whatever specification is chosen for each parts of the model. Being the vectors of parameters separable in the log-likelihood function, it is possible to fit the hurdle model by estimating the parameters of the complementary log-log regression separately from the ones of the truncated Poisson model.

The expected value is given by:

$$E[n_i \mid x_i, w_i] = \sum_{d_i=0}^{1} \sum_{n_i=1}^{\infty} n_i g(n_i, d_i \mid x_i, w_i)$$

(26)
$$= [1 - f_1(w_i^{'}\delta)]\hat{f}_2(x_i^{'}\beta) = \left(1 - e^{-e^{(w_i^{'}\delta)}}\right)\frac{e^{(x_i^{'}\beta)}}{1 - e^{-e^{(x_i^{'}\beta)}}} = \Phi_i \lambda_i,$$

$$\text{where } \Phi_i = \frac{1 - e^{-e^{(w_i^{'}\delta)}}}{1 - e^{-e^{(x_i^{'}\beta)}}}$$

The formula above makes clear that the expected value of the hurdle model differs from the one of the parent model by the factor Φ_i: furthermore the hurdle model would collapse to the parent model if $f_1(\cdot) \equiv f_2(\cdot)$, i.e. $\Phi = 1$. It becomes clear that if the hurdle model is the correct specification, neglecting this structure and estimating just the parent process (i.e. implicitly assuming that $\Phi = 1$) leads not only to inefficient estimates, but also to inconsistent ones.

If the probability of crossing the hurdle is greater than the sum of probabilities of having $N = n_i > 0$ in the parent process, the mean of the hurdle model is greater that the mean of the parent model and we are in presence of underdispersion; we have overdispersion otherwise.

As Winkelmann [1997] points out, it's worth to notice that in hurdle models underdispersion (or overdispersion) is defined at *individual leve*l and not for the sample as a whole; in fact, the variance function can be written as follows:

(27)
$$Var\left[n_i \mid x_i, w_i\right] = E\left[n_i \mid x_i, w_i\right] + \frac{1 - \Phi_i}{\Phi_i}\left(E\left[n_i \mid x_i, w_i\right]\right)^2$$

where the coefficient $\frac{1 - \Phi_i}{\Phi_i}$ clearly varies among individuals. It is therefore useful to stress once again that if the hurdle specification is the correct one, neglecting that the zeros and the positives originate from different processes leads to loss of both efficiency and consistency.

The peculiar structure of hurdle models makes the interpretation of the estimates not easy to grasp at a first sight. A convenient (and general) way to represent the effect of a marginal change in a regressor consists in explicitly splitting the conditional mean into a hurdle part and a part for the positives:

$$(28) \qquad E[N \mid x, w] = \Pr[n > 0 \mid w] E[n \mid n > 0, x]$$

Let z be $z = w \cup z$, the marginal effects can be computed as follows:

$$(29) \qquad \frac{\partial E[N \mid z]}{\partial z} = \left\{ \frac{\partial \Pr[n > 0, z]}{\partial z} E[n \mid n > 0, z] \right\} + \left\{ \frac{\partial E[n \mid n > 0, z]}{\partial z} \Pr[n > 0, z] \right\}$$

where $E[n \mid n > 0, z]$ is the mean of the truncated Poisson and $\Pr[n > 0, z]$ is the truncation probability. The marginal effect is therefore decomposed in two parts: a direct effect due to a couple moving from the condition of being childless to parenthood and an indirect effect on the number of children of those who already had ones. An equivalent expression is given by:

$$(30) \qquad \frac{\dfrac{\partial E[n \mid z]}{E[n \mid z]}}{\partial z} = \frac{\dfrac{\partial \Pr[n > 0, z]}{\Pr[n > 0, z]}}{\partial z} + \frac{\dfrac{\partial E[n \mid n > 0, z]}{E[n \mid n > 0, z]}}{\partial z}$$

Throughout this work, equations (10) and (11) will be widely used to derive and comment results.

3. The NCDS

While finding a suitable econometric model which resembles the structure of the theoretical approach is not a particularly difficult task, testing its predictions with actual data is much more complicated: as discussed earlier in this chapter, difficulties arise from the fact that the theory is on purpose rather "agnostic" about the concept of quality (and, consequently, of minimum quality) and, at the same time, quite demanding in terms of required data due to the time span covered (all the years from the beginning of marriage to the end of the fertile life of the woman and *possibly* even further to the emancipation of last child). Moreover, one would ideally have data on couples' attitudes towards parenthood as well as separate information on both partners, in order to reconstruct the couple's information set from the day of the wedding on.

The British National Child Development Survey (NCDS) has proved to be the best available source. The NCDS is a continuing longitudinal study which takes as its subjects all the people born in one week in England, Scotland and Wales in one week in March 1958. Follow-ups took place in 1965, 1969, 1974, 1981, 1991, 2000 and 2004. Originally designed to examine the social and obstetric factors associated with stillbirth and death in early infancy, NCDS has developed to much more ambitious survey program which tries to collect information on: "*child development from birth to early adolescence, child care, medical care, health, physical statistics, school readiness, home environment, educational progress, parental involvement, cognitive and social growth, family relationships, economic activity, income, training and housing*"[5].

The NCDS provides a huge source of data indeed but the actual usage of the datasets implies a rather steep learning curve: however, researchers at the CLS (Centre for Longitudinal

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[5] *Economic and Social Data* web site, http://www.esds.ac.uk

Studies) in London are constantly working in order to provide a better documentation and an easier using experience. The NCDS discounts the fact that it was originally thought to serve other aims. Those aims are markedly different and much less ambitious from the ones it has developed over time. This led to several changes to the plan of the survey; for instance, variables have been discontinued or been differently coded in different sweeps of the survey. Furthermore, the very fact that the NCDS deals with several aspects of life implies that not every variable has been measured and coded as one could wish for his/her particular purpose. Unfortunately, family income is one of the variables which would have deserved a better and more coherent treatment, especially in earlier sweeps: not always both pre-tax and after-tax income are present and, furthermore, the measure of income does not always refer every time to the same span (year and/or week); in the earlier sweeps, income is even classified into a certain number of categories. Occasionally the measures of income include benefits but not always. This issues may be the main reason why economists are not among the heavy users of this dataset. As we will see later, a careful work on the data has proved to be necessary to build a coherent measure of the observed income and, ultimately, to build up the final dataset.

It is worth to mention that the British Cohort Study (BSC) and the Millennium Cohort Study, which take as their subject children born in 1970 and 2001-2003 respectively and share several features with the NCDS, will allow in the future to build larger datasets and to perform intergenerational comparisons.

4. Sample and strategy

The sample was constructed as follows: only women interviewed in the 2004 follow-up of the NCDS (NCDS7) were taken into account. These 4889 women were 46 at the time of the interview and therefore their fertile period was (presumably) over. Due to the need of collecting information from several follow-ups of the NCDS, only cases interviewed in every wave from NCDS4 to NCDS7 were kept. Consistently with the theoretical model, only women who married only once and who conceived all of their children (if any) with their husbands are considered[6]. Further drops of inconsistent cases, or cases where crucial variables were missing, led to a sample of 2127 women.

Summarizing, the sample is made of 2127 married women who married only once, completed their fertility and had children –if any- with no one but their husbands.

Having already discussed the abstract econometric model and the sources of the data, the actual implementation of the model is described on end.

Since the most recent raising of school leaving age (from 15 to 16) took place in 1973, the compulsory education, in terms of number of years, remained stable over the marital life of the women of our sample, i.e. it is not possible to exploit that reform as a "natural experiment" for testing the effect of a change of the minimum quality threshold on the total fertility.

On the other hand, it is reasonable to assume that parents take account of the *expenditures* they have to afford to provide their children with the minimum quality rather than the *level* of the threshold

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[6] One may wonder how the estimates obtainable from such a sample carry on whenever we consider out-of-wedlock fertility or women who are divorced, separated or married more than once. One of the main assumptions of the static theoretical model is that fertility decision are taken in a specific moment –conventionally at the "birth" of the decision unit, i.e. the day of the wedding- and do not change over time. So, one could argue that - in terms of an empirical application - it may be not *strictly* relevant whether this decision is made at the time of the first or - say- of the third marriage, or when an out-of-wedlock cohabitation begins; what matters is rather that, once made, that decision is respected. Along to this line, the percentage of women who have had children with more than one partner is more likely to tell us whether or not such an hypothesis may hold. This is the case of only 8% of the women in NCDS7: this suggests that our estimates *may* be valid in a wider set of situations. Nevertheless, it is sensible to use a sample consistent with the theory and to generalize the estimation results only *cum granu salis*. In fact, in order to obtain testable predictions, these special cases should be specifically addressed by specific theoretical models (see the previous chapter for references on this topic).

itself[7]. That means that it is possible to estimate the effect of changes in the *marginal expenditures on minimum quality* (denoted as $\pi_{\bar{q}}$ in the previous chapter) on the couple's complete fertility, whenever either the minimum quality in terms of *skills*, its *cost* or both have changed during the period under examination.

In UK, although with some differences between England and Wales on one hand and Scotland on the other, programmes of study and attainment targets are clearly defined (through the *National Curriculum* in England and Wales and through the *Scottish Curriculum* in Scotland) and, at the end of each key stage of compulsory education[8], pupils are required to sit examination papers on several subjects, while other aspects of their achievements are directly assessed by their teachers. We will consider children attainment targets and private costs relative to primary education, which is usually completed by age of 11[9].

