UNIVERSITÀ DEGLI STUDI DI PADOVA

DIPARTIMENTO DI FISICA E ASTRONOMIA "G. GALILEI"

Scuola di Dottorato di Ricerca in Fisica
ciclo XXVIII

The flavour beyond the Standard Model: Grand Unification and Extra Dimensions

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A.A. 2015-2016

Abstract

We approach the flavour problem in the context of SO(10) grand unification and extra dimensions. Allowing the matter fields to propagate in the bulk of an extra dimension compactified on orbifold, one can explain the observed hierarchies of fermion masses and mixing in terms of different localizations of the fields profiles. The fundamental Yukawa couplings can be taken of order one and anarchical, thus allowing for a more natural theory of flavour. This approach has a non trivial realization in the grand unified framework, where fermions of different species are grouped in multiplets with common profile. In this thesis we study the possibilities of implementing this scenario in SO(10) Grand Unified Theories (GUT) with Supersymmetry, considering a GUT-sized extra dimension. A crucial role is played by a mechanism of symmetry breaking in the bulk, responsible of splitting the profiles of the SO(10) matter multiplets. We build different SO(10) models to test this mechanism, taking into account various options for breaking SO(10) down to the Standard Model gauge group and for the field content in the Higgs sector. A full numerical analysis is performed, proving the viability of the models and testing the success rate with respect to the anarchical Yukawa parameters. The models provide predictions for unobserved quantities in the flavour sector, which turn out to be remarkably stable with respect to several model variants.

Riassunto

In questa tesi affrontiamo il cosiddetto problema del sapore (flavour) nel contesto di teorie di grande unificazione in SO(10) e dimensioni extra. Permettendo ai campi di materia di propagarsi all'interno di una dimensione extra compattificata, si possono spiegare le gerarchie caratteristiche delle masse fermioniche e degli angoli di mescolamento. Questo è possibile grazie a differenti localizzazioni dei profili di questi campi, permettendo di avere nelle interazioni Yukawiane parametri di ordine 1 e anarchici. Questo approccio al problema del sapore ha tuttavia una realizzazione non scontata nel contesto di grande unificazione, dove fermioni di specie differenti sono raggruppati in multipletti con profili comuni. In questa tesi vengono studiate le possibilità di implementare tale scenario in Teorie di Grande Unificazione (GUT) in SO(10) in presenza di Supersimmetria, considerando una dimensione extra compattificata alla scala GUT. Un meccanismo di rottura di simmetria nel bulk della dimensione extra ha un ruolo cruciale in questo scenario, essendo responsabile di distinguere i profili dei multipletti di SO(10) che costituiscono i campi di materia. Per testare questo meccanismo, costruiamo diversi modelli in SO(10), tenendo conto di varie possibilità per rompere SO(10) nel gruppo di gauge del Modello Standard e per le rappresentazioni dei campi costituenti il settore di Higgs. Viene svolta un'analisi numerica completa che dimostra l'attuabilità dei nostri modelli e ne testa la probabilità di successo rispetto alla variazione anarchica dei parametri Yukawiani. I modelli forniscono predizioni per alcune quantità non ancora osservate nel settore del flavour, che risultano essere sorprendentemente stabili rispetto a molte varianti.

List of publications

- **Articles Published in Peer-reviewed Journals:**

  - **"A realistic pattern of fermion masses from a five-dimensional SO(10) model"**
    F. Feruglio, K. M. Patel and D. Vicino.
    arXiv:1507.00669 [hep-ph]
    10.1007/JHEP09(2015)040
    JHEP **1509**, 040 (2015)

  - **"Order and Anarchy hand in hand in 5D SO(10)"**
    F. Feruglio, K. M. Patel and D. Vicino.
    arXiv:1407.2913 [hep-ph]
    10.1007/JHEP09(2014)095
    JHEP **1409**, 095 (2014)

- **Conference Proceedings:**

  - **"An ordered anarchy from 5D SO(10)"**
    F. Feruglio, K. M. Patel, D. Vicino
    J.Phys.Conf.Ser. 631 (2015) 012026(2015-07-30)
    DOI: 10.1088/1742-6596/631/1/012026
    Conference: C14-12-02.1

  - **"Order and Anarchy in 5D SO(10)"**
    D. Vicino
    Nucl.Part.Phys.Proc. 265-266 (2015) 317-319(2015-10-06)
    DOI: 10.1016/j.nuclphysbps.2015.06.080
    Conference: C14-09-07.1

Acknowledgements

I would like to express my deep gratitude to Prof. Ferruccio Feruglio, who has patiently guided me during the course of my PhD. Working with him has given me the possibility to gain an invaluable baggage of knowledge. With his clear explanations, utmost kindness and expertise, he has guided me in my scientific endeavour, always inspiring and encouraging me.

A deepest and sincere acknowledgement goes to Dr. Ketan Patel for the active work atmosphere he was able to create and for the special care he has dedicated to me, both as a collaborator and as a friend. His patience in answering my questions and his passionate discussions have helped and stimulated me a lot. I am deeply indebted to him for the results that I could reach throughout this work. His great passion and expertise has been of maximal inspiration to me.

I would also like to thank all the members of the group 4 for the activities shared in these years, especially the persons who have mostly contributed to my scientific background through their passionate teaching or through enlightening discussions: Prof. Gianguido Dall'Agata, Prof. Antonio Masiero, Dr. Massimo Passera, Prof. Stefano Rigolin, Prof. Fabio Zwirner and Dr. Andrea Wulzer.

A heartfelt thank you goes to my family, for the support that I always had, which has been of utmost importance in these years and which I feel deeply indebted for. A final and especially warm thank goes to Alessandro, who has always brought a smile into my life, dedicating constant care and support to me even in the most difficult moments.

Contents

Chapter 1

Introduction

Nowadays the frontier of particle physics is dictated by the great success of the Standard Model, while facing at the same time several questions that this theory leaves unanswered, pointing out its limitations. The formulation of the Standard Model (SM) has required more than 40 years [1–5], through various attempts of explaining in a consistent theoretical framework the characteristics of the fundamental particles and interactions, which the experimental observations have progressively revealed to us. The enormous success of the theory in predicting the experimental data finds nowadays its ultimate confirmation with the discovery of the Higgs boson, announced on the 4th of July of 2012 [6, 7]. This was the last missing piece to authenticate the validity of the SM and it represents the conclusion of an important chapter in the history of particle physics. However, despite this encouraging success, it's evident that the Higgs' discovery has completed just a little portion of a much bigger puzzle that nature proposes to us. Many phenomena that we have experimentally observed are in fact missing in the SM description. The first is the evidence of neutrino oscillations, which have been officially confirmed at the Super-Kamiokande experiment [8] and at the Sudbury Neutrino Observatory [9, 10], leading to the 2015 Nobel Prize for Physics to the leaders of these two collaborations [11]. Neutrinos oscillations prove the existence of neutrino masses, which the SM doesn't account for. Other evidences are the observed existence of dark matter and dark energy and the asymmetric content of baryonic and antibaryonic matter in the universe. From a more theoretical point of view, the SM also suffer of some shortcomings. The specific SM gauge group and the matter representations don't have any theoretical explanation. Other more specific aspects not solved within the SM picture are the hierarchy problem associated to the Higgs mass and the so-called flavour problem. The latter consists in the existence of a large number of parameters describing the fermion masses and mixing angles, which, fixed by experimental measurements, vary in a vast range of magnitudes and exhibit a quite unnatural pattern. The SM don't tell us what is the theoretical reason behind this parameters structure.

It is clear that the SM leaves too much unsaid and it must be part of a more fundamental theory. An extension should be formulated as a theory valid at higher energy, while the SM should arise as a low energy effective theory valid below some cut-off scale.

But how to go beyond? Many extensions of the SM has been formulated, in order to address one or more particular aspects of its deficiencies. In this thesis we study possible frameworks beyond the SM, approaching two main issues: the flavour problem and the origin of the gauge structure. A possible explanation to the latter aspect arises from the Grand Unified Theories (GUTs) proposal. In these theories the SM gauge group is embedded into a larger simple group, unifying the description of all the particle interactions at higher energy. The breaking of such unified group should give rise to the observed low energy structure of gauge interactions and particles properties described by the SM. The idea of such unification is not a casual one, but it finds its first motivation in the behavior of the SM itself. The evolution of the SM gauge couplings at high energy, in fact, shows the tendency of the couplings to get closer around a common value. This behavior may be not casual, but effect of a new underlying gauge structure that could emerge at high energy. More encouraging, when one includes Supersymmetry, another possible extension of the SM, the evolution of the gauge couplings is corrected in such a way to improve this tendency of the gauge couplings, reaching an exact unification at the scale $M_{\mathrm{GUT}} \approx 10^{16}$ GeV. Some bounds, like the limits on proton decay, also suggest that this is a suitable scale for unification. Moreover, some compelling proposals for explaining the smallness of neutrino masses, as the see-saw mechanism, predict the existence of heavy right-handed neutrinos at a similar energy scale. Therefore, many hints seem to tell us that some new physics should exists at the GUT scale and one unified theory may provide reason of all these hints at the same time, correlating for example the gauge couplings unification and the existence of small neutrino masses.

The concept of unification is not of recent development. The first papers on the subject were published more than 40 years ago [12–14]. Since then the idea of unification keeps being of great interest for physicists and many fundamental papers have explored the various features and consequences of grand unification. Unfortunately, the difficulty to test experimentally a theory realized at such a high energy, didn't permit to reach firm and ultimate results about GUTs during this period of time. Indeed, a great level of arbitrariness is left in the model building of this kind of theories, since we cannot be guided by direct observations. This is maybe the most relevant drawback of GUTs. However, one should always follow the basic guiding principle of reproducing the low energy data and, in this prospect, different GUT models can provide compelling explanations to various aspects of the emerging low energy picture.

One of the most attractive proposal in GUT is the SO(10) gauge group. In this framework, not only the gauge couplings are unified, but also the fermions quantum numbers find a common origin. One generation of fermions including all the species, indeed, nicely fits into a **16** spinorial representation of SO(10). Such representation includes also a SM singlet, which can play the role of RH neutrino and easily account for neutrino masses through a see-saw. The beauty and simplicity of this unification, however, has to face with the more cumbersome aspect of flavor. The fermion masses are characterized by specific hierarchies and the quarks and leptons exhibit very different mixing patterns, in a picture that doesn't reveal unification at all. It is not obvious how to reproduce such diversified features of fermions in a completely unified framework and, in practice, this requires a large number of free parameters, with fine tuned values that span several orders of magnitude. Therefore, the problem of flavour seems to arise at the GUT level as well, with no qualitative difference with respect to the SM. Still, some new ingredients are present in the grand unified context, as the predicted correlations between masses and mixing of the various sector, which give some first hints in understanding the flavour structure, but not a simple and satisfactory explanation.

This doctoral thesis focuses exactly on this aspect of SO(10) GUTs, reviewing what are the actual characteristics of the flavour problem in this theory and attempting to improve some aspects of it with novel approaches. In particular, we would like to reach a natural theory of flavour, where fermion masses and mixing are described by a set of order one parameters and the hierarchical structures are generated by some underlying mechanism.

Why to include extra dimensions? Extra dimensions have been historically proposed as extension of the SM to address completely different issues, like in String Theory to include the description of gravity at the quantum level. However, they offer new ingredients useful in other sectors and, among these, one of major interest for us, providing possible solution to the flavour problem. In fact, in presence of a compactified extra dimension (ED) of spatial type, one can explain the hierarchies of fermion masses and mixing in terms of order one parameters. The mechanism responsible of the emerging hierarchies comes from allowing the fermions to propagate in the bulk of the ED, where bulk mass parameters of order one modulate their zero mode profiles into an exponential shape. The Higgs sector is localized on a brane and it gives mass to the fermions through fundamental Yukawa couplings of order one and with anarchical structure. With a suitable assignment of different bulk masses to the fermions, effective Yukawa couplings with realistic hierarchies arise from convoluting the order one Yukawas with the profiles, characterized by exponential suppression or enhancement at the Higgs brane. This scenario is a well-known proposal to address the flavour problem within the SM, where to each fermion representation is assigned an independent bulk mass as free parameter. The implementation of this scenario in SO(10) GUT, however, is not an immediate extension. Because of the unified description of fermions, one can assign only one common bulk mass parameter to all the fermions of a given generation, thus accounting only for possible hierarchies through the three generations, but not distinguishing among the different species. One necessarily needs to split the profiles of the fermions belonging to the same **16** multiplet, differentiating the behavior of its components. This is actually the core problem that we aim to address in this thesis. A mechanism to achieve the goal of profile splitting has been proposed by Kitano and Li in a supersymmetric SO(10) model in 5 dimensions [15]. The main ingredient of this mechanism is the spontaneous breaking of SO(10) down to

SU(5)×U(1)$_X$, responsible of correcting the bulk mass of the **16** with different contributions with respect to the its SU(5) components. This mechanism seems to be quite a unique and efficient possibility to realize the profile splitting in a supersymmetric SO(10) model, and we think it deserved a dedicated study. Indeed, such scenario predicts with a certain rigidity the profiles splitting and it is not obvious whether it is compatible or not with a realistic mass spectrum, accounting for the characteristics of all the fermions. We focus exactly on this aspect, testing the viability of this mechanism when implemented in specific SO(10) models in extra dimensions, that we can build with different features. This study is essentially the content of the two publications related to this doctoral thesis, ref.[16] and [17].

Our analysis confirms the possibility of an efficient and natural theory of flavour, where we can get the diversified fermion masses and mixing in the framework of complete unification and structureless Yukawas. The mechanism used at the core of this theory turns out to be very robust with respect to different frameworks that one can implement, as confirmed by the stability of the predictions that we get in our models. The essential features characterizing the predictions of the models constructed by us are the following:

- compatibility with only large values of $\tan\beta$, related to the t-b-τ unification realized in this framework;

- a more favored normal ordering of neutrino masses, with respect to the inverted ordering;

- very small mass predicted for the lightest neutrino $m_{\nu lightest} \sim meV$, below the actual experimental sensibility;

- very small effective mass predicted for the neutrinoless double beta decay, $|m_{\beta\beta}| \sim$ meV, also below the actual experimental sensibility;;

- very hierarchical spectrum predicted for right handed neutrinos, not compatible with models of standard thermal leptogenesis;

- none preferred value predicted for Dirac and Majorana phases of the lepton sector.

We dedicate attention also to the concept of "anarchy" in the Yukawa couplings, studying the behavior of our models under the random variation of order one Yukawa parameters. We thus point out the conditions to have a good success rate of the models, distinguishing these cases as more natural solutions with respect to others that account for a realistic spectrum but with necessarily fine-tuned parameters.

Let us finally point out that these models are constructed by us in the limit of unbroken N=1 Supersymmetry (SUSY), while a fully realistic approach should include the specification of a SUSY breaking mechanism. We have started a preliminary study dedicated to this aspect, trying to formulate a mechanism of N=1 SUSY breaking based on extra dimensions and compatible with the characteristics of our models. However, the implementation in our models turns out to be not so trivial. After reviewing some existing mechanisms from the literature, we give a contribution of major originality in the Radion mechanism, which, however, is still under exploration by us and will be object of future developments.

The outline of the thesis is the following. Chapter 2 is dedicated to a brief review of the SM, pointing out its main shortcomings and the need for an extension beyond it. We discuss with major concern the problem of flavour and the main approaches to address it, introducing concepts that will remain as basic background throughout all the discussion in the thesis. We give a brief description of Supersymmetry (extended in appendix A) and discuss its most interesting features from the point of view of grand unification. In chapter 3 we develop the framework of grand unification, passing through the introduction of SU(5) and Pati-Salam groups, arriving to describe in major details the SO(10) gauge group and its characterization in the flavour sector. In chapter 4 we review the basics of theories with a compactified extra dimension, describing how the profiles of the various fields arise, in particular the exponential zero-mode profile that will characterize the matter fields. We introduce also the description of Supersymmetry in 5 dimensions making explicit its formulation in terms of the usual 4-dimensional superspace. In Chapters 5 and 6 we collect the concepts of grand unifications and extra dimensions introduced before in the realization of some specific models built by us. We address the flavour problem in supersymmetric SO(10) GUT in 5 dimensions, implementing and testing the Kitano-Li mechanism for the profiles splitting. These chapters essentially report our publications, ref. [16] and [17]. Chapter 7 illustrates, at the level of work in progress, our attempts of implementing a mechanism of N=1 SUSY breaking in extra dimensions, that would complete in a more realistic way the construction of our specific models. We finally report in the last chapter the overall conclusions of our work.

Chapter 2

The Standard Model and Beyond

This is an introductory chapter which aims to motivate the necessity of going beyond the SM theory. We start giving a brief overview of the SM, in order to introduce all the ingredients which are at the basis of the discussion in the following chapters. We focus in particular on the Yukawa sector, outlining the characteristic properties of quark and lepton particles. Particular emphasis will be put on the problem of neutrino masses and on the "flavour puzzle", reviewing the attempts of explaining the observed structures of fermion masses and mixing. After a brief review of other open problems of the SM, we dedicate more space to illustrate the supersymmetric extension of the SM, outlining the basic concepts and results which will be of interest in the further development of the discussion.

2.1 The Standard Model in brief

2.1.1 Gauge group, symmetry breaking and representations

The Standard Model (SM) of particle physics is a gauge quantum field theory which describes the fundamental forces as effect of local symmetries of nature, under which the matter particles transform with peculiar properties. The SM gauge group is the direct product of three local symmetries:

$$G_{SM} \equiv SU(3)_{\rm C} \times SU(2)_{\rm L} \times U(1)_{\rm Y}\,.$$

$\mathrm{SU}(3)_{\rm C}$, with coupling constant g_s, is the group describing the strong interactions. $\mathrm{SU}(2)_{\rm L}\times\mathrm{U}(1)_Y$, with couplings g and g' respectively, describe the electroweak (EW) interactions. The gauge vector bosons of the SM, which mediate these interactions between the particles, live in the adjoint representations of the respective gauge groups and they are:

$$\begin{array}{lcl} \text{8 gluons:} & G^{a=1,\ldots,8} & (8,1)_0 \\ \text{3 weak bosons:} & W^{i=1,2,3} & (1,3)_0 \\ \text{1 hypercharge boson:} & B & (1,1)_0 \end{array} \qquad (2.1)$$

where the quantum numbers on the right describe the representation under G_{SM}. At this level, all the gauge bosons are massless, being the mass terms in the Lagrangian forbidden by the gauge symmetry. The spontaneous symmetry breaking of the EW sector is realized as:

$$\mathrm{SU(3)_C} \times \mathrm{SU(2)_L} \times \mathrm{U(1)}_Y \longrightarrow \mathrm{SU(3)_C} \times \mathrm{U(1)}_{EM}$$

by the VEV of the Higgs scalar field, which has the representation:

$$\text{Higgs scalar:} \qquad H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \qquad (1,2)_{\frac{1}{2}} \tag{2.2}$$

The VEV is aligned as

$$\langle H \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix} \tag{2.3}$$

and the experimentally measured parameter $v \simeq 174$ GeV dictates the scale of the EW breaking ($M_{\mathrm{EW}} \approx 10^2$ GeV). Three generators are broken in this process[1]: T_{1L}, T_{2L} and a linear combination of T_{3L} and Y. In correspondence, three gauge bosons get a mass at the EW scale (Higgs mechanism):

$$W^\pm = (W^1 \pm iW^2)/\sqrt{2} \quad \text{and} \quad Z = \cos\theta_w W^3 - \sin\theta_w B,$$

which mediate charged and neutral current weak interactions, respectively. θ_w is the Weinberg angle which parameterizes the change of basis after the EW symmetry breaking. The linear combination of generators $T_{3L}+Y$ remains unbroken, as it can be easily verified applying it to the VEV (2.3). This combination is generator of the surviving $\mathrm{U(1)_{EM}}$ symmetry, and leads to the electromagnetic charge formula:

$$Q = T_{3L} + Y \tag{2.4}$$

The correspondent vector boson remains massless and it is identified with the photon:

$$\gamma = \sin\theta_w W^3 + \cos\theta_w B,$$

mediator of the electromagnetic interactions. The Weinberg angle is measured at the energy of $M_Z = 91.2$ GeV from the relation between the g and g' couplings:

$$\sin^2\theta_w = g'^2/(g^2 + g'^2) \simeq 0.23 \,.$$

[1] $T_{iL} \equiv \sigma_i$ are the three generators of $\mathrm{SU(2)_L}$, with σ_i the Pauli matrices. Y is the hypercharge, generator of the abelian symmetry $\mathrm{U(1)}_Y$.

The matter content is described by 5 type of fermion fields characterized by specific representations under the SM gauge group:

$$
\begin{array}{lcc}
\text{LH quark doublet:} & Q = \begin{pmatrix} u_1 & u_2 & u_3 \\ d_1 & d_2 & d_3 \end{pmatrix} & (3,2)_{\frac{1}{6}} \\
\\
\text{LH lepton doublet:} & L = \begin{pmatrix} \nu \\ e \end{pmatrix} & (1,2)_{-\frac{1}{2}} \\
\\
\text{RH quark singlets:} & \begin{array}{c} d^c = (d_1^c \;\; d_2^c \;\; d_3^c) \\ u^c = (u_1^c \;\; u_2^c \;\; u_3^c) \end{array} & \begin{array}{c} (\overline{3},1)_{\frac{1}{3}} \\ (\overline{3},1)_{-\frac{2}{3}} \end{array} \\
\\
\text{RH electron singlet:} & e^c & (1,1)_1 \\
\ldots\text{RH neutrino?} & \ldots\nu^c\,? & (1,1)_0
\end{array}
\tag{2.5}
$$

where the $1, 2, 3$ distinguish the colors of the quarks. Each fermion is classified according to the specific interaction. In particular, leptons are non-strongly interacting particles, while the quarks interact strongly and are believed to be the constituents of the hadrons. All of them, with the exclusion of neutrinos, have electromagnetic interaction and Eq.(2.4) gives the correct electromagnetic charges. The representations distinguish also for the chirality of fermions, being the left-handed (LH) components weakly interacting, while the right-handed (RH) components not. Note that RH neutrinos ν^c are not included in the SM, but their existence may be required to explain the observed tiny mass of neutrinos, in some extensions of the SM. They would be completely neutral with respect to the SM interactions.

All of these fermion representations come in three independent copies, one for each family. We distinguish the family of a fermion by an index $i = 1, 2, 3$ that span what we call "flavour" or "generation" space. Two fermions with equal representations are phyisically distinguished for the flavour only because of their different masses, while having identical interaction properties. In this way we distinguish the three up-type quarks u, c, t, down-type quarks d, s, b and charged leptons e, μ, τ and the three neutrinos ν_1, ν_2, ν_3. The various masses of this particles are reported in Table 2.1. As we are going to describe in the next section, also the fermion masses are generated the VEV of the Higgs field.

2.1.2 Yukawa couplings

Due to the chiral structure of the SM, with distinguished representations for LH and RH fermions, it is not possible to introduce bare mass terms in the Lagrangian for quarks and leptons, which would violate the gauge symmetry. Fermion masses arise from the EW symmetry breaking through the Higgs VEV in the so-called Yukawa interactions, which couple the Higgs scalar to a LH and a RH fermion:

$$
\mathcal{L}_{Yuk} = -Y_{uij} Q_i u_j^c H_u - Y_{dij} Q_i d_j^c H_d - Y_{eij} L_i e_j^c H_d + h.c. \tag{2.6}
$$

For future convenience, we have introduced a notation for the Higgs that is familiar in the SUSY framework, where H_u and H_d are independent fields. At the SM level, however, there is only one Higgs and we identify: $H_u \equiv H$ and $H_d \equiv i\sigma_2 H^*$, where H is defined in Eq.(2.2). Notice that H_d has hypercharge $Y = -1/2$.
There are three independent Yukawa couplings: Y_u for up-type quarks, Y_d for down-type quarks and Y_e for charged leptons, corresponding to the SM invariants that one can construct with the given quantum numbers. These couplings account for the degeneracy of the fermions in the flavour space and thus they are described, in whole generality, by 3×3 complex matrices in the family indices i, j. In such a way we describe not only the fermion masses but also the mixing among different families.
The Yukawa matrices can be diagonalized by two unitary transformation acting independently on the LH and the RH fields. From these diagonalizations we make explicit the 9 real parameters describing the masses of quarks and charged leptons, as we are going to discuss.

Quark sector: masses and mixing. In the quark sector, the diagonalized Yukawa matrices are:

$$Y_u^{diag} = U_{u_L}^\dagger Y_u U_{u_R} = diag(y_u, y_c, y_t) \tag{2.7}$$
$$Y_d^{diag} = U_{d_L}^\dagger Y_d U_{d_R} = diag(y_d, y_s, y_b), \tag{2.8}$$

where U_R and U_L are the RH and LH rotations respectively, for the up and down sectors. All the eigenvalues y_i's are real and non-negative, obtained by diagonalizing the hermitian quantities $Y^\dagger Y$ and $YY^\dagger$. These parameters lead to the fermion masses when the Higgs field takes VEV as in Eq.(2.3):

$$m_u = v\, y_u; \quad m_c = v\, y_c; \quad m_t = v\, y_t \tag{2.9}$$
$$m_d = v\, y_d; \quad m_s = v\, y_d; \quad m_b = v\, y_b \tag{2.10}$$

The updated measured values for the masses are reported in Table 2.1. The very hierarchical set of values along the three generations reflects the hierarchy of the Yukawa couplings. Because of the common LH Q doublet in the Yukawa couplings, on which only one rotation is allowed, we cannot diagonalize simultaneously both the up and down matrices. If we choose to work in the down quark mass basis, performing the rotations:

$$Q \to U_{d_L} Q, \quad d^c \to U_{d_R}^\dagger d^c \quad \text{and} \quad u^c \to U_{u_R} u^c \tag{2.11}$$

then we have:

$$Y_d \to Y_d^{diag}; \quad Y_u \to U_{d_L}^\dagger U_{u_L} Y_u^{diag} \equiv V_{CKM}^\dagger Y_u^{diag} \tag{2.12}$$

where the matrix

$$V_{CKM} \equiv U_{u_L} U_{d_L}^\dagger = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \tag{2.13}$$

Quark and lepton masses at M_Z:

$$m_u = 1.8 - 3\text{ MeV}, \quad m_c = 1.250 - 1.300\text{ GeV}, \quad m_t = 171.99 - 174.43\text{ GeV}$$
$$m_d = 4.5 - 5.3,\text{MeV} \quad m_s = 90 - 100\text{ MeV}, \quad m_b = 4.15 - 4.21\text{ GeV}$$
$$m_e = 0.511\text{ MeV}, \quad m_\mu = 105.66\text{ MeV}, \quad m_\tau = 1.78\text{ GeV}$$

Neutrino mass differences:

$$\Delta_S = 7.50 \pm 0,19 \times 10^{-5}\,\text{eV}^2\text{ (NO or IO)}$$
$$\Delta_A = 2.457 \pm 0.047 \times 10^{-3}\,\text{eV}^2\text{ (NO) or }2.449 \pm 0.048 \times 10^{-3}\,\text{eV}^2\text{ (IO)}$$

Quark mixing parameters:

$$|V_{\text{CKM}}| = \begin{pmatrix} 0.97427 \pm 0.00014 & 0.22536 \pm 0.0006 & 0.00355 \pm 0.00015 \\ 0.22522 \pm 0.00061 & 0.97343 \pm 0.0001 & 0.0414 \pm 0.0012 \\ 0.00886^{+0.00033}_{-0.00032} & 0.0405^{+0.0011}_{-0.0012} & 0.99914 \pm 0.00005 \end{pmatrix};$$

$$J_{CP} = 3.02^{+0.21}_{-0.20} \times 10^{-5}$$

Lepton mixing parameters:

$$\sin^2(\theta_{12}) = 0.304 \pm 0.013;$$
$$\sin^2(\theta_{21}) = 0.452 \pm 0.052\text{ (NO) or }0.579 \pm 0.037\text{ (IO)}$$
$$\sin^2(\theta_{13}) = 0.0218 \pm 0.0010\text{ (NO) or }0.0219 \pm 0.0011\text{ (IO)}$$

Table 2.1: Current status of fermions masses and mixing elements [18, 19]. NO (IO) stands for normal (inverted) ordering in neutrino masses.

is known as the Cabibbo-Kobayashi-Maskawa (CKM) matrix and describes the misalignment between the up and the down mass eigenstates. After the change of basis (2.11), this matrix enters in the weak charged current interactions and mediates the flavour changing charged current processes. In these processes an up type quark of flavour i can be transformed into a down-type quark of flavour j from interaction with a $W^\pm$ boson, modulated by the CKM entry V_{ij}.

The CKM matrix is parameterized by 3 angles and, in principle, 6 phases. By redefinition of the various quark fields, one can remove 5 of the 6 phases. The remaining one phase is the only source for CP violation in the quark sector and it can be parameterized by the Jarlskog invariant, defined as

$$J^q_{CP} \equiv Im\{V_{ud} V^*_{us} V^*_{cd} V_{cs}\}. \tag{2.14}$$

The entries of the CKM and the Jarlskog invariant have been experimentally measured and their most updated values are reported in Table 2.1. As it can been seen from the data, the matrix results very hierarchical and close to the identity. A useful parametrization which highlights this hierarchical structure is given in terms of the Wolfenstein parameters:

$$V_{CKM} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}. \tag{2.15}$$

where A, ρ, η are of order 1, while $\lambda \equiv \sin\theta_C \approx \theta_C$ is the sine of the Cabibbo angle θ_C, which is very small: $\lambda \sim 0.223$. One can see that the diagonal entries are close to 1, while the off diagonal ones are very small, proportional to different powers of λ. This hierarchical structure comes from the fact that the misalignment between the up and down quark mass eigenstates is small.

Lepton sector: masses and mixing. As it can be seen in Eq.(2.6), there is only one matrix of Yukawa couplings in the lepton sector. This leads to a mass term only for the charged leptons while neutrinos remain massless. Therefore it is possible to diagonalize the Y_e matrix without introducing any mixing in the weak charged current interactions:

$$Y_e^{diag} = V^\dagger_{e_L} Y_e V_{e_R} = diag(y_e, y_\mu, y_\tau) \tag{2.16}$$

The eigenvalues describe the charged lepton masses, once the Higgs takes VEV:

$$m_e = v\, y_e; \quad m_\mu = v\, y_\mu; \quad m_\tau = v\, y_\tau \tag{2.17}$$

The measured mass values are reported in Table 2.1 and they show a hierarchical structure quite similar to the down quark sector.

The absence of neutrino mass terms in the SM is an evident shortcoming which to face with the experimental observation of neutrino oscillations, proving the existence of lepton mixing and tiny neutrino masses. This is maybe the first straightforward evidence for the necessity of extending the SM. While we are going to face the formal problem of introducing neutrino masses in the next section, 2.1.3, here we assume a Majorana mass term for LH neutrinos and analyze the effect of its misalignment with respect to the charged lepton mass basis. Such mass comes from extending the SM Yukawa sector with the operator of Eq.(2.24) in the next section, which leads to an effective mass term M_{ν_L}

for LH neutrinos. We can diagonalize M_{ν_L} as[2]:

$$M_{\nu_L}^{diag} = V_{\nu_L}^T M_{\nu_L} V_{\nu_L} = diag(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}). \tag{2.18}$$

The diagonal entries parameterize the neutrino masses. Anyway, the experimental data about neutrinos are not complete yet and we know the values of only two mass differences from the solar and the atmospheric neutrino oscillations, which are defined as:

$$\Delta_S = m_2^2 - m_1^2; \quad \Delta_A = \left| m_3^2 - m_2^2 \right| \tag{2.19}$$

The values of $\Delta_{S,A}$ fitted from the experiments are also reported in Table 2.1: their magnitude, $(10^{-2} \div 10^{-3}\text{eV})^2$, is 8-12 orders smaller than the masses of the charged lepton sector and 9-14 orders smaller than the quark sector. Oscillation experiments can only measure squared mass differences, while the absolute mass scale of neutrinos remains unknown. However, there are bounds on this scale given from inderect experimental observations. For the lightest neturino mass we have the bound:

$$m_{\nu lightest} < 2.2 \text{ eV}$$

from the Troitsk experiment [20], based on the tritium β-decay. The future KATRIN experiment [21], if not measuring directly this mass, will improve this bound of one order of magnitude. Other bounds come from cosmology and the most recent data from Planck [22] give:

$$\sum_i m_i < 0.23 \text{ eV}; \quad \text{with} \quad N_{eff} = 3.30 \pm 0.27$$

where N_{eff} is the effective number of neutrinos compatible with the observations.

Note that, with only two measured values from oscillations, one cannot distinguish if neutrinos respect the normal ordering (NO) of the three generations ($m_{\nu_1} < m_{\nu_2} < m_{\nu_3}$) or if they have an inverted ordering (IO) ($m_{\nu_3} < m_{\nu_1} < m_{\nu_2}$), where the third generation is the lightest. Because of this the sign of Δ_A is undefined. The ordering and the mass of the lightest neutrinos currently remain unknown properties of neutrinos and, together with $m_{\nu lightest}$ they can be object of predictions for theories which extend the SM in the neutrino sector.

Similarly to what already seen for the quarks, we cannot diagonalize simultaneously the charged leptons and the neutrino mass matrices. Working in the charged lepton mass basis, then the misalignment with the neutrino eigenbasis is parametrized by the Pontecorvo-Maki-Nakagava-Sakata (PMNS) matrix, with similar origin of the CKM:

$$U_{PMNS} \equiv V_{e_L}^\dagger V_{\nu_L} \tag{2.20}$$

[2]We can consider also the case of Dirac neutrinos, with the Dirac mass term diagonalized as:

$$M_\nu^{diag} = V_{e_L}^\dagger M_\nu^D V_{\nu_R} = diag(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) \ ,$$

where M_ν^D is defined in Eq.(2.23) of the next section.

which describes the flavour changing charged currents in the weak interactions. The PMNS matrix is parametrized by 3 rotation angles θ_{12}, θ_{23}, θ_{13} and, in the case neutrinos are Majorana, by 3 independent phases (the other 3 are absorbed in fields redefinition), as given in the usual parametrization:

$$U_{PMNS} =$$

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix} \cdot \begin{pmatrix} 1 & & \\ & e^{i\frac{\alpha_{21}}{2}} & \\ & & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix}, \tag{2.21}$$

where c_{ij}, s_{ij} stand for $\cos\theta_{ij}$, $\sin\theta_{ij}$. The phase δ_{CP} is known as Dirac phase, since it is there also for Dirac neutrinos, and it is responsible of CP violation. The phases α_{21} and α_{31} are known as Majorana phases, since they appear only if neutrinos are of Majorana type, while they can be eliminated by field redefinitions in case they are Dirac. Majorana phases are other sources of CP-violation.

About the PMNS matrix we have much less informations than the CKM matrix. The three rotation angles has been measured from the observation of solar, atmospheric and reactor neutrino oscillations and the most updated values are reported in table (2.1). The PMNS phases are still unknown. Anyway, current experiments like T2K, NOvA, DUNE, MINOS should provide tight constraints on δ_{CP} in the next future. The Majorana phases are instead much more challenging to measure because the observed phenomenon of neutrino oscillations is not sensible to them.

The measured mixing angles already describe a very different structure with respect to the CKM. Indeed, despite θ_{13}, they are quite large and lead to an overall non hierarchical structure of the PMNS which is very far from the identity matrix.

2.1.3 The problem of neutrino mass.

To account for the existence of non-vanishing neutrino masses the SM must be extended. This can be done either by adding a RH neutrino and a new Yukawa coupling (Dirac neutrinos) or violating the lepton number (L) introducing a Majorana mass term (Majorana neutrinos)

A. Dirac neutrinos. Adding the RH component ν_c one can construct the Dirac Yukawa coupling:

$$-Y_\nu^D L \nu^c H_u \tag{2.22}$$

which, by the VEV of the Higgs, gives mass to neutrinos in the same way of the other fermions:

$$M_\nu^D = Y_\nu^D v \tag{2.23}$$

Anyway, to explain the extremely small mass with respect to the charged sector, the entries of this Yukawa coupling should be very small.

B. Majorana neutrinos. Majorana neutrinos are equal to their antiparticle and are identified by the relation between their chiral components: $\nu_R = C(\overline{\nu}_L)^T$, where C is the charge-conjugation operator. As such they have only two independent components. If we think to the SM as an effective theory, relaxing the requirement of renormalizability, we can generate a mass term for LH Majorana neutrinos through the dimension-5 Weinberg operator [23] of the type:

$$\frac{Y_L}{\Lambda}(LH_u)^T C(LH_u)\,, \tag{2.24}$$

which, through the Higgs VEV, leads to the Majorana mass term $M_{\nu_L}\nu^T C\nu$ with:

$$M_{\nu_L} = Y_L\frac{v^2}{\Lambda_L}. \tag{2.25}$$

Note that this term violates the lepton number L. Since this violation is not observed at low energy, we expect Λ_L, the scale at which L is violated, to be "high". This, differently from the Dirac case, would provide a natural suppression of the neutrino masses. For a natural Yukawa coupling, $Y_\nu \simeq \mathcal{O}(1)$, we would need $\Lambda_L \simeq 10^{13}$ GeV in order to get neutrino masses at the scale of the eV. Anyway, without a real theory which fixes Y_L, we cannot say too much about Λ_L.
Finally, if neutrinos are Majorana and an independent RH neutrino ν^c also exists, we have a Majorana mass term also for the right sector:

$$M_R\nu^{cT}C\nu^c \tag{2.26}$$

Note that, this mass term is renormalizable and M_R is a mass parameter independent from v. It could also be generated from the VEV of a new scalar field, which must be a SM singlet.

Between the two possibilities, Majorana neutrinos offer a much richer scenario to give explanation to the smallness of neutrino mass. Indeed it is possible to think about the effective operator Eq.(2.24) as resulting from the integration of an heavy state with renormalizable coupling, explaining the high scale Λ_L. The UV completion of this effective operator can be realized in three different ways, through the following renormalizable couplings:

$$LH_u\nu^c; \qquad L^T CL\Delta; \qquad LTH_u\,, \tag{2.27}$$

where ν_c is a fermionic singlet ($Y = 0$), identified as the RH neutrino, Δ is a scalar triplet ($Y = 1$) and T a fermionic triplet ($Y = 0$) of $\mathrm{SU(2)_L}$. This choice of the fields indeed preserve the gauge invariance. For these fields there is a mass term at high scale such that, when the fields are integrated out, this mass remains in the effective operator

(2.24), playing the role of the suppression scale Λ_L. This mechanism explains naturally the smallness of neutrino masses and goes under the name of "seesaw mechanism". The three options of fields in (2.27) chosen for the UV completion are classified, respectively, as type-I [24–28], type-II [29–32] and type-III [33] seesaw.

Let us briefly review the case of type-I seesaw, where the role of heavy field is played by the RH neutrino. This is particularly compelling in the context of SO(10) GUT, where RH neutrinos are automatically included in the theory, as we are going to see in the next chapter. Consider M_R to be the RH Majorana mass at high energy and ${M_\nu}^D$ the Dirac mass from the Yukawa coupling. M_ν^D and M_R enter the neutrino mass matrix defined on the basis (ν, ν^c):

$$\begin{pmatrix} 0 & M_\nu^D \\ M_\nu^{D^T} & M_R \end{pmatrix} . \tag{2.28}$$

When we compute the eigenvalues of this matrix in the limit $M_R \gg M_\nu^D$, we get the light eigenvalue:

$$M_{Light} \approx M_\nu^D M_R^{-1} M_\nu^D \tag{2.29}$$

that is known as type-I seesaw formula. This corresponds to the effective neutrino mass which is obtained by integrating out the heavy ν^c.

2.2 What is the flavour puzzle?

Let us now summarize what we have learned from the previous sections. We have depicted the scenario of "flavour" in the SM, where fermion masses and mixing angles arise from the Yukawa couplings. This framework doesn't exhibit the same simplicity of the gauge sector, where the simple Higgs mechanism can efficiently explain the symmetry breaking and the rise of massive vector bosons. Indeed, the existence of different interaction properties for the fermions, distinguished for quark and leptons and for their chiralities, as well as the existence of three families of them, makes the flavour scenario rather complicated. Furthermore, the need to account for neutrino masses and the uncertainty relative to their (Majorana or Dirac) nature make the issue even more intricate and calls for an inevitable extension of the SM. A total number of 20 (+2) independent parameters are needed to describe the flavour sector: 12 independent masses, 3 mixing angles + 1 CP-violating phase in the quark sector, 3 mixing angles + 1 Dirac CP-violating phase (+ 2 possible Majorana phases) in the lepton sector. Among these, 18 parameters have been experimentally measured (cf. Table 2.1). Of the remaining 4 parameters, the absolute scale of neutrino masses is constrained in a limited range, the Dirac CP-violating phase δ_{CP} is going to be constrained from the actual experiments, while the 2 possible Majorana phases are still unknown.

The measured values, furthermore, exhibit a non-trivial structure, which makes the flavour scenario even more intriguing. Specific hierarchies are observed between the three generations of quarks (especially enhanced for the up sector) and charged leptons, with masses spanning over 5 orders of magnitude. The neutrino masses, on the other side, are extremely small, differentiating from the quarks for $9 \div 14$ orders of magnitude. The mixing angles result very hierarchical in the quark sector, while in the lepton sector all the PMNS entries are of order one, except for $|U_{13}| \approx \lambda$ (in correspondence of the small $\theta_{13} < \theta_{12},\, \theta_{23}$). The situation can be summarized as follows[3]:

$$\begin{aligned} m_u : m_c : m_t &\approx \lambda^8 : \lambda^4 : 1; \\ m_d : m_s : m_b &\approx \lambda^5 : \lambda^3 : 1; \qquad\qquad \frac{m_{\nu 2}^2 - m_{\nu 1}^2}{|m_{\nu 3}^2 - m_{\nu 2}^2|} \approx \lambda^2; \\ m_e : m_\mu : m_\tau &\approx \lambda^6 : \lambda^2 : 1; \end{aligned} \tag{2.30}$$

$$V_{CKM} \approx \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}; \qquad U_{PMNS} \approx \begin{pmatrix} 0.8 & 0.5 & 0.2 \\ 0.5 & 0.6 & 0.6 \\ 0.3 & 0.6 & 0.7 \end{pmatrix}, \tag{2.31}$$

where we have parameterized the hierarchies in terms of powers of the Cabibbo angle λ.

Why flavour parameters are arranged in this way? Why masses are so hierarchical? Why the mixing matrices are so different for quarks and leptons? Why neutrino masses are so small? Why all the fermions are arranged in three families?
All these questions compose the so-called "flavour puzzle".
The SM model has been of enormous success in furnishing a description of the flavour sector through the principles of gauge symmetry and the Yukawa couplings, but so far it doesn't provide any answer to the above questions and it does nothing more than using a large set of parameters, without explaining their origin. Indeed the values of these parameters are just dictated by the experiments and no relation among them is predicted by the theory, which is very unsatisfactory. However, we have strong reasons to believe that the SM is not an ultimate theory and some extensions are needed to explain various aspects of nature which are still missing in it (first, as said, the existence of neutrino mass). In light of this necessity, we can ask if such complicated flavour scheme is not given by chance, but it may have origin from some underlying mechanism lying beyond the SM. One can imagine various extensions of the SM trying to explain the flavour puzzle. There are two fundamental approaches to do this. Let us review them briefly, without going into the details of the particular models. For more complete reviews, the reader can refer to [34–36].

In a first approach, one imagines the Yukawa couplings being deduced from first principles, postulating the existence of a fundamental theory which determines the Yukawas uniquely in terms of a small set of input parameters. The attempts in this direction generally propose fundamental theories based on the existence of new symmetries in the flavour space, which give correlations among the entries of the Yukawa matrices, forcing in such a way a particular texture. There are many examples of this kind of models, based on continuous or discrete symmetries (see [35, 37] for reviews). However, despite

[3]Mass values at the scale M_Z.

decades of experimental progresses and effort on the theoretical side, we are still far from a satisfactory fundamental theory of flavour.

An advantage of this approach is the reduction of the number of free parameters, increasing the predictive power of the theory. On the other side, there is the problem of explaining the origin of these additional symmetries, which, in the discrete case, span a vast set of options, from very minimal group like A_4 and S_4 to much more complicated ones. Also, this approach generally distinguishes the description of the quark and the lepton sectors. Indeed, given the huge difference in the quark and in the lepton mixing, and given the peculiar mass scale of neutrinos, one usually describes the two sectors separately, focusing on the exceptional properties shown by neutrinos: this is the case of various discrete symmetries which aim to reproduce the texture of the PMNS matrix as close to the tri-bimaximal structure, where $\theta_{13} = 0$ and θ_{23} is maximal (see for example [37–40]). It is even more challenging to give a common description in terms of flavour symmetries to both the quark and lepton sectors and many models try to address this problem in the context of grand unification (cf. Sect.3.4.7).

In a second approach, in a sense opposite to the first, one imagines to explain the Yukawa matrices in terms of anarchical parameters, not related to any particular symmetry. The purpose of this strategy is to obtain a natural theory of flavour, where the fundamental parameters are all of the same order and not subjected to the fine-tuned choice of particular values. In practice, in this kind of theories, the Yukawa couplings depend on a large number of order-one parameters that are considered as irreducible unknowns. The observed structure of the Yukawas are then reproduced by means of some other mechanism which creates the wanted hierarchies at the effective level. Examples of these mechanisms are given by the Froggatt-Nielsen machanism [41], by the propagation of fermions in extra dimensions (cf. Chap. 4) and also by the simplest version of partial compositeness [42, 43]. The basic idea of these mechanisms is to associate a hierarchical structure in the generation space to the matter fields, instead of imposing assumptions on the Yukawa couplings, which are taken anarchical. The effective Yukawa matrix comes from "sandwiching" the fundamental anarchical Yukawa between two diagonal hierarchical matrices associated to the fields. To roughly give the idea, the observed matrices of Yukawa couplings arise as follows:

$$\mathcal{Y} \approx \begin{pmatrix} \varepsilon^2 & 0 & 0 \\ 0 & \varepsilon & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(1) \\ \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(1) \\ \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(1) \end{pmatrix} \begin{pmatrix} \hat{\varepsilon}^2 & 0 & 0 \\ 0 & \hat{\varepsilon} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.32}$$

where ε and $\hat{\varepsilon}$ are just example of small parameters to express the hierarchy associated to the fields, while the $\mathcal{O}(1)$ matrix is the natural, fundamental Yukawa. In the Froggatt-Nielsen (FN) theory [41], which has been the pioneering work in this approach, the hierarchies (here parameterized by ε, $\hat{\varepsilon}$) are generated as powers of the VEV of a scalar field responsible of the breaking of the abelian flavour symmetry $G_f = U(1)_{FN}$. The powers are different for the various fermion fields according to the FN charge assignment.

In the framework of partial compositeness, instead, the hierarchical parameters ε, $\hat{\varepsilon}$ are associated to the degree of compositenss of the particles.
Finally, in extra dimensional models, when fermions are allowed to propagate in the bulk, the different localizations of the fermion profiles generate hierarchical couplings with the Higgs. This scenario will be discussed in full details in Chapt. 4 and it is the framework in which we will address the flavour problem in the context of grand unification.
The main advantage of this approach based on anarchical Yukawas is of course reducing the range and the fine-tuning in the Yukawa parameters, leading to a more natural theory of flavour. Also, in this approach we can elaborate at the same level quarks and leptons, imposing the ansatz of structureless Yukawas on both the sectors. Generally, the observational differences between the two can be reproduced by a proper choice of the parameters generating the hierarchy factors, without necessity of strong fine-tunings. From this point of view, one can imagine this strategy to be more suitable in the context of unification. On the other side, let us remark a disadvantage with respect to the previous approach, that is the reduction in predictability of the model. Indeed, in this kind of models we don't reduce the number of initial parameters, but we are forced to add some new quantities parameterizing the hierarchies associated to the fields. Moreover, working with anarchical Yukawas of order one, where we don't fix specific numbers but only a typical range of variation, we can't get precise predictions for masses and mixing angles, but only probability distributions obtained by scanning the order-one parameters of the theory.

2.3 Other open problems in the SM

Besides the problem of explaining the flavour structure of the SM and describing the neutrino masses, as anticipated, other shortcomings affect the theory, motivating the need to extend the SM in various directions. Let us briefly summarize what are the main open problems of the SM:

- **The hierarchy problem**. The Higgs mass, experimentally measured at $\simeq 126$ GeV, can receive large radiative corrections from new physics existing at high energy scale. We are sure of the existence of a new physics scale, at least to account for the description of gravity at $M_{Plank} \approx 10^{18}$ GeV. This requires a tremendous amount of fine-tunning, to be realized order by order in the perturbation theory, to make the Higgs stable at the EW scale. One of the most compelling and popular solutions to this problem is given by Supersymmetry (cf. appendix A and next section).

- **Baryon-antibaryon asymmetry**. The problem is related to the observed excess of baryonic matter with respect to antibaryonic matter in the universe. It is a natural assumption that the Big Bang should have produced equal amounts of matter and antimatter. Therefore, the asymmetry must have been generated by some physical law acting differently on the two sectors. This is possible when B and CP violating interactions are active and the universe is out of equilibrium. CP-violating phases which distinguish the interactions for particles and antiparticles are required. In the SM the CP-violating phase of the quark sector resides in the CKM matrix, but it results too small to account for the observed baryon-antibaryon asymmetry. One of the mechanisms that could account for this problem is Leptogenesis [44–47], which assumes an original asymmetry between leptons and antileptons generated in the very early universe, resulting in the dominance of leptons. Certain (non-perturbative) processes, called sphalerons, can convert such lepton asymmetry into a baryon asymmetry.

- **Dark matter and dark energy.** It is an experimentally proved fact that the visible matter described by the SM represents only the $\sim 5\%$ of the total energy content of our universe. A $\sim 25\%$ is composed by dark matter (DM), which is a new kind of matter observed only from the gravitational interaction. DM particles must be introduced as new particles which are neutral with respect to the SM group, or at most weakly interacting, and they necessarily requires a SM extension. The residual $\sim 70\%$ is made of dark energy, responsible of the universe expansion, which is also completely ignored in the SM.

- **Origin of the gauge sector.** The choice of the gauge groups and the representations characterizing the SM is motivated by the experimental observations. Anyway, we could ask, why nature is based on this particular gauge group, factorized in three subgroups describing three different kind of interactions? Why those specific representations are assigned to the matter particles? As we will see (cf. Chapt. 3), an attempt to answer these questions is given by grand unified theory (GUT). Ultimately, in the perspective of unification, one should ask how to include the description of Gravity, that is completely ignored at the level of the SM.

The attempts to solve each of the open problems above have incited the development of a vast variety of possible extensions of the SM. Usually, one possible extension is thought for solving a particular problem of these, but then the new framework of the extended SM can provide new ingredients or hints to address also other open questions. Of course, one aim to have a picture as complete as possible, so that the most compelling extensions of the SM are those accounting for solutions to more open problems. In this thesis we will explore the possibility of a BSM theory, putting at the center of our attention the solution to the origin of the gauge sector, provided by unification, together with the problem of flavour. As side issues we will take into account the hierarchy problem by inclusion of Supersymmetry.

2.4 Supersymmetric extension of the SM

Among the various possible frameworks for extending the SM, Supersymmetry (SUSY) has received the utmost attention over the last four decades. The appeal of SUSY results first of all from the solution provided to the hierarchy problem, and, in the perspective of a gauge-unified extension of the SM, from the prediction of a precise unification of the gauge couplings. There are anyway other results which have motivated SUSY as a very interesting extension of the SM. We can mention, first of all, the fact itself that SUSY provides a unified description of the two known forms of particles: bosons and fermions. Secondly, SUSY models with conserved R-parity, give a suitable candidate for dark matter (the Lightest Supersymmetric Particle). Finally, another attractive feature is that when SUSY is gauged it naturally leads to the description of gravity[4].

While treating in details all these aspects goes definitely beyond the purpose of this thesis, it is worth to dedicate a a section for reviewing, without claim of completeness, the basic formalism and the salient features of SUSY. In appendix A we introduce concepts and notations which will be used in our further discussion and applications. In this section, instead, we present a couple of results which will be of major relevance in the context of Grand Unification, which the next chapter is focused on.

2.4.1 Gauge couplings unification

In any quantum field theory, the coupling parameters are subjected to the renormalization group equations (RGE), that dictate how they evolve with the energy scale, the so-called "running" of the couplings. The effect is due to quantum corrections at the loop level and depends on the particles content at the various energy scales. A remarkable result of SUSY is the unification of the gauge couplings predicted by the RGE with the particle content of the MSSM [49–51]. Let us consider the evolution of a coupling parameter from the energy scale M_1 to the scale M_2, calling μ the energy scale variable in between: $M_1 \leq \mu \leq M_2$. The RGEs depend on the gauge symmetry and the field content at $\mu = M_1$. Let us define $\alpha_i \equiv \frac{g_i^2}{4\pi}$, where g_i are the three couplings of the SM gauge group, g_3 of $SU(3)_C$, g_2 of $SU(2)_L$, g_1 of $U(1)_Y$. The evolution equations are given by the β-function at one loop as:

$$\beta(\alpha_i) \equiv \frac{d\alpha_i}{dt} = \frac{1}{2\pi} b_i \alpha_i^2 \tag{2.33}$$

where $t = \ln \mu$. The β-function coefficients b_i receive contributions from the gauge, the fermion and the scalar field content, which are given, in this respective order, as:

$$b_i = -\frac{11}{3} C_2(R^i_{Adj}) + \frac{2}{3} T(R^i_f) + \frac{1}{6} T(R^i_s) \tag{2.34}$$

[4] In fact local SUSY implies the existence of a gravitational multiplet made of a spin 3/2 particle (gravitino) and a spin 2 particle, which couples to the energy-momentum tensor as in general relativity, allowing the identification with the graviton [48].

where the numerical factors come from the one-loop integrals associated to the three kind of particles and the other quantities parameterize the dependence on the representation of these particles under the i gauge group in terms of some representation group invariants. The first contribution comes from the gauge bosons: $C_2(R)$ is the Casimir[5] of the gauge group representation R, $C_2(R)\mathbf{I}_2 = \Sigma_a T_R^a T_R^a$, and here it is relative to the adjoint. The second and the third contributions come from the fermions and the scalars, respectively, and they are proportional to another invariant[6], the representation index $T(R)$, $T(R)\delta^{ab} = \mathrm{Tr}(T_R^a T_R^b)$, relative to the given representation of fermions R_f or scalars R_s. In Eq.(2.34) it is considered the sum over all the fermions and scalars which transform under the given gauge group, including the sum over the generations and the components relative to the representations under the other groups. Considering the SM field content, we find: $b_3^{SM} = -7$, $b_2^{SM} = -19/6$, $b_1^{SM} = 41/6$. If we consider the MSSM field content, including the SM superpartners, we find the coefficients to be: $b_3 = -3$, $b_2 = +1$ and $b_1 = +33/5$. Taking these coefficients into account, we can integrate the RGE (2.33) between the starting energy scale $M_1 = M_Z$ (Z-boson mass scale) and the final scale M_2, the solutions are:

$$\alpha_1^{-1}(M_Z) = \alpha_1^{-1}(M_2) - \frac{b_1}{2\pi} ln \frac{M_2}{M_Z} \tag{2.35}$$
$$\alpha_2^{-1}(M_Z) = \alpha_2^{-1}(M_2) - \frac{b_2}{2\pi} ln \frac{M_2}{M_Z}$$
$$\alpha_3^{-1}(M_Z) = \alpha_3^{-1}(M_2) - \frac{b_3}{2\pi} ln \frac{M_2}{M_Z}$$
(2.36)

Therefore, known the value of the three couplings at M_Z, RGE predicts the value of the couplings at M_2. If there exist a scale M_{GUT} at which $\alpha_1(M_{\mathrm{GUT}}) = \alpha_2(M_{\mathrm{GUT}}) = \alpha_3(M_{\mathrm{GUT}}) \equiv \alpha_U$ the unification of the gauge couplings is realized at such a scale, that we call Grand Unification scale or GUT scale.

While in the SM there is not perfect convergence of the gauge couplings, but there is a tendency to meet at 10^{15} GeV, the specific β-function coefficients of the MSSM provide an exact unification at the scale $M_{\mathrm{GUT}} \approx 2 \times 10^{16}$ GeV, as it can been seen in Fig. 2.1.

We will inquire in the next chapter, dedicated to grand unified theories, the consequences of this unified framework.

