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CICLO XXV°

An optimal therapeutic treatment for HIV infection with a differential Game approach

Trattamento terapeutico ottimo contro infezione da HIV: formalizzazione in termini di Gioco differenziale

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A te vecchio lupo di mare,
alla Egle e a tutte le nostre famiglie.
Ora so cosa ha provato Dante
nella 6° *bolgia dell'* 8° *cerchio dell'Inferno*
e vi prometto che farò di tutto
per non essere mai così...

Abstract

This thesis is a work focused on the application of the differential game theory to HIV therapeutic treatment, where a player is represented by the immune system of the subject (helped by drug therapy) which opposes the HIV virus. The aim of our study is to determine the optimal therapy that allows to prevent viral replication in his body (rather than the complete eradication of the infection, that remains chronic) so as to reduce the damage caused to the immune system, and allow greater survival and quality of life. To this end, our work is subdivided into 2 phases: in the first one we analyze the related literature in order to identify models and dynamics that describe the behavior of HIV and the behavior of immune system in the presence of this virus. In the second one we propose a generalized model which considers all six classes of antiretroviral drugs and different immune cells dynamics, with the aim of representing as much as possible the real setting of this problem. At a later stage, we validate our model with numerical simulations, determining optimal structured treatment interruption (STI) schedules for medications.

Sommario

Questa tesi è un lavoro focalizzato sull'applicazione della Teoria dei giochi differenziali al trattamento terapeutico dell'HIV, in cui un giocatore è rappresentato dal sistema immunitario del soggetto (supportato dalla terapia farmacologica) al quale si oppone il virus dell'HIV. Lo scopo del nostro studio è quello di determinare la terapia ottimale che consente di prevenire la replicazione virale nel corpo del soggetto stesso, (e non l'eradicazione completa dell'infezione che rimane cronica) così da ridurre i danni provocati al sistema immunitario, e garantirgli una sopravvivenza e una qualità di vita certamente maggiori. A tal fine, il nostro lavoro è stato suddiviso in 2 fasi: nella prima fase abbiamo analizzato la letteratura annessa al fine di individuare modelli e dinamiche che descrivono il comportamento dell'HIV ed il comportamento del sistema immunitario in presenza di questo virus. Nella seconda fase abbiamo proposto un modello generalizzato, che consideri tutte le sei classi di farmaci antiretrovirali e le dinamiche delle diverse cellule immunitarie, con l'obiettivo di approssimare al meglio la realtà di questo problema. Successivamente, siamo passati alla validazione del nostro modello, con simulazioni numeriche, cercando di determinare il piano strutturato ottimo delle interruzioni al trattamento farmacologico (STI).

Contents

Introduction

HIV is a virus whose genetic code consists of a ribonucleic acid, RNA; it belongs to the retroviruses family, characterized by the presence of an enzyme, DNA-polymerase RNA-independent, capable of transcribing the genetic code RNA into DNA. This ability allows the virus to integrate his genome into the one of the cells it infects such that the integrated virus would not be defeated nor by the immune response nor by drugs.

It is now established that HIV is the result of mutations in various strains of SIV (Simian Immunodeficiency Virus). Such mutations made it able to progressively infect humans. The jump of species would have occurred around the 1930s in some regions of West Sub-Saharan Africa. Strain HIV-1 derives from an immunodeficiency virus found in chimpanzee *Pan troglodytes troglodytes* (SIVcpz), on the other hand HIV-2 derives from a virus which affects the monkeys *Sooty Mangabey* (SIVsmm). The former is responsible for the current pandemic worldwide, the latter has a very limited infectivity and pathogenicity and it is mainly limited to its places of origin. Recently it has been isolated in black people from the endemic areas of Western countries.

The strains of HIV-1 can be classified into four groups: the "major" group M, the "outlier" group O and two new groups, N and P. These four groups may represent four separate introductions of simian immunodeficiency virus into humans.

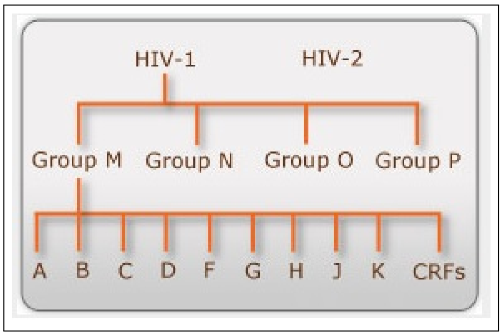

Figure 1: HIV types, groups and subtypes.

Group O appears to be restricted to west-central Africa and group N (a strain discovered in 1998 in Cameroon) is extremely rare. In 2009 a new strain closely relating to gorilla simian immunodeficiency virus was discovered in a Cameroonian woman. It was designated HIV-1 group P [89]. More than 90% of HIV-1 infections belong to HIV-1 group M and, unless specified, our considerations will be related to HIV-1 group M only.

Within group M there are at least nine genetically distinct subtypes of HIV-1. These are subtypes A, B, C, D, F, G, H, J and K [137].

Occasionally, two viruses of different subtypes can meet in the cell of an infected person and mix together their genetic material to create a new hybrid virus (a process similar to sexual reproduction, and sometimes called "viral sex") [14]. Many of these new strains do not survive for long, but those that infect more than one person are known as "circulating recombinant forms" or CRFs. For example, the CRF A/B is a mixture of subtypes A and B.

The classification of HIV strains into subtypes and CRFs is a complex issue and the definitions are subject to changes as new discoveries are made. Some scientists talk about subtypes A1, A2, A3, F1 and F2 instead of A and F, though others regard the former as sub-subtypes.

One of the CRFs is called A/E because it is thought to be a result from hybridization between subtype A and some other "parent" subtype E. However, no one has ever found a pure form of subtype E. Confusingly, many people still refer to the CRF A/E as "subtype E" (in fact it is most correctly called CRF01AE) [54].

A virus isolated in Cyprus was originally placed in a new subtype I, before being reclassified as a recombinant form A/G/I. It is now a common thought that this virus represents an even more complex CRF comprised of subtypes A, G, H, K and unclassified regions, so that the designation "I" is no longer used [33].

"Since the discover of the epidemic, almost 70 million people have been infected with the HIV virus and about 35 million people have died of AIDS. Globally, 34 million [31.4-35.9 million] people were living with HIV at the end of 2011. An estimated 0.8% of adults aged 15-49 years worldwide are living with HIV, although the burden of the epidemic continues to vary considerably between countries and regions. Sub-Saharan Africa remains most severely affected, with nearly 1 in every 20 adults (4.9%) living with HIV and accounting for 69% of the people living with HIV worldwide." [137]

Our body has an immune system that, in a healthy person, can deal with infections due to bacteria and viruses. It has two main ways to get rid of pathogens: the first one is a system that tends to remove all pathogens regardless of their biological nature (such as the acid in the stomach). The second one is a specific immune response adopted to the pathogen agent, and it is characterized by the response given by lymphocytes . In this paper

we focus on the second one.
Lymphocytes, which derive from the bone marrow and mature in the lymphatic system, are activated by a specific antigen, or foreign substance. They are divided in T-lymphocytes and in B-lymphocytes, moreover T lymphocytes are subdivided in T-helper and T-cytotoxic.
The tasks of the lymphocytes varies depending on their type: T-lymphocytes are activated by the antigens present on the surface of macrophage cells, B-Lymphocytes are activated by antigens vacant in the blood. Therefore, while the T-lymphocytes recognize the presence of intracellular pathogens and are effective against cells already infected, B-lymphocytes recognize the extracellular pathogens in the blood, in particular toxins and free bacteria.

The role of T-helper cells helps us to understand the close relationship between the two types of immune response and how important it is that lymphocytes interact amoung each other. Whether it is a Cytotoxic T-cell or a B-lymphocyte, it is necessary the presence of a T-helper cell for their activation; then the Cytotoxic T-cells destroy infected cells, while B-lymphocytes release specific antibodies that trigger chemical processes or recall macrophage cells that phagocytize the antigen. The importance of T-helper cells consists in the adjustment function for both immune systems.

After exposure to an antigen some particular cells remain on our lymphatic and blood system, the so called T-memory and B-memory cells, that will be present for the remainder of life and will facilitate the task of the immune system in the event of a second exposure to a particular agent pathogen. This is essentially why the vaccines are useful in preventing diseases. Unfortunately many viruses mutate frequently, changing for example the external proteins of their coating.

HIV is characterized by its marked tendency to mutate: mutations are errors that individual viral particles make in replicative cycles. Each error leads to the appearance of a virus more or less different from the original one. Mutations are mostly “disadvantageous” for the viral species as mutated viruses tend to disappear. Nevertheless some mutations are “beneficial” and allow mutated viruses to acquire drug resistance and immune response. Its aptitude for changing is one of the most effective mechanisms used by HIV to evade our immune response and treatment.

HIV has different transmission modalities: among these the more important are sexual, blood and vertical (mother to child). The most common is sexual (85%), followed by contact with infected blood or blood products. In developing countries particularly important is vertical transmission: this can occur during pregnancy to trans-placental passage (20-40%), during childbirth (40-70%), and finally during early breastfeeding (15-20%).[137]

HIV can be suppressed by a combination of antiretroviral therapy (ART) consisting of three or more antiretroviral (ARV) drugs. ART does not cure HIV infection but controls viral replication within a person’s body and allows an individual’s immune system to strengthen and regain the capacity to

fight off infections. With ART, people living with HIV can live healthy and productive lives.
More than 9.7 million people living with HIV in low- and middle-income countries were receiving ART at the end of 2012. Of this, about 640 000 were children. This is over 30-fold increase in the number of people receiving ART in developing countries between 2003 and 2012, and close to a 20% increase in just one year (from 8 million in 2011 to 9.7 million in 2012).
In 2012, 62% of the estimated 1.5 million pregnant women living with HIV in low- and middle-income countries received effective antiretroviral drugs to avoid transmission to their children, up from 48% in 2010.[137]

The transmission of HIV from an HIV-positive mother to her child during pregnancy, labour, delivery or breastfeeding is called vertical (as seen above) or mother-to-child transmission (MTCT). In the absence of any intervention HIV transmission rates are between 15-45%. MTCT can be nearly fully prevented if both the mother and the child are provided with antiretroviral drugs throughout the stages when infection could occur.
World Health Organization (WHO) recommends a range of options for prevention of MTCT (PMTCT), which includes providing ARVs to mothers and infants during pregnancy, labour and post-natal period, or offering life-long treatment to HIV-positive pregnant women regardless of their CD4 count. New guidelines for PMTCT has issued in 2013.

Zidovudine (ZDV), alias *Azidothymidine* (AZT), a nucleoside analog reverse-transcriptase inhibitor (NRTI), was the first drug capable of stemming the effects of the syndrome and was used in 1987. The tendency of the virus to develop resistance mutations and the high toxicity of the drug led the researchers to drop out of monotherapy in 1991 and to start using two-drug therapies. Finally, the discovery of protease inhibitors in 1996 [78] has enabled a highly effective new drug protocol, based on three viral inhibitors, which constitutes the current standard therapy, known as HAART [136] (Highly Active Antiretroviral Therapy).
Drugs commonly used in anti-retroviral therapy (ART) have as their objective the reduction and containment of replication of new copies of the virus. Every single day the body of an HIV positive person, if not submitted to ART treatment, produces millions of copies of the virus; HIV uses a specific type of lymphocytes cells (CD4+ T) as factories to reproduce millions of copies of themselves. With some particular proteins, HIV can build copies equal to itself within the infected cells and then make them “sprout” outside, ready to infect other CD4+ T cells. The most important proteins necessary to the survival and replication of the virus are the reverse transcriptase, the protease and the fusion ones. These protein chains are targeted by specific drugs that enter the infected cells and block their activity. Such drugs are named reverse transcriptase inhibitors, protease inhibitors and fusion inhibitors according to the protein they are related to [135].
Actually there are more than 20 approved antiretroviral drugs, divided into 6

different pharmacological classes. Anti-retroviral drugs are broadly classified by the phase of the retrovirus life-cycle that the drug inhibits:

- nucleoside/nucleotide reverse transcriptase inhibitors (NRTI);
- non-nucleoside reverse transcriptase inhibitors (NNRTI);
- protease inhibitors (PI);
- fusion inhibitors (FI);
- coreceptor antagonists;
- integrase inhibitors (II).

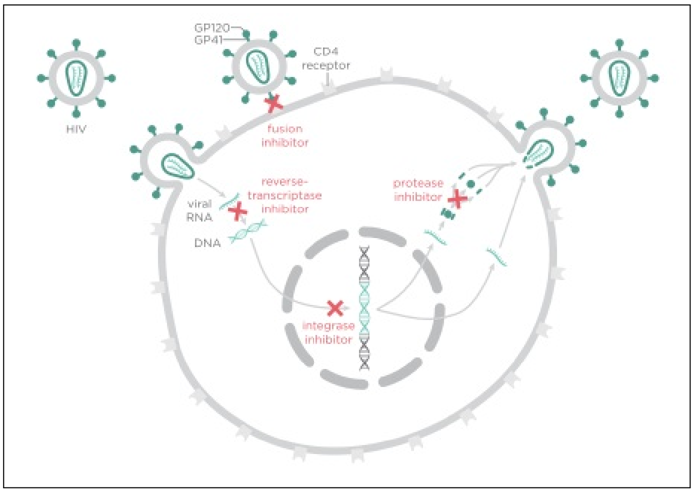

Figure 2: Schematic description of the mechanism of the five classes of currently available antiretroviral drugs against HIV.

Here's a detailed description of the above drugs.

- Nucleoside reverse transcriptase inhibitors (NRTI) and nucleotide reverse transcriptase inhibitors (NtRTI) are nucleoside and nucleotide analogues which inhibit reverse transcription. NRTIs are chain terminators such that once incorporated, work by preventing other nucleosides from also being incorporated because of the absence of a $3'$ OH group; they both act as competitive substrate inhibitors. Examples of NRTIs include *deoxythymidine, zidovudine, stavudine, didanosine, zalcitabine, abacavir, lamivudine, emtricitabine,* and *tenofovir* [23].
- Non-Nucleoside reverse transcriptase inhibitors (NNRTI) inhibit reverse transcriptase by binding to an allosteric site of the enzyme; NNRTIs act as non-competitive inhibitors of reverse transcriptase. NNRTIs affect

the handling of substrate (nucleotides) by reverse transcriptase by binding near the active site and causing "molecular arthritis". NNRTIs can be further classified into 1st generation and 2nd generation NNRTIs. 1st generation NNRTIs are more rigid in structure and resistance can quickly be developed against them. Because 2nd generation NNRTIs have a more flexible structure, they can adjust more readily and resist mutation more effectively. NNRTIs include, for example *nevirapine, delavirdine, efavirenz,* and *rilpivirine* [23].

- Protease inhibitors block the viral protease enzyme necessary to produce mature virions upon budding from the host membrane. Particularly, these drugs prevent the cleavage of gag and gag/pol precursor proteins. Virus particles produced in the presence of protease inhibitors are defective and mostly non-infectious. Examples of HIV protease inhibitors are *Lopinavir, Indinavir, Nelfinavir, Amprenavir* and *Ritonavir.* Maturation inhibitors have a similar effect by binding to gag, but development of two experimental drugs in this class, Bevirimat and Vivecon, was halted in 2010. Resistance to some protease inhibitors is high. Second generation drugs have been developed that are effective against otherwise resistant HIV variants [118].

- Entry inhibitors (or fusion inhibitors) interfere with binding, fusion and entry of HIV-1 to the host cell by blocking one of several targets. *Maraviroc* and *enfuvirtide* are the two currently available agents in this class. Maraviroc works by targeting CCR5, a co-receptor located on human helper T-cells. Caution should be used when administering this drug however due to a possible shift in tropism which allows HIV to target an alternative co-receptor such as CXCR4. In rare cases, individuals may have a mutation in the CCR5 delta gene which results in a nonfunctional CCR5 co-receptor and in turn, a means of resistance or slow progression of the disease. However as mentioned previously, this can be overcome if an HIV variant that targets CXCR4 becomes dominant [51]. To prevent fusion of the virus with the host membrane, enfuvirtide can be used. Enfuvirtide is a peptide drug that must be injected and acts by interacting with the N-terminal heptad repeat of gp41 of HIV to form an inactive hetero six-helix bundle, therefore preventing infection of host cells [3].

- Integrase inhibitors inhibit the enzyme integrase, which is responsible for integration of viral DNA into the DNA of the infected cell. There are several integrase inhibitors currently under clinical trial, and raltegravir became the first to receive FDA approval in October 2007. Raltegravir has two metal binding groups that compete for substrate with two $Mg2+$ ions at the metal binding site of integrase. Another clinically approved integrase inhibitor is Elvitegravir [91].

The initial therapeutic drug regimen is generally based on a combination among 3 of the above mentioned drugs. Typical combinations involve the conjunctive use of either two NRTIs and one NNRTI, or one PI (with or without boosting with Ritonavir) in combination with two NRTIs [64, 91].

When to start antiretroviral therapy is subject to debate [99, 114]. The World Health Organization recommends antiretrovirals in all adolescents, adults and pregnant women with a CD4 count less than $500/\mu l$ with this being especially important in those with counts less than $350/\mu l$ or those with symptoms regardless of CD4 count [137]. This is supported by the fact that beginning treatment at this level reduces the risk of death [103]. The United States in addition recommends them for all HIV-infected people regardless of CD4 count or symptoms; however it makes this recommendation with less confidence for those with higher counts [137]. While the WHO also recommends treatment in those who are co-infected with tuberculosis and those with chronic active hepatitis B [137]. Once treatment is begun, it is recommended that it is continued without breaks or "holidays". Many people are diagnosed only after treatment ideally should have begun. The desired outcome of treatment is a long term plasma HIV-RNA count below 50 copies/mL. Levels to determine if treatment is effective are initially recommended after four weeks and once levels fall below 50 copies/mL checks every three to six months are typically adequate. Inadequate control is deemed to be greater than 400 copies/mL. Based on these criteria treatment is effective in more than 95% of people during the first year [114].

Benefits of treatment include a decreased risk of progression to AIDS and a decreased risk of death.[121] In the developing world treatment also improves physical and mental health [104]. With treatment there is a 70% reduced risk of acquiring tuberculosis. Additional benefits include a decreased risk of transmission of the disease to sexual partners and a decrease in mother-to-child transmission [137]. The effectiveness of treatment depends to a large part on compliance [99]. Reasons for non-adherence include poor access to medical care [76], inadequate social supports, mental illness and drug abuse [56]. The complexity of treatment regimens (due to pill numbers and dosing frequency) and adverse effects may reduce adherence [66]. Even though cost is an important issue with some medications [77], 47% of those who needed them were taking them in low and middle income countries as of 2010 and the rate of adherence is similar in low-income and high-income countries [110].

Specific adverse events are related to the antiretroviral agent taken [65]. Some relatively common adverse events include: lipodystrophy syndrome, dyslipidemia, and diabetes mellitus, especially with protease inhibitors. Other common symptoms include diarrhea [65, 15], and an increased risk of cardiovascular disease [6]. Newer recommended treatments are associated with fewer adverse effects. Certain medications may be associated with birth defects and therefore may be unsuitable for women hoping to have children

[99].
The treatment guidelines specifically for the US are set by the United States Department of Health and Human Services (DHHS). The current guidelines for adults and adolescents were stated on December 1, 2009 [79]. Because HIV disease progression in children is more rapid than in adults, and laboratory parameters are less predictive of risk for disease progression, in particular for young infants, treatment recommendations from the DHHS have been more aggressive in children than in adults; guidelines were published November 3, 2005 [127].
Treatment recommendations for children are slightly different from those for adults. In the developing world, as of 2010, 23% of children who were in need of treatment had access [110]. Both the World Health Organization and the United States recommend treatment for all children less than twelve months of age [137]. The United States recommends in those between one year and five years of age treatment in those with HIV RNA counts of greater than 100 000 $copies/mL$, and in those more than five years treatments when CD4 counts are less than $500/\mu l$.

Administration of antiretroviral drugs should be daily and uninterrupted; it requires care and precision in the observance of schedules, even in relation to meals and to use of other drugs. Faithful adherence to the prescriptions is necessary to maintain effectiveness over time. Omission of a few doses leads to a reduction in the quantities of drugs in plasma, therefore the residual level could be lower than the one necessary to inhibit the virus. This allows the resumption of viral replication that leads inexorably to the emergence of resistant virus. Resistant mutants are destined to persist, making the drug ineffective, even if the intake of drugs is carried out according to the rhythms and the correct doses. Insufficient adherence to the prescriptions and the consequent emergence of resistant mutants are the main causes of treatment failure.
Goal of drug therapy is to prevent viral replication in the body (rather than the complete eradication of the infection, that remains chronic) so as to reduce the damage caused to the immune system and allow greater survival and quality of life. HIV-infected individuals, who may promptly discover their state and rely on expert team, can trust in expectancy and quality of life not dissimilar to those suffering from other chronic diseases, such as hypertension or diabetes [136]. It is rather a crucial issue the study of HIV biological evolution and of its relationship with the immune system, in order to determine a therapy policy that can defeat it.

Chapter 1

HIV modeling: a review

In this work we analyze about a hundred articles of approximately 50 authors who tackle HIV topic throughout the formulation of dynamical systems and in some cases of Optimal Control problems. Our aim is to present an existing review of the examined literature and to classify the related works basing on the following criteria:

1. Does the model considers T lymphocytes infected by the viruses?
2. Does the model presents different types of infected T lymphocytes?
3. Does the model presents different types of T lymphocytes?
4. Does the model presents different types of HIV viruses?
5. Does the model considers the immune effectors/precursors?

In the following sections we describe the analyzed models focusing on their characteristics and presenting a reference model for each group. In Sect. 1.1 we introduce all variables and parameters necessary to describe the models. Sect. 1.2 considers models without infected T cells dynamics, and it is splitted into 4 sub-sections. Whilst Sect. 1.3 treats models with infected T cells dynamics, and it is splitted into 9 sub-sections. We describe the dynamics of the virus and of the immune system and highlight the existing optimal control approach. Finally in Sect. 1.4 we try to achieve a frequency analysis of the different values assigned to the parameters commonly used in the analyzed models. Sect. 1.5 concludes.

1.1 Variables, Controls and Parameters

We unify the notation of variables, controls and parameters and of their units of measurement, so as to facilitate the models' comparison. Trying, at the same time, to be linked to the literature notation as much as possible.

Table 2.2 lists possible actions of immune system or therapy or virus: It should be noted that in some models these parameters are considered as simple parameters, in others as functions and still others controls, in any case their values are between 0 and 1. Each table presents entries in alphabetical order, for readability.

The main variables considered in the analyzed models are Uninfected CD4+ T cell population size (T) and HIV population size (V). Our immune system is constituted by T cells; these once infected by the virus (T^i) may either remain latently infected (T^l), and act as healthy cells (with a slightly different death rate) or activate (T^a) and start to produce new free virions. Models that consider this differentiation of infected T cells use variables following the notation indicated in Table 2.1. Some models allows for differentiation of uninfected T cells and viruses: considering for example macrophages and monocyties (M) or wild-type virus (V_1) and mutant virus (V_2). Some very few variables and parameters are not presented in the tables as they are specific of a given model so that they will be introduced and described only in the related section.

In Optimal Control sub-sections initial and final time are denoted by t_0 and t_f respectively.

Table 1.1: Controls and/or parameters ($\in [0, 1]$).

Symbol	Description
u	Percentage of chemotherapy effect
u_j	Efficacy/dosage of drug j
u_d	Viral suppressing drugs
u_{ib}	Immune boosting
u_{pi}	Efficacy/dosage of protease inhibitor
u_{rti}	Efficacy/dosage of reverse transcriptase inhibitor
u_v	Mutation rate of virus

1.2 Models without infected T cells dynamics

These models date back to the '90s and consider only the dynamics of uninfected T cells (T) and viruses (V), regardless of infected T lymphocytes. Models of this section are splitted into 2 parts: basic models, which do not provide any T cells and viruses differentiation, and models which consider target cells and virions differentiation together with host immune response.

1.2.1 Basic models

A basic group of models is constituted by works of Kirschner et al. [47] and Joshi [40], where T-cells and viruses evolution are described through a

Table 1.2: Variables (Unit of measure: $cells/mm^3$).

Symbol	Description
I^P	Immune Precursor Cytotoxic T Lymphocyte population size
I^E	Immune Effector Cytotoxic T Lymphocyte population size
M	Uninfected macrophage/monocyte population size
M^i	Infected macrophage/monocyte population size
T	Uninfected CD4+ T cell population size
T^i	Infected CD4+ T cell population size
T^l	Latently infected CD4+ T cell population size
T^a	Actively infected CD4+ T cell population size
V	HIV population size
V^I	Infectious HIV population size
V^{NI}	Non-infectious HIV population size

system of two coupled differential equations. Authors do not consider infected T-cells, neither T-cells and viruses differentiation, nor immune response. The dynamics of the reference system are

$$\begin{aligned}\dot{T}(t) &= s_1 - \frac{s_2 V(t)}{b_1 + V(t)} + u_{ib}(t)T(t) - \mu_T T(t) - k_1 V(t) T(t)\\ \dot{V}(t) &= \frac{g\Big(1 - u_d(t)\Big)V(t)}{b_2 + V(t)} - \mu_V V(t) T(t)\ .\end{aligned} \tag{1.1}$$

The term

$$s_1 - \frac{s_2 V(t)}{b_1 + V(t)}$$

represents the source/proliferation of uninfected CD4+ T cells and includes both an external (not plasma) contribution of cells from sources such as the thymus and lymph nodes, and an internal (plasma) contribution from CD4+ T cells differentiation. The immune boosting contributes to increase the number of uninfected T-cells and this is explicated by the term $u_{ib}(t)T(t)$. On the other hand T-cells decrease both by the effect of spontaneous decay and the presence of the HIV virus. These behaviours are formalized by terms $-\mu_T T(t)$ and $-k_1 V(t)T(t)$ respectively.
For what concerns the viruses evolution, the term

$$\frac{g\Big(1 - u_d(t)\Big)V(t)}{b_2 + V(t)}$$

is a source of virus that accounts for viral contributions to the plasma from both external compartments, such as the lymph system as well as virus

Table 1.3: Parameters (Unit of measure: $1/day$).

Symbol	Description
b_E	Maximum birth rate for immune effectors
c	Cytotoxic T Lymphocyte (CTL) activation rate
d_E	Maximum death rate for immune effectors
i_r	Death rate of infected cells due to immune response
$i_{r,j}$	Death rate of infected cells type j due to immune resp.
k	Rate T convert to specific immune reaction cells
k'	Rate T convert to unspecific immune reaction cells
k_2	Rate T^l convert to actively infected
$k_{2,j}$	Rate T^l type j convert to actively infected
k_4	Rate free virus infects mac/mono cells
μ_{I^E}	Natural death rate for immune effectors (CTL)
μ_{I^P}	Natural death rate of CTL precursors
μ_T	Death rate of uninfected CD4+ T cells
μ_{T_j}	Death rate of uninfected CD4+ T type j cells
μ_{T^i}	Death rate of infected CD4+ T cells
$\mu_{T^i_j}$	Death rate of infected CD4+ T type j cells
μ_{T^l}	Death rate of latently infected CD4+ T cells
$\mu_{T^l_j}$	Death rate of latently infected CD4+ T type j cells
μ_{T^a}	Death rate of actively infected CD4+ T cells
$\mu_{T^a_j}$	Death rate of actively infected CD4+ T type j cells
μ_V	Death rate of free virus
π	Growth rate of viruses
π_w	Growth rate of type w viruses
Π_M	Rate of free virus production by infected macrophages
q	Growth rate of CTL eff. due to inf. cells and CTL prec.
r	rate of growth for CD4+ T cell population

produced by infected cells in the plasma [34]. This source is counteracted by the therapy $u_d(t)$. Also viruses die, at a rate $-\mu_v T(t)$ which is proportional to the number of uninfected T-cells. Parameters s_1, s_2, b_1, b_2, g are assumed positive and constant.

Kirschner et al. [47] consider a given exponential function in place of the control $u_{ib}(t)$. In other terms, they assume that the enhancement of the immune system causes an increase of the CD4+ T cells proportional to the population of these cells at the rate $c_1 te^{-c_2 t}/day$. This choice assumes that the drug efficiency decays exponentially, and its effect is not instantaneous. They further assume that the virus growth rate does not depend on the viral suppressing drig control (u_d).

Table 1.4: Parameters.

Symbol	Description	U.m./Value
A	"Weight" on the benefit and cost of therapy	$\in [0, +\infty]$
A_d	"Weight" of viral suppressing drug	$\in [0, +\infty]$
A_{ib}	"Weight" of immune boosting	$\in [0, +\infty]$
E_M	Equilibrium number for mac/mono population	$\frac{cells}{mm^3}$
f	Treat. efficacy reduction/rel. efficacy in virus population	$\in [0, 1]$
f_k	Treat. k efficacy reduction/rel. efficacy in virus population	$\in [0, 1]$
g	Input rate of external viral source	$\frac{cells}{mm^3 \times day}$
g'	Growth rate of virions due to infected cells	$\frac{virions}{cell \times day}$
g'_w	Growth rate of virions type w due to infected cells	$\frac{virions}{cell \times day}$
k_1	Rate CD4 T cells becomes infected by free virus	$\frac{mm^3}{virions \times day}$
$k_{1,w}$	Rate CD4 T cells becomes infected by virus type w	$\frac{mm^3}{virions \times day}$
$k_{1,w}^M$	Rate M cells becomes infected by virus type w	$\frac{mm^3}{virions \times day}$
k_3	Rate infected mac/mono infects CD4+ T cells	$\frac{mm^3}{cells \times day}$
K_b	Saturation constant for immune effector birth	$\frac{cells}{mm^3}$
K_d	Saturation constant for immune effector death	$\frac{cells}{mm^3}$
λ_E	Immune effector production (source) rate	$\frac{cells}{mm^3 \times day}$
N	Number of free virus produced by T infected cells	$\frac{virions}{cell}$
N_j	Number of free virus produced by T_j^i (infected) cells	$\frac{virions}{cell}$
N_j^M	Number of free virus produced by M_j^i (infected) cells	$\frac{virions}{cell}$
p'	Specific immune response rate	$\frac{cells}{mm^3 \times day}$
p'_w	Specific immune response rate against virus w	$\frac{cells}{mm^3 \times day}$
ρ_j	Average number virions infecting a type j cell	$\frac{virions}{cell}$
s'	Unspecific immune response rate	$\frac{cells}{mm^3 \times day}$
s'_w	Unspecific immune response rate against virus w	$\frac{cells}{mm^3 \times day}$
s_1	Source/production of CD4+ T cells	$\frac{cells}{mm^3 \times day}$
$s_{1,j}$	Source/production of CD4+ T type j cells	$\frac{cells}{mm^3 \times day}$
s_2	Source/production of CD4+ T cells	$\frac{cells}{mm^3 \times day}$
$s_{2,j}$	Source/production of CD4+ T type j cells	$\frac{cells}{mm^3 \times day}$
s	Source/production of CD4+ T cells	$\frac{cells}{mm^3 \times day}$
s_j	Source/production of CD4+ T type j cells	$\frac{cells}{mm^3 \times day}$
T_{max}	Maximum CD4+ T cell population level	$\frac{cells}{mm^3}$

Optimal control approach

Joshi in [40] uses the optimal control theory to tackle the HIV therapy by maximizing the following objective function

$$J\big(u_{ib}(t), u_d(t)\big) = \int_{t_0}^{t_f} \Big[T(t) - \big(A_{ib}u_{ib}^2(t) + A_d u_d^2(t)\big)\Big]\, \mathrm{d}t, \qquad (1.2)$$

where parameters A_{ib} and A_d are positive and constant whilst controls u_{ib} and u_d are specified in Table 2.2.

1.2.2 Models with both uninfected T cells and viruses differentiation

De Boer et al. in [24, 25] treat the diversity and virulence thresholds in AIDS. In this context they propose a model which takes into account uninfected T cells and viruses differentiation as follows (for $j = 1, 2, \dots, n$)

$$\begin{aligned}
\dot{T}_j(t) &= s_1 + T_j\left(\frac{s_2 V_j(t)}{1 + V_j(t)} - 1 - n\nu V_j(t) + \nu_j V_j(t)\right) \\
\dot{T}_{n+1}(t) &= s_1 + T_{n+1}\left(\frac{s_2 V_{n+1}(t)}{1 + V_{n+1}(t)} - 1 - n\nu V_j(t) + \nu_{n+1} V_{n+1}(t)\right) \\
\dot{V}_j &= \pi V_j\big(1 - T_j(t)\big) \\
\dot{V}_{n+1} &= \pi V_{n+1}\big(1 - T_{n+1}(t)\big),
\end{aligned} \tag{1.3}$$

where n is the number of viral strains, T_j represents the immune response at different established quasispecies viruses V_j, and T_{n+1} represents the immune response at a new mutant virus V_{n+1}. Parameters ν_j and ν_{n+1} are the "average" virulences of established and mutant strains respectively. Again s_1 and s_2 are positive parameters and π is the growth rate of viruses.

1.2.3 Models with uninfected T cells differentiation and with immune effectors/precursors

Hraba et al. [35] and Zarei et al. [130] consider infected T cells differentiation and they split these cells into immature (T^{imm}) and mature (T^{mat}). Such differentiation constitutes an original topic which is not present in any other model of the analyzed literature. The reference model is the following

$$\begin{aligned}
\dot{T}^{imm}(t) &= s_{1,imm} + \beta\big(T_0^{mat} - T^{mat}(t)\big) - \tau T^{imm}(t) \\
&- k_1 V(t) I^E(t) T^{imm}(t) \\
\dot{T}^{mat}(t) &= \tau T^{imm}(t) - k_1 V(t) I^E(t) T^{mat}(t) - \mu_{T^{mat}} T^{mat}(t) \\
\dot{V}(t) &= \big(1 - u_{rti}(t)\big)\pi V(t) - i_r I^E(t) V(t) \\
\dot{I}^E(t) &= \big(cqV(t) + \alpha I^E(t) V(t)\big)\left(\frac{T^{mat}(t)}{T_0^{mat}}\right) - \mu_{I^E} I^E(t),
\end{aligned} \tag{1.4}$$

where β is the amplifying coefficient of the linear feedback effect of T^{mat} cells decrease on the influx of T^{imm} cells. Parameter τ represents the rate of maturation of T^{imm} cells into T^{mat} cells, finally parameter α is the proliferation rate of immune effector cells (I^E) under the antigenic stimulation by HIV products.

Optimal control approach

Zarei et al. [130] use an optimal control formulation which considers the objective functional

$$J\big(u_{rti}(t)\big) = t_f - \lambda \int_{t_0}^{t_f} u_{rti}^2(t)\,\mathrm{d}t\,, \tag{1.5}$$

where parameter λ is used to set the relative importance between minimizing the systemic side effects on the body and maximizing the survival time t_f.

1.2.4 Models with uninfected T cells differentiation, viruses differentiation and immune effectors/precursors presence

This group of works is constituted essentially by Nowak works [69, 70, 71, 72, 73] in collaboration with different authors, i.e. May, Anderson and McLean.
The peculiarity of these models is the presence of unspecific (i.e. cross-reactive T_g) and strain specific (T_j) immune reactions. Authors assume that each mutant j induces the production of certain immune agents (CD4 T cells). A fraction of these agents is specifically directed only against that particular mutant strain (e.g., via the immunodominant loop). An other fraction is directed against more conserved sites (e.g., the *gag* or *pol* gene products or conserved regions within *env* protein) and hence the agent is able to react with several different mutant strains. Therefore immunological responses to the virus are characterized by both a specific response to individual strains (which direct immunological attack against that specific strain) and a non-specific general response (that acts against all strains).
Each mutant can kill all CD4 T cells regardless of their specificity to a particular mutant.

In [69] Nowak et al. formulated the following model [69] (for $j = 1, 2, \ldots, n$)

$$\begin{aligned}
\dot{T}_j(t) &= cV_j(t)I_j^P(t) + rT_j(t) - k_1\Big(\sum V_j(t)\Big)T_j(t)\\
\dot{V}_j(t) &= V_j(t)\big(\pi - s'T_g(t) - p'T_j(t)\big)\\
\dot{I}_j^P(t) &= C_{I^P} - \mu_{I^P}I_j^P(t) - cV_j(t)I_j^P(t) - k_1\Big(\sum V_j(t)\Big)I_j^P(t),
\end{aligned} \tag{1.6}$$

where C_{I^P} is the production rate of precursors (I_j^P) of immune cells. These precursors are removed at rate μ_{I^P}, and either are infected/killed by the virus (at rate k_1) or turned into activated cells by contact with the viral strain j (at rate c). Notice that activated/target cells T_j can proliferate (at rate r) or can be infected/killed by the virus (at rate k_1). Furthermore observe that these equations do not consider T-cells and viruses natural death rates.

In [70] Nowak et al. assume that target cells (T) can be converted to specific and unspecific immune reaction cells at rates k and k' respectively. Authors also include the population dynamics of the total number of CD4+ cells including those not directed against HIV, denoted by (T). The model, adopted also by Wodarz in [125], is (for $j = 1, 2, \ldots, n$)

$$\begin{aligned}
\dot{T}(t) &= s_1 - \mu_T T(t) - k_1\Big(\sum V_j(t)\Big)T(t) - k\Big(\sum V_j(t)\Big)T(t) \\
&- k'\Big(\sum V_j(t)\Big)T(t) \\
\dot{T}_j(t) &= kV_j(t)T(t) - k_1\Big(\sum V_j(t)\Big)T_j(t) \\
\dot{T}_g(t) &= k'\Big(\sum V_j(t)\Big)T(t) - k_1\Big(\sum V_j(t)\Big)T_g(t) \\
\dot{V}_j(t) &= V_j(t)\left[\pi\Big(T(t) + \Big(\sum T_j(t)\Big) + T_g(t)\Big) - s'T_g(t) - p'T_j(t)\right].
\end{aligned} \tag{1.7}$$

Authors neglect the possibility that mutation leads to mutants already in the system, because the number of immunologically different mutants appears to be very large [68, 74].

Subsequently, Nowak et al. in [71] propose different types of conversion immune reaction rates for each viral strain and formulate a new reproductive rate function. Related differential equations are as follows (for $j = 1, 2, \ldots, n$)

$$\begin{aligned}
\dot{T}(t) &= s_1 - \mu_T T(t) - k_1\Big(\sum V_j(t)\Big)T(t) \\
\dot{T}_j(t) &= kV_j(t)T(t) - k_1\Big(\sum V_j(t)\Big)T_j(t) \\
\dot{T}_g(t) &= k'\Big(\sum V_j(t)\Big)T(t) - k_1\Big(\sum V_j(t)\Big)T_g(t) \\
\dot{V}_j(t) &= \big(\Pi_j + \pi_j T(t)\big)V_j(t) - V_j(t)\big(s_j'T_g(t) - p_j'T_j(t)\big),
\end{aligned} \tag{1.8}$$

where Π_j is a constant background replication rate which denotes replication of the virus in cells other than of the CD4+ type (such as macrophages and monocytes). Notice that, despite the article is very dated, it also allows for uninfected cells differentiation, in fact it considers cells as macrophages and monocytes.

Finally in [73] Nowak et al. consider dynamics of immune responses against multiple HIV epitopes.

1.3 Models with infected T cells dynamics

A large number of works also consider the dynamics of infected T lymphocytes introducing new variables that represent contaminate lymphocytes. In this models it is necessary to consider the rate of infection (k_1) to which healthy T cells are infected by virus. The first two groups of models (Sub-Sect. 1.3.1 and 1.3.2) do not consider any kind of differentiation, while other groups (Sub-Sect. 1.3.3, 1.3.4, 1.3.5, 1.3.6, 1.3.7, 1.3.3, 1.3.2, 1.3.8, 1.3.9) introduce uninfected and infected T-cells differentiation and viruses differentiation.

1.3.1 Basic Models

This sub-section contains models that together with T-cells and virus evolution consider also infected T-cells.
Subsection is splitted into 2 parts: the first one with deterministic models, the second one with stochastic or fuzzy models.

Deterministic models

The work of reference of this group does not consider the latently infected or actively infected T lymphocytes but only infected lymphocytes. Moreover viruses are counteracted by reverse transcriptase inhibitors or protease inhibitors. Connected works are [17, 38, 85, 87, 88, 90]. Equations under "imperfect" RT inhibitors can be written as

$$\begin{aligned} \dot{T}(t) &= s_1 + rT(t)\left(1 - \frac{T(t)}{T_{max}}\right) - \mu_T T(t) - \big(1 - \eta_{rti}(t)\big)\beta T(t)V(t) \\ \dot{T^i}(t) &= \big(1 - \eta_{rti}(t)\big)\beta T(t)V(t) - \mu_{T^i} T^i(t) \\ \dot{V}(t) &= N\mu_{T^i} T^i(t) - \mu_V V(t). \end{aligned} \tag{1.9}$$

PIs on the other hands do not block infection, but rather block the protease enzyme so that the virus particles produced are non infectious. Consequently two types of virus particles come out when PIs are used. The first type is given by the infectious virus particles that still continue to infect CD4+ T cells and the second one is given by the non infectious type. Equations under

PIs can be written as follows

$$
\begin{aligned}
\dot{T}(t) &= s_1 + rT(t)\left(1 - \frac{T(t)}{T_{max}}\right) - \mu_T T(t) - \beta T(t)V(t)\\
\dot{T^i}(t) &= \beta T(t)V(t) - \mu_{T^i} T^i(t)\\
\dot{V_I}(t) &= \big(1 - \eta_{pi}(t)\big) N \mu_{T^i} T^i(t) - \mu_V V_I(t)\\
\dot{V}_{NI}(t) &= \eta_{pi}(t) N \mu_{T^i} T^i(t) - \mu_V V_{NI}(t).
\end{aligned}
\tag{1.10}
$$

In equations (1.9) and (1.10) the inhibitors block infection by reducing the value of β and perfect inhibition occurs when $\beta = 0$. (The estimation of parameters β given in [37] is $\beta = 7.5 \times 10^{-6}/mm^3 \times day$).
Furthermore η_{rti} $(0 \leq \eta_{rti}(t) \leq 1)$ is the effectiveness or combined effectiveness of the RT inhibitors used. Perfect inhibition occurs when $\eta_{rti} = 1$ and there is no inhibition when $\eta_{rti} = 0$.
Parameter η_{pi} $(0 \leq \eta_{pi}(t) \leq 1)$ is the combined effectiveness of the used PIs. Perfect inhibition occurs when $\eta_{pi} = 1$ and there is no inhibition when $\eta_{pi} = 0$.
Note the presence of the multiplicative term $N\mu_{T^i}$ that characterizes models belonging to this group, where N is the number of free virus produced by T infected cells and μ_{T^i} is the death rate of infected CD4+ T cells. Another peculiar feature in this group is given by the term

$$
r\left(1 - \frac{T(t)}{T_{max}}\right)
$$

that represents the production of T-cells due to cloning in the presence of an antigen, taking into account the maximum number of lymphocytes (T_{max}).

Caetano et al. [17] propose a model which considers latently and actively infected T lymphocytes

$$
\begin{aligned}
\dot{T}(t) &= \frac{s_1\theta}{\theta + V(t)} - \mu_T T(t) + rT(t)\left(1 - \frac{T(t) + T^l(t) + T^a(t)}{T_{max}}\right)\\
&- k_{1,0} e^{-\alpha_1 u_1} V(t)T(t)\\
\dot{T^l}(t) &= \omega k_{1,0} e^{-\alpha_1 u_1} V(t)T(t) - \mu_{T^l} T^l(t) - k_{2,0} e^{-\alpha_2 u_2} T^l(t)\\
\dot{T^a}(t) &= (1-\omega) k_{1,0} e^{-\alpha_1 u_1} V(t)T(t) + k_{2,0} e^{-\alpha_2 u_2} T^l(t) - \mu_{T^a} T^a(t)\\
\dot{V}(t) &= \beta_2(\beta_2 - N_0) e^{-\beta_1 t} \mu_{T^a} T^a(t) - k_{1,0} e^{-\alpha_1 u_1} V(t)T(t) - \mu_V V(t).
\end{aligned}
\tag{1.11}
$$

"Basically, T cells are stimulated to proliferate with rate

$$
rT(t)\left(1 - \frac{T(t) + T^l(t) + T^a(t)}{T_{max}}\right)
$$

in the presence of antigen and HIV. Without the presence of HIV, the rate of generation is

$$\frac{s_1\theta}{\theta + V(t)}$$

In the presence of free HIV (V), uninfected cells (T) can be infected to became T^l or T^a cells, depending on probability of the cells to become actively or latently infected with rate ω. The T^l cells can be activated to become T^a cells. The activation rate is k_2. The T^a cells are short living and will normally be killed upon activation with death rate μ_{T^a}. The T, T^l cells and V free viruses have finite life and the death rates in this model are μ_T, μ_{T^l} and μ_V respectively. When T^a cells die, free viruses V are released with rate

$$\beta_2(\beta_2 - N_0)e^{-\beta_1 t}$$

Drugs such as reverse transcriptase inhibitors (zidouvine and lamivudine) and protease inhibitors (saquinavir, indinavir and ritonavir) affect the dynamics via parameters $k_{1,0}$ and $k_{2,0}$."[17]
The term

$$k_{1,0}e^{-\alpha_1 u_1}V(t)T(t)$$

that appears both in uninfected T cells and viruses equations (1.11) is introduced because the authors assume that a virion can infect only a single cell.

Stochastic and Fuzzy models [53]

Papers of Dalal et al. [22], Gregio et al. [32], Tan et al. [105, 106] and Tuckwell et al. [109] present stochastic models, where the evolution of T cells and viruses is similar to the one of the deterministic formulation apart from the assumption of stochasticity for the death rates. Dalal et al. in [22] introduce randomness into the model by replacing the parameters μ_t, μ_{T^i}, μ_V by

$$\mu_T \longrightarrow \mu_T + \sigma_1\dot{B}_1(t) \quad \mu_{T^i} \longrightarrow \mu_{T^i} + \sigma_1\dot{B}_1(t) \quad \mu_V \longrightarrow \mu_V + \sigma_2\dot{B}_2(t).$$

Their model is the following

$$\begin{aligned} \mathrm{d}T(t) &= \Big[s_1 - \mu_T T(t) - k_1\big(1 - u_{rti}(t)\big)T(t)V(t)\Big]\,\mathrm{d}t - \sigma_1 T(t)\,\mathrm{d}B_1(t) \\ \mathrm{d}T^i(t) &= \Big[k_1\big(1 - u_{rti}\big)T(t)V(t) - \mu_{T^i}T^i(t)\Big]\,\mathrm{d}t - \sigma_1 T^i(t)\,\mathrm{d}B_1(t) \\ \mathrm{d}V(t) &= \Big[\big(1 - u_{pi}\big)N\mu_{T^i}T^i(t) - \mu_V V(t) - k_1\big(1 - u_{rti}\big)T(t)V(t)\Big]\,\mathrm{d}t \\ &- \sigma_2 V(t)\mathrm{d}B_2(t), \end{aligned} \tag{1.12}$$

where $B_1(t)$ and $B_2(t)$ are independent standard Brownian motions; σ_1 and σ_2 are the intensities of the noise (in [22] $\sigma_1 = 0.1$ and $\sigma_2 = 0.1$).

Tuckewell et al. [109] propose another stochastic model that considers latently and actively infected T cells too as follows

$$\begin{aligned}
\mathrm{d}T(t) &= \big[s_1 - \mu_T T(t) - k_{1,1})T(t)V(t)\big]\ \mathrm{d}t - \sigma\big[k_{1,1}T(t)V(t)\big]\ \mathrm{d}W(t) \\
\mathrm{d}T^l(t) &= \big[k_{1,1}T(t)V(t) - \mu_{T^l}T^l(t) - k_2 T^l(t)\big]\ \mathrm{d}t \\
&- \sigma\big[k_{1,1}pT(t)V(t)\big]\ \mathrm{d}W(t) \\
\mathrm{d}T^a(t) &= \big[k_{1,1}(1-p)T(t)V(t) - \mu_{T^a}T^a(t) + k_2 T^l(t)\big]\ \mathrm{d}t \\
&- \sigma\big[k_{1,1}(1-p)T(t)V(t)\big]\ \mathrm{d}W(t) \\
\mathrm{d}V(t) &= \big[g' T^l(t) - k_{1,2}T(t)V(t) - \mu_V V(t)\big]\ \mathrm{d}t \\
&- \sigma\big[k_{1,2}T(t)V(t)\big]\ \mathrm{d}W(t).
\end{aligned} \tag{1.13}$$

Parameter p gives the percentage of infected cells which are latent (in [109] $p = 0.1$) and σ is the additional noise term. Note that $k_{1,1}$ and $k_{1,2}$ are the infection rates per virion and uninfected T cells respectively.

In order to model the dynamical systems of HIV under uncertainty Lin et al. [53] and Zarei et al. [131] apply a fuzzy approach. We do not go into the details of these models because both the formulations and the type of parameters they adopt are not comparable to the ones of the classical literature.

Optimal control approach

Caetano et al. in [17] minimize this objective function

$$J\big(u_1(t), u_2(t)\big) = \frac{\gamma_1}{T(t_f)} + \int_{t_0}^{t_f} \Big[A_1\big(1 - e^{-\epsilon_1 u_1^2(t)}\big) + A_2\big(1 - e^{-\epsilon_2 u_2^2(t)}\big) + \frac{\gamma_2}{T(t)}\Big]\ \mathrm{d}t \tag{1.14}$$

where the last term is included to force uninfected T cells to increase with treatment.

1.3.2 Models with immune effectors/precursors

In this sub-section a new immune response is taken into account. The works of Anderson et al. [2], Wodarz et al. [120, 121, 122, 123, 124, 125, 126], Bonhoeffer et al. [7], Kubiak et al. [48], Shim et al. [102], Culshaw et al. [20], Radisavljevic-Gajic [93], Jin et al. [39], Kalams et al. [43], Rosemberg et al.

[96], Saah et al. [97], Schmitz et al. [100], Thomsem et al. [107, 108], and Pannocchia et al. [80] introduce the dynamics of immune effectors/precursors. The reference model is the following

$$
\begin{aligned}
\dot{T}(t) &= s_1 - \mu_T T(t) - k_1(1 - fu(t))V(t)T(t) \\
\dot{T}^i(t) &= k_1(1 - u(t))V(t)T(t) - \mu_{T^i} T^i(t) - i_r I^E(t)T^i(t) \\
\dot{I}^P(t) &= cT(t)T^i(t)I^P(t) - cqT^i(t)I^P(t) - \mu_{I^P} I^P(t) \\
\dot{I}^E(t) &= cqT^i(t)I^P(t) - \mu_{I^E} I^E(t) \\
\dot{V}(t) &= g'T^i(t) - \mu_V V(t).
\end{aligned}
\tag{1.15}
$$

The authors consider infected T cells of a generic type, and include immune effectors (I^E) and/or immune precursors (I^P) in the dynamics.

Bonhoeffer et al. in [7] consider an alternative virus progression, also considering quiescent cells.

Wodarz' papers exploit the assumption that a cell can be infected by two viruses at the same time, more precisely by the wild type virus (T_1^i) only, by the escape mutant virus (T_2^i) only, or finally by both two virus strains simultaneously ($T_{1,2}^i$). The dynamics of these models are

$$
\begin{aligned}
\dot{T}(t) &= s_1 - \mu_T T(t) - k_{1,1}\big(T_1^i(t) + T_{1,2}^i(t)\big)T(t) - k_{1,2}\big(T_2^i(t) + T_{1,2}^i(t)\big)T(t) \\
\dot{T}_1^i(t) &= k_{1,1}\big(T_1^i(t) + T_{1,2}^i(t)\big)T(t) - k_{1,2}\big(T_2^i(t) + T_{1,2}^i(t)\big)T_1^i(t) \\
&- \mu_{T_1^i} T_1^i(t) - \mu_T T_1^i(t) - i_r T_1^i(t)I^E(t) \\
\dot{T}_2^i(t) &= k_{1,2}\big(T_2^i(t) + T_{1,2}^i(t)\big)T(t) - k_{1,1}\big(T_1^i(t) + T_{1,2}^i(t)\big)T_2^i(t) \\
&- \mu_{T_2^i} T_2^i(t) - \mu_T T_2^i(t) \\
\dot{T}_{1,2}^i(t) &= k_{1,1}\big(T_1^i(t) + T_{1,2}^i(t)\big)T_2^i(t) + k_{1,2}\big(T_2^i(t) + T_{1,2}^i(t)\big)T_1^i(t) \\
&- \mu_{T_i^i} T_{1,2}^i(t) - i_r T_{1,2}^i(t)I^E(t) \\
\dot{I}^E(t) &= c\big(T_1^i(t) + T_{1,2}^i(t)\big) - \mu_{I^E} I^E(t).
\end{aligned}
\tag{1.16}
$$

Notice that there is not the virus equation: Wozard et al. claim that explicit inclusion of free virus population does not change the results described here. Cells infected by the wild-type strain die as a result of virus replication at a rate $\mu_{T_1^i}$, and are killed by CTL at a rate i_r. Cells infected with the escape mutant died at a rate $\mu_{T_2^i}$ and are not killed by CTL. Finally, cells infected with both strains are killed by CTL at a rate i_r, and are assumed to be killed by the virus at the rate determined by the faster killing strain, denoted by $\mu_{T_i^i}$ in the model.

At last, Pannocchia et al. [80] introduce a new variable R which represents the index of the aggressiveness of the virus and they formulate the following dynamics

$$\begin{aligned}
\dot{T}(t) &= s_1 - \mu_T T(t) - R(t)V(t)T(t) \\
\dot{T^i}(t) &= R(t)V(t)T(t) - \mu_{T^i}T^i(t) - i_r I^E(t)T^i(t) \\
\dot{I}^P(t) &= cT(t)T^i(t)I^P(t) - cqT^i(t)I^P(t) - \mu_{I^P}I^P(t) \\
\dot{I}^E(t) &= cqT^i(t)I^P(t) - \mu_{I^E}I^E(t) \\
\dot{V}(t) &= g'\big(1 - f_{pi}u_{pi}\big)T^i(t) - \mu_V V(t) \\
\dot{R}(t) &= R_0\big(1 - f_{rti}u_{rti}\big).
\end{aligned} \tag{1.17}$$

In the case of untreated HIV-infected individual, index R increases linearly at a rate proportional to a given constant $R_0 = 1 \times 10^{-8}\ ml/(copies \times day)^2$, (see [80]).
Parameter g' represents the growth rate of virions due to infected cells and in general (see e.g. models in Sub-Sect. 1.3.1) it is assumed to be

$$g' = N\mu_{T^i}.$$

Coefficients f_{rti} and f_{pi} are assumed to take value between 0.5 and 0.9.

Optimal control approach

Culshaw et al. in [20] adopt an optimal control approach maximising the objective function

$$J\big(u(t)\big) = \int_{t_0}^{t_f} \left(T(t) + I^E(t) - \frac{A_{ter}u^2(t)}{2}\right) \mathrm{d}t. \tag{1.18}$$

1.3.3 Models with uninfected T cells differentiation

All models of Sect. 1.2 consider T-cells without any kind of differentiation. This sub-section contains models without immune effectors/precursors, and with no viruses differentiation, but with uninfected T cells differentiation as proposed by Perelson et al. [83, 84, 86], Kirschner et al. [44, 46], Butler et al. [16] and Fister et al. [30].

“Stimulation of the T cell by antigen can lead to the production of new virus particles that bud from the surface of the infected cell. The budding can take place very rapidly, leading to the lysis of the host cell (this seem to be the case in T4 cell infection), or it can take place slowly and spare the host cell, as seen in macrophages an monocytes.

Perelson (1989) and Perelson et al. (1993) modelled these events by considering cells that are uninfected, cells that are latently infected, i.e., that contain the virus but are not producing it, cells that are actively infected, i.e., that are producing virus, and the population of free viral particles. They described the dynamics of these population by the system of ordinary differential equations that we give below".[44]
The reference model used in papers [16, 30, 44, 46, 83, 84, 86] is the following

$$\begin{aligned}
\dot{T}(t) &= \frac{s_1}{1+V(t)} - \mu_T T(t) + rT(t)\left(1 - \frac{T(t)+T^l(t)+T^a(t)}{T_{max}}\right) \\
&- \big(1-u(t)\big)k_1 V(t)T(t) \\
\dot{T^l}(t) &= \big(1-u(t)\big)k_1 V(t)T(t) - \mu_{T^l}T^l(t) - k_2 T^l(t) \\
\dot{T^a}(t) &= k_2 T^l(t) - \mu_{T^a}T^a(t) \\
\dot{V}(t) &= N\mu_{T^a}T^a(t) - k_1 V(t)T(t) - \mu_V V(t).
\end{aligned} \tag{1.19}$$

Observe that again it appears the multiplicative term $N\mu_{T^a}$ as a growth rate.

This model has some common terms with previous formulations. In particular the production of T cells due to cloning in the presence of an antigen term

$$rT(t)\left(1 - \frac{T(t)+T^l(t)+T^a(t)}{T_{max}}\right),$$

already described in Sect. 1.3.1 and the rate of generation of uninfected T cells without the presence of HIV term

$$\frac{s_1}{1+V(t)}$$

similar to the one already described in Sub-Sect. 1.2.1 and 1.3.1.

Kirschner and Perelson in [44] propose an extended variant of (1.19), that includes the population of macrophages and monocytes.The dynamics are

$$\begin{aligned}
\dot{T}(t) &= \frac{s_1\theta}{\theta + V(t)} - \mu_T T(t) + rT(t)\left(1 - \frac{T(t) + T^l(t) + T^a(t)}{T_{max}}\right) \\
&- k_1 V(t)T(t) - k_3 M^a(t)T(t) \\
\dot{T^l}(t) &= k_1 V(t)T(t) + k_3 M^a(t)T(t) - \mu_{T^l} T^l(t) - k_2 T^l(t) \\
\dot{T^a}(t) &= k_2 T^l(t) - \mu_{T^a} T^a(t) \\
\dot{V}(t) &= N z(t)\mu_{T^a} T^a(t) + \Pi_M z(t) M^a(t) - k_1 V(t)T(t) - \mu_V V(t) \\
\dot{M}(t) &= \mu_M E_M - \mu_M M(t) - k_4 V(t) M(t) \\
\dot{M^a}(t) &= k_4 V(t) M(t) - \mu_{M^a} M^a(t),
\end{aligned} \tag{1.20}$$

where $z(t)$ is a term which reduces the effects of viral replication. In particular

$$z(t) = \begin{cases} 1 & \text{outside the treatment period} \\ P & \text{during the time of AZT treatment.} \end{cases}$$

The authors assume that constant P is proportional to the dose of the drug. It can also be interpreted as the effectiveness of the drug in halting viral reproduction.
Notice that there is no latently infected macrophage population, since the virus seems to replicate once inside them, [44].

Optimal control approach

In [16], [30] and [46], an optimal control approach is tackled in order to determine the optimal therapy. The objective function to maximise is

$$J(u(t)) = \int_{t_0}^{t_f} \left[T(t) - \frac{1}{2}Au^2(t)\right] \mathrm{d}t. \tag{1.21}$$

1.3.4 Models with viruses differentiation

Another type of differentiation can be made among the viruses. This sub-section contains models without immune effectors/precursors and uninfected and infected T cells differentiation, but with viruses differentiation. Works of Bonhoeffer et al. [9], Marée et al. [58] and Ribeiro et al. [94, 95] are included in this group. All models in this section are also characterized by the presence of the parameter s that represents a selection coefficient.

Ribiero et al. in [94] present a basic model, and afterwards expand it to consider resistant mutants that differ by two mutation errors during reverse transcriptase phase. Their model considers five variables: uninfected T cells, cells infected by wild type virus, two types of cells infected by one-error mutant and cells infected by two-errors mutant. We report only the reference model, as the extended one is closely related to it.

$$\begin{aligned}
\dot{T}(t) &= s_1 - \mu_T T(t) - k_1\big[T_1^i(t) + (1-s)T_2^i(t)\big]T(t) \\
\dot{T}_1^i(t) &= k_1\big[(1-u_v)T_1^i(t) + (1-s)u_v T_2^i(t)\big]T(t) - \mu_{T^i}T_1^i(t) \\
\dot{T}_2^i(t) &= k_1\big[u_v T_1^i(t) + (1-s)(1-u_v)T_2^i(t)\big]T(t) - \mu_{T^i}T_2^i(t).
\end{aligned} \tag{1.22}$$

Observe that the authors do not consider a separate equation for the free virions, but simply assume that virion abundance is proportional to infected cell abundance.

Ribiero et al. in [95] present a multistrain model linked to the previous one, where the selection coefficient (s_k) changes with the different virus strain, in addition probability of mutation is taken into account. The dynamics are

$$\begin{aligned}
\dot{T}(t) &= s_1 - \mu_T T(t) - T(t)\sum_{j=0}^{l} k_1(1-u)(1-s_j)T_j^i(t) \\
\dot{T}_j^i(t) &= \sum_{k=0}^{l} k_1(1-u)(1-s_k)u_v^{j,k}T_k^i(t) - \mu_{T^i}T_k^i(t),
\end{aligned} \tag{1.23}$$

where s_k is the selection coefficient of k-th virus strain and the matrix $u_v^{j,k}$ describes the probability of mutation of strain k into strain j during reverse transcription. The authors also assume that cells infected by mutant j are produced either by infection of a susceptible cell with mutant j or by mutation of strain k into strain j during infection.

Bonhoeffer et al. in [9] and Marée at al. in [58] use a different model considering wild-type virus (V_1) and mutant virus (V_2) as follows

$$\begin{aligned}
\dot{T}(t) &= \alpha F(t)T^i(t) - \mu_{T^i}T^i(t) \\
\dot{V}_1(t) &= \alpha F(t)V_1(t) - \mu_V V_1 \\
\dot{V}_2(t) &= \alpha(1-s)F(t)V_2(t) - \mu_V V_2,
\end{aligned} \tag{1.24}$$

where $\alpha = g'k_1/\beta$ appears as a generalized replication rate combining infection k_1, production g' and clearance β. Function $F(t)$, $0 \le F(t) \le 1$ represents target cells availability (and/or other factors limiting viral replication).

1.3.5 Models with both uninfected and infected T cells differentiation

This sub-section contains models with uninfected and infected T cells differentiation, without immune effectors/precursors and viruses differentiation. The reference model we describe is used by Adams et al. [1], Banks et al. [4, 5], Neri et al. [67] and Jang et al. [36]. This model entails two co-circulating populations of target cells: T_1 and T_2. In Adams et al. [1], T_2 corresponds to macrophage/monocyte population size.
Observe that also a multidrug treatment is considered and, as in [132, 133, 134], all effects of HAART medication are combined and represented by control variables. The dynamics are

$$\begin{aligned}
\dot{T}_1(t) &= s_{1,1} - \mu_T T_1(t) - k_{1,1}\big(1-u_{rti}(t)\big)V(t)T_1(t)\\
\dot{T}_2(t) &= s_{1,2} - \mu_T T_2(t) - k_{1,2}\big(1-f u_{rti}(t)\big)V(t)T_2(t)\\
\dot{T}_1^i(t) &= k_{1,1}\big(1-u_{rti}(t)\big)V(t)T_1(t) - \mu_{T^i}T_1^i(t) - i_{r,1}I^E(t)T_1^i(t)\\
\dot{T}_2^i(t) &= k_{1,2}\big(1-f u_{rti}(t)\big)V(t)T_2(t) - \mu_{T^i}T_2^i(t) - i_{r,2}I^E(t)T_2^i(t)\\
\dot{V}(t) &= \big(1-u_{pi}(t)\big)N\mu_{T^i}\big(T_1^i(t)+T_2^i(t)\big) - \mu_V V(t)\\
&- \big[k_{1,1}\rho_1\big(1-u_{rti}(t)\big)V(t)T_1(t) + k_{1,2}\rho_2\big(1-f u_{rti}(t)\big)V(t)T_2(t)\big]\\
\dot{I}^E(t) &= \lambda_E + \frac{b_E\big(T_1^i(t)+T_2^i(t)\big)}{\big(T_1^i(t)+T_2^i(t)\big)+K_b}I^E(t) - \frac{d_E\big(T_1^i(t)+T_2^i(t)\big)}{\big(T_1^i(t)+T_2^i(t)\big)+K_d}I^E(t)\\
&- \mu_{I^E}I^E(t).
\end{aligned} \tag{1.25}$$

The term $\big[k_{1,1}\rho_1(1-u_{rti})V(t)T_1(t) + k_{1,2}\rho_2(1-f u_{rti})V(t)T_2(t)\big]$ is included in the $\dot{V}$ equation in order to account for the removal of free virus taking places when free virions infect a T_1 or T_2 cell. The model considers the possibility that more viruses can be responsible for each new infection, that is, a cell can be infected by $\rho_i > 1$ viruses. Observe that again it appears the multiplicative term $N\mu_{T^i}$ as a growth rate.
This specific model presents also immune effectors, but in the others models below, immune effectors are not consider.

Banks et al. [4, 5] also consider infectious and non-infectious viruses, by means of the two variables V_I and V_{NI} respectively. The differential equations with these variables are

$$\begin{aligned}
\dot{V}_I(t) &= (1-u_{pi})N\mu_{T^i}\big(T_1^i(t)+T_2^i(t)\big) - \mu_V V_I(t)\\
&- \big[k_{1,1}\rho_1(1-u_{rti})V_I(t)T_1(t) + k_{1,2}\rho_2(1-f u_{rti})V_I(t)T_2(t)\big]\\
\dot{V}_{NI}(t) &= u_{pi}N\mu_{T^i}\big(T_1^i(t)+T_2^i(t)\big) - \mu_V V_{NI}(t).
\end{aligned} \tag{1.26}$$

Optimal control approach

Two are the optimal control approaches belonging to this group.
Jang et al. [36] minimize the objective function:

$$\begin{aligned} J(u_{rti}, u_{pi}, t) &= \frac{1}{2}\Bigg[\int_{t_0}^{t_f} \big(A_{rti}u_{rti}^2(t) + A_{pi}u_{pi}^2(t)\big)\,\mathrm{d}t \\ &+ Q\big(V(t_f) - \widetilde{V}\big)^2 + S\big(I^E(t_f) - \widetilde{I}^E\big)^2 + Pt_f^2\Bigg], \end{aligned} \quad (1.27)$$

where Q, S, P are the utilities of having a level of virus and immune effectors as close as possible to the steady state. In fact $\widetilde{V}$ and $\widetilde{I}^E$ are virus and immune effectors "healthy" steady states.

Neri et al. [67] suggest this objective function to minimize

$$J(\alpha, \beta) = \int_{t_0}^{t_f} \Big[10\big(I^E(t) - \widetilde{I}^E\big)^2 + \alpha^2 + \beta^2\Big]\mathrm{d}t, \quad (1.28)$$

where $\alpha = (\alpha_1, \alpha_2, \ldots)$ define the RTI medication schedule. Specifically, each α_k of even place gives the length (in days) of the kth ON-medication period whereas each α_k of odd place gives the length of the kth OFF-medication period. The schedule for PI medication is defined in the same way by an integer vector $\beta = (\beta_1, \beta_2, \ldots)$.

1.3.6 Models with both infected T cells and viruses differentiation

This sub-section contains models with infected T cells and viruses differentiation, without immune effectors/precursors and with not uninfected T cells differentiation.
Some authors introduce the game theoretical approach in order to consider the interaction between the immune system/therapy and HIV virus.[128, 129]

Works belonging to this sub-section are: McLean et al. [61, 62], Bonhoeffer et al. [8], Jung et al. [42], Di Mascio et al. [28], Jolly et al. [41], Levy et al. [50], Chen et al. [18], Dang et al. [21], Velichenko et al. [112, 113], Sattentau et al. [98], Luo et al. [55], Wu et al. [128, 129].

In [128] Wu et al. present a basic model where T cells infected by resistant virus release copies of the resistant virus into the bloodstream. Besides, they hypothesize that T cells infected by the sensitive virus, however, can release copies of both sensitive and resistant viruses, where the fraction of resistant viruses u_v is the virus control. The model appears as

$$\begin{array}{rcl} \dot{T}(t) &=& s_1 - \mu_T T(t) - k_{1,s}\big(1-u_{rti}(t)\big)V_s(t)T(t) - k_{1,r}V_r(t)T(t) \\ \dot{T}^i_s(t) &=& k_{1,s}\big(1-u_{rti}(t)\big)V_s(t)T(t) - \mu_{T^i_s}T^i_s(t) \\ \dot{T}^i_r(t) &=& k_{1,r}V_r(t)T(t) - \mu_{T^i_r}T^i_r(t) \\ \dot{V}_s(t) &=& \big(1-u_{pi}(t)\big)\big(1-u_v(t)\big)\pi_s T^i_s(t) - \mu_{V_s}V_s(t) \\ \dot{V}_r(t) &=& \pi_r T^i_r(t) + \pi_s u_v(t)T^i_s(t) - \mu_{V_r}V_r(t), \end{array} \tag{1.29}$$

where $V_s(t)$, $V_r(t)$ are the number of sensitive viruses and mutated viruses, and $T^i_s(t)$, $T^i_r(t)$ are T-cells infected by sensitive or resistant viruses.
In the viruses equations, the source of viruses not depend on $N\mu_{T^i}$ as in previous groups of models but on π_s and π_r (growth rate of sensitive or resistant viruses respectively, see Table 2.4).
In [129] Wu et al. extend the previous model considering 4 types of viral population: wild-type virus with TW10 epitope, mutant virus with only T242N mutation, mutant virus with only G248A mutation, mutant virus with both T242N and G248A mutations; and 3 type of T-cells: unifected T-cells, infected T-cells, Cytotoxic T Lymphocytes (CTL).

Luo et al. [55] present this model

$$\begin{array}{rcl} \dot{T}(t) &=& s_1 - \mu_T T(t) - k_{1,s}\big(1-\xi_{s,1}u_1\big)\big(1-\xi_{s,2}u_2\big)V_s(t)T(t) \\ &-& k_{1,r}\big(1-\xi_{r,1}u_1\big)\big(1-\xi_{r,2}u_2\big)V_r(t)T(t) \\ \dot{T}^i_s(t) &=& k_{1,s}\big(1-\xi_{s,1}u_1\big)\big(1-\xi_{s,2}u_2\big)V_s(t)T(t) - \mu_{T^i_s}T^i_s(t) + \lambda_s \\ \dot{T}^i_r(t) &=& k_{1,r}\big(1-\xi_{r,1}u_1\big)\big(1-\xi_{r,2}u_2\big)V_r(t)T(t) - \mu_{T^i_r}T^i_r(t) + \lambda_r \\ \dot{V}_s(t) &=& N_s\mu_{T^i_s}T^i_s(t) - \mu_{V_s}V_s(t) \\ \dot{V}_r(t) &=& N_r\mu_{T^i_r}T^i_r(t) - \mu_{V_r}V_r(t), \end{array} \tag{1.30}$$

where $\xi_{k,i}$ is the maximum effectiveness of antiviral regimen u_i on virus strain v_k (they use ξ and not f, see Table 2.4). There is also a contribution due to the activation from long-lived reservoir at rate λ_k. Notice that u_i are not controls variables but parameters. This model is arbitrarily scalable to any number of viral species.

Bonhoeffer in [8] gives two variants, the first one considers the presence of two strains of viruses: wild-type virus (V_1) and mutant virus (V_2), so there will be cells infected by the former (named T^i_1) and cells infected by the latter (named T_2^i). Dynamics are

$$\begin{aligned}
\dot{T}(t) &= s_1 - \mu_T T(t) - k_{1,1}V_1(t)T(t) - k_{1,2}V_2(t)T(t) \\
\dot{T}_1^i(t) &= k_{1,1}(1-u_v)V_1(t)T(t) + k_{1,2}u_v V_2(t)T(t) - \mu_{T^i}T_1^i(t) \\
\dot{T}_2^i(t) &= k_{1,1}u_v V_1(t)T(t) + k_{1,2}(1-u_v)V_2(t)T(t) - \mu_{T^i}T_2^i(t) \\
\dot{V}_1(t) &= g_1' T_1^i(t) - \mu_V V_1(t) \\
\dot{V}_2(t) &= g_2' T_2^i(t) - \mu_V V_2(t).
\end{aligned} \tag{1.31}$$

As we have seen in Sub-Sect. 1.3.2, g_1' and g_2' are the growth rates of virions due to infected cells (see Table 2.4) and specifically $g' = N\mu_{T^i}$ as we have seen in Sub-Sect. 1.3.2.

The second model proposed by Bonhoeffer, splits the infected T cells in productively infected cells (T_1^a), latently infected cells (T_1^l), chronic producers (T_2^a), defectively infected (T_2^l) cells, and their dynamics are as follows

$$\begin{aligned}
\dot{T}(t) &= s_1 - \mu_T T(t) - k_1 V(t)T(t) \\
\dot{T}_1^a(t) &= k_1 q_1 V(t)T(t) - \mu_{T_1^a}T_1^a(t) \\
\dot{T}_1^l(t) &= k_1 q_2 V(t)T(t) - \mu_{T_1^l}T_1^l(t) \\
\dot{T}_2^a(t) &= k_1 q_3 V(t)T(t) - \mu_{T_2^a}T_2^a(t) \\
\dot{T}_2^l(t) &= k_1 q_4 V(t)T(t) - \mu_{T_2^l}T_2^l(t) \\
\dot{V}(t) &= g_1' T_1^a(t) + g_3' T_2^a(t) - \mu_V V(t).
\end{aligned} \tag{1.32}$$

Finally Di Mascio et al. in [28] aver that latently infected cells (T^l), if activated, read the viral DNA and may begin viral production, i.e. they convert into T^a cells. This model contains productively infected cells (T^a), long-lived infected cells (M^a), and latently infected cells (T^l). It also incorporates both infectious (V_I) and non-infectious (V_{NI}) virions.
Observe that delays (τ_{rti} and τ_{pi}) are introduced to take into account that antiretroviral drugs are not instantly active (pharmacological delay) and that the pharmacological delay for reverse trancriptase inhibitors (RTI) may be different from the protease inhibitors (PI) one.
They assume for simplicity that the number of uninfected CD4+ T cells (T) and long-lived cells (M) remain constant during the period of observation.

Di Mascio et al. model is

$$\begin{aligned}
\dot{T^a}(t) &= \big[1-u_{rti}h(t-\tau_{rti})\big]k_{1,T}V_I(t)T(t)+k_2T^l(t)-\mu_{T^a}T^a(t)\\
\dot{M^a}(t) &= \big[1-u_{rti}h(t-\tau_{rti})\big]k_{1,M}V_I(t)M(t)-\mu_{M^a}M^a(t)\\
\dot{T^l}(t) &= \big[1-u_{rti}h(t-\tau_{rti})\big]fk_{1,T}V_I(t)T(t)-k_2T^l(t)-\mu_{T^l}T^l(t)\\
\dot{V_I} &= \big[1-u_{pi}h(t-\tau_{pi})\big]N\mu_{T^a}T^a(t)+\big[1-u_{pi}h(t-\tau_{pi})\big]g'_M M^a(t)\\
&- \mu_V V_I(t)\\
\dot{V}_{NI} &= u_{pi}h(t-\tau_{pi})N\mu_{T^a}T^a(t)+u_{pi}h(t-\tau_{pi})g'_M M^a(t)-\mu_V V_{NI}(t),
\end{aligned} \tag{1.33}$$

where $h(t-\tau)$ is a Heavyside function, which is 0 for $t<\tau$ and 1 for $t\geq\tau$.

Velichenko et al. propose in [112, 113] a model which is very similar to the one presented by Wu et al. in [128]. They take into account the action of medicinal drugs on a virus throughout two functions: $\eta_1(t)$ and $\eta_2(t)$, asserting that medicaments do not act on either the resistant group of viruses nor on CD4+ T cells damaged by a resistant virus. Function $\eta_1(t)$ describes the suppression by chemotherapy of the process of infection of CD4+ T cells while $\eta_2(t)$ describes the suppression of the inflow of viruses from the lymphoid system. The dynamics are

$$\begin{aligned}
\dot{T}(t) &= s_1-\frac{s_2V_s(t)}{b_1+V_s(t)}+\frac{\lambda_1}{C+V_s(t)}V_s(t)T(t)-\mu_T T(t)-k_{1,s}\eta_1(t)V_s(t)T(t)\\
&- k_{1,r}V_r(t)T(t)\\
\dot{T}^i_s(t) &= k_{1,s}\eta_1(t)V_s(t)T(t)-\mu_{T^i}T^i_s(t)-\frac{\lambda_2}{C_i+V_s(t)}T^i_s(t)\big(V_s(t)+V_r(t)\big)\\
\dot{T}^i_r(t) &= k_{1,r}V_r(t)T(t)-\mu_{T^i}T^i_r(t)-\frac{\lambda_2}{C_i+V_s(t)}T^i_r(t)\big(V_s(t)+V_r(t)\big)\\
\dot{V}_s(t) &= (1-Q)\frac{\lambda_3}{C_i+V_s(t)}T^i_s(t)\big(V_s(t)+V_r(t)\big)-i_rT(t)V_s(t)\\
&+ \eta_2(t)\frac{G_sV_s(t)}{b_{2,s}+V_s(t)}\\
\dot{V}_r(t) &= \frac{\lambda_3}{C_i+V_s(t)}T^i_r(t)\big(V_s(t)+V_r(t)\big)+Q\frac{\lambda_3}{C_i+V_s(t)}-i_rT(t)V_r(t)\\
&+ \frac{G_r(V)V_r(t)}{b_{2,r}+V_s(t)}.
\end{aligned} \tag{1.34}$$

At a later stage the authors introduce the following formulation for functions $\eta_1(t)$ and $\eta_2(t)$

$$\eta_1(t)=e^{-c_1(t-t_0)} \qquad \eta_2(t)=\max\{e^{-c_2(t-t_0)},c_3\}.$$

They first assume these functions as exogenously given, after that they consider also the treatment process and therefore declare the following motion equations for such variables

$$\begin{aligned} \dot{\eta}_1(t) &= c_1\big[1-\eta_1(t)-u_1\big] \\ \dot{\eta}_2(t) &= c_2\frac{\big[1-\eta_2(t)+u_2(c_3-1)\big]}{1-c_3} \\ \eta_1(0) &= 1 \\ \eta_2(0) &= 1. \end{aligned} \tag{1.35}$$

The threshold function $G_r(V)$ describes the inflow of a resistant from the lymphoid system and it is defined as follows

$$G_r(V) = \begin{cases} 0 & V < V_0 \\ G_s & V \geq V_0 \end{cases}$$

where V_0 is named resistance threshold.

McLean et al. in [61, 62] assume that there exist errors in replication, in the sense that a percentage Q of infections is "faithful" and produces progeny of the parent type, whilst remaining percentage $(1-Q)$ of the infections produces progeny of different parent type. Many mutations will be lethal, and the infective parameters ($k_{1,s}$ and $k_{1,r}$) take this into account.
The reference model is

$$\begin{aligned} \dot{T}(t) &= s_1 - \mu_T T(t) - k_{1,s}(1-u_{rti})T_s(t)T(t) - k_{1,r}(1-fu_{rti})T_r(t)T(t) \\ \dot{T}^i_s(t) &= Qk_{1,s}(1-u_{rti})T_s(t)T(t) + (1-Q)k_{1,r}(1-fu_{rti})T_r(t)T(t) \\ &- \mu_{T^i}T^i_s(t) \\ \dot{T}^i_r(t) &= (1-Q)k_{1,s}(1-u_{rti})T_s(t)T(t) + Qk_{1,r}(1-fu_{rti})T_r(t)T(t) \\ &- \mu_{T^i}T^i_r(t), \end{aligned} \tag{1.36}$$

where it is assumed that, in absence of Zidovine, sensitive strains have a fitness advantage over resistant ones (thus $k_{1,s} > k_{1,r}$). Parameter f is the relative efficacy of Zidouvine at blocking new infections with resistant virus. Notice that there isn't any virus equations and the terms

$$k_{1,s}(1-u_{rti})T_s(t)T(t) \qquad k_{1,r}(1-fu_{rti})T_r(t)T(t)$$

depend on T_s and T_r respectively, rather than on V_s and V_r.

Furthermore in [61] McLean et al. propose a model of interactions between HIV and other pathogens.

Optimal control approach

Velichenko et al. [112, 113] use an optimal control approach minimizing the following objective function

$$J = \int \varphi(t)\mathrm{d}t, \tag{1.37}$$

with

$$\varphi(t) = \begin{cases} \left[A - T(t)\right]^2 & T(t) < A \\ 0 & T(t) \geq A \end{cases}$$

where A is an endogenous threshold [111].

An interesting approach is given within this group of models by Wu et al. [128] who tackle the HIV problem in a differential game context.
The HIV payoff to maximize is

$$J\big(u_v(t)\big) = \int_{t_0}^{t_f} \left[V_s^2(t) + V_r^2(t) - A_v u_v^2(t)\right] \mathrm{d}t, \tag{1.38}$$

while the host payoff to maximize is

$$J\big(u_{rti}(t), u_{pi}(t)\big) = \int_{t_0}^{t_f} \left[T^2(t) - A_{rti}u_{rti}^2(t) - A_{pi}u_{pi}^2(t)\right] \mathrm{d}t. \tag{1.39}$$

The authors prove the existence and the uniqueness of a Nash Equilibrium and find it numerically using the forward-backward sweeping method, proposed by Velichenko et al. in [113].

1.3.7 Models with both uninfected and infected T cells differentiation and viruses differentiation

The most general approach is given by Wein et al. [116, 117], Flint et al. [31] and Kutch [49] who take into account all variables (apart from immune effectors/precursors). The differentiation is given in T-cells (both uninfected and infected) and in viruses. The presence of macrophages differentiation (M, M_1^i, M_2^i) characterizes this model with respect to models presented in Sub-Sect. 1.3.3.

Wein et al. in their works [116, 117] assume that reverse errors in transcription of strain i in cells infected by strain j occur with probability $Q_{i,j}$. Thus the mutation probability $Q_{1,2}$ is the probability that wild-type virus (V_1) is transformed into a partially drug-resistant mutant strain (V_2) during reverse transcriptase. Whilst $Q_{i,i}$ is the frequency of error-free transcription. Parameters α_i denote the fraction of new cells infected by virus strain i that are blocked by the RT inhibitors in the drug regimen, and β_i denote the fraction of newly produced virions of strain i that are rendered non-infectious by protease inhibitors (PI). The dynamics of the reference model are

$$\begin{aligned}
\dot{T}(t) &= s_{1,T} + rT(t)\left(1 - \frac{T(t) + T_1^i(t) + T_2^i(t)}{T_{max}}\right) - \mu_T T(t) \\
&- k_{1,1}(1-\alpha_1)V_1^I(t)T(t) - k_{1,2}(1-\alpha_2)V_2^I(t)T(t) \\
\dot{T}_1^i(t) &= Q_{1,1}k_{1,1}(1-\alpha_1)V_1^I(t)T(t) + Q_{2,1}k_{1,2}(1-\alpha_2)V_2^I(t)T(t) \\
&- \mu_{T_1^i}T_1^i(t) \\
\dot{T}_2^i(t) &= Q_{1,2}k_{1,1}(1-\alpha_1)V_1^I(t)T(t) + Q_{2,2}k_{1,2}(1-\alpha_2)V_2^I(t)T(t) \\
&- \mu_{T_2^i}T_2^i(t) \\
\dot{M}(t) &= s_{1,M} - \mu_M M(t) - k_{1,1}^M(1-\alpha_1)V_1^I(t)M(t) \\
&- k_{1,2}^M(1-\alpha_2)V_2^I(t)M(t) \\
\dot{M}_1^i(t) &= Q_{1,1}k_{1,1}^M(1-\alpha_1)V_1^I(t)M(t) + Q_{2,1}k_{1,2}^M(1-\alpha_2)V_2^I(t)M(t) \\
&- \mu_{M_1^i}M_1^i(t) \\
\dot{M}_2^i(t) &= Q_{1,2}k_{1,1}^M(1-\alpha_1)V_1^I(t)M(t) + Q_{2,2}k_{1,2}^M(1-\alpha_2)V_2^I(t)M(t) \\
&- \mu_{M_2^i}M_2^i(t) \\
\dot{V}_1^I(t) &= (1-\beta_1)N_1\mu_{T_1^i}T_1^i(t) + (1-\beta_1)N_1^M\mu_{M_1^i}M_1^i(t) - \mu_{V_1}V_1^I(t) \\
\dot{V}_2^I(t) &= (1-\beta_2)N_2\mu_{T_2^i}T_2^i(t) + (1-\beta_2)N_2^M\mu_{M_2^i}M_2^i(t) - \mu_{V_2}V_2^I(t) \\
\dot{V}_1(t) &= N_1\mu_{T_1^i}T_1^i(t) + N_1^M\mu_{M_1^i}M_1^i(t) - \mu_{V_1}V_1(t) \\
\dot{V}_2(t) &= N_2\mu_{T_2^i}T_2^i(t) + N_2^M\mu_{M_2^i}M_2^i(t) - \mu_{V_2}V_2(t).
\end{aligned} \tag{1.40}$$

Authors omit latently infected cells, which do not appear to play a large role for them and assume that the strength of immune response is constant over the time period under study. Indeed the model does not explicitly incorporate any immune response to the virus. Authors assume that immune response effects are incorporated into constant parameters that describe the death rate of productively infected short (μ_T) and long-lived (μ_M) cells and the rate of clearance of free virus particle (μ_V).

Kutch in [49] uses a model with latently (T_1^l, T_2^l, T_3^l) and actively (T_1^a, T_2^a, T_3^a) infected T cells and 3 viruses strains: normal viruses (V_1), RTI resistant viruses (V_2), PI resistant viruses (V_3)

$$
\begin{aligned}
\dot{T}(t) &= \frac{s_1}{1+V_1(t)+V_2(t)+V_3(t)} \\
&+ rT(t)\left(1-\frac{T(t)+T_1^l(t)+T_1^a(t)+T_2^l(t)+T_3^a(t)+T_1^l(t)+T_3^a(t)}{T_{max}}\right) \\
&- \mu_T T(t) - k_1\big(1-fu_{rti}(t)\big)V_1(t)T(t) - k_1 V_2(t)T(t) \\
&- k_1\big(1-fu_{rti}(t)\big)V_3(t)T(t) \\
\dot{T}_1^l(t) &= k_1\big(1-fu_{rti}(t)\big)V_1(t)T(t) - \mu_T T_1^l(t) - k_2 T_1^l(t) \\
\dot{T}_1^a(t) &= k_2 T_1^l(t) - \mu_{T^a} T_1^a(t) \\
\dot{V}_1(t) &= N\big(1-fu_{pi}(t)\big)\mu_{T^a} T_1^a(t) - k_1 V_1(t)T(t) - \mu_V V_1(t) \\
\dot{T}_2^l(t) &= k_1 V_2(t)T(t) - \mu_T T_2^l(t) - k_2 T_2^l(t) \\
\dot{T}_2^a(t) &= k_2 T_2^l(t) - \mu_{T^a} T_2^a(t) \\
\dot{V}_2(t) &= N\big(1-fu_{pi}(t)\big)\mu_{T^a} T_2^a(t) - k_1 V_2(t)T(t) - \mu_V V_2(t) \\
\dot{T}_3^l(t) &= k_1\big(1-fu_{rti}(t)\big)V_3(t)T(t) - \mu_T T_3^l(t) - k_2 T_3^l(t) \\
\dot{T}_3^a(t) &= k_2 T_3^l(t) - \mu_{T^a} T_3^a(t) \\
\dot{V}_3(t) &= N\mu_{T^a} T_3^a(t) - k_1 V_3(t)T(t) - \mu_V V_3(t),
\end{aligned}
\tag{1.41}
$$

where parameter f models drug efficacy, just to take into account the effectiveness of the delivery (see Table 2.4).

Optimal control approach

Indeed Kutch in [49] uses an optimal control approach maximizing the following objective function

$$
J\big(u_{rti}(t), u_{pi}(t)\big) = 1000\ T(t_f) + \int_{t_0}^{t_f} \Big[10\big(1-u_{rti}\big)^2 + 10\big(1-u_{rti}\big)^2\Big]\ \mathrm{d}t. \tag{1.42}
$$

1.3.8 Models with uninfected T cells differentiation, viruses differentiation and immune effectors/precursors presence

Wilson et al. in [119] propose a model that considers uninfected T cells and viruses differentiation, and that includes also immune effectors. Criteria characterising this group of models are the same as those already seen in SubSect. 1.2.4, with exception that here infected T lymphocytes are considered. In this model, $v(m,t)$ denotes the population of virus strain identified by a given fitness level m at time t. Authors regard the non-negative parameter m as a continuous variable. For mathematical convergence and to reduce their assumptions, Wilson et al. define the domain of m to be finite, such that $0 \leq m \leq M$. Then the total virus population ($V(t)$) at time t is given by

$$V(t) = \int_0^M v(m,t)\ \mathrm{d}m\ .$$

Analogously $s(m,t)$ is the total number of T lymphocytes with specificity for viral antigen and fitness level between m and $m + \mathrm{d}m$ at time t. Therefore the population of immune cells ($T(t)$) at time t is given by

$$T(t) = \int_0^M s(m,t)\ \mathrm{d}m.$$

Finally the population of immune cells infected at time t is given by

$$T^i(t) = \int_0^M i(m,t)\ \mathrm{d}m$$

where $i(m,t)$ represents the population (at time t) of cells infected by virus with fitness level equal to m.
The model proposed by Wilson et al. is the following

$$\begin{aligned}
\frac{\partial s(m,t)}{\partial t} &= s_1 + \frac{s_2 v(m,t)s(m,t)}{b_1 + v(m,t)} - \mu_T s(m,t) - k_1 V(t)s(m,t) \\
\frac{\partial i(m,t)}{\partial t} &= k_1 V(t)T(t)v(m,t) - \mu_{T^i} i(m,t) - \pi i(m,t) \\
&- \frac{\eta_0 m_0^n}{m_0^n + m^n} s(m,t)i(m,t) \\
\frac{\partial v(m,t)}{\partial t} &= u_V \int_0^M h(m,m')v(m',t)\ \mathrm{d}m' - u_V(1-P_0)v(m,t) \\
&+ N k_2 i(m,t) - \mu_V v(m,t),
\end{aligned} \tag{1.43}$$

where $h(m,m')$ is the probability that viral strain m' mutates to viral strain m. Specifically

$$h(m,m') = \begin{cases} P_0 & \text{if } m = m' \\ \alpha e^{-\beta(m-m')^2} & \text{if } m \neq m' \end{cases}$$

1.3.9 Models with uninfected and infected T cells differentiation, with viruses differentiation and with immune effectors/precursors

One of the most general model in HIV literature has been proposed by Kirschner and Webb in 1996,[45] and considers all criteria seen in Sect. **??**.

The peculiarity of this model is the dependence on the cells age of dynamics. Denoted by $T^i(t,a)$, the density of infected T cells with age of infection a at time t, the total infected T cell population at time t is given by

$$\int_0^{a_{max}} T^i(t,a)\,\mathrm{d}a,$$

where a_{max} is the maximum age of infected T cells. The dynamics are the following

$$\begin{aligned}
\frac{\partial T(t)}{\partial t} &= s_1 + \frac{s_2}{1+V(t)} - \mu_T T(t) + rT(t)\frac{V(t)}{b_1+V(t)} \\
&- k_1 V(t)T(t) + \int_0^{a_1} \gamma(t,a,p)T^i(t,a)\,\mathrm{d}a \\
\frac{\partial T^i(t,a)}{\partial t} + \frac{\partial T^i(t,a)}{\partial a} &= -\mu_{T^i}T^i(t,a) - rT^i(t,a)\frac{V(t)}{b_2+V(t)} \\
&- \gamma(t,a,p)T^i(t,a) \\
\frac{\partial V(t)}{\partial t} &= Nr\frac{V(t)}{b_2+V(t)}\int_{a_1}^{a_{max}} T^i(t,a)\mathrm{d}a - i_r T(t)V(t) \\
&+ \frac{gV(t)}{b_2+V(t)}.
\end{aligned} \tag{1.44}$$

Treatment corresponds to the function $\gamma(t,a,p)$, which is periodic in time t with period p, it depends on the age a of cellular infection andit is given by the expression

$$\gamma(t,a,p) = \begin{cases} \dfrac{\alpha p}{(1-e^{-\beta p})}\,e^{-\beta p} & 0 \le a \le a_1 \text{ and } 0 \le t \le p \\ \dfrac{\alpha p}{(1-e^{-\beta p})}\,e^{-\beta(t-p)} & 0 \le a \le a_1 \text{ and } p \le t \le 2p \\ \vdots & \\ 0 & a > a_1 \end{cases}$$

where α represents the total daily drug dosage in chemotherapy and β indicates the decay rate of AZT based on half-life of 1 hour.

1.4 Parameters Analysis

The aim of this section is to investigate the different values of parameters used in each model and to calculate the frequencies of such values in the cited literature. The most peculiar parameters in the previous models are listed below together with their unit of measurement.

c : Cytotoxic T Lymphocyte (CTL) activation rate $[1/day]$.

g : Input rate of external viral source $[cells/(mm^3 \cdot day)]$.

g' : Growth rate of virions due to infected cells $[cells/(mm^3 \cdot day)]$.

i_r : Death rate of infected cells due to immune response $[1/day]$.

k_1 : Rate CD4 T cells becomes infected by free virus $(mm^3/(virions \cdot day))$.

k_2 : Rate T^l convert to actively infected $[1/day]$.

μ_{IE} : Natural death rate for immune effectors (CTL) $[1/day]$.

μ_{IP} : Natural death rate of CTL precursors $[1/day]$.

μ_T : Death rate of uninfected CD4+ T cells $[1/day]$.

μ_{Ti} : Death rate of infected CD4+ T cells $[1/day]$.

μ_V : Death rate of free virus $[1/day]$.

N : Number of free virus produced by T infected cells $[virions/cell]$.

π : Growth rate of viruses $[1/day]$.

q : Growth rate of CTL eff. due to inf. cells and CTL prec $[1/day]$.

r : Rate of growth for CD4+ T cell population $[1/day]$.

T_{max} : Maximum CD4+ T cell population level $[cells/mm^3]$.

We don't go into details about the evaluation of these values, we only limit ourselves to a statistical analysis on their adoption in the models we presented. Table 1.5 shows us their frequencies

Table 1.5: Frequency of parameter values

Symbol	Use frequency	1° value	partial frequency	2° value	partial frequency
c	13%	0.03-0.3	88%	others	12%
g	12%	25	57%	30	29%
g'	17%	0-36	50%	100-300	50%
i_r	15%	1-2	67%	0.00074-0.05	33%
k_1	67%	2.4×10^{-5}	27%	$2.5 - 8 \times 10^{-4}$	20%
k_2	13%	0.003	75%	0.036-0.3	25%
μ_{I^E}	22%	0.1	92%	others	8%
μ_{I^P}	8%	0.001	100%	others	0%
μ_T	60%	0.01-0.09	44%	< 0.01	17%
μ_{T^i}	70%	0.1-0.9	55%	0.01-0.09	21%
μ_V	52%	1-9	65%	< 1	13%
N	30%	≥ 1000	56%	< 1000	44%
π	12%	1-10	86%	others	14%
q	8%	0.5	75%	others	25%
r	22%	0.03	57%	0.01	14%
s_1	73%	10	52%	1-7	25%
s_2	17%	31.98	50%	1-3	40%
T_{max}	17%	1500	70%	1700	20%

1.5 Review conclusions

We have presented a review of the literature on modelling studies for HIV and AIDS topics. Most models are formulated in terms of dynamical systems and optimal control problems; stochastic and fuzzy models are also proposed although only in few papers. A recent game theory approach has been studied and, in our opinion, it constitutes the more effective approach in order to take into account both the viral activity and the immune system/drugs action.

We think that there should be standard assumptions on parameters' evaluation because we have found some discrepancies among their values. We believe it is necessary a list of all the possible parameters that are involved in models of this type, and a detailed study of the probability distribution of their values, through the medical literature.

Finally there is a need to build more realistic models that incorporate all properties presented in this review, considering all six classes of antiretroviral drugs and determining optimal structured treatment interruption (STI) schedules for medications. Infact administration of antiretroviral drugs should be daily and uninterrupted; it requires care and precision in the observance of schedules, even in relation to meals and to use of other drugs. Faithful adherence to the prescriptions is necessary to maintain effectiveness over

time. Omission of a few doses leads to a reduction in the quantities of drugs in plasma, therefore the residual level could be lower than the one necessary to inhibit the virus. This allows the resumption of viral replication that leads inexorably to the emergence of resistant virus. Resistant mutants are destined to persist, making the drug ineffective, even if the intake of drugs is carried out according to the rhythms and the correct doses. Insufficient adherence to the prescriptions with the consequent emergence of resistant mutants are the main causes of treatment failure.
Therefore it becomes necessary a model that simulates the behavior of the immune system and of the HIV virus, so as to determine what is the optimal therapy to be administered to the patient in order to prevent viral replication in his body (rather than the complete eradication of the infection, that remains chronic) so as to reduce the damage caused to the immune system and to allow greater survival and quality of life. HIV-infected individuals, who may promptly discover their state and rely on expert team, should trust in expectancy and quality of life not dissimilar to those suffering from other chronic diseases, such as hypertension or diabetes [136].
The study of HIV biological evolution and of its relationship with the immune system is rather a crucial issue in order to determine an efficient therapy policy able to defeat it.

Chapter 2

A new model

In Chapter 1 we presented a review of the literature related to HIV modelization in the Optimal Control context and we classified the works basing on the following criteria

- Does the model considers T lymphocytes infected by the viruses?
- Does the model presents different types of infected T lymphocytes?
- Does the model presents different types of T lymphocytes?
- Does the model presents different types of HIV viruses?
- Does the model considers the immune effectors/precursors?

Now we propose a model that considers all 5 criteria above indicated, that is as general as possible, allowing complexity.
Our model present 27 variables (Table 2.1) and 8 controls (Table 2.2), among these we have 6 controls for the therapy and 2 controls for the virus. It is very complex, and it consider in general the dynamics of 4 types of cells:

1. HIV viruses;
2. CD4+ T Helper cells;
3. macrophage cells;
4. immune precursor/effector Cytotoxic T lymphocyte cells.

Each of these categories is subdivided into subcategories. Now we analyze each category.

1. **HIV viruses.** As explained in Chapter 1 there are many different types of HIV, and considers all the different types is not possible due to the computational complexity that would result.
   Therefore, in line with the related literature, as seen in [128], we consider only two types of viruses

   (a) *drug sensitive viruses* (V_s);
   (b) *drug resistant viruses* (V_r).

   Within the subcategory of sensitive viruses there are many strains of virus, and within subcategory of resistant viruses there are many strains of virus. In general, these strain of viruses can be wild type viruses or escape mutant viruses or combination of them.
   The HIV controls are the mutation rates of viruses, and precisely

   - the mutation rate from sensitive to other sensitive viruses ($u^v_{s,s}$);
   - the mutation rate from sensitive to resistant viruses ($u^v_{s,r}$);
   - the mutation rate from resistant to other resistant viruses ($u^v_{r,r}$);
   - the mutation rate from resistant to sensitive viruses ($u^v_{s,r}$);

   where we suppose that $u^v_{s,s} = 1 - u^v_{s,r}$, i.e the mutation rate from sensitive to other sensitive viruses and the mutation rate from sensitive to resistant viruses are complementary. Similarly $u^v_{r,r} = 1 - u^v_{r,s}$.

2. **CD4+ T Helper cells** category is subdivided into

   (a) *generic CD4+ T Helper cells* also said *naive T cells*. They are mature T cells that did not meet yet their antigen (Ag). In fact through the process of lymphocyte recirculation, they circulate continuously between the blood and secondary or periferical lymphoid tissues (such as the lymph nodes, the spleen and mucosa-associated lymphoid tissue as Peyer's patches) then return directly in the bloodstream through the spleen or through the thoracic duct.
   In this way, the naive T cells patrol the body in search of the Ag, coming into contact with thousands of Antigen Presenting Cell (APC or B7 molecules) , so check out all Major Histocompatibility Complex (MHC) peptide complex .
   If the naive T lymphocyte do not encounters the specific Ag, within a relatively short time, it leaves the peripheral lymphoid tissue and recirculates between the blood and the tissues, until it meets the Ag or it dies and it is replaced by a new lymphocyte that expresses different receptors.
   In this way you have a spare of the T cell receptor (TCR) repertoire and the number of circulating lymphocytes is constant.

If naive T lymphocytes recognize the specific Ag (presented by APC cells in the peripheral lymphoid tissue) there is their activation, their proliferation or clonal expansion through 2-4 mitosis per day for 3-5 days and their differentiation into effector cells capable to destroy infected cells or to activate other cells of the immune system.
Finally, lymphocytes leave the lymph nodes trough the efferent lymphatic vessels or they leave the spleen through the trabecular vessels.

In our model this cells, can be

i. *uninfected generic CD4+ T Helper cells* (T_h), these represent healthy naive T cells;

ii. *infected generic CD4+ T Helper cells*, these represent naive T cells infected by viruses. “Stimulation of the T cell by antigen can lead to the production of new virus particles that bud from the surface of the infected cell. The budding can take place very rapidly, leading to the lysis of the host cell (this seem to be the case in T4 cell infection), or it can take place slowly and spare the host cell, as seen in macrophages an monocytes.
Perelson (1989) and Perelson et al. (1993) modelled these events by considering cells that are uninfected, cells that are latently infected, i.e., that contain the virus but are not producing it, cells that are actively infected, i.e., that are producing virus, and the population of free viral particles. They described the dynamics of these population by the system of ordinary differential equations that we give below”.[44]
Therefore we subdivided this category in 4 subcategories [16, 17, 30, 44, 46, 83, 84, 86]

   A. *latently naive T cells, infected by sensitive viruses* ($T^l_{h,s}$);
   B. *actively naive T cells, infected by sensitive viruses* ($T^a_{h,s}$);
   C. *latently naive T cells, infected by resistant viruses* ($T^l_{h,r}$);
   D. *actively naive T cells, infected by resistant viruses* ($T^a_{h,r}$).

(b) *CD4+ T Helper cells strain specific for sensitive viruses*, as we seen in Nowak works [69, 70, 71, 72, 73] the main points are the following

- the immune response to viral infection creates a subpopulation of lymphocytes that are specific for a particular virus strain and that attack only this particular viral strain;
- the immune response to the virus is also characterized by a response that is specific to the virus, but it acts against all

strains. In other words, it acts against the parts of the virus preserved by the mutations;

- each mutation of the initial viral infection can cause the death of all lymphocytes, as these can be directed to parts of the virus variables or preserved.

In our model this cells can be:

i. *uninfected CD4+ T Helper cells strain specific for sensitive viruses* (T_s), these represent healthy CD4+ T Helper cells strain specific for sensitive viruses;

ii. *infected CD4+ T Helper cells strain specific for sensitive viruses*. We subdivided also this category in 4 subcategories [16, 17, 30, 44, 46, 83, 84, 86]

A. *latently CD4+ T Helper cells strain specific for sensitive viruses, infected by sensitive viruses* ($T^l_{s,s}$);

B. *actively CD4+ T Helper cells strain specific for sensitive viruses, infected by sensitive viruses* ($T^a_{s,s}$);

C. *latently CD4+ T Helper cells strain specific for sensitive viruses, infected by resistant viruses* ($T^l_{s,r}$);

D. *actively CD4+ T Helper cells strain specific for sensitive viruses, infected by resistant viruses* ($T^a_{s,r}$).

(c) *CD4+ T Helper cells strain specific for resistant viruses.*
Similarly to what we saw in the previous point in our model this cells can be:

i. *uninfected CD4+ T Helper cells strain specific for resistant viruses* (T_r), these represent healthy CD4+ T Helper cells strain specific for resistant viruses;

ii. *infected CD4+ T Helper cells strain specific for resistant viruses*. We subdivided also this category in 4 subcategories [16, 17, 30, 44, 46, 83, 84, 86]

A. *latently CD4+ T Helper cells strain specific for resistant viruses, infected by sensitive viruses* ($T^l_{r,s}$);

B. *actively CD4+ T Helper cells strain specific for resistant viruses, infected by sensitive viruses* ($T^a_{r,s}$);

C. *latently CD4+ T Helper cells strain specific for resistant viruses, infected by resistant viruses* ($T^l_{r,r}$);

D. *actively CD4+ T Helper cells strain specific for resistant viruses, infected by resistant viruses* ($T^a_{r,r}$).

(d) *CD4+ T Helper cells unspecific for sensitive and resistant viruses.*
Similarly to what we saw in the previous point in our model this cells can be:

i. *uninfected CD4+ T Helper cells unspecific for sensitive and resistant viruses* (T_g), these represent healthy CD4+ T Helper cells unspecific for sensitive and resistant viruses;

ii. *infected CD4+ T Helper cells unspecific for sensitive and resistant viruses.* We subdivided also this category in 4 subcategories [16, 17, 30, 44, 46, 83, 84, 86]

A. *latently CD4+ T Helper cells unspecific for sensitive and resistant viruses, infected by sensitive viruses* ($T^l_{g,s}$);

B. *actively CD4+ T Helper cells unspecific for sensitive and resistant viruses, infected by sensitive viruses* ($T^a_{g,s}$);

C. *latently CD4+ T Helper cells unspecific for sensitive and resistant viruses, infected by resistant viruses* ($T^l_{g,r}$);

D. *actively CD4+ T Helper cells unspecific for sensitive and resistant viruses, infected by resistant viruses* ($T^a_{g,r}$).

3. **Macrophages.** We extend our model by including the macrophage/ monocyte (mac/mono) cell population. According to [63], there are approximately $6000/mm^3$ white blood cells in a healthy human. No more than 5% of these are in mac/mono population. Approximately 10% of these cells are CD4+ [81]. The reasons for including the macrophage/monocyte cell populations are many. HIV is cytopathic in T4 cells; however, macrophages/monocytes survive once infected, and slowly bud new virus particles. They therefore, play a role as a viral source referred to as a reservoir. Notice the fact that infected macrophages appears to be able to infect T4 cells through presentation of antigen [57]. Macrophages have a long life span, so we take the per capita death rate of uninfected cells to be $0.005/day$ [27].
We have assumed that there is no latently infected macrophages population since the virus seems to always replicate inside them [44].
We also assume that macrophages produce virus at a slow constant rate, sparing the host cell, so there is only natural death for these cells and not death by bursting like that for infected T4 cells.
In our model also macrophages can be

(a) *uninfected macrophages* (M_h), these represent healthy macrophage/ monocyte population cells

(b) *infected macrophages*, these represent macrophage/monocyte population cells infected by viruses. We subdivided also this category in 2 subcategories [44]

i. *actively macrophages infected by sensitive viruses* ($M^a_{h,s}$);

ii. *actively macrophages infected by resistant viruses* ($M^a_{h,r}$).

4. **Immune precursor/effector Cytotoxic T lymphocytes.** Cytotoxic T lymphocyte (CTL) are also know as CD8+ lymphocytes or as

T_C cells. They have a lytic capability and eliminate the virus infected cells along with the tumor cells. Most cytotoxic T cells express T-cells receptors (TCRs) that can recognize a specific antigen peptide bound to class I MHC molecules, present on all nucleated cells, and a glycoprotein called CD8, which is attracted to non-variable portions of the class I MHC molecule. The affinity between CD8 and the MHC molecule keeps the T_C cell and the target cell bound closely together during antigen-specific activation. Effector CTLs (in our model I^e) are generated from CTL precursors (in our model I^p). The naive T_C cells are also known as CTL precursors and their activation leads to the differentiation of effector CTLs from them.

2.1 Variables, Controls and Parameters

Here we present variables, controls, and parameters of our model. As already anticipated the model provides

- 27 variables (Table 2.1);
- 8 controls (Table 2.2);
- 98 parameters (Table 2.3 and Table 2.4).

Note that values assigned to all parameters in our instances are listed in Appendix B.

Table 2.1: Variables (Unit of measure: $cells/mm^3$)

Symbol	Description
I^p	Immune Precursor Cytotoxic T Lymphocytes
I^e	Immune Effector Cytotoxic T Lymphocytes
M_h	Uninfected macrophages
$M_{h,j}$	Macrophage infected by viruses j
T_h	Uninfected CD4+ T Helper cells
T_s	Uninfected CD4+ T Helper cells strain specific for V_s
T_r	Uninfected CD4+ T Helper cells strain specific for V_r
T_g	Uninfected CD4+ T Helper cells unspecific for V_s and V_r
$T^l_{i,j}$	Latently infected CD4+ T_i cells infected by virus j
$T^a_{i,j}$	Actively infected CD4+ T_i cells infected by virus j
V_s	HIV drugs sensitive viruses
V_r	HIV drugs resistant viruses

Table 2.2: Controls ($\in [0, 1]$)

Symbol	Description
u_{co}	Dosage of coreceptor CCR5 antagonists
u_{fi}	Dosage of fusion inhibitors
u_{ib}	Immune boosting
u_{ii}	Dosage of integrase inhibitors
u_{pi}	Dosage of protease inhibitors
u_{rt}	Dosage of reverse transcriptase inhibitors
$u^v_{i,j}$	Mutation rate from virus V_i to virus V_j

Table 2.3: Parameters (Unit of measure: $1/day$)

Symbol	Description
c	Cytotoxic T Lymphocyte (CTL) activation rate
i_j	Death rate of cells infected by viruses V_j due to immune response
k	Rate T_h convert to specific immune reaction cells (T_s or T_r)
k'	Rate T_h convert to unspecific immune reaction cells (T_g)
k_2	Rate latently infected cells convert to actively infected cells
μ_i	Natural death rate of type i cells
π_s	Growth rate of sensitive viruses (V_s)
π_r	Growth rate of resistant viruses (V_r)
q	Growth rate of I^e due to infected cells and I^p
r	rate of growth for CD4+ T Helper cell population

Table 2.4: Parameters

Symbol	Description	U.m./Value
A_i	Weight on the benefit i and cost of therapy i	$\in [0, +\infty]$
$B_{i,j}$	Weight of viral mutation from strain i to strain j	$\in [0, +\infty]$
b_2	Half saturation constant	mm^3
g	Input rate of external viral source	$\frac{cells}{mm^3 \times day}$
I_0^e	Initial value of I^e cells	$\frac{cells}{mm^3}$
I_0^p	Initial value of I^p cells	$\frac{cells}{mm^3}$
$k_{1,j}$	Rate CD4 T cells becomes infected by viruses j	$\frac{mm^3}{virions \times day}$
$k_{1,j}^M$	Rate macrophages becomes infected by viruses j	$\frac{mm^3}{virions \times day}$
k_3	Rate infected macrophages infects CD4+ T cells	$\frac{mm^3}{cells \times day}$
M_h^0	Initial value of M_h cells	$\frac{cells}{mm^3}$
$M_{h,j}^{a,0}$	Initial value of actively M_h cells infected by viruses V_j	$\frac{cells}{mm^3}$
θ_i	Scaling parameter for type i cells	
p'_j	Specific immune response rate against viruses V_j	$\frac{cells}{mm^3 \times day}$
$Q_{i,j}$	Percentage of i cells that recognize the virus j	$\in [0, 1]$
ρ_j	Average number of virions j infecting a cell	$\frac{virions}{cell}$
$s_{1,j}$	Source/production of type j cells	$\frac{cells}{mm^3 \times day}$
s'_j	Unspecific immune response rate against viruses V_j	$\frac{cells}{mm^3 \times day}$
T_i^0	Initial value of T_i cells	$\frac{cells}{mm^3}$
$T_{i,j}^{l,0}$	Initial value of latently T_i cells infected by viruses V_j	$\frac{cells}{mm^3}$
$T_{i,j}^{a,0}$	Initial value of actively T_i cells infected by viruses V_j	$\frac{cells}{mm^3}$
T_{max}	Maximum CD4+ T Helper cell population level	$\frac{cells}{mm^3}$
V_j^0	Initial value of viruses V_j	$\frac{cells}{mm^3}$

2.2 Payoff

The immune system/therapy payoff to maximize is

$$\begin{aligned} J_{Ther}\big(u_1(t)\big) &= \int_{t_0}^{t_f} f_{0,1}\big(x(t), u(t), t\big)\ \mathrm{d}t \\ &= \int_{t_0}^{t_f} \Big[M_h(t) + I^p(t) + I^e(t) + \sum_{\forall i \in \Gamma} T_i(t) \\ &- \frac{1}{2} \sum_{\forall k \in \Omega} A_k u_k^2(t)\Big]\ \mathrm{d}t \end{aligned} \tag{2.1}$$

with $\Gamma = \{h, s, r, g\}$ and $\Omega \in \{co, fi, ib, ii, pi, rt\}$.

While the viruses payoff to maximize is

$$\begin{aligned} J_{HIV}\big(u_2(t)\big) &= \int_{t_0}^{t_f} f_{0,2}\big(x(t), u(t), t\big)\ \mathrm{d}t \\ &= \int_{t_0}^{t_f} \Big[V_s(t) + V_r(t) + \sum_{\substack{\forall i \in \Gamma \\ \forall j \in \Theta}} T_{i,j}^l(t) + \sum_{\substack{\forall i \in \Gamma \\ \forall j \in \Theta}} + T_{i,j}^a(t) \\ &- \frac{1}{2}\Big(B_{s,r}\big(1 - u_{s,s}^v(t)\big)^2 + B_{r,s}\big(1 - u_{r,r}^v(t)\big)^2\Big)\Big]\ \mathrm{d}t \end{aligned} \tag{2.2}$$

with $\Gamma = \{h, s, r, g\}$ and $\Theta = \{s, r\}$.
As already said notice that $u_{s,s}^v = 1 - u_{s,r}^v$ and $u_{r,r}^v = 1 - u_{r,s}^v$ because in this model for simplicity we consider only 2 type of viruses: sensitive (V_s) and resistant (V_r) to drugs. Therefore the viruses mutations can be respectively

1. from sensitive to other sensitive viruses;
2. from sensitive to resistant viruses;
3. from resistant to other resistant viruses;
4. from resistant to sensitive viruses.

We suppose that mutations of point 1 and 2 are complementary, that is $u_{s,s}^v = 1 - u_{s,r}^v$.
Analogously we suppose that matutation of point 3 and 4 are complementary, that is $u_{r,r}^v = 1 - u_{r,s}^v$.
In this model the state functions is

$$\begin{aligned} x(t) &= \big(T_h(t), T_s(t), T_r(t), T_g(t), M_h(t), I^p(t), I^e(t), T_{h,s}^l(t), T_{h,s}^a(t), T_{h,r}^l(t), \\ &T_{h,r}^a(t), T_{s,s}^l(t), T_{s,s}^a(t), T_{s,r}^l(t), T_{s,r}^a(t), T_{r,s}^l(t), T_{r,s}^a(t), T_{r,r}^l(t), T_{r,r}^a(t), \\ &T_{g,s}^l(t), T_{g,s}^a(t), T_{g,r}^l(t), T_{g,r}^a(t), M_{h,s}^a(t), M_{h,r}^a(t), V_s(t), V_r(t)\big) \end{aligned}$$

while the controls are given by

$$u_1(t) = \big(u_{co}(t), u_{fi}(t), u_{ib}(t), u_{ii}(t), u_{pi}(t), u_{rt}(t)\big)$$

and

$$u_2(t) = \big(u^v_{s,s}(t), u^v_{r,r}(t)\big)$$

for the immune system - therapy andthe virus, respectively.

2.3 Dynamics

The dynamics for the **uninfected** $CD4+$ **T-Helper cells** are the following

$$\begin{aligned}
\dot{T}_h(t) &= f_1\big(x(t), u(t), t\big) = \\
&= \frac{s_{1,T_h}\theta_{T_h}}{\theta_{T_h} + \big(V_s(t) + V_r(t)\big)} + rT_h(t)\left(1 - \frac{\sum T_i(t) + \sum T^l_{i,j}(t) + \sum T^a_{i,j}(t)}{T_{max}}\right) \\
&+ u_{ib}(t)T_h(t) - \mu_{T_h}T_h(t) \\
&- k_{1,s}\big(1 - u_{fi}(t)\big)\big(1 - u_{co}(t)\big)\big(1 - Q_{T_h,V_s}\big)V_s(t)T_h(t) \\
&- k_{1,r}\big(1 - Q_{T_h,V_r}\big)V_r(t)T_h(t) - k\big(V_s(t) + V_r(t)\big)T_h(t) \\
&- k'\big(V_s(t) + V_r(t)\big)T_h(t) - k_3\big(M^a_{h,s}(t) + M^a_{h,r}(t)\big)T_h(t) \qquad (2.3)
\end{aligned}$$

$$\begin{aligned}
\dot{T}_s(t) &= f_2\big(x(t), u(t), t\big) = \\
&= kV_s(t)T_h(t) + u_{ib}(t)T_s(t) - \mu_{T_s}T_s(t) \\
&- k_{1,s}\big(1 - u_{fi}(t)\big)\big(1 - u_{co}(t)\big)\big(1 - Q_{T_s,V_s}\big)V_s(t)T_s(t) \\
&- k_{1,r}\big(1 - Q_{T_s,V_r}\big)V_r(t)T_s(t) - k_3\big(M^a_{h,s}(t) + M^a_{h,r}(t)\big)T_s(t) \qquad (2.4)
\end{aligned}$$

$$\begin{aligned}
\dot{T}_r(t) &= f_3\big(x(t), u(t), t\big) = \\
&= kV_r(t)T_h(t) + u_{ib}(t)T_r(t) - \mu_{T_r}T_r(t) \\
&- k_{1,s}\big(1 - u_{fi}(t)\big)\big(1 - u_{co}(t)\big)\big(1 - Q_{T_r,V_s}\big)V_s(t)T_r(t) \\
&- k_{1,r}\big(1 - Q_{T_r,V_r}\big)V_r(t)T_r(t) - k_3\big(M^a_{h,s}(t) + M^a_{h,r}(t)\big)T_r(t) \qquad (2.5)
\end{aligned}$$

$$\begin{aligned}
\dot{T}_g(t) &= f_4\big(x(t), u(t), t\big) = \\
&= k'\big(V_s(T) + V_r(t)\big)T_h(t) + u_{ib}(t)T_g(t) - \mu_{T_g}T_g(t) \\
&- k_{1,s}\big(1 - u_{fi}(t)\big)\big(1 - u_{co}(t)\big)\big(1 - Q_{T_g,V_s}\big)V_s(t)T_g(t) \\
&- k_{1,r}\big(1 - Q_{T_g,V_r}\big)V_r(t)T_g(t) - k_3\big(M^a_{h,s}(t) + M^a_{h,r}(t)\big)T_g(t) \qquad (2.6)
\end{aligned}$$

Naive lymphocytes (T_h) dynamics represented by equation (2.3), where the term

$$\frac{s_{1,T_h}\theta_{T_h}}{\theta_{T_h} + \big(V_s(t) + V_r(t)\big)},$$

(proposed by Kirschner et al. [47], Caetano et al. [17] and Joshi [41]) is the source/proliferation of uninfected CD4+ T_h cells and includes both an external (not plasma) contribution of cells from sources, such as the thymus and lymph nodes, and an internal (plasma) contribution from CD4+ T_h cells differentiation.
The term

$$rT_h(t)\left(1-\frac{\sum T_i(t)+\sum T^l_{i,j}(t)+\sum T^a_{i,j}(t)}{T_{max}}\right)$$

represents the production of T_h-cells due to cloning in the presence of an antigen, taking into account the maximum number of lymphocytes (T_{max}).
The element

$$u_{ib}(t)T_h(t)$$

represents the immune boosting that depends on control u_{ib} that allows an increase of the immune barriers proportional to the number of T_h cells.
The term

$$-\mu_{T_h}T_h(t)$$

is the natural death of naive T-cells.
The adder

$$-k_{1,s}\big(1-u_{fi}(t)\big)\big(1-u_{co}(t)\big)\big(1-Q_{T_h,V_s}\big)V_s(t)T_h(t)$$

represents the number of healthy naive cells infected by sensitive viruses, it is proportional to the number of sensitive viruses (V_s) and naive T cells (T_h). Coefficient $k_{1,s}$ is the infection rate of T_h cells by sensitive viruses.
$\big(1-u_{fi}(t)\big)$ and $\big(1-u_{co}(t)\big)$ are the actions of fusion inhibitors and coreceptor antagonists against the infection of healthy cells. These drugs interfere with binding, fusion and entry of HIV to the host cell by blocking one of several target cells. Finally $(1-Q_{T_h,V_s}\big)$ is the portion of generic CD4+ T Helper cells that does not recognize and does not obstruct sensitive viruses.
The same goes for the term

$$-k_{1,r}\big(1-Q_{T_h,V_r}\big)V_r(t)T_h(t)-k\big(V_s(t)+V_r(t)\big)T_h(t)$$

with the difference that fusion inhibitors and coreceptor antagonist can not counteract T_h cells infection by resitant viruses (V_r).
The elements

$$-k\big(V_s(t)+V_r(t)\big)T_h(t)-k'\big(V_s(t)+V_r(t)\big)T_h(t)$$

represent the number of T_h cells converted to T_s, T_r or T_g cells respectively. Of course this number is proportional to the number of sensitive (V_s) and resistant (V_r) viruses.
Finally

$$-k_3\big(M^a_{h,s}(t)+M^a_{h,r}(t)\big)T_h(t)$$

is the number of T_h cells contaminated by infected macrophages.
We now consider healthy T_s cells (2.4). For these types of cells the positive contribute is given only by

$$kV_s(t)T_h(t) + u_{ib}(t)T_s(t).$$

As described above u_{ib} represents the immune boosting and T_s the number of naive T-cells converted in CD4+ T Helper cells strain specific for sensitive viruses (V_s). They have certainly a negative contribute given by the their natural death

$$-\mu_{T_s}T_s(t).$$

Also uninfected CD4+ T Helper cells strain specific for sensitive viruses (T_s) can be infected by sensitive or resistant viruses through the terms

$$\begin{aligned}&- k_{1,s}\big(1 - u_{fi}(t)\big)\big(1 - u_{co}(t)\big)\big(1 - Q_{T_s,V_s}\big)V_s(t)T_s(t)\\&- k_{1,r}\big(1 - Q_{T_s,V_r}\big)V_r(t)T_s(t)\end{aligned}$$

or by infected macrophages across "cell to cell" infection through the term

$$-k_3\big(M^a_{h,s}(t) + M^a_{h,r}(t)\big)T_s(t).$$

Healthy T_r and T_g cells have dynamics (2.5), (2.6) which are similar to the one of healthy T_s cells (2.4).

