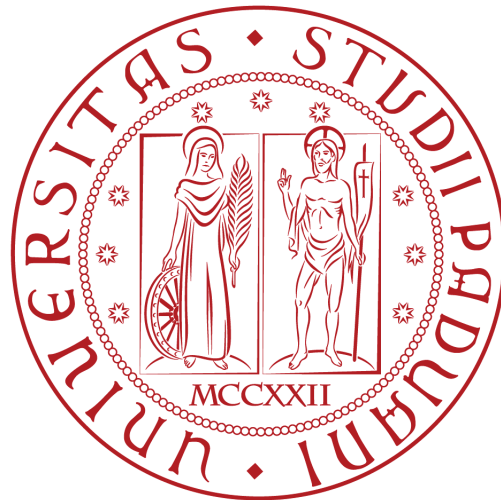Università degli Studi di Padova

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# Scalars beyond the Standard Model: Composite Higgs, dark matter and neutrino masses

Doctoral Dissertation of

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# Abstract

This thesis deals with Composite Higgs (CH) models, dark matter and neutrino masses. In CH models, the Higgs is a pseudo-Goldstone boson of a high-energy strong dynamics. We construct CP-even and CP-odd bosonic effective chiral Lagrangian for a generic symmetric coset $\mathcal{G}/\mathcal{H}$. Assuming that the only sources of custodial symmetry are the ones present in the SM, we study the projection of this Lagrangian into the low-energy SM chiral Lagrangian. This is applied in three particular scenarios: the original $SU(5)/SO(5)$ Georgi-Kaplan model, the minimal custodial-preserving $SO(5)/SO(4)$ model and the minimal $SU(3)/(SU(2) \times U(1))$ model, which intrinsically breaks custodial symmetry.

We furthermore consider an extension of the Standard Model involving two new scalar particles around the TeV scale: a singlet neutral scalar $\phi$, to be eventually identified as the Dark Matter candidate, plus a doubly charged $SU(2)_L$ singlet scalar, $S^{++}$, that can be the source for the non-vanishing neutrino masses and mixings. Assuming an unbroken $\mathbb{Z}_2$ symmetry in the scalar sector, under which only the additional neutral scalar $\phi$ is odd, we write the most general (renormalizable) scalar potential. This model may be regarded as a possible extension of the conventional Higgs portal Dark Matter scenario which in addition accounts for neutrino masses and mixings. This framework cannot completely explain the observed positron excess. However a softening of the discrepancy observed in conventional Higgs portal framework can be obtained, especially when the scale of new physics responsible, for generating neutrino masses and lepton number violating processes, is around 2 TeV.

# Riassunto

Questa tesi si occupa di studiare modelli di Higgs Composto (HC), materia oscura e masse dei neutrini. In modelli di tipo HC, lo scalare di Higgs è uno pseudo-bosone di Goldstone associato che origina dalla rottura di una simmetria forte ad alta energia. Nella tesi costruiamo la Lagrangiana chirale bosonica effettiva, per un generico coset simmetrico $\mathcal{G}/\mathcal{H}$, derivando esplicitamente tutti gli operatori (sia CP-even che CP-odd) che appaiono fino a quattro derivate. Supponendo che l'uniche fonte di rottura di simmetria custodial siano quelle già presente nel Modello Standard (MS), studiamo la proiezione di questa Lagrangiana sulla Lagrangiana chirale di bassa energia del MS. Particolareggiamo questo studio considerando tre scenari particolari: il modello originale di Georgi-Kaplan $SU(5)/SO(5)$, il modello minimale con simmetria custodial, $SO(5)/SO(4)$, ed il modello minimale senza simmetria custodial, $SU(3)/(SU(2) \times U(1))$.

Nella tesi consideriamo inoltre unestensione del MS che coinvolge due nuove particelle scalari con massa alla scala TeV: un singoletto scalare neutro $\phi$, che sarà poi identificato come candidato di materia oscura e un singoletto di $SU(2)_L$ scalare con carica $q = 2$, $S^{++}$, che può essere la fonte per le masse e del mixing dei neutrini. Supponendo l'esistenza di una simmetria $\mathbb{Z}_2$ nel settore scalare, sotto la quale solo $\phi$ è dispari, scriviamo il potenziale scalare (rinormalizzabile) più generale possibile. Il modello si può vedere come una possibile estensione dei modelli con Higgs Portal in cui si tiene anche conto del meccanismo con cui generare le masse e i mixings dei neutrini. Il modello da noi studiato, pur predice un eccesso di positroni, non tale tuttavia da poter spiegare l'eccesso di positroni sperimentalmente osservato. Pur tuttavia si possono ottenere dei limiti meno stringenti rispetto ai normali modelli di Higgs Portal, in particolare se la scala della nuova fisica, responsabile della generazione delle masse dei neutrini e dei processi che violano il numero leptonico, è intorno ai 2 TeV.

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# Chapter 1

# Introduction

There is no denying that the Standard Model of particle physics is one of the most successful scientific theories ever. It has been able not only to explain a wide range of phenomena with a relatively simple and elegant theoretical formulation, but also to make several predictions that have been systematically confirmed.

The last of them was the discovery of a neutral, CP-even scalar particle at the Large Hadron Collider (LHC), announced on July 4th, 2012, with data so far showing no deviations from the SM Higgs boson hypothesis. And yet, maybe as astonishing as its success is its inability to give an explanation to a considerable amount of problems, of both theoretical and experimental nature, such as the lightness of the recently discovered scalar state ("Higgs particle", for brevity), the existence of dark matter or the masses of neutrinos. This thesis deals deal with these questions, studying some proposals that offer solutions to them.

The so-called hierarchy problem is related to the mass of the Higgs particle. If it interacts with physics at some scale $\Lambda$ then it is generically expected that loops of SM particles will induce a quadratically divergent Higgs mass $\sim \Lambda$. Even if the SM doesn't include a description of gravity, (which is another of the reasons why we know it's not the final theory of Nature), it is expected that gravitational effects become relevant at the Planck scale, $M_P \sim 10^{19}$ GeV. The SM cannot explain the disparity between the observed Higgs mass, $m_H \sim 125$ GeV and the value we would generically expect it should have, once gravitational effects are included.

Connected to the hierarchy problem is the mistery of the origin of the electroweak symmetry breaking (EWSB), since both arise from the Higgs potential. The spontaneous breaking of the electroweak (EW) symmetry is implemented in the SM through the Higgs mechanism, so that the Higgs field has a potential which leads it to develop a vacuum expectation value (VEV) (which

is proportional to its mass) that is not invariant under EW transformations. However, the SM provides no explanation of where the potential and its parameters come from.

Maybe one of the most bitter issues with the SM is that it can only account for around the 5% of the content of the Universe. Indeed, from observations of the cosmic microwave background (CMB) it is now known that the energy budget of the Universe is divided in 68.3% dark energy, 26.8% dark matter and 4.9% ordinary matter, which is formed by the particles described by the SM.

First hypothesized in 1933 by Zwicky, the existence of non-baryonic dark matter is today widely accepted by the community. It is able to explain very diverse phenomena at several orders of magnitude, including the original problem of galactic clusters (where there is a mismatch between their observed orbital velocity of galaxy clusters and the one expected by the amount of light emitted by them) or large structure formation in the Universe. An alternative paradigm is the Modified Newtonian Dynamics (MOND), but the dark matter hypothesis is the only scenario that so far can explain observations such as the Bullet Cluster, of which gravitational lensing observations are considered the best evidence for the existence of dark matter.

Another set of evidence which the SM is not able to account for is the oscillation of neutrinos. Since the Super Kamiokande collaboration observed the oscillation of atmospheric neutrinos it is an established fact that neutrinos must be massive. However, in the Standard Model the neutrinos are massless, and therefore some beyond standard model (BSM) physics is needed. It remains unknown whether the nature of neutrinos is Dirac or Majorana, since oscillation experiments are not able to tell us anything about it. Therefore, further observations (such as neutrinoless double beta decay experiments) are needed in order to determine the nature of neutrino masses.

These three problems have been a big motivation for the proposal of beyond Standard Model (BSM) models. With respect to the hierarchy problem some of the best known solutions are Supersymmetry (SUSY) and models with extra dimensions. The lack of any experimental signal confirming these models has renovated the interest on Composite Higgs models (CH), on which this thesis will focus. In these scenarios the Higgs field is supposed to be a Goldstone boson (GB) associated to a global breaking of symmetry. It's a solution to the hierarchy problem inspired by low-energy QCD, where the pions are GBs of the spontaneously broken chiral symmetry, so that their masses are protected by a shift symmetry and, despite being scalars, they are much lighter than the first vector resonance ($m_\pi \sim 135$ MeV $\ll m_\rho \sim 770$ MeV). Any given CH model is characterized by the symmetry breaking pattern, where some global group $\mathcal{G}$ is spontaneously broken to a subgroup $\mathcal{H}$; the assumption is that the Higgs scalar and would-be GBs are Goldstone bosons belonging in the coset $\mathcal{G}/\mathcal{H}$, just as the pions are Goldstones belonging in the $SU(2)_L \times SU(2)_R / SU(2)_V$ coset.

A lot of theoretical effort is going also in trying to understand exactly how the dark matter is composed of. Through numerical simulations we have at the moment reasonable knowledge about how dark matter organizes itself, for example in and around galaxies, but despite all the observational information we have gathered, very little is known about its fundamental nature. One of the most popular proposals (and with which the second part of this thesis will deal) are models where the DM consists of weakly interacting massive particles (WIMPs), where thanks to the so-called "WIMP" miracle a DM candidate interacting with cross-sections of the order of the weak interaction and masses around the 100 GeV lead to the observed relic abundance of dark matter. These models have been particularly attractive in the last decades because SUSY models can provide several WIMP candidates.

Finally, there are several proposals for the origin of neutrino masses, such as the see-saw mechanism, where the masses arise from tree-level diagrams. Another interesting solution is the possibility of the lightness of the neutrinos being due to a loop suppression, where the exchange of new TeV scalar particles allows for the neutrino masses to appear not at tree but at loop level. An appealing feature of these models is that one can expect that these particles could be produced at colliders like the LHC, therefore making them potentially testable in the foreseeable future.

The first part of this thesis deals with a solution for the hierarchy problem of the Higgs scalar, while the second part will propose the existence of two new scalar particles in order to solve the dark matter and neutrino masses problem. It should be noted that not only these two approaches are independent, but apparently contradictory, since a priori the two new scalar states could potentially have a hierarchy problem of their own. This is symptomatic of the current state of affairs in particle physics, where physicists have come up with a myriad of solutions to a series of diverse problems, and still no experimental signal has been able to discriminate among them. Of course it's reasonable that the most popular frame of BSM until the recent years, SUSY, is precisely one that could potentially explain more than one issue at the same time, giving a cohesive theory of what physics beyond the EW scale looks like. Still, the lack of new experimental signs (in the LHC, in dark matter detection, in double-beta decay experiments...) is maybe shifting the perspective to a more bottom-up approach, using tools such as effective Lagrangians, considering the SM as an effective field theory, or more restrictive models, trying to expand the spectrum of particles of the SM as little as possible.

This thesis, based on the works [2] and [3], is structured as follows. Chapter 2 presents the SM Lagrangian, explains the hierarchy problem and shows the characteristics of CH models. Chapter3 considers the chiral formulation of the SM Lagrangian, and Chapter 4 introduces a formalism to write the effective bosonic Lagrangian of generic CH models. Chapter 5 applies this

construction to the $SU(5)/SO(5)$ Georgi-Kaplan model and studies its Goldstone boson potential, while Chapter 6 illustrates the effective Lagrangian for the $SO(5)/SO(4)$ and $SU(3)/(SU(2) \times U(1))$ CH models. The second part of the thesis deals with a model with an extended scalar sector, with Chapter 7 presenting an extension of the SM by a singlet scalar, identified with a dark matter candidate, and doubly charged scalar that provides loop-level masses to neutrinos. The dark matter relic abundance is computed and predictions for direct and indirect detection experiments are presented.

# Chapter 2

# The Standard Model and the hierarchy problem

## 2.1 The Standard Model

The Standard Model (SM) is a model which describes the interactions of all known elementary particles and their interactions, explaining almost all experimental data of particle physics. It is a quantum field theory based on the local symmetries $SU(3)_c$, which describes the strong forces, and $SU(2)_L \times U(1)_Y$, which accounts for the electric and weak ones. The SM can be described in a compact and elegant form through the lagrangian

$$
\begin{aligned}
\mathscr{L}_{SM} = &-\frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} + \\
&+i\bar{Q}_L\not{D}Q_L + i\bar{U}_R\not{D}U_R + i\bar{D}_R\not{D}D_R + i\bar{L}\not{D}L_L + i\bar{E}_R\not{D}E_R + \\
&+D_\mu\Phi^\dagger D^\mu\Phi + \frac{\mu^2}{2}\Phi^\dagger\Phi - \frac{\lambda}{4}\left(\Phi^\dagger\Phi\right)^2 + \\
&+\left[\bar{Q}_L\Phi\mathbf{Y}_D D_R + \bar{Q}_L\tilde{\Phi}\mathbf{Y}_U U_R + \bar{L}_L\Phi\mathbf{Y}_E E_R + \text{h.c.}\right] + \\
&+\frac{g_s^2}{16\pi^2}\theta G^a_{\mu\nu}\tilde{G}^{a\mu\nu}
\end{aligned}
\tag{2.1}
$$

The gauge bosons mediating the $SU(3)_c \times SU(2)_L \times U(1)_Y$ interactions of the SM are, respectively, the 8 gluon fields, $G^a_{\mu\nu}$, plus the 4 electroweak (EW) bosons, $W^a_{\mu\nu}$ and $B_{\mu\nu}$. Their kinetic terms are given by the first line of 2.1, with any given field $F^i_{\mu\nu}$ strength being written as

$$
F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu - ig^i[A^i_\mu, A^i_\nu]
\tag{2.2}
$$

where $g^i$ corresponds $\{g_s, g, g'\}$, the strong and electroweak coupling strengths. The last term of the lagrangian is a $\theta$-term, which gives a non-negligible contribution to the neutron electric dipole moment, which allow to infer a limit of $\theta \ll 10^{-9}$. The fact that this is so small is known as the strong CP problem. The matter content is described by fermionic fields of spin $1/2$, divided in quarks (particles which participate in strong interactions) and leptons (which only interact through the EW force). According to their transformations under the gauge group, the quarks are described by $Q_L \in (3, 2)_{1/6}$, $U_R \in (3, 1)_{2/3}$ and $D_R \in (3, 1)_{-1/3}$, while the leptons transform as $L_L \in (1, 2)_{-1/2}$ and $E_R \in (2, 1)_{-1}$, where the numbers in brakets indicate the transformation properties under $SU(3)_c \times SU(2)_L$ while the subscript corresponds to their $U(1)_Y$ hypercharge. These fermions come in what are known as generations or families, meaning that there are actually 3 identical copies of each of them differing only in their mass. The covariant derivative of any fermions $\Psi$ is given by

$$D_\mu \Psi = \partial_\mu \Psi + \sum_i i g_i A^i \Psi \tag{2.3}$$

Three of the four EW bosons are massive, as opposed to the fourth one being a massless photon and to the eight gluons. This is implemented in the SM via the Higgs mechanism, which describes a spontaneous breaking of the $SU(2)_L \times U(1)_Y$ group into the electromagnetic $U(1)_{em}$. The key ingredient of this setup is the Higgs scalar field, $\Phi \in (1, 2)_{1/2}$, described by

$$\Phi = \begin{pmatrix} \Phi^+ \\ \Phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} i\phi_1 + \phi_2 \\ \phi_0 - i\phi_3 \end{pmatrix} \tag{2.4}$$

with its covariant derivative given by

$$\mathbf{D}_\mu \Phi \equiv \left( \partial_\mu + \frac{i}{2} g' B_\mu + \frac{i}{2} g \tau_i W^i_\mu \right) \Phi \tag{2.5}$$

where $\tau_i$ are the Pauli matrices. The boundness from below of the Higgs potential, $V(\Phi) = -\frac{\mu^2}{2} \Phi^\dagger \Phi + \frac{\lambda}{4} \left( \Phi^\dagger \Phi \right)^2$, recquires that $\lambda \leq 0$. If $\mu^2 < 0$, $\Phi = 0$ is the only minimum of the potential, and therefore the vacuum is symmetric under the EW gauge group, $SU(2)_L \times U(1)_Y$, which remains unbroken. However, if $\mu^2 > 0$, then potential is minimized for $|\Phi| = \sqrt{\mu^2/\lambda} \equiv v/\sqrt{2}$, so that the true vacuum is no invariant under the electroweak group, which is thus spontaneously broken to the electromagnetic gauge group, $U(1)_{em}$. $v \simeq 246$ GeV is known as the EW scale, since it's the characteristic scale of the electroweak interactions, being the vacuum expectation value of the Higgs scalar: every EW observable with mass dimensions must be proportional to some power of $v$. In particular, the $W^\pm$ gauge bosons get a tree-level mass of $m_W = g\, v/2$ after the EWSB and the $Z$ boson a mass of $m_Z = \sqrt{g^2 + g'^2}\, v/2$, with the Higgs mass being $m_H = \lambda v^2/2$,

The excitations around the $U(1)_{em}$ invariant vacuum can be described parametrizing the Higgs field using the matrix

$$\mathbf{U} = e^{i\pi^a(x)\tau_a/v} \tag{2.6}$$

so that

$$\Phi(x) = \frac{v + h(x)}{\sqrt{2}} \mathbf{U}(x) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.7}$$

The excitation $h(x)$ corresponds to the physical Higgs scalar and the $\pi^a(x)$ fields correspond to the would-be Goldstone bosons associated with the symmetry breaking. The latter ones supply the massive EW gauge bosons with their longitudinal modes, a process usually described as the GBs being "eaten" by the vector bosons. The properties (production and decay rates) of the neutral, CP-even scalar particle observed by the CMS and ATLAS collaborations in the LHC [4, 5] so far don't show deviations from the predictions of the SM Higgs mechanism just presented [6], so that the $h(x)$ excitation can be identified with this new state.

## 2.2 Hierarchy problem

The biggest problem of the Standard Model related to the Higgs sector is the fact that the EW scale, or equivalently the Higgs mass, is very sensitive to quantum corrections. This issue, known as the hierarchy or naturalness problem, arises from $m_h$ getting quadratically divergent corrections via loops of SM particles.

The renormalized Higgs mass at a scale $\mu$ is defined as

$$m_h^2(\mu) = m_h^2(\mu_0) + \delta m_h^2 \tag{2.8}$$

where $\mu_0$ is a reference renormalization scale. If we consider the SM as an effective field theory valid up to a scale $\Lambda$, then loops of SM loops induce a correction given by

$$\delta m_h^2 \sim \frac{1}{v^2} \left( 4m_t^2 - 2m_W^2 - m_Z^2 - m_h^2 \right) \Lambda^2 \tag{2.9}$$

$\Lambda$ is the scale at which the SM stops being valid and new physics need to be considered; if there are no non-trivial effects, in general this means that the Higgs mass is sensitive to the highest scale that couples to SM fields. Therefore, if the Higgs field is coupled to quantum gravity in some way, then this scale could be identified with the Planck scale, $\Lambda = M_P \sim 10^{19}$ GeV. In this case, there is a very precise cancellation between the mass parameter $m_h^2(\mu)$ and the bare mass

$m_h^2(\mu_0)$ is required. Setting $m_h^2(\mu_0 = v) \equiv m_h^2 \sim 125$ GeV

$$\frac{\delta m_h^2}{m_h^2} \simeq \left(\frac{\Lambda}{500 \text{ GeV}}\right)^2 \simeq 10^{32} \tag{2.10}$$

Thus, a tuning of 1 in $10^{32}$ is needed to get a mass of the Higgs particle at the observed value, $m_h^2 \sim (125 \text{ GeV})^2$. If this cancellation does not occurr, $m_h$ would generically be expected to be of the order of $M_P$. Even if there were new states coupled to the Higgs at energies above the EW scale there is in general no reason to expect their loop contributions to $\delta m_h^2$ to have any type of cancellation. This fine-tuning is said not to be natural in the sense defined by 't Hooft in [7]: at some scale $E$ a parameter is allowed to be small if setting it to zero increases the symmetry of the system. In that case, parameter is said to be "technically natural". This is the case in the SM with fermion masses, since in the limit of massless fermions the SM lagrangian is invariant under chiral symmetry Similarly, the mass of the gauge bosons is protected by gauge symmetry, receiving a contribution to their mass of order $\sim \log \Lambda$ instead of $\sim \Lambda$.

However, the SM lagrangian with $\mu^2 = 0$ doesn't present any additional symmetry , and thus a small Higgs mass (compared to $\Lambda$) is not natural in the previous sense. The possibility of the existence of a new symmetry which would make a Higgs mass of the order of the weak scale has been a strong theoretical motivation behind many of the proposed BSM scenarios of the past decades. For example: supersymmetry (SUSY models), gauge symmetry (extra-dimensions models) or shift symmetry (Composite Higgs). Each of these scenarios is such that for $\mu = 0$ the lagrangian shows the correspondent additional symmetry. If the scale of the new physics responsible for the extra symmetry is around 1 TeV, i.e., the new states associated to it have a mass $\Lambda \sim 1$ TeV, then the tuning problem in 2.10 is fundamentally solved. This is the reason why it has been generally expected for the past decades that new physics should be seen around this scales, being this reasoning a very strong motivation both from the theoretical and experimental side.

Supersymmetric models are one of the most common BSM scenarios, and low-energy SUSY is one the most popular solution to the hierarchy problem. These weakly coupled theories are based on the existence of a symmetry which relates particles of different spins, so that every boson from the SM must be accompanied by a new fermionic particle, and viceversa. In particular, the superpartner of the Higgs scalar is the so-called Higgsino, which is a spin 1/2 particle. In the limit of exact supersymmetry the mass of the Higgs and the higgsino must be similar, but since the latter is protected by chiral symmetry because of its fermionic nature, $m_h$ becomes protected by this symmetry and it's smallness is therefore natural. Since SUSY must be broken because no superpartner has been so far observed, the Higgs mass will be of the order of the breaking scale,

$\Lambda_{SUSY}$, but any quadratic divergence above it would be cancelled, and therefore $\Lambda_{SUSY} \sim 1$ TeV is an attractive scenario. However, despite offering solutions to additional problems of the SM such as having a Dark Matter candidate or featuring the unification of gauge couplings, the current search for superparticles tends to put constringent constraints on the parameter space. There is nonetheless a big theoretical effort to evade these bounds by constructing non-minimal models which however lose some of the original appeal of the supersymmetric scenarios.

The CH models are based on the assumption that the Higgs scalar is the Goldstone boson of some symmetry breaking, and therefore is protected by a shift symmetry. This is the approach in which this thesis will put its focus and consequently more detail will be given in the following sections. The main idea is to mimic the mechanism that in QCD allows the pions to be light scalar particles, with masses of around 100 MeV, without having a hierarchy problem. In fact, in the original technicolor models [8, 9] that proposed these ideas there was a strong dynamics which ruled the EWSB and there was no Higgs scalar altogether. However, strong phenomenological bounds and the existence of a light scalar eventually lead to expand the technicolor models into the CH ones.

Finally, it is worth mentioning the relaxion mechanism which was proposed by [10] around 2 years ago. The hypothesis is that instead of a new symmetry, what explains the smallness of the EW scale with respect to the Planck scale is a dynamical mechanism. The model essentially consists in the mass of the Higgs undergoes a process of "cosmological relaxation" through the history of the universe from $M_P$ .

## 2.3 Composite Higgs models

With Composite Higgs we refer to a broad set of models where the Higgs particle is the Goldstone boson of some global symmetry breaking. For example, models inspired by QCD which assume the existence of a strong dynamics that forms composite states, among which is the Higgs scalar, as well as potentially new BSM states. Physics beyond the composite typical scale interact directly with the constituents of these composite states, just as in the case of pions and gluons, and thus the mass of the Higgs does not get a contribution from arbitrarily high scales.

In particular, the precise mechanism through which the pions are much lighter than the $\rho$ meson and the rest of QCD resonances is their Goldstone boson nature. The QCD lagrangian has an approximate $SU(2)_L \times SU(2)_R$ chiral lagrangian which is spontaneously broken by a quark-antiquark condensate in the QCD vacuum to $SU(2)_V$. The three pions $\{\pi^{\pm}, \pi^0\}$ are identified with the Goldstone bosons of this symmetry breaking. Similarly, Composite Higgs models assume

the existence of a global symmetry $\mathcal{G}$ which is spontaneously broken by some strong dynamics at a scale $\Lambda_s$ to a subgroup $\mathcal{H}$. The Higgs scalar is identified as one of the $n = \dim \mathcal{G}/\mathcal{H}$ Goldstone bosons associated to this breaking.

The QCD pions are not massless, and are therefore called pseudo-Goldstone bosons, because the chiral symmetry is not exact and is explicitly broken, for example by quark masses and the electromagnetic interactions. Quark masses, which mix left and right quarks, give pions a mass of $m_\pi^2 \sim m_q^2$, whereas QED interactions, which differentiates between charged and neutral pions, split their masses an amount proportional to $\alpha_{\text{EM}}$. In the same way, the Higgs particle is massive, and therefore sources that explicitly break the $\mathcal{G}$ symmetry will be required, such as for example the SM gauge interactions or the Yukawa couplings with SM fermions.

There are two approaches when constructing a CH models. On the one hand, it is possible to try to formulate the equivalent of the QCD interactions, looking for a precise strong dynamics which is confining mechanism so that the proposed fundamental constituents form condensates which lead to a suitable pattern of global symmetry breaking. This approach has clear difficulties because of the intrinsically strong coupling nature of the dynamics. The first works along these lines [11–16] propose the existence of some "ultracolour" interactions, and studied the formation of a condensate at a scale $\Lambda_{UC}$. The idea of studying a possible UV theory of the global symmetry breaking was recovered in [17] where a 5D formulation serves as the weakly interacting dual theory of the strong dynamics. These works were the ones to revive the interest on the Composite Higgs ideas and in the possibility of some strong dynamics being behind the EWSB.

On the other hand there is the option to follow an effective approach similar to the study through the chiral lagrangian of pion interactions, in which the pattern of symmetry breaking fixes all the low-energy dynamics and the knowledge of the precise quark and gluon interactions is not needed. This method is the most widely spread in the context of CH models, with each model specified by their specific $\mathcal{G}/\mathcal{H}$ coset characterizing the global symmetry breaking. The first coset to be proposed was $SU(5)/SO(5)$ in [16], where in addition to the Higgs scalar and the 3 longitudinal SM Goldstones there are other 10 Golstone scalars. In more recent years a lot of attention has been given to the so-called Minimal Composite Higgs Model, with coset $SO(5)/SO(4)$, which is the symmetry breaking pattern associated to the 5D theory proposed in [17].

This last effective approach is studied in this thesis, and unless explicitly said, we will refer to Composite Higgs models to this kind of constructions.

### 2.3.1 Construction of a Composite Higgs model

A CH model is fundamentally characterized by the $\mathcal{G} \to \mathcal{H}$ breaking pattern, and there are a series of conditions that this groups must fulfill in order to reproduce the observed properties of the Higgs scalar and the EWSB.

1. The number of Goldstone bosons produced in the breaking is $n = \dim \mathcal{G}/\mathcal{H}$. Among these, 1 of them is to be identified with the Higgs scalar and other 3 with the SM GB's, so that a first requirement is $n \geq 4$.

2. Since 3 of GB's are to be the longitudinal modes of the EW gauge bosons, the Goldstones must transform non- trivially under $G_{\mathrm{SM}} = SU(2)_L \times U(1)_Y$, so that $G_{\mathrm{SM}} \subset \mathcal{G}$ (the strong sector should not break explicitly the SM gauge group, so that the condition $G_{\mathrm{SM}} \cap \mathcal{G} \neq 0$ is not enough).

3. The unbroken group $\mathcal{H}$ should accomodate a $SU(2)_L \times SU(2)_R$ group which functions as a custodial symmetry in the way described in section 3.2.1. Even if this is not strictly necessary from the EWSB point of view, the strong experimental constraints in the value of $\rho$ makes this an important phenomenological bound.

4. There must be some source of explicit breaking of $\mathcal{G}$ so that the Goldstones can develop a potential in order to trigger the EWSB. This breaking must be additional to the one granted by the gauging of $G_{\mathrm{SM}}$: the parameters $g$ and $g'$ cannot give a negative mass to the GBs[18], and therefore the Higgs scalar cannot have a non-zero VEV.

5. The scale up to which the effective description is valid, $\Lambda_s$ (that can be interpreted as either the scale at which the interaction becomes strong or as the mass of the first resonance), the GB scale $f$ and the EW scale $v$, which is determined dinamically, must have a hierarchy. In particular, [19] requires $4\pi f \geq \Lambda_s$. The relation between $v$ and $f$ is usually expressed through the ratio

$$\xi \equiv \frac{v^2}{f^2} \in [0, 1] \tag{2.11}$$

which is phenomenologically important because it parametrizes the transition from the strong dynamics being decoupled from the SM and the EW being linearly realized ($\xi \to 0, f \to \infty$) or the other limit case in which the $\mathcal{G} \to \mathcal{H}$ symmetry breaking scale coincides with the EW one, which corresponds with the technicolor scenario ($\xi \to 1, f \to v$).

Potentially interesting simple cosets are $SU(N)/SO(N)$, $SO(N)/SO(N-1)$ and $SU(N)/SU(N-1) \times U(1)$. The requirement of $n = \dim \mathcal{G}/\mathcal{H} \geq 4$ implies that the minimum $N$ respectively for

each case is 3, 5 and 3, i.e. the cosets $SU(3)/SO(3)$, $SO(5)/SO(4)$ and $SU(3)/SU(2) \times U(1)$, with 5, 4 and 4 GB's respectively.

The coset $SU(3)/SO(3)$ is clearly not a possible candidate since $SO(3)$ is smaller than the necessary $SU(2) \times U(1)$ gauge subgroup. The next possible coset of this category would be $SU(4)/SO(4)$, but the Goldstones in this case belong to the $(\mathbf{3,3})$ representation of $SU(2)_L \times SU(2)_R$, and therefore there is no Higgs doublet among them. In conclusion, the smallest posible coset of this type is $SU(5)/SO(5)$, which has 14 Goldstones bosons transforming under $SU(2)_L \times SU(2)_R$ as $\mathbf{14} = (\mathbf{3,3}) + (\mathbf{2,2}) + (\mathbf{1,1})$.

The breaking pattern $SO(5)/SO(4)$ is minimal in the sense that the only corresponding GBs transform as a doublet plus additionally having an $\mathcal{H}$ group which can itsel function as a custodial symmetry, since $SO(4) \simeq SU(2)_L \times SU(2)_R$. The problem with the otherwise also minimal $SU(3)/(SU(2) \times U(1))$ is that it lacks a custodial group in the preserved subgroup and therefore it will violate $\rho = 1$ at tree level, and thus having strong experimental constraints.

Of course the list of possible cosets is extremely large, and there have been intense work studying the different advantages of each of them. For example, the $SO(6)/SO(5)$ breaking pattern has, in addition to an $SU(2)_L$ doublet, a SM singlet scalar which has been identified in [20] as a DM candidate, while the $SO(6)/SO(4) \times SO(2)$ can have additional phenomenological implications because it presents two doublets as GBs.

Part of the work of this thesis consists in studying the low-energy effects of these CH models. In particular, in Chapter 4 we will establish a systematic way to construct an effective field theory which captures the dynamics of Goldstone and gauge bosons.

# Chapter 3

# Chiral lagrangian for the Standard Model

Composite Higgs models, as any theory trying to explain the EWSB, follow a top-down approach, making predictions such as the existence of new particles. Thus, one way to test these models is to observe the production of these new states in colliders. However, the fact that no new particle has so far been observed in the LHC has made a bottom-up aproach an attractive way to study the existence of new physics (NP). Deviations from the SM can be studied in a model-independent using Effective Field Theories (EFTs), a tool that allows to study physical effects at a given energy scale (in our case, the EW scale, $v = 246$ GeV) without needing to know the UV behaviour of the putative NP. These low-energy effects are parametrized by an expansion operators composed of SM field invariant under Lorentz and gauge symmetries.

Two different kind of EFTs can be built depending on the assumed nature of the Higgs particle: linear and non-linear (or chiral) EFTs. In the first case the Higgs belongs in an $SU(2)_L$ doublet and the EW symmetry is linearly realized, as it's the case in the SM. This construction, often called the SM Effective Field Theory (SMFET) is useful when the possible NP is supposed to be a weakly coupled theory and the Higgs remains an elementary particle, such as in SUSY. A chiral EFT, on the contrary, is suitable when the Higgs sector is expected to arise from a strongly interacting theory, for example in CH scenarios. In this case the EW is non-linearly realized and the Higgs scalar is not assumed to belong in a weak doublet. The chiral lagrangian will be used in the following chapters, when the high-energy lagrangians describing CH models will be projected into the HEFT.

In this chapter we briefly introduce the SMEFT and then focus in the chiral lagrangian of the SM, which will be useful for us when in subsequent chapters we study the low-energy effects

of CH models. We first present the original non-linear Appelquist-Longhitano-Feruglio (ALF) lagrangian, where the Higgs was supposed to be either non-existing or very heavy (both reasonable assumptions in the 1980s), and then expand it to include the effects of a light lagrangian, a construction usually called Higgs Effective Field Theory (HEFT). In both cases we will deal only with the bosonic sector of the effective lagrangians. We will also present the custodial symmetry, an important phenomenological constraint for BSM models.

## 3.1 The linear Lagrangian (SMEFT)

If the Higgs scalar is considered to belong in a doublet $\Phi$, as in the SM, then an effective lagrangian can be written using the doublet structure as the basic building block. In this case, the EW symmetry is said to be linearly realized. All possible operators invariant operators are written in an expansion corresponding to their canonical mass dimension: so that the lagrangian has dimension 4, operators with dimension higher than 4 are suppressed by powers of $1/\Lambda$, where $\Lambda$ is the energy up to which the EFT is supposed to be valid. Thus, the leading-order lagrangian ($d = 4$) corresponds with the SM lagrangian and NP effects come with a $1/\Lambda$ supressions. Assuming lepton and baryion number conservation, both accidental symmetries of the SM, the SMEFT lagrangian can be written as

$$\mathscr{L}_{\text{linear}} = \mathscr{L}_{SM} + \Delta\mathscr{L}_{\text{linear}} \,, \tag{3.1}$$

where $\mathscr{L}_{SM}$ was defined in 2.1 and

$$\Delta\mathscr{L}_{\text{linear}} = \sum_i \frac{c_i}{\Lambda^2}\mathcal{O}_i^{d=6} + \sum_i \frac{c_i}{\Lambda^4}\mathcal{O}_i^{d=8} + \dots \,, \tag{3.2}$$

with $c_i$ being order one parameters and $\mathcal{O}_i^d$ denoting a complete basis of operators of dimension $d$. The $d = 6$ complete basis was first derived in [21], while [22] corrected some inaccuracies and proposed a different basis that is widespread nowadays. Assuming baryon number conservation there are 59 operators. A different basis which is useful for our purposes is the so-called Hagiwara-Isihara-Szalpski-Zeppenfeld (HISZ) basis [23, 24], since it refers only the bosonic sector. As an example, the $d = 6$ CP violating bosonic basis is given by

$$\begin{aligned} \mathcal{Q}_{\varphi\widetilde{B}} &= \mathbf{B}^*_{\mu\nu}\mathbf{B}^{\mu\nu}\Phi^\dagger\Phi \\ \mathcal{Q}_{\varphi\widetilde{W}} &= \Phi^\dagger\mathbf{W}^*_{\mu\nu}\mathbf{W}^{\mu\nu}\Phi \\ \mathcal{Q}_{\varphi\widetilde{B}W} &= \mathbf{B}^*_{\mu\nu}\Phi^\dagger\mathbf{W}^{\mu\nu}\Phi \,, \end{aligned} \tag{3.3}$$

## 3.2 The chiral lagrangian for the SM

The lagrangian for the Standard Model can be written in an alternative way to the one presented in the previous chapter. Let us parametrize the degrees of freedom of the Higgs field as

$$\mathbf{M} = \frac{1}{\sqrt{2}} \left( \phi_0 \mathbb{1} + i\vec{\tau} \cdot \vec{\phi} \right) = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_0 + i\phi_3 & i\phi_1 + \phi_2 \\ i\phi_1 - \phi_2 & \phi_0 - i\phi_3 \end{pmatrix} \equiv \begin{pmatrix} \tilde{\Phi} & \Phi \end{pmatrix} \tag{3.4}$$

The matrix $\mathbf{M}$ satisfies

$$\frac{1}{2}\text{Tr}\left(\mathbf{M}^\dagger \mathbf{M}\right) = \Phi^\dagger \Phi = \frac{1}{2}\left(\phi_0^2 + \vec{\phi} \cdot \vec{\phi}\right) \tag{3.5}$$

so that the Higgs potential can be written as

$$V\left(\mathbf{M}\right) = \frac{\mu^2}{4}\text{Tr}\left(\mathbf{M}^\dagger \mathbf{M}\right) - \frac{\lambda}{8}\text{Tr}\left(\mathbf{M}^\dagger \mathbf{M}\right)^2 \tag{3.6}$$

The field $\mathbf{M}$ transforms as a bi-doublet of $SU(2)_L \times SU(2)_R$ (that is, in the $\left(\frac{1}{2}, \frac{1}{2}\right)$ representation), and the potential 3.6 is indeed invariant under this symmetry, or similarly under $O(4)$, if we consider the vector $\vec{\varphi} \equiv \left(\phi_0, \vec{\phi}\right)$, since $SU(2)_L \times SU(2)_R \simeq O(4)$. The global $SU(2)_L$ is gauged and identified with the weak group, while the hypercharge group is associated to the $U(1)_Y \subset SU(2)_R$ generated by $\tau^3$.

The Higgs lagrangian with the bi-doublet matrix $\mathbf{M}$ reads as

$$\begin{aligned}
\mathscr{L}_{\mathbf{M}} =& \frac{1}{2}\text{Tr}\left((\mathbf{D}_\mu \mathbf{M})^\dagger \mathbf{D}^\mu \mathbf{M}\right) - \frac{\mu^2}{4}\text{Tr}\left(\mathbf{M}^\dagger \mathbf{M}\right) - \frac{\lambda}{8}\text{Tr}\left(\mathbf{M}^\dagger \mathbf{M}\right)^2 + \\
& + \left[\bar{Q}_L \mathbf{M} \mathcal{Y}_Q Q_R + \bar{L}_L \mathcal{Y}_L \mathbf{M} L_R + \text{h.c}\right]
\end{aligned} \tag{3.7}$$

where

$$\mathbf{D}_\mu \mathbf{M}(x) = \partial_\mu \mathbf{M}(x) + \frac{ig}{2}W_\mu^a(x)\tau_a \mathbf{M}(x) - \frac{ig'}{2}B_\mu(x)\mathbf{M}(x)\tau^3 \tag{3.8}$$

and

$$\mathcal{Y}_Q \equiv \begin{pmatrix} \mathbf{Y}_D & 0 \\ 0 & \mathbf{Y}_U \end{pmatrix} \qquad \mathcal{Y}_L \equiv \begin{pmatrix} 0 & 0 \\ 0 & \mathbf{Y}_E \end{pmatrix} \tag{3.9}$$

$$Q_R \equiv \begin{pmatrix} U_R \\ D_R \end{pmatrix} \qquad L_R \equiv \begin{pmatrix} 0 \\ E_R \end{pmatrix} \tag{3.10}$$

### 3.2.1 Custodial symmetry

Notice that in the limit where $g' = 0$, $\mathbf{Y}_D = \mathbf{Y}_U$ and $\mathbf{Y}_E = 0$, the lagrangian 3.7 is symmetric under $SU(2)_L \times SU(2)_R$.