For concluding, it is worth to do a remark: Eqs.(2.33) consider 1-loop corrections to the propagators of the gauge bosons, and more precise results are available considering higher order loop corrections, which we have neglected for simplicity of discussion. Another approximation that we have done is working under the assumption that all the supersymmetric particles decouple at the same scale, the SUSY breaking scale. In practice this

[5]In particular $C_2(SU(N)_{Adj}) = N$, $C_2(U(1)) = 0$

[6]For the fundamental of SU(N) it is $T(N_{SU(N)}) = \frac{1}{2}$, $T(Y_{U(1)_Y}) = Y^2$

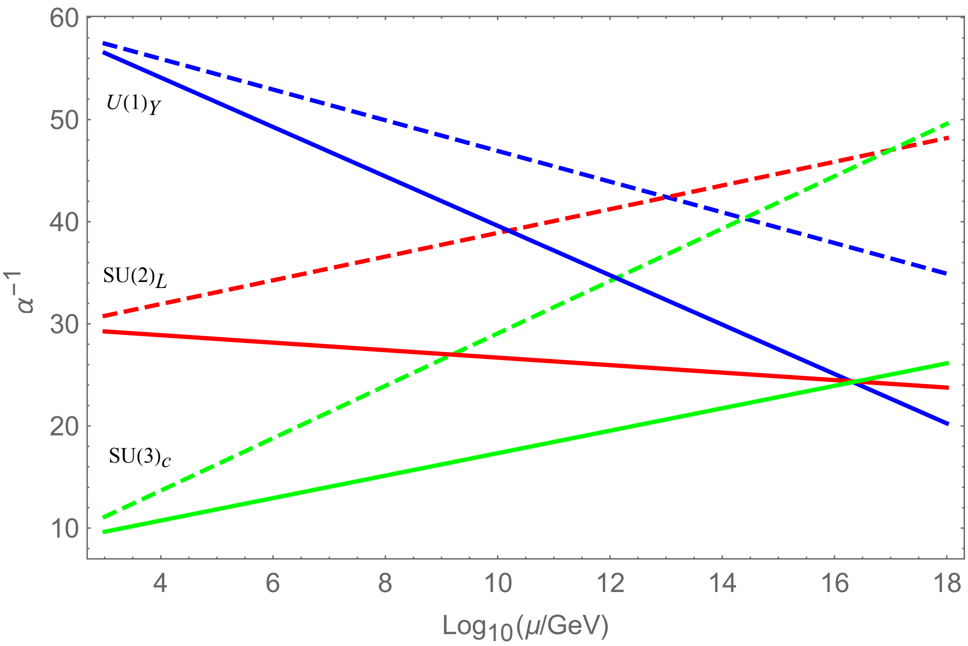

Figure 2.1: One-loop RGE evolution of the three inverse gauge couplings α_i^{-1}, reproduced by the author. The SM in dashed lines, the MSSM in solid lines. In the MSSM case a common threshold of 500 GeV is assumed for the sparticle spectrum.

is not the case, because, according to the SUSY breaking mechanism, the various superpartners decouple from the SM spectrum acquiring different masses. In a fully complete approach one should account for the "threshold corrections" that consider the specific contributions of the SUSY spectrum and are analyzed for example in [52, 53].
It was recently shown in [54] that the actual bounds given on the MSSM spectrum from LHC run I are still compatible with the unification of the gauge couplings in the MSSM. It is worth mentioning that, while the solution to the hierarchy problem requires a spectrum of all the superpartner around the $\sim$ TeV scale, other possibilities exist if one aims to reach the unification of couplings without caring for the hierarchy problem. For example, in the scenario of Split Supersymmetry [55, 56] it is found that only higgsinos and gauginos are required at a low scale, while squark and sleptons are allowed at much higher energy, in order to guarantee the gauge unification, without accounting for the solution to the hierarchy problem.

Let us finally remark that SUSY is not the only possibility to reach the gauge coupling unification, but because of other compelling features of this theory, it has been one of the most studied and considered so far. Other extensions of the SM, introducing new physics content at intermediate scales, can adjust the running of the couplings in the SM in such a way to provide exact unification.

2.4.2 Yukawa couplings unification

Also the Yukawa parameters (fermion masses and mixing angles), which are measured at low energy, are subjected to RGE flow. According to different values of $\tan\beta$ in the MSSM, which enters in distinguishing the masses of the up-quark sector from the down-quark and charged lepton sector, the running of the parameters has different behaviors, pointing out some peculiar structures that the Yukawa should respect at high energy. It's particularly important to study the evolution of the Yukawa couplings up to M_{GUT} if we want to provide a consistent description of the flavour sector in the context of grand unification. The starting values for the running are the experimental values at M_Z (cf. Tab. 2.1). We will take into account the precise predicted quantities in our further applications (cf. Chap. 5-6), but for the moment let us summarize the results of the Yukawa RGE flows [53, 57–62] as follows:

- at the scale M_{GUT} there is a partial unification of the third generation between the down-quark and the charged lepton sector, $Y^d_{33} \simeq Y^e_{33} \to m_b \simeq m_\tau$, which is verified within an uncertainty of 20-30%, depending on the values of $\tan\beta$[7]. For the first and second generations, we don't find unification but relations of the kind: $m_d \simeq 3m_e,\ m_s \simeq \frac{1}{3}m_\mu$. These approximations are known as Georgi-Jarlskog relations.

- Only for large values of $\tan\beta$ (≈ 50) also the up-sector enters to complete the unification of the third generation: $Y^u_{33} \approx Y^d_{33} \approx Y^e_{33}$, that is the so-calles t-b-τ unification.

- Independently of $\tan\beta$, the quark mixing parameters (CKM) are only slightly affected by the running, so that solutions obtained under initial diagonal Yukawa assumptions are suitable in a good approximation. The lepton mixing parameters are slightly affected in the case that the neutrino spectrum is very hierarchical, but they have important corrections in case of a degenerate spectrum.

- For very low values of $\tan\beta$ (≈ 1) the parameters reach the Landau pole much before M_{GUT}, so nothing can be said about their value at such a high energy.

These results are already visible taking into account the leading contributions at 1-loop in the MSSM RGEs. The stability of b-τ unification with respect to $\tan\beta$ is understood by the fact that the masses of down quarks and charged leptons are both controlled by $\cos\beta$. On the other side, the up quarks depend on $\sin\beta$, so the ratio with the other sectors is directly depending on $\tan\beta$. The Yukawa unification, in this case, is reached only for large $\tan\beta$ because this becomes responsible of the hierarchy between the mass of the t and the b, allowing the same Yukawa factor.

[7] While the unification is almost exact for small and large values of $\tan\beta$, at the leading order it is verified within the 20-30% also for intermediate values of $\tan\beta$. If we take into account specific threshold corrections at higher orders, it is possible to reach exact unification also in these cases [53].

We can conclude that the superymmetric extension of the SM leads to a partial unification of the Yukawa couplings, which is instead not predicted within the mere SM and which must be understood in a framework of grand unification.

Chapter 3

Grand Unified Theory, the road to SO(10)

3.1 Introduction and motivations for a Grand Unification

As anticipated in the introduction, we still don't know the origin of the specific theoretical setup of the SM, since the choice of the symmetry group, the assignment of representations and the values of many masses and coupling constants aren't anyhow motivated by the theory, but just dictated by the experimental data. The three fundamental interactions, above the EW breaking scale, are not related to each other in any specific way, and each of the three gauge couplings is a free parameter. It would be desirable to find a more complete theory that includes the SM, while explaining its theoretical structure, correlating the interactions and the many assigned parameters.

While trying to address this question, it is the SM itself, in its puzzling structure, to provide us with a strong hint towards a possible solution: the unification of the gauge couplings. It's a remarkable fact that the three SM gauge couplings, which at the EW scale appear well distinguished and hierarchical, from the RGE exhibit the tendency to converge at much higher energy, $M_{\rm GUT}^{SM} \sim 10^{15}$ GeV. This, while not being a proof of it, naturally suggests for a Grand Unified Theory (GUT), where the forces are described by a unique gauge coupling, relative to a larger simple gauge group which embeds the SM one. While such unification is not exactly verified under the assumption of the mere SM content, various extensions with new physics particles entering the running at intermediate scales can adjust the evolution to exact unification. As seen in the previous chapter, SUSY (or Split SUSY), for example, provides this adjustment.

In this scenario, the electromagnetic, weak and strong interactions are all contained in a larger set of interrelated interactions. Such a theory must include new gauge vector bosons, corresponding to the new generators of the enlarged group. A mechanism of spontaneous symmetry breaking operating at the GUT scale should thus distinguish the three SM gauge couplings, while providing mass of order $M_{\rm GUT}$ for the bosons of the broken generators.

This scenario, however, poses an immediate problem, related to the proton decay. Indeed, some of the additional gauge bosons mediate baryon number violating transitions which, through dimension 6 operators of the type $QQQL/M^2$, can mediate the nucleon decay. The dominant decay mode is $p \to e^+\pi^0$ ($n \to e^+\pi^-$). Since such gauge bosons acquire mass at $M_{\rm GUT}$, this process have an amplitude suppressed by $\sim 1/M^2_{\rm GUT}$, in particular:

$$\mathcal{M}_{p\to e^+\pi^0} \simeq \frac{\alpha_{\rm GUT}}{M^2_{\rm GUT}} \tag{3.1}$$

leading to the proton life-time [63]:

$$\tau_p \simeq \frac{1}{\mathcal{M}^2_{p\to e^+\pi^0} m_p^5} \simeq 4.5 \times 10^{29\pm0.7} \left(\frac{M_{\rm GUT}}{2.1\times10^{14}{\rm GeV}}\right)^4 \quad \text{years} \tag{3.2}$$

where m_p is the proton mass. With the unification scale $M^{SM}_{\rm GUT}$ predicted by the SM, , the expected lifetime is $\tau_p \sim 10^{33}$ years, which violates the present bounds given by the SuperKamiokande experiment, that is $\tau_p \gtrsim 10^{34}$ years [64].

Taking into account Supersymmetry, with the spectrum of the MSSM, the possible scenario of grand unification is very much ameliorated. As illustrated in the previous chapter, the convergence of the gauge couplings in the MSSM turns out to be not approximate, but exactly realized [49–51]. The unification scale is predicted at $M_{\rm GUT} \approx 2\times10^{16}$ GeV, raised enough to make the proton decay rate, mediated by dimension-6 operators, well compatible with the observations, predicting $\tau_p \simeq 10^{37}$ years. While SUSY is not the only possible alternative for a consistent unification, as mentioned, it provides solution also to the hierarchy problem, which arises, in this case, from introducing new degrees of freedom at $M_{\rm GUT}$, that would shift the Higgs mass towards such high energy scale through loop corrections. Therefore, both these reasons have historically given a great incitement to formulate models of grand unification based on SUSY, even if, nowadays, the solution to the hierarchy problem results severely constrained. However, considering SUSY GUTs, one has to take care of other dangerous operators that mediate the proton decay, as we will briefly review in the next section 3.1.1.

Many of the open problems regarding the explanation of the SM gauge structure and the correlations between the parameters, may find possible answer in the framework of unification. Earliest attempts of unification were made by Pati and Salam [12, 13], who have first tried to unify quarks and leptons into two irreducible representations of the group $\rm SU(4)\times SU(2)_L\times SU(2)_R$, the so-called Pati-Salam (PS) group, where lepton number is seen as the fourth color. The PS group, however, is not a simple group and doesn't provide gauge unification. Shortly after, Georgi and Glashow [14] proposed the first possibility of complete gauge unification, pointing out that the SM gauge group can be embedded into the simple Lie group SU(5).

Indeed, the unification group must be a simple Lie group, embedding the SM gauge group as subgroup and its rank is required to be ≥ 4 (the SM one). The smallest groups satisfying these requirements are SU(5), SU(6), SO(10), and E_6 [65]. Furthermore, we have to assure that some irreducible representations of the group contain the exact quantum numbers of the SM fermions and scalar, which are fixed by the branching rules between the unified and the SM group. This excludes, for example, the group SU(6) as candidate. In the following section we will review briefly the SU(5) and the Pati-Salam theories, and then we focus on the more compelling SO(10) GUT, which is the framework of choice for our particular models.

3.1.1 Some general drawbacks of GUT

Difficulties in the breaking sector. Besides the important choice of the unification group, we would like to remark that the specification of the symmetry breaking mechanism is of key importance. Without spontaneous breaking, all the vector bosons would be massless and all the coupling constants would be equal. The symmetry breaking distinguishes between the different interactions: the leptoquark bosons, which couple quarks to leptons and can mediate proton decay, acquire very large masses; the weak interaction bosons acquire much smaller masses at the EW scale; and the photon and gluons remain massless. Anyway, the symmetry breaking pattern can in principle be realized in many different ways. The larger the unified group is, the more are the possible chains of intermediate breaking. The problem is indeed mostly evident in SO(10) with respect to other proposed subgroups. Moreover, if a spontaneous breaking is realized, this requires the presence of scalar fields in large representations of the gauge group, allowing arbitrariness in the construction and also leading to problems like doublet-triplet (DT) splitting and large enhancement of the unified coupling above the scale of grand unification (M_{GUT}).
The dynamics of symmetry breaking usually happens at very high energies, so that it is almost impossible to be tested experimentally. This represents a weak point for GUTs, because it leaves a great number of open possibilities in the model building. Anyway, different breaking chains are of phenomenological interest, as they constrain the low-energy observables in the effective SM flavour structure, giving rise to different relations among the masses of up-quarks, down-quarks, charged leptons and neutrinos.

Proton decay. As already mentioned, GUT theories have to deal with the problem of proton decay. While the presence of SUSY naturally improves this aspect by suppressing enough the dimension-6 operators generated by the exchange of lepto-quark gauge bosons, other kind of operators of dimension-5 and dimension-4 can be responsible of the proton decay in the SUSY GUT framework. For a complete review of these operators we refer the reader to [63, 66].
Dimension-5 operators arise from the exchange between lepton and quarks superfields of color-triplets associated to the Higgs fields, and in particular of the triplet Higgsinos.

The operators are of the type $(QQQL)/M$, suppressed by one power of the triplet mass. The solution requires a mechanism to push the mass of the this triplet very high (cf. Sect.3.2.5), while maintaining the Higgs doublet partner at the EW scale. Dimension-4 operators come from Lepton (L) or Baryon (B) number violating interactions, such as $\sim QL\tilde{q}$ between a quark, a lepton and a scalar superpartner. Such interactions lead to operators of the type $(QQQL)/(m_{\rm SUSY}^2)$, with a neglectable suppression if we take the SUSY breaking scale $m_{\rm SUSY} \sim$ TeV. These operators are very dangerous for the proton decay and the usual way to control them is to forbid the existence of L/B-violating terms in the superpotential. This is done generally by requiring R-parity symmetry. R-parity is not guaranteed as symmetry in all GUTs and in some models it must be imposed by hand. It is the case of SU(5), while some SO(10) models have a naturally implemented R-parity[36]. The study of the proton decay in SUSY GUTs depends on the details of the model and on the SUSY spectrum. In the models of our construction that we will discuss in Chapt.5 and 6 this issue will not be analyzed, as far as we don't specify a mechanism for breaking N=1 SUSY.

Flavour puzzle in the unified framework. Another difficulty arises in GUT due to the partial or complete unification of the matter fields. While, on one side, this gives a nice explanation to the matter quantum numbers, on the other it does not easily account for the different characteristics of masses and mixing observed at low energy. As we will conclude at the end of this chapter, the overall picture of the flavour sector in GUT requires lots of parameters, fine tuned and distributed in a large range of magnitudes, resulting as problematic as in the SM. While the detailed characteristics of the flavour puzzle depend on the specific GUT model, this remains a general issue of grand unification: lots of improvements are needed in order to reach a realistic and natural theory of flavour based on GUT. This is the main purpose of the models we will propose for SO(10) GUT (cf Chapt.5-6), with the novel ingredient of extra dimensions.

3.2 SU(5) GUT

In this section we briefly review the SU(5) GUT originally proposed by Georgi and Glashow [14] as useful tool to understand the further unification into the larger SO(10) group.

The simple group SU(5) is defined as:

$$SU(5) = \{U | U = 5 \times 5 \text{ complex matrix; } U^{\dagger}U = 1; \text{ det}U = 1\} \tag{3.3}$$

Let us analyze the characteristic representations of this group.

3.2.1 Gauge sector

SU(5) has 24 generators, and therefore the Yang-Mills theory has 24 gauge vector bosons, which lives in the adjoint representation **24** of SU(5). SU(5) contains the SM gauge group $\mathrm{SU(3)_C{\times}SU(2)_L{\times}U(1)_Y}$ as a subgroup, and the 12 vector bosons of this subgroup are identified with the electromagnetic, weak and strong interactions of the SM. The other 12 vector bosons mediate new interactions. Such interactions are expected to be very weak because the new bosons gain a large mass at the GUT scale, when SU(5) gets broken.

The **24** can be decomposed under $\mathrm{SU(3)_C{\times}SU(2)_L{\times}U(1)_Y}$ as follows:

$$\mathbf{24} = (8,1)_0 + (1,3)_0 + (3,2)_{-5/6} + (\bar{3},2)_{5/6} + (1,1)_0 \tag{3.4}$$

Let us identify the components of this decomposition, reporting the generators as hermitian traceless 5×5 matrices:

- The $(8,1)_0$ component is the adjoint representation of $\mathrm{SU(3)_C}$, describing the 8 gluons. The generators of the $\mathrm{SU(3)_C}$ subgroup in SU(5) are given by $T_{A=1,\ldots,8}$:

$$T_A = \left(\begin{array}{c|c} \frac{1}{2}\lambda^A & 0 \\ \hline 0 & 0 \end{array}\right), \tag{3.5}$$

  where λ_A are the Gell-Mann matrices of SU(3).

- The $(1,3)_0$ component is the adjoint of the $\mathrm{SU(2)}_L$ group, describing the three weak vector bosons. The generators of $\mathrm{SU(2)}_L$ are given by $T_{A=21,22,23}$:

$$T_A = \left(\begin{array}{c|c} 0 & 0 \\ \hline 0 & \frac{\tau_A}{2} \end{array}\right), \tag{3.6}$$

  where $\tau_A \equiv \sigma_i$ are the Pauli matrices.

- The singlet component $(1,1)_0$ corresponds to the hypercharge Y, that is identified with the remaining generator of SU(5), commuting with the generators of SU(3) and SU(2) and it is given by:

$$T_{24} = \sqrt{\frac{3}{5}}\left(\begin{array}{c|c} -\frac{1}{3}\,I_3 & 0 \\ \hline 0 & \frac{1}{2}\,I_2 \end{array}\right) = \sqrt{\frac{3}{5}}Y \tag{3.7}$$

- The $(3,2)_{-5/6} + (\bar{3},2)_{5/6}$ multiplets are new gauge vector bosons corresponding to the 12 broken generators in $\mathrm{SU(5)/SU(3)_C{\times}SU(2)_L{\times}U(1)_Y}$. These generators, $T_{A=9,\ldots,20}$, have the symmetric and antisymmetric forms:

$$\left(\begin{array}{ccc|cc} & & & 1 & 0 \\ & 0 & & 0 & 0 \\ & & & 0 & 0 \\ \hline 1 & 0 & 0 & & \\ 0 & 0 & 0 & & 0 \end{array}\right); \quad \left(\begin{array}{ccc|cc} & & & -i & 0 \\ & 0 & & 0 & 0 \\ & & & 0 & 0 \\ \hline i & 0 & 0 & & \\ 0 & 0 & 0 & & 0 \end{array}\right); \tag{3.8}$$

with 1 and i in the six possible off-diagonal positions.

The overall normalization is chosen in such a way that all the SU(5) generators satisfy:

$$Tr(T_A T_B) = \frac{1}{2}\delta_{AB}$$

3.2.2 Matter sector

The assignment of the matter representations in the SU(5) model is less straightforward and comes from checking the SM quantum numbers reproduced by the SU(5) representations, that must fully reproduce the known ones for quarks and leptons. Let us analyze the possibilities:

- the antifundamental representation of SU(5) is decomposed under the SM group $\mathrm{SU(3)_C}\times\mathrm{SU(2)_L}\times\mathrm{U(1)_Y}$ as:

$$\bar{\mathbf{5}} = (\bar{3}, 1)_{1/3} + (1, 2)_{-1/2} \tag{3.9}$$

  describing the lepton EW doublet L and the colored down quark singlet, d^c with the quantum numbers as reported in Eq.(2.5). According to the structure of the fundamental representation of the generators introduced above, we identify the $\bar{\mathbf{5}}$ components as:

$$\bar{\mathbf{5}} \equiv \psi_i = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e \\ -\nu \end{pmatrix}_L \tag{3.10}$$

  where, by convention, we have expressed the lepton doublet as $(e \;\; -\nu)^T = i\sigma_2\, L$.

- To construct further representations we consider the tensor product:

$$\mathbf{5} \times \mathbf{5} = \mathbf{10}_a + \mathbf{15}_s$$

  The $\mathbf{10}$ is the antisymmetric part of this *i.e.* it must respect:

$$\mathbf{10}^{ij} = -\mathbf{10}^{ji} \propto \mathbf{5}_1^i \mathbf{5}_2^j - \mathbf{5}_2^j \mathbf{5}_1^i.$$

  This antisymmetric structure, distinguishing the indices i, j with respect to the SU(3) and SU(2) subspaces $i = \{a = 1, ..3, \alpha = 1, 2\}$, leads to:

$$10^{ab} \equiv \epsilon^{abc}(u^c)_c = (\bar{3}, 1)_{-2/3}; \quad 10^{a\alpha} \equiv q^{a\alpha} = (3,\, 2)_{1/6}; \quad 10^{\alpha\beta} \equiv \epsilon^{\alpha\beta} e^c = (1,\, 1)_1; \tag{3.11}$$

  Indeed, the decomposition under the SM is:

$$\mathbf{10} = (3,\, 2)_{1/6} + (\bar{3}, 1)_{-2/3} + (1,\, 1)_1 \tag{3.12}$$

describing the quark colored EW doublet Q, the coloured up quark singlet, u^c and the RH electron singlet e^c, with the quantum numbers as introduced in Eq.(2.5). Making all the components explicit we have:

$$\mathbf{10} \equiv \psi_{ij} = \left(\begin{array}{ccc|cc} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ -u_3^c & 0 & u_1^c & u_2 & d_2 \\ u_2^c & -u_1^c & 0 & u_3 & d_3 \\ \hline -u_1 & -u_2 & -u_3 & 0 & e^c \\ -d_1 & -d_2 & -d_3 & -e^c & 0 \end{array}\right)_L \tag{3.13}$$

where the signs of the various entries are chosen with respect to the antisymmetry property defined above.

SU(5) permits to arrange the SM matter content of one generation in two distinct representations: $\bar{\mathbf{5}}$ and $\mathbf{10}$, thus providing a partial unification of the fermions. A remarkable feature is that these representations unify the quark and lepton sector. With respect to the PS group (cf. Sect.3.3), this quark-lepton unification mixes quark and lepton chiralities in an asymmetric way.
Notice that the RH neutrino sector doesn't arise naturally in this framework: RH neutrinos should be introduced separately as an SU(5) singlet.

3.2.3 Higgs sector and Yukawa couplings

To complete the description of the SM field content, we need a representation for the Higgs EW doublet which couples to the fermions in $\bar{\mathbf{5}}$ and $\mathbf{10}$ via Yukawa interactions. Given the content of these representations, we recognize that the mass terms of the different fermions must be constructed through the bilinears:

$$M_u: \;\; \mathbf{10} \times \mathbf{10} = \bar{\mathbf{5}} + \overline{\mathbf{45}} + \overline{\mathbf{50}}\;; \qquad M_d,\, M_e: \;\; \bar{\mathbf{5}} \times \mathbf{10} = \mathbf{5} + \mathbf{45}\;.$$

Thus, the suitable representations for the Higgs that can couple to these bilinears, guaranteeing the invariance under SU(5), are the fundamental and the antifundamental $\mathbf{5}$ and $\bar{\mathbf{5}}$ or the $\mathbf{45}$ and $\overline{\mathbf{45}}$. Indeed, among the SM components of the $\bar{\mathbf{5}}$ in Eq.(3.9) one can recognize the correct quantum numbers of the Higgs doublet. Similarly, in the SM decomposition of the $\mathbf{45}$:

$$\mathbf{45} = (1,2)_{1/2} + (3,1)_{-1/3} + (3,3)_{-1/3} + (\bar{3},1)_{4/3} + (\bar{3},2)_{-7/6} + (\bar{6},1)_{-1/3} + (8,2)_{1/2} \tag{3.14}$$

the first component has the correct quantum numbers.
The minimal choice is to work only with the smallest representation for the Higgs: $\mathbf{5}_H$. In a SUSY framework, due to the holomorphic nature of the superpotential, we cannot use the complex conjugate representation of the same field, so that we need to distinguish the up-type and down-type Higgs doublets through independent representations. Thus, in SUSY, both the representations $\mathbf{5}_H \supset H_u$ and $\bar{\mathbf{5}}_H \supset H_d$ are needed.

In this framework of minimal supersymmetric SU(5) there are two independent renormalizable Yukawa interactions in the superpotential:

$$W_Y = Y_5\,(\mathbf{10}\,\mathbf{10}\,\mathbf{5}_H) + Y_{\bar{5}}\,(\mathbf{10}\,\bar{\mathbf{5}}\,\bar{\mathbf{5}}_H) \tag{3.15}$$

Of course these terms include the SM Yukawas, as it can be seen from projecting on the SM components:

$$Y_5\,(\mathbf{10}\,\mathbf{10}\,\mathbf{5}_H) \to Y_5\,Q\,u^c\,H_u; \quad Y_{\bar{5}}\,(\mathbf{10}\,\bar{\mathbf{5}}\,\bar{\mathbf{5}}_H) \to Y_{\bar{5}}\,\left(Q\,d^c\,H_d + e^c\,L\,H^d\right)\,, \tag{3.16}$$

leading to the relations:

$$Y_u = Y_5; \qquad Y_d = Y_e^T = Y_{\bar{5}} \tag{3.17}$$

This SU(5) model predicts the same Yukawa coupling for the down quark and the charged lepton sectors and, therefore, the exact equality of their mass spectrum at the GUT scale. Considering the values of the masses evolved by RGE up to the GUT scale, this exact prediction of unification results unrealistic, but it would be quite good as leading order contribution. Indeed, as described in the previous chapter (cf. Sect.2.4.2), $m_b \simeq m_\tau$ is obtained at the GUT scale by the MSSM within the 20-30% of uncertainty [62], while quite a similar hierarchy exists between the first two generations of down quarks and charged leptons[1]: $m_d/m_s \approx m_e/m_\mu$.

This minimal model looks like a first promising attempt in the unification of gauge and matter fields, but it needs some corrections to provide a realistic mass spectrum. These corrections require an extension by the introduction of new Higgs representations or higher dimensional non renormalizable Yukawa couplings.

In the first approach one can introduce the representations $\mathbf{45}_H$ and $\overline{\mathbf{45}}_H$ for a new couple of Higgs doublets [68]. The general Yukawa sector becomes:

$$W_Y = \mathbf{10}\,(\,Y_5\mathbf{5}_H + Y_{45}\mathbf{45}_H)\,\mathbf{10} + \mathbf{10}\,(\,Y_{\bar{5}}\bar{\mathbf{5}}_H + Y_{\overline{45}}\overline{\mathbf{45}}_H)\,\bar{\mathbf{5}} \tag{3.18}$$

From the decomposition in SM components, taking into account the different VEVs and the Clebsch-Gordon coefficients, one finds the relations for the SM Yukawas:

$$v_u\,Y_u = \langle\mathbf{5}\rangle\,Y_5 + \langle\mathbf{45}\rangle\,Y_{45}; \quad v_d\,Y_d = \langle\bar{\mathbf{5}}\rangle\,Y_{\bar{5}} + \langle\bar{\mathbf{45}}\rangle\,Y_{\overline{45}}; \quad v_d Y_e^T = \langle\bar{\mathbf{5}}\rangle\,Y_{\bar{5}} - 3\,\langle\bar{\mathbf{45}}\rangle\,Y_{\overline{45}} \tag{3.19}$$

Notice that the Higgs doublet inside the $\mathbf{45}_H$ couples to the down quark and lepton sectors with coefficients different by a factor -3. This factor is known as Georgi-Jarlskog factor [68] and is essentially due to the fact that the quarks come in 3 colors. This factor is very important, accounting for a distinction between the Yukawa couplings of down quarks and leptons that can correct the prediction on the spectrum in a more realistic way. Notice that a $\mathbf{45}_H$ alone would predict the unrealistic relation $M_d = 3M_e$. The MSSM Higgs will correspond to a linear combination of the doublets in the $\mathbf{5}_H$ and the $\mathbf{45}_H$ and, in order to get a realistic spectrum, we expect in the Yukawa couplings a leading contribution

[1] This ratios are almost independent from RGE effects [67].

from $\mathbf{5}_H$ and a sub-leading contribution from $\mathbf{45}_H$. Such a model can indeed fit the mass spectrum of all the fermions, with adequate selection of the Yukawa parameters, the $Y_{\overline{45}}$ accounting for the subleading corrections to the unified mass spectrum predicted by the minimal model [].

In a second approach, one can extend the minimal model by allowing a non renormalizable dimension-5 operator [69]:

$$\frac{Y'_{\bar{5}}}{\Lambda}\,\mathbf{10}\,\bar{\mathbf{5}}\,\mathbf{24}_H\bar{\mathbf{5}}_H, \tag{3.20}$$

which involves the usual Higgs $\bar{\mathbf{5}}_H$ and the new scalar field $\mathbf{24}_H$, a Higgs-like field responsible of the breaking of SU(5) down to the SM gauge group. Λ is a cut-off scale that we can assume to be M_{Plank}. This solution, at the price of giving up renormalizability, permits to maintain only the minimal Higgs representations and provides naturally suppressed corrections to the leading Yukawas Y_5 and $Y_{\bar{5}}$.

What about neutrinos? Like in the SM, in the minimal SU(5) model, neutrinos remain massless at the renormalizable level. With the present field content one can reproduce neutrino masses only through the higher-dimensional Weinberg operator:

$$\frac{Y_L}{\Lambda}\bar{\mathbf{5}}\mathbf{5}_H\bar{\mathbf{5}}\mathbf{5}_H\ , \tag{3.21}$$

Assuming neutrinos to be of Majorana type, one can introduce an SU(5) singlet for RH neutrinos, and thus adding a Dirac and Majorana mass terms to the Lagrangian (cf Sect.2.1.3), reproducing light neutrino masses by type-I seesaw mechanism.
Another possibility is to add a Higgs-like field $\mathbf{15}_H$ which contains a weak triplet:

$$\mathbf{15} = (1,3)_6 + (3,2)_1 + (6,1)_{-4} \tag{3.22}$$

and interacts with the two LH neutrinos through the new Yukawa coupling:

$$Y_{15}\,\bar{\mathbf{5}}\bar{\mathbf{5}}\,\mathbf{15}_H \tag{3.23}$$

and one can thus implement the type-II seesaw mechanism.
Both the options come with a new set of free parameters, that make these realizations not very compelling.

3.2.4 Symmetry breaking

The model must account for the symmetry breaking mechanism:

$$\mathrm{SU(5)} \to \mathrm{SU(3)_C} \times \mathrm{SU(2)_L} \times \mathrm{U(1)_Y}.$$

Since the rank of SU(5) is the same of the SM, this breaking can happen in one-step. The simplest choice is a spontaneous breaking realized by introducing a scalar field that gets a VEV at $M_{\rm GUT}$. Such a scalar must be in a representation of SU(5) that contains as sub-representation a SM singlet. The smallest candidate with this feature is a **24** of SU(5), with the decomposition given in Eq.(3.4). As anticipated above, this field can enter in correcting the Yukawa sector through the non-renormalizable operator (3.20). The most general renormalizable superpotential that one can write for the minimal Higgs sector $\mathbf{5}_H + \bar{\mathbf{5}}_H + \mathbf{24}_H$ is:

$$W_H = -\mu^2 \bar{\mathbf{5}}_H \mathbf{5}_H - m_{24}^2 \mathbf{24}_H \mathbf{24}_H + \lambda \mathbf{24}_H \mathbf{24}_H \mathbf{24}_H + \eta \bar{\mathbf{5}}_H \mathbf{24}_H \mathbf{5}_H \tag{3.24}$$

This potential counts 4 complex parameters, which reduce to 6 real parameters, redefining the fields by phase transformation. The freedom on these parameters can accommodate the VEV $\langle \mathbf{24} \rangle$, responsible of the SU(5)) breaking, at the GUT scale. The further EW breaking $\mathrm{SU(2)_L \times U(1)_Y \to U(1)}_{EM}$ is realized by the MSSM Higgses $\mathbf{5}_H$, $\bar{\mathbf{5}}_H$. A problem of hierarchy between these parameters arises for distinguishing the MSSM Higgs at the EW scale.

3.2.5 Doublet-Triplet (DT) splitting problem

The representations introduced above for the MSSM Higgs introduce a problem. As it can be seen from the decompositions in Eq.(3.9) for the $\mathbf{5}_H$ and in Eq.(3.14) for the $\mathbf{45}_H$, the EW Higgs doublet is always accompanied by a colored triplet. This is a new field which doesn't fit into the SM content. When we construct the Yukawa couplings of the matter fields, Eqs.(3.15-3.18), we are introducing a coupling not only with the Higgs doublet, but also with this new triplet. Let us denote the triplets as $T = (3,1)_{-1/3} \in \mathbf{5}_H$ and $\overline{T} = (\bar{3},1)_{1/3} \in \bar{\mathbf{5}}_H$, where we have reported the SM quantum numbers. From the Yukawa interactions of Eq.(3.15) we get the terms:

$$W_Y \supset Y_u U^c E^c T + Y_u QQT + Y_d D^c U^c \overline{T} + Y_d QL\overline{T} \tag{3.25}$$

These are B and L violating interactions which open new channels for proton decay, as mentioned in the introduction. It can be shown that the major contribution to the proton decay happens through the exchange of the colored triplet Higgsinos, leading to effective dimension-5 operator [63, 70, 71] of the kind:

$$QQQL/M_T, \tag{3.26}$$

where M_T is the mass of the triplet. Such operator mainly contributes, with the further exchange at one loop of scalar superpartners, to the decay channel $p \to K^+ + \overline{\nu}$, with amplitude of the form [70, 72]:

$$\mathcal{M}_{p\to K^+ + \overline{\nu}} \simeq \frac{Y_u Y_d}{M_T m_{SUSY}} \tag{3.27}$$

Such amplitudes needs to be suppressed by a big value of the mass M_T in order to respect the experimental bound on the decay rate in this channel, $\tau_{p\to K^+ + \overline{\nu}} \gtrsim 5.6 \times 10^{33}$ years

[73]. Taking m_{SUSY} ~1-10 TeV, this would imply $M_T \gtrsim 10^{13\div 14}$ GeV. Of course higher SUSY breaking scales would relax this constraint.

Accounting for the Higgs doublet mass at the EW scale from the potential in Eq.(**??**), M_T would naturally be of the same order, so that the coupling of the operator (3.26) wouldn't have any strong suppression, representing a serious issue for the proton decay.

A solution to this problem requires a mechanism that splits the masses of the doublet and the triplet inside the same Higgs multiplet. The problem goes under the name of "doublet-triplet (DT) splitting problem" [74–77] and it is a non trivial issue of all GUTs which include SU(5) as subgroup. The solution generally requires the introduction of new degrees of freedom, like new Higgs representations with useful group-theoretical properties, which do the job of allowing a mass at the GUT scale only for the triplets, but not for the MSSM doublets. We are going to briefly describe a couple of examples of this mechanisms that will be of major interest in our applications. For a more complete and detailed review of various D-T splitting mechanisms, we refer the reader to [66]. Most of the proposed solutions for SU(5), up to model dependent details, are adaptable to the SO(10) GUT as well.

- Missing partner mechanism. This mechanism is based on a suitable choice of Higgs representations, larger than the minimal one, which accounts for a disparity in the total number of doublets (D) and triplets (T) content, in such a way that at least one triplet has no doublet counterpart. A proper potential is constructed, where two minimal Higgs representations don't have a bare mass term, while they can gain mass at high scale via the interactions with the other heavy Higgs fields. The disparity between D and T implies that all the triplets in the minimal representations gain a heavy mass, while a couple of Higgs doublet is guaranteed to remain light. This mechanism can be implemented in several ways in SU(5) GUT models [78–81] and also in SO(10) [82]. It is also applied in our SO(10) model [16] (cf. Chapt. 5).

- Extra Dimensions. In GUT models realized on compactified ED with orbifolding, an economic solution to the D-T splitting is realized by means of proper boundary conditions imposed on the Higgs multiplet living in the bulk. If the gauge symmetry is broken via ED through orbifolding, opposite parity assignments to the D and T components are possible, inducing the cancellation of the triplet 0-mode (cf. Sect.4.3), while maintaining the 0-mode of the doublet. This results in the existence of a light D, while pushing the mass of T up to the compactification scale, which in these models must be $\gtrsim M_{\rm GUT}$. The first reference for this mechanism is [83], while further applications are made in [84, 85] in the SU(5) context, where all dimension-5 operators mediating the proton decay are automatically vanishing, and in [86, 87] in the SO(10) context. Also in our SO(10) model [17] we will make use of this orbifold property (cf. Chapt. 6).

3.3 The Pati-Salam group

As SU(5) represents the first attempt of fully unifying the gauge group, let us introduce the first attempt of unifying quarks and leptons through the Pati-Salam (PS) group [12, 13]. Both of these routes, indeed, are of fundamental importance for understanding the further developments of unification in the larger group SO(10).
The PS group is defined as:

$$\mathrm{SU}(4)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R} \tag{3.28}$$

This group evidently embeds the SM gauge group, since $\mathrm{SU}(4)_\mathrm{C} \supset \mathrm{SU}(3)_\mathrm{C}\times\mathrm{U}(1)_{\mathrm{B-L}}$, while the hypercharge Y comes from a linear combination of $\mathrm{U}(1)_{\mathrm{B-L}}$ and the diagonal generator $T_{3\mathrm{R}}$ of $\mathrm{SU}(2)_\mathrm{R}$. The PS group is not a simple group, so that it doesn't provide a unification of the gauge couplings. Anyway, it's a first step towards unification, and it can be embedded in a larger simple group as SO(10). Indeed, since $\mathrm{SU}(4) \approx \mathrm{SO}(6)$ and $\mathrm{SU}(2)\times\mathrm{SU}(2)\approx\mathrm{SO}(4)$ (where $\approx$ means a homomorphism), it is easy to see that $\mathrm{PS}\approx\mathrm{SO}(6)\times$Âŋ $\mathrm{SO}(4)\subseteq \mathrm{SO}(10)$. As we will discuss later, the PS group can be an intermediate step in one possible chain for breaking SO(10) and as such it deserves a basic illustration of its features and relevant representations, that will become useful in the next sections.

3.3.1 Fields representations

Matter sector

The quark-lepton unification is realized by recognizing that one family of all the SM fermions fit into the two irreducible representations:

$$(4, 2, 1) + (\overline{4}, 1, 2) \tag{3.29}$$

and, explicitly:

$$Q = (4,2,1) = \begin{pmatrix} u_1 & u_2 & u_3 & \nu \\ d_1 & d_2 & d_3 & e \end{pmatrix}; \quad Q^c = (\overline{4},1,2) = \begin{pmatrix} d_1^c & d_2^c & d_3^c & e^c \\ -u_1^c & -u_2^c & -u_3^c & -\nu^c \end{pmatrix} \tag{3.30}$$

Interestingly, the SM fermions are embedded in two representations that distinguish symmetrically the left and right chiralities, reflecting the feature of the gauge group, which includes the new $SU(2)_\mathrm{R}$ at a par with $SU(2)_\mathrm{L}$. The idea is that this left-right symmetry is broken at lower energy giving rise to the chiral SM content.
Notice that a RH neutrino ν^c is naturally included in $(\overline{4}, 1, 2)$, providing the possibility to implement the neutrino mass via the see-saw mechanism.

Gauge sector

The PS group counts 21 generators and therefore there are 21 gauge vector bosons. These gauge fields live in the adjoint representation of the PS group, that is given by the adjoint representations of the factorized subgroups:

$$(15, 1, 1) + (1, 3, 1) + (1, 1, 3) \tag{3.31}$$

The generators are recognized as follows:

- for the component $(15, 1, 1)$ let us consider the decomposition of $\mathrm{SU}(4)_\mathrm{C} \supset \mathrm{SU}(3)_\mathrm{C}\times\mathrm{U}(1)_\mathrm{B-L}$:

  $$15 = 1_0 + 3_{4/3} + \overline{3}_{-4/3} + 8_0 \tag{3.32}$$

  where we identify the 8_0 describing the 8 SM gluons of $\mathrm{SU}(3)_\mathrm{C}$.

  The singlet 1_0 is associated to $\mathrm{U}(1)_\mathrm{B-L}$. This generator must be normalized in such a way to reproduce correctly 9 the baryon and lepton number of the SU(4) fermion 4-plets in Eq.(3.30:

  $$B - L = \begin{pmatrix} \frac{1}{3} & & & \\ & \frac{1}{3} & & \\ & & \frac{1}{3} & \\ & & & -1 \end{pmatrix} \tag{3.33}$$

  Thus, from this definition, having embedded the fermion fields as in Eq.(3.30), we can easily identify the hypercharge Y as the combination:

  $$Y = T_{3R} + \frac{B-L}{2} \tag{3.34}$$

  in order to reproduce correctly the SM fermions quantum numbers as reported in Eq.(2.5).

  The remaining 6 fields, $\mathrm{SU}(3)_\mathrm{C}$ triplets, $X_{PS} \equiv 3_{4/3}$ and $\overline{X}_{PS} \equiv \overline{3}_{-4/3}$ are new gauge bosons with respect to the SM. These bosons are in principle dangerous mediators of lepto-quark transitions. However, as shown for example in [88], the Lagrangian describing such transitions is invariant under a global U(1) symmetry with given charge assignment to the fields which, in the fermion sector, is identified with $B+L$. Since $B-L$ is also a (gauge) symmetry, both B and L result conserved by the gauge interactions. This is a particular feature of the PS group that preserves the proton from decaying, in spite of the lepto-quark interactions. The generators associated to these bosons must be broken in order to reproduce the low energy SM theory, but their masses don't need, in this case, to respect the strict bound from proton life-time.

- (1,3,1) is the adjoint of $\mathrm{SU}(2)_\mathrm{L}$ describing the 3 bosons of the weak interactions.

- (1,1,3) is the adjoint of $SU(2)_R$. It describes 3 gauge bosons: the one associated to the diagonal generator T_{3R} enters in the definition of the hypercharge, while the other 2 generators must be broken.

Higgs sector and Yukawa couplings

Working in the SUSY framework, we recognize that the couple of EW Higgs doublets $\{H_u, H_d\}$, responsible of the EW symmetry breaking, is described by the representation under $SU(4)\times SU(2)_L\times SU(2)_R$:

$$H_1 = (1,2,2)_{H_1} \tag{3.35}$$

that is the most economic choice. The next suitable representation is:

$$H_2 = (15,2,2)_{H_2} \tag{3.36}$$

which, according to the decomposition (3.32), contains the $SU(3)_C$ singlet. This larger representation, anyway, introduces some new fields other than the wanted EW doublets. Both these representations couple to the bilinear $(\overline{4},2,1)(4,1,2)$ respecting the invariance under the PS group.
It's worth noting that the Higgs doublet H_1, Eq.(3.35), by construction is not affected by the presence of colored triplet, avoiding the D-T splitting problem of SU(5) (and, as we will see, of SO(10)). On the contrary, the higher representation H_2, Eq.(3.36), includes also some triplets.
The minimal PS model, with only one Higgs of the type H_1, would lead to the unified Yukawa coupling:

$$Y_1(\overline{4},1,2)(4,2,1)(1,2,2)_{H_1} \tag{3.37}$$

which implies, at the SM level: $Y_u = Y_d = Y_e = Y_\nu^D$, including also a Dirac mass term for neutrinos. This prediction is not realistic, since, even allowing for different Higgs VEVS v_u and v_d, it would imply the equality between the down-quark and the charged lepton masses, and, even worse, the equality between up-quark and neutrino masses. While the former is acceptable as leading order result (as seen in SU(5)), the latter is completely unrealistic, being wrong by 15 orders of magnitude. Similarly to what already seen in the SU(5) case, the introduction of the second Higgs representation H_2 distinguishes the Yukawa couplings in the lepton sector, because of different Clebsch-Gordon coefficients in the SM decomposition, such that, from the single coupling:

$$Y_2(\overline{4},1,2)(4,2,1)(15,2,2)_{H_2} \tag{3.38}$$

one finds the SM Yukawa relations:

$$Y_d = -3Y_e; \quad Y_u = -3Y_\nu^D$$

, where -3 is the same Georgi-Jarlskog encountered in the SU(5) framework (cf Sect.3.2.3). The combinations of both the representations H_1 and H_2 gives enough freedom in the

Yukawa couplings to fit realistically the quark and the charged-lepton sectors. The neutrino mass, anyway, still needs to be distinguished and adjusted to a lower energy scale. A natural way to do this, having already introduced RH neutrinos as SM singlets, is via a type-I seesaw mechanism. In order to introduce a Majorana mass term for the RH neutrinos, we need to couple a new field with the bilinear $(\overline{4},1,2)\times(\overline{4},1,2)$. From decomposition of the SU(4) tensor product: $\bar{4}\times\bar{4}=6+\overline{10}$, the possibilities to construct an invariant under the PS group are through the representations:

$$(6,1,1)_H,\ (6,1,3)_H,\ (10,1,1)_H,\ (10,1,3)_H$$

Anyway, since this field will take a VEV generating the mass term, we have to choose a representation containing a SM singlet, along which the VEV must be aligned in order not to break the SM group. By considering the decomposition under $\mathrm{SU}(4)_\mathrm{C} \supset \mathrm{SU}(3)_\mathrm{C}\times \mathrm{U}(1)_{\mathrm{B-L}}$:

$$6 = 3_{2/3} + \overline{3}_{-2/3}; \quad 10 = 1_2 + 3_{2/3} + 6_{-2/3}\,. \tag{3.39}$$

Only the SU(3) 10-plet contains a singlet and the combination of charges leads to Y=0 only for $\Delta_R = (10,1,3)_H$. To realize the seesaw mechanism, we need a VEV $\langle\Delta_R\rangle$ at high scale with respect to the Higgs VEV[2] of the Dirac neutrino mass term: $\langle\Delta_R\rangle \gg v_u$. The field Δ_R is also used to break the PS group down to the SM (see next paragraph), naturally motivating the high scale of its VEV.

We finally mention that in some models the L-R symmetry is enhanced by imposing a discrete symmetry which exchanges $\mathrm{SU}(2)_\mathrm{L}\leftrightarrow\mathrm{SU}(2)_\mathrm{R}$ [89, 90]. To respect this symmetry, one has to introduce an explicit Majorana mass term also for the LH neutrinos through an analogous Yukawa coupling of the bilinear $(4,2,1)\times(4,2,1)$ with the field $\Delta_L = (\overline{10},3,1)_H$. This realization is a type-II seesaw mechanism, that in this case is combined with the type I. Since the VEV $\langle\Delta_L\rangle$ of this field breaks $\mathrm{SU}(2)_\mathrm{L}$, it is subjected to strong constraints from electroweak precision tests, implying $\langle\Delta_L\rangle \ll v_u$. From the combination of type I and type II seesaw, the scalar potential that we can write for the fields $\Delta_L, \Delta_R, H_1, H_2$ predicts the relation [28, 32]: $\langle\Delta_L\rangle \approx \mathcal{O}(1)\frac{v_u^2}{\langle\Delta_R\rangle}$, giving a natural suppression to $\langle\Delta_L\rangle$.

3.3.2 Symmetry breaking

The PS group has rank 5, so the breaking chain to the SM must include the breaking of an extra U(1) symmetry. We can imagine the breaking chain to take place in two steps:

$$\begin{aligned} SU(4)_\mathrm{C}\times SU(2)_\mathrm{L}\times SU(2)_\mathrm{R} \ &\overset{<X>}{\longrightarrow}\ SU(3)_\mathrm{C}\times SU(2)_\mathrm{L}\times SU(2)_\mathrm{R}\times U(1)_{\mathrm{B-L}} \\ &\overset{<X'>}{\longrightarrow}\ SU(3)_\mathrm{C}\times SU(2)_\mathrm{L}\times U(1)_\mathrm{Y} \end{aligned} \tag{3.40}$$

While, in general, this scheme can be realized in 2 steps by different representations X and X', it turns out that the single representation $(10,1,3)$, introduced above as Δ_R has

[2] v_u is given by the linear combination of $\langle H_1\rangle$ and $\langle H_2\rangle$: $Y_u v_u \equiv Y_1\langle H_1\rangle + Y_2\langle H_2\rangle$

the correct quantum numbers to achieve both these breakings. Without introducing new fields, we can then perform the breaking by $<\Delta_R>$, combining its utility as Majorana mass term for RH neutrinos. Another possibility is to play the same job with the representation $(4,1,2)_H$, which contains an $\mathrm{SU(3)_C}$ singlet, too, and has a suitable combination of $B-L$ and T_{3R} quantum numbers. Theis field can also be used for generating a Majorana mass term for neutrinos, but via a non renormalizable dimension-5 operator:

$$\sim \frac{1}{\Lambda}(\overline{4},1,2)(\overline{4},1,2)(4,1,2)_H(4,1,2)_H$$

The further EW symmetry breaking is realized by the $\mathrm{SU}(2)_L$ doublet in H_1 and/or H_2 as described above.

3.4 SO(10) GUT

SO(10) includes both SU(5) and $\mathrm{SU(4)\times SU(2)_L\times SU(2)_R}$ as subgroups, so that the main aspects of SO(10) unification are based on the results described in the previous sections for these two groups. The SO(10) theory [91] is particularly attractive because SO(10) is the smallest simple Lie group for which a single irreducible representation (namely the spinorial **16**) can accommodate the entire SM fermion content of one generation. Thus, not only SO(10) accounts for the gauge unification, but also for a complete unification of the matter fields (despite the 3 families replica). Moreover, all the irreducible representations of SO(10) are free from anomaly, unlike the case of SU(5) (and the SM itself), where the representations of the matter fields seem to be carefully combined in such a way to cancel the anomaly.

SO(10) is the special orthogonal group of rotations in a 10-dimensional vector space:

$$SO(10) = \{O|O = 10\times 10 \text{ real matrix};\ O^T O = 1;\ \det O = 1\} \tag{3.41}$$

The orthogonal matrices O act as a rotation leaving invariant the norm of a 10-dimensional real vector field. The choice $\det O = +1$ selects the group of transformations with continuous connection to the identity.
While listing the representations of SO(10), it is useful to perform their decomposition with respect to both the subgroups $\mathrm{SU(4)\times SU(2)_L\times SU(2)_R}$ and $\mathrm{SU(5)\times U(1)_X}$[3], recognizing the representations already introduced in the previous sections.

[3] The extra $\mathrm{U(1)_X}$ is needed to reproduce the rank = 5 of SO(10). As we are going to see from the symmetry breaking scheme, the charge X is defined as a linear combination of T_{3R} and $B-L$ orthogonal to the hypercharge Y.

3.4.1 Gauge sector

The SO(10) group has 45 generators. Among these, 12 describe the SM gauge group, while the remaining 33 must be broken at the high energy M_{GUT}. The gauge fields live in the adjoint representation **45**, that is decomposed as:

$$\mathbf{45} \overset{SU(5)\times U(1)_X}{=} 1_0 + 10_4 + \overline{10}_{-4} + 24_0 \tag{3.42}$$

$$\mathbf{45} \overset{SU(4)_C\times SU(2)_L SU(2)_R}{=} (15,1,1) + (1,3,1) + (1,1,3) + (6,2,2) \tag{3.43}$$

Where it is evident which are the new generators introduced, from comparison with the adjoint representations of the two subgroups already described in Eq.(3.4,3.31): $1_0 + 10_4 + \overline{10}_{-4}$ of SU(5)×U(1)$_{\mathrm{X}}$ and (6,2,2) of the PS group. These new generators must be of course broken. This breaking is the first step of the breaking chain of SO(10), which, as we are going to discuss, can be realized via the SU(5) route or via the PS route.

3.4.2 Matter sector

As mentioned in the introduction, all the SM fermion content of one generation nicely fits into a spinorial representation **16** of SO(10). A useful approach to construct such representation is made in terms of the SU(5) basis, by means of creation an annihilation operators [92], as it is reviewed in appendix B. Another approach, through the explicit construction of the Γ matrices of the Clifford algebra is given in [93]. **16** and $\overline{\mathbf{16}}$ are two irreducible spinorial representations of SO(10), which come from the reducible representation **32** by projecting into opposite chiralities[4] .

We assign to matter fields the representation $\mathbf{16}_{i=1,2,3}$, one for each generation. The **16** decomposes under SU(5)× U(1)$_X$ as:

$$\mathbf{16} = 10_{-1} + \overline{5}_3 + 1_{-5} \tag{3.44}$$

where we recognize all the SU(5) matter representations introduced in Sect.3.2.2, **10** and **5**, plus the singlet 1_{-5}, that naturally completes the **16** components, providing the

[4] Here we refer to the chirality defined on the SO(10) vector space by action of the operator defined in Eq.(B.17) from the Clifford algebra (see appendix B).

description of RH neutrinos:

$$16_{SO(10)} \supset 1_{SU(5)} = \psi_0 = \nu^c; \quad 16_{SO(10)} \supset \overline{5}_{SU(5)} = \psi_i = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e \\ -\nu \end{pmatrix}_L ; \tag{3.45}$$

$$16_{SO(10)} \supset 10_{SU(5)} = \psi_{ij} = \left(\begin{array}{ccc|cc} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ -u_3^c & 0 & u_1^c & u_2 & d_2 \\ u_2^c & -u_1^c & 0 & u_3 & d_3 \\ \hline -u_1 & -u_2 & -u_3 & 0 & e^c \\ -d_1 & -d_2 & -d_3 & -e^c & 0 \end{array}\right)_L \tag{3.46}$$

Under the PS group, the **16** simply decomposes as:

$$\mathbf{16} = (4, 2, 1) + (\overline{4}, 1, 2) \tag{3.47}$$

unifying the representations introduced for the PS group (cf. Sect.3.3.1).

The inclusion of the singlet nicely accounts for the description of RH neutrinos, making SO(10) GUTs a naturally suitable framework to explain the small neutrino masses through a type-I seesaw mechanism.

3.4.3 Anomaly cancellation

The fact that SO(10) is an anomaly-free is an important group property, that would explain the theoretical origin of the anomaly cancellation in the SM. The gauge anomaly for left-handed fermions in a given representation R of a group with generators T_R^a goes like:

$$\frac{1}{2}\mathrm{Tr}\left(\{T_\mathrm{R}^a, T_\mathrm{R}^b\}T_\mathrm{R}^c\right) = A(\mathrm{R})d^{abc}, \tag{3.48}$$

where d^{abc} is a completely symmetric[5] tensor depending on the representation and $A(\mathrm{R})$ is the anomaly coefficient of R. Since $A(\mathrm{R}_1 + \mathrm{R}_2 + \ldots) = A(\mathrm{R}_1) + A(\mathrm{R}_2) + \ldots$, the peculiar representations in the SM, as well as in SU(5) GUT, are such that this sum is equal to zero. An anomaly-free group, instead, has always $A(\mathrm{R}) = 0$. This property can be understood for SO(10) from a group theoretical argument reported in [94]. Considering the SO(10) generators $T_{ij} = -T_{ji}$ in an arbitrary representation, the anomaly is proportional to the invariant tensor

$$\mathrm{Tr}\{T_{ij}, T_{kl}\}T_{mn}\,, \tag{3.49}$$

which must respect the antisymmetry under the exchanges $i \leftrightarrow j$, $k \leftrightarrow l$, $m \leftrightarrow n$ and the symmetry under the exchange of pairs $ij \leftrightarrow kl$, $kl \leftrightarrow mn$ and $ij \leftrightarrow mn$. As shown in [94] such constraints cannot be respected by any tensor (constructed by Kronecker δ's), forcing the anomaly index to be zero.

[5] As it can be seen from the cyclic properties of the trace.

3.4.4 Symmetry Breaking

There exist several intermediate symmetries through which $SO(10)$ can be broken to $\mathrm{SU(3)_C \times SU(2)_L \times U(1)_Y}$. The maximal subgroups of $SO(10)$ are four [65]:

- $\mathrm{SO(6)\times SO(4) \approx SU(4)\times SU(2)_L \times SU(2)_R}$,
- $\mathrm{SU(5)\times U(1)_X}$,
- SO(9),
- SO(7)×SU(2).

Among these, only through the breaking chains of $\mathrm{SU(4)\times SU(2)_L \times SU(2)_R}$ and $\mathrm{SU(5)\times U(1)_X}$ one can obtain the correct quantum numbers for the SM particle content [36]. For a general model, the presence of these intermediate scales of breaking introduce more uncertainty in some predictions (proton lifetime and $\sin^2\theta_w$) compared to the case of a single step breaking, as in the case of $SU(5)$. However, we focus on supersymmetric GUTs where, as mentioned in the introduction, the MSSM particle content guarantees the unification of couplings at $M_{\rm GUT}$ without the need of intermediate scales. In this scenario we assume that all the necessary breaking steps of SO(10) are realized at or above $M_{\rm GUT}$, while below this scale the gauge group is the SM one and the couplings are distinguished as predicted by the running in the MSSM.
The standard approach is to realize a spontaneous symmetry breaking, in more steps, with a suitable choice of scalar fields. Indeed, the scalars with this role must live in a proper representation of SO(10) which contains a singlet of the subgroup that we want to leave unbroken. The VEV will be aligned in the direction of this singlet.
Let us briefly analyze the two possible breaking chains. While we have to deal with the new higher steps of the breaking, the lower steps, breaking SU(5) or the PS group, have already been discussed in the previous sections for the breaking of these two subgroups, so that we need only to find the SO(10) representations including the wanted PS or SU(5) components.

- **The SU(5) breaking chain**. A first possibility is:

$$\begin{aligned} SO(10) &\xrightarrow{\langle 45\rangle,\langle 210\rangle,} SU(5)\times U(1)_X \\ &\xrightarrow{\langle 16\rangle,\langle 126\rangle} SU(5) \\ &\xrightarrow{\langle 45\rangle,\langle 54\rangle,\langle 210\rangle} SU(3)_C\times SU(2)_L\times U(1)_Y \end{aligned} \tag{3.50}$$

as it can be seen from the fields decompositions under $SU(5)\times U(1)_X$:

$$\begin{aligned} \mathbf{45} &= \mathbf{1_0} + 10_4 + \overline{10}_{-4} + \mathbf{24_0} \\ \mathbf{210} &= \mathbf{1_0} + 5_8 + \overline{5}_8 + 104 + \overline{10}_4 + \mathbf{24_0} + 40_4 + \overline{40}_4 + 75_0 \\ \mathbf{16} &= \mathbf{1_{-5}} + \overline{5}_3 + 10_{-1} \\ \mathbf{126} &= \mathbf{1_{-10}} + \overline{5}_{-2} + 10_{-6} + \overline{15}_6 + 45_2 + \overline{50}_{-2} \\ \mathbf{54} &= 15_4 + \overline{15}_{-4} - 4 - \mathbf{24_0} \end{aligned}$$

where we have highlighted in bold the components getting the VEV and driving the breaking at the different steps. As seen, the 24_0 of SU(5) breaks down to the SM gauge group.