The dynamics for the **uninfected macrophages, $CD8^+$ CTL precursors and effectors** are the following:

$$\begin{aligned}\dot{M}_h(t) &= f_5\big(x(t), u(t), t\big) =\\&= s_{1,M_h} - \mu_{M_h}M_h(t) - k^M_{1,s}\big(1 - u_{rt}(t)\big)\big(1 - u_{pi}(t)\big)V_s(t)M_h(t)\\&- k^M_{1,r}V_r(t)M_h(t) \qquad (2.7)\end{aligned}$$

$$\begin{aligned}\dot{I}^p(t) &= f_6\big(x(t), u(t), t\big) =\\&= cI^p(t)\sum T_i(t)\sum T^a_{i,j}(t) - cqI^p(t)\sum T^a_{i,j}(t) - \mu_{I^p}I^p(t) \qquad (2.8)\end{aligned}$$

$$\begin{aligned}\dot{I}^e(t) &= f_7\big(x(t), u(t), t\big) =\\&= cqI^p(t)\sum T^a_{i,j}(t) - \mu_{I^e}I^e(t) \qquad (2.9)\end{aligned}$$

Macrophages dynamic (2.7) is very simple, in fact it has a constant source given by

$$s_{1,M_h},$$

a natural death element

$$-\mu_{M_h} M_h(t),$$

and finally infections terms

$$\begin{aligned}&- k_{1,s}^{M}\big(1-u_{rt}(t)\big)\big(1-u_{pi}(t)\big)V_s(t)M_h(t)\\&- k_{1,r}^{M}V_r(t)M_h(t).\end{aligned}$$

Notice that drugs that counteract the V_s infections in these cells are the reverse transcriptase inhibitors (RTI) and protease inhibitors (PI), where the first ones inhibit reverse transcription, the second ones block the viral protease enzyme necessary to produce mature virions upon budding from the host membrane. These drugs prevent the cleavage of gag and gag/pol precursor proteins.

About immune precursors/effectors (I^p, I^e) dynamics (2.8),(2.9) we extend the model presented by Wodarz and Nowak in [120] by replacing infected T-cells with actively infected T_h, T_s, T_r and T_g cells ($\sum T_{i,j}^a$).
In this regard in the presence of free HIV (V_s or V_r), uninfected cells (T_h) can be infected to became latently or actively infected cells. The latently infected cells ($T_{h,j}^l$ with $j \in \{s,r\}$) can be activated to become actively infected cells ($T_{h,j}^a$ with $j \in \{s,r\}$) with an activation rate denoted by k_2.

The dynamics for **infected T_h cells** are the following

$$\begin{aligned}\dot{T}_{h,s}^l(t) &= f_8\big(x(t),u(t),t\big) =\\&= k_{1,s}\big(1-u_{fi}(t)\big)\big(1-u_{co}(t)\big)\big(1-Q_{T_h,V_s}\big)V_s(t)T_h(t)\\&+ k_3 M_{h,s}^a(t)T_h(t) - \mu_{T_{h,s}^l}T_{h,s}^l(t)\\&- k_2\big(1-u_{rt}(t)\big)\big(1-u_{ii}(t)\big)T_{h,s}^l(t) \qquad (2.10)\end{aligned}$$

$$\begin{aligned}\dot{T}_{h,s}^a(t) &= f_9\big(x(t),u(t),t\big) =\\&= k_2\big(1-u_{rt}(t)\big)\big(1-u_{ii}(t)\big)T_{h,s}^l(t) - \mu_{T_{h,s}^a}T_{h,s}^a(t)\\&- i_s I^e(t)T_{h,s}^a(t) \qquad (2.11)\end{aligned}$$

$$\begin{aligned}\dot{T}_{h,r}^l(t) &= f_{10}\big(x(t),u(t),t\big) =\\&= k_{1,r}\big(1-Q_{T_h,V_r}\big)V_r(t)T_h(t) + k_3 M_{h,r}^a(t)T_h(t)\\&- \mu_{T_{h,r}^l}T_{h,r}^l(t) - k_2 T_{h,r}^l(t) \qquad (2.12)\end{aligned}$$

$$\begin{aligned}\dot{T}_{h,r}^a(t) &= f_{11}\big(x(t),u(t),t\big) =\\&= k_2 T_{h,r}^l(t) - \mu_{T_{h,r}^a}T_{h,r}^a(t) - i_r I^e(t)T_{h,r}^a(t) \qquad (2.13)\end{aligned}$$

The actively infected cells are short living and will normally be killed upon activation with death rate $\mu_{T^a_{h,j}}$, $j \in \{s, r\}$.
In equation (2.10) we represent dynamics of latently T_h cells infected by sensitive viruses. The positives contributes

$$\begin{aligned}&+ k_{1,s}\big(1 - u_{fi}(t)\big)\big(1 - u_{co}(t)\big)\big(1 - Q_{T_h,V_s}\big)V_s(t)T_h(t)\\&+ k_3 M^a_{h,s}(t)T_h(t)\end{aligned}$$

represent healthy T_h-cells infected by V_s and healthy T_h-cells contaminated by actively infected macrophages ($M^a_{h,s}$) respectively.
The term

$$-\mu_{T^l_{h,s}} T^l_{h,s}(t)$$

is the natural death and finally

$$-k_2\big(1 - u_{rt}(t)\big)\big(1 - u_{ii}(t)\big)T^l_{h,s}(t)$$

is the number of infected cells that convert from latently to actively infected. This process is counteract by reverse trascriptase and integrase inhibitors which inhibit reverse trascription and integration of viral DNA respectively.
In equation (2.11) there is a positive contribute

$$k_2\big(1 - u_{rt}(t)\big)\big(1 - u_{ii}(t)\big)T^l_{h,s}(t)$$

as seen in (2.10); this gives the number of infected cells that convert from latently to actively infected.
There are also 2 negative contributes

$$-\mu_{T^a_{h,s}} T^a_{h,s}(t),$$

and

$$-i_s I^e(t) T^a_{h,s}(t).$$

The first one is the natural death of actively infected lymphocytes while the second one is a novelty introduced by our model; it represent the number of actively infected naive T cells ($T^a_{h,s}$) killed by CTL effectors (I^e), at a constant rate i_s. Of course this number is proportional to the number of immune effectors (I^e).
Similar considerations can be observed for equations (2.12) and (2.13) with the variation that there are not drugs that can counteract resistant viruses. Moreover the action of CTL effectors occurs at a constant rate i_r.

The dynamics for **infected T_s cells** are the following

$$\begin{aligned}
\dot{T}^l_{s,s}(t) &= f_{12}\big(x(t),u(t),t\big) = \\
&= k_{1,s}\big(1-u_{fi}(t)\big)\big(1-u_{co}(t)\big)\big(1-Q_{T_s,V_s}\big)V_s(t)T_s(t) \\
&+ k_3 M^a_{h,s}(t)T_s(t) - \mu_{T^l_{s,s}}T^l_{s,s}(t) \\
&- k_2\big(1-u_{rt}(t)\big)\big(1-u_{ii}(t)\big)T^l_{s,s}(t) \qquad (2.14)
\end{aligned}$$

$$\begin{aligned}
\dot{T}^a_{s,s}(t) &= f_{13}\big(x(t),u(t),t\big) = \\
&= k_2\big(1-u_{rt}(t)\big)\big(1-u_{ii}(t)\big)T^l_{s,s}(t) - \mu_{T^a_{s,s}}T^a_{s,s}(t) \\
&- i_s I^e(t)T^a_{s,s}(t) \qquad (2.15)
\end{aligned}$$

$$\begin{aligned}
\dot{T}^l_{s,r}(t) &= f_{14}\big(x(t),u(t),t\big) = \\
&= k_{1,r}\big(1-Q_{T_s,V_r}\big)V_r(t)T_s(t) + k_3 M^a_{h,r}(t)T_s(t) \\
&- \mu_{T^l_{s,r}}T^l_{s,r}(t) - k_2 T^l_{s,r}(t) \qquad (2.16)
\end{aligned}$$

$$\begin{aligned}
\dot{T}^a_{s,r}(t) &= f_{15}\big(x(t),u(t),t\big) = \\
&= k_2 T^l_{s,r}(t) - \mu_{T^a_{s,r}}T^a_{s,r}(t) - i_r I^e(t)T^a_{s,r}(t) \qquad (2.17)
\end{aligned}$$

As said for naive T lymphocytes (T_h) the same holds for equations (2.14), (2.15), (2.16), (2.17) of T_s cells.

The dynamics for **infected T_r cells** are the following

$$\begin{aligned}
\dot{T}^l_{r,s}(t) &= f_{16}\big(x(t),u(t),t\big) = \\
&= k_{1,s}\big(1-u_{fi}(t)\big)\big(1-u_{co}(t)\big)\big(1-Q_{T_r,V_s}\big)V_s(t)T_r(t) \\
&+ k_3 M^a_{h,s}(t)T_r(t) - \mu_{T^l_{r,s}}T^l_{r,s}(t) \\
&- k_2\big(1-u_{rt}(t)\big)\big(1-u_{ii}(t)\big)T^l_{r,s}(t) \qquad (2.18)
\end{aligned}$$

$$\begin{aligned}
\dot{T}^a_{r,s}(t) &= f_{17}\big(x(t),u(t),t\big) = \\
&= k_2\big(1-u_{rt}(t)\big)\big(1-u_{ii}(t)\big)T^l_{r,s}(t) - \mu_{T^a_{r,s}}T^a_{r,s}(t) \\
&- i_s I^e(t)T^a_{r,s}(t) \qquad (2.19)
\end{aligned}$$

$$\begin{aligned}
\dot{T}^l_{r,r}(t) &= f_{18}\big(x(t),u(t),t\big) = \\
&= k_{1,r}\big(1-Q_{T_r,V_r}\big)V_r(t)T_r(t) + k_3 M^a_{h,r}(t)T_r(t) \\
&- \mu_{T^l_{r,r}}T^l_{r,r}(t) - k_2 T^l_{r,r}(t) \qquad (2.20)
\end{aligned}$$

$$\begin{aligned}
\dot{T}^a_{r,r}(t) &= f_{19}\big(x(t),u(t),t\big) = \\
&= k_2 T^l_{r,r}(t) - \mu_{T^a_{r,r}}T^a_{r,r}(t) - i_r I^e(t)T^a_{r,r}(t) \qquad (2.21)
\end{aligned}$$

As said for naive T lymphocytes (T_h) the same holds for equations (2.18), (2.19), (2.20), (2.21) of T_r cells.

The dynamics for **infected T_g cells** are the following

$$\begin{aligned}\dot{T}^l_{g,s}(t) &= f_{20}\big(x(t),u(t),t\big) = \\ &= k_{1,s}\big(1-u_{fi}(t)\big)\big(1-u_{co}(t)\big)\big(1-Q_{T_g,V_s}\big)V_s(t)T_g(t) \\ &+ k_3 M^a_{h,s}(t)T_g(t) - \mu_{T^l_{g,s}}T^l_{g,s}(t) \\ &- k_2\big(1-u_{rt}(t)\big)\big(1-u_{ii}(t)\big)T^l_{g,s}(t) \end{aligned} \qquad (2.22)$$

$$\begin{aligned}\dot{T}^a_{g,s}(t) &= f_{21}\big(x(t),u(t),t\big) = \\ &= k_2\big(1-u_{rt}(t)\big)\big(1-u_{ii}(t)\big)T^l_{g,s}(t) - \mu_{T^a_{g,s}}T^a_{g,s}(t) \\ &- i_s I^e(t)T^a_{g,s}(t) \end{aligned} \qquad (2.23)$$

$$\begin{aligned}\dot{T}^l_{g,r}(t) &= f_{22}\big(x(t),u(t),t\big) = \\ &= k_{1,r}\big(1-Q_{T_g,V_r}\big)V_r(t)T_g(t) + k_3 M^a_{h,r}(t)T_g(t) \\ &- \mu_{T^l_{g,r}}T^l_{g,r}(t) - k_2 T^l_{g,r}(t) \end{aligned} \qquad (2.24)$$

$$\begin{aligned}\dot{T}^a_{g,r}(t) &= f_{23}\big(x(t),u(t),t\big) = \\ &= k_2 T^l_{g,r}(t) - \mu_{T^a_{g,r}}T^a_{g,r}(t) - i_r I^e(t)T^a_{g,r}(t) \end{aligned} \qquad (2.25)$$

As said for naive T lymphocytes (T_h) the same holds for equations (2.22), (2.23), (2.24), (2.25) of T_g cells.

The dynamics for **infected macrophages** are the following

$$\begin{aligned}\dot{M}^a_{h,s}(t) &= f_{24}\big(x(t),u(t),t\big) = \\ &= k^M_{1,s}\big(1-u_{rt}(t)\big)\big(1-u_{pi}(t)\big)V_s(t)M_h(t) - \mu_{M^a_{h,s}}M^a_{h,s} \end{aligned} \qquad (2.26)$$

$$\begin{aligned}\dot{M}^a_{h,r}(t) &= f_{25}\big(x(t),u(t),t\big) = \\ &= k^M_{1,r}V_r(t)M_h(t) - \mu_{M^a_{h,r}}M^a_{h,r} \end{aligned} \qquad (2.27)$$

Notice, we assumed that there is no latently infected macrophage population since the virus seems to replicate once inside them. We also assume that macrophages produce virus at a slow constant rate, sparing the host cell, so there is only natural death, not death by bursting like that for infected T cells [44].
These equations presents two infections rate, the former ($k^M_{1,s}$) related to

sensitive viruses (V_s), the latter related to resistant viruses (V_r).
Finally, we hypothesize that reverse transcriptase and protease inhibitors can counteract macrophages infected by sensitive viruses [116, 117], while there are not drugs to counteract resistant viruses.

The dynamics for **sensitive (V_s) and resistant (V_r) viruses** are the following

$$\begin{aligned}
\dot{V}_s(t) &= f_{26}\big(x(t), u(t), t\big) = \\
&= \frac{g\big(1-u_{pi}(t)\big)V_s(t)}{b_2 + V_s(t)} \\
&+ \pi_s\big(1-u_{pi}(t)\big)u^v_{s,s}(t)\Big(M^a_{h,s}(t) + \sum T^a_{i,s}(t)\Big) \\
&+ \pi_r\big(1-u_{pi}(t)\big)\big(1-u^v_{r,r}(t)\big)\Big(M^a_{h,r}(t) + \sum T^a_{i,r}(t)\Big) \\
&- k_{1,s}\ \rho_s\ \big(1-u_{fi}(t)\big)\big(1-u_{co}(t)\big)\Big[\sum T_i(t)\big(1-Q_{T_i,V_s}\big)\Big]V_s(t) \\
&- k^M_{1,s}\ \rho_s\ \big(1-u_{rt}(t)\big)\big(1-u_{pi}(t)\big)M_h(t)V_s(t) \\
&- \mu_{V_s}V_s(t) - \big(s'_sT_g(t) + p'_sT_s(t)\big)V_s(t) \qquad (2.28)
\end{aligned}$$

$$\begin{aligned}
\dot{V}_r(t) &= f_{27}\big(x(t), u(t), t\big) = \\
&= \frac{gV_r(t)}{b_2 + V_r(t)} \\
&+ \pi_r u^v_{r,r}(t)\Big(M^a_{h,r}(t) + \sum T^a_{i,r}(t)\Big) \\
&+ \pi_s\big(1-u^v_{s,s}(t)\big)\Big(M^a_{h,s}(t) + \sum T^a_{i,s}(t)\Big) \\
&- k_{1,r}\ \rho_r\ \Big[\sum T_i(t)\big(1-Q_{T_i,V_r}\big)\Big]V_r(t) \\
&- k^M_{1,r}\ \rho_r\ M_hV_r(t)\ - \mu_{V_r}V_r(t) - \big(s'_rT_g(t) + p'_rT_r(t)\big)V_r(t) \qquad (2.29)
\end{aligned}$$

Sensitive viruses equation (2.28) presents 3 positive and 4 negative contributes. The 3 positive ones are

$$\begin{aligned}
&+\frac{g\big(1-u_{pi}(t)\big)V_s(t)}{b_2 + V_s(t)}, \\
&+\pi_s\big(1-u_{pi}(t)\big)u^v_{s,s}(t)\Big(M^a_{h,s}(t) + \sum T^a_{i,s}(t)\Big), \\
&+\pi_r\big(1-u_{pi}(t)\big)\big(1-u^v_{r,r}(t)\big)\Big(M^a_{h,r}(t) + \sum T^a_{i,r}(t)\Big).
\end{aligned}$$

The first one is a source of virus that accounts for viral contributions to the plasma from both external compartments, such as the lymph system,

as well as virus produced by infected cells in the plasma [34]. This source is counteracted by the protease inhibitors therapy $u_{pi}(t)$.
The second one represents the growth of sensitive viruses at a constant rate π_s. This growth is counteracted by protease inhibitors, and it is proportional to the number of actively infected cells contaminated by sensitive viruses and to the mutation rate from sensitive to other sensitive viruses ($u^v_{s,s}$).
Finally the third one represents the growth (at a constant rate π_r) of resistant viruses that mutate into sensitive viruses (at a rate $1 - u^v_{r,r} = u^v_{r,s}$). Also this growth is counteracted by protease inhibitors, and it is proportional to the number of actively infected cells, contaminated by resistant viruses.
Parameters b_2, g, π_s, π_r are assumed positive and constant.
The 4 negative contributes are

$$
\begin{aligned}
&- k_{1,s}\ \rho_s\ \big(1 - u_{fi}(t)\big)\big(1 - u_{co}(t)\big)\Big[\sum T_i(t)\big(1 - Q_{T_i,V_s}\big)\Big] V_s(t),\\
&- k^M_{1,s}\ \rho_s\ \big(1 - u_{rt}(t)\big)\big(1 - u_{pi}(t)\big) M_h(t) V_s(t),\\
&- \mu_{V_s} V_s(t),\\
&- \big(s'_s T_g(t) + p'_s T_s(t)\big) V_s(t),
\end{aligned}
$$

where the first and second one indicate that viruses which infect lymphocytes are not free to circulate in the blood, and so they can not infect other cells at the same time. The constant ρ_s indicates the average number of sensitive virions infecting a cell: in our numerical simulations we set this number equal to 1, but the model permits also other values for this parameter, that is a cell could be infected by more viruses.
Also viruses die, see the third negative term.
The fourth negative term represents the number of sensitive virions blocked by specific immune response (represented by T_s cells) and by unspecific immune response (represented by T_g cells).
For what concerns resistant viruses dynamics (2.28), considerations are the same just viewed for sensitive viruses, with the only difference that protease inhibitors can not counteract V_r.

The **boundary conditions** are the following

$$
\begin{array}{rrr}
T_h(t_0) = T_h^0 & T_s(t_0) = T_s^0 & T_r(t_0) = T_r^0 \\
T_g(t_0) = T_g^0 & M_h(t_0) = M_h^0 & I^p(t_0) = I_0^p \\
I^e(t_0) = I_0^e & T_{h,s}^l(t_0) = T_{h,s}^{l,0} & T_{h,s}^a(t_0) = T_{h,s}^{a,0} \\
T_{h,r}^l(t_0) = T_{h,r}^{l,0} & T_{h,r}^a(t_0) = T_{h,r}^{a,0} & T_{s,s}^l(t_0) = T_{s,s}^{l,0} \\
T_{s,s}^a(t_0) = T_{s,a}^{a,0} & T_{s,r}^l(t_0) = T_{s,r}^{l,0} & T_{s,r}^a(t_0) = T_{s,r}^{a,0} \\
T_{r,s}^l(t_0) = T_{r,s}^{l,0} & T_{r,s}^a(t_0) = T_{r,s}^{a,0} & T_{r,r}^l(t_0) = T_{r,r}^{l,0} \\
T_{r,r}^a(t_0) = T_{r,r}^{a,0} & T_{g,s}^l(t_0) = T_{g,s}^{l,0} & T_{g,s}^a(t_0) = T_{g,s}^{a,0} \\
T_{g,r}^l(t_0) = T_{g,r}^{l,0} & T_{g,r}^a(t_0) = T_{g,r}^{a,0} & M_{h,s}^a(t_0) = M_{h,s}^{a,0} \\
M_{h,r}^a(t_0) = M_{h,r}^{a,0} & V_s(t_0) = V_s^0 & V_r(t_0) = V_r^0
\end{array} \tag{2.30}
$$

$$T_i(t_f), M_h(t_f), I^p(t_f), I^e(t_f), T_{i,j}^l(t_f), T_{i,j}^a(t_f), V_j(t_f) \in \mathbb{R}^+ \tag{2.31}$$

$$
\begin{array}{rcl}
u_1(t) & \in & [0,1] \\
u_2(t) & \in & [0,1]
\end{array} \tag{2.32}
$$

And as already said, for the control $u_2(t)$:

$$
\begin{array}{rcl}
u_{s,s}^v(t) + u_{s,r}^v(t) & = & 1 \\
u_{r,r}^v(t) + u_{r,s}^v(t) & = & 1
\end{array} \tag{2.33}
$$

To conclude each therapy control, that represents the various drugs dosage, is assumed to vary within the range $[0,1]$.
The HIV-controls, represent the mutation rates of viruses, lay in the same range.

Observation: One proposed is a generalized model , and it can be applied to any simpler formulation just resetting appropriate parameter values thus overlooking certain variables, such as macrophages (M_h), immune effectors/precursors (I^e, I^p) or CD4+ T Helper cells unspecific for sensitive and resistant viruses (T_g).

One possible approach to this problem would be a predator-prey model (Lotka-Volterra model [115]), as the following

$$
\begin{array}{rcl}
\dot{x} & = & ax - bxy - ex^2 \\
\dot{y} & = & -cy + dxy - fy^2,
\end{array}
$$

where x are the preys, a is the preys growth rate, bxy is the number of contacts per unit of time between preys and predators and finally ex^2 term

reflects the prey internal competition due to the limitation of external resources. Variable y describes the predators: they have a natural decaying rate $-cy$ proportional to their current number; their number increase at a rate dxy proportional to their number y and the number x of their preys; finally fy^2 represents the predator internal competition due to the limited number of available prey.
In our context T-lymphocytes would have the role of predator and HIV the role of prey, at the beginning. After that T-lymphocytes become prey and HIV the predator, which is in its turn preyed by drug therapy.
Such alternate behaviours make the problem different from the predator-prey one.

2.4 Nash Equilibria

Equations (2.1)-(2.29) constitute a differential game between immune system-therapy and HIV. In order to find a Nash Equilibrium, let's introduce the Hamiltonian functions for the two players. For therapy

$$
\begin{aligned}
H_{Ther} & \big(x(t), u_1(t), \lambda_{1,1}, \lambda_{1,2}, \dots, \lambda_{1,27}, t\big) = \\
= & \lambda_{1,0}\ f_{0,1}\big(x(t), u(t), t\big) + \lambda_{1,1}\ f_1\big(x(t), u(t), t\big) + \\
+ & \lambda_{1,2}\ f_2\big(x(t), u(t), t\big) + \dots + \lambda_{1,27}\ f_{27}\big(x(t), u(t), t\big) = \\
= & \lambda_{1,0} \Bigg\{ M_h(t) + I^p(t) + I^e(t) + \sum_{\forall i \in \Gamma} T_i(t) \\
- & \frac{1}{2} \sum_{\forall k \in \Omega} A_k u_k^2(t) \Bigg\} \\
+ & \lambda_{1,1} \Bigg\{ \frac{s_{1,T_h} \theta_{T_h}}{\theta_{T_h} + \big(V_s(t) + V_r(t)\big)} + r T_h(t) \left(1 - \frac{\sum T_i(t) + \sum T^l_{i,j}(t) + \sum T^a_{i,j}(t)}{T_{max}}\right) \\
+ & u_{ib}(t) T_h(t) - \mu_{T_h} T_h(t) \\
- & k_{1,s}\big(1 - u_{fi}(t)\big)\big(1 - u_{co}(t)\big)\big(1 - Q_{T_h,V_s}\big) V_s(t) T_h(t) \\
- & k_{1,r}\big(1 - Q_{T_h,V_r}\big) V_r(t) T_h(t) - k\big(V_s(t) + V_r(t)\big) T_h(t) \\
- & k'\big(V_s(t) + V_r(t)\big) T_h(t) - k_3\big(M^a_{h,s}(t) + M^a_{h,r}(t)\big) T_h(t) \Bigg\} \\
+ & \lambda_{1,2} \Bigg\{ k V_s(t) T_h(t) + u_{ib}(t) T_s(t) - \mu_{T_s} T_s(t) \\
- & k_{1,s}\big(1 - u_{fi}(t)\big)\big(1 - u_{co}(t)\big)\big(1 - Q_{T_s,V_s}\big) V_s(t) T_s(t) \\
- & k_{1,r}\big(1 - Q_{T_s,V_r}\big) V_r(t) T_s(t) - k_3\big(M^a_{h,s}(t) + M^a_{h,r}(t)\big) T_s(t) \Bigg\} \\
+ & \lambda_{1,3} \Bigg\{ k V_r(t) T_h(t) + u_{ib}(t) T_r(t) - \mu_{T_r} T_r(t) \\
- & k_{1,s}\big(1 - u_{fi}(t)\big)\big(1 - u_{co}(t)\big)\big(1 - Q_{T_r,V_s}\big) V_s(t) T_r(t) \\
- & k_{1,r}\big(1 - Q_{T_r,V_r}\big) V_r(t) T_r(t) - k_3\big(M^a_{h,s}(t) + M^a_{h,r}(t)\big) T_r(t) \Bigg\} \\
+ & \lambda_{1,4} \Bigg\{ k'\big(V_s(T) + V_r(t)\big) T_h(t) + u_{ib}(t) T_g(t) - \mu_{T_g} T_g(t) \Bigg\} \\
+ & \lambda_{1,5} \Bigg\{ s_{1,M_h} - \mu_{M_h} M_h(t) - k^M_{1,s}\big(1 - u_{rt}(t)\big)\big(1 - u_{pi}(t)\big) V_s(t) M_h(t) \\
- & k^M_{1,r} V_r(t) M_h(t) \Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad \lambda_{1,6}\left\{cI^p(t)\sum T_i(t)\sum T^a_{i,j}(t) - cqI^p(t)\sum T^a_{i,j}(t) - \mu_{I^p}I^p(t)\right\}\\
&+ \quad \lambda_{1,7}\left\{cqI^p(t)\sum T^a_{i,j}(t) - \mu_{I^e}I^e(t)\right\}\\
&+ \quad \lambda_{1,8}\left\{k_{1,s}\big(1-u_{fi}(t)\big)\big(1-u_{co}(t)\big)\big(1-Q_{T_h,V_s}\big)V_s(t)T_h(t)\right.\\
&+ \quad k_3M^a_{h,s}(t)T_h(t) - \mu_{T^l_{h,s}}T^l_{h,s}(t)\\
&- \quad \left.k_2\big(1-u_{rt}(t)\big)\big(1-u_{ii}(t)\big)T^l_{h,s}(t)\right\}\\
&+ \quad \lambda_{1,9}\left\{k_2\big(1-u_{rt}(t)\big)\big(1-u_{ii}(t)\big)T^l_{h,s}(t) - \mu_{T^a_{h,s}}T^a_{h,s}(t)\right.\\
&- \quad \left.i_sI^e(t)T^a_{h,s}(t)\right\}\\
&+ \quad \lambda_{1,10}\left\{k_{1,r}\big(1-Q_{T_h,V_r}\big)V_r(t)T_h(t) + k_3M^a_{h,r}(t)T_h(t)\right.\\
&- \quad \left.\mu_{T^l_{h,r}}T^l_{h,r}(t) - k_2T^l_{h,r}(t)\right\}\\
&+ \quad \lambda_{1,11}\left\{k_2T^l_{h,r}(t) - \mu_{T^a_{h,r}}T^a_{h,r}(t) - i_rI^e(t)T^a_{h,r}(t)\right\}\\
&+ \quad \lambda_{1,12}\left\{k_{1,s}\big(1-u_{fi}(t)\big)\big(1-u_{co}(t)\big)\big(1-Q_{T_s,V_s}\big)V_s(t)T_s(t)\right.\\
&+ \quad k_3M^a_{h,s}(t)T_s(t) - \mu_{T^l_{s,s}}T^l_{s,s}(t)\\
&- \quad \left.k_2\big(1-u_{rt}(t)\big)\big(1-u_{ii}(t)\big)T^l_{s,s}(t)\right\}\\
&+ \quad \lambda_{1,13}\left\{k_2\big(1-u_{rt}(t)\big)\big(1-u_{ii}(t)\big)T^l_{s,s}(t) - \mu_{T^a_{s,s}}T^a_{s,s}(t)\right.\\
&- \quad \left.i_sI^e(t)T^a_{s,s}(t)\right\}\\
&+ \quad \lambda_{1,14}\left\{k_{1,r}\big(1-Q_{T_s,V_r}\big)V_r(t)T_s(t) + k_3M^a_{h,r}(t)T_s(t)\right.\\
&- \quad \left.\mu_{T^l_{s,r}}T^l_{s,r}(t) - k_2T^l_{s,r}(t)\right\}\\
&+ \quad \lambda_{1,15}\left\{k_2T^l_{s,r}(t) - \mu_{T^a_{s,r}}T^a_{s,r}(t) - i_rI^e(t)T^a_{s,r}(t)\right\}
\end{aligned}
$$

$$
\begin{aligned}
+ \quad & \lambda_{1,16}\Bigg\{k_{1,s}\big(1-u_{fi}(t)\big)\big(1-u_{co}(t)\big)\big(1-Q_{T_r,V_s}\big)V_s(t)T_r(t) \\
+ \quad & k_3 M^a_{h,s}(t)T_r(t) - \mu_{T^l_{r,s}}T^l_{r,s}(t) \\
- \quad & k_2\big(1-u_{rt}(t)\big)\big(1-u_{ii}(t)\big)T^l_{r,s}(t)\Bigg\} \\
+ \quad & \lambda_{1,17}\Bigg\{k_2\big(1-u_{rt}(t)\big)\big(1-u_{ii}(t)\big)T^l_{r,s}(t) - \mu_{T^a_{r,s}}T^a_{r,s}(t) \\
- \quad & i_s I^e(t)T^a_{r,s}(t)\Bigg\} \\
+ \quad & \lambda_{1,18}\Bigg\{k_{1,r}\big(1-Q_{T_r,V_r}\big)V_r(t)T_r(t) + k_3 M^a_{h,r}(t)T_r(t) \\
- \quad & \mu_{T^l_{r,r}}T^l_{r,r}(t) - k_2 T^l_{r,r}(t)\Bigg\} \\
+ \quad & \lambda_{1,19}\Bigg\{k_2 T^l_{r,r}(t) - \mu_{T^a_{r,r}}T^a_{r,r}(t) - i_r I^e(t)T^a_{r,r}(t)\Bigg\} \\
+ \quad & \lambda_{1,20}\Bigg\{k_{1,s}\big(1-u_{fi}(t)\big)\big(1-u_{co}(t)\big)\big(1-Q_{T_g,V_s}\big)V_s(t)T_g(t) \\
+ \quad & k_3 M^a_{h,s}(t)T_g(t) - \mu_{T^l_{g,s}}T^l_{g,s}(t) \\
- \quad & k_2\big(1-u_{rt}(t)\big)\big(1-u_{ii}(t)\big)T^l_{g,s}(t)\Bigg\} \\
+ \quad & \lambda_{1,21}\Bigg\{k_2\big(1-u_{rt}(t)\big)\big(1-u_{ii}(t)\big)T^l_{g,s}(t) - \mu_{T^a_{g,s}}T^a_{g,s}(t) \\
- \quad & i_s I^e(t)T^a_{g,s}(t)\Bigg\} \\
+ \quad & \lambda_{1,22}\Bigg\{k_{1,r}\big(1-Q_{T_g,V_r}\big)V_r(t)T_g(t) + k_3 M^a_{h,r}(t)T_g(t) \\
- \quad & \mu_{T^l_{g,r}}T^l_{g,r}(t) - k_2 T^l_{g,r}(t)\Bigg\} \\
+ \quad & \lambda_{1,23}\Bigg\{k_2 T^l_{g,r}(t) - \mu_{T^a_{g,r}}T^a_{g,r}(t) - i_r I^e(t)T^a_{g,r}(t)\Bigg\} \\
+ \quad & \lambda_{1,24}\Bigg\{k^M_{1,s}\big(1-u_{rt}(t)\big)\big(1-u_{pi}(t)\big)V_s(t)M_h(t) - \mu_{M^a_{h,s}}M^a_{h,s}\Bigg\} \\
+ \quad & \lambda_{1,25}\Bigg\{k^M_{1,r}V_r(t)M_h(t) - \mu_{M^a_{h,r}}M^a_{h,r}\Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad \lambda_{1,26} \left\{ \frac{g\big(1 - u_{pi}(t)\big)V_s(t)}{b_2 + V_s(t)} \right. \\
&+ \quad \pi_s\big(1 - u_{pi}(t)\big)u^v_{s,s}(t)\Big(M^a_{h,s}(t) + \sum T^a_{i,s}(t)\Big) \\
&+ \quad \pi_r\big(1 - u_{pi}(t)\big)\big(1 - u^v_{r,r}(t)\big)\Big(M^a_{h,r}(t) + \sum T^a_{i,r}(t)\Big) \\
&- \quad k_{1,s}\ \rho_s\ \big(1 - u_{fi}(t)\big)\big(1 - u_{co}(t)\big)\Big[\sum T_i(t)\big(1 - Q_{T_i,V_s}\big)\Big]V_s(t) \\
&- \quad k^M_{1,s}\ \rho_s\ \big(1 - u_{rt}(t)\big)\big(1 - u_{pi}(t)\big)M_h(t)V_s(t) \\
&- \quad \left. \mu_{V_s}V_s(t) - \big(s'_sT_g(t) + p'_sT_s(t)\big)V_s(t) \right\} \\
&+ \quad \lambda_{1,27} \left\{ \frac{gV_r(t)}{b_2 + V_r(t)} \right. \\
&+ \quad \pi_r u^v_{r,r}(t)\Big(M^a_{h,r}(t) + \sum T^a_{i,r}(t)\Big) \\
&+ \quad \pi_s\big(1 - u^v_{s,s}(t)\big)\Big(M^a_{h,s}(t) + \sum T^a_{i,s}(t)\Big) \\
&- \quad k_{1,r}\ \rho_r\ \Big[\sum T_i(t)\big(1 - Q_{T_i,V_r}\big)\Big]V_r(t) \\
&- \quad \left. k^M_{1,r}\ \rho_r\ M_hV_r(t)\ - \mu_{V_r}V_r(t) - \big(s'_rT_g(t) + p'_rT_r(t)\big)V_r(t) \right\}
\end{aligned}
\tag{2.34}
$$

And for HIV virus

$$
\begin{aligned}
H_{HIV} \quad & \big(x(t), u_2(t), \lambda_{2,1}, \lambda_{2,2}, \ldots, \lambda_{2,27}, t\big) = \\
= \quad & \lambda_{2,0}\ f_{0,2}\big(x(t), u(t), t\big) + \lambda_{2,1}\ f_1\big(x(t), u(t), t\big) + \\
+ \quad & \lambda_{2,2}\ f_2\big(x(t), u(t), t\big) + \cdots + \lambda_{2,27}\ f_{27}\big(x(t), u(t), t\big)\ = \\
= \quad & \lambda_{2,0} \Bigg\{ V_s(t) + V_r(t) + \sum_{\forall i \in \Gamma\ \forall j \in \Theta} T^l_{i,j}(t) + \sum_{\forall i \in \Gamma\ \forall j \in \Theta} + T^a_{i,j}(t) \\
- \quad & \frac{1}{2}\Big(B_{s,r}\big(1 - u^v_{s,s}(t)\big)^2 + B_{r,s}\big(1 - u^v_{r,r}(t)\big)^2 \Big) \Bigg\} \\
+ \quad & \lambda_{2,1} \Bigg\{ \frac{s_{1,T_h}\theta_{T_h}}{\theta_{T_h} + \big(V_s(t) + V_r(t)\big)} + rT_h(t) \left(1 - \frac{\sum T_i(t) + \sum T^l_{i,j}(t) + \sum T^a_{i,j}(t)}{T_{max}} \right) \\
+ \quad & u_{ib}(t)T_h(t) - \mu_{T_h} T_h(t) \\
- \quad & k_{1,s}\big(1 - u_{fi}(t)\big)\big(1 - u_{co}(t)\big)\big(1 - Q_{T_h,V_s}\big)V_s(t)T_h(t) \\
- \quad & k_{1,r}\big(1 - Q_{T_h,V_r}\big)V_r(t)T_h(t) - k\big(V_s(t) + V_r(t)\big)T_h(t) \\
- \quad & k'\big(V_s(t) + V_r(t)\big)T_h(t) - k_3\big(M^a_{h,s}(t) + M^a_{h,r}(t)\big)T_h(t) \Bigg\} \\
+ \quad & \lambda_{2,2} \Bigg\{ kV_s(t)T_h(t) + u_{ib}(t)T_s(t) - \mu_{T_s}T_s(t) \\
- \quad & k_{1,s}\big(1 - u_{fi}(t)\big)\big(1 - u_{co}(t)\big)\big(1 - Q_{T_s,V_s}\big)V_s(t)T_s(t) \\
- \quad & k_{1,r}\big(1 - Q_{T_s,V_r}\big)V_r(t)T_s(t) - k_3\big(M^a_{h,s}(t) + M^a_{h,r}(t)\big)T_s(t) \Bigg\} \\
+ \quad & \lambda_{2,3} \Bigg\{ kV_r(t)T_h(t) + u_{ib}(t)T_r(t) - \mu_{T_r}T_r(t) \\
- \quad & k_{1,s}\big(1 - u_{fi}(t)\big)\big(1 - u_{co}(t)\big)\big(1 - Q_{T_r,V_s}\big)V_s(t)T_r(t) \\
- \quad & k_{1,r}\big(1 - Q_{T_r,V_r}\big)V_r(t)T_r(t) - k_3\big(M^a_{h,s}(t) + M^a_{h,r}(t)\big)T_r(t) \Bigg\} \\
+ \quad & \lambda_{2,4} \Bigg\{ k'\big(V_s(T) + V_r(t)\big)T_h(t) + u_{ib}(t)T_g(t) - \mu_{T_g}T_g(t) \Bigg\} \\
+ \quad & \lambda_{2,5} \Bigg\{ s_{1,M_h} - \mu_{M_h}M_h(t) - k^M_{1,s}\big(1 - u_{rt}(t)\big)\big(1 - u_{pi}(t)\big)V_s(t)M_h(t) \\
- \quad & k^M_{1,r}V_r(t)M_h(t) \Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
+ &\quad \lambda_{2,6}\left\{cI^p(t)\sum T_i(t)\sum T^a_{i,j}(t) - cqI^p(t)\sum T^a_{i,j}(t) - \mu_{I^p} I^p(t)\right\}\\
+ &\quad \lambda_{2,7}\left\{cqI^p(t)\sum T^a_{i,j}(t) - \mu_{I^e} I^e(t)\right\}\\
+ &\quad \lambda_{2,8}\left\{k_{1,s}\big(1-u_{fi}(t)\big)\big(1-u_{co}(t)\big)\big(1-Q_{T_h,V_s}\big)V_s(t)T_h(t)\right.\\
+ &\quad k_3 M^a_{h,s}(t)T_h(t) - \mu_{T^l_{h,s}} T^l_{h,s}(t)\\
- &\quad \left. k_2\big(1-u_{rt}(t)\big)\big(1-u_{ii}(t)\big)T^l_{h,s}(t)\right\}\\
+ &\quad \lambda_{2,9}\left\{k_2\big(1-u_{rt}(t)\big)\big(1-u_{ii}(t)\big)T^l_{h,s}(t) - \mu_{T^a_{h,s}} T^a_{h,s}(t)\right.\\
- &\quad \left. i_s I^e(t)T^a_{h,s}(t)\right\}\\
+ &\quad \lambda_{2,10}\left\{k_{1,r}\big(1-Q_{T_h,V_r}\big)V_r(t)T_h(t) + k_3 M^a_{h,r}(t)T_h(t)\right.\\
- &\quad \left. \mu_{T^l_{h,r}} T^l_{h,r}(t) - k_2 T^l_{h,r}(t)\right\}\\
+ &\quad \lambda_{2,11}\left\{k_2 T^l_{h,r}(t) - \mu_{T^a_{h,r}} T^a_{h,r}(t) - i_r I^e(t)T^a_{h,r}(t)\right\}\\
+ &\quad \lambda_{2,12}\left\{k_{1,s}\big(1-u_{fi}(t)\big)\big(1-u_{co}(t)\big)\big(1-Q_{T_s,V_s}\big)V_s(t)T_s(t)\right.\\
+ &\quad k_3 M^a_{h,s}(t)T_s(t) - \mu_{T^l_{s,s}} T^l_{s,s}(t)\\
- &\quad \left. k_2\big(1-u_{rt}(t)\big)\big(1-u_{ii}(t)\big)T^l_{s,s}(t)\right\}\\
+ &\quad \lambda_{2,13}\left\{k_2\big(1-u_{rt}(t)\big)\big(1-u_{ii}(t)\big)T^l_{s,s}(t) - \mu_{T^a_{s,s}} T^a_{s,s}(t)\right.\\
- &\quad \left. i_s I^e(t)T^a_{s,s}(t)\right\}\\
+ &\quad \lambda_{2,14}\left\{k_{1,r}\big(1-Q_{T_s,V_r}\big)V_r(t)T_s(t) + k_3 M^a_{h,r}(t)T_s(t)\right.\\
- &\quad \left. \mu_{T^l_{s,r}} T^l_{s,r}(t) - k_2 T^l_{s,r}(t)\right\}\\
+ &\quad \lambda_{2,15}\left\{k_2 T^l_{s,r}(t) - \mu_{T^a_{s,r}} T^a_{s,r}(t) - i_r I^e(t)T^a_{s,r}(t)\right\}
\end{aligned}
$$

$$
\begin{aligned}
+ \quad & \lambda_{2,16} \Big\{ k_{1,s}\big(1-u_{fi}(t)\big)\big(1-u_{co}(t)\big)\big(1-Q_{T_r,V_s}\big)V_s(t)T_r(t) \\
+ \quad & k_3 M^a_{h,s}(t)T_r(t) - \mu_{T^l_{r,s}} T^l_{r,s}(t) \\
- \quad & k_2\big(1-u_{rt}(t)\big)\big(1-u_{ii}(t)\big)T^l_{r,s}(t) \Big\} \\
+ \quad & \lambda_{2,17} \Big\{ k_2\big(1-u_{rt}(t)\big)\big(1-u_{ii}(t)\big)T^l_{r,s}(t) - \mu_{T^a_{r,s}} T^a_{r,s}(t) \\
- \quad & i_s I^e(t) T^a_{r,s}(t) \Big\} \\
+ \quad & \lambda_{2,18} \Big\{ k_{1,r}\big(1-Q_{T_r,V_r}\big)V_r(t)T_r(t) + k_3 M^a_{h,r}(t)T_r(t) \\
- \quad & \mu_{T^l_{r,r}} T^l_{r,r}(t) - k_2 T^l_{r,r}(t) \Big\} \\
+ \quad & \lambda_{2,19} \Big\{ k_2 T^l_{r,r}(t) - \mu_{T^a_{r,r}} T^a_{r,r}(t) - i_r I^e(t) T^a_{r,r}(t) \Big\} \\
+ \quad & \lambda_{2,20} \Big\{ k_{1,s}\big(1-u_{fi}(t)\big)\big(1-u_{co}(t)\big)\big(1-Q_{T_g,V_s}\big)V_s(t)T_g(t) \\
+ \quad & k_3 M^a_{h,s}(t)T_g(t) - \mu_{T^l_{g,s}} T^l_{g,s}(t) \\
- \quad & k_2\big(1-u_{rt}(t)\big)\big(1-u_{ii}(t)\big)T^l_{g,s}(t) \Big\} \\
+ \quad & \lambda_{2,21} \Big\{ k_2\big(1-u_{rt}(t)\big)\big(1-u_{ii}(t)\big)T^l_{g,s}(t) - \mu_{T^a_{g,s}} T^a_{g,s}(t) \\
- \quad & i_s I^e(t) T^a_{g,s}(t) \Big\} \\
+ \quad & \lambda_{2,22} \Big\{ k_{1,r}\big(1-Q_{T_g,V_r}\big)V_r(t)T_g(t) + k_3 M^a_{h,r}(t)T_g(t) \\
- \quad & \mu_{T^l_{g,r}} T^l_{g,r}(t) - k_2 T^l_{g,r}(t) \Big\} \\
+ \quad & \lambda_{2,23} \Big\{ k_2 T^l_{g,r}(t) - \mu_{T^a_{g,r}} T^a_{g,r}(t) - i_r I^e(t) T^a_{g,r}(t) \Big\} \\
+ \quad & \lambda_{2,24} \Big\{ k^M_{1,s}\big(1-u_{rt}(t)\big)\big(1-u_{pi}(t)\big)V_s(t)M_h(t) - \mu_{M^a_{h,s}} M^a_{h,s} \Big\} \\
+ \quad & \lambda_{2,25} \Big\{ k^M_{1,r} V_r(t)M_h(t) - \mu_{M^a_{h,r}} M^a_{h,r} \Big\}
\end{aligned}
$$

$$
\begin{aligned}
+ & \quad \lambda_{2,26} \left\{ \frac{g\big(1-u_{pi}(t)\big)V_s(t)}{b_2+V_s(t)} \right. \\
+ & \quad \pi_s\big(1-u_{pi}(t)\big)u^v_{s,s}(t)\Big(M^a_{h,s}(t)+\sum T^a_{i,s}(t)\Big) \\
+ & \quad \pi_r\big(1-u_{pi}(t)\big)\big(1-u^v_{r,r}(t)\big)\Big(M^a_{h,r}(t)+\sum T^a_{i,r}(t)\Big) \\
- & \quad k_{1,s}\,\rho_s\,\big(1-u_{fi}(t)\big)\big(1-u_{co}(t)\big)\Big[\sum T_i(t)\big(1-Q_{T_i,V_s}\big)\Big]V_s(t) \\
- & \quad k^M_{1,s}\,\rho_s\,\big(1-u_{rt}(t)\big)\big(1-u_{pi}(t)\big)M_h(t)V_s(t) \\
- & \quad \left. \mu_{V_s}V_s(t)-\big(s'_sT_g(t)+p'_sT_s(t)\big)V_s(t) \right\} \\
+ & \quad \lambda_{2,27} \left\{ \frac{gV_r(t)}{b_2+V_r(t)} \right. \\
+ & \quad \pi_r u^v_{r,r}(t)\Big(M^a_{h,r}(t)+\sum T^a_{i,r}(t)\Big) \\
+ & \quad \pi_s\big(1-u^v_{s,s}(t)\big)\Big(M^a_{h,s}(t)+\sum T^a_{i,s}(t)\Big) \\
- & \quad k_{1,r}\,\rho_r\,\Big[\sum T_i(t)\big(1-Q_{T_i,V_r}\big)\Big]V_r(t) \\
- & \quad \left. k^M_{1,r}\,\rho_r\,M_hV_r(t)\;-\mu_{V_r}V_r(t)-\big(s'_rT_g(t)+p'_rT_r(t)\big)V_r(t) \right\}
\end{aligned}
\tag{2.35}
$$

The model is difficult to analyze due to the high number of variables and constraints. Let's simplify it by relaxing the constraints on the state variables (2.31), assuming that

$$
T_i(t_f), M_h(t_f), I^p(t_f), I^e(t_f), T^l_{i,j}(t_f), T^a_{i,j}(t_f), V_j(t_f) \in \mathbb{R} \tag{2.36}
$$

and verify ex-post their validity. So that the transversality conditions are

$$
\begin{aligned}
\lambda_{1,j}(t_f) &= 0, \qquad j=1,2,\ldots,27, \\
\lambda_{2,j}(t_f) &= 0, \qquad j=1,2,\ldots,27.
\end{aligned}
\tag{2.37}
$$

From conditions

$$
\begin{aligned}
\lambda_{i,0} &\in \{0,1\}, & i&=1,2, \\
\big(\lambda_{i,0},\lambda_{i,j}(t)\big) &\neq 0, & i&=1,2 \quad j=1,2,\ldots,27,
\end{aligned}
\tag{2.38}
$$

we obtain that

$$
\lambda_{1,0}=\lambda_{2,0}=1. \tag{2.39}
$$

From the HIV point of view, the necessary conditions for Hamiltonian maximum are

$$\begin{aligned}\frac{\partial H_{HIV}}{\partial u^v_{s,s}(t)} &= \pi_s\Big(M^a_{h,s}(t)+T^a_{g,s}(t)+T^a_{h,s}(t)+T^a_{r,s}(t)+T^a_{s,s}(t)\Big)\cdot\\ &\cdot\ \Big(\lambda_{2,27}(t)+\lambda_{2,26}(t)\big(-1+u_{pi}(t)\big)\Big)-B_{s,r}\big(-1+u^v_{s,s}(t)\big)\\ &= 0\end{aligned} \tag{2.40}$$

$$\begin{aligned}\frac{\partial H_{HIV}}{\partial u^v_{r,r}(t)} &= \pi_r\Big(M^a_{h,r}(t)+T^a_{g,r}(t)+T^a_{h,r}(t)+T^a_{r,r}(t)+T^a_{s,r}(t)\Big)\cdot\\ &\cdot\ \Big(\lambda_{2,27}(t)+\lambda_{2,26}(t)\big(-1+u_{pi}(t)\big)\Big)-B_{r,s}\big(-1+u^v_{r,r}(t)\big)\\ &= 0\end{aligned} \tag{2.41}$$

The co-state equations for HIV are

$$\begin{aligned}\dot\lambda_{2,1}(t) &= -\frac{\partial H_{HIV}}{\partial T_h}\\ \dot\lambda_{2,2}(t) &= -\frac{\partial H_{HIV}}{\partial T_s}\\ &\vdots\\ \dot\lambda_{2,27}(t) &= -\frac{\partial H_{HIV}}{\partial V_r}\end{aligned} \tag{2.42}$$

but we do not make explicit these equations, because they are very long and complex.
About the HIV for the control $u_2(t)=\big(u^v_{s,s}(t),u^v_{r,r}(t)\big)$ the stationary points are

$$\hat u^v_{s,s}(t) = \frac{B_{s,r}-\pi_s\big(M^a_{h,s}(t)+\sum T^a_{i,s}(t)\big)\Big(\lambda_{2,27}(t)+\lambda_{2,26}(t)\big(-1+u_{pi}(t)\big)\Big)}{B_{s,r}} \tag{2.43}$$

$$\hat u^v_{r,r}(t) = \frac{B_{r,s}-\pi_r\big(M^a_{h,r}(t)+\sum T^a_{i,r}(t)\big)\Big(\lambda_{2,27}(t)+\lambda_{2,26}(t)\big(-1+u_{pi}(t)\big)\Big)}{B_{r,s}} \tag{2.44}$$

We consider the 2×2 Hessian matrix for controls $u^v_{s,s}(t)$ and $u^v_{r,r}(t)$

$$\tilde{\mathbf{H}}''_{HIV} = \begin{pmatrix} -B_{s,r} & 0 \\ 0 & -B_{r,s} \end{pmatrix}, \tag{2.45}$$

that is a negative definite matrix ($B_{s,r} > 0$ and $B_{r,s} > 0$).
Therefore the Hamiltonian function for these controls is concave, and the optimal strategy for HIV is

$$u^{v^*}_{s,s}(t) = \min\{1, \max\{0, \hat{u}^v_{s,s}(t)\}\} \tag{2.46}$$

$$u^{v^*}_{r,r}(t) = \min\{1, \max\{0, \hat{u}^v_{r,r}(t)\}\} \tag{2.47}$$

We can see that from the transversality conditions (2.37) it follows that

$$u^{v^*}_{s,s}(t_f) = u^{v^*}_{r,r}(t_f) = 1. \tag{2.48}$$

Since $u^v_{s,r}(t)$ is the mutation rate from sensible viruses to resistant viruses, from equations (2.33) we obtain that $u^v_{s,r}(t) = 1 - u^v_{s,s}(t)$ and $u^{v^*}_{s,r}(t) = 1 - u^{v^*}_{s,s}(t)$. Analogously $u^{v^*}_{r,s}(t) = 1 - u^{v^*}_{r,r}(t)$, therefore

$$u^{v^*}_{s,r}(t_f) = u^{v^*}_{r,s}(t_f) = 0 \tag{2.49}$$

Let's consider now the necessary conditions for the immune system-therapy strategies, that constitute a Nash Equilibrium

$$\begin{aligned}
\frac{\partial H_{Ther}}{\partial u_{co}} &= -A_{co}u_{co}(t) - k_{1,s}\big(-1 + u_{fi}(t)\big)V_s(t) \cdot \\
&\cdot \Big[(-1 + Q_{T_g,V_s})\lambda_{1,20}(t)T_g(t) - (-1 + Q_{T_g,V_s})\lambda_{1,4}(t)T_g(t) \\
&- (-1 + Q_{T_h,V_s})\big(\lambda_{1,1}(t) - \lambda_{1,8}(t)\big)T_h(t) \\
&+ (-1 + Q_{T_r,V_s})\big(\lambda_{1,16}(t) - \lambda_{1,3}(t)\big)T_s(t) \\
&+ (-1 + Q_{T_s,V_s})\big(\lambda_{1,12}(t) - \lambda_{1,2}(t)\big)T_s(t) \\
&+ \rho_s\lambda_{1,26}(t)\big(T_g(t) - Q_{T_g,V_s}T_g(t) + T_h(t) - Q_{T_h,V_s}T_h(t) \\
&+ T_r(t) - Q_{T_r,V_s}T_r(t) + T_s(t) - Q_{T_s,V_s}T_s(t)\big)\Big] \\
&= 0
\end{aligned} \tag{2.50}$$

$$\begin{aligned}
\frac{\partial H_{Ther}}{\partial u_{fi}} &= -A_{fi}u_{fi}(t) - k_{1,s}\big(-1+u_{co}(t)\big)V_s(t)\,\cdot \\
&\cdot\ \Big[(-1+Q_{T_g,V_s})\lambda_{1,20}(t)T_g(t) - (-1+Q_{T_g,V_s})\lambda_{1,4}(t)T_g(t) \\
&-\ (-1+Q_{T_h,V_s})\big(\lambda_{1,1}(t)-\lambda_{1,8}(t)\big)T_h(t) \\
&+\ (-1+Q_{T_r,V_s})\big(\lambda_{1,16}(t)-\lambda_{1,3}(t)\big)T_s(t) \\
&+\ (-1+Q_{T_s,V_s})\big(\lambda_{1,12}(t)-\lambda_{1,2}(t)\big)T_s(t) \\
&+\ \rho_s\lambda_{1,26}(t)\big(T_g(t)-Q_{T_g,V_s}T_g(t)+T_h(t)-Q_{T_h,V_s}T_h(t) \\
&+\ T_r(t)-Q_{T_r,V_s}T_r(t)+T_s(t)-Q_{T_s,V_s}T_s(t)\big)\Big] \\
&= 0 \qquad (2.51)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial H_{Ther}}{\partial u_{ib}} &= \lambda_{1,4}T_g(t)+\lambda_{1,1}T_h(t)+\lambda_{1,3}T_r(t)+\lambda_{1,2}T_s(t)-A_{ib}u_{ib}(t) \\
&= 0 \qquad (2.52)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial H_{Ther}}{\partial u_{ii}} &= -A_{ii}u_{ii}(t) - k_2\big(-1+u_{rt}(t)\big)\,\cdot \\
&\cdot\ \Big[\big(\lambda_{1,20}(t)-\lambda_{1,21}(t)\big)T^l_{g,s}(t)+\big(\lambda_{1,8}(t)-\lambda_{1,9}(t)\big)T^l_{h,s}(t) \\
&+\ \big(\lambda_{1,16}(t)-\lambda_{1,17}(t)\big)T^l_{r,s}(t)+\big(\lambda_{1,12}(t)-\lambda_{1,13}(t)\big)T^l_{s,s}(t)\Big] \\
&= 0 \qquad (2.53)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial H_{Ther}}{\partial u_{pi}} &= A_{pi}u_{pi}(t)+k^M_{1,s}\big(\lambda_{1,24}(t)-\lambda_{1,5}(t)\big)M_h(t)\big(-1+u_{rt}(t)\big)V_s(t) \\
&+\ \lambda_{1,26}(t)\Bigg\{\pi_r\big(M^a_{h,r}(t)+T^a_{g,r}(t)+T^a_{h,r}(t)+T^a_{r,r}(t)+T^a_{s,r}(t)\big)\,\cdot \\
&\cdot\ \big(-1+u^v_{r,r}(t)\big)-\pi_s u^v_{s,s}(t)\big(M^a_{h,r}(t)+T^a_{g,r}(t)+T^a_{h,r}(t) \\
&+\ T^a_{r,r}(t)+T^a_{s,r}(t)\big)+V_s(t)\Bigg[-k^M_{1,s}\rho_s M_h(t)\big(-1+u_{rt}(t)\big) \\
&-\ \frac{g}{b_2+V_s(t)}\Bigg]\Bigg\} \\
&= 0 \qquad (2.54)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial H_{Ther}}{\partial u_{rt}} &= k_2\big(-1+u_{ii}(t)\big)\Big[\big(\lambda_{1,20}(t)-\lambda_{1,21}(t)\big)T^l_{g,s}(t) \\
&+ \big(\lambda_{1,8}(t)-\lambda_{1,9}(t)\big)T^l_{h,s}(t)+\big(\lambda_{1,16}(t)-\lambda_{1,17}(t)\big)T^l_{r,s}(t) \\
&+ \big(\lambda_{1,12}(t)-\lambda_{1,13}(t)\big)T^l_{s,s}(t)\Big]-A_{rt}u_{rt}(t) \\
&+ k^M_{1,s}M_h(t)V_s(t)\big(-1+u_{pi}(t)\big)\big(\lambda_{1,24}(t)-\pi_s\lambda_{1,26}(t) \\
&- \lambda_{1,5}(t)\big) \\
&= 0 \qquad (2.55)
\end{aligned}$$

The co-state equations for immune system and therapy are

$$\begin{aligned}
\dot{\lambda}_{1,1}(t) &= -\frac{\partial H_{Ther}}{\partial T_h} \\
\dot{\lambda}_{1,2}(t) &= -\frac{\partial H_{Ther}}{\partial T_s} \\
&\vdots \\
\dot{\lambda}_{1,27}(t) &= -\frac{\partial H_{Ther}}{\partial V_r} \qquad (2.56)
\end{aligned}$$

but we do not explicit them, because they are very long and complex. From these equations we obtain stationary points for the control $u_1(t) = \big(u_{co}(t), u_{fi}(t), u_{ib}(t), u_{ii}(t), u_{pi}(t), u_{rt}(t)\big)$. Again we do not explicit them.

Similarly the 6×6 Hessian matrix for the controls of immune system-therapy $\big(u_{co}(t), u_{fi}(t), u_{ib}(t), u_{ii}(t), u_{pi}(t), u_{rt}(t)\big)$ is

$$\tilde{\mathbf{H}}''_{Ther} = \begin{pmatrix}
-A_{co} & h_{1,2}(t) & 0 & 0 & 0 & 0 \\
h_{1,2}(t) & -A_{fi} & 0 & 0 & 0 & 0 \\
0 & 0 & -A_{ib} & 0 & 0 & 0 \\
0 & 0 & 0 & -A_{ii} & 0 & h_{4,6}(t) \\
0 & 0 & 0 & 0 & -A_{pi} & h_{5,6}(t) \\
0 & 0 & 0 & h_{4,6}(t) & h_{5,6}(t) & -A_{rt}
\end{pmatrix}. \qquad (2.57)$$

where

$$\begin{aligned}
h_{1,2}(t) &= k_{1,s}V_s(t)\Big\{(Q_{T_h,V_s}-1)\big(\lambda_{1,1}(t)-\lambda_{1,8}(t)\big)T_h(t) \\
&- (Q_{T_s,V_s}-1)\big(\lambda_{1,12}(t)-\lambda_{1,2}(t)\big)T_s(t) \\
&- (Q_{T_r,V_s}-1)\big(\lambda_{1,16}(t)-\lambda_{1,3}(t)\big)T_r(t) \\
&- (Q_{T_g,V_s}-1)\big(\lambda_{1,20}(t)-\lambda_{1,4}(t)\big)T_g(t) \\
&+ \rho_s\lambda_{1,26}(t)\Big[(Q_{T_h,V_s}-1)T_h(t)+(Q_{T_s,V_s}-1)T_s(t) \\
&+ (Q_{T_r,V_s}-1)T_r(t)+(Q_{T_g,V_s}-1)T_g(t)\Big]\Big\}, \qquad (2.58)
\end{aligned}$$

$$\begin{aligned} h_{4,6}(t) &= k_2\Big[\big(-\lambda_{1,12}(t)+\lambda_{1,13}(t)\big)T^l_{s,s}(t) \\ &+ \big(-\lambda_{1,16}(t)+\lambda_{1,17}(t)\big)T^l_{r,s}(t) \\ &+ \big(-\lambda_{1,20}(t)+\lambda_{1,21}(t)\big)T^l_{g,s}(t) \\ &+ \big(-\lambda_{1,8}(t)+\lambda_{1,9}(t)\big)T^l_{h,s}(t)\Big], \end{aligned} \quad (2.59)$$

$$h_{5,6}(t) = k^M_{1,s}\big(\lambda_{1,24}(t)-\rho_s\lambda_{1,26}(t)-\lambda_{1,5}(t)\big)V_s(t)M_h(t). \quad (2.60)$$

Here the principal minor of 1st order of Hessian matrix is

$$-A_{co}$$

that is negative.
The determinant of the principal minor of 2nd order is

$$\det \tilde{\mathbf{H}}''_{Ther,2} = A_{co}\;A_{fi} - h^2_{1,2}(t) \quad (2.61)$$

and is negative if and only if

$$A_{co}\;A_{fi} > h^2_{1,2}(t). \quad (2.62)$$

The determinant of the principal minor of 3rd order is

$$-A_{ib}\cdot\det \tilde{\mathbf{H}}''_{Ther,2}. \quad (2.63)$$

The determinant of the principal minor of 4th order is

$$A_{ii}\cdot A_{ib}\cdot\det \tilde{\mathbf{H}}''_{Ther,2}. \quad (2.64)$$

The determinant of the principal minor of 5th order is

$$-A_{pi}\cdot A_{ii}\cdot A_{ib}\cdot\det \tilde{\mathbf{H}}''_{Ther,2}. \quad (2.65)$$

Finally the determinant of the principal minor of 6th order, such as the determinant of Hessian matrix, is

$$\det(\tilde{\mathbf{H}}''_{Ther}) = \det \tilde{\mathbf{H}}''_{Ther,2}\cdot\big[A_{rt}\;A_{pi}\;A_{ii} - h^2_{4,6}(t)\;A_{pi} - h^2_{5,6}(t)\;A_{ii}\big]. \quad (2.66)$$

Observe that if

$$\det \tilde{\mathbf{H}}''_{Ther,2} = A_{co}\;A_{fi} - h^2_{1,2}(t) > 0 \quad (2.67)$$

and

$$A_{rt}\;A_{pi}\;A_{ii} - h^2_{4,6}(t)\;A_{pi} - h^2_{5,6}(t)\;A_{ii} > 0, \quad (2.68)$$

the Hessian matrix is negative definite, therefore the Hamiltonian function $H_{Ther}\big(x,u_1,u_2,\lambda_1(t),\lambda_2(t),t\big)$[1]. is concave in $\big(x,u_1,u_2\big)$, for all $t\in[t_0,t_f]$. Unfortunately functions $h_{1,2}(t), h_{4,6}(t)$ and $h_{5,6}(t)$ are not apriory known, so we can't apply the Mangasarian and Arrow theorems at this general setting. Hence we will verify these conditions ex-post.

[1] where $\lambda_1(t) = \big(\lambda_{1,1}(t),\lambda_{1,2}(t),\ldots,\lambda_{1,27}(t)\big)$ and $\lambda_2(t) = \big(\lambda_{2,1}(t),\lambda_{2,2}(t),\ldots,\lambda_{2,27}(t)\big)$

2.5 Connection between Nash equilibrium and ESS

A widely used issue in biology is the concept of Evolutionary Stable Strategy (ESS), born from the work of John Maynard Smith and George Robert Price [59, 60]. They first used this approach to explain phenomena such as

- the prevalence of a fair relationship male/female in an animal population [19],
- or relationships between prey and predators and attitudes related to this struggle (Hawk-Dove Game [19]).

Given a population of players, an ESS is a strategy that is adopted by the population (in our case is adopted by HIV population), whose particularity is the stability in the face of attempts, by the population, to change its trajectory.

The introduction of this concept also allows us to justify the use of game theory in a population of virus. Viruses are not able to control their strategy, they reproduce themselves and are subject to the force of natural selection. Even just the fact that the payoff of HIV represents the reproductive success (i.e. the number of virus products) helps us to understand that the concept of ESS will be useful in our consideration.

As suggested by Nowak in [75], let us explore a game with more than 2 strategies. The payoff for the strategy S_i versus strategy S_j is given by $E(S_i, S_j)$.

1. The strategy pair (S_k, S_k) is a strict Nash equilibrium if and only if this is true for both players and
$$E(S_k, S_k) > E(S_i, S_k) \quad \forall i \neq k.$$
2. The strategy pair (S_k, S_k) is a Nash equilibrium if and only if this is true for both players and
$$E(S_k, S_k) \geq E(S_i, S_k) \quad \forall i.$$
3. The strategy pair (S_k, S_k) is a ESS if $\forall i \neq k$ we have
$$E(S_k, S_k) > E(S_i, S_k)$$
or
$$\Big[E(S_k, S_k) = E(S_i, S_k) \quad \text{and} \quad E(S_k, S_i) > E(S_i, S_i)\Big]^2.$$

[2] Maynard Smith's second condition

Note that the ESS guarantees that the selection will oppose any potential invader. The same is true for a strict Nash equilibrium, but not for a Nash equilibrium.
If $E(S_k, E_k) = E(S_j, S_k)$ and $E(S_k, S_j) < E(S_j, S_j)$, then S_k is still a Nash equilibrium, but the selection will favor S_j invading S_k. Thus it makes sense to add fourth definition.

4. The strategy pair (S_k, S_k) is stable against invasion by selection (let us called "weak ESS") if $\forall i \neq k$ we have either

$$E(S_k, S_k) > E(S_i, S_k)$$

or

$$E(S_k, S_k) = E(S_i, S_k) \quad \text{and} \quad E(S_k, S_i) \geq E(S_i, S_i).$$

If a strategy is a strict Nash equilibrium (SNE) then it is also an ESS. If a strategy is an ESS is also a weak ESS (WESS). If a strategy is a weak ESS is also a Nash equilibrium (NE). Thus strict Nash equilibrium implies ESS implies weak ESS implies Nash equilibrium:

$$SNE \Rightarrow ESS \Rightarrow WESS \Rightarrow NE.$$

Back to our model, we want to show that the Nash equilibrium characterized by the additional conditions

$$\left|\frac{\partial u_{ii}}{\partial u^v_{s,s}}\right| + \left|\frac{\partial u_{ii}}{\partial u^v_{r,r}}\right| < 1 \tag{2.69}$$

$$\left|\frac{\partial u_{pi}}{\partial u^v_{s,s}}\right| + \left|\frac{\partial u_{pi}}{\partial u^v_{r,r}}\right| < 1 \tag{2.70}$$

$$\left|\frac{\partial u_{rt}}{\partial u^v_{s,s}}\right| + \left|\frac{\partial u_{rt}}{\partial u^v_{r,r}}\right| < 1 \tag{2.71}$$

$$\left|\frac{\partial u^v_{s,s}}{\partial u_{rt}}\right| < 1 \tag{2.72}$$

$$\left|\frac{\partial u^v_{r,r}}{\partial u_{rt}}\right| < 1 \tag{2.73}$$

is an ESS, and we will prove that proving that it is SNE.
Note that if there is a Nash equilibrium does not rule out that there may be an infinite number of Nash equilibria.

We do not report for each control, the expressions of solution of maximum equations (2.40), (2.41), (2.50), (2.51), (2.52), (2.53), (2.54), (2.55) because they are very large and complex.

Proposition 2.5.1. *Suppose that the couple of strategies (Therapy, HIV) is characterized by the solution of maximum equations* (2.40), (2.41), (2.50), (2.51), (2.52), (2.53), (2.54), (2.55) *and that these strategies are subject to the additional conditions mentioned above* (2.69), (2.70), (2.71),(2.72), (2.73) *then*

1. *The Nash equilibrium is unique.*
2. *From any starting point, iterations defined by strategies converge to this unique Nash equilibrium.*

Proof. We now set up uniqueness and convergence together by showing that, for each control, the expressions of solution of maximum equations (2.40), (2.41), (2.50), (2.51), (2.52), (2.53), (2.54), (2.55) are a contraction mappings. According to Contractions Lemma (Banach Cacciopoli, [26, page 117]), let M be an Euclidean complete space and let $f : M \to M$ be a mapping. In addition, consider any vector norm. Let $d(\cdot)$ be the distance function by the vector norm. We have

$$\begin{aligned} d(f(u), f(v)) &= ||f(u) - f(v)|| \qquad (2.74)\\ &\leq \left|\left|\frac{\partial f}{\partial x}\right|\right| ||(u-v)|| = \left|\left|\frac{\partial f}{\partial x}\right|\right| d(u,v). \qquad (2.75)\end{aligned}$$

The matrix norm used here is induced by the vector norm too. The inequality follows from the property of hte matrix norm. Hence, it is clear that if we have the Jacobian $||\partial f/\partial x|| < 1 - \epsilon$, everywhere for some positive ϵ value, we can let $k = 1 - \epsilon < 1$, and applies the contaction mapping theorem.
We now derive conditions using $||\cdot||_\infty$, for the expressions of solution of maximum equations (2.40), (2.41), (2.50), (2.51), (2.52), (2.53), (2.54), (2.55) to be a contraction map. Its Jacobian is defined by the formula

$$J_{ij} = \frac{\partial u_i}{\partial u_j}.$$

The complete form of Jacobian is given as follows

$$J = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial u_{ii}}{\partial u^v_{s,s}} & \frac{\partial u_{ii}}{\partial u^v_{r,r}} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial u_{pi}}{\partial u^v_{s,s}} & \frac{\partial u_{pi}}{\partial u^v_{r,r}} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial u_{rt}}{\partial u^v_{s,s}} & \frac{\partial u_{rt}}{\partial u^v_{r,r}} \\ 0 & 0 & 0 & 0 & \frac{\partial u^v_{s,s}}{\partial u_{rt}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\partial u^v_{r,r}}{\partial u_{rt}} & 0 & 0 & 0 \end{bmatrix}. \qquad (2.76)$$

It follows, therefore, that

$$||J||_\infty = \max\left\{ \begin{aligned} & \left|\frac{\partial u_{ii}}{\partial u^v_{s,s}}\right| + \left|\frac{\partial u_{ii}}{\partial u^v_{r,r}}\right|, \\ & \left|\frac{\partial u_{pi}}{\partial u^v_{s,s}}\right| + \left|\frac{\partial u_{pi}}{\partial u^v_{r,r}}\right|, \\ & \left|\frac{\partial u_{rt}}{\partial u^v_{s,s}}\right| + \left|\frac{\partial u_{rt}}{\partial u^v_{r,r}}\right|, \\ & \left|\frac{\partial u^v_{s,s}}{\partial u_{rt}}\right|, \quad \left|\frac{\partial u^v_{r,r}}{\partial u_{rt}}\right| \end{aligned} \right\} \tag{2.77}$$

From the hypothesis (2.69), (2.70), (2.71),(2.72), (2.73) we conclude $0 < ||J||_\infty < 1$. Hence the expressions of solution of maximum equations (2.40), (2.41), (2.50), (2.51), (2.52), (2.53), (2.54), (2.55) are contraction mappings, and bothuniqueness and global convergence are guaranteed.

It has been proven that if the best response is unique, then the Nash equilibrium is a strict Nash equilibrium [82]. Thus, it is proven that our Nash equilibrium is a strict Nash equilibrium, which implies an ESS. □

Chapter 3

Numerical Solution

The optimal control problems which permits to calculate the Nash equilibria of the therapy / HIV game, presented in Chapter 2, can be generalized as follows

$$\begin{aligned} &\max_u \int_{t_0}^{t_f} f(t, x(t), u(t))dt \\ \text{subject to} \quad & x'(t) = g(t, x(t), u(t)) \\ & x(t_0) = x_0 \\ & x(t_f) \in \mathbb{R} \\ & u(t) \in [0, 1]^m. \end{aligned} \tag{3.1}$$

Even if an analytical solution would be preferable, in certain conditions we are unfortunately obliged to find a numerical solution, using an algorithm that approximates the optimal control u^*. The necessary condition for optimality are

$$x'(t) = g(t, x(t), u(t)), \quad x(t_0) = x_0, \tag{3.2}$$

$$\lambda'(t) = -\frac{\partial H}{\partial x} = -(f_x(t, x, u) + \lambda(t) g_x(t, x, u)), \quad \lambda(t_f) = 0, \tag{3.3}$$

$$0 = \frac{\partial H}{\partial u} = f_u(t, x, u) + \lambda(t) g_u(t, x, u) \quad \text{in} \quad u^*. \tag{3.4}$$

The third equation, the optimality condition, can be manipulated to find a representation of u^* in terms of t, x and λ. In these terms it is replaced in the differential equations for x, λ, and the first two equations form a problem with boundary values at two different points. The fact that one is initial (for the state) and one is the final (for the adjoint equation) and the fact that the state does not depend on the values of the adjoint function, it gives us the ability to create an efficient and intuitive algorithm: the algorithm Forward-Backward Sweep [52].

Let's break the interval $[t_0, t_f]$ at the points $t_0 = b_1, b_2, \ldots, b_N, b_{N+1} = t_f$;

these points are equally spaced by the same constant step h. The approximation will be a vector $\vec{u} = (u_1, u_2, \ldots, u_{N+1})$, where $u_i \approx u(b_i)$. Taken as an approximation of the state and of the adjoint function the vectors $\vec{x} = (x_1, \ldots, x_{N+1})$ e $\vec{\lambda} = (\lambda_1, \ldots, \lambda_{N+1})$, the steps of the algorithm are the following

Step 1 Make an initial prediction for the $\vec{u}$ on the range.

Step 2 Using the initial condition $x_1 = x(t_0) = x_0$ and the values of $\vec{u}$, solve $\vec{x}$ forward in the time according to his differential equation.

Step 3 Using the transverality condition $\lambda_{N+1} = \lambda(t_f) = 0$ and the values of $\vec{u}$ e $\vec{x}$, solve $\vec{\lambda}$ backward in the time according to his SDE.

Step 4 Update $\vec{u}$ inserting the new calculated values for $\vec{x}$ e $\vec{\lambda}$ in the characterization of optimal control.

Step 5 Convergence control: if the values of variables in this iteration and in the previous are sufficiently close, we output the current value as solution. Otherwise, return to step **Step 2**

Usually the initial choice $\vec{u} \equiv 0$ is sufficient, if you shouldn't divide for u, otherwise it is better to opt for a different initial choice.
In **Step 4**, controls are usually updated through a convex combination between the previous value of the control and the last calculated one. This procedure speeds the convergence. With regard to Step 2 and 3, any solver of differential equations can be applied. A common method is the routine Runge-Kutta of 4^{th} order [92], in this case, given a step h, the equation $x'(t) = f(t, x(t))$ and $x(t)$, the approximation of $x(t+h)$ is

$$x(t+h) \approx x(t) + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{3.5}$$

where

$$\begin{aligned} k_1 &= f(t, x(t)), \\ k_2 &= f(t + \frac{h}{2}, x(t) + \frac{h}{2}k_1), \\ k_3 &= f(t + \frac{h}{2}, x(t) + \frac{h}{2}k_2), \\ k_4 &= f(t + h, x(t) + hk_3). \end{aligned} \tag{3.6}$$

There are a lot of tests for convergence. Often it is sufficient to require that$||u - oldu|| = \sum_{i=1}^{N+1} |u_i - oldu_i|$ is small, where $\vec{u}$ is the vector of estimated values of control during the current iteration and $\vec{oldu}$ the vector of values of the previous iteration. Therefore we require that

$$\frac{||\vec{u} - \vec{oldu}||}{||\vec{u}||} \leq \delta, \tag{3.7}$$

where δ is the accepted tolerance. Let's make a small adjustment to allow to enter the null control ($\vec{u} \equiv 0$), multiplying both sides by $||\vec{u}||$. Therefore we wish that

$$\delta||\vec{u}|| - ||\vec{u} - \vec{oldu}|| \geq 0, \tag{3.8}$$

that is

$$\delta \sum_{i=1}^{N+1} |u_i| - \sum_{i=1}^{N+1} |u_i - oldu_i| \geq 0. \tag{3.9}$$

In our model this request is made only on the state and on the adjoint variables, and not on the control.

Another way to proceed is to explicit u^* with respect to t, x and λ. In this case **Step 1** is not performed on the control, but on the state and adjoint variables; the convergence test runs on these variables, after the vector $\vec{u}$ is calculated separately the vector. This way of acting is not wrong, but it can easily lead to errors in the transcription of the code. if possible, it is advisable to calculate the control u^* and then test the convergence.

In our model unfortunately theory don't help us. Therefore, we try to solve the problem numerically.
We have a system with 81 differential equations of the first order which are strongly non linear, more precisely

- 27 equations on the states;
- 54 equations on the adjoint variables;
- 27 boundary conditions for the states at time t_0;
- 54 boundary conditions for the adjoint variables at time t_f;

therefore we must adopt a FORWARD-BACKARD SWEEP approach [52].

For the realization of the algorithm we have used `Mathematica`, for the symbolic calculus, and `Matlab` for the numeric calculus. The reason for this choice is linked to the fact that `Matlab` is very weak from the point of view of the symbolic computation and `Mathematica` is very weak from the point of view of the numerical calculation. Therefore we have opted to use a combination of them, so that one method could fill in the gaps of the other.

As mentioned in Section 2.4, Hamiltonian concavity with respect to HIV is guaranteed, whereas with respect to therapy controls, this concavity is not guaranteed and it must be checked ex post. In this regard, our algorithm (at each iteration forward, at each iteration backward, and to each of the 4 orders of routine Runge Kutta) assigns to the virus controls the values:

$$\min \left\{1, \max\{0, \hat{u}_2(t)\}\right\}$$

where $\hat{u}_2(t)$ is a stationary point; and it assigns to the therapy controls the value between 0, 1, and $\hat{u}_1(t)$ that maximizes the Hamiltonian function at that time.

The convergence request is given by the equation (3.9) on state and adjoint variables, but in our algorithm we verify also the validity of ESS condition (2.69), (2.70), (2.71), (2.72), (2.73) to ensure uniqueness and convergence of found Nash equilibrium.

3.1 Numerical integration

About numerical calculation of payoff, often the integral

$$I(f) = \int_a^b f(x)\mathrm{d}x, \tag{3.10}$$

(where f is an arbitrary continuous function in $[a, b]$) is not determinable analytically, or even if it is possible, the final expression can be too complex and therefore its numerical evaluation is subject to large errors. Sometimes, as in the case of the approximation, the integrand function $f(x)$ is not known in analytical form, but only in a finite number of points. Therefore it is interesting to have numerical methods that can approximate the value of the integral from a finite number of values of the integrand function. This is possible by using the so-called numerical quadrature formulas.
In this work we propose 3 different methods [92]

1. composite rectangle method;
2. composite trapezoid method;
3. composite Cavalieri-Simpson method.

3.1.1 Composite rectangle formula

A simple procedure to approximate $I(f)$ consists to subdivided $[a, b]$ into sub-intervals

$$I_i = [x_i, x_{i+1}], \qquad i = 0, 1, \dots, N,$$

with

$$x_i = a + ih, \qquad i = 0, 1, \dots, N, \qquad h = (b - a)/N.$$

Thanks to the integrals additivity, we can write

$$I(f) = \int_a^b f(x)\mathrm{d}x = \sum_{i=0}^{N} \int_{I_i} f(x)\mathrm{d}x. \tag{3.11}$$

Now, for all sub-intervals I_i we substitute f integral with the integral of a polynomial $\bar{f}$ that approximates f on I_i. The easiest solution is to use a constant polynomial that interpolate f in the midpoint of sub-interval I_i

$$\bar{x}_i = \frac{x_i + x_{i+1}}{2}$$

and we obtain the composed rectangle formula

$$I_R^C(f) = h\left[\sum_{i=0}^{N} f(\bar{x}_i)\right] + R(f) \tag{3.12}$$

where $R(f)$ is the error and it is possible to demonstrate that if f is continuously differentiable up to the second order we have

$$R(f) = \frac{(b-a)}{24}\, h^2 f''(\xi), \qquad \xi \in [a,b] \tag{3.13}$$

3.1.2 Composite trapezoid formula

Where to approximate the f integral on I_i you use the interpolating polynomial of degree 1 in the extreme nodes x_k and x_{k+1}, we formulate the so-called composite trapezoidal formula

$$I_T^C(f) = \frac{h}{2}\left[f(a) + \sum_{i=1}^{N-1} f(x_i) + f(b)\right] + R(f); \tag{3.14}$$

where $R(f)$ is the error and it is possible to demonstrate that for $C^2(a,b)$ functions

$$R(f) = -\frac{(b-a)}{12}\, h^2 f''(\xi), \qquad \xi \in [a,b] \tag{3.15}$$

3.1.3 Composite Cavalieri-Simpson formula

We proceed in a similar way to what is done to derive the composite trapezoid formula. The only difference is that now, on each interval I_i, the function f will be replaced with the interpolating polynomial of degree 2 in the 3 nodes x_k, $(x_k + x_{k+1})/2$, x_{k+1}. This polynomial has the form

$$I_C^C S(f) = \frac{h}{3}\left[f(a) + 4\sum_{\substack{i=1\\ i \text{ odd}}}^{N-1} f(x_i) + 2\sum_{\substack{i=2\\ i \text{ even}}}^{N-2} f(x_i) + f(b)\right] + R(f); \tag{3.16}$$

where $R(f)$ is the error and it is possible to demonstrate that for all $f \in C^4(a,b)$

$$R(f) = -\frac{(b-a)}{180}\, h^4 f^{(4)}(\xi), \qquad \xi \in [a,b] \tag{3.17}$$

3.2 Various instances

We have proved that concavity of the virus Hamiltonian function is guaranteed by the 2×2 Hessian matrix (2.45). According to the observed instances, in general, the concavity for the Hamiltonian of therapy controls is not assured. However, with different parameter values, we can obtained instances in which this concavity holds.

We first present a situation where Hamiltonian concavity for therapy controls is guaranteed. In this context our algorithm, for every iteration (or in every subinterval in which the integration interval is subdivided) and for each of the four degrees of Runge Kutta routine is more simple and converges very fast. It assigns to the controls the value

$$\min\big\{1, max\{0, \hat{u}(t)\}\big\}, \tag{3.18}$$

(where $\hat{u}(t)$ is a stationary point) and this can be done just because the therapy and HIV Hamiltonians are both concave. Values assigned to all adjoint variables $\lambda_{i,j}(t)$ at time t_0 are 0.1, and this allows therapy Hamiltonian function to be concave.
The instance here presented is characterized by a high drugs toxicity, moreover drugs are very effective, in fact, small doses force it to reduce the virus mutation rates $u^v_{r,s}$ and $u^v_{s,r}$ (for parameters value see Appendix B).

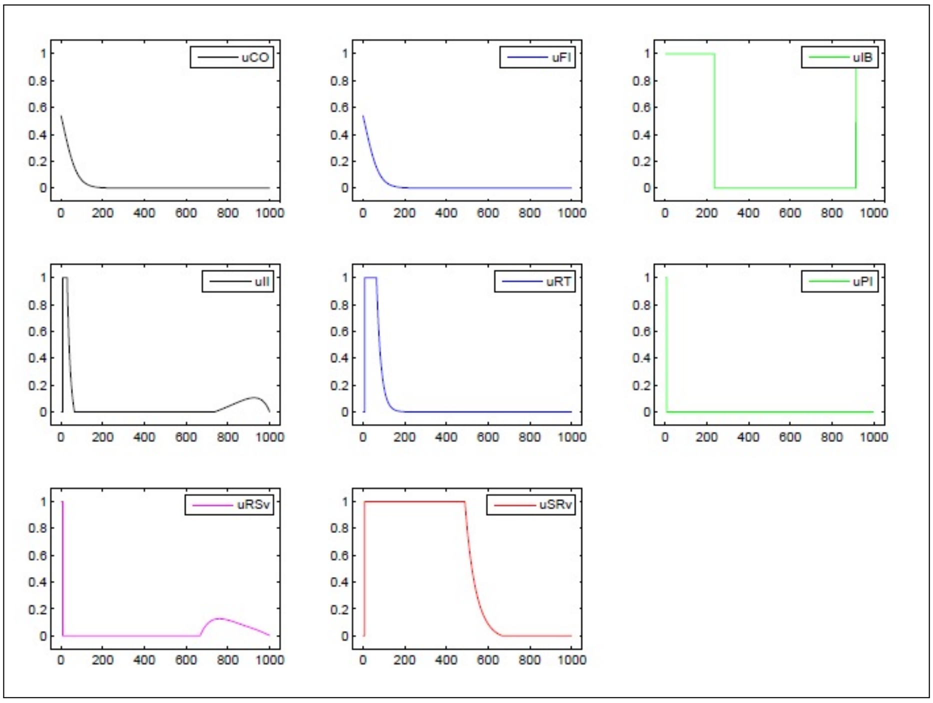

Figure 3.1: 1st instance with concave therapy Hamiltonian.

As shown in Figure 3.1, the first six graph represent therapy dosages and the last two the HIV mutations.
Fusion inhibitors (u_{fi}) and coreceptor antagonist (u_{co}), are used for the same scope that of interfering with binding, fusion and entry of HIV to the host cell and blocking one of several targets, in fact their dosages are the same for all 1000 days.
Also integrase (u_{ii}) and reverse transcriptase inhibitors (u_{rt}) are used in the same phase of virions creation that is the reverse trascription from RNA to DNA. Their dosage are very similar for the first 100 days, but integrase inhibitors are also used around 600 days to cope with rising of mutation rate from resistant to sensitive viruses ($u^v_{r,s}$).
Immune boosting (u_{ib}) seems to have a pattern of cyclic type (as it is the nature of such drugs whose purpose is to strengthen the immune system) but by following an appropriate cyclicity in fact as you can see the second cycle of therapy begins when the viruses mutation rate are now null and void.
Protease inhibitors (u_{pi}) block the viral protease enzyme necessary to produce mature virions upon budding from the host membrane. These drugs are very efficient in fact they are used approximately 25 days.

For what concerns HIV, the mutation rate from sensitive to resistant viruses ($u^v_{s,r}$) is maximum for the first 500 days, then begin to decrease thanks to drug effects and vanishes around 600 days when there is a tail strike of mutation rate from resistant to sensitive viruses ($u^v_{r,s}$) that however is completely canceled by the action of integrase inhibitors (u_{ii}).

The payoff for therapy and HIV are listed in Table 3.1. The high drugs toxicity induces to elevated costs for the immune system, leading to a negative payoff, that musts be interpreted only as costs for the patient.

Table 3.1: 1^{st} instance: payoff.

Numerical integration method	Therapy payoff	HIV payoff
Composite rectangle method	-40479	177.3520
Composite trapezoids method	-40480	177.3520
Composite Cavalieri-Simpson method	-40480	177.3488

In spite of this, analyzing the trends of the graphs of different lymphocytes, we can ensure that the lymphocyte count is, for the entire time interval, around 1000 $cells/mm^3$ (see Figure 3.2).
Therefore our aim is to determine a therapy that allows to prevent viral replication with side effects, for the immune system, that are the lowest possible.

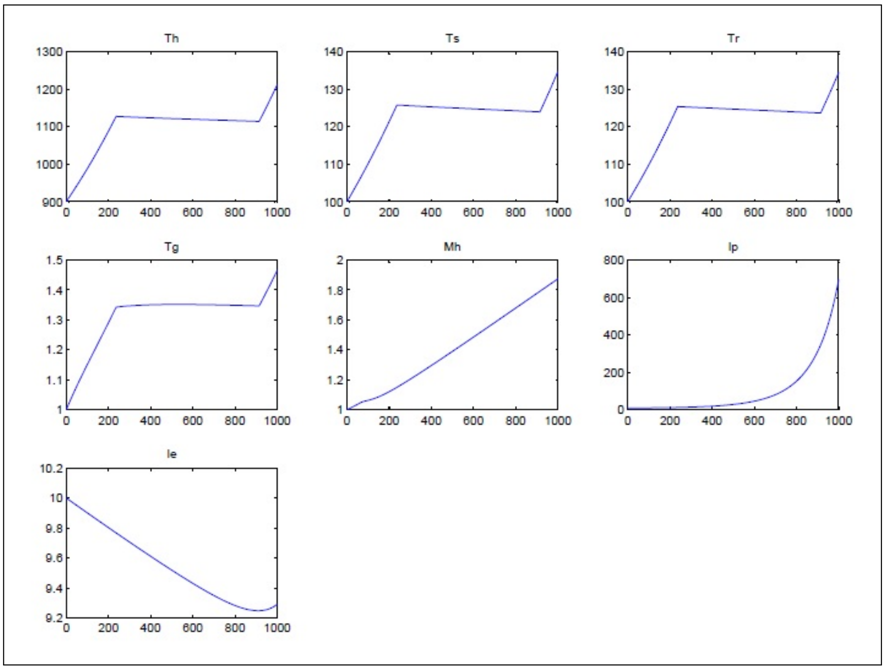

Figure 3.2: 1^{st} instance: lymphocytes, macrophage and CTL dynamics.

The white blood cell count is definitely one of the most important tests to verify the presence of HIV virus: it is an indicator of the health status of a patient and is one of the key factors in determining when to start HAART and prophylaxis for opportunistic infections. The tests usually measure the white blood cells present in the sample and calculate the percentage of CD4+ T-lymphocytes. This value can be variable between different individuals and influenced by factors that affect the white blood cells in general, such as drug use and the presence of acute infections. In general, the percentage of CD4+ T-cells is stable and it can be a parameter for evaluating the immune functions of a patient. Usually, it has a level of 500-1000 $cells/cm^3$ in a healthy patient, below this threshold, we can think of a possible infection to damage the immune system.

Finally, in this instance, the ESS condition (2.69), (2.70), (2.71), (2.72), (2.73) presented in Section 2.5 are met therefore the instance presented has an unique Nash equilibria that is also an evolutionary stable strategy.

Let us consider no-concave therapy instances: in these cases the optimal therapy controls are not necessarily given by equation (3.18) and more local optimal points may exist. In order to find the global optimum, our algorithm in every subinterval in which the integration interval is subdivided and for each of the four degrees of Runge Kutta routine, assigns to therapy controls the value between

$$0, \qquad 1, \qquad \hat{u}(t)$$

(where $\hat{u}(t)$ is a stationary point). This procedure expands the processing time because at each step, it checks therapy controls which maximize the Hamiltonian for 8 times:

- 4 times (one for each of the 4 degrees of Runge Kutta) during forward process, in calculation of variables;

plus

- 4 times (one for each of the 4 degrees of Runge Kutta) during backward process, in calculation of adjoint variables.

Therefore, at each iteration the algorithm will perform a search for the optimal therapy sequence that maximizes the Hamiltonian function 8 times within a $3^6 \times 6$ $(= 729 \times 6)$ matrix
These not concave instances are generally characterized by a low drugs toxicity, therefore, in these cases, the payoff of the player immune system/therapy is much higher. Notice a situation in which the drugs are less effective, therefore, it is required a prolonged intake over time.
Numerical simulation for this instance regards values randomizations at time t_0 of adjoint variables $\lambda_{i,j}(t)$. We first present in Figure 3.3 an instance with all adjoint variables at time t_0 initialized to 0.1.

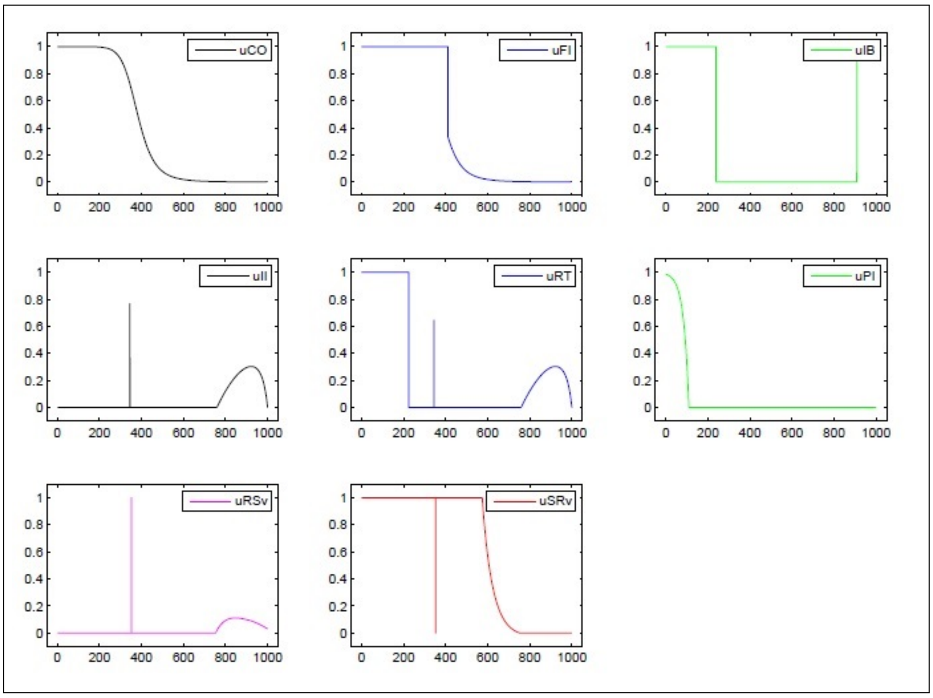

Figure 3.3: 2^{nd} instance with no concave therapy Hamiltonian and $\lambda_{i,j}(t_0) = 0.1$.

HIV $u^v_{s,r}$ mutation rate is maximum for about 600 days then begins to decrease thanks to action of therapy controls u_{co}, u_{fi} and u_{rt}, u_{pi}. Finally it

vanishes around 700 days. Also in this instance there is a final contribution by the mutation rate $u^v_{r,s}$, that is completely torn down by the combined actions of u_{ii} and u_{rt}.
Note that around the 400$^{\text{th}}$ day there is a positive peak for $u^v_{r,s}$ and consequently a negative peak for $u^v_{s,r}$. The mutation rate from resistant to sensitive viruses is immediately counteracted by integrase and reverse trascriptase inhibitors.
Immune boosting (u_{ib}) continues to have a cyclical dosage, whilst protease inhibitors are used only the first 100 days.

The payoff for therapy and HIV are listed in Table 3.2

Table 3.2: 2$^{\text{nd}}$ instance with $\lambda_{i,j}(t_0) = 0.1$: payoff.

Numerical integration method	Therapy payoff	HIV payoff
Composite rectangle method	1411.7	171.9974
Composite trapezoids method	1411.7	171.9975
Composite Cavalieri-Simpson method	1411.5	171.9336

The non-concavity of the Hamiltonian functions of therapy controls, does not allow us to affirm that the just proposed therapy in Figure 2 is the optimal one. It is therefore necessary to perform the randomizations on the initial values of the adjoint variable lambda, in order to identify the values of $\lambda_{i,j}(t_0)$ that make at least a suboptimally therapy.
We made the choice of these initial values in the range $[-100, 100]$, and after multiple randomizations we verified that the higher immune system-therapy payoffs were obtained for initial values of $\lambda_{i,j}(t)$ in the range $[0, 1]$. At this point, we performed a further randomization, this time much more precise, on 100 different instances with adjoint variable values $\lambda_{i,j}(t_0) \in [0, 1]$, getting the below results (Figure 3.4 and Table 3.3).
Recall that these randomizations have high computational times, for example the last numerical simulation took 37182 seconds i.e. about 11 hours.

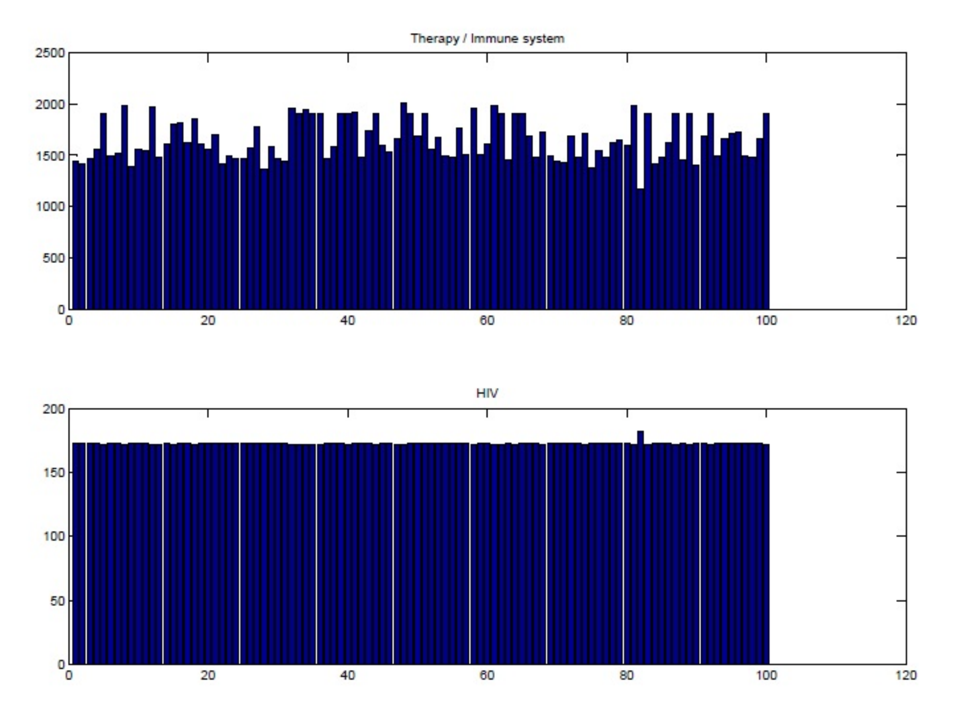

Figure 3.4: Instances randomization for $\lambda_{i,j} \in [0, 1]$.

Table 3.3: Instances randomization: payoff.

Integration method	Therapy max	Therapy mean	Corresponding HIV[1]	HIV mean[2]
Composite rectangle	2002.1	1654.4	171.6968	171.9550
Composite trapezoids	2002.1	1654.4	171.6974	171.9555
Cavalieri-Simpson	2000.2	1653.2	171.3787	171.6368

Finally, we stored the values of $\lambda_{i,j}(t)$ at time t_0 that maximized the randomization and we replace them in the search algorithm of the optimal therapy, getting the following chart.

[1]HIV payoff corresponding to maximum therapy payoff obtained with the randomization.
[2]HIV mean obtained with the randomization.

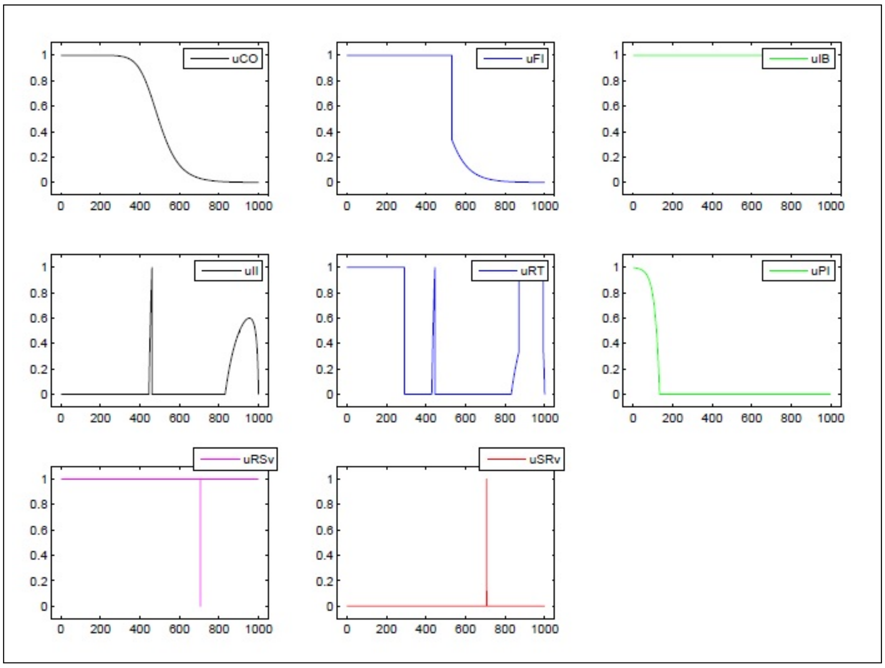

Figure 3.5: 2^{nd} instance with no concave therapy Hamiltonian and $\lambda_{i,j}(t_0)$ assigned using randomization results.

In this instance the HIV mutation rate from sensitive to resistant ($u^v_{s,r}$) viruses is most of the times equal to zero, except for a positive peak around the 700^{th} day. Obviously this peak corresponds to a negative peak by the mutation rate from resistant to sensitive ($u^v_{r,s}$) viruses. This last rate is constantly at the maximum level: this is not a thing to fear because if all the viruses become drug sensitive then the therapeutic treatments would be able to eradicate easily the infection. In addition, the final increment of integrase (u_{ii}) and reverse transcriptase (u_{rt}) inhibitors avoids further positive peaks by $u^v_{s,r}$. Notice that the immune boosting remains constantly equal to 1, because the increase of lymphocytes is not dangerous and can not cause an increase of preys for viruses, as these tend to be all drugs sensitive and therefore easily countered by drugs. The payoff for therapy and HIV of this “no-concave” instance are listed in Table 3.4.

Table 3.4: 2^{nd} instance with $\lambda_{i,j}(t_0)$ assigned using randomization results: sub-optimally payoff.

Numerical integration method	Therapy payoff	HIV payoff
Composite rectangle method	2002.1	171.6968
Composite trapezoids method	2002.1	171.6974
Composite Cavalieri-Simpson method	2000.2	171.3787

As expected, values meet as seen in Table 3.3, and the ESS conditions

(2.69), (2.70), (2.71), (2.72), (2.73) presented in Section 2.5 are not verified, in fact it is not an unique Nash equilibrium and it is not an evolutionary stable strategy.

3.3 Conclusions

In this work we have presented an application of differential Game Theory to a medical-therapeutic context for the HIV treatment, suggested by article [128]. We carried out a literature analysis, to study the different approaches used in this field. Our aim was to formulate a model that could be as close as possible to the real dynamics between HIV and Human immune system, in the context of differential games and, at the same time, that it was solvable. The model consists of 27 state functions and 8 controls (6 for the therapy and 2 for HIV). The optimality necessary conditions to obtain the Nash equilibrium of the game lead us to a system of 81 differential equations (27 equation for state variables and 54 for adjoint variables) of the first order which are strongly non-linear and are subjected to two different types of boundary conditions: at the initial time (t_0) for the state functions and at the final time (t_f) for the adjoint functions.
We designed an algorithm that uses the Runge Kutta routine of 4^{th} order [92], while the question of the boundary conditions at different times for different variables has been entrusted through the Forward-Backward Sweep algorithm [52]. All this in order to determine the optimal therapy in specific instances where it is guaranteed or not the concavity of the immune system-therapy Hamiltonian function. When concavity is not guaranteed, we have found a sub-optimal solution through randomizations of initial values given to the adjoint variables $\left(\lambda_{i,j}(t_0)\right)$.

Although the proposed model is not too complex, the fact that these variables were "intertwined" with each other has made it very difficult to manage, and it was difficult to analyze even the large equations that formed it. One of major problems we encountered was the need to use the software `Mathematica` and `Matlab` in symbiosis. Symbolic computation was carried out by `Mathematica`, while the numerical calculation by `Matlab`. Certainly one tool that would allow optimal management of both the symbolic and numerical calculation would have simplified the issue.
During this work, I realized that although specialist softwares as `Mathematica` and `Matlab` have within them appropriate routines for solving differential equations, they do not provide within these routines, the way to assign, at each step, controls values.

We believe it would be appropriate to validate our model on known clinical situations of patients who have been continuously monitored both with regard to the count of immune cells and the administered therapy.

A possible further extension of this model could be obtained in matching the Game Theory approach with the Fuzzy system issue, paying attention to the unavoidable complexity increase.

Finally it is necessary to go much more in-depth in the study of parameters validation, in order to standardize them and to determine the probability distribution of their values, starting from drugs toxicity costs.

Appendix A

Differential Games

A game is said dynamic if at least one player can use a strategy that evolves at each instant of time, and that can refer to the previous actions, both his own and those of the opponent. In order to analyze a dynamic game, we need to know the order the players adopt to take their choices and what type of information is available to them. Assuming all actions as observable by all players, we will say perfect information game, on the contrary we will say incomplete information game.

Let's consider a game with N players who know at each instant t their past actions up to time $t-1$. A differential game is a dynamic game, played in continuous time. Two distinctive features of differential games are

- a set of variables that characterize the state of the dynamic system at any instant of time during the course of the game;
- a set of differential equations that describes the time evolution of state variables.

Differential games may vary depending on what information each player has, and this depends not only on the information it uses, but also on the information it holds. It cans play using only on the temporal information, typical of the open-loop strategies, or using the entire history of actions, like Markovian strategies, in which decisions are influenced by the time t and the state vector at time t. In our analysis we focused on open-loop strategies.

Let's consider a game in a time range $[0, T]$ with T finite or for an unlimited time span $[0, +\infty)$. The state of the game for any time t is described by the vector $x(t) \in X$ where $X \subseteq \mathbb{R}^n$ is the space of states and the initial state is a fixed constant $x_0 \in X$. At each instant of time $t \in [0, T]$, each player $i \in \{1, 2, \dots, N\}$ chooses a control $u^i(t)$ from a set of admissible controls $U^i(x(t), u^{-i}(t), t) \subseteq \mathbb{R}^{m^i}$, where the vector $u^{-i}(t)$ represents controls of other players, that is

$$u^{-i}(t) = (u^1(t), u^2(t), \dots, u^{i-1}(t), u^{i+1}(t), \dots, u^N(t)). \qquad \text{(A.1)}$$

The state of the system evolves according to the differential equation

$$\dot{x}(t) = f(x(t), u^1(t), u^2(t), \ldots, u^N(t), t), \qquad x(0) = x_0, \tag{A.2}$$

where the function

$$f : \Omega \longrightarrow \mathbb{R}^n \tag{A.3}$$

with

$$\Omega = \{(x, u^1, \ldots, u^N, t) | x \in X, t \in [0, T], u^i \in U^i(x, u^{-i}, t), i = 1, \ldots, N\}. \tag{A.4}$$

Each player $i \in \{1, 2, \ldots, N\}$ wants to maximize its own objective functional

$$J^i(u^i(\cdot)) = \int_0^T e^{-r^i t} F^i(x(t), u^1(t), u^2(t), \ldots, u^N(t), t)dt + e^{-r^i T} S^i(x(T)). \tag{A.5}$$

The function $F^i : \Omega \mapsto \mathbb{R}$ represents the instantaneous payoff of player i, r^i its discount rate, and $S^i : X \mapsto \mathbb{R}$ scrap function, understood as a payoff at the final time T. In the case that $T = +\infty$ we assume that $S(x) = 0$, $\forall x \in X$.

A.1 Nash equilibria

A Nash equilibrium of a differential game is a N-uple of strategies $(\varphi^1, \varphi^2, \ldots, \varphi^N)$ such that, given the equilibrium strategy of the opponent, no player has an incentive to change its strategy. All the opponents of the player i use a Markov strategy $u^j(t) = \varphi^j(x(t), t), j \neq i$, then the player i is in front of an optimal control problem having to maximize the function

$$J^i_{\varphi^{-i}}(u^i(\cdot)) = \int_0^T e^{-r^i t} F^i_{\varphi^{-i}}(x(t), u^1(t), u^2(t), \ldots, u^N(t), t)dt + e^{-r^i T} S^i(x(T)) \tag{A.6}$$

subjects to the conditions

$$\dot{x}(t) = f^i_{\varphi^{-i}}(x(t), u^i(t), t) \qquad x(0) = x_0 \qquad u^i(t) \in U^i_{\varphi^{-i}}(x(t), t), \tag{A.7}$$

where

$$\begin{aligned} F^i_{\varphi^{-i}}(x, u^i, t) =& F^i(x, \varphi^1(x, t), \ldots, \varphi^{i-1}(x, t), u^i, \varphi^{i+1}(x, t), \ldots, \varphi^N(x, t), t), \\ f^i_{\varphi^{-i}}(x, u^i, t) =& f(x, \varphi^1(x, t), \ldots, \varphi^{i-1}(x, t), u^i, \varphi^{i+1}(x, t), \ldots, \varphi^N(x, t), t), \\ U^i_{\varphi^{-i}}(x, u^i, t) =& U^i(x, \varphi^1(x, t), \ldots, \varphi^{i-1}(x, t), \varphi^{i+1}(x, t), \ldots, \varphi^N(x, t), t). \end{aligned} \tag{A.8}$$

Given each $(N-1)$-uple $\varphi^{-i} = (\varphi^1, \ldots, \varphi^{i-1}, \varphi^{i+1}, \ldots, \varphi^N)$ of functions $\varphi^j : X \times [0, T] \mapsto \mathbb{R}^{m^j}$, $j \neq i$,the problem to maximize $J^i_{\varphi^{-i}}(u^i(\cdot))$ is an optimal control problem.

Definition A.1.1. *The N-uple $(\varphi^1, \varphi^2, \ldots, \varphi^N)$ of functions $\varphi^i : X \times [0,T] \mapsto \mathbb{R}^{m^i}$, $i \in \{1, 2, \ldots, N\}$ is said Markovian Nash equilibria if, for all $i \in \{1, 2, \ldots, N\}$, a solution of optimal control $u^i(\cdot)$ that maximizes $J^i_{\varphi^{-i}}(u^i(\cdot))$ exist and it is given by the Markovian strategy $u^i(t) = \varphi^i(x(t), t)$.*

A.2 Necessary condition for Nash equilibria (Pontryagin Maximum Principle)

The Pontryagin Maximum Principle provides necessary optimality conditions for optimal control problems, it is a valuable tool for many applications in several contexts.

Let's define a function H with real values

$$H(x, u, \lambda, t) = F(x, u, t) + \lambda f(u, x, t). \tag{A.9}$$

The domain of the function H is the set

$$\{(x, u, \lambda, t) | x \in X, u \in U(x, t), \lambda \in \mathbb{R}^n, t \in [0, T]\}.$$

The function H is called Hamiltonian function and it plays an important role in the Maximum Principle of Pontryagin. The function λ, associated with the state variable x, is called the adjoint function. Let's also define the maximized Hamiltonian $H^* : X \times \mathbb{R}^n \times [0, T] \mapsto \mathbb{R}$

$$H^*(x, \lambda, t) = \max\{H(x, u, \lambda, t) | u \in U(x, t)\}. \tag{A.10}$$

Before stating the necessary conditions for the optimality of a solution, we introduce the following theorem which instead gives us sufficient conditions for an optimal solution [12, 13, 29, 101].

Theorem A.2.1. *We consider the problem expressed by equations*

$$\dot{x}(t) = f(x(t), u(t), t) \tag{A.11}$$

and the boundary condition

$$x(0) = x_0 \in X, \tag{A.12}$$

where $u(t) = (u_1(t), u_2(t), \ldots, u_m(t)) \in \mathbb{R}^m$ is the vector of actions chosen by the player at time t. The set of eligible actions at time t, given the state of the system x, is given by $U(x, t) \in \mathbb{R}^m$. The constant x_0 is known and the function f is defined on $\Omega = \{(x, u, t) | x \in X, u \in U(x, t), t \in [0, T]\}$ and taking values in $\mathbb{R}^n$, hence $f(x, u, t)$ is an n-dimensional vector. The player objective is to choose the control $u : [0, T] \mapsto \mathbb{R}^m$ that maximize its objective functional

$$J(u(\cdot)) = \int_0^T e^{-rt} F(x(t), u(t), t) dt + e^{-rT} S(x(T)) \tag{A.13}$$

where $r \geq 0$ is the discount rate, $F(x,u,t)$ the instantaneous utility due to the choice of the control, and $S(x(T))$, the final value due to the state $x(T)$.

now we define the Hamiltonian function and maximezed Hamiltonian function. We assume that the state space X is convex and that the scrap function S is of class C^1, and concave. Let $u(\cdot)$ be an admissible control with a corresponding state $x(\cdot)$.

- *If there exists an absolutely continuous function $\lambda : [0,T] \mapsto \mathbb{R}^n$ such that the maximality condition*

$$H(x(t), u(t), \lambda(t), t) = H^*(x(t), \lambda(t), t), \tag{A.14}$$

the adjoint equation

$$\dot{\lambda}(t) = r\lambda(t) - H_x^*(x(t), \lambda(t), t), \tag{A.15}$$

and the transversality conditions

$$\lambda(T) = S'(x(T)) \tag{A.16}$$

are fulfilled, and such that the function $x \mapsto H^*(x, \lambda(t), t)$ is concave and differentiable with respect to the state variable x for all $t \in [0,T]$, then $u(\cdot)$ is an optimal control.

- *If the set of admissible controls, $U(x,t)$ does not depend on x, the result remains true unless you replace the equation* (A.15) *with*

$$\dot{\lambda}(t) = r\lambda(t) - H_x(x(t), u(t), \lambda(t), t). \tag{A.17}$$

If the scrap function $S(x(t)) = 0$ and the discount rate $r = 0$, it gets a problem in the Lagrange form

$$\max J(u) = \int_0^T F(x(t), u(t), t)dt, \tag{A.18}$$

$$\begin{aligned} \text{soggetto a} \quad \dot{x}(t) &= f(x(t), u(t), t), \\ x(t_0) &= x_0, \\ x_i(T) &= x_i^T, \quad i = 1, \ldots, l, \\ x_i(T) &\geq x_i^T, \quad i = l+1, \ldots, r, \\ x_i(T) &\in \mathbb{R}, \quad i = r+1, \ldots, n, \\ u(t) &\in U(x,t) \in \mathbb{R}^m. \end{aligned} \tag{A.19}$$

We define the Hamiltonian functions as follows

$$H(x,u,\lambda,t) = \lambda_0 F(x,u,t) + \sum_{i=1}^{n} \lambda_i(t) f_i(x,u,t), \tag{A.20}$$

and we state the following necessary conditions for the optimality of a solution.

Theorem A.2.2. *Let $u^*(t)$ be an optimal control continuous piecewise, defined on $[0,T]$, which is associated to the state function $x^*(t)$. Then there exists a constant $\lambda_0 \in \mathbb{R}$ and a continuous function of class C^1 piecewise $\lambda : [0,T] \mapsto \mathbb{R}^n$ such that for all $t \in [0,T]$, are valid the following conditions:*

(i) $(\lambda_0, \lambda(t)) \neq 0 \in \mathbb{R}^{n+1}$;

(ii) $u^*(t)$ *maximize* $H(x^*(t), u, \lambda(t), t)$ *for* $u \in U(x,t)$;

(iii) *except for the t where $u^*(t)$ is discontinuous,* $\dot{p}(t) = -\frac{\partial H(x^*(t),u^*(t),\lambda(t),t)}{\partial x}$;

(iv) $\lambda_0 \in \{0,1\}$;

(v) $\lambda_i(T) \in \mathbb{R}, \quad i = 1,\ldots,l,$
$\lambda_i(T) \geq 0$ *e* $\lambda_i(T)(x_i(T) - x_i^T) = 0, \quad i = l+1,\ldots,r,$
$\lambda_i(T) = 0, \quad i = r+1,\ldots,n.$

Appendix B

Parameters

The sets of parameter values we presented in the numerical Section 3.2 are very similar to the ones most commonly used in the related literature (see Section 1.4). We have modified only the death rate of cells infected by viruses V_s or V_r due to immune response (i_s and i_r respectively). We deliberately kept these parameters close to zero, in order to obtain the two instances concave and not concave. The two instances mainly differ from each other for the drugs toxicity parameters, again to consider concave and not concave properties.

Table B.1 shows the values we assigned to drugs toxicity and mutation costs for both instances. Table B.2 shows all common parameter values for both instances. Finally Table B.3 contains boundary conditions.