Potential 3.6 has a minimum in

$$\frac{\langle \mathrm{Tr}\left(\mathbf{M}^\dagger \mathbf{M}\right)\rangle}{2} = \langle \Phi^\dagger \Phi \rangle = \frac{\mu^2}{\lambda} \equiv \frac{v^2}{2} \tag{3.11}$$

at which the electroweak symmetry is spontaneously broken. Setting the alignment $\vec{\varphi}_{\min} = (v, \vec{0})$, it is seen that the vacuum is not invariant under the whole $O(4) \simeq SU(2)_L \times SU(2)_R$, but only under $O(3) \simeq SU(2)_{L+R}$ transformations. That the conserved group is the diagonal group $SU(2)_{L+R}$ is seen by expressing the vacuum as

$$\langle \mathbf{M} \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v & 0 \\ 0 & v \end{pmatrix} \tag{3.12}$$

Since $\mathbf{M}$ transforms as

$$\mathbf{M} \to L\mathbf{M}R^\dagger \tag{3.13}$$

$\langle \mathbf{M} \rangle$ is only invariant if $L = R$. This is known as the $SU(2)_c$ custodial symmetry of the Standard Model, and its importance is due to the fact that it ensures that the relation

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2 \theta} = 1 \tag{3.14}$$

is exact at tree level. Let us consider first the case in which $g' = 0$. In this limit there is no massless photon in the spectrum and the $Z$ particle coincides with the $W^3$ boson. A priori, the masses of the charged $W^\pm$ and of $W^3$ need not be equal; however, the existence of an additional global unbroken symmetry $SU(2)_{L+R}$ under which the weak currents as well as the weak bosons transform as a triple enforces the equality $m_{W^\pm} = m_{W^3}$. Therefore in this limit $\rho = 1$. If $g' \neq 0$, then the relation that holds at tree-level is indeed 3.14.

However, this relationship is broken at loop levels by terms proportional to $g'$ and $(\mathbf{Y}_D - \mathbf{Y}_U)$, so that

$$\rho = 1 + \Delta\rho \tag{3.15}$$

where $\Delta\rho \sim 10^{-3}$ [25] accounts for custodially breaking radiative contributions. For example, loops of Higgs particles induce a term of the form

$$\Delta\rho = \frac{-11 G_F m_Z^2 \sin^2 \theta_W}{24\sqrt{2}\pi^2} \log\left(\frac{m_h^2}{m_Z^2}\right) \tag{3.16}$$

so that, as expected, in the limit $g' \to 0$ ($\sin^2 \theta_W \to 0$), $\Delta\rho = 0$.

It can be shown that in any theory of EW symmetry breaking where the vacuum is symmetric under $U(1)_{\mathrm{em}}$ as well as under an $SU(2)_c$ under which the generators of $SU(2)_L$ transform as a

triplet, the mass matrix of the electroweak bosons in the basis $(W^1_\mu, W^2_\mu, W^3_\mu, B_\mu)$ has the form

$$\mathcal{M}^2 \propto \begin{pmatrix} g^2 & & & \\ & g^2 & & \\ & & g^2 & -gg' \\ & & -gg' & g'^2 \end{pmatrix} \tag{3.17}$$

and thus $\rho = 1$ at tree level, as in the SM.

### 3.2.2 Construction of the Appelquist-Longhitano-Feruglio basis

The existence of a light Higgs scalar was seen to be a problem since the early stages of the construction of the SM [9]. An attractive solution then for the hierarchy problem was to consider that the Higgs particle was either very heavy or simply non-existing, with the EWSB triggered by some strong dynamics. In this section we will follow [26–29] and construct an effective lagrangian for this non-linear scenario, which is usually called the chiral lagrangian. This name is due to the chiral lagrangian for pions of low-energy QCD, which is based on the spontaneous breaking of the chiral group $SU(2)_L \times SU(2)_R$.

In order to capture the essence of the formulation we will focus in the bosonic sector of the EFT, describing the longitudinal and transversal modes of EW gauge bosons. The inclusion of fermions would essentially amount to the addition of a larger number of operators.

Since $m_h^2 \equiv m_\phi^2 = \frac{1}{2}\mu^2 = \frac{1}{2}\lambda v^2$, the limit $m_h \to \infty$ is equivalent to having $\lambda \to \infty$ while keeping $v$ finite. In this case, the potential develops an infinite positive curvature at its minimum,

$$\mathbf{M}\mathbf{M}^\dagger = \frac{\phi_0(x) + \vec{\phi}^2(x)}{2} = v^2 \tag{3.18}$$

Thus, the $\phi_0(x)$ and $\vec{\phi}(x)$ fields are bound to lie in a three-dimensional hypersphere of radius $v$. Defining $\mathbf{U} \equiv \mathbf{M}/v$, the lowest order bosonic lagrangian describing the GBs is now given by

$$\mathcal{L}_0 = \frac{v^2}{4}\text{Tr}\left(\mathbf{D}_\mu \mathbf{U}^\dagger \mathbf{D}^\mu \mathbf{U}\right) \tag{3.19}$$

where

$$\mathbf{D}_\mu \mathbf{U}(x) \equiv \partial_\mu \mathbf{U}(x) + \frac{ig}{2}W^a_\mu(x)\tau_a \mathbf{U} - \frac{ig'}{2}B_\mu(x)\mathbf{U}\tau_3 \tag{3.20}$$

As with the SMEFT, this lagrangian is given by a tower of operators, which are now constructed using the $\mathbf{U}$ matrix and the EW gauge boson fields as the building blocks. However, in the chiral case the expansion is not given by the canonical mass dimension of operators, but it is instead

an expansion in derivatives (or momenta). We will construct a set of operators that allow us to write the bosonic chiral lagrangian of the SM up to four derivatives, the ALF basis [26–29]. It is instructive to show the construction of the basis with some detail, since a parallel procedure will be follow in the next chapter when the effective lagrangian for generic CH models is built.

The invariant operators must be constructed by taking the traces of local operators which transform covariantly under $SU(2)_L \times U(1)_Y$. We can choose in particular to build the operators from a set of objects $O(x)$ which are $SU(2)_L$ covariants and $U(1)_Y$ singlets,

$$O(x) \to O'(x) = LML^\dagger \tag{3.21}$$

with

$$L = e^{i\epsilon^a(x)\tau_a/2} \in SU(2)_L \tag{3.22}$$

Thus, for any given $O(x)$, $\text{Tr}\,(O(x))$ is an $SU(2)_L \times U(1)_Y$ invariant. We introduce the adjoint representation operator $\mathcal{D}_\mu$,

$$\mathcal{D}_\mu O(x) \equiv \partial_\mu O(x) + ig\left[\mathbf{W}_\mu(x), O(x)\right] \tag{3.23}$$

where $\mathbf{W}_\mu(x) \equiv W_\mu^a(x)\tau_a/2$. We introduce the pseudo-scalar and chiral fields $\mathbf{T}$ and $\mathbf{V}_\mu$ transforming in the adjoint of $SU(2)_L$ defined as

$$
\begin{aligned}
\mathbf{T}(x) &\equiv \mathbf{U}(x)\tau_3\mathbf{U}^\dagger(x), & \mathbf{T}(x) &\to L\mathbf{T}(x)L^\dagger \\
\mathbf{V}(x) &\equiv \left(\mathbf{D}_\mu\mathbf{U}(x)\right), \mathbf{U}^\dagger(x) & \mathbf{V}(x) &\to L\mathbf{V}_\mu(x)L^\dagger
\end{aligned}
\tag{3.24}
$$

We define as $G$ the set of objects transforming covariantly, as $O(x)$, obtained by taking covariant derivatives of $\mathbf{T}$, $\mathbf{V}$, $\mathbf{W}_{\mu\nu}$ and $\mathbf{B}_{\mu\nu}$, where

$$
\begin{aligned}
\mathbf{W}_{\mu\nu} &\equiv \partial_\mu\mathbf{W}_\nu - \partial_\nu\mathbf{W}_\mu + ig\left[\mathbf{W}_\mu, \mathbf{W}_\nu\right], \\
\mathbf{B}_{\mu\nu} &\equiv B_{\mu\nu}\mathbf{T} = (\partial_\mu B_\nu - \partial_\nu B_\mu)\mathbf{T}
\end{aligned}
\tag{3.25}
$$

so that

$$
\begin{aligned}
G = \{&\mathbf{T}, \mathbf{V}_\mu, \mathbf{W}_{\mu\nu}, \mathbf{B}_{\mu\nu}; \mathcal{D}_\sigma\mathbf{T}, \mathcal{D}_\sigma\mathbf{V}, \mathcal{D}_\sigma\mathbf{W}_{\mu\nu}, \mathcal{D}_\sigma\mathbf{B}_{\mu\nu}; \ldots \mathcal{D}_\alpha\cdots\mathcal{D}_\sigma\mathbf{T}, \mathcal{D}_\alpha\cdots\mathcal{D}_\sigma\mathbf{V}_\mu, \\
&, \mathcal{D}_\alpha\cdots\mathcal{D}_\sigma\mathbf{W}_{\mu\nu}, \mathcal{D}_\sigma\mathbf{B}_{\mu\nu}\}
\end{aligned}
\tag{3.26}
$$

Any invariant operator can be constructed from traces of sequences of elements of G. It can be shown that indeed $\mathcal{D}_\mu$ behaves as a derivative,

$$\mathcal{D}_\mu\left(O_1(x)O_2(x)\right) = \mathcal{D}_\mu O_1(x)O_2(x) + O_1(x)\mathcal{D}_\mu O_2(x) \tag{3.27}$$

It should be noted that it is not needed to include in $G$ the operator $\tilde{\mathbf{K}}_\mu \equiv \mathbf{U} \left( \mathbf{D}_\mu \mathbf{U} \right)^\dagger$ since the unitarity of $\mathbf{U}$ ensures that

$$\tilde{\mathbf{K}}_\mu = \mathbf{U} \left( \mathbf{D}_\mu \mathbf{U} \right)^\dagger = -\mathbf{V}_\mu \tag{3.28}$$

One can prove that all the elements of G are traceless quite straightforwardly. First of all, we prove that $T$, $V_\mu$ and $W_{\mu\nu}$ are traceless:

$$\begin{aligned}
\text{Tr} \left( \mathbf{T} \right) &= \text{Tr} \left( \mathbf{U}\tau_3 \mathbf{U}^\dagger \right) = 0 \\
\text{Tr} \left( \mathbf{V}_\mu \right) &= \text{Tr} \left( \left( \partial_\mu \mathbf{U} \right) \mathbf{U}^\dagger \right) + ig\text{Tr} \left( \mathbf{W}_\mu \right) - \frac{ig'}{2} B_\mu(x)\text{Tr} \left( \tau_3 \right) = 0 \\
\text{Tr} \left( \mathbf{W}_{\mu\nu} \right) &= \partial_\mu \text{Tr} \left( \mathbf{W}_\nu \right) - \partial_\nu \text{Tr} \left( \mathbf{W}_\mu \right) + ig\text{Tr} \left( [\mathbf{W}_\mu, \mathbf{W}_\nu] \right) = 0
\end{aligned} \tag{3.29}$$

On top of that, the covariant derivative $\mathcal{D}_\mu$ of any traceless element $O \in$ G is traceless:

$$\text{Tr} \left( \mathcal{D}_\mu O \right) = \partial_\mu \text{Tr} \left( O \right) + ig\text{Tr} \left( [\mathbf{W}_\mu, O] \right) = 0 \tag{3.30}$$

Thus, the successive derivatives of $\mathbf{T}$, $\mathbf{V}_\mu$ and $\mathbf{W}_{\mu\nu}$, which form G have null trace. Being SU(2) covariants and traceless, all the elements $O \in G$ can be decomposed as a linear (complex) combination of the Pauli matrices,

$$O = O^a \tau_a \tag{3.31}$$

The Kronecker delta $\delta_{ij}$ and the Levi-Civita symbol $\epsilon_{ijk}$ are the only $SU(2)$ invariant tensors thus, the product of any number of $\sigma$ matrices, being invariants, must be expressable as linear combinations of products of these objects. In conclusion, the traces of products of these matrices can be decomposed in traces of the product of only two and three $\tau_a$ matrices, since $\text{Tr} \left( \tau_i \tau_j \right) = 2\delta_{ij}$ and $\text{Tr} \left( \tau_i \tau_j \tau_k \right) = 2i\epsilon_{ijk}$. Furthermore, the trace of three $\tau$ matrices appears only once, at most, in each product. These are particularities of $SU(2)$, and when in the following chapter we follow a similar procedure to write the effective lagrangians invariant under different groups these kind of reductions might no be possible.

We can then restrict ourselves to traces of products of only 2 or 3 elements in G. In order to build all the operators, we follow the next steps: first, we make a list of all structures of the type $\text{Tr}(\prod_i O_I)$ up to a certain dimension; then, from these we form a basis of independent terms; finally, build all the invariant terms for the lagrangian employing also $B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\nu$.

In general, the operators constructed in this way will not be independent. However, a complete set of independent operators can be found by using the antisymmetry of the field-strenghts $\mathbf{W}_{\mu\nu}$ and $B_{\mu\nu}$, integration by parts and the identities

$$[\mathcal{D}_\mu, \mathcal{D}_\nu] O(x) = ig \left[ \mathbf{W}_{\mu\nu}, O(x) \right] \qquad ; \qquad \forall O(x) \tag{3.32}$$

$$\mathbf{V}_{\mu\nu} \equiv \mathcal{D}_\mu \mathbf{V}_\nu - \mathcal{D}_\nu \mathbf{V}_\mu = ig\mathbf{W}_{\mu\nu} - \frac{ig'}{2} B_{\mu\nu}\mathbf{T} + [\mathbf{V}_\mu, \mathbf{V}_\nu] \tag{3.33}$$

The fundamental fields of the theory, $\mathbf{U}, \mathbf{W}_\mu$ and $B_\mu$, transform under discrete $CP$ transformations as

$$
\begin{aligned}
CP\, \mathbf{U}\,(\vec{x}, t)\, P^{-1}C^{-1} &= \eta\,\tau_2\,\mathbf{U}\,(-\vec{x}, t)\,\tau_2 \\
CP\, \mathbf{W}_\mu\,(\vec{x}, t)\, P^{-1}C^{-1} &= \tau_2\,\mathbf{W}^\mu\,(-\vec{x}, t)\,\tau_2 \\
CP\, B_\mu\,(\vec{x}, t)\, P^{-1}C^{-1} &= \eta' B^\mu\,(-\vec{x}, t)
\end{aligned}
\tag{3.34}
$$

with $\eta = \pm 1$ and $\eta' = \pm 1$. In order to set $\eta'$ we see how $\mathbf{V}_\mu$ transforms under CP:

$$
\begin{aligned}
(\mathbf{D}_\mu \mathbf{U})' &= \partial_\mu \mathbf{U}' + ig\mathbf{W}'_\mu \mathbf{U}' - \frac{ig'}{2} B'_\mu \mathbf{U}' \tau_3 = \\
&= \eta\tau_2 \partial_\mu \mathbf{U}\,(-\vec{x}, t)\,\tau_2 + ig\eta\tau_2 \mathbf{W}_\mu\,(-\vec{x}, t)\,\mathbf{U}\,(-\vec{x}, t) + \frac{ig'}{2}\eta' B_\mu\,(-\vec{x}, t)\,\tau_3 \implies \\
\implies \mathbf{V}'_\mu &= \tau_2\,(\partial_\mu \mathbf{U}\,(-\vec{x}, t))\,\mathbf{U}^\dagger\,(-\vec{x}, t) + ig\tau_2 \mathbf{W}_\mu\,(-\vec{x}, t)\,\tau_2 + \frac{ig'}{2}\eta' B_\mu\,(-\vec{x}, t)\,\tau_3
\end{aligned}
\tag{3.35}
$$

Therefore, if we set $\eta' = -1$ then $\mathbf{V}_\mu$ transforms in a well-defined manner under CP, i.e.

$$
\begin{aligned}
CP\, \mathbf{T}\,(\vec{x}, t)\, P^{-1}C^{-1} &= -\tau_2\,\mathbf{T}\,(-\vec{x}, t)\,\tau_2 \\
CP\, \mathbf{V}_\mu\,(\vec{x}, t)\, P^{-1}C^{-1} &= \tau_2 \mathbf{V}^\mu\,(-\vec{x}, t)\,\tau_2
\end{aligned}
\tag{3.36}
$$

Instead of considering operators $O$ which are $SU(2)_L$ covariants and $U(1)_Y$ invariants we could have chosen to use $SU(2)_L$ invariants and $U(1)_Y$ covariants,

$$\bar{O}(x) \to \bar{O}'(x) = e^{i\epsilon_0(x)\tau_3/2}\bar{O}(x)e^{-i\epsilon_0(x)\tau_3/2} \tag{3.37}$$

Defining $\mathbf{I}_\mu \equiv \mathbf{U}^\dagger \mathbf{D}_\mu \mathbf{U}$ then the set of $SU(2)_L \times U(1)_Y$ invariants would be written by taking traces of elements from the set

$$H = \left\{\tau_3, \mathbf{I}_\mu, \bar{\mathcal{D}}_\sigma \mathbf{I}_\mu, \ldots, \bar{\mathcal{D}}_\alpha \cdots \bar{\mathcal{D}}_\sigma \mathbf{I}_\mu\right\} \tag{3.38}$$

where the covariant derivative $\bar{\mathcal{D}}_\mu$ is defined as

$$\bar{\mathcal{D}}_\mu \bar{O}(x) = \partial_\mu \bar{O}(x) + \frac{ig'}{2} B_\mu(x)\left[\tau_3, \bar{O}(x)\right] \tag{3.39}$$

Due to the unitarity of $\mathbf{U}$, we have that $\mathbf{U}\mathbf{I}_\mu \mathbf{U}^\dagger = \mathbf{V}_\mu$. Furthermore, $\mathbf{U}\bar{O}\mathbf{U}^\dagger \in G$, since transforms like $O$,

$$\mathbf{U}\bar{O}\mathbf{U}^\dagger \to L\left(\mathbf{U}\bar{O}\mathbf{U}^\dagger\right) L^\dagger \tag{3.40}$$

Since the following identity holds

$$\mathbf{U}\left(\bar{\mathcal{D}}_\mu \bar{O}\right)\mathbf{U}^\dagger = \mathcal{D}_\mu\left(\mathbf{U}\bar{O}\mathbf{U}^\dagger\right) - \left[\mathbf{V}_\mu, \mathbf{U}\bar{O}\mathbf{U}^\dagger\right] \tag{3.41}$$

it's clear that invariants of the type $\mathrm{Tr}\left(\bar{O}\right)$ can be expressed as traces of objects $O \in G$ by insertions of $\mathbf{U}\mathbf{U}^\dagger = 1$. Therefore, the complete set of $SU(2)_L \times U(1)_Y$ operators can be constructed as was done in the first place, with using $SU(2)_L$ covariant and $U(1)_Y$ invariant building blocks, without loss of generality. Invariants written in terms of determinants of products of operators need not be considered, since using the Cayley-Hamilton theorem any operator involving determinants can be written in terms of operators constructed by taking the traces of covariant objects.

In conclusion, In order to write the structures from which build the list of operators there are some things to be noted. First, one can avoid using $\mathcal{D}_\mu\mathbf{T}$, since

$$\mathcal{D}_\mu\mathbf{T} = [\mathbf{V}_\mu, \mathbf{T}] \tag{3.42}$$

Additionally, notice that $\mathbf{T}^2 = 1$. Finally, even if the commutator of elements in $G$ will be considered, one can get rid of the anticommutators, since only products of up to 3 elements are considered, and

$$\mathrm{Tr}\left(O\{O_1, O_2\}\right) = O_1^1 O_2^j \delta_{ij}\mathrm{Tr}\left(O\right) = 0 \tag{3.43}$$

Thus in general we will always include the antysimmetric combination whenever the product of 3 operators appears:

$$\mathrm{Tr}\left(O_1 O_2 O_3\right) = \frac{1}{2}\mathrm{Tr}\left([O_1, O_2]O_3\right) + \frac{1}{2}\mathrm{Tr}\left(\{O_1, O_2\}O_3\right) = \frac{1}{2}\mathrm{Tr}\left([O_1, O_2]O_3\right) \tag{3.44}$$

The list of independent structures, organized by the number of derivatives $\mathbf{D}$, is shown in Eq. (3.45), where $(\mathbf{W}_{\mu\nu} \to \mathbf{B}_{\mu\nu})$ indicates all possible structures built by replacing $\mathbf{W}_{\mu\nu}$ by $\mathbf{B}_{\mu\nu}$. In order to construct the list of operators one takes products the structures listed in 3.45. The operators can be classified according to their CP character (even or odd) and how they relate to custodial symmetry. In particular, the name "custodial preserving" will be given to those operators that either respect custodial symmetry or vanish in the limit $g' = 0$ - in other words, operators that break custodial symmetry in the same way that the Standar Model does. The rest of operators, which even in the limit $g' = 0$ violate custodial symmetry, are referred to as "custodial breaking", since they represent BSM custodial breaking effects.

An easy way to identify to which class an operator belong is identifying the presence of the $\mathbf{T}$ operator: if it appears as part of $\mathbf{B}_{\mu\nu}$, accompanying the field $B_{\mu\nu}$ the operator is custodial

$$
\begin{aligned}
\mathbf{D} = \mathbf{1} \quad & \mathrm{Tr}\,(\mathbf{T}\mathbf{V}_\mu) \\
\mathbf{D} = \mathbf{2} \quad & \mathrm{Tr}\,(\mathbf{T}\mathcal{D}_\mu\mathbf{V}_\nu) \qquad \mathrm{Tr}\,(\mathbf{V}_\mu\mathbf{V}_\nu) \\
& \mathrm{Tr}\,(\mathbf{T}\mathbf{W}_{\mu\nu}) \qquad \mathrm{Tr}\,(\mathbf{T}[\mathbf{V}_\mu,\mathbf{V}_\nu]) \qquad\qquad (\mathbf{W}_{\mu\nu}\to\mathbf{B}_{\mu\nu}) \\
\mathbf{D} = \mathbf{3} \quad & \mathrm{Tr}\,(\mathbf{T}\mathcal{D}_\mu\mathcal{D}_\nu\mathbf{V}_\rho) \\
& \mathrm{Tr}\,(\mathbf{V}_\mu\mathcal{D}_\nu\mathbf{V}_\rho) \qquad \mathrm{Tr}\,([\mathbf{T},\mathbf{V}_\mu]\mathcal{D}_\nu\mathbf{V}_\rho) \\
& \mathrm{Tr}\,(\mathbf{V}_\mu\mathcal{D}_\nu\mathbf{V}_\rho) \qquad \mathrm{Tr}\,([\mathbf{T},\mathbf{V}_\mu]\mathbf{W}_{\nu\rho}) \qquad (\mathbf{W}_{\mu\nu}\to\mathbf{B}_{\mu\nu}) \\
& \mathrm{Tr}\,(\mathbf{V}_\mu\mathbf{W}_{\nu\rho}) \qquad \mathrm{Tr}\,(\mathbf{V}_\mu\mathbf{V}_\nu\mathbf{V}_\rho) \\
\mathbf{D} = \mathbf{4} \quad & \mathrm{Tr}\,(\mathbf{T}\mathcal{D}_\mu\mathcal{D}_\nu\mathcal{D}_\rho\mathbf{V}_\rho) \\
& \mathrm{Tr}\,(\mathbf{V}_\mu\mathcal{D}_\nu\mathcal{D}_\rho\mathbf{V}_\sigma) \qquad \mathrm{Tr}\,(\mathbf{W}_{\mu\nu}\mathbf{W}_{\rho\sigma}) \qquad (\mathbf{W}_{\mu\nu}\to\mathbf{B}_{\mu\nu}) \\
& \mathrm{Tr}\,(\mathbf{V}_\mu\mathcal{D}_\nu\mathbf{W}_{\rho\sigma}) \qquad \mathrm{Tr}\,([\mathbf{T},\mathbf{W}_{\mu\nu}]\mathcal{D}_\rho\mathbf{V}_\sigma) \\
& \mathrm{Tr}\,((\mathcal{D}_\mu\mathbf{V}_\nu)(\mathcal{D}_\rho\mathbf{V}_\sigma)) \qquad \mathrm{Tr}\,([\mathbf{V}_\mu,\mathbf{V}_\nu]\mathcal{D}_\rho\mathbf{V}_\sigma) \\
& \mathrm{Tr}\,(\mathcal{D}_\mu\mathbf{V}_\nu\mathbf{W}_{\rho\sigma}) \qquad \mathrm{Tr}\,([\mathbf{V}_\mu,\mathbf{V}_\nu]\mathbf{W}_{\rho\sigma})
\end{aligned}
\tag{3.45}
$$

preserving (since $\tau_3$ can be identified as the generator of $U(1)_Y \subset SU(2)_L$), and custodial violating otherwise. The vector field $\mathbf{V}_\mu$ is a singlet under $SU(2)_R$, and therefore its insertions do not break $SU(2)_L \times SU(2)_R$.

The complete independent basis of CP even bosonic operators up to four derivatives reads as follows, with the definitions $X^*_{\mu\nu} \equiv \epsilon_{\mu\nu\rho\sigma}X^{\rho\sigma}$ for $X = \{B, \mathbf{W}\}$:

**CP-even perators with two derivatives**

$$
\begin{array}{cc}
\underline{\text{Custodial preserving}} & \underline{\text{Custodial breaking}} \\[4pt]
\mathcal{O}_C = -\dfrac{v^2}{4}\mathrm{Tr}(\mathbf{V}^\mu\mathbf{V}_\mu) & \mathcal{O}_T = \dfrac{v^2}{4}\mathrm{Tr}(\mathbf{T}\mathbf{V}_\mu)\mathrm{Tr}(\mathbf{T}\mathbf{V}^\mu)
\end{array}
\tag{3.46}
$$

**CP-even operators with four derivatives**

$$
\begin{array}{ll}
\underline{\text{Custodial preserving}} & \underline{\text{Custodial breaking}} \\[6pt]
\mathcal{O}_B = -\dfrac{1}{4}B_{\mu\nu}B^{\mu\nu} & \mathcal{O}_7 = g^2(\mathrm{Tr}(\mathbf{T}\mathbf{W}_{\mu\nu}))^2 \\[6pt]
\mathcal{O}_W = -\dfrac{1}{2}\mathrm{Tr}(\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}) & \mathcal{O}_8 = ig\,\mathrm{Tr}(\mathbf{T}\mathbf{W}_{\mu\nu})\mathrm{Tr}(\mathbf{T}[\mathbf{V}^\mu,\mathbf{V}^\nu]) \\[6pt]
\mathcal{O}_1 = gg'B_{\mu\nu}\mathrm{Tr}(\mathbf{T}\mathbf{W}^{\mu\nu}) & \mathcal{O}_9 = g\,\mathrm{Tr}(\mathbf{T}\mathbf{V}^\mu)\mathrm{Tr}(\mathbf{V}^\nu\mathbf{W}^*_{\mu\nu}) \\[6pt]
\mathcal{O}_2 = ig'B_{\mu\nu}\mathrm{Tr}(\mathbf{T}[\mathbf{V}^\mu,\mathbf{V}^\nu]) & \mathcal{O}_{10} = \mathrm{Tr}(\mathbf{T}\mathcal{D}_\mu\mathbf{V}^\mu)\mathrm{Tr}(\mathbf{T}\mathcal{D}_\nu\mathbf{V}^\nu) \\[6pt]
\mathcal{O}_3 = ig\,\mathrm{Tr}(\mathbf{W}_{\mu\nu}[\mathbf{V}^\mu,\mathbf{V}^\nu]) & \mathcal{O}_{11} = \mathrm{Tr}([\mathbf{T},\mathbf{V}_\nu]\mathcal{D}_\mu\mathbf{V}^\mu)\mathrm{Tr}(\mathbf{T}\mathbf{V}^\nu) \\[6pt]
\mathcal{O}_4 = (\mathrm{Tr}(\mathbf{V}_\mu\mathbf{V}^\mu))^2 & \mathcal{O}_{12} = \mathrm{Tr}(\mathbf{V}_\mu\mathbf{V}^\mu)(\mathrm{Tr}(\mathbf{T}\mathbf{V}_\nu))^2 \\[6pt]
\mathcal{O}_5 = \mathrm{Tr}((\mathcal{D}_\mu\mathbf{V}^\mu)^2) & \mathcal{O}_{13} = \mathrm{Tr}(\mathbf{V}_\mu\mathbf{V}_\nu)\mathrm{Tr}(\mathbf{T}\mathbf{V}^\mu)\mathrm{Tr}(\mathbf{T}\mathbf{V}^\nu) \\[6pt]
\mathcal{O}_6 = (\mathrm{Tr}(\mathbf{V}_\mu\mathbf{V}_\nu))^2 & \mathcal{O}_{14} = (\mathrm{Tr}(\mathbf{T}\mathbf{V}_\mu)\mathrm{Tr}(\mathbf{T}\mathbf{V}_\nu))^2\,.
\end{array}
\tag{3.47}
$$

The low-energy electroweak chiral CP-even bosonic Lagrangian up to four derivatives is given by

$$\mathscr{L}_{\text{low}} = \mathscr{L}_{\text{low}}^{p^2} + \mathscr{L}_{\text{low}}^{p^4}, \tag{3.48}$$

where $\mathscr{L}_{\text{low}}^{p^2}$ and $\mathscr{L}_{\text{low}}^{p^4}$ contain two and and four-derivative operators,

$$
\begin{aligned}
\mathscr{L}_{\text{low}}^{p^2} &= \mathcal{O}_C + c_T \mathcal{O}_T, \\
\mathscr{L}_{\text{low}}^{p^4} &= \mathcal{O}_B + \mathcal{O}_W + \sum_{i=1}^{14} c_i\, \mathcal{O}_i
\end{aligned}
\tag{3.49}
$$

It is important to notice how $c_T$ parametrizes the explicit breaking of custodial symmetry at leading order given by the operator $\mathcal{O}_T$. The fact that it contributes to the $\rho$ parameter is due to it contributing to the $Z$ mass but not to $m_W$, so that

$$\rho = \frac{m_W^2}{m_Z^2 \cos^2\theta} = \frac{1}{1 - 2c_t} \simeq 1 + 2c_T \tag{3.50}$$

The experimental constraints on $\Delta\rho$ lead to a value of $c_T \sim 10^{-3}$ could lead one to choose to have $O_T$ appear at NLO. Furthermore, it is an example of the strong constraint that $\rho \simeq 1$ puts on generic models of EWSB, imposing an approximate symmetry that in the SM happens to be an accidental one.

The CP-odd counterpart reads as follows:

**CP-odd operators with two derivatives**

| Custodial preserving | Custodial breaking | |
|---|---|---|
| $-$ | $\mathcal{R}_{2D} = i\,\dfrac{v^2}{4}\,\mathrm{Tr}\left(\mathbf{T}\,\mathcal{D}_\mu \mathbf{V}^\mu\right)$ | (3.51) |

**CP-odd operators with four derivatives**

| Custodial preserving | Custodial breaking | |
|---|---|---|
| $\mathcal{R}_{BB^*} = -\dfrac{g'^2}{4} B^*_{\mu\nu} B^{\mu\nu}$ | $\mathcal{R}_2 = g\,\mathrm{Tr}\left(\mathbf{W}^{\mu\nu}\mathbf{V}_\mu\right)\mathrm{Tr}\left(\mathbf{T}\,\mathbf{V}_\nu\right)$ | |
| $\mathcal{R}_{WW^*} = -\dfrac{g^2}{2}\mathrm{Tr}(\mathbf{W}^*_{\mu\nu}\mathbf{W}^{\mu\nu})$ | $\mathcal{R}_3 = 2\,g^2\mathrm{Tr}\left(\mathbf{T}\,\mathbf{W}^*_{\mu\nu}\right)\mathrm{Tr}\left(\mathbf{T}\,\mathbf{W}_{\mu\nu}\right)$ | (3.52) |
| $\mathcal{R}_1 = 2g\,g'\,B^*_{\mu\nu}\mathrm{Tr}\left(\mathbf{T}\mathbf{W}^{\mu\nu}\right)$ | $\mathcal{R}_4 = i\,\mathrm{Tr}\left(\mathbf{V}^\mu\,\mathcal{D}^\nu\mathbf{V}_\nu\right)\mathrm{Tr}\left(\mathbf{T}\,\mathbf{V}_\mu\right)$ | |
| | $\mathcal{R}_5 = i\,\mathrm{Tr}\left(\mathbf{T}\,\mathcal{D}^\mu\mathbf{V}_\mu\right)\mathrm{Tr}\left(\mathbf{V}^\nu\,\mathbf{V}_\nu\right)$ | |
| | $\mathcal{R}_6 = i\,\mathrm{Tr}\left(\mathbf{T}\,\mathcal{D}^\mu\mathbf{V}_\mu\right)\left(\mathrm{Tr}\left(\mathbf{T}\mathbf{V}^\nu\right)\right)^2$ | |

The low-energy electroweak chiral Lagrangian describing the CP-odd bosonic lagrangian up to four derivatives, can instead be written as:

$$\mathscr{L}_{\text{low},\cancel{CP}} = \mathscr{L}_{\text{low},\cancel{CP}}^{p^2} + \mathscr{L}_{\text{low},\cancel{CP}}^{p^4}, \tag{3.53}$$

where $\mathscr{L}_{\text{low},\cancel{CP}}^{p^2}$ and $\mathscr{L}_{\text{low},\cancel{CP}}^{p^4}$ contain two and four-derivative operators, respectively,

$$\mathscr{L}_{\text{low},\cancel{CP}}^{p^2} = c_{2D}\mathcal{R}_{2D} \,,$$

$$\mathscr{L}_{\text{low},\cancel{CP}}^{p^4} = \mathcal{R}_{BB^*} + \mathcal{R}_{WW^*} + \sum_{i=1}^{6} c_i\,\mathcal{R}_i \,, \tag{3.54}$$

## 3.3 The chiral lagrangian with a light Higgs (HEFT)

The ALF basis describes a situation in which the Higgs scalar either doesn't exist or its mass is so big that it can be decoupled from the theory. However, since 2012 the existence of a light scalar of a mass of 125 GeV has been established, and therefore must be included in an effective lagrangian describing the SM. Its effects can be studied by expanding the ALF basis by adding a gauge singlet scalar $h$. This approach is useful since it allows for a description of not only the Higgs scalar, which is embedded in an $SU(2)_L$ doublet, but also of other BSM scenarios. This construction is known in the literature as the Higgs Effective Field Theory (HEFT) [30–33]

The effects of the singlet scalar $h$ are encoded in generic functions $\mathcal{F}(h)$,

$$\mathcal{F}_i(h) = 1 + 2a_i\frac{h}{v} + b_i\frac{h^2}{v^2} + \dots \tag{3.55}$$

which in particular do not depend on derivatives of $h$. The modification with respect to the ALF basis is twofold: on the one hand, new operators which depend on derivatives of $h$ can appear; on the other hand, every term of the lagrangian appears as the product of one operator and a generic function $\mathcal{F}_a$. Their a priori infinite expansion in powers of $h$ encode the fact that in this approach this Higgs-like particle is treated in the same way as the SM Goldstone bosons, $\pi_a$, which appear in the exponential matrix $\mathbf{U}$.