A second possibility is to break directly SO(10) to SU(5) lowering the rank with the component 1_{-5} of **16**, or 1_{-10} of **126**, and then following the same SU(5) breaking down to the SM:

$$\begin{aligned} SO(10) &\xrightarrow{<16>\,<126>,} SU(5) \\ &\xrightarrow{<45>,<54>,<210>} SU(3)_C \times SU(2)_L \times U(1)_Y \end{aligned} \tag{3.51}$$

Finally, one could use directly the $24_0 \in$ **45** for breaking SO(10) to SM$\times$U$(1)_X$ and always a **16** or a **126** to break U$(1)_X$ and lower the rank.

- **The Pati-Salam breaking chain.**
  Also in this case the breaking can be realized with different chains. Let's consider the decompositions of the fields above, now with respect to the PS group:

$$\begin{aligned} \mathbf{45} &= (1,1,3)+(1,3,1)+(15,1,1)+(6,2,2) \\ \mathbf{210} &= (\mathbf{1},\mathbf{1},\mathbf{1})+(15,1,1)+(6,2,2)+(15,3,1)+(15,1,3)+(\overline{10},2,2)+(10,2,2) \\ \mathbf{16} &= (4,2,1)+(\overline{4},1,2) \\ \mathbf{126} &= (6,1,1)+(\overline{10},1,3)+(10,3,1)+(15,2,2) \\ \mathbf{54} &= (\mathbf{1},\mathbf{1},\mathbf{1})+(1,3,3)+(20,1,1)+(6,2,2) \end{aligned}$$

  There most economic chain of breaking is:

$$\begin{aligned} SO(10) &\xrightarrow{<54>,<210>\supset(1,1,1)} SU(4)_C \times SU(2)_L \times SU(2)_R \\ &\xrightarrow{<16>\supset(\overline{4},1,2),<126>\supset(\overline{10},1,3)} SU(3) \times SU(2)_L \times U(1)_Y \end{aligned} \tag{3.52}$$

  though other intermediate steps of breaking can be realized with components of **45** and **210**. Notice that the final breaking is realized with the PS components inside the **16** and **126** already studied before. In this case, the hypercharge Y is given by $Y = T_{3R} + (B-L)/2$, as already pointed out.

Each breaking chain has its characteristic set of fields and a symmetry breaking superpotential must be constructed to account for their VEVs. Here it is implicitly considered that the further EW symmetry breaking is achieved by the SU$(2)_\mathrm{L}$ doublet which are components of **10**, **120** or $\overline{\mathbf{126}}$, or a linear combinations of them, as we are going to discuss (cf. Sect.3.4.5).

3.4.5 Higgs sector and Yukawa couplings

Since all the SM fermion content is included in one **16**, the Yukawa interaction terms should couple the bilinear $\mathbf{16} \times \mathbf{16}$ with a Higgs field in a representation which guarantees

the SO(10) invariance and includes the MSSM weak doublets.
The product of two **16** decomposes as:

$$\mathbf{16}\times\mathbf{16}=\mathbf{10}_s+\mathbf{120}_a+\mathbf{126}_s \tag{3.53}$$

where s and a denote the symmetric and antisymmetric nature of the couplings in the family space, which arises from the group theoretical properties of the corresponding bilinear couplings. At the renormalizable level, the possible representations for the Higgs field are of the three types:

$$\mathbf{10}_H,\quad \mathbf{120}_H,\quad \overline{\mathbf{126}}_H\,. \tag{3.54}$$

It's immediate to see that these representations include the ones already discussed in the SU(5) and the PS subgroups for the Higgs, highlighted in bold in the following decompositions:

$$\begin{aligned}
\mathbf{10}_H &\overset{\mathrm{SU(5)\times U(1)}_X}{=} \mathbf{5_2}+\overline{\mathbf{5}}_{\mathbf{-2}}\\
&\overset{PS}{=} (\mathbf{1},\mathbf{2},\mathbf{2})+(6,1,1)\\[1em]
\mathbf{120}_H &\overset{\mathrm{SU(5)\times U(1)}_X}{=} \mathbf{5_2}+\overline{\mathbf{5}}_{\mathbf{-2}}+10_{-6}+\overline{10}_6+\mathbf{45_2}+\overline{\mathbf{45}}_{\mathbf{-2}}\\
&\overset{PS}{=} (\mathbf{1},\mathbf{2},\mathbf{2})+(10,1,1)+(\overline{10},1,1)+(6,1,3)+(6,3,1)+(\mathbf{15},\mathbf{2},\mathbf{2})\\[1em]
\overline{\mathbf{126}}_H &\overset{\mathrm{SU(5)\times U(1)}_X}{=} 1_{10}+\mathbf{5_2}+\overline{10}_6+15_{-6}+\overline{\mathbf{45}}_{-2}+50_2\\
&\overset{PS}{=} (6,1,1)+(10,1,3)+(\overline{10},3,1)+(\mathbf{15},\mathbf{2},\mathbf{2})
\end{aligned} \tag{3.55}$$

where the SU(5) components **5** and **45** contain the $Y=+1/2$ $\mathrm{SU(2)_L}$ up Higgs doublet, while the SU(5) component $\overline{\mathbf{5}}$ and $\overline{\mathbf{45}}$ contain the $Y=-1/2$ down doublet. In the PS notation, the two doublets are embedded in a $\mathrm{SU(2)_L}$- $\mathrm{SU(2)_R}$ bi-doublet.

Any combination of these three representations can in principle enter the Yukawa couplings. The $\overline{\mathbf{126}}$ representation is particularly interesting because it permits to describe light Majorana neutrinos via a seesaw mechanism either of type I or of type II or by a combination of both. Indeed, $\overline{\mathbf{126}}$ contains also the components $(10,1,3)$ and $(\overline{10},3,1)$ which allow for the seesaw as described in the PS framework (cf. Sect.3.3.1). In terms of SU(5) components of $\overline{\mathbf{126}}$, these roles are played by the singlet 1_{10}, giving Majorana mass to RH neutrinos in type-I seesaw, and by the 15_{-6} giving[6] Majorana mass to LH neutrinos in the type-II seesaw mechanism.
Other models, anyway, utilize non-renormalizable operators to generate RH neutrino masses, with the introduction of a $\overline{\mathbf{16}}_H$:

$$\frac{Y_{16ij}}{\Lambda}\,\mathbf{16}_i\mathbf{16}_j\overline{\mathbf{16}}_H\overline{\mathbf{16}}_H \tag{3.56}$$

[6] This is achieved because 15_{-6} has a $(1,3)_1$ component under $\mathrm{SU(3)_C\times SU(2)_L\times U(1)_Y}$ which couples to two lepton doublets as ${(1,2)_{-1/2}}_L{(1,2)_{-1/2}}_L{(1,3)_1}_H$

where Λ is a cut off scale, say $\Lambda \approx M_{\text{Plank}}$. This is possible because also $\overline{\mathbf{16}}_H$ contains a $SU(5)$ singlet component. In terms of the PS components, this role is played by $(\overline{4},1,2)$ as already pointed out in Sect.3.3.1.

The most general SO(10) renormalizable Lagrangian that we can write for the Yukawa sector looks like:

$$\mathcal{L}_Y = 16_{Fi}\left(Y_{10ij}\mathbf{10}_H + Y_{120ij}\mathbf{120}_H + Y_{126ij}\overline{\mathbf{126}}_H\right)16_{Fj} + \text{h.c.}\,, \tag{3.57}$$

where Y_{10} and Y_{126} are complex symmetric matrices while Y_{120} is complex antisymmetric. From the decomposition with respect to the SM group, we obtain the follow mass matrices [92]:

$$M_u = Y_{10}\left\langle 10^u_{H5}\right\rangle + Y_{120}\left(\left\langle 120_{H5}^{\ u}\right\rangle + \left\langle 120_{H45}^{\ u}\right\rangle\right) + Y_{126}\left\langle \overline{126}^u_{H45}\right\rangle \equiv Y_u v_u \tag{3.58}$$

$$M_d = Y_{10}\left\langle 10_{H\bar{5}}^{\ d}\right\rangle + Y_{120}\left(\left\langle 120_{H\bar{5}}^{\ d}\right\rangle + \left\langle 120_{H\overline{45}}^{\ d}\right\rangle\right) + Y_{126}\left\langle \overline{126}_{H\overline{45}}^{\ d}\right\rangle \equiv Y_d v_d \tag{3.59}$$

$$M_e = Y_{10}\left\langle 10_{H\bar{5}}^{\ d}\right\rangle + Y_{120}\left(\left\langle 120_{H\bar{5}}^{\ d}\right\rangle - 3\left\langle 120_{H\overline{45}}^{\ d}\right\rangle\right) - 3Y_{126}\left\langle \overline{126}_{H\overline{45}}^{\ d}\right\rangle \equiv Y_e v_d \tag{3.60}$$

$$M_\nu^D = Y_{10}\left\langle 10^u_{H5}\right\rangle + Y_{120}\left(\left\langle 120_{H\bar{5}}^{\ u}\right\rangle - 3\left\langle 120_{H\overline{45}}^{\ u}\right\rangle\right) - 3Y_{126}\left\langle \overline{126}_{H45}^{\ u}\right\rangle \tag{3.61}$$

$$M_R = Y_{126}\left\langle \overline{126}_{H1}^{\ u}\right\rangle \tag{3.62}$$

$$M_L = Y_{126}\left\langle \overline{126}_{H15}^{\ u}\right\rangle \tag{3.63}$$

where the VEVs are distinguished by the SU(5) component on the bottom and by the apex u, d referred to the $Y = \pm 1/2$ hypercharge distinguishing the up and down type Higgs of the MSSM.
The first three lines describe the Yukawa sector of the SM, for quarks and charged leptons. Note again the presence of the Georgi-Jarlskog factor -3 differentiating the coupling of leptons from quarks with the 45 component of $\mathbf{120_H}$ and $\overline{\mathbf{126}}_H$: it is the same factor already encountered in the SU(5) unification and in PS relative to the $(15,2,2)_H$, due to a Clebsch-Gordon coefficient emerging from the SM decomposition.
The last three lines, Eqs.(3.61,3.62,3.63), represent the different possible contributions to neutrino masses. M_ν^D is a Dirac mass term, while M_R and M_L are Majorana mass terms for RH and LH neutrinos respectively. M_ν^D, M_R and M_L enter the neutrino mass matrix defined on the basis (ν, ν^c):

$$\begin{pmatrix} M_L & M_\nu^D \\ M_\nu^{DT} & M_R \end{pmatrix}\,. \tag{3.64}$$

As already mentioned in the context of the PS group (cf. Sect.3.3.1) the realization of the type I and type II seesaw requires a huge hierarchy between the VEVs of the fields generating the mass terms. In this SO(10) framework, the VEV $\left\langle \overline{126}_{H1}^{\ u}\right\rangle$ along the SU(5) singlet with non zero U(1)_X is responsible of giving mass to RH neutrinos M_R, but it is also responsible of the rank reduction of SO(10), so that, to preserve the unification, it is assumed $M_R \approx M_{\text{GUT}}$. The VEVs generating M_ν^D are responsible of the EW symmetry

breaking, so it is $M_\nu^D \approx M_{\rm EW} \ll M_R$. Finally, the VEV of the SU(2) triplet $\langle \overline{126}_{H}{}^{u}_{15} \rangle$ is required to be very small [30]: $M_L \approx \mathcal{O}(M_{\rm EW}^2/M_R)$, in analogy to what happens in the left-right PS model.
Therefore, given the hierarchy:

$$M_R \gg M_\nu^D \gg M_L$$

the mass matrix (3.64) can be block-diagonalized, obtaining the following approximated expression for the light neutrino mass matrix:

$$M_\nu = M_L - M_\nu^D M_R^{-1} M_\nu^{D^T} \,, \tag{3.65}$$

that is the seesaw formula with contributions from type-II (first term) and type-I (second term) seesaw.

Let us now consider in more concreteness the possible models. Depending on the Higgs content in the theory, one gets different correlations between the mass matrices of the fermions, as evinced from the mass formulas in Eqs.(3.58-3.63). Among the various possibilities, we should look for the most economic scheme, in order to have a more predictive model. What are then the minimal combinations of Higgs representations needed to have a realistic theory? A minimal model, where only one copy of the $\mathbf{10}_H$ is included, is completely unrealistic. It would describe equal masses for quarks and leptons, up to an overall factor distinguishing $\pm 1/2$ components of the weak isospin doublets, and no mixing. Indeed, this is the same problem we encountered before by means of only $5+\overline{5}$ in SU(5) and (1,2,2) in the PS group. In general, even with the other representations, one single Yukawa matrix can always be diagonalized by rotating the $\mathbf{16}_F$, without thus accounting for the fermion mixings. Therefore, at least two Higgs representations are necessary. Two equal representations are forbidden, because they would predict unrealistic relations between the masses: $M_d = M_e$ with two $\mathbf{10}_H$, $M_d = -3M_e$ with two $\overline{\mathbf{126}}_H$ and $m_1 = 0$, $m_2 = -m_3$ in case of two $\mathbf{120}_H$, due to the antisymmetry of the Y_{120}.
Therefore we conclude that at least two *different* combinations of Higgs representations are necessary. Notice that, in absence of $\overline{\mathbf{126}}_H$, neutrinos would be Dirac and their masses would be unrealistically related to charged leptons. To solve this problem, in models without $\overline{\mathbf{126}}_H$ we have to introduce the representation $\overline{\mathbf{16}}_H$, which plays the role of $\overline{\mathbf{126}}_H$ in the seesaw mechanism. We thus construct a Majorana mass for RH neutrinos through the non-renormalizable operator in Eq.(3.56). On the other side, it is possible to have models where the $\overline{\mathbf{126}}_H$ takes VEV only in some particular directions, for example contributing only to the neutrino sector and not to the one of charged fermions (see [95, 96] or our application in [16]).
Non-minimal renormalizable models, where various combinations of Higgses in $\mathbf{10}$, $\mathbf{120}$ and $\overline{\mathbf{126}}$ representations are introduced, have been shown to fit the fermion mass data well [97–109].
The most compelling and studied choice for the Higgs sector is given by the couple

$\mathbf{10}_H + \overline{\mathbf{126}}_H$, which is the most economic framework including the useful properties of $\overline{\mathbf{126}}_H$ in the neutrino sector. The SUSY model based on this couple of Higgs representations goes under the name of minimal renormalizable supersymmetric SO(10) (MSGUT). The alternative $\mathbf{10}_H + \mathbf{120}_H$ is also possible but, as mentioned, it needs to account for neutrino masses via non renormalizable operators. Furthermore, the antisymmetric structure of Y_{120} counts less free parameters so the fit of charged fermion masses and mixing is more constrained. It has been shown that this kind of model cannot reproduce a realistic scenario of flavour, because it predicts either m_d or m_t unrealistically small [96].

The minimal model: $\mathbf{10}_H + \overline{\mathbf{126}}_H$

The basic structure of the minimal SO(10) MSGUT, based on the $\mathbf{10}_H + \overline{\mathbf{126}}_H$ Higgs representations, was first studied by Clark, Kuo and Nakagawa [110] and Aulakh and Mohapatra [111] in the 80's. However a lot of attention has been dedicated to this model also in more recent years [100, 101, 112–119] due to the observation [120] that the dominance of type-II seesaw can explain the large atmospheric mixing in the leptonic sector through a correlation with the b-τ unification, which is verified within $20-30\%$ corrections in the MSSM [62].

The model is characterized (in addition to $\mathbf{10}_H + \overline{\mathbf{126}}_H$) by the presence of $\mathbf{126}_H$ and $\mathbf{210}_H$ representations in the symmetry breaking sector. $\mathbf{126}_H$ is needed to preserve the D-flatness[7] while $\mathbf{210}_H$ plays the dual role of triggering the second spontaneous breaking, from SU(5) to the SM, and provides the necessary mixing among the $\mathbf{10}_H$ and $\overline{\mathbf{126}}_H$ weak doublet components in the superpotential. Notice that the choice of $\mathbf{210}_H$ for the group breaking, instead of the simpler representations **45** or **54**, is necessary for this mixing since the decomposition of the product $\mathbf{10}\times\overline{\mathbf{126}} = \mathbf{210}+\mathbf{150}$ allows to construct a singlet only with $\mathbf{210}_H$. The tiny VEV of the $\mathrm{SU(2)_L}$ triplet component of $\overline{\mathbf{126}}_H$, responsible for the type-II seesaw, is induced via $\mathbf{210}_H$ couplings as well[8].

Let us focus on the Yukawa sector of the model. The mass matrices resulting from

[7] Indeed, the VEV of $\overline{\mathbf{126}}_H$ leads to a non-vanishing D-term which breaks SUSY at high scale. SUSY can be preserved up to the TeV scale by adding the $\mathbf{126}_H$ and assuming $\langle\mathbf{126}_H\rangle = \langle\overline{\mathbf{126}}_H\rangle$.

[8] The superpotential of the $SO(10)$ Higgs sector is described by [121]:

$$\begin{aligned} W_H &= \frac{M_{210}}{4!}\mathbf{210}_H^2 + \frac{\lambda}{4!}\mathbf{210}_H^3 + \frac{M_{126}}{5!}\mathbf{126}_H\overline{\mathbf{126}}_H \\ &+\frac{\eta}{5!}\mathbf{126}_H\mathbf{210}_H\overline{\mathbf{126}} + M_{10}\mathbf{10}_H^2 \\ &+\frac{1}{4!}\mathbf{210}_H\mathbf{10}_H\left(\alpha\mathbf{126}_H + \overline{\alpha}\overline{\mathbf{126}}_H\right). \end{aligned}$$

in terms of 7 independent complex parameters. The minimization of this scalar potential has been analyzed in Refs. [116, 119]

Eqs.(3.58-3.63), without the $\mathbf{120}_H$, are:

$$M_u = Y_{10}\left\langle 10^u_{H5}\right\rangle + Y_{126}\left\langle \overline{126}^u_{H45}\right\rangle \equiv Y_u v_u \tag{3.66}$$
$$M_d = Y_{10}\left\langle 10_{H\bar{5}}^{d}\right\rangle + Y_{126}\left\langle \overline{126}_{H\overline{45}}^{d}\right\rangle \equiv Y_d v_d \tag{3.67}$$
$$M_e = Y_{10}\left\langle 10_{H\bar{5}}^{d}\right\rangle - 3Y_{126}\left\langle \overline{126}_{H\overline{45}}^{d}\right\rangle \equiv Y_d v_d \tag{3.68}$$
$$M_\nu^D = Y_{10}\left\langle 10^u_{H5}\right\rangle - 3Y_{126}\left\langle \overline{126}_{H\overline{45}}^{u}\right\rangle \tag{3.69}$$
$$M_R = Y_{126}\left\langle \overline{126}_{H1}^{u}\right\rangle \tag{3.70}$$
$$M_L = Y_{126}\left\langle \overline{126}_{H15}^{u}\right\rangle \tag{3.71}$$

Y_{10} and Y_{126} are both 3×3 complex symmetric matrices. Thus, they consist of 6 complex parameters each, for a total of 24 real parameters. Not all of these are physical parameters, but some of them can be absorbed by a rotation of the $\mathbf{16}_F$ matter field[9] chosen for diagonalize, for example the Y_{10} matrix, to which only 3 complex parameters remain associated. Also 3 imaginary phases can be reabsorbed in a phase transformation for each generation of the $\mathbf{16}_F$. The flavour sector in the end is described by a total of 15 parameters. With these parameters it is possible to fit the 18 observables: 9 charged fermion masses, 2 neutrinos mass difference, 3 mixing angles and 1 phase of V_{CKM}, 3 mixing angles of U_{PMNS}. The fit must also involve the parameters in the scalar superpotential, which are responsible of determining the VEVs $v^R = \left\langle \overline{126}_{H1}^{u}\right\rangle$ and $v^L = \left\langle \overline{126}_{H15}^{u}\right\rangle$. Without going into the details, we should report that, allowing an arbitrary superpotential for the Higgs sector, it is possible to obtain a good fit, as verified in various papers [100–102, 113, 115]. The best fits are obtained in a mixed type-I and type-II seesaw scenario, where anyway the type-II is highly sub-dominant with respect to the type-I. However, when the constraints required from gauge unification and proton decay are taken into account, the MSGUT is not able to fit the fermions mass data anymore, failing in the neutrino sector [95, 118, 119]. This is verified with generic combination of type-I and type-II seesaw [102]. Such an incompatibility emerges because the supersymmetric unification forces the seesaw scale to be $\approx M_{\mathrm{GUT}}$, giving a neutrino mass scale smaller than the one required by the atmospheric neutrino oscillation.

While the minimal model can remain a valid framework for non-supersymmetric theories [122–124], the SUSY version needs a suitable extension, resorting a non-minimal Higgs sector. Note that the use of the couple $\mathbf{120}_H + \overline{\mathbf{126}}_H$, replacing the $\mathbf{10}_H$ with $\mathbf{120}_H$, would only worsen the situation because the antisymmetric Yukawa Y_{120} has a reduced number of free parameters with respect to Y_{10} and it leads to the prediction $m_\tau \approx 3m_b$ at MGUT , far from $m_\tau \approx m_b$ predicted in the MSSM. This possibility can in principle work in the non-supersymmetric SO(10), as inquired in [122].

Therefore the most simple choice for extending the MSGUT is to add a $\mathbf{120}_H$ besides the minimal couple $\mathbf{10}_H + \overline{\mathbf{126}}_H$.

[9] The unified description of all the fermionic matter fields inside a **16**, allows to do only one field rotation, differently than the SM case.

The non minimal model: $\mathbf{10}_H + \mathbf{120}_H + \overline{\mathbf{126}}_H$

Adding a $\mathbf{120}_H$ Higgs implies to have new free parameters both in the Yukawa sector and in the scalar potential. At the price of having a less simple model, in this way one hopes to relax the constraints on the parameter space of the MSGUT. Since the $\mathbf{120}_H$ can't have any role in the symmetry breaking from SO(10) down to the SM gauge group, the breaking sector must be driven again from $\overline{\mathbf{126}}_H$+$\mathbf{126}_H$ and $\mathbf{210}_H$, as already established in MSGUT. The superpotential of the Higgs sector responsible of the breaking in this cases mixes[10] also the $\mathbf{120}_H$. In this model, the mass formulas are given by Eqs.(3.58-3.63) in their full content. This non minimal model has been shown to give excellent fit of the fermion masses and mixing with general combination of type-I or type-II seesaw [103, 104, 125, 126], better than the minimal model, with a preference for type-I seesaw dominance. To reduce the parameter space, some models have been constructed with additional assumptions, as done for example in [127] with a parity symmetry or in [103] via spontaneous CP violation.

3.4.6 Doublet-Triplet splitting problem

Let us briefly mention at this point the problems of Doublet-Triplet splitting and proton decay. As already seen in the SU(5) SUSY framework, the Higgs representations $\mathbf{10}_H$, $\mathbf{120}_H$ and $\overline{\mathbf{126}}_H$ contain, besides the MSSM Higgs doublets, also dangerous colored triplets that can mediate the proton decay via dimension-5 operators. Some possible approaches to face this problem have already been listed in the SU(5) context (cf. Sect.3.2.5) and are applicable in the SO(10) framework as well, with suitable choice of the SO(10) representations. It is worth mentioning that in SO(10) another possibility of D-T splitting has been proposed: the Dimopoulos-Wilczek mechanism [74]. In this case the mass splitting is achieved by using the VEV of a 45_H at $M_{\rm GUT}$, responsible of breaking SO(10)$\to SU(5) \times U(1)_X$, specifically aligned in a direction that gives mass only to the triplet. Different implementation of the same mechanism are made in [128, 129]. Detailed calculations have shown, nevertheless, that constraints from proton decay require the effective M_T to be larger than $M_{\rm GUT}$ by at least one order of magnitude. This in turn requires some unnatural condition on the couplings [36].

3.4.7 Facing the flavour puzzle in 4D SO(10): overview

In Sect.3.4.5 we have pointed out the structure of the most general Yukawa Lagrangian for an SO(10) model, Eq.(3.57), and we have analyzed the viability of some concrete

[10] The additional terms with respect to MSGUT are:

$$M_{120}\mathbf{120}_H^2 + \frac{\gamma}{4!}\mathbf{10}_H\mathbf{120}_H\mathbf{210}_H + +\frac{\eta'}{4!}\mathbf{210}_H\mathbf{120}_H\mathbf{120}_H + \frac{1}{4!}\mathbf{210}_H\mathbf{120}_H(\alpha'\mathbf{126}_H + \beta'\overline{\mathbf{126}}_H)$$

models. Despite the particular choice for the representations, a large number of free parameters, entering the Yukawa matrices, are needed to describe the flavour sector. As verified in typical SO(10) GUT models [36, 108–112, 117, 121, 125, 126, 130–151] the best fit parameters of the Yukawa couplings span several orders of magnitude, varying from $\mathcal{O}(10^{-6})$ to $\mathcal{O}(1)$. In general, in these models there is no advantage or qualitative difference with respect to the SM in describing the flavour sector, as pointed out by several dedicated works, see for examples [97–107].
To have a theory of flavour in SO(10) GUT, explaining the origin of the characteristic masses and mixing angles is a very challenging issue. In fact, with respect to the SM, where the masses and mixing matrices can be studied and explained separately for the various fermions, in SO(10) we have to deal with the difficulty of the unified description of fermions. Anyway, despite the large number of free parameters, the fact that SO(10) provides an intrinsic relation between the various Yukawa matrices (absent in the pure SM) is by itself an interesting feature which puts the problem of flavour in a new perspective. Some convenient aspects in this sense, are, for example, the minimal prediction $M_d \approx M_e$ underlined by the SU(5) subgroup and the prediction for large θ_{23} from type II seesaw dominance in the MSGUT.

In a way similar to what already described in section 2.2, regarding the flavour puzzle within the SM, a standard approach to improve the flavour sector consists in establishing the correlations among the masses and mixing parameters by imposing additional assumptions on the texture of the Yukawa matrices, through a class of horizontal symmetries in the flavour space. It is assumed to work in the framework SO(10)$\times G_f$, where G_f is a flavour symmetry group. A lot of attempts have been done in this respect, considering different possible G_f. Let us summarize the main results of these approaches, while for a complete review the reader can refer to [36, 37].

- Models with continuous G_f and lop-sided/asymmetric mass textures, which arise when SO(10) is broken down to the SM gauge group through the SU(5) chain. From the SU(5) relation, $M_d = M_e^T$, there is the possibility of explaining large leptonic mixing angles due to the large mixing induced in the Dirac neutrino mass matrix. Examples of realistic models are realized introducing a U(1) family symmetry [135, 136, 140, 141, 152], a U(2) family symmetry [153–155] and a SU(3) or SO(3) family symmetry [144, 156–159].

- Models with continuous G_f and symmetric mass textures, which naturally arise when SO(10) is broken through the PS symmetry breaking chain. Examples of this kind of models are based on SU(2) family symmetry [160, 161]. Due to the left-right symmetric nature, this type of models tends to be more predictive compared to the previous ones.

- Models with discrete flavour symmetries. In this case permutation groups are used for explaining and predicting the pattern of fermion masses and mixing, especially in the lepton sector. Most of the models with this approach account for a description of the lepton mixing matrix U_{PMNS} close to the tribimaximal structure. In the SO(10) context this realization is non trivial, because the existent symmetry must be badly broken in the quark sector. Examples of models of this kind are [132, 146–148, 162–165], based on various discrete symmetries like A_4, S_4, Z_2 and others, or combinations of them.

The various models have very different predictions regarding some observables that we are (or were) waiting to measure. Such predictions permit to easily rule out the non compatible models as soon as new measurements are available. For example, the measure of a non vanishing $\sin^2\theta_{13}$ and of more precise solar and atmospheric neutrino mixings [166–168] has already ruled out some minimal models based on discrete flavour symmetries. Other predictions concern the CP violating phase, which arises in these models from the complex phases in the VEVs of the scalar fields and from the phases in the Yukawa couplings.

The general advantage of the approach based on flavour symmetries is the reduction of free parameters, being the Yukawa structure is constrained by the symmetry. This gives more predictive power to the models, but there are two general drawbacks: on one side, we are introducing by hand a new symmetry without a well identified origin, and, on the other side, we need to construct complicated frameworks for the suitable breaking of this symmetry.

Another approach to address the flavour problem is based, on the contrary, on the assumption of structureless Yukawa matrices. In this case one aims to make the theory more natural, by means of no particular choice of the flavour parameters, taking Yukawas as made of order 1 random entries. Of course, a realistic scenario must include some alternative mechanism that, underlying the anarchical structure, reproduces the hierarchies observed in the fermion mass data. As already mentioned in the first chapter, a mechanism that can play this role is based on the existence of extra dimensions (ED), where the the diversified masses and mixing arise from different localizations of the fermions in the ED. While the application of this approach to the SM is quite trivial and very well studied [169–174], the implementation in the unified framework of SO(10) GUT is much more challenging. This issue is the main subject of our works and will be presented in details in the following chapters. The advantage of this approach is reducing the range and the fine-tuning in the fundamental Yukawa parameters, making the theory much more natural. On the other side, as disadvantage, this approach reduces the predictive power of the models, since all the predictions are made within the uncertainty of order 1 parameters.

The next chapter is dedicated to explain the technical framework of ED and of SUSY in ED. The following chapters 5 and 6 are dedicated to the implementation of these tools constructing realistic models in the 5D SO(10) SUSY framework.

Chapter 4

Extra Dimensions

Extra Dimensions (ED) are an interesting tool that can address various problems in particle physics. The original idea of extending the space time with ED was introduced by Kaluza and Klein in the context of unification of gravity with the other interactions [175, 176], a concept that remains today at the basics of string theory. Many features of ED have been elaborated following the developments in string theory, introducing concepts like: compactification on different types of geometry characterized by possible symmetries, use of different types of metric, existence of special points like the "branes" of the ED, different localization of matter and gauge degrees of freedom, just to mention some examples. As outcome, ED have become a rich conceptual and mathematical framework where to reconsider in a novel approach various open questions in BSM physics (see [177] for a general review). Many model building possibilities, indeed, come by playing with the features of ED, which can reproduce different phenomenology at the 4D effective level. A well known application comes from theories where the SM fields are localized on a brane, while gravity is free to propagate in the ED, addressing the hierarchy problem by means of a large ED [178, 179] or a small warped ED [180]. Other theories involve the SM fields propagating in the ED, leading to some interesting possibilities like the localization of the profiles [181, 182]. A mass source in the bulk, for example, can modulate exponentially falling profiles for the fermions, with the possibility to generate different localization with respect to the Higgs field: a feature that can be used to explain hierarchical fermion masses in various scenarios [169, 170, 183–185].
In the present work we focus on the non-gravitational framework and we exploit two main tools offered by a compactified ED:

- Introduction of one or more Z_2 symmetries on the ED as new instrument for symmetry breaking (gauge symmetry or SUSY);
- Creation of hierarchical profiles for the matter fields, as new instrument to motivate the existence of hierarchical Yukawa couplings with the Higgs, leading to a natural realization of the flavour scheme, where the fundamental Yukawas are anarchical and of order 1, in units of the typical energy scale of the theory.

This chapter is intended to give a brief review of the basic formalism of ED, introducing the description of gauge fields, scalars and fermions in 5 dimensions, showing how the Kaluza-Klein profiles and in particular the hierarchical 0-mode profiles arise.

4.1 General remarks and notation

We assume to work in 5 dimensions (5D) with one ED of space type. We refer to the ED with the coordinate y or x^5, while x^μ are the usual 4 space-time coordinates. We use the latin index $M = 1, \ldots, 5$ to indicate the 5 coordinates $\{x^M\} \equiv \{x^0, x^1, x^2, x^3, y\}$ (or, in short notation $\{x, y\}$), while the greek index $\mu = 0, \ldots, 3$ for the first 4 space-time coordinates. Any field that is allowed to propagate into the ED is a 5-dimensional fields and depends on (x, y), while a field localized on a specific point of the ED is 4-dimensional, depending only on (x).
We assume the metric to be flat, given by:

$$g_{MN} = (1, -1, -1, -1, -1) \tag{4.1}$$

As any 4-dimensional field theory is constructed on the basilar principle of invariance under the Lorentz group SO(1, 3), a 5-dimensional theory is generally formulated assuming the invariance under the extended Lorentz group SO(1, 4). Notice that a compactification of the ED is by itself responsible of breaking the extended Lorentz symmetry. A general 5D covariant description should be however recovered in the limit of decompactification[1] (for example, on a circle, taking the radius $R \to \infty$). On the other side, some special properties of the compactified ED, as the orbifolding (see next section), explicitly break SO(1,4) and can give rise to explicitly non covariant terms in the Lagrangian along the direction of the 5th coordinate (this happens, for example, on the branes of an orbifold).

4.2 Compactification

The existence of ED is constrained by the evidence that nature is correctly described by a formulation in 4-dimensional space-time, at least within the actual limits of experimental observations. Therefore, for a realistic description of ED, we have to require the ED to be somehow "hidden" with respect to the ordinary distance scale tested by the experiments. In terms of particles phenomenology, this means to set the scale of the ED at least beyond the actual range of validity tested for the SM. One then assumes the ED to be *compactified* on a geometric structure with a characteristic compactification scale R that respects such experimental bounds. The most stringent constraints come from flavour-violating

[1] A common way to describe the compactification is through spontaneous breaking of the space-time symmetries by the VEV of a scalar field. Before getting this VEV, the formulation of the theory should respect the symmetries.

processes. With the SM gauge group, this bound is found to be[2] $\frac{1}{R} \gtrsim 10^3$ TeV [183, 186] for models with flat metric, and $\frac{1}{R} \gtrsim 50$ TeV [43, 172, 187] for models with a warped metric [180]. For the use we are going to do in SO(10) GUT, we will assume the compactification scale $1/R \gtrsim M_{\mathrm{GUT}}$, well above the minimal bounds.

While the fundamental theory at the compactification scale is explicitly described in terms of 5D, at lower energy we have an effective theory in 4D, which is affected by the characteristics of the ED once we integrate it out.
In particular, the 5D Lagrangian density is related to the 4D one by:

$$\mathcal{L}_{4D}\left(\phi(x), \partial_\mu \phi(x)\right) = \int_L dy\, \mathcal{L}_{5D}\left(\Phi(x,y), \partial_M \Phi(x,y)\right);$$

so that, for the action:

$$\mathcal{S}_{5D} = \int dx \int dy\, \mathcal{L}_{5D};$$

Where y, in general, is constrained on an interval of finite length L.

4.2.1 Circle compactification

We can compactify the ED dimension on a circle, that is an interval $[0,\, 2\pi R]$ with periodic boundary conditions on the boundaries: $y = 0 \sim y = 2\pi R$. In this case, for a generic field $\Phi(x,\, y)$ the simplest BC's are the periodic ones[3]:

$$\Phi(x,0) = \Phi(x, 2\pi R)$$

Equivalently, we can obtain a circle from an infinite extra dimension ($-\infty < y < \infty$) imposing the translation symmetry:

$$\tau : y \to y + 2\pi R$$

The periodic boundary condition chosen for the fields read:

$$\Phi(x,\, y) = \Phi(x,\, y + 2\pi R), \tag{4.2}$$

4.2.2 Orbifold compactification

In general, "orbifolding" is a procedure of reducing the fundamental domain of the theory by a set of identifications of the geometric manifold. A way to do this on the line $\mathbb{R}$ is through the discrete Z_2 symmetry, defined by the action on the 5th coordinate:

$$Z_2 :\, y \to -y$$

[2]Note that these bounds are quite flexible according to the level of anarchy assumptions on the Yukawa couplings. See [186] for a review of the problem.

[3]We leave to a further discussion the case of non-trivial periodicity of the fields, like in the Scherk-Schwarz compactification (cf. Sect.7.2).

that maps the line into the half-line: $\mathbb{R} \to \mathbb{R}/Z_2$.
The S^1/Z_2 orbifold is constructed applying Z_2 to the circle. Parameterizing the circle with y values from $-\pi R$ to πR , the y domain of the S^1/Z_2 orbifold extends in the interval $[0, \pi R]$. It's important to stress that the endpoints of the orbifold ($y = 0,\, \pi R$) *do not* transform under Z_2 and hence are called *fixed* points; moreover, they are not identified with each other, unlike the endpoints of S^1.
We can do a further reduction of the fundamental domain by adding a second Z_2' symmetry acting as:

$$Z_2' : \, y' \to -y' \quad \text{with} \quad y' \equiv y - \frac{\pi R}{2}$$

The $S^1/(Z_2 \times Z_2')$ orbifold is constructed applying both Z_2 and Z_2' to the circle and is described in the interval ($y = 0,\, \frac{\pi R}{2}$) .

Introducing these particular discrete symmetries, that are parity transformations, we have to assign the transformation properties of the fields, in order to guarantee the invariance of the Lagrangian $\mathcal{L}_{5D}$. For a generic field we have:

$$\Phi(x, -y) = P\, \Phi(x,\, y), \quad \Phi(x, -y') = P'\, \Phi(x,\, y') \tag{4.3}$$

with $P = \pm 1$ and $P' = \pm 1$, the parity assignments with respect to Z_2 and Z_2', being independent from each other.

The profiles of the fields along the ED are derived as solutions of the equations of motion (e.o.m.) that must respect the boundary conditions given by the above parities. The assignment of these parities is used to make some profiles vanishing and it becomes a useful tool for reproducing the fermions chirality (see later) and to perform the breaking of some symmetries. We'll inquire later about the crucial role of these discrete symmetries within the description of our specific models.

4.3 Kaluza-Klein modes and profiles

The fact of constraining the ED on a compactified structure with periodic boundary conditions has the effect of selecting discrete stationary waves, in correspondence with the quantized 5^{th}-component of the momentum. This set of wave functions can be used as basis for an expansion of the field along y. A 5-dimensional field, that represents a particle allowed to propagate in the extra dimension, will therefore come with a whole "tower" of quantized momenta that, from the 4D point of view, appear like copies of the same particle with different masses in a given discrete spectrum, the so-called Kaluza-Klein (KK) excitations of the particle.
We thus consider the general decomposition of any 5D field:

$$\Phi(x, y) = \sum_{n=-\infty}^{+\infty} \phi^{(n)}(x) f_n(y) \tag{4.4}$$

that we call "KK expansion". The dependence on y and x is factorized in the profiles $f_n(y)$, and in the 4D fields $\phi^{(n)}(x)$, the KK modes. The profiles $f_n(y)$ describe the localization of the particle in the ED and are given by solving the e.o.m., in the procedure of "KK reduction", and imposing the boundary conditions dictated by the Z_2 symmetries and eventual boundary terms.

4.3.1 Scalars and general procedure for KK reduction

Let us consider the case of a scalar field as example to illustrate the KK reduction.
The starting point is always the 5D action, that for a scalar in the bulk is:

$$S_{5D}^{bulk} = \int d^4x \int dy \; \left(\partial^M \Phi^\dagger \partial_M \Phi - M^2 \Phi^\dagger \Phi\right) \tag{4.5}$$

$$= \int d^4x \int dy \; \left(\partial^\mu \Phi^\dagger \partial_\mu \Phi - \partial_5 \Phi^\dagger \partial_5 \Phi - M^2 \Phi^\dagger \Phi\right) \tag{4.6}$$

where M is a 5D bulk mass. Note that, in this 5D framework, the scalars have dimension $[\Phi] = 3/2$ in mass units. We plug in the expansion (4.4), obtaining:

$$S_{5D}^{bulk} = \int d^4x \int dy \; \sum_{m,n} \left[\partial^M \phi^{(n)^*} \partial_M \phi^{(m)} f_n^*(y) f_m(y) - M^2 \phi^{(n)^*} \phi^{(m)} f_n^*(y) f_m(y)\right] \tag{4.7}$$

and then, after integrating over the extra dimension, we *require* to obtain the 4D action:

$$S_{4D} = \int d^4x \; \sum_n \left[\partial^\mu \phi^{(n)^*} \partial_\mu \phi^{(n)} - m_n^2 \phi^{(n)^*} \phi^{(n)}\right] \tag{4.8}$$

that means we are imposing to get, in the effective 4D framework, the infinite set of 4-dimensional KK modes $\phi^{(n)}$ with mass m_n.
Comparing Eq.(4.7) and (4.8) we get two relations for the profiles:

(1) the orthonormality condition:

$$\int dy \; f_n^*(y) f_m(y) = \delta_{mn} \tag{4.9}$$

where the integral is taken over the interval $[-\pi R, \pi R]$ in the circle compactification.

(2) the differential equation in the bulk:

$$\partial_5^2 f_n(y) + \left(m_n^2 - M^2\right) f_n(y) = 0. \tag{4.10}$$

A general solution of (4.10) is:

$$f_n(y) = N_1 \, e^{i\sqrt{m_n^2 - M^2}\, y} + N_2 \, e^{-i\sqrt{m_n^2 - M^2}\, y}$$

with $N_{1,2}$ coefficients to be determined from Eq.(4.9) and the boundary conditions. For a generic mass m_n, the periodic condition $f_n(y) = f_n(y + 2\pi R)$ gives the mass spectrum:

$$m_n^2 = M^2 + \frac{n^2}{R^2}, \qquad n = 1, 2, ... \tag{4.11}$$

and the explicit form of f_n becomes

$$f_n(y) = N_1 e^{i n \frac{y}{R}} + N_2 e^{-i n \frac{y}{R}} \tag{4.12}$$

that is the basis of a Fourier expansion.
Note that, in presence of a non vanishing bulk mass, there is no solution for a massless mode (0-mode), while, in absence of M, the 0-mode exists and it has a flat (constant) profile.

Orbifolding

If we impose a Z_2 parity, working in the S^1/Z_2 orbifold, we have further boundary conditions (BC) to respect, that come from integrating the e.o.m. around the boundaries:

- for Z_2-even fields:

$$\partial_5 f_n(y) = 0 \quad \text{at} \quad y = 0, \pi R \quad \text{(Neumann BC)} \tag{4.13}$$

- for Z_2-odd fields:

$$f_n(y) = 0 \quad \text{at} \quad y = 0, \pi R \quad \text{(Dirichlet BC)} \tag{4.14}$$

As consequence of these BC, only half of the set of KK modes (4.12) survive, with correspondent profiles given by:

- for Z_2-even fields:

$$f_0(y) = \frac{1}{\sqrt{2\pi R}} \quad \text{and} \quad f_{n>0}(y) = \frac{1}{\sqrt{\pi R}} \cos(\frac{n\, y}{R}) \tag{4.15}$$

- for Z_2-odd fields:

$$f_0(y) = 0 \quad \text{and} \quad f_{n>0}(y) = \frac{1}{\sqrt{\pi R}} \sin(\frac{n\, y}{R}) \tag{4.16}$$

Since the orbifold has two special fixed points at the branes, we can modify the action adding some boundary terms defined in the 4D space and exactly localized at the branes. For example we can add two mass terms at the boundaries and the action becomes:

$$S_{5D} = S_{5D}^{bulk} - \int d^4x \int dy \; \left[\delta(y) M_1 \Phi^\dagger \Phi - \delta(y - \pi R) M_2 \Phi^\dagger \Phi\right] \tag{4.17}$$

These two extra terms modify the e.o.m. for the profiles as:

$$\left[\partial_5^2 + \left(m_n^2 - M^2\right) - \delta(y) M_1 - \delta(y - \pi R) M_2\right] f_n(y) = 0 \tag{4.18}$$

and contribute as BC. Integrating Eq.(4.18) around the boundaries, in the intervals $[-\epsilon, +\epsilon]$ and $[\pi R - \epsilon,\, \pi R + \epsilon]$ we get the new BCs:

- for Z_2-even fields:

$$\partial_5 f_n(y) - \frac{M_1}{2} f_n(y) = 0 \qquad \text{at} \qquad y = 0 \tag{4.19}$$

$$\partial_5 f_n(y) + \frac{M_2}{2} f_n(y) = 0 \qquad \text{at} \qquad y = \pi R \tag{4.20}$$

- for Z_2-odd fields, as before: $f_n(y) = 0 \quad \text{at} \quad y = 0, \pi R$

Considering the general solutions (4.10), by imposing these new BC, for a Z_2-even field we can find a particular solution corresponding to a massless 0-mode. Indeed, for $m_n = 0$, Eq.(4.10) becomes:

$$f_n(y) = N_1 e^{My} + N_2 e^{-My}; \qquad 0 \leq y \leq \pi R$$

and the BCs (4.19,4.20) imply that a non vanishing solution for the Z_2-even field exists only if:

$$M_1 = \pm 2M \quad \text{and} \quad M_2 = \mp 2M \tag{4.21}$$

leading, to:

$$f_0(y) \;=\; N_1 e^{M|y|}; \qquad \text{if } M_1 = 2M,\ M_2 = -2M \tag{4.22}$$

$$f_0(y) \;=\; N_2 e^{-M|y|}; \qquad \text{if } M_1 = -2M,\ M_2 = 2M. \tag{4.23}$$

The normalization factors with respect to the interval $[-\pi R, \pi R]$ are:

$$N_1 = \sqrt{\frac{M}{e^{2M\pi R} - 1}}; \qquad N_2 = \sqrt{\frac{-M}{e^{-2M\pi R} - 1}}.$$

The existence of the scalar 0-mode, only for the Z_2-even fields, in presence of a bulk mass M is thus conditioned by the presence of brane mass terms which must be of opposite sign and equal to twice the bulk mass. This "fine-tuned" solution for the 0-mode is of crucial importance for the existence of a 0-mode profile in presence of SUSY, as we will show later.

4.3.2 Fermions

Allowing fermions to propagate in the ED, we have to reconsider the spinorial representations of the extended Lorentz group in the extra space. These representations may be not chiral, leading to a problem of undefined chirality in 4D. In the 5D case, reviewed in appendix C, the simplest irreducible representation of spin-1/2 is a 4-components Dirac spinor, so that we cannot distinguish between the 2 chiral fields described by 2-components Weyl spinors as usual in 4D.

To reproduce correctly the SM as effective theory of a 5D theory, we must restore the concept of chirality in 5D, since the SM is a chiral theory, *i.e.* it distinguishes the transformations of fermions under the gauge group according to the chirality.

The solution comes from the orbifold compactification. Essentially, we replace the chirality by assignment of opposite Z_2-parities to the wanted chiral components of the fields. To see how this works, we have to perform the KK reduction as seen above for scalars, arriving to determine the fermion profiles of the 0-mode and the higher KK modes.
The starting point is the 5D action:

$$S_5D = \int d^4x \int_0^{2\pi R} dy\, \bar{\Psi} \left(i\Gamma^M \partial_M - \hat{M} \right) \Psi \tag{4.24}$$

$$= \int d^4x \int_0^{2\pi R} dy\, \bar{\Psi} \left(i\gamma^\mu \partial_\mu - \gamma_5 \partial_5 - \hat{M} \right) \Psi \tag{4.25}$$

where Γ^M are the Dirac matrices in 5 dimensions (see appendix C) and $\hat{M}$ is a 5D bulk mass parameter. The 5-dimensional spinorial field Psi has the usual KK decomposition:

$$\Psi \equiv \Psi(x, y) = \sum_{n=-\infty}^{\infty} \psi^{(n)}(x) f_n(y) \tag{4.26}$$

with dimension $[\Psi] = 2$ in mass units. The Z_2-parity assignment for distinguishing the chirality has to guarantee the invariance of the Lagrangian. Since the term $\bar{\Psi}\left(\gamma_5\partial_5 + \hat{M}\right)\Psi$ mixes the L and R chiralities of Ψ, and taking into account the odd transformation $\partial_5 \to -\partial_5$, the products $\bar{\Psi}_L\Psi_R$ and $\bar{\Psi}_R\Psi_L$ must be Z_2-odd, that means Ψ_L and Ψ_R must have opposite Z_2 parity.
As consequence, to preserve the full Z_2 invariance, the 5D mass term $\hat{M}$ must be Z_2-odd as well, so we redefine it as $\hat{M} \equiv Msgn(y)$, with M a real constant.
The chiral mixing terms in the Lagrangian tell us that the profiles of the chiralities will be correlated.
From KK reduction procedure, we find the usual orthonormality condition (4.9) and the following e.o.m. for the profiles:

$$(-\partial_5 + M\mathrm{sgn}(y))\, f_{nR}(y) = m_n f_{nL}(y) \tag{4.27}$$

$$(+\partial_5 + M\mathrm{sgn}(y))\, f_{nL}(y) = m_n f_{nR}(y) \tag{4.28}$$

For this system of equations, there are simple solutions for the 0-mode, with $m_n = 0$:

$$f_{0L}(y) = N_L e^{-M|y|}; \qquad -\pi R \leq y \leq \pi R \tag{4.29}$$

$$f_{0R}(y) = N_R e^{M|y|}; \qquad -\pi R \leq y \leq \pi R \tag{4.30}$$

where $N_{L,R}$ are normalization factors:

$$N_L = \sqrt{\frac{-M}{e^{-2M\pi R} - 1}}; \qquad N_R = \sqrt{\frac{M}{e^{2M\pi R} - 1}}.$$

When imposing BC, our request of opposite Z_2-parity forces one of the two profiles to

vanish identically. Only the Z_2-even mode survives, because the Z_2-odd BC implies a vanishing solution at the boundaries, as in Eq.(4.14).
This implies that, for each 5D fermion, the 0-mode exists only for one chirality. Within the SM, a second fermionic field must be introduced independently, with opposite parity assignment, to reproduce the 0-mode of the other chirality, and thus assigning independent transformations under the gauge group.
For further utility in SUSY framework, it is worth noting that the 0-mode solutions (4.29-4.30) coincide with what obtained for scalars in presence of particular boundary masses related to the bulk mass, see Eqs.(4.22-4.23).
Differently from the scalars, we finally notice that, the presence of possible brane mass terms for the fermions would make the 0-mode vanish. Indeed, the action would be:

$$S_{5D} = S_{5D}^{bulk} - \int d^4x \int dy \left(\delta(y) \frac{M_1}{\Lambda} \bar{\Psi}\Psi - \delta(y - \pi R) \frac{M_2}{\Lambda} \bar{\Psi}\Psi \right) \tag{4.31}$$

where Λ is an energy scale. The boundary terms, while not affecting the e.o.m. in the bulk, modify the BCs as:

$$\frac{M_1}{\Lambda}\Psi(0) = 0; \quad \frac{M_2}{\Lambda}\Psi(\pi R) = 0; \tag{4.32}$$

that, applied to Eqs.(4.29-4.30), imply an identically vanishing solution.

The solutions for general m_n to the system of Eqs.(4.27-4.28) leads to the higher KK modes. Performing a second derivative we decouple the system, rearranged into the two second order equations:

$$\partial_5^2 f_{nR}(y) + (m_n^2 - M^2) f_{nR}(y) = 0; \tag{4.33}$$

$$\partial_5^2 f_{nL}(y) + (m_n^2 - M^2) f_{nL}(y) = 0. \tag{4.34}$$

These equations coincide exactly with what found for the scalars, Eqs.(4.10), and lead to the same mass spectrum, with solutions proportional to $\cos(ny/R)$ for the Z_2-even field and to $\sin(ny/R)$ for the Z_2-odd field. Coefficients are determined by reimposing the first order equations, and the BCs.
Notice that, while the 0-mode vanishes for the Z_2-odd field, the rest of its KK tower exists in the bulk and is vanishing only on the branes.

4.3.3 Gauge fields

We now consider the gauge part of 5D action, treating as example the abelian case (5D QED). We have the action:

$$\mathcal{S}_{5D} = \int d^4x \int_0^{2\pi R} dy \, -\frac{1}{4} F_{MN} F^{MN} \tag{4.35}$$

$$= \int d^4x \int_0^{2\pi R} dy \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} F_{\mu 5} F^{\mu 5} \right) \tag{4.36}$$

where

$$F_{MN} \equiv F_{MN}(x,\, y) = \partial_M A_N(x,\, y) - \partial_N A_M(x,\, y) \tag{4.37}$$

The components of the 5D gauge field are:

$$A_M = \{A_\mu,\, A_5\}$$

transforming as a 5D-vector under the Lorentz group SO(1,4). From the 4D point of view, A_μ is a 4D-vector field and A_5 is a scalar. The 5D Lagrangian is constructed to be invariant under the $U(1)$ gauge transformation:

$$A_M(x,\, y) \longrightarrow A_M(x,\, y) + \partial_M \theta(x,\, y) \tag{4.38}$$

With compactification on a circle, $A_M(x,\, y)$ satisfy periodic BC and can be expressed, as usual, in the KK expansion:

$$A_{\mu,5}(x,y) = \sum_{n=-\infty}^{\infty} A_{\mu,5}^{(n)}(x) g_n(y), \tag{4.39}$$

From KK reduction the profiles result in the normalized Fourier basis:

$$g_n(y) = e^{i\,n\frac{y}{R}} \frac{1}{\sqrt{2\pi R}}; \qquad \int_0^{2\pi R} dy g_n^*(y) g_m(y) = \delta_{mn} \tag{4.40}$$

It can be shown that the modes $A_5^{(n)}(x)$ can be eliminated by a gauge transformation [186] and the final Lagrangain, after KK reduction, looks like:

$$\mathcal{L}_{4D} = \sum_n -\frac{1}{4} F_{\mu\nu}^{(n)} F^{(n)\mu\nu} - \frac{1}{2}\frac{n^2}{R^2} |A_\mu^{(n)}|^2 - \frac{1}{2} |\partial_\mu A_5^{(0)}|^2 \tag{4.41}$$

The $A_\mu^{(n)}$ fields, for $n \neq 0$, get the mode-dependent mass:

$$m_n = \frac{n}{R} \tag{4.42}$$

The zero-mode $A_\mu^{(0)}$ remains massless and it is identified with the massless SM photon. For A_5 only the massless 0-mode exist, while the whole KK tower disappear.
This result is not surprising since, according to the Higgs' mechanism, to give mass to a vector boson, a scalar boson must disappear from the theory. The scalars $A_5^{(n \neq 0)}$ are thus the "Goldstone bosons" giving mass to the vector bosons $A_\mu^{(n)}$.

Working on S^1/Z_2 orbifold, the Z_2 invariance requires opposite parities for $A_5(x,y)$ and $A_\mu(x,y)$, in particular: $A_5(x,y)$ is odd, $A_\mu(x,y)$ is even. As happening for scalars and fermions, the orbifolding is selecting one half of the KK tower. The A_μ profiles will be in terms of $\cos(\frac{ny}{R})$, the ones of A_5 in terms of $\sin(\frac{ny}{R})$. The 0-mode of $A_5(x,y)$ get canceled, and, as seen before, the rest of its KK tower disappears through a gauge transformation. Only the $A_\mu(x,y)$ field survives, with a flat 0-mode and the expansion:

$$A_\mu(x,y) = A_\mu^{(0)}(x)\frac{1}{\sqrt{2\pi R}} + \sum_{n=1}^{\infty} A_\mu^{(n)}(x)\frac{1}{\sqrt{\pi R}}\cos\frac{ny}{R} \tag{4.43}$$

Again, the surviving KK modes resembles the same profiles obtained for scalars and fermions, when these are not affected by bulk or brane mass terms.

4.4 Summary and applications

The framework described above can be applied to the SM and it is of particular interest for addressing the structure of the flavour sector. As we have seen, we are able to generate exponentially hierarchical profiles for the 0-mode of the fermion fields, Eqs.(4.29, 4.30), once we introduce a bulk mass source. By assigning different bulk masses for each type of fermions and for each generation, we can create different localization of the fermions along the extra dimension. Introducing the Higgs scalar, that can be propagating in the bulk or be localized on one brane, the Yukawa coupling of the fermions with the Higgs results modulated by the profiles overlap, resembling a hierarchy dictated by the fermions' profiles. This is the key-mechanism used to reproduce hierarchical structure of the Yukawa couplings, starting from anarchical order 1 parameters. This principle, used in various scenarios of the 5D SM (flat ED, RS model), is shown to reproduce correctly the observed hierarchies of masses and mixing angles of the SM quarks and leptons [169–174], but at the price of introducing many bulk mass parameters.

The only scalar field introduced in the SM is the Higgs: in the most simple models it is treated as a 4D field localized at one brane, where the hierarchy expressed by the fermion profiles (and transmitted to Yukawas) is mostly enhanced. In other models the Higgs is allowed to propagate in the bulk and, without assuming any bulk/brane mass, its 0-mode profile is constant, still allowing for a hierarchical overlap with the fermion profiles. The exponential 0-mode solution for scalars of Eq.(4.22) is generally not very interesting in the SM framework, where the only scalar is the Higgs. As see, its existence requires "fine-tuned" relations betweenn bulk and brane parameters. However, this solution will be of crucial importance in the SUSY framework, where the description of scalars and fermions is unified in the chiral multiplet, and these particular "fine-tuned" relations arise naturally from the supersymmetric Lagrangian. In the same way, we matched the existence of common profiles of the higher KK modes for scalars and fermions.