Table B.1: Weights

Symbol	Description	1^{st} instance	2^{nd} instance
A_{co}	Coreceptor antagonists toxicity	10^6	10
A_{fi}	Fusion inhibitors toxicity	10^6	10
A_{ib}	Immune boosting toxicity	1	1
A_{ii}	Integrase inhibitors toxicity	10^2	10
A_{pi}	Protease inhibitors toxicity	2×10^2	10
A_{rt}	Reverse transcriptase inhibitors toxicity	10^6	10
$B_{s,r}$	Mutation costs from sensitive to resistant viruses	10^{-10}	10^{-10}
$B_{r,s}$	Mutation costs from resistant to sensitive viruses	10^{-11}	10^{-11}

Table B.2: Common parameters at both instances

Symbol	Description	Value
b_2	Half saturation constant	$1\ mm^3$
c	Cytotoxic T Lymphocyte (CTL) activation rate	$0.03\ \frac{1}{day}$
g	Input rate of external viral source	$25\ \frac{cells}{mm^3 \times day}$
i_s	Death rate of cells infected by V_s due to immune response	$10^{-308}\ \frac{1}{day}$
i_r	Death rate of cells infected by V_r due to immune response	$10^{-308}\ \frac{1}{day}$
k	Rate T_h convert to specific immune reaction cells (T_s or T_r)	$10^{-6}\ \frac{1}{day}$
k'	Rate T_h convert to unspecific immune reaction cells (T_g)	$10^{-6}\ \frac{1}{day}$
$k_{1,s}$	Rate CD4 T cells becomes infected by V_s	$0.0003\ \frac{mm^3}{virions \times day}$
$k_{1,r}$	Rate CD4 T cells becomes infected by V_r	$0.00005\ \frac{mm^3}{virions \times day}$
$k_{1,s}^M$	Rate macrophages becomes infected by V_s	$0.00467\ \frac{mm^3}{virions \times day}$
$k_{1,r}^M$	Rate macrophages becomes infected by V_r	$0.001\ \frac{mm^3}{virions \times day}$
k_2	Rate latently infected cells convert to actively infected cells	$0.003\ \frac{1}{day}$
k_3	Rate infected macrophages infects CD4+ T cells	$10^{-6}\ \frac{mm^3}{cells \times day}$
μ_{T_h}	Death rate of uninfected CD4+ T_h cells	$0.02\ \frac{1}{day}$
μ_{T_s}	Death rate of uninfected CD4+ T_s cells	$0.02\ \frac{1}{day}$
μ_{T_r}	Death rate of uninfected CD4+ T_r cells	$0.02\ \frac{1}{day}$
μ_{T_g}	Death rate of uninfected CD4+ T_g cells	$0.02\ \frac{1}{day}$
μ_{M_h}	Death rate of uninfected M_h cells	$0.005\ \frac{1}{day}$
μ_{I^p}	Death rate of uninfected I^p cells	$0.001\ \frac{1}{day}$
μ_{I^e}	Death rate of uninfected I^e cells	$0.1\ \frac{1}{day}$
$\mu_{T_{i,s}^l}$	Death rate of latently infected $T_{i,s}^l$ cells	$0.1\ \frac{1}{day}$
$\mu_{T_{i,s}^a}$	Death rate of actively infected $T_{i,s}^a$ cells	$0.24\ \frac{1}{day}$
$\mu_{T_{i,r}^l}$	Death rate of latently infected $T_{i,r}^l$ cells	$0.2\ \frac{1}{day}$
$\mu_{T_{i,r}^a}$	Death rate of actively infected $T_{i,r}^a$ cells	$0.5\ \frac{1}{day}$
$\mu_{M_{h,s}^a}$	Death rate of actively infected $M_{h,s}^a$ cells	$0.04\ \frac{1}{day}$
$\mu_{M_{h,r}^a}$	Death rate of actively infected $M_{h,r}^a$ cells	$0.07\ \frac{1}{day}$
μ_{V_s}	Death rate of sensitive viruses V_s	$2.4\ \frac{1}{day}$
μ_{V_r}	Death rate of resistant viruses V_r	$5\ \frac{1}{day}$
θ_i	Scaling parameter for type i cells	1
π_s	Growth rate of sensitive viruses (V_s)	$8\ \frac{1}{day}$
π_r	Growth rate of resistant viruses (V_r)	$5\ \frac{1}{day}$
p'_s	Specific immune response rate against V_s	$0.020\ \frac{cells}{mm^3 \times day}$
p'_r	Specific immune response rate against V_r	$0.001\ \frac{cells}{mm^3 \times day}$
q	Growth rate of I^e due to infected cells and I^p	$0.5\ \frac{1}{day}$
Q_{T_h,V_s}	Percentage of T_h cells that recognize the virus V_s	0
Q_{T_h,V_r}	Percentage of T_h cells that recognize the virus V_r	0
Q_{T_s,V_s}	Percentage of T_s cells that recognize the virus V_s	0.90
Q_{T_s,V_r}	Percentage of T_s cells that recognize the virus V_r	0
Q_{T_r,V_s}	Percentage of T_r cells that recognize the virus V_s	0
Q_{T_r,V_r}	Percentage of T_r cells that recognize the virus V_r	0.90
Q_{T_g,V_s}	Percentage of T_g cells that recognize the virus V_s	0
Q_{T_g,V_r}	Percentage of T_g cells that recognize the virus V_r	0
r	rate of growth for CD4+ T Helper cell population	$0.03\ \frac{1}{day}$
ρ_s	Average number of virions V_s infecting a cell	$1\ \frac{virions}{cell}$
ρ_r	Average number of virions V_r infecting a cell	$1\ \frac{virions}{cell}$
s_{1,T_h}	Source/production T_h cells	$1\ \frac{cells}{mm^3 \times day}$
s_{1,M_h}	Source/production of macrophages	$1\ \frac{cells}{mm^3 \times day}$
s'_s	Unspecific immune response rate against V_s	$9.7\ \frac{cells}{mm^3 \times day}$
s'_r	Unspecific immune response rate against V_r	$0.0009\ \frac{cells}{mm^3 \times day}$
T_{max}	Maximum CD4+ T Helper cell population level	$1700\ \frac{cells}{mm^3}$

Table B.3: Common boundary conditions at both instances

Symbol	Description	Value $\left(\frac{cells}{mm^3}\right)$
I_0^e	Initial value of I^e cells	10
I_0^p	Initial value of I^p cells	10
M_h^0	Initial value of M_h cells	1
T_h^0	Initial value of T_h cells	900
T_s^0	Initial value of T_s cells	100
T_r^0	Initial value of T_r cells	100
T_g^0	Initial value of T_g cells	1
$T_{i,s}^{l,0}$	Initial value of latently T_i cells infected by viruses V_s	10
$T_{i,s}^{a,0}$	Initial value of actively T_i cells infected by viruses V_s	0
$T_{i,r}^{l,0}$	Initial value of latently T_i cells infected by viruses V_s	10
$T_{i,r}^{a,0}$	Initial value of actively T_i cells infected by viruses V_r	0
$M_{h,s}^{a,0}$	Initial value of actively M_h cells infected by viruses V_s	10
$M_{h,r}^{a,0}$	Initial value of actively M_h cells infected by viruses V_r	10
V_s^0	Initial value of viruses V_s	500
V_r^0	Initial value of viruses V_r	300

Appendix C

The algorithms

C.1 Mathemathica functions for symbolic calculations

In this section we present the `Mathematica` code needed for symbolic calculations.
The last part named "*NDSolve*" shows the statements required to solve the system of 81 differential equations of first order nonlinear mentioned in Chapter 3. Unfortunately, this instruction does not lead to any result, hence the need to intervene with `Matlab` for the numerical computations.

campi vettoriali

X={x1, x2, x3, x4, x5, x6, x7, x8, x9, x10, x11, x12, x13, x14, x15, x16, x17, x18, x19, x20, x21, x22, x23, x24, x25, x26, x27};

XT[t_]:={x1[t], x2[t], x3[t], x4[t], x5[t], x6[t], x7[t], x8[t], x9[t], x10[t], x11[t], x12[t], x13[t], x14[t], x15[t], x16[t], x17[t], x18[t], x19[t], x20[t], x21[t], x22[t], x23[t], x24[t], x25[t], x26[t], x27[t]};

U1={u11, u12, u13, u14, u15, u16};

U1T[t_]:={u11[t], u12[t], u13[t], u14[t], u15[t], u16[t]};

U2={u21, u22};

U2T[t_]:={u21[t], u22[t]};

L1={l11, l12, l13, l14, l15, l16, l17, l18, l19, l110, l111, l112, l113, l114, l115, l116, l117, l118, l119, l120, l121, l122, l123, l124, l125, l126, l127};

L1T[t_]:={l11[t], l12[t], l13[t], l14[t], l15[t], l16[t], l17[t], l18[t], l19[t], l110[t], l111[t], l112[t], l113[t], l114[t], l115[t], l116[t], l117[t], l118[t], l119[t], l120[t], l121[t], l122[t], l123[t], l124[t], l125[t], l126[t], l127[t]};

L2={l21, l22, l23, l24, l25, l26, l27, l28, l29, l210, l211, l212, l213, l214, l215, l216, l217, l218, l219, l220, l221, l222, l223, l224, l225, l226, l227};

L2T[t_]:={l21[t], l22[t], l23[t], l24[t], l25[t], l26[t], l27[t], l28[t], l29[t], l210[t], l211[t], l212[t], l213[t], l214[t], l215[t], l216[t], l217[t], l218[t], l219[t], l220[t], l221[t], l222[t], l223[t], l224[t], l225[t], l226[t], l227[t]};

T[{th_, ts_, tr_, tg_, mh_, ip_, ie_, thsl_, thsa_, thrl_, thra_, tssl_, tssa_, tsrl_, tsra_, trsl_, trsa_, trrl_, trra_, tgsl_, tgsa_, tgrl_, tgra_, mhsa_, mhra_, vs_, vr_}, {uco_, ufi_,uib_,uii_,upi_,urt_}, {ussv_, urrv_}]:=(s1th*oth)/(oth+vs+vr)+

```
r*th*(1-(th+ts+tr+tg+thsl+thrl+tssl+tsrl+trsl+
trrl+tgsl+tgrl+thsa+thra+tssa+tsra+trsa+trra+tgsa+tgra)/(tmax))+uib*th-muth*th-k1s*(1-
ufi)*(1-uco)*(1-qthvs)*vs*th-k1r*(1-qthvr)*vr*th-k*(vs+vr)*th-kprimo*(vs+vr)*th-
k3*(mhsa+mhra)*th;
TS[{th_, ts_, tr_, tg_, mh_, ip_, ie_, thsl_, thsa_, thrl_, thra_, tssl_, tssa_, tsrl_, tsra_, trsl_,
trsa_, trrl_, trra_, tgsl_, tgsa_, tgrl_, tgra_, mhsa_, mhra_, vs_, vr_}, {uco_,
ufi_,uib_,uii_,upi_,urt_}, {ussv_, urrv_}]:=k*vs*th+uib*ts-muts*ts-k1s*(1-ufi)*(1-uco)*(1-
qtsvs)*vs*ts-k1r*(1-qtsvr)*vr*ts-k3*(mhsa+mhra)*ts;
TR[{th_, ts_, tr_, tg_, mh_, ip_, ie_, thsl_, thsa_, thrl_, thra_, tssl_, tssa_, tsrl_, tsra_, trsl_,
trsa_, trrl_, trra_, tgsl_, tgsa_, tgrl_, tgra_, mhsa_, mhra_, vs_, vr_}, {uco_,
ufi_,uib_,uii_,upi_,urt_}, {ussv_, urrv_}]:=k*vr*th+uib*tr-mutr*tr-k1s*(1-ufi)*(1-uco)*(1-
qtrvs)*vs*tr-k1r*(1-qtrvr)*vr*tr-k3*(mhsa+mhra)*tr;
TG[{th_, ts_, tr_, tg_, mh_, ip_, ie_, thsl_, thsa_, thrl_, thra_, tssl_, tssa_, tsrl_, tsra_, trsl_,
trsa_, trrl_, trra_, tgsl_, tgsa_, tgrl_, tgra_, mhsa_, mhra_, vs_, vr_}, {uco_,
ufi_,uib_,uii_,upi_,urt_}, {ussv_, urrv_}]:=kprimo*(vs+vr)*th+uib*tg-mutg*tg-k1s*(1-
ufi)*(1-uco)*(1-qtgvs)*vs*tg-k1r*(1-qtgvr)*vr*tg-k3*(mhsa+mhra)*tg;
MH[{th_, ts_, tr_, tg_, mh_, ip_, ie_, thsl_, thsa_, thrl_, thra_, tssl_, tssa_, tsrl_, tsra_, trsl_,
trsa_, trrl_, trra_, tgsl_, tgsa_, tgrl_, tgra_, mhsa_, mhra_, vs_, vr_}, {uco_,
ufi_,uib_,uii_,upi_,urt_}, {ussv_, urrv_}]:=s1mh-mumh*mh-k1sm*(1-urt)*(1-upi)*vs*mh-
k1rm*vr*mh;
IP[{th_, ts_, tr_, tg_, mh_, ip_, ie_, thsl_, thsa_, thrl_, thra_, tssl_, tssa_, tsrl_, tsra_, trsl_,
trsa_, trrl_, trra_, tgsl_, tgsa_, tgrl_, tgra_, mhsa_, mhra_, vs_, vr_}, {uco_,
ufi_,uib_,uii_,upi_,urt_}, {ussv_,
```

urrv_}]:=c*ip*(th+ts+tr+tg)*(thsa+thra+tssa+tsra+trsa+trra+tgsa+tgra)-

c*q*ip*(thsa+thra+tssa+tsra+trsa+trra+tgsa+tgra)-muip*ip;

IE[{th_, ts_, tr_, tg_, mh_, ip_, ie_, thsl_, thsa_, thrl_, thra_, tssl_, tssa_, tsrl_, tsra_, trsl_,

trsa_, trrl_, trra_, tgsl_, tgsa_, tgrl_, tgra_, mhsa_, mhra_, vs_, vr_}, {uco_,

ufi_,uib_,uii_,upi_,urt_}, {ussv_, urrv_}]:=c*q*ip*(thsa+thra+tssa+tsra+trsa+trra+

tgsa+tgra)-muie*ie;

THSL[{th_, ts_, tr_, tg_, mh_, ip_, ie_, thsl_, thsa_, thrl_, thra_, tssl_, tssa_, tsrl_, tsra_, trsl_,

trsa_, trrl_, trra_, tgsl_, tgsa_, tgrl_, tgra_, mhsa_, mhra_, vs_, vr_}, {uco_,

ufi_,uib_,uii_,upi_,urt_}, {ussv_, urrv_}]:=k1s*(1-ufi)*(1-uco)*(1-

qthvs)*vs*th+k3*mhsa*th-muthsl*thsl-k2*(1-urt)*(1-uii)*thsl;

THSA[{th_, ts_, tr_, tg_, mh_, ip_, ie_, thsl_, thsa_, thrl_, thra_, tssl_, tssa_, tsrl_, tsra_, trsl_,

trsa_, trrl_, trra_, tgsl_, tgsa_, tgrl_, tgra_, mhsa_, mhra_, vs_, vr_}, {uco_,

ufi_,uib_,uii_,upi_,urt_}, {ussv_, urrv_}]:=k2*(1-urt)*(1-uii)*thsl-muthsa*thsa-is*ie*thsa;

THRL[{th_, ts_, tr_, tg_, mh_, ip_, ie_, thsl_, thsa_, thrl_, thra_, tssl_, tssa_, tsrl_, tsra_, trsl_,

trsa_, trrl_, trra_, tgsl_, tgsa_, tgrl_, tgra_, mhsa_, mhra_, vs_, vr_}, {uco_,

ufi_,uib_,uii_,upi_,urt_}, {ussv_, urrv_}]:=k1r*(1-qthvr)*vr*th+k3*mhra*th-muthrl*thrl-

k2*thrl;

THRA[{th_, ts_, tr_, tg_, mh_, ip_, ie_, thsl_, thsa_, thrl_, thra_, tssl_, tssa_, tsrl_, tsra_, trsl_,

trsa_, trrl_, trra_, tgsl_, tgsa_, tgrl_, tgra_, mhsa_, mhra_, vs_, vr_}, {uco_,

ufi_,uib_,uii_,upi_,urt_}, {ussv_, urrv_}]:=k2*thrl-muthra*thra-ir*ie*thra;

TSSL[{th_, ts_, tr_, tg_, mh_, ip_, ie_, thsl_, thsa_, thrl_, thra_, tssl_, tssa_, tsrl_, tsra_, trsl_,

trsa_, trrl_, trra_, tgsl_, tgsa_, tgrl_, tgra_, mhsa_, mhra_, vs_, vr_}, {uco_,

```
ufi_,uib_,uii_,upi_,urt_}, {ussv_, urrv_}]:=k1s*(1-ufi)*(1-uco)*(1-qtsvs)*vs*ts+k3*mhsa*ts-
mutssl*tssl-k2*(1-urt)*(1-uii)*tssl;
TSSA[{th_, ts_, tr_, tg_, mh_, ip_, ie_, thsl_, thsa_, thrl_, thra_, tssl_, tssa_, tsrl_, tsra_, trsl_,
trsa_, trrl_, trra_, tgsl_, tgsa_, tgrl_, tgra_, mhsa_, mhra_, vs_, vr_}, {uco_,
ufi_,uib_,uii_,upi_,urt_}, {ussv_, urrv_}]:=k2*(1-urt)*(1-uii)*tssl-mutssa*tssa-is*ie*tssa;
TSRL[{th_, ts_, tr_, tg_, mh_, ip_, ie_, thsl_, thsa_, thrl_, thra_, tssl_, tssa_, tsrl_, tsra_, trsl_,
trsa_, trrl_, trra_, tgsl_, tgsa_, tgrl_, tgra_, mhsa_, mhra_, vs_, vr_}, {uco_,
ufi_,uib_,uii_,upi_,urt_}, {ussv_, urrv_}]:=k1r*(1-qtsvr)*vr*ts+k3*mhra*ts-mutsrl*tsrl-
k2*tsrl;
TSRA[{th_, ts_, tr_, tg_, mh_, ip_, ie_, thsl_, thsa_, thrl_, thra_, tssl_, tssa_, tsrl_, tsra_, trsl_,
trsa_, trrl_, trra_, tgsl_, tgsa_, tgrl_, tgra_, mhsa_, mhra_, vs_, vr_}, {uco_,
ufi_,uib_,uii_,upi_,urt_}, {ussv_, urrv_}]:=k2*tsrl-mutsra*tsra-ir*ie*tsra;
TRSL[{th_, ts_, tr_, tg_, mh_, ip_, ie_, thsl_, thsa_, thrl_, thra_, tssl_, tssa_, tsrl_, tsra_, trsl_,
trsa_, trrl_, trra_, tgsl_, tgsa_, tgrl_, tgra_, mhsa_, mhra_, vs_, vr_}, {uco_,
ufi_,uib_,uii_,upi_,urt_}, {ussv_, urrv_}]:=k1s*(1-ufi)*(1-uco)*(1-qtrvs)*vs*tr+k3*mhsa*tr-
mutrsl*trsl-k2*(1-urt)*(1-uii)*trsl;
TRSA[{th_, ts_, tr_, tg_, mh_, ip_, ie_, thsl_, thsa_, thrl_, thra_, tssl_, tssa_, tsrl_, tsra_, trsl_,
trsa_, trrl_, trra_, tgsl_, tgsa_, tgrl_, tgra_, mhsa_, mhra_, vs_, vr_}, {uco_,
ufi_,uib_,uii_,upi_,urt_}, {ussv_, urrv_}]:=k2*(1-urt)*(1-uii)*trsl-mutrsa*trsa-is*ie*trsa;
TRRL[{th_, ts_, tr_, tg_, mh_, ip_, ie_, thsl_, thsa_, thrl_, thra_, tssl_, tssa_, tsrl_, tsra_, trsl_,
trsa_, trrl_, trra_, tgsl_, tgsa_, tgrl_, tgra_, mhsa_, mhra_, vs_, vr_}, {uco_,
ufi_,uib_,uii_,upi_,urt_}, {ussv_, urrv_}]:=k1r*(1-qtrvr)*vr*tr+k3*mhra*tr-mutrrl*trrl-
k2*trrl;
```

TRRA[{th_, ts_, tr_, tg_, mh_, ip_, ie_, thsl_, thsa_, thrl_, thra_, tssl_, tssa_, tsrl_, tsra_, trsl_, trsa_, trrl_, trra_, tgsl_, tgsa_, tgrl_, tgra_, mhsa_, mhra_, vs_, vr_}, {uco_, ufi_,uib_,uii_,upi_,urt_}, {ussv_, urrv_}]:=k2*trrl-mutrra*trra-ir*ie*trra;

TGSL[{th_, ts_, tr_, tg_, mh_, ip_, ie_, thsl_, thsa_, thrl_, thra_, tssl_, tssa_, tsrl_, tsra_, trsl_, trsa_, trrl_, trra_, tgsl_, tgsa_, tgrl_, tgra_, mhsa_, mhra_, vs_, vr_}, {uco_, ufi_,uib_,uii_,upi_,urt_}, {ussv_, urrv_}]:=k1s*(1-ufi)*(1-uco)*(1-qtgvs)*vs*tg+k3*mhsa*tg-mutgsl*tgsl-k2*(1-urt)*(1-uii)*tgsl;

TGSA[{th_, ts_, tr_, tg_, mh_, ip_, ie_, thsl_, thsa_, thrl_, thra_, tssl_, tssa_, tsrl_, tsra_, trsl_, trsa_, trrl_, trra_, tgsl_, tgsa_, tgrl_, tgra_, mhsa_, mhra_, vs_, vr_}, {uco_, ufi_,uib_,uii_,upi_,urt_}, {ussv_, urrv_}]:=k2*(1-urt)*(1-uii)*tgsl-mutgsa*tgsa-is*ie*tgsa;

TGRL[{th_, ts_, tr_, tg_, mh_, ip_, ie_, thsl_, thsa_, thrl_, thra_, tssl_, tssa_, tsrl_, tsra_, trsl_, trsa_, trrl_, trra_, tgsl_, tgsa_, tgrl_, tgra_, mhsa_, mhra_, vs_, vr_}, {uco_, ufi_,uib_,uii_,upi_,urt_}, {ussv_, urrv_}]:=k1r*(1-qtgvr)*vr*tg+k3*mhra*tg-mutgrl*tgrl-k2*tgrl;

TGRA[{th_, ts_, tr_, tg_, mh_, ip_, ie_, thsl_, thsa_, thrl_, thra_, tssl_, tssa_, tsrl_, tsra_, trsl_, trsa_, trrl_, trra_, tgsl_, tgsa_, tgrl_, tgra_, mhsa_, mhra_, vs_, vr_}, {uco_, ufi_,uib_,uii_,upi_,urt_}, {ussv_, urrv_}]:=k2*tgrl-mutgra*tgra-ir*ie*tgra;

MHSA[{th_, ts_, tr_, tg_, mh_, ip_, ie_, thsl_, thsa_, thrl_, thra_, tssl_, tssa_, tsrl_, tsra_, trsl_, trsa_, trrl_, trra_, tgsl_, tgsa_, tgrl_, tgra_, mhsa_, mhra_, vs_, vr_}, {uco_, ufi_,uib_,uii_,upi_,urt_}, {ussv_, urrv_}]:=k1sm*(1-urt)*(1-upi)*vs*mh-mumhsa*mhsa;

MHRA[{th_, ts_, tr_, tg_, mh_, ip_, ie_, thsl_, thsa_, thrl_, thra_, tssl_, tssa_, tsrl_, tsra_, trsl_, trsa_, trrl_, trra_, tgsl_, tgsa_, tgrl_, tgra_, mhsa_, mhra_, vs_, vr_}, {uco_, ufi_,uib_,uii_,upi_,urt_}, {ussv_, urrv_}]:=k1rm*vr*mh-mumhra*mhra;

```
VS[{th_, ts_, tr_, tg_, mh_, ip_, ie_, thsl_, thsa_, thrl_, thra_, tssl_, tssa_, tsrl_, tsra_, trsl_,
trsa_, trrl_, trra_, tgsl_, tgsa_, tgrl_, tgra_, mhsa_, mhra_, vs_, vr_}, {uco_,
ufi_,uib_,uii_,upi_,urt_}, {ussv_, urrv_}]:=(g*(1-upi)*vs)/(b2+vs)+ps*(1-
upi)*ussv*(mhsa+thsa+tssa+trsa+tgsa)+pr*(1-upi)*(1-urrv)*(mhra+thra+tsra+trra+tgra)-
k1s*rhos*(1-ufi)*(1-uco)*(th*(1-qthvs)+ ts*(1-qtsvs)+tr*(1-qtrvs)+tg*(1-qtgvs))*vs-
k1sm*rhos*(1-urt)*(1-upi)*mh*vs-muvs*vs-(ssprimo*tg+psprimo*ts)*vs;
VR[{th_, ts_, tr_, tg_, mh_, ip_, ie_, thsl_, thsa_, thrl_, thra_, tssl_, tssa_, tsrl_, tsra_, trsl_,
trsa_, trrl_, trra_, tgsl_, tgsa_, tgrl_, tgra_, mhsa_, mhra_, vs_, vr_}, {uco_,
ufi_,uib_,uii_,upi_,urt_}, {ussv_,
urrv_}]:=(g*vr)/(b2+vr)+pr*urrv*(mhra+thra+tsra+trra+tgra)+ps*(1-
ussv)*(mhsa+thsa+tssa+trsa+tgsa)-k1r*rhor*(th*(1-qthvr)+ts*(1-qtsvr)+tr*(1-qtrvr)+tg*(1-
qtgvr))*vr-k1rm*rhor*mh*vr-muvr*vr-(srprimo*tg+prprimo*tr)*vr;

JT[{th_, ts_, tr_, tg_, mh_, ip_, ie_, thsl_, thsa_, thrl_, thra_, tssl_, tssa_, tsrl_, tsra_, trsl_,
trsa_, trrl_, trra_, tgsl_, tgsa_, tgrl_, tgra_, mhsa_, mhra_, vs_, vr_}, {uco_,
ufi_,uib_,uii_,upi_,urt_}, {ussv_, urrv_}]:=mh+ip+ie+th+ts+tr+tg-
0.5*(aco*uco^2+afi*ufi^2+aib*uib^2+aii*uii^2+api*upi^2+art*urt^2);
JV[{th_, ts_, tr_, tg_, mh_, ip_, ie_, thsl_, thsa_, thrl_, thra_, tssl_, tssa_, tsrl_, tsra_, trsl_,
trsa_, trrl_, trra_, tgsl_, tgsa_, tgrl_, tgra_, mhsa_, mhra_, vs_, vr_}, {uco_,
ufi_,uib_,uii_,upi_,urt_}, {ussv_,
urrv_}]:=vs+vr+thsl+thsa+thrl+thra+tssl+tssa+tsrl+tsra+trsl+trsa+trrl+trra+tgsl+tgsa+tgrl+tgr
a-0.5*(bsr*(1-ussv)^2+brs*(1-urrv)^2);
```

```
HT[X_,U1_,U2_, {l1_, l2_, l3_, l4_, l5_, l6_, l7_, l8_, l9_, l10_, l11_, l12_, l13_, l14_, l15_,
l16_, l17_, l18_, l19_, l20_, l21_, l22_, l23_, l24_, l25_, l26_, l27_}]:=l01 JT[X, U1, U2] + l1
T[X, U1, U2] + l2 TS[X, U1, U2]+ l3 TR[X, U1, U2] + l4 TG[X, U1, U2] + l5 MH[X, U1,
U2] + l6 IP[X, U1, U2] + l7 IE[X, U1, U2] + l8 THSL[X, U1, U2] +  l9 THSA[X, U1, U2] +
l10 THRL[X, U1, U2] +  l11 THRA[X, U1, U2] +  l12 TSSL[X, U1, U2] +  l13 TSSA[X, U1,
U2] +  l14 TSRL[X, U1, U2] +  l15 TSRA[X, U1, U2] + l16 TRSL[X, U1, U2] +  l17
TRSA[X, U1, U2] +  l18 TRRL[X, U1, U2] +  l19 TRRA[X, U1, U2] +   l20 TGSL[X, U1,
U2] +  l21 TGSA[X, U1, U2] +  l22 TGRL[X, U1, U2] +  l23 TGRA[X, U1, U2] +
   l24 MHSA[X, U1, U2] +  l25 MHRA[X, U1, U2] +
   l26 VS[X, U1, U2] + l27 VR[X, U1, U2] ;
HV[X_,U1_,U2_, {l1_, l2_, l3_, l4_, l5_, l6_, l7_, l8_, l9_, l10_, l11_, l12_, l13_, l14_, l15_,
l16_, l17_, l18_, l19_, l20_, l21_, l22_, l23_, l24_, l25_, l26_, l27_}]:=l02 JV[X, U1, U2] +
l1 T[X, U1, U2] + l2 TS[X, U1, U2]+ l3 TR[X, U1, U2] + l4 TG[X, U1, U2] + l5 MH[X, U1,
U2] + l6 IP[X, U1, U2] + l7 IE[X, U1, U2] + l8 THSL[X, U1, U2] +  l9 THSA[X, U1, U2] +
l10 THRL[X, U1, U2] +  l11 THRA[X, U1, U2] +  l12 TSSL[X, U1, U2] +  l13 TSSA[X, U1,
U2] +  l14 TSRL[X, U1, U2] +  l15 TSRA[X, U1, U2] + l16 TRSL[X, U1, U2] +  l17
TRSA[X, U1, U2] +  l18 TRRL[X, U1, U2] +  l19 TRRA[X, U1, U2] +   l20 TGSL[X, U1,
U2] +  l21 TGSA[X, U1, U2] +  l22 TGRL[X, U1, U2] +  l23 TGRA[X, U1, U2] +
   l24 MHSA[X, U1, U2] +  l25 MHRA[X, U1, U2] +
   l26 VS[X, U1, U2] + l27 VR[X, U1, U2] ;
l01=1; l02=1;
EQMAX1[X_, U1_,U2_, L1_]:=Table[D[HT[X, U1, U2, L1],u],{u,U1}]
EQMAX2[X_, U1_,U2_, L1_]:=Table[D[HV[X, U1, U2, L1],u],{u,U2}]
```

hessiano simbolico

```
HESS[H_, U_]:=Flatten[Map[Thread,{D[H,{U},{U}]}],1];

hevcontrolli=HESS[HV[XT[t],U1T[s],U2T[s],L2T[t]],U2T[s]]//MatrixForm;

hesicontrolli=HESS[HT[XT[t],U1T[s],U2T[s],L1T[t]],U1T[s]]//FullSimplify;

H2terapia=Take[hesicontrolli,2,2];

DET2=Det[Take[hesicontrolli,2,2]]//FullSimplify;
```

il cui determinante è:H_2= aco afi - Subscript[a, 12]^2 che è >0 se e solo se Subscript[a, 12]^2<aco afi cioè o $a_{12}< -\sqrt{aco\,afi}$ o $a_{12}>\sqrt{aco\,afi}$.

```
H3terapia=Take[hesicontrolli,3,3];
```

il cui determinante è: H_3= - aib H_2 che è <0 se e solo se H_2 >0 e cioè se e solo se Subscript[a, 12]^2<aco afi cioè o $a_{12}< -\sqrt{aco\,afi}$ o $a_{12}>\sqrt{aco\,afi}$.

```
H4terapia=Take[hesicontrolli,4,4];
```

il cui determinante è H_4= - aii H_3= aii aib H_2 che è >0 se e solo se H_2 >0 e cioè se e solo se Subscript[a, 12]^2<aco afi cioè o $a_{12}< -\sqrt{aco\,afi}$ o $a_{12}>\sqrt{aco\,afi}$.

```
H5terapia=Take[hesicontrolli,5,5];
```

il cui determinante è H_5= - api H_4= - api aii aib H_2 che è <0 se e solo se H_2 >0 e cioè se e solo se Subscript[a, 12]^2<aco afi cioè o $a_{12}< -\sqrt{aco\,afi}$ o $a_{12}>\sqrt{aco\,afi}$.

```
H6terapia=hesicontrolli;
```

il cui determinante è H_6= aib H_2 [art api aii - Subscript[a, 56]^{2}aii - Subscript[a, 46]^2 api] che è >0 se e solo se {H_2 >0 e [art api aii - Subscript[a, 56]^{2}aii - Subscript[a, 46]^2 api]>0} e cioè se e solo se {Subscript[a, 12]^2<aco afi cioè o $a_{12}< -\sqrt{aco\,afi}$ o $a_{12}>\sqrt{aco\,afi}$ e Subscript[a, 56]^{2}aii + Subscript[a, 46]^2 api < art api aii}.

Riassumendo se accadesse che {H_2 >0 e [art api aii - Subscript[a, 56]^{2}aii - Subscript[a, 46]^2 api]>0} allora la matrice hessiana relativa ai controlli della terapia sarebbe definita negativa, pertanto l'hamiltoniana sarebbe concava, e quindi le strategie ottime sarebbero:

Subscript[u, i]*=min {1,max {0,u_i^}} per i=1..6

DET6=Det[H6terapia]//FullSimplify;

equazioni del massimo separate per ESS

sistmaxsep1={EQMAX1[XT[t],U1T[s], U2T[s], L1T[t]][[1]]==0,EQMAX1[XT[t],U1T[s], U2T[s], L1T[t]][[2]]==0, EQMAX1[XT[t],U1T[s], U2T[s], L1T[t]][[3]]==0,EQMAX1[XT[t],U1T[s], U2T[s], L1T[t]][[4]]==0,EQMAX1[XT[t],U1T[s], U2T[s], L1T[t]][[5]]==0,EQMAX1[XT[t],U1T[s], U2T[s], L1T[t]][[6]]==0};

sistmaxsep2={EQMAX2[XT[t],U1T[s], U2T[s], L2T[t]][[1]]==0, EQMAX2[XT[t],U1T[s], U2T[s], L2T[t]][[2]]==0 };

Timing[soleqmaxsep1=NSolve[sistmaxsep1, {U1T[s][[1]], U1T[s][[2]], U1T[s][[3]], U1T[s][[4]], U1T[s][[5]], U1T[s][[6]]}]//Simplify];

Timing[soleqmaxsep2=Solve[sistmaxsep2, {U2T[s][[1]], U2T[s][[2]]}]//FullSimplify];

vettsoless={1/aco k1s x26[t] ((1.` -1.` qthvs) l11[t] x1[t]-1.` (1.` -1.` qthvs) l18[t] x1[t]-1.` (1.` -1.` qtsvs) l112[t] x2[t]+(1.` -1.` qtsvs) l12[t] x2[t]-1.` (1.` -1.` qtrvs) l116[t] x3[t]+(1.` -1.` qtrvs) l13[t] x3[t]-1.` (1.` -1.` qtgvs) l120[t] x4[t]+(1.` -1.` qtgvs) l14[t] x4[t]+rhos l126[t] ((1.` -1.` qthvs) x1[t]+(1.` -1.` qtsvs) x2[t]+(1.` -1.` qtrvs) x3[t]+(1.` -1.` qtgvs) x4[t])) (1.` -(1.` k1s x26[t] (-1.` (1.` -1.` qthvs) l11[t] x1[t]+(1.` -1.` qthvs) l18[t] x1[t]+(1.` -1.` qtsvs) l112[t] x2[t]-1.` (1.` -1.` qtsvs) l12[t] x2[t]+(1.` -1.` qtrvs) l116[t] x3[t]-1.` (1.` -1.` qtrvs) l13[t] x3[t]+(1.` -1.` qtgvs) l120[t] x4[t]-1.` (1.` -1.` qtgvs) l14[t] x4[t]-1.` rhos l126[t] ((1.` -

1.` qthvs) x1[t]+(1.` -1.` qtsvs) x2[t]+(1.` -1.` qtrvs) x3[t]+(1.` -1.` qtgvs) x4[t])) (-1.` aco-1.` k1s x26[t] (-1.` (1.` -1.` qthvs) l11[t] x1[t]+(1.` -1.` qthvs) l18[t] x1[t]+(1.` -1.` qtsvs) l112[t] x2[t]-1.` (1.` -1.` qtsvs) l12[t] x2[t]+(1.` -1.` qtrvs) l116[t] x3[t]-1.` (1.` -1.` qtrvs) l13[t] x3[t]+(1.` -1.` qtgvs) l120[t] x4[t]-1.` (1.` -1.` qtgvs) l14[t] x4[t]-1.` rhos l126[t] ((1.` -1.` qthvs) x1[t]+(1.` -1.` qtsvs) x2[t]+(1.` -1.` qtrvs) x3[t]+(1.` -1.` qtgvs) x4[t]))))/(1.` aco afi-1.` $k1s^2$ $x26[t]^2$ (-1.` (1.` -1.` qthvs) l11[t] x1[t]+(1.` -1.` qthvs) l18[t] x1[t]+(1.` -1.` qtsvs) l112[t] x2[t]-1.` (1.` -1.` qtsvs) l12[t] x2[t]+(1.` -1.` qtrvs) l116[t] x3[t]-1.` (1.` -1.` qtrvs) l13[t] x3[t]+(1.` -1.` qtgvs) l120[t] x4[t]-1.` (1.` -1.` qtgvs) l14[t] x4[t]-1.` rhos l126[t] ((1.` -1.` qthvs) x1[t]+(1.` -1.` qtsvs) x2[t]+(1.` -1.` qtrvs) x3[t]+(1.` -1.` qtgvs) $x4[t]))^2$)),-(k1s x26[t] ((1.` -1.` qthvs) l11[t] x1[t]-1.` (1.` -1.` qthvs) l18[t] x1[t]-1.` (1.` -1.` qtsvs) l112[t] x2[t]+(1.` -1.` qtsvs) l12[t] x2[t]-1.` (1.` -1.` qtrvs) l116[t] x3[t]+(1.` -1.` qtrvs) l13[t] x3[t]-1.` (1.` -1.` qtgvs) l120[t] x4[t]+(1.` -1.` qtgvs) l14[t] x4[t]+rhos l126[t] ((1.` -1.` qthvs) x1[t]+(1.` -1.` qtsvs) x2[t]+(1.` -1.` qtrvs) x3[t]+(1.` -1.` qtgvs) x4[t])) (-1.` aco-1.` k1s x26[t] (-1.` (1.` -1.` qthvs) l11[t] x1[t]+(1.` -1.` qthvs) l18[t] x1[t]+(1.` -1.` qtsvs) l112[t] x2[t]-1.` (1.` -1.` qtsvs) l12[t] x2[t]+(1.` -1.` qtrvs) l116[t] x3[t]-1.` (1.` -1.` qtrvs) l13[t] x3[t]+(1.` -1.` qtgvs) l120[t] x4[t]-1.` (1.` -1.` qtgvs) l14[t] x4[t]-1.` rhos l126[t] ((1.` -1.` qthvs) x1[t]+(1.` -1.` qtsvs) x2[t]+(1.` -1.` qtrvs) x3[t]+(1.` -1.` qtgvs) x4[t]))))/(1.` aco afi-1.` $k1s^2$ $x26[t]^2$ (-1.` (1.` -1.` qthvs) l11[t] x1[t]+(1.` -1.` qthvs) l18[t] x1[t]+(1.` -1.` qtsvs) l112[t] x2[t]-1.` (1.` -1.` qtsvs) l12[t] x2[t]+(1.` -1.` qtrvs) l116[t] x3[t]-1.` (1.` -1.` qtrvs) l13[t] x3[t]+(1.` -1.` qtgvs) l120[t] x4[t]-1.` (1.` -1.` qtgvs) l14[t] x4[t]-1.` rhos l126[t] ((1.` -1.` qthvs) x1[t]+(1.` -1.` qtsvs) x2[t]+(1.` -1.` qtrvs) x3[t]+(1.` -1.` qtgvs) $x4[t]))^2$),(1.` (l11[t] x1[t]+l12[t] x2[t]+l13[t] x3[t]+l14[t] x4[t]))/aib,1/aii (1.` k2 (l112[t] x12[t]-1.` l113[t] x12[t]+l116[t] x16[t]-1.` l117[t] x16[t]+l120[t] x20[t]-1.` l121[t] x20[t]+l18[t] x8[t]-1.` l19[t] x8[t])-(1.` k2 (-1.` l112[t]

x12[t]+l113[t] x12[t]-1.` l116[t] x16[t]+l117[t] x16[t]-1.` l120[t] x20[t]+l121[t] x20[t]-1.`
l18[t] x8[t]+l19[t] x8[t]) (1.` api k2$^{2}$ (l112[t] x12[t]-1.` l113[t] x12[t]+l116[t] x16[t]-1.`
l117[t] x16[t]+l120[t] x20[t]-1.` l121[t] x20[t]+l18[t] x8[t]-1.` l19[t] x8[t]) (-1.` l112[t]
x12[t]+l113[t] x12[t]-1.` l116[t] x16[t]+l117[t] x16[t]-1.` l120[t] x20[t]+l121[t] x20[t]-1.`
l18[t] x8[t]+l19[t] x8[t])-1.` aii (-1.` api (k2 l112[t] x12[t]-1.` k2 l113[t] x12[t]+k2 l116[t]
x16[t]-1.` k2 l117[t] x16[t]+k2 l120[t] x20[t]-1.` k2 l121[t] x20[t]-1.` k1sm l124[t] x26[t]
x5[t]+k1sm rhos l126[t] x26[t] x5[t]+k1sm l15[t] x26[t] x5[t]+k2 l18[t] x8[t]-1.` k2 l19[t]
x8[t])-1.` k1sm (l124[t]-1.` rhos l126[t]-1.` l15[t]) x26[t] x5[t] (-1.` pr l126[t] (1.` -1.` u22[s])
(x11[t]+x15[t]+x19[t]+x23[t]+x25[t])-(1.` g l126[t] x26[t])/(b2+x26[t])-1.` k1sm l124[t]
x26[t] x5[t]+k1sm rhos l126[t] x26[t] x5[t]+k1sm l15[t] x26[t] x5[t]-1.` ps l126[t] u21[s]
(x13[t]+x17[t]+x21[t]+x24[t]+x9[t])))))/(-1.` aii (1.` api art-1.` k1sm$^{2}$ (l124[t]-1.` rhos
l126[t]-1.` l15[t])$^{2}$ x26[t]$^{2}$ x5[t]$^{2}$)+1.` api k2^{2} (-1.` l112[t] x12[t]+l113[t] x12[t]-1.` l116[t]
x16[t]+l117[t] x16[t]-1.` l120[t] x20[t]+l121[t] x20[t]-1.` l18[t] x8[t]+l19[t] x8[t])^{2})),1/api
(1.` (-1.` pr l126[t] (1.` -1.` u22[s]) (x11[t]+x15[t]+x19[t]+x23[t]+x25[t])-(1.` g l126[t]
x26[t])/(b2+x26[t])-1.` k1sm l124[t] x26[t] x5[t]+k1sm rhos l126[t] x26[t] x5[t]+k1sm l15[t]
x26[t] x5[t]-1.` ps l126[t] u21[s] (x13[t]+x17[t]+x21[t]+x24[t]+x9[t]))-(1.` k1sm (l124[t]-1.`
rhos l126[t]-1.` l15[t]) x26[t] x5[t] (1.` api k2^{2} (l112[t] x12[t]-1.` l113[t] x12[t]+l116[t]
x16[t]-1.` l117[t] x16[t]+l120[t] x20[t]-1.` l121[t] x20[t]+l18[t] x8[t]-1.` l19[t] x8[t]) (-1.`
l112[t] x12[t]+l113[t] x12[t]-1.` l116[t] x16[t]+l117[t] x16[t]-1.` l120[t] x20[t]+l121[t]
x20[t]-1.` l18[t] x8[t]+l19[t] x8[t])-1.` aii (-1.` api (k2 l112[t] x12[t]-1.` k2 l113[t] x12[t]+k2
l116[t] x16[t]-1.` k2 l117[t] x16[t]+k2 l120[t] x20[t]-1.` k2 l121[t] x20[t]-1.` k1sm l124[t]
x26[t] x5[t]+k1sm rhos l126[t] x26[t] x5[t]+k1sm l15[t] x26[t] x5[t]+k2 l18[t] x8[t]-1.` k2
l19[t] x8[t])-1.` k1sm (l124[t]-1.` rhos l126[t]-1.` l15[t]) x26[t] x5[t] (-1.` pr l126[t] (1.` -1.`

u22[s]) (x11[t]+x15[t]+x19[t]+x23[t]+x25[t])-(1.` g l126[t] x26[t])/(b2+x26[t])-1.` k1sm
l124[t] x26[t] x5[t]+k1sm rhos l126[t] x26[t] x5[t]+k1sm l15[t] x26[t] x5[t]-1.` ps l126[t]
u21[s] (x13[t]+x17[t]+x21[t]+x24[t]+x9[t])))))/(-1.` aii (1.` api art-1.` k1sm$^{2}$ (l124[t]-1.` rhos
l126[t]-1.` l15[t])$^{2}$ x26[t]$^{2}$ x5[t]$^{2}$)+1.` api k2^{2} (-1.` l112[t] x12[t]+l113[t] x12[t]-1.` l116[t]
x16[t]+l117[t] x16[t]-1.` l120[t] x20[t]+l121[t] x20[t]-1.` l18[t] x8[t]+l19[t] x8[t])^{2})),-(1.` api
k2^{2} (l112[t] x12[t]-1.` l113[t] x12[t]+l116[t] x16[t]-1.` l117[t] x16[t]+l120[t] x20[t]-1.`
l121[t] x20[t]+l18[t] x8[t]-1.` l19[t] x8[t]) (-1.` l112[t] x12[t]+l113[t] x12[t]-1.` l116[t]
x16[t]+l117[t] x16[t]-1.` l120[t] x20[t]+l121[t] x20[t]-1.` l18[t] x8[t]+l19[t] x8[t])-1.` aii (-1.`
api (k2 l112[t] x12[t]-1.` k2 l113[t] x12[t]+k2 l116[t] x16[t]-1.` k2 l117[t] x16[t]+k2 l120[t]
x20[t]-1.` k2 l121[t] x20[t]-1.` k1sm l124[t] x26[t] x5[t]+k1sm rhos l126[t] x26[t] x5[t]+k1sm
l15[t] x26[t] x5[t]+k2 l18[t] x8[t]-1.` k2 l19[t] x8[t])-1.` k1sm (l124[t]-1.` rhos l126[t]-1.`
l15[t]) x26[t] x5[t] (-1.` pr l126[t] (1.` -1.` u22[s]) (x11[t]+x15[t]+x19[t]+x23[t]+x25[t])-(1.`
g l126[t] x26[t])/(b2+x26[t])-1.` k1sm l124[t] x26[t] x5[t]+k1sm rhos l126[t] x26[t]
x5[t]+k1sm l15[t] x26[t] x5[t]-1.` ps l126[t] u21[s] (x13[t]+x17[t]+x21[t]+x24[t]+x9[t]))))/(-
1.` aii (1.` api art-1.` k1sm$^{2}$ (l124[t]-1.` rhos l126[t]-1.` l15[t])$^{2}$ x26[t]$^{2}$ x5[t]$^{2}$)+1.` api k2^{2} (-1.`
l112[t] x12[t]+l113[t] x12[t]-1.` l116[t] x16[t]+l117[t] x16[t]-1.` l120[t] x20[t]+l121[t]
x20[t]-1.` l18[t] x8[t]+l19[t] x8[t])$^{2}$),-(1/bsr)1.` (-1.` bsr+1.` ps l227[t]
(x13[t]+x17[t]+x21[t]+x24[t]+x9[t])-1.` ps l226[t] (1.` -1.` u15[s])
(x13[t]+x17[t]+x21[t]+x24[t]+x9[t])),-(1/brs)1.` (-1.` brs-1.` pr l227[t]
(x11[t]+x15[t]+x19[t]+x23[t]+x25[t])+1.` pr l226[t] (1.` -1.` u15[s])
(x11[t]+x15[t]+x19[t]+x23[t]+x25[t])) };

UESS={U1T[s][[1]], U1T[s][[2]], U1T[s][[3]], U1T[s][[4]], U1T[s][[5]],
U1T[s][[6]],U2T[s][[1]], U2T[s][[2]] };

```
jac=Table[D[vettsoless[[i]], u], {i, 1, 8}, {u,UESS}]//FullSimplify;

condESSperriga=Table[Sum[Abs[jac[[i,j]]],{j,1,8}],{i,1,8}]//Simplify;
```

equazioni del massimo unite per controlli ottimi

```
sistmax={EQMAX1[XT[t],U1T[t], U2T[t], L1T[t]][[1]]==0,EQMAX1[XT[t],U1T[t], U2T[t],
L1T[t]][[2]]==0, EQMAX1[XT[t],U1T[t], U2T[t], L1T[t]][[3]]==0,EQMAX1[XT[t],U1T[t],
U2T[t], L1T[t]][[4]]==0,EQMAX1[XT[t],U1T[t], U2T[t],
L1T[t]][[5]]==0,EQMAX1[XT[t],U1T[t], U2T[t], L1T[t]][[6]]==0,EQMAX2[XT[t],U1T[t],
U2T[t], L2T[t]][[1]]==0, EQMAX2[XT[t],U1T[t], U2T[t], L2T[t]][[2]]==0  };
Timing[soleqmax=NSolve[sistmax, {U1T[t][[1]], U1T[t][[2]], U1T[t][[3]], U1T[t][[4]],
U1T[t][[5]],U1T[t][[6]],U2T[t][[1]], U2T[t][[2]]}]];
soleqmax={{u11[t]->1/aco 1.` (k1s (1.` -1.` qthvs) l11[t] x1[t] x26[t]-1.` k1s (1.` -1.` qthvs)
l18[t] x1[t] x26[t]-1.` k1s (1.` -1.` qtsvs) l112[t] x2[t] x26[t]+k1s (1.` -1.` qtsvs) l12[t] x2[t]
x26[t]-1.` k1s (1.` -1.` qtrvs) l116[t] x26[t] x3[t]+k1s (1.` -1.` qtrvs) l13[t] x26[t] x3[t]-1.` k1s
(1.` -1.` qtgvs) l120[t] x26[t] x4[t]+k1s (1.` -1.` qtgvs) l14[t] x26[t] x4[t]+k1s rhos l126[t]
x26[t] ((1.` -1.` qthvs) x1[t]+(1.` -1.` qtsvs) x2[t]+(1.` -1.` qtrvs) x3[t]+(1.` -1.` qtgvs) x4[t]))-
(1.` (-1.` k1s (1.` -1.` qthvs) l11[t] x1[t] x26[t]+k1s (1.` -1.` qthvs) l18[t] x1[t] x26[t]+k1s (1.`
-1.` qtsvs) l112[t] x2[t] x26[t]-1.` k1s (1.` -1.` qtsvs) l12[t] x2[t] x26[t]+k1s (1.` -1.` qtrvs)
l116[t] x26[t] x3[t]-1.` k1s (1.` -1.` qtrvs) l13[t] x26[t] x3[t]+k1s (1.` -1.` qtgvs) l120[t] x26[t]
x4[t]-1.` k1s (1.` -1.` qtgvs) l14[t] x26[t] x4[t]-1.` k1s rhos l126[t] x26[t] ((1.` -1.` qthvs)
x1[t]+(1.` -1.` qtsvs) x2[t]+(1.` -1.` qtrvs) x3[t]+(1.` -1.` qtgvs) x4[t])) (-1.` aco (k1s (1.` -1.`
qthvs) l11[t] x1[t] x26[t]-1.` k1s (1.` -1.` qthvs) l18[t] x1[t] x26[t]-1.` k1s (1.` -1.` qtsvs)
l112[t] x2[t] x26[t]+k1s (1.` -1.` qtsvs) l12[t] x2[t] x26[t]-1.` k1s (1.` -1.` qtrvs) l116[t] x26[t]
```

x3[t]+k1s (1.` -1.` qtrvs) l13[t] x26[t] x3[t]-1.` k1s (1.` -1.` qtgvs) l120[t] x26[t] x4[t]+k1s (1.` -1.` qtgvs) l14[t] x26[t] x4[t]+k1s rhos l126[t] x26[t] ((1.` -1.` qthvs) x1[t]+(1.` -1.` qtsvs) x2[t]+(1.` -1.` qtrvs) x3[t]+(1.` -1.` qtgvs) x4[t]))-1.` (-1.` k1s (1.` -1.` qthvs) l11[t] x1[t] x26[t]+k1s (1.` -1.` qthvs) l18[t] x1[t] x26[t]+k1s (1.` -1.` qtsvs) l112[t] x2[t] x26[t]-1.` k1s (1.` -1.` qtsvs) l12[t] x2[t] x26[t]+k1s (1.` -1.` qtrvs) l116[t] x26[t] x3[t]-1.` k1s (1.` -1.` qtrvs) l13[t] x26[t] x3[t]+k1s (1.` -1.` qtgvs) l120[t] x26[t] x4[t]-1.` k1s (1.` -1.` qtgvs) l14[t] x26[t] x4[t]-1.` k1s rhos l126[t] x26[t] ((1.` -1.` qthvs) x1[t]+(1.` -1.` qtsvs) x2[t]+(1.` -1.` qtrvs) x3[t]+(1.` -1.` qtgvs) x4[t])) (k1s (1.` -1.` qthvs) l11[t] x1[t] x26[t]-1.` k1s (1.` -1.` qthvs) l18[t] x1[t] x26[t]-1.` k1s (1.` -1.` qtsvs) l112[t] x2[t] x26[t]+k1s (1.` -1.` qtsvs) l12[t] x2[t] x26[t]-1.` k1s (1.` -1.` qtrvs) l116[t] x26[t] x3[t]+k1s (1.` -1.` qtrvs) l13[t] x26[t] x3[t]-1.` k1s (1.` -1.` qtgvs) l120[t] x26[t] x4[t]+k1s (1.` -1.` qtgvs) l14[t] x26[t] x4[t]+k1s rhos l126[t] x26[t] ((1.` -1.` qthvs) x1[t]+(1.` -1.` qtsvs) x2[t]+(1.` -1.` qtrvs) x3[t]+(1.` -1.` qtgvs) x4[t]))))/(aco (1.` aco afi-1.` (-1.` k1s (1.` -1.` qthvs) l11[t] x1[t] x26[t]+k1s (1.` -1.` qthvs) l18[t] x1[t] x26[t]+k1s (1.` -1.` qtsvs) l112[t] x2[t] x26[t]-1.` k1s (1.` -1.` qtsvs) l12[t] x2[t] x26[t]+k1s (1.` -1.` qtrvs) l116[t] x26[t] x3[t]-1.` k1s (1.` -1.` qtrvs) l13[t] x26[t] x3[t]+k1s (1.` -1.` qtgvs) l120[t] x26[t] x4[t]-1.` k1s (1.` -1.` qtgvs) l14[t] x26[t] x4[t]-1.` k1s rhos l126[t] x26[t] ((1.` -1.` qthvs) x1[t]+(1.` -1.` qtsvs) x2[t]+(1.` -1.` qtrvs) x3[t]+(1.` -1.` qtgvs) x4[t]))^2)),u12[t]->-(-1.` aco (k1s (1.` -1.` qthvs) l11[t] x1[t] x26[t]-1.` k1s (1.` -1.` qthvs) l18[t] x1[t] x26[t]-1.` k1s (1.` -1.` qtsvs) l112[t] x2[t] x26[t]+k1s (1.` -1.` qtsvs) l12[t] x2[t] x26[t]-1.` k1s (1.` -1.` qtrvs) l116[t] x26[t] x3[t]+k1s (1.` -1.` qtrvs) l13[t] x26[t] x3[t]-1.` k1s (1.` -1.` qtgvs) l120[t] x26[t] x4[t]+k1s (1.` -1.` qtgvs) l14[t] x26[t] x4[t]+k1s rhos l126[t] x26[t] ((1.` -1.` qthvs) x1[t]+(1.` -1.` qtsvs) x2[t]+(1.` -1.` qtrvs) x3[t]+(1.` -1.` qtgvs) x4[t]))-1.` (-1.` k1s (1.` -1.` qthvs) l11[t] x1[t] x26[t]+k1s (1.` -1.` qthvs) l18[t] x1[t] x26[t]+k1s (1.` -

1.` qtsvs) l112[t] x2[t] x26[t]-1.` k1s (1.` -1.` qtsvs) l12[t] x2[t] x26[t]+k1s (1.` -1.` qtrvs)
l116[t] x26[t] x3[t]-1.` k1s (1.` -1.` qtrvs) l13[t] x26[t] x3[t]+k1s (1.` -1.` qtgvs) l120[t] x26[t]
x4[t]-1.` k1s (1.` -1.` qtgvs) l14[t] x26[t] x4[t]-1.` k1s rhos l126[t] x26[t] ((1.` -1.` qthvs)
x1[t]+(1.` -1.` qtsvs) x2[t]+(1.` -1.` qtrvs) x3[t]+(1.` -1.` qtgvs) x4[t])) (k1s (1.` -1.` qthvs)
l11[t] x1[t] x26[t]-1.` k1s (1.` -1.` qthvs) l18[t] x1[t] x26[t]-1.` k1s (1.` -1.` qtsvs) l112[t]
x2[t] x26[t]+k1s (1.` -1.` qtsvs) l12[t] x2[t] x26[t]-1.` k1s (1.` -1.` qtrvs) l116[t] x26[t]
x3[t]+k1s (1.` -1.` qtrvs) l13[t] x26[t] x3[t]-1.` k1s (1.` -1.` qtgvs) l120[t] x26[t] x4[t]+k1s (1.`
-1.` qtgvs) l14[t] x26[t] x4[t]+k1s rhos l126[t] x26[t] ((1.` -1.` qthvs) x1[t]+(1.` -1.` qtsvs)
x2[t]+(1.` -1.` qtrvs) x3[t]+(1.` -1.` qtgvs) x4[t])))/(1.` aco afi-1.` (-1.` k1s (1.` -1.` qthvs)
l11[t] x1[t] x26[t]+k1s (1.` -1.` qthvs) l18[t] x1[t] x26[t]+k1s (1.` -1.` qtsvs) l112[t] x2[t]
x26[t]-1.` k1s (1.` -1.` qtsvs) l12[t] x2[t] x26[t]+k1s (1.` -1.` qtrvs) l116[t] x26[t] x3[t]-1.` k1s
(1.` -1.` qtrvs) l13[t] x26[t] x3[t]+k1s (1.` -1.` qtgvs) l120[t] x26[t] x4[t]-1.` k1s (1.` -1.`
qtgvs) l14[t] x26[t] x4[t]-1.` k1s rhos l126[t] x26[t] ((1.` -1.` qthvs) x1[t]+(1.` -1.` qtsvs)
x2[t]+(1.` -1.` qtrvs) x3[t]+(1.` -1.` qtgvs) x4[t]))^{2}),u13[t]->(1.` (l11[t] x1[t]+l12[t]
x2[t]+l13[t] x3[t]+l14[t] x4[t]))/aib,u14[t]->1/aii 1.` (k2 l112[t] x12[t]-1.` k2 l113[t]
x12[t]+k2 l116[t] x16[t]-1.` k2 l117[t] x16[t]+k2 l120[t] x20[t]-1.` k2 l121[t] x20[t]+k2 l18[t]
x8[t]-1.` k2 l19[t] x8[t])-1/aii 1.` (-1.` k2 l112[t] x12[t]+k2 l113[t] x12[t]-1.` k2 l116[t]
x16[t]+k2 l117[t] x16[t]-1.` k2 l120[t] x20[t]+k2 l121[t] x20[t]-1.` k2 l18[t] x8[t]+k2 l19[t]
x8[t]) ((1.` (1.` bsr pr l126[t] (x11[t]+x15[t]+x19[t]+x23[t]+x25[t]) (1.` brs-1.` pr l226[t]
(x11[t]+x15[t]+x19[t]+x23[t]+x25[t])+pr l227[t] (x11[t]+x15[t]+x19[t]+x23[t]+x25[t]))-1.`
brs (-1.` bsr (-1.` pr l126[t] (x11[t]+x15[t]+x19[t]+x23[t]+x25[t])-(1.` g l126[t]
x26[t])/(b2+x26[t])-1.` k1sm l124[t] x26[t] x5[t]+k1sm rhos l126[t] x26[t] x5[t]+k1sm l15[t]
x26[t] x5[t])+1.` ps l126[t] (x13[t]+x17[t]+x21[t]+x24[t]+x9[t]) (1.` bsr+ps l226[t]

(x13[t]+x17[t]+x21[t]+x24[t]+x9[t])-1.` ps l227[t]

(x13[t]+x17[t]+x21[t]+x24[t]+x9[t])))))/(brs bsr (k1sm l124[t] x26[t] x5[t]-1.` k1sm rhos

l126[t] x26[t] x5[t]-1.` k1sm l15[t] x26[t] x5[t]))-(1.` (1.` bsr pr^2 l126[t] l226[t]

$(x11[t]+x15[t]+x19[t]+x23[t]+x25[t])^2$-1.` brs (1.` api bsr-1.` ps^2 l126[t] l226[t]

$(x13[t]+x17[t]+x21[t]+x24[t]+x9[t])^2$)) (1.` brs bsr (k1sm l124[t] x26[t] x5[t]-1.` k1sm rhos

l126[t] x26[t] x5[t]-1.` k1sm l15[t] x26[t] x5[t]) (-1.` aii (k2 l112[t] x12[t]-1.` k2 l113[t]

x12[t]+k2 l116[t] x16[t]-1.` k2 l117[t] x16[t]+k2 l120[t] x20[t]-1.` k2 l121[t] x20[t]-1.` k1sm

l124[t] x26[t] x5[t]+k1sm rhos l126[t] x26[t] x5[t]+k1sm l15[t] x26[t] x5[t]+k2 l18[t] x8[t]-

1.` k2 l19[t] x8[t])-1.` (k2 l112[t] x12[t]-1.` k2 l113[t] x12[t]+k2 l116[t] x16[t]-1.` k2 l117[t]

x16[t]+k2 l120[t] x20[t]-1.` k2 l121[t] x20[t]+k2 l18[t] x8[t]-1.` k2 l19[t] x8[t]) (-1.` k2

l112[t] x12[t]+k2 l113[t] x12[t]-1.` k2 l116[t] x16[t]+k2 l117[t] x16[t]-1.` k2 l120[t]

x20[t]+k2 l121[t] x20[t]-1.` k2 l18[t] x8[t]+k2 l19[t] x8[t]))-1.` (1.` aii art-1.` (-1.` k2 l112[t]

x12[t]+k2 l113[t] x12[t]-1.` k2 l116[t] x16[t]+k2 l117[t] x16[t]-1.` k2 l120[t] x20[t]+k2

l121[t] x20[t]-1.` k2 l18[t] x8[t]+k2 l19[t] $x8[t])^2$) (1.` bsr pr l126[t]

(x11[t]+x15[t]+x19[t]+x23[t]+x25[t]) (1.` brs-1.` pr l226[t]

(x11[t]+x15[t]+x19[t]+x23[t]+x25[t])+pr l227[t] (x11[t]+x15[t]+x19[t]+x23[t]+x25[t]))-1.`

brs (-1.` bsr (-1.` pr l126[t] (x11[t]+x15[t]+x19[t]+x23[t]+x25[t])-(1.` g l126[t]

x26[t])/(b2+x26[t])-1.` k1sm l124[t] x26[t] x5[t]+k1sm rhos l126[t] x26[t] x5[t]+k1sm l15[t]

x26[t] x5[t])+1.` ps l126[t] (x13[t]+x17[t]+x21[t]+x24[t]+x9[t]) (1.` bsr+ps l226[t]

(x13[t]+x17[t]+x21[t]+x24[t]+x9[t])-1.` ps l227[t]

(x13[t]+x17[t]+x21[t]+x24[t]+x9[t]))))))/(brs bsr (k1sm l124[t] x26[t] x5[t]-1.` k1sm rhos

l126[t] x26[t] x5[t]-1.` k1sm l15[t] x26[t] x5[t]) (-1.` aii brs bsr (k1sm l124[t] x26[t] x5[t]-1.`

k1sm rhos l126[t] x26[t] x5[t]-1.` k1sm l15[t] x26[t] $x5[t])^2$-1.` (1.` aii art-1.` (-1.` k2 l112[t] x12[t]+k2 l113[t] x12[t]-1.` k2 l116[t] x16[t]+k2 l117[t] x16[t]-1.` k2 l120[t] x20[t]+k2 l121[t] x20[t]-1.` k2 l18[t] x8[t]+k2 l19[t] x8[t])^2) (1.` bsr pr$^2$ l126[t] l226[t] (x11[t]+x15[t]+x19[t]+x23[t]+x25[t])$^2$-1.` brs (1.` api bsr-1.` ps^2 l126[t] l226[t] (x13[t]+x17[t]+x21[t]+x24[t]+x9[t])^2))))),u15[t]->-(1.` brs bsr (k1sm l124[t] x26[t] x5[t]-1.` k1sm rhos l126[t] x26[t] x5[t]-1.` k1sm l15[t] x26[t] x5[t]) (-1.` aii (k2 l112[t] x12[t]-1.` k2 l113[t] x12[t]+k2 l116[t] x16[t]-1.` k2 l117[t] x16[t]+k2 l120[t] x20[t]-1.` k2 l121[t] x20[t]-1.` k1sm l124[t] x26[t] x5[t]+k1sm rhos l126[t] x26[t] x5[t]+k1sm l15[t] x26[t] x5[t]+k2 l18[t] x8[t]-1.` k2 l19[t] x8[t])-1.` (k2 l112[t] x12[t]-1.` k2 l113[t] x12[t]+k2 l116[t] x16[t]-1.` k2 l117[t] x16[t]+k2 l120[t] x20[t]-1.` k2 l121[t] x20[t]+k2 l18[t] x8[t]-1.` k2 l19[t] x8[t]) (-1.` k2 l112[t] x12[t]+k2 l113[t] x12[t]-1.` k2 l116[t] x16[t]+k2 l117[t] x16[t]-1.` k2 l120[t] x20[t]+k2 l121[t] x20[t]-1.` k2 l18[t] x8[t]+k2 l19[t] x8[t]))-1.` (1.` aii art-1.` (-1.` k2 l112[t] x12[t]+k2 l113[t] x12[t]-1.` k2 l116[t] x16[t]+k2 l117[t] x16[t]-1.` k2 l120[t] x20[t]+k2 l121[t] x20[t]-1.` k2 l18[t] x8[t]+k2 l19[t] x8[t])$^2$) (1.` bsr pr l126[t] (x11[t]+x15[t]+x19[t]+x23[t]+x25[t]) (1.` brs-1.` pr l226[t] (x11[t]+x15[t]+x19[t]+x23[t]+x25[t])+pr l227[t] (x11[t]+x15[t]+x19[t]+x23[t]+x25[t]))-1.` brs (-1.` bsr (-1.` pr l126[t] (x11[t]+x15[t]+x19[t]+x23[t]+x25[t])-(1.` g l126[t] x26[t])/(b2+x26[t])-1.` k1sm l124[t] x26[t] x5[t]+k1sm rhos l126[t] x26[t] x5[t]+k1sm l15[t] x26[t] x5[t])+1.` ps l126[t] (x13[t]+x17[t]+x21[t]+x24[t]+x9[t]) (1.` bsr+ps l226[t] (x13[t]+x17[t]+x21[t]+x24[t]+x9[t])-1.` ps l227[t] (x13[t]+x17[t]+x21[t]+x24[t]+x9[t])))))/(-1.` aii brs bsr (k1sm l124[t] x26[t] x5[t]-1.` k1sm rhos l126[t] x26[t] x5[t]-1.` k1sm l15[t] x26[t] x5[t])$^2$-1.` (1.` aii art-1.` (-1.` k2 l112[t] x12[t]+k2 l113[t] x12[t]-1.` k2 l116[t] x16[t]+k2 l117[t] x16[t]-1.` k2 l120[t] x20[t]+k2 l121[t] x20[t]-1.` k2 l18[t] x8[t]+k2 l19[t] x8[t])^2) (1.` bsr pr$^2$ l126[t] l226[t] (x11[t]+x15[t]+x19[t]+x23[t]+x25[t])$^2$-1.` brs (1.` api bsr-

1.` ps$^2$ l126[t] l226[t] (x13[t]+x17[t]+x21[t]+x24[t]+x9[t])$^2$))),u16[t]->-(1.` (1.` bsr pr l126[t]
(x11[t]+x15[t]+x19[t]+x23[t]+x25[t]) (1.` brs-1.` pr l226[t]
(x11[t]+x15[t]+x19[t]+x23[t]+x25[t])+pr l227[t] (x11[t]+x15[t]+x19[t]+x23[t]+x25[t]))-1.`
brs (-1.` bsr (-1.` pr l126[t] (x11[t]+x15[t]+x19[t]+x23[t]+x25[t])-(1.` g l126[t]
x26[t])/(b2+x26[t])-1.` k1sm l124[t] x26[t] x5[t]+k1sm rhos l126[t] x26[t] x5[t]+k1sm l15[t]
x26[t] x5[t])+1.` ps l126[t] (x13[t]+x17[t]+x21[t]+x24[t]+x9[t]) (1.` bsr+ps l226[t]
(x13[t]+x17[t]+x21[t]+x24[t]+x9[t])-1.` ps l227[t]
(x13[t]+x17[t]+x21[t]+x24[t]+x9[t])))))/(brs bsr (k1sm l124[t] x26[t] x5[t]-1.` k1sm rhos
l126[t] x26[t] x5[t]-1.` k1sm l15[t] x26[t] x5[t]))+(1.` (1.` bsr pr^2 l126[t] l226[t]
(x11[t]+x15[t]+x19[t]+x23[t]+x25[t])^2-1.` brs (1.` api bsr-1.` ps^2 l126[t] l226[t]
(x13[t]+x17[t]+x21[t]+x24[t]+x9[t])^2)) (1.` brs bsr (k1sm l124[t] x26[t] x5[t]-1.` k1sm rhos
l126[t] x26[t] x5[t]-1.` k1sm l15[t] x26[t] x5[t]) (-1.` aii (k2 l112[t] x12[t]-1.` k2 l113[t]
x12[t]+k2 l116[t] x16[t]-1.` k2 l117[t] x16[t]+k2 l120[t] x20[t]-1.` k2 l121[t] x20[t]-1.` k1sm
l124[t] x26[t] x5[t]+k1sm rhos l126[t] x26[t] x5[t]+k1sm l15[t] x26[t] x5[t]+k2 l18[t] x8[t]-
1.` k2 l19[t] x8[t])-1.` (k2 l112[t] x12[t]-1.` k2 l113[t] x12[t]+k2 l116[t] x16[t]-1.` k2 l117[t]
x16[t]+k2 l120[t] x20[t]-1.` k2 l121[t] x20[t]+k2 l18[t] x8[t]-1.` k2 l19[t] x8[t]) (-1.` k2
l112[t] x12[t]+k2 l113[t] x12[t]-1.` k2 l116[t] x16[t]+k2 l117[t] x16[t]-1.` k2 l120[t]
x20[t]+k2 l121[t] x20[t]-1.` k2 l18[t] x8[t]+k2 l19[t] x8[t]))-1.` (1.` aii art-1.` (-1.` k2 l112[t]
x12[t]+k2 l113[t] x12[t]-1.` k2 l116[t] x16[t]+k2 l117[t] x16[t]-1.` k2 l120[t] x20[t]+k2
l121[t] x20[t]-1.` k2 l18[t] x8[t]+k2 l19[t] x8[t])$^2$) (1.` bsr pr l126[t]
(x11[t]+x15[t]+x19[t]+x23[t]+x25[t]) (1.` brs-1.` pr l226[t]
(x11[t]+x15[t]+x19[t]+x23[t]+x25[t])+pr l227[t] (x11[t]+x15[t]+x19[t]+x23[t]+x25[t]))-1.`
brs (-1.` bsr (-1.` pr l126[t] (x11[t]+x15[t]+x19[t]+x23[t]+x25[t])-(1.` g l126[t]

x26[t])/(b2+x26[t])-1.` k1sm l124[t] x26[t] x5[t]+k1sm rhos l126[t] x26[t] x5[t]+k1sm l15[t] x26[t] x5[t])+1.` ps l126[t] (x13[t]+x17[t]+x21[t]+x24[t]+x9[t]) (1.` bsr+ps l226[t] (x13[t]+x17[t]+x21[t]+x24[t]+x9[t])-1.` ps l227[t] (x13[t]+x17[t]+x21[t]+x24[t]+x9[t]))))))/(brs bsr (k1sm l124[t] x26[t] x5[t]-1.` k1sm rhos l126[t] x26[t] x5[t]-1.` k1sm l15[t] x26[t] x5[t]) (-1.` aii brs bsr (k1sm l124[t] x26[t] x5[t]-1.` k1sm rhos l126[t] x26[t] x5[t]-1.` k1sm l15[t] x26[t] x5[t])$^2$-1.` (1.` aii art-1.` (-1.` k2 l112[t] x12[t]+k2 l113[t] x12[t]-1.` k2 l116[t] x16[t]+k2 l117[t] x16[t]-1.` k2 l120[t] x20[t]+k2 l121[t] x20[t]-1.` k2 l18[t] x8[t]+k2 l19[t] x8[t])^2) (1.` bsr pr$^2$ l126[t] l226[t] (x11[t]+x15[t]+x19[t]+x23[t]+x25[t])$^2$-1.` brs (1.` api bsr-1.` ps^2 l126[t] l226[t] (x13[t]+x17[t]+x21[t]+x24[t]+x9[t])^2)))),u21[t]->1/bsr 1.` (1.` bsr+ps l226[t] (x13[t]+x17[t]+x21[t]+x24[t]+x9[t])-1.` ps l227[t] (x13[t]+x17[t]+x21[t]+x24[t]+x9[t]))+(1.` ps l226[t] (x13[t]+x17[t]+x21[t]+x24[t]+x9[t]) (1.` brs bsr (k1sm l124[t] x26[t] x5[t]-1.` k1sm rhos l126[t] x26[t] x5[t]-1.` k1sm l15[t] x26[t] x5[t]) (-1.` aii (k2 l112[t] x12[t]-1.` k2 l113[t] x12[t]+k2 l116[t] x16[t]-1.` k2 l117[t] x16[t]+k2 l120[t] x20[t]-1.` k2 l121[t] x20[t]-1.` k1sm l124[t] x26[t] x5[t]+k1sm rhos l126[t] x26[t] x5[t]+k1sm l15[t] x26[t] x5[t]+k2 l18[t] x8[t]-1.` k2 l19[t] x8[t])-1.` (k2 l112[t] x12[t]-1.` k2 l113[t] x12[t]+k2 l116[t] x16[t]-1.` k2 l117[t] x16[t]+k2 l120[t] x20[t]-1.` k2 l121[t] x20[t]+k2 l18[t] x8[t]-1.` k2 l19[t] x8[t]) (-1.` k2 l112[t] x12[t]+k2 l113[t] x12[t]-1.` k2 l116[t] x16[t]+k2 l117[t] x16[t]-1.` k2 l120[t] x20[t]+k2 l121[t] x20[t]-1.` k2 l18[t] x8[t]+k2 l19[t] x8[t]))-1.` (1.` aii art-1.` (-1.` k2 l112[t] x12[t]+k2 l113[t] x12[t]-1.` k2 l116[t] x16[t]+k2 l117[t] x16[t]-1.` k2 l120[t] x20[t]+k2 l121[t] x20[t]-1.` k2 l18[t] x8[t]+k2 l19[t] x8[t])$^2$) (1.` bsr pr l126[t] (x11[t]+x15[t]+x19[t]+x23[t]+x25[t]) (1.` brs-1.` pr l226[t] (x11[t]+x15[t]+x19[t]+x23[t]+x25[t])+pr l227[t] (x11[t]+x15[t]+x19[t]+x23[t]+x25[t]))-1.`

brs (-1.` bsr (-1.` pr l126[t] (x11[t]+x15[t]+x19[t]+x23[t]+x25[t])-(1.` g l126[t]
x26[t])/(b2+x26[t])-1.` k1sm l124[t] x26[t] x5[t]+k1sm rhos l126[t] x26[t] x5[t]+k1sm l15[t]
x26[t] x5[t])+1.` ps l126[t] (x13[t]+x17[t]+x21[t]+x24[t]+x9[t]) (1.` bsr+ps l226[t]
(x13[t]+x17[t]+x21[t]+x24[t]+x9[t])-1.` ps l227[t]
(x13[t]+x17[t]+x21[t]+x24[t]+x9[t]))))))/(bsr (-1.` aii brs bsr (k1sm l124[t] x26[t] x5[t]-1.`
k1sm rhos l126[t] x26[t] x5[t]-1.` k1sm l15[t] x26[t] x5[t])$^{2}$-1.` (1.` aii art-1.` (-1.` k2 l112[t]
x12[t]+k2 l113[t] x12[t]-1.` k2 l116[t] x16[t]+k2 l117[t] x16[t]-1.` k2 l120[t] x20[t]+k2
l121[t] x20[t]-1.` k2 l18[t] x8[t]+k2 l19[t] x8[t])$^{2}$) (1.` bsr pr^{2} l126[t] l226[t]
(x11[t]+x15[t]+x19[t]+x23[t]+x25[t])^{2}-1.` brs (1.` api bsr-1.` ps^{2} l126[t] l226[t]
(x13[t]+x17[t]+x21[t]+x24[t]+x9[t])^{2})))),u22[t]->1/brs 1.` (1.` brs-1.` pr l226[t]
(x11[t]+x15[t]+x19[t]+x23[t]+x25[t])+pr l227[t] (x11[t]+x15[t]+x19[t]+x23[t]+x25[t]))-(1.`
pr l226[t] (x11[t]+x15[t]+x19[t]+x23[t]+x25[t]) (1.` brs bsr (k1sm l124[t] x26[t] x5[t]-1.`
k1sm rhos l126[t] x26[t] x5[t]-1.` k1sm l15[t] x26[t] x5[t]) (-1.` aii (k2 l112[t] x12[t]-1.` k2
l113[t] x12[t]+k2 l116[t] x16[t]-1.` k2 l117[t] x16[t]+k2 l120[t] x20[t]-1.` k2 l121[t] x20[t]-
1.` k1sm l124[t] x26[t] x5[t]+k1sm rhos l126[t] x26[t] x5[t]+k1sm l15[t] x26[t] x5[t]+k2
l18[t] x8[t]-1.` k2 l19[t] x8[t])-1.` (k2 l112[t] x12[t]-1.` k2 l113[t] x12[t]+k2 l116[t] x16[t]-1.`
k2 l117[t] x16[t]+k2 l120[t] x20[t]-1.` k2 l121[t] x20[t]+k2 l18[t] x8[t]-1.` k2 l19[t] x8[t]) (-
1.` k2 l112[t] x12[t]+k2 l113[t] x12[t]-1.` k2 l116[t] x16[t]+k2 l117[t] x16[t]-1.` k2 l120[t]
x20[t]+k2 l121[t] x20[t]-1.` k2 l18[t] x8[t]+k2 l19[t] x8[t]))-1.` (1.` aii art-1.` (-1.` k2 l112[t]
x12[t]+k2 l113[t] x12[t]-1.` k2 l116[t] x16[t]+k2 l117[t] x16[t]-1.` k2 l120[t] x20[t]+k2
l121[t] x20[t]-1.` k2 l18[t] x8[t]+k2 l19[t] x8[t])$^{2}$) (1.` bsr pr l126[t]
(x11[t]+x15[t]+x19[t]+x23[t]+x25[t]) (1.` brs-1.` pr l226[t]
(x11[t]+x15[t]+x19[t]+x23[t]+x25[t])+pr l227[t] (x11[t]+x15[t]+x19[t]+x23[t]+x25[t]))-1.`

```
brs (-1.` bsr (-1.` pr l126[t] (x11[t]+x15[t]+x19[t]+x23[t]+x25[t])-(1.` g l126[t]
x26[t])/(b2+x26[t])-1.` k1sm l124[t] x26[t] x5[t]+k1sm rhos l126[t] x26[t] x5[t]+k1sm l15[t]
x26[t] x5[t])+1.` ps l126[t] (x13[t]+x17[t]+x21[t]+x24[t]+x9[t]) (1.` bsr+ps l226[t]
(x13[t]+x17[t]+x21[t]+x24[t]+x9[t])-1.` ps l227[t]
(x13[t]+x17[t]+x21[t]+x24[t]+x9[t]))))))/(brs (-1.` aii brs bsr (k1sm l124[t] x26[t] x5[t]-1.`
k1sm rhos l126[t] x26[t] x5[t]-1.` k1sm l15[t] x26[t] x5[t])^2-1.` (1.` aii art-1.` (-1.` k2 l112[t]
x12[t]+k2 l113[t] x12[t]-1.` k2 l116[t] x16[t]+k2 l117[t] x16[t]-1.` k2 l120[t] x20[t]+k2
l121[t] x20[t]-1.` k2 l18[t] x8[t]+k2 l19[t] x8[t])^2) (1.` bsr pr^2 l126[t] l226[t]
(x11[t]+x15[t]+x19[t]+x23[t]+x25[t])^2-1.` brs (1.` api bsr-1.` ps^2 l126[t] l226[t]
(x13[t]+x17[t]+x21[t]+x24[t]+x9[t])^2))))}};

U=Flatten[{U1T[t], U2T[t]}];

soleqmax1=U/.soleqmax[[1]];
```

equazioni aggiunte

```
EQAGG1[X_, U1_, U2_, L1_]:=Flatten[Map[Thread,{-D[HT[X,U1,U2,L1],{X}]}]];

EQAGG2[X_, U1_, U2_, L2_]:=Flatten[Map[Thread,{-D[HV[X,U1,U2,L2],{X}]}]];
```

Parametri

```
aco=10.;

afi=10.;

aib=1.;

aii=10.;

api=10.;

art=10.;

bsr=10.^(-10);

brs=10.^(-11);

b2=1.;

c=0.03;

g=25.;

is=10.^(-308.);

ir=10.^(-308.);

k=0.000001;

kprimo=0.000001;

k2=0.003;

k1s=0.0003;

k1r=0.00005;

k1sm=0.00467;

k1rm=0.001;

k3=0.000001;
```

```
muth=0.02;
muts=0.02;
mutr=0.02;
mutg=0.02;
mumh=0.005;
muip=0.001;
muie=0.1;
muthsl=0.1;
muthsa=0.24;
muthrl=0.2;
muthra=0.5;
mutssl=0.1;
mutssa=0.24;
mutsrl=0.2;
mutsra=0.5;
mutrsl=0.1;
mutrsa=0.24;
mutrrl=0.2;
mutrra=0.5;
mutgsl=0.1;
mutgsa=0.24;
mutgrl=0.2;
mutgra=0.5;
```

```
mumhsa=0.04;

mumhra=0.07;

muvs=2.4;

muvr=5.;

oth=1.;

ps=8*10^(-12);

pr=5*10^(-12);

psprimo=0.020;

prprimo=0.001;

q=0.5;

qthvs=0.;

qthvr=0.;

qtsvs=0.90;

qtsvr=0.;

qtrvs=0.;

qtrvr=0.90;

qtgvs=0.80;

qtgvr=0.80;

r=0.03;

rhos=1.;

rhor=1.;

s1th=1.;

s1mh=1.;
```

```
ssprimo=9.7;

srprimo=0.0009;

tmax=1700.;

ie0=10.;

ip0=10.;

mh0=1.;

mha0=1.;

th0= 900.;

ts0= 100.;

tr0= 100.;

tg0=1.;

thsl0=10.;

thsa0=0.;

thrl0=10.;

thra0=0.;

tssl0=10.;

tssa0=0.;

tsrl0=10.;

tsra0=0.;

trsl0=10.;

trsa0=0.;

trrl0=10.;

trra0=0.;
```

```
tgsl0=10.;

tgsa0=0.;

tgrl0=10.;

tgra0=0.;

mhsa0=10.;

mhra0=10.;

vs0=500.;

vr0=300.;

t0=0.;

tf=1000.;

l01=1.;

l02=1.;

cond0={th0,ts0,tr0,tg0,mh0, ip0, ie0,

thsl0,thsa0,thrl0,thra0,tssl0,tssa0,tsrl0,tsra0,trsl0,trsa0,trrl0,trra0,tgsl0,tgsa0,tgrl0,tgra0,mhsa0,

mhra0,vs0, vr0};
```

controlli candidati ottimi e condizioni iniziali

```
{u11[t],u12[t],u13[t],u14[t],u15[t],u16[t],u21[t],u22[t]}=soleqmax1;

condinx=Flatten[Map[Thread, {XT[t0]==cond0}],1];

condfinl1=Flatten[Map[Thread,

{L1T[tf]=={0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.}}],1];

condfinl2=Flatten[Map[Thread,

{L2T[tf]=={0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.}}],1];
```

```
condinfin=Flatten[{condinx, condfinl1, condfinl2}];

condinl1=Flatten[Map[Thread,
{L1T[t0]=={0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.
1,0.1,0.1,0.1,0.1,0.1}}],1];

condinl2=Flatten[Map[Thread,
{L2T[t0]=={0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.
1,0.1,0.1,0.1,0.1,0.1}}],1];

condin=Join[condinx,condinl1, condinl2];
```

equazioni differenziali

```
CV=Flatten[{T[XT[t], U1T[t], U2T[t]],TS[XT[t], U1T[t], U2T[t]],TR[XT[t], U1T[t],
U2T[t]],TG[XT[t], U1T[t], U2T[t]], MH[XT[t], U1T[t], U2T[t]], IP[XT[t], U1T[t], U2T[t]],
IE[XT[t], U1T[t], U2T[t]], THSL[XT[t], U1T[t], U2T[t]],
    THSA[XT[t], U1T[t], U2T[t]], THRL[XT[t], U1T[t], U2T[t]], THRA[XT[t], U1T[t],
U2T[t]], TSSL[XT[t], U1T[t], U2T[t]],
    TSSA[XT[t], U1T[t], U2T[t]], TSRL[XT[t], U1T[t], U2T[t]], TSRA[XT[t], U1T[t],
U2T[t]], TRSL[XT[t], U1T[t], U2T[t]],
    TRSA[XT[t], U1T[t], U2T[t]], TRRL[XT[t], U1T[t], U2T[t]], TRRA[XT[t], U1T[t],
U2T[t]], TGSL[XT[t], U1T[t], U2T[t]],
    TGSA[XT[t], U1T[t], U2T[t]], TGRL[XT[t], U1T[t], U2T[t]], TGRA[XT[t], U1T[t],
U2T[t]],  MHSA[XT[t], U1T[t], U2T[t]],
    MHRA[XT[t], U1T[t], U2T[t]],VS[XT[t], U1T[t], U2T[t]],VR[XT[t], U1T[t], U2T[t]]}];

eqx=Flatten[Map[Thread,{XT'[t]== CV}],1];
```

```
eql1=Flatten[Map[Thread,{L1T'[t]==EQAGG1[XT[t],U1T[t],U2T[t],L1T[t]]}],1];

eql2=Flatten[Map[Thread,{L2T'[t]==EQAGG2[XT[t],U1T[t],U2T[t],L2T[t]]}],1];

eq=Join[eqx,eql1,eql2];

var=Flatten[{X,L1,L2}];
```

NDSolve

```
sol=NDSolve[ Join[eq,condinfin],var,{t,t0,tf}];

sol=NDSolve[ Join[eq,condinfin],var,{t,t0,tf}, AccuracyGoal->0,PrecisionGoal-
>0,MaxStepFraction->1,MaxStepSize->1, MaxSteps->1000 ];
```

C.2 Matlab function for numerical calculations

In this appendix we report the `Matlab` code for the numerical solutions of concave and non-concave instances. As mentioned in Chapter 3, the algorithm has long computational time, therefore, in the case of concave instances we used a simpler algorithm (which will not be reported for typographical reasons) quite similar to this, that at each step assigned to controls the value (3.18)

$$\min\left\{1, max\{0, \hat{u}(t)\}\right\},$$

Finally, even randomizations code is not reported, we remember that it is quite similar to the below code with the difference that each instance is calculated inside a "*for loop*" that assigns for all $\lambda_{i,j}(t_0)$ random values in range $[0, 1]$.

```
%HIV con Runge-Kutta

function [z]=hiv_contr_dentro_alg_no_conca_v8(t0,tf,N,sigma)

tic;

%%%%%%%%%%%%PARAMETRI 2° ISTANZA H_therapy NON CONCAVA

aco = 10.;

afi = 10.;

aib = 1.;

aii = 10.;

api = 10.;

art = 10.;

bsr = 10.^(-10);

brs = 10.^(-11);

b2 = 1;

c = 0.03;

g = 25;

is =10.^(-308.);

ir = 10.^(-308.);

k = 0.000001;

kprimo = 0.000001;

k2 = 0.003;

k1s = 0.0003;

k1r = 0.00005;

k1sm = 0.00467;

k1rm = 0.001;

k3 = 0.000001;

muth = 0.02;

muts = 0.02;

mutr = 0.02;

mutg = 0.02;

mumh = 0.005;

muip = 0.001;

muie = 0.1;

muthsl = 0.1;
```

```
muthsa = 0.24;
muthrl = 0.2;
muthra = 0.5;
%
mutssl = 0.1;
mutssa = 0.24;
mutsrl = 0.2;
mutsra = 0.5;
%
mutrsl = 0.1;
mutrsa = 0.24;
mutrrl = 0.2;
mutrra = 0.5;
%
mutgsl = 0.1;
mutgsa = 0.24;
mutgrl = 0.2;
mutgra = 0.5;
mumhsa = 0.04;
mumhra = 0.07;
muvs = 2.4;
muvr = 5.;
oth = 1.;
ps = 8*10^(-12);   %8*10^(-11.5);
pr = 5*10^(-12);   %5*10^(-11.5);
psprimo = 0.020;
prprimo = 0.001;
q = 0.5;
qthvs = 0.;
qthvr = 0.;
%
qtsvs = 0.90;
qtsvr = 0.;
```

```
%
qtrvs = 0.;
qtrvr = 0.90;
%
qtgvs = 0.80;
qtgvr = 0.80;
r = 0.03;
rhos = 1.;
rhor = 1.;
s1th = 1.;
s1mh = 1.;
ssprimo = 9.7;
srprimo = 0.0009;
tmax = 1700.;
%%%%%%%%%%%%%%%%%%%%%CONDIZIONI INIZIALI
ie0 = 10.;
ip0 = 10.;
mh0 = 1.;
th0 = 900.;
ts0 = 100.;
tr0 = 100.;
tg0 = 1.;
thsl0 = 10.;
thsa0 = 0.;
thrl0 = 10.;
thra0 = 0.;
tssl0 = 10.;
tssa0 = 0.;
tsrl0 = 10.;
tsra0 = 0.;
trsl0 = 10.;
trsa0 = 0.;
trrl0 = 10.;
```

```
trra0 = 0.;

tgsl0 = 10.;

tgsa0 = 0.;

tgrl0 = 10.;

tgra0 = 0.;

mhsa0 = 10.;

mhra0 = 10.;

vs0 = 500.;

vr0 = 300.;


%%%%%%%%%%%%%%%%%INIZIALIZZAZIONE VARIABILI

test=-1;

testess=-1;t=linspace(t0,tf,N+1);

%h=(tf-t0)/N; con questo h il modello a volte diverge;

% conviene usare h=1/N;

h=1/N;

h2=h/2;

a11=0; a12=0; a13=0; a14=0; a15=0; a16=0;

D=[a11,a12,a13,a14,a15,a16;a11,a12,a13,a14,a15,1;a11,a12,a13,a14,a15,0;a11,a12,a13,a14,1
,a16;a11,a12,a13,a14,1,1;a11,a12,a13,a14,1,0;a11,a12,a13,a14,0,a16;a11,a12,a13,a14,0,1;a11
,a12,a13,a14,0,0;a11,a12,a13,1,a15,a16;a11,a12,a13,1,a15,1;a11,a12,a13,1,a15,0;a11,a12,a13
,1,1,a16;a11,a12,a13,1,1,1;a11,a12,a13,1,1,0;a11,a12,a13,1,0,a16;a11,a12,a13,1,0,1;a11,a12,a
13,1,0,0;a11,a12,a13,0,a15,a16;a11,a12,a13,0,a15,1;a11,a12,a13,0,a15,0;a11,a12,a13,0,1,a16;
a11,a12,a13,0,1,1;a11,a12,a13,0,1,0;a11,a12,a13,0,0,a16;a11,a12,a13,0,0,1;a11,a12,a13,0,0,0;
a11,a12,1,a14,a15,a16;a11,a12,1,a14,a15,1;a11,a12,1,a14,a15,0;a11,a12,1,a14,1,a16;a11,a12,
1,a14,1,1;a11,a12,1,a14,1,0;a11,a12,1,a14,0,a16;a11,a12,1,a14,0,1;a11,a12,1,a14,0,0;a11,a12,
1,1,a15,a16;a11,a12,1,1,a15,1;a11,a12,1,1,a15,0;a11,a12,1,1,1,a16;a11,a12,1,1,1,1;a11,a12,1,
1,1,0;a11,a12,1,1,0,a16;a11,a12,1,1,0,1;a11,a12,1,1,0,0;a11,a12,1,0,a15,a16;a11,a12,1,0,a15,
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a16;0,1,a13,0,a15,1;0,1,a13,0,a15,0;0,1,a13,0,1,a16;0,1,a13,0,1,1;0,1,a13,0,1,0;0,1,a13,0,0,a1
6;0,1,a13,0,0,1;0,1,a13,0,0,0;0,1,1,a14,a15,a16;0,1,1,a14,a15,1;0,1,1,a14,a15,0;0,1,1,a14,1,a1
6;0,1,1,a14,1,1;0,1,1,a14,1,0;0,1,1,a14,0,a16;0,1,1,a14,0,1;0,1,1,a14,0,0;0,1,1,1,a15,a16;0,1,1,
1,a15,1;0,1,1,1,a15,0;0,1,1,1,1,a16;0,1,1,1,1,1;0,1,1,1,1,0;0,1,1,1,0,a16;0,1,1,1,0,1;0,1,1,1,0,0;
0,1,1,0,a15,a16;0,1,1,0,a15,1;0,1,1,0,a15,0;0,1,1,0,1,a16;0,1,1,0,1,1;0,1,1,0,1,0;0,1,1,0,0,a16;
0,1,1,0,0,1;0,1,1,0,0,0;0,1,0,a14,a15,a16;0,1,0,a14,a15,1;0,1,0,a14,a15,0;0,1,0,a14,1,a16;0,1,0
,a14,1,1;0,1,0,a14,1,0;0,1,0,a14,0,a16;0,1,0,a14,0,1;0,1,0,a14,0,0;0,1,0,1,a15,a16;0,1,0,1,a15,
1;0,1,0,1,a15,0;0,1,0,1,1,a16;0,1,0,1,1,1;0,1,0,1,1,0;0,1,0,1,0,a16;0,1,0,1,0,1;0,1,0,1,0,0;0,1,0,
0,a15,a16;0,1,0,0,a15,1;0,1,0,0,a15,0;0,1,0,0,1,a16;0,1,0,0,1,1;0,1,0,0,1,0;0,1,0,0,0,a16;0,1,0,
0,0,1;0,1,0,0,0,0;0,0,a13,a14,a15,a16;0,0,a13,a14,a15,1;0,0,a13,a14,a15,0;0,0,a13,a14,1,a16;0
,0,a13,a14,1,1;0,0,a13,a14,1,0;0,0,a13,a14,0,a16;0,0,a13,a14,0,1;0,0,a13,a14,0,0;0,0,a13,1,a1
5,a16;0,0,a13,1,a15,1;0,0,a13,1,a15,0;0,0,a13,1,1,a16;0,0,a13,1,1,1;0,0,a13,1,1,0;0,0,a13,1,0,
a16;0,0,a13,1,0,1;0,0,a13,1,0,0;0,0,a13,0,a15,a16;0,0,a13,0,a15,1;0,0,a13,0,a15,0;0,0,a13,0,1,
a16;0,0,a13,0,1,1;0,0,a13,0,1,0;0,0,a13,0,0,a16;0,0,a13,0,0,1;0,0,a13,0,0,0;0,0,1,a14,a15,a16;
0,0,1,a14,a15,1;0,0,1,a14,a15,0;0,0,1,a14,1,a16;0,0,1,a14,1,1;0,0,1,a14,1,0;0,0,1,a14,0,a16;0,
0,1,a14,0,1;0,0,1,a14,0,0;0,0,1,1,a15,a16;0,0,1,1,a15,1;0,0,1,1,a15,0;0,0,1,1,1,a16;0,0,1,1,1,1;
0,0,1,1,1,0;0,0,1,1,0,a16;0,0,1,1,0,1;0,0,1,1,0,0;0,0,1,0,a15,a16;0,0,1,0,a15,1;0,0,1,0,a15,0;0,0
,1,0,1,a16;0,0,1,0,1,1;0,0,1,0,1,0;0,0,1,0,0,a16;0,0,1,0,0,1;0,0,1,0,0,0;0,0,0,a14,a15,a16;0,0,0,
a14,a15,1;0,0,0,a14,a15,0;0,0,0,a14,1,a16;0,0,0,a14,1,1;0,0,0,a14,1,0;0,0,0,a14,0,a16;0,0,0,a1
4,0,1;0,0,0,a14,0,0;0,0,0,1,a15,a16;0,0,0,1,a15,1;0,0,0,1,a15,0;0,0,0,1,1,a16;0,0,0,1,1,1;0,0,0,
1,1,0;0,0,0,1,0,a16;0,0,0,1,0,1;0,0,0,1,0,0;0,0,0,0,a15,a16;0,0,0,0,a15,1;0,0,0,0,a15,0;0,0,0,0,1
,a16;0,0,0,0,1,1;0,0,0,0,1,0;0,0,0,0,0,a16;0,0,0,0,0,1;0,0,0,0,0,0];

u11s=D(:,1); u12s=D(:,2); u13s=D(:,3); u14s=D(:,4); u15s=D(:,5); u16s=D(:,6);

fHT='-
0.5.*(aco.*u11s.^2+afi.*u12s.^2+aib.*u13s.^2+aii.*u14s.^2+api.*u15s.^2+art.*u16s.^2)+x1(i
)+x2(i)+l112(i).*(-(mutssl.*x12(i))-k2.*(-1+u14s).*(-1+u16s).*x12(i)+k3.*x2(i).*x24(i)-
k1s.*(-1+qtsvs).*(-1+u11s).*(-1+u12s).*x2(i).*x26(i))+l12(i).*(-(muts.*x2(i))+u13s.*x2(i)-
k3.*x2(i).*(x24(i)+x25(i))+k.*x1(i).*x26(i)+k1s.*(-1+qtsvs).*(-1+u11s).*(-
1+u12s).*x2(i).*x26(i)+k1r.*(-1+qtsvr).*x2(i).*x27(i))+l110(i).*(-
((k2+muthrl).*x10(i))+x1(i).*(k3.*x25(i)-k1r.*(-1+qthvr).*x27(i)))+l114(i).*(-
((k2+mutsrl).*x14(i))+x2(i).*(k3.*x25(i)-k1r.*(-1+qtsvr).*x27(i)))+x3(i)+l116(i).*(-
(mutrsl.*x16(i))-k2.*(-1+u14s).*(-1+u16s).*x16(i)+k3.*x24(i).*x3(i)-k1s.*(-1+qtrvs).*(-
1+u11s).*(-1+u12s).*x26(i).*x3(i))+l13(i).*(k.*x1(i).*x27(i)-mutr.*x3(i)+u13s.*x3(i)-
k3.*(x24(i)+x25(i)).*x3(i)+k1s.*(-1+qtrvs).*(-1+u11s).*(-1+u12s).*x26(i).*x3(i)+k1r.*(-
1+qtrvr).*x27(i).*x3(i))+l118(i).*(-((k2+mutrrl).*x18(i))+(k3.*x25(i)-k1r.*(-
1+qtrvr).*x27(i)).*x3(i))+x4(i)+l120(i).*(-(mutgsl.*x20(i))-k2.*(-1+u14s).*(-
1+u16s).*x20(i)+k3.*x24(i).*x4(i)-k1s.*(-1+qtgvs).*(-1+u11s).*(-
1+u12s).*x26(i).*x4(i))+l14(i).*(kprimo.*x1(i).*(x26(i)+x27(i))-mutg.*x4(i)+u13s.*x4(i)-
k3.*(x24(i)+x25(i)).*x4(i)+k1s.*(-1+qtgvs).*(-1+u11s).*(-1+u12s).*x26(i).*x4(i)+k1r.*(-
1+qtgvr).*x27(i).*x4(i))+l122(i).*(-((k2+mutgrl).*x22(i))+(k3.*x25(i)-k1r.*(-
1+qtgvr).*x27(i)).*x4(i))+x5(i)+l124(i).*(-(mumhsa.*x24(i))+k1sm.*(-1+u15s).*(-
1+u16s).*x26(i).*x5(i))+l125(i).*(-(mumhra.*x25(i))+k1rm.*x27(i).*x5(i))+l15(i).*(s1mh-
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(mumh+k1sm.*(-1+u15s).*(-
1+u16s).*x26(i)+k1rm.*x27(i)).*x5(i))+x6(i)+x7(i)+l123(i).*(k2.*x22(i)-
x23(i).*(mutgra+ir.*x7(i)))+l111(i).*(k2.*x10(i)-
x11(i).*(muthra+ir.*x7(i)))+l119(i).*(k2.*x18(i)-
x19(i).*(mutrra+ir.*x7(i)))+l115(i).*(k2.*x14(i)-x15(i).*(mutsra+ir.*x7(i)))+l121(i).*(k2.*(-
1+u14s).*(-1+u16s).*x20(i)-x21(i).*(mutgsa+is.*x7(i)))+l117(i).*(k2.*(-1+u14s).*(-
1+u16s).*x16(i)-x17(i).*(mutrsa+is.*x7(i)))+l113(i).*(k2.*(-1+u14s).*(-1+u16s).*x12(i)-
x13(i).*(mutssa+is.*x7(i)))+l18(i).*(k3.*x1(i).*x24(i)-k1s.*(-1+qthvs).*(-1+u11s).*(-
1+u12s).*x1(i).*x26(i)-muthsl.*x8(i)-k2.*(-1+u14s).*(-1+u16s).*x8(i))+l19(i).*(k2.*(-
1+u14s).*(-1+u16s).*x8(i)-(muthsa+is.*x7(i)).*x9(i))+l16(i).*x6(i).*(-muip-
c.*q.*(x11(i)+x13(i)+x15(i)+x17(i)+x19(i)+x21(i)+x23(i)+x9(i))+c.*(x1(i)+x2(i)+x3(i)+x4(i)
).*(x11(i)+x13(i)+x15(i)+x17(i)+x19(i)+x21(i)+x23(i)+x9(i)))+l17(i).*(-
(muie.*x7(i))+c.*q.*x6(i).*(x11(i)+x13(i)+x15(i)+x17(i)+x19(i)+x21(i)+x23(i)+x9(i)))+l127(
i).*(pr.*u22s.*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))-
muvr.*x27(i)+(g.*x27(i))/(b2+x27(i))+k1r.*rhor.*x27(i).*((-1+qthvr).*x1(i)+(-
1+qtsvr).*x2(i)-x3(i)+qtrvr.*x3(i)-x4(i)+qtgvr.*x4(i))-
x27(i).*(prprimo.*x3(i)+srprimo.*x4(i))-k1rm.*rhor.*x27(i).*x5(i)-ps.*(-
1+u21s).*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i)))+l126(i).*(pr.*(-1+u15s).*(-
1+u22s).*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))-muvs.*x26(i)-(g.*(-
1+u15s).*x26(i))/(b2+x26(i))+k1s.*rhos.*(-1+u11s).*(-1+u12s).*x26(i).*((-
1+qthvs).*x1(i)+(-1+qtsvs).*x2(i)-x3(i)+qtrvs.*x3(i)-x4(i)+qtgvs.*x4(i))-
x26(i).*(psprimo.*x2(i)+ssprimo.*x4(i))-k1sm.*rhos.*(-1+u15s).*(-1+u16s).*x26(i).*x5(i)-
ps.*(-1+u15s).*u21s.*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i)))+l11(i).*(-
(muth.*x1(i))+u13s.*x1(i)-k3.*x1(i).*(x24(i)+x25(i))+k1s.*(-1+qthvs).*(-1+u11s).*(-
1+u12s).*x1(i).*x26(i)+k1r.*(-1+qthvr).*x1(i).*x27(i)-k.*x1(i).*(x26(i)+x27(i))-
kprimo.*x1(i).*(x26(i)+x27(i))+(oth.*s1th)/(oth+x26(i)+x27(i))+r.*x1(i).*(1-
(x1(i)+x10(i)+x11(i)+x12(i)+x13(i)+x14(i)+x15(i)+x16(i)+x17(i)+x18(i)+x19(i)+x2(i)+x20(i
)+x21(i)+x22(i)+x23(i)+x3(i)+x4(i)+x8(i)+x9(i))/tmax))';

fHV='-0.5.*(bsr.*(-1+u21s).^2+brs.*(-
1+u22s).^2)+x10(i)+x11(i)+x12(i)+x13(i)+x14(i)+x15(i)+x16(i)+x17(i)+x18(i)+x19(i)+x20(i
)+x21(i)+x22(i)+x23(i)+x26(i)+l212(i).*(-(mutssl.*x12(i))-k2.*(-1+u14s).*(-
1+u16s).*x12(i)+k3.*x2(i).*x24(i)-k1s.*(-1+qtsvs).*(-1+u11s).*(-
1+u12s).*x2(i).*x26(i))+x27(i)+l22(i).*(-(muts.*x2(i))+u13s.*x2(i)-
k3.*x2(i).*(x24(i)+x25(i))+k.*x1(i).*x26(i)+k1s.*(-1+qtsvs).*(-1+u11s).*(-
1+u12s).*x2(i).*x26(i)+k1r.*(-1+qtsvr).*x2(i).*x27(i))+l210(i).*(-
((k2+muthrl).*x10(i))+x1(i).*(k3.*x25(i)-k1r.*(-1+qthvr).*x27(i)))+l214(i).*(-
((k2+mutsrl).*x14(i))+x2(i).*(k3.*x25(i)-k1r.*(-1+qtsvr).*x27(i)))+l216(i).*(-
(mutrsl.*x16(i))-k2.*(-1+u14s).*(-1+u16s).*x16(i)+k3.*x24(i).*x3(i)-k1s.*(-1+qtrvs).*(-
1+u11s).*(-1+u12s).*x26(i).*x3(i))+l23(i).*(k.*x1(i).*x27(i)-mutr.*x3(i)+u13s.*x3(i)-
k3.*(x24(i)+x25(i)).*x3(i)+k1s.*(-1+qtrvs).*(-1+u11s).*(-1+u12s).*x26(i).*x3(i)+k1r.*(-
1+qtrvr).*x27(i).*x3(i))+l218(i).*(-((k2+mutrrl).*x18(i))+(k3.*x25(i)-k1r.*(-
1+qtrvr).*x27(i)).*x3(i))+l220(i).*(-(mutgsl.*x20(i))-k2.*(-1+u14s).*(-
1+u16s).*x20(i)+k3.*x24(i).*x4(i)-k1s.*(-1+qtgvs).*(-1+u11s).*(-
1+u12s).*x26(i).*x4(i))+l24(i).*(kprimo.*x1(i).*(x26(i)+x27(i))-mutg.*x4(i)+u13s.*x4(i)-
k3.*(x24(i)+x25(i)).*x4(i)+k1s.*(-1+qtgvs).*(-1+u11s).*(-1+u12s).*x26(i).*x4(i)+k1r.*(-
1+qtgvr).*x27(i).*x4(i))+l222(i).*(-((k2+mutgrl).*x22(i))+(k3.*x25(i)-k1r.*(-
1+qtgvr).*x27(i)).*x4(i))+l224(i).*(-(mumhsa.*x24(i))+k1sm.*(-1+u15s).*(-
1+u16s).*x26(i).*x5(i))+l225(i).*(-(mumhra.*x25(i))+k1rm.*x27(i).*x5(i))+l25(i).*(s1mh-
(mumh+k1sm.*(-1+u15s).*(-1+u16s).*x26(i)+k1rm.*x27(i)).*x5(i))+l223(i).*(k2.*x22(i)-
x23(i).*(mutgra+ir.*x7(i)))+l211(i).*(k2.*x10(i)-
x11(i).*(muthra+ir.*x7(i)))+l219(i).*(k2.*x18(i)-
x19(i).*(mutrra+ir.*x7(i)))+l215(i).*(k2.*x14(i)-x15(i).*(mutsra+ir.*x7(i)))+l221(i).*(k2.*(-
1+u14s).*(-1+u16s).*x20(i)-x21(i).*(mutgsa+is.*x7(i)))+l217(i).*(k2.*(-1+u14s).*(-
1+u16s).*x16(i)-x17(i).*(mutrsa+is.*x7(i)))+l213(i).*(k2.*(-1+u14s).*(-1+u16s).*x12(i)-
x13(i).*(mutssa+is.*x7(i)))+x8(i)+l28(i).*(k3.*x1(i).*x24(i)-k1s.*(-1+qthvs).*(-1+u11s).*(-
1+u12s).*x1(i).*x26(i)-muthsl.*x8(i)-k2.*(-1+u14s).*(-1+u16s).*x8(i))+x9(i)+l29(i).*(k2.*(-
1+u14s).*(-1+u16s).*x8(i)-(muthsa+is.*x7(i)).*x9(i))+l26(i).*x6(i).*(-muip-
c.*q.*(x11(i)+x13(i)+x15(i)+x17(i)+x19(i)+x21(i)+x23(i)+x9(i))+c.*(x1(i)+x2(i)+x3(i)+x4(i)
).*(x11(i)+x13(i)+x15(i)+x17(i)+x19(i)+x21(i)+x23(i)+x9(i)))+l27(i).*(-
```

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(muie.*x7(i))+c.*q.*x6(i).*(x11(i)+x13(i)+x15(i)+x17(i)+x19(i)+x21(i)+x23(i)+x9(i)))+l227(
i).*(pr.*u22s.*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))-
muvr.*x27(i)+(g.*x27(i))/(b2+x27(i))+k1r.*rhor.*x27(i).*((-1+qthvr).*x1(i)+(-
1+qtsvr).*x2(i)-x3(i)+qtrvr.*x3(i)-x4(i)+qtgvr.*x4(i))-
x27(i).*(prprimo.*x3(i)+srprimo.*x4(i))-k1rm.*rhor.*x27(i).*x5(i)-ps.*(-
1+u21s).*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i)))+l226(i).*(pr.*(-1+u15s).*(-
1+u22s).*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))-muvs.*x26(i)-(g.*(-
1+u15s).*x26(i))/(b2+x26(i))+k1s.*rhos.*(-1+u11s).*(-1+u12s).*x26(i).*((-
1+qthvs).*x1(i)+(-1+qtsvs).*x2(i)-x3(i)+qtrvs.*x3(i)-x4(i)+qtgvs.*x4(i))-
x26(i).*(psprimo.*x2(i)+ssprimo.*x4(i))-k1sm.*rhos.*(-1+u15s).*(-1+u16s).*x26(i).*x5(i)-
ps.*(-1+u15s).*u21s.*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i)))+l21(i).*(-
(muth.*x1(i))+u13s.*x1(i)-k3.*x1(i).*(x24(i)+x25(i))+k1s.*(-1+qthvs).*(-1+u11s).*(-
1+u12s).*x1(i).*x26(i)+k1r.*(-1+qthvr).*x1(i).*x27(i)-k.*x1(i).*(x26(i)+x27(i))-
kprimo.*x1(i).*(x26(i)+x27(i))+(oth.*s1th)/(oth+x26(i)+x27(i))+r.*x1(i).*(1-
(x1(i)+x10(i)+x11(i)+x12(i)+x13(i)+x14(i)+x15(i)+x16(i)+x17(i)+x18(i)+x19(i)+x2(i)+x20(i
)+x21(i)+x22(i)+x23(i)+x3(i)+x4(i)+x8(i)+x9(i))/tmax))';

x1=zeros(1,N+1); x2=zeros(1,N+1); x3=zeros(1,N+1); x4=zeros(1,N+1);

x5=zeros(1,N+1); x6=zeros(1,N+1); x7=zeros(1,N+1); x8=zeros(1,N+1);

x9=zeros(1,N+1); x10=zeros(1,N+1); x11=zeros(1,N+1); x12=zeros(1,N+1);

x13=zeros(1,N+1); x14=zeros(1,N+1); x15=zeros(1,N+1); x16=zeros(1,N+1);

x17=zeros(1,N+1); x18=zeros(1,N+1); x19=zeros(1,N+1); x20=zeros(1,N+1);

x21=zeros(1,N+1); x22=zeros(1,N+1); x23=zeros(1,N+1); x24=zeros(1,N+1);

x25=zeros(1,N+1); x26=zeros(1,N+1);
x27=zeros(1,N+1);x0=[th0;ts0;tr0;tg0;mh0;ip0;ie0;thsl0;thsa0;thrl0;thra0;tssl0;tssa0;...

    tsrl0;tsra0;trsl0;trsa0;trrl0;trra0;tgsl0;tgsa0;tgrl0;tgra0;mhsa0;...

    mhra0;vs0;vr0;];

x1(:,1)=x0(1); x2(:,1)=x0(2); x3(:,1)=x0(3); x4(:,1)=x0(4); x5(:,1)=x0(5);

x6(:,1)=x0(6); x7(:,1)=x0(7); x8(:,1)=x0(8); x9(:,1)=x0(9); x10(:,1)=x0(10);

x11(:,1)=x0(11); x12(:,1)=x0(12); x13(:,1)=x0(13); x14(:,1)=x0(14);

x15(:,1)=x0(15); x16(:,1)=x0(16); x17(:,1)=x0(17); x18(:,1)=x0(18);

x19(:,1)=x0(19); x20(:,1)=x0(20); x21(:,1)=x0(21); x22(:,1)=x0(22);

x23(:,1)=x0(23); x24(:,1)=x0(24); x25(:,1)=x0(25); x26(:,1)=x0(26);

x27(:,1)=x0(27);


%%%%%%%%lambda al tempo t0 che massimizzano la randomizzazione

l11=0.8693*ones(1,N+1); l12=0.5679 *ones(1,N+1); l13=0.1202 *ones(1,N+1);

l14=0.9703*ones(1,N+1); l15=0.8400*ones(1,N+1); l16=0.0623*ones(1,N+1);

l17=0.2781*ones(1,N+1); l18=0.4096*ones(1,N+1); l19=0.6268*ones(1,N+1);

l110=0.9191*ones(1,N+1); l111=0.4152*ones(1,N+1); l112=0.7115*ones(1,N+1);

l113=0.9709*ones(1,N+1); l114=0.4355*ones(1,N+1); l115=0.6354*ones(1,N+1);
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```
l116=0.9016 *ones(1,N+1); l117=0.8716*ones(1,N+1); l118=0.1427*ones(1,N+1);
l119=0.2006*ones(1,N+1); l120=0.6209*ones(1,N+1); l121=0.7750*ones(1,N+1);
l122=0.2468*ones(1,N+1); l123=0.3695*ones(1,N+1); l124=0.3411*ones(1,N+1);
l125=0.4564*ones(1,N+1); l126=0.1708*ones(1,N+1); l127=0.7456*ones(1,N+1);
%
l21=0.0980*ones(1,N+1); l22=0.8903*ones(1,N+1); l23=0.0821*ones(1,N+1);
l24=0.7815*ones(1,N+1); l25=0.8487*ones(1,N+1); l26=0.4509*ones(1,N+1);
l27=0.6053*ones(1,N+1); l28=0.3545*ones(1,N+1); l29=0.7591*ones(1,N+1);
l210=0.5894*ones(1,N+1); l211=0.6116*ones(1,N+1); l212=0.8432*ones(1,N+1);
l213=0.4570*ones(1,N+1); l214=0.8828*ones(1,N+1); l215=0.4936*ones(1,N+1);
l216=0.6229*ones(1,N+1); l217=0.5622*ones(1,N+1); l218=0.7217*ones(1,N+1);
l219=0.5468*ones(1,N+1); l220=0.7844*ones(1,N+1); l221=0.4464*ones(1,N+1);
l222=0.9119*ones(1,N+1); l223=0.0866*ones(1,N+1); l224=0.9989*ones(1,N+1);
l225=0.7276*ones(1,N+1); l226=0.5584*ones(1,N+1); l227=0.1428*ones(1,N+1);
% % % lambda_1_max =
% % % 0.8693    0.5679    0.1202    0.9703    0.8400
% % % 0.0623    0.2781    0.4096    0.6268    0.9191
% % % 0.4152    0.7115    0.9709    0.4355    0.6354
% % % 0.9016    0.8716    0.1427    0.2006    0.6209
% % % 0.7750    0.2468    0.3695    0.3411    0.4564
% % % 0.1708    0.7456
% % % lambda_2_max =
% % % 0.0980    0.8903    0.0821    0.7815    0.8487
% % % 0.4509    0.6053    0.3545    0.7591    0.5894
% % % 0.6116    0.8432    0.4570    0.8828    0.4936
% % % 0.6229    0.5622    0.7217    0.5468    0.7844
% % % 0.4464    0.9119    0.0866    0.9989    0.7276
% % % 0.5584    0.1428

%%%%%%%%%%%%%%% lambda al tempo t0 tutti uguali a 0.1
% l11=0.1*ones(1,N+1); l12=0.1*ones(1,N+1); l13=0.1*ones(1,N+1);
% l14=0.1*ones(1,N+1); l15=0.1*ones(1,N+1); l16=0.1*ones(1,N+1);
% l17=0.1*ones(1,N+1); l18=0.1*ones(1,N+1); l19=0.1*ones(1,N+1);
```

```
% l110=0.1*ones(1,N+1); l111=0.1*ones(1,N+1); l112=0.1*ones(1,N+1);

% l113=0.1*ones(1,N+1); l114=0.1*ones(1,N+1); l115=0.1*ones(1,N+1);

% l116=0.1*ones(1,N+1); l117=0.1*ones(1,N+1); l118=0.1*ones(1,N+1);

% l119=0.1*ones(1,N+1); l120=0.1*ones(1,N+1); l121=0.1*ones(1,N+1);

% l122=0.1*ones(1,N+1); l123=0.1*ones(1,N+1); l124=0.1*ones(1,N+1);

% l125=0.1*ones(1,N+1); l126=0.1*ones(1,N+1); l127=0.1*ones(1,N+1);

% %

% l21=0.1*ones(1,N+1); l22=0.1*ones(1,N+1); l23=0.1*ones(1,N+1);

% l24=0.1*ones(1,N+1); l25=0.1*ones(1,N+1); l26=0.1*ones(1,N+1);

% l27=0.1*ones(1,N+1); l28=0.1*ones(1,N+1); l29=0.1*ones(1,N+1);

% l210=0.1*ones(1,N+1); l211=0.1*ones(1,N+1); l212=0.1*ones(1,N+1);

% l213=0.1*ones(1,N+1); l214=0.1*ones(1,N+1); l215=0.1*ones(1,N+1);

% l216=0.1*ones(1,N+1); l217=0.1*ones(1,N+1); l218=0.1*ones(1,N+1);

% l219=0.1*ones(1,N+1); l220=0.1*ones(1,N+1); l221=0.1*ones(1,N+1);

% l222=0.1*ones(1,N+1); l223=0.1*ones(1,N+1); l224=0.1*ones(1,N+1);

% l225=0.1*ones(1,N+1); l226=0.1*ones(1,N+1); l227=0.1*ones(1,N+1);x=[x1; x2; x3; x4; x5; x6; x7; x8; x9; x10; x11; x12; x13; x14; x15;...

x16; x17; x18; x19; x20; x21; x22; x23; x24; x25; x26; x27];l1=[l11; l12; l13; l14; l15; l16; l17; l18; l19; l110; l111; l112; l113;...

l114; l115; l116; l117; l118; l119; l120; l121; l122; l123; l124;...

l125; l126; l127];

l2=[l21; l22; l23; l24; l25; l26; l27; l28; l29; l210; l211; l212; l213;...

l214; l215; l216; l217; l218; l219; l220; l221; l222; l223; l224;...

l225; l226; l227];

u1=zeros(6,N+1);

u2=zeros(2,N+1);JSI=zeros(1,N+1); JV=zeros(1,N+1);

ess1=zeros(1,N+1);

ess2=zeros(1,N+1);

ess3=zeros(1,N+1);

ess4=zeros(1,N+1);

ess5=zeros(1,N+1);

conta=0;

%%%%%%%%%%%%%%%%CICLO WHILE

while(test<0  && conta<11)    %si potrebbe anche aggiungere   &&  testess<0)
```

```
conta=conta+1;

oldx=x;

oldl1=l1;

oldl2=l2;

oldu1=u1;

oldu2=u2;

oldJSI=JSI;

oldJV=JV;



%i=1;


%%%%%%%%%%%FORWARD PROCESS

for i=1:N


  %CONTROLLI COEEFICIENTI DI RUNGE KUTTA 1° ORDINE

  a11= (1.*(k1s*(1.-1.*qthvs)*l11(i)*x1(i)*x26(i)-1.*k1s*(1.-
1.*qthvs)*l18(i)*x1(i)*x26(i)-1.*k1s*(1.-1.*qtsvs)*l112(i)*x2(i)*x26(i)+k1s*(1.-
1.*qtsvs)*l12(i)*x2(i)*x26(i)-1.*k1s*(1.-1.*qtrvs)*l116(i)*x26(i)*x3(i)+k1s*(1.-
1.*qtrvs)*l13(i)*x26(i)*x3(i)-1.*k1s*(1.-1.*qtgvs)*l120(i)*x26(i)*x4(i)+k1s*(1.-
1.*qtgvs)*l14(i)*x26(i)*x4(i)+k1s*rhos*l126(i)*x26(i)*((1.-1.*qthvs)*x1(i)+(1.-
1.*qtsvs)*x2(i)+(1.-1.*qtrvs)*x3(i)+(1.-1.*qtgvs)*x4(i))))/aco-(1.*(-1.*k1s*(1.-
1.*qthvs)*l11(i)*x1(i)*x26(i)+k1s*(1.-1.*qthvs)*l18(i)*x1(i)*x26(i)+k1s*(1.-
1.*qtsvs)*l112(i)*x2(i)*x26(i)-1.*k1s*(1.-1.*qtsvs)*l12(i)*x2(i)*x26(i)+k1s*(1.-
1.*qtrvs)*l116(i)*x26(i)*x3(i)-1.*k1s*(1.-1.*qtrvs)*l13(i)*x26(i)*x3(i)+k1s*(1.-
1.*qtgvs)*l120(i)*x26(i)*x4(i)-1.*k1s*(1.-1.*qtgvs)*l14(i)*x26(i)*x4(i)-
1.*k1s*rhos*l126(i)*x26(i)*((1.-1.*qthvs)*x1(i)+(1.-1.*qtsvs)*x2(i)+(1.-1.*qtrvs)*x3(i)+(1.-
1.*qtgvs)*x4(i)))*(-1.*aco*(k1s*(1.-1.*qthvs)*l11(i)*x1(i)*x26(i)-1.*k1s*(1.-
1.*qthvs)*l18(i)*x1(i)*x26(i)-1.*k1s*(1.-1.*qtsvs)*l112(i)*x2(i)*x26(i)+k1s*(1.-
1.*qtsvs)*l12(i)*x2(i)*x26(i)-1.*k1s*(1.-1.*qtrvs)*l116(i)*x26(i)*x3(i)+k1s*(1.-
1.*qtrvs)*l13(i)*x26(i)*x3(i)-1.*k1s*(1.-1.*qtgvs)*l120(i)*x26(i)*x4(i)+k1s*(1.-
1.*qtgvs)*l14(i)*x26(i)*x4(i)+k1s*rhos*l126(i)*x26(i)*((1.-1.*qthvs)*x1(i)+(1.-
1.*qtsvs)*x2(i)+(1.-1.*qtrvs)*x3(i)+(1.-1.*qtgvs)*x4(i)))-1.*(-1.*k1s*(1.-
1.*qthvs)*l11(i)*x1(i)*x26(i)+k1s*(1.-1.*qthvs)*l18(i)*x1(i)*x26(i)+k1s*(1.-
1.*qtsvs)*l112(i)*x2(i)*x26(i)-1.*k1s*(1.-1.*qtsvs)*l12(i)*x2(i)*x26(i)+k1s*(1.-
1.*qtrvs)*l116(i)*x26(i)*x3(i)-1.*k1s*(1.-1.*qtrvs)*l13(i)*x26(i)*x3(i)+k1s*(1.-
1.*qtgvs)*l120(i)*x26(i)*x4(i)-1.*k1s*(1.-1.*qtgvs)*l14(i)*x26(i)*x4(i)-
1.*k1s*rhos*l126(i)*x26(i)*((1.-1.*qthvs)*x1(i)+(1.-1.*qtsvs)*x2(i)+(1.-1.*qtrvs)*x3(i)+(1.-
1.*qtgvs)*x4(i)))*(k1s*(1.-1.*qthvs)*l11(i)*x1(i)*x26(i)-1.*k1s*(1.-
1.*qthvs)*l18(i)*x1(i)*x26(i)-1.*k1s*(1.-1.*qtsvs)*l112(i)*x2(i)*x26(i)+k1s*(1.-
1.*qtsvs)*l12(i)*x2(i)*x26(i)-1.*k1s*(1.-1.*qtrvs)*l116(i)*x26(i)*x3(i)+k1s*(1.-
1.*qtrvs)*l13(i)*x26(i)*x3(i)-1.*k1s*(1.-1.*qtgvs)*l120(i)*x26(i)*x4(i)+k1s*(1.-
1.*qtgvs)*l14(i)*x26(i)*x4(i)+k1s*rhos*l126(i)*x26(i)*((1.-1.*qthvs)*x1(i)+(1.-
1.*qtsvs)*x2(i)+(1.-1.*qtrvs)*x3(i)+(1.-1.*qtgvs)*x4(i)))))/(aco*(1.*aco*afi-1.*(-1.*k1s*(1.-
1.*qthvs)*l11(i)*x1(i)*x26(i)+k1s*(1.-1.*qthvs)*l18(i)*x1(i)*x26(i)+k1s*(1.-
1.*qtsvs)*l112(i)*x2(i)*x26(i)-1.*k1s*(1.-1.*qtsvs)*l12(i)*x2(i)*x26(i)+k1s*(1.-
```

