



UNIVERSITA' DEGLI STUDI DI PADOVA

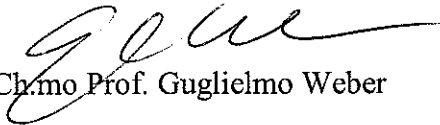
Sede Amministrativa: Università degli Studi di Padova

Dipartimento di Scienze Economiche 'Marco Fanno'

DOTTORATO DI RICERCA IN : ECONOMIA E MANAGEMENT

CICLO: XIX

**ESSAYS ON HEALTH AND SAVINGS:
CROSS – COUNTRY EVIDENCE**

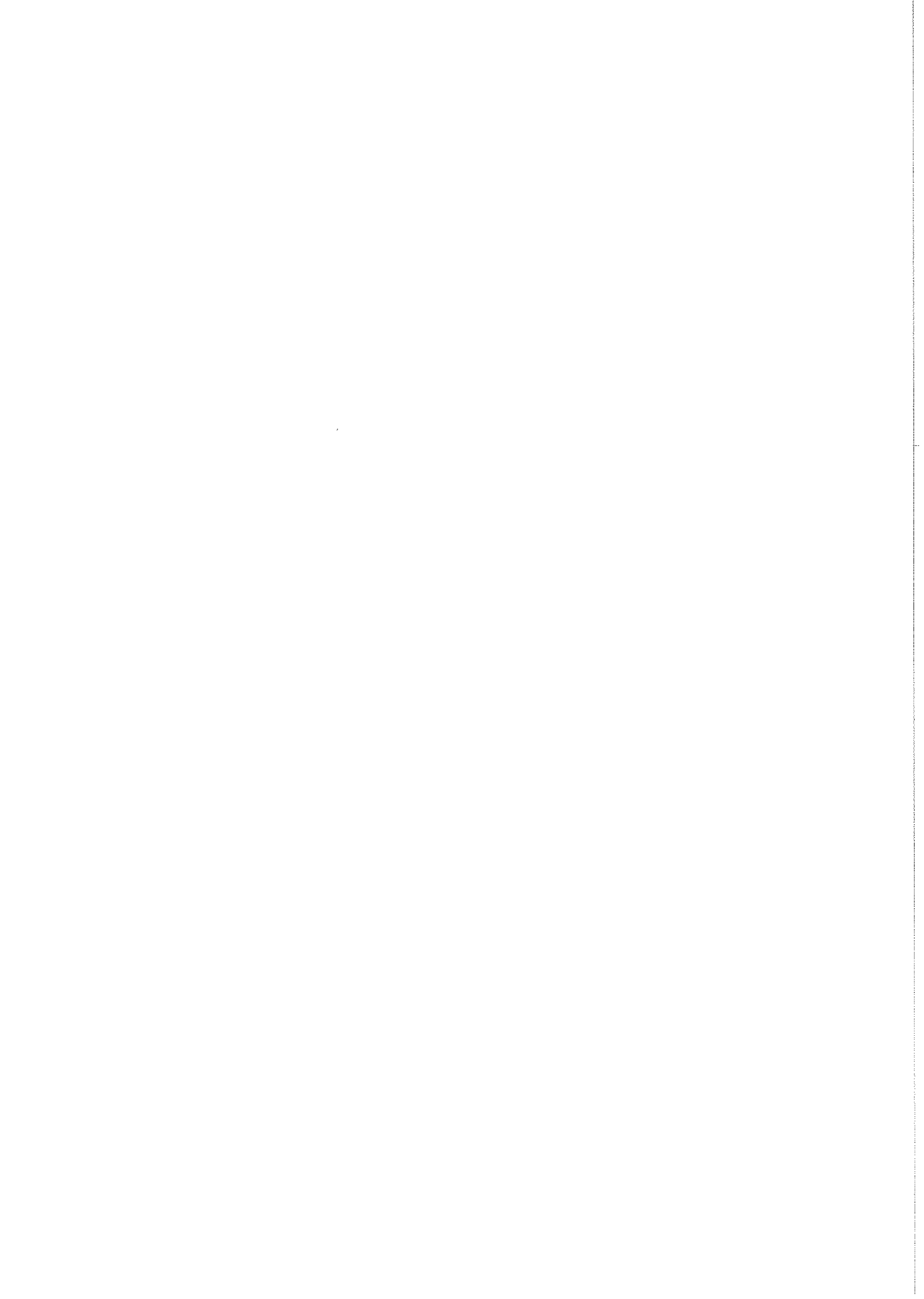

Coordinatore : : Ch.mo Prof. Guglielmo Weber

Supervisore : : Ch.mo Prof. Guglielmo Weber

Dottoranda : Loretti Isabella Dobrescu

A handwritten signature in cursive script, likely belonging to Loretti Isabella Dobrescu.

31 luglio 2008



Acknowledgements

It is a pleasure to thank the many people who made this thesis possible.

First and foremost, this work would not have been possible without the support of my supervisor, Prof. Guglielmo Weber, who shared with me a lot of his expertise and research insights and under whose supervision I chose this topic and began the thesis.

I would like to thank the many people who have taught me economics during the years.

I am grateful to Prof. Laurence J. Kotlikoff, my coauthor for the second chapter of the thesis and my supervisor for the year spent at Boston University, for teaching me the ‘job’ of doing research.

I wish to thank Prof. Randall Ellis (Boston University), for his advices eased the path in dealing properly with the complexity of the insurance topic.

I am indebted to my professors of the Doctoral School in Economics and Management and student colleagues for providing a stimulating environment in which to learn and grow.

I wish to express my warmest gratitude to my colleague and fiancée Alberto Motta, for his professional and emotional support during this ‘economics journey’.

It is difficult to overstate my appreciation to Prof. Constantin Chilarescu, who first brought me into the world of research and with whom I began to learn about economics and mathematics. Not only a great mentor and colleague, he has also been a cornerstone in my professional development.

I am grateful to the technical and administrative personnel in the Economics Department of University of Padua, for helping my many codes to run smoothly and for assisting me in many different ways. Marta Fontanin, Sandra Parrini, Mario Bettella and Riccardo Marcon deserve special mention.

I acknowledge the European Commission through the 6th framework program for the projects SHARE-I3, RII-CT-2006-062193 and COMPARE, CIT-CT-2005-028856, for financially supported me during my last year of research.

I cannot finish without saying how grateful I am to my family, especially to my mother and my grandmother, on whose constant support and encouragement I have always relied.

z

Contents

List of Figures	iii
List of Tables	v
Introduction	1
1 To love or to pay: On consumption, health and health care	5
1.1 Introduction	5
1.2 Basic facts	7
1.2.1 On world demographics	7
1.2.2 On health status and spending evolution over time	9
1.3 The Model	17
1.3.1 Utility function	17
1.3.2 Uncertainty	21
1.3.3 Budget constraint	24
1.3.4 Timing of the model	24
1.3.5 Recursive framework	26
1.4 Data	27
1.5 Calibrations and Estimation Methodology	29
1.6 Results and Model Fit	31
1.7 Conclusions	49
2 Why Have Developed Countries Stopped Saving?	51
2.1 Introduction	51

2.2	The Models	53
2.2.1	Model 1: Uncertain Future Time Preferences	53
2.2.2	Model 2: Uncertain Future Preferences with Time Inconsistent Decision Makers	56
2.2.3	Model 3: Changing Intratemporal Preferences	57
2.3	Data	58
2.4	Estimation	58
2.5	Results	61
2.6	Conclusions	72
3	Stochastic preferences: lessons on government inconsistency	75
3.1	Introduction	75
3.2	The model	77
3.2.1	Solving for optimal consumption through backward recursion	82
3.3	Description of data	86
3.4	Estimation procedure	86
3.4.1	Data simulation	87
3.4.2	SMM	88
3.5	Results	90
3.6	Conclusions	94
	Bibliography	97
	Appendix A: Gauss-Hermite quadrature approach to discretization	104
	Appendix B: Moment Conditions	107
	Appendix C: Numerical simulation, Chapter 1	109
	Appendix D: Data, Chapter 1	110
	Appendix E: The Health Cost Model, Chapter 1	113

List of Figures

1-1	World population from the time perspective	7
1-2	World population in growth rate terms	8
1-3	World population change in time	8
1-4	Health expenditure 1990 vs 2004, as a share of GDP in selected OECD countries (Source: OECD Health data 2006 . Version: June 2006)	9
1-5	Health expenditure per capita, by years of life expectancy in selective OECD countries, 2004 (Source: OECD in Figures 2006-2007, Demography and health)	10
1-6	Detailed financing sources as percentage of overall health expenditures, in selective OECD countries (time series) (Source: OECD health data. Version: October 2007)	13
1-7	Percentage informal help received with personal care within the household (Source: SHARE data 2004)	14
1-8	Percentage individuals receiving formal personal care help within the household (Source: SHARE data 2004; EUROFAMCARE 2004)	14
1-9	Median net total wealth by age (adjusted for purchasing power parity) (Source: SHARE data 2004)	15
1-10	The level of health index, by country (Source: SHARE data 2004)	16
1-11	Health care utilization (number of physicians visits in last 12 months), by age (Source: SHARE data 2004)	17
1-12	Cohesion Coefficient by Country Group and Wealth Percentile	43
1-13	Health transition probabilities conditional on age, previous health status and wealth, Mediterranean countries	44

1-14	Health transition probabilities conditional on age, previous health status and wealth, Central European countries	45
1-15	Health transition probabilities conditional on age, previous health status and wealth, Scandinavian countries	46
1-16	Health Transition for Representative Agent	48
2-1	Time Preference Rate, model 1	67
2-2	Time Preference Rate, model 2	67
2-3	Age-Specific Utility Weights by Age Group, model 1	68
2-4	Age-Specific Utility Weights by Age Group, model 2	68
2-5	Age-Specific Utility Weights by Age Group for Selected Years, U.S.	70
2-6	Age-Specific Utility Weights by Age Group for Selected Years, France	71
2-7	Age-Specific Utility Weights by Age Group for Selected Years, Italy	71
3-1	Age-Specific Utility Weights by Age Group	93
3-2	Stochastic Preference Paths in Time.	94

List of Tables

1.1	Current Health Expenditure by ICHA-HF Health Care Financing, 2004	12
1.2	Choice of Moments	31
1.3	Estimated Structural Parameters, Mediterranean Countries	33
1.4	Estimated Structural Parameters, Central European Countries	34
1.5	Estimated Structural Parameters, Scandinavian Countries	35
1.6	Estimated Moments and Goodness of Fit Test - Representative Agent	36
1.7	Estimated Moments and Goodness of Fit Test - Wealth Subgroups, Mediter- ranean Countries	37
1.8	Estimated Moments and Goodness of Fit Test - Wealth Subgroups, Central Eu- ropean Countries	38
1.9	Estimated Moments and Goodness of Fit Test - Wealth Subgroups, Scandinavian Countries	39
2.1	National saving rate for selected years	52
2.2	Government Spending Parameters	59
2.3	Parameters Estimated - Definitions	60
2.4	Parameters Estimated in Model 1 and Model 3	60
2.5	Parameters Estimated in Model 2	60
2.6	Choice of Moments	60
2.7	Estimated Moments and Goodness of Fit Test, U.S.	62
2.8	Estimated Moments and Goodness of Fit Test, France	63
2.9	Estimated Moments and Goodness of Fit Test, Italy	64

2.10	Parameter Estimates, U.S.	65
2.11	Parameter Estimates, France	65
2.12	Parameter Estimates, Italy	66
2.13	Saving Rates, U.S.	69
2.14	Saving Rates, France	69
2.15	Saving Rates, Italy	69
3.1	Parameters Estimated - Definitions	88
3.2	Parameters Estimated	90
3.3	Choice of Moments	90
3.4	Parameter Estimates	91
3.5	Estimated Moments and Goodness of Fit Test	92
3.6	Benchmark countries in OECD studies	113

Introduction

The main objective of the present work is to shed light on aspects concerning, on the one hand, health, health spending and intra-family cohesion (strength of family ties), and on the other hand, savings, demographics and preferences structure. Briefly, it includes three chapters, the first one dealing with the microeconomic problems of increased medical spending and longer life expectancy, in the general framework of the insurance markets, while the last two address the macroeconomic question of what are the main determinants of the international and intertemporal differences in the national saving rates. Is it demographics, government spending, productivity growth or preferences?

Over the years, there has been an increasingly important debate between economists and policymakers on individuals decisions after retirement, in terms of consumption and insurance, over the life-cycle. The first chapter focuses on features regarding consumption, health and health care expenditures related to age, namely incurred after the age of retirement by single individuals, from a microeconomic point of view. The aim is to explore the effects of uninsurable risk of health expenditures as well as the insurable risk of health status on consumption-insurance choice. Following this research line, I developed a model of consumption of goods and utility of being looked after, taking into consideration the existence of a formal insurance possibility and of an informal insurance arrangement provided by the family, besides out-of-pocket spending, in the integrated framework of the bequest motive. Using European data (SHARE) on three country groups, Mediterranean, Central European and Scandinavian, I estimated aggregate (representative agent) and disaggregate (for wealth subgroups) models, using a simulated life-cycle approach. More specifically, solving numerically the dynamic model by backward recursion and estimating the interest parameters using the Simulated Method of Mo-

ments (SMM), allowed me to simulate each individual's wealth, consumption, formal insurance, medical spending, health and mortality. With the simultaneous consideration of both risks, on health and medical spending, I found that the model can generate increasing precautionary savings (and consequently bequests) with age after retirement, and therefore fits well the data profiles. Furthermore, I found that the older, and so the sickest, the individuals will become, the more weight they are going to attach to the informal care. Finally, results illustrated that, both at the aggregate and disaggregate level, cohesion coefficient, which represents the strength of family ties, displays an increasing age structure. In addition, estimates showed that individuals that benefit of higher cohesion coefficients are likely to display a certain health status transition in the years after retirement, resulting in a higher life expectancy as measured by the survival probabilities.

Chapter 2 concentrate on the fact that national saving rates differ enormously across developed countries. But these differences obscure a common trend, namely a dramatic decline over time. France and Italy, for example, saved over 17 percent of national income in 1970, but less than 7 percent in 2006. Japan saved 30 percent in 1970, but only 8 percent in 2006. And the U.S. saved 9 percent in 1970, but only 2 percent in 2006. What explains these international and intertemporal differences? Is it demographics, government spending, productivity growth, or preferences? Our answer is preferences. Developed societies are placing increasing weight on the welfare of those currently alive, particularly contemporaneous older generations. This conclusion emerges from estimating three models in which society makes consumption and labor supply decisions in light of uncertainty over future government spending, productivity, and social preferences. The three models differ in terms of the nature of preference uncertainty and the extent to which current society can control future societies' spending and labor supply decisions. In the first model, there is only one society considered to rule forever. This society knows its current intertemporal preferences (discount factor) and current intratemporal preferences (age-specific weighting shares in utilities from consumption and leisure). However, it doesn't know the future intertemporal preferences (how its discount factor will evolve). The second model is a time-inconsistency variant of the first. Consequently, we allowed societies to rule for only one period, rather than put a single society forever in charge. Although today's society knows its future preferences, it controls future societies' consumption and leisure allo-

cation decisions only indirectly via the amount of capital it leaves for the next period. In the third model, society has stable intertemporal preferences, but changing intratemporal preferences (time variant age-specific utility weights). The main results of all three models, based on modeling and estimating the discount factor and the utility weights for U.S., France and Italy, confirm the theory that in time society changed the preference structure towards assigning progressively more weight to the present generations with respect to the future ones (declining stochastic discount factor). Moreover, it also shows that in time, as far as preferences within the present generations are concerned, society evolved more and more towards a preference structure which assigns higher level of importance to the old generations with respect to the young ones. Indeed, we found the utility weights to follow a bell-shape pattern just for the first half of the existing age groups, while for the second one the curve is more flat, declining at a slower rate than it rose in the first half. In other words, society tends to allocate less importance to the young generations than to the old ones, and this pattern is accentuated in time.

The last chapter is intended to offer an alternative answer to the question raised in Chapter 2, relative to which are the main factors that determine the national saving rate of a country. It does so by estimating a finite-horizon overlapping generation model of consumption choice for the U.S., where in each period of time, government and households decide together what is to be consumed and saved. The novelty consists in modelling societies preference for young, old and unborn generations separately. Another new feature consists in considering a four stochastic dimension model, where besides uncorrelated technology and government spending shock, I included positively correlated societies preferences for young and for old generations. To this purpose, I estimated a set of parameters consistent with the structure of the modelled economy, evaluating their measurements based on demographics and preferences. It is registered that when an age-specific utility weights structure and stochastic society' preferences towards young and old generations are introduced, the model achieve a good empirical performance. In this respect, results clearly confirm the main idea of Chapter 2 that, in time, society tended progressively attributed more importance to the old generations with respect to the young or future ones and accentuated this tendency over the years. In order to extend this study, further research can be done by replicating the analysis for France and Italy.

Chapter 1

To love or to pay: On consumption, health and health care

1.1 Introduction

In recent years, much attention was given to the matter of intergenerational transfers in the process of wealth accumulation or as a key element in consumption - health expenditure choices, from both theoretical and empirical point of view. The contribution of this chapter is to further extend the discussion with another relevant issue, analyzing whether intergenerational transfers are actually used by the elderly to obtain health care services in case of a health shock.

For this purpose, I will adopt a different view with respect to the one of using intergenerational transfers for consumption smoothing purposes as claimed by Ando and Modigliani [1963], Modigliani [1986], or for the bequest reason, given the imperfections of the annuity market, as in Kotlikoff and Spivak [1981], Kotlikoff et al. [1986], [1987], Eckstein et al. [1983]. The approach of this work is to consider the provision of health care in case of a health shock and analyze how is this related with the bequest motive.

In addition to the health status shocks, this analysis considers another important source of background risk: the presence of uncertain future medical expenses. Uncertainty related to health expenditures, understood as unexpected changes in the out-of-pocket health spending, is not a new concept introduced to explain an individual's decision-making, especially at the end of the life cycle. A well-established literature, counting the works of Hubbard et al., [1994, 1995],

Palumbo, [1999], Dynan et al., [2004], examines how the risk of future health expenditures generates precautionary savings. This setting was proposed due to the fact that uncertain health expenditure is the main source of risk that is large and difficult to be diversified for the elderly, even though most of them are covered by some form of compulsory insurance.