Regarding the gauge fields, instead, their profiles in the most simple case are not modulated by new parameters[4] and the 0-mode is flat, given by a constant that is proportional to the compactification scale. The role of this constant is important, since it provides the

[4] The gauge KK profiles can eventually be modified by a warping factor in the Randall Sundrum model [180] or by other parameters like brane kinetic terms, but in both the cases the 0-mode is not affected.

relation of rescaling between the 5D and the 4D gauge coupling:

$$g4 = \frac{g5}{\sqrt{2\pi R}} \tag{4.44}$$

The higher KK gauge boson excitations are not constant and, when we distinguish the 0-mode fermion profiles, they have a different profile overlap, giving rise to non universal interactions with fermions. Thus, the exchange of KK gauge bosons produce 4-fermions operators which, after rotation from flavour to mass eigenbasis, mediate flavour changing neutral current (FCNC), with a contribution that is dependent from the profiles overlap and from the inverse mass square of the KK gauge bosons, thus dependent on the compactification scale. The current experimental limits on FCNC set a lower bound on the scale of the ED, that is anyway model dependent (see [186] for review).

We finally remark that, in absence of any bulk or brane parameters, the profiles of the gauge bosons and the fermions match exactly for the whole KK tower. As seen for scalars and fermions, this result is important for the existence of a unified profile of the vector multiplet in SUSY, as we are going to elaborate in the next chapter.

4.5 Supersymmetry in 5D

The description of SUSY in ED is interesting for various reasons. On one side, the issue of how to perform the SUSY breaking find new interesting possibilities in the ED ([188–193], just to mention some examples of historical relevance). On the other side, many of the mechanisms based on ED and addressing open problems of the SM (like the flavour), generically rely on the existence of ED itself, without really depending on its size: even ED far above the TeV scale are in principle allowed. As consequence, in models with high scale ED, some new physics is needed to stabilize the electroweak scale and SUSY has so far been considered one of the best candidates to do this job, motivating the study of supersymmetric models also in ED. In our work the main aim of introducing SUSY is concerning more the unification of couplings rather than solving the hierarchy problem of the EW scale. Indeed, the main object of our study is the advantage of a 5D SUSY GUT scenario in reproducing the phenomenology of the flavour sector of the SM, while keeping the specification of a SUSY breaking mechanism as a side topic, to implement afterwards. In this chapter we'll review the necessary formalism to describe a N=1 SUSY theory in 5D in terms of N=2 SUSY in 4D. We will conclude reporting the general action describing the matter and gauge fields supermultiplets, that will be the starting point for the next chapters, where we report in details some specific models.

4.6 N=1 SUSY in 5D, N=2 SUSY in 4D

The description of higher-dimensional SUSY starts from the simple observation that, whatever the higher dimensional theory is, it must contain the ordinary 4D SUSY and

thus have a description in terms of the ordinary 4D superspace. In our specific case, the orbifold compactification breaks at least half of the N=1 5D SUSY, since the set of 5D SUSY transformations generates translations in the y direction, that is not a symmetry of the orbifold [194]. The language of 4D superspace is preferrable to make the surviving N=1 4D SUSY manifest and to express the couplings with the 4D SUSY terms which can be added at the boundaries.
We can guess the superfield content of the 4D theory simply by knowing the total number of SUSY generators in the full theory. As already mentioned, in 5D the smallest spinorial representation is a 4-components Dirac spinor, thus the SUSY generators in 5D must be of this type. Since such spinors are made of 8 real components, there are 8 supercharges, that correspond to the scenario of N=2 SUSY in 4D, thus we expect the same field content. The correspondence between N=1 SUSY in 5D and N=2 SUSY in 4D is the essential link to write a formulation of 5D SUSY in terms of 4D superspace, since we know how any N=2 multiplet is composed of N=1 multiplets (see section A.3 in appendix A).

4.6.1 Field content

Acting with the N=1 SUSY generators in 5D, we can construct gauge and matter multiplets, that we are going to describe. For basic notation of supersymmetry we refer the reader to appendix A.

5D gauge multiplets.

The 5D gauge multiplet contains a vector A^M, a real scalar Σ, and an SU(2)-R doublet of gauginos Λ^i [190, 195]. Furthermore, one requires three real auxiliary fields X^a, which form a triplet of SU(2)-R. These fields are all in the adjoint representation of the gauge group. The SUSY parameter is a symplectic Majorana spinor[5]: ξ^i:

$$\xi_1 = \begin{pmatrix} (\xi_L)_\alpha \\ (\bar{\xi}_R^T)^{\dot{\alpha}} \end{pmatrix}, \xi_2 = \begin{pmatrix} (\xi_R)_\alpha \\ -(\bar{\xi}_L^T)^{\dot{\alpha}} \end{pmatrix}; \tag{4.45}$$

and the transformation laws are given by [190]:

$$\delta_\xi A^M = i\bar{\xi}_i \Gamma^M \Lambda^i \tag{4.46}$$
$$\delta_\xi \Sigma = i\bar{\xi}_i \Lambda^i \tag{4.47}$$
$$\delta_\xi \Lambda^i = \left(\Gamma^{MN} F_{MN} + \Gamma^M D_M \Sigma\right) \xi^i + i \left(X^a \sigma^a\right)^i_j \xi^j \tag{4.48}$$
$$\delta_\xi X^a = \bar{\xi}_i (\sigma^a)^i_j \Gamma^M D_M \Lambda^j + i \left[\Sigma, \bar{\xi}_i (\sigma^a)^i_j \Lambda^j\right], \tag{4.49}$$

where $\Gamma^{MN} = \frac{1}{4}[\Gamma^M, \Gamma^N]$ and $D_M\Sigma = \partial_M \Sigma + i[A_M, \Sigma]$. Λ^i are symplectic Majorana gauginos, as defined in appendix C.2, Eq.(C.6).

[5]see details in appendix C.2

We assume that the surviving 4D SUSY is generated by a set of parameters ξ^i defined by the Weyl spinor ξ_L, with $\xi_R = 0$. The transformation rules of the component fields under this smaller SUSY, using 4D Weyl spinors, are:

$$\begin{aligned}
\delta_{\xi_L} A^\mu &= i\bar{\xi}_L\bar{\sigma}^\mu\lambda_L + i\xi_L\sigma^\mu\bar{\lambda}_L && (4.50)\\
\delta_{\xi_L} A^5 &= -\bar{\xi}_L\bar{\lambda}_R - \xi_L\lambda_R && (4.51)\\
\delta_{\xi_L} \Sigma &= i\bar{\xi}_L\bar{\lambda}_R - i\xi_L\lambda_R && (4.52)\\
\delta_{\xi_L} \lambda_L &= \sigma^{\mu\nu}F_{\mu\nu}\xi_L - iD_5\Sigma\xi_L + iX^3\xi_L && (4.53)\\
\delta_{\xi_L} \lambda_R &= i\sigma^\mu F_{5\mu}\bar{\xi}_L - \sigma^\mu D_\mu\Sigma\bar{\xi}_L + i(X^1 + iX^2)\xi_L && (4.54)\\
\delta_{\xi_L} (X^1 + iX^2) &= 2\bar{\xi}_L\bar{\sigma}^\mu D_\mu\lambda_R - 2i\bar{\xi}_L D_5\bar{\lambda}_L + i[\Sigma, 2\bar{\xi}_L\bar{\lambda}_L] && (4.55)\\
\delta_{\xi_L} X^3 &= \bar{\xi}_L\bar{\sigma}^\mu D_\mu\lambda_L + i\bar{\xi}_L D_5\bar{\lambda}_R - \xi_L\sigma^\mu D_\mu\bar{\lambda}_L - i\xi_L D_5\lambda_R \\
&\quad + i[\Sigma, (\bar{\xi}_L\bar{\lambda}_R + \xi_L\lambda_R)]\,, && (4.56)
\end{aligned}$$

where $\sigma^{\mu\nu} = \frac{1}{4}(\sigma^\mu\bar{\sigma}^\nu - \sigma^\nu\bar{\sigma}^\mu)$.
One can observe [190, 194, 195] that the fields A_μ, $\lambda_1 \equiv \lambda_L$ and $D \equiv (X^3 - D_5\Sigma)$ transform precisely as the components of a 4D vector superfield in the Wess-Zumino (WZ) gauge:

$$V = \theta\sigma^\mu\bar{\theta}A_\mu - i\bar{\theta}^2\theta\lambda_1 + i\theta^2\bar{\theta}\bar{\lambda}_1 + \frac{1}{2}\theta^2\bar{\theta}^2 D \qquad (4.57)$$

Anologously, the fields $(\Sigma + iA_5)$, $\lambda_2 \equiv (-i\sqrt{2}\lambda_R)$ and $F_\chi \equiv (X^1 + iX^2)$ transform as the components of a chiral adjoint superfield in the variable $x' = x + i\theta\sigma^\mu\bar{\theta}$:

$$\chi = \frac{1}{\sqrt{2}}(\Sigma + iA_5) + \sqrt{2}\theta\lambda_2 - \theta^2 F_\chi \qquad (4.58)$$

With these multiplets we will describe the 5D gauge fields (made of the 4D vector components plus the scalar A_5) and the respective gauginos. The abelian gauge transformations of these two superfields are:

$$\begin{aligned}
V &\longrightarrow V + \Lambda + \bar{\Lambda} && (4.59)\\
\chi &\longrightarrow \chi + \sqrt{2}\partial_5\Lambda
\end{aligned}$$

where Λ is a chiral superfield depending, in general, also on y.

5D matter multiplet

The 5D matter multiplet contains an $SU(2)_R$ doublet of scalar fields ϕ^i, a Dirac field Ψ and a doublet of auxiliary fields F_i, that is the field content of an hypermultiplet in N=2 4D SUSY. Such hypermultiplet is described by 2 chiral multiplets in N=1 4D SUSY and, as seen for the vector multiplet, we can verify how this correspondence emerges from the 5D SUSY transformations [190, 195] :

$$\begin{aligned}
\delta_\xi\phi^i &= -\sqrt{2}\epsilon^{ij}\bar{\xi}_j\Psi && (4.60)\\
\delta_\xi\Psi &= i\sqrt{2}\gamma^M D_M\phi^i\epsilon_{ij}\xi^j - \sqrt{2}\Sigma\phi^i\epsilon_{ij}\xi^j + \sqrt{2}F_i\xi^i && (4.61)\\
\delta_\xi F_i &= i\sqrt{2}\bar{\xi}_i\gamma^M D_M\Psi + \sqrt{2}\bar{\xi}_i\Sigma\Psi - 2i\bar{\xi}_i\lambda^j\epsilon_{jk}\phi^k\,. && (4.62)
\end{aligned}$$

As before, we consider the transformation generated by the Weyl spinor ξ_L, with $\xi_R = 0$, and the decomposition of the Dirac spinor into the two Weyl spinors, $\Psi = \left((\psi_L)_\alpha,\ (\overline{\psi}_R)^{\dot{\alpha}}\right)^T$, we then obtain the following transformation rules:

$$\delta_\xi \phi_1 = \sqrt{2}\xi_L \psi_L \tag{4.63}$$
$$\delta_\xi \phi_2 = \sqrt{2}\bar{\xi}_L \overline{\psi}_R \tag{4.64}$$
$$\delta_\xi \psi_L = -\sqrt{2}\left(i\sigma^\mu D_\mu \phi_1\ \bar{\xi}_L + D_5 \phi_2 \xi_L\right) + \sqrt{2}F_1 \xi_L \tag{4.65}$$
$$\delta_\xi \psi_R = \sqrt{2}\left(-i\bar{\sigma}^\mu D_\mu \phi_2 \xi_L + D_5 \phi_1 \bar{\xi}_L\right) - \sqrt{2}F_2 \bar{\xi}_L \tag{4.66}$$
$$\delta_\xi F_1 = i\sqrt{2}\bar{\xi}_L \bar{\sigma}^\mu D_\mu \psi_L - \sqrt{2}\bar{\xi}_L D_5 \overline{\psi}_R + \sqrt{2}\bar{\xi}_L\left(\phi_2 \bar{\lambda}_2 + \phi_1 \bar{\lambda}_1\right) \tag{4.67}$$
$$\delta_\xi F_2 = -i\sqrt{2}\xi_L \sigma^\mu D_\mu \psi_R + \sqrt{2}\xi_L D_5 \psi_L - \sqrt{2}\xi_L\left(\phi_2 \lambda_1 - \phi_1 \lambda_2\right) \tag{4.68}$$

In the 4D superfield formulation, we can see that the component fields are arranged in the two chiral 4d superfields (in the x' basis):

$$\Phi = \phi + \sqrt{2}\theta\psi + \theta^2 F \tag{4.69}$$
$$\Phi^c = \phi^c + \sqrt{2}\theta\psi^c + \theta^2 F^c. \tag{4.70}$$

with:

$$\phi \equiv \phi_1;\ \phi^c \equiv \phi_2^\dagger; \quad \psi \equiv \psi_L;\ \psi^c \equiv \psi_R; \tag{4.71}$$
$$F \equiv (F_1 + D_5\phi_2 - \Sigma\phi_2);\ F^c \equiv (-F_2^\dagger - D_5\phi_1^\dagger - \phi_1^\dagger\Sigma) \tag{4.72}$$

With these multiplets we will describe the matter fields (fermions and their scalar superpartners) propagating in the bulk.

Notice that all the fields above, defined in the 4D superspace, carry an implicit dependence on the 5^{th} coordinate y, that can be thought as a "label" from the 4D point of view. We will consider this dependence implicit from now on. All these fields, indeed, will be subjected to the KK expansion. Their profiles will be determined by the action, that we are going to specify in the next paragraph.

4.6.2 Action

For the fields identified above, we want to write an action in the 4D superspace. The requirement is that, once all the auxiliary fields have been integrated out, this action reduces to the correct components action for the 5D theory, respecting the gauge symmetry and the 5D covariance for all the superfield components. Most importantly, the action, written in terms of $N = 1$ superfields, must respect the $\mathrm{SU}(2)_R$ symmetry, with the rules pointed out in appendix A, section A.3.
As worked out in [194], in terms of the above superfields, Eqs.(4.57, 4.58, 4.69, 4.70), the

action is:

$$
\begin{aligned}
S_5 \;=\; & \int d^5x \left[\frac{1}{4g_5^2}\int d^2\theta\ W^\alpha W_\alpha + h.c.\right. \\
& + \frac{2}{g_5^2}\int d^4\theta\ \left(\partial_5 V - \frac{1}{\sqrt{2}}(\chi+\chi^\dagger)\right)^2 \\
& + \int d^4\theta\ \left(\Phi^\dagger e^{-V}\Phi + \Phi^c e^V \Phi^{c\dagger}\right) \\
& \left. + \int d^2\theta\ \Phi^c\left(\partial_5 + \hat{M} - \frac{1}{\sqrt{2}}\chi\right)\Phi + h.c.\right]
\end{aligned} \tag{4.73}
$$

The first two lines in the action (4.73) describe the 5D super Yang-Mills theory and satisfy the gauge transformation (4.59). The last two lines describe the matter fields and their gauge interactions, including also a bulk mass term (the $\hat{M}$ parameter), in analogy to what we have seen above for scalars and fermions. In appendix D we perform a full check of this Lagrangian, by expanding the superfields, performing the integrals over $\theta, \bar{\theta}$, and integrating out the auxiliary terms.

4.6.3 Orbifolding and N=2 SUSY breaking

The action reported above, Eq.(4.73), is valid for a general compactification of the ED on a circle or a interval. It remains valid also when we compactify the ED on an orbifold S^1/Z_2, if we assign to each of the above superfields (V, χ, Φ, Φ^c) an intrinsic Z_2-parity that guarantees the invariance of the action under Z_2, as seen in the section 4.2.2:

$$V(x,y) = PV(x,-y), \text{ etc., with } P = \pm 1 \ .$$

From the structure of (4.73) it is straightforward to see that we need opposite parity assignment between the components of the same N=2 supermultiplets, due to the presence of the 5^{th} derivative, that transforms as an odd quantity: $\partial_5 \to -\partial_5$. The parity assignment is reported in table 4.6.3. For consistency of this parity assignment, the bulk

N=2 multiplets	**N=1 multiplets**	
	$P=+1$	$P=-1$
Vector multiplet	V	χ
Hypermultiplet	Φ	Φ^c

mass parameter M must be odd under Z_2 and we assume it to be:

$$\hat{M} = M\mathrm{sgn}(y),$$

M being a real constant. This is what we have already seen happening for the fermion bulk mass in (4.24).

This parity assignment, distinguishing the N=1 multiplets inside the same N=2 multiplets, shows explicitly how the orbifolding is responsible of breaking N=2 4D SUSY (or N=1 5D SUSY) down into N=1 4D SUSY. This breaking is due to different BC, that make the fields odd under Z_2 vanishing at the branes. Thus, at the boundaries we are free to add any localized contribution formulated in 4D that is respecting only N=1 SUSY. Inside the bulk, anyway, both the odd and even fields exist, so we have to maintain a description in terms of N=2 SUSY. The only interactions allowed by N=2 SUSY between the chiral multiplets are gauge interactions. Indeed, in Eq.(4.24), the field χ contains the 5^{th} component of the 5D gauge vector, A_5, and its interaction with Φ and Φ^c is controlled by the gauge coupling constant g_5.

4.6.4 Profiles

Each of the above superfields is propagating in the bulk and is subjected to the KK expansion. As already seen for the non supersymmetric case, we can deduce the profiles of the fields from the KK reduction. From the breaking of N=2 SUSY and as far as N=1 4D SUSY remains unbroken, we'll get common profiles for all the field components belonging to the same N=1 supermultiplet.

Gauge multiplets

For the vector multiplet V, with $P = +1$, we get a flat 0-mode profile, while the higher KK profiles are given by $\cos(\frac{ny}{R})$, exactly the expansion found for the vector bosons, Eq.(4.43). These profiles describe the whole vector multiplet, including the gaugino λ_1. This can be consistently checked by making the Lagrangian explicit with respect to the field components, see Eq.(D.6) in appendix D: the gauginos part is the typical Lagrangian for fermions without a bulk mass term, for which we have already checked the existence of these solutions, (cf. Sect.4.3.2).
For the chiral multiplet χ, with $P = -1$, the 0-mode is vanishing, while the higher KK profiles are expressed in terms of $\sin(\frac{ny}{R})$. These profiles are common for the scalar Σ and the second gaugino λ_2, as can be consistently checked from Eq.(D.6) where these fields enter the Lagrangian for scalars and fermions, respectively, without bulk mass term. We can instead get rid of the higher modes A_5^n by performing a supersymmetric gauge transformation, resembling what happening in the non-SUSY case.

Matter multiplets

The matter multiplet is described by the chiral superfields:

$$\Phi(x,y) = \Phi_n(x) f_n(y) \quad \text{and} \quad \Phi^c(x,y) = \Phi^c_n(x) f^c_n(y).$$

In this case, without passing necessarily through the expansion in the field components, we can deduce the e.o.m. of the profiles directly for the superfields:

$$(\hat{M} + \partial_5) f_n(y) = m_n f_n(y); \quad (\hat{M} - \partial_5) f_n^c(y) = m_n f_n^{c*}(y) \tag{4.74}$$

that, including the BC ($f_n(y)$ is Z_2-even, while $f_n^c(y)$ is Z_2-odd) leads to the normalized solution for the 0-mode ($m_n = 0$):

$$f_0(y) = \sqrt{\frac{M}{1 - e^{-2M\pi R}}} e^{-My} \ , \quad f_0^c(y) = 0. \tag{4.75}$$

This is exactly the 0-mode solution found for fermions with bulk mass, Eq.(4.29), and it reflects the fact that the two chiral components ψ and ψ^c of the 5D spinor Ψ are distinguished by the Z_2 parity, leading to a vanishing 0-mode for the Z_2-odd component. The solution (5.2) describes the whole chiral multiplet, thus being valid also for the scalar superpartner of the fermion. As seen in Eq.(4.22-4.23), this kind of solution for the scalar field exist only if the bulk mass is accompanied by brane mass terms in the given relation (4.21). We can verify that this is happening in the scalar piece of Lagrangian obtained by expanding the superfields in the action (4.73). Again, this is reported in Eq.(D.6), where we can see that the scalars ϕ and ϕ^c are accompanied by the term:

$$\partial_5 \hat{M} \cdot \left(\phi^\dagger \phi - \phi^{c\dagger} \phi^c\right) = \left(2M\delta(y) - 2M\delta(y - \pi R)\right) \left(\phi^\dagger \phi - \phi^{c\dagger} \phi^c\right),$$

that provides two brane masses in the wanted relation (4.21), emerging from the Z_2-odd nature of the bulk mass $\hat{M} = M\text{sgn}(y)$.
All the higher modes are given as usual by $\cos(\frac{ny}{R})$ for the even field, and $\sin(\frac{ny}{R})$ for the odd field, with mass spectrum: $m_n^2 = M^2 + \frac{n^2}{R^2}$, as already checked for both the scalar and the fermionic fields. This closes consistently the analysis of the profiles, respecting N=1 SUSY.

The chiral multiplet Φ describes a fermionic matter field and its 0-mode (5.2) profile will be used to create the wanted hierarchies in the Yukawa couplings with the Higgs, by introducing different bulk masses for the various matter multiplets. This mechanism is exactly what already discussed for fermions in section 4.4 within the SM. In the supersymmetric case, the same kind of hierarchical Yukawa structure will emerge also for the sfermions, the fermion superpartners, as far as we maintain N=1 4D SUSY unbroken. The novel characteristic of the SUSY theory is also given by the constraints of N=2 supersymmetric couplings in the bulk, that couples the matter fields to the field χ in a way that can correct the 0-mode profiles of the matter multiplet if χ gets a VEV. This feature will be at the core of our application in SO(10) GUT and will be described in the details in the next section.

Chapter 5

A flavour model in 5D SO(10)

5.1 Introduction

This chapter is dedicated to the building of a specific model within the framework of SO(10) and extra dimensions, essentially based on our publication in ref. [16].

We have illustrated the general characteristics of SO(10) GUTs in Chap. 3, pointing out the compelling features as well as some weak points of this theory. Among the latter, we have shown how a realistic description of the flavour sector results quite complicated, requiring a large set of parameters, variating in a wide range. Improving this aspect of SO(10) GUTs is a very challenging issue and there are several models, in both renormalizable [108–112, 117, 121, 125, 126, 130–134] and non-renormalizable [87, 135–150] versions (see [36, 151] for reviews), where new ingredients are added either to reduce the number of free parameters or to reduce their relevant range. In the first case the predictability of the model is increased, while in the second case the model becomes more natural. Indeed it would be desirable to account for the hierarchies of the charged fermion mass ratios and of the quark mixing angles in terms of an irreducible set of order-one parameters. We have illustrated how extra dimensions (cf. Chap. 4) provide a useful framework in this perspective. The model we are going to describe implements such a framework in SO(10) GUT.

The idea at the core of our model has been originally proposed in ref. [15] by Kitano and Li, who developed a SUSY SO(10) model formulated in five flat space-time dimensions. The fifth dimension is an interval whose length is of the order of the inverse GUT scale. Fermions are hosted in three **16** multiplets living in the full 5-dimensional space-time while Yukawa interactions, described by matrices with order-one elements, are localized at one of the branes. The Yukawa couplings for the fermions of the SM are obtained by convoluting these order-one matrices with the profiles of the fermionic zero-modes. The resulting picture is essentially equivalent to that produced by many models of fermion masses such as those based on Froggatt-Nielsen (FN) $U(1)_{FN}$ flavour symmetries [41] or those relying on the mechanism of partial compositeness [42, 43]. This kind of mechanism is known to work well when applied to the SM in 5D (cf. Chap. 4), anyway its extension in SO(10) is not so trivial. Indeed, in an SO(10) context one would expect fermions in the same **16** representation to share the same zero-mode profile, which in the FN language would correspond to all members of a **16** having the same FN charge. Such a picture is clearly unrealistic since it leads to mass ratios for up and down type quarks of the same order of magnitude. The key point of the Kitano and Li model is that the breaking of SO(10) down to $SU(5)\times U(1)_X$ determines different profiles for the zero-modes of the different SU(5) components inside the **16** multiplet. The model becomes flexible enough to account for the different hierarchies observed in the different charge sectors. Unfortunately the total number of parameters is still very large since a single matrix of Yukawa interactions localized at one brane is insufficient to correctly describe the quark mixing and more than one type of Yukawa interactions are required.

In our proposal we improve in several aspects the model of ref. [15]. Let us summarize the novel features that we introduce: we reformulate the sector of Yukawa interactions in terms of operators of the same dimension, improving the domain of validity of the effective theory; we explicitly address the D-T splitting problem by choosing a Higgs sector to which the missing partner mechanism is applicable; we check numerically the viability of the model, performing a fit to an idealized set of data, obtained by running the observed masses and mixing angles up to the GUT scale; we finally perform a test of naturalness on the model in order to verify the validity of the ansatz of anarchical Yukawas.

It is particularly interesting that the present SO(10) model can give rise to fermion mass matrices similar to the ones obtained in $SU(5)\times U(1)_{FN}$ models [196–199], very effective in reconciling the nearly anarchical pattern of neutrinos with the hierarchical one of charged fermions.

Here we do not aim at a fully realistic model and we deliberately leave apart several important issues, such as gauge coupling unification and the problem of proton decay. Also, to avoid major complications and strong model dependence we work in the limit of exact N=1 SUSY, which is not a completely realistic scenario. The specification of an N=1 SUSY breaking mechanism will be object of a further study.

In the following section, we briefly review the basic framework of Kitano-Li model and explain a mechanism responsible for creating hierarchies among the different fermions. In section 5.3, we provide a modified version of this model and discuss its essential features in details. The fermion mass relations predicted by the model are discussed and their viability is investigated through detailed numerical analysis in Sect.5.4. We finally conclude in section 5.5.

5.2 Flavour hierarchy from extra-dimension and SO(10) GUT

The basic framework of the 5-dimensional (5D) SUSY theory, which the Kitano-Li model [15] is based on, was already presented in section 4.5. Let us report it again with a little change of notation, compatible with [16]. Consider a 5D $N = 1$ SUSY U(1) gauge theory compactified on half a circle S^1/Z_2 [200], which can be conveniently written [194] in terms of 4-dimensional superspace formalism. When decomposed into 4D, the 5D vector supermultiplet contains an $N = 1$ chiral multiplet Φ and a vector multiplet V. In a similar way, the 5D hypermultiplet consists of a pair of $N = 1$ 4D chiral multiplets H and H^c. The U(1) gauge invariant action of interacting hypermultiplet and vector multiplet was already introduced in Eq.(4.73) of the previous chapter and, with the new notation, reads:

$$\begin{aligned} S_5 &= \int dy\; d^4x \left[\left[\int d^4\theta \left(\partial_y V - \frac{1}{\sqrt{2}}(\Phi + \bar{\Phi})\right)^2 + \frac{1}{4}\int (d^2\theta\; W^\alpha W_\alpha + \text{h.c.})\right.\right. \\ &+ \int d^4\theta\; \left(\bar{H}e^{2g_5QV}H + \bar{H}^c e^{-2g_5QV}H^c\right) \\ &+ \left.\left(\int d^2\theta\; H^c\left(\hat{m} + \partial_y - \sqrt{2}g_5Q\Phi\right)H + \text{h.c.}\right)\right], \end{aligned} \tag{5.1}$$

where W^α is a field strength, g_5 is the 5D gauge coupling constant, $\hat{m}$ is the bulk mass and Q is the U(1) charge of the chiral multiplet H. The mass dimensions are: $[\Phi] = [H] = [H^c] = +3/2$, $[\hat{m}] = +1$, $[V] = +1/2$ and $[g_5] = -1/2$. The vector multiplet V and chiral multiplet H transform as even fields under the Z_2 symmetry while the fields Φ and H^c are odd. For consistency, the bulk mass parameter $\hat{m}$ is odd under Z_2 and the simplest choice is $\hat{m}(y) = m\;\text{sgn}(y)$, m being a real constant. The only interactions of the model allowed by $N = 1$ 5D SUSY are gauge interactions. Indeed Φ is related to the fifth component of the 5D gauge multiplet and its interaction with H and H^c is controlled by the gauge coupling constant g_5.

The compactification on S^1/Z_2 breaks 4D $N = 2$ SUSY down to $N = 1$ SUSY, thus allowing for a chiral fermion content. Beyond the bulk action S_5 of Eq.(5.1) there can be contributions strictly localized on the branes, which should only respect $N = 1$ SUSY. Here we discuss the theory in the ideal limit of exact $N = 1$ SUSY and we neglect soft SUSY breaking contributions with a characteristic scale in the range $1 \div 10$ TeV. One can perform the Kaluza-Klein (KK) expansion of 5D bulk fields and obtain the massless spectrum of the 4D theory using the equations of motion and boundary conditions imposed by the Z_2 symmetry on different fields.

For the chiral superfield $H(x, y) = \sum_n H_n(x)f_n(y)$, one finds a localized zero-mode profile

$$f_0(y) = \sqrt{\frac{2m}{1-e^{-2m\pi R}}}e^{-my} \ , \tag{5.2}$$

where R is the compactification radius of the extra dimension. Notice that here, with respect to Eq.(4.75), the normalization is taken on the fundamental interval $[0, \pi R]$, instead of $[-\pi R, \pi R]$. For $m < 0$ ($m > 0$) the zero-mode profile $f_0(y)$ of H is localized at the $y = \pi R$ ($y = 0$) brane. This feature can be exploited to suppress (enhance) the strength of the interactions between such zero mode and fields from a Higgs sector localized at the $y = 0$ brane. In this way the hierarchical pattern observed in fermion masses and mixing angles can be explained without appealing to small ad hoc parameters [181, 182, 201]. The chiral superfield H^c is odd under Z_2 and has no zero modes. The massive KK modes have masses $m_n^2 = m^2 + (n/R)^2$, above the compactification scale $M_{KK} = 1/R$. The vector supermultiplet V has a zero mode constant in y and given by[1] $1/\sqrt{\pi R}$. Thus the gauge coupling constant g_4 of the 4D effective theory is related to g_5 by

$$g_4 = \frac{g_5}{\sqrt{\pi R}} \ . \tag{5.3}$$

The chiral multiplet Φ has no zero mode, but its scalar component can acquire a vacuum expectation value (VEV).

The above framework is used in [15] to construct a grand unified model based on the SO(10) gauge group. In this model the $N = 1$ chiral multiplets H and H^c are replaced by three copies of $\mathbf{16}$ and $\mathbf{16}^c$, transforming as $\mathbf{16}$ and $\overline{\mathbf{16}}$ under SO(10) respectively. The vector supermultiplet, comprising $\mathbf{45}_V$ and $\mathbf{45}_\Phi$, transforms in the adjoint of SO(10). The breaking of SO(10) down to the SM gauge group is realized in several steps. The VEV of the $\mathbf{45}_\Phi$, aligned along the direction of a $\mathrm{U(1)_X}$ subgroup, breaks SO(10) down to SU(5)$\times$ $\mathrm{U(1)_X}$. Since the $\mathbf{45}_\Phi$ field is odd under Z_2, its VEV has a non-trivial profile in the fifth dimension, $\langle \mathbf{45}_\Phi \rangle = v_\Phi^{3/2} sgn(y)$, and generates a D-term for $\mathrm{U(1)_X}$ [194, 201, 202]:

$$-D = \partial_5 \langle \mathbf{45}_\Phi \rangle = 2 v_\Phi^{3/2} [\delta(y) - \delta(y - \pi R)] \ . \tag{5.4}$$

To preserve $N = 1$ SUSY such a D-term can be canceled by introducing on the branes new fields charged under $\mathrm{U(1)_X}$ [194, 202]. In the Kitano-Li model [15] the brane $y = 0$ hosts a pair $(\mathbf{16}_H, \overline{\mathbf{16}}_H)$ of chiral superfields while another pair $(\mathbf{16'}_H, \overline{\mathbf{16}}'_H)$ is introduced at the brane $y = \pi R$. Their VEVs are adjusted to exactly cancel the D-term of Eq.(5.4). In this way, the $\mathrm{U(1)_X}$ subgroup is broken near the GUT scale. For this reason the X generator should be orthogonal to the SM ones and this condition uniquely determines $\mathrm{U(1)_X}$ inside SO(10). The other fields needed on the brane $y = 0$ are a chiral multiplet $\mathbf{45}_H$, which breaks the residual SU(5) symmetry to the SM gauge group (cf. Sect.3.4.4), and $\mathbf{10}_H$, which contains a pair of Higgs doublets. The 5D superpotential of the model is

[1] Normalized on the interval $[0, \pi R]$.

[15]:

$$
\begin{aligned}
\mathcal{W}_{\rm KL} &= \mathbf{16}_i^c \left[\hat{m}_i + \partial_y - \sqrt{2} g_5\, \mathbf{45}_\Phi\right] \mathbf{16}_i \\
&+ \frac{\delta(y)}{\Lambda} \left[Y_{ij} \mathbf{16}_i \mathbf{16}_j \mathbf{10}_H + \frac{(Y_R)_{ij}}{\Lambda} (\mathbf{16}_i \overline{\mathbf{16}}_H)(\mathbf{16}_j \overline{\mathbf{16}}_H) + \frac{Y'_{ij}}{\Lambda} \mathbf{16}_i \mathbf{16}_j \mathbf{10}_H \mathbf{45}_H + ...\right] \\
&+ \delta(y)\; w_0(\mathbf{45}_H, \mathbf{16}_H, \overline{\mathbf{16}}_H, \mathbf{10}_H, ...) \\
&+ \delta(y - \pi R)\; w_\pi(\mathbf{16}'_H, \overline{\mathbf{16}}'_H)\ ,
\end{aligned}
\tag{5.5}
$$

where w_0 and w_π are gauge invariant superpotentials depending only on Higgs supermultiplets and Λ is the cut-off scale of the theory. The basis of $\mathbf{16}_i$ is conveniently chosen so that the bulk mass term of $\mathbf{16}_i$ in $\mathcal{W}_{\rm KL}$ is diagonal. In addition to the fields contained in the above $\mathcal{W}_{\rm KL}$, a solution to doublet-triplet splitting problem through Dimopolous-Wilczeck mechanism [74] in the simplest version requires another $\mathbf{10}_H$, a pair of $\mathbf{16}_H$ and several SO(10) singlet fields [203, 204].

Let us now review the Yukawa sector of the model encoded in the second line of Eq.(5.5). The first term is responsible for fermion masses of Dirac type. This term would predict an unrealistic set of masses in the charged fermion sector and for this reason additional contributions suppressed by more powers of the cut-off scale are needed. One example of such contributions is the third term[2] in the second line. In ref. [15] it is explicitly assumed that all these contributions effectively give rise to Yukawa matrices in each charge sector, $Y_{u,d,\nu,e}$, that can be treated as independent. In the second term of the second line, the VEV of $\mathbf{16}_H$ generates masses of right-handed neutrinos of the order of the GUT scale, inducing tiny masses for the light neutrinos through the type I seesaw mechanism.

The Yukawa couplings for the charged fermion zero modes are obtained by convoluting $Y_{u,d,e}$ with the zero-mode profiles, which in turn are controlled by both the bulk masses m_i and the VEV of $\mathbf{45}_\Phi$. Such a VEV generates different contributions to the bulk masses of the SU(5) components of each $\mathbf{16}$ bulk multiplet, proportional to their $\mathrm{U(1)_X}$ charges. Under SU(5)$\times$ $\mathrm{U(1)_X}$ the $\mathbf{16}$ decomposes as

$$
\mathbf{16} = 10_{-1} + \bar{5}_3 + 1_{-5}\ , \tag{5.6}
$$

where the numbers in subscript represents $\mathrm{U(1)_X}$ charge of a given SU(5) multiplet: Q_X^r. Each SU(5) multiplet gets an effective bulk mass m_i^r ($r = 10, \bar{5}, 1$) given by

$$
m_i^r = m_i - \sqrt{2} g_5 Q_X^r v_\Phi^{3/2}\ , \tag{5.7}
$$

that can be expressed in units of the cut-off scale as

$$
m_i^r = \Lambda\ a_i^r \tag{5.8}
$$

[2]This term contains exactly the operator in Eq.(3.20) expressed in SU(5) components.

in terms of dimensionless quantities

$$a_i^r = \mu_i - Q_X^r k_X \ , \qquad \mu_i = \frac{m_i}{\Lambda} \ , \qquad k_X = \sqrt{2}\frac{g_5 v_\Phi^{3/2}}{\Lambda} \ . \tag{5.9}$$

The Yukawa couplings $\mathcal{Y}_f$ $(f = u, d, e)$ of the charged fermion zero modes are

$$\mathcal{Y}_u = F_{10} Y_u F_{10} \ , \qquad \mathcal{Y}_d = F_{10} Y_d F_{\bar{5}} \ , \qquad \mathcal{Y}_e = F_{\bar{5}} Y_e F_{10} \tag{5.10}$$

where the entries of diagonal matrices F_r are the zero-mode profiles evaluated at the $y = 0$ brane:

$$F_r = \mathtt{diag}(n_1^r, n_2^r, n_3^r) \ , \qquad n_i^r = \sqrt{\frac{2a_i^r}{1 - e^{-2a_i^r c}}} \ , \qquad c = \Lambda \pi R \ . \tag{5.11}$$

The mass matrix of light neutrinos is obtained through the type I seesaw mechanism and is proportional to

$$m_\nu \propto F_{\bar{5}} \ Y_\nu Y_R^{-1} Y_\nu^T \ F_{\bar{5}} \ . \tag{5.12}$$

It was shown in [15] that a suitable choice of the VEV of $\mathbf{45}_\Phi$ can generate the following hierarchy in the profiles:

$$F_{10} \backsimeq \mathtt{diag}(\lambda^4, \ \lambda^2, \ 1), \quad F_{\bar{5}} \backsimeq \mathtt{diag}(\lambda, \ 1, \ 1) \tag{5.13}$$

for 10 and $\bar{5}$ fermions with $\lambda \sim 0.23$. These profiles give rise to realistic hierarchies in fermions masses and mixing angles, including the neutrinos, even if all the Yukawa matrices in Eq.(5.5) have anarchical $\mathcal{O}(1)$ elements. The strong hierarchy in the profiles of F_{10} compared to $F_{\bar{5}}$ provides a qualitative understanding of the extremely hierarchical spectrum of up-type quarks and the less hierarchical down-type quarks and charged leptons. The milder hierarchy in the neutrino masses and emergence of the large mixing angles can also be understood in this way.

5.3 A modified Kitano-Li model

The above framework looks consistent at the qualitative level, but it has not been analyzed on the quantitative grounds to check its viability and its predictability. We would like to address such a question.

5.3.1 Improving of the validity of the effective field theory

In its present version, the model can be only applied to an energy range ending very close to the GUT scale, and the effective description it provides could suffer from large uncertainties coming from the unknown UV completion. Indeed the VEV of $\mathbf{45}_H$ breaks SU(5) into the SM gauge group and can be identified with the GUT scale $M_{\mathrm{GUT}} \approx 2\times 10^{16}$ GeV. The higher-order terms in the second line of Eq.(5.5), as the one proportional to Y', are suppressed by powers of M_{GUT}/Λ and are very small if $\Lambda \gg M_{\mathrm{GUT}}$. In this limit the charged fermion Yukawa interactions on the brane are dominated by the first term, leading to: $Y_u = Y_d = Y_e = Y$. The down-type quarks and charged leptons become exactly degenerate in this limit since the zero-mode profiles cannot distinguish between SM sub-multiplets within 10 and $\bar{5}$. On the other hand the simple GUT scale extrapolation of the currently observed values of the masses of down-type quarks and charged leptons requires $m_b/m_\tau \approx 0.7$, $m_s/m_\mu \approx 0.2$ and $m_d/m_e \approx 2.5$ for almost any value of $\tan\beta$ [53]. Such large corrections, particularly in the first two generations, cannot be induced through the higher-dimensional operators unless $M_{\rm GUT} \sim \Lambda$ is considered and if all the Yukawa couplings in the theory are taken to be $\mathcal{O}(1)$ parameters. Taking the cut-off scale Λ very close to the $M_{\rm GUT}$ questions the validity of the effective field theory approach which underlies the whole construction of the model.

The effective theory description can be restored by assuming $\Lambda \gg M_{\rm GUT}$. The correction in the down-type quarks and charged lepton masses then requires leading-order contribution in Yukawa interactions which can be achieved either by $\overline{\mathbf{126}}_H$ or $\mathbf{120}_H$ or by both.

5.3.2 Field content and superpotential

We choose to complete the Yukawa interactions in the charged sector with a $\mathbf{120}_H$. Unlike $\mathbf{10}_H$ and $\overline{\mathbf{126}}_H$, the Yukawa interactions of $\mathbf{120}_H$ with $\mathbf{16}_i$ are anti-symmetric in generation space and hence they introduce less number of free parameters compared to $\overline{\mathbf{126}}_H$. Keeping this aspect in mind, here we propose a variant of the Kitano-Li model based on $\mathbf{10}_H + \mathbf{120}_H$ fields on the brane, which can account for all the charged fermion masses and mixing angles, as we show through a detailed quantitative analysis in the next section. We would like to remark that this choice is not trivial since in a normal 4-dimensional framework this combination of Higgs fields don't provide a realistic fit of the charged fermions data [96], as we discussed in Sect.3.4.5. The $\mathbf{16}_H$ and $\overline{\mathbf{16}}_H$ on the brane are replaced by $\mathbf{126}_H$ and $\overline{\mathbf{126}}_H$, which pick up a VEV at the GUT scale, solve the D-term problem and generate the masses for right handed (RH) neutrinos through a leading order term in the Yukawa interaction. The $\mathbf{126}_H$ and $\overline{\mathbf{126}}_H$ also play a crucial role in solving the doublet-triplet splitting problem through the missing partner mechanism as described in [81, 82]. We provide a detailed discussions of the model in this section.

We use the same field configuration in the bulk as previously used in [15] and only modify the brane sector considerably. We assume as superpotential of the model

$$\begin{aligned} \mathcal{W} &= \mathbf{16}_i^c \left[\hat{m}_i + \partial_y - \sqrt{2} g_5\, \mathbf{45}_\Phi\right] \mathbf{16}_i \\ &+ \frac{\delta(y)}{\Lambda}\left[Y_{10}^{ij}\mathbf{16}_i\mathbf{16}_j\mathbf{10}_H + Y_{120}^{ij}\mathbf{16}_i\mathbf{16}_j\mathbf{120}_H + Y_{126}^{ij}\mathbf{16}_i\mathbf{16}_j\overline{\mathbf{126}}_H + ...\right] \\ &+ \delta(y) w_0(\mathbf{45}_H, \mathbf{10}_H, \mathbf{126}_H, \overline{\mathbf{126}}_H, \mathbf{120}_H) \\ &+ \delta(y-\pi R) w_\pi(\mathbf{126}'_H, \overline{\mathbf{126}}'_H)\ . \end{aligned} \tag{5.14}$$

As already discussed, the VEV of $\mathbf{45}_\Phi$ breaks the SO(10) symmetry down to SU(5)$\times$

$U(1)_X$ and splits the profiles of the SU(5) sub-multiplets of $\mathbf{16}_i$. We now discuss in detail the roles played by each of the brane fields in this model.

- Under SU(5)× $U(1)_X$ the multiplets $\mathbf{10}_H$ and $\mathbf{120}_H$ decompose as:

$$\begin{aligned} \mathbf{10}_H &= 5_2 + \overline{5}_{-2} \ , \\ \mathbf{120}_H &= 5_2 + \overline{5}_{-2} + 10_{-6} + \overline{10}_6 + 45_2 + \overline{45}_{-2} \ . \end{aligned} \tag{5.15}$$

  The $\mathbf{10}_H$ contains a pair of weak doublets, one in 5 and the other in $\bar{5}$, which transforms as a pair of Higgs doublets in the Minimal Supersymmetric Standard Model (MSSM), while $\mathbf{120}_H$ contains two pairs of such doublets, one pair of doublets residing in 45 and $\overline{45}$ of SU(5). We assume that these doublets get mixed with each other through the couplings in the superpotential w_0 and that only one linear combination of them remains light and plays the role of MSSM Higgs doublets. A natural solution of the doublet-triplet splitting problem leading to such light pair of doublets is offered by the missing partner mechanism in this model, as we discuss it later in detail. Since the light doublets are admixtures of doublets in $\mathbf{10}_H$ and $\mathbf{120}_H$, the Yukawa couplings of charged SM fermions are linear combinations of Y_{10} and Y_{120}. Such relations were derived explicitly in [205] and we write them in the next section. It is well known that a pair of MSSM doublets residing in 45 and $\overline{45}$ of SU(5) distinguishes the Yukawa couplings of down-type quarks from those of the charged leptons (cf. Sect.3.2.3).

- The $\overline{\mathbf{126}}_H$ representation of SO(10) decomposes under SU(5)× $U(1)_X$ as

$$\overline{\mathbf{126}}_H = 1_{10} + 5_2 + \overline{10}_6 + 15_{-6} + \overline{45}_{-2} + 50_2 \ . \tag{5.16}$$

  An analogous decomposition for the $\mathbf{126}_H$ holds. The pair $(\overline{\mathbf{126}}_H, \mathbf{126}_H)$ replaces the pair $(\mathbf{16}_H, \overline{\mathbf{16}}_H)$ used by Kitano and Li and plays a similar role. The VEVs of the SU(5) singlets residing in $\mathbf{126}_H$, $\overline{\mathbf{126}}_H$ are used to cancel the D-term on the branes that arises from the VEV of $\mathbf{45}_\Phi$. The vanishing of the D-term requires [194, 202]

$$\begin{aligned} 0 = -D_{U(1)_X} &= \delta(y) \left[2v_\Phi^{3/2} + 10g_5(|\langle\mathbf{126}_H\rangle|^2 - |\langle\overline{\mathbf{126}}_H\rangle|^2)\right] \\ &- \delta(y-\pi R) \left[2v_\Phi^{3/2} - 10g_5(|\langle\mathbf{126}'_H\rangle|^2 - |\langle\overline{\mathbf{126}}'_H\rangle|^2)\right] \ , \end{aligned} \tag{5.17}$$

  where we identify the gauge coupling constant of $U(1)_X$ with g_5. Clearly, this breaks the $U(1)_X$ symmetry and reduces the rank of the residual gauge symmetry. The VEV of $\overline{\mathbf{126}}_H$ also generates the masses for the RH neutrinos through the Yukawa interaction term proportional to Y_{126} of Eq.(5.14). Note that $\overline{\mathbf{126}}_H$ also contains a pair of weak doublets. However such a pair is assumed to be as heavy as the other submultiplets of $\overline{\mathbf{126}}_H$ as required by the missing partner mechanism for solving the doublet-triplet splitting problem, as we discuss below. In this way, $\overline{\mathbf{126}}_H$ does not contribute to the charged fermion masses.

- The decomposition of $\mathbf{45}_H$ is given by

$$\mathbf{45}_H = 1_0 + 10_4 + \overline{10}_{-4} + 24_0 \ . \tag{5.18}$$

Note that $\mathbf{45}_H$ contains an adjoint of SU(5) and can trigger the SU(5) breaking down to the SM gauge symmetry. This cannot be achieved by $\mathbf{45}_\Phi$ in the bulk because the VEV of the 24-plet of SU(5) residing in $\mathbf{45}_\Phi$ would induce a non-vanishing D-term corresponding to $\mathrm{U(1)_Y}$. Such a D-term cannot be canceled without the breaking of $\mathrm{U(1)_Y}$ and hence we need a $\mathbf{45}_H$ to break SU(5).

5.3.3 The missing partner mechanism

The above Higgs content on the brane naturally solves the doublet-triplet splitting problem through the missing partner mechanism as pointed out in [82]. In this mechanism, a set of "light" fields is considered, with an assumption that they get masses only through interactions with "heavy" fields. In other words, the mechanism assumes the absence of bare mass terms for the light Higgs sector. In the above model, $\mathbf{10}_H$ and $\mathbf{120}_H$ fields can be considered as light, while $\mathbf{126}_H$, $\overline{\mathbf{126}}_H$ and $\mathbf{45}_H$ are considered as the heavy ones. As can be seen from the decomposition under the SM gauge group, the light fields contain three pairs of weak doublets and three pairs of color triplets. The heavy fields contain the same number of triplets but only two pairs of doublets.

The unequal content of doublets and triplets in the heavy sector arises from the 50, $\overline{50}$ of SU(5) residing in $\mathbf{126}_H$, $\overline{\mathbf{126}}_H$ which contain only triplets[3]. One assumes that there is no GUT scale bare mass terms for the light fields so that different sub-multiplets of the light fields get masses through their interactions with $\mathbf{126}_H$, $\overline{\mathbf{126}}_H$ and $\mathbf{45}_H$. Such interactions can arise in w_0, for example

$$w_0 = \mathbf{120}_H\ \mathbf{126}_H\ \mathbf{45}_H + \mathbf{120}_H\ \overline{\mathbf{126}}_H\ \mathbf{45}_H + \frac{1}{\Lambda}\mathbf{10}_H\ \mathbf{126}_H\ \mathbf{45}_H^2 + \frac{1}{\Lambda}\mathbf{10}_H\ \overline{\mathbf{126}}_H\ \mathbf{45}_H^2 + ... \tag{5.19}$$

where for simplicity we omit the coupling constants. The doublets and triplets from the different heavy and light fields get mixed with each other when $\mathbf{45}_H$ takes a VEV. The three triplets from the light fields get mixed with the same number of triplets in the heavy fields and all of them obtain GUT scale masses. On the other hand, one combination of weak doublets in the light sector remains massless since the heavy sector contains only two of such doublets. It is also shown in [82] that the other sub-multiplets in $\mathbf{120}_H$ also get mixed with their counterparts in the heavy fields and all of them become massive.

[3]The decomposition of the 50 of SU(5) under the $\mathrm{SU(3)_C \times SU(2)_L \times U(1)_Y}$ group is:

$$50 = (1,1)_{-2} + (\mathbf{3},\mathbf{1})_{\mathbf{-1/3}} + (\bar{3},2)_{-7/6} + (\bar{6},3)_{-1/3} + (6,1)_{4/3} + (8,2)_{1/2}\ ,$$

where the triplet common to the 5 and 45 representations is highlightened in bold, while there is no doublet.

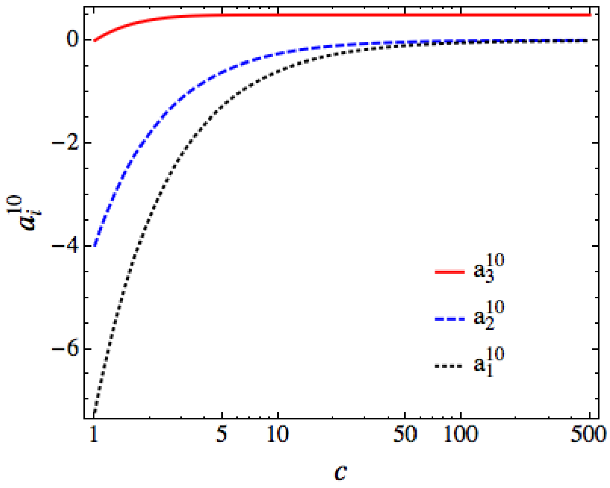

Figure 5.1: The bulk mass parameters a_i^{10} as functions of $c = \pi\Lambda/M_{KK}$ as required from the hierarchy in up-type quark masses. The dotted, dashed and solid lines correspond to a_1^{10}, a_2^{10} and a_3^{10} respectively.

Hence one finds only one linear combination of weak doublets from the $\mathbf{10}_H + \mathbf{120}_H$ which remains light and can be used as the MSSM Higgs doublets.

The above scalar content, *i.e.* $\mathbf{10}_H + \mathbf{120}_H$ fields as the light fields and $\mathbf{45}_H + \mathbf{126}_H + \overline{\mathbf{126}}_H$ as the heavy fields, is the most economic among the other possibilities [82] of light and heavy fields which provide a solution to the doublet-triplet splitting problem through the missing partner mechanism in SO(10). However, in 4D SO(10) theories, $\mathbf{10}_H$ and $\mathbf{120}_H$ alone do not lead to realistic charged fermion masses and quark mixing angles as first pointed out in [96] through a numerical study. The limited numbers of Yukawa couplings were found unable to reproduce appropriate hierarchies in the charged fermion masses. This is not the case in the present model as we show it later explicitly through a detailed numerical analysis. The zero-mode profiles of the different fermions generated from the compactification of an extra dimension in this model relax the tension that exists in pure 4D theories. Before we proceed to a quantitative analysis of the fermion mass spectrum in the above framework, we discuss the range of validity of the effective field theory approach on which this model is based.

5.3.4 Fixing the cut-off scale

The effective Yukawa couplings in 4D and the light neutrino mass matrix are as in the Kitano-Li original model, Eqs.(5.7–5.12), where now $Y_R = Y_{126}$ and $Y_{u,d,\nu,e}$ are linear combinations of Y_{10} and Y_{120}. All the Yukawa couplings $(Y_{10})_{ij}$, $(Y_{120})_{ij}$ and $(Y_{126})_{ij}$ are assumed to be of order one. The behaviour of the fermion zero-modes at $y = 0$ brane can be classified according to the values of the bulk mass parameters and c:

$$\text{for } a_i^r > 0 \text{ and } |a_i^r|c \gtrsim 1, \qquad n_i^r \approx \sqrt{2a_i^r}$$
$$\text{for } a_i^r < 0 \text{ and } |a_i^r|c \gtrsim 1, \qquad n_i^r \approx \sqrt{2|a_i^r|}\; e^{-|a_i^r|c}$$
$$\text{for } a_i^r \lesseqgtr 0 \text{ and } |a_i^r|c < 1, \qquad n_i^r \approx \frac{1}{\sqrt{c}}\;.$$

The parameter $c = \pi\Lambda/M_{KK}$ represents the cut-off scale in units of the KK scale and, to consistently describe the first few KK modes within our effective theory, we take $c \geq 10$. To neglect higher-dimensional operators contributing to fermion masses we will show that a value of c larger than 10 is required. To reproduce the large top Yukawa coupling we have to take $a_3^{10} = \mu_3 + k_X \approx y_t/2$. The hierarchy among the first, second and third generations of quarks can be reproduced by choosing $\mu_3 \approx y_t/2$ and $|k_X|, |\mu_{1,2}| \ll 1$. For example, the values of a_i^{10} required to generate $(n_1^{10}, n_2^{10}, n_3^{10}) = (\lambda^4, \lambda^2, 1)$ are shown in Fig. 5.1, as a function of c. For large c, a_3^{10} approaches to 0.5 while $|a_{1,2}^{10}|$ go like $1/c$. In terms of our input parameters, we approximately have $\mu_3 \approx 0.5$, while $|k_X|$, $|\mu_{1,2}|$ are $\mathcal{O}(1/c)$.

The parameter c, describing the gap between the cut-off scale and the KK scale, characterizes the domain of validity of our effective theory. Here we estimate how large c can be in our model and how small can be the ratio $M_{\rm GUT}/\Lambda$, which controls the non-leading contributions to the Yukawa interactions on the $y = 0$ brane. A relation between c and the GUT scale parameters such as k_X can be derived from the phenomenological requirement $|\mu_{1,2}| \sim |k_X| \approx 1/c$, needed to successfully fit the fermion spectrum. There are several scales relevant to the breaking of the grand unified symmetry SO(10): v_Φ, $\langle \mathbf{126}_H \rangle$, $\langle \overline{\mathbf{126}}_H \rangle$, $\langle \mathbf{45}_H \rangle$. In first approximation we make no distinction among them and we assume $M_{\rm GUT} \approx v_\Phi$. From Eq.(5.9) and using $|k_X| \sim 1/c$, one can express the VEV of the $\mathbf{45}_\Phi$ in terms of c as:

$$\begin{aligned} v_\Phi &= \left(\frac{1}{2g_4^2\pi R}\right)^{1/3}\left(\frac{\Lambda}{c}\right)^{\frac{2}{3}} \\ &= \left(\frac{1}{2g_4^2}\right)^{1/3}\frac{\Lambda}{c}\,, \end{aligned} \tag{5.20}$$

where we have used Eq.(5.3) and $c = \pi R\Lambda$. Considering a dimensionless coupling $g_4 \sim \mathcal{O}(1)$, one finds

$$\frac{M_{\rm GUT}}{\Lambda} \approx \frac{1}{c}\,, \tag{5.21}$$

showing that in the preferred region of parameter space the GUT scale and the KK scale are close to each other. To conveniently suppress the higher order contribution to the Yukawa interactions on the $y = 0$ brane we can take $c = 100$ and the cut-off Λ approximately corresponds to the Planck scale. The bulk mass parameters relevant to our analysis, m_i and v_Φ, are all around the GUT scale, except m_3 which should be relatively close to Λ.

5.4 Fermion mass relations and Numerical Analysis

We now derive the effective mass matrices of fermion zero modes in the model and discuss their viability through a detailed numerical analysis. As noted earlier, the light fields $\mathbf{10}_H$ and $\mathbf{120}_H$ respectively contain one and two pairs of MSSM-like Higgs doublets. As it is arranged by the missing partner mechanism, one pair of their linear combinations remains massless and plays the role of the MSSM Higgs doublets, namely H_u and H_d. Hence each of the doublets $H^1_{u,d} \in \mathbf{10}_H$ and $H^{2,3}_{u,d} \in \mathbf{120}_H$ has a component of H_u or H_d which can conveniently be parametrized in terms of mixing parameters α_i and $\bar{\alpha}_i$ such that $H^i_u = \alpha_i H_u$ and $H^i_d = \bar{\alpha}_i H_d$. The appropriate normalizations of H_u and H_d then require

$$\sum_{i=1}^{3} |\alpha_i|^2 = \sum_{i=1}^{3} |\bar{\alpha}_i|^2 = 1. \tag{5.22}$$

The VEVs of MSSM Higgs doublets are fixed by the electroweak symmetry breaking scale and are denoted by $\langle H_u \rangle = v \sin\beta$ and $\langle H_d \rangle = v \cos\beta$ with $v = 174$ GeV.