```
1.*qtrvs)*l116(i)*x26(i)*x3(i)-1.*k1s*(1.-1.*qtrvs)*l13(i)*x26(i)*x3(i)+k1s*(1.-
1.*qtgvs)*l120(i)*x26(i)*x4(i)-1.*k1s*(1.-1.*qtgvs)*l14(i)*x26(i)*x4(i)-
1.*k1s*rhos*l126(i)*x26(i)*((1.-1.*qthvs)*x1(i)+(1.-1.*qtsvs)*x2(i)+(1.-1.*qtrvs)*x3(i)+(1.-
1.*qtgvs)*x4(i)))^2));        a12=-((-1.*aco*(k1s*(1.-1.*qthvs)*l11(i)*x1(i)*x26(i)-1.*k1s*(1.-
1.*qthvs)*l18(i)*x1(i)*x26(i)-1.*k1s*(1.-1.*qtsvs)*l112(i)*x2(i)*x26(i)+k1s*(1.-
1.*qtsvs)*l12(i)*x2(i)*x26(i)-1.*k1s*(1.-1.*qtrvs)*l116(i)*x26(i)*x3(i)+k1s*(1.-
1.*qtrvs)*l13(i)*x26(i)*x3(i)-1.*k1s*(1.-1.*qtgvs)*l120(i)*x26(i)*x4(i)+k1s*(1.-
1.*qtgvs)*l14(i)*x26(i)*x4(i)+k1s*rhos*l126(i)*x26(i)*((1.-1.*qthvs)*x1(i)+(1.-
1.*qtsvs)*x2(i)+(1.-1.*qtrvs)*x3(i)+(1.-1.*qtgvs)*x4(i)))-1.*(-1.*k1s*(1.-
1.*qthvs)*l11(i)*x1(i)*x26(i)+k1s*(1.-1.*qthvs)*l18(i)*x1(i)*x26(i)+k1s*(1.-
1.*qtsvs)*l112(i)*x2(i)*x26(i)-1.*k1s*(1.-1.*qtsvs)*l12(i)*x2(i)*x26(i)+k1s*(1.-
1.*qtrvs)*l116(i)*x26(i)*x3(i)-1.*k1s*(1.-1.*qtrvs)*l13(i)*x26(i)*x3(i)+k1s*(1.-
1.*qtgvs)*l120(i)*x26(i)*x4(i)-1.*k1s*(1.-1.*qtgvs)*l14(i)*x26(i)*x4(i)-
1.*k1s*rhos*l126(i)*x26(i)*((1.-1.*qthvs)*x1(i)+(1.-1.*qtsvs)*x2(i)+(1.-1.*qtrvs)*x3(i)+(1.-
1.*qtgvs)*x4(i)))*(k1s*(1.-1.*qthvs)*l11(i)*x1(i)*x26(i)-1.*k1s*(1.-
1.*qthvs)*l18(i)*x1(i)*x26(i)-1.*k1s*(1.-1.*qtsvs)*l112(i)*x2(i)*x26(i)+k1s*(1.-
1.*qtsvs)*l12(i)*x2(i)*x26(i)-1.*k1s*(1.-1.*qtrvs)*l116(i)*x26(i)*x3(i)+k1s*(1.-
1.*qtrvs)*l13(i)*x26(i)*x3(i)-1.*k1s*(1.-1.*qtgvs)*l120(i)*x26(i)*x4(i)+k1s*(1.-
1.*qtgvs)*l14(i)*x26(i)*x4(i)+k1s*rhos*l126(i)*x26(i)*((1.-1.*qthvs)*x1(i)+(1.-
1.*qtsvs)*x2(i)+(1.-1.*qtrvs)*x3(i)+(1.-1.*qtgvs)*x4(i))))/(1.*aco*afi-1.*(-1.*k1s*(1.-
1.*qthvs)*l11(i)*x1(i)*x26(i)+k1s*(1.-1.*qthvs)*l18(i)*x1(i)*x26(i)+k1s*(1.-
1.*qtsvs)*l112(i)*x2(i)*x26(i)-1.*k1s*(1.-1.*qtsvs)*l12(i)*x2(i)*x26(i)+k1s*(1.-
1.*qtrvs)*l116(i)*x26(i)*x3(i)-1.*k1s*(1.-1.*qtrvs)*l13(i)*x26(i)*x3(i)+k1s*(1.-
1.*qtgvs)*l120(i)*x26(i)*x4(i)-1.*k1s*(1.-1.*qtgvs)*l14(i)*x26(i)*x4(i)-
1.*k1s*rhos*l126(i)*x26(i)*((1.-1.*qthvs)*x1(i)+(1.-1.*qtsvs)*x2(i)+(1.-1.*qtrvs)*x3(i)+(1.-
1.*qtgvs)*x4(i)))^2));        a13=(1.*(l11(i)*x1(i)+l12(i)*x2(i)+l13(i)*x3(i)+l14(i)*x4(i)))/aib;
a14=(1.*(k2*l112(i)*x12(i)-1.*k2*l113(i)*x12(i)+k2*l116(i)*x16(i)-
1.*k2*l117(i)*x16(i)+k2*l120(i)*x20(i)-1.*k2*l121(i)*x20(i)+k2*l18(i)*x8(i)-
1.*k2*l19(i)*x8(i)))/aii-(1.*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-
1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-
1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i))*((1.*(1.*bsr*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x
25(i))*(1.*brs-
1.*pr*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))+pr*l227(i)*(x11(i)+x15(i)+x19(i)+x23(i
)+x25(i)))-1.*brs*(-1.*bsr*(-1.*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))-
(1.*g*l126(i)*x26(i))/(b2+x26(i))-
1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i))+
1.*ps*l126(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))*(1.*bsr+ps*l226(i)*(x13(i)+x17(i)+x21(i)
+x24(i)+x9(i))-
1.*ps*l227(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))))))/(brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-
1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-1.*k1sm*l15(i)*x26(i)*x5(i)))-
(1.*(1.*bsr*pr^2*l126(i)*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))^2-
1.*brs*(1.*api*bsr-
1.*ps^2*l126(i)*l226(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))^2))*(1.*brs*bsr*(k1sm*l124(i)
*x26(i)*x5(i)-1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-1.*k1sm*l15(i)*x26(i)*x5(i))*(-
1.*aii*(k2*l112(i)*x12(i)-1.*k2*l113(i)*x12(i)+k2*l116(i)*x16(i)-
1.*k2*l117(i)*x16(i)+k2*l120(i)*x20(i)-1.*k2*l121(i)*x20(i)-
1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i)+k
2*l18(i)*x8(i)-1.*k2*l19(i)*x8(i))-1.*(k2*l112(i)*x12(i)-
1.*k2*l113(i)*x12(i)+k2*l116(i)*x16(i)-1.*k2*l117(i)*x16(i)+k2*l120(i)*x20(i)-
1.*k2*l121(i)*x20(i)+k2*l18(i)*x8(i)-1.*k2*l19(i)*x8(i))*(-
1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-
1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i)))-1.*(1.*aii*art-
1.*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-
1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-
1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i))^2)*(1.*bsr*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x2
5(i))*(1.*brs-
1.*pr*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))+pr*l227(i)*(x11(i)+x15(i)+x19(i)+x23(i
)+x25(i)))-1.*brs*(-1.*bsr*(-1.*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))-
(1.*g*l126(i)*x26(i))/(b2+x26(i))-
1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i))+
```

```
1.*ps*l126(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))*(1.*bsr+ps*l226(i)*(x13(i)+x17(i)+x21(i)
+x24(i)+x9(i))-
1.*ps*l227(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i)))))))/(brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-
1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-1.*k1sm*l15(i)*x26(i)*x5(i))*(-
1.*aii*brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-
1.*k1sm*l15(i)*x26(i)*x5(i))^2-1.*(1.*aii*art-1.*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-
1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-
1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i))^2)*(1.*bsr*pr^2*l126(i)*l226(i)*(x11(i)+x15(i)+x19(i)+
x23(i)+x25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*l126(i)*l226(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))^2))))))/aii;        a15= -
((1.*brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-
1.*k1sm*l15(i)*x26(i)*x5(i))*(-1.*aii*(k2*l112(i)*x12(i)-
1.*k2*l113(i)*x12(i)+k2*l116(i)*x16(i)-1.*k2*l117(i)*x16(i)+k2*l120(i)*x20(i)-
1.*k2*l121(i)*x20(i)-
1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i)+k
2*l18(i)*x8(i)-1.*k2*l19(i)*x8(i))-1.*(k2*l112(i)*x12(i)-
1.*k2*l113(i)*x12(i)+k2*l116(i)*x16(i)-1.*k2*l117(i)*x16(i)+k2*l120(i)*x20(i)-
1.*k2*l121(i)*x20(i)+k2*l18(i)*x8(i)-1.*k2*l19(i)*x8(i))*(-
1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-
1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i)))-1.*(1.*aii*art-
1.*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-
1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-
1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i))^2)*(1.*bsr*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x2
5(i))*(1.*brs-
1.*pr*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))+pr*l227(i)*(x11(i)+x15(i)+x19(i)+x23(i
)+x25(i)))-1.*brs*(-1.*bsr*(-1.*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))-
(1.*g*l126(i)*x26(i))/(b2+x26(i))-
1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i))+
1.*ps*l126(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))*(1.*bsr+ps*l226(i)*(x13(i)+x17(i)+x21(i)
+x24(i)+x9(i))-1.*ps*l227(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))))))/(-
1.*aii*brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-
1.*k1sm*l15(i)*x26(i)*x5(i))^2-1.*(1.*aii*art-1.*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-
1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-
1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i))^2)*(1.*bsr*pr^2*l126(i)*l226(i)*(x11(i)+x15(i)+x19(i)+
x23(i)+x25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*l126(i)*l226(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))^2))));        a16=(-
1.*(1.*bsr*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))*(1.*brs-
1.*pr*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))+pr*l227(i)*(x11(i)+x15(i)+x19(i)+x23(i
)+x25(i)))-1.*brs*(-1.*bsr*(-1.*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))-
(1.*g*l126(i)*x26(i))/(b2+x26(i))-
1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i))+
1.*ps*l126(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))*(1.*bsr+ps*l226(i)*(x13(i)+x17(i)+x21(i)
+x24(i)+x9(i))-
1.*ps*l227(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))))))/(brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-
1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-
1.*k1sm*l15(i)*x26(i)*x5(i)))+(1.*(1.*bsr*pr^2*l126(i)*l226(i)*(x11(i)+x15(i)+x19(i)+x23(
i)+x25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*l126(i)*l226(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))^2))*(1.*brs*bsr*(k1sm*l124(i)
*x26(i)*x5(i)-1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-1.*k1sm*l15(i)*x26(i)*x5(i))*(-
1.*aii*(k2*l112(i)*x12(i)-1.*k2*l113(i)*x12(i)+k2*l116(i)*x16(i)-
1.*k2*l117(i)*x16(i)+k2*l120(i)*x20(i)-1.*k2*l121(i)*x20(i)-
1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i)+k
2*l18(i)*x8(i)-1.*k2*l19(i)*x8(i))-1.*(k2*l112(i)*x12(i)-
1.*k2*l113(i)*x12(i)+k2*l116(i)*x16(i)-1.*k2*l117(i)*x16(i)+k2*l120(i)*x20(i)-
1.*k2*l121(i)*x20(i)+k2*l18(i)*x8(i)-1.*k2*l19(i)*x8(i))*(-
1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-
1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i)))-1.*(1.*aii*art-
1.*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-
1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-
1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i))^2)*(1.*bsr*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x2
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5(i))*(1.*brs-
1.*pr*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))+pr*l227(i)*(x11(i)+x15(i)+x19(i)+x23(i
)+x25(i)))-1.*brs*(-1.*bsr*(-1.*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))-
(1.*g*l126(i)*x26(i))/(b2+x26(i))-
1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i))+
1.*ps*l126(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))*(1.*bsr+ps*l226(i)*(x13(i)+x17(i)+x21(i)
+x24(i)+x9(i))-
1.*ps*l227(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i)))))))/(brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-
1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-1.*k1sm*l15(i)*x26(i)*x5(i))*(-
1.*aii*brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-
1.*k1sm*l15(i)*x26(i)*x5(i))^2-1.*(1.*aii*art-1.*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-
1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-
1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i))^2)*(1.*bsr*pr^2*l126(i)*l226(i)*(x11(i)+x15(i)+x19(i)+
x23(i)+x25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*l126(i)*l226(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))^2)))); u21s=
min(1,max(0,(1.*(1.*bsr+ps*l226(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))-
1.*ps*l227(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))))/bsr+(1.*ps*l226(i)*(x13(i)+x17(i)+x21(
i)+x24(i)+x9(i))*(1.*brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-
1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-1.*k1sm*l15(i)*x26(i)*x5(i))*(-
1.*aii*(k2*l112(i)*x12(i)-1.*k2*l113(i)*x12(i)+k2*l116(i)*x16(i)-
1.*k2*l117(i)*x16(i)+k2*l120(i)*x20(i)-1.*k2*l121(i)*x20(i)-
1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i)+k
2*l18(i)*x8(i)-1.*k2*l19(i)*x8(i))-1.*(k2*l112(i)*x12(i)-
1.*k2*l113(i)*x12(i)+k2*l116(i)*x16(i)-1.*k2*l117(i)*x16(i)+k2*l120(i)*x20(i)-
1.*k2*l121(i)*x20(i)+k2*l18(i)*x8(i)-1.*k2*l19(i)*x8(i))*(-
1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-
1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i)))-1.*(1.*aii*art-
1.*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-
1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-
1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i))^2)*(1.*bsr*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x2
5(i))*(1.*brs-
1.*pr*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))+pr*l227(i)*(x11(i)+x15(i)+x19(i)+x23(i
)+x25(i)))-1.*brs*(-1.*bsr*(-1.*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))-
(1.*g*l126(i)*x26(i))/(b2+x26(i))-
1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i))+
1.*ps*l126(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))*(1.*bsr+ps*l226(i)*(x13(i)+x17(i)+x21(i)
+x24(i)+x9(i))-1.*ps*l227(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i)))))))/(bsr*(-
1.*aii*brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-
1.*k1sm*l15(i)*x26(i)*x5(i))^2-1.*(1.*aii*art-1.*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-
1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-
1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i))^2)*(1.*bsr*pr^2*l126(i)*l226(i)*(x11(i)+x15(i)+x19(i)+
x23(i)+x25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*l126(i)*l226(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))^2)))))); u22s=
min(1,max(0,(1.*(1.*brs-
1.*pr*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))+pr*l227(i)*(x11(i)+x15(i)+x19(i)+x23(i
)+x25(i))))/brs-
(1.*pr*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))*(1.*brs*bsr*(k1sm*l124(i)*x26(i)*x5(i
)-1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-1.*k1sm*l15(i)*x26(i)*x5(i))*(-
1.*aii*(k2*l112(i)*x12(i)-1.*k2*l113(i)*x12(i)+k2*l116(i)*x16(i)-
1.*k2*l117(i)*x16(i)+k2*l120(i)*x20(i)-1.*k2*l121(i)*x20(i)-
1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i)+k
2*l18(i)*x8(i)-1.*k2*l19(i)*x8(i))-1.*(k2*l112(i)*x12(i)-
1.*k2*l113(i)*x12(i)+k2*l116(i)*x16(i)-1.*k2*l117(i)*x16(i)+k2*l120(i)*x20(i)-
1.*k2*l121(i)*x20(i)+k2*l18(i)*x8(i)-1.*k2*l19(i)*x8(i))*(-
1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-
1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i)))-1.*(1.*aii*art-
1.*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-
1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-
1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i))^2)*(1.*bsr*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x2
5(i))*(1.*brs-

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1.*pr*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))+pr*l227(i)*(x11(i)+x15(i)+x19(i)+x23(i
)+x25(i)))-1.*brs*(-1.*bsr*(-1.*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))-
(1.*g*l126(i)*x26(i))/(b2+x26(i))-
1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i))+
1.*ps*l126(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))*(1.*bsr+ps*l226(i)*(x13(i)+x17(i)+x21(i)
+x24(i)+x9(i))-1.*ps*l227(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i)))))))/(brs*(-
1.*aii*brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-
1.*k1sm*l15(i)*x26(i)*x5(i))^2-1.*(1.*aii*art-1.*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-
1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-
1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i))^2)*(1.*bsr*pr^2*l126(i)*l226(i)*(x11(i)+x15(i)+x19(i)+
x23(i)+x25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*l126(i)*l226(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))^2))))));


    if(a11<0 || a11>1)
        a11=0;
    end;

    if(a12<0 || a12>1)
        a12=0;
    end;

    if(a13<0 || a13>1)
        a13=0;
    end;

    if(a14<0 || a14>1)
        a14=0;
    end;

    if(a15<0 || a15>1)
        a15=0;
    end;

    if(a16<0 || a16>1)
        a16=0;
    end;
```

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a12,0,1,0,1;0,a12,0,1,0,0;0,a12,0,0,a15,a16;0,a12,0,0,a15,1;0,a12,0,0,a15,0;0,a12,0,0,1,a16;0,
a12,0,0,1,1;0,a12,0,0,1,0;0,a12,0,0,0,a16;0,a12,0,0,0,1;0,a12,0,0,0,0;0,1,a13,a14,a15,a16;0,1,
a13,a14,a15,1;0,1,a13,a14,a15,0;0,1,a13,a14,1,a16;0,1,a13,a14,1,1;0,1,a13,a14,1,0;0,1,a13,a1
4,0,a16;0,1,a13,a14,0,1;0,1,a13,a14,0,0;0,1,a13,1,a15,a16;0,1,a13,1,a15,1;0,1,a13,1,a15,0;0,1,
a13,1,1,a16;0,1,a13,1,1,1;0,1,a13,1,1,0;0,1,a13,1,0,a16;0,1,a13,1,0,1;0,1,a13,1,0,0;0,1,a13,0,a
15,a16;0,1,a13,0,a15,1;0,1,a13,0,a15,0;0,1,a13,0,1,a16;0,1,a13,0,1,1;0,1,a13,0,1,0;0,1,a13,0,0
,a16;0,1,a13,0,0,1;0,1,a13,0,0,0;0,1,1,a14,a15,a16;0,1,1,a14,a15,1;0,1,1,a14,a15,0;0,1,1,a14,1
,a16;0,1,1,a14,1,1;0,1,1,a14,1,0;0,1,1,a14,0,a16;0,1,1,a14,0,1;0,1,1,a14,0,0;0,1,1,1,a15,a16;0,
1,1,1,a15,1;0,1,1,1,a15,0;0,1,1,1,1,a16;0,1,1,1,1,1;0,1,1,1,1,0;0,1,1,1,0,a16;0,1,1,1,0,1;0,1,1,1,
0,0;0,1,1,0,a15,a16;0,1,1,0,a15,1;0,1,1,0,a15,0;0,1,1,0,1,a16;0,1,1,0,1,1;0,1,1,0,1,0;0,1,1,0,0,a
16;0,1,1,0,0,1;0,1,1,0,0,0;0,1,0,a14,a15,a16;0,1,0,a14,a15,1;0,1,0,a14,a15,0;0,1,0,a14,1,a16;0,
1,0,a14,1,1;0,1,0,a14,1,0;0,1,0,a14,0,a16;0,1,0,a14,0,1;0,1,0,a14,0,0;0,1,0,1,a15,a16;0,1,0,1,a
15,1;0,1,0,1,a15,0;0,1,0,1,1,a16;0,1,0,1,1,1;0,1,0,1,1,0;0,1,0,1,0,a16;0,1,0,1,0,1;0,1,0,1,0,0;0,
1,0,0,a15,a16;0,1,0,0,a15,1;0,1,0,0,a15,0;0,1,0,0,1,a16;0,1,0,0,1,1;0,1,0,0,1,0;0,1,0,0,0,a16;0,
1,0,0,0,1;0,1,0,0,0,0;0,0,a13,a14,a15,a16;0,0,a13,a14,a15,1;0,0,a13,a14,a15,0;0,0,a13,a14,1,a
16;0,0,a13,a14,1,1;0,0,a13,a14,1,0;0,0,a13,a14,0,a16;0,0,a13,a14,0,1;0,0,a13,a14,0,0;0,0,a13,
1,a15,a16;0,0,a13,1,a15,1;0,0,a13,1,a15,0;0,0,a13,1,1,a16;0,0,a13,1,1,1;0,0,a13,1,1,0;0,0,a13,
1,0,a16;0,0,a13,1,0,1;0,0,a13,1,0,0;0,0,a13,0,a15,a16;0,0,a13,0,a15,1;0,0,a13,0,a15,0;0,0,a13,
0,1,a16;0,0,a13,0,1,1;0,0,a13,0,1,0;0,0,a13,0,0,a16;0,0,a13,0,0,1;0,0,a13,0,0,0;0,0,1,a14,a15,a
16;0,0,1,a14,a15,1;0,0,1,a14,a15,0;0,0,1,a14,1,a16;0,0,1,a14,1,1;0,0,1,a14,1,0;0,0,1,a14,0,a16
;0,0,1,a14,0,1;0,0,1,a14,0,0;0,0,1,1,a15,a16;0,0,1,1,a15,1;0,0,1,1,a15,0;0,0,1,1,1,a16;0,0,1,1,1
```

,1;0,0,1,1,1,0;0,0,1,1,0,a16;0,0,1,1,0,1;0,0,1,1,0,0;0,0,1,0,a15,a16;0,0,1,0,a15,1;0,0,1,0,a15,0;0,0,1,0,1,a16;0,0,1,0,1,1;0,0,1,0,1,0;0,0,1,0,0,a16;0,0,1,0,0,1;0,0,1,0,0,0;0,0,0,a14,a15,a16;0,0,0,a14,a15,1;0,0,0,a14,a15,0;0,0,0,a14,1,a16;0,0,0,a14,1,1;0,0,0,a14,1,0;0,0,0,a14,0,a16;0,0,0,a14,0,1;0,0,0,a14,0,0;0,0,0,1,a15,a16;0,0,0,1,a15,1;0,0,0,1,a15,0;0,0,0,1,1,a16;0,0,0,1,1,1;0,0,0,1,1,0;0,0,0,1,0,a16;0,0,0,1,0,1;0,0,0,1,0,0;0,0,0,0,a15,a16;0,0,0,0,a15,1;0,0,0,0,a15,0;0,0,0,0,1,a16;0,0,0,0,1,1;0,0,0,0,1,0;0,0,0,0,0,a16;0,0,0,0,0,1;0,0,0,0,0,0];

```
eval_HT=eval(fHT);

[max_HT, id_HT]=nanmax(eval_HT);

u11(1)=D(id_HT,1);
u12(1)=D(id_HT,2);
u13(1)=D(id_HT,3);
u14(1)=D(id_HT,4);
u15(1)=D(id_HT,5);
u16(1)=D(id_HT,6);

u21(1)=u21s;
u22(1)=u22s;

%VARIABILI COEEFICIENTI DI RUNGE KUTTA 1° ORDINE
kv1(1)=-(muth*x1(i))+u13(1)*x1(i)-k3*x1(i)*(x24(i)+x25(i))-k1s*(1-qthvs)*(1-u11(1))*(1-u12(1))*x1(i)*x26(i)-k1r*(1-qthvr)*x1(i)*x27(i)-k*x1(i)*(x26(i)+x27(i))-kprimo*x1(i)*(x26(i)+x27(i))+(oth*s1th)/(oth+x26(i)+x27(i))+r*x1(i)*(1-(x1(i)+x10(i)+x11(i)+x12(i)+x13(i)+x14(i)+x15(i)+x16(i)+x17(i)+x18(i)+x19(i)+x2(i)+x20(i)+x21(i)+x22(i)+x23(i)+x3(i)+x4(i)+x8(i)+x9(i))/tmax);

kv2(1)=-(muts*x2(i))+u13(1)*x2(i)-k3*x2(i)*(x24(i)+x25(i))+k*x1(i)*x26(i)-k1s*(1-qtsvs)*(1-u11(1))*(1-u12(1))*x2(i)*x26(i)-k1r*(1-qtsvr)*x2(i)*x27(i);
kv3(1)=k*x1(i)*x27(i)-mutr*x3(i)+u13(1)*x3(i)-k3*(x24(i)+x25(i))*x3(i)-k1s*(1-qtrvs)*(1-u11(1))*(1-u12(1))*x26(i)*x3(i)-k1r*(1-qtrvr)*x27(i)*x3(i);
kv4(1)=kprimo*x1(i)*(x26(i)+x27(i))-mutg*x4(i)+u13(1)*x4(i)-k3*(x24(i)+x25(i))*x4(i)-k1s*(1-qtgvs)*(1-u11(1))*(1-u12(1))*x26(i)*x4(i)-k1r*(1-qtgvr)*x27(i)*x4(i);
kv5(1)=s1mh-mumh*x5(i)-k1sm*(1-u15(1))*(1-u16(1))*x26(i)*x5(i)-k1rm*x27(i)*x5(i);
kv6(1)=-(muip*x6(i))-
```

```
c*q*x6(i)*(x11(i)+x13(i)+x15(i)+x17(i)+x19(i)+x21(i)+x23(i)+x9(i))+c*(x1(i)+x2(i)+x3(i)+
x4(i))*x6(i)*(x11(i)+x13(i)+x15(i)+x17(i)+x19(i)+x21(i)+x23(i)+x9(i));    kv7(1)=-
(muie*x7(i))+c*q*x6(i)*(x11(i)+x13(i)+x15(i)+x17(i)+x19(i)+x21(i)+x23(i)+x9(i));
kv8(1)=k3*x1(i)*x24(i)+k1s*(1-qthvs)*(1-u11(1))*(1-u12(1))*x1(i)*x26(i)-muthsl*x8(i)-
k2*(1-u14(1))*(1-u16(1))*x8(i);    kv9(1)=k2*(1-u14(1))*(1-u16(1))*x8(i)-muthsa*x9(i)-
is*x7(i)*x9(i);    kv10(1)=-(k2*x10(i))-muthrl*x10(i)+k3*x1(i)*x25(i)+k1r*(1-
qthvr)*x1(i)*x27(i);    kv11(1)=k2*x10(i)-muthra*x11(i)-ir*x11(i)*x7(i);    kv12(1)=-
(mutssl*x12(i))-k2*(1-u14(1))*(1-u16(1))*x12(i)+k3*x2(i)*x24(i)+k1s*(1-qtsvs)*(1-
u11(1))*(1-u12(1))*x2(i)*x26(i);    kv13(1)=k2*(1-u14(1))*(1-u16(1))*x12(i)-
mutssa*x13(i)-is*x13(i)*x7(i);    kv14(1)=-(k2*x14(i))-
mutsrl*x14(i)+k3*x2(i)*x25(i)+k1r*(1-qtsvr)*x2(i)*x27(i);    kv15(1)=k2*x14(i)-
mutsra*x15(i)-ir*x15(i)*x7(i);    kv16(1)=-(mutrsl*x16(i))-k2*(1-u14(1))*(1-
u16(1))*x16(i)+k3*x24(i)*x3(i)+k1s*(1-qtrvs)*(1-u11(1))*(1-u12(1))*x26(i)*x3(i);
kv17(1)=k2*(1-u14(1))*(1-u16(1))*x16(i)-mutrsa*x17(i)-is*x17(i)*x7(i);    kv18(1)=-
(k2*x18(i))-mutrrl*x18(i)+k3*x25(i)*x3(i)+k1r*(1-qtrvr)*x27(i)*x3(i);
kv19(1)=k2*x18(i)-mutrra*x19(i)-ir*x19(i)*x7(i);    kv20(1)=-(mutgsl*x20(i))-k2*(1-
u14(1))*(1-u16(1))*x20(i)+k3*x24(i)*x4(i)+k1s*(1-qtgvs)*(1-u11(1))*(1-
u12(1))*x26(i)*x4(i);    kv21(1)=k2*(1-u14(1))*(1-u16(1))*x20(i)-mutgsa*x21(i)-
is*x21(i)*x7(i);    kv22(1)=-(k2*x22(i))-mutgrl*x22(i)+k3*x25(i)*x4(i)+k1r*(1-
qtgvr)*x27(i)*x4(i);    kv23(1)=k2*x22(i)-mutgra*x23(i)-ir*x23(i)*x7(i);    kv24(1)=-
(mumhsa*x24(i))+k1sm*(1-u15(1))*(1-u16(1))*x26(i)*x5(i);    kv25(1)=-
(mumhra*x25(i))+k1rm*x27(i)*x5(i);    kv26(1)=pr*(1-u15(1))*(1-
u22(1))*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))-muvs*x26(i)+(g*(1-
u15(1))*x26(i))/(b2+x26(i))-k1s*rhos*(1-u11(1))*(1-u12(1))*x26(i)*((1-qthvs)*x1(i)+(1-
qtsvs)*x2(i)+(1-qtrvs)*x3(i)+(1-qtgvs)*x4(i))-x26(i)*(psprimo*x2(i)+ssprimo*x4(i))-
k1sm*rhos*(1-u15(1))*(1-u16(1))*x26(i)*x5(i)+ps*(1-
u15(1))*u21(1)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i));
kv27(1)=pr*u22(1)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))-
muvr*x27(i)+(g*x27(i))/(b2+x27(i))-k1r*rhor*x27(i)*((1-qthvr)*x1(i)+(1-qtsvr)*x2(i)+(1-
qtrvr)*x3(i)+(1-qtgvr)*x4(i))-x27(i)*(prprimo*x3(i)+srprimo*x4(i))-
k1rm*rhor*x27(i)*x5(i)+ps*(1-u21(1))*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i));



% Ora troverete molte variabili di appoggio,
% sono state usate perchè le espressioni simboliche
% prima si sono ricavate da mathematica e poi si sono
% dovute incollare in matlab, facendo tutte le correzioni
% del caso. Il motivo di questa scelta è legato al fatto
% che si è provato a dare in pasto a matlab i sistemi simbolici
% per il calcolo delle soluzioni simboliche, ma questo produceva
% espressioni enormi di difficile lettura e manipolazione,
% anche usando le routines simple, simplify, ecc.
%

%VARIABILI DI APPOGGIO wf per 2° ordine
wf1(i)=x1(i)+h2*kv1(1);
```

```
wf2(i)=x2(i)+h2*kv2(1);
wf3(i)=x3(i)+h2*kv3(1);
wf4(i)=x4(i)+h2*kv4(1);
wf5(i)=x5(i)+h2*kv5(1);
wf6(i)=x6(i)+h2*kv6(1);
wf7(i)=x7(i)+h2*kv7(1);
wf8(i)=x8(i)+h2*kv8(1);
wf9(i)=x9(i)+h2*kv9(1);
wf10(i)=x10(i)+h2*kv10(1);
wf11(i)=x11(i)+h2*kv11(1);
wf12(i)=x12(i)+h2*kv12(1);
wf13(i)=x13(i)+h2*kv13(1);
wf14(i)=x14(i)+h2*kv14(1);
wf15(i)=x15(i)+h2*kv15(1);
wf16(i)=x16(i)+h2*kv16(1);
wf17(i)=x17(i)+h2*kv17(1);
wf18(i)=x18(i)+h2*kv18(1);
wf19(i)=x19(i)+h2*kv19(1);
wf20(i)=x20(i)+h2*kv20(1);
wf21(i)=x21(i)+h2*kv21(1);
wf22(i)=x22(i)+h2*kv22(1);
wf23(i)=x23(i)+h2*kv23(1);
wf24(i)=x24(i)+h2*kv24(1);
wf25(i)=x25(i)+h2*kv25(1);
wf26(i)=x26(i)+h2*kv26(1);
wf27(i)=x27(i)+h2*kv27(1);


%VARIABILI DI APPOGGIO hf per 2° ordine
hf11(i)=0.5*(l11(i)+l11(i+1));
hf12(i)=0.5*(l12(i)+l12(i+1));
hf13(i)=0.5*(l13(i)+l13(i+1));
hf14(i)=0.5*(l14(i)+l14(i+1));
```

```
hf15(i)=0.5*(l15(i)+l15(i+1));
hf16(i)=0.5*(l16(i)+l16(i+1));
hf17(i)=0.5*(l17(i)+l17(i+1));
hf18(i)=0.5*(l18(i)+l18(i+1));
hf19(i)=0.5*(l19(i)+l19(i+1));
hf110(i)=0.5*(l110(i)+l110(i+1));
hf111(i)=0.5*(l111(i)+l111(i+1));
hf112(i)=0.5*(l112(i)+l112(i+1));
hf113(i)=0.5*(l113(i)+l113(i+1));
hf114(i)=0.5*(l114(i)+l114(i+1));
hf115(i)=0.5*(l115(i)+l115(i+1));
hf116(i)=0.5*(l116(i)+l116(i+1));
hf117(i)=0.5*(l117(i)+l117(i+1));
hf118(i)=0.5*(l118(i)+l118(i+1));
hf119(i)=0.5*(l119(i)+l119(i+1));
hf120(i)=0.5*(l120(i)+l120(i+1));
hf121(i)=0.5*(l121(i)+l121(i+1));
hf122(i)=0.5*(l122(i)+l122(i+1));
hf123(i)=0.5*(l123(i)+l123(i+1));
hf124(i)=0.5*(l124(i)+l124(i+1));
hf125(i)=0.5*(l125(i)+l125(i+1));
hf126(i)=0.5*(l126(i)+l126(i+1));
hf127(i)=0.5*(l127(i)+l127(i+1));

hf21(i)=0.5*(l21(i)+l21(i+1));
hf22(i)=0.5*(l22(i)+l22(i+1));
hf23(i)=0.5*(l23(i)+l23(i+1));
hf24(i)=0.5*(l24(i)+l24(i+1));
hf25(i)=0.5*(l25(i)+l25(i+1));
hf26(i)=0.5*(l26(i)+l26(i+1));
hf27(i)=0.5*(l27(i)+l27(i+1));
hf28(i)=0.5*(l28(i)+l28(i+1));
hf29(i)=0.5*(l29(i)+l29(i+1));
```

```
hf210(i)=0.5*(l210(i)+l210(i+1));

hf211(i)=0.5*(l211(i)+l211(i+1));

hf212(i)=0.5*(l212(i)+l212(i+1));

hf213(i)=0.5*(l213(i)+l213(i+1));

hf214(i)=0.5*(l214(i)+l214(i+1));

hf215(i)=0.5*(l215(i)+l215(i+1));

hf216(i)=0.5*(l216(i)+l216(i+1));

hf217(i)=0.5*(l217(i)+l217(i+1));

hf218(i)=0.5*(l218(i)+l218(i+1));

hf219(i)=0.5*(l219(i)+l219(i+1));

hf220(i)=0.5*(l220(i)+l220(i+1));

hf221(i)=0.5*(l221(i)+l221(i+1));

hf222(i)=0.5*(l222(i)+l222(i+1));

hf223(i)=0.5*(l223(i)+l223(i+1));

hf224(i)=0.5*(l224(i)+l224(i+1));

hf225(i)=0.5*(l225(i)+l225(i+1));

hf226(i)=0.5*(l226(i)+l226(i+1));

hf227(i)=0.5*(l227(i)+l227(i+1));




%CONTROLLI COEEFICIENTI DI RUNGE KUTTA 2° ORDINE

a11=(1.*(k1s*(1.-1.*qthvs)*hf11(i)*wf1(i)*wf26(i)-1.*k1s*(1.-
1.*qthvs)*hf18(i)*wf1(i)*wf26(i)-1.*k1s*(1.-1.*qtsvs)*hf112(i)*wf2(i)*wf26(i)+k1s*(1.-
1.*qtsvs)*hf12(i)*wf2(i)*wf26(i)-1.*k1s*(1.-1.*qtrvs)*hf116(i)*wf26(i)*wf3(i)+k1s*(1.-
1.*qtrvs)*hf13(i)*wf26(i)*wf3(i)-1.*k1s*(1.-1.*qtgvs)*hf120(i)*wf26(i)*wf4(i)+k1s*(1.-
1.*qtgvs)*hf14(i)*wf26(i)*wf4(i)+k1s*rhos*hf126(i)*wf26(i)*((1.-1.*qthvs)*wf1(i)+(1.-
1.*qtsvs)*wf2(i)+(1.-1.*qtrvs)*wf3(i)+(1.-1.*qtgvs)*wf4(i))))/aco-(1.*(-1.*k1s*(1.-
1.*qthvs)*hf11(i)*wf1(i)*wf26(i)+k1s*(1.-1.*qthvs)*hf18(i)*wf1(i)*wf26(i)+k1s*(1.-
1.*qtsvs)*hf112(i)*wf2(i)*wf26(i)-1.*k1s*(1.-1.*qtsvs)*hf12(i)*wf2(i)*wf26(i)+k1s*(1.-
1.*qtrvs)*hf116(i)*wf26(i)*wf3(i)-1.*k1s*(1.-1.*qtrvs)*hf13(i)*wf26(i)*wf3(i)+k1s*(1.-
1.*qtgvs)*hf120(i)*wf26(i)*wf4(i)-1.*k1s*(1.-1.*qtgvs)*hf14(i)*wf26(i)*wf4(i)-
1.*k1s*rhos*hf126(i)*wf26(i)*((1.-1.*qthvs)*wf1(i)+(1.-1.*qtsvs)*wf2(i)+(1.-
1.*qtrvs)*wf3(i)+(1.-1.*qtgvs)*wf4(i)))*(-1.*aco*(k1s*(1.-1.*qthvs)*hf11(i)*wf1(i)*wf26(i)-
1.*k1s*(1.-1.*qthvs)*hf18(i)*wf1(i)*wf26(i)-1.*k1s*(1.-
1.*qtsvs)*hf112(i)*wf2(i)*wf26(i)+k1s*(1.-1.*qtsvs)*hf12(i)*wf2(i)*wf26(i)-1.*k1s*(1.-
1.*qtrvs)*hf116(i)*wf26(i)*wf3(i)+k1s*(1.-1.*qtrvs)*hf13(i)*wf26(i)*wf3(i)-1.*k1s*(1.-
1.*qtgvs)*hf120(i)*wf26(i)*wf4(i)+k1s*(1.-
1.*qtgvs)*hf14(i)*wf26(i)*wf4(i)+k1s*rhos*hf126(i)*wf26(i)*((1.-1.*qthvs)*wf1(i)+(1.-
1.*qtsvs)*wf2(i)+(1.-1.*qtrvs)*wf3(i)+(1.-1.*qtgvs)*wf4(i)))-1.*(-1.*k1s*(1.-
1.*qthvs)*hf11(i)*wf1(i)*wf26(i)+k1s*(1.-1.*qthvs)*hf18(i)*wf1(i)*wf26(i)+k1s*(1.-
```

```
1.*qtsvs)*hf112(i)*wf2(i)*wf26(i)-1.*k1s*(1.-1.*qtsvs)*hf12(i)*wf2(i)*wf26(i)+k1s*(1.-
1.*qtrvs)*hf116(i)*wf26(i)*wf3(i)-1.*k1s*(1.-1.*qtrvs)*hf13(i)*wf26(i)*wf3(i)+k1s*(1.-
1.*qtgvs)*hf120(i)*wf26(i)*wf4(i)-1.*k1s*(1.-1.*qtgvs)*hf14(i)*wf26(i)*wf4(i)-
1.*k1s*rhos*hf126(i)*wf26(i)*((1.-1.*qthvs)*wf1(i)+(1.-1.*qtsvs)*wf2(i)+(1.-
1.*qtrvs)*wf3(i)+(1.-1.*qtgvs)*wf4(i)))*(k1s*(1.-1.*qthvs)*hf11(i)*wf1(i)*wf26(i)-
1.*k1s*(1.-1.*qthvs)*hf18(i)*wf1(i)*wf26(i)-1.*k1s*(1.-
1.*qtsvs)*hf112(i)*wf2(i)*wf26(i)+k1s*(1.-1.*qtsvs)*hf12(i)*wf2(i)*wf26(i)-1.*k1s*(1.-
1.*qtrvs)*hf116(i)*wf26(i)*wf3(i)+k1s*(1.-1.*qtrvs)*hf13(i)*wf26(i)*wf3(i)-1.*k1s*(1.-
1.*qtgvs)*hf120(i)*wf26(i)*wf4(i)+k1s*(1.-
1.*qtgvs)*hf14(i)*wf26(i)*wf4(i)+k1s*rhos*hf126(i)*wf26(i)*((1.-1.*qthvs)*wf1(i)+(1.-
1.*qtsvs)*wf2(i)+(1.-1.*qtrvs)*wf3(i)+(1.-1.*qtgvs)*wf4(i)))))/(aco*(1.*aco*afi-1.*(-
1.*k1s*(1.-1.*qthvs)*hf11(i)*wf1(i)*wf26(i)+k1s*(1.-
1.*qthvs)*hf18(i)*wf1(i)*wf26(i)+k1s*(1.-1.*qtsvs)*hf112(i)*wf2(i)*wf26(i)-1.*k1s*(1.-
1.*qtsvs)*hf12(i)*wf2(i)*wf26(i)+k1s*(1.-1.*qtrvs)*hf116(i)*wf26(i)*wf3(i)-1.*k1s*(1.-
1.*qtrvs)*hf13(i)*wf26(i)*wf3(i)+k1s*(1.-1.*qtgvs)*hf120(i)*wf26(i)*wf4(i)-1.*k1s*(1.-
1.*qtgvs)*hf14(i)*wf26(i)*wf4(i)-1.*k1s*rhos*hf126(i)*wf26(i)*((1.-1.*qthvs)*wf1(i)+(1.-
1.*qtsvs)*wf2(i)+(1.-1.*qtrvs)*wf3(i)+(1.-1.*qtgvs)*wf4(i)))^2));        a12= -((-
1.*aco*(k1s*(1.-1.*qthvs)*hf11(i)*wf1(i)*wf26(i)-1.*k1s*(1.-
1.*qthvs)*hf18(i)*wf1(i)*wf26(i)-1.*k1s*(1.-1.*qtsvs)*hf112(i)*wf2(i)*wf26(i)+k1s*(1.-
1.*qtsvs)*hf12(i)*wf2(i)*wf26(i)-1.*k1s*(1.-1.*qtrvs)*hf116(i)*wf26(i)*wf3(i)+k1s*(1.-
1.*qtrvs)*hf13(i)*wf26(i)*wf3(i)-1.*k1s*(1.-1.*qtgvs)*hf120(i)*wf26(i)*wf4(i)+k1s*(1.-
1.*qtgvs)*hf14(i)*wf26(i)*wf4(i)+k1s*rhos*hf126(i)*wf26(i)*((1.-1.*qthvs)*wf1(i)+(1.-
1.*qtsvs)*wf2(i)+(1.-1.*qtrvs)*wf3(i)+(1.-1.*qtgvs)*wf4(i)))-1.*(-1.*k1s*(1.-
1.*qthvs)*hf11(i)*wf1(i)*wf26(i)+k1s*(1.-1.*qthvs)*hf18(i)*wf1(i)*wf26(i)+k1s*(1.-
1.*qtsvs)*hf112(i)*wf2(i)*wf26(i)-1.*k1s*(1.-1.*qtsvs)*hf12(i)*wf2(i)*wf26(i)+k1s*(1.-
1.*qtrvs)*hf116(i)*wf26(i)*wf3(i)-1.*k1s*(1.-1.*qtrvs)*hf13(i)*wf26(i)*wf3(i)+k1s*(1.-
1.*qtgvs)*hf120(i)*wf26(i)*wf4(i)-1.*k1s*(1.-1.*qtgvs)*hf14(i)*wf26(i)*wf4(i)-
1.*k1s*rhos*hf126(i)*wf26(i)*((1.-1.*qthvs)*wf1(i)+(1.-1.*qtsvs)*wf2(i)+(1.-
1.*qtrvs)*wf3(i)+(1.-1.*qtgvs)*wf4(i)))*(k1s*(1.-1.*qthvs)*hf11(i)*wf1(i)*wf26(i)-
1.*k1s*(1.-1.*qthvs)*hf18(i)*wf1(i)*wf26(i)-1.*k1s*(1.-
1.*qtsvs)*hf112(i)*wf2(i)*wf26(i)+k1s*(1.-1.*qtsvs)*hf12(i)*wf2(i)*wf26(i)-1.*k1s*(1.-
1.*qtrvs)*hf116(i)*wf26(i)*wf3(i)+k1s*(1.-1.*qtrvs)*hf13(i)*wf26(i)*wf3(i)-1.*k1s*(1.-
1.*qtgvs)*hf120(i)*wf26(i)*wf4(i)+k1s*(1.-
1.*qtgvs)*hf14(i)*wf26(i)*wf4(i)+k1s*rhos*hf126(i)*wf26(i)*((1.-1.*qthvs)*wf1(i)+(1.-
1.*qtsvs)*wf2(i)+(1.-1.*qtrvs)*wf3(i)+(1.-1.*qtgvs)*wf4(i))))/(1.*aco*afi-1.*(-1.*k1s*(1.-
1.*qthvs)*hf11(i)*wf1(i)*wf26(i)+k1s*(1.-1.*qthvs)*hf18(i)*wf1(i)*wf26(i)+k1s*(1.-
1.*qtsvs)*hf112(i)*wf2(i)*wf26(i)-1.*k1s*(1.-1.*qtsvs)*hf12(i)*wf2(i)*wf26(i)+k1s*(1.-
1.*qtrvs)*hf116(i)*wf26(i)*wf3(i)-1.*k1s*(1.-1.*qtrvs)*hf13(i)*wf26(i)*wf3(i)+k1s*(1.-
1.*qtgvs)*hf120(i)*wf26(i)*wf4(i)-1.*k1s*(1.-1.*qtgvs)*hf14(i)*wf26(i)*wf4(i)-
1.*k1s*rhos*hf126(i)*wf26(i)*((1.-1.*qthvs)*wf1(i)+(1.-1.*qtsvs)*wf2(i)+(1.-
1.*qtrvs)*wf3(i)+(1.-1.*qtgvs)*wf4(i)))^2));        a13=
(1.*(hf11(i)*wf1(i)+hf12(i)*wf2(i)+hf13(i)*wf3(i)+hf14(i)*wf4(i)))/aib;        a14=
(1.*(k2*hf112(i)*wf12(i)-1.*k2*hf113(i)*wf12(i)+k2*hf116(i)*wf16(i)-
1.*k2*hf117(i)*wf16(i)+k2*hf120(i)*wf20(i)-1.*k2*hf121(i)*wf20(i)+k2*hf18(i)*wf8(i)-
1.*k2*hf19(i)*wf8(i)))/aii-(1.*(-1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-
1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-
1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i))*((1.*(1.*bsr*pr*hf126(i)*(wf11(i)+wf15(i)+wf19(i
)+wf23(i)+wf25(i))*(1.*brs-
1.*pr*hf226(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))+pr*hf227(i)*(wf11(i)+wf15(i)+
wf19(i)+wf23(i)+wf25(i)))-1.*brs*(-1.*bsr*(-
1.*pr*hf126(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))-
(1.*g*hf126(i)*wf26(i))/(b2+wf26(i))-
1.*k1sm*hf124(i)*wf26(i)*wf5(i)+k1sm*rhos*hf126(i)*wf26(i)*wf5(i)+k1sm*hf15(i)*wf26
(i)*wf5(i))+1.*ps*hf126(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))*(1.*bsr+ps*hf226(i)*
(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))-
1.*ps*hf227(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))))))/(brs*bsr*(k1sm*hf124(i)*wf2
6(i)*wf5(i)-1.*k1sm*rhos*hf126(i)*wf26(i)*wf5(i)-1.*k1sm*hf15(i)*wf26(i)*wf5(i)))-
(1.*(1.*bsr*pr^2*hf126(i)*hf226(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))^2-
1.*brs*(1.*api*bsr-
1.*ps^2*hf126(i)*hf226(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))^2))*(1.*brs*bsr*(k1s
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m*hf124(i)*wf26(i)*wf5(i)-1.*k1sm*rhos*hf126(i)*wf26(i)*wf5(i)-
1.*k1sm*hf15(i)*wf26(i)*wf5(i))*(-1.*aii*(k2*hf112(i)*wf12(i)-
1.*k2*hf113(i)*wf12(i)+k2*hf116(i)*wf16(i)-1.*k2*hf117(i)*wf16(i)+k2*hf120(i)*wf20(i)-
1.*k2*hf121(i)*wf20(i)-
1.*k1sm*hf124(i)*wf26(i)*wf5(i)+k1sm*rhos*hf126(i)*wf26(i)*wf5(i)+k1sm*hf15(i)*wf26
(i)*wf5(i)+k2*hf18(i)*wf8(i)-1.*k2*hf19(i)*wf8(i))-1.*(k2*hf112(i)*wf12(i)-
1.*k2*hf113(i)*wf12(i)+k2*hf116(i)*wf16(i)-1.*k2*hf117(i)*wf16(i)+k2*hf120(i)*wf20(i)-
1.*k2*hf121(i)*wf20(i)+k2*hf18(i)*wf8(i)-1.*k2*hf19(i)*wf8(i))*(-
1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-
1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i)))-
1.*(1.*aii*art-1.*(-1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-
1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-
1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i))^2)*(1.*bsr*pr*hf126(i)*(wf11(i)+wf15(i)+wf19(i)
+wf23(i)+wf25(i))*(1.*brs-
1.*pr*hf226(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))+pr*hf227(i)*(wf11(i)+wf15(i)+
wf19(i)+wf23(i)+wf25(i)))-1.*brs*(-1.*bsr*(-
1.*pr*hf126(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))-
(1.*g*hf126(i)*wf26(i))/(b2+wf26(i))-
1.*k1sm*hf124(i)*wf26(i)*wf5(i)+k1sm*rhos*hf126(i)*wf26(i)*wf5(i)+k1sm*hf15(i)*wf26
(i)*wf5(i))+1.*ps*hf126(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))*(1.*bsr+ps*hf226(i)*
(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))-
1.*ps*hf227(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i)))))))/(brs*bsr*(k1sm*hf124(i)*wf
26(i)*wf5(i)-1.*k1sm*rhos*hf126(i)*wf26(i)*wf5(i)-1.*k1sm*hf15(i)*wf26(i)*wf5(i))*(-
1.*aii*brs*bsr*(k1sm*hf124(i)*wf26(i)*wf5(i)-1.*k1sm*rhos*hf126(i)*wf26(i)*wf5(i)-
1.*k1sm*hf15(i)*wf26(i)*wf5(i))^2-1.*(1.*aii*art-1.*(-
1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-
1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-
1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i))^2)*(1.*bsr*pr^2*hf126(i)*hf226(i)*(wf11(i)+wf15
(i)+wf19(i)+wf23(i)+wf25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*hf126(i)*hf226(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))^2))))))/aii; a15= -
((1.*brs*bsr*(k1sm*hf124(i)*wf26(i)*wf5(i)-1.*k1sm*rhos*hf126(i)*wf26(i)*wf5(i)-
1.*k1sm*hf15(i)*wf26(i)*wf5(i))*(-1.*aii*(k2*hf112(i)*wf12(i)-
1.*k2*hf113(i)*wf12(i)+k2*hf116(i)*wf16(i)-1.*k2*hf117(i)*wf16(i)+k2*hf120(i)*wf20(i)-
1.*k2*hf121(i)*wf20(i)-
1.*k1sm*hf124(i)*wf26(i)*wf5(i)+k1sm*rhos*hf126(i)*wf26(i)*wf5(i)+k1sm*hf15(i)*wf26
(i)*wf5(i)+k2*hf18(i)*wf8(i)-1.*k2*hf19(i)*wf8(i))-1.*(k2*hf112(i)*wf12(i)-
1.*k2*hf113(i)*wf12(i)+k2*hf116(i)*wf16(i)-1.*k2*hf117(i)*wf16(i)+k2*hf120(i)*wf20(i)-
1.*k2*hf121(i)*wf20(i)+k2*hf18(i)*wf8(i)-1.*k2*hf19(i)*wf8(i))*(-
1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-
1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i)))-
1.*(1.*aii*art-1.*(-1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-
1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-
1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i))^2)*(1.*bsr*pr*hf126(i)*(wf11(i)+wf15(i)+wf19(i)
+wf23(i)+wf25(i))*(1.*brs-
1.*pr*hf226(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))+pr*hf227(i)*(wf11(i)+wf15(i)+
wf19(i)+wf23(i)+wf25(i)))-1.*brs*(-1.*bsr*(-
1.*pr*hf126(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))-
(1.*g*hf126(i)*wf26(i))/(b2+wf26(i))-
1.*k1sm*hf124(i)*wf26(i)*wf5(i)+k1sm*rhos*hf126(i)*wf26(i)*wf5(i)+k1sm*hf15(i)*wf26
(i)*wf5(i))+1.*ps*hf126(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))*(1.*bsr+ps*hf226(i)*
(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))-
1.*ps*hf227(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))))))/(-
1.*aii*brs*bsr*(k1sm*hf124(i)*wf26(i)*wf5(i)-1.*k1sm*rhos*hf126(i)*wf26(i)*wf5(i)-
1.*k1sm*hf15(i)*wf26(i)*wf5(i))^2-1.*(1.*aii*art-1.*(-
1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-
1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-
1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i))^2)*(1.*bsr*pr^2*hf126(i)*hf226(i)*(wf11(i)+wf15
(i)+wf19(i)+wf23(i)+wf25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*hf126(i)*hf226(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))^2)))); a16= (-
1.*(1.*bsr*pr*hf126(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))*(1.*brs-

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1.*pr*hf226(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))+pr*hf227(i)*(wf11(i)+wf15(i)+
wf19(i)+wf23(i)+wf25(i)))-1.*brs*(-1.*bsr*(-
1.*pr*hf126(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))-
(1.*g*hf126(i)*wf26(i))/(b2+wf26(i))-
1.*k1sm*hf124(i)*wf26(i)*wf5(i)+k1sm*rhos*hf126(i)*wf26(i)*wf5(i)+k1sm*hf15(i)*wf26
(i)*wf5(i))+1.*ps*hf126(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))*(1.*bsr+ps*hf226(i)*
(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))-
1.*ps*hf227(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))))))/(brs*bsr*(k1sm*hf124(i)*wf2
6(i)*wf5(i)-1.*k1sm*rhos*hf126(i)*wf26(i)*wf5(i)-
1.*k1sm*hf15(i)*wf26(i)*wf5(i)))+(1.*(1.*bsr*pr^2*hf126(i)*hf226(i)*(wf11(i)+wf15(i)+wf
19(i)+wf23(i)+wf25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*hf126(i)*hf226(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))^2))*(1.*brs*bsr*(k1s
m*hf124(i)*wf26(i)*wf5(i)-1.*k1sm*rhos*hf126(i)*wf26(i)*wf5(i)-
1.*k1sm*hf15(i)*wf26(i)*wf5(i))*(-1.*aii*(k2*hf112(i)*wf12(i)-
1.*k2*hf113(i)*wf12(i)+k2*hf116(i)*wf16(i)-1.*k2*hf117(i)*wf16(i)+k2*hf120(i)*wf20(i)-
1.*k2*hf121(i)*wf20(i)-
1.*k1sm*hf124(i)*wf26(i)*wf5(i)+k1sm*rhos*hf126(i)*wf26(i)*wf5(i)+k1sm*hf15(i)*wf26
(i)*wf5(i)+k2*hf18(i)*wf8(i)-1.*k2*hf19(i)*wf8(i))-1.*(k2*hf112(i)*wf12(i)-
1.*k2*hf113(i)*wf12(i)+k2*hf116(i)*wf16(i)-1.*k2*hf117(i)*wf16(i)+k2*hf120(i)*wf20(i)-
1.*k2*hf121(i)*wf20(i)+k2*hf18(i)*wf8(i)-1.*k2*hf19(i)*wf8(i))*(-
1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-
1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i)))-
1.*(1.*aii*art-1.*(-1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-
1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-
1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i))^2)*(1.*bsr*pr*hf126(i)*(wf11(i)+wf15(i)+wf19(i)
+wf23(i)+wf25(i))*(1.*brs-
1.*pr*hf226(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))+pr*hf227(i)*(wf11(i)+wf15(i)+
wf19(i)+wf23(i)+wf25(i)))-1.*brs*(-1.*bsr*(-
1.*pr*hf126(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))-
(1.*g*hf126(i)*wf26(i))/(b2+wf26(i))-
1.*k1sm*hf124(i)*wf26(i)*wf5(i)+k1sm*rhos*hf126(i)*wf26(i)*wf5(i)+k1sm*hf15(i)*wf26
(i)*wf5(i))+1.*ps*hf126(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))*(1.*bsr+ps*hf226(i)*
(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))-
1.*ps*hf227(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i)))))))/(brs*bsr*(k1sm*hf124(i)*wf
26(i)*wf5(i)-1.*k1sm*rhos*hf126(i)*wf26(i)*wf5(i)-1.*k1sm*hf15(i)*wf26(i)*wf5(i))*(-
1.*aii*brs*bsr*(k1sm*hf124(i)*wf26(i)*wf5(i)-1.*k1sm*rhos*hf126(i)*wf26(i)*wf5(i)-
1.*k1sm*hf15(i)*wf26(i)*wf5(i))^2-1.*(1.*aii*art-1.*(-
1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-
1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-
1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i))^2)*(1.*bsr*pr^2*hf126(i)*hf226(i)*(wf11(i)+wf15
(i)+wf19(i)+wf23(i)+wf25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*hf126(i)*hf226(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))^2)))));        u21s=
min(1,max(0,(1.*(1.*bsr+ps*hf226(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))-
1.*ps*hf227(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))))/bsr+(1.*ps*hf226(i)*(wf13(i)+
wf17(i)+wf21(i)+wf24(i)+wf9(i))*(1.*brs*bsr*(k1sm*hf124(i)*wf26(i)*wf5(i)-
1.*k1sm*rhos*hf126(i)*wf26(i)*wf5(i)-1.*k1sm*hf15(i)*wf26(i)*wf5(i))*(-
1.*aii*(k2*hf112(i)*wf12(i)-1.*k2*hf113(i)*wf12(i)+k2*hf116(i)*wf16(i)-
1.*k2*hf117(i)*wf16(i)+k2*hf120(i)*wf20(i)-1.*k2*hf121(i)*wf20(i)-
1.*k1sm*hf124(i)*wf26(i)*wf5(i)+k1sm*rhos*hf126(i)*wf26(i)*wf5(i)+k1sm*hf15(i)*wf26
(i)*wf5(i)+k2*hf18(i)*wf8(i)-1.*k2*hf19(i)*wf8(i))-1.*(k2*hf112(i)*wf12(i)-
1.*k2*hf113(i)*wf12(i)+k2*hf116(i)*wf16(i)-1.*k2*hf117(i)*wf16(i)+k2*hf120(i)*wf20(i)-
1.*k2*hf121(i)*wf20(i)+k2*hf18(i)*wf8(i)-1.*k2*hf19(i)*wf8(i))*(-
1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-
1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i)))-
1.*(1.*aii*art-1.*(-1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-
1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-
1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i))^2)*(1.*bsr*pr*hf126(i)*(wf11(i)+wf15(i)+wf19(i)
+wf23(i)+wf25(i))*(1.*brs-
1.*pr*hf226(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))+pr*hf227(i)*(wf11(i)+wf15(i)+
wf19(i)+wf23(i)+wf25(i)))-1.*brs*(-1.*bsr*(-
```

```
1.*pr*hf126(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))-
(1.*g*hf126(i)*wf26(i))/(b2+wf26(i))-
1.*k1sm*hf124(i)*wf26(i)*wf5(i)+k1sm*rhos*hf126(i)*wf26(i)*wf5(i)+k1sm*hf15(i)*wf26
(i)*wf5(i))+1.*ps*hf126(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))*(1.*bsr+ps*hf226(i)*
(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))-
1.*ps*hf227(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i)))))))/(bsr*(-
1.*aii*brs*bsr*(k1sm*hf124(i)*wf26(i)*wf5(i)-1.*k1sm*rhos*hf126(i)*wf26(i)*wf5(i)-
1.*k1sm*hf15(i)*wf26(i)*wf5(i))^2-1.*(1.*aii*art-1.*(-
1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-
1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-
1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i))^2)*(1.*bsr*pr^2*hf126(i)*hf226(i)*(wf11(i)+wf15
(i)+wf19(i)+wf23(i)+wf25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*hf126(i)*hf226(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))^2)))))); u22s=
min(1,max(0,(1.*(1.*brs-
1.*pr*hf226(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))+pr*hf227(i)*(wf11(i)+wf15(i)+
wf19(i)+wf23(i)+wf25(i))))/brs-
(1.*pr*hf226(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))*(1.*brs*bsr*(k1sm*hf124(i)*w
f26(i)*wf5(i)-1.*k1sm*rhos*hf126(i)*wf26(i)*wf5(i)-1.*k1sm*hf15(i)*wf26(i)*wf5(i))*(-
1.*aii*(k2*hf112(i)*wf12(i)-1.*k2*hf113(i)*wf12(i)+k2*hf116(i)*wf16(i)-
1.*k2*hf117(i)*wf16(i)+k2*hf120(i)*wf20(i)-1.*k2*hf121(i)*wf20(i)-
1.*k1sm*hf124(i)*wf26(i)*wf5(i)+k1sm*rhos*hf126(i)*wf26(i)*wf5(i)+k1sm*hf15(i)*wf26
(i)*wf5(i)+k2*hf18(i)*wf8(i)-1.*k2*hf19(i)*wf8(i))-1.*(k2*hf112(i)*wf12(i)-
1.*k2*hf113(i)*wf12(i)+k2*hf116(i)*wf16(i)-1.*k2*hf117(i)*wf16(i)+k2*hf120(i)*wf20(i)-
1.*k2*hf121(i)*wf20(i)+k2*hf18(i)*wf8(i)-1.*k2*hf19(i)*wf8(i))*(-
1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-
1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i)))-
1.*(1.*aii*art-1.*(-1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-
1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-
1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i))^2)*(1.*bsr*pr*hf126(i)*(wf11(i)+wf15(i)+wf19(i)
+wf23(i)+wf25(i))*(1.*brs-
1.*pr*hf226(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))+pr*hf227(i)*(wf11(i)+wf15(i)+
wf19(i)+wf23(i)+wf25(i)))-1.*brs*(-1.*bsr*(-
1.*pr*hf126(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))-
(1.*g*hf126(i)*wf26(i))/(b2+wf26(i))-
1.*k1sm*hf124(i)*wf26(i)*wf5(i)+k1sm*rhos*hf126(i)*wf26(i)*wf5(i)+k1sm*hf15(i)*wf26
(i)*wf5(i))+1.*ps*hf126(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))*(1.*bsr+ps*hf226(i)*
(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))-
1.*ps*hf227(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i)))))))/(brs*(-
1.*aii*brs*bsr*(k1sm*hf124(i)*wf26(i)*wf5(i)-1.*k1sm*rhos*hf126(i)*wf26(i)*wf5(i)-
1.*k1sm*hf15(i)*wf26(i)*wf5(i))^2-1.*(1.*aii*art-1.*(-
1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-
1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-
1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i))^2)*(1.*bsr*pr^2*hf126(i)*hf226(i)*(wf11(i)+wf15
(i)+wf19(i)+wf23(i)+wf25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*hf126(i)*hf226(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))^2))))));

        if(a11<0 || a11>1)

        a11=0;

    end;



    if(a12<0 || a12>1)

        a12=0;

    end;
```

```
if(a13<0 || a13>1)
    a13=0;
end;

if(a14<0 || a14>1)
    a14=0;
end;

if(a15<0 || a15>1)
    a15=0;
end;

if(a16<0 || a16>1)
    a16=0;
end;
```

```
    D=[a11,a12,a13,a14,a15,a16;a11,a12,a13,a14,a15,1;a11,a12,a13,a14,a15,0;a11,a12,
a13,a14,1,a16;a11,a12,a13,a14,1,1;a11,a12,a13,a14,1,0;a11,a12,a13,a14,0,a16;a11,a12,a13,a1
4,0,1;a11,a12,a13,a14,0,0;a11,a12,a13,1,a15,a16;a11,a12,a13,1,a15,1;a11,a12,a13,1,a15,0;a1
1,a12,a13,1,1,a16;a11,a12,a13,1,1,1;a11,a12,a13,1,1,0;a11,a12,a13,1,0,a16;a11,a12,a13,1,0,1;
a11,a12,a13,1,0,0;a11,a12,a13,0,a15,a16;a11,a12,a13,0,a15,1;a11,a12,a13,0,a15,0;a11,a12,a1
3,0,1,a16;a11,a12,a13,0,1,1;a11,a12,a13,0,1,0;a11,a12,a13,0,0,a16;a11,a12,a13,0,0,1;a11,a12,
a13,0,0,0;a11,a12,1,a14,a15,a16;a11,a12,1,a14,a15,1;a11,a12,1,a14,a15,0;a11,a12,1,a14,1,a1
6;a11,a12,1,a14,1,1;a11,a12,1,a14,1,0;a11,a12,1,a14,0,a16;a11,a12,1,a14,0,1;a11,a12,1,a14,0,
0;a11,a12,1,1,a15,a16;a11,a12,1,1,a15,1;a11,a12,1,1,a15,0;a11,a12,1,1,1,a16;a11,a12,1,1,1,1;
a11,a12,1,1,1,0;a11,a12,1,1,0,a16;a11,a12,1,1,0,1;a11,a12,1,1,0,0;a11,a12,1,0,a15,a16;a11,a1
2,1,0,a15,1;a11,a12,1,0,a15,0;a11,a12,1,0,1,a16;a11,a12,1,0,1,1;a11,a12,1,0,1,0;a11,a12,1,0,0
,a16;a11,a12,1,0,0,1;a11,a12,1,0,0,0;a11,a12,0,a14,a15,a16;a11,a12,0,a14,a15,1;a11,a12,0,a1
4,a15,0;a11,a12,0,a14,1,a16;a11,a12,0,a14,1,1;a11,a12,0,a14,1,0;a11,a12,0,a14,0,a16;a11,a12
,0,a14,0,1;a11,a12,0,a14,0,0;a11,a12,0,1,a15,a16;a11,a12,0,1,a15,1;a11,a12,0,1,a15,0;a11,a12
,0,1,1,a16;a11,a12,0,1,1,1;a11,a12,0,1,1,0;a11,a12,0,1,0,a16;a11,a12,0,1,0,1;a11,a12,0,1,0,0;a
11,a12,0,0,a15,a16;a11,a12,0,0,a15,1;a11,a12,0,0,a15,0;a11,a12,0,0,1,a16;a11,a12,0,0,1,1;a1
1,a12,0,0,1,0;a11,a12,0,0,0,a16;a11,a12,0,0,0,1;a11,a12,0,0,0,0;a11,1,a13,a14,a15,a16;a11,1,
a13,a14,a15,1;a11,1,a13,a14,a15,0;a11,1,a13,a14,1,a16;a11,1,a13,a14,1,1;a11,1,a13,a14,1,0;a
11,1,a13,a14,0,a16;a11,1,a13,a14,0,1;a11,1,a13,a14,0,0;a11,1,a13,1,a15,a16;a11,1,a13,1,a15,
1;a11,1,a13,1,a15,0;a11,1,a13,1,1,a16;a11,1,a13,1,1,1;a11,1,a13,1,1,0;a11,1,a13,1,0,a16;a11,
1,a13,1,0,1;a11,1,a13,1,0,0;a11,1,a13,0,a15,a16;a11,1,a13,0,a15,1;a11,1,a13,0,a15,0;a11,1,a1
3,0,1,a16;a11,1,a13,0,1,1;a11,1,a13,0,1,0;a11,1,a13,0,0,a16;a11,1,a13,0,0,1;a11,1,a13,0,0,0;a
11,1,1,a14,a15,a16;a11,1,1,a14,a15,1;a11,1,1,a14,a15,0;a11,1,1,a14,1,a16;a11,1,1,a14,1,1;a1
1,1,1,a14,1,0;a11,1,1,a14,0,a16;a11,1,1,a14,0,1;a11,1,1,a14,0,0;a11,1,1,1,a15,a16;a11,1,1,1,a
15,1;a11,1,1,1,a15,0;a11,1,1,1,1,a16;a11,1,1,1,1,1;a11,1,1,1,1,0;a11,1,1,1,0,a16;a11,1,1,1,0,1;
a11,1,1,1,0,0;a11,1,1,0,a15,a16;a11,1,1,0,a15,1;a11,1,1,0,a15,0;a11,1,1,0,1,a16;a11,1,1,0,1,1;
a11,1,1,0,1,0;a11,1,1,0,0,a16;a11,1,1,0,0,1;a11,1,1,0,0,0;a11,1,0,a14,a15,a16;a11,1,0,a14,a15,
```

1;a11,1,0,a14,a15,0;a11,1,0,a14,1,a16;a11,1,0,a14,1,1;a11,1,0,a14,1,0;a11,1,0,a14,0,a16;a11,
1,0,a14,0,1;a11,1,0,a14,0,0;a11,1,0,1,a15,a16;a11,1,0,1,a15,1;a11,1,0,1,a15,0;a11,1,0,1,1,a16;
a11,1,0,1,1,1;a11,1,0,1,1,0;a11,1,0,1,0,a16;a11,1,0,1,0,1;a11,1,0,1,0,0;a11,1,0,0,a15,a16;a11,
1,0,0,a15,1;a11,1,0,0,a15,0;a11,1,0,0,1,a16;a11,1,0,0,1,1;a11,1,0,0,1,0;a11,1,0,0,0,a16;a11,1,
0,0,0,1;a11,1,0,0,0,0;a11,0,a13,a14,a15,a16;a11,0,a13,a14,a15,1;a11,0,a13,a14,a15,0;a11,0,a1
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a13,a14,0,0;a11,0,a13,1,a15,a16;a11,0,a13,1,a15,1;a11,0,a13,1,a15,0;a11,0,a13,1,1,a16;a11,0,
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1,1;a11,0,1,1,1,0;a11,0,1,1,0,a16;a11,0,1,1,0,1;a11,0,1,1,0,0;a11,0,1,0,a15,a16;a11,0,1,0,a15,
1;a11,0,1,0,a15,0;a11,0,1,0,1,a16;a11,0,1,0,1,1;a11,0,1,0,1,0;a11,0,1,0,0,a16;a11,0,1,0,0,1;a1
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a16;1,a12,0,1,1,1;1,a12,0,1,1,0;1,a12,0,1,0,a16;1,a12,0,1,0,1;1,a12,0,1,0,0;1,a12,0,0,a15,a16;
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a12,0,0,0,1;1,a12,0,0,0,0;1,1,a13,a14,a15,a16;1,1,a13,a14,a15,1;1,1,a13,a14,a15,0;1,1,a13,a1
4,1,a16;1,1,a13,a14,1,1;1,1,a13,a14,1,0;1,1,a13,a14,0,a16;1,1,a13,a14,0,1;1,1,a13,a14,0,0;1,1,
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a13,0,1,a16;1,1,a13,0,1,1;1,1,a13,0,1,0;1,1,a13,0,0,a16;1,1,a13,0,0,1;1,1,a13,0,0,0;1,1,1,a14,a
15,a16;1,1,1,a14,a15,1;1,1,1,a14,a15,0;1,1,1,a14,1,a16;1,1,1,a14,1,1;1,1,1,a14,1,0;1,1,1,a14,0
,a16;1,1,1,a14,0,1;1,1,1,a14,0,0;1,1,1,1,a15,a16;1,1,1,1,a15,1;1,1,1,1,a15,0;1,1,1,1,1,a16;1,1,1
,1,1,1;1,1,1,1,1,0;1,1,1,1,0,a16;1,1,1,1,0,1;1,1,1,1,0,0;1,1,1,0,a15,a16;1,1,1,0,a15,1;1,1,1,0,a1
5,0;1,1,1,0,1,a16;1,1,1,0,1,1;1,1,1,0,1,0;1,1,1,0,0,a16;1,1,1,0,0,1;1,1,1,0,0,0;1,1,0,a14,a15,a16
;1,1,0,a14,a15,1;1,1,0,a14,a15,0;1,1,0,a14,1,a16;1,1,0,a14,1,1;1,1,0,a14,1,0;1,1,0,a14,0,a16;1,
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,0,0,1,a16;1,1,0,0,1,1;1,1,0,0,1,0;1,1,0,0,0,a16;1,1,0,0,0,1;1,1,0,0,0,0;1,0,a13,a14,a15,a16;1,0,
a13,a14,a15,1;1,0,a13,a14,a15,0;1,0,a13,a14,1,a16;1,0,a13,a14,1,1;1,0,a13,a14,1,0;1,0,a13,a1
4,0,a16;1,0,a13,a14,0,1;1,0,a13,a14,0,0;1,0,a13,1,a15,a16;1,0,a13,1,a15,1;1,0,a13,1,a15,0;1,0,
a13,1,1,a16;1,0,a13,1,1,1;1,0,a13,1,1,0;1,0,a13,1,0,a16;1,0,a13,1,0,1;1,0,a13,1,0,0;1,0,a13,0,a
15,a16;1,0,a13,0,a15,1;1,0,a13,0,a15,0;1,0,a13,0,1,a16;1,0,a13,0,1,1;1,0,a13,0,1,0;1,0,a13,0,0
,a16;1,0,a13,0,0,1;1,0,a13,0,0,0;1,0,1,a14,a15,a16;1,0,1,a14,a15,1;1,0,1,a14,a15,0;1,0,1,a14,1
,a16;1,0,1,a14,1,1;1,0,1,a14,1,0;1,0,1,a14,0,a16;1,0,1,a14,0,1;1,0,1,a14,0,0;1,0,1,1,a15,a16;1,
0,1,1,a15,1;1,0,1,1,a15,0;1,0,1,1,1,a16;1,0,1,1,1,1;1,0,1,1,1,0;1,0,1,1,0,a16;1,0,1,1,0,1;1,0,1,1,
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0,0,0,0,1;1,0,0,0,0,0;0,a12,a13,a14,a15,a16;0,a12,a13,a14,a15,1;0,a12,a13,a14,a15,0;0,a12,a1
3,a14,1,a16;0,a12,a13,a14,1,1;0,a12,a13,a14,1,0;0,a12,a13,a14,0,a16;0,a12,a13,a14,0,1;0,a12,

```
a13,a14,0,0;0,a12,a13,1,a15,a16;0,a12,a13,1,a15,1;0,a12,a13,1,a15,0;0,a12,a13,1,1,a16;0,a12,
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16;0,a12,a13,0,a15,1;0,a12,a13,0,a15,0;0,a12,a13,0,1,a16;0,a12,a13,0,1,1;0,a12,a13,0,1,0;0,a
12,a13,0,0,a16;0,a12,a13,0,0,1;0,a12,a13,0,0,0;0,a12,1,a14,a15,a16;0,a12,1,a14,a15,1;0,a12,1
,a14,a15,0;0,a12,1,a14,1,a16;0,a12,1,a14,1,1;0,a12,1,a14,1,0;0,a12,1,a14,0,a16;0,a12,1,a14,0,
1;0,a12,1,a14,0,0;0,a12,1,1,a15,a16;0,a12,1,1,a15,1;0,a12,1,1,a15,0;0,a12,1,1,1,a16;0,a12,1,1,
1,1;0,a12,1,1,1,0;0,a12,1,1,0,a16;0,a12,1,1,0,1;0,a12,1,1,0,0;0,a12,1,0,a15,a16;0,a12,1,0,a15,
1;0,a12,1,0,a15,0;0,a12,1,0,1,a16;0,a12,1,0,1,1;0,a12,1,0,1,0;0,a12,1,0,0,a16;0,a12,1,0,0,1;0,a
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,a14,1,1;0,a12,0,a14,1,0;0,a12,0,a14,0,a16;0,a12,0,a14,0,1;0,a12,0,a14,0,0;0,a12,0,1,a15,a16;
0,a12,0,1,a15,1;0,a12,0,1,a15,0;0,a12,0,1,1,a16;0,a12,0,1,1,1;0,a12,0,1,1,0;0,a12,0,1,0,a16;0,
a12,0,1,0,1;0,a12,0,1,0,0;0,a12,0,0,a15,a16;0,a12,0,0,a15,1;0,a12,0,0,a15,0;0,a12,0,0,1,a16;0,
a12,0,0,1,1;0,a12,0,0,1,0;0,a12,0,0,0,a16;0,a12,0,0,0,1;0,a12,0,0,0,0;0,1,a13,a14,a15,a16;0,1,
a13,a14,a15,1;0,1,a13,a14,a15,0;0,1,a13,a14,1,a16;0,1,a13,a14,1,1;0,1,a13,a14,1,0;0,1,a13,a1
4,0,a16;0,1,a13,a14,0,1;0,1,a13,a14,0,0;0,1,a13,1,a15,a16;0,1,a13,1,a15,1;0,1,a13,1,a15,0;0,1,
a13,1,1,a16;0,1,a13,1,1,1;0,1,a13,1,1,0;0,1,a13,1,0,a16;0,1,a13,1,0,1;0,1,a13,1,0,0;0,1,a13,0,a
15,a16;0,1,a13,0,a15,1;0,1,a13,0,a15,0;0,1,a13,0,1,a16;0,1,a13,0,1,1;0,1,a13,0,1,0;0,1,a13,0,0
,a16;0,1,a13,0,0,1;0,1,a13,0,0,0;0,1,1,a14,a15,a16;0,1,1,a14,a15,1;0,1,1,a14,a15,0;0,1,1,a14,1
,a16;0,1,1,a14,1,1;0,1,1,a14,1,0;0,1,1,a14,0,a16;0,1,1,a14,0,1;0,1,1,a14,0,0;0,1,1,1,a15,a16;0,
1,1,1,a15,1;0,1,1,1,a15,0;0,1,1,1,1,a16;0,1,1,1,1,1;0,1,1,1,1,0;0,1,1,1,0,a16;0,1,1,1,0,1;0,1,1,1,
0,0;0,1,1,0,a15,a16;0,1,1,0,a15,1;0,1,1,0,a15,0;0,1,1,0,1,a16;0,1,1,0,1,1;0,1,1,0,1,0;0,1,1,0,0,a
16;0,1,1,0,0,1;0,1,1,0,0,0;0,1,0,a14,a15,a16;0,1,0,a14,a15,1;0,1,0,a14,a15,0;0,1,0,a14,1,a16;0,
1,0,a14,1,1;0,1,0,a14,1,0;0,1,0,a14,0,a16;0,1,0,a14,0,1;0,1,0,a14,0,0;0,1,0,1,a15,a16;0,1,0,1,a
15,1;0,1,0,1,a15,0;0,1,0,1,1,a16;0,1,0,1,1,1;0,1,0,1,1,0;0,1,0,1,0,a16;0,1,0,1,0,1;0,1,0,1,0,0;0,
1,0,0,a15,a16;0,1,0,0,a15,1;0,1,0,0,a15,0;0,1,0,0,1,a16;0,1,0,0,1,1;0,1,0,0,1,0;0,1,0,0,0,a16;0,
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16;0,0,a13,a14,1,1;0,0,a13,a14,1,0;0,0,a13,a14,0,a16;0,0,a13,a14,0,1;0,0,a13,a14,0,0;0,0,a13,
1,a15,a16;0,0,a13,1,a15,1;0,0,a13,1,a15,0;0,0,a13,1,1,a16;0,0,a13,1,1,1;0,0,a13,1,1,0;0,0,a13,
1,0,a16;0,0,a13,1,0,1;0,0,a13,1,0,0;0,0,a13,0,a15,a16;0,0,a13,0,a15,1;0,0,a13,0,a15,0;0,0,a13,
0,1,a16;0,0,a13,0,1,1;0,0,a13,0,1,0;0,0,a13,0,0,a16;0,0,a13,0,0,1;0,0,a13,0,0,0;0,0,1,a14,a15,a
16;0,0,1,a14,a15,1;0,0,1,a14,a15,0;0,0,1,a14,1,a16;0,0,1,a14,1,1;0,0,1,a14,1,0;0,0,1,a14,0,a16
;0,0,1,a14,0,1;0,0,1,a14,0,0;0,0,1,1,a15,a16;0,0,1,1,a15,1;0,0,1,1,a15,0;0,0,1,1,1,a16;0,0,1,1,1
,1;0,0,1,1,1,0;0,0,1,1,0,a16;0,0,1,1,0,1;0,0,1,1,0,0;0,0,1,0,a15,a16;0,0,1,0,a15,1;0,0,1,0,a15,0;
0,0,1,0,1,a16;0,0,1,0,1,1;0,0,1,0,1,0;0,0,1,0,0,a16;0,0,1,0,0,1;0,0,1,0,0,0;0,0,0,a14,a15,a16;0,0
,0,a14,a15,1;0,0,0,a14,a15,0;0,0,0,a14,1,a16;0,0,0,a14,1,1;0,0,0,a14,1,0;0,0,0,a14,0,a16;0,0,0,
a14,0,1;0,0,0,a14,0,0;0,0,0,1,a15,a16;0,0,0,1,a15,1;0,0,0,1,a15,0;0,0,0,1,1,a16;0,0,0,1,1,1;0,0,
0,1,1,0;0,0,0,1,0,a16;0,0,0,1,0,1;0,0,0,1,0,0;0,0,0,0,a15,a16;0,0,0,0,a15,1;0,0,0,0,a15,0;0,0,0,0
,1,a16;0,0,0,0,1,1;0,0,0,0,1,0;0,0,0,0,0,a16;0,0,0,0,0,1;0,0,0,0,0,0];
```

```
eval_HT=eval(fHT);

[max_HT, id_HT]=nanmax(eval_HT);


u11(2)=D(id_HT,1);

u12(2)=D(id_HT,2);

u13(2)=D(id_HT,3);

u14(2)=D(id_HT,4);

u15(2)=D(id_HT,5);

u16(2)=D(id_HT,6);
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u21(2)=u21s;

u22(2)=u22s;

%VARIABILI COEEFICIENTI DI RUNGE KUTTA 2° ORDINE

kv1(2)=-(muth*wf1(i))+u13(2)*wf1(i)-k3*wf1(i)*(wf24(i)+wf25(i))-k1s*(1-qthvs)*(1-u11(2))*(1-u12(2))*wf1(i)*wf26(i)-k1r*(1-qthvr)*wf1(i)*wf27(i)-k*wf1(i)*(wf26(i)+wf27(i))-kprimo*wf1(i)*(wf26(i)+wf27(i))+(oth*s1th)/(oth+wf26(i)+wf27(i))+r*wf1(i)*(1-(wf1(i)+wf10(i)+wf11(i)+wf12(i)+wf13(i)+wf14(i)+wf15(i)+wf16(i)+wf17(i)+wf18(i)+wf19(i)+wf2(i)+wf20(i)+wf21(i)+wf22(i)+wf23(i)+wf3(i)+wf4(i)+wf8(i)+wf9(i))/tmax);

kv2(2)=-(muts*wf2(i))+u13(2)*wf2(i)-k3*wf2(i)*(wf24(i)+wf25(i))+k*wf1(i)*wf26(i)-k1s*(1-qtsvs)*(1-u11(2))*(1-u12(2))*wf2(i)*wf26(i)-k1r*(1-qtsvr)*wf2(i)*wf27(i); kv3(2)=k*wf1(i)*wf27(i)-mutr*wf3(i)+u13(2)*wf3(i)-k3*(wf24(i)+wf25(i))*wf3(i)-k1s*(1-qtrvs)*(1-u11(2))*(1-u12(2))*wf26(i)*wf3(i)-k1r*(1-qtrvr)*wf27(i)*wf3(i); kv4(2)=kprimo*wf1(i)*(wf26(i)+wf27(i))-mutg*wf4(i)+u13(2)*wf4(i)-k3*(wf24(i)+wf25(i))*wf4(i)-k1s*(1-qtgvs)*(1-u11(2))*(1-u12(2))*wf26(i)*wf4(i)-k1r*(1-qtgvr)*wf27(i)*wf4(i); kv5(2)=s1mh-mumh*wf5(i)-k1sm*(1-u15(2))*(1-u16(2))*wf26(i)*wf5(i)-k1rm*wf27(i)*wf5(i); kv6(2)=-(muip*wf6(i))-c*q*wf6(i)*(wf11(i)+wf13(i)+wf15(i)+wf17(i)+wf19(i)+wf21(i)+wf23(i)+wf9(i))+c*(wf1(i)+wf2(i)+wf3(i)+wf4(i))*wf6(i)*(wf11(i)+wf13(i)+wf15(i)+wf17(i)+wf19(i)+wf21(i)+wf23(i)+wf9(i)); kv7(2)=-(muie*wf7(i))+c*q*wf6(i)*(wf11(i)+wf13(i)+wf15(i)+wf17(i)+wf19(i)+wf21(i)+wf23(i)+wf9(i)); kv8(2)=k3*wf1(i)*wf24(i)+k1s*(1-qthvs)*(1-u11(2))*(1-u12(2))*wf1(i)*wf26(i)-muthsl*wf8(i)-k2*(1-u14(2))*(1-u16(2))*wf8(i); kv9(2)=k2*(1-u14(2))*(1-u16(2))*wf8(i)-muthsa*wf9(i)-is*wf7(i)*wf9(i); kv10(2)=-(k2*wf10(i))-muthrl*wf10(i)+k3*wf1(i)*wf25(i)+k1r*(1-qthvr)*wf1(i)*wf27(i); kv11(2)=k2*wf10(i)-muthra*wf11(i)-ir*wf11(i)*wf7(i); kv12(2)=-(mutssl*wf12(i))-k2*(1-u14(2))*(1-u16(2))*wf12(i)+k3*wf2(i)*wf24(i)+k1s*(1-qtsvs)*(1-u11(2))*(1-u12(2))*wf2(i)*wf26(i); kv13(2)=k2*(1-u14(2))*(1-u16(2))*wf12(i)-mutssa*wf13(i)-is*wf13(i)*wf7(i); kv14(2)=-(k2*wf14(i))-mutsrl*wf14(i)+k3*wf2(i)*wf25(i)+k1r*(1-qtsvr)*wf2(i)*wf27(i); kv15(2)=k2*wf14(i)-mutsra*wf15(i)-ir*wf15(i)*wf7(i); kv16(2)=-(mutrsl*wf16(i))-k2*(1-u14(2))*(1-u16(2))*wf16(i)+k3*wf24(i)*wf3(i)+k1s*(1-qtrvs)*(1-u11(2))*(1-u12(2))*wf26(i)*wf3(i); kv17(2)=k2*(1-u14(2))*(1-u16(2))*wf16(i)-mutrsa*wf17(i)-is*wf17(i)*wf7(i); kv18(2)=-(k2*wf18(i))-mutrrl*wf18(i)+k3*wf25(i)*wf3(i)+k1r*(1-qtrvr)*wf27(i)*wf3(i); kv19(2)=k2*wf18(i)-mutrra*wf19(i)-ir*wf19(i)*wf7(i); kv20(2)=-(mutgsl*wf20(i))-k2*(1-u14(2))*(1-u16(2))*wf20(i)+k3*wf24(i)*wf4(i)+k1s*(1-qtgvs)*(1-u11(2))*(1-u12(2))*wf26(i)*wf4(i); kv21(2)=k2*(1-u14(2))*(1-u16(2))*wf20(i)-mutgsa*wf21(i)-is*wf21(i)*wf7(i); kv22(2)=-(k2*wf22(i))-mutgrl*wf22(i)+k3*wf25(i)*wf4(i)+k1r*(1-qtgvr)*wf27(i)*wf4(i); kv23(2)=k2*wf22(i)-mutgra*wf23(i)-ir*wf23(i)*wf7(i); kv24(2)=-(mumhsa*wf24(i))+k1sm*(1-u15(2))*(1-u16(2))*wf26(i)*wf5(i); kv25(2)=-(mumhra*wf25(i))+k1rm*wf27(i)*wf5(i); kv26(2)=pr*(1-u15(2))*(1-u22(2))*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))-muvs*wf26(i)+(g*(1-u15(2))*wf26(i))/(b2+wf26(i))-k1s*rhos*(1-u11(2))*(1-u12(2))*wf26(i)*((1-qthvs)*wf1(i)+(1-qtsvs)*wf2(i)+(1-qtrvs)*wf3(i)+(1-qtgvs)*wf4(i))-wf26(i)*(psprimo*wf2(i)+ssprimo*wf4(i))-k1sm*rhos*(1-u15(2))*(1-u16(2))*wf26(i)*wf5(i)+ps*(1-u15(2))*u21(2)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i)); kv27(2)=pr*u22(2)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))-muvr*wf27(i)+(g*wf27(i))/(b2+wf27(i))-k1r*rhor*wf27(i)*((1-qthvr)*wf1(i)+(1-qtsvr)*wf2(i)+(1-qtrvr)*wf3(i)+(1-qtgvr)*wf4(i))-wf27(i)*(prprimo*wf3(i)+srprimo*wf4(i))-k1rm*rhor*wf27(i)*wf5(i)+ps*(1-u21(2))*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i));

```
%VARIABILI DI APPOGGIO wf per 3° ordine

wf1(i)=x1(i)+h2*kv1(2);

wf2(i)=x2(i)+h2*kv2(2);

wf3(i)=x3(i)+h2*kv3(2);

wf4(i)=x4(i)+h2*kv4(2);

wf5(i)=x5(i)+h2*kv5(2);

wf6(i)=x6(i)+h2*kv6(2);

wf7(i)=x7(i)+h2*kv7(2);

wf8(i)=x8(i)+h2*kv8(2);

wf9(i)=x9(i)+h2*kv9(2);

wf10(i)=x10(i)+h2*kv10(2);

wf11(i)=x11(i)+h2*kv11(2);

wf12(i)=x12(i)+h2*kv12(2);

wf13(i)=x13(i)+h2*kv13(2);

wf14(i)=x14(i)+h2*kv14(2);

wf15(i)=x15(i)+h2*kv15(2);

wf16(i)=x16(i)+h2*kv16(2);

wf17(i)=x17(i)+h2*kv17(2);

wf18(i)=x18(i)+h2*kv18(2);

wf19(i)=x19(i)+h2*kv19(2);

wf20(i)=x20(i)+h2*kv20(2);

wf21(i)=x21(i)+h2*kv21(2);

wf22(i)=x22(i)+h2*kv22(2);

wf23(i)=x23(i)+h2*kv23(2);

wf24(i)=x24(i)+h2*kv24(2);

wf25(i)=x25(i)+h2*kv25(2);

wf26(i)=x26(i)+h2*kv26(2);

wf27(i)=x27(i)+h2*kv27(2);
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```
%VARIABILI DI APPOGGIO hf per 3° ordine

hf11(i)=0.5*(l11(i)+l11(i+1));

hf12(i)=0.5*(l12(i)+l12(i+1));

hf13(i)=0.5*(l13(i)+l13(i+1));

hf14(i)=0.5*(l14(i)+l14(i+1));

hf15(i)=0.5*(l15(i)+l15(i+1));

hf16(i)=0.5*(l16(i)+l16(i+1));

hf17(i)=0.5*(l17(i)+l17(i+1));

hf18(i)=0.5*(l18(i)+l18(i+1));

hf19(i)=0.5*(l19(i)+l19(i+1));

hf110(i)=0.5*(l110(i)+l110(i+1));

hf111(i)=0.5*(l111(i)+l111(i+1));

hf112(i)=0.5*(l112(i)+l112(i+1));

hf113(i)=0.5*(l113(i)+l113(i+1));

hf114(i)=0.5*(l114(i)+l114(i+1));

hf115(i)=0.5*(l115(i)+l115(i+1));

hf116(i)=0.5*(l116(i)+l116(i+1));

hf117(i)=0.5*(l117(i)+l117(i+1));

hf118(i)=0.5*(l118(i)+l118(i+1));

hf119(i)=0.5*(l119(i)+l119(i+1));

hf120(i)=0.5*(l120(i)+l120(i+1));

hf121(i)=0.5*(l121(i)+l121(i+1));

hf122(i)=0.5*(l122(i)+l122(i+1));

hf123(i)=0.5*(l123(i)+l123(i+1));

hf124(i)=0.5*(l124(i)+l124(i+1));

hf125(i)=0.5*(l125(i)+l125(i+1));

hf126(i)=0.5*(l126(i)+l126(i+1));

hf127(i)=0.5*(l127(i)+l127(i+1));


hf21(i)=0.5*(l21(i)+l21(i+1));

hf22(i)=0.5*(l22(i)+l22(i+1));

hf23(i)=0.5*(l23(i)+l23(i+1));

hf24(i)=0.5*(l24(i)+l24(i+1));
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hf25(i)=0.5*(l25(i)+l25(i+1));

hf26(i)=0.5*(l26(i)+l26(i+1));

hf27(i)=0.5*(l27(i)+l27(i+1));

hf28(i)=0.5*(l28(i)+l28(i+1));

hf29(i)=0.5*(l29(i)+l29(i+1));

hf210(i)=0.5*(l210(i)+l210(i+1));

hf211(i)=0.5*(l211(i)+l211(i+1));

hf212(i)=0.5*(l212(i)+l212(i+1));

hf213(i)=0.5*(l213(i)+l213(i+1));

hf214(i)=0.5*(l214(i)+l214(i+1));

hf215(i)=0.5*(l215(i)+l215(i+1));

hf216(i)=0.5*(l216(i)+l216(i+1));

hf217(i)=0.5*(l217(i)+l217(i+1));

hf218(i)=0.5*(l218(i)+l218(i+1));

hf219(i)=0.5*(l219(i)+l219(i+1));

hf220(i)=0.5*(l220(i)+l220(i+1));

hf221(i)=0.5*(l221(i)+l221(i+1));

hf222(i)=0.5*(l222(i)+l222(i+1));

hf223(i)=0.5*(l223(i)+l223(i+1));

hf224(i)=0.5*(l224(i)+l224(i+1));

hf225(i)=0.5*(l225(i)+l225(i+1));

hf226(i)=0.5*(l226(i)+l226(i+1));

hf227(i)=0.5*(l227(i)+l227(i+1));


%CONTROLLI COEEFICIENTI DI RUNGE KUTTA 3° ORDINE

a11=(1.*(k1s*(1.-1.*qthvs)*hf11(i)*wf1(i)*wf26(i)-1.*k1s*(1.-
1.*qthvs)*hf18(i)*wf1(i)*wf26(i)-1.*k1s*(1.-1.*qtsvs)*hf112(i)*wf2(i)*wf26(i)+k1s*(1.-
1.*qtsvs)*hf12(i)*wf2(i)*wf26(i)-1.*k1s*(1.-1.*qtrvs)*hf116(i)*wf26(i)*wf3(i)+k1s*(1.-
1.*qtrvs)*hf13(i)*wf26(i)*wf3(i)-1.*k1s*(1.-1.*qtgvs)*hf120(i)*wf26(i)*wf4(i)+k1s*(1.-
1.*qtgvs)*hf14(i)*wf26(i)*wf4(i)+k1s*rhos*hf126(i)*wf26(i)*((1.-1.*qthvs)*wf1(i)+(1.-
1.*qtsvs)*wf2(i)+(1.-1.*qtrvs)*wf3(i)+(1.-1.*qtgvs)*wf4(i))))/aco-(1.*(-1.*k1s*(1.-
1.*qthvs)*hf11(i)*wf1(i)*wf26(i)+k1s*(1.-1.*qthvs)*hf18(i)*wf1(i)*wf26(i)+k1s*(1.-
1.*qtsvs)*hf112(i)*wf2(i)*wf26(i)-1.*k1s*(1.-1.*qtsvs)*hf12(i)*wf2(i)*wf26(i)+k1s*(1.-
1.*qtrvs)*hf116(i)*wf26(i)*wf3(i)-1.*k1s*(1.-1.*qtrvs)*hf13(i)*wf26(i)*wf3(i)+k1s*(1.-
1.*qtgvs)*hf120(i)*wf26(i)*wf4(i)-1.*k1s*(1.-1.*qtgvs)*hf14(i)*wf26(i)*wf4(i)-
1.*k1s*rhos*hf126(i)*wf26(i)*((1.-1.*qthvs)*wf1(i)+(1.-1.*qtsvs)*wf2(i)+(1.-
1.*qtrvs)*wf3(i)+(1.-1.*qtgvs)*wf4(i)))*(-1.*aco*(k1s*(1.-1.*qthvs)*hf11(i)*wf1(i)*wf26(i)-
1.*k1s*(1.-1.*qthvs)*hf18(i)*wf1(i)*wf26(i)-1.*k1s*(1.-
1.*qtsvs)*hf112(i)*wf2(i)*wf26(i)+k1s*(1.-1.*qtsvs)*hf12(i)*wf2(i)*wf26(i)-1.*k1s*(1.-
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1.*qtrvs)*hf116(i)*wf26(i)*wf3(i)+k1s*(1.-1.*qtrvs)*hf13(i)*wf26(i)*wf3(i)-1.*k1s*(1.-
1.*qtgvs)*hf120(i)*wf26(i)*wf4(i)+k1s*(1.-
1.*qtgvs)*hf14(i)*wf26(i)*wf4(i)+k1s*rhos*hf126(i)*wf26(i)*((1.-1.*qthvs)*wf1(i)+(1.-
1.*qtsvs)*wf2(i)+(1.-1.*qtrvs)*wf3(i)+(1.-1.*qtgvs)*wf4(i)))-1.*(-1.*k1s*(1.-
1.*qthvs)*hf11(i)*wf1(i)*wf26(i)+k1s*(1.-1.*qthvs)*hf18(i)*wf1(i)*wf26(i)+k1s*(1.-
1.*qtsvs)*hf112(i)*wf2(i)*wf26(i)-1.*k1s*(1.-1.*qtsvs)*hf12(i)*wf2(i)*wf26(i)+k1s*(1.-
1.*qtrvs)*hf116(i)*wf26(i)*wf3(i)-1.*k1s*(1.-1.*qtrvs)*hf13(i)*wf26(i)*wf3(i)+k1s*(1.-
1.*qtgvs)*hf120(i)*wf26(i)*wf4(i)-1.*k1s*(1.-1.*qtgvs)*hf14(i)*wf26(i)*wf4(i)-
1.*k1s*rhos*hf126(i)*wf26(i)*((1.-1.*qthvs)*wf1(i)+(1.-1.*qtsvs)*wf2(i)+(1.-
1.*qtrvs)*wf3(i)+(1.-1.*qtgvs)*wf4(i)))*(k1s*(1.-1.*qthvs)*hf11(i)*wf1(i)*wf26(i)-
1.*k1s*(1.-1.*qthvs)*hf18(i)*wf1(i)*wf26(i)-1.*k1s*(1.-
1.*qtsvs)*hf112(i)*wf2(i)*wf26(i)+k1s*(1.-1.*qtsvs)*hf12(i)*wf2(i)*wf26(i)-1.*k1s*(1.-
1.*qtrvs)*hf116(i)*wf26(i)*wf3(i)+k1s*(1.-1.*qtrvs)*hf13(i)*wf26(i)*wf3(i)-1.*k1s*(1.-
1.*qtgvs)*hf120(i)*wf26(i)*wf4(i)+k1s*(1.-
1.*qtgvs)*hf14(i)*wf26(i)*wf4(i)+k1s*rhos*hf126(i)*wf26(i)*((1.-1.*qthvs)*wf1(i)+(1.-
1.*qtsvs)*wf2(i)+(1.-1.*qtrvs)*wf3(i)+(1.-1.*qtgvs)*wf4(i)))))/(aco*(1.*aco*afi-1.*(-
1.*k1s*(1.-1.*qthvs)*hf11(i)*wf1(i)*wf26(i)+k1s*(1.-
1.*qthvs)*hf18(i)*wf1(i)*wf26(i)+k1s*(1.-1.*qtsvs)*hf112(i)*wf2(i)*wf26(i)-1.*k1s*(1.-
1.*qtsvs)*hf12(i)*wf2(i)*wf26(i)+k1s*(1.-1.*qtrvs)*hf116(i)*wf26(i)*wf3(i)-1.*k1s*(1.-
1.*qtrvs)*hf13(i)*wf26(i)*wf3(i)+k1s*(1.-1.*qtgvs)*hf120(i)*wf26(i)*wf4(i)-1.*k1s*(1.-
1.*qtgvs)*hf14(i)*wf26(i)*wf4(i)-1.*k1s*rhos*hf126(i)*wf26(i)*((1.-1.*qthvs)*wf1(i)+(1.-
1.*qtsvs)*wf2(i)+(1.-1.*qtrvs)*wf3(i)+(1.-1.*qtgvs)*wf4(i)))^2));        a12= -((-
1.*aco*(k1s*(1.-1.*qthvs)*hf11(i)*wf1(i)*wf26(i)-1.*k1s*(1.-
1.*qthvs)*hf18(i)*wf1(i)*wf26(i)-1.*k1s*(1.-1.*qtsvs)*hf112(i)*wf2(i)*wf26(i)+k1s*(1.-
1.*qtsvs)*hf12(i)*wf2(i)*wf26(i)-1.*k1s*(1.-1.*qtrvs)*hf116(i)*wf26(i)*wf3(i)+k1s*(1.-
1.*qtrvs)*hf13(i)*wf26(i)*wf3(i)-1.*k1s*(1.-1.*qtgvs)*hf120(i)*wf26(i)*wf4(i)+k1s*(1.-
1.*qtgvs)*hf14(i)*wf26(i)*wf4(i)+k1s*rhos*hf126(i)*wf26(i)*((1.-1.*qthvs)*wf1(i)+(1.-
1.*qtsvs)*wf2(i)+(1.-1.*qtrvs)*wf3(i)+(1.-1.*qtgvs)*wf4(i)))-1.*(-1.*k1s*(1.-
1.*qthvs)*hf11(i)*wf1(i)*wf26(i)+k1s*(1.-1.*qthvs)*hf18(i)*wf1(i)*wf26(i)+k1s*(1.-
1.*qtsvs)*hf112(i)*wf2(i)*wf26(i)-1.*k1s*(1.-1.*qtsvs)*hf12(i)*wf2(i)*wf26(i)+k1s*(1.-
1.*qtrvs)*hf116(i)*wf26(i)*wf3(i)-1.*k1s*(1.-1.*qtrvs)*hf13(i)*wf26(i)*wf3(i)+k1s*(1.-
1.*qtgvs)*hf120(i)*wf26(i)*wf4(i)-1.*k1s*(1.-1.*qtgvs)*hf14(i)*wf26(i)*wf4(i)-
1.*k1s*rhos*hf126(i)*wf26(i)*((1.-1.*qthvs)*wf1(i)+(1.-1.*qtsvs)*wf2(i)+(1.-
1.*qtrvs)*wf3(i)+(1.-1.*qtgvs)*wf4(i)))*(k1s*(1.-1.*qthvs)*hf11(i)*wf1(i)*wf26(i)-
1.*k1s*(1.-1.*qthvs)*hf18(i)*wf1(i)*wf26(i)-1.*k1s*(1.-
1.*qtsvs)*hf112(i)*wf2(i)*wf26(i)+k1s*(1.-1.*qtsvs)*hf12(i)*wf2(i)*wf26(i)-1.*k1s*(1.-
1.*qtrvs)*hf116(i)*wf26(i)*wf3(i)+k1s*(1.-1.*qtrvs)*hf13(i)*wf26(i)*wf3(i)-1.*k1s*(1.-
1.*qtgvs)*hf120(i)*wf26(i)*wf4(i)+k1s*(1.-
1.*qtgvs)*hf14(i)*wf26(i)*wf4(i)+k1s*rhos*hf126(i)*wf26(i)*((1.-1.*qthvs)*wf1(i)+(1.-
1.*qtsvs)*wf2(i)+(1.-1.*qtrvs)*wf3(i)+(1.-1.*qtgvs)*wf4(i))))/(1.*aco*afi-1.*(-1.*k1s*(1.-
1.*qthvs)*hf11(i)*wf1(i)*wf26(i)+k1s*(1.-1.*qthvs)*hf18(i)*wf1(i)*wf26(i)+k1s*(1.-
1.*qtsvs)*hf112(i)*wf2(i)*wf26(i)-1.*k1s*(1.-1.*qtsvs)*hf12(i)*wf2(i)*wf26(i)+k1s*(1.-
1.*qtrvs)*hf116(i)*wf26(i)*wf3(i)-1.*k1s*(1.-1.*qtrvs)*hf13(i)*wf26(i)*wf3(i)+k1s*(1.-
1.*qtgvs)*hf120(i)*wf26(i)*wf4(i)-1.*k1s*(1.-1.*qtgvs)*hf14(i)*wf26(i)*wf4(i)-
1.*k1s*rhos*hf126(i)*wf26(i)*((1.-1.*qthvs)*wf1(i)+(1.-1.*qtsvs)*wf2(i)+(1.-
1.*qtrvs)*wf3(i)+(1.-1.*qtgvs)*wf4(i)))^2));        a13=
(1.*(hf11(i)*wf1(i)+hf12(i)*wf2(i)+hf13(i)*wf3(i)+hf14(i)*wf4(i)))/aib;
a14=(1.*(k2*hf112(i)*wf12(i)-1.*k2*hf113(i)*wf12(i)+k2*hf116(i)*wf16(i)-
1.*k2*hf117(i)*wf16(i)+k2*hf120(i)*wf20(i)-1.*k2*hf121(i)*wf20(i)+k2*hf18(i)*wf8(i)-
1.*k2*hf19(i)*wf8(i)))/aii-(1.*(-1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-
1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-
1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i))*((1.*(1.*bsr*pr*hf126(i)*(wf11(i)+wf15(i)+wf19(i
)+wf23(i)+wf25(i))*(1.*brs-
1.*pr*hf226(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))+pr*hf227(i)*(wf11(i)+wf15(i)+
wf19(i)+wf23(i)+wf25(i)))-1.*brs*(-1.*bsr*(-
1.*pr*hf126(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))-
(1.*g*hf126(i)*wf26(i))/(b2+wf26(i))-
1.*k1sm*hf124(i)*wf26(i)*wf5(i)+k1sm*rhos*hf126(i)*wf26(i)*wf5(i)+k1sm*hf15(i)*wf26
(i)*wf5(i))+1.*ps*hf126(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))*(1.*bsr+ps*hf226(i)*
(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))-
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1.*ps*hf227(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))))))/(brs*bsr*(k1sm*hf124(i)*wf26(i)*wf5(i)-1.*k1sm*rhos*hf126(i)*wf26(i)*wf5(i)-1.*k1sm*hf15(i)*wf26(i)*wf5(i)))-(1.*(1.*bsr*pr^2*hf126(i)*hf226(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))^2-1.*brs*(1.*api*bsr-1.*ps^2*hf126(i)*hf226(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))^2))*(1.*brs*bsr*(k1sm*hf124(i)*wf26(i)*wf5(i)-1.*k1sm*rhos*hf126(i)*wf26(i)*wf5(i)-1.*k1sm*hf15(i)*wf26(i)*wf5(i))*(-1.*aii*(k2*hf112(i)*wf12(i)-1.*k2*hf113(i)*wf12(i)+k2*hf116(i)*wf16(i)-1.*k2*hf117(i)*wf16(i)+k2*hf120(i)*wf20(i)-1.*k2*hf121(i)*wf20(i)-1.*k1sm*hf124(i)*wf26(i)*wf5(i)+k1sm*rhos*hf126(i)*wf26(i)*wf5(i)+k1sm*hf15(i)*wf26(i)*wf5(i)+k2*hf18(i)*wf8(i)-1.*k2*hf19(i)*wf8(i))-1.*(k2*hf112(i)*wf12(i)-1.*k2*hf113(i)*wf12(i)+k2*hf116(i)*wf16(i)-1.*k2*hf117(i)*wf16(i)+k2*hf120(i)*wf20(i)-1.*k2*hf121(i)*wf20(i)+k2*hf18(i)*wf8(i)-1.*k2*hf19(i)*wf8(i))*(-1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i)))-1.*(1.*aii*art-1.*(-1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i))^2)*(1.*bsr*pr*hf126(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))*(1.*brs-1.*pr*hf226(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))+pr*hf227(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i)))-1.*brs*(-1.*bsr*(-1.*pr*hf126(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))-(1.*g*hf126(i)*wf26(i))/(b2+wf26(i))-1.*k1sm*hf124(i)*wf26(i)*wf5(i)+k1sm*rhos*hf126(i)*wf26(i)*wf5(i)+k1sm*hf15(i)*wf26(i)*wf5(i))+1.*ps*hf126(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))*(1.*bsr+ps*hf226(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))-1.*ps*hf227(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i)))))))/(brs*bsr*(k1sm*hf124(i)*wf26(i)*wf5(i)-1.*k1sm*rhos*hf126(i)*wf26(i)*wf5(i)-1.*k1sm*hf15(i)*wf26(i)*wf5(i))*(-1.*aii*brs*bsr*(k1sm*hf124(i)*wf26(i)*wf5(i)-1.*k1sm*rhos*hf126(i)*wf26(i)*wf5(i)-1.*k1sm*hf15(i)*wf26(i)*wf5(i))^2-1.*(1.*aii*art-1.*(-1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i))^2)*(1.*bsr*pr^2*hf126(i)*hf226(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))^2-1.*brs*(1.*api*bsr-1.*ps^2*hf126(i)*hf226(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))^2))))/aii; a15= -((1.*brs*bsr*(k1sm*hf124(i)*wf26(i)*wf5(i)-1.*k1sm*rhos*hf126(i)*wf26(i)*wf5(i)-1.*k1sm*hf15(i)*wf26(i)*wf5(i))*(-1.*aii*(k2*hf112(i)*wf12(i)-1.*k2*hf113(i)*wf12(i)+k2*hf116(i)*wf16(i)-1.*k2*hf117(i)*wf16(i)+k2*hf120(i)*wf20(i)-1.*k2*hf121(i)*wf20(i)-1.*k1sm*hf124(i)*wf26(i)*wf5(i)+k1sm*rhos*hf126(i)*wf26(i)*wf5(i)+k1sm*hf15(i)*wf26(i)*wf5(i)+k2*hf18(i)*wf8(i)-1.*k2*hf19(i)*wf8(i))-1.*(k2*hf112(i)*wf12(i)-1.*k2*hf113(i)*wf12(i)+k2*hf116(i)*wf16(i)-1.*k2*hf117(i)*wf16(i)+k2*hf120(i)*wf20(i)-1.*k2*hf121(i)*wf20(i)+k2*hf18(i)*wf8(i)-1.*k2*hf19(i)*wf8(i))*(-1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i)))-1.*(1.*aii*art-1.*(-1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i))^2)*(1.*bsr*pr*hf126(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))*(1.*brs-1.*pr*hf226(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))+pr*hf227(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i)))-1.*brs*(-1.*bsr*(-1.*pr*hf126(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))-(1.*g*hf126(i)*wf26(i))/(b2+wf26(i))-1.*k1sm*hf124(i)*wf26(i)*wf5(i)+k1sm*rhos*hf126(i)*wf26(i)*wf5(i)+k1sm*hf15(i)*wf26(i)*wf5(i))+1.*ps*hf126(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))*(1.*bsr+ps*hf226(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))-1.*ps*hf227(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))))))/(-1.*aii*brs*bsr*(k1sm*hf124(i)*wf26(i)*wf5(i)-1.*k1sm*rhos*hf126(i)*wf26(i)*wf5(i)-1.*k1sm*hf15(i)*wf26(i)*wf5(i))^2-1.*(1.*aii*art-1.*(-1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-

1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-
1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i))^2)*(1.*bsr*pr^2*hf126(i)*hf226(i)*(wf11(i)+wf15
(i)+wf19(i)+wf23(i)+wf25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*hf126(i)*hf226(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))^2)))); a16=(-
1.*(1.*bsr*pr*hf126(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))*(1.*brs-
1.*pr*hf226(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))+pr*hf227(i)*(wf11(i)+wf15(i)+
wf19(i)+wf23(i)+wf25(i)))-1.*brs*(-1.*bsr*(-
1.*pr*hf126(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))-
(1.*g*hf126(i)*wf26(i))/(b2+wf26(i))-
1.*k1sm*hf124(i)*wf26(i)*wf5(i)+k1sm*rhos*hf126(i)*wf26(i)*wf5(i)+k1sm*hf15(i)*wf26
(i)*wf5(i))+1.*ps*hf126(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))*(1.*bsr+ps*hf226(i)*
(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))-
1.*ps*hf227(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))))))/(brs*bsr*(k1sm*hf124(i)*wf2
6(i)*wf5(i)-1.*k1sm*rhos*hf126(i)*wf26(i)*wf5(i)-
1.*k1sm*hf15(i)*wf26(i)*wf5(i)))+(1.*(1.*bsr*pr^2*hf126(i)*hf226(i)*(wf11(i)+wf15(i)+wf
19(i)+wf23(i)+wf25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*hf126(i)*hf226(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))^2))*(1.*brs*bsr*(k1s
m*hf124(i)*wf26(i)*wf5(i)-1.*k1sm*rhos*hf126(i)*wf26(i)*wf5(i)-
1.*k1sm*hf15(i)*wf26(i)*wf5(i))*(-1.*aii*(k2*hf112(i)*wf12(i)-
1.*k2*hf113(i)*wf12(i)+k2*hf116(i)*wf16(i)-1.*k2*hf117(i)*wf16(i)+k2*hf120(i)*wf20(i)-
1.*k2*hf121(i)*wf20(i)-
1.*k1sm*hf124(i)*wf26(i)*wf5(i)+k1sm*rhos*hf126(i)*wf26(i)*wf5(i)+k1sm*hf15(i)*wf26
(i)*wf5(i)+k2*hf18(i)*wf8(i)-1.*k2*hf19(i)*wf8(i))-1.*(k2*hf112(i)*wf12(i)-
1.*k2*hf113(i)*wf12(i)+k2*hf116(i)*wf16(i)-1.*k2*hf117(i)*wf16(i)+k2*hf120(i)*wf20(i)-
1.*k2*hf121(i)*wf20(i)+k2*hf18(i)*wf8(i)-1.*k2*hf19(i)*wf8(i))*(-
1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-
1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i)))-
1.*(1.*aii*art-1.*(-1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-
1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-
1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i))^2)*(1.*bsr*pr*hf126(i)*(wf11(i)+wf15(i)+wf19(i)
+wf23(i)+wf25(i))*(1.*brs-
1.*pr*hf226(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))+pr*hf227(i)*(wf11(i)+wf15(i)+
wf19(i)+wf23(i)+wf25(i)))-1.*brs*(-1.*bsr*(-
1.*pr*hf126(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))-
(1.*g*hf126(i)*wf26(i))/(b2+wf26(i))-
1.*k1sm*hf124(i)*wf26(i)*wf5(i)+k1sm*rhos*hf126(i)*wf26(i)*wf5(i)+k1sm*hf15(i)*wf26
(i)*wf5(i))+1.*ps*hf126(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))*(1.*bsr+ps*hf226(i)*
(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))-
1.*ps*hf227(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i)))))))/(brs*bsr*(k1sm*hf124(i)*wf
26(i)*wf5(i)-1.*k1sm*rhos*hf126(i)*wf26(i)*wf5(i)-1.*k1sm*hf15(i)*wf26(i)*wf5(i))*(-
1.*aii*brs*bsr*(k1sm*hf124(i)*wf26(i)*wf5(i)-1.*k1sm*rhos*hf126(i)*wf26(i)*wf5(i)-
1.*k1sm*hf15(i)*wf26(i)*wf5(i))^2-1.*(1.*aii*art-1.*(-
1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-
1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-
1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i))^2)*(1.*bsr*pr^2*hf126(i)*hf226(i)*(wf11(i)+wf15
(i)+wf19(i)+wf23(i)+wf25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*hf126(i)*hf226(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))^2)))); u21s=
min(1,max(0,(1.*(1.*bsr+ps*hf226(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))-
1.*ps*hf227(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))))/bsr+(1.*ps*hf226(i)*(wf13(i)+
wf17(i)+wf21(i)+wf24(i)+wf9(i))*(1.*brs*bsr*(k1sm*hf124(i)*wf26(i)*wf5(i)-
1.*k1sm*rhos*hf126(i)*wf26(i)*wf5(i)-1.*k1sm*hf15(i)*wf26(i)*wf5(i))*(-
1.*aii*(k2*hf112(i)*wf12(i)-1.*k2*hf113(i)*wf12(i)+k2*hf116(i)*wf16(i)-
1.*k2*hf117(i)*wf16(i)+k2*hf120(i)*wf20(i)-1.*k2*hf121(i)*wf20(i)-
1.*k1sm*hf124(i)*wf26(i)*wf5(i)+k1sm*rhos*hf126(i)*wf26(i)*wf5(i)+k1sm*hf15(i)*wf26
(i)*wf5(i)+k2*hf18(i)*wf8(i)-1.*k2*hf19(i)*wf8(i))-1.*(k2*hf112(i)*wf12(i)-
1.*k2*hf113(i)*wf12(i)+k2*hf116(i)*wf16(i)-1.*k2*hf117(i)*wf16(i)+k2*hf120(i)*wf20(i)-
1.*k2*hf121(i)*wf20(i)+k2*hf18(i)*wf8(i)-1.*k2*hf19(i)*wf8(i))*(-
1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-
1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i)))-
1.*(1.*aii*art-1.*(-1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-

```
1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-
1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i))^2)*(1.*bsr*pr*hf126(i)*(wf11(i)+wf15(i)+wf19(i)
+wf23(i)+wf25(i))*(1.*brs-
1.*pr*hf226(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))+pr*hf227(i)*(wf11(i)+wf15(i)+
wf19(i)+wf23(i)+wf25(i)))-1.*brs*(-1.*bsr*(-
1.*pr*hf126(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))-
(1.*g*hf126(i)*wf26(i))/(b2+wf26(i))-
1.*k1sm*hf124(i)*wf26(i)*wf5(i)+k1sm*rhos*hf126(i)*wf26(i)*wf5(i)+k1sm*hf15(i)*wf26
(i)*wf5(i))+1.*ps*hf126(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))*(1.*bsr+ps*hf226(i)*
(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))-
1.*ps*hf227(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i)))))))/(bsr*(-
1.*aii*brs*bsr*(k1sm*hf124(i)*wf26(i)*wf5(i)-1.*k1sm*rhos*hf126(i)*wf26(i)*wf5(i)-
1.*k1sm*hf15(i)*wf26(i)*wf5(i))^2-1.*(1.*aii*art-1.*(-
1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-
1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-
1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i))^2)*(1.*bsr*pr^2*hf126(i)*hf226(i)*(wf11(i)+wf15
(i)+wf19(i)+wf23(i)+wf25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*hf126(i)*hf226(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))^2)))))); u22s=
min(1,max(0,(1.*(1.*brs-
1.*pr*hf226(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))+pr*hf227(i)*(wf11(i)+wf15(i)+
wf19(i)+wf23(i)+wf25(i))))/brs-
(1.*pr*hf226(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))*(1.*brs*bsr*(k1sm*hf124(i)*w
f26(i)*wf5(i)-1.*k1sm*rhos*hf126(i)*wf26(i)*wf5(i)-1.*k1sm*hf15(i)*wf26(i)*wf5(i))*(-
1.*aii*(k2*hf112(i)*wf12(i)-1.*k2*hf113(i)*wf12(i)+k2*hf116(i)*wf16(i)-
1.*k2*hf117(i)*wf16(i)+k2*hf120(i)*wf20(i)-1.*k2*hf121(i)*wf20(i)-
1.*k1sm*hf124(i)*wf26(i)*wf5(i)+k1sm*rhos*hf126(i)*wf26(i)*wf5(i)+k1sm*hf15(i)*wf26
(i)*wf5(i)+k2*hf18(i)*wf8(i)-1.*k2*hf19(i)*wf8(i))-1.*(k2*hf112(i)*wf12(i)-
1.*k2*hf113(i)*wf12(i)+k2*hf116(i)*wf16(i)-1.*k2*hf117(i)*wf16(i)+k2*hf120(i)*wf20(i)-
1.*k2*hf121(i)*wf20(i)+k2*hf18(i)*wf8(i)-1.*k2*hf19(i)*wf8(i))*(-
1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-
1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i)))-
1.*(1.*aii*art-1.*(-1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-
1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-
1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i))^2)*(1.*bsr*pr*hf126(i)*(wf11(i)+wf15(i)+wf19(i)
+wf23(i)+wf25(i))*(1.*brs-
1.*pr*hf226(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))+pr*hf227(i)*(wf11(i)+wf15(i)+
wf19(i)+wf23(i)+wf25(i)))-1.*brs*(-1.*bsr*(-
1.*pr*hf126(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))-
(1.*g*hf126(i)*wf26(i))/(b2+wf26(i))-
1.*k1sm*hf124(i)*wf26(i)*wf5(i)+k1sm*rhos*hf126(i)*wf26(i)*wf5(i)+k1sm*hf15(i)*wf26
(i)*wf5(i))+1.*ps*hf126(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))*(1.*bsr+ps*hf226(i)*
(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))-
1.*ps*hf227(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i)))))))/(brs*(-
1.*aii*brs*bsr*(k1sm*hf124(i)*wf26(i)*wf5(i)-1.*k1sm*rhos*hf126(i)*wf26(i)*wf5(i)-
1.*k1sm*hf15(i)*wf26(i)*wf5(i))^2-1.*(1.*aii*art-1.*(-
1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-
1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-
1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i))^2)*(1.*bsr*pr^2*hf126(i)*hf226(i)*(wf11(i)+wf15
(i)+wf19(i)+wf23(i)+wf25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*hf126(i)*hf226(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))^2))))));


    if(a11<0 || a11>1)

        a11=0;

    end;
```

```
if(a12<0 || a12>1)
    a12=0;
end;

if(a13<0 || a13>1)
    a13=0;
end;

if(a14<0 || a14>1)
    a14=0;
end;

if(a15<0 || a15>1)
    a15=0;
end;

if(a16<0 || a16>1)
    a16=0;
end;


    D=[a11,a12,a13,a14,a15,a16;a11,a12,a13,a14,a15,1;a11,a12,a13,a14,a15,0;a11,a12,
a13,a14,1,a16;a11,a12,a13,a14,1,1;a11,a12,a13,a14,1,0;a11,a12,a13,a14,0,a16;a11,a12,a13,a1
4,0,1;a11,a12,a13,a14,0,0;a11,a12,a13,1,a15,a16;a11,a12,a13,1,a15,1;a11,a12,a13,1,a15,0;a1
1,a12,a13,1,1,a16;a11,a12,a13,1,1,1;a11,a12,a13,1,1,0;a11,a12,a13,1,0,a16;a11,a12,a13,1,0,1;
a11,a12,a13,1,0,0;a11,a12,a13,0,a15,a16;a11,a12,a13,0,a15,1;a11,a12,a13,0,a15,0;a11,a12,a1
3,0,1,a16;a11,a12,a13,0,1,1;a11,a12,a13,0,1,0;a11,a12,a13,0,0,a16;a11,a12,a13,0,0,1;a11,a12,
a13,0,0,0;a11,a12,1,a14,a15,a16;a11,a12,1,a14,a15,1;a11,a12,1,a14,a15,0;a11,a12,1,a14,1,a1
6;a11,a12,1,a14,1,1;a11,a12,1,a14,1,0;a11,a12,1,a14,0,a16;a11,a12,1,a14,0,1;a11,a12,1,a14,0,
0;a11,a12,1,1,a15,a16;a11,a12,1,1,a15,1;a11,a12,1,1,a15,0;a11,a12,1,1,1,a16;a11,a12,1,1,1,1;
a11,a12,1,1,1,0;a11,a12,1,1,0,a16;a11,a12,1,1,0,1;a11,a12,1,1,0,0;a11,a12,1,0,a15,a16;a11,a1
2,1,0,a15,1;a11,a12,1,0,a15,0;a11,a12,1,0,1,a16;a11,a12,1,0,1,1;a11,a12,1,0,1,0;a11,a12,1,0,0
,a16;a11,a12,1,0,0,1;a11,a12,1,0,0,0;a11,a12,0,a14,a15,a16;a11,a12,0,a14,a15,1;a11,a12,0,a1
4,a15,0;a11,a12,0,a14,1,a16;a11,a12,0,a14,1,1;a11,a12,0,a14,1,0;a11,a12,0,a14,0,a16;a11,a12
,0,a14,0,1;a11,a12,0,a14,0,0;a11,a12,0,1,a15,a16;a11,a12,0,1,a15,1;a11,a12,0,1,a15,0;a11,a12
,0,1,1,a16;a11,a12,0,1,1,1;a11,a12,0,1,1,0;a11,a12,0,1,0,a16;a11,a12,0,1,0,1;a11,a12,0,1,0,0;a
11,a12,0,0,a15,a16;a11,a12,0,0,a15,1;a11,a12,0,0,a15,0;a11,a12,0,0,1,a16;a11,a12,0,0,1,1;a1
1,a12,0,0,1,0;a11,a12,0,0,0,a16;a11,a12,0,0,0,1;a11,a12,0,0,0,0;a11,1,a13,a14,a15,a16;a11,1,
a13,a14,a15,1;a11,1,a13,a14,a15,0;a11,1,a13,a14,1,a16;a11,1,a13,a14,1,1;a11,1,a13,a14,1,0;a
11,1,a13,a14,0,a16;a11,1,a13,a14,0,1;a11,1,a13,a14,0,0;a11,1,a13,1,a15,a16;a11,1,a13,1,a15,
1;a11,1,a13,1,a15,0;a11,1,a13,1,1,a16;a11,1,a13,1,1,1;a11,1,a13,1,1,0;a11,1,a13,1,0,a16;a11,
1,a13,1,0,1;a11,1,a13,1,0,0;a11,1,a13,0,a15,a16;a11,1,a13,0,a15,1;a11,1,a13,0,a15,0;a11,1,a1
3,0,1,a16;a11,1,a13,0,1,1;a11,1,a13,0,1,0;a11,1,a13,0,0,a16;a11,1,a13,0,0,1;a11,1,a13,0,0,0;a
```

```
11,1,1,a14,a15,a16;a11,1,1,a14,a15,1;a11,1,1,a14,a15,0;a11,1,1,a14,1,a16;a11,1,1,a14,1,1;a1
1,1,1,a14,1,0;a11,1,1,a14,0,a16;a11,1,1,a14,0,1;a11,1,1,a14,0,0;a11,1,1,1,a15,a16;a11,1,1,1,a
15,1;a11,1,1,1,a15,0;a11,1,1,1,1,a16;a11,1,1,1,1,1;a11,1,1,1,1,0;a11,1,1,1,0,a16;a11,1,1,1,0,1;
a11,1,1,1,0,0;a11,1,1,0,a15,a16;a11,1,1,0,a15,1;a11,1,1,0,a15,0;a11,1,1,0,1,a16;a11,1,1,0,1,1;
a11,1,1,0,1,0;a11,1,1,0,0,a16;a11,1,1,0,0,1;a11,1,1,0,0,0;a11,1,0,a14,a15,a16;a11,1,0,a14,a15,
1;a11,1,0,a14,a15,0;a11,1,0,a14,1,a16;a11,1,0,a14,1,1;a11,1,0,a14,1,0;a11,1,0,a14,0,a16;a11,
1,0,a14,0,1;a11,1,0,a14,0,0;a11,1,0,1,a15,a16;a11,1,0,1,a15,1;a11,1,0,1,a15,0;a11,1,0,1,1,a16;
a11,1,0,1,1,1;a11,1,0,1,1,0;a11,1,0,1,0,a16;a11,1,0,1,0,1;a11,1,0,1,0,0;a11,1,0,0,a15,a16;a11,
1,0,0,a15,1;a11,1,0,0,a15,0;a11,1,0,0,1,a16;a11,1,0,0,1,1;a11,1,0,0,1,0;a11,1,0,0,0,a16;a11,1,
0,0,0,1;a11,1,0,0,0,0;a11,0,a13,a14,a15,a16;a11,0,a13,a14,a15,1;a11,0,a13,a14,a15,0;a11,0,a1
3,a14,1,a16;a11,0,a13,a14,1,1;a11,0,a13,a14,1,0;a11,0,a13,a14,0,a16;a11,0,a13,a14,0,1;a11,0,
a13,a14,0,0;a11,0,a13,1,a15,a16;a11,0,a13,1,a15,1;a11,0,a13,1,a15,0;a11,0,a13,1,1,a16;a11,0,
a13,1,1,1;a11,0,a13,1,1,0;a11,0,a13,1,0,a16;a11,0,a13,1,0,1;a11,0,a13,1,0,0;a11,0,a13,0,a15,a
16;a11,0,a13,0,a15,1;a11,0,a13,0,a15,0;a11,0,a13,0,1,a16;a11,0,a13,0,1,1;a11,0,a13,0,1,0;a11
,0,a13,0,0,a16;a11,0,a13,0,0,1;a11,0,a13,0,0,0;a11,0,1,a14,a15,a16;a11,0,1,a14,a15,1;a11,0,1,
a14,a15,0;a11,0,1,a14,1,a16;a11,0,1,a14,1,1;a11,0,1,a14,1,0;a11,0,1,a14,0,a16;a11,0,1,a14,0,
1;a11,0,1,a14,0,0;a11,0,1,1,a15,a16;a11,0,1,1,a15,1;a11,0,1,1,a15,0;a11,0,1,1,1,a16;a11,0,1,1,
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1;a11,0,1,0,a15,0;a11,0,1,0,1,a16;a11,0,1,0,1,1;a11,0,1,0,1,0;a11,0,1,0,0,a16;a11,0,1,0,0,1;a1
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1,0,0,a15,a16;0,1,0,0,a15,1;0,1,0,0,a15,0;0,1,0,0,1,a16;0,1,0,0,1,1;0,1,0,0,1,0;0,1,0,0,0,a16;0,
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0,1,1,0;0,0,0,1,0,a16;0,0,0,1,0,1;0,0,0,1,0,0;0,0,0,0,a15,a16;0,0,0,0,a15,1;0,0,0,0,a15,0;0,0,0,0
,1,a16;0,0,0,0,1,1;0,0,0,0,1,0;0,0,0,0,0,a16;0,0,0,0,0,1;0,0,0,0,0,0];

    eval_HT=eval(fHT);

    [max_HT, id_HT]=nanmax(eval_HT);

    u11(3)=D(id_HT,1);
    u12(3)=D(id_HT,2);
    u13(3)=D(id_HT,3);
    u14(3)=D(id_HT,4);
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u15(3)=D(id_HT,5);

u16(3)=D(id_HT,6);

u21(3)=u21s;

u22(3)=u22s;

%VARIABILI COEEFICIENTI DI RUNGE KUTTA 3° ORDINE

kv1(3)=-(muth*wf1(i))+u13(3)*wf1(i)-k3*wf1(i)*(wf24(i)+wf25(i))-k1s*(1-qthvs)*(1-u11(3))*(1-u12(3))*wf1(i)*wf26(i)-k1r*(1-qthvr)*wf1(i)*wf27(i)-k*wf1(i)*(wf26(i)+wf27(i))-kprimo*wf1(i)*(wf26(i)+wf27(i))+(oth*s1th)/(oth+wf26(i)+wf27(i))+r*wf1(i)*(1-(wf1(i)+wf10(i)+wf11(i)+wf12(i)+wf13(i)+wf14(i)+wf15(i)+wf16(i)+wf17(i)+wf18(i)+wf19(i)+wf2(i)+wf20(i)+wf21(i)+wf22(i)+wf23(i)+wf3(i)+wf4(i)+wf8(i)+wf9(i))/tmax);

kv2(3)=-(muts*wf2(i))+u13(3)*wf2(i)-k3*wf2(i)*(wf24(i)+wf25(i))+k*wf1(i)*wf26(i)-k1s*(1-qtsvs)*(1-u11(3))*(1-u12(3))*wf2(i)*wf26(i)-k1r*(1-qtsvr)*wf2(i)*wf27(i); kv3(3)=k*wf1(i)*wf27(i)-mutr*wf3(i)+u13(3)*wf3(i)-k3*(wf24(i)+wf25(i))*wf3(i)-k1s*(1-qtrvs)*(1-u11(3))*(1-u12(3))*wf26(i)*wf3(i)-k1r*(1-qtrvr)*wf27(i)*wf3(i); kv4(3)=kprimo*wf1(i)*(wf26(i)+wf27(i))-mutg*wf4(i)+u13(3)*wf4(i)-k3*(wf24(i)+wf25(i))*wf4(i)-k1s*(1-qtgvs)*(1-u11(3))*(1-u12(3))*wf26(i)*wf4(i)-k1r*(1-qtgvr)*wf27(i)*wf4(i); kv5(3)=s1mh-mumh*wf5(i)-k1sm*(1-u15(3))*(1-u16(3))*wf26(i)*wf5(i)-k1rm*wf27(i)*wf5(i); kv6(3)=-(muip*wf6(i))-c*q*wf6(i)*(wf11(i)+wf13(i)+wf15(i)+wf17(i)+wf19(i)+wf21(i)+wf23(i)+wf9(i))+c*(wf1(i)+wf2(i)+wf3(i)+wf4(i))*wf6(i)*(wf11(i)+wf13(i)+wf15(i)+wf17(i)+wf19(i)+wf21(i)+wf23(i)+wf9(i)); kv7(3)=-(muie*wf7(i))+c*q*wf6(i)*(wf11(i)+wf13(i)+wf15(i)+wf17(i)+wf19(i)+wf21(i)+wf23(i)+wf9(i)); kv8(3)=k3*wf1(i)*wf24(i)+k1s*(1-qthvs)*(1-u11(3))*(1-u12(3))*wf1(i)*wf26(i)-muthsl*wf8(i)-k2*(1-u14(3))*(1-u16(3))*wf8(i); kv9(3)=k2*(1-u14(3))*(1-u16(3))*wf8(i)-muthsa*wf9(i)-is*wf7(i)*wf9(i); kv10(3)=-(k2*wf10(i))-muthrl*wf10(i)+k3*wf1(i)*wf25(i)+k1r*(1-qthvr)*wf1(i)*wf27(i); kv11(3)=k2*wf10(i)-muthra*wf11(i)-ir*wf11(i)*wf7(i); kv12(3)=-(mutssl*wf12(i))-k2*(1-u14(3))*(1-u16(3))*wf12(i)+k3*wf2(i)*wf24(i)+k1s*(1-qtsvs)*(1-u11(3))*(1-u12(3))*wf2(i)*wf26(i); kv13(3)=k2*(1-u14(3))*(1-u16(3))*wf12(i)-mutssa*wf13(i)-is*wf13(i)*wf7(i); kv14(3)=-(k2*wf14(i))-mutsrl*wf14(i)+k3*wf2(i)*wf25(i)+k1r*(1-qtsvr)*wf2(i)*wf27(i); kv15(3)=k2*wf14(i)-mutsra*wf15(i)-ir*wf15(i)*wf7(i); kv16(3)=-(mutrsl*wf16(i))-k2*(1-u14(3))*(1-u16(3))*wf16(i)+k3*wf24(i)*wf3(i)+k1s*(1-qtrvs)*(1-u11(3))*(1-u12(3))*wf26(i)*wf3(i); kv17(3)=k2*(1-u14(3))*(1-u16(3))*wf16(i)-mutrsa*wf17(i)-is*wf17(i)*wf7(i); kv18(3)=-(k2*wf18(i))-mutrrl*wf18(i)+k3*wf25(i)*wf3(i)+k1r*(1-qtrvr)*wf27(i)*wf3(i); kv19(3)=k2*wf18(i)-mutrra*wf19(i)-ir*wf19(i)*wf7(i); kv20(3)=-(mutgsl*wf20(i))-k2*(1-u14(3))*(1-u16(3))*wf20(i)+k3*wf24(i)*wf4(i)+k1s*(1-qtgvs)*(1-u11(3))*(1-u12(3))*wf26(i)*wf4(i); kv21(3)=k2*(1-u14(3))*(1-u16(3))*wf20(i)-mutgsa*wf21(i)-is*wf21(i)*wf7(i); kv22(3)=-(k2*wf22(i))-mutgrl*wf22(i)+k3*wf25(i)*wf4(i)+k1r*(1-qtgvr)*wf27(i)*wf4(i); kv23(3)=k2*wf22(i)-mutgra*wf23(i)-ir*wf23(i)*wf7(i); kv24(3)=-(mumhsa*wf24(i))+k1sm*(1-u15(3))*(1-u16(3))*wf26(i)*wf5(i); kv25(3)=-(mumhra*wf25(i))+k1rm*wf27(i)*wf5(i); kv26(3)=pr*(1-u15(3))*(1-u22(3))*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))-muvs*wf26(i)+(g*(1-u15(3))*wf26(i))/(b2+wf26(i))-k1s*rhos*(1-u11(3))*(1-u12(3))*wf26(i)*((1-qthvs)*wf1(i)+(1-qtsvs)*wf2(i)+(1-qtrvs)*wf3(i)+(1-qtgvs)*wf4(i))-wf26(i)*(psprimo*wf2(i)+ssprimo*wf4(i))-k1sm*rhos*(1-u15(3))*(1-u16(3))*wf26(i)*wf5(i)+ps*(1-u15(3))*u21(3)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i)); kv27(3)=pr*u22(3)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))-muvr*wf27(i)+(g*wf27(i))/(b2+wf27(i))-k1r*rhor*wf27(i)*((1-qthvr)*wf1(i)+(1-qtsvr)*wf2(i)+(1-qtrvr)*wf3(i)+(1-qtgvr)*wf4(i))-wf27(i)*(prprimo*wf3(i)+srprimo*wf4(i))-k1rm*rhor*wf27(i)*wf5(i)+ps*(1-u21(3))*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i));