One other well known approach that I will integrate is considering the bequest motive. In their paper of 1981, Kotlikoff and Summers started an academic debate when they argued for the dominance of bequest motives for savings based on the estimated level of intergenerational transfers, over the conventional wisdom of life cycle motives for savings. On the other hand, Dynan et al. [2002] argued that in a world in which bequests may be unintentional, "it is not useful or even possible to parse net worth into life-cycle and bequest components on an ex ante basis, because each dollar can effectively serve both purposes". From this point of view, I follow Brown and Finkelstein [2004] in assigning great importance to potentially high health care costs for sickness or long term invalidity as motivators for health insurance and to precautionary savings for bequest motive.

Considering all these elements, I propose a theoretical life-cycle model of consumption of goods and health care services with a realistic simulation exercise in order to explore the effects of uninsurable risk of health expenditures as well as the insurable risk of health status on consumption - health insurance choice, when it simultaneously exists a formal insurance possibility and an informal insurance arrangement for individuals of age 65 and over. I make the strong assumption that there are no country-specific shocks. Generally, by adopting this setting, the model generates increasing precautionary savings (and consequently bequests) with age after retirement, and therefore it fits the data much better than those studies that consider only health status risk. Specifically, for what it will be defined as "strong cohesion coefficient" countries, the level of formal health insurance is low, health expenditures being mainly covered by the family, represented by children or relatives. The contrary is valid for the "weak cohesion coefficient" countries. If this is true, since the first category of countries, that benefit of a high cohesion coefficient, is the one that experiences higher life expectancy, there is an underlying relation between these two facts: reliability on family for health and health spending shocks and high life expectancy. Empirically, what I estimate is precisely the family relation strength parameter, with an age-specific structure, and show that the higher this cohesion coefficient,

the higher the life expectancy, as measured by the age-adjusted survival probabilities.

The remaining part of the chapter is organized as follows: Section 2 presents some figures that will better emphasize the purpose of the work and give an idea on the real situation. Section 3 develops the dynamic programming model of retirement behavior and Section 4 describes the data. In Section 5 I describe the model estimation, using Gauss-Herman quadrature method and the Simulated Method of Moments (SMM). Section 6 presents estimates on preference parameters and beliefs for the structural model. Section 7 concludes.

1.2 Basic facts

1.2.1 On world demographics

The facts show that world population doubled from 3 billion in 1959 to 6 billion by 1999, and this increase occurred over 40 years. The Census Bureau's latest projections imply that population growth will continue into the 21st century, although more slowly. Consequently, as showed by Figure 1-1, world population is projected to grow from 6 billion in 1999 to 9 billion by 2042, an increase of 50 percent that will require under these circumstances 43 years. On the other hand, Figure 1-2 shows world population growth rate that rose from about 1.5 percent per year in 1950-1951 to a maximum of over 2 percent in the early 1960s, mainly due to reductions in mortality. After this peak, growth rates started to decline, due to rising age at marriage as

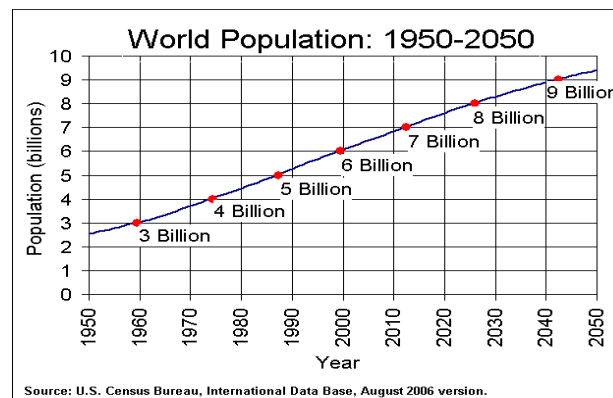


Figure 1-1: World population from the time perspective

well as increasing availability and use of effective contraceptive methods ¹.

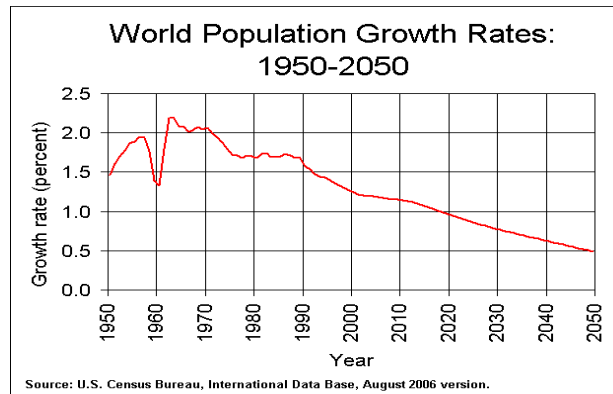


Figure 1-2: World population in growth rate terms

In addition to growth rates, another way to look at population growth is to consider annual changes in total population, as illustrated by Figure 1-3. The annual increase in world population peaked at about 88 million in the late 1980s. This maximum occurred because the world population was higher in the 1980s than in the 1960s, even if the peak for the world population growth rate was registered in 1960s.

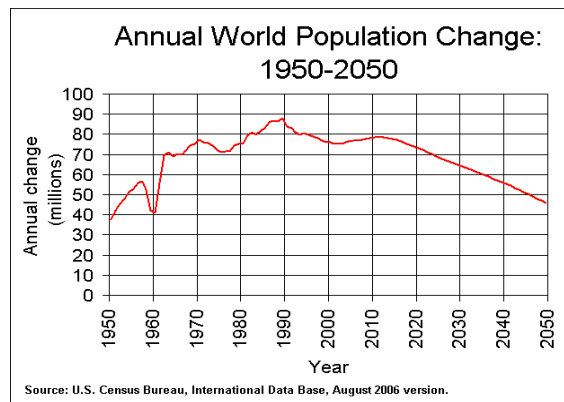


Figure 1-3: World population change in time

¹As shown by Figure 1-2, changes in population growth have not always been steady. The drop in the growth rate from 1959-1960, for instance, was due to the Great Leap Forward in China. During that time, both natural disasters and decreased agricultural output in the wake of massive social reorganization caused China's death rate to rise sharply and its fertility rate to fall by almost half.

1.2.2 On health status and spending evolution over time

It is well known that the ageing process is characterized by a high prevalence of chronic diseases and by the coexistence, in individuals, of multiple morbidities. Although ageing individuals are very heterogeneous in terms of health status, in general, it is true that ageing populations constantly need a large range of health services, from acute care to long term care. In the past decades, these special needs of an increasing number of older persons induced a raise in health services utilization, generating the remarkable upward sloping trend of health care expenditures in industrialized societies.

According to the OECD International Classification of Health Accounts, total expenditure on health is defined as the sum of expenditure on activities that – through application of medical, paramedical, and nursing knowledge and technology – have the goals of promoting health and preventing disease and curing illness and reducing premature mortality, caring for persons affected by chronic illness who require nursing care, caring for persons with health-related impairments, disability, and handicaps who require nursing care, assisting patients to die with dignity, and providing and administering public health and health programmes, health insurance and other funding arrangements. Figure 1-4 provides a country-comparison of health expenditures as share of GDP in 1990 versus 2004, while Figure 1-5 plots the evolution of health expenditure per capita, by health expectancy, in selected OECD countries.

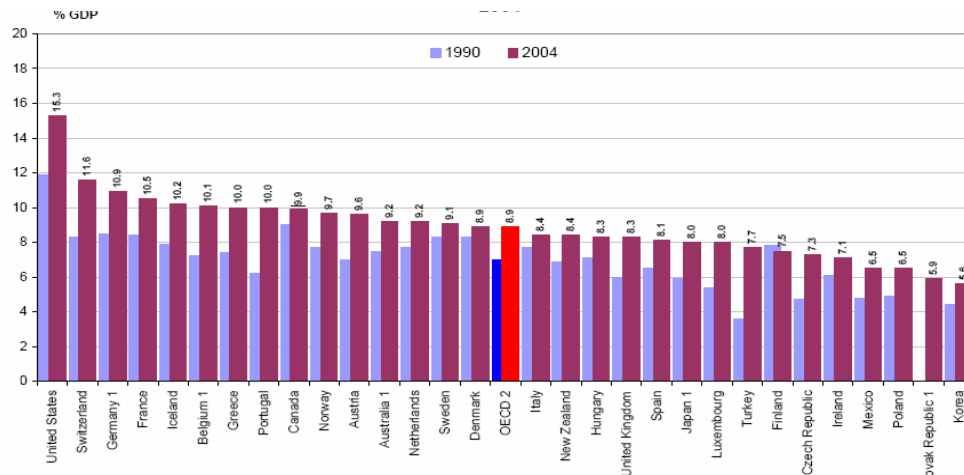


Figure 1-4: Health expenditure 1990 vs 2004, as a share of GDP in selected OECD countries (Source: OECD Health data 2006 . Version: June 2006)

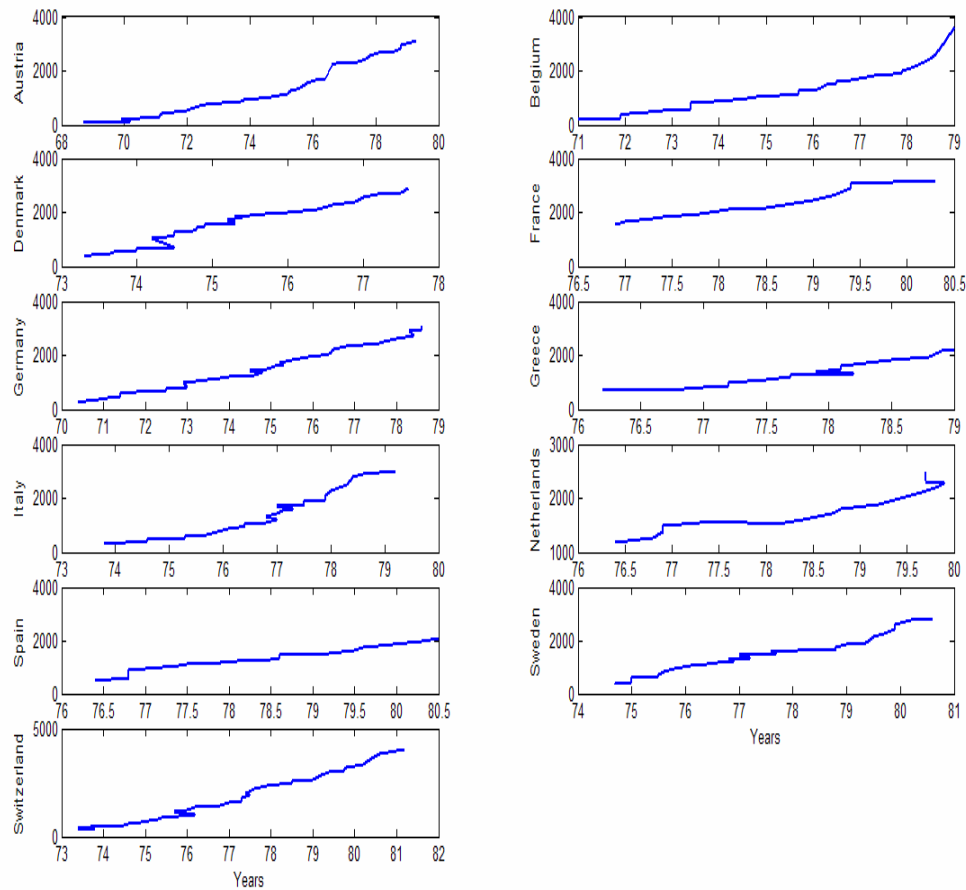


Figure 1-5: Health expenditure per capita, by years of life expectancy in selective OECD countries, 2004 (Source: OECD in Figures 2006-2007, Demography and health)

The crucial question is whether these increased expenditure levels are justified by the level of health care and consequently by the level of health status understood as life expectancy, for instance. Following the tradition in demography, the life expectancy measure is given by the expected remaining years of life (which depends on unknown future mortality rates), that is life expectancy for a hypothetical individual who faces the cross-section of mortality rates from a given year. Looking at per capita health care expenditures by years of life expectancy at birth in Figure 1-5, the situation is not most favorable, in the sense that both life expectancy and health expenditures relevantly raised in the last decades, with a significant impact on wealth accumulation.

Previous work has already demonstrated that health expenditure related to age may constitute an important reason why elderly do not decumulate wealth, as they should be doing according to the life-cycle model. They perceive increased health status risk, which is an increasing function of age, as in Palumbo [1999], and modify the individual preferences in terms of consumption/savings for health care behavior. Given this, the retirees take into consideration the uncertain future health expenditures when deciding the current level of consumption. Uncertain health care expenses introduce random shocks in their decision behavior determining them to engage in precautionary savings and/or appeal to insurance markets. In fact, through the insurance market, it is possible to put into practice the strong incentive of the retirees to share the risks of an unexpected increase in health care costs, after the age of retirement, with other institutions or individuals.

An important distinction must be made between the formal and informal insurance: given the relationship within a family, at the informal level there are far much less problems of adverse selection or moral hazard, because information on the exact health status are detained not only by the retired person but also by the family which supplies the health care. This is not the case in an informational incomplete formal insurance market where health information are unknown to the insurer. Since health status of the retiree is common knowledge for both himself and the family (reduced adverse selection), there remains the problem of incentive (moral hazard) to participate in the informal insurance agreements which is solved as follows: the pensioner has an incentive to share the risks with the family (besides possible private formal health insurance) due to uncertainty on type and magnitude of health expenditures related to age and his risk aversion (due to consumer non-satiation “more is better” – insurance, in this case); the family has an incentive to provide the informal agreement due to either strategic bequest motive induced by the retired person, who is considered to leave bequest only in exchange of health care, or to inter-family altruism motive, both motives for which I will account in the model. Moreover, formal insurance display a strong risk-pooling feature that in general, informal insurance lacks. Insurance companies cover a wide range of individuals and manage to gain through the risk-pooling characteristic of the formal insurance markets. On the contrary, informal insurance is provided by the extended family, mainly by children, and their number is rarely so high as to assure a high level of risk-pooling for the informal insurance they provide.

Table 1.1: Current Health Expenditure by ICHA-HF Health Care Financing, 2004

Category of funding	Austria	Belgium	Denmark	France	Germany	Greece	Italy	Netherlands	Spain	Sweden	Switzerland
HF11: General govern. (excl. social security)	29.7	12.9	84.2	4.9	10.0	42.7	74.5	6.2	64.9	85.4	17.9
HF12: Social security funds	45.6	58.7	0.0	74.5	68.7	3.7	0.1	60.2	5.4	0.0	40.5
HF23: Private household out-of-pocket exp	17.1	22.2	14.2	7.3	11.8	49.6	22.4	8.6	21.9	11.6	31.6
HF24: Private health insurance	4.9	4.9	1.5	12.5	8.7	1.5	0.9	18.6	6.5	2.5	9.0
HF25: All other private funds	2.7	1.4	0.1	0.8	0.8	2.5	2.0	6.3	1.3	0.5	1.0
Type of coverage	Social Insurance	Social Insurance	Public Tax Financed	Social Insurance	Social Insurance	Public Tax Financed	Public Tax Financed	Social Insurance	Public Tax Financed	Public Tax Financed	Private Mandatory Insurance

Source: OECD health data. Version: October 2007

Despite the microeconomic approach of this analysis, seems that the macroeconomic data support the importance of the insurance markets in balancing the health spending. According to the OECD estimates of the System of Health Accounts calculated for 30 countries' health indicators, for the countries considered previously, the categories of funding are detailed in Table 1.1 and account for an informal insurance component².

Consequently, it can be seen that overall, almost 65% of the health expenditures are supported by governmental funds, while the remaining are incurred by the private sector. If the financing situation is instead detailed by specific sources of funding for health expenditures, the situation is dominated in most countries by social security (general government) funds, as presented by Figure 1-6. Notice that there are four types of systems of coverage in the countries analyzed, and most of them have statutory coverage, represented by a public health care system, for more than 90% of the population (Denmark, France, Greece, Italy, Sweden and Switzerland)³. Private health insurance on the other hand, in some countries, is the primary source of health coverage (Austria, Belgium, Germany, Netherlands, and Spain) for a relatively significant proportion of the population that is not eligible for public coverage or that is eligible but chose to opt out the public system.

²The selected eleven OECD countries are the ones considered in this analysis, for which micro data were provided by the SHARE database.

³Paccagnella, Rebba and Weber [2007].

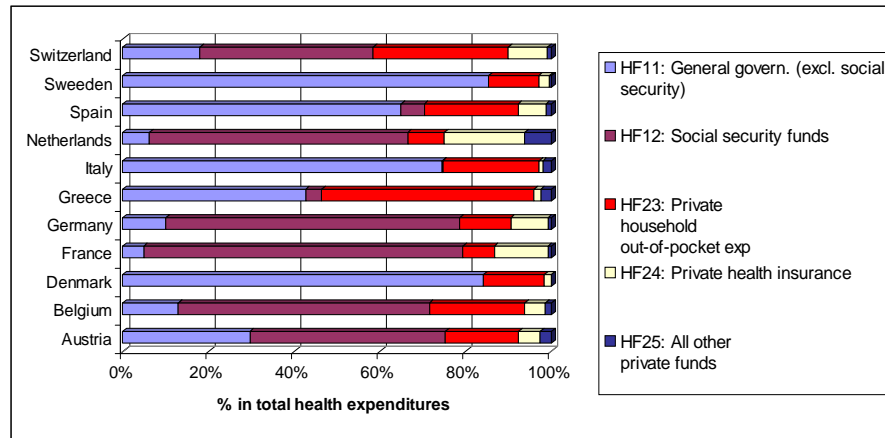


Figure 1-6: Detailed financing sources as percentage of overall health expenditures, in selective OECD countries (time series) (Source: OECD health data. Version: October 2007)

The data used on formal and informal care, voluntary private health insurance, median wealth - age profiles, health status - and health care - age profiles are provided by the first wave of the Survey on Health, Ageing and Retirement in Europe, SHARE⁴.