**CP-even perators with two derivatives**

| Custodial preserving | Custodial breaking | |
|---|---|---|
| $\mathcal{P}_C = -\dfrac{v^2}{4}\text{Tr}(\mathbf{V}^\mu\mathbf{V}_\mu)$ | $\mathcal{P}_T = \dfrac{v^2}{4}\text{Tr}(\mathbf{T}\mathbf{V}_\mu)\text{Tr}(\mathbf{T}\mathbf{V}^\mu)$ | (3.56) |

**CP-even perators with four derivatives**

<div align="center">

Custodial preserving        Custodial breaking

</div>

$$\mathcal{P}_B = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} \qquad\qquad \mathcal{P}_{12} = g^2(\mathrm{Tr}(\mathbf{T}\mathbf{W}_{\mu\nu}))^2$$

$$\mathcal{P}_W = -\frac{1}{2}\mathrm{Tr}(\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}) \qquad\qquad \mathcal{P}_{13} = ig\mathrm{Tr}(\mathbf{T}\mathbf{W}_{\mu\nu})\mathrm{Tr}(\mathbf{T}[\mathbf{V}^\mu,\mathbf{V}^\nu])$$

$$\mathcal{P}_1 = gg'B_{\mu\nu}\mathrm{Tr}(\mathbf{T}\mathbf{W}^{\mu\nu}) \qquad\qquad \mathcal{P}_{14} = g\epsilon_{\mu\nu\rho\lambda}\mathrm{Tr}(\mathbf{T}\mathbf{V}^\mu)\mathrm{Tr}(\mathbf{V}^\nu\mathbf{W}^{\rho\lambda})$$

$$\mathcal{P}_2 = ig'B_{\mu\nu}\mathrm{Tr}(\mathbf{T}[\mathbf{V}^\mu,\mathbf{V}^\nu]) \qquad\qquad \mathcal{P}_{15} = \mathrm{Tr}(\mathbf{T}\mathcal{D}_\mu\mathbf{V}^\mu)\mathrm{Tr}(\mathbf{T}\mathcal{D}_\nu\mathbf{V}^\nu)$$

$$\mathcal{P}_3 = ig\mathrm{Tr}(\mathbf{W}_{\mu\nu}[\mathbf{V}^\mu,\mathbf{V}^\nu]) \qquad\qquad \mathcal{P}_{16} = \mathrm{Tr}([\mathbf{T},\mathbf{V}_\nu]\mathcal{D}_\mu\mathbf{V}^\mu)\mathrm{Tr}(\mathbf{T}\mathbf{V}^\nu)$$

$$\mathcal{P}_4 = ig'B_{\mu\nu}\mathrm{Tr}(\mathbf{T}\mathbf{V}^\mu)\partial^\nu(h/v) \qquad\qquad \mathcal{P}_{17} = ig\mathrm{Tr}(\mathbf{T}\mathbf{W}_{\mu\nu})\mathrm{Tr}(\mathbf{T}\mathbf{V}^\mu)\partial^\nu(h/v)$$

$$\mathcal{P}_5 = ig\mathrm{Tr}(\mathbf{W}_{\mu\nu}\mathbf{V}^\mu)\partial^\nu(h/v) \qquad\qquad \mathcal{P}_{18} = \mathrm{Tr}(\mathbf{T}[\mathbf{V}_\mu,\mathbf{V}_\nu])\mathrm{Tr}(\mathbf{T}\mathbf{V}^\mu)\partial^\nu(h/v)$$

$$\mathcal{P}_6 = (\mathrm{Tr}(\mathbf{V}_\mu\mathbf{V}^\mu))^2 \qquad\qquad \mathcal{P}_{19} = \mathrm{Tr}(\mathbf{T}\mathcal{D}_\mu\mathbf{V}^\mu)\mathrm{Tr}(\mathbf{T}\mathbf{V}_\nu)\partial^\nu(h/v)$$

$$\mathcal{P}_7 = \mathrm{Tr}(\mathbf{V}_\mu\mathbf{V}^\mu)\partial_\nu\partial^\nu(h/v) \qquad\qquad \mathcal{P}_{21} = (\mathrm{Tr}(\mathbf{T}\mathbf{V}_\mu))^2\partial_\nu(h/v)\partial^\nu(h/v)$$

$$\mathcal{P}_8 = \mathrm{Tr}(\mathbf{V}_\mu\mathbf{V}_\nu)\partial^\mu(h/v)\partial^\nu(h/v) \qquad\qquad \mathcal{P}_{22} = \mathrm{Tr}(\mathbf{T}\mathbf{V}_\mu)\mathrm{Tr}(\mathbf{T}\mathbf{V}_\nu)\partial^\mu(h/v)\partial^\nu(h/v)$$

$$\mathcal{P}_9 = \mathrm{Tr}((\mathcal{D}_\mu\mathbf{V}^\mu)^2) \qquad\qquad \mathcal{P}_{23} = \mathrm{Tr}(\mathbf{V}_\mu\mathbf{V}^\mu)(\mathrm{Tr}(\mathbf{T}\mathbf{V}_\nu))^2$$

$$\mathcal{P}_{10} = \mathrm{Tr}(\mathbf{V}_\nu\mathcal{D}_\mu\mathbf{V}^\mu)\partial^\nu(h/v) \qquad\qquad \mathcal{P}_{24} = \mathrm{Tr}(\mathbf{V}_\mu\mathbf{V}_\nu)\mathrm{Tr}(\mathbf{T}\mathbf{V}^\mu)\mathrm{Tr}(\mathbf{T}\mathbf{V}^\nu)$$

$$\mathcal{P}_{11} = (\mathrm{Tr}(\mathbf{V}_\mu\mathbf{V}_\nu))^2 \qquad\qquad \mathcal{P}_{25} = (\mathrm{Tr}(\mathbf{T}\mathbf{V}_\mu))^2\partial_\nu\partial^\nu(h/v)$$

$$\mathcal{P}_{20} = \mathrm{Tr}(\mathbf{V}_\mu\mathbf{V}^\mu)\partial_\nu(h/v)\partial^\nu(h/v) \qquad\qquad \mathcal{P}_{26} = (\mathrm{Tr}(\mathbf{T}\mathbf{V}_\mu)\mathrm{Tr}(\mathbf{T}\mathbf{V}_\nu))^2\,.$$

$$(3.57)$$

To fully encompass the $h$ sector, this list should be extended by a set of four pure-$h$ operators:

**Operators with two derivatives**

$$\mathcal{P}_H = \frac{1}{2}\left(\partial_\mu h\right)^2\,. \tag{3.58}$$

**Operators with four derivatives**

$$\mathcal{P}_{\Box H} = \frac{1}{v^2}\left(\partial_\mu\partial^\mu h\right)^2\,, \qquad\qquad \mathcal{P}_{\Delta H} = \frac{1}{v^3}\left(\partial_\mu h\right)^2\Box h\,,$$

$$\mathcal{P}_{DH} = \frac{1}{v^4}\left((\partial_\mu h)(\partial^\mu h)\right)^2\,. \tag{3.59}$$

In summary, the low-energy electroweak chiral Lagrangian describing the CP-even gauge-Goldstone and the gauge-scalar interactions can thus be written as

$$\mathscr{L}_{\mathrm{low}} = \mathscr{L}_{\mathrm{low}}^{p^2} + \mathscr{L}_{\mathrm{low}}^{p^4}\,, \tag{3.60}$$

<div align="center">

25

</div>

where $\mathscr{L}_{\text{low}}^{p^2}$ and $\mathscr{L}_{\text{low}}^{p^4}$ contain two and and four-derivative operators,

$$
\begin{aligned}
\mathscr{L}_{\text{low}}^{p^2} &= \mathcal{P}_C \mathcal{F}_C(h) + c_T \mathcal{P}_T \mathcal{F}_T(h) + \mathcal{P}_H \mathcal{F}_H(h) \,, \\
\mathscr{L}_{\text{low}}^{p^4} &= \mathcal{P}_B \mathcal{F}_B(h) + \mathcal{P}_W \mathcal{F}_W(h) + \sum_{i=1}^{26} c_i \, \mathcal{P}_i \mathcal{F}_i(h) + \\
&\quad + c_{\Box H} \mathcal{P}_{\Box H} \mathcal{F}_{\Box H}(h) + c_{\Delta H} \mathcal{P}_{\Delta H} \mathcal{F}_{\Delta H}(h) + c_{DH} \mathcal{P}_{DH} \mathcal{F}_{DH}(h) \,,
\end{aligned}
\tag{3.61}
$$

**CP-odd operators with two derivatives**

$$\underline{\text{Custodial preserving}} \qquad\qquad \underline{\text{Custodial breaking}}$$

$$- \qquad\qquad \mathcal{S}_{2D} = i \frac{v^2}{4} \operatorname{Tr}\left(\mathbf{T}\, \mathcal{D}_\mu \mathbf{V}^\mu\right) \tag{3.62}$$

**CP-odd operators with four derivatives**

$$\underline{\text{Custodial preserving}} \qquad\qquad \underline{\text{Custodial breaking}}$$

$$\mathcal{S}_{BB^*} = -\frac{g'^2}{4} B^*_{\mu\nu} B^{\mu\nu} \qquad\qquad \mathcal{S}_4 = g\operatorname{Tr}\left(\mathbf{W}^{\mu\nu}\mathbf{V}_\mu\right)\operatorname{Tr}\left(\mathbf{T}\,\mathbf{V}_\nu\right)$$

$$\mathcal{S}_{WW^*} = -\frac{g^2}{2}\operatorname{Tr}(\mathbf{W}^*_{\mu\nu}\mathbf{W}^{\mu\nu}) \qquad\qquad \mathcal{S}_5 = i\operatorname{Tr}\left(\mathbf{V}^\mu\,\mathbf{V}^\nu\right)\operatorname{Tr}\left(\mathbf{T}\,\mathbf{V}_\mu\right)\partial_\nu(h/v)$$

$$\qquad\qquad\qquad\qquad\qquad \mathcal{S}_6 = i\operatorname{Tr}\left(\mathbf{V}^\mu\,\mathbf{V}_\mu\right)\operatorname{Tr}\left(\mathbf{T}\,\mathbf{V}^\nu\right)\partial_\nu(h/v)$$

$$\mathcal{S}_1 = 2g\,g'\,B^*_{\mu\nu}\operatorname{Tr}\left(\mathbf{T}\mathbf{W}^{\mu\nu}\right) \qquad\qquad \mathcal{S}_7 = g\operatorname{Tr}\left(\mathbf{T}\,[\mathbf{W}^{\mu\nu},\mathbf{V}_\mu]\right)\partial_\nu(h/v)$$

$$\mathcal{S}_2 = 2\,i\,g'\,B^*_{\mu\nu}\operatorname{Tr}\left(\mathbf{T}\,\mathbf{V}^\mu\right)\partial^\nu(h/v) \qquad\qquad \mathcal{S}_8 = 2\,g^2\operatorname{Tr}\left(\mathbf{T}\,\mathbf{W}^*_{\mu\nu}\right)\operatorname{Tr}\left(\mathbf{T}\,\mathbf{W}_{\mu\nu}\right)$$

$$\mathcal{S}_3 = 2\,i\,g\operatorname{Tr}\left(\mathbf{W}^*_{\mu\nu}\,\mathbf{V}_\mu\right)\partial_\nu(h/v) \qquad\qquad \mathcal{S}_9 = 2\,i\,g\operatorname{Tr}\left(\mathbf{W}^*_{\mu\nu}\,\mathbf{T}\right)\operatorname{Tr}\left(\mathbf{T}\,\mathbf{V}_\mu\right)\partial_\nu(h/v)$$

$$\mathcal{S}_{10} = i\operatorname{Tr}\left(\mathbf{V}^\mu\,\mathcal{D}^\nu\mathbf{V}_\nu\right)\operatorname{Tr}\left(\mathbf{T}\,\mathbf{V}_\mu\right)$$

$$\mathcal{S}_{11} = i\operatorname{Tr}\left(\mathbf{T}\,\mathcal{D}^\mu\mathbf{V}_\mu\right)\operatorname{Tr}\left(\mathbf{V}^\nu\,\mathbf{V}_\nu\right)$$

$$\mathcal{S}_{12} = i\operatorname{Tr}\left([\mathbf{V}^\mu,\mathbf{T}]\,\mathcal{D}^\nu\mathbf{V}_\nu\right)\partial_\mu(h/v)$$

$$\mathcal{S}_{13} = i\operatorname{Tr}\left(\mathbf{T}\,\mathcal{D}^\mu\mathbf{V}_\mu\right)\partial^\nu\partial_\nu(h/v)$$

$$\mathcal{S}_{14} = i\operatorname{Tr}\left(\mathbf{T}\,\mathcal{D}^\mu\mathbf{V}_\mu\right)\partial^\nu(h/v)\,\partial_\nu(h/v)$$

$$\mathcal{S}_{15} = i\operatorname{Tr}\left(\mathbf{T}\,\mathbf{V}^\mu\right)\left(\operatorname{Tr}\left(\mathbf{T}\,\mathbf{V}^\nu\right)\right)^2\partial_\mu(h/v)$$

$$\mathcal{S}_{16} = i\operatorname{Tr}\left(\mathbf{T}\,\mathcal{D}^\mu\mathbf{V}_\mu\right)\left(\operatorname{Tr}\left(\mathbf{T}\,\mathbf{V}^\nu\right)\right)^2$$

$$\tag{3.63}$$

The low-energy electroweak chiral Lagrangian describing the CP-odd gauge, gauge-Goldstone and the gauge-Higgs interactions can instead be written as:

$$\mathscr{L}_{\text{low},\cancel{CP}} = \mathscr{L}_{\text{low},\cancel{CP}}^{p^2} + \mathscr{L}_{\text{low},\cancel{CP}}^{p^4} \,, \tag{3.64}$$

where $\mathscr{L}^{p^2}_{\text{low},\cancel{CP}}$ and $\mathscr{L}^{p^4}_{\text{low},\cancel{CP}}$ contain two and four-derivative operators, respectively,

$$\mathscr{L}^{p^2}_{\text{low},\cancel{CP}} = d_{2D}\mathcal{S}_{2D}\mathcal{E}_{2D}(h)\,,$$

$$\mathscr{L}^{p^4}_{\text{low},\cancel{CP}} = \mathcal{S}_{BB^*}\mathcal{E}_{BB^*}(h) + \mathcal{S}_{WW^*}\mathcal{E}_{WW^*}(h) + \sum_{i=1}^{16} d_i\,\mathcal{S}_i\mathcal{E}_i(h)\,,$$

(3.65)

with $\mathcal{E}(h)$ being generic Higgs functions as defined in 3.55

### 3.3.1 Matching the linear and non-linear lagrangians

In order to connect the linear and chiral lagrangian one uses the relation 2.7, which connects the doublet $\Phi$ with the matrix $\mathbf{U}$ and the higgs Scalar $h(x)$. For example, the kinetic term of the SM Higgs doublet results in

$$(\mathbf{D}^\mu\Phi)\,(\mathbf{D}_\mu\Phi)^\dagger = \frac{1}{2}\,(\partial_\mu h)^2 - \frac{v^2}{4}\left(1+\frac{h}{v}\right)^2\text{Tr}\left(\mathbf{V}_\mu\mathbf{V}^\mu\right) = \mathcal{P}_H + \left(1+\frac{h}{v}\right)^2\mathcal{P}_C \qquad (3.66)$$

Thus, $\mathscr{L}^{p^2}_{\text{low}}$ in 3.65 can reproduce the SM prediction with the particular choices of $\mathcal{F}_C(h) = 1$, $c_T = 0$ and $\mathcal{F}_H(h) = (1+h/v)^2$; using the parametrization from 3.55, this in particular means $a_C = b_C = 0$ and $a_H = b_H = 1$, disregarding higher order terms in $h/v$. This dependence in the structure $(1+h/v)^2$ is characteristic of the SM lagrangian and it's a signature of Higgs the belonging in an $SU(2)_L$ doublet. Therefore, BSM extensions in which the EW symmetry is linearly realized will be charaterized by functions $\mathcal{F}_i(h)$ of powers of $(1+h/v)$.

# Chapter 4

# The effective lagrangian for a $\mathcal{G}/\mathcal{H}$ coset

## 4.1 Effective chiral Lagrangian for symmetric cosets

The goal of this chapter is to present a method for writing the high-energy effective Lagrangian for any given Composite Higgs model. As explained in section 2.3, a CH setup can be characterized by a global group $\mathcal{G}$ which is broken down to a subgroup $\mathcal{H}$ by some unspecified strong dynamics at a scale $\Lambda_s$. In addition to the requirements listed in subsection 2.3.1, the group $\mathcal{G}$ will be assumed to be symmetric, which essentially means that there is a parity-like transformation under which the unbroken generators remain invariant while the broken generators change sign. All realistic models, not only of Composite Higgs but also in QCD, show such a transformation.

This work will deal with up to four derivative purely bosonic operators, both even and odd under CP transformations, following respectively [34] and [2]. An application of this construction to the Georgi-Kaplan $SU(5)/SO(5)$ will be shown in Chapter 5, whereas the application to the Minimal Composite Higgs Model $SO(5)/SO(4)$ and the explicitly custodial breaking $SU(3)/SU(2) \times U(1)$ model will be studied in Chapter 6.

### 4.1.1 Non-linear realisations of the $\mathcal{G}/\mathcal{H}$ symmetry breaking

The Goldstone theorem shows that a global symmetry breaking $\mathcal{G} \to \mathcal{H}$ leads to the existence of $n = \dim(\mathcal{G}/\mathcal{H})$ massless bosons, known as Goldstone bosons. Denoting by $T_a$ ($a = 1 \ldots, dim(\mathcal{H})$) the unbroken generators (i.e., the generators of $\mathcal{H}$), and $X_{\hat{a}}$ ($\hat{a} = 1 \ldots, dim(\mathcal{G}/\mathcal{H})$) the broken

generators (i.e., generators of the coset $\mathcal{G}/\mathcal{H}$), the Goldstone bosons are described by the field

$$\Xi(x) = \Xi^{\hat{a}}(x)\, X_{\hat{a}}\,. \tag{4.1}$$

The so-called Callan-Coleman-Weiss-Zumino (CCWZ) construction proves (as it's shown in Appendix A) that the Goldstone bosons associated to the global symmetry breaking $\mathcal{G} \to \mathcal{H}$, can be parametrized by the matrix field

$$\Omega(x) \equiv e^{i\,\Xi(x)/2f}\,, \tag{4.2}$$

which, as is shown in said Appendix, transforms under global $\mathfrak{g} \in \mathcal{G}$ transformations as

$$\Omega(x) \to \mathfrak{g}\,\Omega(x)\,\mathfrak{h}^{-1}(\Xi, \mathfrak{g})\,, \tag{4.3}$$

where $\mathfrak{h}(\Xi, \mathfrak{g}) \in \mathcal{H}$.

Formally, a coset $\mathcal{G}/\mathcal{H}$ is said symmetric if it admits an automorphism $\mathcal{R}$ so that

$$\mathfrak{g} \in \mathcal{G} \to \mathcal{R}(\mathfrak{g}) \equiv \mathfrak{g}_{\mathcal{R}}\,,$$

$$\mathcal{R}: \begin{cases} T_a \to +T_a \\ X_{\hat{a}} \to -X_{\hat{a}} \end{cases} \tag{4.4}$$

The fact that $\mathcal{H}$ is closed, that the structure constants of any compact group are completely antisymmetric and that the coset $\mathcal{G}/\mathcal{H}$ is symmetric imply that the generators satisfy the following conditions:

$$[T,\, T] \propto T\,, \qquad [T,\, X] \propto X\,, \qquad [X,\, X] \propto T\,, \tag{4.5}$$

As already pointed out in Ref. [35], it can be shown that in the presence of such an automorphism the non-linear field transformations of $\Omega(x)$ can also be recast as:

$$\Omega(x) \to \mathfrak{h}\,\Omega(x)\,\mathfrak{g}_{\mathcal{R}}^{-1}\,. \tag{4.6}$$

From Eqs. (4.3) and (4.6), it is thus possible to define for all symmetric cosets a "squared" non-linear field $\boldsymbol{\Sigma}(x)$:

$$\boldsymbol{\Sigma}(x) \equiv \Omega(x)^2\,, \tag{4.7}$$

transforming under $\mathcal{G}$ as,

$$\boldsymbol{\Sigma}(x) \to \mathfrak{g}\,\boldsymbol{\Sigma}(x)\,\mathfrak{g}_{\mathcal{R}}^{-1}\,, \tag{4.8}$$

showing explicitly that the transformation on $\Xi(x)$ is a realisation of $\mathcal{G}$, and that it is linear when restricted to $\mathcal{H}$. Notice that the GB field matrix $\boldsymbol{\Sigma}(x)$ transforms under the grading $\mathcal{R}$ as:

$$\boldsymbol{\Sigma}(x) \to \boldsymbol{\Sigma}(x)^{-1} \,. \tag{4.9}$$

It is then a matter of taste, in a symmetric coset framework, to use $\Omega(x)$ or $\boldsymbol{\Sigma}(x)$ for describing the GBs degrees of freedom and the interactions between the GB fields and the gauge/matter fields. The $\Omega$-representation to derive $\mathcal{H}$-covariant quantities entering the model Lagrangian has been used in several examples. However, when discussing QCD or EW chiral Lagrangians, the $\Sigma$-representation has been more often adopted. To make a straightforward comparison with $\mathscr{L}_{\text{low}}$ introduced in Chapter 3, the $\Sigma$-representation will be kept in the following.

One can introduce the vector chiral field[1]:

$$\widetilde{\mathbf{V}}_\mu = (\partial_\mu \boldsymbol{\Sigma}) \, \boldsymbol{\Sigma}^{-1} \,, \qquad \widetilde{\mathbf{V}}_\mu \to \mathfrak{g} \, \widetilde{\mathbf{V}}_\mu \, \mathfrak{g}^{-1} \,, \tag{4.10}$$

transforming in the adjoint of $\mathcal{G}$.

In a realistic context, however, gauge interactions should be introduced, and to assign quantum numbers it is convenient to formally gauge the full group $\mathcal{G}$. In the symmetric coset case, it is possible to define both the $\mathcal{G}$ gauge fields $\widetilde{\mathbf{S}}_\mu$, and the graded siblings $\widetilde{\mathbf{S}}_\mu^{\mathcal{R}} \equiv \mathcal{R}(\widetilde{\mathbf{S}}_\mu)$, transforming under $\mathcal{G}$, respectively, as:

$$\widetilde{\mathbf{S}}_\mu \to \mathfrak{g} \, \widetilde{\mathbf{S}}_\mu \, \mathfrak{g}^{-1} - \frac{i}{g_S} \mathfrak{g}(\partial_\mu \, \mathfrak{g}^{-1}) \,, \qquad \widetilde{\mathbf{S}}_\mu^{\mathcal{R}} \to \mathfrak{g}_{\mathcal{R}} \, \widetilde{\mathbf{S}}_\mu^{\mathcal{R}} \, \mathfrak{g}_{\mathcal{R}}^{-1} - \frac{i}{g_S} \mathfrak{g}_{\mathcal{R}}(\partial_\mu \, \mathfrak{g}_{\mathcal{R}}^{-1}) \,, \tag{4.11}$$

with $g_S$ denoting the associated gauge coupling constant. The (gauged) version of the chiral vector field $\widetilde{\mathbf{V}}_\mu$ can then be defined as:

$$\widetilde{\mathbf{V}}_\mu = (\mathbf{D}_\mu \boldsymbol{\Sigma}) \, \boldsymbol{\Sigma}^{-1} \,, \tag{4.12}$$

with the covariant derivative of the non-linear field $\boldsymbol{\Sigma}(x)$ being,

$$\mathbf{D}_\mu \boldsymbol{\Sigma} = \partial_\mu \boldsymbol{\Sigma} + i \, g_S (\widetilde{\mathbf{S}}_\mu \boldsymbol{\Sigma} - \boldsymbol{\Sigma} \, \widetilde{\mathbf{S}}_\mu^{\mathcal{R}}) \,. \tag{4.13}$$

The following three $\mathcal{G}$-covariant objects can thus be used as building blocks for the (gauged) effective chiral Lagrangian:

$$\widetilde{\mathbf{V}}_\mu \,, \qquad \widetilde{\mathbf{S}}_{\mu\nu} \quad \text{and} \quad \boldsymbol{\Sigma} \, \widetilde{\mathbf{S}}_{\mu\nu}^{\mathcal{R}} \, \boldsymbol{\Sigma}^{-1} \,. \tag{4.14}$$

---

[1]In order to avoid confusion we will denote with "$\sim$" gauge bosons and chiral fields embedded in $\mathcal{G}$.

The introduction of the graded vector chiral field $\widetilde{\mathbf{V}}_\mu^{\mathcal{R}}$ does not add any further independent structure, as indeed

$$\widetilde{\mathbf{V}}_\mu^{\mathcal{R}} \equiv \mathcal{R}(\widetilde{\mathbf{V}}_\mu) = (\mathbf{D}_\mu \mathbf{\Sigma})^{-1} \mathbf{\Sigma} \qquad \text{with} \qquad \mathbf{\Sigma} \, \widetilde{\mathbf{V}}_\mu^{\mathcal{R}} \, \mathbf{\Sigma}^{-1} = -\widetilde{\mathbf{V}}_\mu \,. \tag{4.15}$$

Under the hypothesis of absence of any custodial symmetry breaking source besides the SM ones, any operator containing the high-energy sibling of the scalar chiral field $\mathbf{T}(x)$,

$$\widetilde{\mathbf{T}} \equiv \mathbf{\Sigma} \, Q_Y \mathbf{\Sigma}^{-1} \tag{4.16}$$

with $Q_Y$ being the embedding in $\mathcal{G}$ of the hypercharge generator, should not enter in the basis, except in one specific case, discussed later on, where the presence of two $\widetilde{\mathbf{T}}$ gives rise to a custodial preserving operator.

### 4.1.2 Basis of independent operators

It is now possible to derive the most general operator basis describing the interactions of the $\mathcal{G}$ gauge fields and of the GBs of a non-linear realisation of the symmetric coset $\mathcal{G}/\mathcal{H}$. We assume that the only sources of custodial symmetry breaking are those present in the SM. Performing an expansion in momenta and considering CP even operators with at most four derivatives, one obtains the following nine independent operators:

**2-momenta CP-even operator**

$$\mathrm{Tr}\left(\widetilde{\mathbf{V}}_\mu \widetilde{\mathbf{V}}^\mu\right) . \tag{4.17}$$

This operator describes the kinetic terms for the GBs and, once the gauge symmetry is broken, results in masses for those GBs associated to the broken generators.

**4-momenta CP-even operators with explicit gauge field strength $\widetilde{\mathbf{S}}_{\mu\nu}$**

$$\mathrm{Tr}\left(\widetilde{\mathbf{S}}_{\mu\nu}\widetilde{\mathbf{S}}^{\mu\nu}\right) , \qquad \mathrm{Tr}\left(\mathbf{\Sigma}\, \widetilde{\mathbf{S}}_{\mu\nu}^{R}\, \mathbf{\Sigma}^{-1}\, \widetilde{\mathbf{S}}^{\mu\nu}\right) , \qquad \mathrm{Tr}\left(\widetilde{\mathbf{S}}_{\mu\nu}\left[\widetilde{\mathbf{V}}^\mu, \widetilde{\mathbf{V}}^\nu\right]\right) . \tag{4.18}$$

The first operator describes the kinetic terms for the gauge bosons $\widetilde{\mathbf{S}}_\mu$. The other two contain gauge-GB and pure-gauge interactions.

**4-momenta CP-even operators without explicit gauge field strength $\widetilde{\mathbf{S}}_{\mu\nu}$**

$$\begin{aligned}
\mathrm{Tr}\left(\widetilde{\mathbf{V}}_\mu \widetilde{\mathbf{V}}^\mu\right) \mathrm{Tr}\left(\widetilde{\mathbf{V}}_\nu \widetilde{\mathbf{V}}^\nu\right) , \qquad & \mathrm{Tr}\left(\widetilde{\mathbf{V}}_\mu \widetilde{\mathbf{V}}_\nu\right) \mathrm{Tr}\left(\widetilde{\mathbf{V}}^\mu \widetilde{\mathbf{V}}^\nu\right) , \\
\mathrm{Tr}\left((\mathcal{D}_\mu \widetilde{\mathbf{V}}^\mu)^2\right) , \qquad \mathrm{Tr}\left(\widetilde{\mathbf{V}}_\mu \widetilde{\mathbf{V}}^\mu \widetilde{\mathbf{V}}_\nu \widetilde{\mathbf{V}}^\nu\right) , \qquad & \mathrm{Tr}\left(\widetilde{\mathbf{V}}_\mu \widetilde{\mathbf{V}}_\nu \widetilde{\mathbf{V}}^\mu \widetilde{\mathbf{V}}^\nu\right) ,
\end{aligned} \tag{4.19}$$

where the adjoint covariant derivative acting on $\widetilde{\mathbf{V}}^\mu$ is defined as

$$\mathcal{D}_\mu \widetilde{\mathbf{V}}^\mu = \partial_\mu \widetilde{\mathbf{V}}^\mu + i\, g_S \left[ \widetilde{\mathbf{S}}_\mu, \widetilde{\mathbf{V}}^\mu \right] .$$

Of course there are a priori many other structures which can be built, but an independent set such as the one listed in Eqs. (4.17)–(4.22) can be found, in a similar fashion as in section 3.2, by integration by parts as well as using the identities

$$[\mathcal{D}_\mu, \mathcal{D}_\nu]\, \mathcal{O}(x) = i g_S \left[ \widetilde{\mathbf{S}}_{\mu\nu}, \mathcal{O}(x) \right] ,$$

$$\widetilde{\mathbf{V}}_{\mu\nu} \equiv \mathcal{D}_\mu \widetilde{\mathbf{V}}_\nu - \mathcal{D}_\nu \widetilde{\mathbf{V}}_\mu = i g_S \widetilde{\mathbf{S}}_{\mu\nu} - i g_S \mathbf{\Sigma} \widetilde{\mathbf{S}}_{\mu\nu}^{\mathcal{R}} \mathbf{\Sigma}^{-1} + [\widetilde{\mathbf{V}}_\mu, \widetilde{\mathbf{V}}_\nu] \qquad (4.20)$$

As in the case of the ALF basis, operators involving determinants are not considered since, using to the Cayley-Hamilton theorem, they can be re-expressed purely in terms of operators built from traces of fields. A particular case is the operator $\mathrm{Tr}((\mathcal{D}_\mu \widetilde{\mathbf{V}}^\mu)\widetilde{\mathbf{V}}_\nu \widetilde{\mathbf{V}}^\nu)$, which is not included in the set of independent operators since it is not invariant under the grading automorphism.

It is worth noticing that in specific $\mathcal{G}/\mathcal{H}$ realisations, some of the operators listed may not be independent. For example the operators with traces of four $\widetilde{\mathbf{V}}^\mu$ appearing in the second line of Eq. (4.22) are redundant in the case $\mathcal{G} = SU(2)_L \times SU(2)_R$ and $\mathcal{H} = SU(2)_V$, as they decompose in products of traces of two $\widetilde{\mathbf{V}}^\mu$. It may not be true in models with larger group $\mathcal{G}$, as it depends on the specific algebra relations of the generators.

Finally, some caution should be also used when fermions are introduced. In this case all operators containing $\mathcal{D}_\mu \widetilde{\mathbf{V}}^\mu$ can be traded, via equations of motion, by operators containing fermions and a careful analysis should be performed to avoid the presence of redundant terms.

The CP-odd independent structures can be found following the same procedure:

**4-momenta CP-odd operators with gauge field strength $\widetilde{\mathbf{S}}_{\mu\nu}$**

$$\mathrm{Tr}\left( \widetilde{\mathbf{S}}_{\mu\nu}^* \widetilde{\mathbf{S}}^{\mu\nu} \right) , \qquad\qquad \mathrm{Tr}\left( \widetilde{\mathbf{S}}_{\mu\nu}^* \, \mathbf{\Sigma}\, \widetilde{\mathbf{S}}^{\mu\nu, R}\, \mathbf{\Sigma}^{-1} \right) , \qquad (4.21)$$

where $\widetilde{\mathbf{S}}_{\mu\nu}^* \equiv \epsilon_{\mu\nu\rho\sigma} \widetilde{\mathbf{S}}^{\rho\sigma}$. The first operator resembles the usual $\theta$ term operator for QCD. The other contains gauge-GB and pure-gauge interactions.

**4-momenta CP-odd operators without gauge field strength $\widetilde{\mathbf{S}}_{\mu\nu}$**

$$\epsilon_{\mu\nu\rho\sigma} \mathrm{Tr}\left( \widetilde{\mathbf{T}} \left[ \widetilde{\mathbf{V}}^\mu, \widetilde{\mathbf{V}}^\nu \right] \right) \mathrm{Tr}\left( \widetilde{\mathbf{T}} \left[ \widetilde{\mathbf{V}}^\rho, \widetilde{\mathbf{V}}^\sigma \right] \right) , \qquad \epsilon_{\mu\nu\rho\sigma} \mathrm{Tr}\left( \widetilde{\mathbf{V}}^\mu \widetilde{\mathbf{V}}^\nu \widetilde{\mathbf{V}}^\rho \widetilde{\mathbf{V}}^\sigma \right) , \qquad (4.22)$$

Other operators with traces of two $\widetilde{\mathbf{V}}$'s are clearly vanishing due to antisymmetry.

The operators listed in Eqs. (4.21)–(4.22) represent the set of independent CP–odd structures that can be introduced in a generic symmetric coset. The operator appearing on the left side of Eq. (4.22) seems a priori custodial symmetry breaking, since it involves the $\widetilde{\mathbf{T}}$ operator. However, as will be seen in the following chapters, its low-energy projection is custodially preserving due to the $SU(2)_L$ algebra. This operator gives a low-energy contribution to the operator $\epsilon_{\mu\nu\rho\sigma}\mathrm{Tr}\left(\mathbf{T}\mathbf{V}^\mu\mathbf{V}^\nu\right)\mathrm{Tr}\left(\mathbf{T}\mathbf{V}^\rho\right)\partial^\sigma h$, which can be expressed as a clearly custodial preserving one:

$$\epsilon_{\mu\nu\rho\sigma}\mathrm{Tr}\left(\mathbf{T}\mathbf{V}^\mu\mathbf{V}^\nu\right)\mathrm{Tr}\left(\mathbf{T}\mathbf{V}^\rho\right)\partial^\sigma h = \frac{2}{3}\mathrm{Tr}\left(\mathbf{V}^\mu\mathbf{V}^\nu\mathbf{V}^\rho\right)\partial^\sigma h \tag{4.23}$$

As can be seen, the appearance of $\mathbf{T}(x)$ precisely in that combination implies that the operator is custodial breaking. It will be shown that indeed the inclusion of the high-energy operator involving two $\widetilde{\mathbf{T}}$ is necessary in order to produce the whole basis of low-energy operators.

Other operators could be easily introduced, apparently giving rise to new independent structures, most notably:

$$\mathrm{Tr}\left(\widetilde{\mathbf{S}}^*_{\mu\nu}\left[\widetilde{\mathbf{V}}^\mu,\widetilde{\mathbf{V}}^\nu\right]\right), \quad \epsilon_{\mu\nu\rho\sigma}\mathrm{Tr}\left((\mathcal{D}^\mu\widetilde{\mathbf{V}}^\nu)(\mathcal{D}^\rho\widetilde{\mathbf{V}}^\sigma)\right), \quad \epsilon_{\mu\nu\rho\sigma}\mathrm{Tr}\left((\mathcal{D}^\mu\widetilde{\mathbf{V}}^\nu)\widetilde{\mathbf{V}}^\rho\widetilde{\mathbf{V}}^\sigma\right),$$

However, making use of the Bianchi identity for the gauge field $\widetilde{\mathbf{S}}_{\mu\nu}$,

$$\epsilon^{\mu\nu\rho}\mathcal{D}_\mu\widetilde{\mathbf{S}}_{\nu\rho} = 0 \tag{4.24}$$

as well of integration by parts and the identities from 4.20, it is possible to show that all these operators do not introduce any independent structure besides the one listed in Eqs. (4.21)–(4.22). It is also worth noticing that in specific $\mathcal{G}/\mathcal{H}$ realizations, some of the operators listed may not be independent. For example the operator with traces of four $\widetilde{\mathbf{V}}^\mu$ appearing on the right hand side of Eq. (4.22) is redundant in all the considered CH models even if it was not the case for its CP-even counterpart.

### 4.1.3 General EW effective Lagrangian for a symmetric $\mathcal{G}/\mathcal{H}$ coset

The list of operators in Eqs. (4.17)–(4.22) is valid on general grounds when formally gauging the full group $\mathcal{G}$. Nevertheless, in most realisations of CH models only the SM gauge group is gauged. Consequently, in the generic gauge field $\widetilde{\mathbf{S}}_\mu$, only the EW components should be retained. While no new operator structures appear in the sector made out exclusively of $\widetilde{\mathbf{V}}_\mu$ fields (see Eqs. (4.17) and (4.22)), all operators where the gauge field strength appears explicitly, such as those in

Eq. (4.18), should be "doubled" by substituting $\widetilde{\mathbf{S}}_\mu$ either with $\widetilde{\mathbf{W}}_\mu$ or $\widetilde{\mathbf{B}}_\mu$, defined by

$$\widetilde{\mathbf{W}}_\mu \equiv W_\mu^a \, Q_L^a \qquad \text{and} \qquad \widetilde{\mathbf{B}}_\mu \equiv B_\mu \, Q_Y \,, \tag{4.25}$$

where $Q_L^a$ and $Q_Y$ denote the embedding in $\mathcal{G}$ of the $SU(2)_L \times U(1)_Y$ generators. It follows that a larger number of invariants can be written in this case. The EW high-energy chiral Lagrangian describing up to four-derivative bosonic interactions is given by

$$\mathscr{L}_{\text{high}} \equiv \mathscr{L}_{CP} + \mathscr{L}_{\cancel{CP}} \tag{4.26}$$

where $\mathscr{L}_{CP}$ and $\mathscr{L}_{\cancel{CP}}$ are the CP-even and CP-odd high-energy Lagrangians respectively. The former contains in total thirteen operators:

$$\mathscr{L}_{CP} = \mathscr{L}_{CP}^{p^2} + \mathscr{L}_{CP}^{p^4} \,, \tag{4.27}$$

where

$$\mathscr{L}_{CP}^{p^2} = \widetilde{\mathcal{A}}_C \,, \tag{4.28}$$

$$\mathscr{L}_{CP}^{p^4} = \widetilde{\mathcal{A}}_B + \widetilde{\mathcal{A}}_W + \tilde{c}_{B\Sigma}\widetilde{\mathcal{A}}_{B\Sigma} + \tilde{c}_{W\Sigma}\widetilde{\mathcal{A}}_{W\Sigma} + \sum_{i=1}^{8} \tilde{c}_i \, \widetilde{\mathcal{A}}_i \,, \tag{4.29}$$

with

$$
\begin{aligned}
\widetilde{\mathcal{A}}_C &= -\frac{f^2}{4}\mathrm{Tr}\left(\widetilde{\mathbf{V}}_\mu \widetilde{\mathbf{V}}^\mu\right) , & \widetilde{\mathcal{A}}_3 &= i\,g\,\mathrm{Tr}\left(\widetilde{\mathbf{W}}_{\mu\nu}\left[\widetilde{\mathbf{V}}^\mu, \widetilde{\mathbf{V}}^\nu\right]\right) , \\
\widetilde{\mathcal{A}}_B &= -\frac{1}{4}\mathrm{Tr}\left(\widetilde{\mathbf{B}}_{\mu\nu}\widetilde{\mathbf{B}}^{\mu\nu}\right) , & \widetilde{\mathcal{A}}_4 &= \mathrm{Tr}\left(\widetilde{\mathbf{V}}_\mu \, \widetilde{\mathbf{V}}^\mu\right)\mathrm{Tr}\left(\widetilde{\mathbf{V}}_\mu \, \widetilde{\mathbf{V}}^\mu\right) , \\
\widetilde{\mathcal{A}}_W &= -\frac{1}{4}\mathrm{Tr}\left(\widetilde{\mathbf{W}}_{\mu\nu}\widetilde{\mathbf{W}}^{\mu\nu}\right) , & \widetilde{\mathcal{A}}_5 &= \mathrm{Tr}\left(\widetilde{\mathbf{V}}_\mu \, \widetilde{\mathbf{V}}_\nu\right)\mathrm{Tr}\left(\widetilde{\mathbf{V}}^\mu \, \widetilde{\mathbf{V}}^\nu\right) , \\
\widetilde{\mathcal{A}}_{B\Sigma} &= g'^2\mathrm{Tr}\left(\mathbf{\Sigma}\widetilde{\mathbf{B}}_{\mu\nu}\mathbf{\Sigma}^{-1}\widetilde{\mathbf{B}}^{\mu\nu}\right) , & \widetilde{\mathcal{A}}_6 &= \mathrm{Tr}\left((\mathcal{D}_\mu \widetilde{\mathbf{V}}^\mu)^2\right) , \\
\widetilde{\mathcal{A}}_{W\Sigma} &= g^2\mathrm{Tr}\left(\mathbf{\Sigma}\widetilde{\mathbf{W}}_{\mu\nu}\mathbf{\Sigma}^{-1}\widetilde{\mathbf{W}}^{\mu\nu}\right) , & \widetilde{\mathcal{A}}_7 &= \mathrm{Tr}\left(\widetilde{\mathbf{V}}_\mu \, \widetilde{\mathbf{V}}^\mu\widetilde{\mathbf{V}}_\nu \, \widetilde{\mathbf{V}}^\nu\right) , \\
\widetilde{\mathcal{A}}_1 &= g\,g'\,\mathrm{Tr}\left(\mathbf{\Sigma}\widetilde{\mathbf{B}}_{\mu\nu}\mathbf{\Sigma}^{-1}\widetilde{\mathbf{W}}^{\mu\nu}\right) , & \widetilde{\mathcal{A}}_8 &= \mathrm{Tr}\left(\widetilde{\mathbf{V}}_\mu \, \widetilde{\mathbf{V}}_\nu\widetilde{\mathbf{V}}^\mu \, \widetilde{\mathbf{V}}^\nu\right) , \\
\widetilde{\mathcal{A}}_2 &= i\,g'\,\mathrm{Tr}\left(\widetilde{\mathbf{B}}_{\mu\nu}\left[\widetilde{\mathbf{V}}^\mu, \widetilde{\mathbf{V}}^\nu\right]\right) , &
\end{aligned}
\tag{4.30}
$$

On the other hand, the CP-odd EW high-energy chiral Lagrangian describing bosonic interactions, up to four derivatives, contains in total six operators:

$$\mathscr{L}_{\cancel{CP}} = \tilde{d}_{WW^*}\widetilde{\mathcal{B}}_{WW^*} + \tilde{d}_{B\Sigma^*}\widetilde{\mathcal{B}}_{B\Sigma^*} + \tilde{d}_{W\Sigma^*}\widetilde{\mathcal{B}}_{W\Sigma^*} + \tilde{d}_1 \, \widetilde{\mathcal{B}}_1 + \tilde{d}_2 \, \widetilde{\mathcal{B}}_2 + \tilde{d}_3 \, \widetilde{\mathcal{B}}_3 \,, \tag{4.31}$$

with

$$
\begin{aligned}
\widetilde{\mathcal{B}}_{WW^*} &= -\frac{g^2}{4}\mathrm{Tr}\left(\widetilde{\mathbf{W}}^*_{\mu\nu}\widetilde{\mathbf{W}}^{\mu\nu}\right) & \widetilde{\mathcal{B}}_1 &= g\,g'\,\mathrm{Tr}\left(\widetilde{\mathbf{W}}^*_{\mu\nu}\boldsymbol{\Sigma}\widetilde{\mathbf{B}}^{\mu\nu}\boldsymbol{\Sigma}^{-1}\right) \\
\widetilde{\mathcal{B}}_{B\Sigma^*} &= g'^2\mathrm{Tr}\left(\widetilde{\mathbf{B}}^*_{\mu\nu}\boldsymbol{\Sigma}\widetilde{\mathbf{B}}^{\mu\nu}\boldsymbol{\Sigma}^{-1}\right) & \widetilde{\mathcal{B}}_2 &= \epsilon_{\mu\nu\rho\sigma}\mathrm{Tr}\left(\widetilde{\mathbf{T}}\left[\widetilde{\mathbf{V}}^\mu,\widetilde{\mathbf{V}}^\nu\right]\right)\mathrm{Tr}\left(\widetilde{\mathbf{T}}\left[\widetilde{\mathbf{V}}^\rho,\widetilde{\mathbf{V}}^\sigma\right]\right) \\
\widetilde{\mathcal{B}}_{W\Sigma^*} &= g^2\mathrm{Tr}\left(\widetilde{\mathbf{W}}^*_{\mu\nu}\boldsymbol{\Sigma}\widetilde{\mathbf{W}}^{\mu\nu}\boldsymbol{\Sigma}^{-1}\right) & \widetilde{\mathcal{B}}_3 &= \epsilon_{\mu\nu\rho\sigma}\mathrm{Tr}\left(\widetilde{\mathbf{V}}^\mu\widetilde{\mathbf{V}}^\nu\widetilde{\mathbf{V}}^\rho\widetilde{\mathbf{V}}^\sigma\right).
\end{aligned}
\tag{4.32}
$$

with the EW covariant derivative in Eq. (4.32) defined as

$$
\mathcal{D}_\mu\widetilde{\mathbf{V}}^\mu = \partial_\mu\widetilde{\mathbf{V}}^\mu + i\,g\left[\widetilde{\mathbf{W}}_\mu,\widetilde{\mathbf{V}}^\mu\right] + i\,g'\left[\widetilde{\mathbf{B}}_\mu,\widetilde{\mathbf{V}}^\mu\right].
\tag{4.33}
$$

The coefficients $\tilde{c}_i$ and $\tilde{d}_i$ are expected to be all of the same order of magnitude, according to the effective field theory approach[2]. NDA [19, 36] applies and indicates that the four-derivative operator coefficients are expected to be of order $f^2/\Lambda_s^2 \gtrsim 1/(4\pi)^2$.