The brane Yukawa couplings of Dirac type fermions are obtained as the linear combinations of only two matrices, Y_{10} and Y_{120}, weighted by the appropriate Clebsch-Gordan (CG) coefficients and Higgs mixing parameters α_i, $\bar{\alpha}_i$ [108, 205]:

$$\begin{aligned} Y_u &= c^u_1 \alpha_1 Y_{10} + (c^u_2 \alpha_2 + c^u_3 \alpha_3) Y_{120}\ , \\ Y_d &= c^d_1 \bar{\alpha}_1 Y_{10} + (c^d_2 \bar{\alpha}_2 + c^d_3 \bar{\alpha}_3) Y_{120}\ , \\ Y_\nu &= c^\nu_1 \alpha_1 Y_{10} + (c^\nu_2 \alpha_2 + c^\nu_3 \alpha_3) Y_{120}\ , \\ Y_e &= c^e_1 \bar{\alpha}_1 Y_{10} + (c^e_2 \bar{\alpha}_2 + c^e_3 \bar{\alpha}_3) Y_{120}\ , \end{aligned} \tag{5.23}$$

where $Y_{u,d,e,\nu}$ are Yukawa matrices for up-type quarks, down-type quarks, charged leptons and Dirac neutrinos. The Y_{10} and Y_{120} are symmetric and anti-symmetric matrices respectively in generation space. The CG coefficients can be read as $c^u_1 = c^d_1 = c^e_1 = c^\nu_1 = 2\sqrt{2}$, $c^u_2 = c^d_2 = c^e_2 = c^\nu_2 = -2\sqrt{2}$ and $-3c^u_3 = -3c^d_3 = c^e_3 = c^\nu_3 = -2i\sqrt{6}$ [108]. A doublet H^3_d residing in $\mathbf{120}_H$ couples to the charged leptons and down-type quarks with different CG coefficients and provides the correction to the wrong mass relation $Y_d = Y_e$ predicted in the presence of only $\mathbf{10}_H$. The above Y_u, Y_d and Y_e are substituted in Eq.(5.10) to obtain the effective Yukawa matrices $\mathcal{Y}_u$, $\mathcal{Y}_d$ and $\mathcal{Y}_e$ of the charged fermion zero modes at the GUT scale.

The RH neutrinos get mass from the Yukawa interactions of $\mathbf{16}$ with $\overline{\mathbf{126}}_H$ when the SU(5) singlet in $\overline{\mathbf{126}}_H$ acquires a VEV. The mass matrix of the RH neutrino zero modes takes the form

$$M_R \equiv v_R\ F_1 Y_{126} F_1\ . \tag{5.24}$$

Note that $v_R \equiv \langle \overline{\mathbf{126}}_H \rangle$ also contributes in canceling the D-terms and is required to be close to the GUT scale in the absence of any fine tuning. The mass matrix for the light neutrinos generated by the canonical seesaw mechanism can be written as

$$M_\nu = -\frac{v^2 \sin^2\beta}{v_R}\ F_{\bar{5}}\ Y_\nu Y^{-1}_{126} Y^T_\nu\ F_{\bar{5}} \equiv -\frac{v^2 \sin^2\beta}{v_R}\ \mathcal{Y}_\nu\ . \tag{5.25}$$

The hierarchy in light neutrino masses is solely governed by the matrix $F_{\overline{5}}$ and, as we will see, it leads to relatively less hierarchical neutrinos in comparison to the charged fermions as arranged by SO(10) breaking in the bulk. Also, the origin of large (but not special values of) lepton mixing angles is apparent in this case. The seesaw mechanism is often seen as the origin of small hierarchies and large mixing in the neutrinos compared to the quark sector. For example, in the mechanism known as seesaw enhancement [206], such a difference can be realized if RH neutrinos have strong hierarchy or they are almost degenerate and Dirac neutrino Yukawas as hierarchical in structure as those of up-type quarks. We would like to emphasize here that hierarchy in the RH neutrino masses in this model is not responsible for enhancement in the leptonic mixing angles as can be seen from Eq.(5.25). In fact since the RH neutrino unifies with other fermions in SO(10), the hierarchy in their masses can be predicted from the common bulk mass parameters once the appropriate profiles of charged fermions are obtained.

In principle the $\mathrm{SU(2)_L}$ triplet contained in $\overline{\mathbf{126}}_H$ can generate an additional contribution to the neutrino masses through the so-called type II seesaw mechanism [30–32]. The coupling of $\overline{\mathbf{126}}_H$ to SO(10) multiplets containing light Higgs doublets induces an effective VEV for such a triplet of order $v^2 \sin^2\beta/M_{\mathrm{GUT}}$. In our model, the missing partner mechanism assumes the absence of a direct coupling among $\overline{\mathbf{126}}_H$ and the two SO(10) multiplets hosting the light Higgs doublets. For example, $\overline{\mathbf{126}}_H$ does not have an SO(10) invariant tree level couplings with two $\mathbf{10}_H$ or with two $\mathbf{120}_H$ or with $\mathbf{10}_H$ and $\mathbf{120}_H$. Further, any such couplings through higher-dimensional operators are assumed to be absent in the missing partner mechanism. Hence, we do not expect type II seesaw contribution to the light neutrino masses and consider only type I seesaw as the mechanism at work for the light neutrino masses in the following analysis.

5.4.1 Fitting the fermion mass spectrum: a viability test

The Yukawa matrices, $\mathcal{Y}_{u,d,e,\nu}$ which follow from Eqs.(5.10,5.23,5.25) are predicted at the GUT scale and we compare them with a representative set of data obtained by extrapolating the measured values of fermion masses and mixing parameters. This strategy has been largely followed in the studies based on varieties of SO(10) models in four dimensions, see [102–107] for examples. It is clear that an extrapolation over more than 14 orders of magnitude is potentially affected by large uncertainties. This is even more true in our model where SUSY breaking effects have been neglected. We are aware that the data we are going to fit at the GUT scale might not faithfully represent the low-energy experimental quantities and that the best fit values of the input parameters we will obtain might considerably change depending on the spectrum of the SUSY particles and the other heavy modes at the GUT scale. We are more interested in the performances of the present model, and we are confident that if it can successfully reproduce a representative set of data, it will also be successful if this set is modified to account for a more realistic framework.

The renormalization group evolution (RGE) of fermion masses in the MSSM primarily depends on the SUSY breaking scale $M_{\rm SUSY}$ and $\tan\beta$. In our numerical study we use, as an idealized set of data at the GUT scale, extrapolated values of the charged fermion masses and the quark mixing parameters from [53] which, in a 2-loop analysis, assumes an effective SUSY breaking scale $M_{\rm SUSY} = 0.5$ TeV and considers different values of $\tan\beta$. To assess the dependence on $\tan\beta$ we will focus on two cases: $\tan\beta = 10$ and $\tan\beta = 50$. For the neutrino masses and mixing angles, we use their low-energy values obtained by one of the recent global fits [166–168] ignoring the RGE effects from the low scale to the GUT scale. The running of the neutrino masses and mixing angles in the MSSM is known to be negligible in case of $\tan\beta \lesssim 30$. It remains small even for large $\tan\beta$ if the neutrino mass spectrum is hierarchical. As we show later in this section, the model analyzed here favours large $\tan\beta$ and strongly hierarchical spectrum for neutrinos with the lightest neutrino mass $\lesssim 0.005$ eV at the GUT scale. For such a hierarchical neutrino spectrum, the RGE running in the neutrino parameters can be considered negligible to a good approximation [207–209]. The GUT scale values of different observables we use in our analysis are listed in Table 5.1. We would like to note that the given extrapolated values do not take into account the threshold corrections in the fermion masses and mixing angles arising from the SUSY breaking. Estimation of such corrections requires precise knowledge of sparticle spectrum which depends on the exact details of SUSY breaking mechanism [210–212] which is not studied here. Further, the threshold corrections may also arise at the GUT scale which can be estimated only if the complete mass spectrum of all the GUT multiplets is known. As it is often done in the similar kind of analysis [102–107], we ignore these effects and assume that if the model under consideration can fit the idealized GUT scale data listed in Table 5.1 then it would also be compatible with the real data.

We now proceed to the details of fitting procedure. Let us first calculate the total number of free parameters in this model. As already mentioned, Y_{10} and Y_{126} are complex symmetric matrices and Y_{120} is a complex anti-symmetric matrix in generation space. Without loss of generality, we can absorb three phases from Y_{10} into the $\mathbf{16}_i$ by a suitable redefinition $\mathbf{16}_i \to e^{i\alpha_i}\mathbf{16}_i$. Hence Y_{10} can be parametrized in terms of 9 real parameters, namely 3 real diagonal elements and 3 complex off-diagonal ones. Y_{126} (Y_{120}) contains 12 (6) real parameters. All the Yukawa couplings are regarded as generic order-one quantities in this model and we constrain them within a narrow range, *i.e.* $0.5 \leq |(Y_{10})_{ij}|,\ |(Y_{126})_{ij}|,\ |(Y_{120})_{ij}| \leq 1.5$ with arbitrary phases. The parameters α_1, $\bar{\alpha}_1$ in Eq.(5.23) can be taken real without loss of generality and can be obtained from $\alpha_{2,3}$, $\bar{\alpha}_{2,3}$ using the normalization condition, Eq.(5.22). This leaves eight real parameters in α_i and $\bar{\alpha}_i$ with the constraints $|\alpha_i|,\ |\bar{\alpha}_i| < 1$, a VEV v_R in Eq. (5.25) and four real parameters in the profiles of zero-mode fermions as described in Eq. (5.9). In total, we have 27 $\mathcal{O}(1)$ real parameters as Yukawa couplings and other 13 real parameters which should correctly reproduce the 18 observables listed in Table 5.1 if the model is viable.

Observables	$\tan\beta = 10$	$\tan\beta = 50$
y_t	0.48 ± 0.02	0.51 ± 0.03
y_b	0.051 ± 0.002	0.37 ± 0.02
y_τ	0.070 ± 0.003	0.51 ± 0.04
m_u/m_c	0.0027 ± 0.0006	0.0027 ± 0.0006
m_d/m_s	0.051 ± 0.007	0.051 ± 0.007
m_e/m_μ	0.0048 ± 0.0002	0.0048 ± 0.0002
m_c/m_t	0.0025 ± 0.0002	0.0023 ± 0.0002
m_s/m_b	0.019 ± 0.002	0.016 ± 0.002
m_μ/m_τ	0.059 ± 0.002	0.050 ± 0.002
$\lvert V_{us}\rvert$	0.227 ± 0.001	
$\lvert V_{cb}\rvert$	0.037 ± 0.001	
$\lvert V_{ub}\rvert$	0.0033 ± 0.0006	
J_{CP}	0.000023 ± 0.000004	
$\Delta_S/10^{-5}$ eV^2	7.54 ± 0.26 (NO or IO)	
$\Delta_A/10^{-3}$ eV^2	2.44 ± 0.08 (NO)	2.40 ± 0.07 (IO)
$\sin^2\theta_{12}$	0.308 ± 0.017 (NO or IO)	
$\sin^2\theta_{23}$	0.425 ± 0.029 (NO)	0.437 ± 0.029 (IO)
$\sin^2\theta_{13}$	0.0234 ± 0.0022 (NO)	0.0239 ± 0.0021 (IO)

Table 5.1: The GUT scale values of the charged fermion masses and quark mixing parameters from [53] that we use in our analysis. The lepton mixing angles and solar and atmospheric mass differences are taken from a global fit analysis [167] ignoring the running effects. NO (IO) stands for the normal (inverted) ordering in the neutrino masses.

The values of the free parameters of the model are estimated using the χ^2 optimization technique which is widely used in [102–107] for similar kind of analysis. We define a χ^2 function

$$\chi^2 = \sum_i^n \left(\frac{P_i(x_1, x_2, .., x_m) - O_i}{\sigma_i}\right)^2 , \tag{5.26}$$

where P_i are the observable quantities derived from Eqs.(5.10, 5.23, 5.25) as complex nonlinear functions of the free parameters of the model. O_i and σ_i are the GUT scale central values and 1σ deviations respectively of the corresponding quantities listed in Table 5.1. The effective Yukawa matrices $\mathcal{Y}_{u,d,e,\nu}$ are numerically diagonalized and we obtain the diagonal Yukawas as the eigenvalues of $\mathcal{Y}_f$ for each sector. For example, the eigenvalues of $\mathcal{Y}_u$ correspond to y_u, y_c, y_t. The absolute values of the third generation Yukawas and appropriate ratios for the first two generations are included in the χ^2 to fit them to their extrapolated experimental values. The quark and lepton mixing parameters are also evaluated in a similar way. For simplicity, we include a ratio Δ_S/Δ_A in χ^2 instead of Δ_S and Δ_A individually. As can be seen from Eq.(5.25), such a ratio and lepton mixing parameters do not depend on v_R. The value of v_R can be obtained later from the fit using the absolute scale of atmospheric neutrino oscillation. This allows us to remove one observable and one free parameter from the fit. The χ^2 function contains only dimensionless quantities. It is then numerically minimized using the downhill-simplex algorithm incorporated in the software tools MINUIT developed by CERN to determine the best fit values of the parameters x_i. From the fitted parameters, one can derive the predictions for the various observables which have not been measured yet such as Dirac CP phase in the leptonic sector, the lightest neutrino mass, the effective mass of neutrinoless double beta decay.

As a preliminary step we fit all the 39 free parameters of the model using 17 observables mentioned above. Even though many of the free parameters are restricted within narrow ranges of $\mathcal{O}(1)$ or should face additional constraints like the one of Eq.(5.22), the number of free parameters are significantly larger than the number of observables. Nevertheless, considering the complex and non-linear dependence of the observable quantities from the input parameters, it is not completely evident that the model can successfully fit the data. We have carried out the χ^2 minimization for two different data set corresponding to $\tan\beta = 10$ and 50. Also, each case is analyzed for different ordering of the neutrino masses, *i.e.* normal (NO) and inverted (IO).

Results. The results of fits are reported in Table 5.2. We obtain very good fit to the data in case of $\tan\beta = 50$ and for both the NO and IO in neutrino masses. In particular, the NO case results into a very good fit in which all the observables from the theory fall well into the experimentally allowed range, as can be seen from Table 5.2. The predictions for various observables obtained at the best fit are also listed in the table. The set of input parameters at the minimum of χ^2 are collected in Appendix E.1 for both the cases. From the best fit in NO case, one obtains a hierarchical profile matrix $F_{10} \approx \texttt{diag.}(\lambda^{3.7}, \lambda^{2.4}, 1)$ for 10-plets and a relatively less hierarchical $F_{\bar{5}} \approx \texttt{diag.}(\lambda^{1.5}, \lambda^{0.9}, 1)$ for $\bar{5}$-plets as it was expected from the SO(10) breaking effects in the bulk. Such an effect is mostly due to the parameter k_X which contributes universally in different flavours. As it can be seen from the best fit values reported in the Appendix, k_X is required to be $\mathcal{O}(\mu_{1,2})$ in order to distinguish between the mass hierarchies among the first and second generations of fermions residing in 10 and $\bar{5}$ of SU(5). As a result, the bulk mass parameter of the third generation μ_3 dominates over k_X and the effective bulk masses a_3^r are nearly equal for both $r = 10$ and $r = \bar{5}$. Thus an approximate $t - b - \tau$ Yukawa unification at the GUT scale is enforced. As it is well known, the bottom and tau Yukawas unify with that of top quark for large $\tan\beta \geq 45$ [58, 59] and hence the model provides a good fit to the data only for $\tan\beta = 50$. We obtain very poor fits for $\tan\beta = 10$ corresponding to $\chi^2_{\rm min} \approx 110$ for NO and $\chi^2_{\rm min} \approx 280$ for IO. The large values of the $\chi^2_{\rm min}$ in these cases are mainly due to the top, bottom and tau Yukawas, which cannot be fitted simultaneously to their extrapolated values.

The best fit obtained for IO and $\tan\beta = 50$ is also shown in the Table 5.2. The

	Normal ordering		Inverted ordering	
Observable	**Fitted value**	**Pull**	**Fitted value**	**Pull**
y_t	0.51	0	0.54	1.00
y_b	0.37	0	0.37	0
y_τ	0.51	0	0.47	-1.00
m_u/m_c	0.0027	0	0.0031	0.67
m_d/m_s	0.051	0	0.045	-0.86
m_e/m_μ	0.0048	0	0.0048	0
m_c/m_t	0.0023	0	0.0023	0
m_s/m_b	0.016	0	0.015	-0.50
m_μ/m_τ	0.050	0	0.049	-0.50
$\|V_{us}\|$	0.227	0	0.227	0
$\|V_{cb}\|$	0.037	0	0.038	1.00
$\|V_{ub}\|$	0.0033	0	0.0030	-0.50
J_{CP}	0.000023	0	0.000021	-0.51
Δ_S/Δ_A	0.0309	0	0.0320	0.73
$\sin^2\theta_{12}$	0.308	0	0.309	0.06
$\sin^2\theta_{23}$	0.425	0	0.435	-0.07
$\sin^2\theta_{13}$	0.0234	0	0.0237	-0.10
χ^2_{min}		≈ 0		≈ 5.75
	Predicted value		**Predicted value**	
$m_{\nu_{\text{lightest}}}$ [meV]	0.08		2.15	
$\|m_{\beta\beta}\|$ [meV]	1.63		30.4	
$\sin\delta^l_{CP}$	0.265		0.510	
M_{N_1} [GeV]	3.85×10^6		1.13×10^4	
M_{N_2} [GeV]	9.31×10^7		3.06×10^6	
M_{N_3} [GeV]	2.19×10^{14}		2.02×10^{13}	
υ_R [GeV]	0.05×10^{16}		0.18×10^{16}	

Table 5.2: Results from numerical fit corresponding to minimized χ^2 for normal (NO) and inverted ordering (IO) in neutrino masses. The fit is carried out for the GUT scale extrapolated data given in Table 5.1 for $\tan\beta = 50$. The input parameters are collected in Appendix E.1.

minimized value of χ^2 is relatively large compared to the one obtained for NO but is acceptable as all the observables are fitted within the 1σ range of their experimental values. Note that the fitted profiles of the light neutrinos, *i.e.* $F_{\bar{5}} \sim \texttt{diag}.(\lambda^{0.8}, \lambda^{0.4}, 1)$, still follows the normal ordering structure (with a slightly less hierarchical structure compared to that of NO case) while $\mathcal{O}(1)$ Yukawas in Y_{126} conspire to create inverted ordering in the neutrino masses. The mismatch between the hierarchies in neutrinos and charged fermions can be attributed more to a tuning of the $\mathcal{O}(1)$ Yukawa couplings, rather than to an effect of the zero-mode profiles. We expect that such a solution is very sensitive to the Yukawa parameters in Y_{126} and that even small deviations from their best fit values can significantly raise the $\chi^2_{\rm min}$. We investigate this issue in the following subsection.

5.4.2 Anarchical Yukawas: a test of naturalness

So far the analysis implies that the model under consideration predicts approximate $t-b-\tau$ Yukawa unification which is compatible only with large values of $\tan\beta$. Moreover both the normal and inverted ordering in the neutrino masses seem to be viable as indicated by the best fit solutions. We do not know yet whether a successful fit in the two cases requires a special tuning of the $\mathcal{O}(1)$ Yukawa parameters or not. In the present approach this question is relevant, since the whole construction is based on the idea of anarchy in the Yukawa sector: the hierarchical pattern of fermion masses and mixing angles is entirely due to the zero-mode profiles, while the Yukawa couplings on the brane have no structure. If special relations between the $\mathcal{O}(1)$ Yukawa parameters were needed in order to reproduce the data, this would represent a fine-tuning problem of our model, which cannot appeal to symmetry or dynamical principles to justify such relations. If, on the contrary, the present model were natural, we would expect that a successful explanation of fermion masses and mixing angles should not depend very much on the specific choices of $\mathcal{O}(1)$ parameters.

To investigate this feature, we have repeated the above analysis with some changes. We randomly varied each of the complex elements in Y_{10}, Y_{126} and Y_{120} such that $|Y_{ij}| \in [0.5, 1.5]$ and $\texttt{arg}(Y_{ij}) \in [0, 2\pi]$, using flat distributions for both. For a given set of random Yukawa couplings, the χ^2 is minimized versus the remaining 12 parameters (4 bulk masses and 8 Higgs mixing parameters α_i, $\bar{\alpha}_i$) using the 17 observables described earlier. Unlike the previous case we investigate 10^5 samples of random $\mathcal{O}(1)$ Yukawas and perform a χ^2 minimization for each case. The analysis is carried out for $\tan\beta = 50$ and for both NO and IO, as only these cases were found in good agreement with the GUT scale data in the previous analysis, where also the Yukawa couplings were fitted.

Results. The results of this procedure are displayed in Fig. 5.2 where we show the distributions of $\chi^2_{\rm min}/\nu$, $\nu = 5$ being the number of independent degrees of freedom (d.o.f.) in the fit. The distribution for NO is clearly peaked at lower values of $\chi^2_{\rm min}/\nu$, with respect to the one for IO. In Table 5.3 we report the number of successful cases for different threshold values of $\chi^2_{\rm min}/\nu$ together with the goodness of fit measured in terms of $p-$values. The threshold $p \geq 0.05$ is often considered as an acceptable value for the statistical validity of a fit. As it can be seen from Table 5.3, in the NO case p is larger than 0.05 in about one percent of the generated samples, while in the IO case only one over total 10^5 samples reaches the modest p-value of 0.05. Hence the NO turns out as a more natural choice in this model.

In the NO case one percent can be regarded as the size of the required tuning to

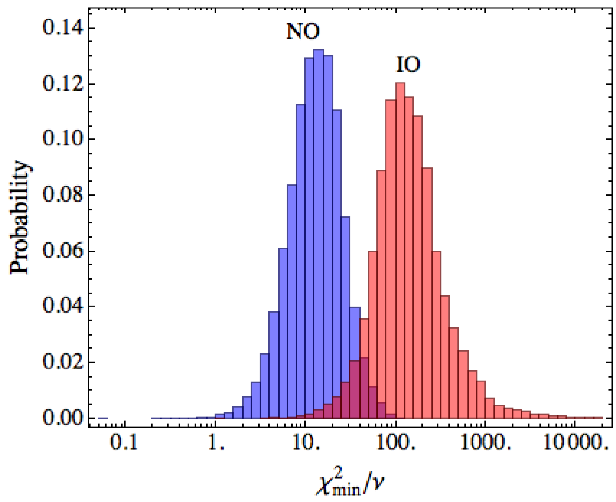

Figure 5.2: The distributions of minimized χ^2/ν for NO and IO in neutrino masses and for $\tan\beta = 50$.

$p-$value	0.50	0.10	**0.05**	0.001
χ^2_{min}/ν (for $\nu = 5$)	≤ 0.87	≤ 1.85	$\leq \mathbf{2.21}$	≤ 4.10
successful cases (NO)	0.1%	0.7%	**1.2%**	5.6%
successful cases (IO)	$< 10^{-3}\%$	$< 10^{-3}\%$	$\mathbf{10^{-3}\%}$	0.01%

Table 5.3: The fraction of successful events obtained for different $p-$values from random samples of $\mathcal{O}(1)$ Yukawa couplings in case of normal and inverted ordering in the neutrino masses.

reproduce the data within the framework of anarchy. It is clear that this number has no absolute meaning and could only be useful if compared with analogous numbers obtained by analyzing other models with a similar approach. A success rate of order 0.01 is a typical outcome in this kind of analysis for the most successful models [196, 213, 214].

The probability distributions for the bulk mass parameters obtained in 1.2% of the NO cases corresponding to $p \geq 0.05$ are shown in Fig. 5.3. One finds $\mu_3 > \mu_2 \geq \mu_1$ in most of the cases as expected and k_X turns out to be $\mathcal{O}(\mu_{1,2})$. A few cases described by smaller peaks in the distributions of μ_1 and μ_2 corresponds to $\mu_1 > \mu_2$. However, such cases are equivalent to the cases with $\mu_1 < \mu_2$ as one can always interchange $\mathbf{16}_1 \leftrightarrow \mathbf{16}_2$ by interchanging μ_1 and μ_2 and also the first and second rows and columns of all the Yukawa coupling matrices on the brane. Such a transformation on Yukawa matrices still preserves their anarchical structure and both these pictures lead to the same physical scenario. As it can be seen from Fig. 5.3, the preferred values of all the bulk masses remain well below the cut-off scale, and they do not endanger the validity of the effective theory. The $k_X < 0$ leads to a relatively weak hierarchy among the down-type quarks and charged leptons in comparison to that in the up-type quarks. From the most probable values of μ_i and k_X of Fig. 5.3 we get,

$$F_{10} \simeq \lambda^{0.4}\,\mathtt{diag}(\lambda^{4.1}, \lambda^{2.2}, 1) \quad \text{and} \quad F_{\bar{5}} \simeq \lambda^{0.3}\,\mathtt{diag}(\lambda^{1.5}, \lambda^{0.7}, 1). \tag{5.27}$$

The above profiles of zero modes provide a quantitative understanding of the differences between the quarks and lepton masses and mixing patterns. The successful cases cor-

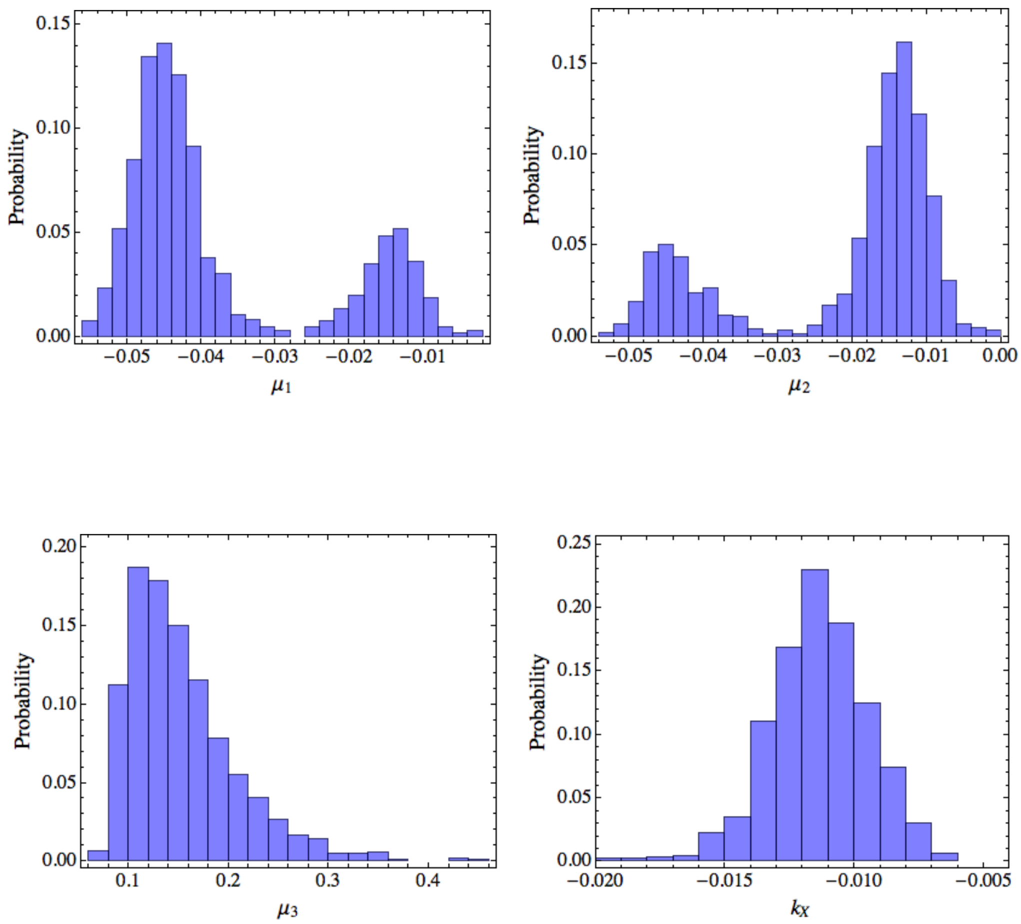

Figure 5.3: The distributions of bulk mass parameters fitted with $\chi^2_{\rm min}/\nu < 2.21$ in case of NO and $\tan\beta = 50$.

responding to $\chi^2_{\rm min}/\nu < 2.21$ can also be used to derive the predictions for the other observables in the lepton sector. The probability distributions for the leptonic Dirac CP phase, the lightest neutrino mass and the effective mass of neutrinoless double beta decay $|m_{\beta\beta}|$ are shown in Fig. 5.4. One finds an almost uniform distribution in δ^l_{CP} and the entire range in CP phase is allowed by the model. The lightest neutrino mass is restricted to be $\lesssim 5$ meV corresponding to a hierarchical neutrino mass spectrum while

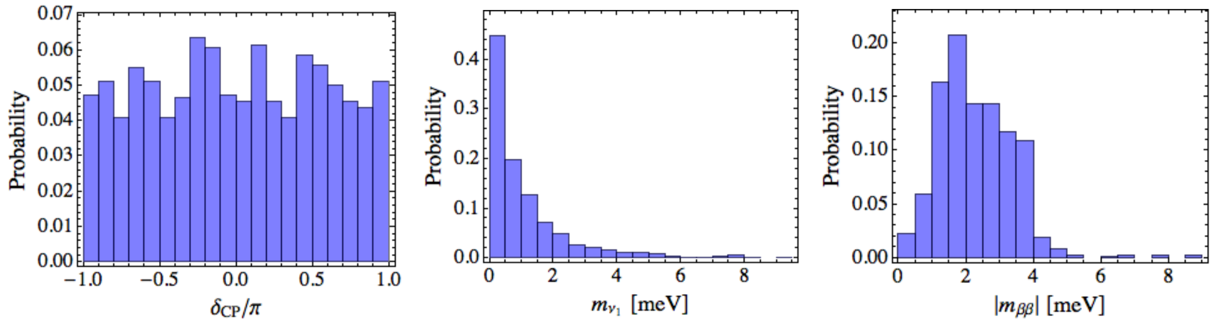

Figure 5.4: The predictions for various observables obtained for $\chi^2_{\rm min}/\nu < 2.21$ in case of NO and $\tan\beta = 50$.

$|m_{\beta\beta}|$ is predicted in the range 0.1-5 meV. Both these predictions are far from the sensitivity of current generation experiments and any positive signal in these experiments would essentially rule out the model.

The predictions for the RH neutrino masses and the VEV of $\overline{\mathbf{126}}_H$ are displayed in Fig. 5.5. The bulk masses of the singlets in $\mathbf{16}_i$ are predicted from the fitted values of μ_i and k_X. From their most probable values, we obtain

$$F_1 \simeq \lambda^{0.6}\ \mathtt{diag}(\lambda^{7.0}, \lambda^{5.0}, 1)\ . \tag{5.28}$$

This results into an extremely hierarchical mass spectrum for the RH neutrinos corresponding to $M_{N_1} \approx \lambda^{15} v_R$, $M_{N_2} \approx \lambda^{11} v_R$ and $M_{N_3} \approx \lambda v_R$, as shown in the left panel of Fig. 5.5. This is understood as effect of the large U(1)$_{\rm X}$ charge of RH neutrinos which generates very large corrections to the bulk mass of the first and the second generation. As explained earlier, the masses of the RH neutrinos do not play any role in the seesaw mechanism but they can be important for leptogenesis. For a hierarchical mass spectrum of RH neutrinos, a successful thermal leptogenesis requires the mass of the lightest RH neutrino $M_{N_1} \gtrsim 10^9$ GeV [45, 46] in a standard flavour independent scheme. When flavour effects are considered, it is possible to generate a sufficient lepton asymmetry through the decay of the next-to-lightest neutrinos if 10^{12} GeV $\gtrsim M_{N_2} \gtrsim 10^9$ GeV and $M_{N_1} \ll 10^9$ GeV as suggested in [215–217]. Both these alternatives cannot be realized in this model, which predicts $M_{N_1} \ll M_{N_2} < 10^9$ GeV.

The scale of atmospheric neutrino oscillation requires the VEV of $\overline{\mathbf{126}}_H$ at least an order of magnitude below $M_{\rm GUT}$, as it can be seen from Fig. 5.5. This is compatible with the D-term cancellation condition, Eq.(5.17). As can be seen from Fig. 5.3, the viable fits to the fermion masses and mixing angles require $v_\Phi^{3/2} = k_X\Lambda/(\sqrt{2}g_5) < 0$. Hence the VEV of $\mathbf{126}_H$ can cancel the D-term in Eq.(5.17) even if $|\langle\overline{\mathbf{126}}_H\rangle| \ll |\langle\mathbf{126}_H\rangle| \sim M_{\rm GUT}$.

We conclude this section with a comment on the choice of the $\mathcal{O}(1)$ Yukawa parameters. For the above analysis, we have randomly chosen them from a flat distribution of points

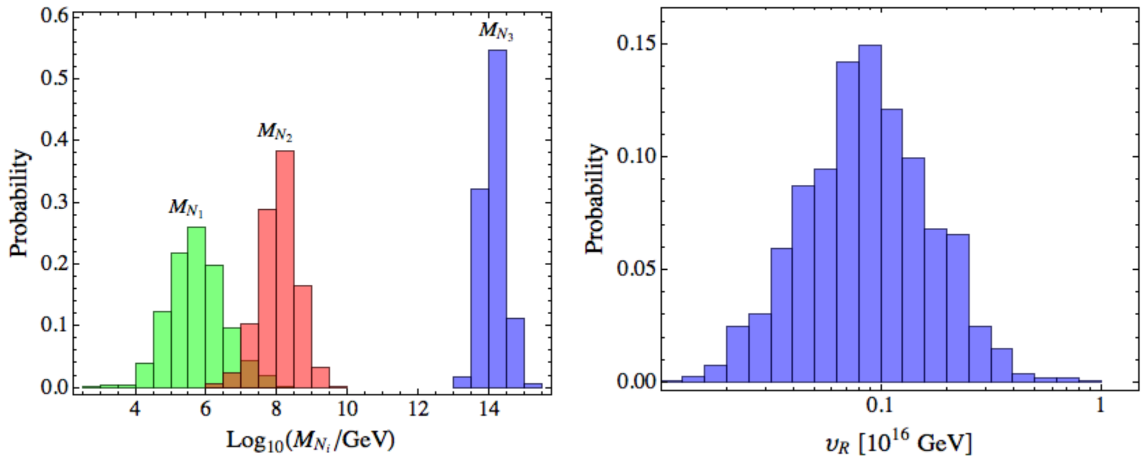

Figure 5.5: The predictions for the masses of RH neutrinos and the VEV of $\overline{\mathbf{126}}_H$ obtained for $\chi^2_{\text{min}}/\nu < 2.21$ in case of NO and $\tan\beta = 50$.

residing on the disc of inner radius 0.5 and outer radius 1.5 in a complex plane. To assess the dependence of our results on the criteria for selecting the Yukawa parameters, we have repeated a similar analysis for random Yukawa couplings residing in the box of vertices $(1+i, -1+i, -1-i, 1-i)$ in a complex plane. The results are shown in Fig. 5.6 where we compare the probability distributions obtained for the two choices of Yukawa parameters. The obtained distributions are almost indistinguishable and the new choice leads to nearly the same results as the old one. The overall results and predictions derived in this subsection are therefore robust and should stand for similar choices of $\mathcal{O}(1)$ parameters.

5.5 Conclusions

We have realized a viable model, where the approach to the flavour problem via anarchical Yukawas, combined with wave-function renormalizations distinguishing generations and fermion species, has been proven successful even within the framework of grand unification. The presence of an extra dimension provides a useful framework to express the hierarchies associated to the wave-function renormalizations in terms of different localizations of the zero-mode profiles. While this realization was well known within the SM, its extension to SO(10) was not trivial at all. Before the SO(10) breaking, all members of a fermion generation, hosted in a **16** representation, have the same zero-mode profile, which depends upon an SO(10) invariant bulk mass term. In the Kitano-Li model SO(10) is broken down to the direct product of SU(5) and $\text{U}(1)_\text{X}$, by the VEV of an SO(10) adjoint that lives in the bulk and has gauge coupling to matter supermultiplets. In this way different bulk

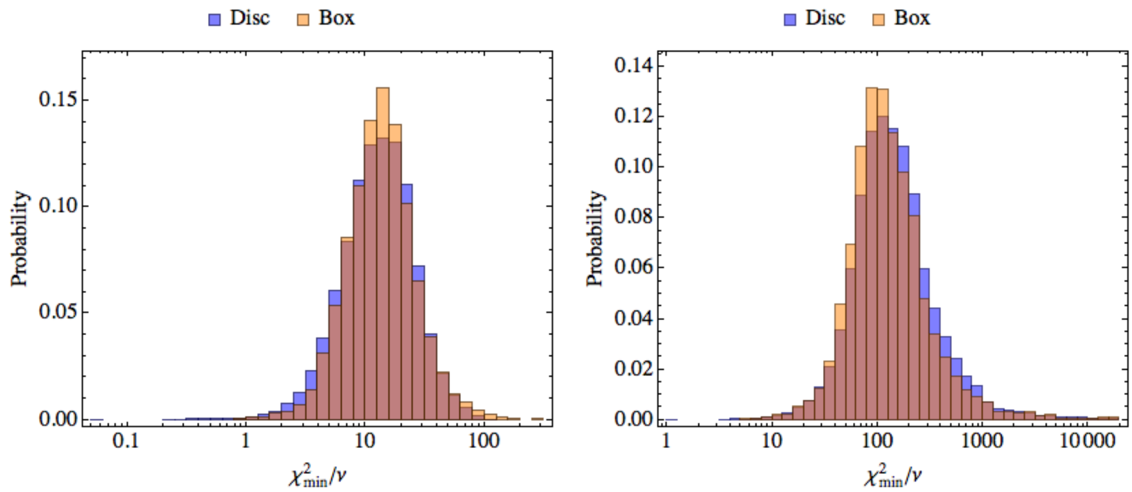

Figure 5.6: A comparison between the minimized χ^2 distributions obtained from the two different distribution of random $\mathcal{O}(1)$ Yukawa couplings (see text for the details). The left (right) panel corresponds to the NO (IO) case and $\tan\beta = 50$.

mass terms for the SU(5) multiplets residing in the same **16** representation are generated, with corresponding different zero-mode profiles.
We have formulated a new model maintaining the core of the Kitano-Li mechanism. To start with, we have modified the original model in such a way that fermion masses and mixing angles are dominated by Yukawa interaction terms of the same dimensionality, while in the original model non-renormalizable operators of different mass dimensions had to provide comparable contributions to achieve a realistic description. We have also included a set of Higgs multiplets that allows for a solution to the doublet-triplet splitting problem through the missing partner mechanism. We have explicitly specified the set of interactions needed in order to implement such a mechanism. Finally, we have tested the validity of the model by realizing a series of fits to an idealized set of 17 data, obtained by naively extrapolating fermion masses and mixing angles from low energy to the GUT scale. Our model depends on 27 anarchical Yukawa couplings, 8 parameters characterizing the light Higgs combinations and 4 parameters that describe the fermion bulk masses. In a first fit we left all parameters to vary freely and we obtained an excellent agreement with data for both the cases of normal and inverted ordered neutrino spectrum, but only for large values of $\tan\beta$. Despite the large number of free parameters, we consider such an agreement not completely trivial, given the fact that 35 of our parameters can vary in a very limited interval close to one and that only the 4 parameters describing the bulk masses are responsible for all the observed hierarchies in the fermion spectrum. In a second stage, to detect a possible fine-tuning among the anarchical Yukawa couplings, we have modified our numerical analysis by first generating a random sample of $\mathcal{O}(1)$ Yukawa

couplings and by subsequently fitting the remaining 12 Higgs and bulk parameters. This procedure have been iterated to obtain a distribution of minimum χ^2 values. We see a clear difference between the cases of normal ordering and inverted ordering in the neutrino masses. While in the inverted ordering case we need about 10^5 samples to reach a p-value close to 0.05, in the normal hierarchy case in about one percent of the cases we have $p > 0.05$. This unambiguously indicates that our model needs a severe fine-tuning of the "anarchical" parameters in the case of inverted ordered neutrino spectrum while the normally ordered one is accommodated much more naturally. We also verified that these results are stable versus changes in the drawing of the anarchical parameters.

Concerning predictions, we have found no preference for any particular value of the leptonic Dirac CP phase. The lightest neutrino mass should lie below 5 meV, corresponding to a hierarchical neutrino mass spectrum while $|m_{\beta\beta}|$ is predicted in the range 0.1-5 meV. Any positive signal in the current generation of experiments aiming at measuring neutrino masses or $|m_{\beta\beta}|$ in the lab would essentially rule out the model. The hierarchy in the right handed neutrino spectrum is very pronounced and the corresponding mass distributions are peaked around 10^6 GeV, 10^8 GeV and 10^{14} GeV. As a consequence, thermal leptogenesis cannot be responsible for the observed baryon asymmetry in our model.

We finally remark that our analysis is limited by the assumption of exact N=1 SUSY. A fully realistic approach would require to specify a SUSY breaking mechanism. This would have allowed to reduce the uncertainty in the extrapolation of fermion masses and mixing angles from low-energy to the GUT scale, at the cost of a much bigger model dependence. We chose to keep this uncertainty working in the most general model-independent framework. Our purpose was testing the general performance of our model, while keeping the question about N=1 SUSY breaking to be developed as a side issue. In Chapt. 7 we will review some attempts to include a SUSY breaking suitable for our particolar model.

Chapter 6

A flavour model in 5D SO(10) through Pati-Salam

This chapter is dedicated to the construction of a second model of SO(10) in 5D and it is based on our publication, ref. [218]. This model is an alternative realization of the framework already presented in the previous chapters to address the flavour problem in SO(10) GUT by means of hierarchical profiles in extra dimensions. In this case we focus also on another cumbersome aspect of SO(10) models, that is the procedure for gauge symmetry breaking (cf. Sect.3.4.4): we simplify this aspect exploiting other tools offered by the presence of extra dimension. Indeed, adding a new spatial dimension compactified on an orbifold S^1/Z_2, one can break the gauge symmetry by selecting appropriate parities of the gauge fields [83]. Only the gauge fields with even parity survive on the 4-dimensional fixed points (or branes) leaving the corresponding gauge symmetry unbroken. In this way, the breaking of SO(10) down to the Pati-Salam (PS) gauge symmetry [219], namely $\mathrm{SU}(4)_{\mathrm{C}}\times\mathrm{SU}(2)_{\mathrm{L}}\times\mathrm{SU}(2)_{\mathrm{R}}$, have been studied in [87, 220, 221]. Once the symmetry is broken through the boundary conditions, one has the freedom to introduce on the branes scalar multiplets transforming only under the unbroken symmetry. This simplifies a lot the field content of the Higgs sector on the brane with reduced symmetry and, as it was shown in [83], it offers an elegant solution to the DT splitting problem.
We combine this idea together with the approach to the flavour problem in GUT, which, as seen, can greatly benefit from the presence of an extra compact dimension. With this purpose we implement again the core mechanism of the Kitano-Li model [15] explained in the previous chapter and successfully tested in our first model [16].

We construct a 5D SO(10) model with N=1 SUSY in which the extra dimension is compactified on an orbifold $S^1/(Z_2 \times Z_2')$ [200]. The reflection under Z_2 breaks one of the SUSY while Z_2' is used to break SO(10) down to the PS gauge symmetry. Thus the effective symmetry on one of the two branes is the PS one with N=1 SUSY. The further breaking of PS to the SM gauge symmetry is implemented by introducing appropriate fields on the brane. Fermions are described by **16** dimensional representations living in the bulk. As a consequence of the breaking of SO(10) down to the PS symmetry the fermion zero modes fall into multiplets of the PS gauge group, namely $(4, 2, 1)$ or $(\bar{4}, 1, 2)$, depending on the Z_2' parity assignment, and a doubling of matter fields per each generation is required. This has the advantage of allowing different profiles for the zero modes of $(4, 2, 1)$ or $(\bar{4}, 1, 2)$ in each generation. At this stage quark-lepton unification inherited from the PS symmetry still holds, and a new independent source of breaking of the PS symmetry is required. This is obtained by the vacuum expectation value (VEV) of an adjoint scalar multiplet that spontaneously breaks SO(10) into SU(5)×U(1)$_\mathrm{X}$ giving rise to a distinct set of zero mode profiles. As seen, such a breaking is flavour blind, introduces only one new parameter and contributes with different weights to lepton and quarks bulk masses. The model presented here provides a simple and viable alternative to the modified Kitano-Li (KL) model constructed by us in [16], based on the framework proposed in [15]. In comparison to that, the current model implements in a simpler way the GUT symmetry breaking and requires representations for the scalar fields with smaller dimensionality. The DT splitting problem does not arise since no color triplet is associated with the weak doublets introduced by us. The simplified scalar spectrum on the brane reduces the number of non-anarchic free parameters in the theory compared to the modified KL model, providing in principle a more predictive framework for the description of the fermion mass spectrum. While the number of independent parameters is still quite large, not allowing for precision tests of the model, we find that all fermion masses and mixing angles can be described with all the fundamental parameters of the theory of $\mathcal{O}(1)$.

The organization of the chapter is as follows. We describe the model including the dynamics on bulk and on the branes in the next section. We then discuss how the fermion mass relations arise in the model in section 6.2. A qualitative comparison between the alternative models is given in this section. In section 6.3, we provide a detailed numerical analysis and a naturalness test of the various options and discuss the results and predictions for the different observables. The study is finally concluded in section 6.4.

6.1 Basic setup of the model

The model is based on a supersymmetric SO(10) grand unified theory in five space-time dimensions [87, 220, 221]. The extra spatial dimension is compactified on an orbifold $S^1/(Z_2 \times Z_2')$ where S^1 represents a circle of radius R. A periodic coordinate y parametrizes the circle and the action of the parity Z_2 (Z_2') is defined by $y \to -y$ ($y' \to -y'$), where $y' \equiv y - \pi R/2$. Points of the circle related by either Z_2 or Z_2' are identified. The interval between the two fixed points $y = 0$ and $y = \pi R/2$ can be considered as the fundamental region. The other fixed points $y = \pi R$ and $y = -\pi R/2$ are identified with the points $y = 0$ and $y = \pi R/2$, respectively. A generic bulk field $\phi(x, y)$ can be categorized by its transformation properties under $Z_2 \times Z_2'$. Denoting by P and P' the parities under Z_2 and Z_2' respectively, a field $\phi_{P,P'}(x, y)$ with given parities (P, P') can be expanded in terms of

Fourier series as follows [87]:

$$
\begin{aligned}
\phi_{++}(x,y) &= \sqrt{\frac{1}{2\pi R}}\phi^{0}_{++}(x) + \sqrt{\frac{1}{\pi R}}\sum_{n=1}^{\infty}\phi^{2n}_{++}(x)\cos\left(\frac{2ny}{R}\right) \ , \\
\phi_{+-}(x,y) &= \sqrt{\frac{1}{\pi R}}\sum_{n=0}^{\infty}\phi^{2n+1}_{+-}(x)\cos\left(\frac{(2n+1)y}{R}\right) \ , \\
\phi_{-+}(x,y) &= \sqrt{\frac{1}{\pi R}}\sum_{n=0}^{\infty}\phi^{2n+1}_{-+}(x)\sin\left(\frac{(2n+1)y}{R}\right) \ , \\
\phi_{--}(x,y) &= \sqrt{\frac{1}{\pi R}}\sum_{n=0}^{\infty}\phi^{2n+2}_{--}(x)\sin\left(\frac{(2n+2)y}{R}\right) \ .
\end{aligned}
\tag{6.1}
$$

Here $n = 0, 1, 2, ...$ denotes the different 4D Kaluza-Klein (KK) modes of a given bulk field. In the free theory, upon the compactification, a 4D component $\phi^k(x)$ acquires a mass k/R, an integer multiple of the compactification scale $1/R$. Only the field with $(P, P') = (+,+)$ contains a massless mode and it is non-vanishing on both the branes. The field ϕ_{+-} (ϕ_{-+}) vanishes on the $y = \pi R/2$ ($y = 0$) brane, while ϕ_{--} vanishes on both the branes.

The theory possesses N=1 SUSY in 5D which corresponds to N=2 SUSY in 4D (cf. Sect.4.6). We utilize the Z_2 symmetry to break N=2 SUSY down to the N=1 SUSY in 4D [200]. In our set-up, the matter and gauge fields propagate in the bulk. We introduce a 16-dimensional hypermultiplet $\mathbf{16}_{\mathcal{H}}$ for each SM generation of fermions and 45-dimensional vector-multiplet $\mathbf{45}_{\mathcal{V}}$ under N=1 SUSY in 5D. As we have seen, in 4D these correspond to a pair of N=1 chiral multiplets for $\mathbf{16}_{\mathcal{H}} \equiv (\mathbf{16}, \mathbf{16}^c)$, and a vector and chiral multiplets for $\mathbf{45}_{\mathcal{V}} \equiv (\mathbf{45}_V, \mathbf{45}_\Phi)$. The breaking of N=2 SUSY down to the N=1 SUSY in 4D is achieved by assigning even Z_2 parity to the $\mathbf{16}$ and $\mathbf{45}_V$ multiplets and odd Z_2 parity to their superpartners $\mathbf{16}^c$ and $\mathbf{45}_\Phi$, as in the previous model.

The Z_2' symmetry is used to break the SO(10) gauge symmetry down to the PS symmetry [87, 220, 221]. The PS gauge symmetry is isomorphic to SO(6)×SO(4) and hence the parity assignments with respect to P' should be appropriately chosen such that the generators of SO(6)×SO(4) remain unbroken. Under SO(6)×SO(4), the two index anti-symmetric SO(10) representation $\mathbf{45}$ decomposes as $(15, 1) + (1, 6) + (6, 4)$. The first two submultiplets, which are the adjoint of SO(6)×SO(4), are taken even and the last one is chosen odd under Z_2'. This assignment breaks SO(10) down to the PS group and set to zero all the gauge fields, other than those of the PS group, on the $y = \pi R/2$ brane. The gauge interactions on this brane respects only the PS gauge symmetry. On the $y = 0$ brane, the full $\mathbf{45}_V$ exists but only the PS gauge fields have massless modes. For these reasons, we call the $y = \pi R/2$ brane "a PS brane" while the $y = 0$ brane "an SO(10) brane".

Once the P' assignments for the gauge fields are chosen as above, the ones for the matter submultiplets follow from the invariance of the gauge interactions. Under the PS symmetry, the SO(10) $\mathbf{16}$-plet decomposes as $(4, 2, 1) + (\bar{4}, 1, 2)$. It can be seen from the

5D N=1	4D N=1	4D N=1 in PS	(P, P')
$\mathbf{45}_{\mathcal{V}}$	$\mathbf{45}_V$	$(15,1,1)+(1,3,1)+(1,1,3)$	$(+,+)$
		$(6,2,2)$	$(+,-)$
	$\mathbf{45}_\Phi$	$(15,1,1)+(1,3,1)+(1,1,3)$	$(-,-)$
		$(6,2,2)$	$(-,+)$
$\mathbf{16}_{\mathcal{H}}$	$\mathbf{16}$	$(4,2,1)$	$(+,+)$
		$(\bar{4},1,2)$	$(+,-)$
	$\mathbf{16}^c$	$(4,1,2)$	$(-,+)$
		$(\bar{4},2,1)$	$(-,-)$
$\mathbf{16}'_{\mathcal{H}}$	$\mathbf{16}'$	$(4,2,1)$	$(+,-)$
		$(\bar{4},1,2)$	$(+,+)$
	$\mathbf{16}'^c$	$(4,1,2)$	$(-,-)$
		$(\bar{4},2,1)$	$(-,+)$

Table 6.1: The parities P and P' of different SO(10) multiplets and their Pati-Salam submultiplets.

gauge interactions that $(4,2,1)$ and $(\bar{4},1,2)$ must have opposite P' charges[1] . Therefore only one of the two possesses zero modes and is different from zero on the $y = \pi R/2$ brane. To accommodate zero modes for a full SM fermion generation we have to double the **16**-plet [87, 220, 221] and assign mutually opposite P' charges for the PS submultiplets. Therefore, we introduce $\mathbf{16}'_{\mathcal{H}}$ per each generation in the bulk with P (P') equal (opposite) to that of the $\mathbf{16}_{\mathcal{H}}$. Notice that this doubling destroys the full quark-lepton unification achieved with only one copy of **16**-plet per generation. We summarize the P and P' assignment of all the bulk fields in Table 6.1.

We now discuss the symmetry breaking pattern in the model. The SO(10) symmetry is broken down to the PS gauge symmetry on the branes by the action of Z'_2. We use the mechanism originally proposed by Kitano-Li in [15] to break the PS symmetry down to the $\mathrm{SU(3)_C\times SU(2)_L\times U(1)_{B-L}\times U(1)_{3R}}$ group. This can be achieved if an SU(5) singlet belonging to $\mathbf{45}_\Phi$ develops a vacuum expectation value (VEV) which breaks SO(10) into $\mathrm{SU(5)\times U(1)_X}$ in the bulk. The residual symmetry on the branes is $\mathrm{SU(3)_C\times SU(2)_L\times U(1)_{B-L}\times U(1)_{3R}}$ which in turn has to be broken into the SM gauge symmetry by introducing appropriate 4D fields on the brane of interest. We will discuss the brane sector and the breaking of $\mathrm{U(1)_{B-L}\times U(1)_{3R}}$ down to $\mathrm{U(1)_Y}$ later in this section. Let's first discuss in details the dynamics in the bulk.

[1]Consider the gauge interaction with the $(6,2,2)_V \subset \mathbf{45}_V$, decomposed under PS:

$$\overline{\mathbf{16}}\,\mathbf{45}_V\,\mathbf{16} \supset \left[(4,1,2)(4,2,1) + (\bar{4},2,1)(\bar{4},1,2)\right](6,2,2)^-_V\,,$$

where $-$ indicates the odd Z_2 parity of the "broken" gauge submultiplet. This forces $(\bar{4},1,2) \subset \mathbf{16}$ to have opposite parity of $(\bar{4},2,1) \subset \overline{\mathbf{16}}$, which is the same of $(4,2,1) \subset \mathbf{16}$

6.1.1 The bulk

The N=1 SUSY in 5D allows only gauge interactions in the bulk [200]. The $\mathbf{45}_\Phi$ interacts with the chiral multiplets $\mathbf{16}$, $\mathbf{16}'$, $\mathbf{16}^c$ and $\mathbf{16}'^c$ through gauge interactions. The superpotential in the bulk is:

$$\mathcal{W}_{\rm bulk} = \mathbf{16}_i^c \left[\hat{m}_i + \partial_y - \sqrt{2} g_5\, \mathbf{45}_\Phi\right] \mathbf{16}_i + \mathbf{16}_i'^c \left[\hat{m}_i' + \partial_y - \sqrt{2} g_5\, \mathbf{45}_\Phi\right] \mathbf{16}_i' \ . \tag{6.2}$$

Here $i = 1,2,3$ denotes three generations of matter. The bulk masses can be chosen real and diagonal without loosing generality and are parametrized by $\hat{m}_i$ and $\hat{m}_i'$. The invariance of $\mathcal{W}_{\rm bulk}$ under $Z_2 \times Z_2'$ makes the bulk masses odd under both the parities and they can be expressed as $\hat{m} = m\ sgn(y)$ and $\hat{m}' = m'\ sgn(y)$, where m and m' are real constants and $sgn(y)$ has period πR. Performing a KK expansion for the matter fields, namely $\mathbf{16}(x,y) = \sum_n \mathbf{16}_n(x) f_n(y)$, after the dimensional reduction one gets for the massless modes [15] :

$$f_0(y) = \sqrt{\frac{2m}{1-e^{-m\pi R}}}\, e^{-my} \quad \text{for } 0 \le y \le \pi R/2 \quad . \tag{6.3}$$

Note that this profile has a different normalization than Eq.(5.2) of the previous model, now taken on the fundamental interval $[0, \pi R/2]$. The $f_0(y)$ is appropriately normalized in the interval $[0, \pi R/2]$. Similar expression for the profiles of the $\mathbf{16}'$ zero modes can be obtained by replacing m with m' in Eq.(6.3). The 4D massless mode is localized at $y = 0$ ($y = \pi R/2$) brane for positive (negative) value of m and its value is exponentially suppressed on the opposite brane.