Figures show that, as far as the overall care is concerned, older individuals living alone are more likely to receive help than those living with others. On the other hand, help may be provided from a network outside the household or within household. The advantage of SHARE dataset is that it provides information on hours and types of formal and informal care. Informal care is measured in hours of care received from the extended family and from non-family, for three types of activities: personal care, help in housekeeping and paperwork. Note that the main source of informal help, for individuals living alone and so not considering the spouse, who is the main provider of informal care otherwise, are one's children (see Figure 1-7).

⁴This paper uses data from release 2 of SHARE 2004. The SHARE data collection has been primarily funded by the European Commission through the 5th framework programme (project QLK6-CT-2001-00360 in the thematic programme Quality of Life). Additional funding came from the US National Institute on Ageing (U01 AG09740-13S2, P01 AG005842, P01 AG08291, P30 AG12815, Y1-AG-4553-01 and OGHA 04-064). Data collection in Austria (through the Austrian Science Foundation, FWF), Belgium (through the Belgian Science Policy Office) and Switzerland (through BBW/OFES/UFES) was nationally funded. The SHARE data collection in Israel was funded by the US National Institute on Aging (R21 AG025169), by the German-Israeli Foundation for Scientific Research and Development (G.I.F.), and by the National Insurance Institute of Israel. Further support by the European Commission through the 6th framework program (projects SHARE-I3, RII-CT-2006-062193, and COMPARE, CIT5-CT-2005-028857) is gratefully acknowledged. For methodological details see Boersch-Supan and Juerges (2005).

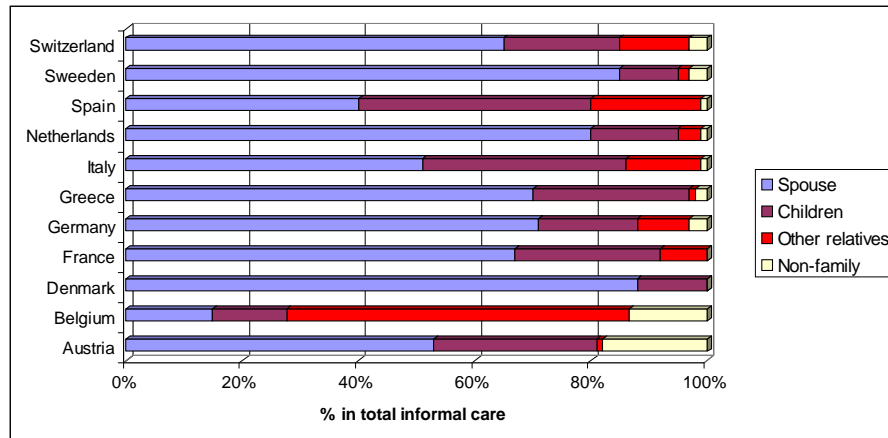


Figure 1-7: Percentage informal help received with personal care within the household (Source: SHARE data 2004)

Moreover, in Northern European countries, that have a wider public support system for elder people, children are not that important as care providers, while non-family (as friends or neighbors) are instead relevant, and this was stated by one-third of single respondents receiving help with personal care or practical tasks in the household. There is also the case that older people in general in these countries are more likely to receive formal help (professional nursing care, paid domestic help and meals of wheels) from outside the household than in the Southern European countries (Figure 1-8), but occasionally rather than frequently.

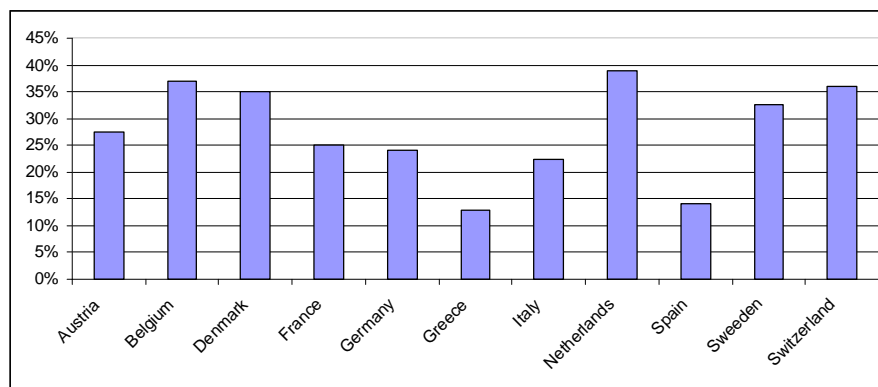


Figure 1-8: Percentage individuals receiving formal personal care help within the household (Source: SHARE data 2004; EUROFAMCARE 2004)

SHARE also provides information on individuals net total wealth, defined as the sum of all financial and real assets, net of liabilities, and which together with the flow of yearly income is a summary indicator of all the available resources. I assume that this measure of net total wealth can be used to finance consumption, to buy health insurance and to pay for the out-of-pocket health expenditures, but also can be left as bequest to children or extended family. Figure 1-9 plots the median net total wealth - age profiles (adjusted for purchasing power parity), across European countries considered in SHARE.

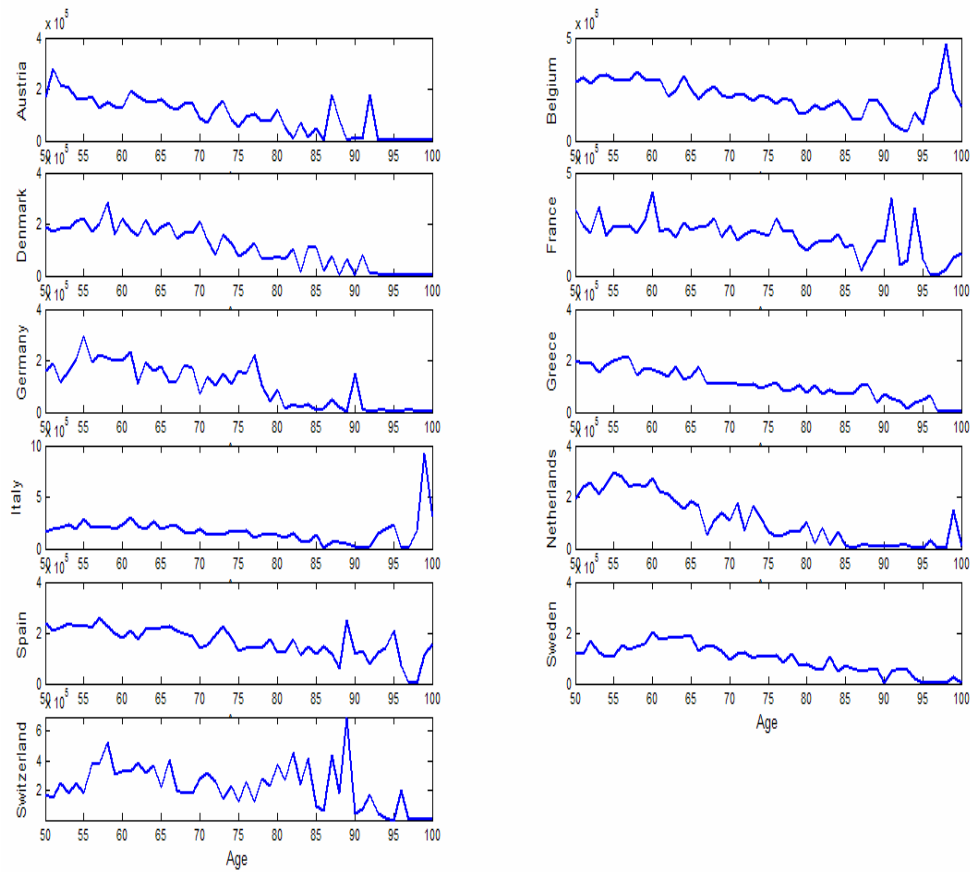


Figure 1-9: Median net total wealth by age (adjusted for purchasing power parity) (Source: SHARE data 2004)

It can easily be noted that there is a high variability of net wealth among countries; this may be due to the structure of the social security system (public/private), the availability of public health care, different intensity of bequest motive, different features of the mortgage markets or

different features of the real and financial assets. There are basically four groups with respect to the net wealth that the elder individuals have: Switzerland, Spain, Italy and Belgium with a high level of wealth (above €140,000), France and Netherlands with a medium - high level of net wealth (€120,000-€140,000), Austria, Denmark and Greece with a medium - low level of net wealth (€100,000-€120,000), and Germany and Sweden with a low level of net wealth (below €100,000). Note that the evaluation of wealth in high - medium - low was made based only on the SHARE sample and does not regard the worldwide levels of wealth.

Another type of information contained in the SHARE dataset is information on health and health care. Figure 1-10 reports health status as the median value of the health index by country. The health index is a continuous variable, computed based on SHARE information on the health status of each individual. Its value captures the prevalence of a large variety of limitations and conditions, as well as their effect on an individual's health status. The range of values lies from zero, which signals the worst health status observed, to unity for the individuals with perfect health. All intermediary values are obtained by reducing the 'perfect health' value by specific amounts (disability weights) related to different conditions/limitations and to their effect on the health status.

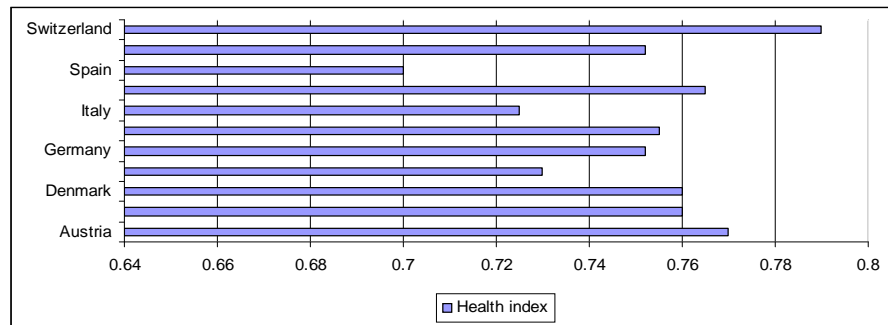


Figure 1-10: The level of health index, by country (Source: SHARE data 2004)

Furthermore, in order to have a clear picture of the health care systems efficiency in SHARE countries, for the elder people, health status needs to be linked to the health care services utilization displayed by the sample of individuals considered (Figure 1-11). As one can notice, the number of medical consultations reported in the last twelve months exhibit a strong relation with age, with seven consultations or more reported by 52% of age 80 - 84 respondents and

42% of age 85+.

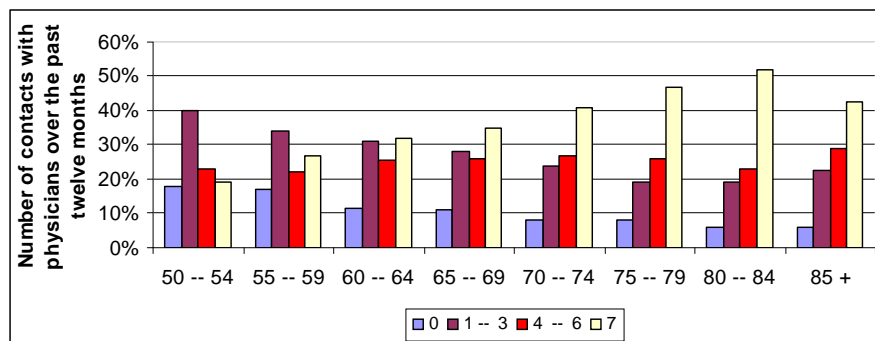


Figure 1-11: Health care utilization (number of physicians visits in last 12 months), by age (source: SHARE data 2004)

1.3 The Model

1.3.1 Utility function

For simplicity, I considered as unit of analysis the household consisting of one single individual who has just retired, which allowed me to concentrate on consumption, health insurance and savings decisions, and not to consider labor supply and retirement choices. The analysis is completed by including the bequest motive, for the purpose of emphasizing the potential effects on medical expense and mortality risk of informal insurance offered by the extended family. I assumed time is discrete and that each period corresponds to one year. The first period of observation occurs when the individual is a years old and entering retirement. The retirement age is assumed to be exogenous and deterministic, with all individuals retiring at age 65. Consequently, the model consists of a series of one-year periods, starting at the age of retirement and ending at the year of death, which is finite and restricted to occur by maximum age A (the maximum effective age is 100). The maximum length of the retirement period therefore is $T = A - a$ (which counts 36 periods). Periods are indexed by t , the number of years in the retirement period, starting at 1 at age a , so that overall $1 \leq t \leq T, t \in \mathbb{N}$. There is a stochastic survival probability $s_t \in [0, 1]$ in year t that evolves in a manner defined below.

Consider an individual seeking to maximize her expected lifetime utility at time t , $t = [1, T]$, (1 is the first period in retirement and T is the last one before she dies), with exponential discounting factor $\beta > 0$, by choosing current and future level of consumption and insurance, both formal and informal. Each period, the individual's utility depends on her health status, $m_t \in [0, 1]$ (that takes values from good, $m_t = 1$, to death, $m_t = 0$), consumption, C_t , and face value of insurance (formal $F_t(f_{t-1})$ and informal I_t), which acts as a positive externality. All the variables mentioned before are functions of time, so they will be indexed by t , while $F_t(f_{t-1})$ is at least of class C^1 (first class of differentiability).

The within-period utility function is given by

$$\begin{aligned} u(m_t, C_t, F(f_t), I_t) & : \mathbb{R}_+ \rightarrow \mathbb{R}_+ & (1.1) \\ u(m_t, C_t, F(f_t), I_t) & = \delta(m_t) \frac{C_t^{1-\gamma} - 1}{1-\gamma} + \epsilon(m_t) \frac{[\alpha F_t(f_{t-1})^\theta + (1-\alpha)I_t^\theta]^{\frac{1-\sigma}{\theta}} - 1}{1-\sigma}, \end{aligned}$$

where $\delta_t(m_t)$ and $\epsilon(m_t)$ describe the health status dependency of utility from consumption of non-durable goods and from pleasure of being looked after respectively, C_t is consumption of non-durable goods in period t , $F_t(f_{t-1})$ is the face value of the formal insurance policy, purchased the previous period (i.e. health expenditures covered by insurance in period t) and I_t is the face value of the informal insurance policy in period t . The parameters $\gamma, \sigma > 0$ are the relative risk aversion parameters for consumption of non-durable and medical goods respectively; the parameter σ increases as individuals become less willing to substitute formal and informal insurance across time (i.e. σ measures the non-separability between formal and informal insurance). I further assumed that consumption and insurance are additively separable in utility and that the utility of being looked after (through formal and/or informal coverage) is a CES embedded in a constant-elastic function, with substitution parameter θ .

More precisely, $\delta_t(m_t)$ determines how a person's utility from consumption of non-durable goods depends on her health status, and is given by

$$\begin{cases} \delta_t(m_t) = 1 + \delta m_t, \quad \delta > 0, & \text{for } 0 < m_t \leq 1 \\ \delta_t(m_t) = 0, & \text{for } m_t = 0 \end{cases}, \quad (1.2)$$

so when dead ($m_t = 0$), health status does not affect utility from consumption, while when healthy ($m_t > 0$) this status has a positive effect on utility (individual enjoys more the consumption of goods when healthy).

On the other hand, parameter $\epsilon(m_t)$ determines how a person's utility from insurance coverage depends on her health status, and is given by

$$\epsilon(m_t) = 1 - m_t, \quad (1.3)$$

so when dead ($m_t = 0$), health status does affect utility from medical care, while when perfectly healthy ($m_t = 1$) it has no effect on utility (healthy individuals do not enjoy any consumption of medical care).

The face value of the formal insurance is given by

$$F_t(f_{t-1}) = \omega f_{t-1} + \bar{f}, \quad \omega \geq 0, \quad f_{t-1} \geq 0, \quad \bar{f} > 0, \quad (1.4)$$

with f_{t-1} as insurance premia paid in period $t - 1$, before period- t health and health expenditures shocks are realized. The total amount paid for the formal insurance f_{t-1} is equal at the limit, with no public insurance provision, to the health care expenditures covered through the health plan $F_t(f_{t-1})$ divided by ω , where ω is the inverse of the loading factor κ ; $\kappa < 1$ allows the possibility of a tax subsidy for the insurance, while $\kappa > 1$ represents the case of the classical administrative costs or adverse selection. Almost all individuals that are 65 or older are eligible for some government provided compulsory health coverage, which supplements the private insurance coverage described in the equation above by ωf_{t-1} . Consequently, \bar{f} can be considered to reflect the minimum level of formal insurance provided by government, given that individuals are opting for a combination of formal and informal insurance. Notice that from a mathematical point of view, \bar{f} is constant and it will play no role in the maximization process; however, it represents the minimum health care consumption floor and will have an impact on parameter estimates.

The face value of informal insurance policy, I_t in the model, represents the money value of the time and/or transfers from the extended family on the individual's behalf. Informal insurance is considered to be function of three variables: the bequest B_t , that the elder individual will

transfer to the extended family after her death, the cohesion coefficient η_t , of the extended family towards the individual, and individual's probability of survive at time $t + 1$, given that she is alive at time t , s_t :

$$I_t = \eta_t(1 - s_t)B_t, \quad \eta_t \in [0, 1], \quad (1.5)$$

with $B_t = a_{t+1}$ representing the wealth the individual will transfer to the next period if alive or leave as bequest if dead. The parameter η_t is allowed to vary with the age of the individual, and also it will vary from one group of countries to another, capturing the degree of family cohesion. As a result, I assumed that cohesion coefficient can be written as

$$\eta_t = \beta_0(1 + \beta_1 * t + \beta_2 * t^2 + \beta_3 * t^3 + \beta_4 * t^4), \quad (1.6)$$

where β_0 represents strictly family cohesion, while the fourth order polynomial in years of retirement captures its age-structure.

I assumed that the market for informal insurance is perfect from the informational point of view, and so the 'premia' paid for the informal insurance equals the face value of the insurance. The intuition is that, each period, family is providing an amount of informal care that equals a fraction of the elder's wealth, weighted by the probability that the individual will die next period and so the bequest will actually be received; under these circumstances, the per-period cost to informal insure equals the 'informal coverage', with benefits being received each period while individual is alive, and costs being paid after her death.

Note that the informal insurance provision scheme does imply a complete lack of commitment of the retired individual to the family, with respect to the amount of bequest she will leave at her death in return to the care received. There is an extended literature (Bernheim, Shleifer, Summers [1985], Venti, Wise [2004], Chiuri, Jappelli [2006]) arguing that illiquid assets can be considered as instruments for commitment to leave bequest. Instead of using this approach, consider the more realistic scenario in which the informal care scheme is function of the whole amount of wealth that can constitute bequest (including liquid and illiquid assets but also the flow of interests, dividends and pension income), adjusted for the individual's probability of dying next period.