On the official web-site of Qualifications and Curriculum Authority[10] it is stated: "*[The Curriculum] must be responsive to changes in society and the economy, and changes in the nature of schooling itself. Teachers, individually and collectively, have to reappraise their teaching in response to the changing needs of their pupils and the impact of economic, social and cultural change"*. It is possible to interpret this statement in two ways: the minimum quality changes in terms of required skills (i.e. its *composition*) due to changes in society but its *level* is stable; both composition *and* level

[7] It is important to stress that we are dealing with *private* expenditures, i.e. all those resources (parental time and money net of any public subsidy) that parents must provide to let their offspring achieve the required minimum attainments.

[8] For further information on study programmes, attainment targets, and descriptions of proficiency levels in UK , see http://curriculum.qca.org.uk/index.aspx and http://www.scotland.gov.uk/Topics/Education/Schools/curriculum

[9] This represents the end of the key-stage 2 in England and Wales. In Scotland pupils transfer to secondary school at age 12 and the Curriculum embraces the whole period 5-14; a transition to the *Curriculum for Excellence*, which considers pupils from the age 3 to 18, is presently taking place.

[10] http://curriculum.qca.org.uk/key-stages-1-and-2/Values-aims-and-purposes/index.aspx

change[11]. If the second interpretation is the right one, it is likely that changes go toward an increase of the minimal required competences. Whatever interpretation one adopts, it can be peacefully assumed that the minimum quality *did not decrease* over time.

Now, whenever the quality of teaching deteriorates but minimal standards remain the same or rise, a larger fraction of overall cost of the minimum quality is put on the family's back. Employing the widely accepted idea that smaller class-sizes allow better communication and teaching and assuming that the average quality of the teachers does not change over time, we proxy the private marginal expenditures on minimum quality with the primary school's pupil/teacher ratios observed by the couple in the year of their wedding[12] at a State level[13]: the higher is the observed ratio, the worse is the expected quality of teaching and, consequently, the more parents have to get directly involved.

[11] For instance, only few decades ago calligraphy was an important subject in primary schools; nowadays an analogous role is played by basic ITC skills, while calligraphy is not even a subject any longer. In such a case, since a new subject *replaces* an obsolete one, the composition of the minimum quality changes, but not (or, at least, not certainly) its level. In turn, the level changes (i.e. rise) whenever new subjects are added *on top* of the existing ones or when the attainment targets become more ambitious.

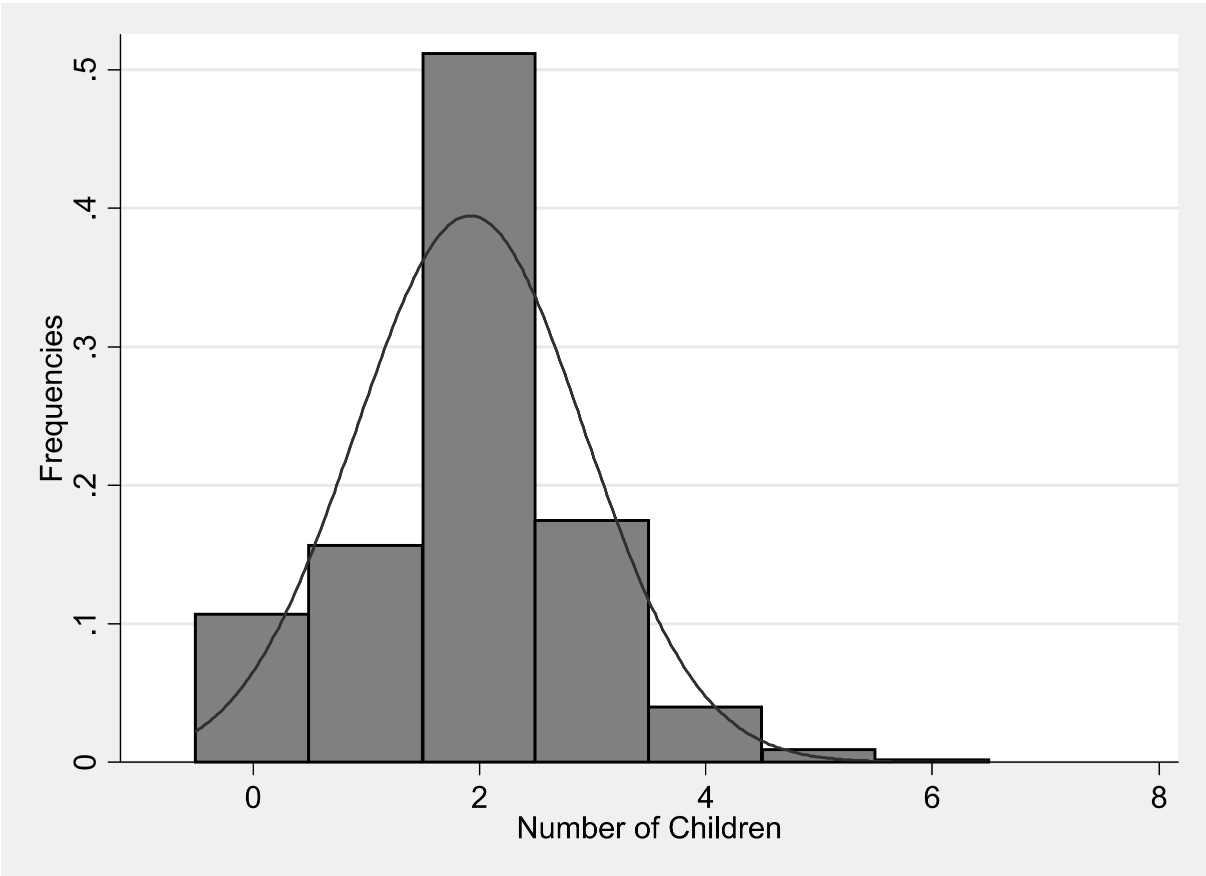
[12] We want to reconstruct the couple's information set at the time when the partners are supposed to make their fertility decisions.

[13] The smaller is the territorial cluster, the more precise are the estimates. Unfortunately only data at State level were available for the whole period under examination (1975-2000).

5. Analysis

The final dataset includes variables derived from NCDS4, NCDS5, NCDS6, and NCDS7. A short description follows.

NC: number of children. It is the dependent variable. More than 50% of the couples have two children while about 10% are childless; the maximum value in our sample is six while parities above three are increasingly rare and the variable clearly exhibits underdispersion, a typical feature of complete fertility data.



Plot 1: Number of children: observed frequencies

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income: it is the couple's discounted (3% per year) stream of income in thousands of Pounds divided by the number of available observations. Each observed income corresponds to the after-tax weekly income regardless of its source, including therefore wages, benefits, rents, etc. According to the year of marriage, up to four observations (1981, 1991, 2000 and 2004) for each couple are available. Strictly speaking, income is not an estimation of the permanent income: in fact, we are dealing with a reduced form model which includes some of the variables that would be involved in a standard estimation of the permanent income (read below).

Variable	Obs	Mean	Std. Dev.
income	2127	.574	.591
age_diff	2127	2.761	3.805
study	2127	12.352	2.122
study_diff	2127	-.0808	2.565
agemar	2127	23.585	4.6
satisfactory	2127	.142	.349
incomplete	2127	.707	.455
rel_attend	2127	.699	.459
ratio	2127	22.484	.881
exposure	1900	18.831	4.366

Table 1: Summary Statistics

inc2: income squared.

***age_diff*:** partners' age difference (age of the husband minus the age of the wife).

study: wife's number of years of full-time education.

***study_diff*:** partners' education difference (as before).

***agemar*:** wife's age at marriage. It is a measure of exposition to the pregnancy risk for the sample as a whole. Most of the women in the sample married in their early twenties but, as the plot below shows, the distribution of the age at first child is less skewed: this suggests that the actual exposition to pregnancy risk does not necessarily coincide with the duration of marriage.

Plot 2: Age at Marriage and Age at First Child (Observed frequencies)

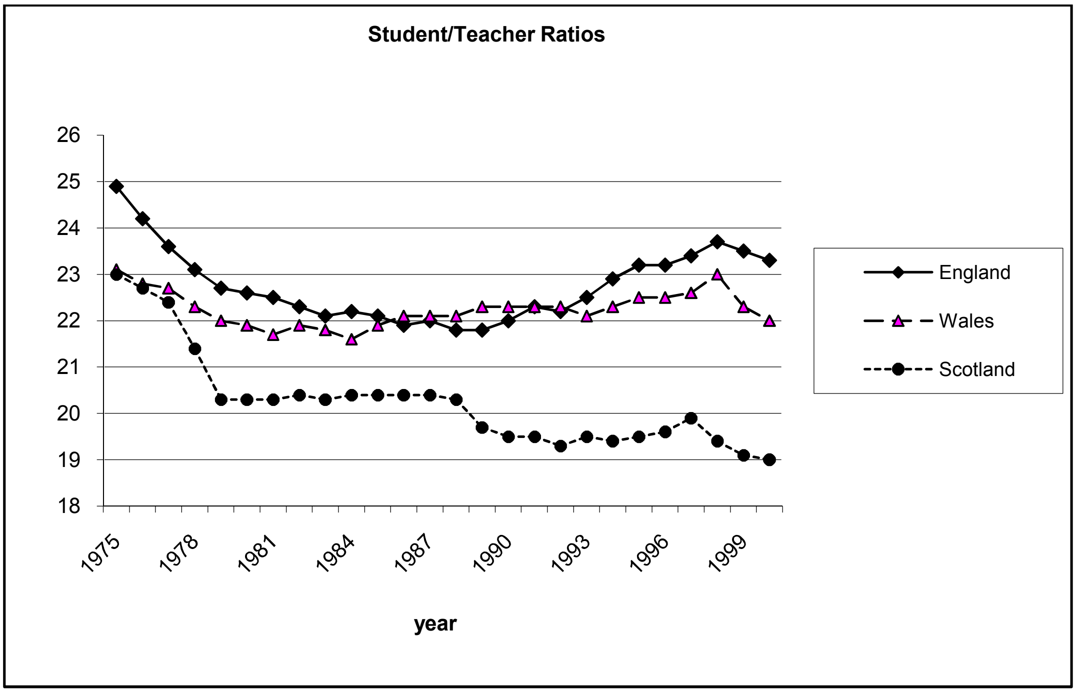
***exposure*:** years of actual exposure to pregnancy risk, i.e. 46 (assumed to be the end of fertile life) minus age at first pregnancy ended with a live birth.