Notice that the first operator listed in Eq. (4.32), $\widetilde{\mathcal{B}}_{WW^*}$, is a topological structure, analogous to the QCD $\theta$–term. Even if this coupling is usually not considered in the SM context, it is included here in the basis for sake of completeness as it could give rise to non-vanishing effects when considering the incorporation of fermions. On the other side, a similar term for the $U(1)_Y$ is identically vanishing.

It is remarkable that, aside from kinetic terms and $\theta$-terms, $\mathscr{L}_{\mathrm{high}}$ contains only fifteen independent operators (ten CP-preserving and five CP-violating), and thus at most ten arbitrary coefficients $\tilde{c}_i$ and five $\tilde{d}_i$ to be determined. They will govern the projection of $\mathscr{L}_{\mathrm{high}}$ into $\mathscr{L}_{\mathrm{low}}$ (in addition to the parameter(s) of the explicit breaking of the global symmetry). It is also worth to note that the gauging of the SM symmetry breaks explicitly the custodial and the grading symmetries. As a result, custodial and/or grading symmetry breaking operators can arise once quantum corrections induced by SM interactions are considered. But this is beyond the scope of this thesis.

In the case $\mathcal{G} = SU(2)_L\times SU(2)_R$ and $\mathcal{H} = SU(2)_V$, the Lagrangian $\mathscr{L}_{\mathrm{high}}$ reduces to the custodial preserving sector of the ALF basis, with the three GBs described by the non-linear realisation of the EWSB mechanism corresponding to the longitudinal degrees of freedom of the SM gauge bosons. In this case, $dim(\mathcal{G}/\mathcal{H}) = 3$ and the $h$ field cannot arise as a GB of the spontaneous $\mathcal{G}$ symmetry breaking. CH models are, instead, built upon cosets with $dim(\mathcal{G}/\mathcal{H}) \geq 4$, the minimal ones being for example $SO(5)/SO(4)$ and $SU(3)/(SU(2) \times U(1))$ for the intrinsically custodial preserving and custodial breaking setups, respectively. The four GBs resulting from the non-linear symmetry breaking mechanism will then correspond to the three would-be SM GBs and

---

[2]The coefficients of the operators $\widetilde{\mathcal{A}}_C$, $\widetilde{\mathcal{A}}_B$ and $\widetilde{\mathcal{A}}_W$ are taken equal to 1, which leads to canonical kinetic terms.

the Higgs particle. In non-minimal models, such as the $SU(5)/SO(5)$ Georgi-Kaplan model, additional GBs appear in the symmetry breaking sector. Either they are light degrees of freedom and then provide interesting candidates for dark matter (see for instance Ref. [20, 37]) or for other exotic particles, or they should become heavy enough through some "ad hoc" global symmetry breaking effect associated to the strong interacting sector [15], leaving a negligible impact on low-energy physics.

# Chapter 5

# The low-energy projection and potential for SU(5)/SO(5)

In this Chapter we will study the Georgi-Kaplan $SU(5)/SO(5)$ model, which was the first Composite Higgs model proposal, analyzed in detail in Ref. [16]. The first part will be devoted to applying the Sigma Decomposition procedure to this particular case, writing the high-energy effective bosonic lagrangian and matching it to the low-energy chiral lagrangian of the HEFT, assuming that the only light Goldstone boson associated to the breaking is the Higgs scalar.

As opposed to the MCHM scenario, which uses the $SO(5)/SO(4)$ coset, the Goldstone spectrum of the Georgi-Kaplan is non-minimal in the sense that, in addition to the 3 would-be SM GBs and the Higgs scalar, there are 10 additional potentially light Goldstones. Therefore, the second part of the chapter will be dedicated to showing that there is a region of the parameter space in which the Higgs scalar is indeed the only light Goldstone boson, proving right the assumption previously done.

### 5.0.1 Spontaneous $SU(5)/SO(5)$ symmetry breaking setup

The spontaneous global breaking of $SU(5) \to SO(5)$ can be seen as the result of a scalar field belonging in the symmetric representation acquiring a vev $\Delta_0$, for example because of a strong dynamics causing some fermions to form a condensate, as it's the case in QCD and the quark-antiquark condensate. Following Ref. [16], this vacuum is in all generality assumed to be a real, symmetric and orthogonal $5 \times 5$ matrix, so that

$$\Delta_0 = \Delta_0^\dagger = \Delta_0^T = \Delta_0^{-1} \tag{5.1}$$

A convenient choice, which facilitates the identification of the $SU(2)_L \times U(1)_Y$ quantum numbers in the $SU(5)$ embedding, is given by

$$\Delta_0 = \begin{pmatrix} 0 & i\sigma_2 & 0 \\ -i\sigma_2 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} . \tag{5.2}$$

It is then possible to describe the massless excitations around the vacuum with a symmetric field $\Delta(x)$, obtained "rotating" the vacuum by means of the GB non-linear field $\Omega(x)$:

$$\Delta(x) = \Omega(x)\,\Delta_0\,\Omega(x)^T , \qquad \Delta(x) \to \mathfrak{g}\,\Delta(x)\,\mathfrak{g}^T . \tag{5.3}$$

The field $\Delta(x)$ describes all fourteen GBs stemming from the $SU(5)/SO(5)$ breaking. Its transformation properties under $SU(5)$, i.e. in the symmetric representation, follow from the invariance of the vacuum under $SO(5)$. Using the following relations between the vacuum $\Delta_0$ and the broken and unbroken generators,

$$\Delta_0\,T_a\,\Delta_0 = -T_a^T , \qquad \Delta_0\,X_{\hat{a}}\,\Delta_0 = X_{\hat{a}}^T , \tag{5.4}$$

and because of the relations in Eq. (5.1), the excitations around the vacuum can be rewritten in terms of the GB field $\mathbf{\Sigma}(x)$:

$$\Delta(x) = \Omega(x)^2\,\Delta_0 \equiv \mathbf{\Sigma}(x)\,\Delta_0 . \tag{5.5}$$

The vector chiral field $\widetilde{\mathbf{V}}_\mu$ is then related to the vacuum excitations,

$$\widetilde{\mathbf{V}}_\mu(x) \equiv (\mathbf{D}_\mu\mathbf{\Sigma}(x))\,\mathbf{\Sigma}^\dagger(x) = (\mathbf{D}_\mu\Delta(x))\,\Delta^*(x) , \tag{5.6}$$

from which it follows that the GB kinetic term can be written as:

$$\mathrm{Tr}\left((\mathbf{D}_\mu\Delta)(\mathbf{D}_\mu\Delta)^*\right) = \mathrm{Tr}\left((\mathbf{D}_\mu\mathbf{\Sigma})(\mathbf{D}_\mu\mathbf{\Sigma})^\dagger\right) = -\mathrm{Tr}\left(\widetilde{\mathbf{V}}_\mu\widetilde{\mathbf{V}}^\mu\right) . \tag{5.7}$$

Considering the fourteen GBs arising from the $SU(5)/SO(5)$ breaking and described by $\Omega(x)$ (or $\mathbf{\Sigma}(x)$), the three would-be SM GBs $\mathcal{X}(x)$ and the scalar singlet field $\varphi(x)$ can be split from the other d.o.f. denoted collectively by $\mathcal{K}(x)$, by decomposing $\Omega(x)$ as [16]:

$$\Omega(x) = e^{i\frac{\varphi(x)}{2f}\mathcal{X}(x)}e^{i\frac{\mathcal{K}(x)}{2f}} . \tag{5.8}$$

As will be shown in section 5.3, there is a region of the parameter space of the potential in which the GBs described by $\mathcal{K}(x)$ are much heavier, so that at energies below $f$ the fields $\Omega(x)$ and

$\mathbf{\Sigma}(x)$ can be approximated by:

$$\Omega(x) \approx e^{i\frac{\varphi(x)}{2f}\mathcal{X}(x)}, \qquad\qquad \mathbf{\Sigma}(x) \approx e^{i\frac{\varphi(x)}{f}\mathcal{X}(x)}. \tag{5.9}$$

Furthermore, the explicit breaking of the global high-energy symmetry is assumed to induce a potential for the singlet field $\varphi(x)$, which eventually acquires dynamically a non-vanishing vev,

$$\frac{\varphi(x)}{f} \equiv \frac{h(x) + \langle\varphi\rangle}{f} = \left(\frac{h(x) + \langle\varphi\rangle}{v}\right)\sqrt{\xi}, \tag{5.10}$$

where $h(x)$ refers to the physical Higgs (denoted often simply as $h$ in what follows).

Denoting by $X$ the broken generator along which the EW symmetry breaking occurs,

$$X = \frac{1}{2}\begin{pmatrix} 0 & 0 & e_1 \\ 0 & 0 & e_2 \\ e_1^T & e_2^T & 0 \end{pmatrix} \qquad \text{with} \quad e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \; e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{5.11}$$

the $SU(5)$ embedding of the SM GB fields can be parametrised as

$$\mathcal{X}(x) = \sqrt{2}\begin{pmatrix} \mathbf{U} & & \\ & \mathbf{U} & \\ & & 1 \end{pmatrix} X \begin{pmatrix} \mathbf{U}^\dagger & & \\ & \mathbf{U}^\dagger & \\ & & 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & \mathbf{U}(x)e_1 \\ 0 & 0 & \mathbf{U}(x)e_2 \\ (\mathbf{U}(x)e_1)^\dagger & (\mathbf{U}(x)e_2)^\dagger & 0 \end{pmatrix},$$
$$\tag{5.12}$$

with $\mathbf{U}(x)$ defined in Eq. (2.6). In the unitary gauge, $\mathcal{X} = \sqrt{2}X$. Given the peculiar structure of the matrix $X$, the $\mathbf{\Sigma}$ field can be written uniquely in terms of linear and quadratic powers of $\mathcal{X}$ because $\mathcal{X}^3 = \mathcal{X}$:

$$\mathbf{\Sigma} \simeq \mathbb{1} + i\sin\left(\frac{\varphi}{f}\right)\mathcal{X} + \left(\cos\left(\frac{\varphi}{f}\right) - 1\right)\mathcal{X}^2. \tag{5.13}$$

The last ingredient needed to fully specify the setup is the embedding of the SM fields in $\mathcal{G}$. Given the choice of vacuum, the $SU(2)_L \times U(1)_Y$ generators can be expressed as

$$Q_L^a = \frac{1}{2}\begin{pmatrix} \sigma_a & & \\ & \sigma_a & \\ & & 0 \end{pmatrix}, \qquad Q_Y = \frac{1}{2}\begin{pmatrix} -\mathbb{1}_2 & & \\ & \mathbb{1}_2 & \\ & & 0 \end{pmatrix}, \tag{5.14}$$

where in these expressions $\sigma_a$ denote the Pauli matrices and the normalisation of the generators is $\text{Tr}(Q_a Q_a) = 1$.

## 5.1 The low-energy effective EW chiral Lagrangian

One can now substitute the explicit expression for $\mathbf{\Sigma}$, $\widetilde{\mathbf{V}}_\mu$, $\widetilde{\mathbf{W}}_\mu$ and $\widetilde{\mathbf{B}}_\mu$ in the operators of the high-energy basis in Eq. (4.32) and obtain $\mathscr{L}_{\text{low}}$ for the Georgi-Kaplan model as a function of the SM would-be GBs, the light scalar singlet field $\varphi(x)$ and the SM gauge fields.

### 5.1.1 The CP-even low-energy projection

For $SU(5)/SO(5)$, the low-energy projection of the custodial preserving CP-even two-derivative operator reads[38]

$$\widetilde{\mathcal{A}}_C \equiv -\frac{f^2}{4} \text{Tr}(\widetilde{\mathbf{V}}_\mu \widetilde{\mathbf{V}}^\mu) = \frac{4}{\xi} \sin^2\left[\frac{\varphi}{2f}\right] \mathcal{P}_C + \mathcal{P}_H \,, \tag{5.15}$$

with $\mathcal{P}_C$ and $\mathcal{P}_H$ being the operators in $\mathscr{L}_{\text{low}}$ defined in Eqs. (3.62) and (3.58), respectively. Having assumed the absence of any sources of custodial breaking besides the SM ones, no other two-derivative operators arise in the low-energy effective chiral Lagrangian.

Besides giving rise to the (correctly normalised) $h$ kinetic term described by $\mathcal{P}_H$, the operator $\widetilde{\mathcal{A}}_C$ intervenes also in the definition of the SM gauge boson masses. To provide a consistent definition for the SM $W$ mass $m_W^2 \equiv g^2 v^2/4$, it is necessary to impose that

$$\xi \equiv \frac{v^2}{f^2} = 4 \sin^2 \frac{\langle \varphi \rangle}{2f} \,, \tag{5.16}$$

providing a strict and model-dependent relation between the EW scale $v$, the vev of the scalar field $\varphi$ and the NP scale $f$. Note that in the $\xi \ll 1$ limit the usual SM result $\langle \varphi \rangle = v$ is recovered. Using Eq. (5.16), the functional dependence on $\varphi/f$ can be nicely translated in terms of the physical $h$ excitation and the EW scale $v$, and the following expressions will be useful later on:

$$\begin{aligned}
\sin\left(\frac{\varphi}{2f}\right) &= \sin\left(\arcsin\left(\frac{v}{2f}\right) + \frac{h}{2f}\right) = \frac{v}{2f} \cos\left(\frac{h}{2f}\right) + \sqrt{1 - \frac{v^2}{4f^2}} \sin\left(\frac{h}{2f}\right) \,, \\
\cos\left(\frac{\varphi}{2f}\right) &= \cos\left(\arcsin\left(\frac{v}{2f}\right) + \frac{h}{2f}\right) = \sqrt{1 - \frac{v^2}{4f^2}} \cos\left(\frac{h}{2f}\right) - \frac{v}{2f} \sin\left(\frac{h}{2f}\right) \,.
\end{aligned} \tag{5.17}$$

The low-energy projection of the four-derivative effective operators of Eq. (4.32) is given in Eq. (5.18). while the remaining two high-energy operators are not independent when focusing only on the light GBs remaining at low-energies: The fact that $\widetilde{\mathcal{A}}_7$ and $\widetilde{\mathcal{A}}_8$ do not give independent contributions as they are linear combinations of other high-energy operators (Eq. (5.19)) is connected with the peculiar structure of the $\mathcal{G}/\mathcal{H}$ breaking and has to be inferred case by case.

$$\widetilde{\mathcal{A}}_B = \mathcal{P}_B \,,$$

$$\widetilde{\mathcal{A}}_W = \mathcal{P}_W \,,$$

$$\widetilde{\mathcal{A}}_{B\Sigma} = - \, 4\, g'^2 \cos^2\left[\frac{\varphi}{2f}\right] \mathcal{P}_B \,,$$

$$\widetilde{\mathcal{A}}_{W\Sigma} = - \, 4\, g^2 \cos^2\left[\frac{\varphi}{2f}\right] \mathcal{P}_W \,,$$

$$\widetilde{\mathcal{A}}_1 = \sin^2\left[\frac{\varphi}{2f}\right] \mathcal{P}_1 \,,$$

$$\widetilde{\mathcal{A}}_2 = \sin^2\left[\frac{\varphi}{2f}\right] \mathcal{P}_2 \, + \, \sqrt{\xi}\sin\left[\frac{\varphi}{f}\right] \mathcal{P}_4 \,,$$

$$\widetilde{\mathcal{A}}_3 = 2\sin^2\left[\frac{\varphi}{2f}\right] \mathcal{P}_3 \, - \, 2\,\sqrt{\xi}\sin\left[\frac{\varphi}{f}\right] \mathcal{P}_5 \,,$$

$$\widetilde{\mathcal{A}}_4 = 4\,\xi^2\mathcal{P}_{DH} \, + \, 16\sin^4\left[\frac{\varphi}{2f}\right] \mathcal{P}_6 \, - \, 16\,\xi\sin^2\left[\frac{\varphi}{2f}\right] \mathcal{P}_{20} \,,$$

$$\widetilde{\mathcal{A}}_5 = 4\,\xi^2\mathcal{P}_{DH} \, - \, 16\,\xi\sin^2\left[\frac{\varphi}{2f}\right] \mathcal{P}_8 \, + \, 16\sin^4\left[\frac{\varphi}{2f}\right] \mathcal{P}_{11} \,,$$

$$\widetilde{\mathcal{A}}_6 = - \, 2\,\xi\,\mathcal{P}_{\Box H} \, - \, \frac{1}{2}\sin^2\left[\frac{\varphi}{f}\right] \mathcal{P}_6 \, + \, 4\,\xi\cos^2\left[\frac{\varphi}{2f}\right] \mathcal{P}_8 \, + \, 4\sin^2\left[\frac{\varphi}{2f}\right] \mathcal{P}_9 +$$

$$- \, 2\sqrt{\xi}\sin\left[\frac{\varphi}{f}\right] (\mathcal{P}_7 - 2\mathcal{P}_{10}) \,,$$

(5.18)

$$\widetilde{\mathcal{A}}_7 = \frac{1}{4}\left(\widetilde{\mathcal{A}}_4 + \widetilde{\mathcal{A}}_5\right) \,, \qquad\qquad \widetilde{\mathcal{A}}_8 = \frac{1}{2}\widetilde{\mathcal{A}}_5 \,. \qquad\qquad (5.19)$$

This specific example is similar to the ALF case, where it can be proven that traces of four $\mathbf{V}_\mu$ can be expressed as products of traces of two $\mathbf{V}_\mu$. In resume, $\mathscr{L}_{\text{low}}$ for the $SU(5)/SO(5)$ scenario considered here depends on only eight independent operators, besides the kinetic terms for gauge bosons and GB fields.

### 5.1.2 The CP-odd low-energy projection

Following the same procedure for the CP-odd operators, the low-energy projection of the four derivative operators of Eq. (4.32) is given in Eq. (5.20). The Higgs-independent part of the operator $\widetilde{\mathcal{B}}_{B\Sigma^*}$ can be safely neglected at low-energy being equivalent to a total derivative and vanishing. On the other side, the Higgs-independent part of the operators $\widetilde{\mathcal{B}}_{WW^*}$ and $\widetilde{\mathcal{B}}_{W\Sigma^*}$ does not vanish as it provides CP-odd non-perturbative contributions. The last operator in the list, $\widetilde{\mathcal{B}}_3$ automatically vanishes in $SU(5)/SO(5)$ model due to the specific properties of the coset generators.

$$
\begin{aligned}
\widetilde{\mathcal{B}}_{WW^*} &= \mathcal{S}_{WW^*}\,, \\
\widetilde{\mathcal{B}}_{B\Sigma^*} &= -4\,\mathcal{S}_{BB^*} + 4\,\sin^2\left[\frac{\varphi}{2f}\right]\mathcal{S}_{BB^*}\,, \\
\widetilde{\mathcal{B}}_{W\Sigma^*} &= -4\,\mathcal{S}_{WW^*} + 4\,\sin^2\left[\frac{\varphi}{2f}\right]\mathcal{S}_{WW^*}\,, \\
\widetilde{\mathcal{B}}_1 &= \frac{1}{2}\sin^2\left[\frac{\varphi}{2f}\right]\mathcal{S}_1\,, \\
\widetilde{\mathcal{B}}_2 &= 4\sin^4\left[\frac{\varphi}{2f}\right](\mathcal{S}_{BB^*} - \mathcal{S}_{WW^*}) + 2\sqrt{\xi}\,\cos\left[\frac{\varphi}{2f}\right]\sin^3\left[\frac{\varphi}{2f}\right](\mathcal{S}_2 + 2\mathcal{S}_3)\,, \\
\widetilde{\mathcal{B}}_3 &= 0\,.
\end{aligned}
\tag{5.20}
$$

Consequently at low-energy only four independent CP-odd perturbative couplings are relevant: the Higgs-dependent parts contained in $\widetilde{\mathcal{B}}_{B\Sigma^*}$, $\widetilde{\mathcal{B}}_{W\Sigma^*}$, $\widetilde{\mathcal{B}}_1$ and $\widetilde{\mathcal{B}}_2$.

## 5.2 Matching the high- and the low-energy Lagrangians

The remnant of the GB nature of the Higgs field can be tracked down to the trigonometric functions that enter into the low-energy EW chiral Lagrangian for the specific CH models: indeed, one given gauge vertex can involve an arbitrary number of $h$ legs, with a suppression in terms of powers of the GB scale $f$. The explicit dependence on the $h$ field is easily recovered using Eq. (5.10) in combination with trigonometric function properties. In the general $\mathscr{L}_{low}$ basis, the dependence on the $h$ field is encoded into the generic functions $\mathcal{F}_i(h)$ in Eq. (3.65) and into some operators which contain derivatives of $h$. The matching between the low-energy EW chiral Lagrangian of the specific CH models and the general $\mathscr{L}_{low}$ basis in Eq. (3.64) allows to identify the products $c_i\,\mathcal{F}_i(h)$ in terms of the high-energy parameters. The existence of peculiar correlations between the low-energy chiral effective operators could indeed provide very valuable information when trying to unveil the nature of the EWSB mechanism [33, 39, 40].

For the specific case of the $SU(5)/SO(5)$ and the terms in its two-derivative Lagrangian it results

$$
\mathcal{F}_C(h) = \frac{4}{\xi}\sin^2\left[\frac{\varphi}{2f}\right]\,, \qquad \mathcal{F}_H(h) = 1\,,
\tag{5.21}
$$

for the custodial preserving sector, while

$$
c_T\mathcal{F}_T(h) = 0
\tag{5.22}
$$

for the custodial breaking term, as expected from a model which was formulated with an embedded custodial symmetry. The expressions for the products $c_i\,\mathcal{F}_i(h)$ and $d_i\,\mathcal{E}_i(h)$, corresponding

| $c_i \mathcal{F}_i(h)$ | $SU(5)/SO(5)$ | $c_i \mathcal{F}_i(h)$ | $SU(5)/SO(5)$ |
|---|---|---|---|
| $\mathcal{F}_C(h)$ | $\frac{4}{\xi}\sin^2\frac{\varphi}{2f}$ | $c_4\mathcal{F}_4(h)$ | $\tilde{c}_2\sqrt{\xi}\sin\frac{\varphi}{f}$ |
| $\mathcal{F}_H(h)$ | $1$ | $c_5\mathcal{F}_5(h)$ | $-2\tilde{c}_3\sqrt{\xi}\sin\frac{\varphi}{f}$ |
| $\mathcal{F}_B(h)$ | $21 - 4g'^2\tilde{c}_{B\Sigma}\cos^2\frac{\varphi}{2f}$ | $c_6\mathcal{F}_6(h)$ | $16\tilde{c}_4\sin^4\frac{\varphi}{2f} - \frac{1}{2}\tilde{c}_6\sin^2\frac{\varphi}{f}$ |
| $\mathcal{F}_W(h)$ | $1 - 4g^2\tilde{c}_{W\Sigma}\cos^2\frac{\varphi}{2f}$ | $c_7\mathcal{F}_7(h)$ | $-2\tilde{c}_6\sqrt{\xi}\sin\frac{\varphi}{f}$ |
| $c_{\square H}\mathcal{F}_{\square H}(h)$ | $-2\tilde{c}_6\xi$ | $c_8\mathcal{F}_8(h)$ | $-16\tilde{c}_5\xi\sin^2\frac{\varphi}{2f} + 4\tilde{c}_6\xi\cos^2\frac{\varphi}{2f}$ |
| $c_{\Delta H}\mathcal{F}_{\Delta H}(h)$ | $-$ | $c_9\mathcal{F}_9(h)$ | $4\tilde{c}_6\sin^2\frac{\varphi}{2f}$ |
| $c_{DH}\mathcal{F}_{DH}(h)$ | $4\left(\tilde{c}_4+\tilde{c}_5\right)\xi^2$ | $c_{10}\mathcal{F}_{10}(h)$ | $4\tilde{c}_6\sqrt{\xi}\sin\frac{\varphi}{f}$ |
| $c_1\mathcal{F}_1(h)$ | $\tilde{c}_1\sin^2\frac{\varphi}{2f}$ | $c_{11}\mathcal{F}_{11}(h)$ | $16\tilde{c}_5\sin^4\frac{\varphi}{2f}$ |
| $c_2\mathcal{F}_2(h)$ | $\tilde{c}_2\sin^2\frac{\varphi}{2f}$ | $c_{20}\mathcal{F}_{20}(h)$ | $-16\tilde{c}_4\xi\sin^2\frac{\varphi}{2f}$ |
| $c_3\mathcal{F}_3(h)$ | $2\tilde{c}_3\sin^2\frac{\varphi}{2f}$ | | |

TABLE 5.1: *Expressions for the products $c_i\,\mathcal{F}_i(h)$ for the CP-even operators of the $SU(5)/SO(5)$ model. As expected, no custodial breaking operator gets any contribution.*

respectively for the CP-even and CP-odd four-derivative Lagrangians, are reported in Tab. 5.1 and Tab. 5.2.

| $d_i\mathcal{E}_i(h)$ | $SU(5)/SO(5)$ |
|---|---|
| $\mathcal{E}_{BB^*}(h)$ | $4\,\tilde{d}_{B\Sigma^*}\sin^2\frac{\varphi}{2f} + 4\,\tilde{d}_2\sin^4\frac{\varphi}{2f}$ |
| $\mathcal{E}_{WW^*}(h)$ | $\tilde{d}_{WW^*} - 4\,\tilde{d}_{W\Sigma^*}\cos^2\frac{\varphi}{2f} - 4\,\tilde{d}_2\sin^4\frac{\varphi}{2f}$ |
| $d_1\mathcal{E}_1(h)$ | $\frac{\tilde{d}_1}{2}\sin^2\frac{\varphi}{2f}$ |
| $d_2\mathcal{E}_2(h)$ | $2\,\tilde{d}_2\sqrt{\xi}\cos\frac{\varphi}{2f}\sin^3\frac{\varphi}{2f}$ |
| $d_3\mathcal{E}_3(h)$ | $4\,\tilde{d}_2\sqrt{\xi}\cos\frac{\varphi}{2f}\sin^3\frac{\varphi}{2f}$ |

TABLE 5.2: *Expressions for the products $c_i\,\mathcal{F}_i(h)$ for the CP-odd operators of the $SU(5)/SO(5)$ model.*

Some relevant conclusions can be inferred from these results:

i) The CP-odd high-energy lagrangian $\mathscr{L}_{\text{high},\cancel{CP}}$ generates all the custodial preserving opera-
tors entering in the low-energy one, $\mathscr{L}_{\text{low},\cancel{CP}}$. The same holds for the CP-even lagrangian

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$\mathscr{L}_{\text{high}}$ except for the pure-$h$ operators from Eq. 3.58 and Eq. (3.59).$\mathcal{P}_{\Delta H}$, $\mathcal{F}_H(h)$, $\mathcal{F}_{\Box H}$ and $\mathcal{F}_{DH}$ be originated due to the GB nature of the $\varphi$ field in that model, which forbids couplings with an odd number of Goldstone bosons, plus the fact that the departure from a pure Goldstone boson nature is through its vev $\langle\varphi\rangle \neq 0$, and not from any source containing derivatives

ii) All low-energy operators, both CP-even and CP-odd, not appearing in Tab. 5.1 or Tab. 5.2 describe effects of non-Standard Model tree-level custodial symmetry breaking, and therefore are not generated by the low-energy $SU(5)/SO(5)$ lagrangian, which has an unbroken $SO(4)$ custodial group.

iii) The choice of a given CH scenario constrains the arbitrary $\mathcal{F}_i(h)$ and $\mathcal{E}_i(h)$ functions to specific trigonometric functions of $h$ and $v$. Furthermore, the sixteen low-energy generic parameters $c_i$ of the CP-even low-energy lagrangian are described by eight high-energy parameters $\tilde{c}_i$.

### 5.2.1 The small $\xi$ limit

It is particularly interesting to consider the $f \gg v$, or equivalently $\xi \ll 1$, limit. In fact in this limit the non-linear CH model should overlap with the case in which the EWSB is linearly realised and the Lagrangian written in terms of the Higgs as an $SU(2)_L$ doublet. For example, taking the operators in Eq. (5.20) for the $SU(5)/SO(5)$ setup and expanding them in Taylor series in $1/f$ as defined in Eq. (5.10), one concludes that at first order in $\xi$,

$$
\begin{aligned}
\widetilde{\mathcal{B}}_{B\Sigma^*} &\approx \xi\,(1+h/v)^2 \mathcal{S}_{BB^*} - 4\,\mathcal{S}_{BB^*} \\
\widetilde{\mathcal{B}}_{W\Sigma^*} &\approx \xi\,(1+h/v)^2 \mathcal{S}_{WW^*} - 4\,\mathcal{S}_{WW^*} \\
\widetilde{\mathcal{B}}_1 &\approx \frac{1}{8}\,\xi\,(1+h/v)^2 \mathcal{S}_1 \\
\widetilde{\mathcal{B}}_2 &\approx 0
\end{aligned}
\tag{5.23}
$$

with $\widetilde{\mathcal{B}}_2$ giving contribution only at order $O(\xi^2)$. This procedure can be repeated with all the terms appearing in Table 5.2, resulting in This should be compared with the effective $d = 6$ CP-odd Lagrangian in Eq. 3.3.There is one-to-one correspondence between these two classes of operators: $\mathcal{Q}_{\varphi\widetilde{B}} \leftrightarrow \widetilde{\mathcal{B}}_{B\Sigma^*}$, $\mathcal{Q}_{\varphi\widetilde{W}} \leftrightarrow \widetilde{\mathcal{B}}_{W\Sigma^*}$ and $\mathcal{Q}_{\varphi\widetilde{B}W} \leftrightarrow \widetilde{\mathcal{B}}_1$. Conversely, $\widetilde{\mathcal{B}}_2$ contributes at low-energy to $\mathcal{S}_2$ and $\mathcal{S}_3$, but only at $\xi^2$, i.e. its linear sibling[1] should have $d = 8$: indeed, by using

---

[1]This is in contrast with Eq. (A.1) in Ref. [33], where two $d = 6$ linear operators have been indicated as siblings of $\mathcal{S}_2$ and $\mathcal{S}_3$. Those operators do contain the interactions of $\mathcal{S}_2$ and $\mathcal{S}_3$, but they are not the lowest dimensional ones.

| $d_i \mathcal{E}_i(h)$ | $SU(5)/SO(5)$ |
|---|---|
| $\mathcal{E}_{BB^*}(h)$ | $\tilde{c}_{B\Sigma^*}\, \xi \left(1 + \dfrac{h}{v}\right)^2 + \mathcal{O}(\xi^2)$ |
| $\mathcal{E}_{WW^*}(h)$ | $\tilde{c}_{WW^*} + \tilde{c}_{W\Sigma^*}\left(-4 + \xi\left(1 + \dfrac{h}{v}\right)^2\right) + \mathcal{O}(\xi^2)$ |
| $d_1 \mathcal{E}_1(h)$ | $\dfrac{\tilde{c}_1}{8}\, \xi \left(1 + \dfrac{h}{v}\right)^2 + \mathcal{O}(\xi^2)$ |
| $d_2 \mathcal{E}_2(h)$ | $\mathcal{O}(\xi^2)$ |
| $d_3 \mathcal{E}_3(h)$ | $\mathcal{O}(\xi^2)$ |

TABLE 5.3: *Expressions for the products $c_i\, \mathcal{F}_i(h)$ for $SU(5)/SO(5)$ in the $\xi \ll 1$ limit.*

integration by parts and the Bianchi identities, it is straightforward to verify that the interactions of $\mathcal{S}_2$ and $\mathcal{S}_3$ are described at the lowest order in the linear expansion by the operators

$$\mathbf{B}^*_{\mu\nu}\left(\Phi^\dagger \overset{\leftrightarrow}{\mathbf{D}}{}^\mu \Phi\right) \mathbf{D}^\nu\left(\Phi^\dagger \Phi\right), \qquad \left(\Phi^\dagger \overset{\leftrightarrow}{\mathbf{D}}{}^\mu \mathbf{W}^*_{\mu\nu} \Phi\right) \mathbf{D}^\nu\left(\Phi^\dagger \Phi\right) \tag{5.24}$$

with $\mathbf{D}_\mu \Phi \equiv \left(\partial_\mu + \frac{i}{2}g' B_\mu + \frac{i}{2}g\sigma_i W^i_\mu\right)\Phi$ and $\Phi^\dagger \overset{\leftrightarrow}{\mathbf{D}}{}^\mu \Phi \equiv \Phi^\dagger \mathbf{D}_\mu \Phi - \mathbf{D}_\mu \Phi^\dagger \Phi$.

The products $c_i\mathcal{F}_i(h)$ corresponding to custodial-breaking operators are suppressed by $\xi^2$ and therefore they are also described in the linear expansion by $d = 8$ operators. However, a complete comparison is not possible in this case, as no $d = 8$ basis has been defined yet, to our knowledge.


## 5.3   Goldstone boson potential

The way the $\Sigma(x)$ matrix has been parametrized in the $SU(5)/SO(5)$ model assumes, as has been stated, that the 10 GBs which are not to be identified with the 3 SM would-be GBs nor with the Higgs scalar are heavy enough and therefore are integrated out. The purpose of this section is to show a particular realization of this model, with an additional $U(1)_A$ subgroup is gauged, in which there is a region of the parameter space where the Higgs scalar is the only light Goldstone boson and all the rest are heavy. Additionally, we show that the Higgs field is the only GB that gets a VEV, so that the EWSB can be triggered.

The gauging of the SM group will break explicitly the global symmetry and therefore generate a potential for the NG bosons, which will then become massive. However, the vacuum will be still SM invariant, since it was proved in [18] that vectors transforming in the unbroken group cannot lead to a potential in which one of the NG bosons develops a VEV, like we would need the Higgs scalar to do.

Then, in order to allow the possibility of having a non-zero VEV for the Higgs, the authors in Ref. [16] gauge a new $U(1)_A$ group which is embedded in the global group $\mathcal{G}$ (contrary to the SM group $SU(2)_L \times U(1)_Y$, which is embedded in $\mathcal{H}$). This gauging also breaks explicitly $SU(5)$, but now it doesn't transform in the unbroken, so the vacuum can be SM-breaking.

Following Georgi and Kaplan, we represent the 14 GB in terms of the unbroken subgroup $SO(4) \simeq SU(2)_L \times SU(2)_R$. In particular, $14 = (3,3) \otimes (2,2) \otimes (1,1)$. Thus,

$$(3,3) \to \boldsymbol{\pi} = \sqrt{\frac{1}{2}} \frac{\pi^{ab}(x)}{f} \begin{pmatrix} \sigma_a \tau_b & \\ & 0 \end{pmatrix} \tag{5.25}$$

where $\tau_a$ represents the Pauli matrices in 2x2 block structure. For the $(2,2)$, which we identify with the d.o.f's of the Higgs doublet, $\boldsymbol{\Phi}$, we have the field

$$(2,2) \to \boldsymbol{\Phi} \equiv \frac{\varphi(x)}{f} \mathcal{X}(x) \tag{5.26}$$

with $\mathcal{X}(x)$ defined in Eq. 5.12. In the unitary gauge, with only physical degrees of freedom, we have $\boldsymbol{\Phi} = \sqrt{2}\frac{\varphi}{f}X$. Finally, the singlet has the following form:

$$(1,1) \to \boldsymbol{\eta} = \sqrt{\frac{1}{10}} \frac{\eta(x)}{f} \begin{pmatrix} \mathbb{I}_4 & \\ & -4 \end{pmatrix} \tag{5.27}$$

Thus, the matrix $\Omega(x)$ 5.8 is given by

$$\Omega(x) = e^{i\boldsymbol{\Phi}/2} e^{i\frac{\mathcal{K}(x)}{2f}} \tag{5.28}$$

where $\mathcal{K}(x) = \boldsymbol{\pi} + \boldsymbol{\eta}$. To second order in the GB momenta, the lagrangian is given by:

$$\mathcal{L} = \frac{1}{4} f^2 \mathrm{Tr}\left(D_\mu \Delta\right)\left(D^\mu \Delta\right)^* - V(\Delta) \tag{5.29}$$

with $\Delta(x) = \Sigma(x)\Delta_0$.

The potential $V(\Delta)$ is generated because of the gauging of SM group as well as the extra $U(1)_A$; these gauge interactions break explicitly the global $SU(5)$, so a potential is generated at the loop level. We use the spurion technique in order to determine the potential. We promote the generators $Q$ to transform formally in the adjoint of $SU(5)$, and define the vector $A_\mu$ appearing in the covariant derivative

$$D_\mu \Delta = \partial_\mu \Delta - iA_\mu \Delta - i\Delta A_\mu^T \tag{5.30}$$

in terms of the the spurions $G_i$:

$$A_\mu = G_{a\alpha}M^\alpha W_\mu^a + G_{Y\alpha}M^\alpha B_\mu + G_{A\alpha}M^\alpha Z'_\mu \equiv G_{La}W_\mu^a + G_Y B_\mu + G_A Z'_\mu \tag{5.31}$$

where $\alpha$ is an index indicating the adjoint representation of $SU(5)$, $M^\alpha$ are matrices in that representation and $W_\mu^a$, $B_\mu$ and $Z'_\mu$ are the gauge vector bosons corresponding to $SU(2)_L$, $U(1)_Y$ and $U(1)_A$ respectively. Using the spurions $G_i$ we can write non-derivative terms which formally preserve the global group. Once we determine the relevant operators, we set the spurions to their "spurion VEV's",

$$G_{La} \to gQ_{La} \qquad G_Y \to g'Q_Y \qquad G_A \to g_A Q_A \tag{5.32}$$

which are precisely the generators of the gauge groups defined as:

$$SU(2)_L \to Q_{La} = \frac{1}{2}\begin{pmatrix} \sigma_a & & \\ & \sigma_a & \\ & & 0 \end{pmatrix} \tag{5.33}$$

$$U(1)_Y \to Q_Y = \frac{1}{2}\begin{pmatrix} \mathbb{I}_2 & & \\ & -\mathbb{I}_2 & \\ & & 0 \end{pmatrix} \tag{5.34}$$

$$U(1)_A \to Q_A = \sqrt{\frac{1}{20}}\begin{pmatrix} \mathbb{I}_4 & \\ & -4 \end{pmatrix} \tag{5.35}$$

Notice that $Q_{La}$ and $Q_Y$ are unbroken generators, while $Q_A$ is a broken one.