Finally, the distribution parameter α , with $\alpha \in [0, 1]$, helps explaining the influence of relative formal/informal insurance share in health care costs, and depends on the health status m_t , ($\alpha(m_t) = a * m_t$, $m_t \in [0, 1]$). I consider $\alpha(m_t)$ as the coefficient that assigns higher importance to the informal care rather than to the formal one, if in poor or fair health states. Notice that, since the individual prefers, in certain circumstances, informal to formal care, this element motivates the introduction of the strategic bequest through which the individual actually purchase informal insurance. Obviously, the individual can decide indirectly how much to informally insure through the amount she decides to leave as bequest at the end of her life to the extended family. She does that by directly choosing consumption and formal insurance premia, while the family provides the informal care according the cohesion measure represented by the parameter η_t .

1.3.2 Uncertainty

The individual faces several sources of risk, treated as completely (health status / survival uncertainty) or partially (medical expense uncertainty) exogenous. The reason is that the focus is on older people that have already shaped their health and lifestyle, but also make choices in terms of their way to respond to medical care uncertainty through insurance. The individual's utility depends on three stochastic variables:

1) Health status uncertainty. I allow the transition probabilities for health status to depend on previous health status and age. The elements of the health status transition matrix are

$$\pi_{k,j,age} = \Pr(m_t = j | m_{t-1} = k, age), \quad k, j \in \{1, 2, 3, 4\}. \quad (1.7)$$

2) Survival uncertainty. Let $s_{m_t,age} = s_t$ denote the probability that an individual is alive at age $t + 1$, conditional on being alive at age t , having time- t health status m_t , and a certain age. This means that the death probability $(1 - s_t)$ in the utility function can equivalently be computed as $(1 - s_t) = \pi(m_t(1))$ where $m_t(1) = death$.

3) Medical expense uncertainty. Besides formal and informal insurance, there is a third possibility to finance the health spending, namely out-of-pocket⁵. Health costs out-of-pocket,

⁵Health costs out-of-pocket were not considered in the utility function since I assumed that if no insurance

hc_t , are defined as the residual of total health care costs considered exogenous h_t , after deducting the coverage (both formal and informal), and a shock ψ_t . I assumed that medical spending depends on health status and age, and moreover is decreasing in formal/informal insurance coverage,

$$hc_t = h_t - (\omega f_{t-1} + \bar{f}) - \eta_t(1 - s_t)B_t + \sigma_{\varepsilon_t} * \psi_t. \quad (1.8)$$

The intuition behind this formulation is that h_t is not a sufficient statistic for health spending out-of-pocket; in order to maintain a certain health status, a continuous investment in health costs is needed. Consequently, the health costs of an individual who passes from poor to good health will exceed the costs of an individual persisting in a good health state. So, first the individual purchases coverage through formal insurance, in the previous period f_{t-1} , as well as through informal insurance $\eta_t(1 - s_t)B_t$, paid after death, then the exogenous health care spending shock is realized and it persists according to an AR(1),

$$\ln(\psi_t) = \rho_\psi \ln(\psi_{t-1}) + \varepsilon_t, \varepsilon_t \sim N(0, \sigma_{\varepsilon_t}^2). \quad (1.9)$$

The issue of health dynamics and death is crucial to the insurance motive, given the high expenses associated with bad health. There are four health states modelled. State 1 is death, and state 2 is a state in which long term care of some form is required (invalidity or poor health). In state 3 individual has medical problems but no need for long term care (fair health). State 4 is the good health state. At the beginning, in period 1, the individual is assumed to start in good health which is consistent with the SHARE data on health status at age 65.

The health state follows a Markov chain with age-varying one-period state transition matrix $P(t)$ described below. In each year, this is a 4×4 matrix. Retirees reaching age A die with probability 1 in the following year. Together with the initial health state, the Markov transition matrices $P(t), t \in [1, T]$, enable the computation of future probabilities attached to all health states, including death. Given the initial health state m_1 , the transition matrix is applied repeatedly to derive the probability $\pi(m_t)$ that a retiree is in one of the four health states at time $t > 1$. In addition, each health status has associated with it a necessary and deterministic

coverage was available, the individual will have to incur the health costs entirely out-of-pocket; however, if prudent, she will have an additional utility from being covered.

health cost, $h_t(m_t)$. Death expenses in state 1 are also deterministic, at level $h_t(m_t(1))$, and are subtracted from the bequest.

Following the calibration exercise used by Ameriks et al. [2005], I considered the same structure of age-dependent adjustment matrices, but I estimated the parameters that allow for the health status shifting. More precisely, the 1-period ahead transition matrix at age $65 + t$ is given by:

$$P(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \pi_{21} & \pi_{22} & \pi_{23} & 1 - \pi_{21} - \pi_{22} - \pi_{23} \\ \pi_{31} & \pi_{32} & \pi_{33} & 1 - \pi_{31} - \pi_{32} - \pi_{33} \\ \pi_{41} & \pi_{42} & \pi_{43} & 1 - \pi_{41} - \pi_{42} - \pi_{43} \end{bmatrix} * A_t,$$

with

$$A_t = \begin{bmatrix} 1 & 0 & 0 & 0 \\ c_1 t^e & 1 - c_1 t^e & 0 & 0 \\ c_1 t^e \frac{1}{1+c_2} & c_1 t^e \frac{c_2}{1+c_2} & 1 - c_1 t^e & 0 \\ c_1 t^e \frac{1}{1+c_2+c_2 c_3} & c_1 t^e \frac{c_2}{1+c_2+c_2 c_3} & c_1 t^e \frac{c_2 c_3}{1+c_2+c_2 c_3} & 1 - c_1 t^e \end{bmatrix}. \quad (1.10)$$

The A_t matrix is the so-called age-adjustment matrix. It shifts probability mass from the left (worse health states and death) towards the right (better health states), relative to the transition matrix at age 65, $P(1)$. The 3 parameters c_1 , c_2 , and c_3 control how fast this shifting occurs. Loosely speaking, parameter c_1 controls the transition from invalidity to death as age increases; c_2 determines how much more likely death is relative to invalidity when in health state fair or good, and c_3 determines how much likely good health state is when in good health. Basically, the exponent c_1 allows for faster than linear shifting as the agent becomes older. Because the system is non-linear, there is no unique solution to the system of 12 equation and 12 parameters. Consequently, I estimated these parameters that control for the speed at which the shifting occurs, together with the persistence and standard deviation of the Markov process characterizing the health spending uncertainty.

1.3.3 Budget constraint

Households enter retirement with wealth $a_1 \geq 0$, and wealth at the beginning of time t is denoted as a_t . Assuming that there is one composite riskless asset in which household can invest and which yields a constant rate of interest r , next period's wealth is given by

$$a_{t+1} = a_t + (y + ra_t) - f_t - C_t - hc_t, \quad (1.11)$$

where $(y + ra_t)$ represents the income flow, which includes constant pension payment as well as wealth interests and dividends.

Associated with this budget rule there is the borrowing constraint

$$a_{t+1} = (1 + r)a_t + y - f_t - C_t - hc_t \geq 0, \forall t. \quad (1.12)$$

Observe that I included in the borrowing constraint the medical expenses, assumed to be realized at the beginning of the period, after health and medical spending shocks are realized. I considered this assumption as more reasonable than the alternative, namely that time- t medical expense shocks are fully unknown when individuals decide whether to hold on to their formal or informal health insurance. Given the timing of medical expenses, under this borrowing constraint an individual with extremely high medical expenses this year could have zero net worth next year.

1.3.4 Timing of the model

The timing of events is the following:

The individual enters period t with health state m_t , wealth state a_t and formal insurance $F_t(f_{t-1})$, bought the previous period. At the beginning of the period, she receives the pension income and pays the formal insurance premia for the next period. Then the health shock is realized and if she is still alive, the medical costs are realized, then she consumes and saves, while if she doesn't survive the next period, funeral costs $h_t(m_t(1))$ are paid and the bequest B_t equals the remaining net resources after accounting for the formal insurance coverage purchased previously, down to a minimum of zero,

$$B_t = \max [a_{t+1}, 0] \geq 0, \forall t. \quad (1.13)$$

One remark on the timing of insurance (formal and informal) provision and payment. While in the formal market, the two moments are successive since the coverage through the insurance becomes active from the next period with respect to the one in which the payment was made by the policy beneficiary, in the informal market, the moments have a considerable lag between them: the old individual benefits of the medical services bought and/or provided by the extended family in the current period, but she will get to pay only later (at the end of her life). In the context of pooling the risk of falling ill within family, there must be a certain level of mutual trust and honesty, given that there are no legal aspects to enforce the informal arrangements within families, or actually there must be an enforceable contract. Assuming that there is a bequest involved (and so the old individual is not consuming all the wealth by the time of his death), which the data will relate to country specific features, there is a higher incentive for the old people to insure informally rather than exclusively through formal insurance markets. On the other hand, the reason for insuring formally is that the medical goods provided in an institutionalized framework are less substitutable with the ones provided by the extended family or, in case the family is actually paying for the professional medical care, less expensive due to the risk-pooling of the formal insurance market, so it is more profitable to pay them through the formal insurance purchased previously and consequently consume more in the current period.

For the family, any optimal allocation involving informal care provision does not involve leisure, since its opportunity cost is certainly higher than the informal care's provision. Moreover, it must also be considered the informal insurance constraint due to which family members cannot transfer to the old individual in case of a health shock more than they actually possess. Given that in the model a period measures a full year, h_t represents the annual health care costs and will be covered through informal insurance up to a maximum amount which accounts for the yearly labour time.

1.3.5 Recursive framework

Assuming the existence of a maximum given the continuity of the functions considered on the compact space determined by the interval of wealth and formal insurance premia, the recursive form is

$$\begin{aligned}
 \underset{C_t, f_t}{Max} V_t(m_t, C_t, F(f_{t-1}), I_t) = \underset{C_t, f_t}{Max} & \left\{ (1 + \delta m_t) \frac{C_t^{1-\gamma} - 1}{1 - \gamma} + \right. \\
 & + (1 - m_t) \frac{\left[\alpha((\omega f_{t-1} + \bar{f}))^\theta + (1 - \alpha)(\eta_t(1 - s_t)B_t)^\theta \right]^{\frac{1-\sigma}{\theta}} - 1}{1 - \sigma} + \\
 & \left. + \beta s_t E_t [V_t(m_{t+1}, C_{t+1}, F(f_t), I_{t+1})] \right\} \tag{1.14}
 \end{aligned}$$

subject to equation (1.12).

An individual's decision thus depends on her state variables, $X_t = (a_t, f_{t-1}, m_t, \psi_t) \in \mathbb{R}_+^4$, her preferences, ϕ , and her beliefs, χ , where

$$\phi = (\gamma, \sigma, \alpha, \omega, r) \in \mathbb{R}_+^5,$$

$$\chi = (\delta, \theta, \eta_t, \beta, \rho_\psi, \sigma_{\varepsilon_t}^2, \rho_{m_t}, \sigma_{m_t}^2, h_t) \in \mathbb{R}^8.$$

From the discrete dynamic optimization principle it follows that the solution to the individual's problem is found in two steps: the first one consists in finding the set of consumption $\{C_t(X_t, \phi, \chi)\}, t \in [1, T]$, formal insurance benefits $\{f_t(X_t, \phi, \chi)\}, t \in [1, T]$ and informal insurance benefits $\{(\eta_t(1 - s_t)a_{t+1})(X_t, \phi, \chi)\}, t \in [1, T]$ rules that solve the system (1.14). Inserting these decision rules into the asset accumulation equation yields next period's wealth, $a_{t+1}(X_t, \phi, \chi)$, for all the values that compose the grid for formal insurance purchased in the previous period. Using the optimal values for wealth, in the second step, the value function is maximized and the optimal value for the formal insurance is found.

I used backward induction to compute value functions and policy functions. The optimization problem is solved by grid search, and the state-space for "wealth" and "formal insurance"

is made discrete. Given that $t \in [1, T]$, the solution of the problem is obtained in a finite number of periods. In the last period, the decision is trivial, with the agent consuming whatever is left, since at time T she has 0 probability to survive the next period. Once the policy function is solved, the corresponding value function in the last period can be obtained and used in computing policy rules for the previous period. This iteration is continued until $t = 1$.

1.4 Data

I estimated the dynamic model using data from the first wave of the Survey of Health, Ageing and Retirement in Europe (SHARE). SHARE is a cross-national microeconomic database, containing information regarding health, socioeconomic status and social and family networks of individuals aged 50 or over. Data refers to household level information regarding health status, economic situation and social support variables and was collected in 2004 (the second data collection wave took place in 2006-2007). SHARE was conducted in eleven countries covering the representatives regions of Europe: Scandinavia (Denmark and Sweden), Central Europe (Austria, France, Germany, Switzerland, Belgium, and the Netherlands) and the Mediterranean (Spain, Italy and Greece).

Details on variables definition as they appear in the dataset, motivation for their selection and correspondent profiles by age, sex, three wealth percentile (25, 50 and 75) and country are in Appendix D. Mainly, the dataset used to estimate the model in Section 1.3. was formed by the annual values of voluntary (supplementary) private insurance (formal insurance henceforth), expenditures on non-durables (consumption henceforth) and total wealth as a comprehensive measure of the financial and real assets, as well as the yearly income flow. Notice that, since the model refers only to single individuals, I dropped the observations regarding the married ones or that have a registered partner.

Based on the existing observations on formal insurance, non-durable expenditure, total wealth and individual observable characteristics, a linear model was used to obtain the predicted values of formal insurance for all individuals that reported wealth and consequently eliminate the missing values registered at the level of this variable. Expenditures on non-durables, on the other hand, consider the amount spent on food at home, on food outside and on telephone bills,

all weighted according to coefficients extrapolated from national datasets of SHARE countries, through an OLS procedure on the same variables. Using the information contained in the first wave of SHARE made it impossible to obtain a temporal dimension for wealth, consumption and formal insurance, that could latter be used to match the simulated series. To overcome this problem, data was further detailed by two individual characteristics, namely age and three total wealth percentile. By selecting wealth-specific groups of individuals, I created life-profiles of identical individuals from the wealth point of view, with ages between 65 and 100. Since the information were obtained based on a cross-sectional dataset, there was much noise within each variable for the profiles obtained. Consequently, these sets of data were further smoothed using a classic 5 years moving average filter and the missing observations at this level were imputed using linear interpolation. Finally, this procedure was repeated for all three groups of countries: Scandinavian (Denmark and Sweden), Central European (Austria, France, Germany, Switzerland, Belgium, and the Netherlands) and the Mediterranean (Spain, Italy and Greece).

There are two econometric issues related to the use of cross-section data. First, in a cross-section, because wages have increased over time (with productivity), older individuals are poorer at every age, and the measured saving profile will overstate asset decumulation over the life cycle. By not accounting for this effect, the model will generate simulated data in which the degree to which elderly people run down their assets is overstated. Second, rich people tend to live longer, so that the average survivor at each age has higher lifetime income than the average deceased individual with the same age. This “mortality bias” tends to overstate asset growth. More than that, as time passes, the surviving people will be, relative to the deceased ones, healthier and knowing that they will live longer, will tend to save more than their deceased counterparts, displaying a slower wealth decumulation during the years in retirement. By not accounting for mortality bias, the model will simulate data that understate the wealth decumulation process.

The solution to both biases is offered by the chosen estimation procedure: using a structural approach, these biases can be accounted for directly, by recreating them with the data generating process. Basically, what this work does is endowing each simulated individual with a certain age, total wealth and initial health status. If older people have lower lifetime wealth in the data, they will have lower total wealth in the simulated data as well. Similarly, the estimated decision rules and the simulated profiles incorporate mortality effects, by different total wealth

percentile, in the same way as the data.

1.5 Calibrations and Estimation Methodology

This section describes parameters estimation procedure and provides a quantitative analysis of its predictions. I am conscious of literature uncertainty regarding the values of many of the model parameters. Due to this reason, I estimated most of the parameters involved in the model and used the literature values for those ones that I did not focus on, but are used just as instruments for the dynamic programming model. The approach is similar to the two-step strategy used by Gourinchas and Parker [2002], Cagetti [2003], and French and Jones [2004]: in the first step calibrate those parameters that can be cleanly identified without explicitly using the model $(\delta, \beta, \rho_{m_t}, \sigma_{m_t}\omega, r, e)$; in the second step estimate the interest parameters $(\sigma, \gamma, \theta, a, \rho_\psi, \sigma_{\varepsilon_t}, c_1, c_2, c_3, \eta_t(\beta_0, \beta_1, \beta_2, \beta_3, \beta_4))$ with the two-stage simulated method of moments (SMM), taking as given the parameters estimated in the first step. The estimation will generate the parameter vector, yielding the simulated life-cycle decision profiles that “best match” the data ones.

Because the underlying motivation is aiming to explain why elderly retain so much wealth and why they insure formally and informally, I matched total wealth, consumption and formal insurance profiles, conditional on age and wealth percentiles; in practice, I considered four samples correspondent to three wealth percentiles (25, 50 and 75) plus an average one, and for each of them I re-created the time series of wealth, consumption and formal insurance for an individual with age between 65 and 100. I used a real risk-free asset return of $(1 + r) = 1.02$, and a corresponding discount factor, β , computed based on data consumption growth rate $\frac{\dot{c}}{c_{t+1}}$ and on the estimated risk aversion, γ . The preference shifter due to health changes δ , was set to 5; however, performing a robustness check ($\delta = 0$) revealed that this has very little effect on the other parameter estimates. For the health status shock, persistence coefficient, ρ_{m_t} , was set to 0.5, while volatility, σ_{m_t} , was set to 0.21417; note that by fixing these two parameters, no constraint did actually bind the health transition matrix, since the age-adjustment elements were estimated based on the data. The grid for wealth, consumption and formal insurance, as well as their starting point were also set to match the data. The parameter ω is the inverse

of the loading factor κ , and for the purpose of this model I considered $\kappa > 1$, that represents the case of the classical administrative costs. I set the parameter κ to its average value within each group of countries that I considered in the analysis, while the country level of the loading factor was provided by the OECD health data.