As already pointed out, the bulk masses do not distinguish the profiles of quarks and leptons of a given generation residing in the $\mathbf{16}$ or $\mathbf{16}'$ and at this stage the observed differences in the quarks and lepton masses and mixing patterns cannot be reproduced. A very crucial correction to this picture can be achieved through the Kitano-Li mechanism [15]. The VEV of $\mathbf{45}_\Phi$ along the SU(5)×U(1)$_{\rm X}$ direction introduces a correction to the bulk masses and distinguishes the profiles of the SU(5) submultiplets. As proposed in [15], this correction, which introduces a single new parameter, modifies the bulk masses according to Eq.(5.7), where the correction depends on the $r = (10, \bar{5}, 1)$ SU(5) representations and is proportional to the corresponding U(1)$_{\rm X}$ charges: $Q_X^{10} = -1$, $Q_X^{\bar{5}} = 3$ and $Q_X^{1} = -5$. Such modification in the bulk masses was argued to be able to generate viable hierarchies in quarks and leptons and this was demonstrated in our specific model [16] through a detailed numerical analysis (cf. Chapt. 5). Expressing the dimensionful quantities in units of the cut-off scale of the theory Λ, we rewrite

$$a_i^r \equiv \frac{m_i^r}{\Lambda} = \mu_i - Q_X^r k_X \ , \tag{6.4}$$

where $\mu_i = m_i/\Lambda$ and $k_X = \sqrt{2} g_5 \langle \mathbf{45}_\Phi \rangle / \Lambda$. As discussed earlier, our Z_2' parity assignment allows massless modes for $(4,2,1) \in \mathbf{16}$, which contains the SM weak doublets of quarks and leptons (Q, L) and for $(\bar{4}, 1, 2) \in \mathbf{16}'$ containing the weak singlet fields (u^c, d^c, e^c, N^c). The different matter fields within PS multiplets receive appropriate corrections from the VEV of $\mathbf{45}_\Phi$ proportional to their $\mathrm{U(1)_X}$ charges:

$$\begin{aligned} a_i^Q &= \mu_i + k_X \ ; \quad a_i^L = \mu_i - 3k_X \ ; \\ a_i^{u^c} &= \mu_i' + k_X \ ; \quad a_i^{d^c} = \mu_i' - 3k_X \ ; \\ a_i^{e^c} &= \mu_i' + k_X \ ; \quad a_i^{N^c} = \mu_i' + 5k_X \ . \end{aligned} \tag{6.5}$$

In conclusion μ_i and μ_i' are responsible of splitting the profiles with respect to the PS submultiplets while k_X with respect to SU(5) submultiplets. The zero mode profiles for the various matter fields can be rewritten from Eq.(6.3) in terms of the dimensioless quantities as:

$$n_i^\alpha(y) \equiv \sqrt{\Lambda} f_{0,i}^\alpha(y) = \sqrt{\frac{2a_i^\alpha}{1 - e^{-a_i^\alpha c}}}\ e^{-a_i^\alpha c \frac{y}{\pi R}} \ , \tag{6.6}$$

where $\alpha = (Q, u^c, d^c, L, e^c, N^c)$ represents MSSM matter fields while $c = \Lambda \pi R$ is a parameter which depends on the relative separation between the compactification scale and cut-off of the theory.

6.1.2 The branes

As already discussed, the N=1 SUSY in 5D forbids Yukawa interactions in the bulk which can be enabled on the branes by introducing a proper Higgs sector. As discussed earlier, on the $y = \pi R/2$ brane only the PS gauge symmetry survives and one can introduce 4D fields filling representations of the PS gauge group. On the contrary, on the $y = 0$ brane full SO(10) multiplets of 4D fields are required. Therefore the PS brane provides a more economic option in terms of the number of 4D fields. More interestingly, for light particles we can introduce only color singlet and electroweak doublet fields on the PS brane, avoiding the DT splitting problem. We introduce 4D chiral multiplets H, H' transforming as $(1, 2, 2)$, $\Sigma \sim (\bar{4}, 1, 2)$, $\overline{\Sigma} \sim (4, 1, 2)$ and $T \sim (1, 1, 3)$ on the PS brane and $\mathbf{16}_H$, $\overline{\mathbf{16}}_H$ on the SO(10) brane. The superpotential is

$$\begin{aligned} \mathcal{W} &= \delta\left(y - \frac{\pi R}{2}\right) \frac{1}{\Lambda}\left[Y_{ij} \mathbf{16}_i \mathbf{16}'_j H + Y'_{ij} \mathbf{16}_i \mathbf{16}'_j H' + \frac{1}{2} Y_{R\,ij} \mathbf{16}'_i \mathbf{16}'_j \frac{\overline{\Sigma}\,\overline{\Sigma}}{\Lambda} + ...\right] \\ &+ \delta\left(y - \frac{\pi R}{2}\right) w_\pi(H, H', \Sigma, \overline{\Sigma}, T) + \delta(y)\ w_0(\mathbf{16}_H, \overline{\mathbf{16}}_H) \ , \end{aligned} \tag{6.7}$$

where it is understood that $\mathbf{16}$ and $\mathbf{16}'$ are decomposed into the PS components. The first line in $\mathcal{W}$ corresponds to the Yukawa interactions responsible for the masses of matter fields, while w_π and w_0 are superpotentials for the chiral multiplets when the matter fields are turned off. The Y and Y' are complex 3×3 matrices while Y_R is a complex symmetric matrix. Below we discuss the roles played by each of the brane fields.

- Σ, $\overline{\Sigma}$ on $y = \pi R/2$ brane
  These fields on $y = \pi R/2$ brane play a multiple role. As discussed earlier, SO(10)

breaks down to $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{3R}$. One can construct two orthogonal linear combinations of the generators of the two U(1)'s which can be identified with the generators of $U(1)_X$ and the SM hypercharge $U(1)_Y$. In our normalization convention, they read

$$
\begin{aligned}
Q_X &= 4\left(T_{3R} - \frac{3}{2}\frac{B-L}{2}\right) , \\
Q_Y &= T_{3R} + \frac{B-L}{2} .
\end{aligned} \tag{6.8}
$$

The fields Σ, $\overline{\Sigma}$ take a VEV along the $U(1)_Y$ direction, trigger the breaking of $U(1)_{B-L} \times U(1)_{3R}$ down to $U(1)_Y$ and contribute to the mechanism by which D-terms are canceled. The VEV of $\mathbf{45}_\Phi$ in the bulk generates D-terms on the branes [194, 201, 202] associated to the $U(1)_X$ gauge symmetry. To preserve SUSY at high scale these D-terms have to be canceled by appropriate dynamics on the branes. The cancellation of the D-term on the $y = \pi R/2$ brane can be achieved by the VEVs of Σ and $\overline{\Sigma}$ with the condition [194, 202]:

$$
D_\pi \equiv 2\langle \mathbf{45}_\Phi \rangle + g_5 Q_X^\Sigma \left(|\langle \Sigma \rangle|^2 - |\langle \overline{\Sigma} \rangle|^2 \right) = 0 . \tag{6.9}
$$

Here $Q_X^\Sigma = -5$ is the charge under $U(1)_X$ of the component of Σ that acquires a VEV. Finally, the VEVs of Σ and $\overline{\Sigma}$ generate the masses for the right-handed neutrinos in the first line in Eq.(6.7), via the non renormalizable operator that we already introduced in Sect.3.3.2 within the PS group.

- $\mathbf{16}_H$, $\overline{\mathbf{16}}_H$ on $y = 0$ brane
  The role of these fields on the $y = 0$ brane is similar to that of Σ and $\overline{\Sigma}$ on the other brane. The VEV of the singlet under $SU(5) \times U(1)_X$ residing in $\mathbf{16}_H$, $\overline{\mathbf{16}}_H$ cancels the D-term on $y = 0$ brane if

$$
D_0 \equiv -2\langle \mathbf{45}_\Phi \rangle + g_5 Q_X^1 \left(|\langle \mathbf{16}_H \rangle|^2 - |\langle \overline{\mathbf{16}}_H \rangle|^2 \right) = 0 , \tag{6.10}
$$

  where $Q_X^1 = -5$ is the $U(1)_X$ charge of the SM singlet in $\mathbf{16}_H$.

- H, H', T on $y = \pi R/2$ brane
  The H and H' are responsible for Dirac type masses of all the fermions. Each of the H and H' contains a pair of Higgs doublets which get mixed through the following terms in w_π in Eq.(6.7):

$$
w_\pi = \frac{M_H}{2} H^2 + \frac{M_{H'}}{2} H'^2 + m H H' + \lambda T H H' + T(\lambda_H H^2 + \lambda_{H'} H'^2) + ... \tag{6.11}
$$

  where dots stand for additional terms involving the Σ, $\overline{\Sigma}$ fields. Decomposing H and H' into electroweak doublets, $H = (H_u, H_d)$ and $H' = (H'_u, H'_d)$, one obtains the following mass term after the electroweak singlet in T acquires a VEV:

$$
(H_u \;\; H'_u) \; \mathcal{M} \begin{pmatrix} H_d \\ H'_d \end{pmatrix} , \qquad \text{with } \; \mathcal{M} = \begin{pmatrix} M_H & m - \lambda\langle T \rangle \\ m + \lambda\langle T \rangle & M_{H'} \end{pmatrix} . \tag{6.12}
$$

Here $M_{H,H'}$ are redefined including the contributions coming from the VEV of T. All the mass parameters are assumed to be much heavier than the electroweak scale, possibly close to the GUT scale. One can arrange a pair of nearly massless Higgs doublets, by enforcing one eigenvalue of $\mathcal{M}$ being much smaller than the other. Such a pair would be an admixture of doublets residing in H and H' and can be written as

$$h_{u,d} = \cos\theta_{u,d} H_{u,d} + \sin\theta_{u,d} H'_{u,d} \tag{6.13}$$

$h_{u,d}$ correspond to the pair of MSSM Higgs doublets[2]. In the limit $\det(\mathcal{M}) = 0$ we can get rid of one variable replacing $M = \frac{m^2 - \lambda^2 \langle T\rangle^2}{M'}$ and the mixing angles read

$$\theta_{u,d} = \frac{1}{2}\tan^{-1}\left(\frac{2M_{H'}(m \mp \lambda\langle T\rangle)}{M_{H'}^2 - (m \mp \lambda\langle T\rangle)^2}\right) . \tag{6.14}$$

The other combinations orthogonal to h_u and h_d obtain masses as large as the GUT scale. Below the GUT scale, the model contains only one pair $h_{u,d}$ which plays the role of MSSM Higgs doublets and triggers electroweak symmetry breaking. Clearly, getting $h_{u,d}$ much lighter than the GUT scale requires a fine-tuning of the parameters in (6.12). As we show in the next section, both H and H' with $\theta_u \neq \theta_d$ are needed to generate viable quark mixing angles. Hence a non-vanishing $\langle T\rangle$ is required. Notice that in principle we could avoid to introduce the field T and use the bilinear $\Sigma\overline{\Sigma}$ with the same role, anyway the contribution from the VEV of $\Sigma\overline{\Sigma}$ would be suppressed by the cut'off scale Λ, resulting in a contribution two orders of magnitude smaller than the GUT scale. As checked by us in the further numerical analysis, a correction of the same order of m in the off-diagonal terms of (6.12) is needed to generate a realistic difference between the up and the down sectors. We finally note that the VEV of T breaks $SU(2)_R$ by keeping $U(1)_{3R}$ unbroken and does not give any additional contribution to the D-terms on the PS brane.

The model involves multiple scales of symmetry breaking.

$$\mathrm{SO(10)} \xrightarrow{1/R} \mathrm{PS} \xrightarrow{\langle \mathbf{45}_\Phi\rangle, \langle T\rangle} \mathrm{SU(3)_C{\times}SU(2)_L{\times}U(1)_{B-L}{\times}U(1)_{3R}} \xrightarrow{\langle\Sigma\rangle, \langle\overline{\Sigma}\rangle} \mathrm{SM}$$

For simplicity, we take all these scales very close to each other and identify them with the GUT scale M_{GUT}. Below the GUT scale the theory looks like the MSSM and we expect standard SUSY gauge coupling unification [49–51]. In order to suppress the higher order corrections in Eq.(6.7), we take $c \equiv \Lambda\pi R \approx \mathcal{O}(100)$ so that the cut-off of the theory, Λ can be lifted up to the Planck scale (see [16] for more discussions on the allowed range of the c parameter). The higher order corrections are at the percent level and remain smaller than experimental uncertainty in the fermion mass data we adopt. The theory provides a predictive framework for fermion masses and mixing angles, to be discussed in details in the following section.

[2] To avoid any source of confusion in the notation, $h_{u,d}$ here replace $H_{u,d}$ of Sect.A.2.

Before ending this section we notice that Yukawa interactions can also be present on the SO(10) brane. A possibility is that all Yukawa interactions are localised at $y = 0$. In this case the dynamics on this brane becomes very similar to the one described in the modified Kitano-Li model discussed by us in [16]. The scalar content on the $y = 0$ brane in [16] consists of $\mathbf{10}_H$, $\mathbf{120}_H$, $\mathbf{126}_H$, $\overline{\mathbf{126}}_H$ and $\mathbf{45}_H$. This combination of fields provides the most economic setup for viable fermion masses and mixing angles, a solution of the DT problem using the missing partner mechanism [79–82] and a consistent GUT symmetry breaking. All these features are already discussed in details in [16] and reported in the previous chapter, we do not repeat them here. In the next sections we will briefly comment on the possibility to adopt the same scalar sector for the $y = 0$ brane in the present setup and we will study its potential in explaining the fermion masses and mixings.

6.2 Fermion masses on the branes

The bulk and brane superpotentials in Eqs.(6.2) and (6.7) encode the information about the fermion masses and mixing angles. As discussed earlier, the Z_2' parity and the VEV of $\mathbf{45}_\Phi$ split the zero-mode profiles of various fermions, while the mixing of H and H' leads to the following effective 4D Yukawa couplings:

$$\mathcal{Y}_u = F_Q\ Y_u\ F_{u^c}\ ;\quad \mathcal{Y}_d = F_Q\ Y_d\ F_{d^c}\ ;\quad \mathcal{Y}_e = F_L\ Y_d\ F_{e^c}\quad \text{and}\quad \mathcal{Y}_\nu = F_L\ Y_u\ F_{N^c}\ , \tag{6.15}$$

where $\mathcal{Y}_{u,d,e,\nu}$ stand for the 3×3 matrices of dimensionless Yukawa couplings of down-type quarks, up-type quarks, charged leptons and Dirac neutrinos, respectively. Notice that we distinguish only $Y_{u,d}$ Yukawas at the fundamental level, while at the effective level the Yukawas are distinguished by the various profiles. The profile matrices are given by

$$F_\alpha = \begin{pmatrix} n_1^\alpha(\pi R/2) & 0 & 0 \\ 0 & n_2^\alpha(\pi R/2) & 0 \\ 0 & 0 & n_3^\alpha(\pi R/2) \end{pmatrix} \quad \text{with } \alpha = (Q, u^c, d^c, L, e^c, N^c) \tag{6.16}$$

where $n_i^\alpha(y)$ are defined in Eq.(6.6). The $Y_{u,d}$ arise from the mixing of MSSM-like Higgs doublets in H and H' and, from Eq.(6.13), can be explicitly represented in terms of fundamental Yukawas as follows:

$$Y_{u,d} = \cos\theta_{u,d} Y + \sin\theta_{u,d} Y'\ . \tag{6.17}$$

Considering the fact that $(F_{u^c})_{33} \approx \mathcal{O}(1) \gg (F_{u^c})_{22}, (F_{u^c})_{11}$ and the same for F_{d^c}, one obtains $\mathcal{Y}_{u,d}\mathcal{Y}_{u,d}^\dagger \approx F_Q Y_{u,d} Y_{u,d}^\dagger F_Q^\dagger$. A common Yukawa $Y_u = Y_d$ leads to an unrealistic scenario of nearly vanishing quark mixing angles. Therefore we require (a) at least two pairs of Higgs doublets allowing for different Y and Y' and (b) unequal mixing $\theta_u \neq \theta_d$ to ensure that Y_u and Y_d are different. The latter condition is satisfied in our model by a $\mathrm{SU(2)_R}$ triplet field T as shown in Eq.(6.13). After the electroweak symmetry breaking through the VEVs of $H_{u,d}$, one obtains the mass matrices:

$$M_{d,e} \equiv v\cos\beta\ \mathcal{Y}_{d,e} \quad \text{and} \quad M_u \equiv v\sin\beta\ \mathcal{Y}_u\ , \tag{6.18}$$

where $\tan\beta \equiv \langle h_u\rangle/\langle h_d\rangle$ and $v \equiv \sqrt{\langle h_u\rangle^2 + \langle h_d\rangle^2} = 174$ GeV.

The RH neutrinos receive masses through the $\mathrm{U(1)_{B-L}}$ breaking VEVs of $\overline{\Sigma}$ and are given as:

$$M_R \equiv v_R\ F_{N^c}\ Y_R\ F_{N^c}\ , \tag{6.19}$$

where $v_R \equiv \langle\overline{\Sigma}\rangle^2/\Lambda$ represents the seesaw scale. If the cut-off of the theory is raised to the Planck scale, the seesaw mechanism takes place two order of magnitude below the GUT scale, the right scale to generate viable neutrino masses. The light neutrinos gain masses through the type-I seesaw mechanism and their mass matrix can be expressed as

$$M_\nu \equiv -\frac{v^2\sin^2\beta}{v_R} F_L\ (Y_u Y_R^{-1} Y_u^T)\ F_L\ . \tag{6.20}$$

The model contains 24 complex parameters of $\mathcal{O}(1)$ (9 each in Y and Y' and 6 in Y_R) as the fundamental Yukawa couplings. In addition, it has two Higgs mixing angles $\theta_{u,d}$ and 7 bulk mass parameters μ_i, μ_i' and k_X.

The bulk masses in Eq.(6.5) can generate different hierarchies in F_Q and F_L, which in turn explain the observed differences in the quark and lepton mixing patterns and mass hierarchies. The SO(10) breaking by Z_2' distinguishes the profiles of left and right handed fields but it still maintains the quark-lepton unification. A milder hierarchy among neutrino masses and large lepton mixing angles result from the VEV of $\mathbf{45}_\Phi$, which distinguishes profiles of different SU(5) submultiplets within the $\mathbf{16}$ and $\mathbf{16}'$. This model differs from the one presented in [16] in the following ways:

- In comparison to [16], the current model has three more bulk masses. This provides more freedom in the profiles of zero-mode fermions. For example, the effective SU(5) symmetry in the profiles is broken once $m_i \neq m_i'$ and, unlike in the previous model, one can distinguish between the masses of down-type quarks and charged leptons even if $Y_d = Y_e$.

- An important difference with respect to [16] is the simplification of the Higgs sector on the brane. In [16], consistent fermion masses and a solution of the DT splitting problem through the missing partner mechanism required $\mathbf{10}_H + \mathbf{120}_H$ Higgs representations, which contain three pairs of MSSM-like Higgs doublets. In the current model, only two pairs are required and this reduces the Higgs mixing parameters from eight to two.

- The scalars introduced on the PS brane are in representations of smaller dimensionality compared to the brane sector fields in [16]. In particular, realistic Yukawa couplings only require a pair of $(1,2,2)$ fields on the PS brane. The DT splitting is automatically solved since no colour triplets are present in the relevant Higgs multiplets. However we need to arrange only one pair of light doublets and this requires an appropriate potential with a fine-tuning, as explained in the last section.

As recalled at the end of the previous section, all Yukawa couplings can be also localised on the SO(10) brane at $y = 0$. We can adopt the same scalar sector as in the model discussed in [16], remarking however a couple of differences with respect to our previous model. There are three more bulk masses in the current setup due to the doubling of matter fields in **16** and **16′** and the Yukawa matrix Y_{10} (Y_{120}) is not symmetric (anti-symmetric) in generation space, with several new parameters of $\mathcal{O}(1)$. Clearly, this model does not provide any improvement in comparison to the old model as far as the field content and dynamics on the brane are concerned. It is however characterized by more parameters, which provide more flexibility in reproducing the correct pattern of fermion masses and mixing angles. We will provide a quantitative analysis of this improvement in the next section.

6.3 Numerical analysis and results

We now discuss in detail the viability of the model in explaining the observed data of fermion masses and mixing parameters and analyze its prediction for the observables which have not been measured yet. Our approach is similar to the one followed by us earlier in [16]. We take an idealized set of data for fermion masses and mixing parameters extrapolated at the GUT scale in the MSSM and check the viability of the model in reproducing them. As in [16], we use the results obtained in [53] for the charged fermion masses and quark mixing parameters. The extrapolation was carried out in the MSSM assuming a SUSY breaking scale of about 500 GeV, and for different values of $\tan\beta$. We perform the viability analysis for two representative values of $\tan\beta$, 10 and 50. After our previous analysis, the results of the global fit of neutrino oscillation data have been updated [19] taking into account the most recent data available till the summer 2014. We take these updated low-energy values of neutrino mass squared differences and lepton mixing angles, neglecting RGE corrections. Such an approximation is valid if neutrino masses are hierarchical [207–209] and indeed this is realized in our model as we will show in this section. Following the widely adopted strategy in this kind of analysis [102–107], the data we use are the result of a specific extrapolation and should be taken as a representative set of GUT scale inputs. The actual data depends on features such as the SUSY breaking scale, SUSY scale threshold corrections, which can be estimated only when the exact mechanism of SUSY breaking is known [210–212]. Again, keeping these uncertainties in mind, we believe that if a given model can fit a representative set of data very well, then it will be able to reproduce with a similar accuracy and success the actual data, by slightly varying the underlying parameters. We summarize the various observables and their input values in Table 6.2. We employ χ^2 minimization technique to fit the free parameters, defined as in the previous model in Eq.(5.26) and adopting the same optimization technique.

Observables	$\tan\beta = 10$	$\tan\beta = 50$
y_t	0.48 ± 0.02	0.51 ± 0.03
y_b	0.051 ± 0.002	0.37 ± 0.02
y_τ	0.070 ± 0.003	0.51 ± 0.04
m_u/m_c	0.0027 ± 0.0006	0.0027 ± 0.0006
m_d/m_s	0.051 ± 0.007	0.051 ± 0.007
m_e/m_μ	0.0048 ± 0.0002	0.0048 ± 0.0002
m_c/m_t	0.0025 ± 0.0002	0.0023 ± 0.0002
m_s/m_b	0.019 ± 0.002	0.016 ± 0.002
m_μ/m_τ	0.059 ± 0.002	0.050 ± 0.002
$\|V_{us}\|$	0.227 ± 0.001	
$\|V_{cb}\|$	0.037 ± 0.001	
$\|V_{ub}\|$	0.0033 ± 0.0006	
J_{CP}	0.000023 ± 0.000004	
$\Delta_S/10^{-5}$ eV^2	7.50 ± 0.19 (NO or IO)	
$\Delta_A/10^{-3}$ eV^2	2.457 ± 0.047 (NO)	2.449 ± 0.048 (IO)
$\sin^2\theta_{12}$	0.304 ± 0.013 (NO or IO)	
$\sin^2\theta_{23}$	0.452 ± 0.052 (NO)	0.579 ± 0.037 (IO)
$\sin^2\theta_{13}$	0.0218 ± 0.0010 (NO)	0.0219 ± 0.0011 (IO)

Table 6.2: The GUT scale values of the charged fermion masses and quark mixing parameters from [53] and neutrino masses and mixing parameters from an up-to-date global fit analysis [19]. NO (IO) stands for the normal (inverted) ordering in the neutrino masses.

6.3.1 Results for the PS brane

We first analyze the Yukawa interactions on the PS brane. The compatibility of the model with anarchic Yukawa structure is tested in two ways. We first fit an idealized data set to the model by minimizing the χ^2 with respect to all the free parameters. The range of $\mathcal{O}(1)$ Yukawa couplings is restricted to be $|Y_{ij}|,\ |Y'_{ij}|,\ |Y_{Rij}| \in [0.5,\ 1.5]$ keeping the phases in the full range $[0,\ 2\pi]$. The aim of this exercise is to assess whether our model can accommodate the data or not. We carry out this exercise assuming normal (NO) or inverted ordering (IO) in the light neutrino masses and each of the two cases is analyzed for two values of $\tan\beta$. We get poor fits for small $\tan\beta$ corresponding to minimized χ^2 values ~ 100 and ~ 300 for NO and IO cases respectively. The results for $\tan\beta = 50$ are displayed in Table 6.3 for which we get good fits for both NO and IO cases. As it can be seen, all the data are fitted with negligible deviations from their central values. The model parameters obtained at the minimum of χ^2 are listed in the Appendix E.2. The basic features of the best fit results are similar to the ones obtained in the previous model [16]. The observed hierarchies of quark and lepton masses requires $|k_X| \sim |\mu_{2,1}|, |\mu'_{2,1}| \ll |\mu_3|, |\mu'_3|$. This in turn enforces a common bulk mass for quarks and

Observable	Normal ordering Fitted value	Pull	Inverted ordering Fitted value	Pull
y_t	0.51	0	0.52	0.33
y_b	0.37	0	0.38	0.50
y_τ	0.51	0	0.51	0
m_u/m_c	0.0027	0	0.0028	0.17
m_d/m_s	0.051	0	0.052	0.14
m_e/m_μ	0.0048	0	0.0048	0
m_c/m_t	0.0023	0	0.0023	0
m_s/m_b	0.016	0	0.017	0.50
m_μ/m_τ	0.050	0	0.050	0
$\lvert V_{us}\rvert$	0.227	0	0.227	0
$\lvert V_{cb}\rvert$	0.037	0	0.037	0
$\lvert V_{ub}\rvert$	0.0033	0	0.0030	-0.50
J_{CP}	0.000023	0	0.000023	0
Δ_S/Δ_A	0.0305	0	0.0305	0
$\sin^2\theta_{12}$	0.304	0	0.304	0
$\sin^2\theta_{23}$	0.452	0	0.442	-0.20
$\sin^2\theta_{13}$	0.0218	0	0.0218	-0.10
$\chi^2_{\rm min}$		≈ 0		≈ 0.96
	Predicted value		**Predicted value**	
$m_{\nu_{\rm lightest}}$ [meV]	3.9		10.6	
$\lvert m_{\beta\beta}\rvert$ [meV]	4.96		48.2	
$\sin\delta^l_{CP}$	-0.39		-0.89	
M_{N_1} [GeV]	190		7.12	
M_{N_2} [GeV]	8.02×10^5		6.75×10^5	
M_{N_3} [GeV]	1.43×10^{14}		1.38×10^{14}	
υ_R [GeV]	0.04×10^{16}		0.056×10^{16}	

Table 6.3: Results from numerical fit corresponding to minimized χ^2 for normal (NO) and inverted ordering (IO) in neutrino masses. The fit is carried out for the GUT scale extrapolated data given in Table 6.2 for $\tan\beta = 50$. The input parameters are collected in the Appendix E.2.

leptons of the third generation and leads to approximate Yukawa unification $y_t \sim y_b \sim y_\tau$, which prefers large $\tan\beta$ [58, 59].

We now discuss the second kind of approach in which we do not fit the fundamental Yukawa couplings of the theory. We treat them as free $\mathcal{O}(1)$ parameters and restrict their absolute values within the range 0.5 - 1.5, allowing arbitrary phases. For given values of these couplings, we minimize the χ^2 function with respect to the bulk masses and Higgs mixing angles. We repeat this procedure many times, each time generating randomly a new set of Yukawa couplings. We fit 17 observables with respect to 9 free parameters (7 bulk masses and 2 Higgs mixing angles), leaving $\nu = 8$ degrees of freedom (dof). The analysis is performed for $\tan\beta = 50$ and for NO and IO in the neutrino masses. The results are displayed in Fig. 6.1 where we plot the normalized distribution of the minimum χ^2/ν. One can see a clear preference for the NO with respect to the IO. Even though one obtains a good best fit for IO case in Table 6.3, this analysis shows that the solution requires more fine-tuning in the underlying Yukawas compared to the one obtained for NO. The χ^2 thresholds corresponding to a given probability value p and the

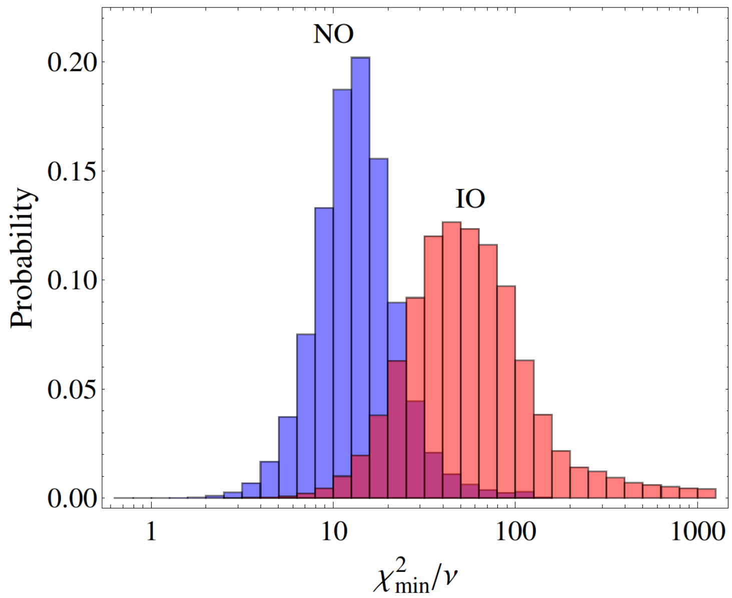

Figure 6.1: The probability distributions of minimized χ^2/ν for NO (blue) and IO (red) in neutrino masses and for $\tan\beta = 50$.

number of cases satisfying the thresholds for different $p-$values are listed in Table 6.4. For $p \geq 0.001$, we find 0.5% cases providing the acceptable values of the $\chi^2_{\rm min} \leq 26.12$.

$p-$value	0.10	0.05	0.01	0.001
$\chi^2_{\rm min}$ (for $\nu = 8$)	≤ 13.36	≤ 15.51	≤ 20.09	≤ 26.12
successful cases (NO)	0.03%	0.05%	0.15%	0.48%
successful cases (IO)	$< 10^{-3}$%	$< 10^{-3}$%	$< 10^{-3}$%	0.005%

Table 6.4: The rate of successful events obtained for different $p-$values from random samples of $\mathcal{O}(1)$ Yukawa couplings in case of normal and inverted ordering in the neutrino masses.

The distributions of the bulk mass parameters and physical predictions for the NO case with $p > 0.001$ are given in Fig. 6.2 and 6.3 respectively.

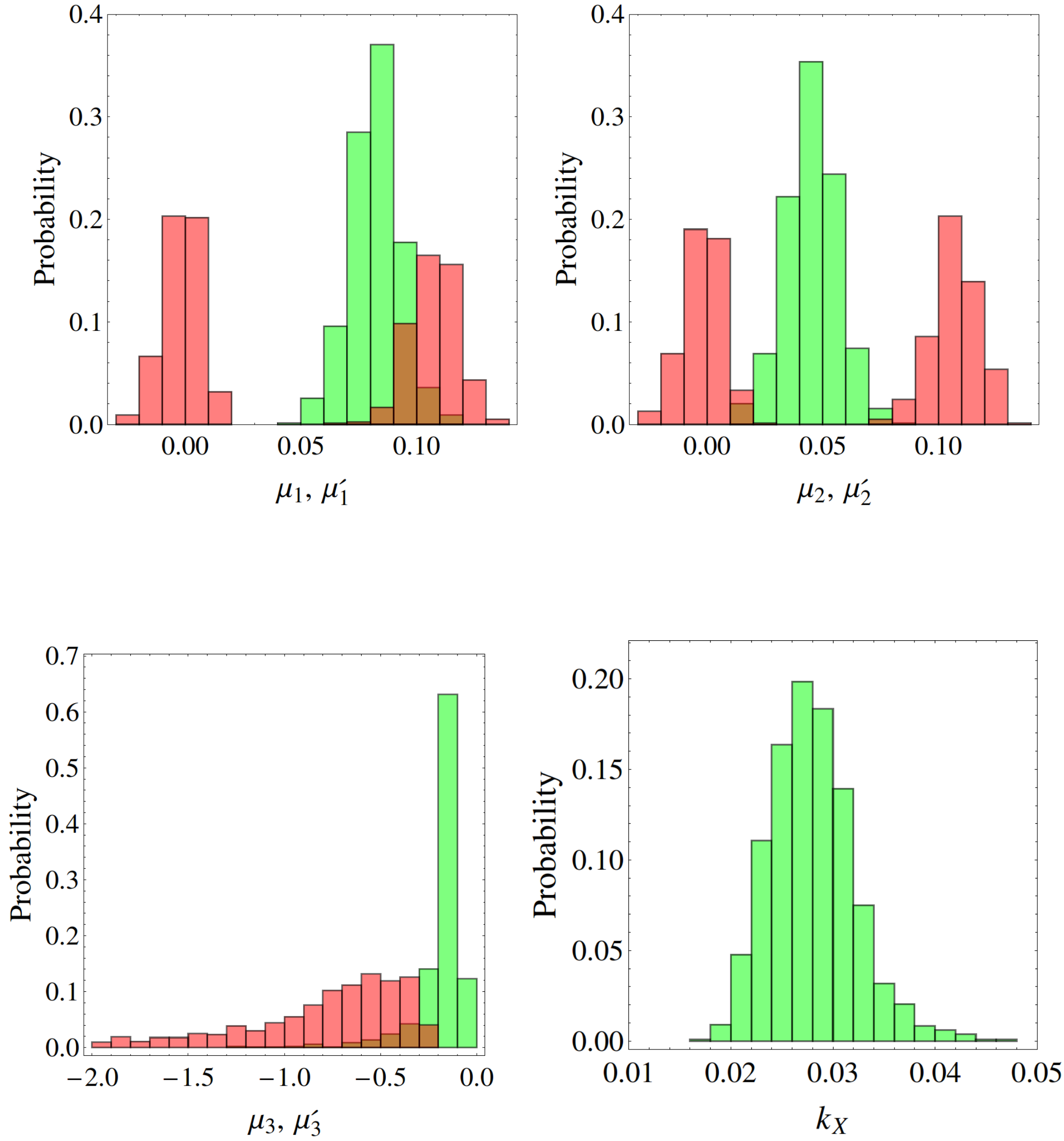

Figure 6.2: The distributions of bulk mass parameters fitted with $\chi^2_{\rm min}/\nu < 3.27$ (or $p > 0.001$) in case of NO and $\tan\beta = 50$. The green (red) distribution corresponds to unprimed (primed) bulk mass parameters.

One finds preference for positive bulk masses for the first and second generations, which are localized close to the $y = 0$ brane. The third generation is localized on the PS brane with a negative bulk mass. From the distributions shown in Fig. 6.2, it is clear that the SO(10) breaking by Z'_2, which distinguishes μ_i and μ'_i, is crucial in generating realistic fermion masses in this model. This is particularly true for the first two generations where

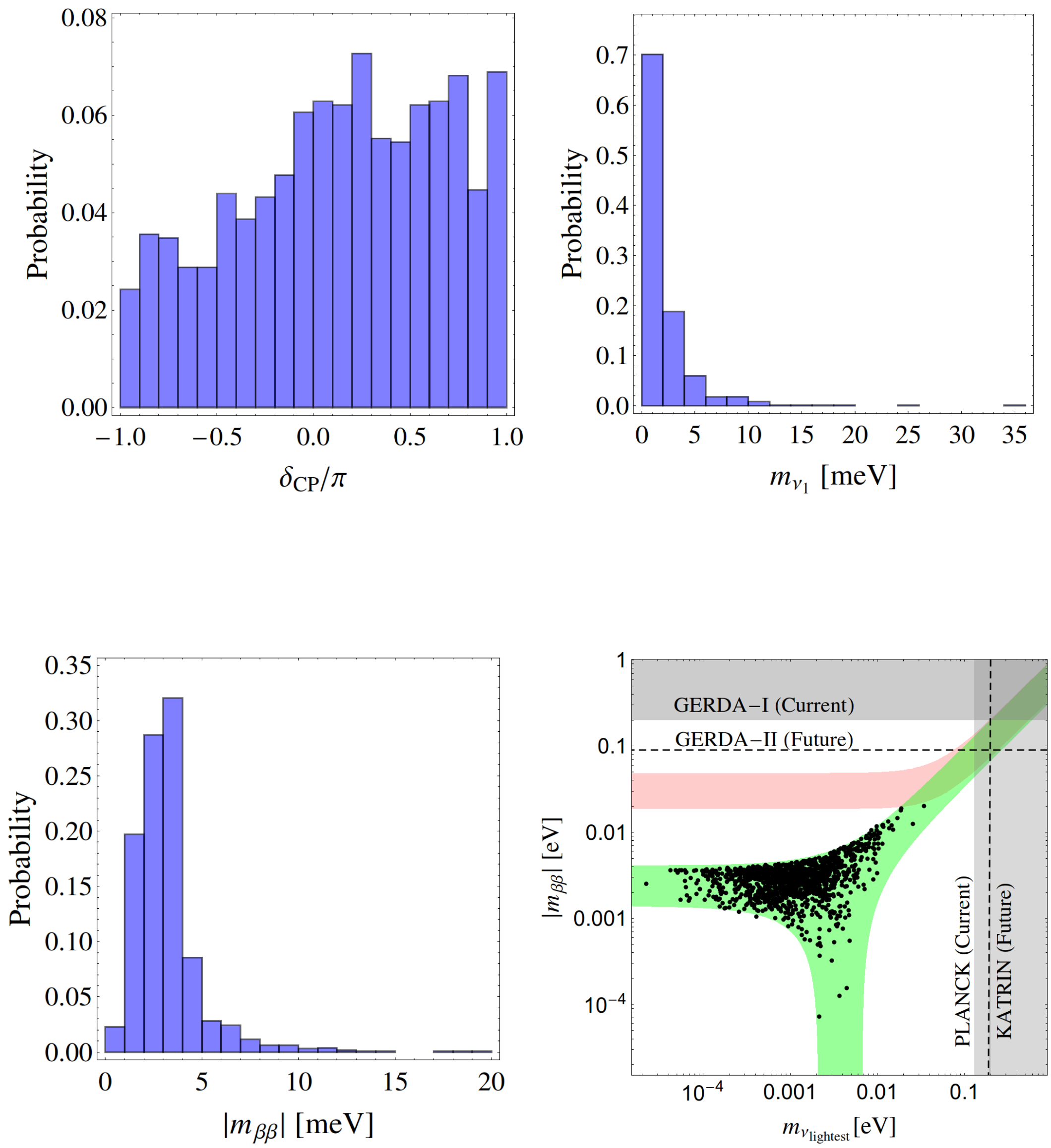

Figure 6.3: The Yukawa interactions of PS brane: prediction for various observables obtained for $p > 0.001$ (corresponding to $\chi^2_{\rm min}/\nu < 3.27$ for $\nu = 8$) in case of normally ordered neutrino masses and $\tan\beta = 50$. The black points in the bottom-right panel are model predictions while the green (red) regions are the allowed ranges for $|m_{\beta\beta}|$ and the lightest neutrino mass in case of NO (IO). The different horizontal and vertical grey bands correspond to the currently excluded regions by GERDA-I [222] and Planck Cosmic Microwave Background measurements and galaxy clustering information from the Baryon Oscillation Spectroscopic Survey [223]. The dashed lines indicate the near future reach of GERDA-II and KATRIN [21] experiments.

difference between μ_i and μ'_i is significant. Notice that this difference is the only source of breaking of the mass degeneracy between the charged leptons and down-type quarks in this model. The k_X parameter is required to be positive and of the order of the bulk masses of the first two generations. Among the observable quantities in the lepton sector, the lightest neutrino mass is predicted to be below 10 meV corresponding to strongly hierarchical neutrinos. The effective mass of the neutrinoless double beta decay $|m_{\beta\beta}|$ lies in the range 1-5 meV, which is beyond the reach of the current generation of experiments. Future detection of neutrino masses well above 0.05 eV and/or of $|m_{\beta\beta}|$ well above the range 1-5 meV would rule out the present model. Since the CP violation is coming from anarchic $\mathcal{O}(1)$ Yukawas, we get no particular preference for the Dirac CP phase in the lepton sector. The model do not favour specific values also for the Majorana CP phases as revealed from the correlations between the $|m_{\beta\beta}|$ and the lightest neutrino mass in the bottom-right panel in Fig. 6.3.

Since the RH neutrinos are accommodated in **16**-plets, their masses are predicted once the masses and mixing angles of remaining fermions are fitted. The predictions are displayed in Fig. 6.4. The spectrum of RH neutrinos turns out to be very hierarchical.

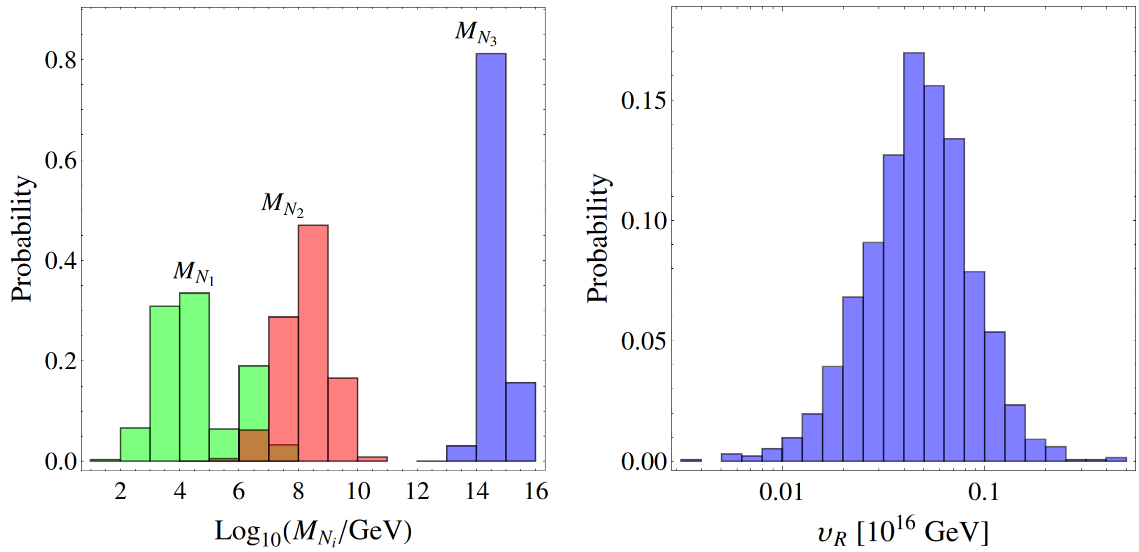

Figure 6.4: The Yukawa interactions of PS brane: prediction for the masses of RH neutrinos and $v_R = \langle\overline{\Sigma}\rangle^2/\Lambda$ obtained for $p > 0.001$ (corresponding to $\chi^2_{\text{min}}/\nu < 3.27$ for $\nu = 8$) in case of normally ordered neutrino masses and $\tan\beta = 50$.

As happening in the previous model, this is a consequence of the large $\text{U}(1)_\text{X}$ charge of RH neutrinos, which corrects with large contributions the bulk masses of the first and second generations making $N_{1,2}$ more sharply localized on $y = 0$ brane compared to the other fermions.

Since $k_X \ll |\mu'_3|$, the third generation RH neutrino remains localized on the PS brane and one gets $M_{N_3} \approx v_R = \langle\overline{\Sigma}\rangle^2/\Lambda$. We obtain relatively light spectrum for the first two generation RH neutrinos corresponding to $M_{N_2} \in [10^7, 10^{10}]$ GeV and $M_{N_1} \in [10^3, 10^5]$ GeV. This is in contrasts to generic 4D SO(10) GUT models [105, 106] where they turn out to be relatively heavier. We also obtain the prediction for v_R after correctly fixing the scale of solar and atmospheric neutrinos. This is shown in Fig. 6.4. One finds $\langle\overline{\Sigma}\rangle \approx M_{\rm GUT}$ from the preferred values of v_R which is of the same order as required by the cancellation of the D-term in Eq.(6.9). Note that $|\langle\overline{\Sigma}\rangle| > |\langle\Sigma\rangle| \sim M_{\rm GUT}$ is required since $k_X = \sqrt{2}g_5\langle\mathbf{45}_\Phi\rangle/\Lambda$ is positive.

The spectrum of RH neutrinos is strongly hierarchical in our model. In the standard thermal leptogenesis [44] scenario, the final lepton asymmetry is dominated by the lepton number violating decays of the lightest RH neutrino. In this case the successful leptogenesis generically requires [45–47]

$$M_{N_1} \geq 3 \times 10^9 \text{ GeV} . \tag{6.21}$$

Clearly, this condition is not respected in our model. To further assess the viability of this scenario, we perform a global fit imposing Eq.(6.21) in our model. We get $\chi^2_{\rm min} \sim 150$ ruling out strongly the possibility of the N_1-dominated leptogenesis. An alternative is to consider N_2 or N_3-dominated leptogenesis, where the lepton flavour effects play an important role [224]. In this case, the lepton asymmetry is mainly generated by N_2 or N_3 decays. The lepton doublets produced in such decays get completely incoherent in flavour space before the wash-out by the light RH neutrinos becomes active [225–229]. The wash-out acts individually on each flavour asymmetry and it is less efficient. In this case a certain combination of flavour asymmetry remains protected from the light RH neutrinos wash-out [224]. We have checked this possibility in our model using the best fit solution reported in Table 6.3 and in the Appendix E.2. We find that N_2 is too light to create a sufficient asymmetry, while most of the asymmetry generated by N_3 is eventually washed out by N_2 and N_1, since these particles have sufficiently large couplings with lepton doublets and Higgs. Therefore, our preliminary investigations performed on the best fit solution indicate that leptogenesis cannot be successfully realized in this model. However a detailed analysis of this issue performing a global χ^2 fit including the constraints imposed by flavoured leptogenesis would be required before ruling out leptogenesis in our model, which goes beyond the scope of the present work.

6.3.2 Results for the SO(10) brane

We now investigate the naturalness of anarchic Yukawas on the SO(10) brane, as briefly discussed at the end of sections 2 and 3. The fermion mass relations are similar to the one already derived for the modified KL model in [16]. With respect to the modified KL model, we have three more bulk masses and several new Yukawa couplings in this model. We obtain good global fits for both NO and IO, when $\tan\beta = 50$. Therefore we perform the second type of analysis in which we fit the 7 bulk mass parameters and 8 Higgs mixing parameters, essentially the one in Eq.(5.22) of the previous model, by taking a flat random distribution for all the $\mathcal{O}(1)$ anarchical parameters. The ranges of these parameters is chosen as in the previous case. Because of the new parameters coming from the Higgs mixing, with respect to the PS brane, we now have only $\nu = 2$ degrees of freedom.

To compare this case to the previous one, we plot the distributions of χ^2/ν for both of them and for NO in neutrino masses in Fig. 6.5. As it can be seen, both the distri-

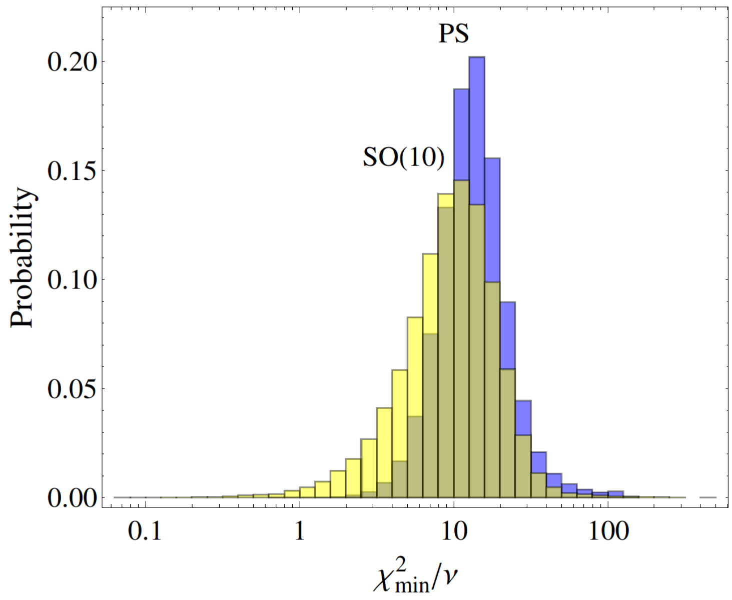

Figure 6.5: A comparison between the Yukawa interactions on PS ($y = \pi R/2$) and SO(10) ($y = 0$) branes. The distributions are obtained for the normal ordering in the neutrino masses and for $\tan\beta = 50$.

butions peak around similar values of χ^2/ν. The SO(10) case however has a relatively broader distribution leading to more successful cases for a given p-value. We get 7%, 15% and 30% successful cases for p-values greater than 0.05, 0.01 and 0.001 respectively (the corresponding thresholds for $\chi^2_{\rm min}$ for $\nu = 2$ dof are 5.99, 9.21 and 13.82). The substantial increase in the success rate in this case compared to that with Yukawas on the PS brane is attributed to the fact that we have six more mixing parameters providing more freedom in fitting the fermion masses and mixing angles starting from random Yukawa couplings. A similar improvements can be seen by comparing the success rates of this case with those of the modified KL model in [16]. The improved success rates in this case is due to three more bulk mass parameters, which allows better fitting of the data.

The predictions for the various observables in the successful cases, corresponding to the $p \geq 0.001$, are displayed in Figs. 6.6 and 6.7. All the predictions are very similar to

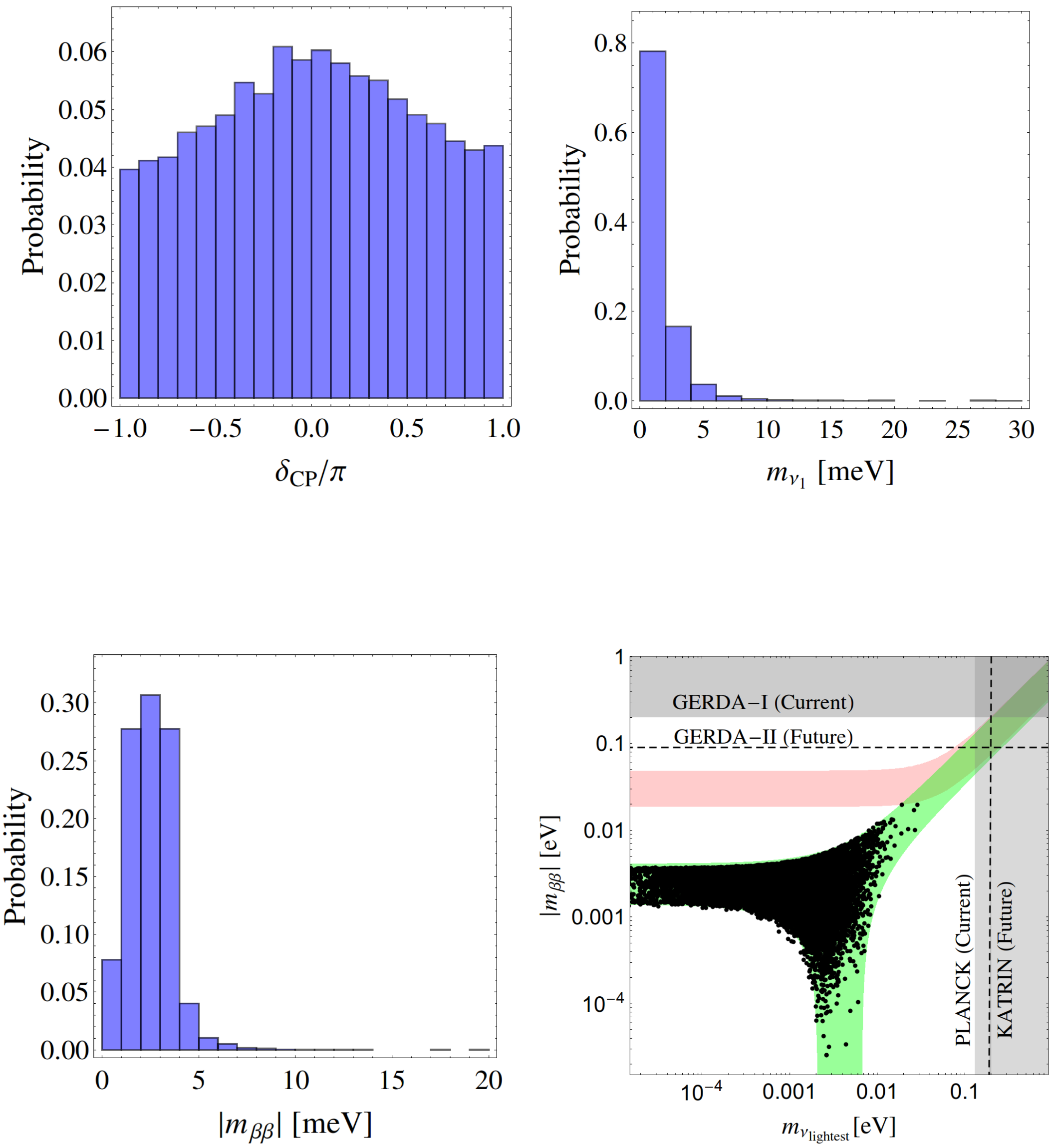

Figure 6.6: The Yukawa interactions on SO(10) brane: prediction for various observables obtained for the successful cases corresponding to $p > 0.001$ (or $\chi^2_{\rm min}/\nu < 6.91$ for $\nu = 2$) in case of normally ordered neutrino masses and $\tan\beta = 50$. See Fig. 6.3 for detailed description.

those obtained in the case of Yukawas on the PS brane and modified KL model in [16]. This shows that these predictions depend almost entirely on the dynamics of the bulk that, generating different zero-mode profiles, distinguishes the various fermion sectors. On the contrary, details of the brane interactions affects only very mildly our results. The main difference arising from the brane interactions in the different cases is the number of free $\mathcal{O}(1)$ parameters and Higgs mixing parameters. Our study shows that when the

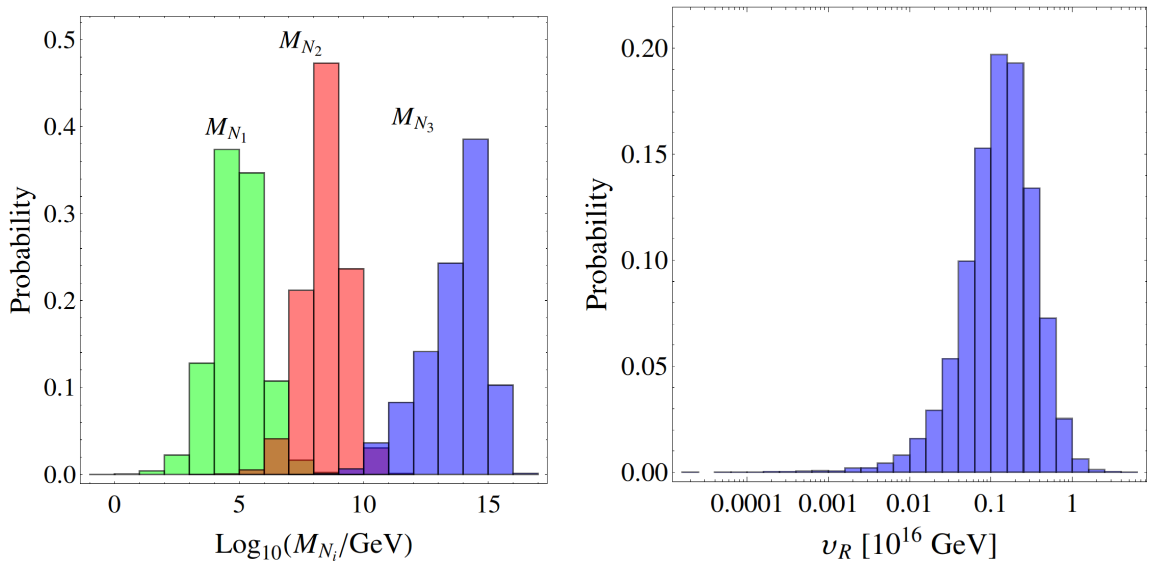

Figure 6.7: The Yukawa interactions on SO(10) brane: prediction for for the masses of RH neutrinos and $v_R = \langle\overline{126}_H\rangle$ obtained for the successful cases corresponding to $p > 0.001$ (or $\chi^2_{\text{min}}/\nu < 6.91$ for $\nu = 2$) in case of normally ordered neutrino masses and $\tan\beta = 50$.

number of bulk mass parameters and Higgs mixing parameters increases also the rate of success, normalized to the number of degrees of freedom, increases.

6.4 Conclusion and discussion

We have proposed a new model that confirms the possibility of implementing the approach of anarchical Yukawas within the framework of SO(10) in extra dimensions. In a previous work we relied on a spontaneous breaking of the grand unified symmetry, at the cost of introducing large SO(10) representations for the symmetry breaking sector with a non-trivial mechanism to solve the doublet-triplet splitting problem. In the present work we have fully exploited the capabilities of the higher-dimensional construction, which allows for gauge symmetry breaking through compactification and offers a more economic solution to the doublet-triplet splitting problem. Since, compared to our previous model, the new construction significantly alters the allowed bulk masses and the Yukawa interactions, we think it deserved an accurate study of its properties, to assess whether the description of fermion masses and mixing angles remains the same or it undergoes major modifications.

We propose a supersymmetric SO(10) model formulated in five dimension. The extra dimension is compactified on an orbifold $S^1/(Z_2 \times Z_2')$ and plays a key role in breaking the symmetries of the model. The compactification breaks N=2 SUSY down to

N=1 SUSY in 4D and, at the same time, breaks SO(10) down to the Pati-Salam group $SU(4)_C \times SU(2)_L \times SU(2)_R$. A further reduction of the gauge symmetry is realized spontaneously, through a symmetry breaking sector including an SO(10) adjoint, automatically present in this 5D construction, and additional brane multiplets included with the purpose of canceling the D-terms of the theory. Below the GUT scale the residual gauge symmetry is that of the SM, which can be finally broken down to $SU(3)_C \times U(1)_{em}$ by a set of electroweak doublets localized on the PS brane. Matter multiplets, introduced in **16** representations of the GUT group as bulk fields, develop profiles for the zero-modes that are localized in specific regions of the extra dimensions, according to their different bulk masses. As in the original Kitano-Li model, a universal parameter, proportional to the VEV of the adjoint of SO(10), allows to distinguish the different SU(5) components inside a **16** representation. Moreover, our framework allows for independent bulk masses for electroweak singlets and doublets of the various generations: this important feature is effect of the symmetry breaking of SO(10) to the PS group through boundary conditions. Anyway, while on one side this allows more flexibility in fitting the data, on the other it has the defect of spoiling the complete unification of the matter multiplet, forcing us two introduce two fields **16** and **16′**. Yukawa interactions can be localized either on the SO(10) or on the PS brane. While we briefly commented on the first possibility, in our study we mainly concentrated on the PS case, since it offers the possibility of introducing an economic Higgs sector, which in particular automatically solves the DT splitting problem.