Thus, the leading order contributions to the potential are:

$$\begin{aligned} V(\Delta) &\equiv V_L + V_Y + V_A \\ V_L &\equiv kf^4 \text{Tr}(G_{La}\Delta)(G_L^a\Delta)^* \to kf^4 g^2 \text{Tr}(Q_{La}\Delta)(Q_L^a\Delta)^* \\ V_Y &\equiv kf^4 \text{Tr}(G_Y\Delta)(G_Y\Delta)^* \to kf^4 g'^2 \text{Tr}(Q_Y\Delta)(Q_Y\Delta)^* \\ V_A &\equiv kf^4 \text{Tr}(G_A\Delta)(G_A\Delta)^* \to kf^4 g_A^2 \text{Tr}(Q_A\Delta)(Q_A\Delta)^* \end{aligned} \tag{5.36}$$

The reason for this structure (in particular, the reason that there are no operators with only one spurion and that they don't mix) is that we impose the symmetries that appear in the lagrangian with only covariant derivatives. Apart from the $SU(5)$ global symmetry, we have three accidental symmetries: the $SU(2)$ under which the index $a$ behaves like a triplet and two accidental $\mathbb{Z}_2$ which allow to flip independently $G_{Y,A} \to -G_{Y,A}$.

In order to express the lagrangian in terms of the matrix $\Sigma(x)$ we recall 5.4 , so that $\Sigma\Delta_0 = \Delta_0\Sigma^T$. Thus,

$$\text{Tr}\left[(Q_{La}\Delta)(Q_L^a\Delta)^*\right] = -\text{Tr}\left[Q_{La}\Sigma Q_L^a\Sigma^\dagger\right]$$

$$\text{Tr}\left[(Q_Y\Delta)(Q_Y\Delta)^*\right] = -\text{Tr}\left[Q_Y\Sigma Q_Y\Sigma^\dagger\right] \tag{5.37}$$

$$\text{Tr}\left[(Q_A\Delta)(Q_A\Delta)^*\right] = \text{Tr}\left[Q_A\Sigma Q_A\Sigma^\dagger\right]$$

where we also have taken into account that the generators are hermitian. Finally,

$$V(\Sigma) = kf^4\left(-g^2\text{Tr}\left[Q_{La}\Sigma Q_L^a\Sigma^\dagger\right] - g'^2\text{Tr}\left[Q_Y\Sigma Q_Y\Sigma^\dagger\right] + g_A^2\text{Tr}\left[Q_A\Sigma Q_A\Sigma^\dagger\right]\right) \tag{5.38}$$

An alternative method is to use the Coleman-Weinberg formula for the 1-loop correctian for a scalar potential due to gauge boson loops. The largest correction from the Coleman-Weinberg potential is given in terms of the mass matrix of the gauge bosons $M_V^2(\Delta)$ ([41]), which can be read from the kinetic term in the lagrangian,

$$V(\Delta) = kf^2\text{Tr}[M_V^2(\Delta)] \tag{5.39}$$

From the lagrangian (5.29) and the covariant derivative (5.31) (where we are not using anymore spurions, but the real values of the generators), we see that the mass term for the $i$ gauge boson is given by

$$(M_V^2)_i = f^2g_i^2\left(Q_i\Delta + \Delta Q_i^T\right)\left(Q_i^*\Delta^* + \Delta^*Q_i^\dagger\right) = f^2g_i^2\left(Q_i\Delta Q_i^*\Delta^* + \mathbb{I} + \text{h.c.}\right) \tag{5.40}$$

Therefore, up to a constant, we find that

$$V(\Delta) = kf^4\sum_i\text{Tr}\left[(Q_i\Delta)(Q_i\Delta)^*\right] \tag{5.41}$$

which is exactly the expression we had found using the spurion technique.

Vafa and Witten's result [18] tells us that only the GB's getting contributions from $V_A$ can get a VEV; in particular, the Goldstones that get a correction to its mass, allowing it to be negative. To see what scalars can get a VEV $V_A$ we try to express it in terms of commutators of $Q_A$ and the GB's matrices:

$$\text{Tr}\left[Q_A\Sigma Q_A\Sigma^\dagger\right] = \text{Tr}\left[Q_A^2\Sigma^\dagger\Sigma\right] + \text{Tr}\left[Q_A\Sigma^\dagger[Q_A, \Sigma]\right]$$

$$= 1 + \text{Tr}\left[Q_A\Sigma^\dagger[Q_A, \Sigma]\right]$$

where we have used the normalization condition $\text{Tr}(X_a X_b) = \delta_{ab}$ for the broken generators. We expand now the commutator in the last expression, getting

$$[Q_A, \Sigma] = e^{i\mathbf{\Phi}/2} e^{i\frac{\mathcal{K}(x)}{f}} \left[Q_A, e^{i\mathbf{\Phi}/2}\right] + \left[Q_A, e^{i\mathbf{\Phi}/2}\right] e^{i\frac{\mathcal{K}(x)}{f}} e^{i\mathbf{\Phi}/2} \tag{5.42}$$

We have used the fact that $Q_A$ commutes with all Goldstones matrices except with the Higgs: $[Q_A, \boldsymbol{\pi}] = [Q_A, \boldsymbol{\eta}] = 0$, $[Q_A, \mathbf{\Phi}] = \frac{\sqrt{5}}{2}\mathbf{\Phi}$. Thus,

$$V_A = kf^4 g_A^2 \left( 1 + \text{Tr}\left( Q_A e^{-i\mathbf{\Phi}/2} [Q_A, e^{i\mathbf{\Phi}/2}] \right) + \text{Tr}\left( Q_A \Sigma^\dagger [Q_A, e^{i\mathbf{\Phi}/2}] e^{i\frac{\mathcal{K}(x)}{f}} e^{i\mathbf{\Phi}/2} \right) \right) \tag{5.43}$$

We see then that only the Higgs will get a contribution to its quadratic term in the potential from $U(1)_A$, since the terms in $V_A$ involving the other Goldstones need the insertion of $\varphi$ scalars, as it's clear from the last piece of the previous equation. In other words: the GB's that can get a VEV are those represented by broken operators which don't commute with $Q_A$ (the Higgs, in this case). The minimization of the potential will give us the $\Sigma$ matrix which rotate our original, SM-preserving, fake vacuum $\Delta_0$ to the real vacuum of the theory. We will compute the potential to second order in the extra 10 GB and to all orders in the Higgs; we can do this last thing due to the properties of matrix $X$. In particular, since $X^3 = X$,

$$e^{i\mathbf{\Phi}/2} = \mathbb{I} + i\sqrt{1 - \cos\frac{\varphi}{f}} X + \left( \sqrt{1 + \cos\left(\frac{\varphi}{f}\right)} - 1 \right) X^2 \tag{5.44}$$

The form of the potential is not very informative, but we will show how indeed $\varphi$ is the only Goldstone to get a non-zero VEV. For this, we compute the masses and see if any of them can be negative. We define

$$\mu_i \equiv \frac{d^2 V}{d\psi_i^2}_{|\psi_a = \psi_b = \ldots = 0} \tag{5.45}$$

where by $\psi_i^2$ we mean the mass eigenstates, which will be combinations of Goldstones. We expect a massless state, which is "eaten" by the $Z'$ gauge boson. The interesting fact is that the only $\mu^2$ that depends on the coupling $g_A$ is the one of the Higgs; all the rest only depend on $g$ and $g'$, and this will lead to the fact that the only one that can take a negative value is $\mu_\varphi^2$.

$$\mu_a^2 = \mu_b^2 = \mu_c^2 = \mu_d^2 = \mu_e^2 = \mu_f^2 = kf^2 \left( 2g'^2 + 4g^2 \right) \tag{5.46}$$

$$\mu_g^2 = \mu_h^2 = \mu_j^2 = 4kf^2 g^2 \tag{5.47}$$

$$\mu_{eaten}^2 = 0 \tag{5.48}$$

$$\mu_\varphi^2 = \frac{kf^2}{2}(g' + 3g^2 - 5g_A^2) \tag{5.49}$$

Therefore, the Higgs can develop a VEV if, for $g_A \neq 1$,

$$c_0 \equiv \frac{3g^2 + g'^2}{5g_A^2} < 1 \tag{5.50}$$

Since the contribution to the Higgs potential of $g_A$ has a different sign to that of the SM bosons, only when the coupling force $U(1)_A$ is strong enough in comparison to $SU(2) \times U(1)_Y$ can the potential change and give a VEV to the Higgs. We also see clearly that for $g_A = 0$ there is no EW-symmetry breaking.

To second order in the all the GB's but the Higgs, the potential $V_A$ (5.43) becomes

$$V_A = \frac{kf^2 g_A^2}{64}\left\{10Y_1^2\left[2\cos\left(\frac{\varphi}{f}\right) - \cos\left(\frac{2\varphi}{f}\right) - 1\right] - Y_2^2\left[\cos\left(\frac{2\varphi}{f}\right) - 1\right] + 40\cos\left(\frac{2\varphi}{f}\right)\right\}$$

where $\Psi_{1,2}$ are polynomials quadratic in $\pi_{a,b}$ and $\eta$,

$$Y_1^2 \equiv (\pi_1 - \pi_6)^2 + (\pi_5 + \pi_7)^2 + (\pi_2 + \pi_9)^2 \tag{5.51}$$

$$Y_2^2 \equiv 5\eta^2 + 2\sqrt{5}\eta(\pi_3 + \pi_4 - \pi_8) + (\pi_3 + \pi_4 - \pi_8)^2 \tag{5.52}$$

We confirm explicitly what we had proven before: the only Goldstone that gets a non-vanishing contribution from $g_A^2$ to $\mu_i$ is the Higgs: all the rest of GB's have always at least a $\varphi^2$ term, as we can see if we expand $V_A$ around $\varphi = 0$:

$$V_A \simeq \frac{kf^2 g_A^2}{64}\left\{\frac{3}{2f^2}Y_1^2\varphi^2 + \frac{2}{f^2}Y_2^2\varphi^2 - \frac{80}{f^2}\varphi^2\right\} \tag{5.53}$$

From the Coleman-Weinberg framework we understand that the coefficient in the potential to a generic term $g_A^2\psi_i...\psi_n$ is equal to the coefficient of the diagram in which the Goldstones $\psi_i...\psi_n$ contribute to the mass of the $Z'$ boson. We see then that the fact that $[Q_A, \varphi] \neq 0$ implies that only the Higgs is allowed to have a vertex like $\varphi\varphi Z'Z'$, while the rest of Goldstone need at least the insertion of two Higgs scalars, $\eta\eta\varphi\varphi Z'Z'$. To have a more explicit expression, we set $\pi_{ab} = \eta = 0$, getting the following potential for the Higgs scalar:

$$V(\varphi) = -\frac{1}{4}f^4 k\left(-5g_A^2\cos^2\left(\frac{\varphi}{f}\right) + 2\left(g'^2 + 3g^2\right)\cos\left(\frac{\varphi}{f}\right)\right) \tag{5.54}$$

The condition that must be fulfilled is:

$$\frac{dV(\varphi)}{d\varphi} = \frac{1}{2}f^3 k\left(g'^2 + 3g^2 - 5g_A^2\cos\left(\frac{\langle\varphi\rangle}{f}\right)\right)\sin\left(\frac{\langle\varphi\rangle}{f}\right) = 0 \tag{5.55}$$
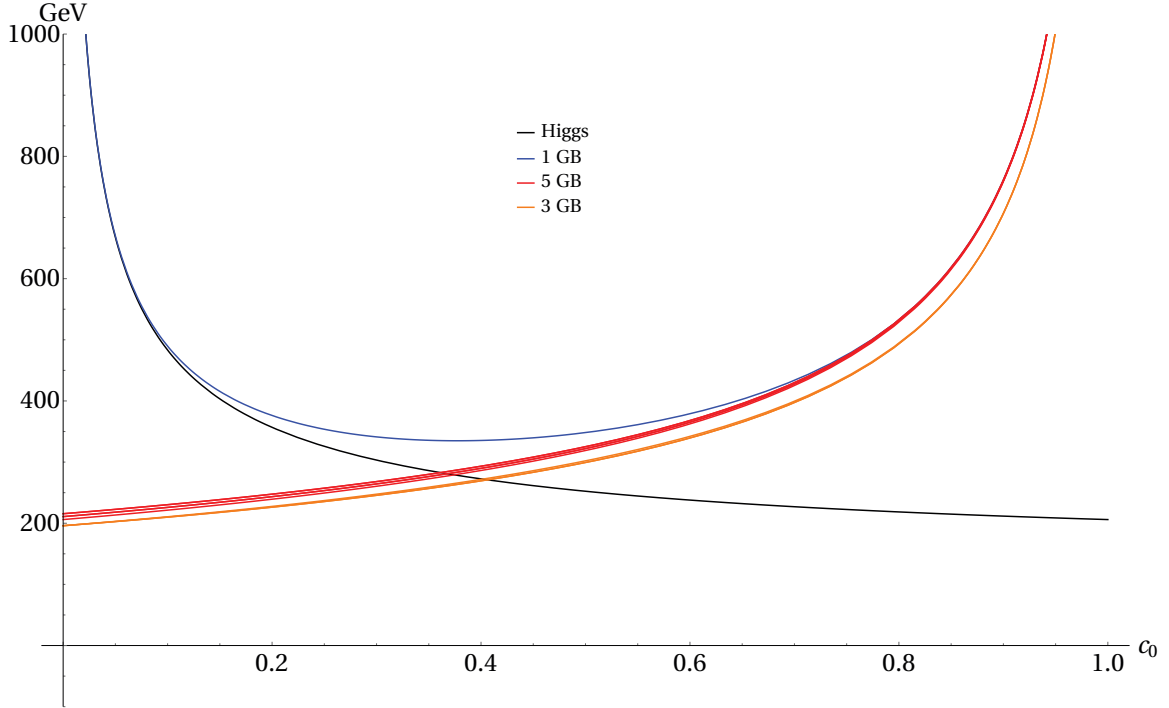
FIGURE 5.1: Mass spectrum of the Goldstone bosons

We have a maximum at 0 and a minimum at

$$c_0 \equiv \cos\left(\frac{\langle\varphi\rangle}{f}\right) = \frac{3g^2 + g'^2}{5g_A^2} \tag{5.56}$$

That they are maximum and minimum respectively can be checked from the second derivative of the potential:

$$\frac{d^2 V(\varphi)}{d\varphi^2} = \frac{5g_A^2 f^2 k}{2}\left(c_0 \cos\left(\frac{\langle\varphi\rangle}{f}\right) - \cos\left(\frac{\langle 2\varphi\rangle}{f}\right)\right) \tag{5.57}$$

so that

$$\begin{cases} \dfrac{d^2 V(\varphi)}{d\varphi^2}\bigg|_0 = \dfrac{5g_A^2 f^2 k}{2}(c_0 - 1) < 0 \implies c_0 < 1 \\ \dfrac{d^2 V(\varphi)}{d\varphi^2}\bigg|_{c_0} = m_\varphi^2 = \dfrac{5g_A^2 f^2 k}{2}(1 - c_0^2) > 0 \implies c_0 < 1 \end{cases} \tag{5.58}$$

At $c_0 = $ both points collapse to an inflexion point. It's important that one must impose the condition $g_A \neq 0$ to find the non-zero solution, so the situation where $g_A$ is not gauged must be directly studied by setting $g_A = 0$ in the potential (which gives $\langle\varphi\rangle = 0$, as expected), not in $c_0$. To get numerical expressions we impose the model to reproduce the mass of the $W$:

$$m_W^2 = \frac{g^2 f^2}{2}(1 - c_0) \tag{5.59}$$

Thus, the only parameter in the model is $g_A$, or $c_0$. We find now the masses of the rest of Goldstones; the analytical expressions are complicated and not particularly enlightening, so we plot them in the following graph. We plot the GB masses in Fig. 5.1, giving the observed values to $m_W^2$, $g$ and $g'$. We get the same GB masses as in Ref. [16]. The Higgs mass cannot go below 200 GeV for any value of $c_0$, so this model is ruled out. However, the original motivation of this computation was to justify the hypothesis done in the previos section in order to write the effective lagrangian of the $SU(5)/SO(5)$: that it is possible that out of the fourteen GBs the Higgs scalar is the only one to get a VEV, and that it can be considered much lighter than all the rest, which get a mass of order $f$.

# Chapter 6

# The low-energy projection SO(5)/SO(4) for SU(3)/SU(2)XU(1)

In this Chapter we will write the high-energy effective bosonic Lagrangians for the MCHM model $SO(5)/SO(4)$ as well as the custodial breaking model $SU(3)/(SU(2) \times U(1))$, and then study the corresponding projections into the low-energy effective lagrangian in a similar fashion as was done in the previous chapter for the Georgi-Kaplan model. The projection of the MCHM will be shown to be similar to that of the $SU(5)/SO(5)$ case. Since this coset corresponds to the most studied CH setup in the recent years, we will compare the basis here constructed with the one proposed in the literature, checking that indeed the predictions for the low-energy lagrangian are identical.

## 6.1 The minimal $SO(5)/SO(4)$ composite Higgs model

Most of the recent literature in CH models deals with the minimal $SO(5)/SO(4)$ [17] setup. The features that make this model appealing are its custodial symmetry approximate conservation and its minimality in terms of number of GBs that arise from the global symmetry breaking: only four to be associated with the SM would-be GBs and the Higgs field.

### 6.1.1 Spontaneous $SO(5)/SO(4)$ symmetry breaking setup

Just as in section 5.0.1, the spontaneous $SO(5)/SO(4)$ symmetry breaking can be obtained giving a vev to a scalar field either in a fundamental or in the symmetric adjoint representation. Many

studies of MCHM, as in [17], choose the vacuum in the fundamental representation. In order to keep the argument as similar as was done for the $SU(5)/SO(5)$, the vacuum is here taken in the symmetric representation. Similarly, the vacuum can be taken in all generality to be a real, symmetric and orthogonal $5 \times 5$ matrix satisfying Eq. (5.1), and a convenient choice is to set

$$\Delta_0 = \begin{pmatrix} \mathbb{1}_4 & 0 \\ 0 & -1 \end{pmatrix}. \tag{6.1}$$

As in the previous case, it is then possible to describe the massless excitations around the vacuum with a symmetric field $\Delta(x)$ obtained "rotating" the vacuum with the GB non-linear field $\Omega(x)$: Eq. (5.3) also holds here, with $\mathfrak{g}$ being now a transformation of $SO(5)$. $\Delta(x)$ transforms in the adjoint of $SO(5)$, as a consequence of the invariance of the vacuum under $SO(4)$, and describes only four GBs.

The relations between the vacuum $\Delta_0$ and the broken and unbroken generators presented in Eq. (5.4) are valid also for this model, and because of the relations in Eq. (5.1) the excitations around the vacuum can be reparametrised in the $\Sigma$-representation as in Eq. (5.5), where now $\Omega(x)$ and $\boldsymbol{\Sigma}(x)$ are given by

$$\Omega(x) = e^{i \frac{\varphi(x)}{2f} \mathcal{X}(x)}, \qquad \boldsymbol{\Sigma}(x) = e^{i \frac{\varphi(x)}{f} \mathcal{X}(x)}. \tag{6.2}$$

The $SO(5)/SO(4)$ generators can be written in a compact form as

$$(X_{\hat{a}})_{ij} = \frac{i}{\sqrt{2}} \left( \delta_{i5} \delta_{j\hat{a}} - \delta_{j5} \delta_{i\hat{a}} \right), \qquad \hat{a} = 1, \ldots, 4, \tag{6.3}$$

and denoting the broken generator along which the EW symmetry breaking occurs as $X_{\hat{4}}$,

$$X_{\hat{4}} = \frac{i}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -e_2 \\ 0 & e_2^T & 0 \end{pmatrix}, \tag{6.4}$$

the GB non-linear field reads

$$\mathcal{X}(x) = -\frac{i}{\sqrt{2}} \operatorname{Tr} \left( \mathbf{U} \sigma_{\hat{a}} \right) X_{\hat{a}}, \qquad \hat{a} = 1, \ldots, 4, \tag{6.5}$$

where $\sigma_{\hat{a}} \equiv \{\sigma_1, \sigma_2, \sigma_3, i\mathbb{1}_2\}$ and which reduces to $\mathcal{X} = \sqrt{2} X_{\hat{4}}$ in the unitary gauge. Alike to the case of the Georgi-Kaplan model, the field $\boldsymbol{\Sigma}$ takes the simple form in terms of linear and quadratic powers of $\mathcal{X}$ shown in Eq. (5.13). Finally, with this convention the embedding of the

$SU(2)_L \times U(1)_Y$ generators in $SO(5)$ reads

$$
Q_L^1 = \frac{1}{2}\begin{pmatrix} & -i\sigma_1 & \\ i\sigma_1 & & \\ & & 0 \end{pmatrix}, \qquad Q_L^2 = \frac{1}{2}\begin{pmatrix} & i\sigma_3 & \\ -i\sigma_3 & & \\ & & 0 \end{pmatrix},
$$
$$
Q_L^3 = \frac{1}{2}\begin{pmatrix} \sigma_2 & & \\ & \sigma_2 & \\ & & 0 \end{pmatrix}, \qquad Q_Y = \frac{1}{2}\begin{pmatrix} \sigma_2 & & \\ & -\sigma_2 & \\ & & 0 \end{pmatrix}.
$$

$$(6.6)$$

### 6.1.2 The low-energy effective EW chiral Lagrangian

Having chosen the explicit realisation of the $SO(5)/SO(4)$ symmetry breaking mechanism and the representation of the embedding of the SM group charges into $SO(5)$, the substitution of the explicit expressions for $\boldsymbol{\Sigma}$, $\widetilde{\mathbf{V}}_\mu$, $\widetilde{\mathbf{W}}_\mu$ and $\widetilde{\mathbf{B}}_\mu$ into the operators of the high-energy basis in Eq. (4.32) produces $\mathscr{L}_{\text{low}}$ for the minimal $SO(5)/SO(4)$ CH model, as a function of the SM would-be GBs and the light scalar resonance $\varphi$.

The low-energy projection of the $SO(5)/SO(4)$ Lagrangian turns out to be exactly the same as that for the $SU(5)/SO(5)$ model. This result depends on the strict connection between $SO(5)$ and $SU(5)$, as indeed the GB matrix fields of the two theories are linked by a unitary global transformation, once decoupling the extra GBs arising in the $SU(5) \to SO(5)$ breaking. Moreover, the gauging of the SM symmetry represents an explicit breaking of the global symmetries and it produces the effect of washing out the differences between the two preserved subgroups, once focusing only on the SM particle spectrum. This also suggests that any model with the minimal number of GBs that can be arranged in a doublet of $SU(2)_L$ and approximate custodial symmetry will yield the same low-energy effective chiral Lagrangian regardless of the specific ultraviolet completion.

## 6.2 The $SU(3)/(SU(2) \times U(1))$ composite Higgs model

As a final example, the $SU(3)/(SU(2) \times U(1))$ CH model is now considered. As only four GBs arise from the breaking of the global symmetry, also this model is minimal. However, contrary to the previously discussed CH models, the preserved subgroup $\mathcal{H}$ does not contain the custodial $SO(4)$ term and therefore no (approximate) custodial symmetry is embeddable in this model. This feature disfavours phenomenologically the $SU(3)/(SU(2) \times U(1))$ CH model as large tree-level contributions to the $T$ parameter occur. Nevertheless, the study of its low-energy projection

is instructive in order to discuss the custodial breaking operators of the effective Lagrangian $\mathcal{L}_{\text{low}}$ in Eq. (3.64). Indeed, although in the initial high-energy $SU(3)/(SU(2) \times U(1))$ Lagrangian no extra sources of custodial breaking (besides the SM ones) are introduced, these operators appear at tree-level in the low-energy effective Lagrangian.

### 6.2.1  Spontaneous $SU(3)/(SU(2) \times U(1))$ symmetry breaking setup

An appropriate choice for the vacuum that breaks $SU(3) \to SU(2) \times U(1)$ is given by the following hermitian and orthogonal matrix:

$$\Delta_0 = \begin{pmatrix} \mathbb{1}_2 & 0 \\ 0 & -1 \end{pmatrix}, \tag{6.7}$$

that satisfies the relations in Eq. (5.1). As in the previous cases, it is then possible to describe the massless excitations around the vacuum with a unitary field $\Delta(x)$ obtained "rotating" the vacuum with the GB non-linear field $\Omega(x)$:

$$\Delta(x) = \Omega(x) \, \Delta_0 \, \Omega(x)^\dagger, \qquad \Delta(x) \to \mathfrak{g} \, \Delta(x) \, \mathfrak{g}^\dagger. \tag{6.8}$$

As the vacuum is invariant under $SU(2) \times U(1)$ transformations, $\Delta(x)$ belongs to the adjoint of $SU(3)$. Being $dim(SU(3)/(SU(2) \times U(1))) = 4$, the field $\Delta(x)$ describes the dynamics of only four GBs, which will be then identified with the longitudinal components of the SM gauge bosons and the physical Higgs particle. Using the following relations between the vacuum $\Delta_0$ and the broken and unbroken generators,

$$\Delta_0 \, T_a \, \Delta_0 = T_a, \qquad \Delta_0 \, X_{\hat{a}} \, \Delta_0 = -X_{\hat{a}}, \tag{6.9}$$

and because of the relations in Eq. (5.1), the excitations around the vacuum can be arranged in the $\Sigma$-representation as in Eq. (5.5) with $\Omega$ and $\Sigma$ given as in Eq. (6.2). Choosing the following direction of EW symmetry breaking,

$$X = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & e_2 \\ e_2^T & 0 \end{pmatrix}, \tag{6.10}$$

it is possible to write the $SU(3)$ embedding of the SM GB fields as

$$\mathcal{X}(x) = \sqrt{2} \begin{pmatrix} \mathbf{U}(x) & \\ & 1 \end{pmatrix} X \begin{pmatrix} \mathbf{U}(x)^\dagger & \\ & 1 \end{pmatrix} = \begin{pmatrix} 0 & \mathbf{U}(x)e_2 \\ (\mathbf{U}(x)e_2)^\dagger & 0 \end{pmatrix}, \tag{6.11}$$

reducing to $\mathcal{X} = \sqrt{2}X$ in the unitary gauge. As for the two models previously analysed, the GB field matrix $\mathbf{\Sigma}$ can be expressed in terms of $\mathcal{X}$ as in Eq. (5.13). Finally the $SU(3)$-embedding of the $SU(2)_L \times U(1)_Y$ generators are given by

$$Q_L^a = \frac{1}{2} \begin{pmatrix} \sigma_a & \\ & 0 \end{pmatrix}, \qquad Q_Y = \frac{1}{6} \begin{pmatrix} \mathbb{1}_2 & \\ & -2 \end{pmatrix}, \tag{6.12}$$

with $\mathrm{Tr}(Q_L^a Q_L^a) = 1$ and $\mathrm{Tr}(Q_Y Q_Y) = 1/6$.

### 6.2.2 The low-energy effective CP-even EW chiral Lagrangian

By substituting the explicit expressions for $\mathbf{\Sigma}$, $\widetilde{\mathbf{V}}_\mu$, $\widetilde{\mathbf{W}}_\mu$ and $\widetilde{\mathbf{B}}_\mu$ into the operators of the high-energy basis in Eq. (4.32), $\mathscr{L}_{\text{low}}$ is obtained for the $SU(3)/(SU(2) \times U(1))$ model as a function of the SM would-be GBs and the light physical Higgs $\varphi$.

#### 6.2.2.1 The two-derivative low-energy projection

The low-energy projection of this CH model, where the custodial symmetry is not approximately conserved, underlines some peculiarities that can be already seen in the resulting expression for the dimension-two operator $\widetilde{\mathcal{A}}_C$:

$$\widetilde{\mathcal{A}}_C = -\frac{f^2}{4} \mathrm{Tr}(\widetilde{\mathbf{V}}_\mu \widetilde{\mathbf{V}}^\mu) = \mathcal{P}_H + \frac{4}{\xi} \sin^2\left[\frac{\varphi}{2f}\right] \mathcal{P}_C + \frac{2}{\xi} \sin^4\left[\frac{\varphi}{2f}\right] \mathcal{P}_T. \tag{6.13}$$

It projects at low-energy not only into the $h$ and GBs kinetic terms as expected, but also into the two-derivative custodial violating operator $\mathcal{P}_T$ in Eq. (3.62).

Alike to the situation for the models previously studied, $\widetilde{\mathcal{A}}_C$ contains the term that describes the masses of the gauge bosons once the EW symmetry is broken. Requiring consistency with the definition of the $W$-mass, the link given in Eq. (5.16) among the EW scale $v$, the Higgs VEV $\langle \varphi \rangle$ and the strong dynamic scale $f$ also follows here.

### 6.2.2.2   The four-derivative low-energy projection

The low-energy projection of the four-derivative operators listed in Eq. (4.32) results in the following decomposition for the $SU(3)/(SU(2) \times U(1))$ model:

$$
\begin{aligned}
\widetilde{\mathcal{A}}_B =& \frac{2}{3} \mathcal{P}_B \,, \\
\widetilde{\mathcal{A}}_W =& \mathcal{P}_W \,, \\
\widetilde{\mathcal{A}}_{B\Sigma} =& -\frac{g'^2}{6}\left(1 + 3\cos\left[\frac{2\varphi}{f}\right]\right)\mathcal{P}_B \,, \\
\widetilde{\mathcal{A}}_{W\Sigma} =& -2\,g^2\,\cos\left[\frac{\varphi}{f}\right]\mathcal{P}_W + \sin^4\left[\frac{\varphi}{2f}\right]\mathcal{P}_{12} \,, \\
\widetilde{\mathcal{A}}_1 =& \frac{1}{4}\sin^2\left[\frac{\varphi}{f}\right]\mathcal{P}_1 \,, \\
\widetilde{\mathcal{A}}_2 =& \frac{1}{4}\sin^2\left[\frac{\varphi}{f}\right]\mathcal{P}_2 + \frac{\sqrt{\xi}}{2}\sin\left[\frac{2\varphi}{f}\right]\mathcal{P}_4 \,, \\
\widetilde{\mathcal{A}}_3 =& \frac{1}{2}\sin^2\left[\frac{\varphi}{f}\right]\mathcal{P}_3 - 2\sqrt{\xi}\sin\left[\frac{\varphi}{f}\right]\mathcal{P}_5 + 2\sin^4\left[\frac{\varphi}{2f}\right]\mathcal{P}_{13} + 2\sqrt{\xi}\sin\left[\frac{\varphi}{f}\right]\sin^2\left[\frac{\varphi}{2f}\right]\mathcal{P}_{17} \,, \\
\widetilde{\mathcal{A}}_4 =& 4\,\xi^2\,\mathcal{P}_{DH} + 16\sin^4\left[\frac{\varphi}{2f}\right]\mathcal{P}_6 - 16\,\xi\sin^2\left[\frac{\varphi}{2f}\right]\mathcal{P}_{20} + 8\,\xi\sin^4\left[\frac{\varphi}{2f}\right]\mathcal{P}_{21}+ \\
& -16\sin^6\left[\frac{\varphi}{2f}\right]\mathcal{P}_{23} + 4\sin^8\left[\frac{\varphi}{2f}\right]\mathcal{P}_{26} \,, \\
\widetilde{\mathcal{A}}_5 =& 4\,\xi^2\,\mathcal{P}_{DH} - 16\,\xi\sin^2\left[\frac{\varphi}{2f}\right]\mathcal{P}_8 + 16\sin^4\left[\frac{\varphi}{2f}\right]\mathcal{P}_{11} + 8\,\xi\sin^4\left[\frac{\varphi}{2f}\right]\mathcal{P}_{22}+ \\
& -16\sin^6\left[\frac{\varphi}{2f}\right]\mathcal{P}_{24} + 4\sin^8\left[\frac{\varphi}{2f}\right]\mathcal{P}_{26} \,, \\
\widetilde{\mathcal{A}}_6 =& -2\,\xi\,\mathcal{P}_{\Box h} - \frac{1}{2}\sin^2\left[\frac{\varphi}{f}\right]\mathcal{P}_6 - 2\sqrt{\xi}\sin\left[\frac{\varphi}{f}\right](\mathcal{P}_7 - 2\mathcal{P}_{10}) + 4\xi\cos^2\left[\frac{\varphi}{2f}\right]\mathcal{P}_8+ \\
& +4\sin^2\left[\frac{\varphi}{2f}\right]\mathcal{P}_9 - 2\sin^4\left[\frac{\varphi}{2f}\right](\mathcal{P}_{15} - 2\mathcal{P}_{16}) - 2\xi\left(1 + 2\cos\left[\frac{\varphi}{f}\right]\right)\sin^2\left[\frac{\varphi}{2f}\right]\mathcal{P}_{22}+ \\
& +2\sqrt{\xi}\sin\left[\frac{\varphi}{f}\right]\sin^2\left[\frac{\varphi}{2f}\right](\mathcal{P}_{18} - 2\mathcal{P}_{19} + \mathcal{P}_{25}) + \sin^2\left[\frac{\varphi}{f}\right]\sin^2\left[\frac{\varphi}{2f}\right]\mathcal{P}_{23}+ \\
& -4\sin^6\left[\frac{\varphi}{2f}\right]\mathcal{P}_{24} + 2\sin^8\left[\frac{\varphi}{2f}\right]\mathcal{P}_{26} \,, \\
\widetilde{\mathcal{A}}_7 =& 2\,\xi^2\,\mathcal{P}_{DH} + 8\sin^4\left[\frac{\varphi}{2f}\right]\mathcal{P}_6 - 4\xi\sin^2\left[\frac{\varphi}{2f}\right]\mathcal{P}_8 - 2\sqrt{\xi}\sin\left[\frac{\varphi}{f}\right]\sin^2\left[\frac{\varphi}{2f}\right]\mathcal{P}_{18}+ \\
& -4\xi\sin^2\left[\frac{\varphi}{2f}\right]\mathcal{P}_{20} - 2\xi\cos\left[\frac{\varphi}{f}\right]\sin^2\left[\frac{\varphi}{2f}\right]\mathcal{P}_{21} + 2\xi\sin^2\left[\frac{\varphi}{2f}\right]\mathcal{P}_{22}- \\
& -2\left(3 - \cos\left[\frac{\varphi}{f}\right]\right)\sin^4\left[\frac{\varphi}{2f}\right]\mathcal{P}_{23} + \sin^2\left[\frac{\varphi}{f}\right]\sin^2\left[\frac{\varphi}{2f}\right]\mathcal{P}_{24} + 2\sin\left[\frac{\varphi}{2f}\right]^8\mathcal{P}_{26}
\end{aligned}
\tag{6.14}
$$

The remaining operator in the list in Eq. (4.32) is not independent in this case, as it can be expressed as the combination

$$\widetilde{\mathcal{A}}_8 = \frac{1}{2}\widetilde{\mathcal{A}}_4 + \widetilde{\mathcal{A}}_5 - 2\widetilde{\mathcal{A}}_7 , \tag{6.15}$$

which in summary implies that the low-energy physical consequences of this model depend on nine arbitrary coefficients.

### 6.2.2.3 The low-energy effective CP-odd EW chiral Lagrangian

The low-energy Lagrangian $\mathscr{L}_{\text{low},\cancel{CP}}$ is obtained by substituting the explicit expressions for $\mathbf{\Sigma}(x)$, $\widetilde{\mathbf{V}}_\mu$, $\widetilde{\mathbf{W}}_\mu$ and $\widetilde{\mathbf{B}}_\mu$ in the operators of the high-energy basis in Eq. (4.32). The low-energy projection of the four-derivative operators listed in Eq. (4.32) results in the following decomposition for the $SU(3)/(SU(2) \times U(1))$ model[1]:

$$
\begin{aligned}
\widetilde{\mathcal{B}}_{WW^*} &= \frac{1}{2}\mathcal{S}_{WW^*} , \\
\widetilde{\mathcal{B}}_{B\Sigma^*} &= -\frac{1}{6}\left(1 + 3\cos\left[\frac{2\varphi}{f}\right]\right)\mathcal{S}_{BB^*} , \\
\widetilde{\mathcal{B}}_{W\Sigma^*} &= -2\cos\left[\frac{\varphi}{f}\right]\mathcal{S}_{WW^*} + \frac{1}{2}\sin^4\left[\frac{\varphi}{2f}\right]\mathcal{S}_8 , \\
\widetilde{\mathcal{B}}_1 &= \frac{1}{8}\sin^2\left[\frac{\varphi}{f}\right]\mathcal{S}_1 , \\
\widetilde{\mathcal{B}}_2 &= \frac{1}{4}\sin^4\left[\frac{\varphi}{f}\right](\mathcal{S}_{BB^*} - \mathcal{S}_{WW^*}) + \frac{1}{4}\sqrt{\xi}\cos\left[\frac{\varphi}{f}\right]\sin^3\left[\frac{\varphi}{f}\right](\mathcal{S}_2 + 2\mathcal{S}_3) , \\
\widetilde{\mathcal{B}}_3 &= 0 .
\end{aligned}
\tag{6.16}
$$

Notice, in particular, the presence of the custodial violating operator $\mathcal{S}_8$ in the decomposition of the $\widetilde{\mathcal{B}}_{W\Sigma^*}$ operator, that corresponds to a tree-level source of custodial symmetry breaking. Notice however that it does not contribute to the $T$ parameter and therefore no constraint can be put on its coefficient.

## 6.3 Matching the high- and the low-energy Lagrangians

Following the same procedure as in Section 5.2 for the Georgi-Kaplan model, we match the low-energy lagrangian of the $SO(5)/SO(4)$ and $SU(3)/(SU(2) \times U(1))$ models with the general $\mathscr{L}_{low}$ basis.

---

[1]This projection is consistent with the one in Ref. [34] in Eq. (6.8), where a typo is present on the first two operators: the correct values are $\widetilde{\mathcal{A}}_B = \mathcal{P}_B/6$, and $\widetilde{\mathcal{A}}_W = \mathcal{P}_W/2$.

### 6.3.1   The $SO(5)/SO(4)$ model

As it was shown in section 6.1.2, the low-energy projection of the MCHM lagrangian is exactly the same as the one for the Georgi-Kaplan model in the situation where 10 of the GB's associated to the $SU(5)/SO(5)$ are heavy and only the Higgs scalar is light. Therefore, the expressions for the $c_i \mathcal{F}(h)$ and $d_I \mathcal{E}(h)$ products for the $SO(5)/SO(4)$ model are those reported in Tab. 5.1 and Tab. 5.2. Thus, all the comments done in regards with those tables in Section 5.2 apply to the matching of the MCHM model as well.