***satisfactory*:** dummy variable; it is one whether the wife either agrees or strongly agrees with the sentence: "*people can have a satisfactory relationship without children*". This variable and the next one come from the "What do you think" attitude self-completion questionnaire (NCDS5).

These attitudes were not recorded in the exact year of marriage but, consistently with the theoretical approach, it is assumed that such opinions do not change over time. For the same reason, the same attitudes are also attached to the husband.

incomplete*:** dummy variable; it is one whether the wife either disagrees or strongly disagrees with the sentence: **"a marriage without children is not fully complete".* This variable and the previous one are needed to infer the couple's attitudes toward parenthood.

***ratio*:** students/teachers ratio in primary schools in the year of marriage at a country level. This is a proxy of marginal expenditures on minimum quality.



Plot 3: Pupil/Teacher Ratios by Year and Country

England, Wales and Scotland are self-explanatory geographical dummies. Approximately 85% of the sample resides in England, 9% in Scotland and the rest 6% in Wales.

rel_attend*:** dummy variable; it equals one if the wife answers either "less than once a month" or "never" to the question: **"how often, if at all, do you attend services or meetings connected with your religion?".* It is zero also if the respondent declared to belong to no religion at all. No other religion-related variable is used and, notably, no variable indicating the actual religion: the reason is that almost the whole sample is made of Christians and that the differences among the diverse Christian confessions did not prove to be statistically significant at any conventional level.

Estimates

The table below illustrates the hurdle model estimates where the zeros are drawn from a complementary log-log distribution and the positives from a truncated Poisson.

The regressors of the complementary log-log and the ones of the truncated Poisson do not perfectly coincide for obvious exclusion restrictions. In particular, the exposure to pregnancy risk coincides with the entire duration of marriage, if we consider the sample as a whole, while it is possible to be more precise if we consider couples who gave birth to at least one child. Furthermore, attitudes towards parenthood, i.e. whether a marriage without children can be considered "complete" or make sense at all, play a role in determining the probability of becoming parents but are irrelevant once the hurdle is crossed, since they do not deliver any further information about the desired family size.

As stated before, the interpretation of the coefficient is not easy to grasp at a first sight: equation (11) makes clear that a marginal change in any independent variable alters both the probability of crossing the hurdle and the total number of children chosen by the couple. In order to gain a solid intuition of the results, it is necessary to study the overall effect of marginal changes in independent variables on the fertility outcomes of some representative *ad hoc* chosen individuals. However, it is possible to obtain a deeper insight of the effect of each independent variable by separately examining the two sets of results. The discussion of the hurdle models estimates will be therefore divided into three sections: the first two sections will deal with the effects of marginal changes in the regressors on the probability of becoming parents and on the total number of children chosen by the couple respectively; the third one will recompose the previous results and provide an interpretation of the estimates of the complete hurdle model.

Number of Children (NC): Hurdle Model Estimates		
	Zeros (compl. log-log)	**Positives (truncated Poisson)**
Income	-0.771*** (0.28)	0.0937** (0.055)
inc2	0.361** (0.14)	-0.00701 (0.0065)
age_diff	-0.0345*** (0.0081)	-0.0114** (0.0058)
Study	0.0438** (0.018)	0.0377*** (0.010)
study_diff	0.0339** (0.014)	0.0205** (0.0082)
Agemar	-0.0714*** (0.0085)	
Satisfactory	-0.660*** (0.084)	
Incomplete	-0.189*** (0.073)	
rel_attend	0,088194444 (0.070)	-0.108** (0.043)
Ratio	-0.267*** (0.056)	-0.176*** (0.035)
Wales	-0.0613 (0.15)	-0.200** (0.086)
Scotland	-0.522*** (0.15)	-0.392*** (0.098)
Constant	8.679***	1.254
Exposure		1

Log-likelihood: -3030.132

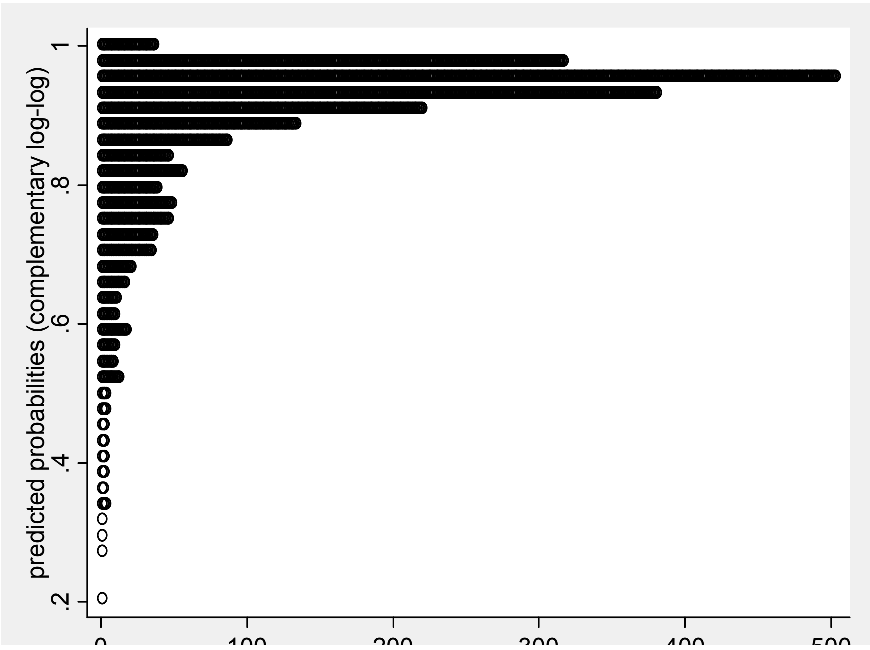
Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

From now on, the expressions "average" and "median couple" will be used to relate to the ideal reference units where each regressor is set at the sample mean and median respectively.

Moreover, the marginal effect of the observed income is computed taking into account that both *income* and *inc2* are included in the regression equations.

<u>Complementary log-log regression estimates</u>. Through the complementary log-log regression we estimate the probability of "crossing the hurdle" (i.e. the chance of having at least one child). From the summary statistics we know that 10.7% of the couples in the sample are childless. It is possible compare this number with the complementary log-log estimates: the "average couple" has 92.7% probability of having children and the chances of becoming parents are even higher for the "median couple" (94.3%). It is crucial to notice that while couples living in England and Wales do not exhibit significant differences in their probability crossing the hurdle, Scottish couples have significantly less chances of doing th same. For instance, take two identical "average couples" who differ only because one resides in England while the other in Scotland: while the English couple has a predicted probability of having children of 93.6%, the Scottish one has a probability 20% of remaining childless. Nevertheless, it can be still peacefully affirmed that the overwhelming majority of married couples in UK is likely to have at least one child. Keeping in mind this result, it is interesting to investigate the effect of a (marginal) change in income on the probability of crossing the hurdle.



Plot 4: Predicted probabilities of crossing the hurdle

Complementary log-log: marginal effects		
	Average Couple	**Median Couple**
income	-8,18%	-7,13%
age_diff	-0,79%	-0,56%
study	1,01%	0,71%
study_diff	0,78%	0,55%
agemar	-1,64%	-1,16%
*Satisfactory**	-19,08%	-17,02%
*Incomplete**	-4,12%	-2,55%
*rel_attend**	0,29%	0,21%
ratio	-6,14%	-4,35%
*Wales**	-1,46%	-1,05%
*Scotland**	-14,91%	-12,54%

*Marginal effects are for a discrete change of the dummy variable from 0 to 1

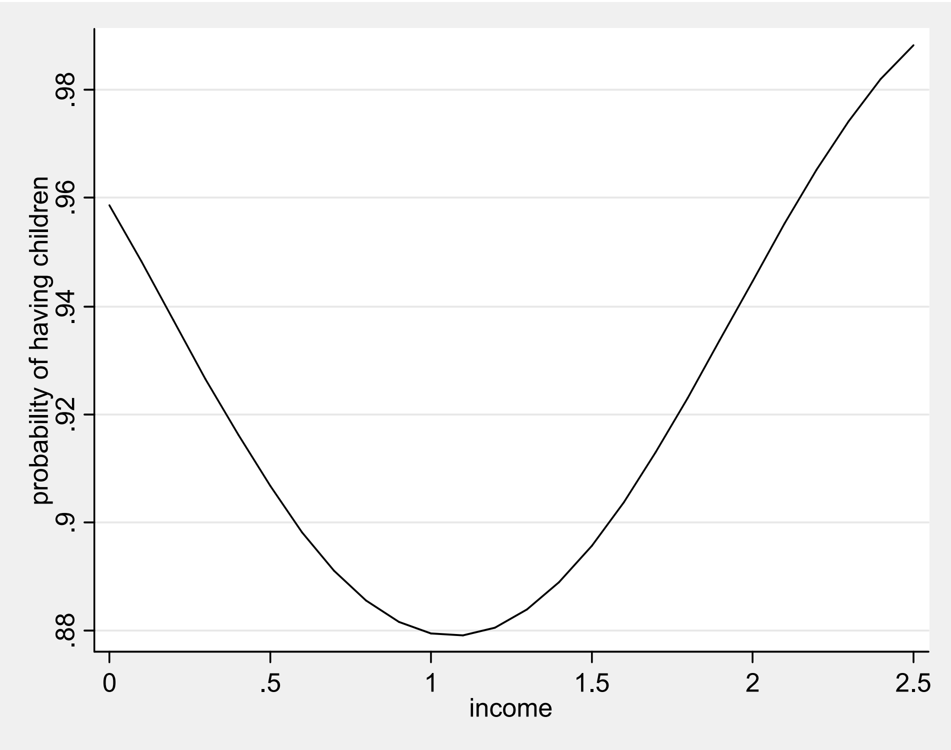
Marginal effect of *income* computed taking into account the squared term *inc2*.