Our model, with Yukawa interactions on the PS branes, has seven parameters controlling the bulk masses and two Higgs-mixing parameters, plus a large number of $\mathcal{O}(1)$ Yukawa couplings. By fitting an idealized set of data, extrapolated at the GUT scale from the observed fermion masses and mixing angles, we find that the agreement is not trivial and requires a large value of $\tan\beta$. Moreover the case of inverted ordering in the neutrino mass spectrum requires much more fine-tuning in the Yukawa couplings than the case of normal ordering. The lightest neutrino mass is predicted to be below 10 meV and the effective mass of the neutrinoless double beta decay $|m_{\beta\beta}|$ lies in the range 1-5 meV. The model can be falsified by the observation of either a non vanishing neutrino mass at KATRIN [21] or $|m_{\beta\beta}|$ at the next generation of experiments. We find no preference for the Dirac CP phase of the lepton sector and the spectrum of RH neutrinos is predicted to be very hierarchical, which unfortunately is incompatible with the generation of the observed baryon asymmetry through thermal leptogenesis.

All these predictions remain essentially unchanged with respect to our previous model, showing a remarkable robustness of the dynamics in the bulk with respect to various dynamics which can be implemented on the branes. Indeed, it is the Kitano-Li mechanism, responsible of splitting the profiles of the different SU(5) components of a **16** in the bulk, to dictate essentially the features of these predictions. All the other characteristics of the model such as the number of independent Yukawa couplings on the branes, the number of Higgs mixing parameters, the additional possibility of distinguishing weak doublets and singlets through the bulk masses, seem to play a secondary role which, at most, can influence the success rate of the model when statistical tests are performed.

Chapter 7

N=1 SUSY breaking through extra dimension?

The title of this chapter is voluntarily put with a question mark. Indeed, we don't aim to present a complete discussion about the mechanisms to break $N = 1$ Supersymmetry in extra dimensions, but only to review, at the level work in progress, the possibilities of implementing a mechanism of this kind in the particular models we have built (cf. Chapt. 5-6).

As we have already pointed out, the analysis performed in our models is not complete, as far as we have not discussed the breaking of $N = 1$ SUSY. This would have allowed a more realistic scenario, taking into account threshold corrections in the running of the couplings and thus extrapolating with smaller uncertainty the fermion masses and mixing angles at the GUT scale. On the other side, the specification of the SUSY breaking mechanism would have introduced a much bigger model dependence, which we have preferred to avoid so far. Also, as consequence of this shortcoming, we were not able to analyze the rich related phenomenology of flavour and CP violations, both in the quark and in the lepton sector, which heavily depends on the chosen mechanism for SUSY breaking.
This brief chapter collects our first attempts of doing this step further, looking for a SUSY breaking mechanism suitable to the scenario of our particular models.

Many mechanisms proposed in the literature are based on spontaneous breaking of SUSY, which can be achieved through the VEV of the D-term or the F-term: the former was proposed in 1974 by Fayet and Iliopoulos [230], while the letter by O'Raifeartaigh in 1975 [231]. Realizing the spontaneous SUSY breaking in a phenomenologically viable model, however, has been proven very difficult. As a consequence, in many of the supersymmetric models studied nowadays the SUSY breaking is considered to occur in a "hidden" sector and to be subsequently transmitted to our "visible" sector by some mediator. Well studied possibilities are: the gravity mediation [232–235], the gauge mediation [236–238] and the anomaly mediation [239–241]. Anyway, given the special presence of an extra dimension in our models, it would be very advantageous if we could perform the SUSY breaking exploiting the tools offered by extra dimensions, avoiding to add too many new degrees of freedom for describing other independent mechanisms. We are thus going to review some mechanisms of SUSY breaking through extra dimensions that we have considered as possible candidates. Even if we can find many example in literature of such mechanisms, the implementation in our particular models turns out to be not trivial, as we are going to show.

7.1 A SUSY breaking on the Higgs brane

It is well known that a generic breaking of SUSY can induce flavour changing and CP violating processes that are strongly constrained by the experiments (see [242, 243] for a review). This happens as consequence of the general misalignment of the soft squark and slepton mass matrices with respect to the relative fermion mass basis. Flavour-violating processes between fermions would thus be mediated by the exchange of MSSM superpartners[1].

The most common approach to this problem is assuming the SUSY breaking and its mediation to the MSSM to be flavour universal. Or at least, such flavour universality should arise in the low energy limit. A scenario where this requirement of flavour universality is a bit relaxed, introducing flavour dependent breaking parameters, is anyway possible if the model can account for suitable suppressions. In ref.[244] Nomura et al. take into account the possibility that the same mechanism responsible of the hierarchical suppressions in the Yukawa couplings can be responsible of a similar suppression in the mediation of the SUSY breaking. In this way the CP and flavour violating parameters would exhibit a non trivial flavour structure with a necessary correlation to the Yukawa parameters. This kind of scenario goes under the name of "flavorful Supersymmetry". As shown in [244] such kind of models can evade the experimental bounds if the flavourful contributions to the SUSY breaking are subdominant with respect to other flavour universal ones.

A realization in our 5-dimensional model is given by implementing the SUSY breaking sector on the same brane hosting the Higgs sector, where the matter fields modulate the Yukawa couplings with their exponential profiles. A similar scenario is considered in specific models in [245, 246].

Let us assume as hidden breaking sector a SM singlet field X, located on the Higgs brane, responsible of breaking SUSY through the VEV of its F-term, F_X. This field couples to all the MSSM superfields through non-renormalizable operators in the Kahler potential and in the superpotential of the theory. These operators are the effective description of a mechanism mediating the SUSY breaking and parameterized by some UV completion

[1] As a remark, let us remind that in general extra dimensional models with fermions in the bulk, new flavour violation is mediated also by the exchange of Kaluza-Klein modes. However, in our GUT-sized extra dimension, the KK modes are far too heavy to contribute significantly. Therefore, here we refer only to the flavour violation induced by the MSSM superpartners.

at higher energy scale M_*, which is taken as cut-off of the theory: $M_* \approx \Lambda > M_{\rm GUT}$. Without specifying this mechanism, we are interested in the contribution to the soft breaking parameters appearing at the effective level. In particular, let us assume X and the Higgs sector on the $y = 0$ brane (cf. Chapt. 5), and consider the contribution to gaugino and sfermion masses which result from terms of the form:

$$\mathcal{L}^{4D}_{soft} \supset \int dy\ \delta(y)\ \left[\int d^2\theta\ \eta_a \frac{X}{\Lambda} W^{a\alpha} W^a_\alpha + \int d^4\theta\ k^\Phi_{ij} \frac{X^\dagger X}{\Lambda^2} \Phi^\dagger_i \Phi_j \right] \tag{7.1}$$

where Φ stands generically for a matter chiral superfield and η_a and k^Φ_{ij} are supposed to be natural parameters of order one. The first term is contributing to gaugino masses, while the second is generating the squark and slepton masses. Taking into account the particular shape of the matter fields, the effective sfermion mass matrix turns out to be of the form:

$$\tilde{m}^{\Phi\,2}_{ij} \approx n^\Phi_i n^\Phi_j \left(\frac{F_X}{\Lambda}\right)^2 \tag{7.2}$$

where the n^Φ_i factors correspond to the profiles of the matter fields evaluated at the brane, as defined for example in Eq.(5.11) of our first model. These are the same factors entering the effective Yukawa couplings, cf. Eq.(5.10). Such factors determine a flavour dependent suppression according to the profile parameters[2] fixed by fitting the fermion masses and mixing angles. Therefore, the pattern obtained in the soft breaking parameters by combining these suppression factors is similar to the hierarchical structure of the Yukawas. Notice that the matter fields enter the coupling in Eq.(7.1) with different combinations with respect to the Yukawa coupling, in particular there is not the chiral structure typical of the fermions. The resulting suppression structure in the sfermion masses is thus related, but not identical, to the Yukawa's one.

The soft masses result highly flavour non-universal. Such non-universal contributions are admitted without contradicting the CP and flavour violation bound from low energy data, as it is shown in [244, 245], but they cannot be the dominant contribution for the soft masses.

Let us analyze the situation in our particular models. Without going into the characteristic details of the various fermions representations, take as example the quark sector: we have seen from the analysis in our models that it is required in general to be $n^\Phi_3 \gg n^\Phi_2 \gtrsim n^\Phi_1$ in order to reproduce a realistic spectrum. This implies that the mass matrix $\tilde{m}^\Phi_{ij} \propto n^\Phi_i n^\Phi_j$ has a structure very different from the identity, since the off diagonal contributions can be enhanced more than, or as much as, the diagonal entries: $\tilde{m}^\Phi_{33} > \tilde{m}^\Phi_{23} \approx \tilde{m}^\Phi_{13} > \tilde{m}^\Phi_{11} \approx \tilde{m}^\Phi_{22} \approx \tilde{m}^\Phi_{12}$. With such hierarchical structure, even if the off-diagonal terms are suppressed, the rotation to the fermion mass basis would create

[2] Working in SO(10), this suppression would be distinguished for the components of the **16** according to the splitting obtained for the profiles (cf. Chapt. 5, 6). Specifying such a feature is not relevant within the present discussion. In [244, 245] it is supposed to have a given suppression factor for each of the SM fermion representations.

large mixing contributions. To make the off-diagonal terms acceptable, it is needed to enhance the diagonal entries of the matrix with a flavour universal contribution, in order to resemble a structure approximately similar to the identity, which would be "stable enough" under the change of basis. Let us remark that this consideration is made at a very qualitative level, without a full numerical check, but it is anyway supported by the results in [244].

In such perspective one could consider the flavour universal contributions to the squark and slepton masses arising automatically at the low energy from the contribution of gaugino masses in the RGE evolution [61, 247]. The gaugino masses arising from the first term in Eq.(7.1), given the flat profile of the gaugino 0-mode, turn out to be of the same order of magnitude of the third generation of squarks,

$$M_a = \eta_a \left(\frac{F_X}{\Lambda} \right) \approx \tilde{m}^{\Phi}_{33} \,.$$

This contribution is a possibility that we could explore in our model. Anyway, its viability is not guaranteed and should be tested by a dedicated study that analyzes the effects of the SUSY breaking potential on all the soft parameters. In particular, we should check if the trilinear scalar interactions ($W \supset XQ_iU_jH_u + XQ_iD_jH_d + \dots$), which are also constrained by flavour violation bounds, are adequately suppressed or they require other suppression assumptions. For example, in the model of ref.[245] these operators are forbidden by a U(1) symmetry. Such symmetry has the consequence of forbidding also the gaugino mass term, for which an extra source of SUSY breaking mediation is necessary.

We conclude that in our models we can implement the SUSY breaking on a brane, exploiting the characteristic profiles of the matter fields, which introduce useful suppression in the flavour-violating parameters. However, this maybe be not a complete solution of the problem. The flavour violation constraints on the low energy theory require a dominant flavour universal contribution, which can arise or by gauginos in the RGE or from an extra universal mediation. Therefore, keeping in mind this result about flavorful SUSY, we proceed exploring other specific mechanisms of SUSY breaking in 5D that could provide such flavour universal contribution, or at least a diagonal enhancement, to the squark and slepton masses.

7.2 Scherk-Schwarz mechanism

This mechanism explains the breaking of SUSY by boundary conditions in compact extra dimensions. This approach is completely different from the idea of spontaneous SUSY breaking in a hidden sector, as it provides an explicit breaking for the fields propagating in the bulk of an ED. The original proposal made by Scherk and Schwarz [189] has later been reviewed by many authors, for example [192, 200, 248–250]. The simplest version of the Scherk-Schwarz (SS) mechanism is realized on a S^1/Z_2 orbifold and consists in applying non trivial periodic boundary conditions to the fields propagating in the bulk. The boundary conditions of the fields are assigned through the following general transformations:

$$\phi(x, \tau(y)) \equiv \phi(x, y + 2\pi R) = T\phi(x, y) \; ; \tag{7.3}$$

$$\phi(x, \zeta(y)) \equiv \phi(x, -y) = Z\phi(x, y) \tag{7.4}$$

Where $Z = \pm 1$ is the Z_2 parity and T is an element of a global symmetry of the theory. So far in our models we have imposed the condition (7.3) by taking T as the identity transformation, but this is not the most general general choice, in presence of global symmetries. The identification on the coordinate: $\tau\zeta\tau(y) = \zeta(y)$ implies the following consistency condition on the orbifold:

$$TZT = Z \tag{7.5}$$

The transformation T must thus be chosen respecting this condition. For example, assume that the Lagrangian possesses an SU(2) global symmetry in the bulk and consider a field transforming in the fundamental. If we identify $Z = \sigma^3$, the expression for T allowed by Eq.(7.5) is:

$$T = e^{2\pi i(\alpha_1 \sigma^1 + \alpha_2 \sigma^2)} \tag{7.6}$$

where σ^i are the Pauli matrices and α_i are real parameters. The T transformation (7.6) is called "twist", since it mixes the two field components at the boundaries.

We have seen that 5D N=1 SUSY in the bulk corresponds to N=2 SUSY in 4D (cf. Sect.4.6.1). Very interestingly, to N=2 SUSY is associated a global SU(2)$_R$ symmetry, under which some field components are singlets, while others are grouped in doublets. Expressing the N=2 hypermultiplets and vector multiplets in terms of N=1 SUSY, the components of the N=1 supermultiplets are not independent, since components of different multiplets belong to the same SU(2)$_R$ representation (cf. Sect.A.3 in appendix A). Following the notation of section 4.6.1, we have in particular:

- the gauginos doublet:
$$\Lambda_{\mathbf{2}} = \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix}$$
from the (on shell) vector multiplet $V = (A_\mu, \lambda_1)$ and the chiral multiplet $\chi = (A_5 + \Sigma, \lambda_2)$;

- the scalars doublet:
$$\Phi_{\mathbf{2}} = \begin{pmatrix} \phi \\ \phi^{c\dagger} \end{pmatrix},$$
from the (on shell) matter chiral multiplets $\Phi = (\phi, \psi)$ and $\Phi^c = (\phi^c, \psi^c)$.

All the remaining components (A_M, Σ, ψ, ψ^c) are SU(2)$_R$ singlets.
If we consider the matter fields propagating in the orbifold *without* bulk masses, the

$SU(2)_R$ is indeed a global symmetry of the 5-dimensional Lagrangian, that is the one of Eq.(4.73).

The SS mechanism exploits this symmetry in order to assign the twisted boundary condition (7.6) to the $SU(2)_R$ doublets defined above, while all the other singlet components have the usual periodic boundary conditions. This fact itself evidently breaks N=1 SUSY. The effect of this special BC turns out in a mass term arising only for the twisted fields.

Let us remark that the gauge fields are always propagating in the bulk, being the gauge symmetry associated to the whole 5-dimensional space time, so that with this mechanism gauginos always receive mass from twisted BC. We can assume, without loss of generality, the twist:

$$T = e^{i2\pi\alpha\sigma^2} \tag{7.7}$$

and the gaugino mass turns out to be:

$$M_{1/2} = \frac{\alpha}{R} \tag{7.8}$$

According to the particular problem that one aims to solve, it is possible to choose the locations of the matter and of the Higgs superfields on the bulk or on the branes. Let us focus on the case of our model, with Higgs on a brane and matter multiplets in the bulk, in which one aims to generate exactly the sfermion masses through twisted BC.
Consider for the moment the absence of bulk masses, so that the profiles of matter fields are not exponentials, but the zero-mode of ϕ is flat, without distinctions for the three generations. If Φ is a generic matter superfield and α_Φ parametrizes its twist, we get, similarly to the gaugino case, the sfermion mass:

$$\tilde{m}^{\Phi 2} = \frac{\alpha_\Phi^2}{R^2} \tag{7.9}$$

In SO(10) all the sfermions, unified in one field $\Phi = 16$, are subjected to a unique twist. The masses (7.9) are identical for the three generations. Anyway, this is true as far the profiles are not distinguished for the three generations, in particular in absence of bulk masses. We have to analyze how the presence of bulk masses, a key ingredient of our flavour models, is going to modified this result. As we are going to discuss in the next section, the effect is not trivial.

Both the gaugino mass (7.8) and the sfermion mass (7.9) are proportional to the KK scale $M_{KK} \equiv 1/R$. Since in our models $M_{KK} > M_{\rm GUT}$, this would imply an unacceptable breaking of SUSY above the GUT scale. If we want bring these masses down to $1 \div 10$ TeV we have to accept a fine-tuning of the parameters α, α_Φ of $13 \div 14$ orders of magnitude. The effect of the hierarchical matter profiles may change this requirement for squarks and sleptons.

7.2.1 Scherk-Schwarz on orbifold with bulk masses

Let us consider our specific models where the profiles of the matter fields are modulated by a bulk mass M distinguished for the three generations[3]. The starting action is Eq.(4.73) and the Z_2 parity assignment to the $\Phi_{\mathbf{2}}$ doublet corresponds to $Z = \sigma^3$. The equation of motion is obtained by decomposing this action into the fields components (cf. Eq.(D.6)). The following Lagrangian results for the scalars, expressed in terms of the doublet $\Phi_{\mathbf{2}}$:

$$\mathcal{L}^{5D}_{\Phi_{\mathbf{2}}} = \partial_M \Phi^\dagger_{\mathbf{2}} \partial^M \Phi_{\mathbf{2}} - M^2 \Phi^\dagger_{\mathbf{2}} \Phi_{\mathbf{2}} + \Phi^\dagger_{\mathbf{2}} \sigma^3 \Phi_{\mathbf{2}} \partial_5 M \tag{7.10}$$

A key feature of Eq.(7.10) is the presence of the boundary term $\sigma^3 \partial_5 M$, where $\partial_5 M = 2M\,(\delta(y) - \delta(y - \pi R))$ arises from the Z_2-odd nature of the bulk mass M. We observe that this term is responsible of the breaking of the global $\mathrm{SU}(2)_R$ symmetry into a $\mathrm{U}(1)_R$ symmetry along the σ^3 direction on the branes. Such breaking doesn't depend on the value of M, but it is due only to the presence of σ^3 in the Lagrangian. Along the remaining σ_1 and σ_2 directions, on which we have defined the twist T, only a discrete subgroup survives, corresponding to the discretized values of the transformation parameter , $\alpha = n$ in Eq.(7.7). Such discretization, necessary to guarantee the invariance of the Lagrangian, reduces the twisting choice simply to $T = \pm I$, where I is the identity. The particular Z_2 parity of the bulk mass in the orbifold seems to forbid the possibility of a continuous twisting and thus the selection of a small value for α_Φ in Eq.(7.9), which would be needed if we want to account for a SUSY spectrum at the $\sim$ TeV scale. However, Eq.(7.9) may not be valid in presence of exponential profiles, for which a dedicated analysis is needed.

Let us explore the only non trivial case, $T = -I$. Imposing the KK reduction (cf. Sect.4.3.1), corresponding to the description of 4-dimensional scalar modes with mass $\tilde{m}^2$, we find the equation of motion for the profiles:

$$\left[\partial_5^2 + \Omega^2 + \sigma^3 \partial_5 M\right] \Phi_{\mathbf{2}}(y) = 0 \tag{7.11}$$

where $\Omega^2 = \tilde{m}^2 - M^2$ contains the mass spectrum $\tilde{m}^2$, which we want to make explicit. Note that now $\Phi_{\mathbf{2}}(y) \equiv (\phi_1(y), \phi_2(y))^T$ is used to represent only the y-dependent profile in the KK expansion and for convenience we have redefined: $\phi_1(y) \equiv \phi(y)$, $\phi_2(y) \equiv \phi^{c\dagger}(y)$. The solution of Eq.(7.11) in the bulk is subjected to the boundary conditions:

$$\begin{array}{ll} \text{(anti)periodicity:} & \Phi_{\mathbf{2}}(2\pi R) = -\Phi_{\mathbf{2}}(0) \\ Z_2 \text{ parity and continuity at } (0, \pi R): & \phi_2(2\pi R) = \phi_2(0) = 0 \\ \text{Discontinuity of the derivative at } (0, \pi R): & \partial_5 \phi_1(0^+) + M\phi_1(0) = 0 \\ & \partial_5 \phi_1(\pi R^-) - M\phi_1(\pi R) = 0 \end{array} \tag{7.12}$$

A non vanishing solution to Eqs.(7.11-7.12) requires the particular condition on the parameters:

$$\cot(\Omega \pi R) = \frac{M}{\Omega} \tag{7.13}$$

[3]It's irrelevant, for the purpose of our discussion, to consider the specific bulk mass splitting that we have realized in the context of SO(10), distinguishing the components of the **16**.

which finally gives the relation for the mass spectrum. It is worth mentioning that, as we have explicitly verified, solving the system of Eqs.(7.11-7.12) for a general twist T leads to a non-vanishing solution only for the cases $T = \pm I$, consistently with the symmetry of the Lagrangian pointed out above. Eq.(7.13) cannot be solved analytically. However, we can find approximated solutions in two interesting regions of the mass spectrum, which are:

$$\text{for } \tilde{m} \ll M : \qquad \tilde{m}^2 \approx 4M^2 \frac{e^{-2M\pi R}}{(1+e^{-2M\pi R})^2} \,, \tag{7.14}$$

$$\text{for } \tilde{m} > M : \qquad \tilde{m}^2 \approx M^2 + \frac{(n+\frac{1}{2})}{R^2} \,, \quad \text{with } n \text{ an integer.} \tag{7.15}$$

The first equation describes the mass of the lightest mode, which is the mass "gained" by the previous massless mode. The second equation describes the shifted mass for the KK modes (described by the integer n) which should be compared with Eq.(4.11). Since we are interested in the low energy spectrum, let us analyze the first limit, Eq.(7.14):

- in the case $|M| \ll M_{KK} = 1/R$, which corresponds to a delocalization of the profiles from the brane, we have $\tilde{m}^2 \approx M^2$: the diagonal contribution to the sfermion mass matrix would have a hierarchy between the generations resembling the hierarchy of the bulk masses, but without exponential suppression. This case is however excluded by the conditions of our models.

- Eq.(7.14) doesn't depend on the change of sign of M. This means that the spectrum doesn't depend on where the profiles are localized, but only on "how much" localized they are. Such feature is a significant difference with respect to the fermion mass spectrum emerging from the Yukawa couplings, where the different localization between the third and the first two generations creates a huge hierarchy. We thus expect the hierarchy of the sfermions to be very different from the fermions.

- In the case $|M| > M_{KK} = 1/R$, which corresponds to very localized profiles, the contribution to the soft masses is exponentially suppressed. This feature may be interesting to lower down the fundamental breaking scale $1/R$. In the context of our model this happens for the third generation, $M_3 \approx \Lambda \approx 10^2 M_{KK}$, for which the emerging contribution to the soft spectrum is absolutely neglactable. The first two generations, instead, have $M_{1,2} \approx M_{KK}$. The suppression is not exponential in this case and we find $\tilde{m}_{1,2} \approx 10^{-2} M_{KK}$, which is still a very heavy soft mass.

Our conclusion is that it seems impossible to account for a $\sim$ TeV spectrum implementing the SS mechanism given the particular features of our model. The ingredients mainly responsible of this difficulty are: (1) the compactification at high scale $M_{KK} \gtrsim M_{\rm GUT}$, (2) the localization of the matter profiles strictly fixed by fitting the flavour observable in the fermion sector. Note that the problem of the high gaugino mass from Eq.(7.8) remains, as α is fixed to be 1 or another integer. Even if we could generate a proper suppression in the sfermion sector, the RGE contributions from such high gaugino masses would delete it.

Let us now do a remark. Our analysis of the orbifold in presence of bulk masses seems essentially to forbid the usual realization of the SS mechanism, since the continuous twisting T is replaced by the only possibilities $\pm I$. This essentially corresponds to introduce a second Z_2' parity on the orbifold. In fact, from the consistency condition (7.5), we can identify

$$Z' = TZ \tag{7.16}$$

as a second parity transformation ($Z'Z' = I$). The resulting spectrum (7.14-7.15) indeed corresponds to the result found in models with double orbifolding $S^1/(Z_2 \times Z_2')$, with the Z_2' assignment given by our identification (7.16), see for example [251].

However, other works from the literature seem to lead to different realization of the SS mechanism in presence of bulk masses. For example in [252] the soft spectrum is derived relating the SS to the Hosotani mechanism [253]. This leads to a different result for the spectrum where, in particular, the necessary discretization of the transformation parameter does not emerge, in apparent contradiction with our analysis. The fundamental difference with respect to our approach seems to be the missing requirement for the invariance of the Lagrangian under the global symmetry of T. Apart of the particular application of the SS mechanism to our models, understanding such contradiction is an interesting issue that deserves some attention for future applications.

7.3 Radion mechanism

Let us now consider the Radion mechanism, for which we have realized an independent analysis that leads to some original results.
In this mechanism the compactification of the ED is assumed to happen by stabilization of its radius, which is generated as VEV of a dynamical component of the metric. The metric is:

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu - R^2 dy^2 \tag{7.17}$$

The scalar field R is called *radion*. Note that in this notation the filed R has dimension $[R] = 0$ in mass units. The 4-dimensional metric is taken constant and flat, only the $(5,5)$ component is dynamical: $g_{55} = -R^2$. Working in SUSY, the radion belongs to an N=1 chiral superfield T containing also: the fifth component of the gravi-photon B_5, the fifth component of the right-handed gravitino ψ^{c5} and a complex auxiliary field F [254]:

$$\begin{aligned} T &= R + iB_5 + \sqrt{2}\theta\psi^{c5} + i\theta\sigma^\mu\bar{\theta}\partial_\mu (R + iB_5) - \theta^2 F \\ &\quad -i\frac{1}{\sqrt{2}}\theta^2\partial_\mu\psi^{c5}\sigma^\mu\bar{\theta} - \frac{1}{4}\theta^2\bar{\theta}^2\partial_\mu\partial^\mu (R + iB_5) \end{aligned} \tag{7.18}$$

An interesting possibility is to induce the SUSY breaking through the F-term of the radion superfield. Note that the dimension of F is $[F] = 1$ in mass units. As originally pointed out in [254], it is possible to write the 5D action in terms of N=1 superfields expressing the interaction of the radion with the gauge and matter fields. The action in presence of bulk mass for the matter superfields turns out to be:

$$\begin{aligned} S_5 \;=\; & \int d^5x \left[\frac{1}{4g_5^2} \int d^2\theta \; T W^\alpha W_\alpha + h.c. \right. \\ & + \frac{2}{g_5^2} \int d^4\theta \; \frac{1}{T+T^\dagger} \left(\partial_5 V - \frac{1}{\sqrt{2}} (\chi + \chi^\dagger) \right)^2 \\ & + \int d^4\theta \; \frac{1}{2} (T+T^\dagger) \left(\Phi^\dagger e^{-V} \Phi + \Phi^c e^V \Phi^{c\dagger} \right) \\ & \left. + \int d^2\theta \; \Phi^c \left(\partial_5 + MT - \frac{1}{\sqrt{2}} \chi \right) \Phi + h.c. \right] , \end{aligned} \tag{7.19}$$

where the gauge and matter field content is in the same notation of Eq.(4.73) in section 4.5. Eq.(7.19) corresponds to the one reported by ref.[254] with an additional bulk mass contribution inserted by us. It can be seen from the expansion of the superfields and replacement of the auxiliary fields that this action leads to the correct Langragian for the field components in a 5D-covariant form, with respect to the metric (7.17), when the radion T takes a VEV and with the necessary rescaling of the component fields:

$$\Sigma \longrightarrow R\Sigma; \qquad \lambda_2 \longrightarrow -iR\lambda_2 \tag{7.20}$$

Eq.(7.19) resembles Eq.(4.73) for $R = 1$ and $F = 0$.

In [254] the scalar spectrum from this action is computed in absence of the bulk mass M. Remarkably, the resulting spectrum corresponds to the SS mechanism, reproducing Eqs.(7.8,7.9) identifying the twisting parameter as[4] $\alpha = F/(2R)$. This correspondence between the SS and the radion mechanism seems to be well extablished [252, 254, 255]. However, it is not very clear in the literature how this correspondence keeps being valid in presence of a bulk mass on orbifold. Since, according to our analysis of the previous section, the SS mechanism itself changes completely its features in presence of bulk mass, the relation between the two mechanisms may not be guaranteed in such conditions.

Let us compute the scalar spectrum. To make the notation more clear, at this level we assume $\langle R \rangle = 1$, while we keep in its generality F, from which the mass spectrum will depend explicitly. From Eq.(7.19) we find the following equation of motion for the scalars:

$$\left(\partial_5^2 + \Omega^2 - \frac{|F|^2}{4} + F \frac{\sigma^1 - i\sigma^2}{2} \partial_5 - F^* \frac{\sigma^1 + i\sigma^2}{2} \partial_5 + \sigma^3 \partial_5 M \right) \Phi_{\mathbf{2}} = 0 \tag{7.21}$$

where again $\Omega^2 = \tilde{m}^2 - M^2$ and $\Phi_{\mathbf{2}}(y) \equiv (\phi_1(y), \phi_2(y))^T$ is the doublet defined before, with the same Z_2 assignment $Z = \sigma^3$. The solution in the bulk is subjected to the following

[4] R here is the physical value of the radius.

boundary conditions[5]:

$$\begin{array}{ll} \text{periodicity:} & \Phi_{\mathbf{2}}(2\pi R) = \Phi_{\mathbf{2}}(0) \\ Z_2 \text{ parity and continuity at } (0,\pi R): & \phi_2(2\pi R) = \phi_2(0) = 0 \\ \text{Discontinuity of the derivative at } (0,\pi R): & \partial_5\phi_1(0^+) + M\phi_1(0) = 0 \\ & \partial_5\phi_1(\pi R^-) + M\phi_1(\pi R) = 0 \end{array} \tag{7.22}$$

For $F = 0$ this system reduces to the known Eq.(4.18) for the scalars and admits a zero-mode solution with $\tilde{m} = 0$. A non vanishing solution for the system of Eqs.(7.21-7.22), which accounts for general values of F, is given by the condition for the spectrum:

$$\sin^2 \frac{F\pi R}{2} = \frac{\tilde{m}^2}{\tilde{m}^2 - M^2} \sin^2 \Omega\pi R \tag{7.23}$$

This equation cannot be solved analytically, but we can find the approximated solutions:

$$\text{for } \tilde{m} \ll M \text{ and } F \ll 1/R: \qquad \tilde{m}^2 \approx M^2 \frac{F^2\pi^2R^2}{4\sinh^2 M\pi R}\,, \tag{7.24}$$

$$\text{for } \tilde{m} > M: \qquad \tilde{m}^2 \approx M^2 + \left(\frac{F}{2} + \frac{n}{R}\right)^2 \,. \tag{7.25}$$

The first equation describes the mass of the lightest mode, "gained" from the zero-mode. The second equation describes the shifted mass for the KK modes described by the integer n. Considering Eq.(7.24) in the low energy limit, we can make the following considerations:

- The spectrum doesn't depend on the sign of M, so that also in this case there is no distinction with respect to where the fields are localized, but only with respect to "how much" localized they are, differently from the fermion masses emerging by the Yukawa couplings.

- In the case $M \gg M_{KK} = 1/R$ the factor $1/(\sinh^2 M\pi R)$ gives a huge exponential suppression. In our model the contribution to the spectrum of the third generation, with $M_3 \approx 10^2 M_{KK}$ results absolutely negligible, whatever the value of F is. For the first two generations of our particular model, with $M_{1,2} \approx M_{KK}$, the suppression is less important and it gives $\tilde{m}^2_{1,2} \approx 10^{-2}F$. One has to rely on a severe fine-tuning on the VEV $F \approx 10^{-12} M_{KK}$ to obtain a soft sfermion spectrum around the TeV scale for the first two generations.

- Differently from the SS result found in the previous section, where the twisting parameter was necessarily discretized and was not appearing explicitly in the final spectrum, we find no evidence of necessary discretization of F, and we can in principle adjust this value to accommodate the wanted contribution in the soft spectrum.

The derivation of the scalar spectrum reported here is an original contribution by us in the explicit context of the Radion mechanism with bulk masses. It is interesting to compare our result with the literature. Remarkably, Eq.(7.23) and the approximated solutions

[5]Note that R from now on indicates the physical radius with dimension $[R] = -1$ in mass units.

(7.24,7.25) coincide exactly with the result reported by [252] in the context of the SS mechanism. This would apparently confirm the matching between radion and Scherk-Schwarz also in presence of bulk masses. However, our derivation for the Scherk-Schwarz case, which is in open contradiction with [252], doesn't confirm such correspondence. This issue, as said before, requires a dedicated study. Finally, computing the spectrum of gauginos, which have not hierarchical profiles, we find the result:

$$M_{1/2} = \frac{F}{2R} \tag{7.26}$$

which matches with the result obtained from the SS mechanism Eq.(7.8) identifying $\alpha = F/2R$, and it confirms the equivalence of the two mechanisms in absence of bulk mass. Also in this case, it is evident the need for a very small value of F in order to suppress the gaugino masses. The value $F \approx 10^{-12} M_{KK}$ chosen above for the sfermions would lead consistently to $M_{1/2} \approx 10$ TeV.
All the sfermions would get a universal contribution from this gaugino mass in the RGE to low energy, generating a mass also for the third generation, which in our particular picture doesn't receive contribution directly from the radion.

Of course a more realistic discussion of this framework would also need the specification of a potential for the radion field. This requires to complete the description in the context of gauged Supergravity [256–260], which goes beyond the qualitative purpose of this discussion.

Let us remark that the considerations we made above are a first naive analysis of the behavior in our model. In particular, we are neglecting the role played in our model by the VEV in the U(1) direction of the scalar component Σ of the gauge chiral multiplet χ (cf. Eq.(4.58) in section 4.5). In a normal framework in absence of the radion field, we have seen that the effect of this VEV is simply a correction to the bulk mass proportional to the U(1) charge, which is used in our SO(10) models to create different effective bulk masses for the SU(5) submultiplets (cf. Eq.(5.7) in chapter 5). However, it is not guaranteed that, in presence of the radion, the effect is still simply a bulk mass correction. Indeed, in the second line of Eq.(7.19) the interaction of χ with T seems to give rise to a new contribution to the scalar spectrum from the VEV of Σ, which cannot be reabsorbed in a redefinition of the bulk mass. Since we are still checking the existence and the effect of this contribution, we prefer to keep this discussion for a future development.

7.4 Conclusion

We have discussed some possibilities of implementing in our models the breaking of N=1 SUSY, involving the tools of extra dimension. While the breaking through a hidden sector on a brane would be admitted, the characteristic hierarchy of our matter profiles implies the necessity of a dominant flavour universal contribution, in order to respect the

constraints from flavour violation. The possibility of providing such contribution by the gaugino masses generated by the same breaking sector, entering the RGE of squark and slepton masses, is in principle viable. However, such possibility should be carefully tested in a dedicated study, where flavour effects on other sensitive soft breaking parameters (like trilinear scalar couplings) are taken into account.
Searching for other possible contributions, we have analyzed a couple of methods: the explicit SUSY breaking by Scherk-Schwarz mechanism and the spontaneous SUSY breaking by radion field. None of them, at a first sight, seems to accommodate easily the requirements of our specific models in a natural way. The main reasons of the difficulty arising in our models reside in the very high scale of compactification ($M_{KK} \gtrsim M_{\rm GUT}$) and in the given hierarchy of the matter field profiles, which is strictly fixed by fitting the fermion spectrum.
As a side result, in our study we have found a different behavior between the Scherk-Schwarz and the Radion mechanism in presence of bulk masses, while they correspond exactly to the same mechanism in absence of bulk mass. This result contradicts other studies like ref.[252], where the SS mechanism is approached through different tools. Such apparent contradiction needs to be clarified and can be object of our future developments.

We finally remark that the considerations that we have made for each mechanism are done here only at a qualitative level, without a full study or numerical analysis. The only principles that have essentially guided our discussion are: the requirement that the sfermion mass matrices respect the flavour constraints and the necessity to maintain such masses qualitatively around 1-10 TeV, in order not to spoil the gauge unification provided by SUSY [54]. The present discussion, in fact, represents just a first step in understanding the possibilities to include the SUSY breaking in our specific context of work, with the aim to get a qualitative idea of the direction in which we should address a dedicated study of the problem.

Conclusions

Grand unified theories, proposed more than forty years ago, provide an elegant synthesis of electroweak and strong interactions, which greatly clarifies some of the crucial aspects of the SM such as particle classification, quantization of the electric charge and diversification of the gauge coupling constants. The unification realized by the SO(10) group is exceptionally attractive for its simplicity in representing the fermion sector and providing automatic gauge anomaly cancellation. In SO(10) GUT one generation of fermions fits in a single spinorial representation of the gauge group, including also a right-handed neutrino, which can naturally account for small neutrino masses through the see-saw mechanism. The simplicity of this unified description of all the fermions is an impressive and compelling feature of SO(10) GUTs. However, such a simple picture must deal with the well-known problem of describing fermion masses and mixing angles, which, in the low-energy data, do not reflect at all such a complete particle unification. While it is certainly possible to accommodate the observed fermion spectrum by exploiting the most general Yukawa interactions allowed by the theory, not much is gained with respect to the Standard Model picture, since lots of parameters and a huge hierarchy in the Yukawa couplings are needed to reproduce the data. Therefore, what is known as "flavour puzzle" in the SM, seems to exist with similar importance also at the level of grand unification.

In this thesis we have dedicated some space to show how such complicated framework of Yukawa couplings arises. We have pointed out the limitations of the minimal SO(10) model in reproducing realistic data, motivating the necessity to go beyond the minimality, introducing three different Higgs representations. A new feature of the Yukawa couplings in SO(10) GUT is the existence of correlations between the parameters of the up, down and charged lepton Yukawas, arising from the decomposition of the SO(10) multiplets into the SM components. From these correlations some promising features arise, for example the possibility to account for similar masses of down quarks and charged leptons, which are grouped in the same SU(5) submultiplet. This may be a first step in understanding the structure underlying the flavour sector from a unified point of view, and to this aspect we devote particular care.

Among the various approaches to improve the description of the flavour sector, we have focused our attention on the requirement of naturalness, thus reducing the range and the fine-tuning of the Yukawa parameters. A situation where all the fundamental Yukawa couplings are anarchical and of order one can be nicely reconciled with the observed fermion masses and mixing angles by appealing to wave-function renormalizations that distinguish generations and fermion species. This framework naturally occurs in models with an extra spatial dimension, through the different localization of the profiles of the fermion zero-modes. Yukawa interactions are defined on one brane and the hierarchy among fermion masses depends exponentially on bulk mass parameters. This kind of framework can easily accommodate a realistic fermion spectrum in the SM, where one has the freedom to adjust all the fermion profiles with independent parameters. In the SO(10) picture, however, this realization is not obvious at all because the unified description in one **16** forces to have a common profile for all the fermion species of one generation, leading to a completely unrealistic spectrum. Some mechanism able to split the profiles with respect to the **16** subcomponents is necessary. In the supersymmetric version of SO(10) this is provided by the Kitano-Li mechanism: the VEV of an adjoint scalar, naturally included in the 5-dimensional gauge field, is responsible of breaking SO(10) in the bulk, providing a correction to the bulk mass parameters. As a consequence, the **16** profile gets splitted into its three SU(5) components **10**, $\overline{\mathbf{5}}$, **1**. Remarkably, the resulting framework resembles the structure of SU(5) grand unification, enhancing a similar description of down quarks and charged leptons. The splitting of the **16** profile allows for more flexibility in reproducing the mass spectrum, at the cost of adding only one new parameter, the VEV of the field. With respect to the extra dimension scenario imposed at the level of the SM, where one independent bulk mass is allowed for each fermion representation, this mechanism results much more economic in reproducing the flavour pattern, since the distinction of the profiles is predicted by the gauge group structure and totally only 4 bulk mass parameters are needed. This nice feature is due to the constraints of the supersymmetric construction.

We have built two specific supersymmetric SO(10) models in 5D, with the purpose of testing the viability of such a framework. While the generic ingredients of this construction are well-defined, a considerable freedom is left in the model building, depending on the specific implementation of the idea.
The first SO(10) model built by us includes the Kitano-Li mechanism in the bulk and a specific Higgs sector on a brane, suitable for the Yukawa couplings, the breaking of SO(10) and the doublet-triplet splitting, which is a delicate issue of GUTs. In a second model we exploit much more the potentiality offered by the extra dimension, providing an explicit breaking of SO(10) through boundary conditions. A doubling of the fermion representations and thus a further splitting of the profiles, distinguished in left and right chiralities, emerge as necessary consequence of this explicit breaking. While this makes the model more flexible in reproducing the flavour observables, on the other hand it spoils the full unification of fermions. The Kitano-Li mechanism is still present as essential ingredient in the bulk, while the Higgs sector on the brane is much more simplified due to the smaller gauge group and automatically avoids the doublet-triplet splitting problem.

The comparison between the two models leads to interesting conclusions. Both the models provide a good fit of fermion masses and mixing angles only for large values of $\tan\beta$ (=50) and for both the ordering of neutrino masses, normal or inverted. The inverted ordering, however, turns out to be a very fine-tuned solution in both the models, as verified computing a naturalness test, by random variation of the Yukawa parameters. The predictions for the unobserved quantities of the flavour sector are given in terms of distributions with respect to the random Yukawa entries of order one. The results turn out to be almost identical in the two models. The lightest neutrino mass is predicted below 10 meV and an effective mass of the neutrinoless double beta decay in the range 1-5 meV. These values are below the sensibility of the actual and next future experiments, so that no direct proof of these results is actually possible, but, on the other side, an experimental signal would immediately falsify the models. No particular preference is found for the Dirac CP phase of the lepton sector. The spectrum of right handed neutrinos is predicted to be strongly hierarchical, with the first generation at 10^3-10^5 GeV, the second at 10^7-10^{10} GeV and the third at 10^{14}-10^{15} GeV. Such hierarchy of the spectrum is not compatible with models of Standard Thermal Leptogenesis, where to generate lepton asymmetry the lightest right handed neutrino must be above $\sim 10^9$ GeV. Flavour effects which may accommodate a lepton asymmetry generated by the second generation are, at first sight, also excluded. If possible, they would require a very fine-tuned selection of the Yukawa couplings, which should be eventually tested in a dedicated study, beyond the scope of the present work.

The fact that these predictions are essentially unchanged from one model to the other is remarkable. The two models have a common mechanism for the profile splitting in the bulk, while they significantly differ in the field content on the branes. Given this difference, a common behavior in the predictions was not granted a priori. This result has an important meaning, essentially revealing that the phenomenology of the model is mostly dictated by the dynamics in the bulk, while not being sensible to the dynamics chosen on the brane. This proves the robustness of the Kitano-Li mechanism in the flavour sector as a model independent process, which is a notable feature for a mechanism operating at a high energy scale, where the freedom in the model building is generally huge. Indeed, all the features of the flavour observables, as well as the predictions above, can be well explained by the setup induced on the model by the Kitano-Li mechanism. The large mixing angles of the lepton sector and the moderate hierarchy among neutrino masses is attributed to a nearly equal wave function renormalization of the three generations of $\bar{5}$. The observed hierarchies in the charged lepton sector and in the quark sectors are mostly due to the different renormalization affecting the three generations of 10. The different rescaling of $\bar{5}$-plets and 10-plets implies that mass ratios in the up-quark sector are nearly the square of the corresponding mass ratios in the down-quark and charged-lepton sectors, which is approximately true. The more natural behavior of the model with normal ordering of neutrino masses is motivated by the fact that left handed neutrinos have the same renormalization of down quarks and charged leptons in the $\bar{5}$-plets, which respect a normal hierarchy. An inverted ordering is found as very fine-tuned solution, since the

Yukawa parameters must be carefully selected to conspire against the natural tendency of order dictated by the charged sector. The compatibility with only large values of $\tan\beta$ is explained by the unification of the third generation in a nearly equal profile for the three SU(5) multiplets. Finally, the Kitano-Li mechanism acts on the hierarchies, that means it modulates the “magnitudes” of the fields profiles convoluted with the Yukawa couplings, but it doesn't have any influence on the imaginary phases of the fields and the couplings of this scenario. Therefore, the undetermined predictions for the Dirac and Majorana phases of the lepton sector, characterized by flat distributions, are explained by the fact that the source of this phases arises entirely from the random variation in the Yukawa couplings.

On the other hand, regarding the dynamics on the brane, we have verified that, while it is not affecting the overall predictions, it greatly influences the success rate of the model with respect to the random variation of Yukawa couplings. An increased number of free parameters in general improve the success rate. This happens both considering the parameters of the Higgs sector and even more increasing the parameters of the fermions profiles, which have a major influence because of the exponential dependence. Therefore, special attention should be posed on the brane dynamics if one wants to improve the stability of the model with respect to the anarchical variation of the Yukawa couplings. Relaxing the anarchy assumption, the role of the branes is less important and the model still works with some fine-tuning in the Yukawas. In any case, one achieves a successful description of the flavour sector in terms of fundamental parameters of order one, which is already a remarkable result.

To conclude, our analysis confirms that the idea of anarchical Yukawa couplings can be successfully implemented even in the context of an SO(10) grand unified theory, providing a more natural description of the flavour sector. While different implementations are possible, the required conditions are that SO(10) is broken down to SU(5) as a first step and that the anarchical Yukawa couplings of the different charge sectors are not entirely dominated by a single SO(10) Higgs multiplet. Remarkably, a more economic choice of the Higgs sector is possible with respect to SO(10) models realized in the usual 4-dimensional spacetime. On the weak side, as in all the models based on a large number of independent order one parameters, it is not possible to plan precision tests of these ideas to fully exploit the accuracy of the experimental data.

Finally, let us remark that the proposed models aim to a general test of the above idea, while missing some aspects, like the specification of a mechanism for N=1 SUSY breaking, which would introduce a strong model dependence. However, the discussion of N=1 SUSY breaking is needed for a fully realistic approach, reducing the uncertainty on the values of the flavour observables extrapolated at the GUT scale. As final aim of this study, we have tried to specify a way to break N=1 SUSY looking for a mechanism that could exploit the existence of the extra dimension. While various SUSY breaking mechanisms in extra dimensions are known, those analyzed by us have characteristics not completely suitable for our specific models. The major obstacles come from the compactification scale being above the GUT scale and from the existence of exponential profiles for the matter fields, which are crucial to reproduce the fermion spectrum, while they induce not suitable hierarchies in the mass matrix for sleptons and squarks. The most simple way seems to add a hidden sector mediating the SUSY breaking localized on the Higgs brane, where a suppression of flavour violating terms is naturally accounted for. However such possibility requires a dominant universal contribution and we should carefully test if this can be simply provided by gaugino masses through RGE evolution or if extra contributions are needed.

This aspect of the SUSY breaking deserves for sure major attention in order to complete our study. In the meanwhile, such preliminary analysis has lead to an original contribution by us, computing the scalar spectrum induced by the Radion mechanism in presence of bulk masses. Some new interesting issues that we could explore are also pointed out, as the consistency of the Scherk-Schwarz mechanism in presence of bulk mass, its eventual relation with the Radion mechanism and the role of a new term entering the scalar spectrum in the Radion mechanism, when we consider the VEV of the scalar field from the gauge sector.

Appendix A

Basics of Supersymmetry

In this appendix we present the basic concepts and notations of Supersymmetry, on which the exposition of this thesis is partially based on. Without claim of completeness, for an exhaustive review of the topic we refer the reader to [261, 262], to which this exposition is mainly inspired.

A.1 SUSY Algebra.

SUSY is a symmetry which transforms boson states into fermionic states (and vice versa). It is introduced as new symmetry of the spacetime, extending the Poincaré algebra. Such kind of additional spacetime symmetries are highly restricted by the Coleman-Mandula theorem [263] and its Haag-Lopuszanski-Sohnius extension [264]. The Coleman-Mandula theorem implies that the most general Lie algebra for the symmetries of the S-matrix[1] in a relativistic quantum field theory is the direct product of the Poincaré algebra with a compact Lie group, which describes the internal symmetries (for example, the gauge symmetries). This means that any generator of the internal symmetry commutes with the Poincaré generators. This is true for generators of bosonic type, with algebra defined by commutation relations. It has been shown [264], however, that generators of fermionic type, which follow rules of anticommutations, provide a non trivial extension of the Poincaré algebra.

The generators of SUSY, transforming bosons into fermions and vice versa, carry a spin 1/2 and are indeed of fermionic type. In case we introduce a single pair (N=1) of SUSY generators[2] Q_α and $\overline{Q}_{\dot\alpha}$, which are 2-components Majorana spinors, the commuta-

[1] The S-matrix is the evolution operator connecting two asymptotyc particle states. The exact assumptions of the Coleman-Mandula theorem on the S-matrix are [261]: (1) the S matrix is based on a local, relativistic quantum field theory in fourdimensional spacetime, (2) there are only a finite number of different particles associated with one-particle states of a given mass, and (3) there is an energy gap between the vacuum and the one particle states.

[2] Q_α and $\overline{Q}_{\dot\alpha}$, with $\alpha, \dot\alpha = 1, 2$ are spinorial representations of the Poincaré group transforming respectively as $(\frac{1}{2}, 0)$ and $(0, \frac{1}{2})$ of $SU(2) \times SU(2) \approx SL(2)C$. They are related by: $Q^\dagger_\alpha \equiv \overline{Q}_{\dot\alpha}$.

tion/anticommutation rules of the algebra are:

$$
\begin{aligned}
[P_\mu, Q_\alpha] &= 0\,, \qquad [P_\mu, \overline{Q}_{\dot\alpha}] = 0\,, \\
[M_{\mu\nu}, Q_\alpha] &= i(\sigma_{\mu\nu})_\alpha^{\ \beta} Q_\beta\,, \qquad [M_{\mu\nu}, \overline{Q}^{\dot\alpha}] = i(\overline{\sigma}_{\mu\nu})^{\dot\alpha}_{\ \dot\beta}\overline{Q}^{\dot\beta}\,, \\
\{Q_\alpha, Q_\beta\} &= 0\,, \qquad \{\overline{Q}_{\dot\alpha}, \overline{Q}_{\dot\beta}\} = 0 \\
\{Q_\alpha, \overline{Q}_{\dot\beta}\} &= 2\sigma^\mu_{\alpha\dot\beta} P_\mu\,.
\end{aligned}
\tag{A.1}
$$

where P_μ is the momentum operator, generator of translations in spacetime, $M_{\mu\nu}$ are generators of the Lorentz transformations and $\sigma^{\mu\nu} = \frac{1}{4}\left[\sigma^\mu, \overline{\sigma}^\nu\right]$.

A.2 Superspace formalism and MSSM

Superspace and Superfields.

An irreducible representation of SUSY is called supermultiplet and contains an equal number of fermions and bosons. All the known SM particles are thus accompanied by a superpartner residing in the same multiplet: a fermion if the known particle is a boson and vice versa. A convenient way to describe supermultiplets is through the superfield formalism. This formalism consists in defining the fields on the "superspace", which is an extensions of the 4-dimensional spacetime where, besides the coordinates x^μ, we include two coordinates θ and $\overline{\theta}$, which behave like Majorana spinors whose 2 components are Grassmann (anticommuting) variables[3]. We indicate the superspace with the set coordinates $(x^\mu, \theta_\alpha, \overline{\theta}_{\dot\alpha})$. Fields defined on this superspace can be expanded as a finite series of Grassmann variables. The most general expansion is:

$$
\begin{aligned}
S(x,\theta,\overline{\theta}) &= f(x) + \theta\psi(x) + \overline{\theta}\overline{\chi}(x) + \theta\theta\, m(x) + \overline{\theta\theta}\, n(x) \\
&+ \theta\sigma^\mu\overline{\theta}\, A_\mu(x) + \theta\theta\,\overline{\theta}\overline{\lambda}(x) + \overline{\theta\theta}\,\theta\rho(x) + \theta\theta\,\overline{\theta\theta}\, d(x)\,,
\end{aligned}
\tag{A.2}
$$

where we recognize the bosonic scalar fields $f(x), m(x), n(x), d(x)$, the bosonic vector field $A_\mu(x)$ and the fermionic (Weyl) fields $\psi(x)$, $\overline{\chi}(x)$, $\rho(x)$, $\overline{\lambda}(x)$, for a total of 8 complex (16 real) fermionic as much as bosonic degrees of freedom.
On the superspace the SUSY generators are defined as:

$$
\begin{aligned}
Q_\alpha &= -i\frac{\partial}{\partial\theta^\alpha} - \sigma^\mu_{\alpha\dot\beta}\overline{\theta}^{\dot\beta}\partial_\mu \\
\overline{Q}_{\dot\alpha} &= i\frac{\partial}{\partial\overline{\theta}^{\dot\alpha}} + \theta^\beta\sigma^\mu_{\beta\dot\alpha}\partial_\mu\,.
\end{aligned}
\tag{A.3}
$$

And the SUSY variation of the superfield is:

$$
\delta_{\epsilon,\overline{\epsilon}} S = \left(i\epsilon Q + i\overline{\epsilon}\overline{Q}\right) S \tag{A.4}
$$

where ϵ, $\overline{\epsilon}$ are Weyl spinors parameters. One can see that acting on the superfield (A.2) with Q as defined in Eq.(A.3), multiplying or deriving by the Grassmann coordinate one

[3] $\{\theta_\alpha, \theta_\beta\} = 0 = \{\overline{\theta}_{\dot\alpha}, \overline{\theta}_{\dot\beta}\}$

gets transitions between superpartners.
A general superfield contains too many component fields to correspond to an irreducible representation of $N = 1$ SUSY. One can lower down the number of components by imposing SUSY invariant conditions. In such a way we identify two important type of fields:

- **Chiral superfields.** A chiral superfield ϕ and an antichiral superfield $\overline{\phi}$ are identified by the conditions[4]

$$\overline{D}_{\dot{\alpha}}\Phi = 0, \quad D_\alpha\overline{\Phi} = 0 \tag{A.5}$$

which give, defining the variable[5] $y^\mu = x^\mu + i\theta\sigma^\mu\overline{\theta}$:

$$\Phi(y,\theta) = \phi(y) + \sqrt{2}\theta\psi(y) - \theta\theta F(y) \tag{A.6}$$
$$\overline{\Phi}(y,\theta) = \overline{\phi}(y) + \sqrt{2}\overline{\theta}\,\overline{\psi}(y) - \overline{\theta}\,\overline{\theta}\,\overline{F}(y) \tag{A.7}$$

The SUSY transformations of the components result:

$$\begin{aligned} \delta\phi &= \sqrt{2}\epsilon\psi \\ \delta\psi &= \sqrt{2}i\partial_\mu\phi\sigma^\mu\overline{\epsilon} - \sqrt{2}F\epsilon \\ \delta F &= \sqrt{2}i\partial_\mu\psi\sigma^\mu\overline{\epsilon}\,. \end{aligned} \tag{A.8}$$

Physically, a chiral superfield describes one complex scalar ϕ and one Weyl fermion ψ, while the F scalar turns out to be an auxiliary field. In multiplet notation, this superfield on shell describes a chiral multiplet including the helicity states $(0, 1/2)$ and its CPT conjugate $(-1/2, 0)$. We denote it in short by the component contents as $\Phi \equiv (\phi, \psi)$. This representation is associated to all the SM fermions, accompanied by their scalar superpartners (sfermions).

- **Vector superfields.** Vector superfields are identified by the reality condition:

$$V(x,\theta,\overline{\theta}) = V^\dagger(x,\theta,\overline{\theta}) \tag{A.9}$$

We can reduce further the number of components by making use of the abelian gauge transformation: $V \to V + \Phi + \Phi^\dagger$, where Φ is a chiral superfield, implying $A_\mu \to A_\mu + \partial_\mu(2Im(\phi))$. With the proper choice of Φ (Wess-Zumino gauge) one reduces the vector superfield to:

$$V_{\rm WZ} = \theta\sigma^\mu\overline{\theta}A_\mu(x) + i\theta\theta\,\overline{\theta}\,\overline{\lambda}(x) - i\overline{\theta\theta}\,\theta\lambda(x) + \frac{1}{2}\theta\theta\overline{\theta\theta}D(x)\ . \tag{A.10}$$

[4]We introdue the covariant derivatives D_α and $\overline{D}_{\dot{\alpha}}$ which anticommute with the SUSY generators Q and $\overline{Q}$:

$$D_\alpha = \frac{\partial}{\partial_{\theta^\alpha}} + i\sigma^\mu_{\alpha\dot{\beta}}\overline{\theta}^{\dot{\beta}}\partial_\mu$$
$$\overline{D}_{\dot{\alpha}} = \frac{\partial}{\partial_{\overline{\theta}^{\dot{\alpha}}}} + i\theta^\beta\sigma^\mu_{\beta\dot{\alpha}}\partial_\mu$$

[5]such a variable leads to a compact notation of the superfield. By Taylor expanding in terms of x, θ and $\overline{\theta}$ one gets the full expression of the superfields involving also the derivatives of the field components.

A vector superfield describes a vector boson A_μ, a Majorana fermion λ, and a scalar D which turns out to be an auxiliary field. This field content corresponds on shell to the irreducible vector multiplet, made of the helicity states $(1/2, 1)$ plus its CPT conjugate $(-1, -1/2)$, that we indicate in short as $V \equiv (A_\mu, \lambda)$. The vector superfield is the representation associated to the gauge vector bosons and the fermion superpartner takes the name of gaugino.

MSSM field content.

With the two type of superfields presented above, chiral and vector, one can describe all the SM particles plus the relative superpartners, which are, in all respects, new particles extending the SM. The overall field content corresponds to the so-called Minimal Supersymmetric Standard Model (MSSM) and it is reported in Tab. A.1 and Tab. A.2. We have a chiral multiplet for each of the fermion representations introduced in the SM, distinguishing the LH and RH components. In the Higgs sector, we have two chiral multiplets distinguishing the Higgs representations of up and down type. This double representation is needed in SUSY for two reasons:

A. With only one Higgs multiplet the gauge symmetry would suffer of anomaly, and would be inconsistent as quantum field theory.[6]

B. Because of the holomorphic structure of the superpotential (see next paragraph) two independent $\mathrm{SU(2)_L}$-doublet representations of the Higgs are needed to form the Yukawa couplings. A $Y = 1/2$ Higgs (H_u) necessary to give masses to the up-type quarks, a $Y = -1/2$ Higgs (H_d) to give masses to the down-type quarks and to the charged leptons.