### 6.3.2   The $SU(3)/(SU(2) \times U(1))$ model

The $\mathcal{F}_C(h)$ and $\mathcal{F}_H(h)$ functions of the two-derivative low-energy chiral Lagrangian Eq. (3.65) stemming from the high-energy $SU(3)/(SU(2) \times U(1))$ model turn out to be

$$\mathcal{F}_C(h) = \frac{4}{\xi} \sin^2 \left[ \frac{\varphi}{2f} \right] , \qquad \mathcal{F}_H(h) = 1 , \qquad (6.17)$$

for the custodial preserving sector, and thus equal to that for $SU(5)/SO(5)$ and $SO(5)/SO(4)$ in Eq. (5.21). This suggests that they are universal for composite models in which the Higgs is embedded as a $SU(2)_L$ doublet. For the custodial breaking sector, instead, it results

$$c_T \mathcal{F}_T(h) = \frac{2}{\xi} \sin^4 \left[ \frac{\varphi}{2f} \right] , \qquad (6.18)$$

and in this case the coefficient $c_T$ is not a free parameter, but is fixed by the high-energy operator $\widetilde{\mathcal{A}}_C$. In consequence, the experimental bounds on the $T$ parameter [42] translate into strong constraints on the parameter $\xi$ and on the strong dynamics scale $f$:

$$\alpha_{\text{em}} \Delta T = \frac{\xi}{4} \qquad \Longrightarrow \qquad \xi \lesssim 0.014 , \qquad f \gtrsim 2 \, \text{TeV} . \qquad (6.19)$$

For the terms in the four-derivative Lagrangian, the expressions for the products $c_i \, \mathcal{F}_i(h)$ corresponding to custodial invariant operators are reported in Tab. 6.1 (third column), while those corresponding to custodial-breaking ones are collected in Tab. 6.2.

Contrary to the case of the two models previously analysed, all custodial preserving and all custodial breaking operators entering the low-energy Lagrangian $\mathscr{L}_{\text{low}}$ in Eq. (3.65) are generated from the high-energy one for the $SU(3)/(SU(2) \times U(1))$ CH model, with the exception of the operator $\mathcal{P}_{\Delta H}$ in Eq. (3.59) and $\mathcal{F}_H(h)$, $\mathcal{F}_{\Box H}$ and $\mathcal{F}_{DH}$. On the other side, also in this case the a priori many arbitrary combinations $c_i \mathcal{F}_i(h)$ can be written in terms of the small set of nine

| $c_i \mathcal{F}_i(h)$ | $SU(3)/SU(2) \times U(1)$ | $c_i \mathcal{F}_i(h)$ | $SU(3)/SU(2) \times U(1)$ |
|---|---|---|---|
| $\mathcal{F}_C(h)$ | $\frac{4}{\xi} \sin^2 \frac{\varphi}{2f}$ | $c_4 \mathcal{F}_4(h)$ | $\frac{\tilde{c}_2}{2} \sqrt{\xi} \sin \frac{2\varphi}{f}$ |
| $\mathcal{F}_H(h)$ | $1$ | $c_5 \mathcal{F}_5(h)$ | $-2\tilde{c}_3 \sqrt{\xi} \sin \frac{\varphi}{f} 2\tilde{c}_6 \xi \sin^2 \frac{\varphi}{2f} \left(1 + 2\cos\frac{\varphi}{f}\right)$ |
| $\mathcal{F}_B(h)$ | $1 - g'^2 \frac{\tilde{c}_{B\Sigma}}{6} \left(1 + 3\cos\frac{2\varphi}{f}\right)$ | $c_6 \mathcal{F}_6(h)$ | $8(2\tilde{c}_4 + \tilde{c}_7) \sin^4 \frac{\varphi}{2f} - \frac{1}{2}\tilde{c}_6 \sin^2 \frac{\varphi}{f}$ |
| $\mathcal{F}_W(h)$ | $1 - 2g^2 \tilde{c}_{W\Sigma} \cos \frac{\varphi}{f}$ | $c_7 \mathcal{F}_7(h)$ | $-2\tilde{c}_6 \sqrt{\xi} \sin \frac{\varphi}{f}$ |
| $c_{\Box H} \mathcal{F}_{\Box H}(h)$ | $-2\tilde{c}_6 \xi$ | $c_8 \mathcal{F}_8(h)$ | $-4(4\tilde{c}_5 + \tilde{c}_7)\xi \sin^2 \frac{\varphi}{2f} + 4\tilde{c}_6 \xi \cos^2 \frac{\varphi}{2f}$ |
| $c_{\Delta H} \mathcal{F}_{\Delta H}(h)$ | $-$ | $c_9 \mathcal{F}_9(h)$ | $4\tilde{c}_6 \sin^2 \frac{\varphi}{2f}$ |
| $c_{DH} \mathcal{F}_{DH}(h)$ | $2 \left(2\tilde{c}_4 + 2\tilde{c}_5 + \tilde{c}_7\right) \xi^2$ | $c_{10} \mathcal{F}_{10}(h)$ | $4\tilde{c}_6 \sqrt{\xi} \sin \frac{\varphi}{f}$ |
| $c_1 \mathcal{F}_1(h)$ | $\frac{\tilde{c}_1}{4} \sin^2 \frac{\varphi}{f}$ | $c_{11} \mathcal{F}_{11}(h)$ | $16\tilde{c}_5 \sin^4 \frac{\varphi}{2f}$ |
| $c_2 \mathcal{F}_2(h)$ | $\frac{\tilde{c}_2}{4} \sin^2 \frac{\varphi}{f}$ | $c_{20} \mathcal{F}_{20}(h)$ | $-4(4\tilde{c}_4 + \tilde{c}_7)\xi \sin^2 \frac{\varphi}{2f}$ |
| $c_3 \mathcal{F}_3(h)$ | $\frac{\tilde{c}_3}{2} \sin^2 \frac{\varphi}{f}$ | | |

TABLE 6.1: *Expressions for the products $c_i \mathcal{F}_i(h)$ for the CP-even, custodial preserving operators of the $SU(3)/(SU(2) \times U(1))$ CH model.*

high-energy parameters $\tilde{c}_i$.

In summary, a quite universal pattern is suggested by our results as to the form of the $c_i \mathcal{F}_i(h)$ functions, at least for the custodial preserving sector. Tab. 6.1 encompasses the main results and allows a direct comparison of the low-energy impact of the models considered (as well as of the BSM physics expected from linear realisations of EWSB). Not only $\mathcal{F}_C(h)$ coincides exactly for all three chiral models considered, see Eqs. (5.21) and (6.17), but the $c_i \mathcal{F}_i(h)$ functions for *all* four-derivative chiral operators do as well, except for the couplings which involve gauge field-strengths for which the intrinsically custodial-invariant groups and $SU(3)/(SU(2) \times U(1))$ differ simply by a rescaling of the scale $f$ and multiplicative factors, see Tab. 6.1.

The functions, appearing in the low-energy basis in each of the CH models considered, encode the dependence on the $h$ field: they turn out to be trigonometric due to the GB nature of the Higgs field in these setups. In the general $\mathscr{L}_{\text{low,CP}}$ basis, this dependence is encoded into the generic functions $\mathcal{F}_i(h)$ in Eq. (3.65) and into some operators which contain derivatives of $h$. It is then possible to identify the products $c_i \mathcal{F}_i(h)$ in terms of the high-energy parameters, by comparing

| $c_i\mathcal{F}_i(h)$ | $SU(3)/(SU(2)\times U(1))$ | $c_i\mathcal{F}_i(h)$ | $SU(3)/(SU(2)\times U(1))$ |
|---|---|---|---|
| $c_T\mathcal{F}_T(h)$ | $\frac{2}{\xi}\sin^4\frac{\varphi}{2f}$ | $c_{21}\mathcal{F}_{21}(h)$ | $8\tilde{c}_4\xi\sin^4\frac{\varphi}{2f} - 2\tilde{c}_7\xi\cos\frac{\varphi}{f}\sin^2\frac{\varphi}{2f}$ |
| $c_{12}\mathcal{F}_{12}(h)$ | $\tilde{c}_{W\Sigma}\sin^4\frac{\varphi}{2f}$ | $c_{22}\mathcal{F}_{22}(h)$ | $8\tilde{c}_5\xi\sin^4\frac{\varphi}{2f} + 2\xi\tilde{c}_7\sin^2\frac{\varphi}{2f} - 2\tilde{c}_6\xi\sin^2\frac{\varphi}{2f}\left(1+2\cos\frac{\varphi}{f}\right)$ |
| $c_{13}\mathcal{F}_{13}(h)$ | $2\tilde{c}_3\sin^4\frac{\varphi}{2f}$ | $c_{23}\mathcal{F}_{23}(h)$ | $-16\tilde{c}_4\sin^6\frac{\varphi}{2f} + \tilde{c}_6\sin^2\frac{\varphi}{2f}\sin^2\frac{\varphi}{f} + 2\tilde{c}_7\sin^4\frac{\varphi}{2f}\left(\cos\frac{\varphi}{f}-3\right)$ |
| $c_{15}\mathcal{F}_{15}(h)$ | $-2\tilde{c}_6\sin^4\frac{\varphi}{2f}$ | | |
| $c_{16}\mathcal{F}_{16}(h)$ | $4\tilde{c}_6\sin^4\frac{\varphi}{2f}$ | $c_{24}\mathcal{F}_{24}(h)$ | $-4(4\tilde{c}_5+\tilde{c}_6)\sin^6\frac{\varphi}{2f} + \tilde{c}_7\sin^2\frac{\varphi}{2f}\sin^2\frac{\varphi}{f}$ |
| $c_{17}\mathcal{F}_{17}(h)$ | $2\tilde{c}_3\sqrt{\xi}\sin^2\frac{\varphi}{2f}\sin\frac{\varphi}{f}$ | $c_{25}\mathcal{F}_{25}(h)$ | $2\tilde{c}_6\sqrt{\xi}\sin^2\frac{\varphi}{2f}\sin\frac{\varphi}{f}$ |
| $c_{18}\mathcal{F}_{18}(h)$ | $2(\tilde{c}_6-\tilde{c}_7)\sqrt{\xi}\sin^2\frac{\varphi}{2f}\sin\frac{\varphi}{f}$ | $c_{26}\mathcal{F}_{26}(h)$ | $2(2(\tilde{c}_4+\tilde{c}_5)+\tilde{c}_6+\tilde{c}_7)\sin^8\frac{\varphi}{2f}$ |
| $c_{19}\mathcal{F}_{19}(h)$ | $-4\tilde{c}_6\sqrt{\xi}\sin^2\frac{\varphi}{2f}\sin\frac{\varphi}{f}$ | | |

TABLE 6.2: *Expressions for the products $c_i\,\mathcal{F}_i(h)$ for the CP-even custodial symmetry breaking operators of $SU(3)/(SU(2)\times U(1))$ CH model. No analogous contributions are present for the $SU(5)/SO(5)$ and $SO(5)/SO(4)$ model.*

the low-energy EW chiral Lagrangian of the specific CH models and the general $\mathscr{L}_{\text{low},\cancel{CP}}$. This is useful to point out specific correlations between couplings that could help investigating the nature of the EWSB mechanism [33, 39, 40].

Table 6.3 reports the expression of the products $c_i\,\mathcal{F}_i(h)$ for the three distinct CH setups considered before, only for the operators of the low-energy basis that indeed receive contributions.

| $d_i\mathcal{E}_i(h)$ | $SU(3)/(SU(2)\times U(1))$ |
|---|---|
| $\mathcal{E}_{BB^*}(h)$ | $4\,\tilde{d}_{B\Sigma^*}\sin^2\frac{\varphi}{2f}\cos^2\frac{\varphi}{2f} + 4\,\tilde{d}_2\sin^4\frac{\varphi}{2f}\cos^4\frac{\varphi}{2f}$ |
| $\mathcal{E}_{WW^*}(h)$ | $\frac{\tilde{d}_{WW^*}}{2} - 2\tilde{d}_{W\Sigma^*}(1+2\cos^2\frac{\varphi}{2f}) - 4\,\tilde{d}_2\sin^4\frac{\varphi}{2f}\cos^4\frac{\varphi}{2f}$ |
| $d_1\mathcal{E}_1(h)$ | $\frac{\tilde{d}_1}{2}\sin^2\frac{\varphi}{2f}\cos^2\frac{\varphi}{2f}$ |
| $d_2\mathcal{E}_2(h)$ | $2\,\tilde{d}_2\sqrt{\xi}\cos^3\frac{\varphi}{2f}\sin^3\frac{\varphi}{2f}\left(2\cos^2\frac{\varphi}{2f}-1\right)$ |
| $d_3\mathcal{E}_3(h)$ | $4\,\tilde{d}_2\sqrt{\xi}\cos^3\frac{\varphi}{2f}\sin^3\frac{\varphi}{2f}\left(2\cos^2\frac{\varphi}{2f}-1\right)$ |
| $d_8\mathcal{E}_8(h)$ | $\frac{\tilde{d}_{W\Sigma^*}}{2}\sin^4\frac{\varphi}{2f}$ |

TABLE 6.3: *Expressions for the products $d_i\,\mathcal{E}_i(h)$ for the CP-odd $SU(3)/(SU(2)\times U(1))$ operators. Notice that $d_8\mathcal{E}_8(h)$ corresponds to a custodial breaking operator.*

From Tab. 6.3 it can be inferred that, besides the custodial preserving operators, only one custodial breaking operator of the low-energy basis, $\mathcal{S}_8$, receives contributions from the projection. As for the previous CH models, the interactions described by $\mathcal{S}_2$ and $\mathcal{S}_3$ turn out to be correlated. Again, notice that the arbitrary functions $\mathcal{F}_i(h)$ of the generic low-energy effective chiral Lagrangian become now a constrained set, as a consequence of having chosen a specific CH model.

## 6.4 Comparison with other basis

A remarkable difference between the high-energy basis in the CP-even and CP-odd cases is that in the latter there is an operator, namely $\widetilde{\mathcal{B}}_2$, which requires the introduction of the custodially breaking operator $\widetilde{\mathbf{T}}$, even if in a particular combination so that the low-energy projection is custodially preserving. Notice that, without the inclusion of this operator, the CP-odd low-energy projections of $SU(5)/SO(5)$ and $SO(5)/SO(4)$ do not get contributions to the operators $\mathcal{S}_2$ or $\mathcal{S}_3$. This would be suspicious, since in the CP-even case all possible custodially preserving operators get contributions in the decomposition. The definite proof that $\widetilde{\mathcal{B}}_2$, and thus $\widetilde{\mathbf{T}}$, are needed came from comparing our results with those of [43]; using the CCWZ formalism, briefly presented in Appendix A, the CP-odd basis is given by

$$
\begin{aligned}
O_{--,1} &= \epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}\left([d_\mu,d_\nu]_L[d_\rho,d_\sigma]_L - [d_\mu,d_\nu]_R[d_\rho,d_\sigma]_R\right) \\
O_{--,2} &= i\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}\left((E^L_{\mu\nu} - E^R_{\mu\nu})[d_\rho,d_\sigma]\right) \\
O_{-+} &= 4\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}\left(\nabla_\mu d_\nu \nabla_\rho d_\sigma\right)
\end{aligned}
$$

The decomposition of the CCWZ CP-odd basis in terms of the low-energy chiral lagrangian is the following:

$$
\begin{aligned}
O_{--,1} &= \frac{1}{2}\left(9\cos\left(\frac{\varphi}{2f}\right) - \cos\left(\frac{3\varphi}{2f}\right)\right)(S_{BB*} - S_{WW*}) - \frac{3}{2}\sqrt{\xi}\sin^3\left(\frac{\varphi}{2f}\right)\left(\frac{S_2}{2} + S_3\right) \\
O_{--,2} &= \frac{1}{8}\left(34\cos\left(\frac{\varphi}{2f}\right) - 2\cos\left(\frac{3\varphi}{2f}\right)\right)(S_{BB*} - S_{WW*}) - \frac{3}{4}\sqrt{\xi}\sin^3\left(\frac{\varphi}{2f}\right)\left(\frac{S_2}{2} + S_3\right) \\
O_{-+} &= -2\sin^2\left(\frac{\varphi}{2f}\right)\left(S_{BB*} + S_{WW*} + \frac{1}{4}S_1\right)
\end{aligned}
$$

In order to check this non-trivial decomposition we do the following consistency: consider the operator

$$
O_6^- = i\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}\left((f^L_{\mu\nu} - f^R_{\mu\nu})[d_\rho,d_\sigma]\right) \tag{6.20}
$$

The decomposition is

$$O_6^- = -\frac{1}{2} \sin\left(\frac{\varphi}{2f}\right) \sin\left(\frac{\varphi}{f}\right) (S_{BB*} - S_{WW*}) + \frac{3}{4}\sqrt{\xi} \sin^3\left(\frac{\varphi}{2f}\right) \left(\frac{S_2}{2} + S_3\right)$$

Taking into account $E_{\mu\nu} = f_{\mu\nu}^+ - i[d_\mu, d_\nu]$, we find that $O_6^- = O_{--,2} - O_{--,1}$. Let's check this:

$$O_{--,2} - O_{--,1} = -\frac{1}{4}\left(\cos\left(\frac{\varphi}{2f}\right) - \cos\left(\frac{3\varphi}{2f}\right)\right)(S_{BB*} - S_{WW*}) + \frac{3}{4}\sqrt{\xi}\sin^3\left(\frac{\varphi}{2f}\right)\left(\frac{S_2}{2} + S_3\right) =$$

$$= -\frac{1}{2}\sin\left(\frac{\varphi}{2f}\right)\sin\left(\frac{\varphi}{f}\right)(S_{BB*} - S_{WW*}) + \frac{3}{4}\sqrt{\xi}\sin^3\left(\frac{\varphi}{2f}\right)\left(\frac{S_2}{2} + S_3\right) = O_6^-$$

Therefore, it is clear that indeed the CCWZ operators give contribution to the $\mathcal{S}_2$ and $\mathcal{S}_3$ operators. This was key to discover that the custodially breaking high-energy building block $\widetilde{\mathbf{T}}$ was necessary.

## 6.5 Conclusions

In this first part of the thesis we have presented a systematic way of constructing the effective chiral lagrangian for a generic CH model, characterized by a coset $\mathcal{G}/\mathcal{H}$, analyzing the bosonic operators with at most four derivatives: seven independent operators (apart from the kinetic term) in the CP-even case and four in the CP-odd one. Restricting the gauge group to $SU(2)_L \times U(1)_Y$ and considering the gauging of the hypercharge as the only source of custodial symmetry breaking, the CP-even basis amounts to ten operators, with the CP-odd one having six, including the topological term, a result that is independent of the specific choice of $\mathcal{G}$ or of the representation of $SU(2)_L$ to which the Higgs particle belongs to, as long as it is a Goldstone boson.

This procedure was then particularized in Chapters 5 and 6 to the $SU(5)/SO(5)$, $SO(5)/SO(4)$ and $SU(3)/(SU(2)\times U(1))$ cosets. The specific algebra of each model leads to relationships among some operators, such as in the $SU(5)/SO(5)$ CP-even case, where the independent number of operators is reduced to nine. It was also pointed out that, in the case that the only light GBs in the Georgi-Kaplan is the Higgs scalar, its low-energy projection coincides with the one of $SO(5)/SO(4)$, since in this case the preserved subgroups in the two models turn out to be isomorphic. It was shown in Chapter 5 that indeed, in the particular realization of the original Georgi-Kaplan model, where an additional $U(1)_A$ group is gauged, there is a part of the parameter space where indeed the Higgs scalar is much lighter than all the rest of the Goldstones.

# Chapter 7

# Higgs portal dark matter and neutrino mass and mixing with a doubly charged scalar

In this chapter we shall focus on a particularly economical loop model of Majorana neutrino mass and mixing [44], in which the low energy effective theory involves just one extra new particle: a doubly charged EW singlet scalar $S$ (denoting both $S^{++}$ and its antiparticle $S^{--}$). It is already known that such a model can lead to an interesting complementarity between low energy charged lepton flavour violation processes, and high energy collider physics, depending on whether the doubly charged scalar $S$ appears as a virtual or real particle [45]. However such a model cannot account for DM, since the doubly charged scalar $S$ decays promptly into either pairs of like-sign charged leptons or $W$ bosons. Here we shall extend the model slightly by introducing an additional neutral scalar $\phi$ and assume an unbroken $Z_2$ symmetry in the scalar sector, under which only the additional neutral scalar $\phi$ is odd, which then becomes a stable DM candidate. The model may be regarded as an extension of the so-called Higgs portal scenario [46], in the presence of a doubly charged scalar which accounts for neutrino mass and mixing. The resulting framework presented here, involving both $S$ and $\phi$, then merges two apparently unrelated features: the existence of a new physics sector at the TeV scale, providing naturally small neutrino masses, and the existence of a good DM candidate.

# 7.1 Dark matter

Astronomers have long known that galaxies and clusters would fly apart unless they were held together by the gravitational pull of much more material than we actually see. The strength of the case built up gradually. The argument that clusters of galaxies would be unbound without dark matter dates back to Zwicky (1937) and others in the 1930s. Kahn and Woltjer (1959) pointed out that the motion of Andromeda towards us implied that there must be dark matter in our Local Group of galaxies. But the dynamical evidence for massive halos (or coronae) around individual galaxies firmed up rather later (e.g. Roberts and Rots 1973, Rubin, Thonnard and Ford 1978).

The amount of dark matter, and how it is distributed, is now far better established than it was when those papers were written. The immense advances in delineated dark matter in clusters and in individual galaxies are manifest in the programme for this meeting. The rapid current progress stems from the con- fluence of several new kinds of data within the same few-year interval: optical surveys of large areas and high redshifts, CMB fluctuation measurements, sharp X-ray images, and so forth. The progress has not been solely observational. Over the last 20 years, a compelling theoretical perspective for the emergence of cosmic structure has been developed. The expanding universe is unstable to the growth of structure, in the sense that regions that start off very slightly overdense have their expansion slowed by their excess gravity, and evolve into conspicuous density contrasts. According to this cold dark matter (CDM) model, the present-day structure of galaxies and clusters is moulded by the gravitational aggregation of nonbaryonic matter, which is an essential ingredient of the early universe (Pagels and Primack 1982, Peebles 1982, Blumenthal et al. 1984, Davis et al. 1985). These models have been firmed up by vastly improved simulations, rendered possible by burgeoning computer power. And astronomers can now compare these virtual universes with the real one, not just at the present era but (by observing very distant objects) can probe back towards the formative stages when the first galaxies emerged.

## 7.1.1 Case for WIMPs

All these observations require new fundamental particle physics, since no particle with the required properties to be DM has been detected so far. Weakly Interacting Massive Particles (WIMPs ) form a particularly interesting generic class of new-particle candidates because they naturally provide about the inferred amount of this nonbaryonic dark matter, a result dubbed the WIMP miracle. WIMPs would be produced thermally in the early Universe. Because they interact only weakly, their annihilation rate would become insignifi- cant as the Universe expands, thus freezing out a relic abundance of the particles. The expected WIMP density would be the same as that

of the nonbaryonic dark matter if the WIMP velocity-averaged annihilation cross section is 1 pb, so that the WIMP mass is 100 GeV, which roughly coincides with the EW scale.

Whatever physics solves the hierarchy problem associated with this symmetry breaking gives rise to additional particles. If an appropriate (often independently motivated) discrete symmetry exists, the lightest such particle is stable. This particle is then weakly interacting, massive, and stable it is a WIMP. Thus, particle theorists are almost justified in saying that the problem of electroweak symmetry breaking predicts the existence of WIMP dark matter. Although the argument for WIMP dark matter is generic, supersymmetry dominates the discussion as a particularly well-motivated model. They may be produced and detected (indirectly) at accelerators such as the Large Hadron Collider. Relic WIMPs may be detected indirectly when they clump in massive astrophysical objects, increasing their annihilation rate enough that their annihilation products may be detectable- Many potential indirect signals are ambiguous, with alternate astrophysical explanations. Some potential indirect signals, however, would be compelling. Annihilation in the Sun or Earth would produce higher-energy neutrinos than any other known process. These neutrinos could be observed in neutrino telescopes such as IceCube or ANTARES. Either FERMI or ground-based air Cerenkov telescopes may detect distinctive gamma-ray features from the galactic center or from sub-halos. Relic WIMPs may also be detected directly when they scatter off nuclei in terrestrial detectors.

This chapter will introduce a scalar singlet WIMP in the context of the so-called Higgs portal scenario, where the only direct coupling of the dark matter particle is through SM singlet operator $|HH^{\dagger}|$.

## 7.2 Neutrino masses

The existence of neutrinos was first proposed by Pauli in 1930, a desperate remedy in order to solve the missing energy problem in beta decays. The Super-Kamiokande experiment in 1998 showed that neutrinos oscillate, and therefore have mass. It observed a deficit of muon neutrinos produced when cosmic rays impact the atmosphere. The oscillation hypothesis implied that some of these muon neutrinos where oscillating into undetected tau neutrinos. However,neutrinos are always left-handed in the SM, unlike quarks and charged leptons, and are therefore predicted to be massless. If right-handed neutrinos were to be added to the Standard Model, then neutrinos could have a Dirac mass, and the theory would also predict the existence of antineutrinos. Another possibility is that neutrinos are their own antiparticle, so that they would have a Majorana mass term.

Explaining the BSM origin of neutrino masses has attracted lots of theoretical efforts. A popular solution is the see-saw mechanism [47, 48], which introduces heavy Majorana right-handed neutrinos. This thesis will focus on loop neutrino mass models, where loops of additional heavy states induce a small mas to neutrinos. They often characterised by additional Higgs doublets and singlets. These extra scalar states can in principle be detected indirectly, via low energy high precision experiments due to their contribution to charged lepton flavour violation (LFV) or neutrinoless double beta decay ($0\nu\beta\beta$), providing a test of the underlying theory of neutrino mass. For example, in the original Zee-Babu model [49–51], involving one singly and one doubly charged extra scalar singlet, neutrino masses arise via a two-loop diagram. The loop model of Ma [52] involves an inert Higgs doublet, odd under a discrete symmetry, which does not develop a vacuum expectation value (VEV) but has Yukawa couplings to some of the leptons (involving right-handed neutrinos) and in turn couples to another Higgs doublet which gets a VEV, allowing neutrino mass via a one-loop diagram. The inert Higgs doublet is a Dark Matter candidate, hence the name Scotogenic [52]. More recently a Cocktail of the Zee-Babu and Ma models has been proposed [53, 54] involving an extra inert Higgs doublet and a doubly charged Higgs singlet but no right-handed neutrinos, where neutrino masses arise due to a three-loop diagram involving also W-bosons. In summary, although such loop models do provide a natural explanation for the smallness of neutrino mass and are phenomenologically rich, having predictions for LFV as well new Higgs discovery at the LHC [55, 56], they do involve rather many new particles and parameters and are rather computationally complicated, as compared for example to seesaw models.

## 7.3 The effective model with a doubly charged scalar

In this section we review the effective Lagrangian model presented in [44], in which the SM is extended by adding one new scalar particle: a complex $SU(2)_L$ singlet, hypercharge $Y = 2$ (hence electric charge $Q = 2$) state $S^{++}$ and its antiparticle $S^{--}$, both doubly charged and denoted in the following as $S$ and $S^\dagger$ respectively.

The doubly charged scalar field $S$ has an effective coupling to the SM $W^\pm$ bosons as well as to same-sign right-handed charged SM leptons, giving rise to a rich phenomenology. In addition to contributing to flavour violating leptonic processes, to leptonic dipole moments and to leptonic radiative decays, the scalar $S$ allows a 2-loop diagram which is responsible for providing all mass (and mixings) to neutrinos. It is shown in [44] that the lowest mass dimension at which the vertex $SWW$ can be realised is by effective operators of dimension $d = 7$. The relevant operator, in the
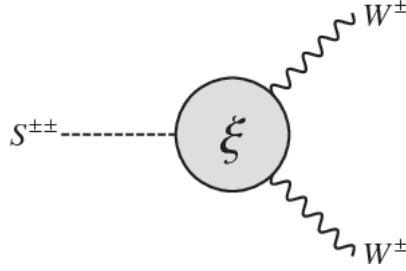
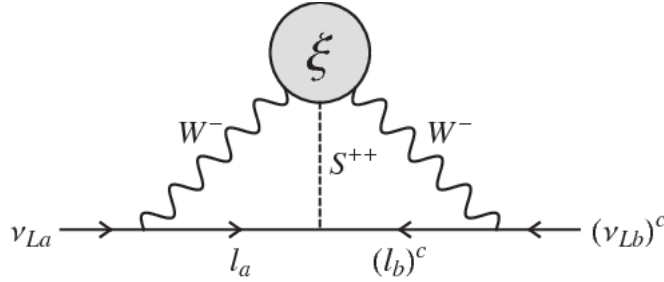FIGURE 7.1: Effective vertex that connects the doubly charged singlet scalar to two WW-bosons.



FIGURE 7.2: Two-loop diagram for the neutrino mass in the effective model.

unitary gauge, for the generation of neutrino masses is:

$$\mathscr{L}_{SWW} = -\frac{g^2 \xi v^4}{4\Lambda^3} \left( S\, W^\mu W_\mu + h.c. \right) \tag{7.1}$$

being $\xi$ an order $\mathcal{O}(1)$ dimensionless parameter and $\Lambda$ the new physics scale above which the effective theory breaks. The coupling of $S$ to same-sign RH leptons is given by

$$\mathscr{L}_{Sll} = f_{ab}\, S^\dagger\, \bar{l}_a P_L l_b^c + h.c. \tag{7.2}$$

with $f_{ab}$ dimensionless parameters. There are strong experimental constraints on the $f_{ab}$ parameter space, basically due to the flavour violating couplings of the charged scalar $S$ with leptons, the strongest bound proceeding from $\mu \to e\gamma$ and $\mu \to 3e$. A detailed analysis of these bounds can be found in [44, 45, 56].

The simultaneous presence of the $SWW$ and $Sll$ vertices generate a 2-loop contribution to the neutrino masses, that schematically can be written as

$$\mathcal{M}_\nu^{2-\text{loop}} = 2\,\xi\, f_{ab}(1 + \delta_{ab})\frac{m_a\, m_b\, m_S^2}{\Lambda^3}\, \tilde{\mathcal{I}}(m_W, m_S, \mu) \tag{7.3}$$

where $m_S$ is the S particle mass, $m_i$ is the mass of the $l_i$ lepton, $\delta_{ab}$ is the Kronecker delta and $\tilde{\mathcal{I}}(m_W, m_S, \mu)$ is the two loop integral calculated in [44]. Apart from the usual contribution to $0\nu\beta\beta$ due to massive neutrinos in presence of a lepton number violating interaction, this
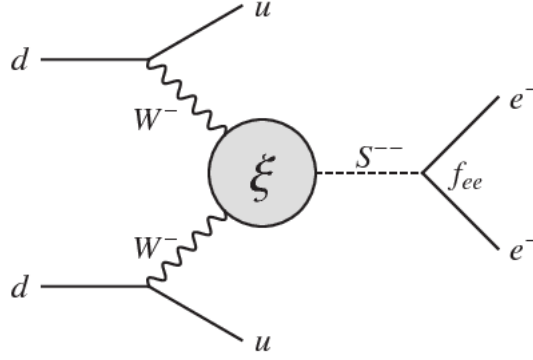
FIGURE 7.3: The non-standard contribution to $0\nu\beta\beta$.

model also produce an additional non-standard contribution to it, since the doubly charged scalar $S$ can couple both to $W^- W^-$ and $e^- e^-$. Taking into account the newest GERDA results of $T_{1/2}^{0\nu\beta\beta}(\text{Ge}) > 2.1 \cdot 10^{25}$ at 90% C.L., [57], one obtains

$$\frac{\xi f_{ee}}{M_S^2 \Lambda^3} < \frac{4.0 \cdot 10^{-3}}{\text{TeV}^5} \, . \tag{7.4}$$

In general it is not an easy task to fulfil all the flavour/dipole bounds and obtain a realistic description for neutrino masses and mixing compatible with the $0\nu\beta\beta$ decay bounds. In [44] a detailed analysis has been performed that highlighted the presence of three typical regions where this may happen, hereafter denoted as "Benchmark Scenarios":

1. Benchmark Scenario A: $f_{ee} \simeq 0$ and $f_{e\tau} \simeq 0$. In this region the additional contribution to the $0\nu\beta\beta$ essentially vanishes. A normal hierarchy between the neutrino masses with the lightest one around 5 meV is obtained;

2. Benchmark Scenario B: $f_{ee} \simeq 0$ and $f_{e\mu} \simeq -(f_{\mu\tau}^*/f_{\mu\mu}^*)f_{e\tau}$. In this region one still has a vanishing additional contribution to the $0\nu\beta\beta$ and a normal ordered neutrino masses with the lightest one around 5 meV. However the constraint relating $f_{e\mu}$ and $f_{e\tau}$ makes this scenario more predictive (falsifiable) in what concerns lepton flavour violation;

3. Benchmark Scenario C: $f_{e\mu} \simeq -(f_{\mu\tau}^*/f_{\mu\mu}^*)f_{e\tau}$. In this region one can assume large values for the $f_{ee}$ coupling. However not to enter in conflict with the GERDA limit on $0\nu\beta\beta$ of Eq. (7.4) one has to push the cutoff scale $\Lambda$ to several TeV, not a desirable thing from the collider phenomenology point of view.

For the analysis presented in the following sections we will use the best fit benchmark point for each of the three scenarios reported by [44]:

1. Benchmark Point A: $m_S = 164.5$ GeV, $\Lambda = 905.9$ GeV , $\xi = 5.02$;

2. Benchmark Point B: $m_S = 364.6$ GeV, $\Lambda = 2505.1$ GeV, $\xi = 6.38$;

3. Benchmark Point C: $m_S = 626.0$ GeV, $\Lambda = 5094.7$ GeV, $\xi = 3.39$.

## 7.4  Higgs portal DM with a doubly charged scalar

In order to account for DM, we now introduce a further particle into the scheme of the previous section, namely an electrically neutral real scalar $\phi$. An unbroken $\mathbb{Z}_2$ symmetry is assumed, under which the field $\phi$ is odd, while all the other particles are even. The motivation of such a setup is twofold: firstly, as already discussed, the presence of an extra doubly charged scalar can provide an economical mechanism for triggering light neutrino masses and mixing [44, 45, 51, 53]. Secondly, the new neutral scalar can account for DM. Possible UV completions of this model could be pursued along the lines of [51, 53]. Here we shall not try to construct an ultraviolet complete model, but continue to consider the effective theory below the cut-off $\Lambda$, where the theory has a rather minimal particle content, with the goal of understanding DM in this extended model. In particular, we shall discuss how the presence of an extended scalar sector can potentially modify the limits and the predictions obtained in the standard DM Higgs portal scenario [46]. In this section we study the DM signatures of the effective Lagrangian model described in the previous section.

The most general (renormalizable) scalar potential for the model at hand is given by

$$
\begin{aligned}
V &= \mu^2 |H^\dagger H| + \lambda |H^\dagger H|^2 + \frac{1}{2}\mu_\phi^2 \phi^2 + \frac{1}{4}\lambda_\phi \phi^4 + \mu_S^2 S^\dagger S + \lambda_S (S^\dagger S)^2 + \\
&\quad + \frac{1}{2}\lambda_{\phi H}\phi^2 \, |H^\dagger H| + \lambda_{SH}(S^\dagger S) \, |H^\dagger H| + \frac{1}{2}\lambda_{\phi S}\phi^2 \, (S^\dagger S)
\end{aligned}
\tag{7.5}
$$

where $H$ is the usual SM Higgs doublet, $S$ the doubly charged scalar and $\phi$ the additional neutral scalar, odd under the unbroken $Z_2$ symmetry, that will play eventually the role of stable DM. In addition to the SM Higgs sector parameters, $\mu$ and $\lambda$, compatibly with the assumption of an unbroken $\mathbb{Z}_2$ symmetry, one can introduce seven additional dimensionless parameters: a quadratic and a quartic self–interacting couplings, respectively for the neutral and charged exotic scalars, plus three parameters associated to the quartic mixings between all the neutral and charged scalars. We assume that the ElectroWeak Symmetry Breaking (EWSB) is associated exclusively to the Higgs sector, i.e. $\mu^2 < 0$ is assumed while $\mu_\phi^2$, $\mu_S^2 > 0$ are considered. The masses of the

exotic scalars, then, read

$$m_\phi^2 \equiv \mu_\phi^2 + \frac{1}{2}\lambda_{\phi H}v^2 \quad , \quad m_S^2 \equiv \mu_S^2 + \frac{1}{2}\lambda_{SH}v^2 \tag{7.6}$$

It is interesting to compare the predictions of this model with the ones of the minimal Higgs portal DM, which is described by the potential of Eq. (7.5) once the doubly charged scalar is decoupled from the theory, i.e. $m_S \gg m_H, m_\phi$ or by setting $\lambda_{\phi S} = 0 = \lambda_{SH}$[1]. The phenomenology of such a minimal Higgs portal DM model has been extensively studied in [46, 58–81]. Here we are interested in how the presence of the doubly charged scalar can affect Higgs portal DM.

### 7.4.1  Relic abundance

In order to obtain the DM relic abundance one has to solve the following Boltzman equation:

$$\frac{dY}{dT} = \sqrt{\frac{\pi g_*(T)}{45}}M_P \langle \sigma v \rangle \left(Y(T)^2 - Y_{eq}(T)^2\right) \tag{7.7}$$

where $Y(T)$ is the DM abundance, $Y_{eq}(T)$ is the equilibrium thermal abundance, $g_*$ is the effective number of degrees of freedom, $M_P$ is the Planck mass and $\langle \sigma v \rangle$ is the thermally averaged annihilation cross section, which must include all relevant annihilation processes:

$$\langle \sigma v \rangle = \int_{4m_\phi^2}^{\infty} \frac{s\sqrt{s - 4m_\phi^2}K_1(\sqrt{s}/T)\sigma v_{rel}}{16Tm_\phi^4 K_2^2(m_\phi/T)}ds \tag{7.8}$$

where $K_1$ and $K_2$ are modified Bessel functions of the second kind. The present DM abundance, $Y(T_0)$, is obtained by integrating Eq. (7.7) down to the today temperature $T_0$. Then, the DM relic density is

$$\Omega_{DM}h^2 = 2.74 \times 10^8 \frac{m_\phi}{\text{GeV}}Y(T_0). \tag{7.9}$$

We computed these quantities using the publicly available version of micrOMEGAs [82, 83].