	Changes in predicted probability				
Median couple					
	min->max	0->1	-+1/2:	-+sd/2:	base values
INCOME	2.3%	-5.8%	-6,0%	-4%	0.46
age_diff	-26,3%	-0,5%	-0,6%	-2,1%	2
Study	6,7%	1,3%	0,7%	1,5%	11
study_diff	11,1%	0,5%	0,6%	1,4%	0
Agemar	-46,2%	0,0%	-1,2%	-5,4%	22
Satisfactory		-17,1%			0
Incomplete		-2,6%			1
rel_attend		0,2%			1
Ratio	-22,0%	0,0%	-4,4%	-3,9%	22,5
Wales		-1,1%			0
Scotland		-12,6%			0
	no children	at least one child			
Pr(yx)	0,057	0,943			
Average couple					
	min->max	0->1	-+1/2:	-+sd/2:	base values
INCOME	3.1%	-6.8%	-6.2%	-4.0%	0.574
age_diff	-25,5%	-0,5%	-0,6%	-2,1%	2,761
Study	7,7%	1,4%	0,7%	1,5%	12,353
study_diff	11,0%	0,5%	0,6%	1,4%	-0,081
Agemar	-42,7%	0,0%	-1,2%	-5,4%	23,585
Satisfactory		-17,0%			0
Incomplete		-2,5%			1
rel_attend		0,2%			1
Ratio	-22,1%	0,0%	-4,4%	-3,8%	22,484
Wales		-1,1%			0
Scotland		-12,5%			0
	no children	at least one child			
Pr(yx)	0,057	0,943			

Pr(yx):	probability of observing each y for specified x values
Min->Max:	change in predicted probability as x changes from its minimum to its maximum
0->1:	change in predicted probability as x changes from 0 to 1
-+1/2:	change in predicted probability as x changes from 1/2 unit below base value to 1/2 unit above
-+sd/2:	change in predicted probability as x changes from 1/2 standard dev below base to 1/2 standard dev above

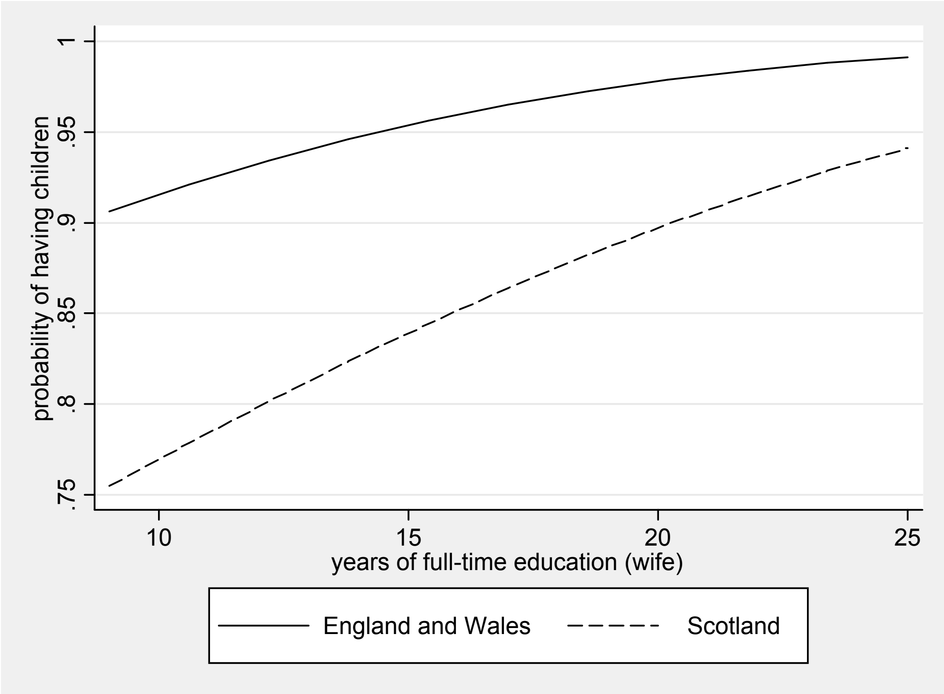
Both *income* and *inc2* are significant at least at a 95% level and have opposite signs; they draw therefore a parabolic path.

The fact that the sign of *income* is negative may be counterintuive at a first glance but it is a possibility explicitly considered by the theory illustrated in the previous chapter: the key is that couples may have different preferences or attitudes towards the consumption of private goods (i.e. goods not related to children) depending on the very fact that they are parents or not. The consumption of goods attainable with a higher income may be somewhat hindered by the presence of children and also *compete* with expenditures on children: we may think of holydays, dinners in fancy restaurants and luxury goods in general. The estimates suggests us that a share of married couples may fall under the spell of a higher "private" consumption as their income rise and decide therefore not to have children. Nevertheless, private consumption and children-related goods become less and less rival as income increases further and finally the probability of having children approaches one for very wealthy couples. The reversal takes place at a value of income of 1.06, a level not reached by 92.3% of the couples in the sample. That means that an increase in income lowers the probability of becoming parents for about 9 couples out of 10, being the 10th already relatively wealthy. On the other hand, it is important to recall that, even if



Plot 4: Predicted probability of crossing the hurdle and income

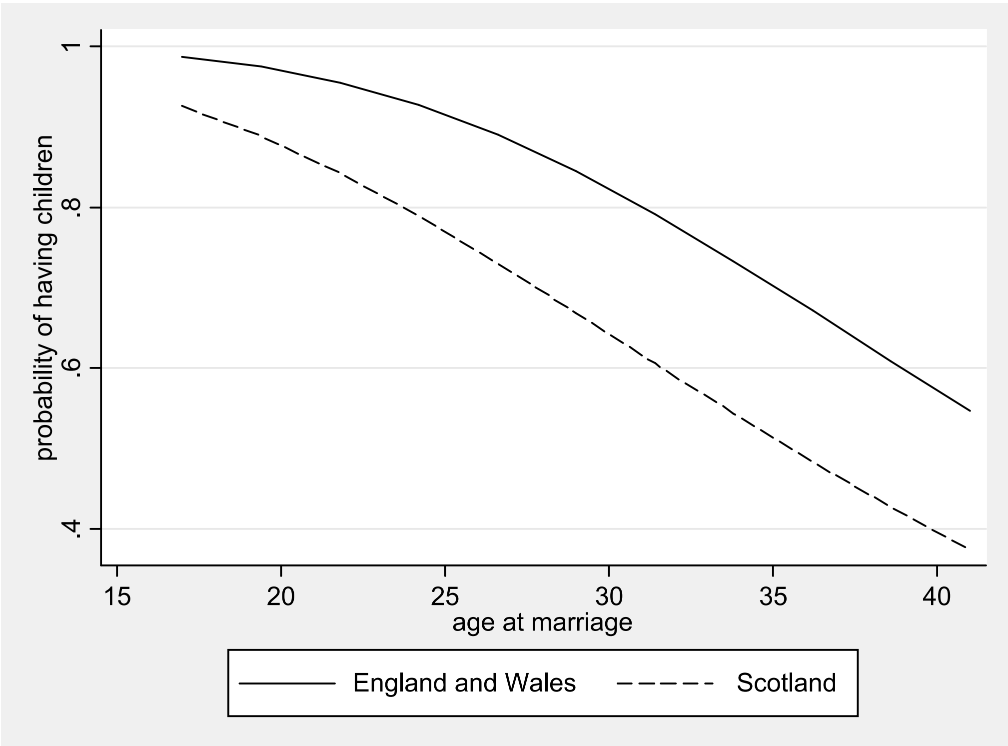
the probability of crossing the hurdle declines with income, it remains - on average - firmly above 80%. It is crucial to stress once again that the measures of



Plot 5: Predicated probabilities and wife's education

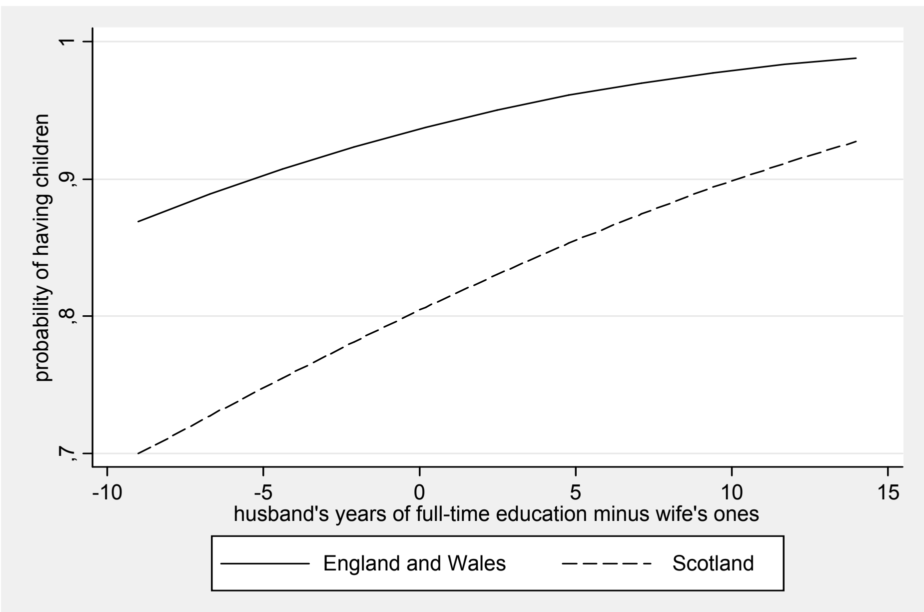
income are one of the weakest point of NCDS. This lack of precision *may* have an impact on the estimates of other variables which one would use as independent variables in the estimation of the permanent income, such as *study* (the years of wife's full-time education) and *study-diff* (the partners' education difference). Even though this study would greatly benefit of more and better measures on income, it is certainly true that these estimates provide a reliable "compass", useful to understand the underlying dynamics of the impact of observed income in child-bearing decisions. Incidentally it happens that this result is also consistent with macro-data showing a negative correlation between income and fertility rates. In the light of here-proposed theory, one should not draw the wrong conclusion that children are "inferior goods"; in fact, it will be shown that, consistently with the idea that children are "normal goods", *income* is significant and has a positive sign in the truncated Poisson regression; the result should be interpreted as evidence that childless couples enjoy a higher marginal utility of (own) consumption than couples with children and therefore an increase in income can negatively affect the decision of becoming parent. This interpretation would not have been possible without the piece-wise utility function approach proposed in the previous chapter.