The combination of weak isospin and hypercharge let us identify the Higgs components according to their electromagnetic charges as:

$$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}, \quad H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}, \tag{A.11}$$

To each component corresponds the relative superpartner, the Higgsinos (see Tab. A.1). Both H_u and H_d fields enter the EW symmetry breaking, getting a VEV along the neutral component: $v_u = \langle H_u^0 \rangle$ and $v_d = \langle H_d^0 \rangle$ respectively. The neutral scalar corresponding to

[6] The conditions for cancellation of gauge anomalies include $\mathrm{Tr}[T_{L3}^2 Y] = 0$ and $\mathrm{Tr}[Y^3] = 0$, where the traces run over all the fermionic degrees of freedom in the theory. In the SM this condition is satisfied by the known representations of quarks and leptons, somehow miraculously. Now a new Weyl fermion, partner of a Higgs scalar in one chiral supermultiplet, enters this trace as a weak isodoublet with hypercharge $Y = 1/2$ or $Y = -1/2$. In either case, such a fermion alone will give a non-zero contribution to the trace, spoiling the anomaly cancellation. This is avoided by introducing two Higgs supermultiplets, one with each of $Y = \pm 1/2$, so that the two contributions to the anomaly get canceled.

Names		spin 0	spin 1/2	SU(3)$_C$×SU(2)$_L$×U(1)$_Y$
squarks, quarks	Q	$(\widetilde{u}\ \ \widetilde{d})$	$(u\ \ d)$	$(3,2)_{\frac{1}{6}}$
(×3 families)	U^c	$\widetilde{u}^c$	u^c	$(\overline{3},1)_{-\frac{2}{3}}$
	D^c	$\widetilde{d}^c$	d^c	$(\overline{3},1)_{\frac{1}{3}}$
sleptons, leptons	L	$(\widetilde{\nu}\ \widetilde{e})$	$(\nu\ \ e_L)$	$(1,2)_{-\frac{1}{2}}$
(×3 families)	E^c	$\widetilde{e}^c$	e^c	$(1,1)_1$
Higgs, higgsinos	H_u	$(H_u^+\ \ H_u^0)$	$(\widetilde{H}_u^+\ \ \widetilde{H}_u^0)$	$(1,1)_{\frac{1}{2}}$
	H_d	$(H_d^0\ \ H_d^-)$	$(\widetilde{H}_d^0\ \ \widetilde{H}_d^-)$	$(1,2)_{-\frac{1}{2}}$

Table A.1: Chiral supermultiplets in the MSSM. The spin-0 fields are complex scalars, and the spin-1/2 fields are LH two-component Weyl fermions. The superparticles are denoted by a $\sim$.

Names	spin 1/2	spin 1	SU(3)$_C$×SU(2)$_L$×U(1)$_Y$
gluino, gluon	$\widetilde{g}$	g	$(8,1)_0$
winos, W bosons	$\widetilde{W}^\pm\ \ \widetilde{W}^0$	$W^\pm\ \ W^0$	$(1,3)_0$
bino, B boson	$\widetilde{B}^0$	B^0	$(1,1)_0$

Table A.2: Gauge vector supermultiplets in the MSSM.

the physical SM Higgs boson, Eq.(2.2), is given by a linear combination of H_u^0 and H_d^0, and the same for its VEV, which gives the condition:

$$v_u^2 + v_d^2 = v^2 \tag{A.12}$$

where $v \approx 174$ GeV. The linear combination can be parameterized in terms of the β angle defined as:

$$\tan\beta \equiv v_u/v_d \tag{A.13}$$

that leads to $v_u = v\sin\beta$ and $v_d = v\cos\beta$. The value of $\tan\beta$ is not experimentally measured yet and it must be considered as a free parameter of the MSSM. Many predictions of the theory, as the mass spectrum and the evolution of the parameters from the renormalization group equations (RGE) keep dependence from β and they are usually reported for two opposite values of low $\tan\beta = 10$ and large $\tan\beta = 50$, that favors the predominance of v_u.

Let us remark some features of the supermultiplets components:

(1) while SUSY does not commute with the Lorentz transformations, it commutes with

all the internal symmetries; as a result, the gauge representations are common for both the fermions and the bosons in the same supermultiplet.
(2) From Eq.(A.1) it follows $[Q_\alpha, P^2] = 0$. Since the operator $P^2 = P_\mu P^\mu$ when applied to a particle gives its mass squared, this implies that particles related by SUSY transformation have the same mass if SUSY remains unbroken. Thus, all the particles in the same supermultiplet would have a common mass. Since this is not observed in nature, SUSY has to be a broken symmetry.

Superymmetric Lagrangian

The Lagrangian density in a SUSY theory is built with the language of superfields and superspace and, in particular, it includes the integration over θ and $\overline{\theta}$ coordinates. The key point in this construction, which must be SUSY invariant, is recognizing that the F-term of a chiral superfield and the D-term of the vector superfield transform under SUSY into themselves plus a total derivative: these structures are then suitable for the Lagrangian, which is defined up to total derivatives. By recognizing the properties of products of superfields, which are still superfields of chiral or vector type, one can construct the Lagrangian in terms of two main building blocks made of field combinations: the superpotential, that transforms like a chiral superfield, of which we keep the F-term, and the Kähler potential, that transforms like a vector superfield, of which we keep the D-term. The superpotential is an holomorphic function $W(\Phi_i)$, product of one or more chiral superfields. As such, it depends only on Φ_i and not on $\Phi_i^\dagger$ and can describe only a potential of the component fields, like mass terms and Yukawa couplings, but not kinetic terms. Because of the integration properties over the Grassmann spinors, only the F-term (*i.*e. the term in $\theta\theta$) survives as contribution to the Lagrangian:

$$\mathcal{L}_F = \int d^2\theta \, W(\Phi_i) + \int d^2\overline{\theta} \, W^\dagger(\Phi_i^\dagger) = [W(\Phi_i)]_{\theta\theta} + \left[W^\dagger(\Phi_i^\dagger)\right]_{\overline{\theta\theta}} \tag{A.14}$$

The Kähler potential $K(\Phi_i^\dagger, \Phi_j)$ is made of the product of chiral and antichiral superfields, in the simplest case $\Phi_i^\dagger \Phi_i$, and it is real. The integration over the Grassmann variables leads in this case only the D-term (*i.*e. the term in $\theta\theta\overline{\theta\theta}$) to survive in the Lagrangian. In the most simple case we have:

$$\mathcal{L}_D = \int d^2\theta d^2\overline{\theta} \Phi_i^\dagger \Phi_i = \left[\Phi_i^\dagger \Phi_i\right]_{\theta\theta\overline{\theta\theta}} \tag{A.15}$$

When expanded into the field components this leads to the kinetic terms for the scalars and the fermions inside Φ_i.
From these basic principles and including the gauge interactions, we find the action for a SUSY theory made of various chiral superfields Φ_i (describing matter multiplets plus the Higgs multiplets) and the gauge fields $V = V_a T^a$, with one vector superfield for each

generator T^a of the internal symmetry group [7]:

$$S = \int d^4x \int d^2\theta d^2\bar{\theta}\, \Phi^\dagger e^V \Phi + \int d^4x \int d^2\theta \left(W(\Phi) + \mathrm{Tr}\, W^\alpha W_\alpha\right) + h.c. \tag{A.16}$$

where the first term gives the gauge invariant kinetic term for the matter and Higgs fields; $W(\Phi)$ is the superpotential containing the Higgs potential and the Yukawa couplings; the third term leads to the kinetic term of gauge bosons and gauginos and it is constructed through the definition of the chiral superfield: $W_\alpha \equiv -\frac{1}{4}\overline{D}^2 D_\alpha V$. In terms of the component fields the Lagrangian reads:

$$\mathcal{L} = \mathcal{L}_{gauge} + \mathcal{L}_{matter} + \mathcal{L}_{int} - V(\phi_i, \phi_i^\dagger) \tag{A.17}$$

where, integrating out the auxiliary fields F_i and $D = D_a T^a$, we find:

$$\begin{aligned}
\mathcal{L}_{gauge} &= -\frac{1}{4} F_a^{\mu\nu} F^a_{\mu\nu} + i\bar{\lambda}^a \sigma^\mu D_\mu \lambda_a \\
\mathcal{L}_{matter} &= |D_\mu \phi_i|^2 + \bar{\psi}_i \sigma^\mu i D_\mu \psi_i \\
\mathcal{L}_{int} &= \sqrt{2} g \bar{\lambda} \psi_i \phi_i^\dagger + \psi_i \psi_j \frac{\partial^2 W}{\partial \phi_i \partial \phi_j} \\
V(\phi_i, \phi_i^\dagger) &= \sum_i \left| \frac{\partial W}{\partial \phi_i} \right|^2 + \frac{g^2}{2} \sum_{a,i} \left| \phi_i^\dagger T^a_{R_i} \phi_i \right|^2
\end{aligned} \tag{A.18}$$

where D_μ stands for the covariant derivative with respect to the gauge group. The last line describes the scalar potential which has two contributions, the first coming from the superpotential (F-term), the second coming from the gauge interactions (D-term).

MSSM superpotential

The above action represents the MSSM action when the chiral superfields Φ_i are interpreted with the content of Tab. A.1 and the gauge vector superfields with the content of Tab. A.2. Let us specify the MSSM superpotential, which consists of two parts:

$$W_{MSSM} = W_Y + W_\mu\,, \tag{A.19}$$

where, in terms of the chiral superfields of Tab. A.1:

$$W_Y = Y^u_{ij} Q_i \overline{u}_j H_u + Y^d_{ij} Q_i \overline{d}_j H_d + Y^e_{ij} L_i \overline{e}_j H_d; \tag{A.20}$$

$$W_\mu = \mu\, H_u H_d. \tag{A.21}$$

The W_Y includes all the SM Yukawa interaction terms written in terms of superfields, the matrices $Y_{u,d,e}$ are exactly identified with the SM ones in Eq.(2.6). In the supersymmetric action the SM Yukawa terms for the fermion fields are reproduced through the second

[7] For simplicity of description here we ignore the possibility of Fayet-Iliopoulos terms associated to the U(1) abelian factors of the gauge group, given by the D-term relative to the U(1) generators T^A: $g\xi_A D^A$.

derivative of the superpotential in $\mathcal{L}_{int}$ of Eq.(A.18). This superpotential leads to equivalent mass terms (parametrized by the same Yukawa matrices) for squarks and sleptons in $V(\Phi_i, \Phi_i^\dagger)$, without considering the contribution of soft SUSY breaking mass terms.
The second part of the superpotential, W_μ, is the so called μ term, which enters in the scalar potential determining the tree-level contribution to the Higgs mass.
Let us finally mention a third contribution that could enter the MSSM superpotential, respecting SUSY and the gauge symmetry:

$$W_{\Delta B, \Delta L} = \lambda_{ijk} L_i L_j \overline{e}_k + \lambda'_{ijk} L_i Q_j \overline{d}_k + \lambda''_{ijk} \overline{u}_i \overline{d}_j \overline{d}_k + \mu'_i L_i H_d \tag{A.22}$$

These terms violate baryon (B) or lepton number (L), which are accidental symmetries of the SM. From such terms we can construct effective dimension-4 operators which are dangerous mediator of the proton decay. To avoid this problem, these terms are forbidden by imposing an R-parity symmetry, defined as:

$$R = (-1)^{3(B-L)+2S}, \tag{A.23}$$

where B and L are the baryon and lepton numbers and S is the spin of the particle. Such parity is respected by the superpotential contributions in Eqs.(A.20.A.21) Interestingly, the definition (A.23) assignes even R-parity to the known SM particles and odd R-parity to their superpartners. This has the important experimental implication that, for R-parity conserving theories, the superpartners must be always produced in pairs and the lightest superpartner must be a stable particle. This is generally called the LSP (Lightest Supersymmetric Particle), which, if neutral under strong and electomagnetic interactions, turns out to be a suitable candidate for dark matter.

SUSY breaking and soft terms

As anticipated, SUSY has to be a broken symmetry, otherwise we should observe the superpartners with the same masses of the SM particles. Different mechanisms can provide SUSY breaking with different predictions for the superpartners mass spectrum. Many models have been proposed, based on the usual spontaneous symmetry breaking or on new breaking mechanisms. In these models it is always needed to extend the MSSM to include new particles and interactions at higher energy scales. We cannot know yet which mechanism is the correct one, until we cannot confirm its prediction detecting the superpartners experimentally. Anyway, from a practical point of view, it is useful to take into account the description of the SUSY breaking and parameterize our ignorance about the exact mechanism by just introducing extra terms which explicitly break SUSY in the effective MSSM Lagrangian. These terms are called "soft" because they don't introduce quadratically divergent corrections to the Higgs mass, a requirement that preserves the solution to the hierarchy problem (cf. next paragraph). This means in particular that dimensionless SUSY-breaking couplings should be absent. The possible terms of this kind are:

$$\mathcal{L}_{\rm soft} \;=\; -\left(\frac{1}{2}M_a\,\lambda^a\lambda^a + \frac{1}{6}a^{ijk}\phi_i\phi_j\phi_k - (m^2)^{ij}\phi_i\phi_j + t^i\phi_i\right) + h.c. \qquad \text{(A.24)}$$

They consist of gaugino masses M_a for each gauge group, scalar squared-mass terms $(m^2)^{ij}$, and 3-scalars couplings a^{ijk}. These soft terms clearly break SUSY and are capable of giving masses to all the scalars and gauginos, even if the gauge bosons and fermions in chiral supermultiplets are massless (or relatively light).
Some mechanisms of SUSY breaking are reviewed in Chapt. 7.

Solution to the hierarchy problem.

The most compelling feature of a SUSY theory comes from the ultraviolet behavior. An extremely important result is that, in the exact SUSY limit, the parameters of the superpotential $W(\Phi)$ do not receive any (finite or infinite) correction from Feynman diagrams at any loop level. This is known as the non-renormalization theorem [265] and it is at the core of the solution to the Higgs mass hierarchy problem. Indeed, this fact prevents the Higgs mass from receiving radiative corrections which, depending quadratically on the scale of new physics, would push the Higgs mass up to the Planck scale, destabilizing the SM. In the SUSY version of the SM, the MSSM, no radiative corrections in the limit of exact SUSY destabilize the Higgs mass, which can therefore be set at the tree level. This happens essentially because the loop contributions from the SM fermions are exactly canceled by equal and opposite contributions from the scalar superpartners. However, since SUSY must be broken, one has to ensure that the SUSY breaking terms do not spoil the non-renormalization theorem. SUSY breaking is indeed parametrized by "soft" terms, which don't introduce quadratic divergences, but only finite contributions. The loop corrections to the Higgs mass thus depend on the parameters of soft breaking (essentially the masses of sfermions and gauginos), which must be less than a TeV so that the weak scale remains stabilized. In such a scenario, to solve the hierarchy problem, one would expect the mass spectrum of the superpartners at the TeV scale, anyway the actual constraints from LHC are severely constraining this possibility, questioning the primary role of SUSY as solution to the hierarchy problem.

A.3 N=2 SUSY

Let us briefly go back to the more formal aspects of SUSY, introducing the general case of $N > 1$ SUSY, which will be of interest in view of further applications in extra dimensions (cf. Sect.4.5). Consider to introduce N couples of SUSY generators Q^A_α, $\overline{Q}^A_{\dot\alpha}$, with $A = 1, \ldots N$. The algebra distinguishes from the case $N = 1$, Eq.(A.1), only in the following rules:

$$\{Q^A_\alpha, Q^B_\beta\} = \epsilon_{\alpha\beta}Z^{AB}, \qquad \{\overline{Q}^A_{\dot\alpha}, \overline{Q}^B_{\dot\beta}\} = \epsilon_{\alpha\beta}(Z^{AB})^*, \quad \{Q^A_\alpha, \overline{Q}^B_{\dot\beta}\} = 2\sigma^\mu_{\alpha\dot\beta}P_\mu\delta^{AB}\,. \quad \text{(A.25)}$$

The $Z^{AB} = -Z^{BA}$ are central charges, commuting with all generators of the full algebra. In the simplest extended case, $N = 2$, there is just one central charge $Z \equiv Z^{12}$. Anyway, when considering massless representations, it is shown that all central charges vanishes [261].

The largest possible symmetry group which can act non trivially on the Q^A generators is a global $\mathrm{U(N)}_R$ symmetry, called R-Symmetry.

In the case N=2 this reduces to an $\mathrm{SU}(2)_R$ symmetry, under which the generators Q^1_α, Q^2_α transform in the fundamental representation. The multiplets are constructed by the action of two generators pairs on a vacuum of given helicity and they can be expressed as direct sum of N=1 supermultiplets. Without introducing a N=2 superspace approach, which goes beyond the purpose of this presentation, one can describe the N=2 multiplets using the N=1 superspace formalism imposing the requirements given by the $\mathrm{SU}(2)_R$ symmetry. One can construct N=2 vector and hypermultipets which are decomposed in N=1 multiplets as (in short notation):

$$\text{N=2 vector multiplet:} \qquad V = (\lambda_\alpha, A_\mu, D) \;\oplus\; \Phi = (\phi, \psi_\alpha, F)$$

$$\text{N=2 hypermultiplet:} \qquad H_1 = (h_1, \psi_{1\alpha}, F_1) \;\oplus\; H_2^c = (\bar{h}_2, \psi_{2\dot{\alpha}}, F_2)$$

The N=2 vector multiplet is made of an N=1 vector multiplet V and an N=1 chiral multiplet Φ transforming in the same representation under internal symmetries. The N=2 hypermultiplet is made of two N=1 chiral multiplets, H_1 and H_2, which transform in complex conjugate representations under internal symmetries (in the notation above, H_1 and H_2^c transform in the same way). From the construction of these multiplets by acting with the N=2 SUSY generators, it can be shown that some components of these fields transform non trivially under $\mathrm{SU}(2)_R$, in particular:

- in the vector multiplet, all the bosonic fields are singlets, while $(\lambda_\alpha, \psi_\alpha)$ transform as a doublet;
- in the hypermultiplet, the scalars $(h_1, \bar{h}_2)$ form a doublet, while all the other components are singlet.

The Lagrangian for N=2 SUSY can be formulated in terms of the above $N = 1$ superfields. The emerging structure is the one found for N=1 SUSY with the N=1 multiplets above, but with some restrictions due to the $\mathrm{SU}(2)_R$ symmetry. In particular, one finds the following "rules" imposed by $\mathrm{SU}(2)_R$:

- no superpotential can be written for the chiral superfield Φ of the vector multiplet;
- for the hypermultiplets, one can see that the superpotential of the chiral multiplets H_1 and H_2^c cannot contain trilinear couplings, since no cubic $\mathrm{SU}(2)_R$ invariant is possible. The only admitted trilinear coupling is a gauge interaction with the chiral superfield Φ: $W \supset g\, H_2^c \Phi H_1$. Finally, a mass term from the bilinear coupling is admitted: $W \supset m\, H_2^c H_1$.

In section 4.5 these “rules” will dictate the structure of the Lagrangian for N=1 SUSY in 5 dimensions, which can be described as N=2 SUSY in a 4-dimensional superspace.

Appendix B

Spinorial representations of SO(10)

In this technical appendix we review the construction of the $SO(10)$ spinorial representations, by the convenient choice of illustrating the $SO(10)$ algebra in the $SU(5)$ basis [36, 92].

B.0.1 The Algebra of $SO(2n)$

Consider a set of n operators ξ_i $(i = 1, ..., n)$, and their hermitian conjugates, $\xi_i^\dagger$, satisfying

$$\{\xi_i, \xi_j^\dagger\} = \delta_{ij}, \qquad \{\xi_i, \xi_j\} = 0, \tag{B.1}$$

where { , } denotes an anti-commutator and [,] denotes a commutator. The operators K^i_j defined as

$$K^i_j \equiv \xi_i^\dagger \xi_j \tag{B.2}$$

satisfy the algebra of the $U(n)$ group

$$[K^i_j, K^m_n] = \delta^m_j K^i_n - \delta^i_n K^m_j. \tag{B.3}$$

We can then define the following $2n$ operators, Γ_μ $(\mu = 1, ..., 2n)$

$$\begin{aligned} \Gamma_{2j-1} &= -i\ (\ \xi_j - \xi_j^\dagger\ ) \\ \Gamma_{2j} &= (\ \xi_j + \xi_j^\dagger\ ), \qquad j = 1, ..., n. \end{aligned} \tag{B.4}$$

The Γ_μ form the Clifford algebra of rank $2n$

$$\{\Gamma_\mu, \Gamma_\nu\} = 2\delta_{\mu\nu} \tag{B.5}$$

and they can then be used to construct the generators of $SO(2n)$ as follows:

$$\Sigma_{\mu\nu} = \frac{1}{2i}\ [\Gamma_\mu, \Gamma_\nu]. \tag{B.6}$$

The dimensionality of the spinor representation of $SO(2n)$ is 2^n. In terms of the $SU(n)$ basis, the spinor representation of $SO(2n)$ can then be constructed by,

$$|0> \quad \sim \quad 1 \tag{B.7}$$

$$\xi_i^\dagger \; |0> \quad \sim \quad n \tag{B.8}$$

$$\xi_i^\dagger \; \xi_j^\dagger \; |0> \quad \sim \quad \frac{n(n-1)}{2} \tag{B.9}$$

$$\xi_i^\dagger \; \xi_j^\dagger \; \xi_l^\dagger \; |0> \quad \sim \quad \frac{n(n-1)(n-2)}{6} \tag{B.10}$$

$$..... \tag{B.11}$$

$$\xi_1^\dagger \; ... \; \xi_n^\dagger \; |0> \quad \sim \quad n \tag{B.12}$$

where $|0>$ is the $SU(n)$ invariant vacuum state. The spinor representation can then be split into two 2^{n-1}-dimensional representations by a chiral projection operator. Let us define

$$\Gamma_0 \equiv i^n \Gamma_1 \, \Gamma_2 \, ... \, \Gamma_{2n} \tag{B.13}$$

and the number operator

$$n_i \equiv \xi_i^\dagger \xi_i. \tag{B.14}$$

Γ_0 then can be written as

$$\begin{aligned} \Gamma_0 &= [\xi_1, \xi_1^\dagger] \, [\xi_2, \xi_2^\dagger] \, ... \, [\xi_n, \xi_n^\dagger] \\ &= \prod_{i=1}^{n} (1 - 2n_i) \\ &= (-1)^n. \end{aligned} \tag{B.15}$$

To arrive at the last step, we have used the property of the number operator $n_i^2 = n_i$ to get $1 - 2n_i = (-1)^{n_i}$ and $n = \sum_i n_i$. One can then check that

$$[\Sigma_{\mu\nu}, \Gamma_0] = 0. \tag{B.16}$$

The chirality projection operator is therefore defined by

$$\frac{1}{2}(1 \pm \Gamma_0). \tag{B.17}$$

Consider the case $n = 5$ and define a column vector $|\psi>$:

$$\begin{aligned} |\psi> \; &= \; |0> \psi_0 + \xi_j^\dagger |0> \psi_j + \frac{1}{2} \xi_j^\dagger \xi_k^\dagger |0> \psi_{jk} + \frac{1}{12} \epsilon^{ijklm} \xi_k^\dagger \xi_l^\dagger \xi_m^\dagger |0> \overline{\psi}_{ji} \\ &+ \frac{1}{24} \epsilon^{jklmn} \xi_k^\dagger \xi_l^\dagger \xi_m^\dagger \xi_n^\dagger |0> \overline{\psi}_j + \xi_1^\dagger \xi_2^\dagger \xi_3^\dagger \xi_4^\dagger \xi_5^\dagger |0> \overline{\psi}_0 \end{aligned} \tag{B.18}$$

where $\overline{\psi}$ is not the complex conjugate of ψ but an independent vector. This can be generalized to any n if we write

$$\psi = \begin{pmatrix} \psi_0 & \psi_i & \psi_{ij} & \overline{\psi}_{ij} & \overline{\psi}_i & \overline{\psi}_0 \end{pmatrix}^T . \tag{B.19}$$

The spinor representation is then split under the chirality projection operator as

$$\psi = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} \tag{B.20}$$

where

$$\psi_\pm = \frac{1}{2}(1 \pm \Gamma_0)\, \psi \tag{B.21}$$

and

$$\psi_+ = \begin{pmatrix} \psi_0 \\ \psi_{ij} \\ \overline{\psi}_j \end{pmatrix}, \qquad \psi_- = \begin{pmatrix} \overline{\psi}_0 \\ \overline{\psi}_{ij} \\ \psi_j \end{pmatrix}. \tag{B.22}$$

In the case of $n = 5$, $\overline{\psi}_i$ and ψ_{ij} are $\overline{5}$ and 10-dimensional representations of $SU(5)$ and ψ_0 is the singlet, the usual representations of the Georgi and Glashow theory [14]. All the SM fermions are assigned to ψ_+.

Appendix C

Spinors in 5D

C.1 Dirac spinors and the chirality problem

In 4D, Lorentz generators in the spinor representation are $\sigma_{\mu\nu} \equiv \frac{1}{4}[\gamma_\mu, \gamma_\nu]$, where γ_μ are the Dirac matrices, satisfying the Clifford algebra relation: $\{\gamma_\mu, \gamma_\nu\} = 2\eta_{\mu\nu}$. In our notation[1]:

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix};$$

where: $\sigma^\mu = \{\mathbf{1}_2, \sigma^i\}$; $\bar{\sigma}^\mu = \{\mathbf{1}_2, -\sigma^i\}$ and σ^i are Pauli's matrices.
We use the γ^5 definition:

$$\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -\mathbf{1} & 0 \\ 0 & \mathbf{1} \end{pmatrix}.$$

A fermion is represented by a 4-components Dirac spinor, that is reducible to the two irreducible representations $(1/2, 0)$, $(0, 1/2)$[2], the 2-components Weyl spinors χ_L and χ_R:

$$\Psi = \begin{pmatrix} \chi_L \\ \chi_R \end{pmatrix}$$

We can also write:

$$\Psi = \psi_L + \psi_R$$

in terms of the 4-components spinors: $\psi_L = \frac{1-\gamma^5}{2}\Psi$ and $\psi_R = \frac{1+\gamma^5}{2}\Psi$, that correspond to the chirality eigenstates under the action of γ_5:

$$\gamma_5\psi_L = \psi_L, \quad \gamma_5\psi_R = -\psi_R$$

[1]Notice that:
$\gamma^\mu = \eta^{\mu\nu}\gamma_\nu \Rightarrow \gamma^0 = \gamma_0$; $\gamma^i = -\gamma_i$ and $\gamma^5 = -\gamma_5$.
[2]Notation for the $SU(2) \times SU(2)$ representations

This shows how, in 4D, the chirality arises as intrinsic property of the representation.

To describe fermions in 5D we need a representation of the 5D Clifford algebra:

$$\{\Gamma_M, \Gamma_N\} = 2g_{MN} \tag{C.1}$$

A possible representation is provided by:

$$\Gamma_\mu \equiv \gamma_\mu, \quad \Gamma_5 \equiv -i\gamma_5 \tag{C.2}$$

The Lorentz generators are represented by:

$$\Sigma_{MN} \equiv \frac{1}{4}\left[\Gamma_M, \Gamma_N\right] \tag{C.3}$$

In addition to γ_μ, γ_5 is now included, *i.e.*, precisely the parity transformation that assigns the chirality in 4D. In 5D we cannot write an extra gamma matrix that plays the role of a parity operator, a charachteristic that is unique to even dimensional representations of the Clifford algebra. As a consequence, the simplest *irreducible* representation in 5D is the 4-components Dirac spinor, rather than a Weyl spinor, so that the notion of chirality remains undefined.

C.2 Symplectic Majorana Spinors

Symplectic Majorana Spinors are 4-components spinors which satisfy the reality condition:

$$\Lambda^i = \varepsilon^{ij}\Lambda_j^c \tag{C.4}$$

where $\Lambda^c = C\overline{\Lambda}^T$ and C is the charge conjugation operator that must satisfy:

$$C^{-1}\Gamma_M C = -\Gamma_M^T; \qquad M = \mu,\, 5$$

We use the explicit form:

$$C = -i\gamma^2\gamma^0 = i\begin{pmatrix} \sigma^2 & 0 \\ 0 & -\sigma^2 \end{pmatrix} \tag{C.5}$$

The lower indices $i, j = 1, 2$ are spinorial indices, transforming as doublets of an $\mathrm{SU}(2)_R$ symmetry. The tensor ε ($\varepsilon^{12} = \varepsilon_{21} = 1$) can be used to raise or lower the indices.

The spinors in (C.4) are 4-components Dirac spinors, and the condition (C.4) implies a particular relation between their components (two 4D Weyl spinors). In terms of Weyl components, they read: $\Lambda^i = \begin{pmatrix} \lambda^i{}_\alpha \\ \varepsilon^{ij}(\bar{\lambda}_j^T)^{\dot{\alpha}} \end{pmatrix}$

$$\Lambda_1 = \begin{pmatrix} (\lambda_1)_\alpha \\ (\bar{\lambda}_2^T)^{\dot{\alpha}} \end{pmatrix}, \Lambda_2 = \begin{pmatrix} (\lambda_2)_\alpha \\ -(\bar{\lambda}_1^T)^{\dot{\alpha}} \end{pmatrix}; \ \overline{\Lambda}_1 = \begin{pmatrix} ({\lambda_2}^T)_\alpha \\ (\bar{\lambda}_1)^{\dot{\alpha}} \end{pmatrix}, \overline{\Lambda}_2 = \begin{pmatrix} -({\lambda_2}^T)_\alpha \\ (\bar{\lambda}_1)^{\dot{\alpha}} \end{pmatrix}; \tag{C.6}$$

Where we have introduced directly the notation in terms of the gauginos, in place of the chiral components: $\lambda_1 \equiv \lambda_R$ and $\lambda_2 \equiv \lambda_L$. We can verify explicitly that the spinors defined in (C.6) satisfy identically the reality condition (C.4). Indeed, remembering that: $\bar{\Lambda} = \Lambda^\dagger\gamma^0$ and $(\bar{\lambda}^{\dot{\alpha}})^\dagger = \lambda^\alpha$, $(\lambda_\alpha)^\dagger = \bar{\lambda}_{\dot{\alpha}}$:

$$\Lambda_j^c = \begin{pmatrix} \varepsilon_{jk}(i\sigma^2\lambda^k)_\alpha \\ -(i\sigma^2\bar{\lambda}_j^T)^{\dot{\alpha}} \end{pmatrix} \tag{C.7}$$

that inserted in Eq.(C.4) gives:

$$\lambda_\alpha^i = (i\sigma^2\lambda^i)_\alpha = -\varepsilon_{\alpha\beta}(\lambda^i)^\beta = (\lambda^i)^\beta\varepsilon_{\beta\alpha}; \tag{C.8}$$

$$(\bar{\lambda}_j^T)^{\dot{\alpha}} = -(i\sigma^2\bar{\lambda}_j^T)^{\dot{\alpha}} = -\varepsilon^{\dot{\alpha}\dot{\beta}}(\bar{\lambda}_j^T)_{\dot{\beta}} = (\bar{\lambda}_j^T)_{\dot{\beta}}\varepsilon^{\dot{\beta}\dot{\alpha}} \tag{C.9}$$

(C.10)

Appendix D

5D covariant Lagrangian in terms of 4D superfields

The action (4.73) describes the 5D supersymmetric theory in terms of 4D superspace language. We have seen how this language is particularly useful for expressing the couplings with the 4D boundary terms, anyway there is a price to pay, that is the loss of explicit 5D covariance. In this appendix we expand fully the action (4.73) in order to check the consistency of the explicit 5D-covariant terms emerging for each field component.
Expanding Eq.(4.73) in the superfield components and integrating over θ, $\bar{\theta}$ we obtain the Lagrangian:

$$
\begin{aligned}
\mathcal{L}_{5D} = & \frac{1}{4g_5^2}\left(-\frac{1}{2}F^{\mu\nu}F_{\mu\nu} - 2i\lambda_1\sigma^\mu\partial_\mu\bar{\lambda}_1 + i\frac{1}{4}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} + h.c.\right) \\
& + \frac{1}{g_5^2}\left(\frac{1}{2}\partial_5 A^\mu\partial_5 A_\mu - \partial_5 A^\mu\partial_\mu A_5 + \frac{1}{2}\partial_\mu A_5\partial^\mu A_5 + \frac{1}{2}\partial_\mu\Sigma\partial^\mu\Sigma\right. \\
& \left. + \partial_5\lambda_1\cdot\lambda_2 - \partial_5\bar{\lambda}_1\cdot\bar{\lambda}_2 - i\lambda_2\sigma^\mu\partial_\mu\bar{\lambda}_2\right) \\
& D_\mu\phi^\dagger\cdot D^\mu\phi - i\overline{\psi}\bar{\sigma}^\mu D_\mu\psi + i\frac{1}{\sqrt{2}}\left(\phi^\dagger\lambda_1^a T_a\psi - \overline{\psi}\bar{\lambda}_1^a T_a\phi\right) \\
& + D_\mu\phi^c\cdot D^\mu\phi^{c\dagger} - i\psi^c\sigma^\mu D_\mu\bar{\psi}^c - i\frac{1}{\sqrt{2}}\left(\psi^c\lambda_1^a T_a\phi^{c\dagger} - \phi^c T_a\overline{\psi}^c\bar{\lambda}_1^a\right) \\
& + \psi^c\left(\partial_5 + M' - i\frac{A_5}{2}\right)\psi + \left(\partial_5 + M'^* + i\frac{A_5}{2}\right)\overline{\psi}\cdot\overline{\psi}^c \\
& - \frac{1}{\sqrt{2}}\left(\psi^c\lambda_2^a T_a\phi + \phi^c\lambda_2^a T_a\psi + \phi^\dagger\bar{\lambda}_2^a T_a\overline{\psi}^c + \overline{\psi}\bar{\lambda}_2^a T_a\phi^{c\dagger}\right) \\
& + \mathcal{L}_{F_{\phi,\phi^c,\chi},D}
\end{aligned}
\quad \text{(D.1)}
$$

With:

$$\begin{aligned}\mathcal{L}_{F_{\phi,\phi^c,\chi},D} &= F_\phi^\dagger F_\phi + F_{\phi^c} F_{\phi^c}^\dagger - \left[\phi^c\left(\partial_5 + M' - i\frac{A_5}{2}\right)F_\phi\right.\\ &\left.+F_{\phi^c}\left(\partial_5 + M' - i\frac{A_5}{2}\right)\phi + h.c.\right]\\ &+\left(\phi^c\frac{F_\chi}{\sqrt{2}}\phi + h.c.\right) + \frac{1}{Rg_5^2}F_\chi^\dagger F_\chi\\ &+\frac{R}{2g_5^2}D^2 - \frac{1}{g_5^2}\Sigma\partial_5 D - \frac{R}{2}\left(\phi^\dagger D\phi - \phi^c D\phi^{c\dagger}\right)\end{aligned} \tag{D.2}$$

where we have isolated the terms in the auxiliary fields F and D, in order to find their e.o.m.

Note that we have redefined the bulk mass parameter including the field Σ, as:

$$M' \equiv M - \frac{\Sigma}{2} \tag{D.3}$$

The e.o.m. for the auxiliary fields result:

$$\begin{aligned}\frac{\delta\mathcal{L}}{\delta F_\phi^\dagger} &= F_\phi - \left(-\partial_5 + M'^* + i\frac{A_5}{2}\right)\phi^{c\dagger} = 0\\ \frac{\delta\mathcal{L}}{\delta F_{\phi^c}^\dagger} &= F_{\phi^c} - \left(\partial_5 + M'^* + i\frac{A_5}{2}\right)\phi^\dagger = 0\\ \frac{\delta\mathcal{L}}{\delta F_\chi^{a\dagger}} &= \frac{1}{g_5^2}F_\chi^a + \frac{\phi^\dagger T^a\phi^{c\dagger}}{\sqrt{2}}\\ \frac{\delta\mathcal{L}}{\delta D^a} &= \frac{1}{g_5^2}D^a + \frac{1}{g_5^2}\partial_5\Sigma^a - \frac{1}{2}\left(\phi^\dagger T^a\phi - \phi^c T^a\phi^{c\dagger}\right) = 0\end{aligned} \tag{D.4}$$

leading to the solutions:

$$\begin{aligned}F_\phi &= \left(-\partial_5 + M'^* + i\frac{A_5}{2}\right)\phi^{c\dagger}\\ F_{\phi^c} &= -\left(\partial_5 + M'^* + i\frac{A_5}{2}\right)\phi^\dagger\\ F_\chi^a &= -g_5^2\frac{\phi^\dagger T^a\phi^{c\dagger}}{\sqrt{2}}\\ D^a &= -\partial_5\Sigma^a + \frac{g_5^2}{2}\left(\phi^\dagger T^a\phi - \phi^c T^a\phi^{c\dagger}\right)\end{aligned} \tag{D.5}$$

Replacing the solutions (D.5) in the action, we get the explicit dependence on the physical fields. Rearranging some expressions through integrating by parts, we finally get the whole

Lagrangian:

$$
\begin{aligned}
\mathcal{L}_{5D} = & -\frac{1}{4g_5^2}F^{\mu\nu}F_{\mu\nu} + \frac{1}{g_5^2}\frac{1}{2}F_{5\mu}F_5^{\mu} \\
& -\frac{1}{g_5^2}i\lambda_1\sigma^{\mu}\partial_{\mu}\bar{\lambda}_1 - \frac{1}{g_5^2}i\lambda_2\sigma^{\mu}\partial_{\mu}\bar{\lambda}_2 + \frac{1}{g_5^2}\left(\partial_5\lambda_1\cdot\lambda_2 + \partial_5\bar{\lambda}_1\cdot\bar{\lambda}_2\right) \\
& +\frac{1}{2g_5^2}\frac{1}{2}\partial_{\mu}\Sigma\partial^{\mu}\Sigma - \frac{1}{2g_5^2}\partial_5\Sigma^a\partial_5\Sigma_a \\
& -i\,\overline{\psi}\bar{\sigma}^{\mu}D_{\mu}\psi - i\,\overline{\psi}^c\bar{\sigma}^{\mu}D_{\mu}\psi^c + \psi^c\left(D_5 + M'\right)\psi + \overline{\psi}\left(-D_5 + M'^*\right)\overline{\psi}^c \\
& + D_{\mu}\phi^{\dagger}\cdot D^{\mu}\phi + D_{\mu}\phi^c\cdot D^{\mu}\phi^{c\dagger} \\
& -\phi^c\left(-D_5^2 + |M'|^2 + \partial_5 M^*\right)\phi^{c\dagger} \\
& -\phi^{\dagger}\left(-D_5^2 + |M'|^2 - \partial_5 M\right)\phi \\
& -\frac{g_5^2}{2}\left(\phi^c T^a\phi\cdot\phi^{\dagger}T_a\phi^{c\dagger}\right) - \frac{1}{8}g_5^2\left(\phi^{\dagger}T^a\phi - \phi^c T_a\phi^{c\dagger}\right)^2 \\
& +i\frac{1}{\sqrt{2}}\left(\phi^{\dagger}\lambda_1^a T_a\psi - \overline{\psi}\bar{\lambda}_1^a T_a\phi\right) - i\frac{1}{\sqrt{2}}\left(\psi^c\lambda_1^a T_a\phi^{c\dagger} - \phi^c T_a\overline{\psi}^c\bar{\lambda}_1^a\right) \\
& +i\frac{1}{\sqrt{2}}\left(\psi^c\lambda_2^a T_a\phi + \phi^c\lambda_2^a T_a\psi - \phi^{\dagger}\bar{\lambda}_2^a T_a\overline{\psi}^c - \overline{\psi}\bar{\lambda}_2^a T_a\phi^{c\dagger}\right)
\end{aligned} \tag{D.6}
$$

Where we have used the definition of the covariant derivative along the 5^{th} direction:

$$D_5 = \partial_5 - i\frac{A_5^a T^a}{2}$$

T^a are the generators of the given gauge group, in the representation chosen for the fields on which the derivative acts. We can easily recognize now the terms describing, in order: gauge bosons, gauginos, gauge scalar, fermions, sfermions and their respective interactions. We reintroduce the 5D notation, with the Γ_M matrices of Eq.(C.2), identifying the Dirac spinor:

$$\Psi = \begin{pmatrix} \psi \\ \overline{\psi}^c \end{pmatrix} \qquad \overline{\Psi} = \Psi^{\dagger}\gamma_0 = \left(\psi^c\overline{\psi}\right)$$

and the symplectic Majorana spinors defined in Eq.(C.6). We can finally read the Laganginan in the explicit 5D-covariant form:

$$
\begin{aligned}
\mathcal{L}_{5D} = & -\frac{1}{4g_5^2}F^{MN}F_{MN} - \frac{1}{g_5^2}i\bar{\Lambda}_i\tilde{\Gamma}^M\partial_M\Lambda_i \\
& + \frac{1}{2g_5^2}\partial_M\Sigma\partial^M\Sigma \\
& - i\,\overline{\Psi}\tilde{\Gamma}^M D_M\Psi + \overline{\Psi}\begin{pmatrix} M' & 0 \\ 0 & M' \end{pmatrix}\Psi \\
& + D_M\phi^\dagger D^M\phi + D_M\phi^c D^M\phi^{c\dagger} \\
& - \phi^c\left(|M'|^2 + \partial_5 M\right)\phi^{c\dagger} \\
& - \phi^\dagger\left(|M'|^2 - \partial_5 M\right)\phi \\
& - \frac{g_5^2}{2}\left(\phi^c T^a\phi\cdot\phi^\dagger T_a\phi^{c\dagger}\right) - \frac{1}{8}g_5^2\left(\phi^\dagger T^a\phi - \phi^c T_a\phi^{c\dagger}\right)^2 \\
& - \frac{1}{\sqrt{2}}\left(\phi^\dagger\bar{\Lambda}_2\Psi - \phi^c\bar{\Lambda}_1\Psi\right) + h.c.
\end{aligned}
\tag{D.7}
$$

Appendix E

Parameters of the best fit solutions in our 5D SO(10) models

E.1 Model from Chapt. 5

We give here the input parameters obtained for the best fit solutions presented in the Table 5.2 corresponding to normal and inverted neutrino mass spectrum and $\tan\beta = 50$.

E.1.1 Normal ordering

The best fit values of the Yukawa matrices and various parameters appearing in Eqs. (5.23, 6.20) at $\chi^2_{\rm min} \approx 0$ are listed below.

$$
\begin{aligned}
Y_{10} &= \begin{pmatrix} 0.78314 & 1.05610 e^{1.79809 i} & 0.92306 e^{-0.19874 i} \\ 1.05610 e^{1.79809 i} & 1.49012 & 1.09077 e^{1.02405 i} \\ 0.92306 e^{-0.19874 i} & 1.09077 e^{1.02405 i} & 0.96156 \end{pmatrix}, \\
Y_{120} &= \begin{pmatrix} 0 & 1.04750 e^{-2.22311 i} & 0.50164 e^{3.02587 i} \\ -1.04750 e^{-2.22311 i} & 0 & 0.78048 e^{-0.43312 i} \\ -0.50164 e^{3.02587 i} & -0.78048 e^{-0.43312 i} & 0 \end{pmatrix}, \\
Y_{126} &= \begin{pmatrix} 1.49976 e^{1.45362 i} & 0.51701 e^{-1.20768 i} & 1.25349 e^{-1.82494 i} \\ 0.51701 e^{-1.20768 i} & 0.50067 e^{-0.49914 i} & 0.91593 e^{0.05375 i} \\ 1.25349 e^{-1.82494 i} & 0.91593 e^{0.05375 i} & 1.38243 e^{-1.37536 i} \end{pmatrix}.
\end{aligned} \tag{E.1}
$$

$$
\begin{aligned}
\alpha_2 &= 0.87373\, e^{-2.05269 i}\,, \quad \alpha_3 = 0.06975\, e^{-1.21499 i}\,, \\
\overline{\alpha}_2 &= 0.81115\, e^{-1.00683 i}\,, \quad \overline{\alpha}_3 = 0.50212\, e^{-1.20195 i}\,.
\end{aligned} \tag{E.2}
$$

The bulk mass parameters are:

$$\{\mu_1,\ \mu_2,\ \mu_3,\ k_X\} \quad = \quad \{-0.03732,\ -0.01565,\ 0.20467,\ -0.01031\}\ . \tag{E.3}$$

From the above parameters and using Eq.(5.11), one obtains the following zero-mode profiles of various SU(5) multiplets at $y = 0$.

$$F_{10} = \lambda^{0.3}\begin{pmatrix} \lambda^{3.7} & 0 & 0 \\ 0 & \lambda^{2.4} & 0 \\ 0 & 0 & 1 \end{pmatrix},\ F_{\bar{5}} = \lambda^{0.3}\begin{pmatrix} \lambda^{1.5} & 0 & 0 \\ 0 & \lambda^{0.9} & 0 \\ 0 & 0 & 1 \end{pmatrix},\ F_{1} = \lambda^{0.4}\begin{pmatrix} \lambda^{6.2} & 0 & 0 \\ 0 & \lambda^{4.8} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{E.4}$$

Further, the localization of zero-mode profiles of different generations of 10, $\bar{5}$ and 1 can be obtained by replacing $m \to m_i^r$ in Eq.(5.2) and are displayed in Fig. E.1.

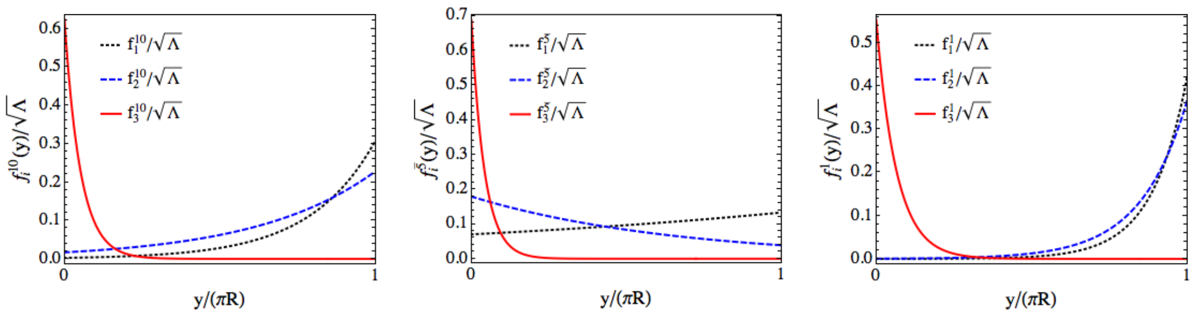

Figure E.1: The localized zero-mode profiles of different SU(5) matter multiplets for the best fit solution obtained in case of normal ordering in the neutrino masses.

E.1.2 Inverted ordering

The best fit values of the Yukawa matrices and various parameters appearing in Eqs. (5.23, 6.20) at $\chi^2_{\min} \approx 5.75$ are listed below.

$$\begin{aligned}
Y_{10} &= \begin{pmatrix} 0.50232 & 1.12746e^{-0.07927i} & 1.25370e^{-0.46501i} \\ 1.12746e^{-0.07927i} & 0.74605 & 1.49999e^{0.94538i} \\ 1.25370e^{-0.46501i} & 1.49999e^{0.94538i} & 1.38633 \end{pmatrix}, \\
Y_{120} &= \begin{pmatrix} 0 & 0.53486e^{2.05559i} & 1.29570e^{2.53388i} \\ -0.53486e^{2.05559i} & 0 & 0.58945e^{-0.03658i} \\ -1.29570e^{2.53388i} & -0.58945e^{-0.03658i} & 0 \end{pmatrix}, \\
Y_{126} &= \begin{pmatrix} 1.49999e^{0.16531i} & 0.50005e^{1.43459i} & 0.58661e^{-2.24612i} \\ 0.50005e^{1.43459i} & 0.50007e^{-0.86236i} & 1.02973e^{-1.58869i} \\ 0.58661e^{-2.24612i} & 1.02973e^{-1.58869i} & 0.96577e^{-0.16857i} \end{pmatrix}.
\end{aligned} \tag{E.5}$$

$$\begin{aligned}
\alpha_2 &= 0.04681\ e^{1.46923i}\ , \quad \alpha_3 = 0.07100\ e^{3.10679i}\ , \\
\overline{\alpha}_2 &= 0.87191\ e^{0.46323i}\ , \quad \overline{\alpha}_3 = 0.36100\ e^{-1.36707i}\ .
\end{aligned} \tag{E.6}$$

The bulk mass parameters are:

$$\{\mu_1,\ \mu_2,\ \mu_3,\ k_X\} = \{-0.040351,\ -0.01099,\ 0.085029,\ -0.01668\}\ . \tag{E.7}$$

From the above parameters and using Eq.(5.11), one obtains the following zero-mode profiles of various SU(5) multiplets at $y = 0$.

$$F_{10} = \lambda^{0.7} \begin{pmatrix} \lambda^{3.9} & 0 & 0 \\ 0 & \lambda^{2.2} & 0 \\ 0 & 0 & 1 \end{pmatrix},\ F_{\bar{5}} = \lambda^{0.4} \begin{pmatrix} \lambda^{0.8} & 0 & 0 \\ 0 & \lambda^{0.4} & 0 \\ 0 & 0 & 1 \end{pmatrix},\ F_1 = \lambda^{1.5} \begin{pmatrix} \lambda^{7.4} & 0 & 0 \\ 0 & \lambda^{5.5} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{E.8}$$

Further, the localization of zero-mode profiles of different generations of 10, $\bar{5}$ and 1 fermions can be obtained by replacing $m \to m_i^r$ in Eq.(5.2) and are displayed in Fig. E.2.

E.2 Model from Chapt. 6

We provide the set of input parameters obtained for the best fit solutions corresponding to normal and inverted neutrino mass spectrum and $\tan\beta = 50$ as presented in the Table 6.3.

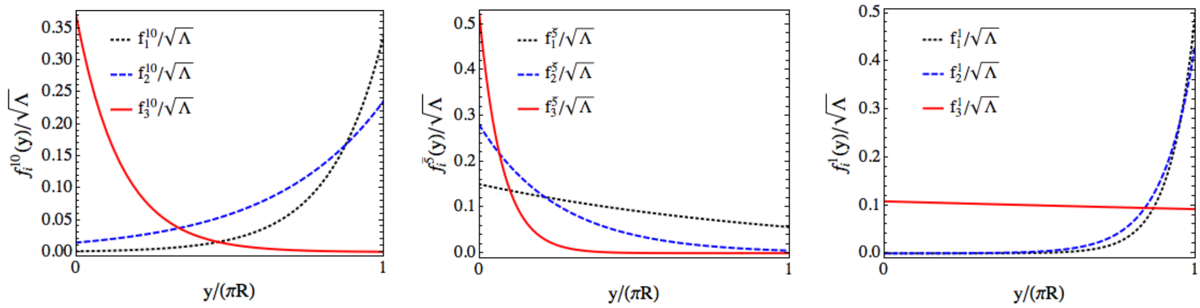

Figure E.2: The localized zero-mode profiles of different SU(5) matter multiplets for the best fit solution obtained in case of inverted ordering in the neutrino masses.

E.2.1 Normal ordering

The values of the Yukawa matrices and bulk masses appearing in Eqs.(6.15, 6.18, 6.20) at $\chi^2_{\rm min} \approx 0$ are as the following. We have removed some unphysical phases by redefining the fields.

$$
\begin{aligned}
Y_u &= \begin{pmatrix} 0.55863\ e^{-0.49590i} & 0.94275 & 1.23911\ e^{-1.10433i} \\ 0.74927 & 1.49374 & 0.66883 \\ 0.50804\ e^{0.22131i} & 0.50000 & 1.26156\ e^{-0.86038i} \end{pmatrix}, \\
Y_d &= \begin{pmatrix} 0.64691\ e^{-0.51014i} & 0.71998\ e^{-0.81349i} & 0.52244\ e^{2.72841i} \\ 0.80610\ e^{1.57886i} & 0.57351\ e^{0.23467i} & 0.50398\ e^{0.47936i} \\ 1.01632\ e^{-0.85648i} & 0.59252\ e^{-1.77531i} & 0.63639\ e^{-2.92490i} \end{pmatrix}, \\
Y_R &= \begin{pmatrix} 1.10716\ e^{0.17875i} & 0.70519\ e^{0.94555i} & 0.81595\ e^{-0.75271i} \\ 0.70519 e^{0.94555i} & 1.30773\ e^{2.93543i} & 1.07719\ e^{-0.17411i} \\ 0.81595 e^{-0.75271i} & 1.07719\ e^{-0.17411i} & 0.71443\ e^{1.37417i} \end{pmatrix}.
\end{aligned} \tag{E.9}
$$

The corresponding bulk mass parameters are:

$$
\begin{aligned}
\{\mu_1,\ \mu_2,\ \mu_3\} &= \{0.049590,\ 0.020895,\ -0.139245\}\ , \\
\{\mu'_1,\ \mu'_2,\ \mu'_3\} &= \{0.066244,\ -0.013373,\ -0.463361\}\ , \\
k_X &= 0.042394\ .
\end{aligned} \tag{E.10}
$$

From the above parameters the profile matrices in Eq.(6.16) for various SM fermions can be expressed in terms of powers of the Cabibbo angle λ as below.

$$F_Q = \lambda^{0.6} \begin{pmatrix} \lambda^{3.1} & 0 & 0 \\ 0 & \lambda^{2.3} & 0 \\ 0 & 0 & 1 \end{pmatrix} , \quad F_{d^c} = \frac{1}{\lambda^{0.1}} \begin{pmatrix} \lambda^{0.8} & 0 & 0 \\ 0 & \lambda^{0.5} & 0 \\ 0 & 0 & 1 \end{pmatrix} ,$$

$$F_L = \lambda^{0.2} \begin{pmatrix} \lambda^{0.4} & 0 & 0 \\ 0 & \lambda^{0.3} & 0 \\ 0 & 0 & 1 \end{pmatrix} , \quad F_{N^c} = \lambda^{0.2} \begin{pmatrix} \lambda^{9.4} & 0 & 0 \\ 0 & \lambda^{6.8} & 0 \\ 0 & 0 & 1 \end{pmatrix} ,$$

$$F_{u^c} = F_{e^c} = \lambda^{0.1} \begin{pmatrix} \lambda^{4.2} & 0 & 0 \\ 0 & \lambda^{1.9} & 0 \\ 0 & 0 & 1 \end{pmatrix} . \tag{E.11}$$

E.2.2 Inverted ordering

The values of the Yukawa matrices and bulk masses appearing in Eqs.(6.15, 6.18, 6.20) at $\chi^2_{\min} \approx 0.96$ are as the following. We have removed some unphysical phases by redefining the fields.

$$Y_u = \begin{pmatrix} 1.05063\, e^{-2.27438i} & 0.50197 & 0.50108\, e^{0.65794i} \\ 1.28888 & 0.95572 & 0.95749 \\ 1.32079\, e^{1.96363i} & 0.84379 & 1.03615\, e^{-1.69586i} \end{pmatrix} ,$$

$$Y_d = \begin{pmatrix} 0.51388\, e^{-2.46719i} & 0.50192\, e^{1.08880i} & 0.72278\, e^{1.00274i} \\ 1.47850\, e^{-1.34548i} & 0.63988\, e^{1.91581i} & 0.62270\, e^{0.06790i} \\ 0.68440\, e^{-1.92037i} & 0.52781\, e^{1.82283i} & 0.50618\, e^{1.03128i} \end{pmatrix} ,$$

$$Y_R = \begin{pmatrix} 1.32057\, e^{-1.64402i} & 1.34754\, e^{-2.56275i} & 0.62345\, e^{1.12638i} \\ 1.34754\, e^{-2.56275i} & 1.44530\, e^{1.87202i} & 0.57696\, e^{-0.04777i} \\ 0.62345\, e^{1.12638i} & 0.57696\, e^{-0.04777i} & 0.62830\, e^{2.31181i} \end{pmatrix} . \tag{E.12}$$

The corresponding bulk mass parameters are:

$$\begin{aligned} \{\mu_1,\ \mu_2,\ \mu_3\} &= \{0.056934,\, 0.023583,\, -0.212866\} , \\ \{\mu'_1,\ \mu'_2,\ \mu'_3\} &= \{0.088673,\, -0.025229,\, -0.421995\} , \\ k_X &= 0.045419 . \end{aligned} \tag{E.13}$$

From the above parameters the profile matrices in Eq.(6.16) for various SM fermions can be expressed in terms of powers of the Cabibbo angle λ as below.

$$F_Q = \lambda^{0.4}\begin{pmatrix}\lambda^{3.6} & 0 & 0\\ 0 & \lambda^{2.6} & 0\\ 0 & 0 & 1\end{pmatrix}, \quad F_{d^c} = \frac{1}{\lambda^{0.04}}\begin{pmatrix}\lambda^{0.8} & 0 & 0\\ 0 & \lambda^{0.4} & 0\\ 0 & 0 & 1\end{pmatrix},$$
$$F_L = \lambda^{0.1}\begin{pmatrix}\lambda^{0.5} & 0 & 0\\ 0 & \lambda^{0.4} & 0\\ 0 & 0 & 1\end{pmatrix}, \quad F_{N^c} = \lambda^{0.3}\begin{pmatrix}\lambda^{10.6} & 0 & 0\\ 0 & \lambda^{6.9} & 0\\ 0 & 0 & 1\end{pmatrix},$$
$$F_{u^c} = F_{e^c} = \lambda^{0.1}\begin{pmatrix}\lambda^{4.9} & 0 & 0\\ 0 & \lambda^{1.6} & 0\\ 0 & 0 & 1\end{pmatrix}. \tag{E.14}$$

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