In Fig. 7.4 we plot the allowed parameter space in the $(m_\phi, \lambda_{\phi H})$ plane for which the DM relic abundance coincides with the observed value, $\Omega_{DM}h^2 = 0.1198$ [1]. The four plots correspond to four different values of $m_S = 250, 500, 750, 1000$ GeV, respectively. In each plot of Fig. 7.4, the full black curve represents the Higgs portal case (i.e. $\lambda_{\phi S} = 0 = \lambda_{SH}$). Then, for each plot the dashed, dot–dashed, dotted and dot–dot–dashed curve are obtained for representative choices of

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[1]Notice, however, that the decoupling limit is only approximately reached by setting one of the tree level parameters, for example $\lambda_{\phi S}$, to zero, while keeping the other two finite. In fact in this case one can generate a one–loop contribution to $\lambda_{\phi S}$ through the $\lambda_{\phi H}$ and $\lambda_{SH}$ vertices. We will come back later on this point.
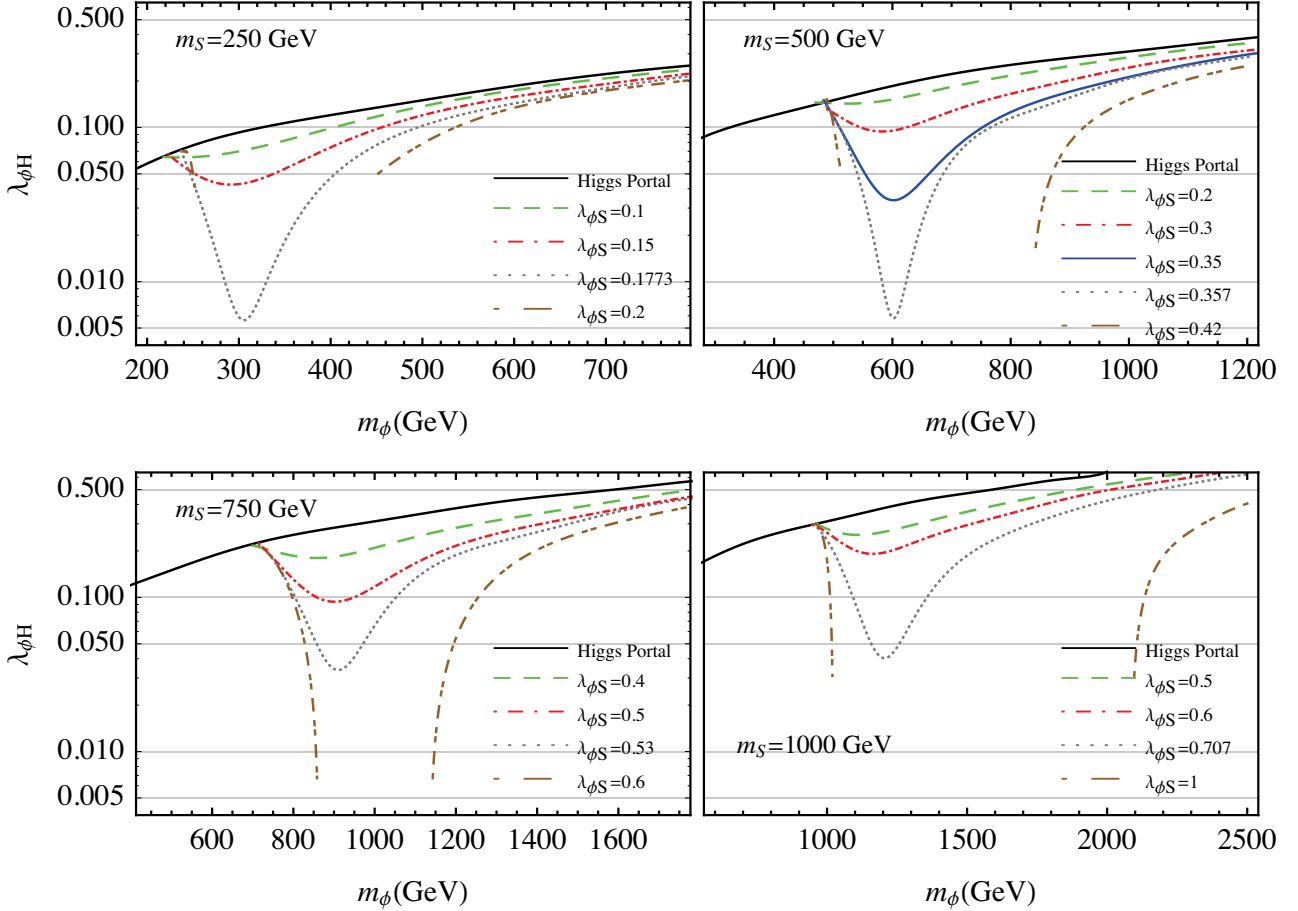
FIGURE 7.4: Allowed parameter space, in the $(m_\phi, \lambda_{\phi H})$ plane, for which the DM relic abundance reproduces the observed value, $\Omega_{DM} h^2 = 0.1198$ [1]. The four plots correspond to four different values of $m_S = 250, 500, 750, 1000$ GeV, respectively. In each plot several choices for $\lambda_{\phi S}$ are shown.

$\lambda_{\phi S}$, which value is shown in the legenda. There is no significative dependence from the choosen value of $\lambda_{SH}$, which has been conventionally taken $\lambda_{SH} = 1$.[2]

When the DM particle, $\phi$, is lighter than the doubly charged scalar, $S$, the process $\phi\phi \to SS^\dagger$ is not efficient, and the relic abundance results as in the pure Higgs portal case, i.e. via the pair $\phi\phi$ annihilating into an off-shell Higgs scalar mediating the processes $\phi\phi \to W^+W^-, ZZ, HH, t\bar{t}$ or at tree level into a pair of Higgs scalars. We'll refer collectively to these processes as $\phi\phi \to SM$.

The same happens when $m_\phi \gg m_S$. However, in the intermediate region, $m_\phi \approx m_S$, the process $\phi\phi \to SS^\dagger$ becomes efficient and, accordingly, $\lambda_{\phi H}$ needs to be suppressed, depending on the chosen value for $\lambda_{\phi S}$, in order to reproduce the correct amount of DM relic density. In particular,

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[2]On the one hand, since the $\phi\phi$ annihilation occurs almost at rest, $m_S > m_H$ in all the considered scenarios, the $H \to SS^\dagger$ decay is not relevant. On the other hand, the contributions coming from $SS^\dagger \to H$ are suppressed by a loop factor.

for large enough $\lambda_{\phi S}$ and for specific values of the $m_\phi$ mass all the relic abundance can be produced exclusively via the coupling $\lambda_{\phi S}$, with $\lambda_{\phi H}$ approaching zero. This is why the dot–dot–dashed (brown) curve exists only in the "small" and "large" $m_\phi$ region. For such values of $\lambda_{\phi S}$, in the intermediate $m_\phi$ range, DM is overproduced, and the set of parameter chosen is not allowed. This fact can be clearly seen in the $m_S = 750$ GeV (lower-left) plot: for $\lambda_{\phi S} = 0.6$ and $850 \lesssim m_\phi \lesssim 1150$ GeV one can never reproduce the correct amount of DM density.

A comment is in order regarding the possibility of setting $\lambda_{\phi H}{=}0$. In the framework at hand, $\lambda_{\phi H}$ can receive loop contributions, through diagrams involving $\lambda_{\phi S}$ and $\lambda_{HS}$ couplings. It can be shown that, for a temperature $T > m_\phi/20$, $\langle \sigma v \rangle$ (and therefore $\lambda_{\phi H}$) doesn't have an impact on $Y(T)$, since for those temperatures the DM abundance is equal to the equilibrium abundance, $Y(T) = Y_{eq}(T)$. When $T \sim m_\phi/20$, the DM particle freezes-out, and the relic abundance depends indeed on $\langle \sigma v \rangle$. As a conclusion, the typical energies in which the loop is relevant is when $p^2 \sim (m_\phi/20)^2$. Setting the renormalization scale at $2m_S$, at one loop one obtains:

$$\lambda_{\phi H}^{ren} \simeq \lambda_{\phi H} + \frac{1}{16\pi^2} \lambda_{\phi S} \lambda_{SH} \log \frac{m_\phi}{40 m_S} \tag{7.10}$$

As typically the loop contribution is few $10^{-3}$, one cannot extrapolate the tree level analysis to values of $\lambda_{\phi H}$ below few $10^{-3}$. In plotting our results we always work with $\lambda_{\phi H} \geq 0.005$.

All these comments are clearly summarised in Fig. 7.5 where the parameter space which yields the correct relic abundance, in the $(\lambda_{\phi S}, \lambda_{H\phi})$ plane, is shown for $m_S = 500$ GeV. The light-red region summarises the region, allowed by relic density data, for the relevant couplings of our DM model. Inside the filled region for definiteness we have also shown few dashed lines for various $m_\phi$ values. For $m_\phi \leq m_S$ one typically spans the lower–left region of the parameter space, while for $m_\phi \geq m_S$ one spans the upper and the right part of the filled area. In particular, we clearly see the existence of a critical value: $\lambda_{\phi S}^{crit} = 0.357$ for this specific $m_S$ case. For $\lambda_{\phi S} \leq \lambda_{\phi S}^{crit}$ it is always possible to find values for $\lambda_{\phi H}$ and $\lambda_{\phi S}$ in order to satisfy the relic abundance bound, independently of the $m_\phi$ mass. In fact one always cross all different colours dashed lines. For $\lambda_{\phi S} \geq \lambda_{\phi S}^{crit}$, only for specific ranges of $m_\phi$ one can find a solution.

It should be noted that the annihilation of two DM particles, $\phi$, moving at non-relativistic velocities could be enhanced due to the Sommerfeld effect [84, 85]. In our case, this effect can only be medidated by a t-channel Higgs exchange. However, it was shown in [86] that such an enhancement is relevant only for $m_\phi > 2$ TeV, which is outside the range of masses considered in the present analysis.
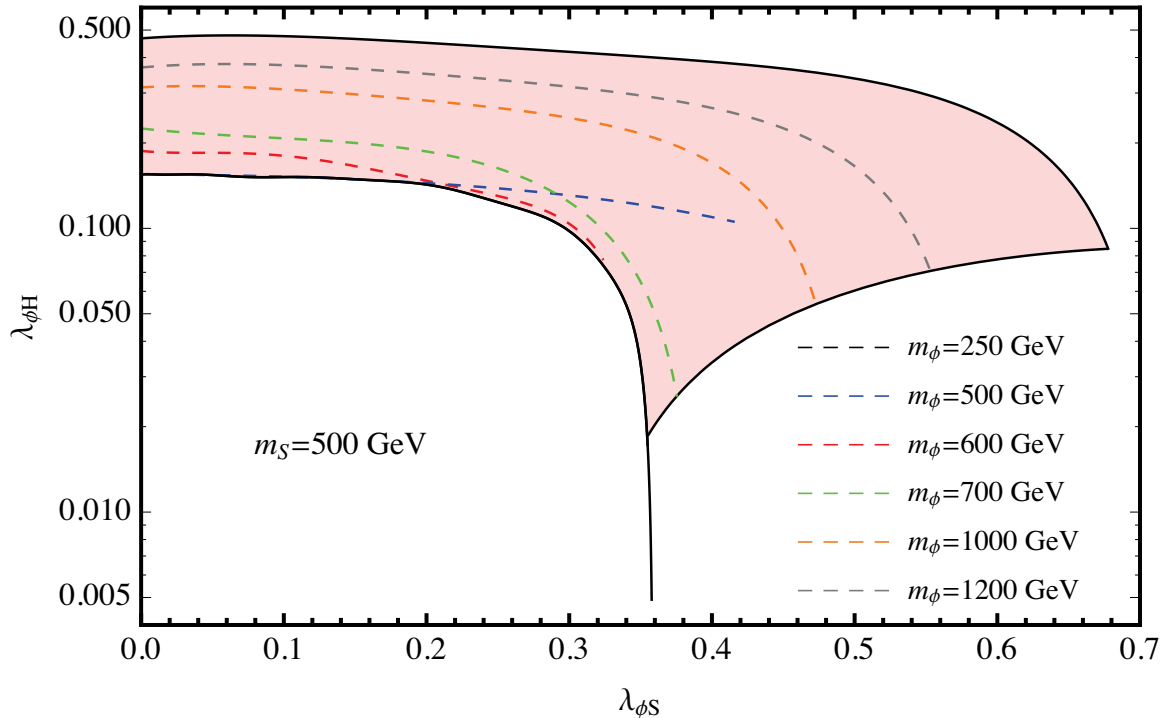
FIGURE 7.5: Allowed parameter space in the $(\lambda_{\phi S}, \lambda_{H\phi})$ plane for $m_S = 500$ GeV and $m_\phi \in (500, 1500)$GeV. The value for $m_\phi = 250$ GeV is shown in order to illustrate the fact that, for $m_\phi$ below $m_S$, the coupling $\lambda_{\phi S}$ has no impact.

### 7.4.2   Direct detection

Direct detection experiments can significantly constrain the allowed parameter space for DM models. Experiments like LUX [87] and XENON [88, 89] can detect the DM particle scattering with the nucleons of the detector material, which in the both cases is Xenon. In our model, as well as in the pure Higgs portal case, this interaction mainly occurs via exchange of a Higgs scalar. The spin-independent cross-section is given by

$$\sigma_{SI} = \frac{f_N^2 \mu^2 m_N^2}{4\pi m_H^4 m_\phi^2} \lambda_{\phi H}^2 \tag{7.11}$$

with $m_N$ the nucleon mass, $\mu = m_\phi m_N / (m_\phi + m_N)$ the DM-nucleon reduced mass and $f_N \sim 0.3 \pm 0.03$ the hadron matrix element [80]. In Fig. 7.6 we show the limits in the plane $(\lambda_{\phi H}, m_\phi)$ from the current constraints of LUX (dashed black line) and the predicted sensitivity of XENON 1T (dot dashed black line). Both the pure Higgs portal and our model can escape the LUX limit. However, while the Higgs portal scenario will be for sure inside the XENON 1T sensitivity region, our model can for all considered values of $m_S$, ranging from 250 GeV and 1000 GeV, escape the direct detection (even taking into account the uncertainty of $\mathcal{O}(10\%)$ in the determination of $f_N$). As explicitly shown in Figs. 7.4 and 7.5, the presence of the new coupling $\lambda_{\phi S}$ can allow values
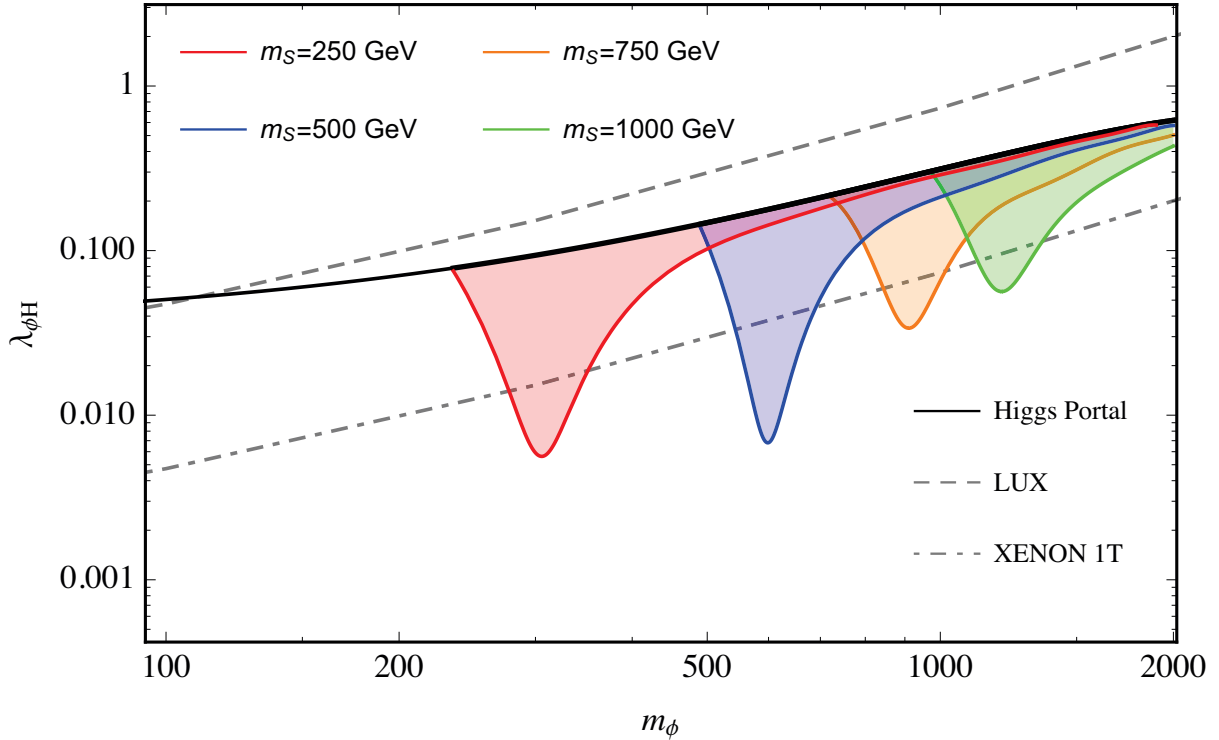
FIGURE 7.6: Allowed parameter space in the ($\lambda_{H\phi}$, $m_\phi$) plane for four different $m_S$ values ($m_S = 250, 500, 750, 1000$ GeV). Dashed and dot–dashed black lines represent the exclusion limits from direct detection by LUX and (the prediction for) XENON 1T, respectively.

for $\lambda_{\phi H}$ below XENON 1T sensitivity. This feature is almost independent from the chosen $m_\phi$ and $m_S$ values, in the $\approx 1$ TeV region.

### 7.4.3 Indirect detection

The DM particles in the galaxy can annihilate and yield several indirect signatures, such as positrons, antiprotons and photons. The detection of these cosmic rays is one of the most promising ways to identify DM existence [90–94]. We will focus here in particular in the observed spectrum of positrons and antiprotons. The production rate of these particles at a position $\vec{x}$ with an energy $E$ is usually expressed as [95]

$$Q_a(\vec{x}, E) = \frac{1}{2} \langle \sigma v \rangle \left( \frac{\rho(\vec{x})}{m_\phi} \right)^2 f_a(E) \tag{7.12}$$

where $\sigma v$ is defined in Eq. (7.8), $\rho(\vec{x})$ is the DM density and $f_a(E) = dN_a/dE$ is the energy distribution of the species $a$ produced in a single annihilation event.

The region of diffusion of cosmic rays is represented by a disk of thickness $2L \simeq (2-30)$ kpc and radius $R \simeq 20$ kpc. The galactic disk is modelled as an infinitely thin disk lying in the middle with half-width $h = 100$ pc and radius $R$. The charged particles, generated from DM annihilation, propagate in a turbulent regime through the strong galactic magnetic field and are deflected by its irregularities. Monte Carlo simulations show that this motion can be described by an energy dependent diffusion term $K(E)$. On top of that, these particles can lose their energy via inverse Compton scattering on interstellar medium, through Coulomb scattering or adiabatically. This energy loss rate is denoted by $b(E)$. Furthermore these particles can be wiped away by galactic convection, with a velocity $V_C \simeq (5-15)$ km/s [93]. Finally, one has also to account for the annihilation rate $\Gamma_{ann}$ induced by the interaction of the charged particles with ordinary matter in the galactic disk. Taking into account all these effects, the equation that describes the evolution of the energy distribution of charged particles reads:

$$\frac{\partial}{\partial z}(V_C \psi_a) - \nabla \cdot (K(E)\nabla \psi_a) - \frac{\partial}{\partial E}(b(E)\psi_a) - 2h\delta(z)\Gamma_{ann}\psi_a = Q_a(\vec{x}, E) \qquad (7.13)$$

where $z$ is the height in cylindrical coordinates adapted to the disk diffusion model, $\psi_a = dn/dE$ is the number density of particles per unit volume and energy. We use the default settings of micrOMEGAS [95] to numerically evaluate the propagation of positrons and antiprotons that originate from DM annihilation.

### 7.4.3.1 Positrons

The energy spectrum of positrons originated from DM annihilation is obtained by solving the diffusion-loss equation keeping only the two dominant contributions: space diffusion and energy losses,

$$-\nabla \cdot (K(E)\nabla \psi_{e^+}) - \frac{\partial}{\partial E}(b(E)\psi_{e^+}) = Q_{e^+}(\mathrm{x}, E) \qquad (7.14)$$

In addition to the $e^+$ flux from the DM decay, there exists a secondary positron flux from interactions between cosmic rays and nuclei in the interstellar medium. This positron background $\Phi_{e^+}^{bg}$ can be well approximated as [96, 97]

$$\Phi_{e^+}^{bg}(E) = \frac{4.5 \cdot 10^{-4}\, E^{0.7}}{1 + 650E^{2.3} + 1500E^{4.2}} [\mathrm{GeV}^{-1}\mathrm{m}^{-2}\mathrm{s}^{-1}\mathrm{sr}^{-1}] \qquad (7.15)$$

In order to have a better understanding of the $\phi\phi$ annihilation rates we plot the relevant branching ratios in Fig. 7.7 using the micrOMEGAs tool.
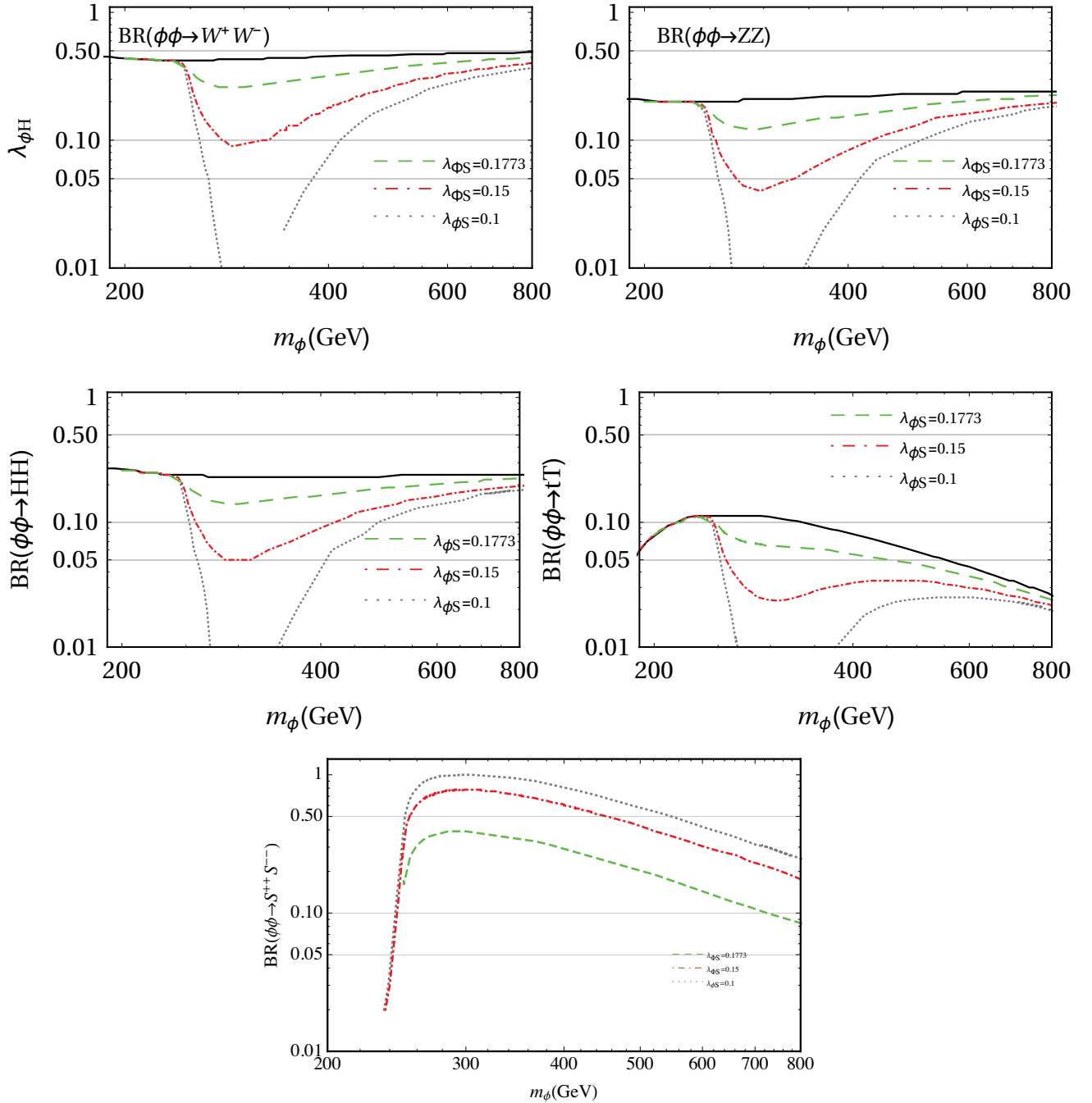
FIGURE 7.7: Branching ratios for $\phi\phi$ to annihilate into various states versus the DM mass with $m_S = 250$ GeV. The value of $\lambda_{\phi H}$ is fixed so that the relic density matches the observed value. In each plot several choices for $\lambda_{\phi S}$ are shown. The black curves correspond to the pure Higgs portal predictions.

In the left column of Fig. 7.8 we show the positron flux as function of the positron energy, for the three benchmark points mentioned in Section 7.3. In each of the three left plots, the dashed (orange) line represents the expected background of Eq. (7.15), while the dot–dashed (black) line
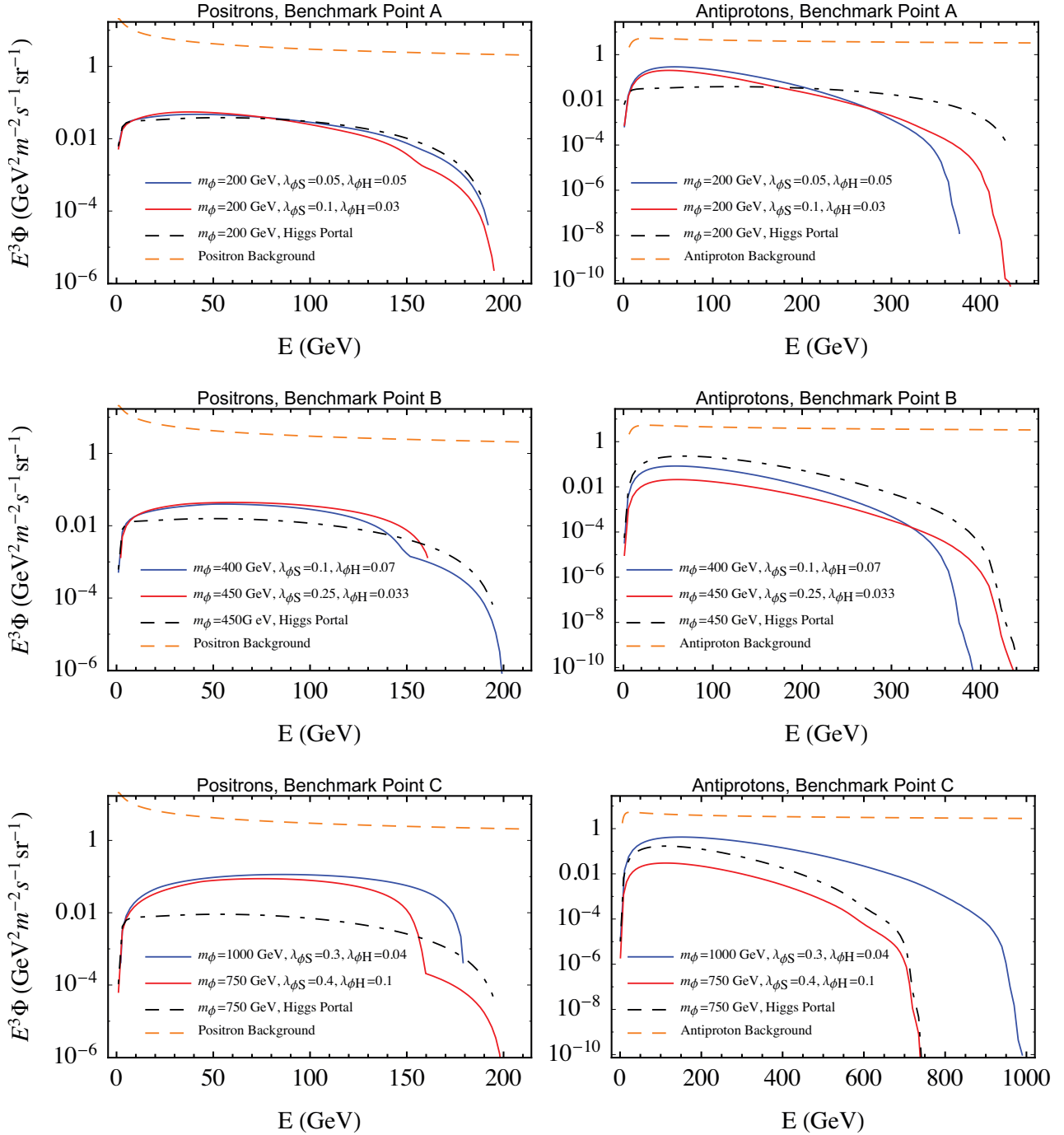
FIGURE 7.8: Predicted positron (left column plots) and antiproton (right column plots) fluxes for the chosen benchmark points A,B and C respectively.

is the prediction for the Higgs portal case. The (blue and red) continuous lines represent our model expectations for two different choices of parameters, reported in each plot legenda, which give the correct relic abundance.

The Higgs portal prediction is always at least two orders of magnitude below the astrophysical positron background. For the set of parameters defining Benchmark Point A, the expected positron flux almost coincides with the Higgs portal scenario one. This is due to the fact that for such values of $m_\phi$ and $m_S$, the $\phi\phi \to SS^\dagger$ channel is still suppressed compared to the usual $\phi\phi \to SM$ one. Morever, for this Benchmark Point the S coupling to electrons and positrons $f_{ee} \approx 0$. For Benchmark Point B (middle left plot) the $\phi\phi \to SS^\dagger$ channel becomes more effective compared to the $\phi\phi \to SM$ one, suppressed by the large $m_\phi$ mass. Still one has $f_{ee} \approx 0$, the dominant $S$ decays being in $WW$ and $e\tau$ (see [44]). This result in a positron flux two or three times higher than in the Higgs Portal scenario. Finally, for Benchmark Point C (lower left plot) the $\phi\phi \to SS^\dagger$ channel becomes dominant. Moreover, in this case one has sizeable $f_{ee}$, letting $S$ mostly decays in positrons. In this region of the parameter space, our model positron flux is one order if magnitude higher compared with the standard Higgs Portal scenario, even if still one order below the expected background.

### 7.4.3.2 Antiprotons

The propagation of antiprotons originated from DM annihilation, neglecting the energy loss term, can be described as [98]

$$- K_{\bar{p}}\nabla^2\psi_{\bar{p}} + V_C\frac{\partial}{\partial z}\psi_{\bar{p}} + 2h\delta(z)\Gamma_{ann}\psi_{\bar{p}} = Q_{\bar{p}}(\mathrm{x}, E) \tag{7.16}$$

The astrophysical antiproton background $\Phi_{\bar{p}}^{bg}$ can be written as

$$\Phi_{\bar{p}}^{bg} = \frac{0.9E^{-0.9}}{14 + 30E^{-1.85} + 0.08E^{2.3}}[\mathrm{GeV}^{-1}\mathrm{m}^{-2}\mathrm{s}^{-1}\mathrm{sr}^{-1}] \tag{7.17}$$

The plots on the right column of Fig. 7.8 show that, as in the positron case, the antiproton flux predicted in the Higgs portal scenario (dot–dashed black line) is roughly two orders of magnitude below the astrophysical background (dashed orange curve).

The doubly charged particle has a largest branching fraction to $W$s in Benchmark Point A than in the rest of cases [44]. Thus, even if in this region of parameter space the $\phi\phi \to SM$ process still dominates, the flux of antiprotons for $E \leq 200$ GeV is higher than the one predicted in the Higgs Portal case. However, for Benchamrk Points B and C, where the $S$ scalar decays mostly to leptons ($e\tau$ and $ee$, respectively), the increasing relevance of the $\phi\phi \to SS^\dagger$ process makes the $\bar{p}$ flux smaller than the corresponding Higgs Portal case for the same $m_\phi$ mass (compare the black and red lines in middle and bottom left plots of Fig. 7.8, respectively for $m_\phi = 450$ GeV and $m_\phi = 750$ GeV).

In our model one can obtain a larger flux by increasing the $\phi$ mass. For example for Benchmark Point C (bottom left plot in Fig. 7.8), one can obtain a rather larger contribution to the antiproton flux selecting $m_\phi = 1000$ GeV, but still one order of magnitude smaller than the expected $\bar{p}$ background.

### 7.4.3.3   Photons

The possibility of indirect detection via photons deserves a comment. The continuum spectrum of photons isn't expected to change significantly from the Higgs portal scenario, analogously to the positrons and antiprotons case. Therefore, most of the analysis done in [80] would apply as well to our model. Moreover, also the gamma rays excess in the continuum spectrum from the Galactic Centre, claimed by Fermi-LAT, cannot be explained in the Higgs portal scenario [80]. Since the relevant region points at a mass of around 50 GeV, the introduction of an heavy doubly charged $S$ scalar would not significanlty change this conclusion.

A more interesting phenomenological aspect would be the production of monochromatic gamma-ray lines and their observation/exclusion by Fermi-LAT experiment. Since the DM candidate is neutral, the production of a monochromatic gamma-ray line must be loop-mediated. The $\phi\phi \to \gamma\gamma$ annihilation process mediated by a doubly charged $S$ loop has been studied in [99], trying to explain the 130 GeV gamma-ray line [100]. In order to reach the observed sensitivity the author of [99] has to enhance the $\phi\phi \to \gamma\gamma$ cross section by considering $S$ embedded in a multiplet of an additional $SU(N)$ strong symmetry. This leads to an $N_C$ dependence of the amplitude. For $\lambda_{\Phi S} \sim \mathcal{O}(1)$ and $m_S \sim 250$ GeV one needs $N_C \sim 9$ to reach the ovserved rate. In our scenario we don't have a similar enhancement.

## 7.5   Conclusions

In this second part of the thesis we have considered an extension of the Standard Model involving two new scalar particles around the TeV scale: a singlet neutral scalar $\phi$, that plays the role of the Dark Matter candidate plus a doubly charged $SU(2)_L$ singlet scalar, $S^{++}$, that is the source for the non-vanishing neutrino masses and mixings. In this framework, besides being able to explain naturally the smallness of neutrino masses with the new physics at the TeV scale, it could be possible to identify DM scenarios which extend the conventional Higgs portal one. We have studied the allowed parameter space for our model, compatible with the present DM relic density. Moreover we have identified possible signatures from direct and indirect DM detection experiments. In general our results indicate that it would is possible, within our framework, to

evade XENON 1T exclusion limits for a significant region of the parameter space. However, we also show that, even if the positron and antiproton flux, originating from DM annihilation, is higher than the standard Higgs portal one, it is still about an order of magnitude lower then the observed background.

In conclusion, our model may be regarded as an extension of the minimal Higgs portal DM scenario with a doubly charged scalar which can account for neutrino mass and mixing. The presence of the doubly charged scalar $S$ introduces a new portal coupling of the DM particle $\phi$ to $S$, namely $\lambda_{\phi S}$, in addition to the usual Higgs portal coupling of $\phi$ to the Higgs doublet $H$, $\lambda_{H\phi}$. The new portal coupling $\lambda_{\phi S}$ becomes important when $m_\phi$ exceeds $m_S$, since then it allows the DM particle to annihilate into pairs of doubly charged scalars, as an alternative to the usual DM annihilation into Higgs pairs. This in turn reduces the coupling $\lambda_{H\phi}$, consistent with the desired relic density, making DM harder to detect by direct detection experiments.

# Chapter 8

# Conclusions

In this thesis we have dealt with three of the main problems our current understanding of particle physics is facing: the hierarchy problem, the existence of dark matter and the origin of neutrino masses. We have approached the first problem using the Composite Higgs model hypothesis, while for the last two a model with an extended scalar sector was proposed.

The first part of this thesis followed the work [2], constructing the chiral lagrangian for the SM, where the Higgs scalar does no longer need to necessarily belong in a doublet, behaving like a Goldstone boson. This is particularly suitable for CH models for example, where the Higgs particle is a GB associated to some global symmetry breaking pattern $\mathcal{G} \rightarrow \mathcal{H}$. With the EFT approach in mind, we wrote the effective bosonic lagrangian of a generic $\mathcal{G}/\mathcal{H}$ coset, determining both the CP-even and CP-odd basis (which was determined in the work [2]) when the EW group $SU(2)_L \times U(1)_Y$ is gauged and no additional sources of custodial breaking are considered. Then this lagrangian was particularized for three specific CH scenarios: the Georgi-Kaplan $SU(5)/SO(5)$ model, the Minimal Composite Higgs Model $SO(5)/SO(4)$ and $SU(3)/(SU(2) \times U(1))$. Since this last one is explicitly custodially breaking, it is useful in order to check that, in absence of custodially breaking terms in the high-energy lagrangian, the fact that it doesn't have a preserved $SO(4)$ is enough and low-energy custodially breaking operators are indeed induced. The results also confirm the powers of $\xi$ predicted in Ref. [33] as weights for each operator of the low-energy effective chiral Lagrangian, allowing an immediate comparison with linear expansions for an elementary Higgs. One can point out that the differences stem from the $h$ dependence: functions of $\sin\left[(\langle\varphi\rangle + h)/2f\right]$ for the CH models and powers of $(v + h)/2$ for the linear realisation. When $\xi \ll 1$, the trigonometric dependence on $h$ reduces exactly to the linear one, as $\sin^2(\varphi/f) = \xi(1 + h/v)^2 + \mathcal{O}(\xi^2)$, neglecting the higher order terms in $\xi$. This result suggests that the use of the linear expansion to construct CH model Lagrangians can be justified in this limit. On

the other hand, if $\xi$ is not so small, the deviations from the linear structure $(1 + h/v)$ could be significant and therefore comparing observables with different Higgs legs could disentangle an elementary from a composite Higgs scenario. It was shown in Chapter 5 that indeed, in the particular realization of the original Georgi-Kaplan model, where an additional $U(1)_A$ group is gauged, there is a part of the parameter space where indeed the Higgs scalar is much lighter than all the rest of the Goldstones.

In the second part we considered an extension of the Standard Model, presented in the work [3], involving two new scalar particles around the TeV scale: a singlet neutral scalar $\phi$, identified as the Dark Matter candidate, plus a doubly charged $SU(2)_L$ singlet scalar, $S^{++}$. The latter one interacts with same sign right-handed SM leptons as well as with the $W^+$ bosons (through an effective operator), so that neutrinos receive a 2-loop masses. Thus, the model may be regarded as a possible extension of the conventional Higgs portal Dark Matter scenario which also accounts for neutrino mass and mixing. Assuming an unbroken $Z_2$ symmetry in the scalar sector, under which only the additional neutral scalar $\phi$ is odd, we wrote the most general (renormalizable) scalar potential, and from it compute the DM relic abundance, the direct detection rates and the fluxes of positrons and protons. In this framework, besides being able to explain naturally the smallness of neutrino masses with the new physics at the TeV scale, it would be possible to identify DM scenarios which extend the conventional Higgs portal one. We have studied the allowed parameter space for our model, compatible with the present DM relic density. Moreover we have identified possible signatures from direct and indirect DM detection experiments. In general our results indicate that it would is possible, within our framework, to evade XENON 1T exclusion limits for a significant region of the parameter space. However, we also show that, even if the positron and antiproton flux, originating from DM annihilation, is higher than the standard Higgs portal one, it is still about an order of magnitude lower then the observed background. Our model may be regarded as an extension of the minimal Higgs portal DM scenario with a doubly charged scalar which can account for neutrino mass and mixing. The presence of the doubly charged scalar $S$ introduces a new portal coupling of the DM particle $\phi$ to $S$, namely $\lambda_{\phi S}$, in addition to the usual Higgs portal coupling of $\phi$ to the Higgs doublet $H$, $\lambda_{H\phi}$. The new portal coupling $\lambda_{\phi S}$ becomes important when $m_\phi$ exceeds $m_S$, since then it allows the DM particle to annihilate into pairs of doubly charged scalars, as an alternative to the usual DM annihilation into Higgs pairs. This in turn reduces the coupling $\lambda_{H\phi}$, consistent with the desired relic density, making DM harder to detect by direct detection experiments.

One could think of possible models where both scenarios here considered could converge. There are CH models such as $SO(6)/SO(5)$ in which an additional singlet scalar can be identified as the dark matter particle, for example. In any instance, this shows that not only more experimental observations are required, but that we also need to find scenarios that are at the same time

economical in terms of the new parameters and particles added and able to solve more than one of the problems the SM is facing. This interplay between the construction of new models and expectancy of experimental signatures, characteristic in general of the scientific method, is maybe particularly emphasized in this period of particle physics.