```
%VARIABILI DI APPOGGIO wf per 4° ordine
wf1(i)=x1(i)+h*kv1(3);
wf2(i)=x2(i)+h*kv2(3);
wf3(i)=x3(i)+h*kv3(3);
wf4(i)=x4(i)+h*kv4(3);
wf5(i)=x5(i)+h*kv5(3);
wf6(i)=x6(i)+h*kv6(3);
wf7(i)=x7(i)+h*kv7(3);
wf8(i)=x8(i)+h*kv8(3);
wf9(i)=x9(i)+h*kv9(3);
wf10(i)=x10(i)+h*kv10(3);
wf11(i)=x11(i)+h*kv11(3);
wf12(i)=x12(i)+h*kv12(3);
wf13(i)=x13(i)+h*kv13(3);
wf14(i)=x14(i)+h*kv14(3);
wf15(i)=x15(i)+h*kv15(3);
wf16(i)=x16(i)+h*kv16(3);
wf17(i)=x17(i)+h*kv17(3);
wf18(i)=x18(i)+h*kv18(3);
wf19(i)=x19(i)+h*kv19(3);
wf20(i)=x20(i)+h*kv20(3);
wf21(i)=x21(i)+h*kv21(3);
wf22(i)=x22(i)+h*kv22(3);
wf23(i)=x23(i)+h*kv23(3);
wf24(i)=x24(i)+h*kv24(3);
wf25(i)=x25(i)+h*kv25(3);
wf26(i)=x26(i)+h*kv26(3);
wf27(i)=x27(i)+h*kv27(3);


%VARIABILI DI APPOGGIO hf per 4° ordine
hf11(i)=(l11(i+1));
hf12(i)=(l12(i+1));
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```
hf13(i)=(l13(i+1));
hf14(i)=(l14(i+1));
hf15(i)=(l15(i+1));
hf16(i)=(l16(i+1));
hf17(i)=(l17(i+1));
hf18(i)=(l18(i+1));
hf19(i)=(l19(i+1));
hf110(i)=(l110(i+1));
hf111(i)=(l111(i+1));
hf112(i)=(l112(i+1));
hf113(i)=(l113(i+1));
hf114(i)=(l114(i+1));
hf115(i)=(l115(i+1));
hf116(i)=(l116(i+1));
hf117(i)=(l117(i+1));
hf118(i)=(l118(i+1));
hf119(i)=(l119(i+1));
hf120(i)=(l120(i+1));
hf121(i)=(l121(i+1));
hf122(i)=(l122(i+1));
hf123(i)=(l123(i+1));
hf124(i)=(l124(i+1));
hf125(i)=(l125(i+1));
hf126(i)=(l126(i+1));
hf127(i)=(l127(i+1));

hf21(i)=(l21(i+1));
hf22(i)=(l22(i+1));
hf23(i)=(l23(i+1));
hf24(i)=(l24(i+1));
hf25(i)=(l25(i+1));
hf26(i)=(l26(i+1));
hf27(i)=(l27(i+1));
```

```
hf28(i)=(l28(i+1));

hf29(i)=(l29(i+1));

hf210(i)=(l210(i+1));

hf211(i)=(l211(i+1));

hf212(i)=(l212(i+1));

hf213(i)=(l213(i+1));

hf214(i)=(l214(i+1));

hf215(i)=(l215(i+1));

hf216(i)=(l216(i+1));

hf217(i)=(l217(i+1));

hf218(i)=(l218(i+1));

hf219(i)=(l219(i+1));

hf220(i)=(l220(i+1));

hf221(i)=(l221(i+1));

hf222(i)=(l222(i+1));

hf223(i)=(l223(i+1));

hf224(i)=(l224(i+1));

hf225(i)=(l225(i+1));

hf226(i)=(l226(i+1));

hf227(i)=(l227(i+1));



%CONTROLLI COEEFICIENTI DI RUNGE KUTTA 4° ORDINE

a11= (1.*(k1s*(1.-1.*qthvs)*hf11(i)*wf1(i)*wf26(i)-1.*k1s*(1.-
1.*qthvs)*hf18(i)*wf1(i)*wf26(i)-1.*k1s*(1.-1.*qtsvs)*hf112(i)*wf2(i)*wf26(i)+k1s*(1.-
1.*qtsvs)*hf12(i)*wf2(i)*wf26(i)-1.*k1s*(1.-1.*qtrvs)*hf116(i)*wf26(i)*wf3(i)+k1s*(1.-
1.*qtrvs)*hf13(i)*wf26(i)*wf3(i)-1.*k1s*(1.-1.*qtgvs)*hf120(i)*wf26(i)*wf4(i)+k1s*(1.-
1.*qtgvs)*hf14(i)*wf26(i)*wf4(i)+k1s*rhos*hf126(i)*wf26(i)*((1.-1.*qthvs)*wf1(i)+(1.-
1.*qtsvs)*wf2(i)+(1.-1.*qtrvs)*wf3(i)+(1.-1.*qtgvs)*wf4(i))))/aco-(1.*(-1.*k1s*(1.-
1.*qthvs)*hf11(i)*wf1(i)*wf26(i)+k1s*(1.-1.*qthvs)*hf18(i)*wf1(i)*wf26(i)+k1s*(1.-
1.*qtsvs)*hf112(i)*wf2(i)*wf26(i)-1.*k1s*(1.-1.*qtsvs)*hf12(i)*wf2(i)*wf26(i)+k1s*(1.-
1.*qtrvs)*hf116(i)*wf26(i)*wf3(i)-1.*k1s*(1.-1.*qtrvs)*hf13(i)*wf26(i)*wf3(i)+k1s*(1.-
1.*qtgvs)*hf120(i)*wf26(i)*wf4(i)-1.*k1s*(1.-1.*qtgvs)*hf14(i)*wf26(i)*wf4(i)-
1.*k1s*rhos*hf126(i)*wf26(i)*((1.-1.*qthvs)*wf1(i)+(1.-1.*qtsvs)*wf2(i)+(1.-
1.*qtrvs)*wf3(i)+(1.-1.*qtgvs)*wf4(i)))*(-1.*aco*(k1s*(1.-1.*qthvs)*hf11(i)*wf1(i)*wf26(i)-
1.*k1s*(1.-1.*qthvs)*hf18(i)*wf1(i)*wf26(i)-1.*k1s*(1.-
1.*qtsvs)*hf112(i)*wf2(i)*wf26(i)+k1s*(1.-1.*qtsvs)*hf12(i)*wf2(i)*wf26(i)-1.*k1s*(1.-
1.*qtrvs)*hf116(i)*wf26(i)*wf3(i)+k1s*(1.-1.*qtrvs)*hf13(i)*wf26(i)*wf3(i)-1.*k1s*(1.-
1.*qtgvs)*hf120(i)*wf26(i)*wf4(i)+k1s*(1.-
1.*qtgvs)*hf14(i)*wf26(i)*wf4(i)+k1s*rhos*hf126(i)*wf26(i)*((1.-1.*qthvs)*wf1(i)+(1.-
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1.*qtsvs)*wf2(i)+(1.-1.*qtrvs)*wf3(i)+(1.-1.*qtgvs)*wf4(i)))-1.*(-1.*k1s*(1.-
1.*qthvs)*hf11(i)*wf1(i)*wf26(i)+k1s*(1.-1.*qthvs)*hf18(i)*wf1(i)*wf26(i)+k1s*(1.-
1.*qtsvs)*hf112(i)*wf2(i)*wf26(i)-1.*k1s*(1.-1.*qtsvs)*hf12(i)*wf2(i)*wf26(i)+k1s*(1.-
1.*qtrvs)*hf116(i)*wf26(i)*wf3(i)-1.*k1s*(1.-1.*qtrvs)*hf13(i)*wf26(i)*wf3(i)+k1s*(1.-
1.*qtgvs)*hf120(i)*wf26(i)*wf4(i)-1.*k1s*(1.-1.*qtgvs)*hf14(i)*wf26(i)*wf4(i)-
1.*k1s*rhos*hf126(i)*wf26(i)*((1.-1.*qthvs)*wf1(i)+(1.-1.*qtsvs)*wf2(i)+(1.-
1.*qtrvs)*wf3(i)+(1.-1.*qtgvs)*wf4(i)))*(k1s*(1.-1.*qthvs)*hf11(i)*wf1(i)*wf26(i)-
1.*k1s*(1.-1.*qthvs)*hf18(i)*wf1(i)*wf26(i)-1.*k1s*(1.-
1.*qtsvs)*hf112(i)*wf2(i)*wf26(i)+k1s*(1.-1.*qtsvs)*hf12(i)*wf2(i)*wf26(i)-1.*k1s*(1.-
1.*qtrvs)*hf116(i)*wf26(i)*wf3(i)+k1s*(1.-1.*qtrvs)*hf13(i)*wf26(i)*wf3(i)-1.*k1s*(1.-
1.*qtgvs)*hf120(i)*wf26(i)*wf4(i)+k1s*(1.-
1.*qtgvs)*hf14(i)*wf26(i)*wf4(i)+k1s*rhos*hf126(i)*wf26(i)*((1.-1.*qthvs)*wf1(i)+(1.-
1.*qtsvs)*wf2(i)+(1.-1.*qtrvs)*wf3(i)+(1.-1.*qtgvs)*wf4(i)))))/(aco*(1.*aco*afi-1.*(-
1.*k1s*(1.-1.*qthvs)*hf11(i)*wf1(i)*wf26(i)+k1s*(1.-
1.*qthvs)*hf18(i)*wf1(i)*wf26(i)+k1s*(1.-1.*qtsvs)*hf112(i)*wf2(i)*wf26(i)-1.*k1s*(1.-
1.*qtsvs)*hf12(i)*wf2(i)*wf26(i)+k1s*(1.-1.*qtrvs)*hf116(i)*wf26(i)*wf3(i)-1.*k1s*(1.-
1.*qtrvs)*hf13(i)*wf26(i)*wf3(i)+k1s*(1.-1.*qtgvs)*hf120(i)*wf26(i)*wf4(i)-1.*k1s*(1.-
1.*qtgvs)*hf14(i)*wf26(i)*wf4(i)-1.*k1s*rhos*hf126(i)*wf26(i)*((1.-1.*qthvs)*wf1(i)+(1.-
1.*qtsvs)*wf2(i)+(1.-1.*qtrvs)*wf3(i)+(1.-1.*qtgvs)*wf4(i)))^2));      a12= -((-
1.*aco*(k1s*(1.-1.*qthvs)*hf11(i)*wf1(i)*wf26(i)-1.*k1s*(1.-
1.*qthvs)*hf18(i)*wf1(i)*wf26(i)-1.*k1s*(1.-1.*qtsvs)*hf112(i)*wf2(i)*wf26(i)+k1s*(1.-
1.*qtsvs)*hf12(i)*wf2(i)*wf26(i)-1.*k1s*(1.-1.*qtrvs)*hf116(i)*wf26(i)*wf3(i)+k1s*(1.-
1.*qtrvs)*hf13(i)*wf26(i)*wf3(i)-1.*k1s*(1.-1.*qtgvs)*hf120(i)*wf26(i)*wf4(i)+k1s*(1.-
1.*qtgvs)*hf14(i)*wf26(i)*wf4(i)+k1s*rhos*hf126(i)*wf26(i)*((1.-1.*qthvs)*wf1(i)+(1.-
1.*qtsvs)*wf2(i)+(1.-1.*qtrvs)*wf3(i)+(1.-1.*qtgvs)*wf4(i)))-1.*(-1.*k1s*(1.-
1.*qthvs)*hf11(i)*wf1(i)*wf26(i)+k1s*(1.-1.*qthvs)*hf18(i)*wf1(i)*wf26(i)+k1s*(1.-
1.*qtsvs)*hf112(i)*wf2(i)*wf26(i)-1.*k1s*(1.-1.*qtsvs)*hf12(i)*wf2(i)*wf26(i)+k1s*(1.-
1.*qtrvs)*hf116(i)*wf26(i)*wf3(i)-1.*k1s*(1.-1.*qtrvs)*hf13(i)*wf26(i)*wf3(i)+k1s*(1.-
1.*qtgvs)*hf120(i)*wf26(i)*wf4(i)-1.*k1s*(1.-1.*qtgvs)*hf14(i)*wf26(i)*wf4(i)-
1.*k1s*rhos*hf126(i)*wf26(i)*((1.-1.*qthvs)*wf1(i)+(1.-1.*qtsvs)*wf2(i)+(1.-
1.*qtrvs)*wf3(i)+(1.-1.*qtgvs)*wf4(i)))*(k1s*(1.-1.*qthvs)*hf11(i)*wf1(i)*wf26(i)-
1.*k1s*(1.-1.*qthvs)*hf18(i)*wf1(i)*wf26(i)-1.*k1s*(1.-
1.*qtsvs)*hf112(i)*wf2(i)*wf26(i)+k1s*(1.-1.*qtsvs)*hf12(i)*wf2(i)*wf26(i)-1.*k1s*(1.-
1.*qtrvs)*hf116(i)*wf26(i)*wf3(i)+k1s*(1.-1.*qtrvs)*hf13(i)*wf26(i)*wf3(i)-1.*k1s*(1.-
1.*qtgvs)*hf120(i)*wf26(i)*wf4(i)+k1s*(1.-
1.*qtgvs)*hf14(i)*wf26(i)*wf4(i)+k1s*rhos*hf126(i)*wf26(i)*((1.-1.*qthvs)*wf1(i)+(1.-
1.*qtsvs)*wf2(i)+(1.-1.*qtrvs)*wf3(i)+(1.-1.*qtgvs)*wf4(i))))/(1.*aco*afi-1.*(-1.*k1s*(1.-
1.*qthvs)*hf11(i)*wf1(i)*wf26(i)+k1s*(1.-1.*qthvs)*hf18(i)*wf1(i)*wf26(i)+k1s*(1.-
1.*qtsvs)*hf112(i)*wf2(i)*wf26(i)-1.*k1s*(1.-1.*qtsvs)*hf12(i)*wf2(i)*wf26(i)+k1s*(1.-
1.*qtrvs)*hf116(i)*wf26(i)*wf3(i)-1.*k1s*(1.-1.*qtrvs)*hf13(i)*wf26(i)*wf3(i)+k1s*(1.-
1.*qtgvs)*hf120(i)*wf26(i)*wf4(i)-1.*k1s*(1.-1.*qtgvs)*hf14(i)*wf26(i)*wf4(i)-
1.*k1s*rhos*hf126(i)*wf26(i)*((1.-1.*qthvs)*wf1(i)+(1.-1.*qtsvs)*wf2(i)+(1.-
1.*qtrvs)*wf3(i)+(1.-1.*qtgvs)*wf4(i)))^2));      a13=
(1.*(hf11(i)*wf1(i)+hf12(i)*wf2(i)+hf13(i)*wf3(i)+hf14(i)*wf4(i)))/aib;
a14=(1.*(k2*hf112(i)*wf12(i)-1.*k2*hf113(i)*wf12(i)+k2*hf116(i)*wf16(i)-
1.*k2*hf117(i)*wf16(i)+k2*hf120(i)*wf20(i)-1.*k2*hf121(i)*wf20(i)+k2*hf18(i)*wf8(i)-
1.*k2*hf19(i)*wf8(i)))/aii-(1.*(-1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-
1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-
1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i))*((1.*(1.*bsr*pr*hf126(i)*(wf11(i)+wf15(i)+wf19(i
)+wf23(i)+wf25(i))*(1.*brs-
1.*pr*hf226(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))+pr*hf227(i)*(wf11(i)+wf15(i)+
wf19(i)+wf23(i)+wf25(i)))-1.*brs*(-1.*bsr*(-
1.*pr*hf126(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))-
(1.*g*hf126(i)*wf26(i))/(b2+wf26(i))-
1.*k1sm*hf124(i)*wf26(i)*wf5(i)+k1sm*rhos*hf126(i)*wf26(i)*wf5(i)+k1sm*hf15(i)*wf26
(i)*wf5(i))+1.*ps*hf126(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))*(1.*bsr+ps*hf226(i)*
(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))-
1.*ps*hf227(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))))))/(brs*bsr*(k1sm*hf124(i)*wf2
6(i)*wf5(i)-1.*k1sm*rhos*hf126(i)*wf26(i)*wf5(i)-1.*k1sm*hf15(i)*wf26(i)*wf5(i)))-
(1.*(1.*bsr*pr^2*hf126(i)*hf226(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))^2-
```

1.*brs*(1.*api*bsr-
1.*ps^2*hf126(i)*hf226(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))^2))*(1.*brs*bsr*(k1sm*hf124(i)*wf26(i)*wf5(i)-1.*k1sm*rhos*hf126(i)*wf26(i)*wf5(i)-
1.*k1sm*hf15(i)*wf26(i)*wf5(i))*(-1.*aii*(k2*hf112(i)*wf12(i)-
1.*k2*hf113(i)*wf12(i)+k2*hf116(i)*wf16(i)-1.*k2*hf117(i)*wf16(i)+k2*hf120(i)*wf20(i)-
1.*k2*hf121(i)*wf20(i)-
1.*k1sm*hf124(i)*wf26(i)*wf5(i)+k1sm*rhos*hf126(i)*wf26(i)*wf5(i)+k1sm*hf15(i)*wf26(i)*wf5(i)+k2*hf18(i)*wf8(i)-1.*k2*hf19(i)*wf8(i))-1.*(k2*hf112(i)*wf12(i)-
1.*k2*hf113(i)*wf12(i)+k2*hf116(i)*wf16(i)-1.*k2*hf117(i)*wf16(i)+k2*hf120(i)*wf20(i)-
1.*k2*hf121(i)*wf20(i)+k2*hf18(i)*wf8(i)-1.*k2*hf19(i)*wf8(i))*(-
1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-
1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i)))-
1.*(1.*aii*art-1.*(-1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-
1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-
1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i))^2)*(1.*bsr*pr*hf126(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))*(1.*brs-
1.*pr*hf226(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))+pr*hf227(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i)))-1.*brs*(-1.*bsr*(-
1.*pr*hf126(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))-
(1.*g*hf126(i)*wf26(i))/(b2+wf26(i))-
1.*k1sm*hf124(i)*wf26(i)*wf5(i)+k1sm*rhos*hf126(i)*wf26(i)*wf5(i)+k1sm*hf15(i)*wf26(i)*wf5(i))+1.*ps*hf126(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))*(1.*bsr+ps*hf226(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))-
1.*ps*hf227(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i)))))))/(brs*bsr*(k1sm*hf124(i)*wf26(i)*wf5(i)-1.*k1sm*rhos*hf126(i)*wf26(i)*wf5(i)-1.*k1sm*hf15(i)*wf26(i)*wf5(i))*(-
1.*aii*brs*bsr*(k1sm*hf124(i)*wf26(i)*wf5(i)-1.*k1sm*rhos*hf126(i)*wf26(i)*wf5(i)-
1.*k1sm*hf15(i)*wf26(i)*wf5(i))^2-1.*(1.*aii*art-1.*(-
1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-
1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-
1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i))^2)*(1.*bsr*pr^2*hf126(i)*hf226(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*hf126(i)*hf226(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))^2))))))/aii; a15=-
((1.*brs*bsr*(k1sm*hf124(i)*wf26(i)*wf5(i)-1.*k1sm*rhos*hf126(i)*wf26(i)*wf5(i)-
1.*k1sm*hf15(i)*wf26(i)*wf5(i))*(-1.*aii*(k2*hf112(i)*wf12(i)-
1.*k2*hf113(i)*wf12(i)+k2*hf116(i)*wf16(i)-1.*k2*hf117(i)*wf16(i)+k2*hf120(i)*wf20(i)-
1.*k2*hf121(i)*wf20(i)-
1.*k1sm*hf124(i)*wf26(i)*wf5(i)+k1sm*rhos*hf126(i)*wf26(i)*wf5(i)+k1sm*hf15(i)*wf26(i)*wf5(i)+k2*hf18(i)*wf8(i)-1.*k2*hf19(i)*wf8(i))-1.*(k2*hf112(i)*wf12(i)-
1.*k2*hf113(i)*wf12(i)+k2*hf116(i)*wf16(i)-1.*k2*hf117(i)*wf16(i)+k2*hf120(i)*wf20(i)-
1.*k2*hf121(i)*wf20(i)+k2*hf18(i)*wf8(i)-1.*k2*hf19(i)*wf8(i))*(-
1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-
1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i)))-
1.*(1.*aii*art-1.*(-1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-
1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-
1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i))^2)*(1.*bsr*pr*hf126(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))*(1.*brs-
1.*pr*hf226(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))+pr*hf227(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i)))-1.*brs*(-1.*bsr*(-
1.*pr*hf126(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))-
(1.*g*hf126(i)*wf26(i))/(b2+wf26(i))-
1.*k1sm*hf124(i)*wf26(i)*wf5(i)+k1sm*rhos*hf126(i)*wf26(i)*wf5(i)+k1sm*hf15(i)*wf26(i)*wf5(i))+1.*ps*hf126(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))*(1.*bsr+ps*hf226(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))-
1.*ps*hf227(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))))))/(-
1.*aii*brs*bsr*(k1sm*hf124(i)*wf26(i)*wf5(i)-1.*k1sm*rhos*hf126(i)*wf26(i)*wf5(i)-
1.*k1sm*hf15(i)*wf26(i)*wf5(i))^2-1.*(1.*aii*art-1.*(-
1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-
1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-
1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i))^2)*(1.*bsr*pr^2*hf126(i)*hf226(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))^2-1.*brs*(1.*api*bsr-

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1.*ps^2*hf126(i)*hf226(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))^2)))); a16=(-
1.*(1.*bsr*pr*hf126(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))*(1.*brs-
1.*pr*hf226(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))+pr*hf227(i)*(wf11(i)+wf15(i)+
wf19(i)+wf23(i)+wf25(i)))-1.*brs*(-1.*bsr*(-
1.*pr*hf126(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))-
(1.*g*hf126(i)*wf26(i))/(b2+wf26(i))-
1.*k1sm*hf124(i)*wf26(i)*wf5(i)+k1sm*rhos*hf126(i)*wf26(i)*wf5(i)+k1sm*hf15(i)*wf26
(i)*wf5(i))+1.*ps*hf126(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))*(1.*bsr+ps*hf226(i)*
(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))-
1.*ps*hf227(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))))))/(brs*bsr*(k1sm*hf124(i)*wf2
6(i)*wf5(i)-1.*k1sm*rhos*hf126(i)*wf26(i)*wf5(i)-
1.*k1sm*hf15(i)*wf26(i)*wf5(i)))+(1.*(1.*bsr*pr^2*hf126(i)*hf226(i)*(wf11(i)+wf15(i)+wf
19(i)+wf23(i)+wf25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*hf126(i)*hf226(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))^2))*(1.*brs*bsr*(k1s
m*hf124(i)*wf26(i)*wf5(i)-1.*k1sm*rhos*hf126(i)*wf26(i)*wf5(i)-
1.*k1sm*hf15(i)*wf26(i)*wf5(i))*(-1.*aii*(k2*hf112(i)*wf12(i)-
1.*k2*hf113(i)*wf12(i)+k2*hf116(i)*wf16(i)-1.*k2*hf117(i)*wf16(i)+k2*hf120(i)*wf20(i)-
1.*k2*hf121(i)*wf20(i)-
1.*k1sm*hf124(i)*wf26(i)*wf5(i)+k1sm*rhos*hf126(i)*wf26(i)*wf5(i)+k1sm*hf15(i)*wf26
(i)*wf5(i)+k2*hf18(i)*wf8(i)-1.*k2*hf19(i)*wf8(i))-1.*(k2*hf112(i)*wf12(i)-
1.*k2*hf113(i)*wf12(i)+k2*hf116(i)*wf16(i)-1.*k2*hf117(i)*wf16(i)+k2*hf120(i)*wf20(i)-
1.*k2*hf121(i)*wf20(i)+k2*hf18(i)*wf8(i)-1.*k2*hf19(i)*wf8(i))*(-
1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-
1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i)))-
1.*(1.*aii*art-1.*(-1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-
1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-
1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i))^2)*(1.*bsr*pr*hf126(i)*(wf11(i)+wf15(i)+wf19(i)
+wf23(i)+wf25(i))*(1.*brs-
1.*pr*hf226(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))+pr*hf227(i)*(wf11(i)+wf15(i)+
wf19(i)+wf23(i)+wf25(i)))-1.*brs*(-1.*bsr*(-
1.*pr*hf126(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))-
(1.*g*hf126(i)*wf26(i))/(b2+wf26(i))-
1.*k1sm*hf124(i)*wf26(i)*wf5(i)+k1sm*rhos*hf126(i)*wf26(i)*wf5(i)+k1sm*hf15(i)*wf26
(i)*wf5(i))+1.*ps*hf126(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))*(1.*bsr+ps*hf226(i)*
(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))-
1.*ps*hf227(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i)))))))/(brs*bsr*(k1sm*hf124(i)*wf
26(i)*wf5(i)-1.*k1sm*rhos*hf126(i)*wf26(i)*wf5(i)-1.*k1sm*hf15(i)*wf26(i)*wf5(i))*(-
1.*aii*brs*bsr*(k1sm*hf124(i)*wf26(i)*wf5(i)-1.*k1sm*rhos*hf126(i)*wf26(i)*wf5(i)-
1.*k1sm*hf15(i)*wf26(i)*wf5(i))^2-1.*(1.*aii*art-1.*(-
1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-
1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-
1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i))^2)*(1.*bsr*pr^2*hf126(i)*hf226(i)*(wf11(i)+wf15
(i)+wf19(i)+wf23(i)+wf25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*hf126(i)*hf226(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))^2)))); u21s=
min(1,max(0,(1.*(1.*bsr+ps*hf226(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))-
1.*ps*hf227(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))))/bsr+(1.*ps*hf226(i)*(wf13(i)+
wf17(i)+wf21(i)+wf24(i)+wf9(i))*(1.*brs*bsr*(k1sm*hf124(i)*wf26(i)*wf5(i)-
1.*k1sm*rhos*hf126(i)*wf26(i)*wf5(i)-1.*k1sm*hf15(i)*wf26(i)*wf5(i))*(-
1.*aii*(k2*hf112(i)*wf12(i)-1.*k2*hf113(i)*wf12(i)+k2*hf116(i)*wf16(i)-
1.*k2*hf117(i)*wf16(i)+k2*hf120(i)*wf20(i)-1.*k2*hf121(i)*wf20(i)-
1.*k1sm*hf124(i)*wf26(i)*wf5(i)+k1sm*rhos*hf126(i)*wf26(i)*wf5(i)+k1sm*hf15(i)*wf26
(i)*wf5(i)+k2*hf18(i)*wf8(i)-1.*k2*hf19(i)*wf8(i))-1.*(k2*hf112(i)*wf12(i)-
1.*k2*hf113(i)*wf12(i)+k2*hf116(i)*wf16(i)-1.*k2*hf117(i)*wf16(i)+k2*hf120(i)*wf20(i)-
1.*k2*hf121(i)*wf20(i)+k2*hf18(i)*wf8(i)-1.*k2*hf19(i)*wf8(i))*(-
1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-
1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i)))-
1.*(1.*aii*art-1.*(-1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-
1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-
1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i))^2)*(1.*bsr*pr*hf126(i)*(wf11(i)+wf15(i)+wf19(i)
+wf23(i)+wf25(i))*(1.*brs-
```

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1.*pr*hf226(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))+pr*hf227(i)*(wf11(i)+wf15(i)+
wf19(i)+wf23(i)+wf25(i)))-1.*brs*(-1.*bsr*(-
1.*pr*hf126(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))-
(1.*g*hf126(i)*wf26(i))/(b2+wf26(i))-
1.*k1sm*hf124(i)*wf26(i)*wf5(i)+k1sm*rhos*hf126(i)*wf26(i)*wf5(i)+k1sm*hf15(i)*wf26
(i)*wf5(i))+1.*ps*hf126(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))*(1.*bsr+ps*hf226(i)*
(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))-
1.*ps*hf227(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i)))))))/(bsr*(-
1.*aii*brs*bsr*(k1sm*hf124(i)*wf26(i)*wf5(i)-1.*k1sm*rhos*hf126(i)*wf26(i)*wf5(i)-
1.*k1sm*hf15(i)*wf26(i)*wf5(i))^2-1.*(1.*aii*art-1.*(-
1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-
1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-
1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i))^2)*(1.*bsr*pr^2*hf126(i)*hf226(i)*(wf11(i)+wf15
(i)+wf19(i)+wf23(i)+wf25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*hf126(i)*hf226(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))^2))))));       u22s=
min(1,max(0,(1.*(1.*brs-
1.*pr*hf226(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))+pr*hf227(i)*(wf11(i)+wf15(i)+
wf19(i)+wf23(i)+wf25(i))))/brs-
(1.*pr*hf226(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))*(1.*brs*bsr*(k1sm*hf124(i)*w
f26(i)*wf5(i)-1.*k1sm*rhos*hf126(i)*wf26(i)*wf5(i)-1.*k1sm*hf15(i)*wf26(i)*wf5(i))*(-
1.*aii*(k2*hf112(i)*wf12(i)-1.*k2*hf113(i)*wf12(i)+k2*hf116(i)*wf16(i)-
1.*k2*hf117(i)*wf16(i)+k2*hf120(i)*wf20(i)-1.*k2*hf121(i)*wf20(i)-
1.*k1sm*hf124(i)*wf26(i)*wf5(i)+k1sm*rhos*hf126(i)*wf26(i)*wf5(i)+k1sm*hf15(i)*wf26
(i)*wf5(i)+k2*hf18(i)*wf8(i)-1.*k2*hf19(i)*wf8(i))-1.*(k2*hf112(i)*wf12(i)-
1.*k2*hf113(i)*wf12(i)+k2*hf116(i)*wf16(i)-1.*k2*hf117(i)*wf16(i)+k2*hf120(i)*wf20(i)-
1.*k2*hf121(i)*wf20(i)+k2*hf18(i)*wf8(i)-1.*k2*hf19(i)*wf8(i))*(-
1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-
1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i)))-
1.*(1.*aii*art-1.*(-1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-
1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-
1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i))^2)*(1.*bsr*pr*hf126(i)*(wf11(i)+wf15(i)+wf19(i)
+wf23(i)+wf25(i))*(1.*brs-
1.*pr*hf226(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))+pr*hf227(i)*(wf11(i)+wf15(i)+
wf19(i)+wf23(i)+wf25(i)))-1.*brs*(-1.*bsr*(-
1.*pr*hf126(i)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))-
(1.*g*hf126(i)*wf26(i))/(b2+wf26(i))-
1.*k1sm*hf124(i)*wf26(i)*wf5(i)+k1sm*rhos*hf126(i)*wf26(i)*wf5(i)+k1sm*hf15(i)*wf26
(i)*wf5(i))+1.*ps*hf126(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))*(1.*bsr+ps*hf226(i)*
(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))-
1.*ps*hf227(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i)))))))/(brs*(-
1.*aii*brs*bsr*(k1sm*hf124(i)*wf26(i)*wf5(i)-1.*k1sm*rhos*hf126(i)*wf26(i)*wf5(i)-
1.*k1sm*hf15(i)*wf26(i)*wf5(i))^2-1.*(1.*aii*art-1.*(-
1.*k2*hf112(i)*wf12(i)+k2*hf113(i)*wf12(i)-1.*k2*hf116(i)*wf16(i)+k2*hf117(i)*wf16(i)-
1.*k2*hf120(i)*wf20(i)+k2*hf121(i)*wf20(i)-
1.*k2*hf18(i)*wf8(i)+k2*hf19(i)*wf8(i))^2)*(1.*bsr*pr^2*hf126(i)*hf226(i)*(wf11(i)+wf15
(i)+wf19(i)+wf23(i)+wf25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*hf126(i)*hf226(i)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i))^2))))));

    if(a11<0 || a11>1)

        a11=0;

    end;



    if(a12<0 || a12>1)

        a12=0;

    end;
```

```
    if(a13<0 || a13>1)
        a13=0;
    end;

    if(a14<0 || a14>1)
        a14=0;
    end;

    if(a15<0 || a15>1)
        a15=0;
    end;

    if(a16<0 || a16>1)
        a16=0;
    end;

        D=[a11,a12,a13,a14,a15,a16;a11,a12,a13,a14,a15,1;a11,a12,a13,a14,a15,0;a11,a12,
a13,a14,1,a16;a11,a12,a13,a14,1,1;a11,a12,a13,a14,1,0;a11,a12,a13,a14,0,a16;a11,a12,a13,a1
4,0,1;a11,a12,a13,a14,0,0;a11,a12,a13,1,a15,a16;a11,a12,a13,1,a15,1;a11,a12,a13,1,a15,0;a1
1,a12,a13,1,1,a16;a11,a12,a13,1,1,1;a11,a12,a13,1,1,0;a11,a12,a13,1,0,a16;a11,a12,a13,1,0,1;
a11,a12,a13,1,0,0;a11,a12,a13,0,a15,a16;a11,a12,a13,0,a15,1;a11,a12,a13,0,a15,0;a11,a12,a1
3,0,1,a16;a11,a12,a13,0,1,1;a11,a12,a13,0,1,0;a11,a12,a13,0,0,a16;a11,a12,a13,0,0,1;a11,a12,
a13,0,0,0;a11,a12,1,a14,a15,a16;a11,a12,1,a14,a15,1;a11,a12,1,a14,a15,0;a11,a12,1,a14,1,a1
6;a11,a12,1,a14,1,1;a11,a12,1,a14,1,0;a11,a12,1,a14,0,a16;a11,a12,1,a14,0,1;a11,a12,1,a14,0,
0;a11,a12,1,1,a15,a16;a11,a12,1,1,a15,1;a11,a12,1,1,a15,0;a11,a12,1,1,1,a16;a11,a12,1,1,1,1;
a11,a12,1,1,1,0;a11,a12,1,1,0,a16;a11,a12,1,1,0,1;a11,a12,1,1,0,0;a11,a12,1,0,a15,a16;a11,a1
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,a16;0,1,1,a14,1,1;0,1,1,a14,1,0;0,1,1,a14,0,a16;0,1,1,a14,0,1;0,1,1,a14,0,0;0,1,1,1,a15,a16;0,
1,1,1,a15,1;0,1,1,1,a15,0;0,1,1,1,1,a16;0,1,1,1,1,1;0,1,1,1,1,0;0,1,1,1,0,a16;0,1,1,1,0,1;0,1,1,1,
0,0;0,1,1,0,a15,a16;0,1,1,0,a15,1;0,1,1,0,a15,0;0,1,1,0,1,a16;0,1,1,0,1,1;0,1,1,0,1,0;0,1,1,0,0,a
16;0,1,1,0,0,1;0,1,1,0,0,0;0,1,0,a14,a15,a16;0,1,0,a14,a15,1;0,1,0,a14,a15,0;0,1,0,a14,1,a16;0,
1,0,a14,1,1;0,1,0,a14,1,0;0,1,0,a14,0,a16;0,1,0,a14,0,1;0,1,0,a14,0,0;0,1,0,1,a15,a16;0,1,0,1,a
15,1;0,1,0,1,a15,0;0,1,0,1,1,a16;0,1,0,1,1,1;0,1,0,1,1,0;0,1,0,1,0,a16;0,1,0,1,0,1;0,1,0,1,0,0;0,
1,0,0,a15,a16;0,1,0,0,a15,1;0,1,0,0,a15,0;0,1,0,0,1,a16;0,1,0,0,1,1;0,1,0,0,1,0;0,1,0,0,0,a16;0,
1,0,0,0,1;0,1,0,0,0,0;0,0,a13,a14,a15,a16;0,0,a13,a14,a15,1;0,0,a13,a14,a15,0;0,0,a13,a14,1,a
16;0,0,a13,a14,1,1;0,0,a13,a14,1,0;0,0,a13,a14,0,a16;0,0,a13,a14,0,1;0,0,a13,a14,0,0;0,0,a13,
1,a15,a16;0,0,a13,1,a15,1;0,0,a13,1,a15,0;0,0,a13,1,1,a16;0,0,a13,1,1,1;0,0,a13,1,1,0;0,0,a13,
1,0,a16;0,0,a13,1,0,1;0,0,a13,1,0,0;0,0,a13,0,a15,a16;0,0,a13,0,a15,1;0,0,a13,0,a15,0;0,0,a13,
0,1,a16;0,0,a13,0,1,1;0,0,a13,0,1,0;0,0,a13,0,0,a16;0,0,a13,0,0,1;0,0,a13,0,0,0;0,0,1,a14,a15,a
16;0,0,1,a14,a15,1;0,0,1,a14,a15,0;0,0,1,a14,1,a16;0,0,1,a14,1,1;0,0,1,a14,1,0;0,0,1,a14,0,a16
;0,0,1,a14,0,1;0,0,1,a14,0,0;0,0,1,1,a15,a16;0,0,1,1,a15,1;0,0,1,1,a15,0;0,0,1,1,1,a16;0,0,1,1,1
,1;0,0,1,1,1,0;0,0,1,1,0,a16;0,0,1,1,0,1;0,0,1,1,0,0;0,0,1,0,a15,a16;0,0,1,0,a15,1;0,0,1,0,a15,0;
0,0,1,0,1,a16;0,0,1,0,1,1;0,0,1,0,1,0;0,0,1,0,0,a16;0,0,1,0,0,1;0,0,1,0,0,0;0,0,0,a14,a15,a16;0,0
,0,a14,a15,1;0,0,0,a14,a15,0;0,0,0,a14,1,a16;0,0,0,a14,1,1;0,0,0,a14,1,0;0,0,0,a14,0,a16;0,0,0,
a14,0,1;0,0,0,a14,0,0;0,0,0,1,a15,a16;0,0,0,1,a15,1;0,0,0,1,a15,0;0,0,0,1,1,a16;0,0,0,1,1,1;0,0,
0,1,1,0;0,0,0,1,0,a16;0,0,0,1,0,1;0,0,0,1,0,0;0,0,0,0,a15,a16;0,0,0,0,a15,1;0,0,0,0,a15,0;0,0,0,0
,1,a16;0,0,0,0,1,1;0,0,0,0,1,0;0,0,0,0,0,a16;0,0,0,0,0,1;0,0,0,0,0,0];

    eval_HT=eval(fHT);

    [max_HT, id_HT]=nanmax(eval_HT);

    u11(4)=D(id_HT,1);
    u12(4)=D(id_HT,2);
    u13(4)=D(id_HT,3);
    u14(4)=D(id_HT,4);
    u15(4)=D(id_HT,5);
    u16(4)=D(id_HT,6);

    u21(4)=u21s;
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u22(4)=u22s;

%VARIABILI COEEFICIENTI DI RUNGE KUTTA 4° ORDINE

kv1(4)=-(muth*wf1(i))+u13(4)*wf1(i)-k3*wf1(i)*(wf24(i)+wf25(i))-k1s*(1-qthvs)*(1-u11(4))*(1-u12(4))*wf1(i)*wf26(i)-k1r*(1-qthvr)*wf1(i)*wf27(i)-k*wf1(i)*(wf26(i)+wf27(i))-kprimo*wf1(i)*(wf26(i)+wf27(i))+(oth*s1th)/(oth+wf26(i)+wf27(i))+r*wf1(i)*(1-(wf1(i)+wf10(i)+wf11(i)+wf12(i)+wf13(i)+wf14(i)+wf15(i)+wf16(i)+wf17(i)+wf18(i)+wf19(i)+wf2(i)+wf20(i)+wf21(i)+wf22(i)+wf23(i)+wf3(i)+wf4(i)+wf8(i)+wf9(i))/tmax);

kv2(4)=-(muts*wf2(i))+u13(4)*wf2(i)-k3*wf2(i)*(wf24(i)+wf25(i))+k*wf1(i)*wf26(i)-k1s*(1-qtsvs)*(1-u11(4))*(1-u12(4))*wf2(i)*wf26(i)-k1r*(1-qtsvr)*wf2(i)*wf27(i); kv3(4)=k*wf1(i)*wf27(i)-mutr*wf3(i)+u13(4)*wf3(i)-k3*(wf24(i)+wf25(i))*wf3(i)-k1s*(1-qtrvs)*(1-u11(4))*(1-u12(4))*wf26(i)*wf3(i)-k1r*(1-qtrvr)*wf27(i)*wf3(i); kv4(4)=kprimo*wf1(i)*(wf26(i)+wf27(i))-mutg*wf4(i)+u13(4)*wf4(i)-k3*(wf24(i)+wf25(i))*wf4(i)-k1s*(1-qtgvs)*(1-u11(4))*(1-u12(4))*wf26(i)*wf4(i)-k1r*(1-qtgvr)*wf27(i)*wf4(i); kv5(4)=s1mh-mumh*wf5(i)-k1sm*(1-u15(4))*(1-u16(4))*wf26(i)*wf5(i)-k1rm*wf27(i)*wf5(i); kv6(4)=-(muip*wf6(i))-c*q*wf6(i)*(wf11(i)+wf13(i)+wf15(i)+wf17(i)+wf19(i)+wf21(i)+wf23(i)+wf9(i))+c*(wf1(i)+wf2(i)+wf3(i)+wf4(i))*wf6(i)*(wf11(i)+wf13(i)+wf15(i)+wf17(i)+wf19(i)+wf21(i)+wf23(i)+wf9(i)); kv7(4)=-(muie*wf7(i))+c*q*wf6(i)*(wf11(i)+wf13(i)+wf15(i)+wf17(i)+wf19(i)+wf21(i)+wf23(i)+wf9(i)); kv8(4)=k3*wf1(i)*wf24(i)+k1s*(1-qthvs)*(1-u11(4))*(1-u12(4))*wf1(i)*wf26(i)-muthsl*wf8(i)-k2*(1-u14(4))*(1-u16(4))*wf8(i); kv9(4)=k2*(1-u14(4))*(1-u16(4))*wf8(i)-muthsa*wf9(i)-is*wf7(i)*wf9(i); kv10(4)=-(k2*wf10(i))-muthrl*wf10(i)+k3*wf1(i)*wf25(i)+k1r*(1-qthvr)*wf1(i)*wf27(i); kv11(4)=k2*wf10(i)-muthra*wf11(i)-ir*wf11(i)*wf7(i); kv12(4)=-(mutssl*wf12(i))-k2*(1-u14(4))*(1-u16(4))*wf12(i)+k3*wf2(i)*wf24(i)+k1s*(1-qtsvs)*(1-u11(4))*(1-u12(4))*wf2(i)*wf26(i); kv13(4)=k2*(1-u14(4))*(1-u16(4))*wf12(i)-mutssa*wf13(i)-is*wf13(i)*wf7(i); kv14(4)=-(k2*wf14(i))-mutsrl*wf14(i)+k3*wf2(i)*wf25(i)+k1r*(1-qtsvr)*wf2(i)*wf27(i); kv15(4)=k2*wf14(i)-mutsra*wf15(i)-ir*wf15(i)*wf7(i); kv16(4)=-(mutrsl*wf16(i))-k2*(1-u14(4))*(1-u16(4))*wf16(i)+k3*wf24(i)*wf3(i)+k1s*(1-qtrvs)*(1-u11(4))*(1-u12(4))*wf26(i)*wf3(i); kv17(4)=k2*(1-u14(4))*(1-u16(4))*wf16(i)-mutrsa*wf17(i)-is*wf17(i)*wf7(i); kv18(4)=-(k2*wf18(i))-mutrrl*wf18(i)+k3*wf25(i)*wf3(i)+k1r*(1-qtrvr)*wf27(i)*wf3(i); kv19(4)=k2*wf18(i)-mutrra*wf19(i)-ir*wf19(i)*wf7(i); kv20(4)=-(mutgsl*wf20(i))-k2*(1-u14(4))*(1-u16(4))*wf20(i)+k3*wf24(i)*wf4(i)+k1s*(1-qtgvs)*(1-u11(4))*(1-u12(4))*wf26(i)*wf4(i); kv21(4)=k2*(1-u14(4))*(1-u16(4))*wf20(i)-mutgsa*wf21(i)-is*wf21(i)*wf7(i); kv22(4)=-(k2*wf22(i))-mutgrl*wf22(i)+k3*wf25(i)*wf4(i)+k1r*(1-qtgvr)*wf27(i)*wf4(i); kv23(4)=k2*wf22(i)-mutgra*wf23(i)-ir*wf23(i)*wf7(i); kv24(4)=-(mumhsa*wf24(i))+k1sm*(1-u15(4))*(1-u16(4))*wf26(i)*wf5(i); kv25(4)=-(mumhra*wf25(i))+k1rm*wf27(i)*wf5(i); kv26(4)=pr*(1-u15(4))*(1-u22(4))*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))-muvs*wf26(i)+(g*(1-u15(4))*wf26(i))/(b2+wf26(i))-k1s*rhos*(1-u11(4))*(1-u12(4))*wf26(i)*((1-qthvs)*wf1(i)+(1-qtsvs)*wf2(i)+(1-qtrvs)*wf3(i)+(1-qtgvs)*wf4(i))-wf26(i)*(psprimo*wf2(i)+ssprimo*wf4(i))-k1sm*rhos*(1-u15(4))*(1-u16(4))*wf26(i)*wf5(i)+ps*(1-u15(4))*u21(4)*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i)); kv27(4)=pr*u22(4)*(wf11(i)+wf15(i)+wf19(i)+wf23(i)+wf25(i))-muvr*wf27(i)+(g*wf27(i))/(b2+wf27(i))-k1r*rhor*wf27(i)*((1-qthvr)*wf1(i)+(1-qtsvr)*wf2(i)+(1-qtrvr)*wf3(i)+(1-qtgvr)*wf4(i))-wf27(i)*(prprimo*wf3(i)+srprimo*wf4(i))-k1rm*rhor*wf27(i)*wf5(i)+ps*(1-u21(4))*(wf13(i)+wf17(i)+wf21(i)+wf24(i)+wf9(i));

%RUNGE KUTTA 4° ORDINE (EULERO) PER LE VARIABILI

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    x1(i+1)=x1(i)+(h/6)*(kv1(1)+2*kv1(2)+2*kv1(3)+kv1(4));
    x2(i+1)=x2(i)+(h/6)*(kv2(1)+2*kv2(2)+2*kv2(3)+kv2(4));
    x3(i+1)=x3(i)+(h/6)*(kv3(1)+2*kv3(2)+2*kv3(3)+kv3(4));
    x4(i+1)=x4(i)+(h/6)*(kv4(1)+2*kv4(2)+2*kv4(3)+kv4(4));
    x5(i+1)=x5(i)+(h/6)*(kv5(1)+2*kv5(2)+2*kv5(3)+kv5(4));
    x6(i+1)=x6(i)+(h/6)*(kv6(1)+2*kv6(2)+2*kv6(3)+kv6(4));
    x7(i+1)=x7(i)+(h/6)*(kv7(1)+2*kv7(2)+2*kv7(3)+kv7(4));
    x8(i+1)=x8(i)+(h/6)*(kv8(1)+2*kv8(2)+2*kv8(3)+kv8(4));
    x9(i+1)=x9(i)+(h/6)*(kv9(1)+2*kv9(2)+2*kv9(3)+kv9(4));
    x10(i+1)=x10(i)+(h/6)*(kv10(1)+2*kv10(2)+2*kv10(3)+kv10(4));
    x11(i+1)=x11(i)+(h/6)*(kv11(1)+2*kv11(2)+2*kv11(3)+kv11(4));
    x12(i+1)=x12(i)+(h/6)*(kv12(1)+2*kv12(2)+2*kv12(3)+kv12(4));
    x13(i+1)=x13(i)+(h/6)*(kv13(1)+2*kv13(2)+2*kv13(3)+kv13(4));
    x14(i+1)=x14(i)+(h/6)*(kv14(1)+2*kv14(2)+2*kv14(3)+kv14(4));
    x15(i+1)=x15(i)+(h/6)*(kv15(1)+2*kv15(2)+2*kv15(3)+kv15(4));
    x16(i+1)=x16(i)+(h/6)*(kv16(1)+2*kv16(2)+2*kv16(3)+kv16(4));
    x17(i+1)=x17(i)+(h/6)*(kv17(1)+2*kv17(2)+2*kv17(3)+kv17(4));
    x18(i+1)=x18(i)+(h/6)*(kv18(1)+2*kv18(2)+2*kv18(3)+kv18(4));
    x19(i+1)=x19(i)+(h/6)*(kv19(1)+2*kv19(2)+2*kv19(3)+kv19(4));
    x20(i+1)=x20(i)+(h/6)*(kv20(1)+2*kv20(2)+2*kv20(3)+kv20(4));
    x21(i+1)=x21(i)+(h/6)*(kv21(1)+2*kv21(2)+2*kv21(3)+kv21(4));
    x22(i+1)=x22(i)+(h/6)*(kv22(1)+2*kv22(2)+2*kv22(3)+kv22(4));
    x23(i+1)=x23(i)+(h/6)*(kv23(1)+2*kv23(2)+2*kv23(3)+kv23(4));
    x24(i+1)=x24(i)+(h/6)*(kv24(1)+2*kv24(2)+2*kv24(3)+kv24(4));
    x25(i+1)=x25(i)+(h/6)*(kv25(1)+2*kv25(2)+2*kv25(3)+kv25(4));
    x26(i+1)=x26(i)+(h/6)*(kv26(1)+2*kv26(2)+2*kv26(3)+kv26(4));
    x27(i+1)=x27(i)+(h/6)*(kv27(1)+2*kv27(2)+2*kv27(3)+kv27(4));

end %%%%%%%%%%%%%%%%%%%%%%%%end for FORWARD PROCESS
```

```
%%%%%%%%%%%%%%%BACKWARD PROCESS

for j=1:N

    i=N+2-j;


    %CONTROLLI COEEFICIENTI DI RUNGE KUTTA 1° ORDINE

    a11= (1.*(k1s*(1.-1.*qthvs)*l11(i)*x1(i)*x26(i)-1.*k1s*(1.-
1.*qthvs)*l18(i)*x1(i)*x26(i)-1.*k1s*(1.-1.*qtsvs)*l112(i)*x2(i)*x26(i)+k1s*(1.-
1.*qtsvs)*l12(i)*x2(i)*x26(i)-1.*k1s*(1.-1.*qtrvs)*l116(i)*x26(i)*x3(i)+k1s*(1.-
1.*qtrvs)*l13(i)*x26(i)*x3(i)-1.*k1s*(1.-1.*qtgvs)*l120(i)*x26(i)*x4(i)+k1s*(1.-
1.*qtgvs)*l14(i)*x26(i)*x4(i)+k1s*rhos*l126(i)*x26(i)*((1.-1.*qthvs)*x1(i)+(1.-
1.*qtsvs)*x2(i)+(1.-1.*qtrvs)*x3(i)+(1.-1.*qtgvs)*x4(i)))))/aco-(1.*(-1.*k1s*(1.-
1.*qthvs)*l11(i)*x1(i)*x26(i)+k1s*(1.-1.*qthvs)*l18(i)*x1(i)*x26(i)+k1s*(1.-
1.*qtsvs)*l112(i)*x2(i)*x26(i)-1.*k1s*(1.-1.*qtsvs)*l12(i)*x2(i)*x26(i)+k1s*(1.-
1.*qtrvs)*l116(i)*x26(i)*x3(i)-1.*k1s*(1.-1.*qtrvs)*l13(i)*x26(i)*x3(i)+k1s*(1.-
1.*qtgvs)*l120(i)*x26(i)*x4(i)-1.*k1s*(1.-1.*qtgvs)*l14(i)*x26(i)*x4(i)-
1.*k1s*rhos*l126(i)*x26(i)*((1.-1.*qthvs)*x1(i)+(1.-1.*qtsvs)*x2(i)+(1.-1.*qtrvs)*x3(i)+(1.-
1.*qtgvs)*x4(i)))*(-1.*aco*(k1s*(1.-1.*qthvs)*l11(i)*x1(i)*x26(i)-1.*k1s*(1.-
1.*qthvs)*l18(i)*x1(i)*x26(i)-1.*k1s*(1.-1.*qtsvs)*l112(i)*x2(i)*x26(i)+k1s*(1.-
1.*qtsvs)*l12(i)*x2(i)*x26(i)-1.*k1s*(1.-1.*qtrvs)*l116(i)*x26(i)*x3(i)+k1s*(1.-
1.*qtrvs)*l13(i)*x26(i)*x3(i)-1.*k1s*(1.-1.*qtgvs)*l120(i)*x26(i)*x4(i)+k1s*(1.-
1.*qtgvs)*l14(i)*x26(i)*x4(i)+k1s*rhos*l126(i)*x26(i)*((1.-1.*qthvs)*x1(i)+(1.-
1.*qtsvs)*x2(i)+(1.-1.*qtrvs)*x3(i)+(1.-1.*qtgvs)*x4(i)))-1.*(-1.*k1s*(1.-
1.*qthvs)*l11(i)*x1(i)*x26(i)+k1s*(1.-1.*qthvs)*l18(i)*x1(i)*x26(i)+k1s*(1.-
1.*qtsvs)*l112(i)*x2(i)*x26(i)-1.*k1s*(1.-1.*qtsvs)*l12(i)*x2(i)*x26(i)+k1s*(1.-
1.*qtrvs)*l116(i)*x26(i)*x3(i)-1.*k1s*(1.-1.*qtrvs)*l13(i)*x26(i)*x3(i)+k1s*(1.-
1.*qtgvs)*l120(i)*x26(i)*x4(i)-1.*k1s*(1.-1.*qtgvs)*l14(i)*x26(i)*x4(i)-
1.*k1s*rhos*l126(i)*x26(i)*((1.-1.*qthvs)*x1(i)+(1.-1.*qtsvs)*x2(i)+(1.-1.*qtrvs)*x3(i)+(1.-
1.*qtgvs)*x4(i)))*(k1s*(1.-1.*qthvs)*l11(i)*x1(i)*x26(i)-1.*k1s*(1.-
1.*qthvs)*l18(i)*x1(i)*x26(i)-1.*k1s*(1.-1.*qtsvs)*l112(i)*x2(i)*x26(i)+k1s*(1.-
1.*qtsvs)*l12(i)*x2(i)*x26(i)-1.*k1s*(1.-1.*qtrvs)*l116(i)*x26(i)*x3(i)+k1s*(1.-
1.*qtrvs)*l13(i)*x26(i)*x3(i)-1.*k1s*(1.-1.*qtgvs)*l120(i)*x26(i)*x4(i)+k1s*(1.-
1.*qtgvs)*l14(i)*x26(i)*x4(i)+k1s*rhos*l126(i)*x26(i)*((1.-1.*qthvs)*x1(i)+(1.-
1.*qtsvs)*x2(i)+(1.-1.*qtrvs)*x3(i)+(1.-1.*qtgvs)*x4(i)))))/(aco*(1.*aco*afi-1.*(-1.*k1s*(1.-
1.*qthvs)*l11(i)*x1(i)*x26(i)+k1s*(1.-1.*qthvs)*l18(i)*x1(i)*x26(i)+k1s*(1.-
1.*qtsvs)*l112(i)*x2(i)*x26(i)-1.*k1s*(1.-1.*qtsvs)*l12(i)*x2(i)*x26(i)+k1s*(1.-
1.*qtrvs)*l116(i)*x26(i)*x3(i)-1.*k1s*(1.-1.*qtrvs)*l13(i)*x26(i)*x3(i)+k1s*(1.-
1.*qtgvs)*l120(i)*x26(i)*x4(i)-1.*k1s*(1.-1.*qtgvs)*l14(i)*x26(i)*x4(i)-
1.*k1s*rhos*l126(i)*x26(i)*((1.-1.*qthvs)*x1(i)+(1.-1.*qtsvs)*x2(i)+(1.-1.*qtrvs)*x3(i)+(1.-
1.*qtgvs)*x4(i)))^2));        a12= -((-1.*aco*(k1s*(1.-1.*qthvs)*l11(i)*x1(i)*x26(i)-
1.*k1s*(1.-1.*qthvs)*l18(i)*x1(i)*x26(i)-1.*k1s*(1.-1.*qtsvs)*l112(i)*x2(i)*x26(i)+k1s*(1.-
1.*qtsvs)*l12(i)*x2(i)*x26(i)-1.*k1s*(1.-1.*qtrvs)*l116(i)*x26(i)*x3(i)+k1s*(1.-
1.*qtrvs)*l13(i)*x26(i)*x3(i)-1.*k1s*(1.-1.*qtgvs)*l120(i)*x26(i)*x4(i)+k1s*(1.-
1.*qtgvs)*l14(i)*x26(i)*x4(i)+k1s*rhos*l126(i)*x26(i)*((1.-1.*qthvs)*x1(i)+(1.-
1.*qtsvs)*x2(i)+(1.-1.*qtrvs)*x3(i)+(1.-1.*qtgvs)*x4(i)))-1.*(-1.*k1s*(1.-
1.*qthvs)*l11(i)*x1(i)*x26(i)+k1s*(1.-1.*qthvs)*l18(i)*x1(i)*x26(i)+k1s*(1.-
1.*qtsvs)*l112(i)*x2(i)*x26(i)-1.*k1s*(1.-1.*qtsvs)*l12(i)*x2(i)*x26(i)+k1s*(1.-
1.*qtrvs)*l116(i)*x26(i)*x3(i)-1.*k1s*(1.-1.*qtrvs)*l13(i)*x26(i)*x3(i)+k1s*(1.-
1.*qtgvs)*l120(i)*x26(i)*x4(i)-1.*k1s*(1.-1.*qtgvs)*l14(i)*x26(i)*x4(i)-
1.*k1s*rhos*l126(i)*x26(i)*((1.-1.*qthvs)*x1(i)+(1.-1.*qtsvs)*x2(i)+(1.-1.*qtrvs)*x3(i)+(1.-
1.*qtgvs)*x4(i)))*(k1s*(1.-1.*qthvs)*l11(i)*x1(i)*x26(i)-1.*k1s*(1.-
1.*qthvs)*l18(i)*x1(i)*x26(i)-1.*k1s*(1.-1.*qtsvs)*l112(i)*x2(i)*x26(i)+k1s*(1.-
1.*qtsvs)*l12(i)*x2(i)*x26(i)-1.*k1s*(1.-1.*qtrvs)*l116(i)*x26(i)*x3(i)+k1s*(1.-
1.*qtrvs)*l13(i)*x26(i)*x3(i)-1.*k1s*(1.-1.*qtgvs)*l120(i)*x26(i)*x4(i)+k1s*(1.-
1.*qtgvs)*l14(i)*x26(i)*x4(i)+k1s*rhos*l126(i)*x26(i)*((1.-1.*qthvs)*x1(i)+(1.-
1.*qtsvs)*x2(i)+(1.-1.*qtrvs)*x3(i)+(1.-1.*qtgvs)*x4(i))))/(1.*aco*afi-1.*(-1.*k1s*(1.-
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1.*qthvs)*l11(i)*x1(i)*x26(i)+k1s*(1.-1.*qthvs)*l18(i)*x1(i)*x26(i)+k1s*(1.-1.*qtsvs)*l112(i)*x2(i)*x26(i)-1.*k1s*(1.-1.*qtsvs)*l12(i)*x2(i)*x26(i)+k1s*(1.-1.*qtrvs)*l116(i)*x26(i)*x3(i)-1.*k1s*(1.-1.*qtrvs)*l13(i)*x26(i)*x3(i)+k1s*(1.-1.*qtgvs)*l120(i)*x26(i)*x4(i)-1.*k1s*(1.-1.*qtgvs)*l14(i)*x26(i)*x4(i)-1.*k1s*rhos*l126(i)*x26(i)*((1.-1.*qthvs)*x1(i)+(1.-1.*qtsvs)*x2(i)+(1.-1.*qtrvs)*x3(i)+(1.-1.*qtgvs)*x4(i)))^2)); a13= (1.*(l11(i)*x1(i)+l12(i)*x2(i)+l13(i)*x3(i)+l14(i)*x4(i)))/aib; a14=(1.*(k2*l112(i)*x12(i)-1.*k2*l113(i)*x12(i)+k2*l116(i)*x16(i)-1.*k2*l117(i)*x16(i)+k2*l120(i)*x20(i)-1.*k2*l121(i)*x20(i)+k2*l18(i)*x8(i)-1.*k2*l19(i)*x8(i)))/aii-(1.*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i))*((1.*(1.*bsr*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))*(1.*brs-1.*pr*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))+pr*l227(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i)))-1.*brs*(-1.*bsr*(-1.*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))-(1.*g*l126(i)*x26(i))/(b2+x26(i))-1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i))+1.*ps*l126(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))*(1.*bsr+ps*l226(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))-1.*ps*l227(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))))))/(brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-1.*k1sm*l15(i)*x26(i)*x5(i)))-(1.*(1.*bsr*pr^2*l126(i)*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))^2-1.*brs*(1.*api*bsr-1.*ps^2*l126(i)*l226(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))^2))*(1.*brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-1.*k1sm*l15(i)*x26(i)*x5(i))*(-1.*aii*(k2*l112(i)*x12(i)-1.*k2*l113(i)*x12(i)+k2*l116(i)*x16(i)-1.*k2*l117(i)*x16(i)+k2*l120(i)*x20(i)-1.*k2*l121(i)*x20(i)-1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i)+k2*l18(i)*x8(i)-1.*k2*l19(i)*x8(i))-1.*(k2*l112(i)*x12(i)-1.*k2*l113(i)*x12(i)+k2*l116(i)*x16(i)-1.*k2*l117(i)*x16(i)+k2*l120(i)*x20(i)-1.*k2*l121(i)*x20(i)+k2*l18(i)*x8(i)-1.*k2*l19(i)*x8(i))*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i)))-1.*(1.*aii*art-1.*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i))^2)*(1.*bsr*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))*(1.*brs-1.*pr*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))+pr*l227(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i)))-1.*brs*(-1.*bsr*(-1.*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))-(1.*g*l126(i)*x26(i))/(b2+x26(i))-1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i))+1.*ps*l126(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))*(1.*bsr+ps*l226(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))-1.*ps*l227(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i)))))))/(brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-1.*k1sm*l15(i)*x26(i)*x5(i))*(-1.*aii*brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-1.*k1sm*l15(i)*x26(i)*x5(i))^2-1.*(1.*aii*art-1.*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i))^2)*(1.*bsr*pr^2*l126(i)*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))^2-1.*brs*(1.*api*bsr-1.*ps^2*l126(i)*l226(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))^2)))))/aii; a15= -((1.*brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-1.*k1sm*l15(i)*x26(i)*x5(i))*(-1.*aii*(k2*l112(i)*x12(i)-1.*k2*l113(i)*x12(i)+k2*l116(i)*x16(i)-1.*k2*l117(i)*x16(i)+k2*l120(i)*x20(i)-1.*k2*l121(i)*x20(i)-1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i)+k2*l18(i)*x8(i)-1.*k2*l19(i)*x8(i))-1.*(k2*l112(i)*x12(i)-1.*k2*l113(i)*x12(i)+k2*l116(i)*x16(i)-1.*k2*l117(i)*x16(i)+k2*l120(i)*x20(i)-1.*k2*l121(i)*x20(i)+k2*l18(i)*x8(i)-1.*k2*l19(i)*x8(i))*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i)))-1.*(1.*aii*art-

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1.*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-
1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-
1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i))^2)*(1.*bsr*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x2
5(i))*(1.*brs-
1.*pr*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))+pr*l227(i)*(x11(i)+x15(i)+x19(i)+x23(i
)+x25(i)))-1.*brs*(-1.*bsr*(-1.*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))-
(1.*g*l126(i)*x26(i))/(b2+x26(i))-
1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i))+
1.*ps*l126(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))*(1.*bsr+ps*l226(i)*(x13(i)+x17(i)+x21(i)
+x24(i)+x9(i))-1.*ps*l227(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))))))/(-
1.*aii*brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-
1.*k1sm*l15(i)*x26(i)*x5(i))^2-1.*(1.*aii*art-1.*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-
1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-
1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i))^2)*(1.*bsr*pr^2*l126(i)*l226(i)*(x11(i)+x15(i)+x19(i)+
x23(i)+x25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*l126(i)*l226(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))^2))));        a16= (-
1.*(1.*bsr*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))*(1.*brs-
1.*pr*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))+pr*l227(i)*(x11(i)+x15(i)+x19(i)+x23(i
)+x25(i)))-1.*brs*(-1.*bsr*(-1.*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))-
(1.*g*l126(i)*x26(i))/(b2+x26(i))-
1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i))+
1.*ps*l126(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))*(1.*bsr+ps*l226(i)*(x13(i)+x17(i)+x21(i)
+x24(i)+x9(i))-
1.*ps*l227(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))))))/(brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-
1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-
1.*k1sm*l15(i)*x26(i)*x5(i)))+(1.*(1.*bsr*pr^2*l126(i)*l226(i)*(x11(i)+x15(i)+x19(i)+x23(
i)+x25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*l126(i)*l226(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))^2))*(1.*brs*bsr*(k1sm*l124(i)
*x26(i)*x5(i)-1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-1.*k1sm*l15(i)*x26(i)*x5(i))*(-
1.*aii*(k2*l112(i)*x12(i)-1.*k2*l113(i)*x12(i)+k2*l116(i)*x16(i)-
1.*k2*l117(i)*x16(i)+k2*l120(i)*x20(i)-1.*k2*l121(i)*x20(i)-
1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i)+k
2*l18(i)*x8(i)-1.*k2*l19(i)*x8(i))-1.*(k2*l112(i)*x12(i)-
1.*k2*l113(i)*x12(i)+k2*l116(i)*x16(i)-1.*k2*l117(i)*x16(i)+k2*l120(i)*x20(i)-
1.*k2*l121(i)*x20(i)+k2*l18(i)*x8(i)-1.*k2*l19(i)*x8(i))*(-
1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-
1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i)))-1.*(1.*aii*art-
1.*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-
1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-
1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i))^2)*(1.*bsr*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x2
5(i))*(1.*brs-
1.*pr*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))+pr*l227(i)*(x11(i)+x15(i)+x19(i)+x23(i
)+x25(i)))-1.*brs*(-1.*bsr*(-1.*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))-
(1.*g*l126(i)*x26(i))/(b2+x26(i))-
1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i))+
1.*ps*l126(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))*(1.*bsr+ps*l226(i)*(x13(i)+x17(i)+x21(i)
+x24(i)+x9(i))-
1.*ps*l227(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i)))))))/(brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-
1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-1.*k1sm*l15(i)*x26(i)*x5(i))*(-
1.*aii*brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-
1.*k1sm*l15(i)*x26(i)*x5(i))^2-1.*(1.*aii*art-1.*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-
1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-
1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i))^2)*(1.*bsr*pr^2*l126(i)*l226(i)*(x11(i)+x15(i)+x19(i)+
x23(i)+x25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*l126(i)*l226(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))^2))));        u21s=
min(1,max(0,(1.*(1.*bsr+ps*l226(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))-
1.*ps*l227(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))))/bsr+(1.*ps*l226(i)*(x13(i)+x17(i)+x21(
i)+x24(i)+x9(i))*(1.*brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-
1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-1.*k1sm*l15(i)*x26(i)*x5(i))*(-
1.*aii*(k2*l112(i)*x12(i)-1.*k2*l113(i)*x12(i)+k2*l116(i)*x16(i)-
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1.*k2*l117(i)*x16(i)+k2*l120(i)*x20(i)-1.*k2*l121(i)*x20(i)-
1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i)+k
2*l18(i)*x8(i)-1.*k2*l19(i)*x8(i))-1.*(k2*l112(i)*x12(i)-
1.*k2*l113(i)*x12(i)+k2*l116(i)*x16(i)-1.*k2*l117(i)*x16(i)+k2*l120(i)*x20(i)-
1.*k2*l121(i)*x20(i)+k2*l18(i)*x8(i)-1.*k2*l19(i)*x8(i))*(-
1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-
1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i)))-1.*(1.*aii*art-
1.*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-
1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-
1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i))^2)*(1.*bsr*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x2
5(i))*(1.*brs-
1.*pr*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))+pr*l227(i)*(x11(i)+x15(i)+x19(i)+x23(i
)+x25(i)))-1.*brs*(-1.*bsr*(-1.*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))-
(1.*g*l126(i)*x26(i))/(b2+x26(i))-
1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i))+
1.*ps*l126(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))*(1.*bsr+ps*l226(i)*(x13(i)+x17(i)+x21(i)
+x24(i)+x9(i))-1.*ps*l227(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i)))))))/(bsr*(-
1.*aii*brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-
1.*k1sm*l15(i)*x26(i)*x5(i))^2-1.*(1.*aii*art-1.*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-
1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-
1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i))^2)*(1.*bsr*pr^2*l126(i)*l226(i)*(x11(i)+x15(i)+x19(i)+
x23(i)+x25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*l126(i)*l226(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))^2))))));        u22s=
min(1,max(0,(1.*(1.*brs-
1.*pr*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))+pr*l227(i)*(x11(i)+x15(i)+x19(i)+x23(i
)+x25(i))))/brs-
(1.*pr*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))*(1.*brs*bsr*(k1sm*l124(i)*x26(i)*x5(i
)-1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-1.*k1sm*l15(i)*x26(i)*x5(i))*(-
1.*aii*(k2*l112(i)*x12(i)-1.*k2*l113(i)*x12(i)+k2*l116(i)*x16(i)-
1.*k2*l117(i)*x16(i)+k2*l120(i)*x20(i)-1.*k2*l121(i)*x20(i)-
1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i)+k
2*l18(i)*x8(i)-1.*k2*l19(i)*x8(i))-1.*(k2*l112(i)*x12(i)-
1.*k2*l113(i)*x12(i)+k2*l116(i)*x16(i)-1.*k2*l117(i)*x16(i)+k2*l120(i)*x20(i)-
1.*k2*l121(i)*x20(i)+k2*l18(i)*x8(i)-1.*k2*l19(i)*x8(i))*(-
1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-
1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i)))-1.*(1.*aii*art-
1.*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-
1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-
1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i))^2)*(1.*bsr*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x2
5(i))*(1.*brs-
1.*pr*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))+pr*l227(i)*(x11(i)+x15(i)+x19(i)+x23(i
)+x25(i)))-1.*brs*(-1.*bsr*(-1.*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))-
(1.*g*l126(i)*x26(i))/(b2+x26(i))-
1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i))+
1.*ps*l126(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))*(1.*bsr+ps*l226(i)*(x13(i)+x17(i)+x21(i)
+x24(i)+x9(i))-1.*ps*l227(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i)))))))/(brs*(-
1.*aii*brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-
1.*k1sm*l15(i)*x26(i)*x5(i))^2-1.*(1.*aii*art-1.*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-
1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-
1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i))^2)*(1.*bsr*pr^2*l126(i)*l226(i)*(x11(i)+x15(i)+x19(i)+
x23(i)+x25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*l126(i)*l226(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))^2))))));

    if(a11<0 || a11>1)

        a11=0;

    end;
```

```
if(a12<0 || a12>1)
    a12=0;
end;

if(a13<0 || a13>1)
    a13=0;
end;

if(a14<0 || a14>1)
    a14=0;
end;

if(a15<0 || a15>1)
    a15=0;
end;

if(a16<0 || a16>1)
    a16=0;
end;