As expected, that the younger is the bride, the higher is the probability that the couple will have children: for the average couple, each year of delay of marriage causes a reduction of 1,6% percent points in the probability of having children; for the median couple, the reduction is of 1,2% percent points. Everything else being equal (sample mean), a women marrying at 17 has a huge 42,7% more chances of becoming mother than one marrying at 41. The effect is even stronger if we consider the median couple (where the woman married at 22 and the probability of becoming parents is 94,3%): we notice that delaying the age at marriage from 17 to 41 years reduces the probability of having children of 46,2%. The plot below shows how the estimated probabilities of having children for the average couple change when the age at marriage of the bride rises: the difference between women residing in Scotland with women living either in England or in Wales is always noticeable (at least minus 12,5% for the "average" couple) and becomes larger as the age at marriage rises. Please note that *agemar* enters into the complementary log-log equation while it does not the truncated Poisson regression equation. The reason is that we have more information about couples



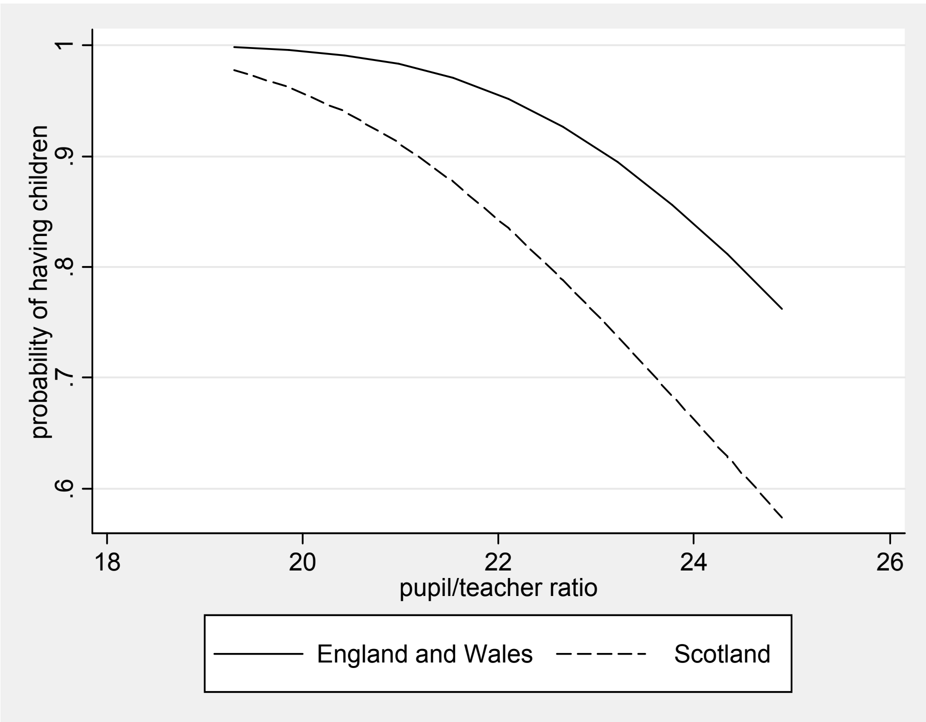
Plot 6: Predicted probability of crossing the hurdle and wife's age at marriage

with children - notably the age at which the woman delivered her first child: this allows us to be more precise (or at least less rough) in determining the time-span of pregnancy risk and use this quantity as exposition parameter in the truncated Poisson regression. When we consider the sample as a whole, we have to rely to the duration of the marriage instead. Now we can turn to the effect of education of husband and wife on the probability of becoming parents. Before 1973, in UK full-time education was generally compulsory from 5 to 14 with some organizational differences among England, Wales and Scotland. Summary statistics of our sample show that the average woman's full-time education is indeed very close to 12 years (12.4) while, on average, men study two more years. The marginal effect of woman's years of full-time education is positive: for the average couple, a further year of wife's education increases the probability of having children of 1,1% (0,7% for the median couple); for the same woman, shifting from minimum to the maximum observed education causes a change in the probability of 7.7% (6.7% for median values). It can be surprising to find a positive effect of woman's education on the probability of becoming mother as it is often argued that parenthood has a higher opportunity cost for more educated women. First of all, it is important to recognize that woman's education is positively correlated with the couple's permanent income; secondly, the theory predicts that the cost of producing child quality (and of reaching the required minimum quality standard) is lower for more educated women. Our estimates would therefore suggest that the positive effect of a reduced cost in delivering quality to children *may*



Plot 7: Predicted probabilities and partners' education differences

overcome the negative opportunity costs of parenthood. However, given the role of partners' education in determining the household's permanent income, the last hypothesis has to be taken *cum granu salis*. Similar arguments hold for the effect of husband's education. Whenever the man is less educated than the woman, the effect on the probability of parenthood is negative, while it is positive if he is more educated than his wife. *Ceteris paribus* (sample mean), the marginal effect of one year of difference in full-time education on the probability of becoming parents is 0.8%, i.e. a little bit smaller than the one observed for women's education. Nevertheless, shifting from the minimal to the maximal difference increases the probability of about 11% both for the average and the median couples. Once again, keeping in mind the probable positive correlation of this variable with the household's permanent income, it is possible to argue that a more educated father is indeed able to deliver quality to his children at a fraction of the cost incurred by a less educated one. These effects induced by *study* and *study_diff* follow the same pattern in England/Wales and Scotland and in both cases the distance between the curves tend to reduce as the education rises but these variables seem to play a more important role in Scotland than in the rest of UK.



Plot 8: Probability of crossing the hurdle and marginal expenditures on minimum quality

If the husband is younger than the wife, the couple has slightly more chances of being having children: everything else being equal (sample mean), every further year of difference between husband and wife decreases the probability of becoming parents of 0.8% percentage points. This result may be read together with the one we obtained for *income*: it is reasonable to guess that a older man is enjoying a higher (personal) income and therefore he may be less partial to parenthood.

The two dummy variables *satisfactory* and *incomplete* register the couple's attitudes toward children and, more precisely, its opinion about the role of children in a marriage. Not surprisingly, people who think that a relationship (a romantic relationship in general, not specifically a marriage) can be satisfactory without children tend to have a lower probability of becoming parents[14]: the marginal effect is to lower the chance of parenthood by 19% for the average couple. People who do not agree that a marriage without children in incomplete tend also to have lower probability of becoming parent but the marginal effect is less strong (-4,12%). There is a weak positive correlation (0.188) between agreeing with the first proposition and disagreeing with the second. Arguably the fact that the first question is posed in a positive and very general manner allows a more sincere answer than the second one, which is formulated in a negative way and focuses specifically on married couples. The agreement with the second statement may therefore have sounded somewhat "offensive" towards those who cannot have children and not very "politically correct" indeed: this is the main reason why both variables have been included in the regression. Interestingly, these attitudes seem to play a much more important role in determining the probability of becoming parents than the active participation to a religious community, even though almost one third of the couples declares to attend religious rites at least once a month: in fact, the Wald test does not reject the null hypothesis that the

[14] *Satisfactory* is potentially endogenous since also couples who *cannot* have children may give a positive answer. It is therefore likely that the estimates for that variable are a bit positively biased. Similar arguments hold for *incomplete*.

coefficient of *rel_attend* is zero. After all, just by looking at the summary statistics, it is possible to notice that more than 90% of the couples have children while only one third is religious: this suggests that the urge to reproduce is largely driven by extra-religious factors. Nevertheless, as it will be shown in a while, the religiousness of the couple does indeed play a role in determining the number of children once the hurdle is crossed (arguably this is due to induced behaviours in terms of births control).