# Appendix A

# CCWZ construction. Non-linear realization of symmetries

## A.1 The CCWZ construction

The Callan-Coleman-Wess-Zumino (CCWZ) formalism [35, 101] is a construction that allow to understand some properties of spontaneously broken theories without having the knowledge of the underlying x mechanisms behind them0. It can be used to write general low-energy effective Lagrangians of theories characterized by a generic $\mathcal{G} \to \mathcal{H}$ breaking pattern. This means that the theory, either weakly or strongly coupled, is invariant under linearly realized transformations of the compact Lie group $\mathcal{G}$, with the vacuum state invariant only under a subgroup $\mathcal{H} \subset \mathcal{G}$.

We denote by $T_a$ (with $a = 1, \ldots, \dim(\mathcal{H})$) the generators of $\mathcal{H}$ and by $X_{\hat{a}}$ the generators of the coset $\mathcal{G}/\mathcal{H}$, in a way such that $(T_a, X_{\hat{a}})$ form an orthonormal basis of $\mathcal{G}$. A generic element $\mathfrak{g} \in \mathcal{G}$ can be decomposed a

$$\mathfrak{g} = e^{iA_{\hat{a}} X^{\hat{a}}} e^{iV_a T^a} \tag{A.1}$$

The Goldstone boson degrees of freedom can be parametrized by the unitary matrix

$$\Omega(x) \equiv e^{i\,\Xi(x)/2f} , \tag{A.2}$$

where $\Xi(x) \equiv \Xi_{\hat{a}} X^{\hat{a}}$. It is interesting to study how the GB field $\Xi(x)$ transforms under a generic transformation $\mathfrak{g} \in \mathcal{G}$.

we obtain

$$\mathfrak{g}\,\Omega(x) = e^{iA_{\hat{a}} X^{\hat{a}}} e^{iV_a T^a} e^{i\Xi_{\hat{a}} X^{\hat{a}}/2f} \equiv e^{i\Xi'_{\hat{a}} X^{\hat{a}}/2f} e^{iV'_a(\xi) T^a} \equiv \Omega'(x)\,\mathfrak{h}\,(\mathfrak{g}, \Xi) \tag{A.3}$$

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where $\mathfrak{h} \in \mathcal{H}$. Therefore, the GB matrix $\Omega$ transforms under general $\mathcal{G}$ transformations as

$$\Omega(x) \to \mathfrak{g}\, \Omega(x)\, \mathfrak{h}^{-1}\, (\Xi, \mathfrak{g}) \tag{A.4}$$

Thus, the Goldstone field $\Xi$ actually transforms non-linearly under $\mathcal{G}$,

$$\Xi'_{\hat{a}} = \Xi_{\hat{a}} + 2f A_{\hat{a}} + \dots \tag{A.5}$$

where the dots stand for higher order terms in $\Xi_{\hat{a}}$ or the parameters $A_{\hat{a}}$, $V_{\hat{a}}$. This transformation assigns one $\Xi'(x)$ field to each $\mathfrak{g}$ element of $\mathcal{G}$, which we can denote by $\Xi^{\mathfrak{g}}(x)$. We find then that this transformation on the Goldstone bosons provides a representation of the whole group $\mathcal{G}$, since it respects the multiplication rule:

$$\Xi^{(\mathfrak{g}_1 \mathfrak{g}_2)} = (\Xi^{\mathfrak{g}_2})^{\mathfrak{g}_1} \tag{A.6}$$

This is the reason why it's said that this is a non-linear representation of $\mathcal{G}$, as opposed to the usual linear group representations where the transformation rule is simply given by a constant matrix acting on the field variables. It is also because of this that the spontaneously broken group is said to be "non-linearly" realized rather than broken.

It is also worth noticing that if the transformation is restricted to $\mathcal{H}$, then A.4 reads

$$\Omega(x) \to \mathfrak{h}\, \Omega(x) \mathfrak{h}^{-1} \tag{A.7}$$

In other words, the preserved subgroup $\mathcal{H}$ acts linearly on the Goldstone bosons.

## A.2 The high-energy effective chiral Lagrangian

The CCWZ method for constructing the the effective lagrangian describing the dynamics of the Goldstone bosons implies using objects transform under the adjoint representation of the unbroken subgroup $\mathcal{H}$, as opposed to the Sigma Decomposition procedure described in Chapter 4, where the building blocks are covariants under the whole group $\mathcal{G}$. $\mathcal{H}$ covariant objects can be easily built from the fields defined in Chapter 4 using the $\Omega$ matrix:

$$
\begin{aligned}
s_{\mu\nu} &\equiv \Omega^{-1}\, \widetilde{\mathbf{S}}_{\mu\nu}\, \Omega\,, & v_\mu &\equiv \Omega^{-1}\, \widetilde{\mathbf{V}}_\mu\, \Omega = \Omega^{-1}\mathbf{D}_\mu\Omega - \Omega\mathbf{D}_\mu\Omega^{-1}\,, \\
s^{\mathcal{R}}_{\mu\nu} &\equiv \Omega\, \widetilde{\mathbf{S}}^{\mathcal{R}}_{\mu\nu}\, \Omega^{-1}\,, & v^{\mathcal{R}}_\mu &\equiv \Omega\, \widetilde{\mathbf{V}}^{\mathcal{R}}_\mu\, \Omega^{-1} = \Omega\mathbf{D}_\mu\Omega^{-1} - \Omega^{-1}\mathbf{D}_\mu\Omega\,.
\end{aligned} \tag{A.8}
$$

It is useful to define the object

$$\Omega^{-1}\mathbf{D}_\mu\Omega \equiv \frac{v_\mu}{2} + i\,p_\mu = \frac{v_\mu^{\hat{a}}}{2}\,X_{\hat{a}} + i\,p_\mu^a\,T_a\,, \tag{A.9}$$

with $v_\mu$ and $p_\mu$ transforming under $\mathcal{H}$ as:

$$v_\mu \to \mathfrak{h}\,v_\mu\,\mathfrak{h}^{-1}\,, \qquad\qquad p_\mu \to \mathfrak{h}\,(p_\mu - i\partial_\mu)\,\mathfrak{h}^{-1}\,. \tag{A.10}$$

Thus, $v_\mu$ transforms homogenously under $\mathcal{H}$ and $p_\mu$ behaves as a connection, so that it can be used to construct covariant derivatives:

$$\nabla_\mu v_\nu = \mathbf{D}_\mu v_\nu + i[p_\mu, v_\nu]\,. \tag{A.11}$$

Since $v_\mu^{\mathcal{R}} = -v_\mu$, the building blocks required to construct the effective Lagrangian are $\{v_\mu,\ s_{\mu\nu},\ s_{\mu\nu}^{\mathcal{R}}\}$. With them it is then possible to write a basis of operators equivalent to the one writte in Chapter 4.

## A.3   The CCWZ construction for the $SO(5)/SO(4)$ model

In Refs. [38, 102] an $SO(5)/SO(4)$ model built using the CCWZ construction is presented. The only particularity that must be taken into account in order to be able to compare the basis they present with our results is that the SM gauge group is contained in an $SO(4)'$ subgroup which is rotated an angle $\theta$ from the unbroken $\mathcal{H} = SO(4)$ group. This misalignment is parametrized by a rotation matrix $R_\theta$, so that the matrix from which the building blocks for the lagrangian are built is

$$U = \Omega\,R_\theta^\dagger\,, \tag{A.12}$$

Using $U$ instead of $\Omega$ one can follow the same steps as in A.2, defining the object

$$-\,i\,U^{-1}\mathbf{D}_\mu U \equiv d_\mu + e_\mu = d_\mu^{\hat{a}}\,X_{\hat{a}} + e_\mu^a\,T_a\,, \tag{A.13}$$

where we are following the notation used in Refs. [38, 102]. Under $SO(4)$ transformations,

$$d_\mu \to \mathfrak{h}\,d^\mu\,\mathfrak{h}^{-1}\,, \qquad\qquad e_\mu \to \mathfrak{h}\,(e_\mu - i\,\partial_\mu)\,\mathfrak{h}^{-1}\,. \tag{A.14}$$

As before, an extended covariant derivative of $d_\mu$ can be built using the the object $e_\mu$,

$$\nabla_\mu d_\nu = \mathbf{D}_\mu d_\nu + i\,[e_\mu, d_\nu]\,, \tag{A.15}$$

as well as a field strength,

$$e_{\mu\nu} \equiv \partial_\mu e_\nu - \partial_\nu e_\mu + i\,[e_\mu, e_\nu]\,, \tag{A.16}$$

The covariant derivative $\mathbf{D}_\mu d_\nu$ is defined using the gauge fields $F_\mu$ associated to the gauged $SO(4)' \subset SO(5)$:

$$\mathbf{D}_\mu d_\nu = \partial_\mu d_\nu + i\,g_S\,F_\mu\,d_\nu\,. \tag{A.17}$$

The field strength corresponding to this gauge group corresponds to the rotated $\widetilde{\mathbf{S}}_{\mu\nu}$ field, i.e. $F_{\mu\nu} = R_\theta \widetilde{\mathbf{S}}_{\mu\nu} R_\theta^\dagger$. From this we can define a field $f_{\mu\nu}$ transforming as a $\mathcal{H}$ covariant, as well as its corresponding graded field,

$$
\begin{aligned}
f_{\mu\nu} &= \Omega^{-1}\,F_{\mu\nu}\,\Omega\,, & f_{\mu\nu} &\to \mathfrak{h}\,f_{\mu\nu}\,\mathfrak{h}^{-1}\,, \\
f_{\mu\nu}^{\mathcal{R}} &= \Omega\,F_{\mu\nu}^{\mathcal{R}}\,\Omega^{-1}\,, & f_{\mu\nu}^{\mathcal{R}} &\to \mathfrak{h}\,f_{\mu\nu}^{\mathcal{R}}\,\mathfrak{h}^{-1}\,,
\end{aligned} \tag{A.18}
$$

It is useful to express $f_{\mu\nu}$ in terms of its broken and unbroken components, defined respectively as

$$f_{\mu\nu}^+ = \frac{f_{\mu\nu} + f_{\mu\nu}^{\mathcal{R}}}{2}\,, \qquad f_{\mu\nu}^- = \frac{f_{\mu\nu} - f_{\mu\nu}^{\mathcal{R}}}{2}\,. \tag{A.19}$$

so that

$$f_{\mu\nu} = f_{\mu\nu,a}^+ T^a + f_{\mu\nu,\hat{a}}^- X^{\hat{a}} \tag{A.20}$$

The preserved field strength $f_{\mu\nu}^+$ is related to the field $e_{\mu\nu}$ by the identity

$$e_{\mu\nu} = f_{\mu\nu}^+ - i\,[d_\mu, d_\nu]\,, \tag{A.21}$$

so that the operators of the effective lagrangian can be written either using the set of building blocks $\{f_{\mu\nu}^+, f_{\mu\nu}^-, d_\mu\}$ or $\{e_{\mu\nu}, f_\mu^-\nu, d_\mu\}$. Since $SO(4)$ is isomorphic to $SU(2)_L \times SU(2)_R$, every object built so far can be expressed via their left and right components. This is useful since the SM gauging induces a breaking of this group, so that left and right transforming objects can receive different contributions. It is useful to define the following fields:

$$
\begin{aligned}
f_{\mu\nu}^+ &= f_{\mu\nu}^L + f_{\mu\nu}^R\,, & \hat{f}_{\mu\nu}^+ &= f_{\mu\nu}^L - f_{\mu\nu}^R\,, \\
e_{\mu\nu} &= e_{\mu\nu}^L + e_{\mu\nu}^R\,, & \hat{e}_{\mu\nu} &= e_{\mu\nu}^L - e_{\mu\nu}^R\,,
\end{aligned} \tag{A.22}
$$

so that, in particular, the following identities hold

$$e_{\mu\nu}^L = f_{\mu\nu}^L - i\,[d_\mu, d_\nu]_L\,, \qquad e_{\mu\nu}^R = f_{\mu\nu}^R - i\,[d_\mu, d_\nu]_R\,. \tag{A.23}$$

Defining the structures

$$
\begin{aligned}
\mathtt{L}^2 &= \mathrm{Tr}[f_{\mu\nu}^L f_L^{\mu\nu}]\,, & \mathtt{R}^2 &= \mathrm{Tr}[f_{\mu\nu}^R f_R^{\mu\nu}] \\
\mathtt{LD_L} &= i\,\mathrm{Tr}\left(f_{\mu\nu}^L\,[d_\mu, d_\nu]_L\right)\,, & \mathtt{RD_R} &= i\,\mathrm{Tr}\left(f_{\mu\nu}^R\,[d_\mu, d_\nu]_R\right) \\
\mathtt{D_L^2} &= \mathrm{Tr}\left([d_\mu, d_\nu]_L\,[d_\mu, d_\nu]_L\right)\,, & \mathtt{D_R^2} &= \mathrm{Tr}\left([d_\mu, d_\nu]_R\,[d_\mu, d_\nu]_R\right) \\
\mathtt{B}^2 &= \mathrm{Tr}[f_{\mu\nu}^- f_-^{\mu\nu}]\,,
\end{aligned}
\tag{A.24}
$$

a CP-even basis in these formulation is given by [34]

$$
\begin{aligned}
\mathcal{L}^{(2)} &= \frac{f^2}{4}\mathrm{Tr}\left(d_\mu d^\mu\right) = \mathtt{L}^2 + \mathtt{R}^2 + \mathtt{B}^2\,, \\
\mathcal{O}_k &= \mathrm{Tr}[f_{\mu\nu}f^{\mu\nu}] = \,, \\
\mathcal{O}_1 &= \mathrm{Tr}\left(d_\mu d^\mu\right)\mathrm{Tr}\left(d_\nu d^\nu\right)\,, \\
\mathcal{O}_2 &= \mathrm{Tr}\left(d_\mu d_\nu\right)\mathrm{Tr}\left(d^\mu d^\nu\right)\,, \\
\mathcal{O}_4^+ &= \left(\mathtt{LD_L} + \mathtt{RD_R}\right)\,, \\
\mathcal{O}_4^- &= \left(\mathtt{LD_L} - \mathtt{RD_R}\right)\,, \\
\mathcal{O}_5^+ &= \mathtt{B}^2\,, \\
\mathcal{O}_5^- &= \mathtt{L}^2 - \mathtt{R}^2\,,
\end{aligned}
\tag{A.25}
$$

whereas a CP-odd basis is given by [43]

$$
\begin{aligned}
\mathcal{O}_{--,1} &= \epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}\left([d_\mu, d_\nu]_L[d_\rho, d_\sigma]_L - [d_\mu, d_\nu]_R[d_\rho, d_\sigma]_R\right)\,, \\
\mathcal{O}_{--,2} &= i\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}\left((e_{\mu\nu}^L - e_{\mu\nu}^R)[d_\rho, d_\sigma]\right)\,, \\
\mathcal{O}_{-+} &= \epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}\left(f_{\mu\nu}^- f_{\rho\sigma}^-\right)\,.
\end{aligned}
\tag{A.26}
$$

# Bibliography

[1] P. A. R. Ade et al. Planck 2015 results. XIII. Cosmological parameters. *Astron. Astrophys.*, 594:A13, 2016. doi: 10.1051/0004-6361/201525830.

[2] I. M. Hierro, L. Merlo, and S. Rigolin. Sigma Decomposition: The CP-Odd Lagrangian. *JHEP*, 04:016, 2016. doi: 10.1007/JHEP04(2016)016.

[3] I. M. Hierro, S. F. King, and S. Rigolin. Higgs portal dark matter and neutrino mass and mixing with a doubly charged scalar. 2016. doi: 10.1016/j.physletb.2017.03.037.

[4] Georges Aad et al. Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC. *Phys. Lett.*, B716:1–29, 2012. doi: 10.1016/j.physletb.2012.08.020.

[5] Serguei Chatrchyan et al. Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC. *Phys. Lett.*, B716:30–61, 2012. doi: 10.1016/j.physletb.2012.08.021.

[6] Chiara Mariotti and Giampiero Passarino. Higgs boson couplings: measurements and theoretical interpretation. *Int. J. Mod. Phys.*, A32(04):1730003, 2017. doi: 10.1142/S0217751X17300034.

[7] Gerard 't Hooft. Naturalness, chiral symmetry, and spontaneous chiral symmetry breaking. *NATO Sci. Ser. B*, 59:135–157, 1980. doi: 10.1007/978-1-4684-7571-5_9.

[8] Steven Weinberg. Implications of Dynamical Symmetry Breaking. *Phys. Rev.*, D13:974–996, 1976. doi: 10.1103/PhysRevD.13.974.

[9] Leonard Susskind. Dynamics of Spontaneous Symmetry Breaking in the Weinberg-Salam Theory. *Phys. Rev.*, D20:2619–2625, 1979. doi: 10.1103/PhysRevD.20.2619.

[10] Peter W. Graham, David E. Kaplan, and Surjeet Rajendran. Cosmological Relaxation of the Electroweak Scale. *Phys. Rev. Lett.*, 115(22):221801, 2015. doi: 10.1103/PhysRevLett.115.221801.

[11] Savas Dimopoulos and John Preskill. Massless Composites With Massive Constituents. *Nucl. Phys.*, B199:206–222, 1982. doi: 10.1016/0550-3213(82)90345-5.

[12] David B. Kaplan and Howard Georgi. SU(2) x U(1) Breaking by Vacuum Misalignment. *Phys. Lett.*, B136:183–186, 1984. doi: 10.1016/0370-2693(84)91177-8.

[13] David B. Kaplan, Howard Georgi, and Savas Dimopoulos. Composite Higgs Scalars. *Phys. Lett.*, B136:187–190, 1984. doi: 10.1016/0370-2693(84)91178-X.

[14] Howard Georgi, David B. Kaplan, and Peter Galison. Calculation of the Composite Higgs Mass. *Phys. Lett.*, B143:152–154, 1984. doi: 10.1016/0370-2693(84)90823-2.

[15] Howard Georgi and David B. Kaplan. Composite Higgs and Custodial SU(2). *Phys. Lett.*, B145:216–220, 1984. doi: 10.1016/0370-2693(84)90341-1.

[16] Michael J. Dugan, Howard Georgi, and David B. Kaplan. Anatomy of a Composite Higgs Model. *Nucl. Phys.*, B254:299–326, 1985. doi: 10.1016/0550-3213(85)90221-4.

[17] Kaustubh Agashe, Roberto Contino, and Alex Pomarol. The Minimal composite Higgs model. *Nucl. Phys.*, B719:165–187, 2005. doi: 10.1016/j.nuclphysb.2005.04.035.

[18] C. Vafa and Edward Witten. Restrictions on Symmetry Breaking in Vector-Like Gauge Theories. *Nucl. Phys.*, B234:173–188, 1984. doi: 10.1016/0550-3213(84)90230-X.

[19] Aneesh Manohar and Howard Georgi. Chiral Quarks and the Nonrelativistic Quark Model. *Nucl. Phys.*, B234:189–212, 1984. doi: 10.1016/0550-3213(84)90231-1.

[20] David Marzocca and Alfredo Urbano. Composite Dark Matter and LHC Interplay. *JHEP*, 07:107, 2014. doi: 10.1007/JHEP07(2014)107.

[21] W. Buchmuller and D. Wyler. Effective Lagrangian Analysis of New Interactions and Flavor Conservation. *Nucl. Phys.*, B268:621–653, 1986. doi: 10.1016/0550-3213(86)90262-2.

[22] B. Grzadkowski, M. Iskrzynski, M. Misiak, and J. Rosiek. Dimension-Six Terms in the Standard Model Lagrangian. *JHEP*, 10:085, 2010. doi: 10.1007/JHEP10(2010)085.

[23] Kaoru Hagiwara, S. Ishihara, R. Szalapski, and D. Zeppenfeld. Low-energy effects of new interactions in the electroweak boson sector. *Phys. Rev.*, D48:2182–2203, 1993. doi: 10.1103/PhysRevD.48.2182.

[24] Kaoru Hagiwara, T. Hatsukano, S. Ishihara, and R. Szalapski. Probing nonstandard bosonic interactions via W boson pair production at lepton colliders. *Nucl. Phys.*, B496:66–102, 1997. doi: 10.1016/S0550-3213(97)00208-3.

[25] Marco Ciuchini, Enrico Franco, Satoshi Mishima, Maurizio Pierini, Laura Reina, and Luca Silvestrini. Update of the electroweak precision fit, interplay with Higgs-boson signal strengths and model-independent constraints on new physics. *Nucl. Part. Phys. Proc.*, 273-275:2219–2225, 2016. doi: 10.1016/j.nuclphysbps.2015.09.361.

[26] Thomas Appelquist and Claude W. Bernard. Strongly Interacting Higgs Bosons. *Phys. Rev.*, D22:200, 1980. doi: 10.1103/PhysRevD.22.200.

[27] Anthony C. Longhitano. Heavy Higgs Bosons in the Weinberg-Salam Model. *Phys. Rev.*, D22:1166, 1980. doi: 10.1103/PhysRevD.22.1166.

[28] Anthony C. Longhitano. Low-Energy Impact of a Heavy Higgs Boson Sector. *Nucl. Phys.*, B188:118–154, 1981. doi: 10.1016/0550-3213(81)90109-7.

[29] F. Feruglio. The Chiral approach to the electroweak interactions. *Int. J. Mod. Phys.*, A8: 4937–4972, 1993. doi: 10.1142/S0217751X93001946.

[30] Benjamin Grinstein and Michael Trott. A Higgs-Higgs bound state due to new physics at a TeV. *Phys. Rev.*, D76:073002, 2007. doi: 10.1103/PhysRevD.76.073002.

[31] Roberto Contino, Christophe Grojean, Mauro Moretti, Fulvio Piccinini, and Riccardo Rattazzi. Strong Double Higgs Production at the LHC. *JHEP*, 05:089, 2010. doi: 10.1007/JHEP05(2010)089.

[32] R. Alonso, M. B. Gavela, L. Merlo, S. Rigolin, and J. Yepes. The Effective Chiral Lagrangian for a Light Dynamical "Higgs Particle". *Phys. Lett.*, B722:330–335, 2013. doi: 10.1016/j.physletb.2013.04.037,10.1016/j.physletb.2013.09.028. [Erratum: Phys. Lett.B726,926(2013)].

[33] M. B. Gavela, J. Gonzalez-Fraile, M. C. Gonzalez-Garcia, L. Merlo, S. Rigolin, and J. Yepes. CP violation with a dynamical Higgs. *JHEP*, 10:044, 2014. doi: 10.1007/JHEP10(2014)044.

[34] Rodrigo Alonso, Ilaria Brivio, Belen Gavela, Luca Merlo, and Stefano Rigolin. Sigma Decomposition. *JHEP*, 12:034, 2014. doi: 10.1007/JHEP12(2014)034.

[35] Sidney R. Coleman, J. Wess, and Bruno Zumino. Structure of phenomenological Lagrangians. 1. *Phys. Rev.*, 177:2239–2247, 1969. doi: 10.1103/PhysRev.177.2239.

[36] Elizabeth E. Jenkins, Aneesh V. Manohar, and Michael Trott. Naive Dimensional Analysis Counting of Gauge Theory Amplitudes and Anomalous Dimensions. *Phys. Lett.*, B726: 697–702, 2013. doi: 10.1016/j.physletb.2013.09.020.

[37] Michele Frigerio, Alex Pomarol, Francesco Riva, and Alfredo Urbano. Composite Scalar Dark Matter. *JHEP*, 07:015, 2012. doi: 10.1007/JHEP07(2012)015.

[38] Roberto Contino, David Marzocca, Duccio Pappadopulo, and Riccardo Rattazzi. On the effect of resonances in composite Higgs phenomenology. *JHEP*, 10:081, 2011. doi: 10.1007/JHEP10(2011)081.

[39] I. Brivio, T. Corbett, O. J. P. boli, M. B. Gavela, J. Gonzalez-Fraile, M. C. Gonzalez-Garcia, L. Merlo, and S. Rigolin. Disentangling a dynamical Higgs. *JHEP*, 03:024, 2014. doi: 10.1007/JHEP03(2014)024.

[40] I. Brivio, O. J. P. boli, M. B. Gavela, M. C. Gonzalez-Garcia, L. Merlo, and S. Rigolin. Higgs ultraviolet softening. *JHEP*, 12:004, 2014. doi: 10.1007/JHEP12(2014)004.

[41] Sidney R. Coleman and Erick J. Weinberg. Radiative Corrections as the Origin of Spontaneous Symmetry Breaking. *Phys. Rev.*, D7:1888–1910, 1973. doi: 10.1103/PhysRevD.7.1888.

[42] J. Beringer et al. Review of Particle Physics (RPP). *Phys. Rev.*, D86:010001, 2012. doi: 10.1103/PhysRevD.86.010001.

[43] Giuliano Panico and Andrea Wulzer. The Composite Nambu-Goldstone Higgs. *Lect. Notes Phys.*, 913:pp.1–316, 2016. doi: 10.1007/978-3-319-22617-0.

[44] Stephen F. King, Alexander Merle, and Luca Panizzi. Effective theory of a doubly charged singlet scalar: complementarity of neutrino physics and the LHC. *JHEP*, 11:124, 2014. doi: 10.1007/JHEP11(2014)124.

[45] Tanja Geib, Stephen F. King, Alexander Merle, Jose Miguel No, and Luca Panizzi. Probing the Origin of Neutrino Masses and Mixings via Doubly Charged Scalars: Complementarity of the Intensity and the Energy Frontiers. *Phys. Rev.*, D93(7):073007, 2016. doi: 10.1103/PhysRevD.93.073007.

[46] Brian Patt and Frank Wilczek. Higgs-field portal into hidden sectors. 2006.

[47] Murray Gell-Mann, Pierre Ramond, and Richard Slansky. Complex Spinors and Unified Theories. *Conf. Proc.*, C790927:315–321, 1979.

[48] Rabindra N. Mohapatra and Goran Senjanovic. Neutrino Mass and Spontaneous Parity Violation. *Phys. Rev. Lett.*, 44:912, 1980. doi: 10.1103/PhysRevLett.44.912.

[49] A. Zee. Quantum Numbers of Majorana Neutrino Masses. *Nucl. Phys.*, B264:99–110, 1986. doi: 10.1016/0550-3213(86)90475-X.

[50] K. S. Babu. Model of 'Calculable' Majorana Neutrino Masses. *Phys. Lett.*, B203:132–136, 1988. doi: 10.1016/0370-2693(88)91584-5.

[51] K. S. Babu and C. Macesanu. Two loop neutrino mass generation and its experimental consequences. *Phys. Rev.*, D67:073010, 2003. doi: 10.1103/PhysRevD.67.073010.

[52] Ernest Ma. Verifiable radiative seesaw mechanism of neutrino mass and dark matter. *Phys. Rev.*, D73:077301, 2006. doi: 10.1103/PhysRevD.73.077301.

[53] Michael Gustafsson, Jose Miguel No, and Maximiliano A. Rivera. Predictive Model for Radiatively Induced Neutrino Masses and Mixings with Dark Matter. *Phys. Rev. Lett.*, 110(21):211802, 2013. doi: 10.1103/PhysRevLett.110.211802,10.1103/PhysRevLett.112.259902. [Erratum: Phys. Rev. Lett.112,no.25,259902(2014)].

[54] Michael Gustafsson, Jose M. No, and Maximiliano A. Rivera. Radiative neutrino mass generation linked to neutrino mixing and 0-decay predictions. *Phys. Rev.*, D90(1):013012, 2014. doi: 10.1103/PhysRevD.90.013012.

[55] Miguel Nebot, Josep F. Oliver, David Palao, and Arcadi Santamaria. Prospects for the Zee-Babu Model at the CERN LHC and low energy experiments. *Phys. Rev.*, D77:093013, 2008. doi: 10.1103/PhysRevD.77.093013.

[56] Juan Herrero-Garcia, Miguel Nebot, Nuria Rius, and Arcadi Santamaria. The ZeeBabu model revisited in the light of new data. *Nucl. Phys.*, B885:542–570, 2014. doi: 10.1016/j.nuclphysb.2014.06.001.

[57] M. Agostini et al. Results on Neutrinoless Double-$\beta$ Decay of $^{76}$Ge from Phase I of the GERDA Experiment. *Phys. Rev. Lett.*, 111(12):122503, 2013. doi: 10.1103/PhysRevLett.111.122503.

[58] Vanda Silveira and A. Zee. SCALAR PHANTOMS. *Phys. Lett.*, B161:136–140, 1985. doi: 10.1016/0370-2693(85)90624-0.

[59] John McDonald. Gauge singlet scalars as cold dark matter. *Phys. Rev.*, D50:3637–3649, 1994. doi: 10.1103/PhysRevD.50.3637.

[60] C. P. Burgess, Maxim Pospelov, and Tonnis ter Veldhuis. The Minimal model of nonbaryonic dark matter: A Singlet scalar. *Nucl. Phys.*, B619:709–728, 2001. doi: 10.1016/S0550-3213(01)00513-2.

[61] Hooman Davoudiasl, Ryuichiro Kitano, Tianjun Li, and Hitoshi Murayama. The New minimal standard model. *Phys. Lett.*, B609:117–123, 2005. doi: 10.1016/j.physletb.2005.01.026.

[62] S. W. Ham, Y. S. Jeong, and S. K. Oh. Electroweak phase transition in an extension of the standard model with a real Higgs singlet. *J. Phys.*, G31(8):857–871, 2005. doi: 10.1088/0954-3899/31/8/017.

[63] Donal O'Connell, Michael J. Ramsey-Musolf, and Mark B. Wise. Minimal Extension of the Standard Model Scalar Sector. *Phys. Rev.*, D75:037701, 2007. doi: 10.1103/PhysRevD.75. 037701.

[64] Xiao-Gang He, Tong Li, Xue-Qian Li, and Ho-Chin Tsai. Scalar dark matter effects in Higgs and top quark decays. *Mod. Phys. Lett.*, A22:2121–2129, 2007. doi: 10.1142/S0217732307025376.

[65] Stefano Profumo, Michael J. Ramsey-Musolf, and Gabe Shaughnessy. Singlet Higgs phenomenology and the electroweak phase transition. *JHEP*, 08:010, 2007. doi: 10.1088/ 1126-6708/2007/08/010.

[66] Vernon Barger, Paul Langacker, Mathew McCaskey, Michael J. Ramsey-Musolf, and Gabe Shaughnessy. LHC Phenomenology of an Extended Standard Model with a Real Scalar Singlet. *Phys. Rev.*, D77:035005, 2008. doi: 10.1103/PhysRevD.77.035005.

[67] Xiao-Gang He, Tong Li, Xue-Qian Li, Jusak Tandean, and Ho-Chin Tsai. Constraints on Scalar Dark Matter from Direct Experimental Searches. *Phys. Rev.*, D79:023521, 2009. doi: 10.1103/PhysRevD.79.023521.

[68] Eduardo Ponton and Lisa Randall. TeV Scale Singlet Dark Matter. *JHEP*, 04:080, 2009. doi: 10.1088/1126-6708/2009/04/080.

[69] Rose Natalie Lerner and John McDonald. Gauge singlet scalar as inflaton and thermal relic dark matter. *Phys. Rev.*, D80:123507, 2009. doi: 10.1103/PhysRevD.80.123507.

[70] Marco Farina, Duccio Pappadopulo, and Alessandro Strumia. CDMS stands for Constrained Dark Matter Singlet. *Phys. Lett.*, B688:329–331, 2010. doi: 10.1016/j.physletb.2010.04.025.

[71] Abhijit Bandyopadhyay, Sovan Chakraborty, Ambar Ghosal, and Debasish Majumdar. Constraining Scalar Singlet Dark Matter with CDMS, XENON and DAMA and Prediction for Direct Detection Rates. *JHEP*, 11:065, 2010. doi: 10.1007/JHEP11(2010)065.

[72] Vernon Barger, Yu Gao, Mathew McCaskey, and Gabe Shaughnessy. Light Higgs Boson, Light Dark Matter and Gamma Rays. *Phys. Rev.*, D82:095011, 2010. doi: 10.1103/ PhysRevD.82.095011.

[73] Wan-Lei Guo and Yue-Liang Wu. The Real singlet scalar dark matter model. *JHEP*, 10: 083, 2010. doi: 10.1007/JHEP10(2010)083.

[74] Jose R. Espinosa, Thomas Konstandin, and Francesco Riva. Strong Electroweak Phase Transitions in the Standard Model with a Singlet. *Nucl. Phys.*, B854:592–630, 2012. doi: 10.1016/j.nuclphysb.2011.09.010.

[75] Stefano Profumo, Lorenzo Ubaldi, and Carroll Wainwright. Singlet Scalar Dark Matter: monochromatic gamma rays and metastable vacua. *Phys. Rev.*, D82:123514, 2010. doi: 10.1103/PhysRevD.82.123514.

[76] Abdelhak Djouadi, Adam Falkowski, Yann Mambrini, and Jeremie Quevillon. Direct Detection of Higgs-Portal Dark Matter at the LHC. *Eur. Phys. J.*, C73(6):2455, 2013. doi: 10.1140/epjc/s10052-013-2455-1.

[77] Yann Mambrini, Michel H. G. Tytgat, Gabrijela Zaharijas, and Bryan Zaldivar. Complementarity of Galactic radio and collider data in constraining WIMP dark matter models. *JCAP*, 1211:038, 2012. doi: 10.1088/1475-7516/2012/11/038.

[78] A. Drozd, B. Grzadkowski, and Jose Wudka. Multi-Scalar-Singlet Extension of the Standard Model - the Case for Dark Matter and an Invisible Higgs Boson. *JHEP*, 04:006, 2012. doi: 10.1007/JHEP04(2012)006,10.1007/JHEP11(2014)130. [Erratum: JHEP11,130(2014)].

[79] Bohdan Grzadkowski and Jose Wudka. Pragmatic approach to the little hierarchy problem: the case for Dark Matter and neutrino physics. *Phys. Rev. Lett.*, 103:091802, 2009. doi: 10.1103/PhysRevLett.103.091802.

[80] James M. Cline, Kimmo Kainulainen, Pat Scott, and Christoph Weniger. Update on scalar singlet dark matter. *Phys. Rev.*, D88:055025, 2013. doi: 10.1103/PhysRevD.92.039906,10.1103/PhysRevD.88.055025. [Erratum: Phys. Rev.D92,no.3,039906(2015)].

[81] Farinaldo S. Queiroz, Kuver Sinha, and Alessandro Strumia. Leptoquarks, Dark Matter, and Anomalous LHC Events. *Phys. Rev.*, D91(3):035006, 2015. doi: 10.1103/PhysRevD.91.035006.

[82] G. Belanger, F. Boudjema, A. Pukhov, and A. Semenov. micrOMEGAs: Version 1.3. *Comput. Phys. Commun.*, 174:577–604, 2006. doi: 10.1016/j.cpc.2005.12.005.

[83] G. Belanger, F. Boudjema, A. Pukhov, and A. Semenov. MicrOMEGAs 2.0: A Program to calculate the relic density of dark matter in a generic model. *Comput. Phys. Commun.*, 176:367–382, 2007. doi: 10.1016/j.cpc.2006.11.008.

[84] Junji Hisano, Shigeki Matsumoto, Minoru Nagai, Osamu Saito, and Masato Senami. Non-perturbative effect on thermal relic abundance of dark matter. *Phys. Lett.*, B646:34–38, 2007. doi: 10.1016/j.physletb.2007.01.012.

[85] Jonathan L. Feng, Manoj Kaplinghat, and Hai-Bo Yu. Sommerfeld Enhancements for Thermal Relic Dark Matter. *Phys. Rev.*, D82:083525, 2010. doi: 10.1103/PhysRevD.82. 083525.

[86] Huayong Han and Sibo Zheng. Higgs-portal Scalar Dark Matter: Scattering Cross Section and Observable Limits. *Nucl. Phys.*, B914:248–256, 2017. doi: 10.1016/j.nuclphysb.2016. 11.015.

[87] D. S. Akerib et al. First results from the LUX dark matter experiment at the Sanford Underground Research Facility. *Phys. Rev. Lett.*, 112:091303, 2014. doi: 10.1103/PhysRevLett. 112.091303.

[88] E. Aprile et al. Dark Matter Results from 225 Live Days of XENON100 Data. *Phys. Rev. Lett.*, 109:181301, 2012. doi: 10.1103/PhysRevLett.109.181301.

[89] E. Aprile et al. Physics reach of the XENON1T dark matter experiment. *JCAP*, 1604(04): 027, 2016. doi: 10.1088/1475-7516/2016/04/027.

[90] P. Salati, F. Donato, and N. Fornengo. Indirect Dark Matter Detection with Cosmic Antimatter. 2010.

[91] Troy A. Porter, Robert P. Johnson, and Peter W. Graham. Dark Matter Searches with Astroparticle Data. *Ann. Rev. Astron. Astrophys.*, 49:155–194, 2011. doi: 10.1146/ annurev-astro-081710-102528.

[92] Carlos E. Yaguna. Gamma rays from the annihilation of singlet scalar dark matter. *JCAP*, 0903:003, 2009. doi: 10.1088/1475-7516/2009/03/003.

[93] A. Goudelis, Y. Mambrini, and C. Yaguna. Antimatter signals of singlet scalar dark matter. *JCAP*, 0912:008, 2009. doi: 10.1088/1475-7516/2009/12/008.

[94] Chiara Arina and Michel H. G. Tytgat. Constraints on Light WIMP candidates from the Isotropic Diffuse Gamma-Ray Emission. *JCAP*, 1101:011, 2011. doi: 10.1088/1475-7516/ 2011/01/011.

[95] G. Belanger, F. Boudjema, P. Brun, A. Pukhov, S. Rosier-Lees, P. Salati, and A. Semenov. Indirect search for dark matter with micrOMEGAs2.4. *Comput. Phys. Commun.*, 182: 842–856, 2011. doi: 10.1016/j.cpc.2010.11.033.

[96] Andrew W. Strong, Igor V. Moskalenko, and Olaf Reimer. Diffuse galactic continuum gamma rays. A Model compatible with EGRET data and cosmic-ray measurements. *Astrophys. J.*, 613:962–976, 2004. doi: 10.1086/423193.

[97] Edward A. Baltz and Joakim Edsjo. Positron propagation and fluxes from neutralino annihilation in the halo. *Phys. Rev.*, D59:023511, 1998. doi: 10.1103/PhysRevD.59.023511.

[98] Qing-Hong Cao, Chuan-Ren Chen, and Ti Gong. Leptophilic Dark Matter and AMS-02 Cosmic-ray Positron Flux. *Chin. J. Phys.*, 55:10, 2016. doi: 10.1016/j.cjph.2016.11.006.

[99] James M. Cline. 130 GeV dark matter and the Fermi gamma-ray line. *Phys. Rev.*, D86: 015016, 2012. doi: 10.1103/PhysRevD.86.015016.

[100] W. B. Atwood et al. The Large Area Telescope on the Fermi Gamma-ray Space Telescope Mission. *Astrophys. J.*, 697:1071–1102, 2009. doi: 10.1088/0004-637X/697/2/1071.

[101] Curtis G. Callan, Jr., Sidney R. Coleman, J. Wess, and Bruno Zumino. Structure of phenomenological Lagrangians. 2. *Phys. Rev.*, 177:2247–2250, 1969. doi: 10.1103/PhysRev. 177.2247.

[102] Aleksandr Azatov, Roberto Contino, Andrea Di Iura, and Jamison Galloway. New Prospects for Higgs Compositeness in $h \to Z\gamma$. *Phys. Rev.*, D88(7):075019, 2013. doi: 10.1103/ PhysRevD.88.075019.