    D=[a11,a12,a13,a14,a15,a16;a11,a12,a13,a14,a15,1;a11,a12,a13,a14,a15,0;a11,a12,
a13,a14,1,a16;a11,a12,a13,a14,1,1;a11,a12,a13,a14,1,0;a11,a12,a13,a14,0,a16;a11,a12,a13,a1
4,0,1;a11,a12,a13,a14,0,0;a11,a12,a13,1,a15,a16;a11,a12,a13,1,a15,1;a11,a12,a13,1,a15,0;a1
1,a12,a13,1,1,a16;a11,a12,a13,1,1,1;a11,a12,a13,1,1,0;a11,a12,a13,1,0,a16;a11,a12,a13,1,0,1;
a11,a12,a13,1,0,0;a11,a12,a13,0,a15,a16;a11,a12,a13,0,a15,1;a11,a12,a13,0,a15,0;a11,a12,a1
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a13,0,0,0;a11,a12,1,a14,a15,a16;a11,a12,1,a14,a15,1;a11,a12,1,a14,a15,0;a11,a12,1,a14,1,a1
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0;a11,a12,1,1,a15,a16;a11,a12,1,1,a15,1;a11,a12,1,1,a15,0;a11,a12,1,1,1,a16;a11,a12,1,1,1,1;
a11,a12,1,1,1,0;a11,a12,1,1,0,a16;a11,a12,1,1,0,1;a11,a12,1,1,0,0;a11,a12,1,0,a15,a16;a11,a1
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,a16;a11,a12,1,0,0,1;a11,a12,1,0,0,0;a11,a12,0,a14,a15,a16;a11,a12,0,a14,a15,1;a11,a12,0,a1
4,a15,0;a11,a12,0,a14,1,a16;a11,a12,0,a14,1,1;a11,a12,0,a14,1,0;a11,a12,0,a14,0,a16;a11,a12
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```

```
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0,0,0,a15,a16;1,0,0,0,a15,1;1,0,0,0,a15,0;1,0,0,0,1,a16;1,0,0,0,1,1;1,0,0,0,1,0;1,0,0,0,0,a16;1,
0,0,0,0,1;1,0,0,0,0,0;0,a12,a13,a14,a15,a16;0,a12,a13,a14,a15,1;0,a12,a13,a14,a15,0;0,a12,a1
3,a14,1,a16;0,a12,a13,a14,1,1;0,a12,a13,a14,1,0;0,a12,a13,a14,0,a16;0,a12,a13,a14,0,1;0,a12,
a13,a14,0,0;0,a12,a13,1,a15,a16;0,a12,a13,1,a15,1;0,a12,a13,1,a15,0;0,a12,a13,1,1,a16;0,a12,
a13,1,1,1;0,a12,a13,1,1,0;0,a12,a13,1,0,a16;0,a12,a13,1,0,1;0,a12,a13,1,0,0;0,a12,a13,0,a15,a
16;0,a12,a13,0,a15,1;0,a12,a13,0,a15,0;0,a12,a13,0,1,a16;0,a12,a13,0,1,1;0,a12,a13,0,1,0;0,a
12,a13,0,0,a16;0,a12,a13,0,0,1;0,a12,a13,0,0,0;0,a12,1,a14,a15,a16;0,a12,1,a14,a15,1;0,a12,1
,a14,a15,0;0,a12,1,a14,1,a16;0,a12,1,a14,1,1;0,a12,1,a14,1,0;0,a12,1,a14,0,a16;0,a12,1,a14,0,
1;0,a12,1,a14,0,0;0,a12,1,1,a15,a16;0,a12,1,1,a15,1;0,a12,1,1,a15,0;0,a12,1,1,1,a16;0,a12,1,1,
1,1;0,a12,1,1,1,0;0,a12,1,1,0,a16;0,a12,1,1,0,1;0,a12,1,1,0,0;0,a12,1,0,a15,a16;0,a12,1,0,a15,
1;0,a12,1,0,a15,0;0,a12,1,0,1,a16;0,a12,1,0,1,1;0,a12,1,0,1,0;0,a12,1,0,0,a16;0,a12,1,0,0,1;0,a
12,1,0,0,0;0,a12,0,a14,a15,a16;0,a12,0,a14,a15,1;0,a12,0,a14,a15,0;0,a12,0,a14,1,a16;0,a12,0
,a14,1,1;0,a12,0,a14,1,0;0,a12,0,a14,0,a16;0,a12,0,a14,0,1;0,a12,0,a14,0,0;0,a12,0,1,a15,a16;
0,a12,0,1,a15,1;0,a12,0,1,a15,0;0,a12,0,1,1,a16;0,a12,0,1,1,1;0,a12,0,1,1,0;0,a12,0,1,0,a16;0,
a12,0,1,0,1;0,a12,0,1,0,0;0,a12,0,0,a15,a16;0,a12,0,0,a15,1;0,a12,0,0,a15,0;0,a12,0,0,1,a16;0,
a12,0,0,1,1;0,a12,0,0,1,0;0,a12,0,0,0,a16;0,a12,0,0,0,1;0,a12,0,0,0,0;0,1,a13,a14,a15,a16;0,1,
a13,a14,a15,1;0,1,a13,a14,a15,0;0,1,a13,a14,1,a16;0,1,a13,a14,1,1;0,1,a13,a14,1,0;0,1,a13,a1
4,0,a16;0,1,a13,a14,0,1;0,1,a13,a14,0,0;0,1,a13,1,a15,a16;0,1,a13,1,a15,1;0,1,a13,1,a15,0;0,1,
a13,1,1,a16;0,1,a13,1,1,1;0,1,a13,1,1,0;0,1,a13,1,0,a16;0,1,a13,1,0,1;0,1,a13,1,0,0;0,1,a13,0,a
15,a16;0,1,a13,0,a15,1;0,1,a13,0,a15,0;0,1,a13,0,1,a16;0,1,a13,0,1,1;0,1,a13,0,1,0;0,1,a13,0,0
,a16;0,1,a13,0,0,1;0,1,a13,0,0,0;0,1,1,a14,a15,a16;0,1,1,a14,a15,1;0,1,1,a14,a15,0;0,1,1,a14,1
,a16;0,1,1,a14,1,1;0,1,1,a14,1,0;0,1,1,a14,0,a16;0,1,1,a14,0,1;0,1,1,a14,0,0;0,1,1,1,a15,a16;0,
1,1,1,a15,1;0,1,1,1,a15,0;0,1,1,1,1,a16;0,1,1,1,1,1;0,1,1,1,1,0;0,1,1,1,0,a16;0,1,1,1,0,1;0,1,1,1,
0,0;0,1,1,0,a15,a16;0,1,1,0,a15,1;0,1,1,0,a15,0;0,1,1,0,1,a16;0,1,1,0,1,1;0,1,1,0,1,0;0,1,1,0,0,a
16;0,1,1,0,0,1;0,1,1,0,0,0;0,1,0,a14,a15,a16;0,1,0,a14,a15,1;0,1,0,a14,a15,0;0,1,0,a14,1,a16;0,
1,0,a14,1,1;0,1,0,a14,1,0;0,1,0,a14,0,a16;0,1,0,a14,0,1;0,1,0,a14,0,0;0,1,0,1,a15,a16;0,1,0,1,a
15,1;0,1,0,1,a15,0;0,1,0,1,1,a16;0,1,0,1,1,1;0,1,0,1,1,0;0,1,0,1,0,a16;0,1,0,1,0,1;0,1,0,1,0,0;0,
1,0,0,a15,a16;0,1,0,0,a15,1;0,1,0,0,a15,0;0,1,0,0,1,a16;0,1,0,0,1,1;0,1,0,0,1,0;0,1,0,0,0,a16;0,
1,0,0,0,1;0,1,0,0,0,0;0,0,a13,a14,a15,a16;0,0,a13,a14,a15,1;0,0,a13,a14,a15,0;0,0,a13,a14,1,a
16;0,0,a13,a14,1,1;0,0,a13,a14,1,0;0,0,a13,a14,0,a16;0,0,a13,a14,0,1;0,0,a13,a14,0,0;0,0,a13,
1,a15,a16;0,0,a13,1,a15,1;0,0,a13,1,a15,0;0,0,a13,1,1,a16;0,0,a13,1,1,1;0,0,a13,1,1,0;0,0,a13,
1,0,a16;0,0,a13,1,0,1;0,0,a13,1,0,0;0,0,a13,0,a15,a16;0,0,a13,0,a15,1;0,0,a13,0,a15,0;0,0,a13,
0,1,a16;0,0,a13,0,1,1;0,0,a13,0,1,0;0,0,a13,0,0,a16;0,0,a13,0,0,1;0,0,a13,0,0,0;0,0,1,a14,a15,a
16;0,0,1,a14,a15,1;0,0,1,a14,a15,0;0,0,1,a14,1,a16;0,0,1,a14,1,1;0,0,1,a14,1,0;0,0,1,a14,0,a16
;0,0,1,a14,0,1;0,0,1,a14,0,0;0,0,1,1,a15,a16;0,0,1,1,a15,1;0,0,1,1,a15,0;0,0,1,1,1,a16;0,0,1,1,1
,1;0,0,1,1,1,0;0,0,1,1,0,a16;0,0,1,1,0,1;0,0,1,1,0,0;0,0,1,0,a15,a16;0,0,1,0,a15,1;0,0,1,0,a15,0;
0,0,1,0,1,a16;0,0,1,0,1,1;0,0,1,0,1,0;0,0,1,0,0,a16;0,0,1,0,0,1;0,0,1,0,0,0;0,0,0,a14,a15,a16;0,0
,0,a14,a15,1;0,0,0,a14,a15,0;0,0,0,a14,1,a16;0,0,0,a14,1,1;0,0,0,a14,1,0;0,0,0,a14,0,a16;0,0,0,
a14,0,1;0,0,0,a14,0,0;0,0,0,1,a15,a16;0,0,0,1,a15,1;0,0,0,1,a15,0;0,0,0,1,1,a16;0,0,0,1,1,1;0,0,
0,1,1,0;0,0,0,1,0,a16;0,0,0,1,0,1;0,0,0,1,0,0;0,0,0,0,a15,a16;0,0,0,0,a15,1;0,0,0,0,a15,0;0,0,0,0
,1,a16;0,0,0,0,1,1;0,0,0,0,1,0;0,0,0,0,0,a16;0,0,0,0,0,1;0,0,0,0,0,0];

eval_HT=eval(fHT);

[max_HT, id_HT]=nanmax(eval_HT);

u11(1)=D(id_HT,1);
u12(1)=D(id_HT,2);
u13(1)=D(id_HT,3);
u14(1)=D(id_HT,4);
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u15(1)=D(id_HT,5);

u16(1)=D(id_HT,6);


u21(1)=u21s;

u22(1)=u22s;


%LAMBDA1(1)

ka11(1)=-1-k*l12(i)*x26(i)+k1s*(1-qthvs)*rhos*l126(i)*(1-u11(1))*(1-u12(1))*x26(i)-
l18(i)*(k3*x24(i)+k1s*(1-qthvs)*(1-u11(1))*(1-u12(1))*x26(i))+k1r*(1-
qthvr)*rhor*l127(i)*x27(i)-k*l13(i)*x27(i)-kprimo*l14(i)*(x26(i)+x27(i))-
l110(i)*(k3*x25(i)+k1r*(1-qthvr)*x27(i))-
c*l16(i)*x6(i)*(x11(i)+x13(i)+x15(i)+x17(i)+x19(i)+x21(i)+x23(i)+x9(i))-l11(i)*(-
muth+u13(1)-(r*x1(i))/tmax-k3*(x24(i)+x25(i))-k1s*(1-qthvs)*(1-u11(1))*(1-u12(1))*x26(i)-
k1r*(1-qthvr)*x27(i)-k*(x26(i)+x27(i))-kprimo*(x26(i)+x27(i))+r*(1-
(x1(i)+x10(i)+x11(i)+x12(i)+x13(i)+x14(i)+x15(i)+x16(i)+x17(i)+x18(i)+x19(i)+x2(i)+x20(i
)+x21(i)+x22(i)+x23(i)+x3(i)+x4(i)+x8(i)+x9(i))/tmax));       ka12(1)=-
1+(r*l11(i)*x1(i))/tmax-l112(i)*(k3*x24(i)+k1s*(1-qtsvs)*(1-u11(1))*(1-u12(1))*x26(i))-
l126(i)*(-(psprimo*x26(i))-k1s*(1-qtsvs)*rhos*(1-u11(1))*(1-u12(1))*x26(i))+k1r*(1-
qtsvr)*rhor*l127(i)*x27(i)-l12(i)*(-muts+u13(1)-k3*(x24(i)+x25(i))-k1s*(1-qtsvs)*(1-
u11(1))*(1-u12(1))*x26(i)-k1r*(1-qtsvr)*x27(i))-l114(i)*(k3*x25(i)+k1r*(1-qtsvr)*x27(i))-
c*l16(i)*x6(i)*(x11(i)+x13(i)+x15(i)+x17(i)+x19(i)+x21(i)+x23(i)+x9(i));       ka13(1)=-
1+(r*l11(i)*x1(i))/tmax+k1s*(1-qtrvs)*rhos*l126(i)*(1-u11(1))*(1-u12(1))*x26(i)-
l116(i)*(k3*x24(i)+k1s*(1-qtrvs)*(1-u11(1))*(1-u12(1))*x26(i))-l13(i)*(-mutr+u13(1)-
k3*(x24(i)+x25(i))-k1s*(1-qtrvs)*(1-u11(1))*(1-u12(1))*x26(i)-k1r*(1-qtrvr)*x27(i))-
l118(i)*(k3*x25(i)+k1r*(1-qtrvr)*x27(i))-l127(i)*(-(prprimo*x27(i))-k1r*(1-
qtrvr)*rhor*x27(i))-
c*l16(i)*x6(i)*(x11(i)+x13(i)+x15(i)+x17(i)+x19(i)+x21(i)+x23(i)+x9(i));       ka14(1)=-
1+(r*l11(i)*x1(i))/tmax-l120(i)*(k3*x24(i)+k1s*(1-qtgvs)*(1-u11(1))*(1-u12(1))*x26(i))-
l126(i)*(-(ssprimo*x26(i))-k1s*(1-qtgvs)*rhos*(1-u11(1))*(1-u12(1))*x26(i))-l14(i)*(-
mutg+u13(1)-k3*(x24(i)+x25(i))-k1s*(1-qtgvs)*(1-u11(1))*(1-u12(1))*x26(i)-k1r*(1-
qtgvr)*x27(i))-l122(i)*(k3*x25(i)+k1r*(1-qtgvr)*x27(i))-l127(i)*(-(k1r*(1-
qtgvr)*rhor*x27(i))-srprimo*x27(i))-
c*l16(i)*x6(i)*(x11(i)+x13(i)+x15(i)+x17(i)+x19(i)+x21(i)+x23(i)+x9(i));       ka15(1)=-1-
k1sm*l124(i)*(1-u15(1))*(1-u16(1))*x26(i)+k1sm*rhos*l126(i)*(1-u15(1))*(1-
u16(1))*x26(i)-k1rm*l125(i)*x27(i)+k1rm*rhor*l127(i)*x27(i)-l15(i)*(-mumh-k1sm*(1-
u15(1))*(1-u16(1))*x26(i)-k1rm*x27(i));       ka16(1)=-1-
c*q*l17(i)*(x11(i)+x13(i)+x15(i)+x17(i)+x19(i)+x21(i)+x23(i)+x9(i))-l16(i)*(-muip-
c*q*(x11(i)+x13(i)+x15(i)+x17(i)+x19(i)+x21(i)+x23(i)+x9(i))+c*(x1(i)+x2(i)+x3(i)+x4(i))*
(x11(i)+x13(i)+x15(i)+x17(i)+x19(i)+x21(i)+x23(i)+x9(i)));       ka17(1)=-
1+muie*l17(i)+ir*l111(i)*x11(i)+is*l113(i)*x13(i)+ir*l115(i)*x15(i)+is*l117(i)*x17(i)+ir*l1
19(i)*x19(i)+is*l121(i)*x21(i)+ir*l123(i)*x23(i)+is*l19(i)*x9(i);       ka18(1)=-(l18(i)*(-
muthsl-k2*(1-u14(1))*(1-u16(1))))-k2*l19(i)*(1-u14(1))*(1-u16(1))+(r*l11(i)*x1(i))/tmax;
ka19(1)=-(ps*l127(i)*(1-u21(1)))-ps*l126(i)*(1-u15(1))*u21(1)+(r*l11(i)*x1(i))/tmax-
c*q*l17(i)*x6(i)-l16(i)*(-(c*q*x6(i))+c*(x1(i)+x2(i)+x3(i)+x4(i))*x6(i))-l19(i)*(-muthsa-
is*x7(i));       ka110(1)=-((-k2-muthrl)*l110(i))-k2*l111(i)+(r*l11(i)*x1(i))/tmax;
ka111(1)=-(pr*l126(i)*(1-u15(1))*(1-u22(1)))-pr*l127(i)*u22(1)+(r*l11(i)*x1(i))/tmax-
c*q*l17(i)*x6(i)-l16(i)*(-(c*q*x6(i))+c*(x1(i)+x2(i)+x3(i)+x4(i))*x6(i))-l111(i)*(-muthra-
ir*x7(i));       ka112(1)=-(l112(i)*(-mutssl-k2*(1-u14(1))*(1-u16(1))))-k2*l113(i)*(1-
u14(1))*(1-u16(1))+(r*l11(i)*x1(i))/tmax;       ka113(1)=-(ps*l127(i)*(1-u21(1)))-
ps*l126(i)*(1-u15(1))*u21(1)+(r*l11(i)*x1(i))/tmax-c*q*l17(i)*x6(i)-l16(i)*(-
(c*q*x6(i))+c*(x1(i)+x2(i)+x3(i)+x4(i))*x6(i))-l113(i)*(-mutssa-is*x7(i));       ka114(1)=-((-
k2-mutsrl)*l114(i))-k2*l115(i)+(r*l11(i)*x1(i))/tmax;       ka115(1)=-(pr*l126(i)*(1-
u15(1))*(1-u22(1)))-pr*l127(i)*u22(1)+(r*l11(i)*x1(i))/tmax-c*q*l17(i)*x6(i)-l16(i)*(-
(c*q*x6(i))+c*(x1(i)+x2(i)+x3(i)+x4(i))*x6(i))-l115(i)*(-mutsra-ir*x7(i));       ka116(1)=-
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(l116(i)*(-mutrsl-k2*(1-u14(1))*(1-u16(1))))-k2*l117(i)*(1-u14(1))*(1-u16(1))+(r*l11(i)*x1(i))/tmax; ka117(1)=-(ps*l127(i)*(1-u21(1)))-ps*l126(i)*(1-u15(1))*u21(1)+(r*l11(i)*x1(i))/tmax-c*q*l17(i)*x6(i)-l16(i)*(-(c*q*x6(i))+c*(x1(i)+x2(i)+x3(i)+x4(i))*x6(i))-l117(i)*(-mutrsa-is*x7(i)); ka118(1)=-((-k2-mutrrl)*l118(i))-k2*l119(i)+(r*l11(i)*x1(i))/tmax; ka119(1)=-(pr*l126(i)*(1-u15(1))*(1-u22(1)))-pr*l127(i)*u22(1)+(r*l11(i)*x1(i))/tmax-c*q*l17(i)*x6(i)-l16(i)*(-(c*q*x6(i))+c*(x1(i)+x2(i)+x3(i)+x4(i))*x6(i))-l119(i)*(-mutrra-ir*x7(i)); ka120(1)=-(l120(i)*(-mutgsl-k2*(1-u14(1))*(1-u16(1))))-k2*l121(i)*(1-u14(1))*(1-u16(1))+(r*l11(i)*x1(i))/tmax; ka121(1)=-(ps*l127(i)*(1-u21(1)))-ps*l126(i)*(1-u15(1))*u21(1)+(r*l11(i)*x1(i))/tmax-c*q*l17(i)*x6(i)-l16(i)*(-(c*q*x6(i))+c*(x1(i)+x2(i)+x3(i)+x4(i))*x6(i))-l121(i)*(-mutgsa-is*x7(i)); ka122(1)=-((-k2-mutgrl)*l122(i))-k2*l123(i)+(r*l11(i)*x1(i))/tmax; ka123(1)=-(pr*l126(i)*(1-u15(1))*(1-u22(1)))-pr*l127(i)*u22(1)+(r*l11(i)*x1(i))/tmax-c*q*l17(i)*x6(i)-l16(i)*(-(c*q*x6(i))+c*(x1(i)+x2(i)+x3(i)+x4(i))*x6(i))-l123(i)*(-mutgra-ir*x7(i)); ka124(1)=mumhsa*l124(i)-ps*l127(i)*(1-u21(1))-ps*l126(i)*(1-u15(1))*u21(1)+k3*l11(i)*x1(i)-k3*l18(i)*x1(i)-k3*l112(i)*x2(i)+k3*l12(i)*x2(i)-k3*l116(i)*x3(i)+k3*l13(i)*x3(i)-k3*l120(i)*x4(i)+k3*l14(i)*x4(i); ka125(1)=mumhra*l125(i)-pr*l126(i)*(1-u15(1))*(1-u22(1))-pr*l127(i)*u22(1)+k3*l11(i)*x1(i)-k3*l110(i)*x1(i)-k3*l114(i)*x2(i)+k3*l12(i)*x2(i)-k3*l118(i)*x3(i)+k3*l13(i)*x3(i)-k3*l122(i)*x4(i)+k3*l14(i)*x4(i); ka126(1)=-(k1s*(1-qthvs)*l18(i)*(1-u11(1))*(1-u12(1))*x1(i))-k1s*(1-qtsvs)*l112(i)*(1-u11(1))*(1-u12(1))*x2(i)-l12(i)*(k*x1(i)-k1s*(1-qtsvs)*(1-u11(1))*(1-u12(1))*x2(i))-l11(i)*(-(k*x1(i))-kprimo*x1(i)-k1s*(1-qthvs)*(1-u11(1))*(1-u12(1))*x1(i)-(oth*s1th)/(oth+x26(i)+x27(i))^2)-k1s*(1-qtrvs)*l116(i)*(1-u11(1))*(1-u12(1))*x3(i)+k1s*(1-qtrvs)*l13(i)*(1-u11(1))*(1-u12(1))*x3(i)-k1s*(1-qtgvs)*l120(i)*(1-u11(1))*(1-u12(1))*x4(i)-l14(i)*(kprimo*x1(i)-k1s*(1-qtgvs)*(1-u11(1))*(1-u12(1))*x4(i))-k1sm*l124(i)*(1-u15(1))*(1-u16(1))*x5(i)+k1sm*l15(i)*(1-u15(1))*(1-u16(1))*x5(i)-l126(i)*(-muvs-psprimo*x2(i)-(g*(1-u15(1))*x26(i))/(b2+x26(i))^2+(g*(1-u15(1)))/(b2+x26(i))-ssprimo*x4(i)-k1s*rhos*(1-u11(1))*(1-u12(1))*((1-qthvs)*x1(i)+(1-qtsvs)*x2(i)+(1-qtrvs)*x3(i)+(1-qtgvs)*x4(i))-k1sm*rhos*(1-u15(1))*(1-u16(1))*x5(i)); ka127(1)=-(k1r*(1-qthvr)*l110(i)*x1(i))-k1r*(1-qtsvr)*l114(i)*x2(i)+k1r*(1-qtsvr)*l12(i)*x2(i)-l11(i)*(-(k*x1(i))-kprimo*x1(i)-k1r*(1-qthvr)*x1(i)-(oth*s1th)/(oth+x26(i)+x27(i))^2)-k1r*(1-qtrvr)*l118(i)*x3(i)-l13(i)*(k*x1(i)-k1r*(1-qtrvr)*x3(i))-k1r*(1-qtgvr)*l122(i)*x4(i)-l14(i)*(kprimo*x1(i)-k1r*(1-qtgvr)*x4(i))-k1rm*l125(i)*x5(i)+k1rm*l15(i)*x5(i)-l127(i)*(-muvr-(g*x27(i))/(b2+x27(i))^2+g/(b2+x27(i))-prprimo*x3(i)-srprimo*x4(i)-k1r*rhor*((1-qthvr)*x1(i)+(1-qtsvr)*x2(i)+(1-qtrvr)*x3(i)+(1-qtgvr)*x4(i))-k1rm*rhor*x5(i));

%LAMBDA2(1)

ka21(1)=-(k*l22(i)*x26(i))+k1s*(1-qthvs)*rhos*l226(i)*(1-u11(1))*(1-u12(1))*x26(i)-l28(i)*(k3*x24(i)+k1s*(1-qthvs)*(1-u11(1))*(1-u12(1))*x26(i))+k1r*(1-qthvr)*rhor*l227(i)*x27(i)-k*l23(i)*x27(i)-kprimo*l24(i)*(x26(i)+x27(i))-l210(i)*(k3*x25(i)+k1r*(1-qthvr)*x27(i))-c*l26(i)*x6(i)*(x11(i)+x13(i)+x15(i)+x17(i)+x19(i)+x21(i)+x23(i)+x9(i))-l21(i)*(-muth+u13(1)-(r*x1(i))/tmax-k3*(x24(i)+x25(i))-k1s*(1-qthvs)*(1-u11(1))*(1-u12(1))*x26(i)-k1r*(1-qthvr)*x27(i)-k*(x26(i)+x27(i))-kprimo*(x26(i)+x27(i))+r*(1-(x1(i)+x10(i)+x11(i)+x12(i)+x13(i)+x14(i)+x15(i)+x16(i)+x17(i)+x18(i)+x19(i)+x2(i)+x20(i)+x21(i)+x22(i)+x23(i)+x3(i)+x4(i)+x8(i)+x9(i))/tmax)); ka22(1)=(r*l21(i)*x1(i))/tmax-l212(i)*(k3*x24(i)+k1s*(1-qtsvs)*(1-u11(1))*(1-u12(1))*x26(i))-l226(i)*(-(psprimo*x26(i))-k1s*(1-qtsvs)*rhos*(1-u11(1))*(1-u12(1))*x26(i))+k1r*(1-qtsvr)*rhor*l227(i)*x27(i)-l22(i)*(-muts+u13(1)-k3*(x24(i)+x25(i))-k1s*(1-qtsvs)*(1-u11(1))*(1-u12(1))*x26(i)-k1r*(1-qtsvr)*x27(i))-l214(i)*(k3*x25(i)+k1r*(1-qtsvr)*x27(i))-c*l26(i)*x6(i)*(x11(i)+x13(i)+x15(i)+x17(i)+x19(i)+x21(i)+x23(i)+x9(i)); ka23(1)=(r*l21(i)*x1(i))/tmax+k1s*(1-qtrvs)*rhos*l226(i)*(1-u11(1))*(1-u12(1))*x26(i)-l216(i)*(k3*x24(i)+k1s*(1-qtrvs)*(1-u11(1))*(1-u12(1))*x26(i))-l23(i)*(-mutr+u13(1)-k3*(x24(i)+x25(i))-k1s*(1-qtrvs)*(1-u11(1))*(1-u12(1))*x26(i)-k1r*(1-qtrvr)*x27(i))-l218(i)*(k3*x25(i)+k1r*(1-qtrvr)*x27(i))-l227(i)*(-(prprimo*x27(i))-k1r*(1-qtrvr)*rhor*x27(i))-

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c*l26(i)*x6(i)*(x11(i)+x13(i)+x15(i)+x17(i)+x19(i)+x21(i)+x23(i)+x9(i));
ka24(1)=(r*l21(i)*x1(i))/tmax-l220(i)*(k3*x24(i)+k1s*(1-qtgvs)*(1-u11(1))*(1-
u12(1))*x26(i))-l226(i)*(-(ssprimo*x26(i))-k1s*(1-qtgvs)*rhos*(1-u11(1))*(1-
u12(1))*x26(i))-l24(i)*(-mutg+u13(1)-k3*(x24(i)+x25(i))-k1s*(1-qtgvs)*(1-u11(1))*(1-
u12(1))*x26(i)-k1r*(1-qtgvr)*x27(i))-l222(i)*(k3*x25(i)+k1r*(1-qtgvr)*x27(i))-l227(i)*(-
(k1r*(1-qtgvr)*rhor*x27(i))-srprimo*x27(i))-
c*l26(i)*x6(i)*(x11(i)+x13(i)+x15(i)+x17(i)+x19(i)+x21(i)+x23(i)+x9(i));    ka25(1)=-
(k1sm*l224(i)*(1-u15(1))*(1-u16(1))*x26(i))+k1sm*rhos*l226(i)*(1-u15(1))*(1-
u16(1))*x26(i)-k1rm*l225(i)*x27(i)+k1rm*rhor*l227(i)*x27(i)-l25(i)*(-mumh-k1sm*(1-
u15(1))*(1-u16(1))*x26(i)-k1rm*x27(i));    ka26(1)=-
(c*q*l27(i)*(x11(i)+x13(i)+x15(i)+x17(i)+x19(i)+x21(i)+x23(i)+x9(i)))-l26(i)*(-muip-
c*q*(x11(i)+x13(i)+x15(i)+x17(i)+x19(i)+x21(i)+x23(i)+x9(i))+c*(x1(i)+x2(i)+x3(i)+x4(i))*
(x11(i)+x13(i)+x15(i)+x17(i)+x19(i)+x21(i)+x23(i)+x9(i)));
ka27(1)=muie*l27(i)+ir*l211(i)*x11(i)+is*l213(i)*x13(i)+ir*l215(i)*x15(i)+is*l217(i)*x17(i
)+ir*l219(i)*x19(i)+is*l221(i)*x21(i)+ir*l223(i)*x23(i)+is*l29(i)*x9(i);    ka28(1)=-1-
l28(i)*(-muthsl-k2*(1-u14(1))*(1-u16(1)))-k2*l29(i)*(1-u14(1))*(1-
u16(1))+(r*l21(i)*x1(i))/tmax;    ka29(1)= -1-ps*l227(i)*(1-u21(1))-ps*l226(i)*(1-
u15(1))*u21(1)+(r*l21(i)*x1(i))/tmax-c*q*l27(i)*x6(i)-l26(i)*(-
(c*q*x6(i))+c*(x1(i)+x2(i)+x3(i)+x4(i))*x6(i))-l29(i)*(-muthsa-is*x7(i));    ka210(1)=-1-(-
k2-muthrl)*l210(i)-k2*l211(i)+(r*l21(i)*x1(i))/tmax;    ka211(1)= -1-pr*l226(i)*(1-
u15(1))*(1-u22(1))-pr*l227(i)*u22(1)+(r*l21(i)*x1(i))/tmax-c*q*l27(i)*x6(i)-l26(i)*(-
(c*q*x6(i))+c*(x1(i)+x2(i)+x3(i)+x4(i))*x6(i))-l211(i)*(-muthra-ir*x7(i));    ka212(1)=-1-
l212(i)*(-mutssl-k2*(1-u14(1))*(1-u16(1)))-k2*l213(i)*(1-u14(1))*(1-
u16(1))+(r*l21(i)*x1(i))/tmax;    ka213(1)=-1-ps*l227(i)*(1-u21(1))-ps*l226(i)*(1-
u15(1))*u21(1)+(r*l21(i)*x1(i))/tmax-c*q*l27(i)*x6(i)-l26(i)*(-
(c*q*x6(i))+c*(x1(i)+x2(i)+x3(i)+x4(i))*x6(i))-l213(i)*(-mutssa-is*x7(i));    ka214(1)=-1-
(-k2-mutsrl)*l214(i)-k2*l215(i)+(r*l21(i)*x1(i))/tmax;    ka215(1)=-1-pr*l226(i)*(1-
u15(1))*(1-u22(1))-pr*l227(i)*u22(1)+(r*l21(i)*x1(i))/tmax-c*q*l27(i)*x6(i)-l26(i)*(-
(c*q*x6(i))+c*(x1(i)+x2(i)+x3(i)+x4(i))*x6(i))-l215(i)*(-mutsra-ir*x7(i));    ka216(1)=-1-
l216(i)*(-mutrsl-k2*(1-u14(1))*(1-u16(1)))-k2*l217(i)*(1-u14(1))*(1-
u16(1))+(r*l21(i)*x1(i))/tmax;    ka217(1)=-1-ps*l227(i)*(1-u21(1))-ps*l226(i)*(1-
u15(1))*u21(1)+(r*l21(i)*x1(i))/tmax-c*q*l27(i)*x6(i)-l26(i)*(-
(c*q*x6(i))+c*(x1(i)+x2(i)+x3(i)+x4(i))*x6(i))-l217(i)*(-mutrsa-is*x7(i));    ka218(1)=-1-
(-k2-mutrrl)*l218(i)-k2*l219(i)+(r*l21(i)*x1(i))/tmax;    ka219(1)=-1-pr*l226(i)*(1-
u15(1))*(1-u22(1))-pr*l227(i)*u22(1)+(r*l21(i)*x1(i))/tmax-c*q*l27(i)*x6(i)-l26(i)*(-
(c*q*x6(i))+c*(x1(i)+x2(i)+x3(i)+x4(i))*x6(i))-l219(i)*(-mutrra-ir*x7(i));    ka220(1)=-1-
l220(i)*(-mutgsl-k2*(1-u14(1))*(1-u16(1)))-k2*l221(i)*(1-u14(1))*(1-
u16(1))+(r*l21(i)*x1(i))/tmax;    ka221(1)=-1-ps*l227(i)*(1-u21(1))-ps*l226(i)*(1-
u15(1))*u21(1)+(r*l21(i)*x1(i))/tmax-c*q*l27(i)*x6(i)-l26(i)*(-
(c*q*x6(i))+c*(x1(i)+x2(i)+x3(i)+x4(i))*x6(i))-l221(i)*(-mutgsa-is*x7(i));    ka222(1)=-1-
(-k2-mutgrl)*l222(i)-k2*l223(i)+(r*l21(i)*x1(i))/tmax;    ka223(1)=-1-pr*l226(i)*(1-
u15(1))*(1-u22(1))-pr*l227(i)*u22(1)+(r*l21(i)*x1(i))/tmax-c*q*l27(i)*x6(i)-l26(i)*(-
(c*q*x6(i))+c*(x1(i)+x2(i)+x3(i)+x4(i))*x6(i))-l223(i)*(-mutgra-ir*x7(i));
ka224(1)=mumhsa*l224(i)-ps*l227(i)*(1-u21(1))-ps*l226(i)*(1-
u15(1))*u21(1)+k3*l21(i)*x1(i)-k3*l28(i)*x1(i)-k3*l212(i)*x2(i)+k3*l22(i)*x2(i)-
k3*l216(i)*x3(i)+k3*l23(i)*x3(i)-k3*l220(i)*x4(i)+k3*l24(i)*x4(i);
ka225(1)=mumhra*l225(i)-pr*l226(i)*(1-u15(1))*(1-u22(1))-
pr*l227(i)*u22(1)+k3*l21(i)*x1(i)-k3*l210(i)*x1(i)-k3*l214(i)*x2(i)+k3*l22(i)*x2(i)-
k3*l218(i)*x3(i)+k3*l23(i)*x3(i)-k3*l222(i)*x4(i)+k3*l24(i)*x4(i);    ka226(1)=-1-
k1s*(1-qthvs)*l28(i)*(1-u11(1))*(1-u12(1))*x1(i)-k1s*(1-qtsvs)*l212(i)*(1-u11(1))*(1-
u12(1))*x2(i)-l22(i)*(k*x1(i)-k1s*(1-qtsvs)*(1-u11(1))*(1-u12(1))*x2(i))-l21(i)*(-(k*x1(i))-
kprimo*x1(i)-k1s*(1-qthvs)*(1-u11(1))*(1-u12(1))*x1(i)-(oth*s1th)/(oth+x26(i)+x27(i))^2)-
k1s*(1-qtrvs)*l216(i)*(1-u11(1))*(1-u12(1))*x3(i)+k1s*(1-qtrvs)*l23(i)*(1-u11(1))*(1-
u12(1))*x3(i)-k1s*(1-qtgvs)*l220(i)*(1-u11(1))*(1-u12(1))*x4(i)-l24(i)*(kprimo*x1(i)-
k1s*(1-qtgvs)*(1-u11(1))*(1-u12(1))*x4(i))-k1sm*l224(i)*(1-u15(1))*(1-
u16(1))*x5(i)+k1sm*l25(i)*(1-u15(1))*(1-u16(1))*x5(i)-l226(i)*(-muvs-psprimo*x2(i)-
(g*(1-u15(1))*x26(i))/(b2+x26(i))^2+(g*(1-u15(1)))/(b2+x26(i))-ssprimo*x4(i)-k1s*rhos*(1-
u11(1))*(1-u12(1))*((1-qthvs)*x1(i)+(1-qtsvs)*x2(i)+(1-qtrvs)*x3(i)+(1-qtgvs)*x4(i))-
k1sm*rhos*(1-u15(1))*(1-u16(1))*x5(i));    ka227(1)=-1-k1r*(1-qthvr)*l210(i)*x1(i)-
k1r*(1-qtsvr)*l214(i)*x2(i)+k1r*(1-qtsvr)*l22(i)*x2(i)-l21(i)*(-(k*x1(i))-kprimo*x1(i)-
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k1r*(1-qthvr)*x1(i)-(oth*s1th)/(oth+x26(i)+x27(i))^2)-k1r*(1-qtrvr)*l218(i)*x3(i)-
l23(i)*(k*x1(i)-k1r*(1-qtrvr)*x3(i))-k1r*(1-qtgvr)*l222(i)*x4(i)-l24(i)*(kprimo*x1(i)-
k1r*(1-qtgvr)*x4(i))-k1rm*l225(i)*x5(i)+k1rm*l25(i)*x5(i)-l227(i)*(-muvr-
(g*x27(i))/(b2+x27(i))^2+g/(b2+x27(i))-prprimo*x3(i)-srprimo*x4(i)-k1r*rhor*((1-
qthvr)*x1(i)+(1-qtsvr)*x2(i)+(1-qtrvr)*x3(i)+(1-qtgvr)*x4(i))-k1rm*rhor*x5(i));

    %VARIABILI DI APPOGGIO wb per 2° ordine
    wb1(i)=0.5*(x1(i)+x1(i-1));
    wb2(i)=0.5*(x2(i)+x2(i-1));
    wb3(i)=0.5*(x3(i)+x3(i-1));
    wb4(i)=0.5*(x4(i)+x4(i-1));
    wb5(i)=0.5*(x5(i)+x5(i-1));
    wb6(i)=0.5*(x6(i)+x6(i-1));
    wb7(i)=0.5*(x7(i)+x7(i-1));
    wb8(i)=0.5*(x8(i)+x8(i-1));
    wb9(i)=0.5*(x9(i)+x9(i-1));
    wb10(i)=0.5*(x10(i)+x10(i-1));
    wb11(i)=0.5*(x11(i)+x11(i-1));
    wb12(i)=0.5*(x12(i)+x12(i-1));
    wb13(i)=0.5*(x13(i)+x13(i-1));
    wb14(i)=0.5*(x14(i)+x14(i-1));
    wb15(i)=0.5*(x15(i)+x15(i-1));
    wb16(i)=0.5*(x16(i)+x16(i-1));
    wb17(i)=0.5*(x17(i)+x17(i-1));
    wb18(i)=0.5*(x18(i)+x18(i-1));
    wb19(i)=0.5*(x19(i)+x19(i-1));
    wb20(i)=0.5*(x20(i)+x20(i-1));
    wb21(i)=0.5*(x21(i)+x21(i-1));
    wb22(i)=0.5*(x22(i)+x22(i-1));
    wb23(i)=0.5*(x23(i)+x23(i-1));
    wb24(i)=0.5*(x24(i)+x24(i-1));
    wb25(i)=0.5*(x25(i)+x25(i-1));
    wb26(i)=0.5*(x26(i)+x26(i-1));
    wb27(i)=0.5*(x27(i)+x27(i-1));
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%VARIABILI DI APPOGGIO hb per 2° ordine

hb11(i)=l11(i)-h2*ka11(1);
hb12(i)=l12(i)-h2*ka12(1);
hb13(i)=l13(i)-h2*ka13(1);
hb14(i)=l14(i)-h2*ka14(1);
hb15(i)=l15(i)-h2*ka15(1);
hb16(i)=l16(i)-h2*ka16(1);
hb17(i)=l17(i)-h2*ka17(1);
hb18(i)=l18(i)-h2*ka18(1);
hb19(i)=l19(i)-h2*ka19(1);
hb110(i)=l110(i)-h2*ka110(1);
hb111(i)=l111(i)-h2*ka111(1);
hb112(i)=l112(i)-h2*ka112(1);
hb113(i)=l113(i)-h2*ka113(1);
hb114(i)=l114(i)-h2*ka114(1);
hb115(i)=l115(i)-h2*ka115(1);
hb116(i)=l116(i)-h2*ka116(1);
hb117(i)=l117(i)-h2*ka117(1);
hb118(i)=l118(i)-h2*ka118(1);
hb119(i)=l119(i)-h2*ka119(1);
hb120(i)=l120(i)-h2*ka120(1);
hb121(i)=l121(i)-h2*ka121(1);
hb122(i)=l122(i)-h2*ka122(1);
hb123(i)=l123(i)-h2*ka123(1);
hb124(i)=l124(i)-h2*ka124(1);
hb125(i)=l125(i)-h2*ka125(1);
hb126(i)=l126(i)-h2*ka126(1);
hb127(i)=l127(i)-h2*ka127(1);


hb21(i)=l21(i)-h2*ka21(1);
hb22(i)=l22(i)-h2*ka22(1);
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hb23(i)=l23(i)-h2*ka23(1);

hb24(i)=l24(i)-h2*ka24(1);

hb25(i)=l25(i)-h2*ka25(1);

hb26(i)=l26(i)-h2*ka26(1);

hb27(i)=l27(i)-h2*ka27(1);

hb28(i)=l28(i)-h2*ka28(1);

hb29(i)=l29(i)-h2*ka29(1);

hb210(i)=l210(i)-h2*ka210(1);

hb211(i)=l211(i)-h2*ka211(1);

hb212(i)=l212(i)-h2*ka212(1);

hb213(i)=l213(i)-h2*ka213(1);

hb214(i)=l214(i)-h2*ka214(1);

hb215(i)=l215(i)-h2*ka215(1);

hb216(i)=l216(i)-h2*ka216(1);

hb217(i)=l217(i)-h2*ka217(1);

hb218(i)=l218(i)-h2*ka218(1);

hb219(i)=l219(i)-h2*ka219(1);

hb220(i)=l220(i)-h2*ka220(1);

hb221(i)=l221(i)-h2*ka221(1);

hb222(i)=l222(i)-h2*ka222(1);

hb223(i)=l223(i)-h2*ka223(1);

hb224(i)=l224(i)-h2*ka224(1);

hb225(i)=l225(i)-h2*ka225(1);

hb226(i)=l226(i)-h2*ka226(1);

hb227(i)=l227(i)-h2*ka227(1);



%CONTROLLI COEEFICIENTI DI RUNGE KUTTA 2° ORDINE

a11= (1.*(k1s*(1.-1.*qthvs)*hb11(i)*wb1(i)*wb26(i)-1.*k1s*(1.-
1.*qthvs)*hb18(i)*wb1(i)*wb26(i)-1.*k1s*(1.-1.*qtsvs)*hb112(i)*wb2(i)*wb26(i)+k1s*(1.-
1.*qtsvs)*hb12(i)*wb2(i)*wb26(i)-1.*k1s*(1.-1.*qtrvs)*hb116(i)*wb26(i)*wb3(i)+k1s*(1.-
1.*qtrvs)*hb13(i)*wb26(i)*wb3(i)-1.*k1s*(1.-1.*qtgvs)*hb120(i)*wb26(i)*wb4(i)+k1s*(1.-
1.*qtgvs)*hb14(i)*wb26(i)*wb4(i)+k1s*rhos*hb126(i)*wb26(i)*((1.-1.*qthvs)*wb1(i)+(1.-
1.*qtsvs)*wb2(i)+(1.-1.*qtrvs)*wb3(i)+(1.-1.*qtgvs)*wb4(i))))/aco-(1.*(-1.*k1s*(1.-
1.*qthvs)*hb11(i)*wb1(i)*wb26(i)+k1s*(1.-1.*qthvs)*hb18(i)*wb1(i)*wb26(i)+k1s*(1.-
1.*qtsvs)*hb112(i)*wb2(i)*wb26(i)-1.*k1s*(1.-1.*qtsvs)*hb12(i)*wb2(i)*wb26(i)+k1s*(1.-
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1.*qtrvs)*hb116(i)*wb26(i)*wb3(i)-1.*k1s*(1.-1.*qtrvs)*hb13(i)*wb26(i)*wb3(i)+k1s*(1.-
1.*qtgvs)*hb120(i)*wb26(i)*wb4(i)-1.*k1s*(1.-1.*qtgvs)*hb14(i)*wb26(i)*wb4(i)-
1.*k1s*rhos*hb126(i)*wb26(i)*((1.-1.*qthvs)*wb1(i)+(1.-1.*qtsvs)*wb2(i)+(1.-
1.*qtrvs)*wb3(i)+(1.-1.*qtgvs)*wb4(i)))*(-1.*aco*(k1s*(1.-
1.*qthvs)*hb11(i)*wb1(i)*wb26(i)-1.*k1s*(1.-1.*qthvs)*hb18(i)*wb1(i)*wb26(i)-
1.*k1s*(1.-1.*qtsvs)*hb112(i)*wb2(i)*wb26(i)+k1s*(1.-1.*qtsvs)*hb12(i)*wb2(i)*wb26(i)-
1.*k1s*(1.-1.*qtrvs)*hb116(i)*wb26(i)*wb3(i)+k1s*(1.-1.*qtrvs)*hb13(i)*wb26(i)*wb3(i)-
1.*k1s*(1.-1.*qtgvs)*hb120(i)*wb26(i)*wb4(i)+k1s*(1.-
1.*qtgvs)*hb14(i)*wb26(i)*wb4(i)+k1s*rhos*hb126(i)*wb26(i)*((1.-1.*qthvs)*wb1(i)+(1.-
1.*qtsvs)*wb2(i)+(1.-1.*qtrvs)*wb3(i)+(1.-1.*qtgvs)*wb4(i)))-1.*(-1.*k1s*(1.-
1.*qthvs)*hb11(i)*wb1(i)*wb26(i)+k1s*(1.-1.*qthvs)*hb18(i)*wb1(i)*wb26(i)+k1s*(1.-
1.*qtsvs)*hb112(i)*wb2(i)*wb26(i)-1.*k1s*(1.-1.*qtsvs)*hb12(i)*wb2(i)*wb26(i)+k1s*(1.-
1.*qtrvs)*hb116(i)*wb26(i)*wb3(i)-1.*k1s*(1.-1.*qtrvs)*hb13(i)*wb26(i)*wb3(i)+k1s*(1.-
1.*qtgvs)*hb120(i)*wb26(i)*wb4(i)-1.*k1s*(1.-1.*qtgvs)*hb14(i)*wb26(i)*wb4(i)-
1.*k1s*rhos*hb126(i)*wb26(i)*((1.-1.*qthvs)*wb1(i)+(1.-1.*qtsvs)*wb2(i)+(1.-
1.*qtrvs)*wb3(i)+(1.-1.*qtgvs)*wb4(i)))*(k1s*(1.-1.*qthvs)*hb11(i)*wb1(i)*wb26(i)-
1.*k1s*(1.-1.*qthvs)*hb18(i)*wb1(i)*wb26(i)-1.*k1s*(1.-
1.*qtsvs)*hb112(i)*wb2(i)*wb26(i)+k1s*(1.-1.*qtsvs)*hb12(i)*wb2(i)*wb26(i)-1.*k1s*(1.-
1.*qtrvs)*hb116(i)*wb26(i)*wb3(i)+k1s*(1.-1.*qtrvs)*hb13(i)*wb26(i)*wb3(i)-1.*k1s*(1.-
1.*qtgvs)*hb120(i)*wb26(i)*wb4(i)+k1s*(1.-
1.*qtgvs)*hb14(i)*wb26(i)*wb4(i)+k1s*rhos*hb126(i)*wb26(i)*((1.-1.*qthvs)*wb1(i)+(1.-
1.*qtsvs)*wb2(i)+(1.-1.*qtrvs)*wb3(i)+(1.-1.*qtgvs)*wb4(i)))))/(aco*(1.*aco*afi-1.*(-
1.*k1s*(1.-1.*qthvs)*hb11(i)*wb1(i)*wb26(i)+k1s*(1.-
1.*qthvs)*hb18(i)*wb1(i)*wb26(i)+k1s*(1.-1.*qtsvs)*hb112(i)*wb2(i)*wb26(i)-1.*k1s*(1.-
1.*qtsvs)*hb12(i)*wb2(i)*wb26(i)+k1s*(1.-1.*qtrvs)*hb116(i)*wb26(i)*wb3(i)-1.*k1s*(1.-
1.*qtrvs)*hb13(i)*wb26(i)*wb3(i)+k1s*(1.-1.*qtgvs)*hb120(i)*wb26(i)*wb4(i)-1.*k1s*(1.-
1.*qtgvs)*hb14(i)*wb26(i)*wb4(i)-1.*k1s*rhos*hb126(i)*wb26(i)*((1.-
1.*qthvs)*wb1(i)+(1.-1.*qtsvs)*wb2(i)+(1.-1.*qtrvs)*wb3(i)+(1.-1.*qtgvs)*wb4(i)))^2));
a12= -((-1.*aco*(k1s*(1.-1.*qthvs)*hb11(i)*wb1(i)*wb26(i)-1.*k1s*(1.-
1.*qthvs)*hb18(i)*wb1(i)*wb26(i)-1.*k1s*(1.-1.*qtsvs)*hb112(i)*wb2(i)*wb26(i)+k1s*(1.-
1.*qtsvs)*hb12(i)*wb2(i)*wb26(i)-1.*k1s*(1.-1.*qtrvs)*hb116(i)*wb26(i)*wb3(i)+k1s*(1.-
1.*qtrvs)*hb13(i)*wb26(i)*wb3(i)-1.*k1s*(1.-1.*qtgvs)*hb120(i)*wb26(i)*wb4(i)+k1s*(1.-
1.*qtgvs)*hb14(i)*wb26(i)*wb4(i)+k1s*rhos*hb126(i)*wb26(i)*((1.-1.*qthvs)*wb1(i)+(1.-
1.*qtsvs)*wb2(i)+(1.-1.*qtrvs)*wb3(i)+(1.-1.*qtgvs)*wb4(i)))-1.*(-1.*k1s*(1.-
1.*qthvs)*hb11(i)*wb1(i)*wb26(i)+k1s*(1.-1.*qthvs)*hb18(i)*wb1(i)*wb26(i)+k1s*(1.-
1.*qtsvs)*hb112(i)*wb2(i)*wb26(i)-1.*k1s*(1.-1.*qtsvs)*hb12(i)*wb2(i)*wb26(i)+k1s*(1.-
1.*qtrvs)*hb116(i)*wb26(i)*wb3(i)-1.*k1s*(1.-1.*qtrvs)*hb13(i)*wb26(i)*wb3(i)+k1s*(1.-
1.*qtgvs)*hb120(i)*wb26(i)*wb4(i)-1.*k1s*(1.-1.*qtgvs)*hb14(i)*wb26(i)*wb4(i)-
1.*k1s*rhos*hb126(i)*wb26(i)*((1.-1.*qthvs)*wb1(i)+(1.-1.*qtsvs)*wb2(i)+(1.-
1.*qtrvs)*wb3(i)+(1.-1.*qtgvs)*wb4(i)))*(k1s*(1.-1.*qthvs)*hb11(i)*wb1(i)*wb26(i)-
1.*k1s*(1.-1.*qthvs)*hb18(i)*wb1(i)*wb26(i)-1.*k1s*(1.-
1.*qtsvs)*hb112(i)*wb2(i)*wb26(i)+k1s*(1.-1.*qtsvs)*hb12(i)*wb2(i)*wb26(i)-1.*k1s*(1.-
1.*qtrvs)*hb116(i)*wb26(i)*wb3(i)+k1s*(1.-1.*qtrvs)*hb13(i)*wb26(i)*wb3(i)-1.*k1s*(1.-
1.*qtgvs)*hb120(i)*wb26(i)*wb4(i)+k1s*(1.-
1.*qtgvs)*hb14(i)*wb26(i)*wb4(i)+k1s*rhos*hb126(i)*wb26(i)*((1.-1.*qthvs)*wb1(i)+(1.-
1.*qtsvs)*wb2(i)+(1.-1.*qtrvs)*wb3(i)+(1.-1.*qtgvs)*wb4(i))))/(1.*aco*afi-1.*(-1.*k1s*(1.-
1.*qthvs)*hb11(i)*wb1(i)*wb26(i)+k1s*(1.-1.*qthvs)*hb18(i)*wb1(i)*wb26(i)+k1s*(1.-
1.*qtsvs)*hb112(i)*wb2(i)*wb26(i)-1.*k1s*(1.-1.*qtsvs)*hb12(i)*wb2(i)*wb26(i)+k1s*(1.-
1.*qtrvs)*hb116(i)*wb26(i)*wb3(i)-1.*k1s*(1.-1.*qtrvs)*hb13(i)*wb26(i)*wb3(i)+k1s*(1.-
1.*qtgvs)*hb120(i)*wb26(i)*wb4(i)-1.*k1s*(1.-1.*qtgvs)*hb14(i)*wb26(i)*wb4(i)-
1.*k1s*rhos*hb126(i)*wb26(i)*((1.-1.*qthvs)*wb1(i)+(1.-1.*qtsvs)*wb2(i)+(1.-
1.*qtrvs)*wb3(i)+(1.-1.*qtgvs)*wb4(i)))^2));
a13=(1.*(hb11(i)*wb1(i)+hb12(i)*wb2(i)+hb13(i)*wb3(i)+hb14(i)*wb4(i)))/aib; a14=
(1.*(k2*hb112(i)*wb12(i)-1.*k2*hb113(i)*wb12(i)+k2*hb116(i)*wb16(i)-
1.*k2*hb117(i)*wb16(i)+k2*hb120(i)*wb20(i)-1.*k2*hb121(i)*wb20(i)+k2*hb18(i)*wb8(i)-
1.*k2*hb19(i)*wb8(i)))/aii-(1.*(-1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-
1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-
1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-
1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i))*((1.*(1.*bsr*pr*hb126(i)*(wb11(i)+wb15(i)+wb
19(i)+wb23(i)+wb25(i))*(1.*brs-

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1.*pr*hb226(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))+pr*hb227(i)*(wb11(i)+wb15(
i)+wb19(i)+wb23(i)+wb25(i)))-1.*brs*(-1.*bsr*(-
1.*pr*hb126(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))-
(1.*g*hb126(i)*wb26(i))/(b2+wb26(i))-
1.*k1sm*hb124(i)*wb26(i)*wb5(i)+k1sm*rhos*hb126(i)*wb26(i)*wb5(i)+k1sm*hb15(i)*w
b26(i)*wb5(i))+1.*ps*hb126(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))*(1.*bsr+ps*hb
226(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))-
1.*ps*hb227(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))))))/(brs*bsr*(k1sm*hb124(i)*
wb26(i)*wb5(i)-1.*k1sm*rhos*hb126(i)*wb26(i)*wb5(i)-
1.*k1sm*hb15(i)*wb26(i)*wb5(i)))-
(1.*(1.*bsr*pr^2*hb126(i)*hb226(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))^2-
1.*brs*(1.*api*bsr-
1.*ps^2*hb126(i)*hb226(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))^2))*(1.*brs*bsr*(k
1sm*hb124(i)*wb26(i)*wb5(i)-1.*k1sm*rhos*hb126(i)*wb26(i)*wb5(i)-
1.*k1sm*hb15(i)*wb26(i)*wb5(i))*(-1.*aii*(k2*hb112(i)*wb12(i)-
1.*k2*hb113(i)*wb12(i)+k2*hb116(i)*wb16(i)-
1.*k2*hb117(i)*wb16(i)+k2*hb120(i)*wb20(i)-1.*k2*hb121(i)*wb20(i)-
1.*k1sm*hb124(i)*wb26(i)*wb5(i)+k1sm*rhos*hb126(i)*wb26(i)*wb5(i)+k1sm*hb15(i)*w
b26(i)*wb5(i)+k2*hb18(i)*wb8(i)-1.*k2*hb19(i)*wb8(i))-1.*(k2*hb112(i)*wb12(i)-
1.*k2*hb113(i)*wb12(i)+k2*hb116(i)*wb16(i)-
1.*k2*hb117(i)*wb16(i)+k2*hb120(i)*wb20(i)-1.*k2*hb121(i)*wb20(i)+k2*hb18(i)*wb8(i)-
1.*k2*hb19(i)*wb8(i))*(-1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-
1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-
1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i)))-
1.*(1.*aii*art-1.*(-1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-
1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-
1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-
1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i))^2)*(1.*bsr*pr*hb126(i)*(wb11(i)+wb15(i)+wb19
(i)+wb23(i)+wb25(i))*(1.*brs-
1.*pr*hb226(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))+pr*hb227(i)*(wb11(i)+wb15(
i)+wb19(i)+wb23(i)+wb25(i)))-1.*brs*(-1.*bsr*(-
1.*pr*hb126(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))-
(1.*g*hb126(i)*wb26(i))/(b2+wb26(i))-
1.*k1sm*hb124(i)*wb26(i)*wb5(i)+k1sm*rhos*hb126(i)*wb26(i)*wb5(i)+k1sm*hb15(i)*w
b26(i)*wb5(i))+1.*ps*hb126(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))*(1.*bsr+ps*hb
226(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))-
1.*ps*hb227(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i)))))))/(brs*bsr*(k1sm*hb124(i)*
wb26(i)*wb5(i)-1.*k1sm*rhos*hb126(i)*wb26(i)*wb5(i)-
1.*k1sm*hb15(i)*wb26(i)*wb5(i))*(-1.*aii*brs*bsr*(k1sm*hb124(i)*wb26(i)*wb5(i)-
1.*k1sm*rhos*hb126(i)*wb26(i)*wb5(i)-1.*k1sm*hb15(i)*wb26(i)*wb5(i))^2-
1.*(1.*aii*art-1.*(-1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-
1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-
1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-
1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i))^2)*(1.*bsr*pr^2*hb126(i)*hb226(i)*(wb11(i)+w
b15(i)+wb19(i)+wb23(i)+wb25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*hb126(i)*hb226(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))^2))))))/aii;
a15= -((1.*brs*bsr*(k1sm*hb124(i)*wb26(i)*wb5(i)-
1.*k1sm*rhos*hb126(i)*wb26(i)*wb5(i)-1.*k1sm*hb15(i)*wb26(i)*wb5(i))*(-
1.*aii*(k2*hb112(i)*wb12(i)-1.*k2*hb113(i)*wb12(i)+k2*hb116(i)*wb16(i)-
1.*k2*hb117(i)*wb16(i)+k2*hb120(i)*wb20(i)-1.*k2*hb121(i)*wb20(i)-
1.*k1sm*hb124(i)*wb26(i)*wb5(i)+k1sm*rhos*hb126(i)*wb26(i)*wb5(i)+k1sm*hb15(i)*w
b26(i)*wb5(i)+k2*hb18(i)*wb8(i)-1.*k2*hb19(i)*wb8(i))-1.*(k2*hb112(i)*wb12(i)-
1.*k2*hb113(i)*wb12(i)+k2*hb116(i)*wb16(i)-
1.*k2*hb117(i)*wb16(i)+k2*hb120(i)*wb20(i)-1.*k2*hb121(i)*wb20(i)+k2*hb18(i)*wb8(i)-
1.*k2*hb19(i)*wb8(i))*(-1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-
1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-
1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i)))-
1.*(1.*aii*art-1.*(-1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-
1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-
1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-
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1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i))^2)*(1.*bsr*pr*hb126(i)*(wb11(i)+wb15(i)+wb19
(i)+wb23(i)+wb25(i))*(1.*brs-
1.*pr*hb226(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))+pr*hb227(i)*(wb11(i)+wb15(
i)+wb19(i)+wb23(i)+wb25(i)))-1.*brs*(-1.*bsr*(-
1.*pr*hb126(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))-
(1.*g*hb126(i)*wb26(i))/(b2+wb26(i))-
1.*k1sm*hb124(i)*wb26(i)*wb5(i)+k1sm*rhos*hb126(i)*wb26(i)*wb5(i)+k1sm*hb15(i)*w
b26(i)*wb5(i))+1.*ps*hb126(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))*(1.*bsr+ps*hb
226(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))-
1.*ps*hb227(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))))))/(-
1.*aii*brs*bsr*(k1sm*hb124(i)*wb26(i)*wb5(i)-1.*k1sm*rhos*hb126(i)*wb26(i)*wb5(i)-
1.*k1sm*hb15(i)*wb26(i)*wb5(i))^2-1.*(1.*aii*art-1.*(-
1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-
1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-
1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-
1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i))^2)*(1.*bsr*pr^2*hb126(i)*hb226(i)*(wb11(i)+w
b15(i)+wb19(i)+wb23(i)+wb25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*hb126(i)*hb226(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))^2)))); a16= (-
1.*(1.*bsr*pr*hb126(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))*(1.*brs-
1.*pr*hb226(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))+pr*hb227(i)*(wb11(i)+wb15(
i)+wb19(i)+wb23(i)+wb25(i)))-1.*brs*(-1.*bsr*(-
1.*pr*hb126(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))-
(1.*g*hb126(i)*wb26(i))/(b2+wb26(i))-
1.*k1sm*hb124(i)*wb26(i)*wb5(i)+k1sm*rhos*hb126(i)*wb26(i)*wb5(i)+k1sm*hb15(i)*w
b26(i)*wb5(i))+1.*ps*hb126(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))*(1.*bsr+ps*hb
226(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))-
1.*ps*hb227(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))))))/(brs*bsr*(k1sm*hb124(i)*
wb26(i)*wb5(i)-1.*k1sm*rhos*hb126(i)*wb26(i)*wb5(i)-
1.*k1sm*hb15(i)*wb26(i)*wb5(i)))+(1.*(1.*bsr*pr^2*hb126(i)*hb226(i)*(wb11(i)+wb15(i)
+wb19(i)+wb23(i)+wb25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*hb126(i)*hb226(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))^2))*(1.*brs*bsr*(k
1sm*hb124(i)*wb26(i)*wb5(i)-1.*k1sm*rhos*hb126(i)*wb26(i)*wb5(i)-
1.*k1sm*hb15(i)*wb26(i)*wb5(i))*(-1.*aii*(k2*hb112(i)*wb12(i)-
1.*k2*hb113(i)*wb12(i)+k2*hb116(i)*wb16(i)-
1.*k2*hb117(i)*wb16(i)+k2*hb120(i)*wb20(i)-1.*k2*hb121(i)*wb20(i)-
1.*k1sm*hb124(i)*wb26(i)*wb5(i)+k1sm*rhos*hb126(i)*wb26(i)*wb5(i)+k1sm*hb15(i)*w
b26(i)*wb5(i)+k2*hb18(i)*wb8(i)-1.*k2*hb19(i)*wb8(i))-1.*(k2*hb112(i)*wb12(i)-
1.*k2*hb113(i)*wb12(i)+k2*hb116(i)*wb16(i)-
1.*k2*hb117(i)*wb16(i)+k2*hb120(i)*wb20(i)-1.*k2*hb121(i)*wb20(i)+k2*hb18(i)*wb8(i)-
1.*k2*hb19(i)*wb8(i))*(-1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-
1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-
1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i)))-
1.*(1.*aii*art-1.*(-1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-
1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-
1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-
1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i))^2)*(1.*bsr*pr*hb126(i)*(wb11(i)+wb15(i)+wb19
(i)+wb23(i)+wb25(i))*(1.*brs-
1.*pr*hb226(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))+pr*hb227(i)*(wb11(i)+wb15(
i)+wb19(i)+wb23(i)+wb25(i)))-1.*brs*(-1.*bsr*(-
1.*pr*hb126(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))-
(1.*g*hb126(i)*wb26(i))/(b2+wb26(i))-
1.*k1sm*hb124(i)*wb26(i)*wb5(i)+k1sm*rhos*hb126(i)*wb26(i)*wb5(i)+k1sm*hb15(i)*w
b26(i)*wb5(i))+1.*ps*hb126(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))*(1.*bsr+ps*hb
226(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))-
1.*ps*hb227(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i)))))))/(brs*bsr*(k1sm*hb124(i)*
wb26(i)*wb5(i)-1.*k1sm*rhos*hb126(i)*wb26(i)*wb5(i)-
1.*k1sm*hb15(i)*wb26(i)*wb5(i))*(-1.*aii*brs*bsr*(k1sm*hb124(i)*wb26(i)*wb5(i)-
1.*k1sm*rhos*hb126(i)*wb26(i)*wb5(i)-1.*k1sm*hb15(i)*wb26(i)*wb5(i))^2-
1.*(1.*aii*art-1.*(-1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-
1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-
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1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-
1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i))^2)*(1.*bsr*pr^2*hb126(i)*hb226(i)*(wb11(i)+w
b15(i)+wb19(i)+wb23(i)+wb25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*hb126(i)*hb226(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))^2)))); u21s=
min(1,max(0,(1.*(1.*bsr+ps*hb226(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))-
1.*ps*hb227(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))))/bsr+(1.*ps*hb226(i)*(wb13(i
)+wb17(i)+wb21(i)+wb24(i)+wb9(i))*(1.*brs*bsr*(k1sm*hb124(i)*wb26(i)*wb5(i)-
1.*k1sm*rhos*hb126(i)*wb26(i)*wb5(i)-1.*k1sm*hb15(i)*wb26(i)*wb5(i))*(-
1.*aii*(k2*hb112(i)*wb12(i)-1.*k2*hb113(i)*wb12(i)+k2*hb116(i)*wb16(i)-
1.*k2*hb117(i)*wb16(i)+k2*hb120(i)*wb20(i)-1.*k2*hb121(i)*wb20(i)-
1.*k1sm*hb124(i)*wb26(i)*wb5(i)+k1sm*rhos*hb126(i)*wb26(i)*wb5(i)+k1sm*hb15(i)*w
b26(i)*wb5(i)+k2*hb18(i)*wb8(i)-1.*k2*hb19(i)*wb8(i))-1.*(k2*hb112(i)*wb12(i)-
1.*k2*hb113(i)*wb12(i)+k2*hb116(i)*wb16(i)-
1.*k2*hb117(i)*wb16(i)+k2*hb120(i)*wb20(i)-1.*k2*hb121(i)*wb20(i)+k2*hb18(i)*wb8(i)-
1.*k2*hb19(i)*wb8(i))*(-1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-
1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-
1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i)))-
1.*(1.*aii*art-1.*(-1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-
1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-
1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-
1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i))^2)*(1.*bsr*pr*hb126(i)*(wb11(i)+wb15(i)+wb19
(i)+wb23(i)+wb25(i))*(1.*brs-
1.*pr*hb226(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))+pr*hb227(i)*(wb11(i)+wb15(
i)+wb19(i)+wb23(i)+wb25(i)))-1.*brs*(-1.*bsr*(-
1.*pr*hb126(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))-
(1.*g*hb126(i)*wb26(i))/(b2+wb26(i))-
1.*k1sm*hb124(i)*wb26(i)*wb5(i)+k1sm*rhos*hb126(i)*wb26(i)*wb5(i)+k1sm*hb15(i)*w
b26(i)*wb5(i))+1.*ps*hb126(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))*(1.*bsr+ps*hb
226(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))-
1.*ps*hb227(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i)))))))/(bsr*(-
1.*aii*brs*bsr*(k1sm*hb124(i)*wb26(i)*wb5(i)-1.*k1sm*rhos*hb126(i)*wb26(i)*wb5(i)-
1.*k1sm*hb15(i)*wb26(i)*wb5(i))^2-1.*(1.*aii*art-1.*(-
1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-
1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-
1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-
1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i))^2)*(1.*bsr*pr^2*hb126(i)*hb226(i)*(wb11(i)+w
b15(i)+wb19(i)+wb23(i)+wb25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*hb126(i)*hb226(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))^2)))))); u22s=
min(1,max(0,(1.*(1.*brs-
1.*pr*hb226(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))+pr*hb227(i)*(wb11(i)+wb15(
i)+wb19(i)+wb23(i)+wb25(i))))/brs-
(1.*pr*hb226(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))*(1.*brs*bsr*(k1sm*hb124(i)
*wb26(i)*wb5(i)-1.*k1sm*rhos*hb126(i)*wb26(i)*wb5(i)-
1.*k1sm*hb15(i)*wb26(i)*wb5(i))*(-1.*aii*(k2*hb112(i)*wb12(i)-
1.*k2*hb113(i)*wb12(i)+k2*hb116(i)*wb16(i)-
1.*k2*hb117(i)*wb16(i)+k2*hb120(i)*wb20(i)-1.*k2*hb121(i)*wb20(i)-
1.*k1sm*hb124(i)*wb26(i)*wb5(i)+k1sm*rhos*hb126(i)*wb26(i)*wb5(i)+k1sm*hb15(i)*w
b26(i)*wb5(i)+k2*hb18(i)*wb8(i)-1.*k2*hb19(i)*wb8(i))-1.*(k2*hb112(i)*wb12(i)-
1.*k2*hb113(i)*wb12(i)+k2*hb116(i)*wb16(i)-
1.*k2*hb117(i)*wb16(i)+k2*hb120(i)*wb20(i)-1.*k2*hb121(i)*wb20(i)+k2*hb18(i)*wb8(i)-
1.*k2*hb19(i)*wb8(i))*(-1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-
1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-
1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i)))-
1.*(1.*aii*art-1.*(-1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-
1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-
1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-
1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i))^2)*(1.*bsr*pr*hb126(i)*(wb11(i)+wb15(i)+wb19
(i)+wb23(i)+wb25(i))*(1.*brs-
1.*pr*hb226(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))+pr*hb227(i)*(wb11(i)+wb15(
i)+wb19(i)+wb23(i)+wb25(i)))-1.*brs*(-1.*bsr*(-

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1.*pr*hb126(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))-
(1.*g*hb126(i)*wb26(i))/(b2+wb26(i))-
1.*k1sm*hb124(i)*wb26(i)*wb5(i)+k1sm*rhos*hb126(i)*wb26(i)*wb5(i)+k1sm*hb15(i)*w
b26(i)*wb5(i))+1.*ps*hb126(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))*(1.*bsr+ps*hb
226(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))-
1.*ps*hb227(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i)))))))/(brs*(-
1.*aii*brs*bsr*(k1sm*hb124(i)*wb26(i)*wb5(i)-1.*k1sm*rhos*hb126(i)*wb26(i)*wb5(i)-
1.*k1sm*hb15(i)*wb26(i)*wb5(i))^2-1.*(1.*aii*art-1.*(-
1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-
1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-
1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-
1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i))^2)*(1.*bsr*pr^2*hb126(i)*hb226(i)*(wb11(i)+w
b15(i)+wb19(i)+wb23(i)+wb25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*hb126(i)*hb226(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))^2))))));

    if(a11<0 || a11>1)

        a11=0;

    end;


    if(a12<0 || a12>1)

        a12=0;

    end;


    if(a13<0 || a13>1)

        a13=0;

    end;


    if(a14<0 || a14>1)

        a14=0;

    end;


    if(a15<0 || a15>1)

        a15=0;

    end;


    if(a16<0 || a16>1)

        a16=0;

    end;
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a13,a14,a15,1;0,1,a13,a14,a15,0;0,1,a13,a14,1,a16;0,1,a13,a14,1,1;0,1,a13,a14,1,0;0,1,a13,a1
4,0,a16;0,1,a13,a14,0,1;0,1,a13,a14,0,0;0,1,a13,1,a15,a16;0,1,a13,1,a15,1;0,1,a13,1,a15,0;0,1,
a13,1,1,a16;0,1,a13,1,1,1;0,1,a13,1,1,0;0,1,a13,1,0,a16;0,1,a13,1,0,1;0,1,a13,1,0,0;0,1,a13,0,a
15,a16;0,1,a13,0,a15,1;0,1,a13,0,a15,0;0,1,a13,0,1,a16;0,1,a13,0,1,1;0,1,a13,0,1,0;0,1,a13,0,0
,a16;0,1,a13,0,0,1;0,1,a13,0,0,0;0,1,1,a14,a15,a16;0,1,1,a14,a15,1;0,1,1,a14,a15,0;0,1,1,a14,1
,a16;0,1,1,a14,1,1;0,1,1,a14,1,0;0,1,1,a14,0,a16;0,1,1,a14,0,1;0,1,1,a14,0,0;0,1,1,1,a15,a16;0,
1,1,1,a15,1;0,1,1,1,a15,0;0,1,1,1,1,a16;0,1,1,1,1,1;0,1,1,1,1,0;0,1,1,1,0,a16;0,1,1,1,0,1;0,1,1,1,
0,0;0,1,1,0,a15,a16;0,1,1,0,a15,1;0,1,1,0,a15,0;0,1,1,0,1,a16;0,1,1,0,1,1;0,1,1,0,1,0;0,1,1,0,0,a
16;0,1,1,0,0,1;0,1,1,0,0,0;0,1,0,a14,a15,a16;0,1,0,a14,a15,1;0,1,0,a14,a15,0;0,1,0,a14,1,a16;0,
1,0,a14,1,1;0,1,0,a14,1,0;0,1,0,a14,0,a16;0,1,0,a14,0,1;0,1,0,a14,0,0;0,1,0,1,a15,a16;0,1,0,1,a
15,1;0,1,0,1,a15,0;0,1,0,1,1,a16;0,1,0,1,1,1;0,1,0,1,1,0;0,1,0,1,0,a16;0,1,0,1,0,1;0,1,0,1,0,0;0,
1,0,0,a15,a16;0,1,0,0,a15,1;0,1,0,0,a15,0;0,1,0,0,1,a16;0,1,0,0,1,1;0,1,0,0,1,0;0,1,0,0,0,a16;0,
1,0,0,0,1;0,1,0,0,0,0;0,0,a13,a14,a15,a16;0,0,a13,a14,a15,1;0,0,a13,a14,a15,0;0,0,a13,a14,1,a
16;0,0,a13,a14,1,1;0,0,a13,a14,1,0;0,0,a13,a14,0,a16;0,0,a13,a14,0,1;0,0,a13,a14,0,0;0,0,a13,
1,a15,a16;0,0,a13,1,a15,1;0,0,a13,1,a15,0;0,0,a13,1,1,a16;0,0,a13,1,1,1;0,0,a13,1,1,0;0,0,a13,
1,0,a16;0,0,a13,1,0,1;0,0,a13,1,0,0;0,0,a13,0,a15,a16;0,0,a13,0,a15,1;0,0,a13,0,a15,0;0,0,a13,
0,1,a16;0,0,a13,0,1,1;0,0,a13,0,1,0;0,0,a13,0,0,a16;0,0,a13,0,0,1;0,0,a13,0,0,0;0,0,1,a14,a15,a
16;0,0,1,a14,a15,1;0,0,1,a14,a15,0;0,0,1,a14,1,a16;0,0,1,a14,1,1;0,0,1,a14,1,0;0,0,1,a14,0,a16
;0,0,1,a14,0,1;0,0,1,a14,0,0;0,0,1,1,a15,a16;0,0,1,1,a15,1;0,0,1,1,a15,0;0,0,1,1,1,a16;0,0,1,1,1
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,1;0,0,1,1,1,0;0,0,1,1,0,a16;0,0,1,1,0,1;0,0,1,1,0,0;0,0,1,0,a15,a16;0,0,1,0,a15,1;0,0,1,0,a15,0;
0,0,1,0,1,a16;0,0,1,0,1,1;0,0,1,0,1,0;0,0,1,0,0,a16;0,0,1,0,0,1;0,0,1,0,0,0;0,0,0,a14,a15,a16;0,0
,0,a14,a15,1;0,0,0,a14,a15,0;0,0,0,a14,1,a16;0,0,0,a14,1,1;0,0,0,a14,1,0;0,0,0,a14,0,a16;0,0,0,
a14,0,1;0,0,0,a14,0,0;0,0,0,1,a15,a16;0,0,0,1,a15,1;0,0,0,1,a15,0;0,0,0,1,1,a16;0,0,0,1,1,1;0,0,
0,1,1,0;0,0,0,1,0,a16;0,0,0,1,0,1;0,0,0,1,0,0;0,0,0,0,a15,a16;0,0,0,0,a15,1;0,0,0,0,a15,0;0,0,0,0
,1,a16;0,0,0,0,1,1;0,0,0,0,1,0;0,0,0,0,0,a16;0,0,0,0,0,1;0,0,0,0,0,0];

    eval_HT=eval(fHT);

    [max_HT, id_HT]=nanmax(eval_HT);

    u11(2)=D(id_HT,1);
    u12(2)=D(id_HT,2);
    u13(2)=D(id_HT,3);
    u14(2)=D(id_HT,4);
    u15(2)=D(id_HT,5);
    u16(2)=D(id_HT,6);

    u21(2)=u21s;
    u22(2)=u22s;

    %LAMBDA1(2)
    ka11(2)=-1-k*hb12(i)*wb26(i)+k1s*(1-qthvs)*rhos*hb126(i)*(1-u11(2))*(1-
u12(2))*wb26(i)-hb18(i)*(k3*wb24(i)+k1s*(1-qthvs)*(1-u11(2))*(1-
u12(2))*wb26(i))+k1r*(1-qthvr)*rhor*hb127(i)*wb27(i)-k*hb13(i)*wb27(i)-
kprimo*hb14(i)*(wb26(i)+wb27(i))-hb110(i)*(k3*wb25(i)+k1r*(1-qthvr)*wb27(i))-
c*hb16(i)*wb6(i)*(wb11(i)+wb13(i)+wb15(i)+wb17(i)+wb19(i)+wb21(i)+wb23(i)+wb9(i))-
hb11(i)*(-muth+u13(2)-(r*wb1(i))/tmax-k3*(wb24(i)+wb25(i))-k1s*(1-qthvs)*(1-u11(2))*(1-
u12(2))*wb26(i)-k1r*(1-qthvr)*wb27(i)-k*(wb26(i)+wb27(i))-
kprimo*(wb26(i)+wb27(i))+r*(1-
(wb1(i)+wb10(i)+wb11(i)+wb12(i)+wb13(i)+wb14(i)+wb15(i)+wb16(i)+wb17(i)+wb18(i)+
wb19(i)+wb2(i)+wb20(i)+wb21(i)+wb22(i)+wb23(i)+wb3(i)+wb4(i)+wb8(i)+wb9(i))/tmax))
;       ka12(2)=-1+(r*hb11(i)*wb1(i))/tmax-hb112(i)*(k3*wb24(i)+k1s*(1-qtsvs)*(1-
u11(2))*(1-u12(2))*wb26(i))-hb126(i)*(-(psprimo*wb26(i))-k1s*(1-qtsvs)*rhos*(1-
u11(2))*(1-u12(2))*wb26(i))+k1r*(1-qtsvr)*rhor*hb127(i)*wb27(i)-hb12(i)*(-muts+u13(2)-
k3*(wb24(i)+wb25(i))-k1s*(1-qtsvs)*(1-u11(2))*(1-u12(2))*wb26(i)-k1r*(1-
qtsvr)*wb27(i))-hb114(i)*(k3*wb25(i)+k1r*(1-qtsvr)*wb27(i))-
c*hb16(i)*wb6(i)*(wb11(i)+wb13(i)+wb15(i)+wb17(i)+wb19(i)+wb21(i)+wb23(i)+wb9(i));
ka13(2)=-1+(r*hb11(i)*wb1(i))/tmax+k1s*(1-qtrvs)*rhos*hb126(i)*(1-u11(2))*(1-
u12(2))*wb26(i)-hb116(i)*(k3*wb24(i)+k1s*(1-qtrvs)*(1-u11(2))*(1-u12(2))*wb26(i))-
hb13(i)*(-mutr+u13(2)-k3*(wb24(i)+wb25(i))-k1s*(1-qtrvs)*(1-u11(2))*(1-u12(2))*wb26(i)-
k1r*(1-qtrvr)*wb27(i))-hb118(i)*(k3*wb25(i)+k1r*(1-qtrvr)*wb27(i))-hb127(i)*(-
(prprimo*wb27(i))-k1r*(1-qtrvr)*rhor*wb27(i))-
c*hb16(i)*wb6(i)*(wb11(i)+wb13(i)+wb15(i)+wb17(i)+wb19(i)+wb21(i)+wb23(i)+wb9(i));
ka14(2)=-1+(r*hb11(i)*wb1(i))/tmax-hb120(i)*(k3*wb24(i)+k1s*(1-qtgvs)*(1-u11(2))*(1-
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u12(2))*wb26(i))-hb126(i)*(-(ssprimo*wb26(i))-k1s*(1-qtgvs)*rhos*(1-u11(2))*(1-u12(2))*wb26(i))-hb14(i)*(-mutg+u13(2)-k3*(wb24(i)+wb25(i))-k1s*(1-qtgvs)*(1-u11(2))*(1-u12(2))*wb26(i)-k1r*(1-qtgvr)*wb27(i))-hb122(i)*(k3*wb25(i)+k1r*(1-qtgvr)*wb27(i))-hb127(i)*(-(k1r*(1-qtgvr)*rhor*wb27(i))-srprimo*wb27(i))-c*hb16(i)*wb6(i)*(wb11(i)+wb13(i)+wb15(i)+wb17(i)+wb19(i)+wb21(i)+wb23(i)+wb9(i)); ka15(2)=-1-k1sm*hb124(i)*(1-u15(2))*(1-u16(2))*wb26(i)+k1sm*rhos*hb126(i)*(1-u15(2))*(1-u16(2))*wb26(i)-k1rm*hb125(i)*wb27(i)+k1rm*rhor*hb127(i)*wb27(i)-hb15(i)*(-mumh-k1sm*(1-u15(2))*(1-u16(2))*wb26(i)-k1rm*wb27(i)); ka16(2)=-1-c*q*hb17(i)*(wb11(i)+wb13(i)+wb15(i)+wb17(i)+wb19(i)+wb21(i)+wb23(i)+wb9(i))-hb16(i)*(-muip-c*q*(wb11(i)+wb13(i)+wb15(i)+wb17(i)+wb19(i)+wb21(i)+wb23(i)+wb9(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*(wb11(i)+wb13(i)+wb15(i)+wb17(i)+wb19(i)+wb21(i)+wb23(i)+wb9(i))); ka17(2)=-1+muie*hb17(i)+ir*hb111(i)*wb11(i)+is*hb113(i)*wb13(i)+ir*hb115(i)*wb15(i)+is*hb117(i)*wb17(i)+ir*hb119(i)*wb19(i)+is*hb121(i)*wb21(i)+ir*hb123(i)*wb23(i)+is*hb19(i)*wb9(i); ka18(2)=-(hb18(i)*(-muthsl-k2*(1-u14(2))*(1-u16(2))))-k2*hb19(i)*(1-u14(2))*(1-u16(2))+(r*hb11(i)*wb1(i))/tmax; ka19(2)=-(ps*hb127(i)*(1-u21(2)))-ps*hb126(i)*(1-u15(2))*u21(2)+(r*hb11(i)*wb1(i))/tmax-c*q*hb17(i)*wb6(i)-hb16(i)*(-(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-hb19(i)*(-muthsa-is*wb7(i)); ka110(2)=-((-k2-muthrl)*hb110(i))-k2*hb111(i)+(r*hb11(i)*wb1(i))/tmax; ka111(2)=-(pr*hb126(i)*(1-u15(2))*(1-u22(2)))-pr*hb127(i)*u22(2)+(r*hb11(i)*wb1(i))/tmax-c*q*hb17(i)*wb6(i)-hb16(i)*(-(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-hb111(i)*(-muthra-ir*wb7(i)); ka112(2)=-(hb112(i)*(-mutssl-k2*(1-u14(2))*(1-u16(2))))-k2*hb113(i)*(1-u14(2))*(1-u16(2))+(r*hb11(i)*wb1(i))/tmax; ka113(2)=-(ps*hb127(i)*(1-u21(2)))-ps*hb126(i)*(1-u15(2))*u21(2)+(r*hb11(i)*wb1(i))/tmax-c*q*hb17(i)*wb6(i)-hb16(i)*(-(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-hb113(i)*(-mutssa-is*wb7(i)); ka114(2)=-((-k2-mutsrl)*hb114(i))-k2*hb115(i)+(r*hb11(i)*wb1(i))/tmax; ka115(2)=-(pr*hb126(i)*(1-u15(2))*(1-u22(2)))-pr*hb127(i)*u22(2)+(r*hb11(i)*wb1(i))/tmax-c*q*hb17(i)*wb6(i)-hb16(i)*(-(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-hb115(i)*(-mutsra-ir*wb7(i)); ka116(2)=-(hb116(i)*(-mutrsl-k2*(1-u14(2))*(1-u16(2))))-k2*hb117(i)*(1-u14(2))*(1-u16(2))+(r*hb11(i)*wb1(i))/tmax; ka117(2)=-(ps*hb127(i)*(1-u21(2)))-ps*hb126(i)*(1-u15(2))*u21(2)+(r*hb11(i)*wb1(i))/tmax-c*q*hb17(i)*wb6(i)-hb16(i)*(-(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-hb117(i)*(-mutrsa-is*wb7(i)); ka118(2)=-((-k2-mutrrl)*hb118(i))-k2*hb119(i)+(r*hb11(i)*wb1(i))/tmax; ka119(2)=-(pr*hb126(i)*(1-u15(2))*(1-u22(2)))-pr*hb127(i)*u22(2)+(r*hb11(i)*wb1(i))/tmax-c*q*hb17(i)*wb6(i)-hb16(i)*(-(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-hb119(i)*(-mutrra-ir*wb7(i)); ka120(2)=-(hb120(i)*(-mutgsl-k2*(1-u14(2))*(1-u16(2))))-k2*hb121(i)*(1-u14(2))*(1-u16(2))+(r*hb11(i)*wb1(i))/tmax; ka121(2)=-(ps*hb127(i)*(1-u21(2)))-ps*hb126(i)*(1-u15(2))*u21(2)+(r*hb11(i)*wb1(i))/tmax-c*q*hb17(i)*wb6(i)-hb16(i)*(-(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-hb121(i)*(-mutgsa-is*wb7(i)); ka122(2)=-((-k2-mutgrl)*hb122(i))-k2*hb123(i)+(r*hb11(i)*wb1(i))/tmax; ka123(2)=-(pr*hb126(i)*(1-u15(2))*(1-u22(2)))-pr*hb127(i)*u22(2)+(r*hb11(i)*wb1(i))/tmax-c*q*hb17(i)*wb6(i)-hb16(i)*(-(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-hb123(i)*(-mutgra-ir*wb7(i)); ka124(2)=mumhsa*hb124(i)-ps*hb127(i)*(1-u21(2))-ps*hb126(i)*(1-u15(2))*u21(2)+k3*hb11(i)*wb1(i)-k3*hb18(i)*wb1(i)-k3*hb112(i)*wb2(i)+k3*hb12(i)*wb2(i)-k3*hb116(i)*wb3(i)+k3*hb13(i)*wb3(i)-k3*hb120(i)*wb4(i)+k3*hb14(i)*wb4(i); ka125(2)=mumhra*hb125(i)-pr*hb126(i)*(1-u15(2))*(1-u22(2))-pr*hb127(i)*u22(2)+k3*hb11(i)*wb1(i)-k3*hb110(i)*wb1(i)-k3*hb114(i)*wb2(i)+k3*hb12(i)*wb2(i)-k3*hb118(i)*wb3(i)+k3*hb13(i)*wb3(i)-k3*hb122(i)*wb4(i)+k3*hb14(i)*wb4(i); ka126(2)=-(k1s*(1-qthvs)*hb18(i)*(1-u11(2))*(1-u12(2))*wb1(i))-k1s*(1-qtsvs)*hb112(i)*(1-u11(2))*(1-u12(2))*wb2(i)-hb12(i)*(k*wb1(i)-k1s*(1-qtsvs)*(1-u11(2))*(1-u12(2))*wb2(i))-hb11(i)*(-(k*wb1(i))-kprimo*wb1(i)-k1s*(1-qthvs)*(1-u11(2))*(1-u12(2))*wb1(i)-(oth*s1th)/(oth+wb26(i)+wb27(i))^2)-k1s*(1-qtrvs)*hb116(i)*(1-u11(2))*(1-u12(2))*wb3(i)+k1s*(1-qtrvs)*hb13(i)*(1-u11(2))*(1-u12(2))*wb3(i)-k1s*(1-qtgvs)*hb120(i)*(1-u11(2))*(1-u12(2))*wb4(i)-hb14(i)*(kprimo*wb1(i)-k1s*(1-qtgvs)*(1-u11(2))*(1-u12(2))*wb4(i))-k1sm*hb124(i)*(1-u15(2))*(1-u16(2))*wb5(i)+k1sm*hb15(i)*(1-u15(2))*(1-u16(2))*wb5(i)-hb126(i)*(-muvs-

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psprimo*wb2(i)-(g*(1-u15(2))*wb26(i))/(b2+wb26(i))^2+(g*(1-u15(2)))/(b2+wb26(i))-ssprimo*wb4(i)-k1s*rhos*(1-u11(2))*(1-u12(2))*((1-qthvs)*wb1(i)+(1-qtsvs)*wb2(i)+(1-qtrvs)*wb3(i)+(1-qtgvs)*wb4(i))-k1sm*rhos*(1-u15(2))*(1-u16(2))*wb5(i)); ka127(2)=-(k1r*(1-qthvr)*hb110(i)*wb1(i))-k1r*(1-qtsvr)*hb114(i)*wb2(i)+k1r*(1-qtsvr)*hb12(i)*wb2(i)-hb11(i)*(-(k*wb1(i))-kprimo*wb1(i)-k1r*(1-qthvr)*wb1(i)-(oth*s1th)/(oth+wb26(i)+wb27(i))^2)-k1r*(1-qtrvr)*hb118(i)*wb3(i)-hb13(i)*(k*wb1(i)-k1r*(1-qtrvr)*wb3(i))-k1r*(1-qtgvr)*hb122(i)*wb4(i)-hb14(i)*(kprimo*wb1(i)-k1r*(1-qtgvr)*wb4(i))-k1rm*hb125(i)*wb5(i)+k1rm*hb15(i)*wb5(i)-hb127(i)*(-muvr-(g*wb27(i))/(b2+wb27(i))^2+g/(b2+wb27(i))-prprimo*wb3(i)-srprimo*wb4(i)-k1r*rhor*((1-qthvr)*wb1(i)+(1-qtsvr)*wb2(i)+(1-qtrvr)*wb3(i)+(1-qtgvr)*wb4(i))-k1rm*rhor*wb5(i));

%LAMBDA2(2)

ka21(2)=-(k*hb22(i)*wb26(i))+k1s*(1-qthvs)*rhos*hb226(i)*(1-u11(2))*(1-u12(2))*wb26(i)-hb28(i)*(k3*wb24(i)+k1s*(1-qthvs)*(1-u11(2))*(1-u12(2))*wb26(i))+k1r*(1-qthvr)*rhor*hb227(i)*wb27(i)-k*hb23(i)*wb27(i)-kprimo*hb24(i)*(wb26(i)+wb27(i))-hb210(i)*(k3*wb25(i)+k1r*(1-qthvr)*wb27(i))-c*hb26(i)*wb6(i)*(wb11(i)+wb13(i)+wb15(i)+wb17(i)+wb19(i)+wb21(i)+wb23(i)+wb9(i))-hb21(i)*(-muth+u13(2)-(r*wb1(i))/tmax-k3*(wb24(i)+wb25(i))-k1s*(1-qthvs)*(1-u11(2))*(1-u12(2))*wb26(i)-k1r*(1-qthvr)*wb27(i)-k*(wb26(i)+wb27(i))-kprimo*(wb26(i)+wb27(i))+r*(1-(wb1(i)+wb10(i)+wb11(i)+wb12(i)+wb13(i)+wb14(i)+wb15(i)+wb16(i)+wb17(i)+wb18(i)+wb19(i)+wb2(i)+wb20(i)+wb21(i)+wb22(i)+wb23(i)+wb3(i)+wb4(i)+wb8(i)+wb9(i))/tmax)); ka22(2)=(r*hb21(i)*wb1(i))/tmax-hb212(i)*(k3*wb24(i)+k1s*(1-qtsvs)*(1-u11(2))*(1-u12(2))*wb26(i))-hb226(i)*(-(psprimo*wb26(i))-k1s*(1-qtsvs)*rhos*(1-u11(2))*(1-u12(2))*wb26(i))+k1r*(1-qtsvr)*rhor*hb227(i)*wb27(i)-hb22(i)*(-muts+u13(2)-k3*(wb24(i)+wb25(i))-k1s*(1-qtsvs)*(1-u11(2))*(1-u12(2))*wb26(i)-k1r*(1-qtsvr)*wb27(i))-hb214(i)*(k3*wb25(i)+k1r*(1-qtsvr)*wb27(i))-c*hb26(i)*wb6(i)*(wb11(i)+wb13(i)+wb15(i)+wb17(i)+wb19(i)+wb21(i)+wb23(i)+wb9(i)); ka23(2)=(r*hb21(i)*wb1(i))/tmax+k1s*(1-qtrvs)*rhos*hb226(i)*(1-u11(2))*(1-u12(2))*wb26(i)-hb216(i)*(k3*wb24(i)+k1s*(1-qtrvs)*(1-u11(2))*(1-u12(2))*wb26(i))-hb23(i)*(-mutr+u13(2)-k3*(wb24(i)+wb25(i))-k1s*(1-qtrvs)*(1-u11(2))*(1-u12(2))*wb26(i)-k1r*(1-qtrvr)*wb27(i))-hb218(i)*(k3*wb25(i)+k1r*(1-qtrvr)*wb27(i))-hb227(i)*(-(prprimo*wb27(i))-k1r*(1-qtrvr)*rhor*wb27(i))-c*hb26(i)*wb6(i)*(wb11(i)+wb13(i)+wb15(i)+wb17(i)+wb19(i)+wb21(i)+wb23(i)+wb9(i)); ka24(2)=(r*hb21(i)*wb1(i))/tmax-hb220(i)*(k3*wb24(i)+k1s*(1-qtgvs)*(1-u11(2))*(1-u12(2))*wb26(i))-hb226(i)*(-(ssprimo*wb26(i))-k1s*(1-qtgvs)*rhos*(1-u11(2))*(1-u12(2))*wb26(i))-hb24(i)*(-mutg+u13(2)-k3*(wb24(i)+wb25(i))-k1s*(1-qtgvs)*(1-u11(2))*(1-u12(2))*wb26(i)-k1r*(1-qtgvr)*wb27(i))-hb222(i)*(k3*wb25(i)+k1r*(1-qtgvr)*wb27(i))-hb227(i)*(-(k1r*(1-qtgvr)*rhor*wb27(i))-srprimo*wb27(i))-c*hb26(i)*wb6(i)*(wb11(i)+wb13(i)+wb15(i)+wb17(i)+wb19(i)+wb21(i)+wb23(i)+wb9(i)); ka25(2)=-(k1sm*hb224(i)*(1-u15(2))*(1-u16(2))*wb26(i))+k1sm*rhos*hb226(i)*(1-u15(2))*(1-u16(2))*wb26(i)-k1rm*hb225(i)*wb27(i)+k1rm*rhor*hb227(i)*wb27(i)-hb25(i)*(-mumh-k1sm*(1-u15(2))*(1-u16(2))*wb26(i)-k1rm*wb27(i)); ka26(2)=-(c*q*hb27(i)*(wb11(i)+wb13(i)+wb15(i)+wb17(i)+wb19(i)+wb21(i)+wb23(i)+wb9(i)))-hb26(i)*(-muip-c*q*(wb11(i)+wb13(i)+wb15(i)+wb17(i)+wb19(i)+wb21(i)+wb23(i)+wb9(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*(wb11(i)+wb13(i)+wb15(i)+wb17(i)+wb19(i)+wb21(i)+wb23(i)+wb9(i)));
ka27(2)=muie*hb27(i)+ir*hb211(i)*wb11(i)+is*hb213(i)*wb13(i)+ir*hb215(i)*wb15(i)+is*hb217(i)*wb17(i)+ir*hb219(i)*wb19(i)+is*hb221(i)*wb21(i)+ir*hb223(i)*wb23(i)+is*hb29(i)*wb9(i); ka28(2)=-1-hb28(i)*(-muthsl-k2*(1-u14(2))*(1-u16(2)))-k2*hb29(i)*(1-u14(2))*(1-u16(2))+(r*hb21(i)*wb1(i))/tmax; ka29(2)= -1-ps*hb227(i)*(1-u21(2))-ps*hb226(i)*(1-u15(2))*u21(2)+(r*hb21(i)*wb1(i))/tmax-c*q*hb27(i)*wb6(i)-hb26(i)*(-(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-hb29(i)*(-muthsa-is*wb7(i));
ka210(2)=-1-(-k2-muthrl)*hb210(i)-k2*hb211(i)+(r*hb21(i)*wb1(i))/tmax; ka211(2)= -
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1-pr*hb226(i)*(1-u15(2))*(1-u22(2))-pr*hb227(i)*u22(2)+(r*hb21(i)*wb1(i))/tmax-c*q*hb27(i)*wb6(i)-hb26(i)*(-(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-hb211(i)*(-muthra-ir*wb7(i)); ka212(2)=-1-hb212(i)*(-mutssl-k2*(1-u14(2))*(1-u16(2)))-k2*hb213(i)*(1-u14(2))*(1-u16(2))+(r*hb21(i)*wb1(i))/tmax; ka213(2)=-1-ps*hb227(i)*(1-u21(2))-ps*hb226(i)*(1-u15(2))*u21(2)+(r*hb21(i)*wb1(i))/tmax-c*q*hb27(i)*wb6(i)-hb26(i)*(-(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-hb213(i)*(-mutssa-is*wb7(i)); ka214(2)=-1-(-k2-mutsrl)*hb214(i)-k2*hb215(i)+(r*hb21(i)*wb1(i))/tmax; ka215(2)=-1-pr*hb226(i)*(1-u15(2))*(1-u22(2))-pr*hb227(i)*u22(2)+(r*hb21(i)*wb1(i))/tmax-c*q*hb27(i)*wb6(i)-hb26(i)*(-(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-hb215(i)*(-mutsra-ir*wb7(i)); ka216(2)=-1-hb216(i)*(-mutrsl-k2*(1-u14(2))*(1-u16(2)))-k2*hb217(i)*(1-u14(2))*(1-u16(2))+(r*hb21(i)*wb1(i))/tmax; ka217(2)=-1-ps*hb227(i)*(1-u21(2))-ps*hb226(i)*(1-u15(2))*u21(2)+(r*hb21(i)*wb1(i))/tmax-c*q*hb27(i)*wb6(i)-hb26(i)*(-(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-hb217(i)*(-mutrsa-is*wb7(i)); ka218(2)=-1-(-k2-mutrrl)*hb218(i)-k2*hb219(i)+(r*hb21(i)*wb1(i))/tmax; ka219(2)=-1-pr*hb226(i)*(1-u15(2))*(1-u22(2))-pr*hb227(i)*u22(2)+(r*hb21(i)*wb1(i))/tmax-c*q*hb27(i)*wb6(i)-hb26(i)*(-(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-hb219(i)*(-mutrra-ir*wb7(i)); ka220(2)=-1-hb220(i)*(-mutgsl-k2*(1-u14(2))*(1-u16(2)))-k2*hb221(i)*(1-u14(2))*(1-u16(2))+(r*hb21(i)*wb1(i))/tmax; ka221(2)=-1-ps*hb227(i)*(1-u21(2))-ps*hb226(i)*(1-u15(2))*u21(2)+(r*hb21(i)*wb1(i))/tmax-c*q*hb27(i)*wb6(i)-hb26(i)*(-(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-hb221(i)*(-mutgsa-is*wb7(i)); ka222(2)=-1-(-k2-mutgrl)*hb222(i)-k2*hb223(i)+(r*hb21(i)*wb1(i))/tmax; ka223(2)=-1-pr*hb226(i)*(1-u15(2))*(1-u22(2))-pr*hb227(i)*u22(2)+(r*hb21(i)*wb1(i))/tmax-c*q*hb27(i)*wb6(i)-hb26(i)*(-(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-hb223(i)*(-mutgra-ir*wb7(i)); ka224(2)=mumhsa*hb224(i)-ps*hb227(i)*(1-u21(2))-ps*hb226(i)*(1-u15(2))*u21(2)+k3*hb21(i)*wb1(i)-k3*hb28(i)*wb1(i)-k3*hb212(i)*wb2(i)+k3*hb22(i)*wb2(i)-k3*hb216(i)*wb3(i)+k3*hb23(i)*wb3(i)-k3*hb220(i)*wb4(i)+k3*hb24(i)*wb4(i); ka225(2)=mumhra*hb225(i)-pr*hb226(i)*(1-u15(2))*(1-u22(2))-pr*hb227(i)*u22(2)+k3*hb21(i)*wb1(i)-k3*hb210(i)*wb1(i)-k3*hb214(i)*wb2(i)+k3*hb22(i)*wb2(i)-k3*hb218(i)*wb3(i)+k3*hb23(i)*wb3(i)-k3*hb222(i)*wb4(i)+k3*hb24(i)*wb4(i); ka226(2)=-1-k1s*(1-qthvs)*hb28(i)*(1-u11(2))*(1-u12(2))*wb1(i)-k1s*(1-qtsvs)*hb212(i)*(1-u11(2))*(1-u12(2))*wb2(i)-hb22(i)*(k*wb1(i)-k1s*(1-qtsvs)*(1-u11(2))*(1-u12(2))*wb2(i))-hb21(i)*(-(k*wb1(i))-kprimo*wb1(i)-k1s*(1-qthvs)*(1-u11(2))*(1-u12(2))*wb1(i)-(oth*s1th)/(oth+wb26(i)+wb27(i))^2)-k1s*(1-qtrvs)*hb216(i)*(1-u11(2))*(1-u12(2))*wb3(i)+k1s*(1-qtrvs)*hb23(i)*(1-u11(2))*(1-u12(2))*wb3(i)-k1s*(1-qtgvs)*hb220(i)*(1-u11(2))*(1-u12(2))*wb4(i)-hb24(i)*(kprimo*wb1(i)-k1s*(1-qtgvs)*(1-u11(2))*(1-u12(2))*wb4(i))-k1sm*hb224(i)*(1-u15(2))*(1-u16(2))*wb5(i)+k1sm*hb25(i)*(1-u15(2))*(1-u16(2))*wb5(i)-hb226(i)*(-muvs-psprimo*wb2(i)-(g*(1-u15(2))*wb26(i))/(b2+wb26(i))^2+(g*(1-u15(2)))/(b2+wb26(i))-ssprimo*wb4(i)-k1s*rhos*(1-u11(2))*(1-u12(2))*((1-qthvs)*wb1(i)+(1-qtsvs)*wb2(i)+(1-qtrvs)*wb3(i)+(1-qtgvs)*wb4(i))-k1sm*rhos*(1-u15(2))*(1-u16(2))*wb5(i)); ka227(2)=-1-k1r*(1-qthvr)*hb210(i)*wb1(i)-k1r*(1-qtsvr)*hb214(i)*wb2(i)+k1r*(1-qtsvr)*hb22(i)*wb2(i)-hb21(i)*(-(k*wb1(i))-kprimo*wb1(i)-k1r*(1-qthvr)*wb1(i)-(oth*s1th)/(oth+wb26(i)+wb27(i))^2)-k1r*(1-qtrvr)*hb218(i)*wb3(i)-hb23(i)*(k*wb1(i)-k1r*(1-qtrvr)*wb3(i))-k1r*(1-qtgvr)*hb222(i)*wb4(i)-hb24(i)*(kprimo*wb1(i)-k1r*(1-qtgvr)*wb4(i))-k1rm*hb225(i)*wb5(i)+k1rm*hb25(i)*wb5(i)-hb227(i)*(-muvr-(g*wb27(i))/(b2+wb27(i))^2+g/(b2+wb27(i))-prprimo*wb3(i)-srprimo*wb4(i)-k1r*rhor*((1-qthvr)*wb1(i)+(1-qtsvr)*wb2(i)+(1-qtrvr)*wb3(i)+(1-qtgvr)*wb4(i))-k1rm*rhor*wb5(i));

%VARIABILI DI APPOGGIO per 3° ordine

wb1(i)=0.5*(x1(i)+x1(i-1));

wb2(i)=0.5*(x2(i)+x2(i-1));

wb3(i)=0.5*(x3(i)+x3(i-1));

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wb4(i)=0.5*(x4(i)+x4(i-1));

wb5(i)=0.5*(x5(i)+x5(i-1));

wb6(i)=0.5*(x6(i)+x6(i-1));

wb7(i)=0.5*(x7(i)+x7(i-1));

wb8(i)=0.5*(x8(i)+x8(i-1));

wb9(i)=0.5*(x9(i)+x9(i-1));

wb10(i)=0.5*(x10(i)+x10(i-1));

wb11(i)=0.5*(x11(i)+x11(i-1));

wb12(i)=0.5*(x12(i)+x12(i-1));

wb13(i)=0.5*(x13(i)+x13(i-1));

wb14(i)=0.5*(x14(i)+x14(i-1));

wb15(i)=0.5*(x15(i)+x15(i-1));

wb16(i)=0.5*(x16(i)+x16(i-1));

wb17(i)=0.5*(x17(i)+x17(i-1));

wb18(i)=0.5*(x18(i)+x18(i-1));

wb19(i)=0.5*(x19(i)+x19(i-1));

wb20(i)=0.5*(x20(i)+x20(i-1));

wb21(i)=0.5*(x21(i)+x21(i-1));

wb22(i)=0.5*(x22(i)+x22(i-1));

wb23(i)=0.5*(x23(i)+x23(i-1));

wb24(i)=0.5*(x24(i)+x24(i-1));

wb25(i)=0.5*(x25(i)+x25(i-1));

wb26(i)=0.5*(x26(i)+x26(i-1));

wb27(i)=0.5*(x27(i)+x27(i-1));



%VARIABILI DI APPOGGIO hb per 3° ordine

hb11(i)=l11(i)-h2*ka11(2);

hb12(i)=l12(i)-h2*ka12(2);

hb13(i)=l13(i)-h2*ka13(2);

hb14(i)=l14(i)-h2*ka14(2);

hb15(i)=l15(i)-h2*ka15(2);

hb16(i)=l16(i)-h2*ka16(2);
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hb17(i)=l17(i)-h2*ka17(2);
hb18(i)=l18(i)-h2*ka18(2);
hb19(i)=l19(i)-h2*ka19(2);
hb110(i)=l110(i)-h2*ka110(2);
hb111(i)=l111(i)-h2*ka111(2);
hb112(i)=l112(i)-h2*ka112(2);
hb113(i)=l113(i)-h2*ka113(2);
hb114(i)=l114(i)-h2*ka114(2);
hb115(i)=l115(i)-h2*ka115(2);
hb116(i)=l116(i)-h2*ka116(2);
hb117(i)=l117(i)-h2*ka117(2);
hb118(i)=l118(i)-h2*ka118(2);
hb119(i)=l119(i)-h2*ka119(2);
hb120(i)=l120(i)-h2*ka120(2);
hb121(i)=l121(i)-h2*ka121(2);
hb122(i)=l122(i)-h2*ka122(2);
hb123(i)=l123(i)-h2*ka123(2);
hb124(i)=l124(i)-h2*ka124(2);
hb125(i)=l125(i)-h2*ka125(2);
hb126(i)=l126(i)-h2*ka126(2);
hb127(i)=l127(i)-h2*ka127(2);


hb21(i)=l21(i)-h2*ka21(2);
hb22(i)=l22(i)-h2*ka22(2);
hb23(i)=l23(i)-h2*ka23(2);
hb24(i)=l24(i)-h2*ka24(2);
hb25(i)=l25(i)-h2*ka25(2);
hb26(i)=l26(i)-h2*ka26(2);
hb27(i)=l27(i)-h2*ka27(2);
hb28(i)=l28(i)-h2*ka28(2);
hb29(i)=l29(i)-h2*ka29(2);
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hb210(i)=l210(i)-h2*ka210(2);

hb211(i)=l211(i)-h2*ka211(2);

hb212(i)=l212(i)-h2*ka212(2);

hb213(i)=l213(i)-h2*ka213(2);

hb214(i)=l214(i)-h2*ka214(2);

hb215(i)=l215(i)-h2*ka215(2);

hb216(i)=l216(i)-h2*ka216(2);

hb217(i)=l217(i)-h2*ka217(2);

hb218(i)=l218(i)-h2*ka218(2);

hb219(i)=l219(i)-h2*ka219(2);

hb220(i)=l220(i)-h2*ka220(2);

hb221(i)=l221(i)-h2*ka221(2);

hb222(i)=l222(i)-h2*ka222(2);

hb223(i)=l223(i)-h2*ka223(2);

hb224(i)=l224(i)-h2*ka224(2);

hb225(i)=l225(i)-h2*ka225(2);

hb226(i)=l226(i)-h2*ka226(2);

hb227(i)=l227(i)-h2*ka227(2);



%CONTROLLI COEEFICIENTI DI RUNGE KUTTA 3° ORDINE

a11= (1.*(k1s*(1.-1.*qthvs)*hb11(i)*wb1(i)*wb26(i)-1.*k1s*(1.-
1.*qthvs)*hb18(i)*wb1(i)*wb26(i)-1.*k1s*(1.-1.*qtsvs)*hb112(i)*wb2(i)*wb26(i)+k1s*(1.-
1.*qtsvs)*hb12(i)*wb2(i)*wb26(i)-1.*k1s*(1.-1.*qtrvs)*hb116(i)*wb26(i)*wb3(i)+k1s*(1.-
1.*qtrvs)*hb13(i)*wb26(i)*wb3(i)-1.*k1s*(1.-1.*qtgvs)*hb120(i)*wb26(i)*wb4(i)+k1s*(1.-
1.*qtgvs)*hb14(i)*wb26(i)*wb4(i)+k1s*rhos*hb126(i)*wb26(i)*((1.-1.*qthvs)*wb1(i)+(1.-
1.*qtsvs)*wb2(i)+(1.-1.*qtrvs)*wb3(i)+(1.-1.*qtgvs)*wb4(i))))/aco-(1.*(-1.*k1s*(1.-
1.*qthvs)*hb11(i)*wb1(i)*wb26(i)+k1s*(1.-1.*qthvs)*hb18(i)*wb1(i)*wb26(i)+k1s*(1.-
1.*qtsvs)*hb112(i)*wb2(i)*wb26(i)-1.*k1s*(1.-1.*qtsvs)*hb12(i)*wb2(i)*wb26(i)+k1s*(1.-
1.*qtrvs)*hb116(i)*wb26(i)*wb3(i)-1.*k1s*(1.-1.*qtrvs)*hb13(i)*wb26(i)*wb3(i)+k1s*(1.-
1.*qtgvs)*hb120(i)*wb26(i)*wb4(i)-1.*k1s*(1.-1.*qtgvs)*hb14(i)*wb26(i)*wb4(i)-
1.*k1s*rhos*hb126(i)*wb26(i)*((1.-1.*qthvs)*wb1(i)+(1.-1.*qtsvs)*wb2(i)+(1.-
1.*qtrvs)*wb3(i)+(1.-1.*qtgvs)*wb4(i)))*(-1.*aco*(k1s*(1.-
1.*qthvs)*hb11(i)*wb1(i)*wb26(i)-1.*k1s*(1.-1.*qthvs)*hb18(i)*wb1(i)*wb26(i)-
1.*k1s*(1.-1.*qtsvs)*hb112(i)*wb2(i)*wb26(i)+k1s*(1.-1.*qtsvs)*hb12(i)*wb2(i)*wb26(i)-
1.*k1s*(1.-1.*qtrvs)*hb116(i)*wb26(i)*wb3(i)+k1s*(1.-1.*qtrvs)*hb13(i)*wb26(i)*wb3(i)-
1.*k1s*(1.-1.*qtgvs)*hb120(i)*wb26(i)*wb4(i)+k1s*(1.-
1.*qtgvs)*hb14(i)*wb26(i)*wb4(i)+k1s*rhos*hb126(i)*wb26(i)*((1.-1.*qthvs)*wb1(i)+(1.-
1.*qtsvs)*wb2(i)+(1.-1.*qtrvs)*wb3(i)+(1.-1.*qtgvs)*wb4(i)))-1.*(-1.*k1s*(1.-
1.*qthvs)*hb11(i)*wb1(i)*wb26(i)+k1s*(1.-1.*qthvs)*hb18(i)*wb1(i)*wb26(i)+k1s*(1.-
1.*qtsvs)*hb112(i)*wb2(i)*wb26(i)-1.*k1s*(1.-1.*qtsvs)*hb12(i)*wb2(i)*wb26(i)+k1s*(1.-
1.*qtrvs)*hb116(i)*wb26(i)*wb3(i)-1.*k1s*(1.-1.*qtrvs)*hb13(i)*wb26(i)*wb3(i)+k1s*(1.-
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1.*qtgvs)*hb120(i)*wb26(i)*wb4(i)-1.*k1s*(1.-1.*qtgvs)*hb14(i)*wb26(i)*wb4(i)-
1.*k1s*rhos*hb126(i)*wb26(i)*((1.-1.*qthvs)*wb1(i)+(1.-1.*qtsvs)*wb2(i)+(1.-
1.*qtrvs)*wb3(i)+(1.-1.*qtgvs)*wb4(i)))*(k1s*(1.-1.*qthvs)*hb11(i)*wb1(i)*wb26(i)-
1.*k1s*(1.-1.*qthvs)*hb18(i)*wb1(i)*wb26(i)-1.*k1s*(1.-
1.*qtsvs)*hb112(i)*wb2(i)*wb26(i)+k1s*(1.-1.*qtsvs)*hb12(i)*wb2(i)*wb26(i)-1.*k1s*(1.-
1.*qtrvs)*hb116(i)*wb26(i)*wb3(i)+k1s*(1.-1.*qtrvs)*hb13(i)*wb26(i)*wb3(i)-1.*k1s*(1.-
1.*qtgvs)*hb120(i)*wb26(i)*wb4(i)+k1s*(1.-
1.*qtgvs)*hb14(i)*wb26(i)*wb4(i)+k1s*rhos*hb126(i)*wb26(i)*((1.-1.*qthvs)*wb1(i)+(1.-
1.*qtsvs)*wb2(i)+(1.-1.*qtrvs)*wb3(i)+(1.-1.*qtgvs)*wb4(i)))))/(aco*(1.*aco*afi-1.*(-
1.*k1s*(1.-1.*qthvs)*hb11(i)*wb1(i)*wb26(i)+k1s*(1.-
1.*qthvs)*hb18(i)*wb1(i)*wb26(i)+k1s*(1.-1.*qtsvs)*hb112(i)*wb2(i)*wb26(i)-1.*k1s*(1.-
1.*qtsvs)*hb12(i)*wb2(i)*wb26(i)+k1s*(1.-1.*qtrvs)*hb116(i)*wb26(i)*wb3(i)-1.*k1s*(1.-
1.*qtrvs)*hb13(i)*wb26(i)*wb3(i)+k1s*(1.-1.*qtgvs)*hb120(i)*wb26(i)*wb4(i)-1.*k1s*(1.-
1.*qtgvs)*hb14(i)*wb26(i)*wb4(i)-1.*k1s*rhos*hb126(i)*wb26(i)*((1.-
1.*qthvs)*wb1(i)+(1.-1.*qtsvs)*wb2(i)+(1.-1.*qtrvs)*wb3(i)+(1.-1.*qtgvs)*wb4(i)))^2));
a12=-((-1.*aco*(k1s*(1.-1.*qthvs)*hb11(i)*wb1(i)*wb26(i)-1.*k1s*(1.-
1.*qthvs)*hb18(i)*wb1(i)*wb26(i)-1.*k1s*(1.-1.*qtsvs)*hb112(i)*wb2(i)*wb26(i)+k1s*(1.-
1.*qtsvs)*hb12(i)*wb2(i)*wb26(i)-1.*k1s*(1.-1.*qtrvs)*hb116(i)*wb26(i)*wb3(i)+k1s*(1.-
1.*qtrvs)*hb13(i)*wb26(i)*wb3(i)-1.*k1s*(1.-1.*qtgvs)*hb120(i)*wb26(i)*wb4(i)+k1s*(1.-
1.*qtgvs)*hb14(i)*wb26(i)*wb4(i)+k1s*rhos*hb126(i)*wb26(i)*((1.-1.*qthvs)*wb1(i)+(1.-
1.*qtsvs)*wb2(i)+(1.-1.*qtrvs)*wb3(i)+(1.-1.*qtgvs)*wb4(i)))-1.*(-1.*k1s*(1.-
1.*qthvs)*hb11(i)*wb1(i)*wb26(i)+k1s*(1.-1.*qthvs)*hb18(i)*wb1(i)*wb26(i)+k1s*(1.-
1.*qtsvs)*hb112(i)*wb2(i)*wb26(i)-1.*k1s*(1.-1.*qtsvs)*hb12(i)*wb2(i)*wb26(i)+k1s*(1.-
1.*qtrvs)*hb116(i)*wb26(i)*wb3(i)-1.*k1s*(1.-1.*qtrvs)*hb13(i)*wb26(i)*wb3(i)+k1s*(1.-
1.*qtgvs)*hb120(i)*wb26(i)*wb4(i)-1.*k1s*(1.-1.*qtgvs)*hb14(i)*wb26(i)*wb4(i)-
1.*k1s*rhos*hb126(i)*wb26(i)*((1.-1.*qthvs)*wb1(i)+(1.-1.*qtsvs)*wb2(i)+(1.-
1.*qtrvs)*wb3(i)+(1.-1.*qtgvs)*wb4(i)))*(k1s*(1.-1.*qthvs)*hb11(i)*wb1(i)*wb26(i)-
1.*k1s*(1.-1.*qthvs)*hb18(i)*wb1(i)*wb26(i)-1.*k1s*(1.-
1.*qtsvs)*hb112(i)*wb2(i)*wb26(i)+k1s*(1.-1.*qtsvs)*hb12(i)*wb2(i)*wb26(i)-1.*k1s*(1.-
1.*qtrvs)*hb116(i)*wb26(i)*wb3(i)+k1s*(1.-1.*qtrvs)*hb13(i)*wb26(i)*wb3(i)-1.*k1s*(1.-
1.*qtgvs)*hb120(i)*wb26(i)*wb4(i)+k1s*(1.-
1.*qtgvs)*hb14(i)*wb26(i)*wb4(i)+k1s*rhos*hb126(i)*wb26(i)*((1.-1.*qthvs)*wb1(i)+(1.-
1.*qtsvs)*wb2(i)+(1.-1.*qtrvs)*wb3(i)+(1.-1.*qtgvs)*wb4(i))))/(1.*aco*afi-1.*(-1.*k1s*(1.-
1.*qthvs)*hb11(i)*wb1(i)*wb26(i)+k1s*(1.-1.*qthvs)*hb18(i)*wb1(i)*wb26(i)+k1s*(1.-
1.*qtsvs)*hb112(i)*wb2(i)*wb26(i)-1.*k1s*(1.-1.*qtsvs)*hb12(i)*wb2(i)*wb26(i)+k1s*(1.-
1.*qtrvs)*hb116(i)*wb26(i)*wb3(i)-1.*k1s*(1.-1.*qtrvs)*hb13(i)*wb26(i)*wb3(i)+k1s*(1.-
1.*qtgvs)*hb120(i)*wb26(i)*wb4(i)-1.*k1s*(1.-1.*qtgvs)*hb14(i)*wb26(i)*wb4(i)-
1.*k1s*rhos*hb126(i)*wb26(i)*((1.-1.*qthvs)*wb1(i)+(1.-1.*qtsvs)*wb2(i)+(1.-
1.*qtrvs)*wb3(i)+(1.-1.*qtgvs)*wb4(i)))^2));        a13=
(1.*(hb11(i)*wb1(i)+hb12(i)*wb2(i)+hb13(i)*wb3(i)+hb14(i)*wb4(i)))/aib;        a14=
(1.*(k2*hb112(i)*wb12(i)-1.*k2*hb113(i)*wb12(i)+k2*hb116(i)*wb16(i)-
1.*k2*hb117(i)*wb16(i)+k2*hb120(i)*wb20(i)-1.*k2*hb121(i)*wb20(i)+k2*hb18(i)*wb8(i)-
1.*k2*hb19(i)*wb8(i)))/aii-(1.*(-1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-
1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-
1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-
1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i))*((1.*(1.*bsr*pr*hb126(i)*(wb11(i)+wb15(i)+wb
19(i)+wb23(i)+wb25(i))*(1.*brs-
1.*pr*hb226(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))+pr*hb227(i)*(wb11(i)+wb15(
i)+wb19(i)+wb23(i)+wb25(i)))-1.*brs*(-1.*bsr*(-
1.*pr*hb126(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))-
(1.*g*hb126(i)*wb26(i))/(b2+wb26(i))-
1.*k1sm*hb124(i)*wb26(i)*wb5(i)+k1sm*rhos*hb126(i)*wb26(i)*wb5(i)+k1sm*hb15(i)*w
b26(i)*wb5(i))+1.*ps*hb126(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))*(1.*bsr+ps*hb
226(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))-
1.*ps*hb227(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))))))/(brs*bsr*(k1sm*hb124(i)*
wb26(i)*wb5(i)-1.*k1sm*rhos*hb126(i)*wb26(i)*wb5(i)-
1.*k1sm*hb15(i)*wb26(i)*wb5(i)))-
(1.*(1.*bsr*pr^2*hb126(i)*hb226(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))^2-
1.*brs*(1.*api*bsr-
1.*ps^2*hb126(i)*hb226(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))^2))*(1.*brs*bsr*(k
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1sm*hb124(i)*wb26(i)*wb5(i)-1.*k1sm*rhos*hb126(i)*wb26(i)*wb5(i)-
1.*k1sm*hb15(i)*wb26(i)*wb5(i))*(-1.*aii*(k2*hb112(i)*wb12(i)-
1.*k2*hb113(i)*wb12(i)+k2*hb116(i)*wb16(i)-
1.*k2*hb117(i)*wb16(i)+k2*hb120(i)*wb20(i)-1.*k2*hb121(i)*wb20(i)-
1.*k1sm*hb124(i)*wb26(i)*wb5(i)+k1sm*rhos*hb126(i)*wb26(i)*wb5(i)+k1sm*hb15(i)*w
b26(i)*wb5(i)+k2*hb18(i)*wb8(i)-1.*k2*hb19(i)*wb8(i))-1.*(k2*hb112(i)*wb12(i)-
1.*k2*hb113(i)*wb12(i)+k2*hb116(i)*wb16(i)-
1.*k2*hb117(i)*wb16(i)+k2*hb120(i)*wb20(i)-1.*k2*hb121(i)*wb20(i)+k2*hb18(i)*wb8(i)-
1.*k2*hb19(i)*wb8(i))*(-1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-
1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-
1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i)))-
1.*(1.*aii*art-1.*(-1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-
1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-
1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-
1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i))^2)*(1.*bsr*pr*hb126(i)*(wb11(i)+wb15(i)+wb19
(i)+wb23(i)+wb25(i))*(1.*brs-
1.*pr*hb226(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))+pr*hb227(i)*(wb11(i)+wb15(
i)+wb19(i)+wb23(i)+wb25(i)))-1.*brs*(-1.*bsr*(-
1.*pr*hb126(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))-
(1.*g*hb126(i)*wb26(i))/(b2+wb26(i))-
1.*k1sm*hb124(i)*wb26(i)*wb5(i)+k1sm*rhos*hb126(i)*wb26(i)*wb5(i)+k1sm*hb15(i)*w
b26(i)*wb5(i))+1.*ps*hb126(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))*(1.*bsr+ps*hb
226(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))-
1.*ps*hb227(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i)))))))/(brs*bsr*(k1sm*hb124(i)*
wb26(i)*wb5(i)-1.*k1sm*rhos*hb126(i)*wb26(i)*wb5(i)-
1.*k1sm*hb15(i)*wb26(i)*wb5(i))*(-1.*aii*brs*bsr*(k1sm*hb124(i)*wb26(i)*wb5(i)-
1.*k1sm*rhos*hb126(i)*wb26(i)*wb5(i)-1.*k1sm*hb15(i)*wb26(i)*wb5(i))^2-
1.*(1.*aii*art-1.*(-1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-
1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-
1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-
1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i))^2)*(1.*bsr*pr^2*hb126(i)*hb226(i)*(wb11(i)+w
b15(i)+wb19(i)+wb23(i)+wb25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*hb126(i)*hb226(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))^2))))))/aii;
a15= -((1.*brs*bsr*(k1sm*hb124(i)*wb26(i)*wb5(i)-
1.*k1sm*rhos*hb126(i)*wb26(i)*wb5(i)-1.*k1sm*hb15(i)*wb26(i)*wb5(i))*(-
1.*aii*(k2*hb112(i)*wb12(i)-1.*k2*hb113(i)*wb12(i)+k2*hb116(i)*wb16(i)-
1.*k2*hb117(i)*wb16(i)+k2*hb120(i)*wb20(i)-1.*k2*hb121(i)*wb20(i)-
1.*k1sm*hb124(i)*wb26(i)*wb5(i)+k1sm*rhos*hb126(i)*wb26(i)*wb5(i)+k1sm*hb15(i)*w
b26(i)*wb5(i)+k2*hb18(i)*wb8(i)-1.*k2*hb19(i)*wb8(i))-1.*(k2*hb112(i)*wb12(i)-
1.*k2*hb113(i)*wb12(i)+k2*hb116(i)*wb16(i)-
1.*k2*hb117(i)*wb16(i)+k2*hb120(i)*wb20(i)-1.*k2*hb121(i)*wb20(i)+k2*hb18(i)*wb8(i)-
1.*k2*hb19(i)*wb8(i))*(-1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-
1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-
1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i)))-
1.*(1.*aii*art-1.*(-1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-
1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-
1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-
1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i))^2)*(1.*bsr*pr*hb126(i)*(wb11(i)+wb15(i)+wb19
(i)+wb23(i)+wb25(i))*(1.*brs-
1.*pr*hb226(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))+pr*hb227(i)*(wb11(i)+wb15(
i)+wb19(i)+wb23(i)+wb25(i)))-1.*brs*(-1.*bsr*(-
1.*pr*hb126(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))-
(1.*g*hb126(i)*wb26(i))/(b2+wb26(i))-
1.*k1sm*hb124(i)*wb26(i)*wb5(i)+k1sm*rhos*hb126(i)*wb26(i)*wb5(i)+k1sm*hb15(i)*w
b26(i)*wb5(i))+1.*ps*hb126(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))*(1.*bsr+ps*hb
226(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))-
1.*ps*hb227(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))))))/(-
1.*aii*brs*bsr*(k1sm*hb124(i)*wb26(i)*wb5(i)-1.*k1sm*rhos*hb126(i)*wb26(i)*wb5(i)-
1.*k1sm*hb15(i)*wb26(i)*wb5(i))^2-1.*(1.*aii*art-1.*(-
1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-
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1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-
1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-
1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i))^2)*(1.*bsr*pr^2*hb126(i)*hb226(i)*(wb11(i)+w
b15(i)+wb19(i)+wb23(i)+wb25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*hb126(i)*hb226(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))^2))));        a16=(-
1.*(1.*bsr*pr*hb126(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))*(1.*brs-
1.*pr*hb226(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))+pr*hb227(i)*(wb11(i)+wb15(
i)+wb19(i)+wb23(i)+wb25(i)))-1.*brs*(-1.*bsr*(-
1.*pr*hb126(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))-
(1.*g*hb126(i)*wb26(i))/(b2+wb26(i))-
1.*k1sm*hb124(i)*wb26(i)*wb5(i)+k1sm*rhos*hb126(i)*wb26(i)*wb5(i)+k1sm*hb15(i)*w
b26(i)*wb5(i))+1.*ps*hb126(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))*(1.*bsr+ps*hb
226(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))-
1.*ps*hb227(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))))))/(brs*bsr*(k1sm*hb124(i)*
wb26(i)*wb5(i)-1.*k1sm*rhos*hb126(i)*wb26(i)*wb5(i)-
1.*k1sm*hb15(i)*wb26(i)*wb5(i)))+(1.*(1.*bsr*pr^2*hb126(i)*hb226(i)*(wb11(i)+wb15(i)
+wb19(i)+wb23(i)+wb25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*hb126(i)*hb226(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))^2))*(1.*brs*bsr*(k
1sm*hb124(i)*wb26(i)*wb5(i)-1.*k1sm*rhos*hb126(i)*wb26(i)*wb5(i)-
1.*k1sm*hb15(i)*wb26(i)*wb5(i))*(-1.*aii*(k2*hb112(i)*wb12(i)-
1.*k2*hb113(i)*wb12(i)+k2*hb116(i)*wb16(i)-
1.*k2*hb117(i)*wb16(i)+k2*hb120(i)*wb20(i)-1.*k2*hb121(i)*wb20(i)-
1.*k1sm*hb124(i)*wb26(i)*wb5(i)+k1sm*rhos*hb126(i)*wb26(i)*wb5(i)+k1sm*hb15(i)*w
b26(i)*wb5(i)+k2*hb18(i)*wb8(i)-1.*k2*hb19(i)*wb8(i))-1.*(k2*hb112(i)*wb12(i)-
1.*k2*hb113(i)*wb12(i)+k2*hb116(i)*wb16(i)-
1.*k2*hb117(i)*wb16(i)+k2*hb120(i)*wb20(i)-1.*k2*hb121(i)*wb20(i)+k2*hb18(i)*wb8(i)-
1.*k2*hb19(i)*wb8(i))*(-1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-
1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-
1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i)))-
1.*(1.*aii*art-1.*(-1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-
1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-
1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-
1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i))^2)*(1.*bsr*pr*hb126(i)*(wb11(i)+wb15(i)+wb19
(i)+wb23(i)+wb25(i))*(1.*brs-
1.*pr*hb226(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))+pr*hb227(i)*(wb11(i)+wb15(
i)+wb19(i)+wb23(i)+wb25(i)))-1.*brs*(-1.*bsr*(-
1.*pr*hb126(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))-
(1.*g*hb126(i)*wb26(i))/(b2+wb26(i))-
1.*k1sm*hb124(i)*wb26(i)*wb5(i)+k1sm*rhos*hb126(i)*wb26(i)*wb5(i)+k1sm*hb15(i)*w
b26(i)*wb5(i))+1.*ps*hb126(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))*(1.*bsr+ps*hb
226(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))-
1.*ps*hb227(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i)))))))/(brs*bsr*(k1sm*hb124(i)*
wb26(i)*wb5(i)-1.*k1sm*rhos*hb126(i)*wb26(i)*wb5(i)-
1.*k1sm*hb15(i)*wb26(i)*wb5(i))*(-1.*aii*brs*bsr*(k1sm*hb124(i)*wb26(i)*wb5(i)-
1.*k1sm*rhos*hb126(i)*wb26(i)*wb5(i)-1.*k1sm*hb15(i)*wb26(i)*wb5(i))^2-
1.*(1.*aii*art-1.*(-1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-
1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-
1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-
1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i))^2)*(1.*bsr*pr^2*hb126(i)*hb226(i)*(wb11(i)+w
b15(i)+wb19(i)+wb23(i)+wb25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*hb126(i)*hb226(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))^2))));        u21s=
min(1,max(0,(1.*(1.*bsr+ps*hb226(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))-
1.*ps*hb227(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))))/bsr+(1.*ps*hb226(i)*(wb13(i
)+wb17(i)+wb21(i)+wb24(i)+wb9(i))*(1.*brs*bsr*(k1sm*hb124(i)*wb26(i)*wb5(i)-
1.*k1sm*rhos*hb126(i)*wb26(i)*wb5(i)-1.*k1sm*hb15(i)*wb26(i)*wb5(i))*(-
1.*aii*(k2*hb112(i)*wb12(i)-1.*k2*hb113(i)*wb12(i)+k2*hb116(i)*wb16(i)-
1.*k2*hb117(i)*wb16(i)+k2*hb120(i)*wb20(i)-1.*k2*hb121(i)*wb20(i)-
1.*k1sm*hb124(i)*wb26(i)*wb5(i)+k1sm*rhos*hb126(i)*wb26(i)*wb5(i)+k1sm*hb15(i)*w
b26(i)*wb5(i)+k2*hb18(i)*wb8(i)-1.*k2*hb19(i)*wb8(i))-1.*(k2*hb112(i)*wb12(i)-
1.*k2*hb113(i)*wb12(i)+k2*hb116(i)*wb16(i)-
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1.*k2*hb117(i)*wb16(i)+k2*hb120(i)*wb20(i)-1.*k2*hb121(i)*wb20(i)+k2*hb18(i)*wb8(i)-1.*k2*hb19(i)*wb8(i))*(-1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i)))-1.*(1.*aii*art-1.*(-1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i))^2)*(1.*bsr*pr*hb126(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))*(1.*brs-1.*pr*hb226(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))+pr*hb227(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i)))-1.*brs*(-1.*bsr*(-1.*pr*hb126(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))-(1.*g*hb126(i)*wb26(i))/(b2+wb26(i))-1.*k1sm*hb124(i)*wb26(i)*wb5(i)+k1sm*rhos*hb126(i)*wb26(i)*wb5(i)+k1sm*hb15(i)*wb26(i)*wb5(i))+1.*ps*hb126(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))*(1.*bsr+ps*hb226(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))-1.*ps*hb227(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i)))))))/(bsr*(-1.*aii*brs*bsr*(k1sm*hb124(i)*wb26(i)*wb5(i)-1.*k1sm*rhos*hb126(i)*wb26(i)*wb5(i)-1.*k1sm*hb15(i)*wb26(i)*wb5(i))^2-1.*(1.*aii*art-1.*(-1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i))^2)*(1.*bsr*pr^2*hb126(i)*hb226(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))^2-1.*brs*(1.*api*bsr-1.*ps^2*hb126(i)*hb226(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))^2))))));

u22s= min(1,max(0,(1.*(1.*brs-1.*pr*hb226(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))+pr*hb227(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))))/brs-(1.*pr*hb226(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))*(1.*brs*bsr*(k1sm*hb124(i)*wb26(i)*wb5(i)-1.*k1sm*rhos*hb126(i)*wb26(i)*wb5(i)-1.*k1sm*hb15(i)*wb26(i)*wb5(i))*(-1.*aii*(k2*hb112(i)*wb12(i)-1.*k2*hb113(i)*wb12(i)+k2*hb116(i)*wb16(i)-1.*k2*hb117(i)*wb16(i)+k2*hb120(i)*wb20(i)-1.*k2*hb121(i)*wb20(i)-1.*k1sm*hb124(i)*wb26(i)*wb5(i)+k1sm*rhos*hb126(i)*wb26(i)*wb5(i)+k1sm*hb15(i)*wb26(i)*wb5(i)+k2*hb18(i)*wb8(i)-1.*k2*hb19(i)*wb8(i))-1.*(k2*hb112(i)*wb12(i)-1.*k2*hb113(i)*wb12(i)+k2*hb116(i)*wb16(i)-1.*k2*hb117(i)*wb16(i)+k2*hb120(i)*wb20(i)-1.*k2*hb121(i)*wb20(i)+k2*hb18(i)*wb8(i)-1.*k2*hb19(i)*wb8(i))*(-1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i)))-1.*(1.*aii*art-1.*(-1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i))^2)*(1.*bsr*pr*hb126(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))*(1.*brs-1.*pr*hb226(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))+pr*hb227(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i)))-1.*brs*(-1.*bsr*(-1.*pr*hb126(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))-(1.*g*hb126(i)*wb26(i))/(b2+wb26(i))-1.*k1sm*hb124(i)*wb26(i)*wb5(i)+k1sm*rhos*hb126(i)*wb26(i)*wb5(i)+k1sm*hb15(i)*wb26(i)*wb5(i))+1.*ps*hb126(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))*(1.*bsr+ps*hb226(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))-1.*ps*hb227(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i)))))))/(brs*(-1.*aii*brs*bsr*(k1sm*hb124(i)*wb26(i)*wb5(i)-1.*k1sm*rhos*hb126(i)*wb26(i)*wb5(i)-1.*k1sm*hb15(i)*wb26(i)*wb5(i))^2-1.*(1.*aii*art-1.*(-1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i))^2)*(1.*bsr*pr^2*hb126(i)*hb226(i)*(wb11(i)+w

```
b15(i)+wb19(i)+wb23(i)+wb25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*hb126(i)*hb226(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))^2))))));

    if(a11<0 || a11>1)
        a11=0;
    end;

    if(a12<0 || a12>1)
        a12=0;
    end;

    if(a13<0 || a13>1)
        a13=0;
    end;

    if(a14<0 || a14>1)
        a14=0;
    end;

    if(a15<0 || a15>1)
        a15=0;
    end;

    if(a16<0 || a16>1)
        a16=0;
    end;

        D=[a11,a12,a13,a14,a15,a16;a11,a12,a13,a14,a15,1;a11,a12,a13,a14,a15,0;a11,a12,
a13,a14,1,a16;a11,a12,a13,a14,1,1;a11,a12,a13,a14,1,0;a11,a12,a13,a14,0,a16;a11,a12,a13,a1
4,0,1;a11,a12,a13,a14,0,0;a11,a12,a13,1,a15,a16;a11,a12,a13,1,a15,1;a11,a12,a13,1,a15,0;a1
1,a12,a13,1,1,a16;a11,a12,a13,1,1,1;a11,a12,a13,1,1,0;a11,a12,a13,1,0,a16;a11,a12,a13,1,0,1;
a11,a12,a13,1,0,0;a11,a12,a13,0,a15,a16;a11,a12,a13,0,a15,1;a11,a12,a13,0,a15,0;a11,a12,a1
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a13,0,0,0;a11,a12,1,a14,a15,a16;a11,a12,1,a14,a15,1;a11,a12,1,a14,a15,0;a11,a12,1,a14,1,a1
6;a11,a12,1,a14,1,1;a11,a12,1,a14,1,0;a11,a12,1,a14,0,a16;a11,a12,1,a14,0,1;a11,a12,1,a14,0,
0;a11,a12,1,1,a15,a16;a11,a12,1,1,a15,1;a11,a12,1,1,a15,0;a11,a12,1,1,1,a16;a11,a12,1,1,1,1;
a11,a12,1,1,1,0;a11,a12,1,1,0,a16;a11,a12,1,1,0,1;a11,a12,1,1,0,0;a11,a12,1,0,a15,a16;a11,a1
2,1,0,a15,1;a11,a12,1,0,a15,0;a11,a12,1,0,1,a16;a11,a12,1,0,1,1;a11,a12,1,0,1,0;a11,a12,1,0,0
,a16;a11,a12,1,0,0,1;a11,a12,1,0,0,0;a11,a12,0,a14,a15,a16;a11,a12,0,a14,a15,1;a11,a12,0,a1
4,a15,0;a11,a12,0,a14,1,a16;a11,a12,0,a14,1,1;a11,a12,0,a14,1,0;a11,a12,0,a14,0,a16;a11,a12
```

,0,a14,0,1;a11,a12,0,a14,0,0;a11,a12,0,1,a15,a16;a11,a12,0,1,a15,1;a11,a12,0,1,a15,0;a11,a12
,0,1,1,a16;a11,a12,0,1,1,1;a11,a12,0,1,1,0;a11,a12,0,1,0,a16;a11,a12,0,1,0,1;a11,a12,0,1,0,0;a
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11,1,a13,a14,0,a16;a11,1,a13,a14,0,1;a11,1,a13,a14,0,0;a11,1,a13,1,a15,a16;a11,1,a13,1,a15,
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1,a13,1,0,1;a11,1,a13,1,0,0;a11,1,a13,0,a15,a16;a11,1,a13,0,a15,1;a11,1,a13,0,a15,0;a11,1,a1
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11,1,1,a14,a15,a16;a11,1,1,a14,a15,1;a11,1,1,a14,a15,0;a11,1,1,a14,1,a16;a11,1,1,a14,1,1;a1
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a11,1,1,0,1,0;a11,1,1,0,0,a16;a11,1,1,0,0,1;a11,1,1,0,0,0;a11,1,0,a14,a15,a16;a11,1,0,a14,a15,
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,a16;1,1,1,a14,0,1;1,1,1,a14,0,0;1,1,1,1,a15,a16;1,1,1,1,a15,1;1,1,1,1,a15,0;1,1,1,1,1,a16;1,1,1
,1,1,1;1,1,1,1,1,0;1,1,1,1,0,a16;1,1,1,1,0,1;1,1,1,1,0,0;1,1,1,0,a15,a16;1,1,1,0,a15,1;1,1,1,0,a1
5,0;1,1,1,0,1,a16;1,1,1,0,1,1;1,1,1,0,1,0;1,1,1,0,0,a16;1,1,1,0,0,1;1,1,1,0,0,0;1,1,0,a14,a15,a16
;1,1,0,a14,a15,1;1,1,0,a14,a15,0;1,1,0,a14,1,a16;1,1,0,a14,1,1;1,1,0,a14,1,0;1,1,0,a14,0,a16;1,
1,0,a14,0,1;1,1,0,a14,0,0;1,1,0,1,a15,a16;1,1,0,1,a15,1;1,1,0,1,a15,0;1,1,0,1,1,a16;1,1,0,1,1,1;
1,1,0,1,1,0;1,1,0,1,0,a16;1,1,0,1,0,1;1,1,0,1,0,0;1,1,0,0,a15,a16;1,1,0,0,a15,1;1,1,0,0,a15,0;1,1
,0,0,1,a16;1,1,0,0,1,1;1,1,0,0,1,0;1,1,0,0,0,a16;1,1,0,0,0,1;1,1,0,0,0,0;1,0,a13,a14,a15,a16;1,0,

```
a13,a14,a15,1;1,0,a13,a14,a15,0;1,0,a13,a14,1,a16;1,0,a13,a14,1,1;1,0,a13,a14,1,0;1,0,a13,a1
4,0,a16;1,0,a13,a14,0,1;1,0,a13,a14,0,0;1,0,a13,1,a15,a16;1,0,a13,1,a15,1;1,0,a13,1,a15,0;1,0,
a13,1,1,a16;1,0,a13,1,1,1;1,0,a13,1,1,0;1,0,a13,1,0,a16;1,0,a13,1,0,1;1,0,a13,1,0,0;1,0,a13,0,a
15,a16;1,0,a13,0,a15,1;1,0,a13,0,a15,0;1,0,a13,0,1,a16;1,0,a13,0,1,1;1,0,a13,0,1,0;1,0,a13,0,0
,a16;1,0,a13,0,0,1;1,0,a13,0,0,0;1,0,1,a14,a15,a16;1,0,1,a14,a15,1;1,0,1,a14,a15,0;1,0,1,a14,1
,a16;1,0,1,a14,1,1;1,0,1,a14,1,0;1,0,1,a14,0,a16;1,0,1,a14,0,1;1,0,1,a14,0,0;1,0,1,1,a15,a16;1,
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0,0,0,a15,a16;1,0,0,0,a15,1;1,0,0,0,a15,0;1,0,0,0,1,a16;1,0,0,0,1,1;1,0,0,0,1,0;1,0,0,0,0,a16;1,
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a13,a14,0,0;0,a12,a13,1,a15,a16;0,a12,a13,1,a15,1;0,a12,a13,1,a15,0;0,a12,a13,1,1,a16;0,a12,
a13,1,1,1;0,a12,a13,1,1,0;0,a12,a13,1,0,a16;0,a12,a13,1,0,1;0,a12,a13,1,0,0;0,a12,a13,0,a15,a
16;0,a12,a13,0,a15,1;0,a12,a13,0,a15,0;0,a12,a13,0,1,a16;0,a12,a13,0,1,1;0,a12,a13,0,1,0;0,a
12,a13,0,0,a16;0,a12,a13,0,0,1;0,a12,a13,0,0,0;0,a12,1,a14,a15,a16;0,a12,1,a14,a15,1;0,a12,1
,a14,a15,0;0,a12,1,a14,1,a16;0,a12,1,a14,1,1;0,a12,1,a14,1,0;0,a12,1,a14,0,a16;0,a12,1,a14,0,
1;0,a12,1,a14,0,0;0,a12,1,1,a15,a16;0,a12,1,1,a15,1;0,a12,1,1,a15,0;0,a12,1,1,1,a16;0,a12,1,1,
1,1;0,a12,1,1,1,0;0,a12,1,1,0,a16;0,a12,1,1,0,1;0,a12,1,1,0,0;0,a12,1,0,a15,a16;0,a12,1,0,a15,
1;0,a12,1,0,a15,0;0,a12,1,0,1,a16;0,a12,1,0,1,1;0,a12,1,0,1,0;0,a12,1,0,0,a16;0,a12,1,0,0,1;0,a
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,a14,1,1;0,a12,0,a14,1,0;0,a12,0,a14,0,a16;0,a12,0,a14,0,1;0,a12,0,a14,0,0;0,a12,0,1,a15,a16;
0,a12,0,1,a15,1;0,a12,0,1,a15,0;0,a12,0,1,1,a16;0,a12,0,1,1,1;0,a12,0,1,1,0;0,a12,0,1,0,a16;0,
a12,0,1,0,1;0,a12,0,1,0,0;0,a12,0,0,a15,a16;0,a12,0,0,a15,1;0,a12,0,0,a15,0;0,a12,0,0,1,a16;0,
a12,0,0,1,1;0,a12,0,0,1,0;0,a12,0,0,0,a16;0,a12,0,0,0,1;0,a12,0,0,0,0;0,1,a13,a14,a15,a16;0,1,
a13,a14,a15,1;0,1,a13,a14,a15,0;0,1,a13,a14,1,a16;0,1,a13,a14,1,1;0,1,a13,a14,1,0;0,1,a13,a1
4,0,a16;0,1,a13,a14,0,1;0,1,a13,a14,0,0;0,1,a13,1,a15,a16;0,1,a13,1,a15,1;0,1,a13,1,a15,0;0,1,
a13,1,1,a16;0,1,a13,1,1,1;0,1,a13,1,1,0;0,1,a13,1,0,a16;0,1,a13,1,0,1;0,1,a13,1,0,0;0,1,a13,0,a
15,a16;0,1,a13,0,a15,1;0,1,a13,0,a15,0;0,1,a13,0,1,a16;0,1,a13,0,1,1;0,1,a13,0,1,0;0,1,a13,0,0
,a16;0,1,a13,0,0,1;0,1,a13,0,0,0;0,1,1,a14,a15,a16;0,1,1,a14,a15,1;0,1,1,a14,a15,0;0,1,1,a14,1
,a16;0,1,1,a14,1,1;0,1,1,a14,1,0;0,1,1,a14,0,a16;0,1,1,a14,0,1;0,1,1,a14,0,0;0,1,1,1,a15,a16;0,
1,1,1,a15,1;0,1,1,1,a15,0;0,1,1,1,1,a16;0,1,1,1,1,1;0,1,1,1,1,0;0,1,1,1,0,a16;0,1,1,1,0,1;0,1,1,1,
0,0;0,1,1,0,a15,a16;0,1,1,0,a15,1;0,1,1,0,a15,0;0,1,1,0,1,a16;0,1,1,0,1,1;0,1,1,0,1,0;0,1,1,0,0,a
16;0,1,1,0,0,1;0,1,1,0,0,0;0,1,0,a14,a15,a16;0,1,0,a14,a15,1;0,1,0,a14,a15,0;0,1,0,a14,1,a16;0,
1,0,a14,1,1;0,1,0,a14,1,0;0,1,0,a14,0,a16;0,1,0,a14,0,1;0,1,0,a14,0,0;0,1,0,1,a15,a16;0,1,0,1,a
15,1;0,1,0,1,a15,0;0,1,0,1,1,a16;0,1,0,1,1,1;0,1,0,1,1,0;0,1,0,1,0,a16;0,1,0,1,0,1;0,1,0,1,0,0;0,
1,0,0,a15,a16;0,1,0,0,a15,1;0,1,0,0,a15,0;0,1,0,0,1,a16;0,1,0,0,1,1;0,1,0,0,1,0;0,1,0,0,0,a16;0,
1,0,0,0,1;0,1,0,0,0,0;0,0,a13,a14,a15,a16;0,0,a13,a14,a15,1;0,0,a13,a14,a15,0;0,0,a13,a14,1,a
16;0,0,a13,a14,1,1;0,0,a13,a14,1,0;0,0,a13,a14,0,a16;0,0,a13,a14,0,1;0,0,a13,a14,0,0;0,0,a13,
1,a15,a16;0,0,a13,1,a15,1;0,0,a13,1,a15,0;0,0,a13,1,1,a16;0,0,a13,1,1,1;0,0,a13,1,1,0;0,0,a13,
1,0,a16;0,0,a13,1,0,1;0,0,a13,1,0,0;0,0,a13,0,a15,a16;0,0,a13,0,a15,1;0,0,a13,0,a15,0;0,0,a13,
0,1,a16;0,0,a13,0,1,1;0,0,a13,0,1,0;0,0,a13,0,0,a16;0,0,a13,0,0,1;0,0,a13,0,0,0;0,0,1,a14,a15,a
16;0,0,1,a14,a15,1;0,0,1,a14,a15,0;0,0,1,a14,1,a16;0,0,1,a14,1,1;0,0,1,a14,1,0;0,0,1,a14,0,a16
;0,0,1,a14,0,1;0,0,1,a14,0,0;0,0,1,1,a15,a16;0,0,1,1,a15,1;0,0,1,1,a15,0;0,0,1,1,1,a16;0,0,1,1,1
,1;0,0,1,1,1,0;0,0,1,1,0,a16;0,0,1,1,0,1;0,0,1,1,0,0;0,0,1,0,a15,a16;0,0,1,0,a15,1;0,0,1,0,a15,0;
0,0,1,0,1,a16;0,0,1,0,1,1;0,0,1,0,1,0;0,0,1,0,0,a16;0,0,1,0,0,1;0,0,1,0,0,0;0,0,0,a14,a15,a16;0,0
,0,a14,a15,1;0,0,0,a14,a15,0;0,0,0,a14,1,a16;0,0,0,a14,1,1;0,0,0,a14,1,0;0,0,0,a14,0,a16;0,0,0,
a14,0,1;0,0,0,a14,0,0;0,0,0,1,a15,a16;0,0,0,1,a15,1;0,0,0,1,a15,0;0,0,0,1,1,a16;0,0,0,1,1,1;0,0,
0,1,1,0;0,0,0,1,0,a16;0,0,0,1,0,1;0,0,0,1,0,0;0,0,0,0,a15,a16;0,0,0,0,a15,1;0,0,0,0,a15,0;0,0,0,0
,1,a16;0,0,0,0,1,1;0,0,0,0,1,0;0,0,0,0,0,a16;0,0,0,0,0,1;0,0,0,0,0,0];
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```
eval_HT=eval(fHT);
```

```
[max_HT, id_HT]=nanmax(eval_HT);

u11(3)=D(id_HT,1);
u12(3)=D(id_HT,2);
u13(3)=D(id_HT,3);
u14(3)=D(id_HT,4);
u15(3)=D(id_HT,5);
u16(3)=D(id_HT,6);

u21(3)=u21s;
u22(3)=u22s;