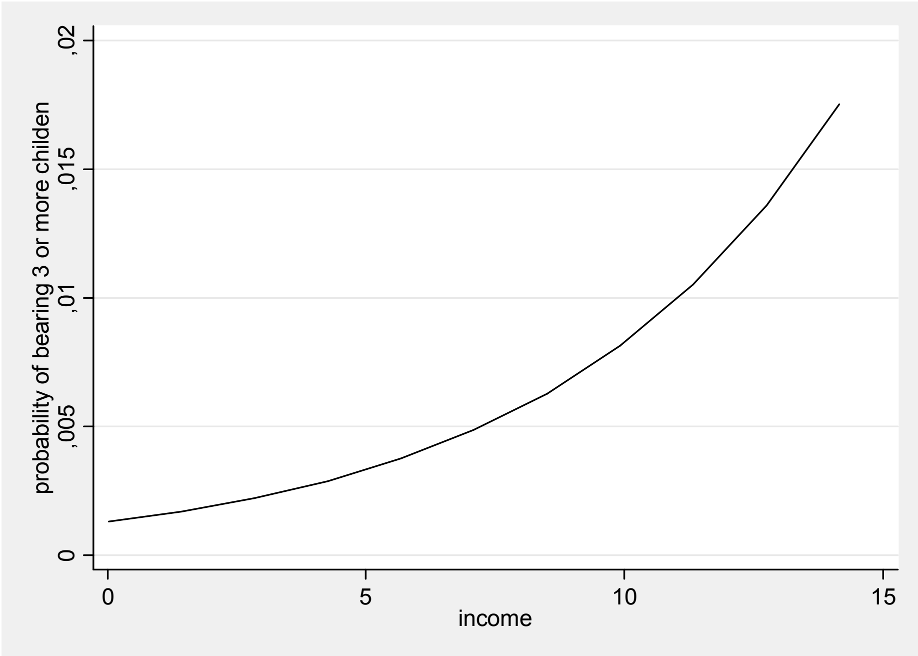
The variable *ratio* proxies the marginal expenditures on minimum quality (denoted as $\pi_q = p\overline{q}$ in the previous chapter) and is defined as pupil/teacher ratio in the primary at the year of marriage at a country level. As predicted by the theory, the effect of a rise of the pupil/teacher ratio is clearly very negative: the marginal effect for the average couple of a rise in *ratio* is a decrease of 6,1 percentage points in the probability of having children for the average couple (-4.35 points for the median one). No matter if we take into account the average or the median couple, it turns out that shifting from the lowest ratio (19.3) to the highest one (24.9) causes a decline of about 22 percent points in the probability of having children. It is important to notice (see above the plot of summary statistics) that *ratio* does not exhibit a monotone trend but a slightly parabolic one: in turn, the correlation between the age at marriage and *ratio* (-0.42) is not particularly problematic in terms of multicollinearity. Similarly to what happens with the other explanatory variables, the difference between the predicted probabilities for Wales and England on one side and Scotland on the other one tend to increase as the pupil/teacher ratio increases: for instance, taking two identical couples, one living in Wales and the other in Scotland, both facing the highest value for *ratio* (all other variables are set to the sample mean), the predicted probability of becoming parents is 74.3% in the first case and 57.5% in the second, i.e. a difference of almost 17 percent points; if we take the minimum value of *ratio*, the difference in the predicted probabilities is just of 2 percent points (99.8% for Wales and 97.8% for Scotland). The estimation results confirm the theory predicting a substantial decrease in the probability of having children as the expenditures on minimum quality rise (this was described as a shift to the left of the "critical" minimum quality threshold in the previous chapter).

b. Truncated Poisson regression estimates. The very first thing to notice is that about half of the couples in the sample have two children; the zero-truncated Poisson alone tends to underestimate this modality while it overestimate the number of couples having just one child; it does a much better job for higher parities[15]. The theory predicts a positive effect of income on

Truncated Poisson: marginal effects				
	Average Couple		**Median Couple**	
NC	*Coef.*	*Std. Err.*	*Coef.*	*Std. Err.*
income	0,153	0,088	0,150	0,086
age_diff	-0,020	0,010	-0,020	0,010
study	0,068	0,019	0,065	0,017
study_diff	0,037	0,015	0,035	0,014
rel_attend	-0,198	0,080	-0,197	0,080
ratio	-0,316	0,062	-0,304	0,063
Wales	-0,329	0,130	-0,313	0,124
Scotland	-0,603	0,129	-0,559	0,122

the number of children chosen by the couple. This prediction is confirmed by the zero-truncated Poisson regression estimates: the one-tail Wald test confirms that the effect of *income* is positive at a 95% significance level; at the same time, *inc2* is not significant at any conventional level. Keeping in mind the *caveat* about the quality of the measures of income, it turns out from our

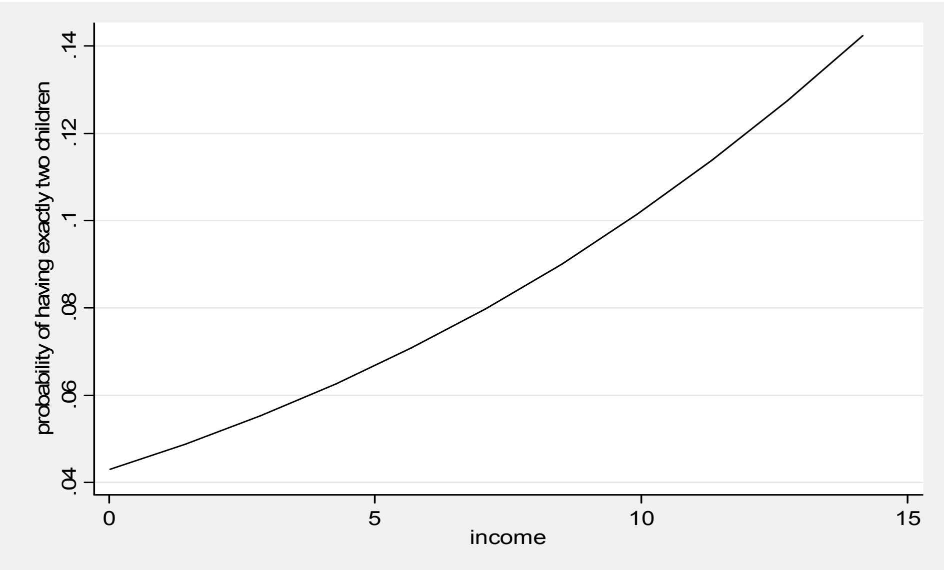
[15] Recall that the frequencies predicted by the zero-truncated Poisson do not coincide with those predicted by the complete hurdle model because they are not weighted by the probability of crossing the hurdle.



Plot 9: Probability of bearing 3 or more children

estimation that the effect of the observed income on the decision about the total number of children is not particularly strong: for instance, if we take into account an English religious couple (*rel_attend* is zero) whose other characteristics are the same of the median couple, we see that shifting from the minimum to the maximum observed income boosts the probability of having two children of 10 percent points (from 5% to 15%) but the effect is low or even negligible for higher parities. We would have very similar figures if we took a couple with average characteristics instead. In general, the ability of *income* to explain a large number of children is somewhat feeble, as the plots show.

Taking the average couple into account, a further year of full-time education of the woman leads to an increase of 0,07 children everything else being equal. For the average English couple, moving from the minimum to the maximum observed number of years of woman's full-time education reduces the probability of having just one child of about 4 percent points (from 95.3% to 91.6%) while the probability of having two children increases of 3.4 percent points, from 4.5% to 7.9%.



Plot 4: Probability of having 2 children

Similarly, there is also an increase in the probability of higher parities as the education of the woman increases. Given that *study* is likely to capture part of the permanent income effect, this result can be also interpreted as we did before for the complementary log-log estimates: more educated women may be able to endow their children with the same level of quality at a lower cost than less educated ones can do; even if bearing and educating children has a higher opportunity cost for these women, this cost may be completely offset and finally overcome by what they save by producing quality for their children more efficiently. This result suggests an intuition that would benefit of a more in-depth investigation, if only suitable data

Predicted probabilities (median couple, religious)		
Conditional probabilities of having:	**income**	
	min	max
1 child	0,95	0,83
2 children	0,05	0,15
3 children	0,00	0,02
4 children	0,00	0,00

Predicted Probabilities (England, religious, rest: average)		
Conditional probabilities of having:	**pupil/teacher ratio**	
	min	max
1 child	0,910	0,965
2 children	0,084	0,034
3 children	0,005	0,001

were available: a woman is willing to buy childcare in the market as long as the its price is lower than the opportunity cost of her time *and* if its quality is *at least* as good as the one she could produce by herself. Similar arguments hold for *study_diff*, the difference of years of full-time education between husband and wife. If we take the average couple into account, each year of difference causes an increase of 0,04 children in the demanded number of children.