I assumed the retirees at age 65 in good health, following the health distribution of the sample selected ($m_1 = m(4) = 1$). I realize that the sample is affected by mortality bias in the sense discussed at the end of the data section. However, since I had actually estimated the health transition matrix based on a sample of 100 simulated individuals that face a cross-section of mortality rates in a given year, the model managed to recreate the health distribution of the real data.

Each period of the model represents one year and individuals die with probability one at age 100 ($T = 36$). The construction of the transition matrix for the health care stochastic process is described in Appendix E. To model the medical costs associated to each health state, I identified the mean annual funeral, long-term care and curative and rehabilitation costs for the seniors using data provided by the OECD statistics. For the bequest, I considered the same range of possible values as for wealth, and used the same grid of values. For the adjustment health status transition matrix, I considered the parameter e to be held fixed at $e = 1.5$ as in Ameriks et al. [2005].

To compute optimal strategies, I first discretize the state space using the Gauss - Hermite quadrature method⁶. The model is then solved by backwards induction, from the age of 65 to 100. For each wealth group, I computed the life-cycle history for a higher number of artificial individuals (100 different individuals for each of the three wealth percentiles and for the representative agent models, using random draws for the two stochastic variables). To each of these individuals, I assigned a value of the state vector $X_t = (a_t, f_{t-1}, m_t, \psi_t)$ which endows them with a value of wealth, health coverage, health status and health costs consistent with the stochastic processes described in Section 1.3.2.

The Simulated Method of Moments (SMM) technique used for this work is the standard one. Solving numerically the model and considering the stochastic structure of the solutions, allowed the simulation of each individual's wealth, consumption, formal insurance and mortality.

⁶For further details on this procedure please see Appendix A.

Table 1.2: Choice of Moments

$$\left\{ \begin{array}{lll} \sigma_{\ln(A_t)}, & \sigma_{\ln(C_t)}, & \sigma_{\ln(C_t/A_t)}, \\ \text{corr}(A_t, C_t), & \text{corr}(A_t, F_t), & \text{corr}(A_t, C_t/A_t), \\ \text{corr}(C_t, F_t), & \text{corr}(C_t, C_t/A_t), & \text{corr}(A_t, A_{t-1}), \\ \text{corr}(A_t, A_{t-2}), & \text{corr}(C_t, C_{t-1}), & \text{corr}(C_t, C_{t-2}), \\ \text{corr}(F_t, F_{t-1}), & \text{corr}(C_t/A_t, C_{t-1}/A_{t-1}), & \text{corr}(C_t/A_t, C_{t-2}/A_{t-2}) \end{array} \right\}$$

I then computed profiles from the artificial histories and take moments of each simulated profile. Comparing the mean of the artificial moments vectors with the ones computed from the real data, parameters were adjusted until the difference between the data and simulated moments was minimized⁷.

The interpretation of the associated χ^2 -test statistic or corresponding *p-value* is whether or not the true data moments are equal to the realized data moments, given the stochastic processes for which the true time series is just one realization. Consequently, this hypothesis fits perfectly the set of parameters to be estimated, that included on the one hand, standard deviations and persistence coefficient for the distribution of health expenditures stochastic process, and on the other hand, the three health status shifting parameters.

The actual choice of moments for the SMM is still an open issue in the literature. In order to ease the interpretation and restrain the set of moments that would potentially be too large, the model limited itself to considering measures of variability, instantaneous correlation coefficients, and persistence. In particular, using SMM, I restricted the estimation to a set of three variables, namely total wealth, consumption, and formal insurance and estimated the model using a set of fifteen true and simulated moments (m_T/m_N), showed in Table 1.2.

1.6 Results and Model Fit

Given the model presented in Section 1.3. and the calibrated parameters described in the estimation methodology, value functions and decisions rules were computed numerically using

⁷The first stage takes place under condition that the weighting matrix $W_T = I_T$. Obtaining the estimates from this stage will allow us to repeat the procedure and use at the second stage the weighting matrix W_T consistently estimated using the estimator proposed by Newey and West (1994), to obtain the final estimates. This matrix, heuristically, it gives more weight to moments that are precisely estimated in the data. For a detailed description of the method see Appendix B.

backward recursion. This section presents the estimation results and discusses the implications of the fitted structural models in behavioral terms, both for the representative agent and for specific levels of wealth individuals. Tables 1.3 - 1.5 report the estimates of the aggregate structural parameters for the twelve models, with health and health expenditure uncertainty.

Considering our estimates, it can be addressed the issue of how well the stochastic model fits the life-cycle wealth, consumption and formal insurance profiles. Comparing the moments, it is expected the simulated variables profiles at the disaggregated level to fit much better the real ones than the average profiles, displaying values of the overidentification test statistics are less elevated. While this is the case for the Scandinavian countries models, it is not equally true for the Mediterranean wealth-specific models, that register a worse fit than the representative agent model or for the Central European group where only the 75th wealth percentile model outperform the representative agent one.

The structural parameters of the models with significant goodness of fit with real data are estimated quite precisely. The overidentifying restrictions implied by the models pass a χ^2 -test at standard significance levels. Thus, we are unable to reject the null hypothesis that the sets of unconditional moments in the model and in the data are the same. However, for the models that do not display a high goodness of fit with empirical data, even though the models are formally rejected, the life cycle profiles generated for the most part resemble to the life-cycle profiles displayed by true data. These weak significance levels registered for some models are also due to the real data profiles. For instance, in the case of Scandinavian countries, institutionalized individuals that enter nursing homes are excluded by sample design. As a result, it can be seen that the moments of the real data are quite different with respect to Mediterranean and Central European data.

Table 1.6 presents results of the match of the set of fifteen moments considered, as well as model's fit for the average simulated individual, as described by the variables in the data, while Tables 1.7 - 1.9 show results for the disaggregated models (that consider different wealth subgroups).

As results illustrate, the simulated wealth profile of the fitted models track the actual wealth in a good proportion and it can be considered to produce good predictions, more precise for the disaggregated models than for the aggregated ones for Scandinavian and partially Central

Table 1.3: Estimated Structural Parameters, Mediterranean Countries

<i>Param.</i>	<i>25th wealth percentile</i>	<i>50th wealth percentile</i>	<i>75th wealth percentile</i>	<i>Represent. agent</i>
σ	-0.0169 (0.0352)	-0.7135 (0.0096)	-5.5296 (0.0990)	-0.0009 (0.0013)
γ	3.4731 (0.1269)*	4.7171 (0.1282)**	3.9661 (0.0388)**	5.6015 (0.2793)*
θ	0.1006 (0.0059)*	-0.5426 (0.0125)	0.5465 (0.0058)**	-0.0012 (0.0075)
a	0.3931 (0.0015)**	0.2300 (0.0296)*	-2.0994 (0.0015)	0.0025 ($0.10 * 10^{-4}$)**
ρ_ψ	1.2937 (0.1499)*	0.8501 (0.0080)**	-0.7148 (0.0045)	0.9500 (0.0279)**
σ_{ε_t}	0.1697 (0.0035)**	0.1120 (0.0609)*	0.0453 (0.0007)**	3.6617 (0.0172)**
c_1	0.0010 (0.0152)	0.00088 (0.0378)	0.00065 (0.0287)	0.0024 (0.0001)*
c_2	1.1142 (0.0036)**	1.0902 (0.0308)**	$1 + 10^{-7}$ (0.0626)**	10.1230 (0.0097)**
c_3	0.1634 (0.0036)**	0.0012 (0.0006)	10^{-7} (0.0056)	0.5470 (0.1821)
β_0	1.5690 (0.2015)*	2.8535 (0.0337)**	0.3902 (0.0109)*	1.1453 (0.0105)**
$\beta_1 * 10^{-2}$	-0.0517 (0.0115)	0.3247 (0.2555)	-0.0051 (0.0062)	-0.0388 (0.0937)
$\beta_2 * 10^{-4}$	0.0120 (0.0495)	-0.0092 (0.7421)	$0.85 * 10^{-4}$ ($0.96 * 10^{-6}$)**	$-0.268 * 10^{-4}$ ($0.77 * 10^{-6}$)
$\beta_3 * 10^{-4}$	0.0082 (0.0725)	0.0015 (0.0004)	0.1507 (0.0038)**	0.0037 (0.0513)*
$\beta_4 * 10^{-8}$	0.1993 (0.0030)**	1.6948 (0.0040)**	$-0.43 * 10^{-4}$ ($7.41 * 10^{-6}$)	$0.69 * 10^{-4}$ ($4.26 * 10^{-6}$)*

Standard errors are in parentheses below each estimated parameter. (*) indicates significance at 5%, while (**) stands for significance at 1%.

Table 1.4: Estimated Structural Parameters, Central European Countries

<i>Param.</i>	<i>25th wealth percentile</i>	<i>50th wealth percentile</i>	<i>75th wealth percentile</i>	<i>Represent. agent</i>
σ	1.7638 (0.0121) **	-0.0069 (0.0085)	-1.4470 (0.0197)	-0.0011 (0.0188)
γ	2.4325 (6.714)	6.1664 (0.0287) **	8.4024 (0.0030)*	6.4460 (0.6250)*
θ	-1.2733 (0.1743)	-0.7040 (0.0008)	0.0739 (0.0003)	-0.0113 (0.0121)
a	-1.8572 (0.1202)	-0.0009 (0.0651)	4.4950 (1.3605)	-0.0100 (0.0144)
ρ_ψ	2.0432 (0.0108) **	1.4332 (0.0408) **	0.9331 (0.0066) **	0.9472 (0.0164) **
σ_{ε_t}	0.2192 (0.0102)*	0.1141 (0.0051) **	0.0731 (0.0008) **	3.0756 (0.6404)
c_1	0.00173 (0.0030)	0.00172 (0.0005)*	0.00017 (1.4810 ⁻⁵)*	0.0015 (2.67 * 10 ⁻⁴) **
c_2	0.3123 (0.7190)	0.2883 (0.0571)*	0.2620 (0.0462)	0.6067 (0.4303)
c_3	0.1230 (0.0017) **	0.0008 (0.0007)*	2.2850 (0.0266) **	0.0039 (3.55 * 10 ⁻⁴) **
β_0	-1.4336 (5.3515)	-3.5498 (0.7818)	-1.6974 (0.0164)	0.7259 (0.0137) **
$\beta_1 * 10^{-2}$	-0.0215 (0.1024)	0.0680 (0.0008) **	0.1300 (0.0046) **	-0.7195 (0.0204)
$\beta_2 * 10^{-4}$	-7 * 10 ⁻⁵ (0.319)	0.0024 (2.4 * 10 ⁻³) **	-1.5 * 10 ⁻⁵ (0.0408)	-2.06 * 10 ⁻⁴ (0.0696)
$\beta_3 * 10^{-4}$	0.0242 (0.0009) **	0.0012 (1.8 * 10 ⁻⁴)*	0.1634 (0.0378)*	0.0614 (0.0022) **
$\beta_4 * 10^{-8}$	0.0239 (0.0009) **	10.2053 (0.5738) **	0.0725 (0.0085)*	2.50 * 10 ⁻⁴ (0.0055)

Standard errors are in parentheses below each estimated parameter. (*) indicates significance at 5%, while (**) stands for significance at 1%.

Table 1.5: Estimated Structural Parameters, Scandinavian Countries

<i>Param.</i>	<i>25th wealth percentile</i>	<i>50th wealth percentile</i>	<i>75th wealth percentile</i>	<i>Represent. agent</i>
σ	11.3468 (6.5991)	7.9393 (0.0064) **	0.4968 (0.0034) **	-0.9787 (0.1164)
γ	2.0872 (0.2420)*	8.7925 (44.5089)	9.0303 (0.8525)*	7.7751 (3.2682)
θ	-2.1693 (0.6424)	-3.4009 (0.0075)	-0.8630 (0.0196)	-2.2705 (0.0433)
a	0.2135 (0.0726)*	-1.6057 (0.0016)	-0.0804 (0.0211)	0.8859 (0.0111) **
ρ_ψ	2.1500 (0.0766) **	1.4540 (0.0018) **	0.9906 (0.0017) **	1.0077 (0.0098) **
σ_{ε_t}	0.2291 (0.0023) **	0.7538 (0.0036) **	0.1549 (0.0011) **	2.4373 (0.1833)*
c_1	0.0008 ($3 * 10^{-5}$)*	0.0042 (0.2650)	0.0032 (0.0002)*	0.0020 (0.0041)
c_2	0.0632 (0.0012) **	0.0101 (0.1039)	0.0250 (0.0075)	0.0411 (0.0024)*
c_3	1.3987 (0.018) **	0.0024 (0.105)	0.0758 (0.0022) **	0.0051 (0.0193)
β_0	1.4956 (0.0045) **	-2.5466 (2.2644)	-1.2619 (0.0012)	-2.1472 (2.8192)
$\beta_1 * 10^{-2}$	-0.0654 (0.0100)	5.2107 (2.5801)	2.3081 (0.0033)*	3.5298 (0.0617)
$\beta_2 * 10^{-4}$	0.0083 (0.0352)	$-9.4 * 10^{-4}$ ($2.66 * 10^{-4}$)	$7.5 * 10^{-5}$ ($2.26 * 10^{-3}$)	$2.242 * 10^{-2}$ (0.0044)
$\beta_3 * 10^{-4}$	0.0156 ($1.1 * 10^{-4}$) **	0.0565 (0.0460)	0.0182 (0.0093)*	0.2340 (0.9837)
$\beta_4 * 10^{-8}$	0.4282 (0.0029) **	-0.0162 ($1.56 * 10^{-2}$)	-0.0064 (0.2652)	-0.0518 (0.0019)

Standard errors are in parentheses below each estimated parameter. (*) indicates significance at 5%, while (**) stands for significance at 1%.

Table 1.6: Estimated Moments and Goodness of Fit Test - Representative Agent

<i>Moments</i>	<i>Med.</i>	<i>Gr.</i>	<i>Centr.</i>	<i>Gr.</i>	<i>Scan.</i>	<i>Gr.</i>
	<i>Sim.</i>	<i>Emp.</i>	<i>Sim.</i>	<i>Emp.</i>	<i>Sim.</i>	<i>Emp.</i>
$\sigma_{\ln(A_t)}$	0.22	0.25	0.25	0.24	0.55	0.49
$\sigma_{\ln(C_t)}$	0.13	0.15	0.19	0.26	0.10	0.23
$\sigma_{\ln\left(\frac{C_t}{A_t}\right)}$	0.24	0.25	0.24	0.14	0.50	0.29
$corr(A_t, C_t)$	0.35	0.28	0.50	0.85	0.67	0.88
$corr(A_t, F_t)$	0.85	0.84	0.63	0.61	0.98	0.92
$corr\left(A_t, \frac{C_t}{A_t}\right)$	-0.74	-0.79	-0.61	-0.78	-0.96	-0.91
$corr(C_t, F_t)$	0.27	0.28	0.34	0.32	0.59	0.74
$corr\left(C_t, \frac{C_t}{A_t}\right)$	0.23	0.28	0.29	0.45	-0.45	-0.73
$corr(A_t, A_{t-1})$	0.79	0.92	0.79	0.87	0.78	0.96
$corr(A_t, A_{t-2})$	0.54	0.85	0.54	0.63	0.52	0.91
$corr(C_t, C_{t-1})$	0.72	0.51	0.76	0.82	0.80	0.89
$corr(C_t, C_{t-2})$	0.60	0.24	0.65	0.48	0.70	0.76
$corr(F_t, F_{t-1})$	0.81	0.97	0.80	0.98	0.76	0.98
$corr\left(C_t/A_t, \frac{C_{t-1}}{A_{t-1}}\right)$	0.77	0.78	0.64	0.72	0.67	0.95
$corr\left(C_t/A_t, \frac{C_{t-2}}{A_{t-2}}\right)$	0.51	0.51	0.34	0.46	0.31	0.88
J_T	0.15		0.32		0.87	
$\chi^2(1)$	3.15		6.72		18.27	
$p - value$	0.0759		0.0095		0.0002	

Table 1.7: Estimated Moments and Goodness of Fit Test - Wealth Subgroups, Mediterranean Countries

<i>Moments</i>	25th		50th		75th	
	<i>Artif.</i>	<i>per. Emp.</i>	<i>Artif.</i>	<i>per. Emp.</i>	<i>Artif.</i>	<i>per. Emp.</i>
$\sigma_{\ln(A_t)}$	0.35	0.36	0.31	0.33	0.20	0.24
$\sigma_{\ln(C_t)}$	0.36	0.38	0.28	0.25	0.25	0.25
$\sigma_{\ln\left(\frac{C_t}{A_t}\right)}$	0.62	0.61	0.39	0.33	0.24	0.21
$\text{corr}(A_t, C_t)$	-0.32	-0.38	0.28	0.37	0.58	0.66
$\text{corr}(A_t, F_t)$	0.87	0.26	0.95	0.89	0.81	0.79
$\text{corr}\left(A_t, \frac{C_t}{A_t}\right)$	-0.69	-0.77	-0.60	-0.59	-0.38	-0.38
$\text{corr}(C_t, F_t)$	-0.07	-0.12	0.06	0.07	0.40	0.41
$\text{corr}\left(C_t, \frac{C_t}{A_t}\right)$	0.79	0.77	0.49	0.47	0.42	0.42
$\text{corr}(A_t, A_{t-1})$	0.79	0.85	0.80	0.94	0.78	0.91
$\text{corr}(A_t, A_{t-2})$	0.54	0.64	0.55	0.88	0.52	0.81
$\text{corr}(C_t, C_{t-1})$	0.62	0.68	0.88	0.91	0.87	0.87
$\text{corr}(C_t, C_{t-2})$	0.50	0.47	0.81	0.73	0.78	0.69
$\text{corr}(F_t, F_{t-1})$	0.67	0.94	0.77	0.97	0.79	0.95
$\text{corr}\left(C_t/A_t, C_{t-1}/A_{t-1}\right)$	0.79	0.80	0.84	0.91	0.66	0.78
$\text{corr}\left(C_t/A_t, C_{t-2}/A_{t-2}\right)$	0.64	0.59	0.65	0.81	0.38	0.34
J_T	0.55		0.32		0.16	
$\chi^2(1)$	11.52		6.82		3.36	
$p - \text{value}$	0.0007		0.0100		0.0670	