%LAMBDA1(3)
ka11(3)=-1-k*hb12(i)*wb26(i)+k1s*(1-qthvs)*rhos*hb126(i)*(1-u11(3))*(1-u12(3))*wb26(i)-hb18(i)*(k3*wb24(i)+k1s*(1-qthvs)*(1-u11(3))*(1-u12(3))*wb26(i))+k1r*(1-qthvr)*rhor*hb127(i)*wb27(i)-k*hb13(i)*wb27(i)-kprimo*hb14(i)*(wb26(i)+wb27(i))-hb110(i)*(k3*wb25(i)+k1r*(1-qthvr)*wb27(i))-c*hb16(i)*wb6(i)*(wb11(i)+wb13(i)+wb15(i)+wb17(i)+wb19(i)+wb21(i)+wb23(i)+wb9(i))-hb11(i)*(-muth+u13(3)-(r*wb1(i))/tmax-k3*(wb24(i)+wb25(i))-k1s*(1-qthvs)*(1-u11(3))*(1-u12(3))*wb26(i)-k1r*(1-qthvr)*wb27(i)-k*(wb26(i)+wb27(i))-kprimo*(wb26(i)+wb27(i))+r*(1-(wb1(i)+wb10(i)+wb11(i)+wb12(i)+wb13(i)+wb14(i)+wb15(i)+wb16(i)+wb17(i)+wb18(i)+wb19(i)+wb2(i)+wb20(i)+wb21(i)+wb22(i)+wb23(i)+wb3(i)+wb4(i)+wb8(i)+wb9(i))/tmax));	ka12(3)=-1+(r*hb11(i)*wb1(i))/tmax-hb112(i)*(k3*wb24(i)+k1s*(1-qtsvs)*(1-u11(3))*(1-u12(3))*wb26(i))-hb126(i)*(-(psprimo*wb26(i))-k1s*(1-qtsvs)*rhos*(1-u11(3))*(1-u12(3))*wb26(i))+k1r*(1-qtsvr)*rhor*hb127(i)*wb27(i)-hb12(i)*(-muts+u13(3)-k3*(wb24(i)+wb25(i))-k1s*(1-qtsvs)*(1-u11(3))*(1-u12(3))*wb26(i)-k1r*(1-qtsvr)*wb27(i))-hb114(i)*(k3*wb25(i)+k1r*(1-qtsvr)*wb27(i))-c*hb16(i)*wb6(i)*(wb11(i)+wb13(i)+wb15(i)+wb17(i)+wb19(i)+wb21(i)+wb23(i)+wb9(i));
ka13(3)=-1+(r*hb11(i)*wb1(i))/tmax+k1s*(1-qtrvs)*rhos*hb126(i)*(1-u11(3))*(1-u12(3))*wb26(i)-hb116(i)*(k3*wb24(i)+k1s*(1-qtrvs)*(1-u11(3))*(1-u12(3))*wb26(i))-hb13(i)*(-mutr+u13(3)-k3*(wb24(i)+wb25(i))-k1s*(1-qtrvs)*(1-u11(3))*(1-u12(3))*wb26(i)-k1r*(1-qtrvr)*wb27(i))-hb118(i)*(k3*wb25(i)+k1r*(1-qtrvr)*wb27(i))-hb127(i)*(-(prprimo*wb27(i))-k1r*(1-qtrvr)*rhor*wb27(i))-c*hb16(i)*wb6(i)*(wb11(i)+wb13(i)+wb15(i)+wb17(i)+wb19(i)+wb21(i)+wb23(i)+wb9(i));
ka14(3)=-1+(r*hb11(i)*wb1(i))/tmax-hb120(i)*(k3*wb24(i)+k1s*(1-qtgvs)*(1-u11(3))*(1-u12(3))*wb26(i))-hb126(i)*(-(ssprimo*wb26(i))-k1s*(1-qtgvs)*rhos*(1-u11(3))*(1-u12(3))*wb26(i))-hb14(i)*(-mutg+u13(3)-k3*(wb24(i)+wb25(i))-k1s*(1-qtgvs)*(1-u11(3))*(1-u12(3))*wb26(i)-k1r*(1-qtgvr)*wb27(i))-hb122(i)*(k3*wb25(i)+k1r*(1-qtgvr)*wb27(i))-hb127(i)*(-(k1r*(1-qtgvr)*rhor*wb27(i))-srprimo*wb27(i))-c*hb16(i)*wb6(i)*(wb11(i)+wb13(i)+wb15(i)+wb17(i)+wb19(i)+wb21(i)+wb23(i)+wb9(i));
ka15(3)=-1-k1sm*hb124(i)*(1-u15(3))*(1-u16(3))*wb26(i)+k1sm*rhos*hb126(i)*(1-u15(3))*(1-u16(3))*wb26(i)-k1rm*hb125(i)*wb27(i)+k1rm*rhor*hb127(i)*wb27(i)-hb15(i)*(-mumh-k1sm*(1-u15(3))*(1-u16(3))*wb26(i)-k1rm*wb27(i));	ka16(3)=-1-c*q*hb17(i)*(wb11(i)+wb13(i)+wb15(i)+wb17(i)+wb19(i)+wb21(i)+wb23(i)+wb9(i))-hb16(i)*(-muip-c*q*(wb11(i)+wb13(i)+wb15(i)+wb17(i)+wb19(i)+wb21(i)+wb23(i)+wb9(i))+c*(wb1(i)+wb
```

2(i)+wb3(i)+wb4(i))*(wb11(i)+wb13(i)+wb15(i)+wb17(i)+wb19(i)+wb21(i)+wb23(i)+wb9(i))); ka17(3)=-1+muie*hb17(i)+ir*hb111(i)*wb11(i)+is*hb113(i)*wb13(i)+ir*hb115(i)*wb15(i)+is*hb117(i)*wb17(i)+ir*hb119(i)*wb19(i)+is*hb121(i)*wb21(i)+ir*hb123(i)*wb23(i)+is*hb19(i)*wb9(i); ka18(3)=-(hb18(i)*(-muthsl-k2*(1-u14(3))*(1-u16(3))))-k2*hb19(i)*(1-u14(3))*(1-u16(3))+(r*hb11(i)*wb1(i))/tmax; ka19(3)=-(ps*hb127(i)*(1-u21(3)))-ps*hb126(i)*(1-u15(3))*u21(3)+(r*hb11(i)*wb1(i))/tmax-c*q*hb17(i)*wb6(i)-hb16(i)*(-(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-hb19(i)*(-muthsa-is*wb7(i)); ka110(3)=-((-k2-muthrl)*hb110(i))-k2*hb111(i)+(r*hb11(i)*wb1(i))/tmax; ka111(3)=-(pr*hb126(i)*(1-u15(3))*(1-u22(3)))-pr*hb127(i)*u22(3)+(r*hb11(i)*wb1(i))/tmax-c*q*hb17(i)*wb6(i)-hb16(i)*(-(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-hb111(i)*(-muthra-ir*wb7(i)); ka112(3)=-(hb112(i)*(-mutssl-k2*(1-u14(3))*(1-u16(3))))-k2*hb113(i)*(1-u14(3))*(1-u16(3))+(r*hb11(i)*wb1(i))/tmax; ka113(3)=-(ps*hb127(i)*(1-u21(3)))-ps*hb126(i)*(1-u15(3))*u21(3)+(r*hb11(i)*wb1(i))/tmax-c*q*hb17(i)*wb6(i)-hb16(i)*(-(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-hb113(i)*(-mutssa-is*wb7(i)); ka114(3)=-((-k2-mutsrl)*hb114(i))-k2*hb115(i)+(r*hb11(i)*wb1(i))/tmax; ka115(3)=-(pr*hb126(i)*(1-u15(3))*(1-u22(3)))-pr*hb127(i)*u22(3)+(r*hb11(i)*wb1(i))/tmax-c*q*hb17(i)*wb6(i)-hb16(i)*(-(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-hb115(i)*(-mutsra-ir*wb7(i)); ka116(3)=-(hb116(i)*(-mutrsl-k2*(1-u14(3))*(1-u16(3))))-k2*hb117(i)*(1-u14(3))*(1-u16(3))+(r*hb11(i)*wb1(i))/tmax; ka117(3)=-(ps*hb127(i)*(1-u21(3)))-ps*hb126(i)*(1-u15(3))*u21(3)+(r*hb11(i)*wb1(i))/tmax-c*q*hb17(i)*wb6(i)-hb16(i)*(-(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-hb117(i)*(-mutrsa-is*wb7(i)); ka118(3)=-((-k2-mutrrl)*hb118(i))-k2*hb119(i)+(r*hb11(i)*wb1(i))/tmax; ka119(3)=-(pr*hb126(i)*(1-u15(3))*(1-u22(3)))-pr*hb127(i)*u22(3)+(r*hb11(i)*wb1(i))/tmax-c*q*hb17(i)*wb6(i)-hb16(i)*(-(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-hb119(i)*(-mutrra-ir*wb7(i)); ka120(3)=-(hb120(i)*(-mutgsl-k2*(1-u14(3))*(1-u16(3))))-k2*hb121(i)*(1-u14(3))*(1-u16(3))+(r*hb11(i)*wb1(i))/tmax; ka121(3)=-(ps*hb127(i)*(1-u21(3)))-ps*hb126(i)*(1-u15(3))*u21(3)+(r*hb11(i)*wb1(i))/tmax-c*q*hb17(i)*wb6(i)-hb16(i)*(-(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-hb121(i)*(-mutgsa-is*wb7(i)); ka122(3)=-((-k2-mutgrl)*hb122(i))-k2*hb123(i)+(r*hb11(i)*wb1(i))/tmax; ka123(3)=-(pr*hb126(i)*(1-u15(3))*(1-u22(3)))-pr*hb127(i)*u22(3)+(r*hb11(i)*wb1(i))/tmax-c*q*hb17(i)*wb6(i)-hb16(i)*(-(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-hb123(i)*(-mutgra-ir*wb7(i)); ka124(3)=mumhsa*hb124(i)-ps*hb127(i)*(1-u21(3))-ps*hb126(i)*(1-u15(3))*u21(3)+k3*hb11(i)*wb1(i)-k3*hb18(i)*wb1(i)-k3*hb112(i)*wb2(i)+k3*hb12(i)*wb2(i)-k3*hb116(i)*wb3(i)+k3*hb13(i)*wb3(i)-k3*hb120(i)*wb4(i)+k3*hb14(i)*wb4(i); ka125(3)=mumhra*hb125(i)-pr*hb126(i)*(1-u15(3))*(1-u22(3))-pr*hb127(i)*u22(3)+k3*hb11(i)*wb1(i)-k3*hb110(i)*wb1(i)-k3*hb114(i)*wb2(i)+k3*hb12(i)*wb2(i)-k3*hb118(i)*wb3(i)+k3*hb13(i)*wb3(i)-k3*hb122(i)*wb4(i)+k3*hb14(i)*wb4(i); ka126(3)=-(k1s*(1-qthvs)*hb18(i)*(1-u11(3))*(1-u12(3))*wb1(i))-k1s*(1-qtsvs)*hb112(i)*(1-u11(3))*(1-u12(3))*wb2(i)-hb12(i)*(k*wb1(i)-k1s*(1-qtsvs)*(1-u11(3))*(1-u12(3))*wb2(i))-hb11(i)*(-(k*wb1(i))-kprimo*wb1(i)-k1s*(1-qthvs)*(1-u11(3))*(1-u12(3))*wb1(i)-(oth*s1th)/(oth+wb26(i)+wb27(i))^2)-k1s*(1-qtrvs)*hb116(i)*(1-u11(3))*(1-u12(3))*wb3(i)+k1s*(1-qtrvs)*hb13(i)*(1-u11(3))*(1-u12(3))*wb3(i)-k1s*(1-qtgvs)*hb120(i)*(1-u11(3))*(1-u12(3))*wb4(i)-hb14(i)*(kprimo*wb1(i)-k1s*(1-qtgvs)*(1-u11(3))*(1-u12(3))*wb4(i))-k1sm*hb124(i)*(1-u15(3))*(1-u16(3))*wb5(i)+k1sm*hb15(i)*(1-u15(3))*(1-u16(3))*wb5(i)-hb126(i)*(-muvs-psprimo*wb2(i)-(g*(1-u15(3))*wb26(i))/(b2+wb26(i))^2+(g*(1-u15(3)))/(b2+wb26(i))-ssprimo*wb4(i)-k1s*rhos*(1-u11(3))*(1-u12(3))*((1-qthvs)*wb1(i)+(1-qtsvs)*wb2(i)+(1-qtrvs)*wb3(i)+(1-qtgvs)*wb4(i))-k1sm*rhos*(1-u15(3))*(1-u16(3))*wb5(i)); ka127(3)=-(k1r*(1-qthvr)*hb110(i)*wb1(i))-k1r*(1-qtsvr)*hb114(i)*wb2(i)+k1r*(1-qtsvr)*hb12(i)*wb2(i)-hb11(i)*(-(k*wb1(i))-kprimo*wb1(i)-k1r*(1-qthvr)*wb1(i)-(oth*s1th)/(oth+wb26(i)+wb27(i))^2)-k1r*(1-qtrvr)*hb118(i)*wb3(i)-hb13(i)*(k*wb1(i)-k1r*(1-qtrvr)*wb3(i))-k1r*(1-qtgvr)*hb122(i)*wb4(i)-hb14(i)*(kprimo*wb1(i)-k1r*(1-qtgvr)*wb4(i))-k1rm*hb125(i)*wb5(i)+k1rm*hb15(i)*wb5(i)-hb127(i)*(-muvr-(g*wb27(i))/(b2+wb27(i))^2+g/(b2+wb27(i))-prprimo*wb3(i)-srprimo*wb4(i)-k1r*rhor*((1-qthvr)*wb1(i)+(1-qtsvr)*wb2(i)+(1-qtrvr)*wb3(i)+(1-qtgvr)*wb4(i))-k1rm*rhor*wb5(i));

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%LAMBDA2(3)

ka21(3)=-(k*hb22(i)*wb26(i))+k1s*(1-qthvs)*rhos*hb226(i)*(1-u11(3))*(1-
u12(3))*wb26(i)-hb28(i)*(k3*wb24(i)+k1s*(1-qthvs)*(1-u11(3))*(1-
u12(3))*wb26(i))+k1r*(1-qthvr)*rhor*hb227(i)*wb27(i)-k*hb23(i)*wb27(i)-
kprimo*hb24(i)*(wb26(i)+wb27(i))-hb210(i)*(k3*wb25(i)+k1r*(1-qthvr)*wb27(i))-
c*hb26(i)*wb6(i)*(wb11(i)+wb13(i)+wb15(i)+wb17(i)+wb19(i)+wb21(i)+wb23(i)+wb9(i))-
hb21(i)*(-muth+u13(3)-(r*wb1(i))/tmax-k3*(wb24(i)+wb25(i))-k1s*(1-qthvs)*(1-u11(3))*(1-
u12(3))*wb26(i)-k1r*(1-qthvr)*wb27(i)-k*(wb26(i)+wb27(i))-
kprimo*(wb26(i)+wb27(i))+r*(1-
(wb1(i)+wb10(i)+wb11(i)+wb12(i)+wb13(i)+wb14(i)+wb15(i)+wb16(i)+wb17(i)+wb18(i)+
wb19(i)+wb2(i)+wb20(i)+wb21(i)+wb22(i)+wb23(i)+wb3(i)+wb4(i)+wb8(i)+wb9(i))/tmax))
;       ka22(3)=(r*hb21(i)*wb1(i))/tmax-hb212(i)*(k3*wb24(i)+k1s*(1-qtsvs)*(1-u11(3))*(1-
u12(3))*wb26(i))-hb226(i)*(-(psprimo*wb26(i))-k1s*(1-qtsvs)*rhos*(1-u11(3))*(1-
u12(3))*wb26(i))+k1r*(1-qtsvr)*rhor*hb227(i)*wb27(i)-hb22(i)*(-muts+u13(3)-
k3*(wb24(i)+wb25(i))-k1s*(1-qtsvs)*(1-u11(3))*(1-u12(3))*wb26(i)-k1r*(1-
qtsvr)*wb27(i))-hb214(i)*(k3*wb25(i)+k1r*(1-qtsvr)*wb27(i))-
c*hb26(i)*wb6(i)*(wb11(i)+wb13(i)+wb15(i)+wb17(i)+wb19(i)+wb21(i)+wb23(i)+wb9(i));
ka23(3)=(r*hb21(i)*wb1(i))/tmax+k1s*(1-qtrvs)*rhos*hb226(i)*(1-u11(3))*(1-
u12(3))*wb26(i)-hb216(i)*(k3*wb24(i)+k1s*(1-qtrvs)*(1-u11(3))*(1-u12(3))*wb26(i))-
hb23(i)*(-mutr+u13(3)-k3*(wb24(i)+wb25(i))-k1s*(1-qtrvs)*(1-u11(3))*(1-u12(3))*wb26(i)-
k1r*(1-qtrvr)*wb27(i))-hb218(i)*(k3*wb25(i)+k1r*(1-qtrvr)*wb27(i))-hb227(i)*(-
(prprimo*wb27(i))-k1r*(1-qtrvr)*rhor*wb27(i))-
c*hb26(i)*wb6(i)*(wb11(i)+wb13(i)+wb15(i)+wb17(i)+wb19(i)+wb21(i)+wb23(i)+wb9(i));
ka24(3)=(r*hb21(i)*wb1(i))/tmax-hb220(i)*(k3*wb24(i)+k1s*(1-qtgvs)*(1-u11(3))*(1-
u12(3))*wb26(i))-hb226(i)*(-(ssprimo*wb26(i))-k1s*(1-qtgvs)*rhos*(1-u11(3))*(1-
u12(3))*wb26(i))-hb24(i)*(-mutg+u13(3)-k3*(wb24(i)+wb25(i))-k1s*(1-qtgvs)*(1-
u11(3))*(1-u12(3))*wb26(i)-k1r*(1-qtgvr)*wb27(i))-hb222(i)*(k3*wb25(i)+k1r*(1-
qtgvr)*wb27(i))-hb227(i)*(-(k1r*(1-qtgvr)*rhor*wb27(i))-srprimo*wb27(i))-
c*hb26(i)*wb6(i)*(wb11(i)+wb13(i)+wb15(i)+wb17(i)+wb19(i)+wb21(i)+wb23(i)+wb9(i));
ka25(3)=-(k1sm*hb224(i)*(1-u15(3))*(1-u16(3))*wb26(i))+k1sm*rhos*hb226(i)*(1-
u15(3))*(1-u16(3))*wb26(i)-k1rm*hb225(i)*wb27(i)+k1rm*rhor*hb227(i)*wb27(i)-
hb25(i)*(-mumh-k1sm*(1-u15(3))*(1-u16(3))*wb26(i)-k1rm*wb27(i));       ka26(3)=-
(c*q*hb27(i)*(wb11(i)+wb13(i)+wb15(i)+wb17(i)+wb19(i)+wb21(i)+wb23(i)+wb9(i)))-
hb26(i)*(-muip-
c*q*(wb11(i)+wb13(i)+wb15(i)+wb17(i)+wb19(i)+wb21(i)+wb23(i)+wb9(i))+c*(wb1(i)+wb
2(i)+wb3(i)+wb4(i))*(wb11(i)+wb13(i)+wb15(i)+wb17(i)+wb19(i)+wb21(i)+wb23(i)+wb9(i
)));
ka27(3)=muie*hb27(i)+ir*hb211(i)*wb11(i)+is*hb213(i)*wb13(i)+ir*hb215(i)*wb15(i)+is*h
b217(i)*wb17(i)+ir*hb219(i)*wb19(i)+is*hb221(i)*wb21(i)+ir*hb223(i)*wb23(i)+is*hb29(i)
*wb9(i);       ka28(3)=-1-hb28(i)*(-muthsl-k2*(1-u14(3))*(1-u16(3)))-k2*hb29(i)*(1-
u14(3))*(1-u16(3))+(r*hb21(i)*wb1(i))/tmax;       ka29(3)= -1-ps*hb227(i)*(1-u21(3))-
ps*hb226(i)*(1-u15(3))*u21(3)+(r*hb21(i)*wb1(i))/tmax-c*q*hb27(i)*wb6(i)-hb26(i)*(-
(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-hb29(i)*(-muthsa-is*wb7(i));
ka210(3)=-1-(-k2-muthrl)*hb210(i)-k2*hb211(i)+(r*hb21(i)*wb1(i))/tmax;       ka211(3)= -
1-pr*hb226(i)*(1-u15(3))*(1-u22(3))-pr*hb227(i)*u22(3)+(r*hb21(i)*wb1(i))/tmax-
c*q*hb27(i)*wb6(i)-hb26(i)*(-(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-
hb211(i)*(-muthra-ir*wb7(i));       ka212(3)=-1-hb212(i)*(-mutssl-k2*(1-u14(3))*(1-
u16(3)))-k2*hb213(i)*(1-u14(3))*(1-u16(3))+(r*hb21(i)*wb1(i))/tmax;       ka213(3)=-1-
ps*hb227(i)*(1-u21(3))-ps*hb226(i)*(1-u15(3))*u21(3)+(r*hb21(i)*wb1(i))/tmax-
c*q*hb27(i)*wb6(i)-hb26(i)*(-(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-
hb213(i)*(-mutssa-is*wb7(i));       ka214(3)=-1-(-k2-mutsrl)*hb214(i)-
k2*hb215(i)+(r*hb21(i)*wb1(i))/tmax;       ka215(3)=-1-pr*hb226(i)*(1-u15(3))*(1-u22(3))-
pr*hb227(i)*u22(3)+(r*hb21(i)*wb1(i))/tmax-c*q*hb27(i)*wb6(i)-hb26(i)*(-
(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-hb215(i)*(-mutsra-ir*wb7(i));
ka216(3)=-1-hb216(i)*(-mutrsl-k2*(1-u14(3))*(1-u16(3)))-k2*hb217(i)*(1-u14(3))*(1-
u16(3))+(r*hb21(i)*wb1(i))/tmax;       ka217(3)=-1-ps*hb227(i)*(1-u21(3))-ps*hb226(i)*(1-
u15(3))*u21(3)+(r*hb21(i)*wb1(i))/tmax-c*q*hb27(i)*wb6(i)-hb26(i)*(-
(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-hb217(i)*(-mutrsa-is*wb7(i));
ka218(3)=-1-(-k2-mutrrl)*hb218(i)-k2*hb219(i)+(r*hb21(i)*wb1(i))/tmax;       ka219(3)=-1-
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pr*hb226(i)*(1-u15(3))*(1-u22(3))-pr*hb227(i)*u22(3)+(r*hb21(i)*wb1(i))/tmax-c*q*hb27(i)*wb6(i)-hb26(i)*(-(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-hb219(i)*(-mutrra-ir*wb7(i)); ka220(3)=-1-hb220(i)*(-mutgsl-k2*(1-u14(3))*(1-u16(3)))-k2*hb221(i)*(1-u14(3))*(1-u16(3))+(r*hb21(i)*wb1(i))/tmax; ka221(3)=-1-ps*hb227(i)*(1-u21(3))-ps*hb226(i)*(1-u15(3))*u21(3)+(r*hb21(i)*wb1(i))/tmax-c*q*hb27(i)*wb6(i)-hb26(i)*(-(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-hb221(i)*(-mutgsa-is*wb7(i)); ka222(3)=-1-(-k2-mutgrl)*hb222(i)-k2*hb223(i)+(r*hb21(i)*wb1(i))/tmax; ka223(3)=-1-pr*hb226(i)*(1-u15(3))*(1-u22(3))-pr*hb227(i)*u22(3)+(r*hb21(i)*wb1(i))/tmax-c*q*hb27(i)*wb6(i)-hb26(i)*(-(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-hb223(i)*(-mutgra-ir*wb7(i)); ka224(3)=mumhsa*hb224(i)-ps*hb227(i)*(1-u21(3))-ps*hb226(i)*(1-u15(3))*u21(3)+k3*hb21(i)*wb1(i)-k3*hb28(i)*wb1(i)-k3*hb212(i)*wb2(i)+k3*hb22(i)*wb2(i)-k3*hb216(i)*wb3(i)+k3*hb23(i)*wb3(i)-k3*hb220(i)*wb4(i)+k3*hb24(i)*wb4(i); ka225(3)=mumhra*hb225(i)-pr*hb226(i)*(1-u15(3))*(1-u22(3))-pr*hb227(i)*u22(3)+k3*hb21(i)*wb1(i)-k3*hb210(i)*wb1(i)-k3*hb214(i)*wb2(i)+k3*hb22(i)*wb2(i)-k3*hb218(i)*wb3(i)+k3*hb23(i)*wb3(i)-k3*hb222(i)*wb4(i)+k3*hb24(i)*wb4(i); ka226(3)=-1-k1s*(1-qthvs)*hb28(i)*(1-u11(3))*(1-u12(3))*wb1(i)-k1s*(1-qtsvs)*hb212(i)*(1-u11(3))*(1-u12(3))*wb2(i)-hb22(i)*(k*wb1(i)-k1s*(1-qtsvs)*(1-u11(3))*(1-u12(3))*wb2(i))-hb21(i)*(-(k*wb1(i))-kprimo*wb1(i)-k1s*(1-qthvs)*(1-u11(3))*(1-u12(3))*wb1(i)-(oth*s1th)/(oth+wb26(i)+wb27(i))^2)-k1s*(1-qtrvs)*hb216(i)*(1-u11(3))*(1-u12(3))*wb3(i)+k1s*(1-qtrvs)*hb23(i)*(1-u11(3))*(1-u12(3))*wb3(i)-k1s*(1-qtgvs)*hb220(i)*(1-u11(3))*(1-u12(3))*wb4(i)-hb24(i)*(kprimo*wb1(i)-k1s*(1-qtgvs)*(1-u11(3))*(1-u12(3))*wb4(i))-k1sm*hb224(i)*(1-u15(3))*(1-u16(3))*wb5(i)+k1sm*hb25(i)*(1-u15(3))*(1-u16(3))*wb5(i)-hb226(i)*(-muvs-psprimo*wb2(i)-(g*(1-u15(3))*wb26(i))/(b2+wb26(i))^2+(g*(1-u15(3)))/(b2+wb26(i))-ssprimo*wb4(i)-k1s*rhos*(1-u11(3))*(1-u12(3))*((1-qthvs)*wb1(i)+(1-qtsvs)*wb2(i)+(1-qtrvs)*wb3(i)+(1-qtgvs)*wb4(i))-k1sm*rhos*(1-u15(3))*(1-u16(3))*wb5(i)); ka227(3)=-1-k1r*(1-qthvr)*hb210(i)*wb1(i)-k1r*(1-qtsvr)*hb214(i)*wb2(i)+k1r*(1-qtsvr)*hb22(i)*wb2(i)-hb21(i)*(-(k*wb1(i))-kprimo*wb1(i)-k1r*(1-qthvr)*wb1(i)-(oth*s1th)/(oth+wb26(i)+wb27(i))^2)-k1r*(1-qtrvr)*hb218(i)*wb3(i)-hb23(i)*(k*wb1(i)-k1r*(1-qtrvr)*wb3(i))-k1r*(1-qtgvr)*hb222(i)*wb4(i)-hb24(i)*(kprimo*wb1(i)-k1r*(1-qtgvr)*wb4(i))-k1rm*hb225(i)*wb5(i)+k1rm*hb25(i)*wb5(i)-hb227(i)*(-muvr-(g*wb27(i))/(b2+wb27(i))^2+g/(b2+wb27(i))-prprimo*wb3(i)-srprimo*wb4(i)-k1r*rhor*((1-qthvr)*wb1(i)+(1-qtsvr)*wb2(i)+(1-qtrvr)*wb3(i)+(1-qtgvr)*wb4(i))-k1rm*rhor*wb5(i));

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%VARIABILI DI APPOGGIO per 4° ordine

wb1(i)=(x1(i-1));

wb2(i)=(x2(i-1));

wb3(i)=(x3(i-1));

wb4(i)=(x4(i-1));

wb5(i)=(+x5(i-1));

wb6(i)=(x6(i-1));

wb7(i)=(x7(i-1));

wb8(i)=(x8(i-1));

wb9(i)=(x9(i-1));

wb10(i)=(x10(i-1));
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wb11(i)=(x11(i-1));
wb12(i)=(x12(i-1));
wb13(i)=(x13(i-1));
wb14(i)=(x14(i-1));
wb15(i)=(x15(i-1));
wb16(i)=(x16(i-1));
wb17(i)=(x17(i-1));
wb18(i)=(x18(i-1));
wb19(i)=(x19(i-1));
wb20(i)=(x20(i-1));
wb21(i)=(x21(i-1));
wb22(i)=(x22(i-1));
wb23(i)=(x23(i-1));
wb24(i)=(x24(i-1));
wb25(i)=(x25(i-1));
wb26(i)=(x26(i-1));
wb27(i)=(x27(i-1));


%VARIABILI DI APPOGGIO hb per 4° ordine
hb11(i)=l11(i)-h*ka11(3);
hb12(i)=l12(i)-h*ka12(3);
hb13(i)=l13(i)-h*ka13(3);
hb14(i)=l14(i)-h*ka14(3);
hb15(i)=l15(i)-h*ka15(3);
hb16(i)=l16(i)-h*ka16(3);
hb17(i)=l17(i)-h*ka17(3);
hb18(i)=l18(i)-h*ka18(3);
hb19(i)=l19(i)-h*ka19(3);
hb110(i)=l110(i)-h*ka110(3);
hb111(i)=l111(i)-h*ka111(3);
hb112(i)=l112(i)-h*ka112(3);
hb113(i)=l113(i)-h*ka113(3);
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hb114(i)=l114(i)-h*ka114(3);
hb115(i)=l115(i)-h*ka115(3);
hb116(i)=l116(i)-h*ka116(3);
hb117(i)=l117(i)-h*ka117(3);
hb118(i)=l118(i)-h*ka118(3);
hb119(i)=l119(i)-h*ka119(3);
hb120(i)=l120(i)-h*ka120(3);
hb121(i)=l121(i)-h*ka121(3);
hb122(i)=l122(i)-h*ka122(3);
hb123(i)=l123(i)-h*ka123(3);
hb124(i)=l124(i)-h*ka124(3);
hb125(i)=l125(i)-h*ka125(3);
hb126(i)=l126(i)-h*ka126(3);
hb127(i)=l127(i)-h*ka127(3);



hb21(i)=l21(i)-h*ka21(3);
hb22(i)=l22(i)-h*ka22(3);
hb23(i)=l23(i)-h*ka23(3);
hb24(i)=l24(i)-h*ka24(3);
hb25(i)=l25(i)-h*ka25(3);
hb26(i)=l26(i)-h*ka26(3);
hb27(i)=l27(i)-h*ka27(3);
hb28(i)=l28(i)-h*ka28(3);
hb29(i)=l29(i)-h*ka29(3);
hb210(i)=l210(i)-h*ka210(3);
hb211(i)=l211(i)-h*ka211(3);
hb212(i)=l212(i)-h*ka212(3);
hb213(i)=l213(i)-h*ka213(3);
hb214(i)=l214(i)-h*ka214(3);
hb215(i)=l215(i)-h*ka215(3);
hb216(i)=l216(i)-h*ka216(3);
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hb217(i)=l217(i)-h*ka217(3);

hb218(i)=l218(i)-h*ka218(3);

hb219(i)=l219(i)-h*ka219(3);

hb220(i)=l220(i)-h*ka220(3);

hb221(i)=l221(i)-h*ka221(3);

hb222(i)=l222(i)-h*ka222(3);

hb223(i)=l223(i)-h*ka223(3);

hb224(i)=l224(i)-h*ka224(3);

hb225(i)=l225(i)-h*ka225(3);

hb226(i)=l226(i)-h*ka226(3);

hb227(i)=l227(i)-h*ka227(3);



%CONTROLLI COEEFICIENTI DI RUNGE KUTTA 4° ORDINE

a11= (1.*(k1s*(1.-1.*qthvs)*hb11(i)*wb1(i)*wb26(i)-1.*k1s*(1.-
1.*qthvs)*hb18(i)*wb1(i)*wb26(i)-1.*k1s*(1.-1.*qtsvs)*hb112(i)*wb2(i)*wb26(i)+k1s*(1.-
1.*qtsvs)*hb12(i)*wb2(i)*wb26(i)-1.*k1s*(1.-1.*qtrvs)*hb116(i)*wb26(i)*wb3(i)+k1s*(1.-
1.*qtrvs)*hb13(i)*wb26(i)*wb3(i)-1.*k1s*(1.-1.*qtgvs)*hb120(i)*wb26(i)*wb4(i)+k1s*(1.-
1.*qtgvs)*hb14(i)*wb26(i)*wb4(i)+k1s*rhos*hb126(i)*wb26(i)*((1.-1.*qthvs)*wb1(i)+(1.-
1.*qtsvs)*wb2(i)+(1.-1.*qtrvs)*wb3(i)+(1.-1.*qtgvs)*wb4(i))))/aco-(1.*(-1.*k1s*(1.-
1.*qthvs)*hb11(i)*wb1(i)*wb26(i)+k1s*(1.-1.*qthvs)*hb18(i)*wb1(i)*wb26(i)+k1s*(1.-
1.*qtsvs)*hb112(i)*wb2(i)*wb26(i)-1.*k1s*(1.-1.*qtsvs)*hb12(i)*wb2(i)*wb26(i)+k1s*(1.-
1.*qtrvs)*hb116(i)*wb26(i)*wb3(i)-1.*k1s*(1.-1.*qtrvs)*hb13(i)*wb26(i)*wb3(i)+k1s*(1.-
1.*qtgvs)*hb120(i)*wb26(i)*wb4(i)-1.*k1s*(1.-1.*qtgvs)*hb14(i)*wb26(i)*wb4(i)-
1.*k1s*rhos*hb126(i)*wb26(i)*((1.-1.*qthvs)*wb1(i)+(1.-1.*qtsvs)*wb2(i)+(1.-
1.*qtrvs)*wb3(i)+(1.-1.*qtgvs)*wb4(i)))*(-1.*aco*(k1s*(1.-
1.*qthvs)*hb11(i)*wb1(i)*wb26(i)-1.*k1s*(1.-1.*qthvs)*hb18(i)*wb1(i)*wb26(i)-
1.*k1s*(1.-1.*qtsvs)*hb112(i)*wb2(i)*wb26(i)+k1s*(1.-1.*qtsvs)*hb12(i)*wb2(i)*wb26(i)-
1.*k1s*(1.-1.*qtrvs)*hb116(i)*wb26(i)*wb3(i)+k1s*(1.-1.*qtrvs)*hb13(i)*wb26(i)*wb3(i)-
1.*k1s*(1.-1.*qtgvs)*hb120(i)*wb26(i)*wb4(i)+k1s*(1.-
1.*qtgvs)*hb14(i)*wb26(i)*wb4(i)+k1s*rhos*hb126(i)*wb26(i)*((1.-1.*qthvs)*wb1(i)+(1.-
1.*qtsvs)*wb2(i)+(1.-1.*qtrvs)*wb3(i)+(1.-1.*qtgvs)*wb4(i)))-1.*(-1.*k1s*(1.-
1.*qthvs)*hb11(i)*wb1(i)*wb26(i)+k1s*(1.-1.*qthvs)*hb18(i)*wb1(i)*wb26(i)+k1s*(1.-
1.*qtsvs)*hb112(i)*wb2(i)*wb26(i)-1.*k1s*(1.-1.*qtsvs)*hb12(i)*wb2(i)*wb26(i)+k1s*(1.-
1.*qtrvs)*hb116(i)*wb26(i)*wb3(i)-1.*k1s*(1.-1.*qtrvs)*hb13(i)*wb26(i)*wb3(i)+k1s*(1.-
1.*qtgvs)*hb120(i)*wb26(i)*wb4(i)-1.*k1s*(1.-1.*qtgvs)*hb14(i)*wb26(i)*wb4(i)-
1.*k1s*rhos*hb126(i)*wb26(i)*((1.-1.*qthvs)*wb1(i)+(1.-1.*qtsvs)*wb2(i)+(1.-
1.*qtrvs)*wb3(i)+(1.-1.*qtgvs)*wb4(i)))*(k1s*(1.-1.*qthvs)*hb11(i)*wb1(i)*wb26(i)-
1.*k1s*(1.-1.*qthvs)*hb18(i)*wb1(i)*wb26(i)-1.*k1s*(1.-
1.*qtsvs)*hb112(i)*wb2(i)*wb26(i)+k1s*(1.-1.*qtsvs)*hb12(i)*wb2(i)*wb26(i)-1.*k1s*(1.-
1.*qtrvs)*hb116(i)*wb26(i)*wb3(i)+k1s*(1.-1.*qtrvs)*hb13(i)*wb26(i)*wb3(i)-1.*k1s*(1.-
1.*qtgvs)*hb120(i)*wb26(i)*wb4(i)+k1s*(1.-
1.*qtgvs)*hb14(i)*wb26(i)*wb4(i)+k1s*rhos*hb126(i)*wb26(i)*((1.-1.*qthvs)*wb1(i)+(1.-
1.*qtsvs)*wb2(i)+(1.-1.*qtrvs)*wb3(i)+(1.-1.*qtgvs)*wb4(i)))))/(aco*(1.*aco*afi-1.*(-
1.*k1s*(1.-1.*qthvs)*hb11(i)*wb1(i)*wb26(i)+k1s*(1.-
1.*qthvs)*hb18(i)*wb1(i)*wb26(i)+k1s*(1.-1.*qtsvs)*hb112(i)*wb2(i)*wb26(i)-1.*k1s*(1.-
1.*qtsvs)*hb12(i)*wb2(i)*wb26(i)+k1s*(1.-1.*qtrvs)*hb116(i)*wb26(i)*wb3(i)-1.*k1s*(1.-
1.*qtrvs)*hb13(i)*wb26(i)*wb3(i)+k1s*(1.-1.*qtgvs)*hb120(i)*wb26(i)*wb4(i)-1.*k1s*(1.-
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1.*qtgvs)*hb14(i)*wb26(i)*wb4(i)-1.*k1s*rhos*hb126(i)*wb26(i)*((1.-
1.*qthvs)*wb1(i)+(1.-1.*qtsvs)*wb2(i)+(1.-1.*qtrvs)*wb3(i)+(1.-1.*qtgvs)*wb4(i)))^2));
a12= -((-1.*aco*(k1s*(1.-1.*qthvs)*hb11(i)*wb1(i)*wb26(i)-1.*k1s*(1.-
1.*qthvs)*hb18(i)*wb1(i)*wb26(i)-1.*k1s*(1.-1.*qtsvs)*hb112(i)*wb2(i)*wb26(i)+k1s*(1.-
1.*qtsvs)*hb12(i)*wb2(i)*wb26(i)-1.*k1s*(1.-1.*qtrvs)*hb116(i)*wb26(i)*wb3(i)+k1s*(1.-
1.*qtrvs)*hb13(i)*wb26(i)*wb3(i)-1.*k1s*(1.-1.*qtgvs)*hb120(i)*wb26(i)*wb4(i)+k1s*(1.-
1.*qtgvs)*hb14(i)*wb26(i)*wb4(i)+k1s*rhos*hb126(i)*wb26(i)*((1.-1.*qthvs)*wb1(i)+(1.-
1.*qtsvs)*wb2(i)+(1.-1.*qtrvs)*wb3(i)+(1.-1.*qtgvs)*wb4(i)))-1.*(-1.*k1s*(1.-
1.*qthvs)*hb11(i)*wb1(i)*wb26(i)+k1s*(1.-1.*qthvs)*hb18(i)*wb1(i)*wb26(i)+k1s*(1.-
1.*qtsvs)*hb112(i)*wb2(i)*wb26(i)-1.*k1s*(1.-1.*qtsvs)*hb12(i)*wb2(i)*wb26(i)+k1s*(1.-
1.*qtrvs)*hb116(i)*wb26(i)*wb3(i)-1.*k1s*(1.-1.*qtrvs)*hb13(i)*wb26(i)*wb3(i)+k1s*(1.-
1.*qtgvs)*hb120(i)*wb26(i)*wb4(i)-1.*k1s*(1.-1.*qtgvs)*hb14(i)*wb26(i)*wb4(i)-
1.*k1s*rhos*hb126(i)*wb26(i)*((1.-1.*qthvs)*wb1(i)+(1.-1.*qtsvs)*wb2(i)+(1.-
1.*qtrvs)*wb3(i)+(1.-1.*qtgvs)*wb4(i)))*(k1s*(1.-1.*qthvs)*hb11(i)*wb1(i)*wb26(i)-
1.*k1s*(1.-1.*qthvs)*hb18(i)*wb1(i)*wb26(i)-1.*k1s*(1.-
1.*qtsvs)*hb112(i)*wb2(i)*wb26(i)+k1s*(1.-1.*qtsvs)*hb12(i)*wb2(i)*wb26(i)-1.*k1s*(1.-
1.*qtrvs)*hb116(i)*wb26(i)*wb3(i)+k1s*(1.-1.*qtrvs)*hb13(i)*wb26(i)*wb3(i)-1.*k1s*(1.-
1.*qtgvs)*hb120(i)*wb26(i)*wb4(i)+k1s*(1.-
1.*qtgvs)*hb14(i)*wb26(i)*wb4(i)+k1s*rhos*hb126(i)*wb26(i)*((1.-1.*qthvs)*wb1(i)+(1.-
1.*qtsvs)*wb2(i)+(1.-1.*qtrvs)*wb3(i)+(1.-1.*qtgvs)*wb4(i))))/(1.*aco*afi-1.*(-1.*k1s*(1.-
1.*qthvs)*hb11(i)*wb1(i)*wb26(i)+k1s*(1.-1.*qthvs)*hb18(i)*wb1(i)*wb26(i)+k1s*(1.-
1.*qtsvs)*hb112(i)*wb2(i)*wb26(i)-1.*k1s*(1.-1.*qtsvs)*hb12(i)*wb2(i)*wb26(i)+k1s*(1.-
1.*qtrvs)*hb116(i)*wb26(i)*wb3(i)-1.*k1s*(1.-1.*qtrvs)*hb13(i)*wb26(i)*wb3(i)+k1s*(1.-
1.*qtgvs)*hb120(i)*wb26(i)*wb4(i)-1.*k1s*(1.-1.*qtgvs)*hb14(i)*wb26(i)*wb4(i)-
1.*k1s*rhos*hb126(i)*wb26(i)*((1.-1.*qthvs)*wb1(i)+(1.-1.*qtsvs)*wb2(i)+(1.-
1.*qtrvs)*wb3(i)+(1.-1.*qtgvs)*wb4(i)))^2));
a13=(1.*(hb11(i)*wb1(i)+hb12(i)*wb2(i)+hb13(i)*wb3(i)+hb14(i)*wb4(i)))/aib;
a14=(1.*(k2*hb112(i)*wb12(i)-1.*k2*hb113(i)*wb12(i)+k2*hb116(i)*wb16(i)-
1.*k2*hb117(i)*wb16(i)+k2*hb120(i)*wb20(i)-1.*k2*hb121(i)*wb20(i)+k2*hb18(i)*wb8(i)-
1.*k2*hb19(i)*wb8(i)))/aii-(1.*(-1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-
1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-
1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-
1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i))*((1.*(1.*bsr*pr*hb126(i)*(wb11(i)+wb15(i)+wb
19(i)+wb23(i)+wb25(i))*(1.*brs-
1.*pr*hb226(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))+pr*hb227(i)*(wb11(i)+wb15(
i)+wb19(i)+wb23(i)+wb25(i)))-1.*brs*(-1.*bsr*(-
1.*pr*hb126(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))-
(1.*g*hb126(i)*wb26(i))/(b2+wb26(i))-
1.*k1sm*hb124(i)*wb26(i)*wb5(i)+k1sm*rhos*hb126(i)*wb26(i)*wb5(i)+k1sm*hb15(i)*w
b26(i)*wb5(i))+1.*ps*hb126(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))*(1.*bsr+ps*hb
226(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))-
1.*ps*hb227(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))))))/(brs*bsr*(k1sm*hb124(i)*
wb26(i)*wb5(i)-1.*k1sm*rhos*hb126(i)*wb26(i)*wb5(i)-
1.*k1sm*hb15(i)*wb26(i)*wb5(i)))-
(1.*(1.*bsr*pr^2*hb126(i)*hb226(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))^2-
1.*brs*(1.*api*bsr-
1.*ps^2*hb126(i)*hb226(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))^2))*(1.*brs*bsr*(k
1sm*hb124(i)*wb26(i)*wb5(i)-1.*k1sm*rhos*hb126(i)*wb26(i)*wb5(i)-
1.*k1sm*hb15(i)*wb26(i)*wb5(i))*(-1.*aii*(k2*hb112(i)*wb12(i)-
1.*k2*hb113(i)*wb12(i)+k2*hb116(i)*wb16(i)-
1.*k2*hb117(i)*wb16(i)+k2*hb120(i)*wb20(i)-1.*k2*hb121(i)*wb20(i)-
1.*k1sm*hb124(i)*wb26(i)*wb5(i)+k1sm*rhos*hb126(i)*wb26(i)*wb5(i)+k1sm*hb15(i)*w
b26(i)*wb5(i)+k2*hb18(i)*wb8(i)-1.*k2*hb19(i)*wb8(i))-1.*(k2*hb112(i)*wb12(i)-
1.*k2*hb113(i)*wb12(i)+k2*hb116(i)*wb16(i)-
1.*k2*hb117(i)*wb16(i)+k2*hb120(i)*wb20(i)-1.*k2*hb121(i)*wb20(i)+k2*hb18(i)*wb8(i)-
1.*k2*hb19(i)*wb8(i))*(-1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-
1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-
1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i)))-
1.*(1.*aii*art-1.*(-1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-
1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-
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1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-
1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i))^2)*(1.*bsr*pr*hb126(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))*(1.*brs-
1.*pr*hb226(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))+pr*hb227(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i)))-1.*brs*(-1.*bsr*(-
1.*pr*hb126(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))-
(1.*g*hb126(i)*wb26(i))/(b2+wb26(i))-
1.*k1sm*hb124(i)*wb26(i)*wb5(i)+k1sm*rhos*hb126(i)*wb26(i)*wb5(i)+k1sm*hb15(i)*wb26(i)*wb5(i))+1.*ps*hb126(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))*(1.*bsr+ps*hb226(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))-
1.*ps*hb227(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i)))))))/(brs*bsr*(k1sm*hb124(i)*wb26(i)*wb5(i)-1.*k1sm*rhos*hb126(i)*wb26(i)*wb5(i)-
1.*k1sm*hb15(i)*wb26(i)*wb5(i))*(-1.*aii*brs*bsr*(k1sm*hb124(i)*wb26(i)*wb5(i)-
1.*k1sm*rhos*hb126(i)*wb26(i)*wb5(i)-1.*k1sm*hb15(i)*wb26(i)*wb5(i))^2-
1.*(1.*aii*art-1.*(-1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-
1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-
1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-
1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i))^2)*(1.*bsr*pr^2*hb126(i)*hb226(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*hb126(i)*hb226(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))^2))))))/aii;
a15= -((1.*brs*bsr*(k1sm*hb124(i)*wb26(i)*wb5(i)-
1.*k1sm*rhos*hb126(i)*wb26(i)*wb5(i)-1.*k1sm*hb15(i)*wb26(i)*wb5(i))*(-
1.*aii*(k2*hb112(i)*wb12(i)-1.*k2*hb113(i)*wb12(i)+k2*hb116(i)*wb16(i)-
1.*k2*hb117(i)*wb16(i)+k2*hb120(i)*wb20(i)-1.*k2*hb121(i)*wb20(i)-
1.*k1sm*hb124(i)*wb26(i)*wb5(i)+k1sm*rhos*hb126(i)*wb26(i)*wb5(i)+k1sm*hb15(i)*wb26(i)*wb5(i)+k2*hb18(i)*wb8(i)-1.*k2*hb19(i)*wb8(i))-1.*(k2*hb112(i)*wb12(i)-
1.*k2*hb113(i)*wb12(i)+k2*hb116(i)*wb16(i)-
1.*k2*hb117(i)*wb16(i)+k2*hb120(i)*wb20(i)-1.*k2*hb121(i)*wb20(i)+k2*hb18(i)*wb8(i)-
1.*k2*hb19(i)*wb8(i))*(-1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-
1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-
1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i)))-
1.*(1.*aii*art-1.*(-1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-
1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-
1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-
1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i))^2)*(1.*bsr*pr*hb126(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))*(1.*brs-
1.*pr*hb226(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))+pr*hb227(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i)))-1.*brs*(-1.*bsr*(-
1.*pr*hb126(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))-
(1.*g*hb126(i)*wb26(i))/(b2+wb26(i))-
1.*k1sm*hb124(i)*wb26(i)*wb5(i)+k1sm*rhos*hb126(i)*wb26(i)*wb5(i)+k1sm*hb15(i)*wb26(i)*wb5(i))+1.*ps*hb126(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))*(1.*bsr+ps*hb226(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))-
1.*ps*hb227(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))))))/(-
1.*aii*brs*bsr*(k1sm*hb124(i)*wb26(i)*wb5(i)-1.*k1sm*rhos*hb126(i)*wb26(i)*wb5(i)-
1.*k1sm*hb15(i)*wb26(i)*wb5(i))^2-1.*(1.*aii*art-1.*(-
1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-
1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-
1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-
1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i))^2)*(1.*bsr*pr^2*hb126(i)*hb226(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*hb126(i)*hb226(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))^2)))); a16=(-
1.*(1.*bsr*pr*hb126(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))*(1.*brs-
1.*pr*hb226(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))+pr*hb227(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i)))-1.*brs*(-1.*bsr*(-
1.*pr*hb126(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))-
(1.*g*hb126(i)*wb26(i))/(b2+wb26(i))-
1.*k1sm*hb124(i)*wb26(i)*wb5(i)+k1sm*rhos*hb126(i)*wb26(i)*wb5(i)+k1sm*hb15(i)*wb26(i)*wb5(i))+1.*ps*hb126(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))*(1.*bsr+ps*hb226(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))-

```
1.*ps*hb227(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))))))/(brs*bsr*(k1sm*hb124(i)*
wb26(i)*wb5(i)-1.*k1sm*rhos*hb126(i)*wb26(i)*wb5(i)-
1.*k1sm*hb15(i)*wb26(i)*wb5(i)))+(1.*(1.*bsr*pr^2*hb126(i)*hb226(i)*(wb11(i)+wb15(i)
+wb19(i)+wb23(i)+wb25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*hb126(i)*hb226(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))^2))*(1.*brs*bsr*(k
1sm*hb124(i)*wb26(i)*wb5(i)-1.*k1sm*rhos*hb126(i)*wb26(i)*wb5(i)-
1.*k1sm*hb15(i)*wb26(i)*wb5(i))*(-1.*aii*(k2*hb112(i)*wb12(i)-
1.*k2*hb113(i)*wb12(i)+k2*hb116(i)*wb16(i)-
1.*k2*hb117(i)*wb16(i)+k2*hb120(i)*wb20(i)-1.*k2*hb121(i)*wb20(i)-
1.*k1sm*hb124(i)*wb26(i)*wb5(i)+k1sm*rhos*hb126(i)*wb26(i)*wb5(i)+k1sm*hb15(i)*w
b26(i)*wb5(i)+k2*hb18(i)*wb8(i)-1.*k2*hb19(i)*wb8(i))-1.*(k2*hb112(i)*wb12(i)-
1.*k2*hb113(i)*wb12(i)+k2*hb116(i)*wb16(i)-
1.*k2*hb117(i)*wb16(i)+k2*hb120(i)*wb20(i)-1.*k2*hb121(i)*wb20(i)+k2*hb18(i)*wb8(i)-
1.*k2*hb19(i)*wb8(i))*(-1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-
1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-
1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i)))-
1.*(1.*aii*art-1.*(-1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-
1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-
1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-
1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i))^2)*(1.*bsr*pr*hb126(i)*(wb11(i)+wb15(i)+wb19
(i)+wb23(i)+wb25(i))*(1.*brs-
1.*pr*hb226(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))+pr*hb227(i)*(wb11(i)+wb15(
i)+wb19(i)+wb23(i)+wb25(i)))-1.*brs*(-1.*bsr*(-
1.*pr*hb126(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))-
(1.*g*hb126(i)*wb26(i))/(b2+wb26(i))-
1.*k1sm*hb124(i)*wb26(i)*wb5(i)+k1sm*rhos*hb126(i)*wb26(i)*wb5(i)+k1sm*hb15(i)*w
b26(i)*wb5(i))+1.*ps*hb126(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))*(1.*bsr+ps*hb
226(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))-
1.*ps*hb227(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i)))))))/(brs*bsr*(k1sm*hb124(i)*
wb26(i)*wb5(i)-1.*k1sm*rhos*hb126(i)*wb26(i)*wb5(i)-
1.*k1sm*hb15(i)*wb26(i)*wb5(i))*(-1.*aii*brs*bsr*(k1sm*hb124(i)*wb26(i)*wb5(i)-
1.*k1sm*rhos*hb126(i)*wb26(i)*wb5(i)-1.*k1sm*hb15(i)*wb26(i)*wb5(i))^2-
1.*(1.*aii*art-1.*(-1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-
1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-
1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-
1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i))^2)*(1.*bsr*pr^2*hb126(i)*hb226(i)*(wb11(i)+w
b15(i)+wb19(i)+wb23(i)+wb25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*hb126(i)*hb226(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))^2))));    u21s=
min(1,max(0,(1.*(1.*bsr+ps*hb226(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))-
1.*ps*hb227(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))))/bsr+(1.*ps*hb226(i)*(wb13(i
)+wb17(i)+wb21(i)+wb24(i)+wb9(i))*(1.*brs*bsr*(k1sm*hb124(i)*wb26(i)*wb5(i)-
1.*k1sm*rhos*hb126(i)*wb26(i)*wb5(i)-1.*k1sm*hb15(i)*wb26(i)*wb5(i))*(-
1.*aii*(k2*hb112(i)*wb12(i)-1.*k2*hb113(i)*wb12(i)+k2*hb116(i)*wb16(i)-
1.*k2*hb117(i)*wb16(i)+k2*hb120(i)*wb20(i)-1.*k2*hb121(i)*wb20(i)-
1.*k1sm*hb124(i)*wb26(i)*wb5(i)+k1sm*rhos*hb126(i)*wb26(i)*wb5(i)+k1sm*hb15(i)*w
b26(i)*wb5(i)+k2*hb18(i)*wb8(i)-1.*k2*hb19(i)*wb8(i))-1.*(k2*hb112(i)*wb12(i)-
1.*k2*hb113(i)*wb12(i)+k2*hb116(i)*wb16(i)-
1.*k2*hb117(i)*wb16(i)+k2*hb120(i)*wb20(i)-1.*k2*hb121(i)*wb20(i)+k2*hb18(i)*wb8(i)-
1.*k2*hb19(i)*wb8(i))*(-1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-
1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-
1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i)))-
1.*(1.*aii*art-1.*(-1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-
1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-
1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-
1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i))^2)*(1.*bsr*pr*hb126(i)*(wb11(i)+wb15(i)+wb19
(i)+wb23(i)+wb25(i))*(1.*brs-
1.*pr*hb226(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))+pr*hb227(i)*(wb11(i)+wb15(
i)+wb19(i)+wb23(i)+wb25(i)))-1.*brs*(-1.*bsr*(-
1.*pr*hb126(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))-
(1.*g*hb126(i)*wb26(i))/(b2+wb26(i))-
```

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1.*k1sm*hb124(i)*wb26(i)*wb5(i)+k1sm*rhos*hb126(i)*wb26(i)*wb5(i)+k1sm*hb15(i)*w
b26(i)*wb5(i))+1.*ps*hb126(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))*(1.*bsr+ps*hb
226(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))-
1.*ps*hb227(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i)))))))/(bsr*(-
1.*aii*brs*bsr*(k1sm*hb124(i)*wb26(i)*wb5(i)-1.*k1sm*rhos*hb126(i)*wb26(i)*wb5(i)-
1.*k1sm*hb15(i)*wb26(i)*wb5(i))^2-1.*(1.*aii*art-1.*(-
1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-
1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-
1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-
1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i))^2)*(1.*bsr*pr^2*hb126(i)*hb226(i)*(wb11(i)+w
b15(i)+wb19(i)+wb23(i)+wb25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*hb126(i)*hb226(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))^2)))))); u22s=
min(1,max(0,(1.*(1.*brs-
1.*pr*hb226(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))+pr*hb227(i)*(wb11(i)+wb15(
i)+wb19(i)+wb23(i)+wb25(i))))/brs-
(1.*pr*hb226(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))*(1.*brs*bsr*(k1sm*hb124(i)
*wb26(i)*wb5(i)-1.*k1sm*rhos*hb126(i)*wb26(i)*wb5(i)-
1.*k1sm*hb15(i)*wb26(i)*wb5(i))*(-1.*aii*(k2*hb112(i)*wb12(i)-
1.*k2*hb113(i)*wb12(i)+k2*hb116(i)*wb16(i)-
1.*k2*hb117(i)*wb16(i)+k2*hb120(i)*wb20(i)-1.*k2*hb121(i)*wb20(i)-
1.*k1sm*hb124(i)*wb26(i)*wb5(i)+k1sm*rhos*hb126(i)*wb26(i)*wb5(i)+k1sm*hb15(i)*w
b26(i)*wb5(i)+k2*hb18(i)*wb8(i)-1.*k2*hb19(i)*wb8(i))-1.*(k2*hb112(i)*wb12(i)-
1.*k2*hb113(i)*wb12(i)+k2*hb116(i)*wb16(i)-
1.*k2*hb117(i)*wb16(i)+k2*hb120(i)*wb20(i)-1.*k2*hb121(i)*wb20(i)+k2*hb18(i)*wb8(i)-
1.*k2*hb19(i)*wb8(i))*(-1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-
1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-
1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i)))-
1.*(1.*aii*art-1.*(-1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-
1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-
1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-
1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i))^2)*(1.*bsr*pr*hb126(i)*(wb11(i)+wb15(i)+wb19
(i)+wb23(i)+wb25(i))*(1.*brs-
1.*pr*hb226(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))+pr*hb227(i)*(wb11(i)+wb15(
i)+wb19(i)+wb23(i)+wb25(i)))-1.*brs*(-1.*bsr*(-
1.*pr*hb126(i)*(wb11(i)+wb15(i)+wb19(i)+wb23(i)+wb25(i))-
(1.*g*hb126(i)*wb26(i))/(b2+wb26(i))-
1.*k1sm*hb124(i)*wb26(i)*wb5(i)+k1sm*rhos*hb126(i)*wb26(i)*wb5(i)+k1sm*hb15(i)*w
b26(i)*wb5(i))+1.*ps*hb126(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))*(1.*bsr+ps*hb
226(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))-
1.*ps*hb227(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i)))))))/(brs*(-
1.*aii*brs*bsr*(k1sm*hb124(i)*wb26(i)*wb5(i)-1.*k1sm*rhos*hb126(i)*wb26(i)*wb5(i)-
1.*k1sm*hb15(i)*wb26(i)*wb5(i))^2-1.*(1.*aii*art-1.*(-
1.*k2*hb112(i)*wb12(i)+k2*hb113(i)*wb12(i)-
1.*k2*hb116(i)*wb16(i)+k2*hb117(i)*wb16(i)-
1.*k2*hb120(i)*wb20(i)+k2*hb121(i)*wb20(i)-
1.*k2*hb18(i)*wb8(i)+k2*hb19(i)*wb8(i))^2)*(1.*bsr*pr^2*hb126(i)*hb226(i)*(wb11(i)+w
b15(i)+wb19(i)+wb23(i)+wb25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*hb126(i)*hb226(i)*(wb13(i)+wb17(i)+wb21(i)+wb24(i)+wb9(i))^2))))));

    if(a11<0 || a11>1)

        a11=0;

    end;



    if(a12<0 || a12>1)

        a12=0;
```

```
end;

if(a13<0 || a13>1)
    a13=0;
end;

if(a14<0 || a14>1)
    a14=0;
end;

if(a15<0 || a15>1)
    a15=0;
end;

if(a16<0 || a16>1)
    a16=0;
end;

    D=[a11,a12,a13,a14,a15,a16;a11,a12,a13,a14,a15,1;a11,a12,a13,a14,a15,0;a11,a12,
a13,a14,1,a16;a11,a12,a13,a14,1,1;a11,a12,a13,a14,1,0;a11,a12,a13,a14,0,a16;a11,a12,a13,a1
4,0,1;a11,a12,a13,a14,0,0;a11,a12,a13,1,a15,a16;a11,a12,a13,1,a15,1;a11,a12,a13,1,a15,0;a1
1,a12,a13,1,1,a16;a11,a12,a13,1,1,1;a11,a12,a13,1,1,0;a11,a12,a13,1,0,a16;a11,a12,a13,1,0,1;
a11,a12,a13,1,0,0;a11,a12,a13,0,a15,a16;a11,a12,a13,0,a15,1;a11,a12,a13,0,a15,0;a11,a12,a1
3,0,1,a16;a11,a12,a13,0,1,1;a11,a12,a13,0,1,0;a11,a12,a13,0,0,a16;a11,a12,a13,0,0,1;a11,a12,
a13,0,0,0;a11,a12,1,a14,a15,a16;a11,a12,1,a14,a15,1;a11,a12,1,a14,a15,0;a11,a12,1,a14,1,a1
6;a11,a12,1,a14,1,1;a11,a12,1,a14,1,0;a11,a12,1,a14,0,a16;a11,a12,1,a14,0,1;a11,a12,1,a14,0,
0;a11,a12,1,1,a15,a16;a11,a12,1,1,a15,1;a11,a12,1,1,a15,0;a11,a12,1,1,1,a16;a11,a12,1,1,1,1;
a11,a12,1,1,1,0;a11,a12,1,1,0,a16;a11,a12,1,1,0,1;a11,a12,1,1,0,0;a11,a12,1,0,a15,a16;a11,a1
2,1,0,a15,1;a11,a12,1,0,a15,0;a11,a12,1,0,1,a16;a11,a12,1,0,1,1;a11,a12,1,0,1,0;a11,a12,1,0,0
,a16;a11,a12,1,0,0,1;a11,a12,1,0,0,0;a11,a12,0,a14,a15,a16;a11,a12,0,a14,a15,1;a11,a12,0,a1
4,a15,0;a11,a12,0,a14,1,a16;a11,a12,0,a14,1,1;a11,a12,0,a14,1,0;a11,a12,0,a14,0,a16;a11,a12
,0,a14,0,1;a11,a12,0,a14,0,0;a11,a12,0,1,a15,a16;a11,a12,0,1,a15,1;a11,a12,0,1,a15,0;a11,a12
,0,1,1,a16;a11,a12,0,1,1,1;a11,a12,0,1,1,0;a11,a12,0,1,0,a16;a11,a12,0,1,0,1;a11,a12,0,1,0,0;a
11,a12,0,0,a15,a16;a11,a12,0,0,a15,1;a11,a12,0,0,a15,0;a11,a12,0,0,1,a16;a11,a12,0,0,1,1;a1
1,a12,0,0,1,0;a11,a12,0,0,0,a16;a11,a12,0,0,0,1;a11,a12,0,0,0,0;a11,1,a13,a14,a15,a16;a11,1,
a13,a14,a15,1;a11,1,a13,a14,a15,0;a11,1,a13,a14,1,a16;a11,1,a13,a14,1,1;a11,1,a13,a14,1,0;a
11,1,a13,a14,0,a16;a11,1,a13,a14,0,1;a11,1,a13,a14,0,0;a11,1,a13,1,a15,a16;a11,1,a13,1,a15,
1;a11,1,a13,1,a15,0;a11,1,a13,1,1,a16;a11,1,a13,1,1,1;a11,1,a13,1,1,0;a11,1,a13,1,0,a16;a11,
1,a13,1,0,1;a11,1,a13,1,0,0;a11,1,a13,0,a15,a16;a11,1,a13,0,a15,1;a11,1,a13,0,a15,0;a11,1,a1
3,0,1,a16;a11,1,a13,0,1,1;a11,1,a13,0,1,0;a11,1,a13,0,0,a16;a11,1,a13,0,0,1;a11,1,a13,0,0,0;a
11,1,1,a14,a15,a16;a11,1,1,a14,a15,1;a11,1,1,a14,a15,0;a11,1,1,a14,1,a16;a11,1,1,a14,1,1;a1
1,1,1,a14,1,0;a11,1,1,a14,0,a16;a11,1,1,a14,0,1;a11,1,1,a14,0,0;a11,1,1,1,a15,a16;a11,1,1,1,a
15,1;a11,1,1,1,a15,0;a11,1,1,1,1,a16;a11,1,1,1,1,1;a11,1,1,1,1,0;a11,1,1,1,0,a16;a11,1,1,1,0,1;
a11,1,1,1,0,0;a11,1,1,0,a15,a16;a11,1,1,0,a15,1;a11,1,1,0,a15,0;a11,1,1,0,1,a16;a11,1,1,0,1,1;
```

a11,1,1,0,1,0;a11,1,1,0,0,a16;a11,1,1,0,0,1;a11,1,1,0,0,0;a11,1,0,a14,a15,a16;a11,1,0,a14,a15,
1;a11,1,0,a14,a15,0;a11,1,0,a14,1,a16;a11,1,0,a14,1,1;a11,1,0,a14,1,0;a11,1,0,a14,0,a16;a11,
1,0,a14,0,1;a11,1,0,a14,0,0;a11,1,0,1,a15,a16;a11,1,0,1,a15,1;a11,1,0,1,a15,0;a11,1,0,1,1,a16;
a11,1,0,1,1,1;a11,1,0,1,1,0;a11,1,0,1,0,a16;a11,1,0,1,0,1;a11,1,0,1,0,0;a11,1,0,0,a15,a16;a11,
1,0,0,a15,1;a11,1,0,0,a15,0;a11,1,0,0,1,a16;a11,1,0,0,1,1;a11,1,0,0,1,0;a11,1,0,0,0,a16;a11,1,
0,0,0,1;a11,1,0,0,0,0;a11,0,a13,a14,a15,a16;a11,0,a13,a14,a15,1;a11,0,a13,a14,a15,0;a11,0,a1
3,a14,1,a16;a11,0,a13,a14,1,1;a11,0,a13,a14,1,0;a11,0,a13,a14,0,a16;a11,0,a13,a14,0,1;a11,0,
a13,a14,0,0;a11,0,a13,1,a15,a16;a11,0,a13,1,a15,1;a11,0,a13,1,a15,0;a11,0,a13,1,1,a16;a11,0,
a13,1,1,1;a11,0,a13,1,1,0;a11,0,a13,1,0,a16;a11,0,a13,1,0,1;a11,0,a13,1,0,0;a11,0,a13,0,a15,a
16;a11,0,a13,0,a15,1;a11,0,a13,0,a15,0;a11,0,a13,0,1,a16;a11,0,a13,0,1,1;a11,0,a13,0,1,0;a11
,0,a13,0,0,a16;a11,0,a13,0,0,1;a11,0,a13,0,0,0;a11,0,1,a14,a15,a16;a11,0,1,a14,a15,1;a11,0,1,
a14,a15,0;a11,0,1,a14,1,a16;a11,0,1,a14,1,1;a11,0,1,a14,1,0;a11,0,1,a14,0,a16;a11,0,1,a14,0,
1;a11,0,1,a14,0,0;a11,0,1,1,a15,a16;a11,0,1,1,a15,1;a11,0,1,1,a15,0;a11,0,1,1,1,a16;a11,0,1,1,
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,1,a16;0,0,0,0,1,1;0,0,0,0,1,0;0,0,0,0,0,a16;0,0,0,0,0,1;0,0,0,0,0,0];

eval_HT=eval(fHT);

[max_HT, id_HT]=nanmax(eval_HT);

u11(4)=D(id_HT,1);
u12(4)=D(id_HT,2);
u13(4)=D(id_HT,3);
u14(4)=D(id_HT,4);
u15(4)=D(id_HT,5);
u16(4)=D(id_HT,6);
```

u21(4)=u21s;

u22(4)=u22s;

%LAMBDA1(4)

ka11(4)=-1-k*hb12(i)*wb26(i)+k1s*(1-qthvs)*rhos*hb126(i)*(1-u11(4))*(1-u12(4))*wb26(i)-hb18(i)*(k3*wb24(i)+k1s*(1-qthvs)*(1-u11(4))*(1-u12(4))*wb26(i))+k1r*(1-qthvr)*rhor*hb127(i)*wb27(i)-k*hb13(i)*wb27(i)-kprimo*hb14(i)*(wb26(i)+wb27(i))-hb110(i)*(k3*wb25(i)+k1r*(1-qthvr)*wb27(i))-c*hb16(i)*wb6(i)*(wb11(i)+wb13(i)+wb15(i)+wb17(i)+wb19(i)+wb21(i)+wb23(i)+wb9(i))-hb11(i)*(-muth+u13(4)-(r*wb1(i))/tmax-k3*(wb24(i)+wb25(i))-k1s*(1-qthvs)*(1-u11(4))*(1-u12(4))*wb26(i)-k1r*(1-qthvr)*wb27(i)-k*(wb26(i)+wb27(i))-kprimo*(wb26(i)+wb27(i))+r*(1-(wb1(i)+wb10(i)+wb11(i)+wb12(i)+wb13(i)+wb14(i)+wb15(i)+wb16(i)+wb17(i)+wb18(i)+wb19(i)+wb2(i)+wb20(i)+wb21(i)+wb22(i)+wb23(i)+wb3(i)+wb4(i)+wb8(i)+wb9(i))/tmax)); ka12(4)=-1+(r*hb11(i)*wb1(i))/tmax-hb112(i)*(k3*wb24(i)+k1s*(1-qtsvs)*(1-u11(4))*(1-u12(4))*wb26(i))-hb126(i)*(-(psprimo*wb26(i))-k1s*(1-qtsvs)*rhos*(1-u11(4))*(1-u12(4))*wb26(i))+k1r*(1-qtsvr)*rhor*hb127(i)*wb27(i)-hb12(i)*(-muts+u13(4)-k3*(wb24(i)+wb25(i))-k1s*(1-qtsvs)*(1-u11(4))*(1-u12(4))*wb26(i)-k1r*(1-qtsvr)*wb27(i))-hb114(i)*(k3*wb25(i)+k1r*(1-qtsvr)*wb27(i))-c*hb16(i)*wb6(i)*(wb11(i)+wb13(i)+wb15(i)+wb17(i)+wb19(i)+wb21(i)+wb23(i)+wb9(i)); ka13(4)=-1+(r*hb11(i)*wb1(i))/tmax+k1s*(1-qtrvs)*rhos*hb126(i)*(1-u11(4))*(1-u12(4))*wb26(i)-hb116(i)*(k3*wb24(i)+k1s*(1-qtrvs)*(1-u11(4))*(1-u12(4))*wb26(i))-hb13(i)*(-mutr+u13(4)-k3*(wb24(i)+wb25(i))-k1s*(1-qtrvs)*(1-u11(4))*(1-u12(4))*wb26(i)-k1r*(1-qtrvr)*wb27(i))-hb118(i)*(k3*wb25(i)+k1r*(1-qtrvr)*wb27(i))-hb127(i)*(-(prprimo*wb27(i))-k1r*(1-qtrvr)*rhor*wb27(i))-c*hb16(i)*wb6(i)*(wb11(i)+wb13(i)+wb15(i)+wb17(i)+wb19(i)+wb21(i)+wb23(i)+wb9(i)); ka14(4)=-1+(r*hb11(i)*wb1(i))/tmax-hb120(i)*(k3*wb24(i)+k1s*(1-qtgvs)*(1-u11(4))*(1-u12(4))*wb26(i))-hb126(i)*(-(ssprimo*wb26(i))-k1s*(1-qtgvs)*rhos*(1-u11(4))*(1-u12(4))*wb26(i))-hb14(i)*(-mutg+u13(4)-k3*(wb24(i)+wb25(i))-k1s*(1-qtgvs)*(1-u11(4))*(1-u12(4))*wb26(i)-k1r*(1-qtgvr)*wb27(i))-hb122(i)*(k3*wb25(i)+k1r*(1-qtgvr)*wb27(i))-hb127(i)*(-(k1r*(1-qtgvr)*rhor*wb27(i))-srprimo*wb27(i))-c*hb16(i)*wb6(i)*(wb11(i)+wb13(i)+wb15(i)+wb17(i)+wb19(i)+wb21(i)+wb23(i)+wb9(i)); ka15(4)=-1-k1sm*hb124(i)*(1-u15(4))*(1-u16(4))*wb26(i)+k1sm*rhos*hb126(i)*(1-u15(4))*(1-u16(4))*wb26(i)-k1rm*hb125(i)*wb27(i)+k1rm*rhor*hb127(i)*wb27(i)-hb15(i)*(-mumh-k1sm*(1-u15(4))*(1-u16(4))*wb26(i)-k1rm*wb27(i)); ka16(4)=-1-c*q*hb17(i)*(wb11(i)+wb13(i)+wb15(i)+wb17(i)+wb19(i)+wb21(i)+wb23(i)+wb9(i))-hb16(i)*(-muip-c*q*(wb11(i)+wb13(i)+wb15(i)+wb17(i)+wb19(i)+wb21(i)+wb23(i)+wb9(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*(wb11(i)+wb13(i)+wb15(i)+wb17(i)+wb19(i)+wb21(i)+wb23(i)+wb9(i))); ka17(4)=-1+muie*hb17(i)+ir*hb111(i)*wb11(i)+is*hb113(i)*wb13(i)+ir*hb115(i)*wb15(i)+is*hb117(i)*wb17(i)+ir*hb119(i)*wb19(i)+is*hb121(i)*wb21(i)+ir*hb123(i)*wb23(i)+is*hb19(i)*wb9(i); ka18(4)=-(hb18(i)*(-muthsl-k2*(1-u14(4))*(1-u16(4))))-k2*hb19(i)*(1-u14(4))*(1-u16(4))+(r*hb11(i)*wb1(i))/tmax; ka19(4)=-(ps*hb127(i)*(1-u21(4)))-ps*hb126(i)*(1-u15(4))*u21(4)+(r*hb11(i)*wb1(i))/tmax-c*q*hb17(i)*wb6(i)-hb16(i)*(-(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-hb19(i)*(-muthsa-is*wb7(i)); ka110(4)=-((-k2-muthrl)*hb110(i))-k2*hb111(i)+(r*hb11(i)*wb1(i))/tmax; ka111(4)=-(pr*hb126(i)*(1-u15(4))*(1-u22(4)))-pr*hb127(i)*u22(4)+(r*hb11(i)*wb1(i))/tmax-c*q*hb17(i)*wb6(i)-hb16(i)*(-(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-hb111(i)*(-muthra-ir*wb7(i)); ka112(4)=-(hb112(i)*(-mutssl-k2*(1-u14(4))*(1-u16(4))))-k2*hb113(i)*(1-u14(4))*(1-u16(4))+(r*hb11(i)*wb1(i))/tmax; ka113(4)=-(ps*hb127(i)*(1-u21(4)))-ps*hb126(i)*(1-u15(4))*u21(4)+(r*hb11(i)*wb1(i))/tmax-c*q*hb17(i)*wb6(i)-hb16(i)*(-(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-

hb113(i)*(-mutssa-is*wb7(i)); ka114(4)=-((-k2-mutsrl)*hb114(i))-k2*hb115(i)+(r*hb11(i)*wb1(i))/tmax; ka115(4)=-(pr*hb126(i)*(1-u15(4))*(1-u22(4)))-pr*hb127(i)*u22(4)+(r*hb11(i)*wb1(i))/tmax-c*q*hb17(i)*wb6(i)-hb16(i)*(-(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-hb115(i)*(-mutsra-ir*wb7(i)); ka116(4)=-(hb116(i)*(-mutrsl-k2*(1-u14(4))*(1-u16(4))))-k2*hb117(i)*(1-u14(4))*(1-u16(4))+(r*hb11(i)*wb1(i))/tmax; ka117(4)=-(ps*hb127(i)*(1-u21(4)))-ps*hb126(i)*(1-u15(4))*u21(4)+(r*hb11(i)*wb1(i))/tmax-c*q*hb17(i)*wb6(i)-hb16(i)*(-(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-hb117(i)*(-mutrsa-is*wb7(i)); ka118(4)=-((-k2-mutrrl)*hb118(i))-k2*hb119(i)+(r*hb11(i)*wb1(i))/tmax; ka119(4)=-(pr*hb126(i)*(1-u15(4))*(1-u22(4)))-pr*hb127(i)*u22(4)+(r*hb11(i)*wb1(i))/tmax-c*q*hb17(i)*wb6(i)-hb16(i)*(-(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-hb119(i)*(-mutrra-ir*wb7(i)); ka120(4)=-(hb120(i)*(-mutgsl-k2*(1-u14(4))*(1-u16(4))))-k2*hb121(i)*(1-u14(4))*(1-u16(4))+(r*hb11(i)*wb1(i))/tmax; ka121(4)=-(ps*hb127(i)*(1-u21(4)))-ps*hb126(i)*(1-u15(4))*u21(4)+(r*hb11(i)*wb1(i))/tmax-c*q*hb17(i)*wb6(i)-hb16(i)*(-(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-hb121(i)*(-mutgsa-is*wb7(i)); ka122(4)=-((-k2-mutgrl)*hb122(i))-k2*hb123(i)+(r*hb11(i)*wb1(i))/tmax; ka123(4)=-(pr*hb126(i)*(1-u15(4))*(1-u22(4)))-pr*hb127(i)*u22(4)+(r*hb11(i)*wb1(i))/tmax-c*q*hb17(i)*wb6(i)-hb16(i)*(-(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-hb123(i)*(-mutgra-ir*wb7(i)); ka124(4)=mumhsa*hb124(i)-ps*hb127(i)*(1-u21(4))-ps*hb126(i)*(1-u15(4))*u21(4)+k3*hb11(i)*wb1(i)-k3*hb18(i)*wb1(i)-k3*hb112(i)*wb2(i)+k3*hb12(i)*wb2(i)-k3*hb116(i)*wb3(i)+k3*hb13(i)*wb3(i)-k3*hb120(i)*wb4(i)+k3*hb14(i)*wb4(i); ka125(4)=mumhra*hb125(i)-pr*hb126(i)*(1-u15(4))*(1-u22(4))-pr*hb127(i)*u22(4)+k3*hb11(i)*wb1(i)-k3*hb110(i)*wb1(i)-k3*hb114(i)*wb2(i)+k3*hb12(i)*wb2(i)-k3*hb118(i)*wb3(i)+k3*hb13(i)*wb3(i)-k3*hb122(i)*wb4(i)+k3*hb14(i)*wb4(i); ka126(4)=-(k1s*(1-qthvs)*hb18(i)*(1-u11(4))*(1-u12(4))*wb1(i))-k1s*(1-qtsvs)*hb112(i)*(1-u11(4))*(1-u12(4))*wb2(i)-hb12(i)*(k*wb1(i)-k1s*(1-qtsvs)*(1-u11(4))*(1-u12(4))*wb2(i))-hb11(i)*(-(k*wb1(i))-kprimo*wb1(i)-k1s*(1-qthvs)*(1-u11(4))*(1-u12(4))*wb1(i)-(oth*s1th)/(oth+wb26(i)+wb27(i))^2)-k1s*(1-qtrvs)*hb116(i)*(1-u11(4))*(1-u12(4))*wb3(i)+k1s*(1-qtrvs)*hb13(i)*(1-u11(4))*(1-u12(4))*wb3(i)-k1s*(1-qtgvs)*hb120(i)*(1-u11(4))*(1-u12(4))*wb4(i)-hb14(i)*(kprimo*wb1(i)-k1s*(1-qtgvs)*(1-u11(4))*(1-u12(4))*wb4(i))-k1sm*hb124(i)*(1-u15(4))*(1-u16(4))*wb5(i)+k1sm*hb15(i)*(1-u15(4))*(1-u16(4))*wb5(i)-hb126(i)*(-muvs-psprimo*wb2(i)-(g*(1-u15(4))*wb26(i))/(b2+wb26(i))^2+(g*(1-u15(4)))/(b2+wb26(i))-ssprimo*wb4(i)-k1s*rhos*(1-u11(4))*(1-u12(4))*((1-qthvs)*wb1(i)+(1-qtsvs)*wb2(i)+(1-qtrvs)*wb3(i)+(1-qtgvs)*wb4(i))-k1sm*rhos*(1-u15(4))*(1-u16(4))*wb5(i)); ka127(4)=-(k1r*(1-qthvr)*hb110(i)*wb1(i))-k1r*(1-qtsvr)*hb114(i)*wb2(i)+k1r*(1-qtsvr)*hb12(i)*wb2(i)-hb11(i)*(-(k*wb1(i))-kprimo*wb1(i)-k1r*(1-qthvr)*wb1(i)-(oth*s1th)/(oth+wb26(i)+wb27(i))^2)-k1r*(1-qtrvr)*hb118(i)*wb3(i)-hb13(i)*(k*wb1(i)-k1r*(1-qtrvr)*wb3(i))-k1r*(1-qtgvr)*hb122(i)*wb4(i)-hb14(i)*(kprimo*wb1(i)-k1r*(1-qtgvr)*wb4(i))-k1rm*hb125(i)*wb5(i)+k1rm*hb15(i)*wb5(i)-hb127(i)*(-muvr-(g*wb27(i))/(b2+wb27(i))^2+g/(b2+wb27(i))-prprimo*wb3(i)-srprimo*wb4(i)-k1r*rhor*((1-qthvr)*wb1(i)+(1-qtsvr)*wb2(i)+(1-qtrvr)*wb3(i)+(1-qtgvr)*wb4(i))-k1rm*rhor*wb5(i));
%LAMBDA2(4)

ka21(4)=-(k*hb22(i)*wb26(i))+k1s*(1-qthvs)*rhos*hb226(i)*(1-u11(4))*(1-u12(4))*wb26(i)-hb28(i)*(k3*wb24(i)+k1s*(1-qthvs)*(1-u11(4))*(1-u12(4))*wb26(i))+k1r*(1-qthvr)*rhor*hb227(i)*wb27(i)-k*hb23(i)*wb27(i)-kprimo*hb24(i)*(wb26(i)+wb27(i))-hb210(i)*(k3*wb25(i)+k1r*(1-qthvr)*wb27(i))-c*hb26(i)*wb6(i)*(wb11(i)+wb13(i)+wb15(i)+wb17(i)+wb19(i)+wb21(i)+wb23(i)+wb9(i))-hb21(i)*(-muth+u13(4)-(r*wb1(i))/tmax-k3*(wb24(i)+wb25(i))-k1s*(1-qthvs)*(1-u11(4))*(1-u12(4))*wb26(i)-k1r*(1-qthvr)*wb27(i)-k*(wb26(i)+wb27(i))-kprimo*(wb26(i)+wb27(i))+r*(1-(wb1(i)+wb10(i)+wb11(i)+wb12(i)+wb13(i)+wb14(i)+wb15(i)+wb16(i)+wb17(i)+wb18(i)+wb19(i)+wb2(i)+wb20(i)+wb21(i)+wb22(i)+wb23(i)+wb3(i)+wb4(i)+wb8(i)+wb9(i))/tmax)); ka22(4)=(r*hb21(i)*wb1(i))/tmax-hb212(i)*(k3*wb24(i)+k1s*(1-qtsvs)*(1-u11(4))*(1-u12(4))*wb26(i))-hb226(i)*(-(psprimo*wb26(i))-k1s*(1-qtsvs)*rhos*(1-u11(4))*(1-u12(4))*wb26(i))+k1r*(1-qtsvr)*rhor*hb227(i)*wb27(i)-hb22(i)*(-muts+u13(4)-k3*(wb24(i)+wb25(i))-k1s*(1-qtsvs)*(1-u11(4))*(1-u12(4))*wb26(i)-k1r*(1-

qtsvr)*wb27(i))-hb214(i)*(k3*wb25(i)+k1r*(1-qtsvr)*wb27(i))-
c*hb26(i)*wb6(i)*(wb11(i)+wb13(i)+wb15(i)+wb17(i)+wb19(i)+wb21(i)+wb23(i)+wb9(i));
ka23(4)=(r*hb21(i)*wb1(i))/tmax+k1s*(1-qtrvs)*rhos*hb226(i)*(1-u11(4))*(1-
u12(4))*wb26(i)-hb216(i)*(k3*wb24(i)+k1s*(1-qtrvs)*(1-u11(4))*(1-u12(4))*wb26(i))-
hb23(i)*(-mutr+u13(4)-k3*(wb24(i)+wb25(i))-k1s*(1-qtrvs)*(1-u11(4))*(1-u12(4))*wb26(i)-
k1r*(1-qtrvr)*wb27(i))-hb218(i)*(k3*wb25(i)+k1r*(1-qtrvr)*wb27(i))-hb227(i)*(-
(prprimo*wb27(i))-k1r*(1-qtrvr)*rhor*wb27(i))-
c*hb26(i)*wb6(i)*(wb11(i)+wb13(i)+wb15(i)+wb17(i)+wb19(i)+wb21(i)+wb23(i)+wb9(i));
ka24(4)=(r*hb21(i)*wb1(i))/tmax-hb220(i)*(k3*wb24(i)+k1s*(1-qtgvs)*(1-u11(4))*(1-
u12(4))*wb26(i))-hb226(i)*(-(ssprimo*wb26(i))-k1s*(1-qtgvs)*rhos*(1-u11(4))*(1-
u12(4))*wb26(i))-hb24(i)*(-mutg+u13(4)-k3*(wb24(i)+wb25(i))-k1s*(1-qtgvs)*(1-
u11(4))*(1-u12(4))*wb26(i)-k1r*(1-qtgvr)*wb27(i))-hb222(i)*(k3*wb25(i)+k1r*(1-
qtgvr)*wb27(i))-hb227(i)*(-(k1r*(1-qtgvr)*rhor*wb27(i))-srprimo*wb27(i))-
c*hb26(i)*wb6(i)*(wb11(i)+wb13(i)+wb15(i)+wb17(i)+wb19(i)+wb21(i)+wb23(i)+wb9(i));
ka25(4)=-(k1sm*hb224(i)*(1-u15(4))*(1-u16(4))*wb26(i))+k1sm*rhos*hb226(i)*(1-
u15(4))*(1-u16(4))*wb26(i)-k1rm*hb225(i)*wb27(i)+k1rm*rhor*hb227(i)*wb27(i)-
hb25(i)*(-mumh-k1sm*(1-u15(4))*(1-u16(4))*wb26(i)-k1rm*wb27(i)); ka26(4)=-
(c*q*hb27(i)*(wb11(i)+wb13(i)+wb15(i)+wb17(i)+wb19(i)+wb21(i)+wb23(i)+wb9(i)))-
hb26(i)*(-muip-
c*q*(wb11(i)+wb13(i)+wb15(i)+wb17(i)+wb19(i)+wb21(i)+wb23(i)+wb9(i))+c*(wb1(i)+wb
2(i)+wb3(i)+wb4(i))*(wb11(i)+wb13(i)+wb15(i)+wb17(i)+wb19(i)+wb21(i)+wb23(i)+wb9(i
)));
ka27(4)=muie*hb27(i)+ir*hb211(i)*wb11(i)+is*hb213(i)*wb13(i)+ir*hb215(i)*wb15(i)+is*h
b217(i)*wb17(i)+ir*hb219(i)*wb19(i)+is*hb221(i)*wb21(i)+ir*hb223(i)*wb23(i)+is*hb29(i)
wb9(i); ka28(4)=-1-hb28(i)(-muthsl-k2*(1-u14(4))*(1-u16(4)))-k2*hb29(i)*(1-
u14(4))*(1-u16(4))+(r*hb21(i)*wb1(i))/tmax; ka29(4)= -1-ps*hb227(i)*(1-u21(4))-
ps*hb226(i)*(1-u15(4))*u21(4)+(r*hb21(i)*wb1(i))/tmax-c*q*hb27(i)*wb6(i)-hb26(i)*(-
(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-hb29(i)*(-muthsa-is*wb7(i));
ka210(4)=-1-(-k2-muthrl)*hb210(i)-k2*hb211(i)+(r*hb21(i)*wb1(i))/tmax; ka211(4)= -
1-pr*hb226(i)*(1-u15(4))*(1-u22(4))-pr*hb227(i)*u22(4)+(r*hb21(i)*wb1(i))/tmax-
c*q*hb27(i)*wb6(i)-hb26(i)*(-(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-
hb211(i)*(-muthra-ir*wb7(i)); ka212(4)=-1-hb212(i)*(-mutssl-k2*(1-u14(4))*(1-
u16(4)))-k2*hb213(i)*(1-u14(4))*(1-u16(4))+(r*hb21(i)*wb1(i))/tmax; ka213(4)=-1-
ps*hb227(i)*(1-u21(4))-ps*hb226(i)*(1-u15(4))*u21(4)+(r*hb21(i)*wb1(i))/tmax-
c*q*hb27(i)*wb6(i)-hb26(i)*(-(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-
hb213(i)*(-mutssa-is*wb7(i)); ka214(4)=-1-(-k2-mutsrl)*hb214(i)-
k2*hb215(i)+(r*hb21(i)*wb1(i))/tmax; ka215(4)=-1-pr*hb226(i)*(1-u15(4))*(1-u22(4))-
pr*hb227(i)*u22(4)+(r*hb21(i)*wb1(i))/tmax-c*q*hb27(i)*wb6(i)-hb26(i)*(-
(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-hb215(i)*(-mutsra-ir*wb7(i));
ka216(4)=-1-hb216(i)*(-mutrsl-k2*(1-u14(4))*(1-u16(4)))-k2*hb217(i)*(1-u14(4))*(1-
u16(4))+(r*hb21(i)*wb1(i))/tmax; ka217(4)=-1-ps*hb227(i)*(1-u21(4))-ps*hb226(i)*(1-
u15(4))*u21(4)+(r*hb21(i)*wb1(i))/tmax-c*q*hb27(i)*wb6(i)-hb26(i)*(-
(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-hb217(i)*(-mutrsa-is*wb7(i));
ka218(4)=-1-(-k2-mutrrl)*hb218(i)-k2*hb219(i)+(r*hb21(i)*wb1(i))/tmax; ka219(4)=-1-
pr*hb226(i)*(1-u15(4))*(1-u22(4))-pr*hb227(i)*u22(4)+(r*hb21(i)*wb1(i))/tmax-
c*q*hb27(i)*wb6(i)-hb26(i)*(-(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-
hb219(i)*(-mutrra-ir*wb7(i)); ka220(4)=-1-hb220(i)*(-mutgsl-k2*(1-u14(4))*(1-
u16(4)))-k2*hb221(i)*(1-u14(4))*(1-u16(4))+(r*hb21(i)*wb1(i))/tmax; ka221(4)=-1-
ps*hb227(i)*(1-u21(4))-ps*hb226(i)*(1-u15(4))*u21(4)+(r*hb21(i)*wb1(i))/tmax-
c*q*hb27(i)*wb6(i)-hb26(i)*(-(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-
hb221(i)*(-mutgsa-is*wb7(i)); ka222(4)=-1-(-k2-mutgrl)*hb222(i)-
k2*hb223(i)+(r*hb21(i)*wb1(i))/tmax; ka223(4)=-1-pr*hb226(i)*(1-u15(4))*(1-u22(4))-
pr*hb227(i)*u22(4)+(r*hb21(i)*wb1(i))/tmax-c*q*hb27(i)*wb6(i)-hb26(i)*(-
(c*q*wb6(i))+c*(wb1(i)+wb2(i)+wb3(i)+wb4(i))*wb6(i))-hb223(i)*(-mutgra-ir*wb7(i));
ka224(4)=mumhsa*hb224(i)-ps*hb227(i)*(1-u21(4))-ps*hb226(i)*(1-
u15(4))*u21(4)+k3*hb21(i)*wb1(i)-k3*hb28(i)*wb1(i)-
k3*hb212(i)*wb2(i)+k3*hb22(i)*wb2(i)-k3*hb216(i)*wb3(i)+k3*hb23(i)*wb3(i)-
k3*hb220(i)*wb4(i)+k3*hb24(i)*wb4(i); ka225(4)=mumhra*hb225(i)-pr*hb226(i)*(1-
u15(4))*(1-u22(4))-pr*hb227(i)*u22(4)+k3*hb21(i)*wb1(i)-k3*hb210(i)*wb1(i)-
k3*hb214(i)*wb2(i)+k3*hb22(i)*wb2(i)-k3*hb218(i)*wb3(i)+k3*hb23(i)*wb3(i)-

```
k3*hb222(i)*wb4(i)+k3*hb24(i)*wb4(i);    ka226(4)=-1-k1s*(1-qthvs)*hb28(i)*(1-
u11(4))*(1-u12(4))*wb1(i)-k1s*(1-qtsvs)*hb212(i)*(1-u11(4))*(1-u12(4))*wb2(i)-
hb22(i)*(k*wb1(i)-k1s*(1-qtsvs)*(1-u11(4))*(1-u12(4))*wb2(i))-hb21(i)*(-(k*wb1(i))-
kprimo*wb1(i)-k1s*(1-qthvs)*(1-u11(4))*(1-u12(4))*wb1(i)-
(oth*s1th)/(oth+wb26(i)+wb27(i))^2)-k1s*(1-qtrvs)*hb216(i)*(1-u11(4))*(1-
u12(4))*wb3(i)+k1s*(1-qtrvs)*hb23(i)*(1-u11(4))*(1-u12(4))*wb3(i)-k1s*(1-
qtgvs)*hb220(i)*(1-u11(4))*(1-u12(4))*wb4(i)-hb24(i)*(kprimo*wb1(i)-k1s*(1-qtgvs)*(1-
u11(4))*(1-u12(4))*wb4(i))-k1sm*hb224(i)*(1-u15(4))*(1-
u16(4))*wb5(i)+k1sm*hb25(i)*(1-u15(4))*(1-u16(4))*wb5(i)-hb226(i)*(-muvs-
psprimo*wb2(i)-(g*(1-u15(4))*wb26(i))/(b2+wb26(i))^2+(g*(1-u15(4)))/(b2+wb26(i))-
ssprimo*wb4(i)-k1s*rhos*(1-u11(4))*(1-u12(4))*((1-qthvs)*wb1(i)+(1-qtsvs)*wb2(i)+(1-
qtrvs)*wb3(i)+(1-qtgvs)*wb4(i))-k1sm*rhos*(1-u15(4))*(1-u16(4))*wb5(i));    ka227(4)=-
1-k1r*(1-qthvr)*hb210(i)*wb1(i)-k1r*(1-qtsvr)*hb214(i)*wb2(i)+k1r*(1-
qtsvr)*hb22(i)*wb2(i)-hb21(i)*(-(k*wb1(i))-kprimo*wb1(i)-k1r*(1-qthvr)*wb1(i)-
(oth*s1th)/(oth+wb26(i)+wb27(i))^2)-k1r*(1-qtrvr)*hb218(i)*wb3(i)-hb23(i)*(k*wb1(i)-
k1r*(1-qtrvr)*wb3(i))-k1r*(1-qtgvr)*hb222(i)*wb4(i)-hb24(i)*(kprimo*wb1(i)-k1r*(1-
qtgvr)*wb4(i))-k1rm*hb225(i)*wb5(i)+k1rm*hb25(i)*wb5(i)-hb227(i)*(-muvr-
(g*wb27(i))/(b2+wb27(i))^2+g/(b2+wb27(i))-prprimo*wb3(i)-srprimo*wb4(i)-k1r*rhor*((1-
qthvr)*wb1(i)+(1-qtsvr)*wb2(i)+(1-qtrvr)*wb3(i)+(1-qtgvr)*wb4(i))-k1rm*rhor*wb5(i));


%RUNGE KUTTA 4° ORDINE PER LE VARIABILI AGG lambda 1

l11(i-1)=l11(i)+(h/6)*(ka11(1)+2*ka11(2)+2*ka11(3)+ka11(4));

l12(i-1)=l12(i)+(h/6)*(ka12(1)+2*ka12(2)+2*ka12(3)+ka12(4));

l13(i-1)=l13(i)+(h/6)*(ka13(1)+2*ka13(2)+2*ka13(3)+ka13(4));

l14(i-1)=l14(i)+(h/6)*(ka14(1)+2*ka14(2)+2*ka14(3)+ka14(4));

l15(i-1)=l15(i)+(h/6)*(ka15(1)+2*ka15(2)+2*ka15(3)+ka15(4));

l16(i-1)=l16(i)+(h/6)*(ka16(1)+2*ka16(2)+2*ka16(3)+ka16(4));

l17(i-1)=l17(i)+(h/6)*(ka17(1)+2*ka17(2)+2*ka17(3)+ka17(4));

l18(i-1)=l18(i)+(h/6)*(ka18(1)+2*ka18(2)+2*ka18(3)+ka18(4));

l19(i-1)=l19(i)+(h/6)*(ka19(1)+2*ka19(2)+2*ka19(3)+ka19(4));

l110(i-1)=l110(i)+(h/6)*(ka110(1)+2*ka110(2)+2*ka110(3)+ka110(4));

l111(i-1)=l111(i)+(h/6)*(ka111(1)+2*ka111(2)+2*ka111(3)+ka111(4));

l112(i-1)=l112(i)+(h/6)*(ka112(1)+2*ka112(2)+2*ka112(3)+ka112(4));

l113(i-1)=l113(i)+(h/6)*(ka113(1)+2*ka113(2)+2*ka113(3)+ka113(4));

l114(i-1)=l114(i)+(h/6)*(ka114(1)+2*ka114(2)+2*ka114(3)+ka114(4));

l115(i-1)=l115(i)+(h/6)*(ka115(1)+2*ka115(2)+2*ka115(3)+ka115(4));

l116(i-1)=l116(i)+(h/6)*(ka116(1)+2*ka116(2)+2*ka116(3)+ka116(4));

l117(i-1)=l117(i)+(h/6)*(ka117(1)+2*ka117(2)+2*ka117(3)+ka117(4));

l118(i-1)=l118(i)+(h/6)*(ka118(1)+2*ka118(2)+2*ka118(3)+ka118(4));

l119(i-1)=l119(i)+(h/6)*(ka119(1)+2*ka119(2)+2*ka119(3)+ka119(4));

l120(i-1)=l120(i)+(h/6)*(ka120(1)+2*ka120(2)+2*ka120(3)+ka120(4));
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l121(i-1)=l121(i)+(h/6)*(ka121(1)+2*ka121(2)+2*ka121(3)+ka121(4));
l122(i-1)=l122(i)+(h/6)*(ka122(1)+2*ka122(2)+2*ka122(3)+ka122(4));
l123(i-1)=l123(i)+(h/6)*(ka123(1)+2*ka123(2)+2*ka123(3)+ka123(4));
l124(i-1)=l124(i)+(h/6)*(ka124(1)+2*ka124(2)+2*ka124(3)+ka124(4));
l125(i-1)=l125(i)+(h/6)*(ka125(1)+2*ka125(2)+2*ka125(3)+ka125(4));
l126(i-1)=l126(i)+(h/6)*(ka126(1)+2*ka126(2)+2*ka126(3)+ka126(4));
l127(i-1)=l127(i)+(h/6)*(ka127(1)+2*ka127(2)+2*ka127(3)+ka127(4));


%RUNGE KUTTA 4° ORDINE  PER LE VARIABILI AGG lambda 2
l21(i-1)=l21(i)+(h/6)*(ka21(1)+2*ka21(2)+2*ka21(3)+ka21(4));
l22(i-1)=l22(i)+(h/6)*(ka22(1)+2*ka22(2)+2*ka22(3)+ka22(4));
l23(i-1)=l23(i)+(h/6)*(ka23(1)+2*ka23(2)+2*ka23(3)+ka23(4));
l24(i-1)=l24(i)+(h/6)*(ka24(1)+2*ka24(2)+2*ka24(3)+ka24(4));
l25(i-1)=l25(i)+(h/6)*(ka25(1)+2*ka25(2)+2*ka25(3)+ka25(4));
l26(i-1)=l26(i)+(h/6)*(ka26(1)+2*ka26(2)+2*ka26(3)+ka26(4));
l27(i-1)=l27(i)+(h/6)*(ka27(1)+2*ka27(2)+2*ka27(3)+ka27(4));
l28(i-1)=l28(i)+(h/6)*(ka28(1)+2*ka28(2)+2*ka28(3)+ka28(4));
l29(i-1)=l29(i)+(h/6)*(ka29(1)+2*ka29(2)+2*ka29(3)+ka29(4));
l210(i-1)=l210(i)+(h/6)*(ka210(1)+2*ka210(2)+2*ka210(3)+ka210(4));
l211(i-1)=l211(i)+(h/6)*(ka211(1)+2*ka211(2)+2*ka211(3)+ka211(4));
l212(i-1)=l212(i)+(h/6)*(ka212(1)+2*ka212(2)+2*ka212(3)+ka212(4));
l213(i-1)=l213(i)+(h/6)*(ka213(1)+2*ka213(2)+2*ka213(3)+ka213(4));
l214(i-1)=l214(i)+(h/6)*(ka214(1)+2*ka214(2)+2*ka214(3)+ka214(4));
l215(i-1)=l215(i)+(h/6)*(ka215(1)+2*ka215(2)+2*ka215(3)+ka215(4));
l216(i-1)=l216(i)+(h/6)*(ka216(1)+2*ka216(2)+2*ka216(3)+ka216(4));
l217(i-1)=l217(i)+(h/6)*(ka217(1)+2*ka217(2)+2*ka217(3)+ka217(4));
l218(i-1)=l218(i)+(h/6)*(ka218(1)+2*ka218(2)+2*ka218(3)+ka218(4));
l219(i-1)=l219(i)+(h/6)*(ka219(1)+2*ka219(2)+2*ka219(3)+ka219(4));
l220(i-1)=l220(i)+(h/6)*(ka220(1)+2*ka220(2)+2*ka220(3)+ka220(4));
l221(i-1)=l221(i)+(h/6)*(ka221(1)+2*ka221(2)+2*ka221(3)+ka221(4));
l222(i-1)=l222(i)+(h/6)*(ka222(1)+2*ka222(2)+2*ka222(3)+ka222(4));
l223(i-1)=l223(i)+(h/6)*(ka223(1)+2*ka223(2)+2*ka223(3)+ka223(4));
```

```
  l224(i-1)=l224(i)+(h/6)*(ka224(1)+2*ka224(2)+2*ka224(3)+ka224(4));
  l225(i-1)=l225(i)+(h/6)*(ka225(1)+2*ka225(2)+2*ka225(3)+ka225(4));
  l226(i-1)=l226(i)+(h/6)*(ka226(1)+2*ka226(2)+2*ka226(3)+ka226(4));
  l227(i-1)=l227(i)+(h/6)*(ka227(1)+2*ka227(2)+2*ka227(3)+ka227(4));

end %%%%%%%%%%%%%end for BACKWARD PROCESS

x=[x1; x2; x3; x4; x5; x6; x7; x8; x9; x10; x11; x12; x13; x14; x15;...
x16; x17; x18; x19; x20; x21; x22; x23; x24; x25; x26; x27];
l1=[l11; l12; l13; l14; l15; l16; l17; l18; l19; l110; l111; l112; l113;...
l114; l115; l116; l117; l118; l119; l120; l121; l122; l123; l124;...
l125; l126; l127];
l2=[l21; l22; l23; l24; l25; l26; l27; l28; l29; l210; l211; l212; l213;...
l214; l215; l216; l217; l218; l219; l220; l221; l222; l223; l224;...
l225; l226; l227];   tempx=sigma*sum(abs(x'))-sum((abs(oldx-x))');
testx=min(min(tempx));

templ1=sigma*sum(abs(l1'))-sum((abs(oldl1-l1))');
testl1=min(min(templ1));

templ2=sigma*sum(abs(l2'))-sum((abs(oldl2-l2))');
testl2=min(min(templ2));

      display(conta);
test=min(testx, min(testl1, testl2))
```

```
% questa parte riguarda i pcontrolliparziali non serve

% ma serve a me per vedere cosa succede durante le iterazioni

% si può togliere per avere più velocità nelle elaborazioni

%           %####################      controlli ottimali parziali +

%           %####################    + calcolo concavità parziale   +

%           %####################    + calcolo payoff parziali      +

%           %####################    + calcolo ESS 1, 2, 3, 4, 5

%           for i=1:N+1

%

%              a11= (1.*(k1s*(1.-1.*qthvs)*l11(i)*x1(i)*x26(i)-1.*k1s*(1.-
1.*qthvs)*l18(i)*x1(i)*x26(i)-1.*k1s*(1.-1.*qtsvs)*l112(i)*x2(i)*x26(i)+k1s*(1.-
1.*qtsvs)*l12(i)*x2(i)*x26(i)-1.*k1s*(1.-1.*qtrvs)*l116(i)*x26(i)*x3(i)+k1s*(1.-
1.*qtrvs)*l13(i)*x26(i)*x3(i)-1.*k1s*(1.-1.*qtgvs)*l120(i)*x26(i)*x4(i)+k1s*(1.-
1.*qtgvs)*l14(i)*x26(i)*x4(i)+k1s*rhos*l126(i)*x26(i)*((1.-1.*qthvs)*x1(i)+(1.-
1.*qtsvs)*x2(i)+(1.-1.*qtrvs)*x3(i)+(1.-1.*qtgvs)*x4(i))))/aco-(1.*(-1.*k1s*(1.-
1.*qthvs)*l11(i)*x1(i)*x26(i)+k1s*(1.-1.*qthvs)*l18(i)*x1(i)*x26(i)+k1s*(1.-
1.*qtsvs)*l112(i)*x2(i)*x26(i)-1.*k1s*(1.-1.*qtsvs)*l12(i)*x2(i)*x26(i)+k1s*(1.-
1.*qtrvs)*l116(i)*x26(i)*x3(i)-1.*k1s*(1.-1.*qtrvs)*l13(i)*x26(i)*x3(i)+k1s*(1.-
1.*qtgvs)*l120(i)*x26(i)*x4(i)-1.*k1s*(1.-1.*qtgvs)*l14(i)*x26(i)*x4(i)-
1.*k1s*rhos*l126(i)*x26(i)*((1.-1.*qthvs)*x1(i)+(1.-1.*qtsvs)*x2(i)+(1.-1.*qtrvs)*x3(i)+(1.-
1.*qtgvs)*x4(i)))*(-1.*aco*(k1s*(1.-1.*qthvs)*l11(i)*x1(i)*x26(i)-1.*k1s*(1.-
1.*qthvs)*l18(i)*x1(i)*x26(i)-1.*k1s*(1.-1.*qtsvs)*l112(i)*x2(i)*x26(i)+k1s*(1.-
1.*qtsvs)*l12(i)*x2(i)*x26(i)-1.*k1s*(1.-1.*qtrvs)*l116(i)*x26(i)*x3(i)+k1s*(1.-
1.*qtrvs)*l13(i)*x26(i)*x3(i)-1.*k1s*(1.-1.*qtgvs)*l120(i)*x26(i)*x4(i)+k1s*(1.-
1.*qtgvs)*l14(i)*x26(i)*x4(i)+k1s*rhos*l126(i)*x26(i)*((1.-1.*qthvs)*x1(i)+(1.-
1.*qtsvs)*x2(i)+(1.-1.*qtrvs)*x3(i)+(1.-1.*qtgvs)*x4(i)))-1.*(-1.*k1s*(1.-
1.*qthvs)*l11(i)*x1(i)*x26(i)+k1s*(1.-1.*qthvs)*l18(i)*x1(i)*x26(i)+k1s*(1.-
1.*qtsvs)*l112(i)*x2(i)*x26(i)-1.*k1s*(1.-1.*qtsvs)*l12(i)*x2(i)*x26(i)+k1s*(1.-
1.*qtrvs)*l116(i)*x26(i)*x3(i)-1.*k1s*(1.-1.*qtrvs)*l13(i)*x26(i)*x3(i)+k1s*(1.-
1.*qtgvs)*l120(i)*x26(i)*x4(i)-1.*k1s*(1.-1.*qtgvs)*l14(i)*x26(i)*x4(i)-
1.*k1s*rhos*l126(i)*x26(i)*((1.-1.*qthvs)*x1(i)+(1.-1.*qtsvs)*x2(i)+(1.-1.*qtrvs)*x3(i)+(1.-
1.*qtgvs)*x4(i)))*(k1s*(1.-1.*qthvs)*l11(i)*x1(i)*x26(i)-1.*k1s*(1.-
1.*qthvs)*l18(i)*x1(i)*x26(i)-1.*k1s*(1.-1.*qtsvs)*l112(i)*x2(i)*x26(i)+k1s*(1.-
1.*qtsvs)*l12(i)*x2(i)*x26(i)-1.*k1s*(1.-1.*qtrvs)*l116(i)*x26(i)*x3(i)+k1s*(1.-
1.*qtrvs)*l13(i)*x26(i)*x3(i)-1.*k1s*(1.-1.*qtgvs)*l120(i)*x26(i)*x4(i)+k1s*(1.-
1.*qtgvs)*l14(i)*x26(i)*x4(i)+k1s*rhos*l126(i)*x26(i)*((1.-1.*qthvs)*x1(i)+(1.-
1.*qtsvs)*x2(i)+(1.-1.*qtrvs)*x3(i)+(1.-1.*qtgvs)*x4(i)))))/(aco*(1.*aco*afi-1.*(-1.*k1s*(1.-
1.*qthvs)*l11(i)*x1(i)*x26(i)+k1s*(1.-1.*qthvs)*l18(i)*x1(i)*x26(i)+k1s*(1.-
1.*qtsvs)*l112(i)*x2(i)*x26(i)-1.*k1s*(1.-1.*qtsvs)*l12(i)*x2(i)*x26(i)+k1s*(1.-
1.*qtrvs)*l116(i)*x26(i)*x3(i)-1.*k1s*(1.-1.*qtrvs)*l13(i)*x26(i)*x3(i)+k1s*(1.-
1.*qtgvs)*l120(i)*x26(i)*x4(i)-1.*k1s*(1.-1.*qtgvs)*l14(i)*x26(i)*x4(i)-
1.*k1s*rhos*l126(i)*x26(i)*((1.-1.*qthvs)*x1(i)+(1.-1.*qtsvs)*x2(i)+(1.-1.*qtrvs)*x3(i)+(1.-
1.*qtgvs)*x4(i)))^2));

%

%              a12=-((-1.*aco*(k1s*(1.-1.*qthvs)*l11(i)*x1(i)*x26(i)-1.*k1s*(1.-
1.*qthvs)*l18(i)*x1(i)*x26(i)-1.*k1s*(1.-1.*qtsvs)*l112(i)*x2(i)*x26(i)+k1s*(1.-
1.*qtsvs)*l12(i)*x2(i)*x26(i)-1.*k1s*(1.-1.*qtrvs)*l116(i)*x26(i)*x3(i)+k1s*(1.-
1.*qtrvs)*l13(i)*x26(i)*x3(i)-1.*k1s*(1.-1.*qtgvs)*l120(i)*x26(i)*x4(i)+k1s*(1.-
1.*qtgvs)*l14(i)*x26(i)*x4(i)+k1s*rhos*l126(i)*x26(i)*((1.-1.*qthvs)*x1(i)+(1.-
```

```
1.*qtsvs)*x2(i)+(1.-1.*qtrvs)*x3(i)+(1.-1.*qtgvs)*x4(i)))-1.*(-1.*k1s*(1.-
1.*qthvs)*l11(i)*x1(i)*x26(i)+k1s*(1.-1.*qthvs)*l18(i)*x1(i)*x26(i)+k1s*(1.-
1.*qtsvs)*l112(i)*x2(i)*x26(i)-1.*k1s*(1.-1.*qtsvs)*l12(i)*x2(i)*x26(i)+k1s*(1.-
1.*qtrvs)*l116(i)*x26(i)*x3(i)-1.*k1s*(1.-1.*qtrvs)*l13(i)*x26(i)*x3(i)+k1s*(1.-
1.*qtgvs)*l120(i)*x26(i)*x4(i)-1.*k1s*(1.-1.*qtgvs)*l14(i)*x26(i)*x4(i)-
1.*k1s*rhos*l126(i)*x26(i)*((1.-1.*qthvs)*x1(i)+(1.-1.*qtsvs)*x2(i)+(1.-1.*qtrvs)*x3(i)+(1.-
1.*qtgvs)*x4(i)))*(k1s*(1.-1.*qthvs)*l11(i)*x1(i)*x26(i)-1.*k1s*(1.-
1.*qthvs)*l18(i)*x1(i)*x26(i)-1.*k1s*(1.-1.*qtsvs)*l112(i)*x2(i)*x26(i)+k1s*(1.-
1.*qtsvs)*l12(i)*x2(i)*x26(i)-1.*k1s*(1.-1.*qtrvs)*l116(i)*x26(i)*x3(i)+k1s*(1.-
1.*qtrvs)*l13(i)*x26(i)*x3(i)-1.*k1s*(1.-1.*qtgvs)*l120(i)*x26(i)*x4(i)+k1s*(1.-
1.*qtgvs)*l14(i)*x26(i)*x4(i)+k1s*rhos*l126(i)*x26(i)*((1.-1.*qthvs)*x1(i)+(1.-
1.*qtsvs)*x2(i)+(1.-1.*qtrvs)*x3(i)+(1.-1.*qtgvs)*x4(i))))/(1.*aco*afi-1.*(-1.*k1s*(1.-
1.*qthvs)*l11(i)*x1(i)*x26(i)+k1s*(1.-1.*qthvs)*l18(i)*x1(i)*x26(i)+k1s*(1.-
1.*qtsvs)*l112(i)*x2(i)*x26(i)-1.*k1s*(1.-1.*qtsvs)*l12(i)*x2(i)*x26(i)+k1s*(1.-
1.*qtrvs)*l116(i)*x26(i)*x3(i)-1.*k1s*(1.-1.*qtrvs)*l13(i)*x26(i)*x3(i)+k1s*(1.-
1.*qtgvs)*l120(i)*x26(i)*x4(i)-1.*k1s*(1.-1.*qtgvs)*l14(i)*x26(i)*x4(i)-
1.*k1s*rhos*l126(i)*x26(i)*((1.-1.*qthvs)*x1(i)+(1.-1.*qtsvs)*x2(i)+(1.-1.*qtrvs)*x3(i)+(1.-
1.*qtgvs)*x4(i)))^2));

%

%           a13=(1.*(l11(i)*x1(i)+l12(i)*x2(i)+l13(i)*x3(i)+l14(i)*x4(i)))/aib;

%

%           a14=(1.*(k2*l112(i)*x12(i)-1.*k2*l113(i)*x12(i)+k2*l116(i)*x16(i)-
1.*k2*l117(i)*x16(i)+k2*l120(i)*x20(i)-1.*k2*l121(i)*x20(i)+k2*l18(i)*x8(i)-
1.*k2*l19(i)*x8(i)))/aii-(1.*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-
1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-
1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i))*((1.*(1.*bsr*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x
25(i))*(1.*brs-
1.*pr*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))+pr*l227(i)*(x11(i)+x15(i)+x19(i)+x23(i
)+x25(i)))-1.*brs*(-1.*bsr*(-1.*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))-
(1.*g*l126(i)*x26(i))/(b2+x26(i))-
1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i))+
1.*ps*l126(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))*(1.*bsr+ps*l226(i)*(x13(i)+x17(i)+x21(i)
+x24(i)+x9(i))-
1.*ps*l227(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))))))/(brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-
1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-1.*k1sm*l15(i)*x26(i)*x5(i)))-
(1.*(1.*bsr*pr^2*l126(i)*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))^2-
1.*brs*(1.*api*bsr-
1.*ps^2*l126(i)*l226(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))^2))*(1.*brs*bsr*(k1sm*l124(i)
*x26(i)*x5(i)-1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-1.*k1sm*l15(i)*x26(i)*x5(i))*(-
1.*aii*(k2*l112(i)*x12(i)-1.*k2*l113(i)*x12(i)+k2*l116(i)*x16(i)-
1.*k2*l117(i)*x16(i)+k2*l120(i)*x20(i)-1.*k2*l121(i)*x20(i)-
1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i)+k
2*l18(i)*x8(i)-1.*k2*l19(i)*x8(i))-1.*(k2*l112(i)*x12(i)-
1.*k2*l113(i)*x12(i)+k2*l116(i)*x16(i)-1.*k2*l117(i)*x16(i)+k2*l120(i)*x20(i)-
1.*k2*l121(i)*x20(i)+k2*l18(i)*x8(i)-1.*k2*l19(i)*x8(i))*(-
1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-
1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i)))-1.*(1.*aii*art-
1.*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-
1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-
1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i))^2)*(1.*bsr*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x2
5(i))*(1.*brs-
1.*pr*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))+pr*l227(i)*(x11(i)+x15(i)+x19(i)+x23(i
)+x25(i)))-1.*brs*(-1.*bsr*(-1.*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))-
(1.*g*l126(i)*x26(i))/(b2+x26(i))-
1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i))+
1.*ps*l126(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))*(1.*bsr+ps*l226(i)*(x13(i)+x17(i)+x21(i)
```

```
+x24(i)+x9(i))-
1.*ps*l227(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i)))))))/(brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-
1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-1.*k1sm*l15(i)*x26(i)*x5(i))*(-
1.*aii*brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-
1.*k1sm*l15(i)*x26(i)*x5(i))^2-1.*(1.*aii*art-1.*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-
1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-
1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i))^2)*(1.*bsr*pr^2*l126(i)*l226(i)*(x11(i)+x15(i)+x19(i)+
x23(i)+x25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*l126(i)*l226(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))^2))))))/aii;

%

%	a15= -((1.*brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-
1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-1.*k1sm*l15(i)*x26(i)*x5(i))*(-
1.*aii*(k2*l112(i)*x12(i)-1.*k2*l113(i)*x12(i)+k2*l116(i)*x16(i)-
1.*k2*l117(i)*x16(i)+k2*l120(i)*x20(i)-1.*k2*l121(i)*x20(i)-
1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i)+k
2*l18(i)*x8(i)-1.*k2*l19(i)*x8(i))-1.*(k2*l112(i)*x12(i)-
1.*k2*l113(i)*x12(i)+k2*l116(i)*x16(i)-1.*k2*l117(i)*x16(i)+k2*l120(i)*x20(i)-
1.*k2*l121(i)*x20(i)+k2*l18(i)*x8(i)-1.*k2*l19(i)*x8(i))*(-
1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-
1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i)))-1.*(1.*aii*art-
1.*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-
1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-
1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i))^2)*(1.*bsr*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x2
5(i))*(1.*brs-
1.*pr*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))+pr*l227(i)*(x11(i)+x15(i)+x19(i)+x23(i
)+x25(i)))-1.*brs*(-1.*bsr*(-1.*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))-
(1.*g*l126(i)*x26(i))/(b2+x26(i))-
1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i))+
1.*ps*l126(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))*(1.*bsr+ps*l226(i)*(x13(i)+x17(i)+x21(i)
+x24(i)+x9(i))-1.*ps*l227(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))))))/(-
1.*aii*brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-
1.*k1sm*l15(i)*x26(i)*x5(i))^2-1.*(1.*aii*art-1.*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-
1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-
1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i))^2)*(1.*bsr*pr^2*l126(i)*l226(i)*(x11(i)+x15(i)+x19(i)+
x23(i)+x25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*l126(i)*l226(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))^2))));

%

%	a16=(-1.*(1.*bsr*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))*(1.*brs-
1.*pr*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))+pr*l227(i)*(x11(i)+x15(i)+x19(i)+x23(i
)+x25(i)))-1.*brs*(-1.*bsr*(-1.*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))-
(1.*g*l126(i)*x26(i))/(b2+x26(i))-
1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i))+
1.*ps*l126(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))*(1.*bsr+ps*l226(i)*(x13(i)+x17(i)+x21(i)
+x24(i)+x9(i))-
1.*ps*l227(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))))))/(brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-
1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-
1.*k1sm*l15(i)*x26(i)*x5(i)))+(1.*(1.*bsr*pr^2*l126(i)*l226(i)*(x11(i)+x15(i)+x19(i)+x23(
i)+x25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*l126(i)*l226(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))^2))*(1.*brs*bsr*(k1sm*l124(i)
*x26(i)*x5(i)-1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-1.*k1sm*l15(i)*x26(i)*x5(i))*(-
1.*aii*(k2*l112(i)*x12(i)-1.*k2*l113(i)*x12(i)+k2*l116(i)*x16(i)-
1.*k2*l117(i)*x16(i)+k2*l120(i)*x20(i)-1.*k2*l121(i)*x20(i)-
1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i)+k
2*l18(i)*x8(i)-1.*k2*l19(i)*x8(i))-1.*(k2*l112(i)*x12(i)-
1.*k2*l113(i)*x12(i)+k2*l116(i)*x16(i)-1.*k2*l117(i)*x16(i)+k2*l120(i)*x20(i)-
1.*k2*l121(i)*x20(i)+k2*l18(i)*x8(i)-1.*k2*l19(i)*x8(i))*(-
```

```
1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-
1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i)))-1.*(1.*aii*art-
1.*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-
1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-
1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i))^2)*(1.*bsr*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x2
5(i))*(1.*brs-
1.*pr*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))+pr*l227(i)*(x11(i)+x15(i)+x19(i)+x23(i
)+x25(i)))-1.*brs*(-1.*bsr*(-1.*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))-
(1.*g*l126(i)*x26(i))/(b2+x26(i))-
1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i))+
1.*ps*l126(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))*(1.*bsr+ps*l226(i)*(x13(i)+x17(i)+x21(i)
+x24(i)+x9(i))-
1.*ps*l227(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i)))))))/(brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-
1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-1.*k1sm*l15(i)*x26(i)*x5(i))*(-
1.*aii*brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-
1.*k1sm*l15(i)*x26(i)*x5(i))^2-1.*(1.*aii*art-1.*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-
1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-
1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i))^2)*(1.*bsr*pr^2*l126(i)*l226(i)*(x11(i)+x15(i)+x19(i)+
x23(i)+x25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*l126(i)*l226(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))^2))));

%

%              u21s= min(1,max(0,(1.*(1.*bsr+ps*l226(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))-
1.*ps*l227(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))))/bsr+(1.*ps*l226(i)*(x13(i)+x17(i)+x21(
i)+x24(i)+x9(i))*(1.*brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-
1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-1.*k1sm*l15(i)*x26(i)*x5(i))*(-
1.*aii*(k2*l112(i)*x12(i)-1.*k2*l113(i)*x12(i)+k2*l116(i)*x16(i)-
1.*k2*l117(i)*x16(i)+k2*l120(i)*x20(i)-1.*k2*l121(i)*x20(i)-
1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i)+k
2*l18(i)*x8(i)-1.*k2*l19(i)*x8(i))-1.*(k2*l112(i)*x12(i)-
1.*k2*l113(i)*x12(i)+k2*l116(i)*x16(i)-1.*k2*l117(i)*x16(i)+k2*l120(i)*x20(i)-
1.*k2*l121(i)*x20(i)+k2*l18(i)*x8(i)-1.*k2*l19(i)*x8(i))*(-
1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-
1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i)))-1.*(1.*aii*art-
1.*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-
1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-
1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i))^2)*(1.*bsr*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x2
5(i))*(1.*brs-
1.*pr*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))+pr*l227(i)*(x11(i)+x15(i)+x19(i)+x23(i
)+x25(i)))-1.*brs*(-1.*bsr*(-1.*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))-
(1.*g*l126(i)*x26(i))/(b2+x26(i))-
1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i))+
1.*ps*l126(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))*(1.*bsr+ps*l226(i)*(x13(i)+x17(i)+x21(i)
+x24(i)+x9(i))-1.*ps*l227(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i)))))))/(bsr*(-
1.*aii*brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-
1.*k1sm*l15(i)*x26(i)*x5(i))^2-1.*(1.*aii*art-1.*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-
1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-
1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i))^2)*(1.*bsr*pr^2*l126(i)*l226(i)*(x11(i)+x15(i)+x19(i)+
x23(i)+x25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*l126(i)*l226(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))^2))))));

%

%              u22s= min(1,max(0,(1.*(1.*brs-
1.*pr*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))+pr*l227(i)*(x11(i)+x15(i)+x19(i)+x23(i
)+x25(i))))/brs-
(1.*pr*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))*(1.*brs*bsr*(k1sm*l124(i)*x26(i)*x5(i
)-1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-1.*k1sm*l15(i)*x26(i)*x5(i))*(-
1.*aii*(k2*l112(i)*x12(i)-1.*k2*l113(i)*x12(i)+k2*l116(i)*x16(i)-
```

```
1.*k2*l117(i)*x16(i)+k2*l120(i)*x20(i)-1.*k2*l121(i)*x20(i)-
1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i)+k
2*l18(i)*x8(i)-1.*k2*l19(i)*x8(i))-1.*(k2*l112(i)*x12(i)-
1.*k2*l113(i)*x12(i)+k2*l116(i)*x16(i)-1.*k2*l117(i)*x16(i)+k2*l120(i)*x20(i)-
1.*k2*l121(i)*x20(i)+k2*l18(i)*x8(i)-1.*k2*l19(i)*x8(i))*(-
1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-
1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i)))-1.*(1.*aii*art-
1.*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-
1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-
1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i))^2)*(1.*bsr*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x2
5(i))*(1.*brs-
1.*pr*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))+pr*l227(i)*(x11(i)+x15(i)+x19(i)+x23(i
)+x25(i)))-1.*brs*(-1.*bsr*(-1.*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))-
(1.*g*l126(i)*x26(i))/(b2+x26(i))-
1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i))+
1.*ps*l126(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))*(1.*bsr+ps*l226(i)*(x13(i)+x17(i)+x21(i)
+x24(i)+x9(i))-1.*ps*l227(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i)))))))/(brs*(-
1.*aii*brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-
1.*k1sm*l15(i)*x26(i)*x5(i))^2-1.*(1.*aii*art-1.*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-
1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-
1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i))^2)*(1.*bsr*pr^2*l126(i)*l226(i)*(x11(i)+x15(i)+x19(i)+
x23(i)+x25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*l126(i)*l226(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))^2))));

%
%        if(a11<0 || a11>1)
%            a11=0;
%        end;
%
%        if(a12<0 || a12>1)
%            a12=0;
%        end;
%
%        if(a13<0 || a13>1)
%            a13=0;
%        end;
%
%        if(a14<0 || a14>1)
%            a14=0;
%        end;
%
%        if(a15<0 || a15>1)
%            a15=0;
%        end;
```