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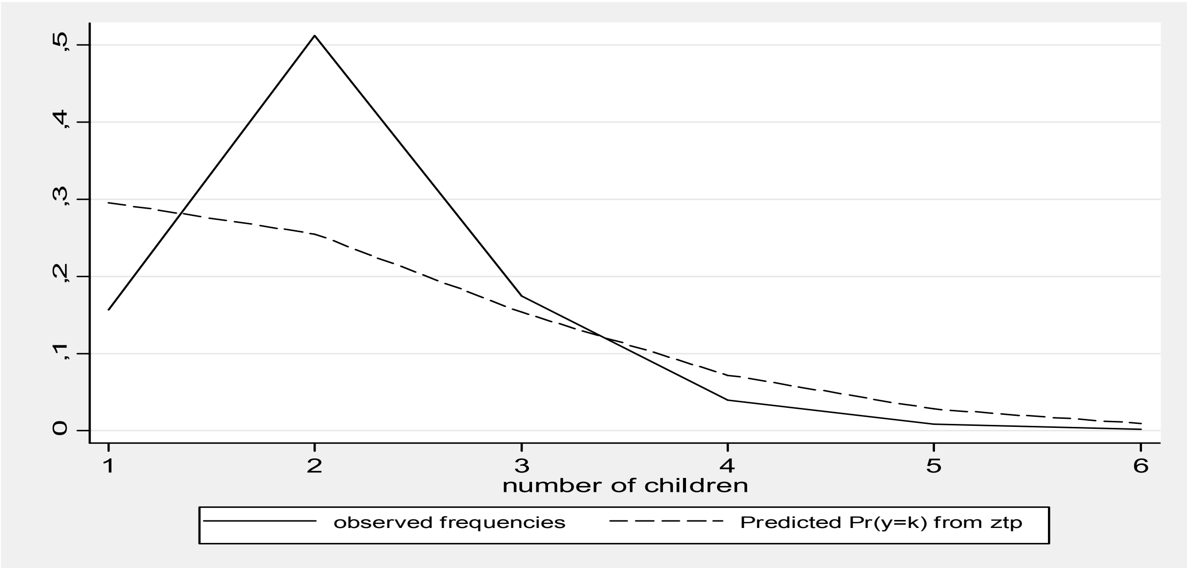
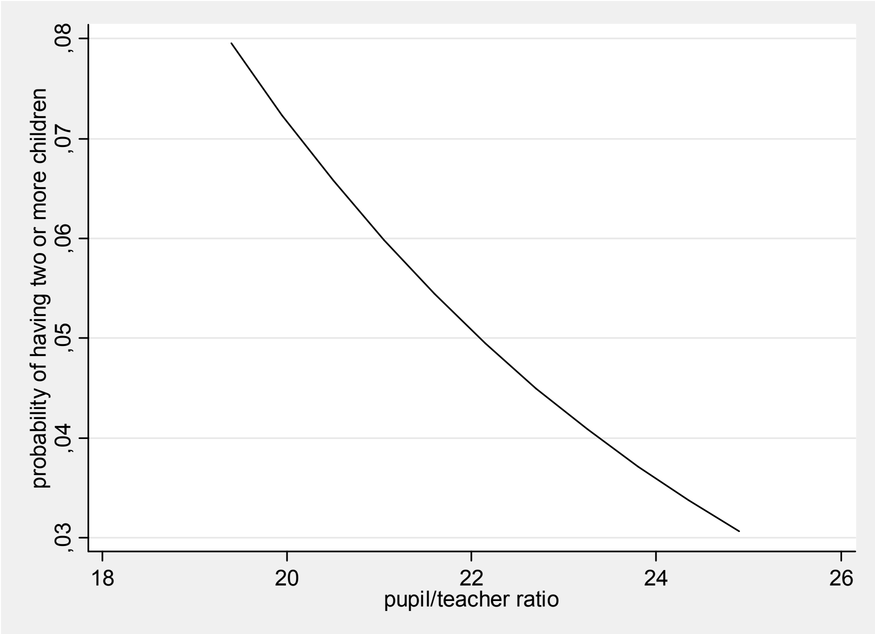


Plot 10: Observed and predicted frequencies (zero-truncated Poisson)

As we have experienced with the complementary log-log regression, the effect of the partners's age difference is slightly negative: each year of difference causes a reduction of 0,02 in the demanded number of children (all other variables set at the sample mean).

Predicted probabilities (England, not religious, rest average)		
Conditional probabilities of having:	**study**	
	min	max
1 child	0,953	0,916
2 children	0,045	0,079
3 children	0,001	0,005

In contrast with what we obtained before on the probability of becoming parents, the religiosity of the parents do play a role in determining the total number of children demanded by the couple. An average participating at least once a month to religious rites has 0,2 more children than a non-religious one. The fact that being religious does not play any significant role in determining the probability of becoming parents but favourably influences the total number of children demanded by the couple can be interpreted as follows: the choice of becoming parents is largely determined by factors that may be labelled as "extra-religious" and it is a choice made by the almost nine couples out of ten; on the other hand, embracing a faith may have a deep impact on factors such as the frequency of coitus, the use of contraceptive methods and the attitude towards abortion: all these aspects may contribute together to higher fertility rates.



Plot 11: Effect of the marginal expenditures on minimum quality on total fertility

English couples have significantly more children than Welsh and Scottish ones: everything else being equal (sample mean), a couple of Cardiff has 18.2% less children and a couple of Glasgow even almost one third less (-32.5%).

The effect of *ratio*, the variable that proxies the expenses on minimum quality, is strongly negative: using the average couple as a benchmark, a one unit change in the pupil/teacher ratio causes a decrease of 0,32 of the demanded number of children. Shifting from the minimum to the maximum observed pupils/teacher ratio (everything else held constant and set at the sample mean), causes a decrease of the probability of having two children of 5 percent points, while the probability of having just one increases by the same proportion. This provides a strong evidence of the negative role of the expenditures for minimum quality on the total fertility. More importantly, taking the average couple into account, the marginal effect of an increase of the expenditures on minimum quality is about two times the one caused by an increase of observed income (0.15): whether both permanent income and expenditures on minimum quality grow (the latter increase may be caused by either a higher level of minimum quality, a higher price of quality or both), fertility can remain stable only if the increase of income overcomes the increase for expenditures on minimum quality.

Marginal changes in predicted counts (ztp)					
Average couple					
	min->max	0->1	-+1/2:	-+sd/2:	base value
income	0.008	0,008	0.008	0.005	0,574
age_diff	-0,033	-0,001	-0,001	-0,004	2,619
study	0,070	0,002	0,004	0,008	12,343
study_diff	0,048	0,002	0,002	0,005	-0,053
rel_attend		-0,011			1
ratio	-0,103	-0,825	-0,017	-0,015	22,486
Wales		-0,018			0
Scotland		-0,031			0
Median couple					
	min->max	0->1	-+1/2:	-+sd/2:	base value
income	0.007	0.008	0.008	0.005	0,46
age_diff	-0,031	-0,001	-0,001	-0,004	2
study	0,070	0,002	0,003	0,007	11
study_diff	0,046	0,002	0,002	0,005	0
rel_attend		-0,010			1
ratio	-0,098	-0,785	-0,016	-0,014	22,5
Wales		-0,017			0
Scotland		-0,030			0

Whenever the underlying tendency in a society is such that expenditures on minimum quality rise faster than income, the positive effect of income on the overall rate of fertility is completely sterilized and even subdued: neglecting the existence of a minimum quality standard would therefore lead to the wrong conclusion that income has a negative impact on fertility, i.e. that children are "inferior goods".

c. Recomposing the results: hurdle model estimates. As equation (11) shows, the overall marginal effect of a change in the covariate *x* on the expected number of children born to a couple depends both on the change in the probability of becoming parents and, once the hurdle has been crossed, on the change in the total number of children induced by that change in *x*. The previous sub-sections disentangled these two components of the hurdle model in order to: 1) explicitly show the "weight" of each component in the overall effect; 2) parallel the empirical approach with the theoretical model. Once again, it will be necessary to choose some representative "types" to comment the results.

The following table shows the estimated probabilities of having *n* children. The hurdle model predicts almost perfectly the extremes of the distribution, i.e. the "zeros", the "fives" and the "sixes"; it makes an excellent job with the "threes" and the "fours" as well, just overestimating them a bit.

Predicted probabilities from hurdle model

Number of children	Mean	Std. Dev.	Min	Max
0	0,107	0,111	0,000	0,797
1	0,332	0,075	0,113	0,699
2	0,283	0,034	0,067	0,324
3	0,169	0,041	0,006	0,236
4	0,079	0,031	0,000	0,188
5	0,030	0,017	0,000	0,122
6	0,010	0,007	0,000	0,066

The estimation is unfortunately not equally satisfying for the "ones" and the "twos", with a significant share of the latter being attributed to the formers. This resembles the situation depicted the previous subsection, when we analyzed the results from the zero-truncated Poisson regression; by chance though, the weights from the complementary log-log regression lower the share of overestimated “ones” considerably.

The reduced ability of the hurdle model to distinguish between the “ones” and the “twos” may have at least three not mutually exclusive causes: first of all, the already discussed lack of better (and more) data on household’s income; secondly, the fact that we are dealing with a reduced form model: instead of the permanent income, we are using the observed flow of income and some education-related variables; the latter are therefore likely to reflect part of the effect of the permanent income; finally, the nature of dependent variable itself: the hurdle model can account for underdispersion but it must be recognized that the number of children in our sample is *strongly* underdispersed, with one single modality (the twos) representing over 50% of the statistic units. The hurdle model predicts therefore a distribution which looks smoother than the observed one.

Hurdle Model: marginal effects (median couple)

Predicted rate: NC= 1,66

	Marginal effect		
	number of children	*percentage*	*base value*
INCOME	0,042	2,49%	0,46
age_diff	-0,028	-1,67%	2
Study	0,073	4,37%	11
study_diff	0,042	2,54%	0
agemar	-0,019	-1,17%	22
satisfactory	-0,285	-17,05%	0
incomplete	-0,043	-2,56%	1
rel_attend	-0,181	-10,87%	1
Ratio	-0,359	-21,47%	22,5
Wales	-0,312	-18,67%	0
Scotland	-0,736	-44,05%	0

If we have a look at the overall marginal effects for the average couple from the hurdle model, it is possible to see that all variables related to the permanent income have a positive sign. The sign of the observed flow of income is positive, i.e. the positive effect predicted by the zero-truncated Poisson regression overcomes the negative impact on the probability of becoming parents.

Nevertheless, from a quantitative point of view, the effect of a change in the flow of observed income is negligible. Interestingly the effect of a change in observed income is stronger for very rich couples: in fact, as we have seen before, the probability of having children for those couples is virtually one and therefore the marginal effect of a rise in income is very close to the one we get from the truncated Poisson alone.For instance, if the previous couple had an income of 3, the marginal effect of a rise of observed income on the demanded number of children would be 5,4% instead of 2,49%.

Hurdle Model: marginal effects (average couple)

Predicted rate: NC= 1,76

	Marginal effect		
	number of children	*percentage*	*base value*
INCOME	0,017	0,98%	0,574
age_diff	-0,031	-1,78%	2,761
study	0,079	4,47%	12,353
study_diff	0,046	2,63%	-0,081
agemar	-0,025	-1,44%	23,585
satisfactory	-0,336	-19,08%	0
incomplete	-0,057	-3,25%	1
rel_attend	-0,187	-10,61%	1
ratio	-0,390	-22,17%	22,484
Wales	-0,327	-18,57%	0
Scotland	-0,795	-45,20%	0

On the other hand, if we consider a “poor” couple whose income equals the 10^{th} percentile of the distribution, the positive effect from the zero-truncated Poisson and the negative one in terms of probability neutralize each other and there is no change in the demanded number of children.