Table 1.8: Estimated Moments and Goodness of Fit Test - Wealth Subgroups, Central European Countries

<i>Moments</i>	<i>25th</i>	<i>per.</i>	<i>50th</i>	<i>per.</i>	<i>75th</i>	<i>per.</i>
	<i>Artif.</i>	<i>Emp.</i>	<i>Artif.</i>	<i>Emp.</i>	<i>Artif.</i>	<i>Emp.</i>
$\sigma_{\ln(A_t)}$	0.40	0.40	0.32	0.38	0.34	0.33
$\sigma_{\ln(C_t)}$	0.45	0.45	0.20	0.24	0.05	0.14
$\sigma_{\ln\left(\frac{C_t}{A_t}\right)}$	0.40	0.25	0.35	0.37	0.35	0.37
$\text{corr}(A_t, C_t)$	0.74	0.87	0.28	0.20	-0.15	-0.24
$\text{corr}(A_t, F_t)$	0.97	0.90	0.96	0.84	0.86	0.79
$\text{corr}\left(A_t, \frac{C_t}{A_t}\right)$	-0.33	0.02	-0.72	-0.84	-0.94	-0.95
$\text{corr}(C_t, F_t)$	0.73	0.84	0.19	0.86	0.09	0.10
$\text{corr}\left(C_t, \frac{C_t}{A_t}\right)$	0.33	0.48	0.35	0.26	0.36	0.30
$\text{corr}(A_t, A_{t-1})$	0.78	0.95	0.80	0.88	0.78	0.85
$\text{corr}(A_t, A_{t-2})$	0.52	0.89	0.55	0.63	0.52	0.50
$\text{corr}(C_t, C_{t-1})$	0.94	0.94	0.87	0.94	0.67	0.68
$\text{corr}(C_t, C_{t-2})$	0.88	0.84	0.80	0.84	0.53	0.50
$\text{corr}(F_t, F_{t-1})$	0.74	0.98	0.74	0.98	0.73	0.94
$\text{corr}\left(C_t/A_t, \frac{C_{t-1}}{A_{t-1}}\right)$	0.61	0.76	0.83	0.93	0.79	0.82
$\text{corr}\left(C_t/A_t, \frac{C_{t-2}}{A_{t-2}}\right)$	0.30	0.31	0.62	0.78	0.55	0.40
J_T	0.45		0.40		0.1	
$\chi^2(1)$	9.45		8.52		2.1	
$p - \text{value}$	0.0020		0.0035		0.147	

Table 1.9: Estimated Moments and Goodness of Fit Test - Wealth Subgroups, Scandinavian Countries

<i>Moments</i>	<i>25th</i>	<i>per.</i>	<i>50th</i>	<i>per.</i>	<i>75th</i>	<i>per.</i>
	<i>Artif.</i>	<i>Emp.</i>	<i>Artif.</i>	<i>Emp.</i>	<i>Artif.</i>	<i>Emp.</i>
$\sigma_{\ln(A_t)}$	0.45	0.44	0.57	0.53	0.80	0.73
$\sigma_{\ln(C_t)}$	0.18	0.18	0.04	0.27	0.04	0.27
$\sigma_{\ln\left(\frac{C_t}{A_t}\right)}$	0.54	0.53	0.54	0.29	0.77	0.49
$\text{corr}(A_t, C_t)$	-0.23	-0.26	0.71	0.93	0.69	0.92
$\text{corr}(A_t, F_t)$	0.76	0.47	0.99	0.96	0.99	0.80
$\text{corr}\left(A_t, \frac{C_t}{A_t}\right)$	-0.84	-0.82	-0.99	-0.85	-0.99	-0.85
$\text{corr}(C_t, F_t)$	0.05	0.20	0.66	0.87	0.68	0.74
$\text{corr}\left(C_t, \frac{C_t}{A_t}\right)$	0.61	0.60	-0.62	-0.72	-0.62	-0.79
$\text{corr}(A_t, A_{t-1})$	0.78	0.89	0.78	0.97	0.77	0.95
$\text{corr}(A_t, A_{t-2})$	0.52	0.70	0.52	0.93	0.52	0.86
$\text{corr}(C_t, C_{t-1})$	0.59	0.81	0.89	0.93	0.86	0.89
$\text{corr}(C_t, C_{t-2})$	0.45	0.45	0.81	0.82	0.78	0.74
$\text{corr}(F_t, F_{t-1})$	0.58	0.98	0.76	0.98	0.77	0.98
$\text{corr}\left(C_t/A_t, \frac{C_{t-1}}{A_{t-1}}\right)$	0.81	0.88	0.73	0.94	0.74	0.92
$\text{corr}\left(C_t/A_t, \frac{C_{t-2}}{A_{t-2}}\right)$	0.67	0.67	0.42	0.86	0.43	0.92
J_T	0.36		0.74		0.74	
$\chi^2(1)$	7.56		15.51		15.54	
$p - \text{value}$	0.0060		$0.08 * 10^{-3}$		$0.05 * 10^{-3}$	

European countries rather than for the Mediterranean ones. Analyzing the second order moments, it is easily noticed that the simulated wealth profiles are flatter than the actual profiles, but this might reflect a weakness of the data, rather than the model.

The simulations showed, for all twelve estimated models, that the profiles generated by the data display quite high coefficients of relative risk aversion associated to consumption, γ . With a lower correspondent discount factor, β , individuals are not willing to save, and this is showed by a drop in wealth for the first three quarters of the retirement period. However, due to the fact that health transition probabilities predict that health worsen with age, simulated wealth turn back to increasing in the last quarter of the period, since uncertainty on the health status and health expenditures that the individual will incur gives her an incentive to insure. The same situation is registered for simulated consumption profiles that monotonically fall during the entire period except for the last quarter; even if neither monotonicity nor smoothness of the decline are displayed by the actual data (which follows the profiles displayed by wealth), they do record a decreasing trend. This general tendency is consistent with most empirical studies of old-age consumption, which suggests that consumption falls with age (Banks et al. [1998]).

Turning finally to the formal insurance profile, it seems that it registers a monotonically decreasing path: the lower the level of wealth, the lower the formal coverage purchased for the next period, and this effect can be noticed for all wealth percentiles, especially in the first part of the time framework; in the last years, even though wealth increases, formal coverage continues to decrease. In general, besides wealth, what determine the values of formal insurance are not just differences in mean medical expenses, but, from the variations point of view, more important are differences in persistence coefficient and variance of the medical spending risk. If health insurance reduces health cost volatility, risk averse individuals may value health insurance at well beyond the cost paid, but since they are at the end of their life, they may value more informal insurance.

I estimated the structural parameters of equation (1.9), allowing for differences in medical expense from one health status to another, function of the age. Results showed that medical expenses for the elderly are high, while they tend to be more persistent for the poor (persistence coefficient ρ_ψ higher than one) and median wealth individuals rather than for the rich, situation registered for all three categories of countries. In addition, poor individuals in Mediterranean

countries experience less persistent medical expenses than their Central European match that, at their turn, register less persistent health spending than their Scandinavian counterparts. The same situation is registered at the level of median and rich individuals. On the contrary, the average elderly register almost the same medical spending persistence across all three countries groups. Notice that the estimates of the health spending risk are not understated because the measure of medical expenditures risk included the compulsory formal insurance provided by the government. Moreover, I found that health spending volatility (variance of log medical expenses σ_{ε_t}) is higher for poor rather than for median wealth individuals, and for median wealth individuals rather than for the rich. This is registered for both Mediterranean and Central European countries; consequently, since the poor are the ones that experience worse health with age, they will also register higher and more volatile medical expenses with respect to median and high wealth individuals. Not the same is registered within the Scandinavian group models, where, although poor have more volatile medical spending than the rich, median wealth individuals have the highest spending variability. Furthermore, poor / median / rich Mediterraneans display a lower variability than poor / median / rich Central Europeans who register a lower variability than poor / median / rich Scandinavians respectively. In terms of representative agent, it is however clear that Northern individuals are facing less volatile health spending than their Central European and Southern correspondents.

Within the same countries group, poor are more risk averse to medical care (higher σ) than the rich; Mediterraneans display lower risk aversion coefficients than Central Europeans, who respectively register lower risk aversion than Scandinavians. This finding is consistent with the higher risk registered for out-of-pocket health expenditures in Scandinavian countries rather than in the Mediterranean ones and for poor rather than the rich, so it is not surprising that we find a high risk aversion for poor and Northern Europeans. For the representative agent model, I found the opposite results on medical spending volatility with respect to the wealth-specific models; however, the findings on the representative agent model's risk aversion continue to follow these results, with Scandinavians displaying the lowest risk aversion (due to less volatility).

The estimated coefficient of the relative risk aversion for consumption of non-durables, γ , displays values between 3.2 and 5.6 for the Mediterranean countries, 2.4 and 8.4 in Central

European ones and between 2.1 and 9.1 for the Scandinavian ones. Standard values for the coefficient of relative risk aversion parameter in life-cycle models are between 2-6. These high values in some cases reflect the relationship of age and wealth with the relative risk aversion, which indicates its tendency to increase with age at any given level of wealth. Based on the life-cycle of risky asset positions, some research has argued that older investors are more risk averse (Morin and Suarez [1983]), but there is debate about their findings (Wang and Hanna [1997] and Bajtelsmit and Bernasek [2001]). It should be noted that wealth does not include housing, and, although there are no minimum wealth and consumption levels that were specifically taken into consideration in the model, I considered a minimum formal insurance provided compulsory by the government and I calibrated the model to fit wealth and consumption data when constructing wealth profiles. While the absolute risk aversion decreases with wealth, there is not such a clear consensus on the relative risk aversion tendency to increase or decrease. In this case, for all groups of countries, poor individuals display lower risk aversion than the median ones; for Central European and Scandinavian countries, median wealth individuals are less risk averse than the rich but more risk averse than the representative agents. On the other hand, rich Mediterranean are less risk averse than median wealth agents, that, at their turn are less risk averse than representative individuals. Scandinavians are more risk averse than Central Europeans, that are more risk averse than Mediterranean, except for the poor that are less risk averse. Consequently, those with low wealth will not tend to save for consumption of non-durables, while their consumption of medical goods will never drop under a certain threshold, even in presence of high negative health spending shock.

Substitution coefficient θ is found to register besides the traditional values $\theta \in (0, 1)$, also relatively high negative values, which imply a Leontief (no-substitution) function of medical services (for the poor and median agents in Central Europe and for Scandinavian individuals in particular); as expected, poor substitute less than rich and Scandinavians less than Central Europeans and than Mediterranean.

Figure 1-12 shows relative cohesion coefficients: the top four graphs refer to Mediterranean countries, the central ones to Central Europe, and the last ones to the Scandinavian group.

Although in all cases, the parameters display an increasing structure of age, at the country level models, Mediterranean countries experience higher cohesion coefficient than Central Eu-

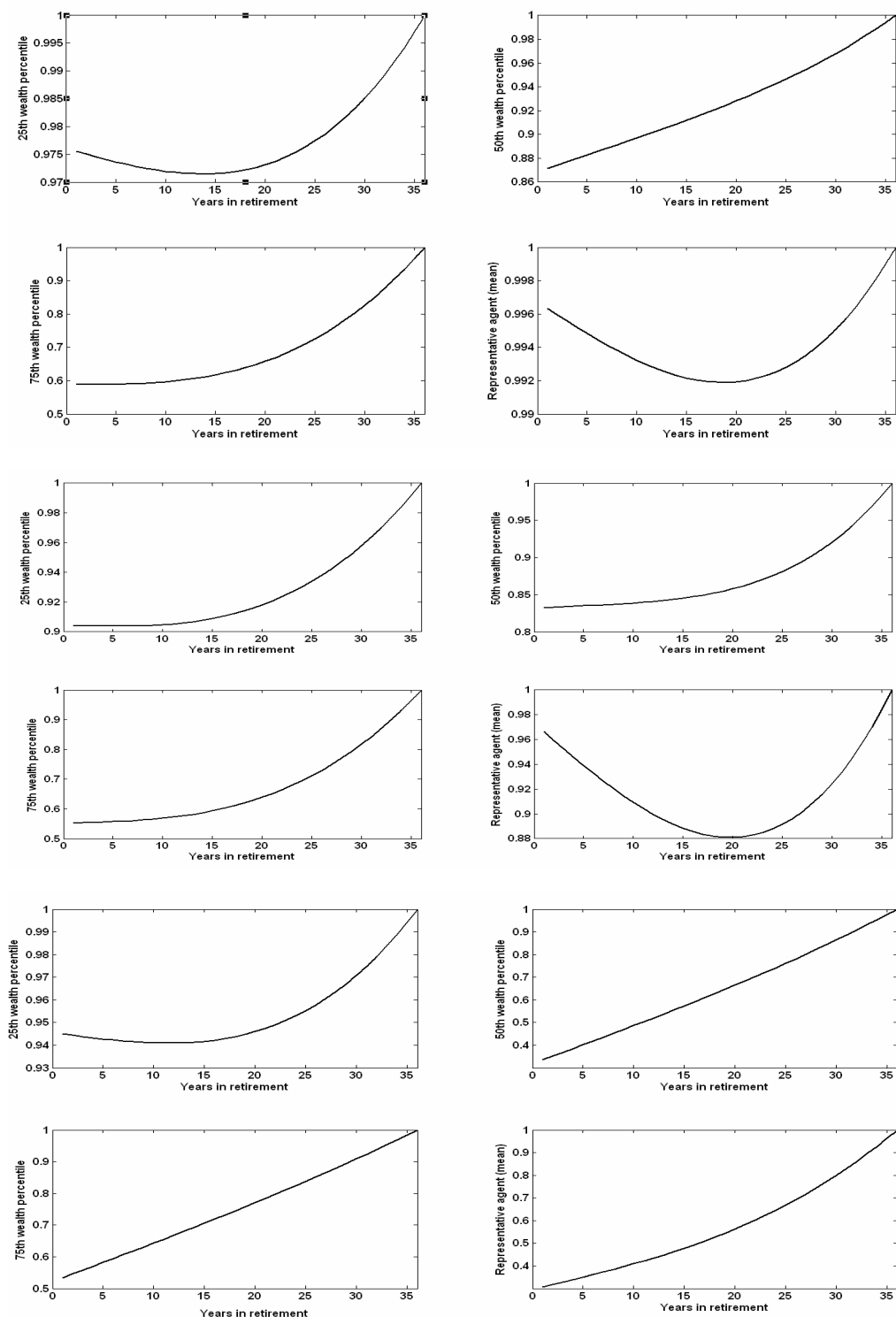


Figure 1-12: Cohesion Coefficient by Country Group and Wealth Percentile

ropean ones. These last ones, in turn, have a superior coefficient with respect to Scandinavian countries, which is consistent with the sociological explanations provided in Reher [1998]. Analyzing wealth specific models, the correspondence is maintained, with cohesion among poor higher than among the median agents, which is further higher than among the rich. Consequently, Southern Europeans usually benefit of a higher cohesion coefficient than their Central European counterparts, who further register a better situation (higher η s) with respect to the Northern Europeans, regardless the wealth level. The only exception are poor Central Europeans who display less cohesion than poor individuals in Northern European countries.

Figures 1-13 - 1-15 present health transition probability matrix conditional on age, previous health status and wealth for the three country groups.

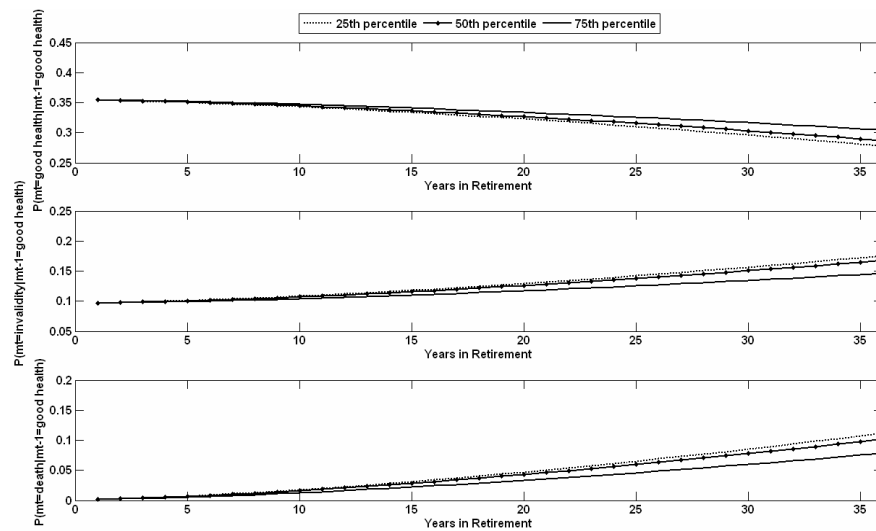


Figure 1-13: Health transition probabilities conditional on age, previous health status and wealth, Mediterranean countries

For Mediterranean countries, the lowest two panels in Figures 1-13 show that for individuals in good health last year, the probability of death within one year rises from 0.20% at age 65 to 10.19% at age 100, while the probability of poor health (invalidity) is about 9.70% at age 65 and increases to 17.53% for the poor and to 14.65% for the rich at age 100. Rich people with poor health are less likely to die than poor people: being in the 75th wealth percentile instead of the 25th percentile lowers the probability of dying by 40.74% at age 80. On the other hand, invalidity is a very persistent health status: a 70-year-old having poor health one year ago has

55.79% chances of having poor health also this year, and it falls with age, as probability of death increases. Moreover, rich people are less likely to die than poor but also are more likely to maintain and to return to good health: high wealth percentile individuals display a higher probability of persisting in a good health state and to return to it if in fair or poor health state.