```
%
%          if(a16<0 || a16>1)
%                a16=0;
%          end;
%
%
%
           D=[a11,a12,a13,a14,a15,a16;a11,a12,a13,a14,a15,1;a11,a12,a13,a14,a15,0;a11,a12,
a13,a14,1,a16;a11,a12,a13,a14,1,1;a11,a12,a13,a14,1,0;a11,a12,a13,a14,0,a16;a11,a12,a13,a1
4,0,1;a11,a12,a13,a14,0,0;a11,a12,a13,1,a15,a16;a11,a12,a13,1,a15,1;a11,a12,a13,1,a15,0;a1
1,a12,a13,1,1,a16;a11,a12,a13,1,1,1;a11,a12,a13,1,1,0;a11,a12,a13,1,0,a16;a11,a12,a13,1,0,1;
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a12,0,0,1,1;0,a12,0,0,1,0;0,a12,0,0,0,a16;0,a12,0,0,0,1;0,a12,0,0,0,0;0,1,a13,a14,a15,a16;0,1,
a13,a14,a15,1;0,1,a13,a14,a15,0;0,1,a13,a14,1,a16;0,1,a13,a14,1,1;0,1,a13,a14,1,0;0,1,a13,a1
4,0,a16;0,1,a13,a14,0,1;0,1,a13,a14,0,0;0,1,a13,1,a15,a16;0,1,a13,1,a15,1;0,1,a13,1,a15,0;0,1,
a13,1,1,a16;0,1,a13,1,1,1;0,1,a13,1,1,0;0,1,a13,1,0,a16;0,1,a13,1,0,1;0,1,a13,1,0,0;0,1,a13,0,a
15,a16;0,1,a13,0,a15,1;0,1,a13,0,a15,0;0,1,a13,0,1,a16;0,1,a13,0,1,1;0,1,a13,0,1,0;0,1,a13,0,0
,a16;0,1,a13,0,0,1;0,1,a13,0,0,0;0,1,1,a14,a15,a16;0,1,1,a14,a15,1;0,1,1,a14,a15,0;0,1,1,a14,1
,a16;0,1,1,a14,1,1;0,1,1,a14,1,0;0,1,1,a14,0,a16;0,1,1,a14,0,1;0,1,1,a14,0,0;0,1,1,1,a15,a16;0,
1,1,1,a15,1;0,1,1,1,a15,0;0,1,1,1,1,a16;0,1,1,1,1,1;0,1,1,1,1,0;0,1,1,1,0,a16;0,1,1,1,0,1;0,1,1,1,
0,0;0,1,1,0,a15,a16;0,1,1,0,a15,1;0,1,1,0,a15,0;0,1,1,0,1,a16;0,1,1,0,1,1;0,1,1,0,1,0;0,1,1,0,0,a

```
16;0,1,1,0,0,1;0,1,1,0,0,0;0,1,0,a14,a15,a16;0,1,0,a14,a15,1;0,1,0,a14,a15,0;0,1,0,a14,1,a16;0,1,0,a14,1,1;0,1,0,a14,1,0;0,1,0,a14,0,a16;0,1,0,a14,0,1;0,1,0,a14,0,0;0,1,0,1,a15,a16;0,1,0,1,a15,1;0,1,0,1,a15,0;0,1,0,1,1,a16;0,1,0,1,1,1;0,1,0,1,1,0;0,1,0,1,0,a16;0,1,0,1,0,1;0,1,0,1,0,0;0,1,0,0,a15,a16;0,1,0,0,a15,1;0,1,0,0,a15,0;0,1,0,0,1,a16;0,1,0,0,1,1;0,1,0,0,1,0;0,1,0,0,0,a16;0,1,0,0,0,1;0,1,0,0,0,0;0,0,a13,a14,a15,a16;0,0,a13,a14,a15,1;0,0,a13,a14,a15,0;0,0,a13,a14,1,a16;0,0,a13,a14,1,1;0,0,a13,a14,1,0;0,0,a13,a14,0,a16;0,0,a13,a14,0,1;0,0,a13,a14,0,0;0,0,a13,1,a15,a16;0,0,a13,1,a15,1;0,0,a13,1,a15,0;0,0,a13,1,1,a16;0,0,a13,1,1,1;0,0,a13,1,1,0;0,0,a13,1,0,a16;0,0,a13,1,0,1;0,0,a13,1,0,0;0,0,a13,0,a15,a16;0,0,a13,0,a15,1;0,0,a13,0,a15,0;0,0,a13,0,1,a16;0,0,a13,0,1,1;0,0,a13,0,1,0;0,0,a13,0,0,a16;0,0,a13,0,0,1;0,0,a13,0,0,0;0,0,1,a14,a15,a16;0,0,1,a14,a15,1;0,0,1,a14,a15,0;0,0,1,a14,1,a16;0,0,1,a14,1,1;0,0,1,a14,1,0;0,0,1,a14,0,a16;0,0,1,a14,0,1;0,0,1,a14,0,0;0,0,1,1,a15,a16;0,0,1,1,a15,1;0,0,1,1,a15,0;0,0,1,1,1,a16;0,0,1,1,1,1;0,0,1,1,1,0;0,0,1,1,0,a16;0,0,1,1,0,1;0,0,1,1,0,0;0,0,1,0,a15,a16;0,0,1,0,a15,1;0,0,1,0,a15,0;0,0,1,0,1,a16;0,0,1,0,1,1;0,0,1,0,1,0;0,0,1,0,0,a16;0,0,1,0,0,1;0,0,1,0,0,0;0,0,0,a14,a15,a16;0,0,0,a14,a15,1;0,0,0,a14,a15,0;0,0,0,a14,1,a16;0,0,0,a14,1,1;0,0,0,a14,1,0;0,0,0,a14,0,a16;0,0,0,a14,0,1;0,0,0,a14,0,0;0,0,0,1,a15,a16;0,0,0,1,a15,1;0,0,0,1,a15,0;0,0,0,1,1,a16;0,0,0,1,1,1;0,0,0,1,1,0;0,0,0,1,0,a16;0,0,0,1,0,1;0,0,0,1,0,0;0,0,0,0,a15,a16;0,0,0,0,a15,1;0,0,0,0,a15,0;0,0,0,0,1,a16;0,0,0,0,1,1;0,0,0,0,1,0;0,0,0,0,0,a16;0,0,0,0,0,1;0,0,0,0,0,0];

%

%

%        eval_HT=eval(fHT);

%

%

%        [max_HT, id_HT]=nanmax(eval_HT);

%

%

%        u1(1,i)=D(id_HT,1);

%        u1(2,i)=D(id_HT,2);

%        u1(3,i)=D(id_HT,3);

%        u1(4,i)=D(id_HT,4);

%        u1(5,i)=D(id_HT,5);

%        u1(6,i)=D(id_HT,6);

%

%        u2(1,i)=u21s;

%        u2(2,i)=u22s;

%

%      %#####   verifica concavità parziale h2 e h6 dei controlli relativi alla terapia

%        H2terapia(1,i)=1.*aco*afi-k1s^2*l11(i)^2*x1(i)^2*x26(i)^2+2*k1s^2*qthvs*l11(i)^2*x1(i)^2*x26(i)^2-k1s^2*qthvs^2*l11(i)^2*x1(i)^2*x26(i)^2-2*k1s^2*rhos*l11(i)*l126(i)*x1(i)^2*x26(i)^2+4*k1s^2*qthvs*rhos*l11(i)*l126(i)*x1(i)^2*x26(i)^2-2*k1s^2*qthvs^2*rhos*l11(i)*l126(i)*x1(i)^2*x26(i)^2-k1s^2*rhos^2*l126(i)^2*x1(i)^2*x26(i)^2+2*k1s^2*qthvs*rhos^2*l126(i)^2*x1(i)^2*x26(i)^2-
```

k1s^2*qthvs^2*rhos^2*l126(i)^2*x1(i)^2*x26(i)^2+2*k1s^2*l11(i)*l18(i)*x1(i)^2*x26(i)^2-
4*k1s^2*qthvs*l11(i)*l18(i)*x1(i)^2*x26(i)^2+2*k1s^2*qthvs^2*l11(i)*l18(i)*x1(i)^2*x26(i)^2+2*k1s^2*rhos*l126(i)*l18(i)*x1(i)^2*x26(i)^2-
4*k1s^2*qthvs*rhos*l126(i)*l18(i)*x1(i)^2*x26(i)^2+2*k1s^2*qthvs^2*rhos*l126(i)*l18(i)*x1(i)^2*x26(i)^2-
k1s^2*l18(i)^2*x1(i)^2*x26(i)^2+2*k1s^2*qthvs*l18(i)^2*x1(i)^2*x26(i)^2-
k1s^2*qthvs^2*l18(i)^2*x1(i)^2*x26(i)^2+2*k1s^2*l11(i)*l112(i)*x1(i)*x2(i)*x26(i)^2-
2*k1s^2*qthvs*l11(i)*l112(i)*x1(i)*x2(i)*x26(i)^2-
2*k1s^2*qtsvs*l11(i)*l112(i)*x1(i)*x2(i)*x26(i)^2+2*k1s^2*qthvs*qtsvs*l11(i)*l112(i)*x1(i)*x2(i)*x26(i)^2-
2*k1s^2*l11(i)*l12(i)*x1(i)*x2(i)*x26(i)^2+2*k1s^2*qthvs*l11(i)*l12(i)*x1(i)*x2(i)*x26(i)^2+2*k1s^2*qtsvs*l11(i)*l12(i)*x1(i)*x2(i)*x26(i)^2-
2*k1s^2*qthvs*qtsvs*l11(i)*l12(i)*x1(i)*x2(i)*x26(i)^2-
2*k1s^2*rhos*l11(i)*l126(i)*x1(i)*x2(i)*x26(i)^2+2*k1s^2*qthvs*rhos*l11(i)*l126(i)*x1(i)*x2(i)*x26(i)^2+2*k1s^2*qtsvs*rhos*l11(i)*l126(i)*x1(i)*x2(i)*x26(i)^2-
2*k1s^2*qthvs*qtsvs*rhos*l11(i)*l126(i)*x1(i)*x2(i)*x26(i)^2+2*k1s^2*rhos*l112(i)*l126(i)*x1(i)*x2(i)*x26(i)^2-2*k1s^2*qthvs*rhos*l112(i)*l126(i)*x1(i)*x2(i)*x26(i)^2-
2*k1s^2*qtsvs*rhos*l112(i)*l126(i)*x1(i)*x2(i)*x26(i)^2+2*k1s^2*qthvs*qtsvs*rhos*l112(i)*l126(i)*x1(i)*x2(i)*x26(i)^2-
2*k1s^2*rhos*l12(i)*l126(i)*x1(i)*x2(i)*x26(i)^2+2*k1s^2*qthvs*rhos*l12(i)*l126(i)*x1(i)*x2(i)*x26(i)^2+2*k1s^2*qtsvs*rhos*l12(i)*l126(i)*x1(i)*x2(i)*x26(i)^2-
2*k1s^2*qthvs*qtsvs*rhos*l12(i)*l126(i)*x1(i)*x2(i)*x26(i)^2-
2*k1s^2*rhos^2*l126(i)^2*x1(i)*x2(i)*x26(i)^2+2*k1s^2*qthvs*rhos^2*l126(i)^2*x1(i)*x2(i)*x26(i)^2+2*k1s^2*qtsvs*rhos^2*l126(i)^2*x1(i)*x2(i)*x26(i)^2-
2*k1s^2*qthvs*qtsvs*rhos^2*l126(i)^2*x1(i)*x2(i)*x26(i)^2-
2*k1s^2*l112(i)*l18(i)*x1(i)*x2(i)*x26(i)^2+2*k1s^2*qthvs*l112(i)*l18(i)*x1(i)*x2(i)*x26(i)^2+2*k1s^2*qtsvs*l112(i)*l18(i)*x1(i)*x2(i)*x26(i)^2-
2*k1s^2*qthvs*qtsvs*l112(i)*l18(i)*x1(i)*x2(i)*x26(i)^2+2*k1s^2*l12(i)*l18(i)*x1(i)*x2(i)*x26(i)^2-2*k1s^2*qthvs*l12(i)*l18(i)*x1(i)*x2(i)*x26(i)^2-
2*k1s^2*qtsvs*l12(i)*l18(i)*x1(i)*x2(i)*x26(i)^2+2*k1s^2*qthvs*qtsvs*l12(i)*l18(i)*x1(i)*x2(i)*x26(i)^2+2*k1s^2*rhos*l126(i)*l18(i)*x1(i)*x2(i)*x26(i)^2-
2*k1s^2*qthvs*rhos*l126(i)*l18(i)*x1(i)*x2(i)*x26(i)^2-
2*k1s^2*qtsvs*rhos*l126(i)*l18(i)*x1(i)*x2(i)*x26(i)^2+2*k1s^2*qthvs*qtsvs*rhos*l126(i)*l18(i)*x1(i)*x2(i)*x26(i)^2-
k1s^2*l112(i)^2*x2(i)^2*x26(i)^2+2*k1s^2*qtsvs*l112(i)^2*x2(i)^2*x26(i)^2-
k1s^2*qtsvs^2*l112(i)^2*x2(i)^2*x26(i)^2+2*k1s^2*l112(i)*l12(i)*x2(i)^2*x26(i)^2-
4*k1s^2*qtsvs*l112(i)*l12(i)*x2(i)^2*x26(i)^2+2*k1s^2*qtsvs^2*l112(i)*l12(i)*x2(i)^2*x26(i)^2-k1s^2*l12(i)^2*x2(i)^2*x26(i)^2+2*k1s^2*qtsvs*l12(i)^2*x2(i)^2*x26(i)^2-
k1s^2*qtsvs^2*l12(i)^2*x2(i)^2*x26(i)^2+2*k1s^2*rhos*l112(i)*l126(i)*x2(i)^2*x26(i)^2-
4*k1s^2*qtsvs*rhos*l112(i)*l126(i)*x2(i)^2*x26(i)^2+2*k1s^2*qtsvs^2*rhos*l112(i)*l126(i)*x2(i)^2*x26(i)^2-
2*k1s^2*rhos*l12(i)*l126(i)*x2(i)^2*x26(i)^2+4*k1s^2*qtsvs*rhos*l12(i)*l126(i)*x2(i)^2*x26(i)^2-2*k1s^2*qtsvs^2*rhos*l12(i)*l126(i)*x2(i)^2*x26(i)^2-
k1s^2*rhos^2*l126(i)^2*x2(i)^2*x26(i)^2+2*k1s^2*qtsvs*rhos^2*l126(i)^2*x2(i)^2*x26(i)^2-
k1s^2*qtsvs^2*rhos^2*l126(i)^2*x2(i)^2*x26(i)^2+2*k1s^2*l11(i)*l116(i)*x1(i)*x26(i)^2*x3(i)-2*k1s^2*qthvs*l11(i)*l116(i)*x1(i)*x26(i)^2*x3(i)-
2*k1s^2*qtrvs*l11(i)*l116(i)*x1(i)*x26(i)^2*x3(i)+2*k1s^2*qthvs*qtrvs*l11(i)*l116(i)*x1(i)*x26(i)^2*x3(i)-
2*k1s^2*rhos*l11(i)*l126(i)*x1(i)*x26(i)^2*x3(i)+2*k1s^2*qthvs*rhos*l11(i)*l126(i)*x1(i)*x26(i)^2*x3(i)+2*k1s^2*qtrvs*rhos*l11(i)*l126(i)*x1(i)*x26(i)^2*x3(i)-
2*k1s^2*qthvs*qtrvs*rhos*l11(i)*l126(i)*x1(i)*x26(i)^2*x3(i)+2*k1s^2*rhos*l116(i)*l126(i)*x1(i)*x26(i)^2*x3(i)-2*k1s^2*qthvs*rhos*l116(i)*l126(i)*x1(i)*x26(i)^2*x3(i)-
2*k1s^2*qtrvs*rhos*l116(i)*l126(i)*x1(i)*x26(i)^2*x3(i)+2*k1s^2*qthvs*qtrvs*rhos*l116(i)*l126(i)*x1(i)*x26(i)^2*x3(i)-
2*k1s^2*rhos^2*l126(i)^2*x1(i)*x26(i)^2*x3(i)+2*k1s^2*qthvs*rhos^2*l126(i)^2*x1(i)*x26(i)^2*x3(i)+2*k1s^2*qtrvs*rhos^2*l126(i)^2*x1(i)*x26(i)^2*x3(i)-
2*k1s^2*qthvs*qtrvs*rhos^2*l126(i)^2*x1(i)*x26(i)^2*x3(i)-
2*k1s^2*l11(i)*l13(i)*x1(i)*x26(i)^2*x3(i)+2*k1s^2*qthvs*l11(i)*l13(i)*x1(i)*x26(i)^2*x3(i)+2*k1s^2*qtrvs*l11(i)*l13(i)*x1(i)*x26(i)^2*x3(i)-

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2*k1s^2*qthvs*qtrvs*l11(i)*l13(i)*x1(i)*x26(i)^2*x3(i)-
2*k1s^2*rhos*l126(i)*l13(i)*x1(i)*x26(i)^2*x3(i)+2*k1s^2*qthvs*rhos*l126(i)*l13(i)*x1(i)
*x26(i)^2*x3(i)+2*k1s^2*qtrvs*rhos*l126(i)*l13(i)*x1(i)*x26(i)^2*x3(i)-
2*k1s^2*qthvs*qtrvs*rhos*l126(i)*l13(i)*x1(i)*x26(i)^2*x3(i)-
2*k1s^2*l116(i)*l18(i)*x1(i)*x26(i)^2*x3(i)+2*k1s^2*qthvs*l116(i)*l18(i)*x1(i)*x26(i)^2*
x3(i)+2*k1s^2*qtrvs*l116(i)*l18(i)*x1(i)*x26(i)^2*x3(i)-
2*k1s^2*qthvs*qtrvs*l116(i)*l18(i)*x1(i)*x26(i)^2*x3(i)+2*k1s^2*rhos*l126(i)*l18(i)*x1(i
)*x26(i)^2*x3(i)-2*k1s^2*qthvs*rhos*l126(i)*l18(i)*x1(i)*x26(i)^2*x3(i)-
2*k1s^2*qtrvs*rhos*l126(i)*l18(i)*x1(i)*x26(i)^2*x3(i)+2*k1s^2*qthvs*qtrvs*rhos*l126(i)
*l18(i)*x1(i)*x26(i)^2*x3(i)+2*k1s^2*l13(i)*l18(i)*x1(i)*x26(i)^2*x3(i)-
2*k1s^2*qthvs*l13(i)*l18(i)*x1(i)*x26(i)^2*x3(i)-
2*k1s^2*qtrvs*l13(i)*l18(i)*x1(i)*x26(i)^2*x3(i)+2*k1s^2*qthvs*qtrvs*l13(i)*l18(i)*x1(i)*
x26(i)^2*x3(i)-
2*k1s^2*l112(i)*l116(i)*x2(i)*x26(i)^2*x3(i)+2*k1s^2*qtrvs*l112(i)*l116(i)*x2(i)*x26(i)^
2*x3(i)+2*k1s^2*qtsvs*l112(i)*l116(i)*x2(i)*x26(i)^2*x3(i)-
2*k1s^2*qtrvs*qtsvs*l112(i)*l116(i)*x2(i)*x26(i)^2*x3(i)+2*k1s^2*l116(i)*l12(i)*x2(i)*x2
6(i)^2*x3(i)-2*k1s^2*qtrvs*l116(i)*l12(i)*x2(i)*x26(i)^2*x3(i)-
2*k1s^2*qtsvs*l116(i)*l12(i)*x2(i)*x26(i)^2*x3(i)+2*k1s^2*qtrvs*qtsvs*l116(i)*l12(i)*x2(i
)*x26(i)^2*x3(i)+2*k1s^2*rhos*l112(i)*l126(i)*x2(i)*x26(i)^2*x3(i)-
2*k1s^2*qtrvs*rhos*l112(i)*l126(i)*x2(i)*x26(i)^2*x3(i)-
2*k1s^2*qtsvs*rhos*l112(i)*l126(i)*x2(i)*x26(i)^2*x3(i)+2*k1s^2*qtrvs*qtsvs*rhos*l112(i
)*l126(i)*x2(i)*x26(i)^2*x3(i)+2*k1s^2*rhos*l116(i)*l126(i)*x2(i)*x26(i)^2*x3(i)-
2*k1s^2*qtrvs*rhos*l116(i)*l126(i)*x2(i)*x26(i)^2*x3(i)-
2*k1s^2*qtsvs*rhos*l116(i)*l126(i)*x2(i)*x26(i)^2*x3(i)+2*k1s^2*qtrvs*qtsvs*rhos*l116(i
)*l126(i)*x2(i)*x26(i)^2*x3(i)-
2*k1s^2*rhos*l12(i)*l126(i)*x2(i)*x26(i)^2*x3(i)+2*k1s^2*qtrvs*rhos*l12(i)*l126(i)*x2(i)
*x26(i)^2*x3(i)+2*k1s^2*qtsvs*rhos*l12(i)*l126(i)*x2(i)*x26(i)^2*x3(i)-
2*k1s^2*qtrvs*qtsvs*rhos*l12(i)*l126(i)*x2(i)*x26(i)^2*x3(i)-
2*k1s^2*rhos^2*l126(i)^2*x2(i)*x26(i)^2*x3(i)+2*k1s^2*qtrvs*rhos^2*l126(i)^2*x2(i)*x2
6(i)^2*x3(i)+2*k1s^2*qtsvs*rhos^2*l126(i)^2*x2(i)*x26(i)^2*x3(i)-
2*k1s^2*qtrvs*qtsvs*rhos^2*l126(i)^2*x2(i)*x26(i)^2*x3(i)+2*k1s^2*l112(i)*l13(i)*x2(i)*
x26(i)^2*x3(i)-2*k1s^2*qtrvs*l112(i)*l13(i)*x2(i)*x26(i)^2*x3(i)-
2*k1s^2*qtsvs*l112(i)*l13(i)*x2(i)*x26(i)^2*x3(i)+2*k1s^2*qtrvs*qtsvs*l112(i)*l13(i)*x2(i
)*x26(i)^2*x3(i)-
2*k1s^2*l12(i)*l13(i)*x2(i)*x26(i)^2*x3(i)+2*k1s^2*qtrvs*l12(i)*l13(i)*x2(i)*x26(i)^2*x3(
i)+2*k1s^2*qtsvs*l12(i)*l13(i)*x2(i)*x26(i)^2*x3(i)-
2*k1s^2*qtrvs*qtsvs*l12(i)*l13(i)*x2(i)*x26(i)^2*x3(i)-
2*k1s^2*rhos*l126(i)*l13(i)*x2(i)*x26(i)^2*x3(i)+2*k1s^2*qtrvs*rhos*l126(i)*l13(i)*x2(i)
*x26(i)^2*x3(i)+2*k1s^2*qtsvs*rhos*l126(i)*l13(i)*x2(i)*x26(i)^2*x3(i)-
2*k1s^2*qtrvs*qtsvs*rhos*l126(i)*l13(i)*x2(i)*x26(i)^2*x3(i)-
k1s^2*l116(i)^2*x26(i)^2*x3(i)^2+2*k1s^2*qtrvs*l116(i)^2*x26(i)^2*x3(i)^2-
k1s^2*qtrvs^2*l116(i)^2*x26(i)^2*x3(i)^2+2*k1s^2*rhos*l116(i)*l126(i)*x26(i)^2*x3(i)^2-
4*k1s^2*qtrvs*rhos*l116(i)*l126(i)*x26(i)^2*x3(i)^2+2*k1s^2*qtrvs^2*rhos*l116(i)*l126(i
)*x26(i)^2*x3(i)^2-
k1s^2*rhos^2*l126(i)^2*x26(i)^2*x3(i)^2+2*k1s^2*qtrvs*rhos^2*l126(i)^2*x26(i)^2*x3(i)^
2-
k1s^2*qtrvs^2*rhos^2*l126(i)^2*x26(i)^2*x3(i)^2+2*k1s^2*l116(i)*l13(i)*x26(i)^2*x3(i)^2
-
4*k1s^2*qtrvs*l116(i)*l13(i)*x26(i)^2*x3(i)^2+2*k1s^2*qtrvs^2*l116(i)*l13(i)*x26(i)^2*x
3(i)^2-
2*k1s^2*rhos*l126(i)*l13(i)*x26(i)^2*x3(i)^2+4*k1s^2*qtrvs*rhos*l126(i)*l13(i)*x26(i)^2
*x3(i)^2-2*k1s^2*qtrvs^2*rhos*l126(i)*l13(i)*x26(i)^2*x3(i)^2-
k1s^2*l13(i)^2*x26(i)^2*x3(i)^2+2*k1s^2*qtrvs*l13(i)^2*x26(i)^2*x3(i)^2-
k1s^2*qtrvs^2*l13(i)^2*x26(i)^2*x3(i)^2+2*k1s^2*l11(i)*l120(i)*x1(i)*x26(i)^2*x4(i)-
2*k1s^2*qtgvs*l11(i)*l120(i)*x1(i)*x26(i)^2*x4(i)-
2*k1s^2*qthvs*l11(i)*l120(i)*x1(i)*x26(i)^2*x4(i)+2*k1s^2*qtgvs*qthvs*l11(i)*l120(i)*x1
(i)*x26(i)^2*x4(i)-
2*k1s^2*rhos*l11(i)*l126(i)*x1(i)*x26(i)^2*x4(i)+2*k1s^2*qtgvs*rhos*l11(i)*l126(i)*x1(i)
*x26(i)^2*x4(i)+2*k1s^2*qthvs*rhos*l11(i)*l126(i)*x1(i)*x26(i)^2*x4(i)-
2*k1s^2*qtgvs*qthvs*rhos*l11(i)*l126(i)*x1(i)*x26(i)^2*x4(i)+2*k1s^2*rhos*l120(i)*l126(
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i)*x1(i)*x26(i)^2*x4(i)-2*k1s^2*qtgvs*rhos*l120(i)*l126(i)*x1(i)*x26(i)^2*x4(i)-
2*k1s^2*qthvs*rhos*l120(i)*l126(i)*x1(i)*x26(i)^2*x4(i)+2*k1s^2*qtgvs*qthvs*rhos*l120(
i)*l126(i)*x1(i)*x26(i)^2*x4(i)-
2*k1s^2*rhos^2*l126(i)^2*x1(i)*x26(i)^2*x4(i)+2*k1s^2*qtgvs*rhos^2*l126(i)^2*x1(i)*x2
6(i)^2*x4(i)+2*k1s^2*qthvs*rhos^2*l126(i)^2*x1(i)*x26(i)^2*x4(i)-
2*k1s^2*qtgvs*qthvs*rhos^2*l126(i)^2*x1(i)*x26(i)^2*x4(i)-
2*k1s^2*l11(i)*l14(i)*x1(i)*x26(i)^2*x4(i)+2*k1s^2*qtgvs*l11(i)*l14(i)*x1(i)*x26(i)^2*x4
(i)+2*k1s^2*qthvs*l11(i)*l14(i)*x1(i)*x26(i)^2*x4(i)-
2*k1s^2*qtgvs*qthvs*l11(i)*l14(i)*x1(i)*x26(i)^2*x4(i)-
2*k1s^2*rhos*l126(i)*l14(i)*x1(i)*x26(i)^2*x4(i)+2*k1s^2*qtgvs*rhos*l126(i)*l14(i)*x1(i)
*x26(i)^2*x4(i)+2*k1s^2*qthvs*rhos*l126(i)*l14(i)*x1(i)*x26(i)^2*x4(i)-
2*k1s^2*qtgvs*qthvs*rhos*l126(i)*l14(i)*x1(i)*x26(i)^2*x4(i)-
2*k1s^2*l120(i)*l18(i)*x1(i)*x26(i)^2*x4(i)+2*k1s^2*qtgvs*l120(i)*l18(i)*x1(i)*x26(i)^2*
x4(i)+2*k1s^2*qthvs*l120(i)*l18(i)*x1(i)*x26(i)^2*x4(i)-
2*k1s^2*qtgvs*qthvs*l120(i)*l18(i)*x1(i)*x26(i)^2*x4(i)+2*k1s^2*rhos*l126(i)*l18(i)*x1(i
)*x26(i)^2*x4(i)-2*k1s^2*qtgvs*rhos*l126(i)*l18(i)*x1(i)*x26(i)^2*x4(i)-
2*k1s^2*qthvs*rhos*l126(i)*l18(i)*x1(i)*x26(i)^2*x4(i)+2*k1s^2*qtgvs*qthvs*rhos*l126(i)
*l18(i)*x1(i)*x26(i)^2*x4(i)+2*k1s^2*l14(i)*l18(i)*x1(i)*x26(i)^2*x4(i)-
2*k1s^2*qtgvs*l14(i)*l18(i)*x1(i)*x26(i)^2*x4(i)-
2*k1s^2*qthvs*l14(i)*l18(i)*x1(i)*x26(i)^2*x4(i)+2*k1s^2*qtgvs*qthvs*l14(i)*l18(i)*x1(i)
*x26(i)^2*x4(i)-
2*k1s^2*l112(i)*l120(i)*x2(i)*x26(i)^2*x4(i)+2*k1s^2*qtgvs*l112(i)*l120(i)*x2(i)*x26(i)^
2*x4(i)+2*k1s^2*qtsvs*l112(i)*l120(i)*x2(i)*x26(i)^2*x4(i)-
2*k1s^2*qtgvs*qtsvs*l112(i)*l120(i)*x2(i)*x26(i)^2*x4(i)+2*k1s^2*l12(i)*l120(i)*x2(i)*x2
6(i)^2*x4(i)-2*k1s^2*qtgvs*l12(i)*l120(i)*x2(i)*x26(i)^2*x4(i)-
2*k1s^2*qtsvs*l12(i)*l120(i)*x2(i)*x26(i)^2*x4(i)+2*k1s^2*qtgvs*qtsvs*l12(i)*l120(i)*x2(
i)*x26(i)^2*x4(i)+2*k1s^2*rhos*l112(i)*l126(i)*x2(i)*x26(i)^2*x4(i)-
2*k1s^2*qtgvs*rhos*l112(i)*l126(i)*x2(i)*x26(i)^2*x4(i)-
2*k1s^2*qtsvs*rhos*l112(i)*l126(i)*x2(i)*x26(i)^2*x4(i)+2*k1s^2*qtgvs*qtsvs*rhos*l112(i
)*l126(i)*x2(i)*x26(i)^2*x4(i)-
2*k1s^2*rhos*l12(i)*l126(i)*x2(i)*x26(i)^2*x4(i)+2*k1s^2*qtgvs*rhos*l12(i)*l126(i)*x2(i)
*x26(i)^2*x4(i)+2*k1s^2*qtsvs*rhos*l12(i)*l126(i)*x2(i)*x26(i)^2*x4(i)-
2*k1s^2*qtgvs*qtsvs*rhos*l12(i)*l126(i)*x2(i)*x26(i)^2*x4(i)+2*k1s^2*rhos*l120(i)*l126(
i)*x2(i)*x26(i)^2*x4(i)-2*k1s^2*qtgvs*rhos*l120(i)*l126(i)*x2(i)*x26(i)^2*x4(i)-
2*k1s^2*qtsvs*rhos*l120(i)*l126(i)*x2(i)*x26(i)^2*x4(i)+2*k1s^2*qtgvs*qtsvs*rhos*l120(i
)*l126(i)*x2(i)*x26(i)^2*x4(i)-
2*k1s^2*rhos^2*l126(i)^2*x2(i)*x26(i)^2*x4(i)+2*k1s^2*qtgvs*rhos^2*l126(i)^2*x2(i)*x2
6(i)^2*x4(i)+2*k1s^2*qtsvs*rhos^2*l126(i)^2*x2(i)*x26(i)^2*x4(i)-
2*k1s^2*qtgvs*qtsvs*rhos^2*l126(i)^2*x2(i)*x26(i)^2*x4(i)+2*k1s^2*l112(i)*l14(i)*x2(i)*
x26(i)^2*x4(i)-2*k1s^2*qtgvs*l112(i)*l14(i)*x2(i)*x26(i)^2*x4(i)-
2*k1s^2*qtsvs*l112(i)*l14(i)*x2(i)*x26(i)^2*x4(i)+2*k1s^2*qtgvs*qtsvs*l112(i)*l14(i)*x2(
i)*x26(i)^2*x4(i)-
2*k1s^2*l12(i)*l14(i)*x2(i)*x26(i)^2*x4(i)+2*k1s^2*qtgvs*l12(i)*l14(i)*x2(i)*x26(i)^2*x4
(i)+2*k1s^2*qtsvs*l12(i)*l14(i)*x2(i)*x26(i)^2*x4(i)-
2*k1s^2*qtgvs*qtsvs*l12(i)*l14(i)*x2(i)*x26(i)^2*x4(i)-
2*k1s^2*rhos*l126(i)*l14(i)*x2(i)*x26(i)^2*x4(i)+2*k1s^2*qtgvs*rhos*l126(i)*l14(i)*x2(i)
*x26(i)^2*x4(i)+2*k1s^2*qtsvs*rhos*l126(i)*l14(i)*x2(i)*x26(i)^2*x4(i)-
2*k1s^2*qtgvs*qtsvs*rhos*l126(i)*l14(i)*x2(i)*x26(i)^2*x4(i)-
2*k1s^2*l116(i)*l120(i)*x26(i)^2*x3(i)*x4(i)+2*k1s^2*qtgvs*l116(i)*l120(i)*x26(i)^2*x3(i
)*x4(i)+2*k1s^2*qtrvs*l116(i)*l120(i)*x26(i)^2*x3(i)*x4(i)-
2*k1s^2*qtgvs*qtrvs*l116(i)*l120(i)*x26(i)^2*x3(i)*x4(i)+2*k1s^2*rhos*l116(i)*l126(i)*x
26(i)^2*x3(i)*x4(i)-2*k1s^2*qtgvs*rhos*l116(i)*l126(i)*x26(i)^2*x3(i)*x4(i)-
2*k1s^2*qtrvs*rhos*l116(i)*l126(i)*x26(i)^2*x3(i)*x4(i)+2*k1s^2*qtgvs*qtrvs*rhos*l116(i
)*l126(i)*x26(i)^2*x3(i)*x4(i)+2*k1s^2*rhos*l120(i)*l126(i)*x26(i)^2*x3(i)*x4(i)-
2*k1s^2*qtgvs*rhos*l120(i)*l126(i)*x26(i)^2*x3(i)*x4(i)-
2*k1s^2*qtrvs*rhos*l120(i)*l126(i)*x26(i)^2*x3(i)*x4(i)+2*k1s^2*qtgvs*qtrvs*rhos*l120(i
)*l126(i)*x26(i)^2*x3(i)*x4(i)-
2*k1s^2*rhos^2*l126(i)^2*x26(i)^2*x3(i)*x4(i)+2*k1s^2*qtgvs*rhos^2*l126(i)^2*x26(i)^2
*x3(i)*x4(i)+2*k1s^2*qtrvs*rhos^2*l126(i)^2*x26(i)^2*x3(i)*x4(i)-
2*k1s^2*qtgvs*qtrvs*rhos^2*l126(i)^2*x26(i)^2*x3(i)*x4(i)+2*k1s^2*l120(i)*l13(i)*x26(i)

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^2*x3(i)*x4(i)-2*k1s^2*qtgvs*l120(i)*l13(i)*x26(i)^2*x3(i)*x4(i)-
2*k1s^2*qtrvs*l120(i)*l13(i)*x26(i)^2*x3(i)*x4(i)+2*k1s^2*qtgvs*qtrvs*l120(i)*l13(i)*x26
(i)^2*x3(i)*x4(i)-
2*k1s^2*rhos*l126(i)*l13(i)*x26(i)^2*x3(i)*x4(i)+2*k1s^2*qtgvs*rhos*l126(i)*l13(i)*x26(i
)^2*x3(i)*x4(i)+2*k1s^2*qtrvs*rhos*l126(i)*l13(i)*x26(i)^2*x3(i)*x4(i)-
2*k1s^2*qtgvs*qtrvs*rhos*l126(i)*l13(i)*x26(i)^2*x3(i)*x4(i)+2*k1s^2*l116(i)*l14(i)*x26(
i)^2*x3(i)*x4(i)-2*k1s^2*qtgvs*l116(i)*l14(i)*x26(i)^2*x3(i)*x4(i)-
2*k1s^2*qtrvs*l116(i)*l14(i)*x26(i)^2*x3(i)*x4(i)+2*k1s^2*qtgvs*qtrvs*l116(i)*l14(i)*x26
(i)^2*x3(i)*x4(i)-
2*k1s^2*rhos*l126(i)*l14(i)*x26(i)^2*x3(i)*x4(i)+2*k1s^2*qtgvs*rhos*l126(i)*l14(i)*x26(i
)^2*x3(i)*x4(i)+2*k1s^2*qtrvs*rhos*l126(i)*l14(i)*x26(i)^2*x3(i)*x4(i)-
2*k1s^2*qtgvs*qtrvs*rhos*l126(i)*l14(i)*x26(i)^2*x3(i)*x4(i)-
2*k1s^2*l13(i)*l14(i)*x26(i)^2*x3(i)*x4(i)+2*k1s^2*qtgvs*l13(i)*l14(i)*x26(i)^2*x3(i)*x4
(i)+2*k1s^2*qtrvs*l13(i)*l14(i)*x26(i)^2*x3(i)*x4(i)-
2*k1s^2*qtgvs*qtrvs*l13(i)*l14(i)*x26(i)^2*x3(i)*x4(i)-
k1s^2*l120(i)^2*x26(i)^2*x4(i)^2+2*k1s^2*qtgvs*l120(i)^2*x26(i)^2*x4(i)^2-
k1s^2*qtgvs^2*l120(i)^2*x26(i)^2*x4(i)^2+2*k1s^2*rhos*l120(i)*l126(i)*x26(i)^2*x4(i)^2-
4*k1s^2*qtgvs*rhos*l120(i)*l126(i)*x26(i)^2*x4(i)^2+2*k1s^2*qtgvs^2*rhos*l120(i)*l126(
i)*x26(i)^2*x4(i)^2-
k1s^2*rhos^2*l126(i)^2*x26(i)^2*x4(i)^2+2*k1s^2*qtgvs*rhos^2*l126(i)^2*x26(i)^2*x4(i)
^2-
k1s^2*qtgvs^2*rhos^2*l126(i)^2*x26(i)^2*x4(i)^2+2*k1s^2*l120(i)*l14(i)*x26(i)^2*x4(i)^
2-
4*k1s^2*qtgvs*l120(i)*l14(i)*x26(i)^2*x4(i)^2+2*k1s^2*qtgvs^2*l120(i)*l14(i)*x26(i)^2*x
4(i)^2-
2*k1s^2*rhos*l126(i)*l14(i)*x26(i)^2*x4(i)^2+4*k1s^2*qtgvs*rhos*l126(i)*l14(i)*x26(i)^2
*x4(i)^2-2*k1s^2*qtgvs^2*rhos*l126(i)*l14(i)*x26(i)^2*x4(i)^2-
k1s^2*l14(i)^2*x26(i)^2*x4(i)^2+2*k1s^2*qtgvs*l14(i)^2*x26(i)^2*x4(i)^2-
k1s^2*qtgvs^2*l14(i)^2*x26(i)^2*x4(i)^2;

%

%        H6terapia(1,i)=-1.*aib*(1.*aii*k1sm^2*(l124(i)-rhos*l126(i)-
l15(i))^2*x26(i)^2*(1.*aco*afi-k1s^2*x26(i)^2*((-1+qthvs)*l11(i)*x1(i)-(-
1+qthvs)*l18(i)*x1(i)-(-1+qtsvs)*(l112(i)-l12(i))*x2(i)-(-1+qtrvs)*(l116(i)-l13(i))*x3(i)-(-
1+qtgvs)*(l120(i)-l14(i))*x4(i)+rhos*l126(i)*((-1+qthvs)*x1(i)+(-1+qtsvs)*x2(i)+(-
1+qtrvs)*x3(i)+(-1+qtgvs)*x4(i)))^2)*x5(i)^2-1.*api*(1.*aii*art*(1.*aco*afi-
k1s^2*x26(i)^2*((-1+qthvs)*l11(i)*x1(i)-(-1+qthvs)*l18(i)*x1(i)-(-1+qtsvs)*(l112(i)-
l12(i))*x2(i)-(-1+qtrvs)*(l116(i)-l13(i))*x3(i)-(-1+qtgvs)*(l120(i)-
l14(i))*x4(i)+rhos*l126(i)*((-1+qthvs)*x1(i)+(-1+qtsvs)*x2(i)+(-1+qtrvs)*x3(i)+(-
1+qtgvs)*x4(i)))^2)-k2^2*(1.*aco*afi-k1s^2*x26(i)^2*((-1+qthvs)*l11(i)*x1(i)-(-
1+qthvs)*l18(i)*x1(i)-(-1+qtsvs)*(l112(i)-l12(i))*x2(i)-(-1+qtrvs)*(l116(i)-l13(i))*x3(i)-(-
1+qtgvs)*(l120(i)-l14(i))*x4(i)+rhos*l126(i)*((-1+qthvs)*x1(i)+(-1+qtsvs)*x2(i)+(-
1+qtrvs)*x3(i)+(-1+qtgvs)*x4(i)))^2)*((-l112(i)+l113(i))*x12(i)+(-l116(i)+l117(i))*x16(i)+(-
l120(i)+l121(i))*x20(i)+(-l18(i)+l19(i))*x8(i))^2));

%

%        H2terapia_2(1,i)=1.*aco*afi-((k1s*x26(i)*((-1+qthvs)*l11(i)*x1(i)-(-
1+qthvs)*l18(i)*x1(i)-(-1+qtsvs)*(l112(i)-l12(i))*x2(i)-(-1+qtrvs)*(l116(i)-l13(i))*x3(i)-(-
1+qtgvs)*(l120(i)-l14(i))*x4(i)+rhos*l126(i)*((-1+qthvs)*x1(i)+(-1+qtsvs)*x2(i)+(-
1+qtrvs)*x3(i)+(-1+qtgvs)*x4(i))))^2);

%

%

%        %################    calcolo Payoff parziali

%        %JTER
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```
%       JSI(i)=-
0.5*(aco*u1(1,i)^2+afi*u1(2,i)^2+aib*u1(3,i)^2+aii*u1(4,i)^2+api*u1(5,i)^2+art*u1(6,i)^2)+
x1(i)+x2(i)+x3(i)+x4(i)+x5(i)+x6(i)+x7(i);

%

%       %JV

%       JV(i)=-0.5*(bsr*(1-u2(1,i))^2+brs*(1-
u2(2,i))^2)+x10(i)+x11(i)+x12(i)+x13(i)+x14(i)+x15(i)+x16(i)+x17(i)+x18(i)+x19(i)+x20(i)
+x21(i)+x22(i)+x23(i)+x26(i)+x27(i)+x8(i)+x9(i);

%

%

%       %#################### Calcolo ESS 1, 2, 3, 4, 5

%       ess1(i)=1.*abs((k1sm*k2*pr*l126(i)*(l124(i)-1.*rhos*l126(i)-
1.*l15(i))*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))*x26(i)*x5(i)*((-
1.*l112(i)+l113(i))*x12(i)+(-1.*l116(i)+l117(i))*x16(i)+(-1.*l120(i)+l121(i))*x20(i)+(-
1.*l18(i)+l19(i))*x8(i)))/(-1.*aii*(1.*api*art-1.*k1sm^2*(l124(i)-1.*rhos*l126(i)-
1.*l15(i))^2*x26(i)^2*x5(i)^2)+1.*api*k2^2*((-1.*l112(i)+l113(i))*x12(i)+(-
1.*l116(i)+l117(i))*x16(i)+(-1.*l120(i)+l121(i))*x20(i)+(-
1.*l18(i)+l19(i))*x8(i))^2))+1.*abs((k1sm*k2*ps*l126(i)*(l124(i)-1.*rhos*l126(i)-
1.*l15(i))*x26(i)*x5(i)*((-1.*l112(i)+l113(i))*x12(i)+(-1.*l116(i)+l117(i))*x16(i)+(-
1.*l120(i)+l121(i))*x20(i)+(-1.*l18(i)+l19(i))*x8(i))*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i)))/(-
1.*aii*(1.*api*art-1.*k1sm^2*(l124(i)-1.*rhos*l126(i)-
1.*l15(i))^2*x26(i)^2*x5(i)^2)+1.*api*k2^2*((-1.*l112(i)+l113(i))*x12(i)+(-
1.*l116(i)+l117(i))*x16(i)+(-1.*l120(i)+l121(i))*x20(i)+(-1.*l18(i)+l19(i))*x8(i))^2));

%

%               ess2(i)=abs((pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))*(1.-
(1.*aii*k1sm^2*(l124(i)-1.*rhos*l126(i)-1.*l15(i))^2*x26(i)^2*x5(i)^2)/(-1.*aii*(1.*api*art-
1.*k1sm^2*(l124(i)-1.*rhos*l126(i)-1.*l15(i))^2*x26(i)^2*x5(i)^2)+1.*api*k2^2*((-
1.*l112(i)+l113(i))*x12(i)+(-1.*l116(i)+l117(i))*x16(i)+(-1.*l120(i)+l121(i))*x20(i)+(-
1.*l18(i)+l19(i))*x8(i))^2)))/api)+abs((ps*l126(i)*(-1.+(1.*aii*k1sm^2*(l124(i)-
1.*rhos*l126(i)-1.*l15(i))^2*x26(i)^2*x5(i)^2)/(-1.*aii*(1.*api*art-1.*k1sm^2*(l124(i)-
1.*rhos*l126(i)-1.*l15(i))^2*x26(i)^2*x5(i)^2)+1.*api*k2^2*((-1.*l112(i)+l113(i))*x12(i)+(-
1.*l116(i)+l117(i))*x16(i)+(-1.*l120(i)+l121(i))*x20(i)+(-
1.*l18(i)+l19(i))*x8(i))^2))*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i)))/api);

%

%       ess3(i)=1.*abs((aii*k1sm*pr*l126(i)*(l124(i)-1.*rhos*l126(i)-
1.*l15(i))*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))*x26(i)*x5(i))/(-1.*aii*(1.*api*art-
1.*k1sm^2*(l124(i)-1.*rhos*l126(i)-1.*l15(i))^2*x26(i)^2*x5(i)^2)+1.*api*k2^2*((-
1.*l112(i)+l113(i))*x12(i)+(-1.*l116(i)+l117(i))*x16(i)+(-1.*l120(i)+l121(i))*x20(i)+(-
1.*l18(i)+l19(i))*x8(i))^2))+1.*abs((aii*k1sm*ps*l126(i)*(l124(i)-1.*rhos*l126(i)-
1.*l15(i))*x26(i)*x5(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i)))/(-1.*aii*(1.*api*art-
1.*k1sm^2*(l124(i)-1.*rhos*l126(i)-1.*l15(i))^2*x26(i)^2*x5(i)^2)+1.*api*k2^2*((-
1.*l112(i)+l113(i))*x12(i)+(-1.*l116(i)+l117(i))*x16(i)+(-1.*l120(i)+l121(i))*x20(i)+(-
1.*l18(i)+l19(i))*x8(i))^2));

%

%       ess4(i)=1.*abs((ps*l226(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i)))/bsr);

%

%       ess5(i)=1.*abs((pr*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i)))/brs);
```

```
%       if (ess1(i)>=1 || ess2(i)>=1 || ess3(i)>=1 || ess4(i)>=1 || ess5(i)>=1 )
%          testess=-1;
%       else
%          testess=1;
%       end;
%
%
%          end; %##########end controlli ottimali parziali
%              %#########+ concavità parz + payoff parz + ESS
%
%
%          %########### calcolo e visualizzazione payoff parziali
%          pay_SI_parziali=h*sum(-0.5*(aco*u1(1,:).^2+afi*u1(2,:).^2+aib*u1(3,:).^2+aii*u1(4,:).^2+api*u1(5,:).^2+art*u1(6,:).^2)+x1(1,:)+x2(1,:)+x3(1,:)+x4(1,:)+x5(1,:)+x6(1,:)+x7(1,:))
%          pay_SI_parziali_2=h*sum(JSI)
%
%          pay_V_parziali=h*sum(-0.5*(bsr*(1-u2(1,:)).^2+brs*(1-u2(2,:)).^2)+x10(1,:)+x11(1,:)+x12(1,:)+x13(1,:)+x14(1,:)+x15(1,:)+x16(1,:)+x17(1,:)+x18(1,:)+x19(1,:)+x20(1,:)+x21(1,:)+x22(1,:)+x23(1,:)+x26(1,:)+x27(1,:)+x8(1,:)+x9(1,:))
%          pay_V_parziali_2=h*sum(JV)
%
%          %############visualizzazione testESS parziale
%          display(testess);
end; %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%end  WHILE ESTERNO
%####################     controlli ottimali FINALI +
%####################    + calcolo concavità FINALI  +
%####################    + calcolo payoff FINALI      +
%####################    + calcolo ESS 1, 2, 3, 4, 5
for i=1:N+1
   a11= (1.*(k1s*(1.-1.*qthvs)*l11(i)*x1(i)*x26(i)-1.*k1s*(1.-1.*qthvs)*l18(i)*x1(i)*x26(i)-1.*k1s*(1.-1.*qtsvs)*l112(i)*x2(i)*x26(i)+k1s*(1.-1.*qtsvs)*l12(i)*x2(i)*x26(i)-1.*k1s*(1.-1.*qtrvs)*l116(i)*x26(i)*x3(i)+k1s*(1.-1.*qtrvs)*l13(i)*x26(i)*x3(i)-1.*k1s*(1.-1.*qtgvs)*l120(i)*x26(i)*x4(i)+k1s*(1.-1.*qtgvs)*l14(i)*x26(i)*x4(i)+k1s*rhos*l126(i)*x26(i)*((1.-1.*qthvs)*x1(i)+(1.-1.*qtsvs)*x2(i)+(1.-1.*qtrvs)*x3(i)+(1.-1.*qtgvs)*x4(i))))/aco-(1.*(-1.*k1s*(1.-1.*qthvs)*l11(i)*x1(i)*x26(i)+k1s*(1.-1.*qthvs)*l18(i)*x1(i)*x26(i)+k1s*(1.-1.*qtsvs)*l112(i)*x2(i)*x26(i)-1.*k1s*(1.-1.*qtsvs)*l12(i)*x2(i)*x26(i)+k1s*(1.-
```

1.*qtrvs)*l116(i)*x26(i)*x3(i)-1.*k1s*(1.-1.*qtrvs)*l13(i)*x26(i)*x3(i)+k1s*(1.-
1.*qtgvs)*l120(i)*x26(i)*x4(i)-1.*k1s*(1.-1.*qtgvs)*l14(i)*x26(i)*x4(i)-
1.*k1s*rhos*l126(i)*x26(i)*((1.-1.*qthvs)*x1(i)+(1.-1.*qtsvs)*x2(i)+(1.-1.*qtrvs)*x3(i)+(1.-
1.*qtgvs)*x4(i)))*(-1.*aco*(k1s*(1.-1.*qthvs)*l11(i)*x1(i)*x26(i)-1.*k1s*(1.-
1.*qthvs)*l18(i)*x1(i)*x26(i)-1.*k1s*(1.-1.*qtsvs)*l112(i)*x2(i)*x26(i)+k1s*(1.-
1.*qtsvs)*l12(i)*x2(i)*x26(i)-1.*k1s*(1.-1.*qtrvs)*l116(i)*x26(i)*x3(i)+k1s*(1.-
1.*qtrvs)*l13(i)*x26(i)*x3(i)-1.*k1s*(1.-1.*qtgvs)*l120(i)*x26(i)*x4(i)+k1s*(1.-
1.*qtgvs)*l14(i)*x26(i)*x4(i)+k1s*rhos*l126(i)*x26(i)*((1.-1.*qthvs)*x1(i)+(1.-
1.*qtsvs)*x2(i)+(1.-1.*qtrvs)*x3(i)+(1.-1.*qtgvs)*x4(i)))-1.*(-1.*k1s*(1.-
1.*qthvs)*l11(i)*x1(i)*x26(i)+k1s*(1.-1.*qthvs)*l18(i)*x1(i)*x26(i)+k1s*(1.-
1.*qtsvs)*l112(i)*x2(i)*x26(i)-1.*k1s*(1.-1.*qtsvs)*l12(i)*x2(i)*x26(i)+k1s*(1.-
1.*qtrvs)*l116(i)*x26(i)*x3(i)-1.*k1s*(1.-1.*qtrvs)*l13(i)*x26(i)*x3(i)+k1s*(1.-
1.*qtgvs)*l120(i)*x26(i)*x4(i)-1.*k1s*(1.-1.*qtgvs)*l14(i)*x26(i)*x4(i)-
1.*k1s*rhos*l126(i)*x26(i)*((1.-1.*qthvs)*x1(i)+(1.-1.*qtsvs)*x2(i)+(1.-1.*qtrvs)*x3(i)+(1.-
1.*qtgvs)*x4(i)))*(k1s*(1.-1.*qthvs)*l11(i)*x1(i)*x26(i)-1.*k1s*(1.-
1.*qthvs)*l18(i)*x1(i)*x26(i)-1.*k1s*(1.-1.*qtsvs)*l112(i)*x2(i)*x26(i)+k1s*(1.-
1.*qtsvs)*l12(i)*x2(i)*x26(i)-1.*k1s*(1.-1.*qtrvs)*l116(i)*x26(i)*x3(i)+k1s*(1.-
1.*qtrvs)*l13(i)*x26(i)*x3(i)-1.*k1s*(1.-1.*qtgvs)*l120(i)*x26(i)*x4(i)+k1s*(1.-
1.*qtgvs)*l14(i)*x26(i)*x4(i)+k1s*rhos*l126(i)*x26(i)*((1.-1.*qthvs)*x1(i)+(1.-
1.*qtsvs)*x2(i)+(1.-1.*qtrvs)*x3(i)+(1.-1.*qtgvs)*x4(i)))))/(aco*(1.*aco*afi-1.*(-1.*k1s*(1.-
1.*qthvs)*l11(i)*x1(i)*x26(i)+k1s*(1.-1.*qthvs)*l18(i)*x1(i)*x26(i)+k1s*(1.-
1.*qtsvs)*l112(i)*x2(i)*x26(i)-1.*k1s*(1.-1.*qtsvs)*l12(i)*x2(i)*x26(i)+k1s*(1.-
1.*qtrvs)*l116(i)*x26(i)*x3(i)-1.*k1s*(1.-1.*qtrvs)*l13(i)*x26(i)*x3(i)+k1s*(1.-
1.*qtgvs)*l120(i)*x26(i)*x4(i)-1.*k1s*(1.-1.*qtgvs)*l14(i)*x26(i)*x4(i)-
1.*k1s*rhos*l126(i)*x26(i)*((1.-1.*qthvs)*x1(i)+(1.-1.*qtsvs)*x2(i)+(1.-1.*qtrvs)*x3(i)+(1.-
1.*qtgvs)*x4(i)))^2));

a12=-((-1.*aco*(k1s*(1.-1.*qthvs)*l11(i)*x1(i)*x26(i)-1.*k1s*(1.-
1.*qthvs)*l18(i)*x1(i)*x26(i)-1.*k1s*(1.-1.*qtsvs)*l112(i)*x2(i)*x26(i)+k1s*(1.-
1.*qtsvs)*l12(i)*x2(i)*x26(i)-1.*k1s*(1.-1.*qtrvs)*l116(i)*x26(i)*x3(i)+k1s*(1.-
1.*qtrvs)*l13(i)*x26(i)*x3(i)-1.*k1s*(1.-1.*qtgvs)*l120(i)*x26(i)*x4(i)+k1s*(1.-
1.*qtgvs)*l14(i)*x26(i)*x4(i)+k1s*rhos*l126(i)*x26(i)*((1.-1.*qthvs)*x1(i)+(1.-
1.*qtsvs)*x2(i)+(1.-1.*qtrvs)*x3(i)+(1.-1.*qtgvs)*x4(i)))-1.*(-1.*k1s*(1.-
1.*qthvs)*l11(i)*x1(i)*x26(i)+k1s*(1.-1.*qthvs)*l18(i)*x1(i)*x26(i)+k1s*(1.-
1.*qtsvs)*l112(i)*x2(i)*x26(i)-1.*k1s*(1.-1.*qtsvs)*l12(i)*x2(i)*x26(i)+k1s*(1.-
1.*qtrvs)*l116(i)*x26(i)*x3(i)-1.*k1s*(1.-1.*qtrvs)*l13(i)*x26(i)*x3(i)+k1s*(1.-
1.*qtgvs)*l120(i)*x26(i)*x4(i)-1.*k1s*(1.-1.*qtgvs)*l14(i)*x26(i)*x4(i)-
1.*k1s*rhos*l126(i)*x26(i)*((1.-1.*qthvs)*x1(i)+(1.-1.*qtsvs)*x2(i)+(1.-1.*qtrvs)*x3(i)+(1.-
1.*qtgvs)*x4(i)))*(k1s*(1.-1.*qthvs)*l11(i)*x1(i)*x26(i)-1.*k1s*(1.-
1.*qthvs)*l18(i)*x1(i)*x26(i)-1.*k1s*(1.-1.*qtsvs)*l112(i)*x2(i)*x26(i)+k1s*(1.-
1.*qtsvs)*l12(i)*x2(i)*x26(i)-1.*k1s*(1.-1.*qtrvs)*l116(i)*x26(i)*x3(i)+k1s*(1.-
1.*qtrvs)*l13(i)*x26(i)*x3(i)-1.*k1s*(1.-1.*qtgvs)*l120(i)*x26(i)*x4(i)+k1s*(1.-
1.*qtgvs)*l14(i)*x26(i)*x4(i)+k1s*rhos*l126(i)*x26(i)*((1.-1.*qthvs)*x1(i)+(1.-
1.*qtsvs)*x2(i)+(1.-1.*qtrvs)*x3(i)+(1.-1.*qtgvs)*x4(i))))/(1.*aco*afi-1.*(-1.*k1s*(1.-
1.*qthvs)*l11(i)*x1(i)*x26(i)+k1s*(1.-1.*qthvs)*l18(i)*x1(i)*x26(i)+k1s*(1.-
1.*qtsvs)*l112(i)*x2(i)*x26(i)-1.*k1s*(1.-1.*qtsvs)*l12(i)*x2(i)*x26(i)+k1s*(1.-
1.*qtrvs)*l116(i)*x26(i)*x3(i)-1.*k1s*(1.-1.*qtrvs)*l13(i)*x26(i)*x3(i)+k1s*(1.-
1.*qtgvs)*l120(i)*x26(i)*x4(i)-1.*k1s*(1.-1.*qtgvs)*l14(i)*x26(i)*x4(i)-
1.*k1s*rhos*l126(i)*x26(i)*((1.-1.*qthvs)*x1(i)+(1.-1.*qtsvs)*x2(i)+(1.-1.*qtrvs)*x3(i)+(1.-
1.*qtgvs)*x4(i)))^2));

a13=(1.*(l11(i)*x1(i)+l12(i)*x2(i)+l13(i)*x3(i)+l14(i)*x4(i)))/aib;

a14=(1.*(k2*l112(i)*x12(i)-1.*k2*l113(i)*x12(i)+k2*l116(i)*x16(i)-
1.*k2*l117(i)*x16(i)+k2*l120(i)*x20(i)-1.*k2*l121(i)*x20(i)+k2*l18(i)*x8(i)-
1.*k2*l19(i)*x8(i)))/aii-(1.*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-
1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-
1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i))*((1.*(1.*bsr*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))*(1.*brs-
1.*pr*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))+pr*l227(i)*(x11(i)+x15(i)+x19(i)+x23(i

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)+x25(i)))-1.*brs*(-1.*bsr*(-1.*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))-
(1.*g*l126(i)*x26(i))/(b2+x26(i))-
1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i))+
1.*ps*l126(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))*(1.*bsr+ps*l226(i)*(x13(i)+x17(i)+x21(i)
+x24(i)+x9(i))-
1.*ps*l227(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))))))/(brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-
1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-1.*k1sm*l15(i)*x26(i)*x5(i)))-
(1.*(1.*bsr*pr^2*l126(i)*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))^2-
1.*brs*(1.*api*bsr-
1.*ps^2*l126(i)*l226(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))^2))*(1.*brs*bsr*(k1sm*l124(i)
*x26(i)*x5(i)-1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-1.*k1sm*l15(i)*x26(i)*x5(i))*(-
1.*aii*(k2*l112(i)*x12(i)-1.*k2*l113(i)*x12(i)+k2*l116(i)*x16(i)-
1.*k2*l117(i)*x16(i)+k2*l120(i)*x20(i)-1.*k2*l121(i)*x20(i)-
1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i)+k
2*l18(i)*x8(i)-1.*k2*l19(i)*x8(i))-1.*(k2*l112(i)*x12(i)-
1.*k2*l113(i)*x12(i)+k2*l116(i)*x16(i)-1.*k2*l117(i)*x16(i)+k2*l120(i)*x20(i)-
1.*k2*l121(i)*x20(i)+k2*l18(i)*x8(i)-1.*k2*l19(i)*x8(i))*(-
1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-
1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i)))-1.*(1.*aii*art-
1.*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-
1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-
1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i))^2)*(1.*bsr*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x2
5(i))*(1.*brs-
1.*pr*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))+pr*l227(i)*(x11(i)+x15(i)+x19(i)+x23(i
)+x25(i)))-1.*brs*(-1.*bsr*(-1.*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))-
(1.*g*l126(i)*x26(i))/(b2+x26(i))-
1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i))+
1.*ps*l126(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))*(1.*bsr+ps*l226(i)*(x13(i)+x17(i)+x21(i)
+x24(i)+x9(i))-
1.*ps*l227(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i)))))))/(brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-
1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-1.*k1sm*l15(i)*x26(i)*x5(i))*(-
1.*aii*brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-
1.*k1sm*l15(i)*x26(i)*x5(i))^2-1.*(1.*aii*art-1.*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-
1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-
1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i))^2)*(1.*bsr*pr^2*l126(i)*l226(i)*(x11(i)+x15(i)+x19(i)+
x23(i)+x25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*l126(i)*l226(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))^2))))))/aii;

  a15= -((1.*brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-
1.*k1sm*l15(i)*x26(i)*x5(i))*(-1.*aii*(k2*l112(i)*x12(i)-
1.*k2*l113(i)*x12(i)+k2*l116(i)*x16(i)-1.*k2*l117(i)*x16(i)+k2*l120(i)*x20(i)-
1.*k2*l121(i)*x20(i)-
1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i)+k
2*l18(i)*x8(i)-1.*k2*l19(i)*x8(i))-1.*(k2*l112(i)*x12(i)-
1.*k2*l113(i)*x12(i)+k2*l116(i)*x16(i)-1.*k2*l117(i)*x16(i)+k2*l120(i)*x20(i)-
1.*k2*l121(i)*x20(i)+k2*l18(i)*x8(i)-1.*k2*l19(i)*x8(i))*(-
1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-
1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i)))-1.*(1.*aii*art-
1.*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-
1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-
1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i))^2)*(1.*bsr*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x2
5(i))*(1.*brs-
1.*pr*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))+pr*l227(i)*(x11(i)+x15(i)+x19(i)+x23(i
)+x25(i)))-1.*brs*(-1.*bsr*(-1.*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))-
(1.*g*l126(i)*x26(i))/(b2+x26(i))-
1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i))+
1.*ps*l126(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))*(1.*bsr+ps*l226(i)*(x13(i)+x17(i)+x21(i)
+x24(i)+x9(i))-1.*ps*l227(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))))))/(-
1.*aii*brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-
1.*k1sm*l15(i)*x26(i)*x5(i))^2-1.*(1.*aii*art-1.*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-
```

1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-
1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i))^2)*(1.*bsr*pr^2*l126(i)*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*l126(i)*l226(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))^2))));

a16=(-1.*(1.*bsr*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))*(1.*brs-
1.*pr*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))+pr*l227(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i)))-1.*brs*(-1.*bsr*(-1.*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))-(1.*g*l126(i)*x26(i))/(b2+x26(i))-
1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i))+
1.*ps*l126(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))*(1.*bsr+ps*l226(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))-
1.*ps*l227(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))))))/(brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-
1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-
1.*k1sm*l15(i)*x26(i)*x5(i)))+(1.*(1.*bsr*pr^2*l126(i)*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*l126(i)*l226(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))^2))*(1.*brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-1.*k1sm*l15(i)*x26(i)*x5(i))*(-
1.*aii*(k2*l112(i)*x12(i)-1.*k2*l113(i)*x12(i)+k2*l116(i)*x16(i)-
1.*k2*l117(i)*x16(i)+k2*l120(i)*x20(i)-1.*k2*l121(i)*x20(i)-
1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i)+k2*l18(i)*x8(i)-1.*k2*l19(i)*x8(i))-1.*(k2*l112(i)*x12(i)-
1.*k2*l113(i)*x12(i)+k2*l116(i)*x16(i)-1.*k2*l117(i)*x16(i)+k2*l120(i)*x20(i)-
1.*k2*l121(i)*x20(i)+k2*l18(i)*x8(i)-1.*k2*l19(i)*x8(i))*(-
1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-
1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i)))-1.*(1.*aii*art-
1.*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-
1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-
1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i))^2)*(1.*bsr*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))*(1.*brs-
1.*pr*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))+pr*l227(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i)))-1.*brs*(-1.*bsr*(-1.*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))-(1.*g*l126(i)*x26(i))/(b2+x26(i))-
1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i))+
1.*ps*l126(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))*(1.*bsr+ps*l226(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))-
1.*ps*l227(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i)))))))/(brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-
1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-1.*k1sm*l15(i)*x26(i)*x5(i))*(-
1.*aii*brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-
1.*k1sm*l15(i)*x26(i)*x5(i))^2-1.*(1.*aii*art-1.*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-
1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-
1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i))^2)*(1.*bsr*pr^2*l126(i)*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*l126(i)*l226(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))^2))));

u21s= min(1,max(0,(1.*(1.*bsr+ps*l226(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))-
1.*ps*l227(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))))/bsr+(1.*ps*l226(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))*(1.*brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-
1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-1.*k1sm*l15(i)*x26(i)*x5(i))*(-
1.*aii*(k2*l112(i)*x12(i)-1.*k2*l113(i)*x12(i)+k2*l116(i)*x16(i)-
1.*k2*l117(i)*x16(i)+k2*l120(i)*x20(i)-1.*k2*l121(i)*x20(i)-
1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i)+k2*l18(i)*x8(i)-1.*k2*l19(i)*x8(i))-1.*(k2*l112(i)*x12(i)-
1.*k2*l113(i)*x12(i)+k2*l116(i)*x16(i)-1.*k2*l117(i)*x16(i)+k2*l120(i)*x20(i)-
1.*k2*l121(i)*x20(i)+k2*l18(i)*x8(i)-1.*k2*l19(i)*x8(i))*(-
1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-
1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i)))-1.*(1.*aii*art-
1.*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-
1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-
1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i))^2)*(1.*bsr*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x2

```
5(i))*(1.*brs-
1.*pr*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))+pr*l227(i)*(x11(i)+x15(i)+x19(i)+x23(i
)+x25(i)))-1.*brs*(-1.*bsr*(-1.*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))-
(1.*g*l126(i)*x26(i))/(b2+x26(i))-
1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i))+
1.*ps*l126(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))*(1.*bsr+ps*l226(i)*(x13(i)+x17(i)+x21(i)
+x24(i)+x9(i))-1.*ps*l227(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i)))))))/(bsr*(-
1.*aii*brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-
1.*k1sm*l15(i)*x26(i)*x5(i))^2-1.*(1.*aii*art-1.*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-
1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-
1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i))^2)*(1.*bsr*pr^2*l126(i)*l226(i)*(x11(i)+x15(i)+x19(i)+
x23(i)+x25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*l126(i)*l226(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))^2))))));

    u22s= min(1,max(0,(1.*(1.*brs-
1.*pr*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))+pr*l227(i)*(x11(i)+x15(i)+x19(i)+x23(i
)+x25(i))))/brs-
(1.*pr*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))*(1.*brs*bsr*(k1sm*l124(i)*x26(i)*x5(i
)-1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-1.*k1sm*l15(i)*x26(i)*x5(i))*(-
1.*aii*(k2*l112(i)*x12(i)-1.*k2*l113(i)*x12(i)+k2*l116(i)*x16(i)-
1.*k2*l117(i)*x16(i)+k2*l120(i)*x20(i)-1.*k2*l121(i)*x20(i)-
1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i)+k
2*l18(i)*x8(i)-1.*k2*l19(i)*x8(i))-1.*(k2*l112(i)*x12(i)-
1.*k2*l113(i)*x12(i)+k2*l116(i)*x16(i)-1.*k2*l117(i)*x16(i)+k2*l120(i)*x20(i)-
1.*k2*l121(i)*x20(i)+k2*l18(i)*x8(i)-1.*k2*l19(i)*x8(i))*(-
1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-
1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i)))-1.*(1.*aii*art-
1.*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-
1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-
1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i))^2)*(1.*bsr*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x2
5(i))*(1.*brs-
1.*pr*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))+pr*l227(i)*(x11(i)+x15(i)+x19(i)+x23(i
)+x25(i)))-1.*brs*(-1.*bsr*(-1.*pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))-
(1.*g*l126(i)*x26(i))/(b2+x26(i))-
1.*k1sm*l124(i)*x26(i)*x5(i)+k1sm*rhos*l126(i)*x26(i)*x5(i)+k1sm*l15(i)*x26(i)*x5(i))+
1.*ps*l126(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))*(1.*bsr+ps*l226(i)*(x13(i)+x17(i)+x21(i)
+x24(i)+x9(i))-1.*ps*l227(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i)))))))/(brs*(-
1.*aii*brs*bsr*(k1sm*l124(i)*x26(i)*x5(i)-1.*k1sm*rhos*l126(i)*x26(i)*x5(i)-
1.*k1sm*l15(i)*x26(i)*x5(i))^2-1.*(1.*aii*art-1.*(-1.*k2*l112(i)*x12(i)+k2*l113(i)*x12(i)-
1.*k2*l116(i)*x16(i)+k2*l117(i)*x16(i)-1.*k2*l120(i)*x20(i)+k2*l121(i)*x20(i)-
1.*k2*l18(i)*x8(i)+k2*l19(i)*x8(i))^2)*(1.*bsr*pr^2*l126(i)*l226(i)*(x11(i)+x15(i)+x19(i)+
x23(i)+x25(i))^2-1.*brs*(1.*api*bsr-
1.*ps^2*l126(i)*l226(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i))^2))))));


    if(a11<0 || a11>1)

            a11=0;

    end;


    if(a12<0 || a12>1)

            a12=0;

    end;
```

```
if(a13<0 || a13>1)
        a13=0;
end;

if(a14<0 || a14>1)
        a14=0;
end;

if(a15<0 || a15>1)
        a15=0;
end;

if(a16<0 || a16>1)
        a16=0;
end;

        D=[a11,a12,a13,a14,a15,a16;a11,a12,a13,a14,a15,1;a11,a12,a13,a14,a15,0;a11,a12,
a13,a14,1,a16;a11,a12,a13,a14,1,1;a11,a12,a13,a14,1,0;a11,a12,a13,a14,0,a16;a11,a12,a13,a1
4,0,1;a11,a12,a13,a14,0,0;a11,a12,a13,1,a15,a16;a11,a12,a13,1,a15,1;a11,a12,a13,1,a15,0;a1
1,a12,a13,1,1,a16;a11,a12,a13,1,1,1;a11,a12,a13,1,1,0;a11,a12,a13,1,0,a16;a11,a12,a13,1,0,1;
a11,a12,a13,1,0,0;a11,a12,a13,0,a15,a16;a11,a12,a13,0,a15,1;a11,a12,a13,0,a15,0;a11,a12,a1
3,0,1,a16;a11,a12,a13,0,1,1;a11,a12,a13,0,1,0;a11,a12,a13,0,0,a16;a11,a12,a13,0,0,1;a11,a12,
a13,0,0,0;a11,a12,1,a14,a15,a16;a11,a12,1,a14,a15,1;a11,a12,1,a14,a15,0;a11,a12,1,a14,1,a1
6;a11,a12,1,a14,1,1;a11,a12,1,a14,1,0;a11,a12,1,a14,0,a16;a11,a12,1,a14,0,1;a11,a12,1,a14,0,
0;a11,a12,1,1,a15,a16;a11,a12,1,1,a15,1;a11,a12,1,1,a15,0;a11,a12,1,1,1,a16;a11,a12,1,1,1,1;
a11,a12,1,1,1,0;a11,a12,1,1,0,a16;a11,a12,1,1,0,1;a11,a12,1,1,0,0;a11,a12,1,0,a15,a16;a11,a1
2,1,0,a15,1;a11,a12,1,0,a15,0;a11,a12,1,0,1,a16;a11,a12,1,0,1,1;a11,a12,1,0,1,0;a11,a12,1,0,0
,a16;a11,a12,1,0,0,1;a11,a12,1,0,0,0;a11,a12,0,a14,a15,a16;a11,a12,0,a14,a15,1;a11,a12,0,a1
4,a15,0;a11,a12,0,a14,1,a16;a11,a12,0,a14,1,1;a11,a12,0,a14,1,0;a11,a12,0,a14,0,a16;a11,a12
,0,a14,0,1;a11,a12,0,a14,0,0;a11,a12,0,1,a15,a16;a11,a12,0,1,a15,1;a11,a12,0,1,a15,0;a11,a12
,0,1,1,a16;a11,a12,0,1,1,1;a11,a12,0,1,1,0;a11,a12,0,1,0,a16;a11,a12,0,1,0,1;a11,a12,0,1,0,0;a
11,a12,0,0,a15,a16;a11,a12,0,0,a15,1;a11,a12,0,0,a15,0;a11,a12,0,0,1,a16;a11,a12,0,0,1,1;a1
1,a12,0,0,1,0;a11,a12,0,0,0,a16;a11,a12,0,0,0,1;a11,a12,0,0,0,0;a11,1,a13,a14,a15,a16;a11,1,
a13,a14,a15,1;a11,1,a13,a14,a15,0;a11,1,a13,a14,1,a16;a11,1,a13,a14,1,1;a11,1,a13,a14,1,0;a
11,1,a13,a14,0,a16;a11,1,a13,a14,0,1;a11,1,a13,a14,0,0;a11,1,a13,1,a15,a16;a11,1,a13,1,a15,
1;a11,1,a13,1,a15,0;a11,1,a13,1,1,a16;a11,1,a13,1,1,1;a11,1,a13,1,1,0;a11,1,a13,1,0,a16;a11,
1,a13,1,0,1;a11,1,a13,1,0,0;a11,1,a13,0,a15,a16;a11,1,a13,0,a15,1;a11,1,a13,0,a15,0;a11,1,a1
3,0,1,a16;a11,1,a13,0,1,1;a11,1,a13,0,1,0;a11,1,a13,0,0,a16;a11,1,a13,0,0,1;a11,1,a13,0,0,0;a
11,1,1,a14,a15,a16;a11,1,1,a14,a15,1;a11,1,1,a14,a15,0;a11,1,1,a14,1,a16;a11,1,1,a14,1,1;a1
1,1,1,a14,1,0;a11,1,1,a14,0,a16;a11,1,1,a14,0,1;a11,1,1,a14,0,0;a11,1,1,1,a15,a16;a11,1,1,1,a
15,1;a11,1,1,1,a15,0;a11,1,1,1,1,a16;a11,1,1,1,1,1;a11,1,1,1,1,0;a11,1,1,1,0,a16;a11,1,1,1,0,1;
a11,1,1,1,0,0;a11,1,1,0,a15,a16;a11,1,1,0,a15,1;a11,1,1,0,a15,0;a11,1,1,0,1,a16;a11,1,1,0,1,1;
a11,1,1,0,1,0;a11,1,1,0,0,a16;a11,1,1,0,0,1;a11,1,1,0,0,0;a11,1,0,a14,a15,a16;a11,1,0,a14,a15,
1;a11,1,0,a14,a15,0;a11,1,0,a14,1,a16;a11,1,0,a14,1,1;a11,1,0,a14,1,0;a11,1,0,a14,0,a16;a11,
1,0,a14,0,1;a11,1,0,a14,0,0;a11,1,0,1,a15,a16;a11,1,0,1,a15,1;a11,1,0,1,a15,0;a11,1,0,1,1,a16;
```

```
a11,1,0,1,1,1;a11,1,0,1,1,0;a11,1,0,1,0,a16;a11,1,0,1,0,1;a11,1,0,1,0,0;a11,1,0,0,a15,a16;a11,
1,0,0,a15,1;a11,1,0,0,a15,0;a11,1,0,0,1,a16;a11,1,0,0,1,1;a11,1,0,0,1,0;a11,1,0,0,0,a16;a11,1,
0,0,0,1;a11,1,0,0,0,0;a11,0,a13,a14,a15,a16;a11,0,a13,a14,a15,1;a11,0,a13,a14,a15,0;a11,0,a1
3,a14,1,a16;a11,0,a13,a14,1,1;a11,0,a13,a14,1,0;a11,0,a13,a14,0,a16;a11,0,a13,a14,0,1;a11,0,
a13,a14,0,0;a11,0,a13,1,a15,a16;a11,0,a13,1,a15,1;a11,0,a13,1,a15,0;a11,0,a13,1,1,a16;a11,0,
a13,1,1,1;a11,0,a13,1,1,0;a11,0,a13,1,0,a16;a11,0,a13,1,0,1;a11,0,a13,1,0,0;a11,0,a13,0,a15,a
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1,0,0,a15,a16;0,1,0,0,a15,1;0,1,0,0,a15,0;0,1,0,0,1,a16;0,1,0,0,1,1;0,1,0,0,1,0;0,1,0,0,0,a16;0,
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a14,0,1;0,0,0,a14,0,0;0,0,0,1,a15,a16;0,0,0,1,a15,1;0,0,0,1,a15,0;0,0,0,1,1,a16;0,0,0,1,1,1;0,0,
0,1,1,0;0,0,0,1,0,a16;0,0,0,1,0,1;0,0,0,1,0,0;0,0,0,0,a15,a16;0,0,0,0,a15,1;0,0,0,0,a15,0;0,0,0,0
,1,a16;0,0,0,0,1,1;0,0,0,0,1,0;0,0,0,0,0,a16;0,0,0,0,0,1;0,0,0,0,0,0];

eval_HT=eval(fHT);

[max_HT, id_HT]=nanmax(eval_HT);

u1(1,i)=D(id_HT,1);
u1(2,i)=D(id_HT,2);
u1(3,i)=D(id_HT,3);
u1(4,i)=D(id_HT,4);
u1(5,i)=D(id_HT,5);
u1(6,i)=D(id_HT,6);
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u2(1,i)=u21s;

u2(2,i)=u22s;




%################## verifica concavità FINALE h2 e h6 dei controlli relativi
alla terapia

H2terapia(1,i)=1.*aco*afi-
k1s^2*l11(i)^2*x1(i)^2*x26(i)^2+2*k1s^2*qthvs*l11(i)^2*x1(i)^2*x26(i)^2-
k1s^2*qthvs^2*l11(i)^2*x1(i)^2*x26(i)^2-
2*k1s^2*rhos*l11(i)*l126(i)*x1(i)^2*x26(i)^2+4*k1s^2*qthvs*rhos*l11(i)*l126(i)*x1(i)^2*
x26(i)^2-2*k1s^2*qthvs^2*rhos*l11(i)*l126(i)*x1(i)^2*x26(i)^2-
k1s^2*rhos^2*l126(i)^2*x1(i)^2*x26(i)^2+2*k1s^2*qthvs*rhos^2*l126(i)^2*x1(i)^2*x26(i)
^2-
k1s^2*qthvs^2*rhos^2*l126(i)^2*x1(i)^2*x26(i)^2+2*k1s^2*l11(i)*l18(i)*x1(i)^2*x26(i)^2-
4*k1s^2*qthvs*l11(i)*l18(i)*x1(i)^2*x26(i)^2+2*k1s^2*qthvs^2*l11(i)*l18(i)*x1(i)^2*x26(
i)^2+2*k1s^2*rhos*l126(i)*l18(i)*x1(i)^2*x26(i)^2-
4*k1s^2*qthvs*rhos*l126(i)*l18(i)*x1(i)^2*x26(i)^2+2*k1s^2*qthvs^2*rhos*l126(i)*l18(i)*
x1(i)^2*x26(i)^2-
k1s^2*l18(i)^2*x1(i)^2*x26(i)^2+2*k1s^2*qthvs*l18(i)^2*x1(i)^2*x26(i)^2-
k1s^2*qthvs^2*l18(i)^2*x1(i)^2*x26(i)^2+2*k1s^2*l11(i)*l112(i)*x1(i)*x2(i)*x26(i)^2-
2*k1s^2*qthvs*l11(i)*l112(i)*x1(i)*x2(i)*x26(i)^2-
2*k1s^2*qtsvs*l11(i)*l112(i)*x1(i)*x2(i)*x26(i)^2+2*k1s^2*qthvs*qtsvs*l11(i)*l112(i)*x1(
i)*x2(i)*x26(i)^2-
2*k1s^2*l11(i)*l12(i)*x1(i)*x2(i)*x26(i)^2+2*k1s^2*qthvs*l11(i)*l12(i)*x1(i)*x2(i)*x26(i)
^2+2*k1s^2*qtsvs*l11(i)*l12(i)*x1(i)*x2(i)*x26(i)^2-
2*k1s^2*qthvs*qtsvs*l11(i)*l12(i)*x1(i)*x2(i)*x26(i)^2-
2*k1s^2*rhos*l11(i)*l126(i)*x1(i)*x2(i)*x26(i)^2+2*k1s^2*qthvs*rhos*l11(i)*l126(i)*x1(i)
*x2(i)*x26(i)^2+2*k1s^2*qtsvs*rhos*l11(i)*l126(i)*x1(i)*x2(i)*x26(i)^2-
2*k1s^2*qthvs*qtsvs*rhos*l11(i)*l126(i)*x1(i)*x2(i)*x26(i)^2+2*k1s^2*rhos*l112(i)*l126(
i)*x1(i)*x2(i)*x26(i)^2-2*k1s^2*qthvs*rhos*l112(i)*l126(i)*x1(i)*x2(i)*x26(i)^2-
2*k1s^2*qtsvs*rhos*l112(i)*l126(i)*x1(i)*x2(i)*x26(i)^2+2*k1s^2*qthvs*qtsvs*rhos*l112(i
)*l126(i)*x1(i)*x2(i)*x26(i)^2-
2*k1s^2*rhos*l12(i)*l126(i)*x1(i)*x2(i)*x26(i)^2+2*k1s^2*qthvs*rhos*l12(i)*l126(i)*x1(i)
*x2(i)*x26(i)^2+2*k1s^2*qtsvs*rhos*l12(i)*l126(i)*x1(i)*x2(i)*x26(i)^2-
2*k1s^2*qthvs*qtsvs*rhos*l12(i)*l126(i)*x1(i)*x2(i)*x26(i)^2-
2*k1s^2*rhos^2*l126(i)^2*x1(i)*x2(i)*x26(i)^2+2*k1s^2*qthvs*rhos^2*l126(i)^2*x1(i)*x2
(i)*x26(i)^2+2*k1s^2*qtsvs*rhos^2*l126(i)^2*x1(i)*x2(i)*x26(i)^2-
2*k1s^2*qthvs*qtsvs*rhos^2*l126(i)^2*x1(i)*x2(i)*x26(i)^2-
2*k1s^2*l112(i)*l18(i)*x1(i)*x2(i)*x26(i)^2+2*k1s^2*qthvs*l112(i)*l18(i)*x1(i)*x2(i)*x26
(i)^2+2*k1s^2*qtsvs*l112(i)*l18(i)*x1(i)*x2(i)*x26(i)^2-
2*k1s^2*qthvs*qtsvs*l112(i)*l18(i)*x1(i)*x2(i)*x26(i)^2+2*k1s^2*l12(i)*l18(i)*x1(i)*x2(i)
*x26(i)^2-2*k1s^2*qthvs*l12(i)*l18(i)*x1(i)*x2(i)*x26(i)^2-
2*k1s^2*qtsvs*l12(i)*l18(i)*x1(i)*x2(i)*x26(i)^2+2*k1s^2*qthvs*qtsvs*l12(i)*l18(i)*x1(i)*
x2(i)*x26(i)^2+2*k1s^2*rhos*l126(i)*l18(i)*x1(i)*x2(i)*x26(i)^2-
2*k1s^2*qthvs*rhos*l126(i)*l18(i)*x1(i)*x2(i)*x26(i)^2-
2*k1s^2*qtsvs*rhos*l126(i)*l18(i)*x1(i)*x2(i)*x26(i)^2+2*k1s^2*qthvs*qtsvs*rhos*l126(i)
*l18(i)*x1(i)*x2(i)*x26(i)^2-
k1s^2*l112(i)^2*x2(i)^2*x26(i)^2+2*k1s^2*qtsvs*l112(i)^2*x2(i)^2*x26(i)^2-
k1s^2*qtsvs^2*l112(i)^2*x2(i)^2*x26(i)^2+2*k1s^2*l112(i)*l12(i)*x2(i)^2*x26(i)^2-
4*k1s^2*qtsvs*l112(i)*l12(i)*x2(i)^2*x26(i)^2+2*k1s^2*qtsvs^2*l112(i)*l12(i)*x2(i)^2*x2
6(i)^2-k1s^2*l12(i)^2*x2(i)^2*x26(i)^2+2*k1s^2*qtsvs*l12(i)^2*x2(i)^2*x26(i)^2-
k1s^2*qtsvs^2*l12(i)^2*x2(i)^2*x26(i)^2+2*k1s^2*rhos*l112(i)*l126(i)*x2(i)^2*x26(i)^2-
4*k1s^2*qtsvs*rhos*l112(i)*l126(i)*x2(i)^2*x26(i)^2+2*k1s^2*qtsvs^2*rhos*l112(i)*l126(i
)*x2(i)^2*x26(i)^2-
2*k1s^2*rhos*l12(i)*l126(i)*x2(i)^2*x26(i)^2+4*k1s^2*qtsvs*rhos*l12(i)*l126(i)*x2(i)^2*
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x26(i)^2-2*k1s^2*qtsvs^2*rhos*l12(i)*l126(i)*x2(i)^2*x26(i)^2-
k1s^2*rhos^2*l126(i)^2*x2(i)^2*x26(i)^2+2*k1s^2*qtsvs*rhos^2*l126(i)^2*x2(i)^2*x26(i)^
2-
k1s^2*qtsvs^2*rhos^2*l126(i)^2*x2(i)^2*x26(i)^2+2*k1s^2*l11(i)*l116(i)*x1(i)*x26(i)^2*
x3(i)-2*k1s^2*qthvs*l11(i)*l116(i)*x1(i)*x26(i)^2*x3(i)-
2*k1s^2*qtrvs*l11(i)*l116(i)*x1(i)*x26(i)^2*x3(i)+2*k1s^2*qthvs*qtrvs*l11(i)*l116(i)*x1(i
)*x26(i)^2*x3(i)-
2*k1s^2*rhos*l11(i)*l126(i)*x1(i)*x26(i)^2*x3(i)+2*k1s^2*qthvs*rhos*l11(i)*l126(i)*x1(i)
*x26(i)^2*x3(i)+2*k1s^2*qtrvs*rhos*l11(i)*l126(i)*x1(i)*x26(i)^2*x3(i)-
2*k1s^2*qthvs*qtrvs*rhos*l11(i)*l126(i)*x1(i)*x26(i)^2*x3(i)+2*k1s^2*rhos*l116(i)*l126(i
)*x1(i)*x26(i)^2*x3(i)-2*k1s^2*qthvs*rhos*l116(i)*l126(i)*x1(i)*x26(i)^2*x3(i)-
2*k1s^2*qtrvs*rhos*l116(i)*l126(i)*x1(i)*x26(i)^2*x3(i)+2*k1s^2*qthvs*qtrvs*rhos*l116(i
)*l126(i)*x1(i)*x26(i)^2*x3(i)-
2*k1s^2*rhos^2*l126(i)^2*x1(i)*x26(i)^2*x3(i)+2*k1s^2*qthvs*rhos^2*l126(i)^2*x1(i)*x2
6(i)^2*x3(i)+2*k1s^2*qtrvs*rhos^2*l126(i)^2*x1(i)*x26(i)^2*x3(i)-
2*k1s^2*qthvs*qtrvs*rhos^2*l126(i)^2*x1(i)*x26(i)^2*x3(i)-
2*k1s^2*l11(i)*l13(i)*x1(i)*x26(i)^2*x3(i)+2*k1s^2*qthvs*l11(i)*l13(i)*x1(i)*x26(i)^2*x3
(i)+2*k1s^2*qtrvs*l11(i)*l13(i)*x1(i)*x26(i)^2*x3(i)-
2*k1s^2*qthvs*qtrvs*l11(i)*l13(i)*x1(i)*x26(i)^2*x3(i)-
2*k1s^2*rhos*l126(i)*l13(i)*x1(i)*x26(i)^2*x3(i)+2*k1s^2*qthvs*rhos*l126(i)*l13(i)*x1(i)
*x26(i)^2*x3(i)+2*k1s^2*qtrvs*rhos*l126(i)*l13(i)*x1(i)*x26(i)^2*x3(i)-
2*k1s^2*qthvs*qtrvs*rhos*l126(i)*l13(i)*x1(i)*x26(i)^2*x3(i)-
2*k1s^2*l116(i)*l18(i)*x1(i)*x26(i)^2*x3(i)+2*k1s^2*qthvs*l116(i)*l18(i)*x1(i)*x26(i)^2*
x3(i)+2*k1s^2*qtrvs*l116(i)*l18(i)*x1(i)*x26(i)^2*x3(i)-
2*k1s^2*qthvs*qtrvs*l116(i)*l18(i)*x1(i)*x26(i)^2*x3(i)+2*k1s^2*rhos*l126(i)*l18(i)*x1(i
)*x26(i)^2*x3(i)-2*k1s^2*qthvs*rhos*l126(i)*l18(i)*x1(i)*x26(i)^2*x3(i)-
2*k1s^2*qtrvs*rhos*l126(i)*l18(i)*x1(i)*x26(i)^2*x3(i)+2*k1s^2*qthvs*qtrvs*rhos*l126(i)
*l18(i)*x1(i)*x26(i)^2*x3(i)+2*k1s^2*l13(i)*l18(i)*x1(i)*x26(i)^2*x3(i)-
2*k1s^2*qthvs*l13(i)*l18(i)*x1(i)*x26(i)^2*x3(i)-
2*k1s^2*qtrvs*l13(i)*l18(i)*x1(i)*x26(i)^2*x3(i)+2*k1s^2*qthvs*qtrvs*l13(i)*l18(i)*x1(i)*
x26(i)^2*x3(i)-
2*k1s^2*l112(i)*l116(i)*x2(i)*x26(i)^2*x3(i)+2*k1s^2*qtrvs*l112(i)*l116(i)*x2(i)*x26(i)^
2*x3(i)+2*k1s^2*qtsvs*l112(i)*l116(i)*x2(i)*x26(i)^2*x3(i)-
2*k1s^2*qtrvs*qtsvs*l112(i)*l116(i)*x2(i)*x26(i)^2*x3(i)+2*k1s^2*l116(i)*l12(i)*x2(i)*x2
6(i)^2*x3(i)-2*k1s^2*qtrvs*l116(i)*l12(i)*x2(i)*x26(i)^2*x3(i)-
2*k1s^2*qtsvs*l116(i)*l12(i)*x2(i)*x26(i)^2*x3(i)+2*k1s^2*qtrvs*qtsvs*l116(i)*l12(i)*x2(i
)*x26(i)^2*x3(i)+2*k1s^2*rhos*l112(i)*l126(i)*x2(i)*x26(i)^2*x3(i)-
2*k1s^2*qtrvs*rhos*l112(i)*l126(i)*x2(i)*x26(i)^2*x3(i)-
2*k1s^2*qtsvs*rhos*l112(i)*l126(i)*x2(i)*x26(i)^2*x3(i)+2*k1s^2*qtrvs*qtsvs*rhos*l112(i
)*l126(i)*x2(i)*x26(i)^2*x3(i)+2*k1s^2*rhos*l116(i)*l126(i)*x2(i)*x26(i)^2*x3(i)-
2*k1s^2*qtrvs*rhos*l116(i)*l126(i)*x2(i)*x26(i)^2*x3(i)-
2*k1s^2*qtsvs*rhos*l116(i)*l126(i)*x2(i)*x26(i)^2*x3(i)+2*k1s^2*qtrvs*qtsvs*rhos*l116(i
)*l126(i)*x2(i)*x26(i)^2*x3(i)-
2*k1s^2*rhos*l12(i)*l126(i)*x2(i)*x26(i)^2*x3(i)+2*k1s^2*qtrvs*rhos*l12(i)*l126(i)*x2(i)
*x26(i)^2*x3(i)+2*k1s^2*qtsvs*rhos*l12(i)*l126(i)*x2(i)*x26(i)^2*x3(i)-
2*k1s^2*qtrvs*qtsvs*rhos*l12(i)*l126(i)*x2(i)*x26(i)^2*x3(i)-
2*k1s^2*rhos^2*l126(i)^2*x2(i)*x26(i)^2*x3(i)+2*k1s^2*qtrvs*rhos^2*l126(i)^2*x2(i)*x2
6(i)^2*x3(i)+2*k1s^2*qtsvs*rhos^2*l126(i)^2*x2(i)*x26(i)^2*x3(i)-
2*k1s^2*qtrvs*qtsvs*rhos^2*l126(i)^2*x2(i)*x26(i)^2*x3(i)+2*k1s^2*l112(i)*l13(i)*x2(i)*
x26(i)^2*x3(i)-2*k1s^2*qtrvs*l112(i)*l13(i)*x2(i)*x26(i)^2*x3(i)-
2*k1s^2*qtsvs*l112(i)*l13(i)*x2(i)*x26(i)^2*x3(i)+2*k1s^2*qtrvs*qtsvs*l112(i)*l13(i)*x2(i
)*x26(i)^2*x3(i)-
2*k1s^2*l12(i)*l13(i)*x2(i)*x26(i)^2*x3(i)+2*k1s^2*qtrvs*l12(i)*l13(i)*x2(i)*x26(i)^2*x3(
i)+2*k1s^2*qtsvs*l12(i)*l13(i)*x2(i)*x26(i)^2*x3(i)-
2*k1s^2*qtrvs*qtsvs*l12(i)*l13(i)*x2(i)*x26(i)^2*x3(i)-
2*k1s^2*rhos*l126(i)*l13(i)*x2(i)*x26(i)^2*x3(i)+2*k1s^2*qtrvs*rhos*l126(i)*l13(i)*x2(i)
*x26(i)^2*x3(i)+2*k1s^2*qtsvs*rhos*l126(i)*l13(i)*x2(i)*x26(i)^2*x3(i)-
2*k1s^2*qtrvs*qtsvs*rhos*l126(i)*l13(i)*x2(i)*x26(i)^2*x3(i)-
k1s^2*l116(i)^2*x26(i)^2*x3(i)^2+2*k1s^2*qtrvs*l116(i)^2*x26(i)^2*x3(i)^2-
k1s^2*qtrvs^2*l116(i)^2*x26(i)^2*x3(i)^2+2*k1s^2*rhos*l116(i)*l126(i)*x26(i)^2*x3(i)^2-
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4*k1s^2*qtrvs*rhos*l116(i)*l126(i)*x26(i)^2*x3(i)^2+2*k1s^2*qtrvs^2*rhos*l116(i)*l126(i)*x26(i)^2*x3(i)^2-k1s^2*rhos^2*l126(i)^2*x26(i)^2*x3(i)^2+2*k1s^2*qtrvs*rhos^2*l126(i)^2*x26(i)^2*x3(i)^2-k1s^2*qtrvs^2*rhos^2*l126(i)^2*x26(i)^2*x3(i)^2+2*k1s^2*l116(i)*l13(i)*x26(i)^2*x3(i)^2-4*k1s^2*qtrvs*l116(i)*l13(i)*x26(i)^2*x3(i)^2+2*k1s^2*qtrvs^2*l116(i)*l13(i)*x26(i)^2*x3(i)^2-2*k1s^2*rhos*l126(i)*l13(i)*x26(i)^2*x3(i)^2+4*k1s^2*qtrvs*rhos*l126(i)*l13(i)*x26(i)^2*x3(i)^2-2*k1s^2*qtrvs^2*rhos*l126(i)*l13(i)*x26(i)^2*x3(i)^2-k1s^2*l13(i)^2*x26(i)^2*x3(i)^2+2*k1s^2*qtrvs*l13(i)^2*x26(i)^2*x3(i)^2-k1s^2*qtrvs^2*l13(i)^2*x26(i)^2*x3(i)^2+2*k1s^2*l11(i)*l120(i)*x1(i)*x26(i)^2*x4(i)-2*k1s^2*qtgvs*l11(i)*l120(i)*x1(i)*x26(i)^2*x4(i)-2*k1s^2*qthvs*l11(i)*l120(i)*x1(i)*x26(i)^2*x4(i)+2*k1s^2*qtgvs*qthvs*l11(i)*l120(i)*x1(i)*x26(i)^2*x4(i)-2*k1s^2*rhos*l11(i)*l126(i)*x1(i)*x26(i)^2*x4(i)+2*k1s^2*qtgvs*rhos*l11(i)*l126(i)*x1(i)*x26(i)^2*x4(i)+2*k1s^2*qthvs*rhos*l11(i)*l126(i)*x1(i)*x26(i)^2*x4(i)-2*k1s^2*qtgvs*qthvs*rhos*l11(i)*l126(i)*x1(i)*x26(i)^2*x4(i)+2*k1s^2*rhos*l120(i)*l126(i)*x1(i)*x26(i)^2*x4(i)-2*k1s^2*qtgvs*rhos*l120(i)*l126(i)*x1(i)*x26(i)^2*x4(i)-2*k1s^2*qthvs*rhos*l120(i)*l126(i)*x1(i)*x26(i)^2*x4(i)+2*k1s^2*qtgvs*qthvs*rhos*l120(i)*l126(i)*x1(i)*x26(i)^2*x4(i)-2*k1s^2*rhos^2*l126(i)^2*x1(i)*x26(i)^2*x4(i)+2*k1s^2*qtgvs*rhos^2*l126(i)^2*x1(i)*x26(i)^2*x4(i)+2*k1s^2*qthvs*rhos^2*l126(i)^2*x1(i)*x26(i)^2*x4(i)-2*k1s^2*qtgvs*qthvs*rhos^2*l126(i)^2*x1(i)*x26(i)^2*x4(i)-2*k1s^2*l11(i)*l14(i)*x1(i)*x26(i)^2*x4(i)+2*k1s^2*qtgvs*l11(i)*l14(i)*x1(i)*x26(i)^2*x4(i)+2*k1s^2*qthvs*l11(i)*l14(i)*x1(i)*x26(i)^2*x4(i)-2*k1s^2*qtgvs*qthvs*l11(i)*l14(i)*x1(i)*x26(i)^2*x4(i)-2*k1s^2*rhos*l126(i)*l14(i)*x1(i)*x26(i)^2*x4(i)+2*k1s^2*qtgvs*rhos*l126(i)*l14(i)*x1(i)*x26(i)^2*x4(i)+2*k1s^2*qthvs*rhos*l126(i)*l14(i)*x1(i)*x26(i)^2*x4(i)-2*k1s^2*qtgvs*qthvs*rhos*l126(i)*l14(i)*x1(i)*x26(i)^2*x4(i)-2*k1s^2*l120(i)*l18(i)*x1(i)*x26(i)^2*x4(i)+2*k1s^2*qtgvs*l120(i)*l18(i)*x1(i)*x26(i)^2*x4(i)+2*k1s^2*qthvs*l120(i)*l18(i)*x1(i)*x26(i)^2*x4(i)-2*k1s^2*qtgvs*qthvs*l120(i)*l18(i)*x1(i)*x26(i)^2*x4(i)+2*k1s^2*rhos*l126(i)*l18(i)*x1(i)*x26(i)^2*x4(i)-2*k1s^2*qtgvs*rhos*l126(i)*l18(i)*x1(i)*x26(i)^2*x4(i)-2*k1s^2*qthvs*rhos*l126(i)*l18(i)*x1(i)*x26(i)^2*x4(i)+2*k1s^2*qtgvs*qthvs*rhos*l126(i)*l18(i)*x1(i)*x26(i)^2*x4(i)+2*k1s^2*l14(i)*l18(i)*x1(i)*x26(i)^2*x4(i)-2*k1s^2*qtgvs*l14(i)*l18(i)*x1(i)*x26(i)^2*x4(i)-2*k1s^2*qthvs*l14(i)*l18(i)*x1(i)*x26(i)^2*x4(i)+2*k1s^2*qtgvs*qthvs*l14(i)*l18(i)*x1(i)*x26(i)^2*x4(i)-2*k1s^2*l112(i)*l120(i)*x2(i)*x26(i)^2*x4(i)+2*k1s^2*qtgvs*l112(i)*l120(i)*x2(i)*x26(i)^2*x4(i)+2*k1s^2*qtsvs*l112(i)*l120(i)*x2(i)*x26(i)^2*x4(i)-2*k1s^2*qtgvs*qtsvs*l112(i)*l120(i)*x2(i)*x26(i)^2*x4(i)+2*k1s^2*l12(i)*l120(i)*x2(i)*x26(i)^2*x4(i)-2*k1s^2*qtgvs*l12(i)*l120(i)*x2(i)*x26(i)^2*x4(i)-2*k1s^2*qtsvs*l12(i)*l120(i)*x2(i)*x26(i)^2*x4(i)+2*k1s^2*qtgvs*qtsvs*l12(i)*l120(i)*x2(i)*x26(i)^2*x4(i)+2*k1s^2*rhos*l112(i)*l126(i)*x2(i)*x26(i)^2*x4(i)-2*k1s^2*qtgvs*rhos*l112(i)*l126(i)*x2(i)*x26(i)^2*x4(i)-2*k1s^2*qtsvs*rhos*l112(i)*l126(i)*x2(i)*x26(i)^2*x4(i)+2*k1s^2*qtgvs*qtsvs*rhos*l112(i)*l126(i)*x2(i)*x26(i)^2*x4(i)-2*k1s^2*rhos*l12(i)*l126(i)*x2(i)*x26(i)^2*x4(i)+2*k1s^2*qtgvs*rhos*l12(i)*l126(i)*x2(i)*x26(i)^2*x4(i)+2*k1s^2*qtsvs*rhos*l12(i)*l126(i)*x2(i)*x26(i)^2*x4(i)-2*k1s^2*qtgvs*qtsvs*rhos*l12(i)*l126(i)*x2(i)*x26(i)^2*x4(i)+2*k1s^2*rhos*l120(i)*l126(i)*x2(i)*x26(i)^2*x4(i)-2*k1s^2*qtgvs*rhos*l120(i)*l126(i)*x2(i)*x26(i)^2*x4(i)-2*k1s^2*qtsvs*rhos*l120(i)*l126(i)*x2(i)*x26(i)^2*x4(i)+2*k1s^2*qtgvs*qtsvs*rhos*l120(i)*l126(i)*x2(i)*x26(i)^2*x4(i)-2*k1s^2*rhos^2*l126(i)^2*x2(i)*x26(i)^2*x4(i)+2*k1s^2*qtgvs*rhos^2*l126(i)^2*x2(i)*x26(i)^2*x4(i)+2*k1s^2*qtsvs*rhos^2*l126(i)^2*x2(i)*x26(i)^2*x4(i)-2*k1s^2*qtgvs*qtsvs*rhos^2*l126(i)^2*x2(i)*x26(i)^2*x4(i)+2*k1s^2*l112(i)*l14(i)*x2(i)*x26(i)^2*x4(i)-2*k1s^2*qtgvs*l112(i)*l14(i)*x2(i)*x26(i)^2*x4(i)-2*k1s^2*qtsvs*l112(i)*l14(i)*x2(i)*x26(i)^2*x4(i)+2*k1s^2*qtgvs*qtsvs*l112(i)*l14(i)*x2(i)*x26(i)^2*x4(i)-

2*k1s^2*l12(i)*l14(i)*x2(i)*x26(i)^2*x4(i)+2*k1s^2*qtgvs*l12(i)*l14(i)*x2(i)*x26(i)^2*x4(i)+2*k1s^2*qtsvs*l12(i)*l14(i)*x2(i)*x26(i)^2*x4(i)-2*k1s^2*qtgvs*qtsvs*l12(i)*l14(i)*x2(i)*x26(i)^2*x4(i)-2*k1s^2*rhos*l126(i)*l14(i)*x2(i)*x26(i)^2*x4(i)+2*k1s^2*qtgvs*rhos*l126(i)*l14(i)*x2(i)*x26(i)^2*x4(i)+2*k1s^2*qtsvs*rhos*l126(i)*l14(i)*x2(i)*x26(i)^2*x4(i)-2*k1s^2*qtgvs*qtsvs*rhos*l126(i)*l14(i)*x2(i)*x26(i)^2*x4(i)-2*k1s^2*l116(i)*l120(i)*x26(i)^2*x3(i)*x4(i)+2*k1s^2*qtgvs*l116(i)*l120(i)*x26(i)^2*x3(i)*x4(i)+2*k1s^2*qtrvs*l116(i)*l120(i)*x26(i)^2*x3(i)*x4(i)-2*k1s^2*qtgvs*qtrvs*l116(i)*l120(i)*x26(i)^2*x3(i)*x4(i)+2*k1s^2*rhos*l116(i)*l126(i)*x26(i)^2*x3(i)*x4(i)-2*k1s^2*qtgvs*rhos*l116(i)*l126(i)*x26(i)^2*x3(i)*x4(i)-2*k1s^2*qtrvs*rhos*l116(i)*l126(i)*x26(i)^2*x3(i)*x4(i)+2*k1s^2*qtgvs*qtrvs*rhos*l116(i)*l126(i)*x26(i)^2*x3(i)*x4(i)+2*k1s^2*rhos*l120(i)*l126(i)*x26(i)^2*x3(i)*x4(i)-2*k1s^2*qtgvs*rhos*l120(i)*l126(i)*x26(i)^2*x3(i)*x4(i)-2*k1s^2*qtrvs*rhos*l120(i)*l126(i)*x26(i)^2*x3(i)*x4(i)+2*k1s^2*qtgvs*qtrvs*rhos*l120(i)*l126(i)*x26(i)^2*x3(i)*x4(i)-2*k1s^2*rhos^2*l126(i)^2*x26(i)^2*x3(i)*x4(i)+2*k1s^2*qtgvs*rhos^2*l126(i)^2*x26(i)^2*x3(i)*x4(i)+2*k1s^2*qtrvs*rhos^2*l126(i)^2*x26(i)^2*x3(i)*x4(i)-2*k1s^2*qtgvs*qtrvs*rhos^2*l126(i)^2*x26(i)^2*x3(i)*x4(i)+2*k1s^2*l120(i)*l13(i)*x26(i)^2*x3(i)*x4(i)-2*k1s^2*qtgvs*l120(i)*l13(i)*x26(i)^2*x3(i)*x4(i)-2*k1s^2*qtrvs*l120(i)*l13(i)*x26(i)^2*x3(i)*x4(i)+2*k1s^2*qtgvs*qtrvs*l120(i)*l13(i)*x26(i)^2*x3(i)*x4(i)-2*k1s^2*rhos*l126(i)*l13(i)*x26(i)^2*x3(i)*x4(i)+2*k1s^2*qtgvs*rhos*l126(i)*l13(i)*x26(i)^2*x3(i)*x4(i)+2*k1s^2*qtrvs*rhos*l126(i)*l13(i)*x26(i)^2*x3(i)*x4(i)-2*k1s^2*qtgvs*qtrvs*rhos*l126(i)*l13(i)*x26(i)^2*x3(i)*x4(i)+2*k1s^2*l116(i)*l14(i)*x26(i)^2*x3(i)*x4(i)-2*k1s^2*qtgvs*l116(i)*l14(i)*x26(i)^2*x3(i)*x4(i)-2*k1s^2*qtrvs*l116(i)*l14(i)*x26(i)^2*x3(i)*x4(i)+2*k1s^2*qtgvs*qtrvs*l116(i)*l14(i)*x26(i)^2*x3(i)*x4(i)-2*k1s^2*rhos*l126(i)*l14(i)*x26(i)^2*x3(i)*x4(i)+2*k1s^2*qtgvs*rhos*l126(i)*l14(i)*x26(i)^2*x3(i)*x4(i)+2*k1s^2*qtrvs*rhos*l126(i)*l14(i)*x26(i)^2*x3(i)*x4(i)-2*k1s^2*qtgvs*qtrvs*rhos*l126(i)*l14(i)*x26(i)^2*x3(i)*x4(i)-2*k1s^2*l13(i)*l14(i)*x26(i)^2*x3(i)*x4(i)+2*k1s^2*qtgvs*l13(i)*l14(i)*x26(i)^2*x3(i)*x4(i)+2*k1s^2*qtrvs*l13(i)*l14(i)*x26(i)^2*x3(i)*x4(i)-2*k1s^2*qtgvs*qtrvs*l13(i)*l14(i)*x26(i)^2*x3(i)*x4(i)-k1s^2*l120(i)^2*x26(i)^2*x4(i)^2+2*k1s^2*qtgvs*l120(i)^2*x26(i)^2*x4(i)^2-k1s^2*qtgvs^2*l120(i)^2*x26(i)^2*x4(i)^2+2*k1s^2*rhos*l120(i)*l126(i)*x26(i)^2*x4(i)^2-4*k1s^2*qtgvs*rhos*l120(i)*l126(i)*x26(i)^2*x4(i)^2+2*k1s^2*qtgvs^2*rhos*l120(i)*l126(i)*x26(i)^2*x4(i)^2-k1s^2*rhos^2*l126(i)^2*x26(i)^2*x4(i)^2+2*k1s^2*qtgvs*rhos^2*l126(i)^2*x26(i)^2*x4(i)^2-k1s^2*qtgvs^2*rhos^2*l126(i)^2*x26(i)^2*x4(i)^2+2*k1s^2*l120(i)*l14(i)*x26(i)^2*x4(i)^2-4*k1s^2*qtgvs*l120(i)*l14(i)*x26(i)^2*x4(i)^2+2*k1s^2*qtgvs^2*l120(i)*l14(i)*x26(i)^2*x4(i)^2-2*k1s^2*rhos*l126(i)*l14(i)*x26(i)^2*x4(i)^2+4*k1s^2*qtgvs*rhos*l126(i)*l14(i)*x26(i)^2*x4(i)^2-2*k1s^2*qtgvs^2*rhos*l126(i)*l14(i)*x26(i)^2*x4(i)^2-k1s^2*l14(i)^2*x26(i)^2*x4(i)^2+2*k1s^2*qtgvs*l14(i)^2*x26(i)^2*x4(i)^2-k1s^2*qtgvs^2*l14(i)^2*x26(i)^2*x4(i)^2;

H6terapia(1,i)=-1.*aib*(1.*aii*k1sm^2*(l124(i)-rhos*l126(i)-l15(i))^2*x26(i)^2*(1.*aco*afi-k1s^2*x26(i)^2*((-1+qthvs)*l11(i)*x1(i)-(-1+qthvs)*l18(i)*x1(i)-(-1+qtsvs)*(l112(i)-l12(i))*x2(i)-(-1+qtrvs)*(l116(i)-l13(i))*x3(i)-(-1+qtgvs)*(l120(i)-l14(i))*x4(i)+rhos*l126(i)*((-1+qthvs)*x1(i)+(-1+qtsvs)*x2(i)+(-1+qtrvs)*x3(i)+(-1+qtgvs)*x4(i)))^2)*x5(i)^2-1.*api*(1.*aii*art*(1.*aco*afi-k1s^2*x26(i)^2*((-1+qthvs)*l11(i)*x1(i)-(-1+qthvs)*l18(i)*x1(i)-(-1+qtsvs)*(l112(i)-l12(i))*x2(i)-(-1+qtrvs)*(l116(i)-l13(i))*x3(i)-(-1+qtgvs)*(l120(i)-l14(i))*x4(i)+rhos*l126(i)*((-1+qthvs)*x1(i)+(-1+qtsvs)*x2(i)+(-1+qtrvs)*x3(i)+(-1+qtgvs)*x4(i)))^2)-k2^2*(1.*aco*afi-k1s^2*x26(i)^2*((-1+qthvs)*l11(i)*x1(i)-(-1+qthvs)*l18(i)*x1(i)-(-1+qtsvs)*(l112(i)-l12(i))*x2(i)-(-1+qtrvs)*(l116(i)-l13(i))*x3(i)-(-

1+qtgvs)*(l120(i)-l14(i))*x4(i)+rhos*l126(i)*((-1+qthvs)*x1(i)+(-1+qtsvs)*x2(i)+(-1+qtrvs)*x3(i)+(-1+qtgvs)*x4(i)))^2)*((-l112(i)+l113(i))*x12(i)+(-l116(i)+l117(i))*x16(i)+(-l120(i)+l121(i))*x20(i)+(-l18(i)+l19(i))*x8(i))^2));

H2terapia_2(1,i)=1.*aco*afi-((k1s*x26(i)*((-1+qthvs)*l11(i)*x1(i)-(-1+qthvs)*l18(i)*x1(i)-(-1+qtsvs)*(l112(i)-l12(i))*x2(i)-(-1+qtrvs)*(l116(i)-l13(i))*x3(i)-(-1+qtgvs)*(l120(i)-l14(i))*x4(i)+rhos*l126(i)*((-1+qthvs)*x1(i)+(-1+qtsvs)*x2(i)+(-1+qtrvs)*x3(i)+(-1+qtgvs)*x4(i))))^2);

detH2(i)=det([-1.*aco,k1s*x26(i)*((-1+qthvs)*l11(i)*x1(i)-(-1+qthvs)*l18(i)*x1(i)-(-1+qtsvs)*(l112(i)-l12(i))*x2(i)-(-1+qtrvs)*(l116(i)-l13(i))*x3(i)-(-1+qtgvs)*(l120(i)-l14(i))*x4(i)+rhos*l126(i)*((-1+qthvs)*x1(i)+(-1+qtsvs)*x2(i)+(-1+qtrvs)*x3(i)+(-1+qtgvs)*x4(i)));k1s*x26(i)*((-1+qthvs)*l11(i)*x1(i)-(-1+qthvs)*l18(i)*x1(i)-(-1+qtsvs)*(l112(i)-l12(i))*x2(i)-(-1+qtrvs)*(l116(i)-l13(i))*x3(i)-(-1+qtgvs)*(l120(i)-l14(i))*x4(i)+rhos*l126(i)*((-1+qthvs)*x1(i)+(-1+qtsvs)*x2(i)+(-1+qtrvs)*x3(i)+(-1+qtgvs)*x4(i))),-1.*afi]);

detH6(i)=det([-1.*aco,k1s*x26(i)*((-1+qthvs)*l11(i)*x1(i)-(-1+qthvs)*l18(i)*x1(i)-(-1+qtsvs)*(l112(i)-l12(i))*x2(i)-(-1+qtrvs)*(l116(i)-l13(i))*x3(i)-(-1+qtgvs)*(l120(i)-l14(i))*x4(i)+rhos*l126(i)*((-1+qthvs)*x1(i)+(-1+qtsvs)*x2(i)+(-1+qtrvs)*x3(i)+(-1+qtgvs)*x4(i))),0,0,0,0;k1s*x26(i)*((-1+qthvs)*l11(i)*x1(i)-(-1+qthvs)*l18(i)*x1(i)-(-1+qtsvs)*(l112(i)-l12(i))*x2(i)-(-1+qtrvs)*(l116(i)-l13(i))*x3(i)-(-1+qtgvs)*(l120(i)-l14(i))*x4(i)+rhos*l126(i)*((-1+qthvs)*x1(i)+(-1+qtsvs)*x2(i)+(-1+qtrvs)*x3(i)+(-1+qtgvs)*x4(i))),-1.*afi,0,0,0,0;0,0,-1.*aib,0,0,0;0,0,0,-1.*aii,0,k2*((-l112(i)+l113(i))*x12(i)+(-l116(i)+l117(i))*x16(i)+(-l120(i)+l121(i))*x20(i)+(-l18(i)+l19(i))*x8(i));0,0,0,0,-1.*api,k1sm*(l124(i)-rhos*l126(i)-l15(i))*x26(i)*x5(i);0,0,0,k2*((-l112(i)+l113(i))*x12(i)+(-l116(i)+l117(i))*x16(i)+(-l120(i)+l121(i))*x20(i)+(-l18(i)+l19(i))*x8(i)),k1sm*(l124(i)-rhos*l126(i)-l15(i))*x26(i)*x5(i),-1.*art]);

%################# calcolo Payoff FINALI

%JTERAPIA

JSI(i)=-0.5*(aco*u1(1,i)^2+afi*u1(2,i)^2+aib*u1(3,i)^2+aii*u1(4,i)^2+api*u1(5,i)^2+art*u1(6,i)^2)+x1(i)+x2(i)+x3(i)+x4(i)+x5(i)+x6(i)+x7(i);

%JVIRUS

JV(i)=-0.5*(bsr*(1-u2(1,i))^2+brs*(1-u2(2,i))^2)+x10(i)+x11(i)+x12(i)+x13(i)+x14(i)+x15(i)+x16(i)+x17(i)+x18(i)+x19(i)+x20(i)+x21(i)+x22(i)+x23(i)+x26(i)+x27(i)+x8(i)+x9(i);

%#################### Calcolo ESS 1, 2, 3, 4, 5 FINALI

ess1(i)=1.*abs((k1sm*k2*pr*l126(i)*(l124(i)-1.*rhos*l126(i)-1.*l15(i))*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))*x26(i)*x5(i)*((-1.*l112(i)+l113(i))*x12(i)+(-1.*l116(i)+l117(i))*x16(i)+(-1.*l120(i)+l121(i))*x20(i)+(-1.*l18(i)+l19(i))*x8(i)))/(-1.*aii*(1.*api*art-1.*k1sm^2*(l124(i)-1.*rhos*l126(i)-1.*l15(i))^2*x26(i)^2*x5(i)^2)+1.*api*k2^2*((-1.*l112(i)+l113(i))*x12(i)+(-1.*l116(i)+l117(i))*x16(i)+(-1.*l120(i)+l121(i))*x20(i)+(-

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1.*l18(i)+l19(i))*x8(i))^2))+1.*abs((k1sm*k2*ps*l126(i)*(l124(i)-1.*rhos*l126(i)-
1.*l15(i))*x26(i)*x5(i)*((-1.*l112(i)+l113(i))*x12(i)+(-1.*l116(i)+l117(i))*x16(i)+(-
1.*l120(i)+l121(i))*x20(i)+(-1.*l18(i)+l19(i))*x8(i))*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i)))/(-
1.*aii*(1.*api*art-1.*k1sm^2*(l124(i)-1.*rhos*l126(i)-
1.*l15(i))^2*x26(i)^2*x5(i)^2)+1.*api*k2^2*((-1.*l112(i)+l113(i))*x12(i)+(-
1.*l116(i)+l117(i))*x16(i)+(-1.*l120(i)+l121(i))*x20(i)+(-1.*l18(i)+l19(i))*x8(i))^2));

    ess2(i)=abs((pr*l126(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))*(1.-
(1.*aii*k1sm^2*(l124(i)-1.*rhos*l126(i)-1.*l15(i))^2*x26(i)^2*x5(i)^2)/(-1.*aii*(1.*api*art-
1.*k1sm^2*(l124(i)-1.*rhos*l126(i)-1.*l15(i))^2*x26(i)^2*x5(i)^2)+1.*api*k2^2*((-
1.*l112(i)+l113(i))*x12(i)+(-1.*l116(i)+l117(i))*x16(i)+(-1.*l120(i)+l121(i))*x20(i)+(-
1.*l18(i)+l19(i))*x8(i))^2)))/api)+abs((ps*l126(i)*(-1.+(1.*aii*k1sm^2*(l124(i)-
1.*rhos*l126(i)-1.*l15(i))^2*x26(i)^2*x5(i)^2)/(-1.*aii*(1.*api*art-1.*k1sm^2*(l124(i)-
1.*rhos*l126(i)-1.*l15(i))^2*x26(i)^2*x5(i)^2)+1.*api*k2^2*((-1.*l112(i)+l113(i))*x12(i)+(-
1.*l116(i)+l117(i))*x16(i)+(-1.*l120(i)+l121(i))*x20(i)+(-
1.*l18(i)+l19(i))*x8(i))^2))*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i)))/api);

    ess3(i)=1.*abs((aii*k1sm*pr*l126(i)*(l124(i)-1.*rhos*l126(i)-
1.*l15(i))*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i))*x26(i)*x5(i))/(-1.*aii*(1.*api*art-
1.*k1sm^2*(l124(i)-1.*rhos*l126(i)-1.*l15(i))^2*x26(i)^2*x5(i)^2)+1.*api*k2^2*((-
1.*l112(i)+l113(i))*x12(i)+(-1.*l116(i)+l117(i))*x16(i)+(-1.*l120(i)+l121(i))*x20(i)+(-
1.*l18(i)+l19(i))*x8(i))^2))+1.*abs((aii*k1sm*ps*l126(i)*(l124(i)-1.*rhos*l126(i)-
1.*l15(i))*x26(i)*x5(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i)))/(-1.*aii*(1.*api*art-
1.*k1sm^2*(l124(i)-1.*rhos*l126(i)-1.*l15(i))^2*x26(i)^2*x5(i)^2)+1.*api*k2^2*((-
1.*l112(i)+l113(i))*x12(i)+(-1.*l116(i)+l117(i))*x16(i)+(-1.*l120(i)+l121(i))*x20(i)+(-
1.*l18(i)+l19(i))*x8(i))^2));

    ess4(i)=1.*abs((ps*l226(i)*(x13(i)+x17(i)+x21(i)+x24(i)+x9(i)))/bsr);

    ess5(i)=1.*abs((pr*l226(i)*(x11(i)+x15(i)+x19(i)+x23(i)+x25(i)))/brs);

    if (ess1(i)>=1 || ess2(i)>=1 || ess3(i)>=1 || ess4(i)>=1 || ess5(i)>=1 )

        testess=-1;

    else

        testess=1;

    end;




end;  %%%%%%%%%%end controlli ottimali FINALI

         %%%%%%%% + concavità FINALI + payoff FINALI + ESS FINALI

 %%%%%%%%%%%%%%%GRAFICI

y=zeros(95,N+1);

for i=1:N+1

y(1,i)=t(i);

y(2:28,i)=x(1:27,i);

y(29:55,i)=l1(1:27,i);

y(56:82,i)=l2(1:27,i);
```

```
y(83:88,i)=u1(1:6,i);
y(89,i)=1-u2(1,i);
y(90,i)=1-u2(2,i);
y(91,i)=H2terapia(1,i);
y(92,i)=H6terapia(1,i);
y(93,i)=H2terapia_2(1,i);
y(94,i)=detH2(i);
y(95,i)=detH6(i);
end;
figure(1);
subplot(3,3,1);
plot(y(1,:),y(83,:), '- k');
legend ('uCO');
axis([t0-(tf/20) tf+(tf/20) -0.1 1.1]);
subplot(3,3,2);
plot(y(1,:),y(84,:), '- b');
legend ('uFI');
axis([t0-(tf/20) tf+(tf/20) -0.1 1.1]);
subplot(3,3,3);
plot(y(1,:),y(85,:), '- g');
legend('uIB');
axis([t0-(tf/20) tf+(tf/20) -0.1 1.1]);
subplot(3,3,4);
plot(y(1,:),y(86,:), '- k');
legend ('uII');
axis([t0-(tf/20) tf+(tf/20) -0.1 1.1]);
subplot(3,3,5);
plot(y(1,:),y(88,:), '- b');
legend ('uRT');
axis([t0-(tf/20) tf+(tf/20) -0.1 1.1]);
subplot(3,3,6);
plot(y(1,:),y(87,:), '- g');
legend ('uPI');
```

```
axis([t0-(tf/20) tf+(tf/20) -0.1 1.1]);
subplot(3,3,7);
plot(y(1,:),y(90,:), '- m');
legend ('uRSv');
axis([t0-(tf/20) tf+(tf/20) -0.1 1.1]);
subplot(3,3,8);
plot(y(1,:),y(89,:), '- r');
legend ('uSRv');
axis([t0-(tf/20) tf+(tf/20) -0.1 1.1]);figure(2);
subplot(3,3,1);
plot(y(1,:), y(2,:));
title('Th')
subplot(3,3,2);
plot(y(1,:), y(3,:));
title('Ts');
subplot(3,3,3);
plot(y(1,:), y(4,:));
title('Tr');
subplot(3,3,4);
plot(y(1,:), y(5,:));
title('Tg');
subplot(3,3,5);
plot(y(1,:), y(6,:));
title('Mh');
subplot(3,3,6);
plot(y(1,:), y(7,:));
title('Ip');
subplot(3,3,7);
plot(y(1,:), y(8,:));
title('Ie');
subplot(3,3,8);
plot(y(1,:), y(9,:));
title('Thsl');
```

```
subplot(3,3,9);
plot(y(1,:), y(10,:));
title('Thsa');figure(3);
subplot(3,3,1);
plot(y(1,:), y(11,:));
title('Thrl');
subplot(3,3,2);
plot(y(1,:), y(12,:));
title('Thra');
subplot(3,3,3);
plot(y(1,:), y(13,:));
title('Tssl');
subplot(3,3,4);
plot(y(1,:), y(14,:));
title('Tssa');
subplot(3,3,5);
plot(y(1,:), y(15,:));
title('Tsrl');
subplot(3,3,6);
plot(y(1,:), y(16,:));
title('Tsra');
subplot(3,3,7);
plot(y(1,:), y(17,:));
title('Trsl');
subplot(3,3,8);
plot(y(1,:), y(18,:));
title('Trsa');
subplot(3,3,9);
plot(y(1,:), y(19,:));
title('Trrl');
figure(4);
subplot(3,3,1);
plot(y(1,:), y(20,:));
```

```
title('Trra');
subplot(3,3,2);
plot(y(1,:), y(21,:));
title('Tgsl');
subplot(3,3,3);
plot(y(1,:), y(22,:));
title('Tgsa');
subplot(3,3,4);
plot(y(1,:), y(23,:));
title('Tgrl');
subplot(3,3,5);
plot(y(1,:), y(24,:));
title('Tgra');
subplot(3,3,6);
plot(y(1,:), y(25,:));
title('Mhsa');
subplot(3,3,7);
plot(y(1,:), y(26,:));
title('Mhra');
subplot(3,3,8);
plot(y(1,:), y(27,:));
title('Vs');
subplot(3,3,9);
plot(y(1,:), y(28,:));
title('Vr');figure(5);
subplot(3,2,1);
plot(y(1,:), y(91,:));
title('minore d ordine 2');
subplot(3,2,2);
plot(y(1,:), y(92,:));
title('minore d ordine 6');
subplot(3,2,3);
plot(y(1,:), y(94,:));
```

```
title('minore d ordine 2');

subplot(3,2,4);

plot(y(1,:), y(95,:));

title('minore d ordine 6');%% calcolo e visualizzazione payoff FINALI

%%%%%%%%%%%%%%%%%%%%%% J therapy

for i=1:N

        JSI_RETT(i)=-
0.5*(aco*((u1(1,i)+u1(1,i+1))/2)^2+afi*((u1(2,i)+u1(2,i+1))/2)^2+aib*((u1(3,i)+u1(3,i+1))/2)
^2+aii*((u1(4,i)+u1(4,i+1))/2)^2+api*((u1(5,i)+u1(5,i+1))/2)^2+art*((u1(6,i)+u1(6,i+1))/2)^2
)+((x1(i)+x1(i+1))/2)+((x2(i)+x2(i+1))/2)+((x3(i)+x3(i+1))/2)+((x4(i)+x4(i+1))/2)+((x5(i)+x
5(i+1))/2)+((x6(i)+x6(i+1))/2)+((x7(i)+x7(i+1))/2);

  JV_RETT(i)=-0.5*(bsr*(1-((u2(1,i)+u2(1,i+1))/2))^2+brs*(1-
((u2(2,i)+u2(2,i+1))/2))^2)+x10(i)+((x11(i)+x11(i+1))/2)+((x12(i)+x12(i+1))/2)+((x13(i)+x1
3(i+1))/2)+((x14(i)+x14(i+1))/2)+((x15(i)+x15(i+1))/2)+((x16(i)+x16(i+1))/2)+((x17(i)+x17(
i+1))/2)+((x18(i)+x18(i+1))/2)+((x19(i)+x19(i+1))/2)+((x20(i)+x20(i+1))/2)+((x21(i)+x21(i+
1))/2)+((x22(i)+x22(i+1))/2)+((x23(i)+x23(i+1))/2)+((x26(i)+x26(i+1))/2)+((x27(i)+x27(i+1)
)/2)+((x8(i)+x8(i+1))/2)+((x9(i)+x9(i+1))/2);

end;

pay_SI=h*sum(-
0.5*(aco*u1(1,:).^2+afi*u1(2,:).^2+aib*u1(3,:).^2+aii*u1(4,:).^2+api*u1(5,:).^2+art*u1(6,:).
^2)+x1(1,:)+x2(1,:)+x3(1,:)+x4(1,:)+x5(1,:)+x6(1,:)+x7(1,:))

pay_SI_2=h*sum(JSI)

pay_SI_rett_comp=h*sum(JSI_RETT)

pay_SI_trap_comp=h2*(JSI(1)+JSI(N+1))+h*sum(JSI(2:N))

pay_SI_cav_simp=h/3*(JSI(1)+JSI(N+1))+4/3*h*sum(JSI(3:2:N))+2/3*h*sum(JSI(2:2:N))

%%%%%%%%%%%%%%%%%%%%    J Virus

pay_V=h*sum(-0.5*(bsr*(1-u2(1,:)).^2+brs*(1-
u2(2,:)).^2)+x10(1,:)+x11(1,:)+x12(1,:)+x13(1,:)+x14(1,:)+x15(1,:)+x16(1,:)+x17(1,:)+x18(1,
:)+x19(1,:)+x20(1,:)+x21(1,:)+x22(1,:)+x23(1,:)+x26(1,:)+x27(1,:)+x8(1,:)+x9(1,:))

pay_V_2=h*sum(JV)

pay_V_rett_comp=h*sum(JV_RETT)

pay_V_trap_comp=h2*(JV(1)+JV(N+1))+h*sum(JV(2:N))

pay_V_cav_simp=h/3*(JV(1)+JV(N+1))+4/3*h*sum(JV(3:2:N))+2/3*h*sum(JV(2:2:N))

%%%%%%%%%%%%%%visualizzazione testESS  FINALE

display(testess);

isto_y=[pay_SI pay_V];

isto_nan_y=[pay_SI_nan pay_V_nan];

subplot(3,2,5);
```

```
bar(isto_y);
z=conta;
toc;
beep
beep
beep
beep
beep
beep
beep
beep
beep
beep
end
```

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