It appears that a couple demands significantly more children in response to a rise in income only if that change is big enough to induce a positive change in the probability of becoming parents: obviously , this happens if the couple's income already lies on the right of the vertex of the probability parabola; if the couple's income lies on the left of the vertex instead (and this is the situation of about 90% of the statistic units in our sample), the increase in income must be quite marked in order to induce a positive change in the probability of having children. Even considering all the already discussed *caveat* about the quality of NCDS measures of income, these result suggest that (too) small changes in income do not significantly affect the demand of children; therefore policy makers willing to support higher fertility rates should carefully consider whether (modest) lump-sum transfers are an adequate strategy to reach their goal: the risk is that just a small fraction of already rich people reacts as expected.

The effects from the two education-related variables are somewhat stronger than the ones observed for the observed flow of income: each year of study above the mean for the woman, increases the number of children born by 4.5 percent points (average couple) while each year of man's education above the one of his wife (the average difference is very close to zero) has also a positive effect (2,6%). Taking into account the median couple, we obtain almost the same marginal effects for both variables. If we consider another couple identical to the median one but where the woman has just 9 years of full-time education, the marginal effect of a further year of study goes up to 9,5%. If the woman is very cultivated instead, *ceteris paribus* gives birth to 2.8 children while the effect of a further year of education is virtually zero. Keeping the median couple as benchmark but considering man with ten years of education more than his wife, a further year of study increases the expected number of children by 3,9%. All in all, the marginal effect of *study_diff* follows a path very similar to the one of *study*.

Given that education (of both partners) is positively related to the permanent income of the couple, these results are in line with the theoretic prediction of a positive effect of a rise of permanent income on the number of children born to a couple. Throughout this work, it has been often hypothesized that parents' education may have a positive effect on fertility *per se*, i.e. by reducing the cost of producing quality for the children. In particular, it has been argued that whether the "savings" in producing (minimum) quality overcome the opportunity cost of mother's time, the role of female education on fertility could be, *ceteris paribus,* positive. Unfortunately it is not possible with the available data to disentangle the effect of woman's education on the level of permanent income of the couple from the one on the production of quality. This is certainly an issue deserving an in-depth investigation in a future study. Similar arguments hold for the education of the male, even though, in this case, the effect on the permanent income should be play the major role, given the implicit assumption that the mother's time is the most important input in the production of children-quality, at least in their first years of life. In order to summarize these remarks and to provide a stronger link with the theory, recall that *income*, *study* and *study_diff* contribute all together to the permanent income, which we indicated with *Y* in the budget constraint; *study* and *study_diff* may also reduce the "fixed cost" of quality $p_q(\overline{q} + Q)$, i.e. those expenditures on quality largely independent from the number of children. The intuition behind is that more educated parents need to put less effort to produce children quality than less educated ones. Nevertheless, disentangling this component of cost from the total expenditures on children seems to lose importance as the size of the family decline and increasing shares of quality are produced using inputs bought in the market.

Since the variables *incomplete* and *satisfactory*, which register the couple's attitudes toward children, do not enter in the zero-truncated Poisson regression, their overall marginal effects coincide with the ones already seen and discussed with the complementary log-log estimates and no further remark is therefore needed.

Religious couples, i.e. couples participating to religious rites at least once a month, bear significantly more children: both taking the average and the median couple as benchmark, the effect is of increasing the demand of children by almost 11 percent points. As we argued before, this result may be the consequence of more conservative attitudes toward contraception and abortion of religious couples. Recall that no distinction is made here among different religions, since the overwhelming majority of the couples in the sample belong to a Christian confession and no statistically significant difference among people devoted to different faiths emerged.

The State of residence plays a major role in determining the total number of children born to a couple. *Ceteris paribus,* English couples have significantly more children than otherwise identical Welsh and Scottish ones. The difference is particularly marked between English couples and Scottish ones, with the latter having about 45% children less, no matter if we take the average or the median couple as benchmark. This result reflects the significantly lower probability of a Scottish couple to bear children at all: indeed, the fact that there is no significant difference in terms of probability of having children between a Welsh and an English couple reduces their difference in the total number of children born to "just" 18,6% (notwithstanding the benchmark).

As predicted by the theory, the variable *ratio*, which represents the proxy of the marginal expenditures on minimum quality, has a very strong impact on the number of children born to a couple. If we consider the median couple, when the pupil/teacher ratio rises from 19 to 25, the reduction in the number of children consists of 2,25 units (from 3,08 to 0,82, i.e. 73.5%). Everything else being equal (median couple), a shift from 19 to 20 causes a reduction of 17,9%; while the decrease is of 22,4% when the ratio shifts from 25 to 26. Similar results can be achieved taking different benchmark couples: for instance, if the benchmark is the average couple, then moving from a ratio of 19 to one of 20 causes a reduction of 17,8% in the number of children born. The marginal effect at a ratio of 25 is a bit higher for the average couple than before(-25,3%); the effect of going abruptly from 19 to 25 is to reduce births from 3.26 to 0.83, i.e. a drop of about 74,5%. It is clear that non-linear nature of the hurdle model implies an inverse relationship between percentages and absolute values, i.e. as the ratio increases, the marginal effect expressed in percentage points also rises while the marginal effect in terms of absolute numbers declines. In this particular context, the focus is on the number of children: it is therefore important to stress that the stronger effects on fertility take place when the expenditures for minimum quality are "low" and are later increased (either because the government decides to raise the minimum level $\overline{q}$ or acts in such a way that the couples have to face a higher p). Therefore, everything else being equal, the model predicts that rising the mandatory years of education from - say - one to five has a much deeper impact on the fertility rate than a rise from five to ten.

This result provides a very strong evidence toward the importance of the role of the minimum quality threshold in the determination of the number of children born to a couple. As predicted by the theory, a rise of minimum level of quality has a markedly negative impact on both the probability of having children and, once the decision of becoming parents has been made, on the total number of children born to the couple. It is necessary to stress once again that rather than directly analyzing the effect of changes of the threshold $\overline{q}$, the nature of the data suggested the alternative strategy of studying the effect of changes of marginal expenditures on minimum quality π_q, proxied by the pupil/teacher ratio in primary schools.

What is particular important to notice is that the magnitude of this effect is markedly greater than the ones observed for the flow of income and for the other permanent income-related variables. For that reason, it seems that, in order to "sterilize" the negative effect on fertility due to an increase in the marginal expenditures on minimum quality, the rise in permanent income has to be more than proportional. Most Western Countries have experienced after World War II an increase of both income and mandatory years of education but also a significant decline on fertility rates ; more in general, the labour market has increasingly required higher level of proficiency even for entry-level jobs. The here-proposed model offers an alternative interpretation of this phenomenon: the decline of fertility can be the consequence of an insufficient growth of income, which could not compensate the contemporary increase of the minimum quality threshold.

6. Conclusions

This chapter proposed a reduced form model of complete fertility inspired by the minimum quality threshold hypothesis introduced in the previous chapter.

The inner structure of the theoretical model, which hypothesize da two-steps process in child-bearing decisions, finds a straightforward econometric counterpart in the family of hurdle count data models. In particular, the probability of deciding to bear children has been estimated by a complementary log-log regression, while the decision on their actual number has been crossed has been estimated by a zero-truncated Poisson regression.

Rather than estimating the effect of a change of the minimum quality threshold on the number of children born to a couple, it has been chosen to rely on the marginal expenditures on minimum quality. This choice is motivated by the assumption that parents are likely to be more influenced by those expenditures than from the level of threshold itself. Furthermore, expenditures change more often than the quality threshold and are easier to locate. In fact, this approach requires just that the minimum quality does not decrease over time. Taking the attainment targets at the end of the primary school as minimum quality threshold, it is possible to proxy the marginal expenditures on minimum quality with the pupil/teacher ratio observed by the parents in their State when they got married. Assuming that the average quality of the teachers does not change over time, the idea is that a higher ratio corresponds to a lower quality of teaching: if the skills that pupils have to develop do not decline accordingly, parents have to invest more of their time and money to allow their offspring to achieve the basic requirements.

The hurdle model predicts satisfactorily the extremes of the distribution and the frequencies of higher parities but fails to correctly allocate the “ones” and “twos”: while the sum of their frequencies is predicted very accurately, the model inflate the “ones” at the expenses of the “twos”. The most likely reason is that the original data are too heavily underdispersed. Furthermore, it is likely that better data on income would have helped to reduce this gap.

As predicted by the theory, there is a strong evidence toward the negative role of expenditures on minimum quality both on the probability of becoming parents and on the overall number of children born to the couple. It important to notice is that the magnitude of a marginal change in the expenditures on minimum quality (-22% for the average couple) is markedly greater than the ones observed for the observed income (only 1%) and for the other permanent income-related variables (4,5% for each further year of wife's full-time education and 2,6% for each further year of difference in partners' education). For that reason, in order to "sterilize" the negative effect on fertility due to an increase in the marginal expenditures on minimum quality, the rise in permanent income should be more than proportional: this result is due to the fact that an increase in income corresponds to a decline in the probability of becoming parents for nine couple out of ten. The drop in the probability is explained by the fact that a higher income allows a higher level of own consumption to those couples who decide to remain childless.

Therefore, being the marginal expenditures on minimum quality the product of the cost of quality and the level of quality threshold, a government aiming to raise the minimum quality without affecting too severely the fertility rate should try to reduce the private costs of minimum quality rather than subsidizing fertility through *una tantum* lump-sum transfers.

The proposed model provides therefore a new and different interpretation of the relationship between general education targets and effects on fertility rates.

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