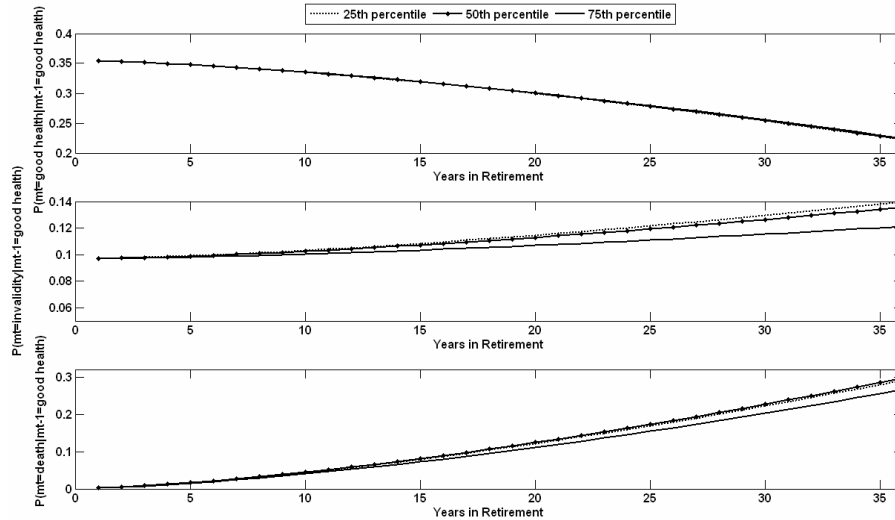


Figure 1-14: Health transition probabilities conditional on age, previous health status and wealth, Central European countries

On the other hand, healthy poor Central Europeans have 7.50% more chances to die than the healthy rich at age 65, but only 9.52% at age 90 (see Figures 1-14). Rich are less likely to die than poor, but are also less likely to pass into invalidity. Overall, probability of death within one year if in good health increases from 0.27% at age 65 to almost 30% at age 100. Regardless of wealth levels, individuals in these countries tend to persist less than their Mediterranean counterparts in good health, but the fact that rich persist more than poor is maintained. Rich Central Europeans display a lower probability to become invalid than poor in good health, but each category is more likely to die than its Mediterranean match.

In Scandinavian countries, the probability of death when in good health rises with age, surprisingly faster for rich and median wealth people than for the poor as showed by Figure 1-15; furthermore, it must be noticed that the chances that death occurs when in good health are extremely high, both in general (0.57% at age 65 and 89.91% for 100 years old) and with respect to the Mediterranean and Central European countries. Not the same is registered for poor health

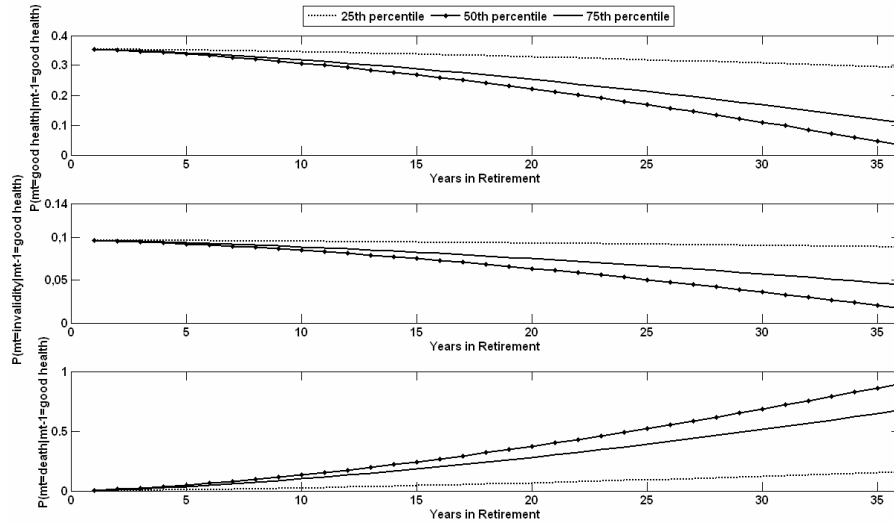


Figure 1-15: Health transition probabilities conditional on age, previous health status and wealth, Scandinavian countries

(invalidity), where healthy 65 years old Northern Europeans are as likely to become invalid as the Southern or Central Europeans. Staying or returning healthy is less likely as age increase, with poor having almost 20% more chances than the rich. Overall though, Scandinavians are more likely to die than both Mediterranean and Central Europeans.

To summarize, although health deteriorates with age across all country groups, Mediterranean display both higher life expectancy (lower probability of death) and higher probability of poor health than Central Europeans. Scandinavians, on the other hand, are the ones that among all, register the smallest number of expected years of life, having also the lowest probability of becoming invalid if in good health.

Consider now the realistic result that the older, and so the sicker, individuals will become, the more weight they are going to attach to care, and in particular to the informal care. Note that this is true for all the models in which the parameter a that accounts for the informal care dependency on health is significant. Results showed that the ones that benefit of high cohesion level, and so, of increased informal insurance from their family are likely to display higher life expectancies, as measured by the survival probability, than their counterparts in opposite situation. Indeed in the case of both wealth-specific and representative agent models, Southern rather than Northern Europeans experience higher coefficients of cohesion and consequently will

rely more on informal insurance. On the other hand, these categories are the ones that register also a longer life expectancy as showed by health transition results in Figure 1-13 - 1-16: while in Southern Europe, individuals are more likely to become invalid than to die, in the North, their counterparts register the opposite situation. As it can be seen, although maintaining a good health is becoming less possible as one moves to the end of life, the probability of death for healthy individuals is higher in Scandinavian countries than in Central Europe, and higher in the latter than in the Mediterranean. However, healthy individuals in the Northern group of countries have less chances of becoming invalid than their Central European and Mediterranean correspondents. The same reasoning holds within the Scandinavian countries group for the 25th and 75th wealth percentile: poor rather than rich display higher cohesion coefficient and are also more likely to have a higher life expectancy. Not the same is registered for Mediterranean and Central European countries, where rich are slightly less likely to die than poor. One explanation for this fact is that wealth discrepancies are more accentuated in Southern and Central Europe than in the Northern Europe which moreover has also a wider and more efficient public health coverage system. In this context, for Southern and Central European countries, the intuition would work in the opposite way: informal care can account for the small difference in the life expectancies of poor with respect to the rich, which would otherwise be higher. Consequently, Mediterranean countries can be defined as "strong cohesion coefficient" countries, health expenditures being mainly covered by the family, in an informal manner. The same situation at a more moderate level ("medium cohesion coefficient") is registered for the Central European zone, while the contrary is valid for, say, "weak cohesion coefficient", meaning for the Scandinavian countries. Moreover, Mediterranean countries, that benefit of a high age-specific cohesion coefficient, are experiencing higher life expectancy, while the opposite is valid for Scandinavian countries. As age increases, maintaining a good health is becoming more difficult and individuals progressively need more medical care. Given that cohesion coefficient increases with age, informal care will also rise, with a direct impact on the life expectancy of the elderly: individuals benefitting of a higher cohesion coefficient are likely to display a higher life expectancy, as measured by the age-adjusted survival probabilities.

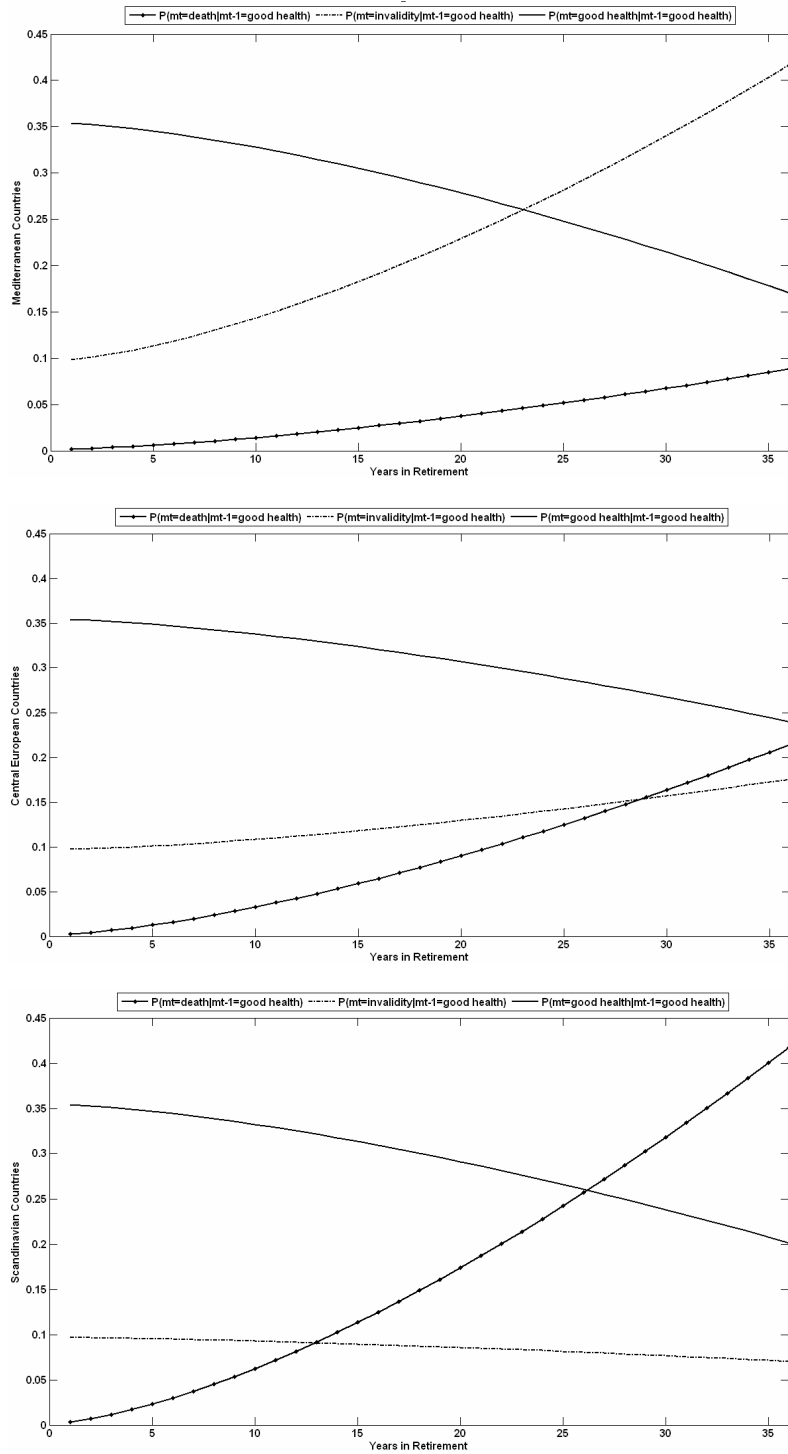


Figure 1-16: Health Transition for Representative Agent

1.7 Conclusions

There has been an increasingly important debate between economists and policymakers, on the individual decisions after retirement, in terms of consumption, saving and insurance, over the life-cycle. The present work contributes to the analysis of individual behavior from this perspective and sheds light on the situation in which there are two possible options available: a formal framework in which the insurance is provided by the conventional market and an informal arrangement that regards the health care supplied by the extended family. Under the realistic assumption, confirmed by the results, that the older, and so the sickest, the individuals will become, the more weight they are going to attach to the informal care, the intuition of the model presented was that people that benefit of high level of informal insurance due to high level of cohesion with the family are likely to display higher life expectancies, as measured by the survival probability, than their counterparts in opposite situation.

To test the validity of this intuition, I developed a simulated life-cycle model designed to outline the decision of the elder people with respect to consumption, health and health care. One of the novelty of this work is that it considers the issue of health status uncertainty and simultaneously the health expenditures uncertainty in the framework of formal insurance and of informal arrangements provided by the extended family. Moreover, I used European data and a more flexible functional form, and as a result I found that medical expenses are very high and volatile, they rise fast with age and that at advanced ages, informal medical care is preferred. On this line, the persistent component of the health expenditures is taken to match each health status within the data and the formal coverage is assured by the payment of the correspondent premia the previous period. In this context, I introduced the bequest motive as a direct response to the uncertainty faced by the individual, but as indirect variable choice. Individuals are considered to be subject to four possible health states modelled accordingly, while the health shock is adjusted to account for age in the transition probabilities from one health status to another. What this work does is carefully estimating the health transition probabilities by age as function of health and wealth percentile and finds large variations along the three dimensions and between country groups.

Even if there is a well-established literature that examines how the risk of future health and future health expenditures generates savings, this is the first attempt to propose a realistic

simulation and estimate the effects of health and health spending risks on the insurance choice, understood both as formal and informal, for older Europeans. Using cross-sectional data from SHARE, profiles were generated for wealth, consumption and formal insurance and matched to the real counterpart profiles. The models fitted quite well and yielded relevant estimates for the analysis. Moreover, with idiosyncratic health expenditures shock, it is natural that they generate declining wealth after retirement; however, health uncertainty provides motive for savings and so for informal insurance. Consequently, I found that the sources of heterogeneity that I considered have a significant role explaining the elderly's saving behavior, with a very high level of medical expenses at very advance ages being a key factor for the need to keep large amount of wealth to insure against this risk.

Taking into consideration indirectly the bequest motive, the main and final aim was to estimate both the structure and level of the cohesion coefficient and the correspondent age-adjustment elements that would impact on the survival probability. Finally, the estimates obtained indicate that the coefficient of cohesion has an age dimension. For the 25th and 75th wealth percentile individuals, it slightly decrease with age for the first part of their retirement years, which makes them decumulate wealth, and then raise in the last part of the period (more for poor than for rich), which increases wealth accumulation (higher age means lower probability of survival or higher probability of receiving bequest). The median and average individuals however will register a monotonically rising cohesion coefficient. Results showed that, both at the aggregate and disaggregate level, the estimates generally reflect that cohesion parameter is one of the determinants of individuals health status transition in the years after retirement, with the final result that a higher cohesion coefficient can be associated with a higher survival probability of the elderly.

The main conclusion is that in order to correctly evaluate any policy reform affecting the elderly's saving decisions in Europe, one needs to account for and model accurately the strength of family ties, and also consider the level of medical expenses by age and wealth in relation to country-specific family cohesion coefficient.

Chapter 2

Why Have Developed Countries Stopped Saving?

2.1 Introduction

National saving rates differ enormously across developed countries. But these differences mask a common trend – a dramatic decline in national saving rates over time. Table 2.1 documents this phenomenon. It shows national saving rates for the U.S., Japan, U.K., France, Italy, Spain, and Canada for selected years from 1970 through 2006. With the exception of Canada, each country's saving rate plummeted over this period. France, for example, saved 17.3% of national income in 1970. In 2006 it saved only 6.6%. Italy saved 17.4% in 1970, but only 4.2% rate in 2006. The U.S. saved at a 9.5% rate in 1970, but almost nothing in 2006.

What explains these differences across countries and over time? Is it changes in demographics, preferences, government spending or economic conditions? To address this question, we estimate a model in which the government and household sector jointly make labor supply and consumption decisions. This societal decision-making framework is motivated by Green and Kotlikoff's [2006] demonstration that economics draws no distinction between private and public property. Instead, the government and household sectors effectively play the role of two people stranded on an island, each of whom can claim, via "official", "legal" or informal proclamation, to own all or part of the island's resources, including his own and the other party's time. But such claims have no economic basis or import. What each person ends up consuming in

Table 2.1: National saving rate for selected years

<i>National savings rate (%)</i>	1970	1980	1990	2000	2006
<i>Canada</i>	12.0	12.3	6.2	12.7	12.5
<i>France</i>	17.3	11.2	9.7	10.5	6.6
<i>Italy</i>	17.4	12.7	8.3	7.1	4.2
<i>Japan</i>	30.5	20.7	20.4	9.6	7.8
<i>Spain</i>	15.9	9.2	10.9	10.1	7.6
<i>United Kingdom</i>	14.0	5.7	3.6	4.2	4.5
<i>United States</i>	9.5	8.6	4.8	6.8	2.1

Source: Computations were based on World Economic Outlook Database, International Monetary Fund, April 2007

goods and leisure depends on fundamental factors, including the ability to threaten and cajole.

Our one-good, closed-economy model assumes that the government and the public (society) resolve their conflicts and capitalize on their opportunities by agreeing to maximize a social welfare function. This function equals the expected discounted flow of utility from the public's consumption and leisure. Each period's consumption and leisure decisions are made in light of uncertain future levels of productivity and government spending as well as uncertain future social preferences.

We model social preference uncertainty in three ways. In model 1, current society is in charge forever. And it knows its current intertemporal preferences (rate of time preference) and current intratemporal preferences (relative weighting of different age groups' utilities from consumption and leisure). What it doesn't know is its future intertemporal preferences (how its rate of time preference will evolve). Model 2 is a time-inconsistency variant of 1. But rather than posit a single society forever in charge, we permit the society in charge to change each period. Although today's society knows its future preferences, it controls future societies' consumption and leisure allocation decisions only indirectly via the amount of capital it leaves behind. In model 3, society has stable intertemporal preferences, but changing intratemporal preferences.

We use the method of moments to estimate parameter values of the dynamic programs associated with each of the three models for the U.S., France, and Italy. The three sets of

results make good economic sense for each of the countries. Unfortunately, data limitations precludes nesting the models. But each model points to one central driving force underlying the decline in national saving rates – a shift in societal preferences coming either in the form of a rising rate of social time preference or a shifting structure of age-specific utility weights that favors the contemporaneous old.

2.2 The Models

2.2.1 Model 1: Uncertain Future Time Preferences

The economy's single good is produced via

$$Y_t = Z_t K_t^\alpha \left(A_t \sum_{a=0}^{100} e_a P_{a,t} n_{a,t} \right)^{1-\alpha}, \quad (2.1)$$

where α is capital share in production, $A_t = (1 + \mu)A_{t-1}$ captures labor-augmenting technical progress, occurring at rate μ , Z_t is time- t multifactor productivity, e_a is the earning ability (efficiency units) of an individual age a , and $P_{a,t}$ counts the population age a at time t . Each individual has one unit of time available each period.

The economy's capital stock, K , evolves according to

$$K_{t+1} = (1 - d)K_t + Z_t K_t^\alpha \left(A_t \sum_{a=0}^{100} e_a P_{a,t} n_{a,t} \right)^{1-\alpha} - \sum_{a=0}^{100} P_{a,t} c_{a,t} - A_t g_t, \quad (2.2)$$

where d is the depreciation rate, $c_{a,t}$ and $n_{a,t}$ are the consumption and labor supply of age- a agents at time t , and g_t is the level of government spending scaled by the level of labor-augmenting technical progress.

The term e_a captures the earnings ability (efficiency units) of age- a workers. This term is zero for workers under age 15 and over age 75; otherwise, e_a satisfies¹

$$e_a = e^{4.47 + 0.0033*(a-15) - 0.000067*(a-15)^2}. \quad (2.3)$$

¹For further details see Fehr, H., Jokisch, S., Kotlikoff, L.J., (2007).

Multifactor productivity, Z_t , and scale government spending, g_t , deviate around stationary long-term values according to the the following processes:

$$\ln Z_t = \rho_Z \ln Z_{t-1} + \varepsilon_t, \text{ with } \varepsilon_t \sim N(0, \sigma_{\varepsilon_t}^2), \quad (2.4)$$

$$\ln g_t = (1 - \rho_g) \ln \bar{g} + \rho_g \ln g_{t-1} + \eta_t, \text{ with } \eta_t \sim N(0, \sigma_{\eta_t}^2). \quad (2.5)$$

Society cares about the utility from consumption and leisure of those agents now alive and those yet to be born. At any point in time, the weight applied to contemporaneous agents' utilities in the social welfare function depends on their ages. Current consumption and labor supply decisions are made in light of uncertainty about future productivity, government spending and rates of time preference.

Society's expected utility at time t is

$$V_t = \sum_{a=0}^{100} P_{a,t} \theta_a u(c_{a,t}, n_{a,t}) + E_t \sum_{\tau=1}^{\infty} \prod_{s=t}^{t+\tau-1} \beta_s \left(\sum_{a=0}^{100} P_{a,t+\tau} \theta_a u(c_{a,t+\tau}, n_{a,t+\tau}) \right), \quad (2.6)$$

where the θ_a parameters are the aforementioned utility weights, the function $u(., .)$ is assumed to be of addilog form

$$u(c, n) = \frac{c^{1-\gamma} - 1}{1-\gamma} + b \frac{(1-n)^{1-\sigma} - 1}{1-\sigma}, \quad (2.7)$$

and β_s , is the time- s discount factor. Society knows β_t , but is uncertain about future values of β_s for $s > t$. Because today's society controls all future allocations, the issue here is one of uncertain future desires, not changing decision makers; i.e., the problem here involves preference uncertainty, not time inconsistency.

The discount factor obeys

$$\ln \beta_t = (1 - \rho_\beta) \ln \bar{\beta} + \rho_\beta \ln \beta_{t-1} + \epsilon_t \text{ with } \epsilon_t \sim N(0, \sigma_{\epsilon_t}^2). \quad (2.8)$$

As with Z_t and g_t , the β_t follows an autoregressive process that fluctuates around a long-run stationary value, and its lagged value represents another state variable. Finally, utility weights are modeled via a third-order polynomial, i.e.,

$$\theta_a = \lambda_0 + \lambda_1 * age + \lambda_2 * age^2 + \lambda_3 * age^3. \quad (2.9)$$

Society's solves the following program:

$$V_t(Z_t, g_t, \beta_t, K_t) = \max_{C_{a,t}, n_{a,t}} \left\{ \sum_{a=0}^{100} \theta_a P_{a,t} u(c_{a,t}, n_{a,t}) + \beta_t E_t [V_{t+1}(Z_{t+1}, g_{t+1}, B_{t+1}, K_{t+1})] \right\} \quad (2.10)$$

subject to (2.2).

Optimality requires

$$c_{a,t}^{-\gamma} = \frac{\theta_{a+1}}{\theta_a} \beta_t E_t c_{a+1,t+1}^{-\gamma} (1 + r_{t+1}), \quad (2.11)$$

$$(1 - n_{a,t})^{-\sigma} = \frac{e_a w_t}{b} c_{a,t}^{-\gamma}, \quad (2.12)$$

$$\frac{c_{a,t}}{c_{a+1,t}} = \left[\frac{\theta_a}{\theta_{a+1}} \right]^{\frac{1}{\gamma}}, \quad (2.13)$$

where r_t and w_t are time- t marginal products of capital and labor.

We solve this and the other models via backward induction starting in our assumed terminal year 2100. Using a later terminal year makes no material difference to parameter estimates. Expectations are formed using Gaussian quadrature².

The key parameters of interest are the initial (1950) value of β , the rate ρ_β which determines

²For a detailed description of Gaussian quadrature, see Appendix A.

β s convergence, on average, to its long-run value, and the long-run value of $\beta, \bar{\beta}$. An initial value β significantly above $\bar{\beta}$ coupled with a fast convergence (a value of ρ_β close to 0) would provide evidence of society placing less and less weight on the future in determining current consumption and leisure.

2.2.2 Model 2: Uncertain Future Preferences with Time Inconsistent Decision Makers

In this model, today's society has stable preferences and knows, therefore, how it now values and will value future consumption and leisure allocations. But it doesn't directly control future allocations. Instead, each period's allocations are made by the prevailing society (the decision makers in charge in the period) whose time-preferences will generally differ from that of current society. The precise levels of such future time-preference factors is unknown to current society. But current society knows that these preference factors will evolve according to (2.8). It also knows that its sole manner of influencing future allocations is via the amount of capital it transmits to the next society, which, in turn, influences what the next society will leave to the following society, and so on.

Formally, each society selects an allocation strategy taking the strategies of other societies as given. This strategy is a map from the state $\nabla_t = \{t, g_t, Z_t, \beta_t, K_t\}$ to the choice variables $\{c_{a,t}, n_{a,t}\}$ for $a \in [0, \dots, 100]$. The fixed point in the strategy space, which guarantees that all strategies are optimal given the strategies of the other players, is a Nash equilibrium.

Time- t society chooses $\{c_{a,t}, n_{a,t}\}$ for all $a \in [0, \dots, 100]$ to maximize

$$W_t = \sum_{a=0}^{100} P_{a,t} \theta_a U(c_{a,t}, n_{a,t}) + \tag{2.14}$$

$$+ E_t \sum_{\tau=1}^{\infty} \beta_t^\tau \left(\sum_{a=0}^{100} P_{a,t+\tau} \theta_a U(c_{a,t+\tau}^*(\nabla_{t+\tau}), n_{a,t+\tau}^*(\nabla_{t+\tau})) \right),$$

subject to (2.2) and conditional on its state variables ∇_t . Note that $c_{a,t+\tau}^*(\nabla_{t+\tau})$ and $n_{a,t+\tau}^*(\nabla_{t+\tau})$ denote the optimal choice that the time- $(t + \tau)$ future society will make contingent on the prevailing state variables $\nabla_{t+\tau}$.

We also solve this problem recursively, starting at date T . First we work out the society T 's allocation decisions as functions of the state variables in the last period, ∇_T . Next, we determine society $(T - 1)$'s allocation decisions as functions of ∇_{T-1} . In making its decisions, the $(T - 1)$ society considers not only its welfare from period $(T - 1)$ allocations, which it directly controls, but also the expected value of its future welfare (discounted using its own time-preference rate) from period T decisions made by society T . The $(T - 2)$ society has a similar problem to that of the $(T - 1)$ society except that it must consider how two future societies will allocate consumption and leisure and so on.

We use Monte Carlo simulations to determine how a society prevailing at time s makes its decisions. Specifically, for a given state variables at time s , ∇_s , and each candidate time- s allocation (consumption and leisure choices), we form the average of current and future realized utility outcomes generated by the simulations to determine how much expected utility the candidate allocation generates. The allocation with the highest expected utility constitutes the optimal time- s decision. The Monte Carlo simulations entail taking draws of future paths of time-preference rates, productivity levels, and levels of scaled government consumption and using the previously determined allocation decisions of future societies to determine the consumption and leisure values that will be chosen along any path.

Again, we assess a shift in social time preference in terms of the degree to which the long-run value of β lies below its initial value as well as the speed at which societal time preference converges, on average, to its long-run value.

2.2.3 Model 3: Changing Intra-temporal Preferences

This model features stable intertemporal preferences, but incorporates changes over time in age-specific utility weights, which now obey

$$\theta_{a,t} = \lambda_0 + \lambda_1 * age + \lambda_2 * age^2 + \lambda_3 * age^3 + \tau_0 * t + \tau_1 * t * age^2 + \tau_2 * t^2 * age. \quad (2.15)$$

In estimating this model, our focus is τ_0 , τ_1 , and τ_2 , which determine the extent to which intra-temporal preferences shift over time toward older generations.

2.3 Data

Our U.S. data consists of a) 1950-2004 annual National Income and Product Account chain-weighted observations of GDP, private consumption, domestic investment, and government discretionary spending, b) annual U.S. Census counts of population by age for 1950-2004, and c) U.S. Census projections of population by single age for 2005-2100. Our French and Italian macro data for 1950 through 2004 come from the Penn World Tables. These countries' single-age demographic data come from special tabulations of the 2006 release of United Nations's World Population Prospects: The 2006 Revision. The UN projects populations only through 2050. We employed a fourth-order polynomial to interpolate from our 1950-2050 data the single-age population counts from 2051 through 2100³.

2.4 Estimation

To limit unknown parameters, we assume a 5% annual rate of depreciation, normalize the 1950 value of Z at 1, obtain the 1950 value of K from data on fixed reproducible tangible wealth⁴ and determine the persistence coefficient ρ_g and standard deviation η_t for government expenditure in equation (2.5), as showed in Table 2.2, from a VAR(1) on total government expenditure adjusted for labour-augmenting technical progress⁵. Computations were based on NIPA Tables (for USA data) and Penn World Tables (for France and Italy data).

A summary of parameter definitions is offered in Table 2.3. We use the Simulated Method of Moments (SMM) (McFadden [1989] and Pakes and Pollard [1989]) to estimate the parameters listed in Tables 2.4 and 2.5 conditional on ten different assumed initial (1950) values of β and choose the one that generates the data for which SMM results best fit their empirical

³Given the data points, the aim of polynomial interpolation is to find the polynomial that fits exactly through these points. In practice, we generated the fourth order polynomial in time for the 1950-2050 period. We further used the estimated polynomial coefficients to obtain the single-age projected counts of population.

⁴The net foreign asset position was obtained by subtracting the foreign investments in the country from the investments abroad. For the U.S., data was obtained from BEA, while for France and Italy, since no data were available, we interpolated within the grid for capital in order to obtain the U.S. correspondent initial capital point for these countries.

⁵We obtained adjusted government expenditure by simply dividing the total amount of government spending at time t by $(1 + \mu)^t$.

Table 2.2: Government Spending Parameters

<i>VAR(1) Parameters</i>	ρ_g	η_t
<i>United States</i>	0.6386	0.0343
<i>France</i>	0.9112	0.0422
<i>Italy</i>	0.9464	0.0348

counterparts. We choose this method of estimating the initial value of β for the following reason. As indicated, the current value of β is a state variable. In our dynamic program, we limit our grid for β to ten possible values ranging from 1 to 10 possible values. Were we instead to attempt to estimate β for 1950 along with other parameters listed in Table 2.4, we would surely compute a value different from that on our grid, i.e., treating β as a continuous, rather than discrete, unknown parameter would be inconsistent with the assumptions underlying the dynamic program used to calculate β . Table 2.6 lists our choice of moments.

In implementing SMM, we simulate $N = 20$ paths of the economy and collect for each path the simulated values of each variable; we compute the set of moments conditional on the initial values of the state variables ∇_0 and of the parameters ϕ_0 and minimize the weighted sum of squared deviations of simulated moments from their corresponding empirical counterparts; remember that this sum can be written as

$$J_T = \arg \min_{\tilde{\phi}} [m_T - \frac{1}{N} m_N(\nabla_0, \phi_0)]' W [m_T - \frac{1}{N} m_N(\nabla_0, \phi_0)], \quad (2.16)$$

where m_T represents data moments and $m_N(\nabla_0, \phi_0)$ is the set of moments of each of the N simulated paths of the artificial economy. W is the weighting or distance matrix that almost surely converges to $W = S^{-1}$, where S is the limit, as $NT \rightarrow \infty$, constant full-rank matrix of the covariance of the estimation errors⁶.

⁶As described in Andrews (1991), an optimal weighting matrix is obtained as the inverse of the variance-covariance matrix of the moment conditions evaluated at a set of first-step estimates, in which W is set equal to the identity matrix. This matrix is consistently estimated using the estimator proposed by Newey and West (1994), which places more weight on moments that are more precisely estimated. Implementing this method entails fitting the moments of the simulated series to their real data counterparts under the condition of $W = I$ and then using estimates from this stage to form the weighting matrix $W = S^{-1}$ for use in a second and final stage estimation of (2.16).

Table 2.3: Parameters Estimated - Definitions

<i>Parameter</i>	<i>Definition</i>
γ	risk aversion parameter for consumption
σ	risk aversion parameter for leisure
β_0	discount factor initial state
$\bar{\beta}$	beta process mean
ρ_β	beta process persistence coefficient
σ_{ϵ_t}	std. deviation of the beta stochastic process
α	share of capital in the production function
β	constant discount factor
ρ_Z	technology shock persistence coefficient
σ_{ϵ_t}	std. deviation of technology shock
μ	labour-augmenting technical progress rate
λ_0	intercept for the age-specific utility weights
λ_1	age parameter for the age-specific U weights
λ_2	age ² parameter for the age-specific U weights
λ_3	age ³ parameter for the age-specific U weights
τ_0	time parameter for the age-specific U weights
τ_1	time*age ² parameter for the age-specific U weights
τ_2	time ² *age parameter for the age-specific U weights

Table 2.4: Parameters Estimated in Model 1 and Model 3

$$\{ \gamma, \sigma, \beta_0, \bar{\beta}, \rho_\beta, \sigma_{\epsilon_t}, \alpha, \rho_Z, \sigma_{\epsilon_t}, \mu, \lambda_0, \lambda_1, \lambda_2, \lambda_3 \}$$

Table 2.5: Parameters Estimated in Model 2

$$\{ \gamma, \sigma, \beta, \alpha, \rho_Z, \sigma_{\epsilon_t}, \mu, \beta_0, \lambda_1, \lambda_2, \lambda_3, \tau_0, \tau_1, \tau_2 \}$$

Table 2.6: Choice of Moments

$$\left\{ \begin{array}{lll} \sigma_{\ln(Y_t)}, & \sigma_{\ln(C_t)}, & \sigma_{\ln(I_t)}, \\ \sigma_{\ln(C_t/Y_t)}, & \text{corr}(Y_t, C_t), & \text{corr}(Y_t, I_t), \\ \text{corr}(C_t, I_t), & \text{corr}(Y_t, Y_{t-1}), & \text{corr}(Y_t, Y_{t-2}), \\ \text{corr}(C_t, C_{t-1}), & \text{corr}(I_t, I_{t-1}), & \text{corr}(C_t/Y_t, C_{t-1}/Y_{t-1}), \\ \text{corr}(C_t, C_{t-2}), & \text{corr}(I_t, I_{t-2}), & \text{corr}(C_t/Y_t, C_{t-2}/Y_{t-2}) \end{array} \right\}$$

2.5 Results

Tables 2.7- 2.9 compare, for each country, the three models' simulated moments with their empirical counterparts. Consider first the U.S. results reported in Table 2.7. Each model easily passes a χ^2 -test of overidentifying restrictions. This reflects the close match between simulated and actual moments shown in the table. Based on p -values, model 1 appears to fit best. For France and Italy, model 2, although it fits certain moments quite well, is rejected. But models 1 and 3 perform quite well. Both pass a χ^2 -test at standard significance levels and have similar and substantial p -values.

Tables 2.10- 2.12 present parameter estimates. A quick glance across models for a given country and across countries for a given model shows that the parameter estimates are economically remarkably reasonable and generally quite similar across models and countries. Consider, for example, the estimates for γ , σ , and μ . Our estimated γ coefficients are 1.93 for the U.S., 2.06 for Italy, and 2.59 for France. Our estimates of σ are 5.47 for the U.S., 4.82 for France, and 5.05 for Italy. The rate of labor-augmenting technical change, μ , is 2.1%, while capital's share, α , is roughly 30% for all countries. The value for ρ_Z – the autoregressive coefficient for multifactor productivity – exceeds 1 in all three models, which is not surprising given that we have not normalized our data.

These estimates are reassuring, but our main focus is changes over time in the discount factor and age-specific societal utility weights. With that in mind, let's consider model 1's findings regarding the evolution of the discount factor. As the first columns of the three tables indicate, β_0 exceeds $\bar{\beta}$ for all three countries. This means that, over our sample period, society is becoming ever more present-oriented⁷. Figures 2-1 and 2-2 show changes over time in all three countries' time preference rates, calculated as $(1 - \beta)/\beta^8$, for model 1 and 2. As it can be seen, time preference rate slowly raises in time, as the discount factor converges to its mean value.

⁷Note that the values of β_0 and $\bar{\beta}$ both exceed 1. Given that the model we are estimating has a finite horizon (year 2100), this presents no problem with respect to an explosive value of the expected utility maximand. Furthermore, given secular growth in consumption, we would expect a discount factor above 1. As discussed in Jonsson and Klein (1996) and Cooley and Prescott (1995), a discount factor in excess of 1 can be consistent with long-run secular growth and infinite horizon utility. One simply needs to normalize the model for labor-augmenting technical change and note that the normalized discount factor is less than 1; i.e., that the normalized model has a finite maximand. Instead of adopting this approach, we preferred to estimate the labor-augmenting technical change rate as a parameter.

⁸Figures plot the simulated AR(1) processes for β ; for simplicity, the error term is neglected.

Table 2.7: Estimated Moments and Goodness of Fit Test, U.S.

<i>Moments</i>	<i>Model 1</i>	<i>Model 2</i>	<i>Model 3</i>	<i>Empirical</i>
$\sigma_{\ln(Y_t)}$	0.430	0.430	0.425	0.534
$\sigma_{\ln(C_t)}$	0.499	0.499	0.500	0.571
$\sigma_{\ln(I_t)}$	0.568	0.566	0.546	0.612
$\sigma_{\ln(C_t/Y_t)}$	0.102	0.103	0.102	0.040
$corr(Y_t, C_t)$	0.992	0.992	0.993	0.999
$corr(Y_t, I_t)$	0.972	0.973	0.965	0.985
$corr(C_t, I_t)$	0.966	0.966	0.964	0.980
$corr(Y_t, Y_{t-1})$	0.997	0.996	0.996	0.999
$corr(Y_t, Y_{t-2})$	0.995	0.995	0.994	0.998
$corr(C_t, C_{t-1})$	0.999	0.999	0.999	0.999
$corr(C_t, C_{t-2})$	0.999	0.999	0.999	0.998
$corr(I_t, I_{t-1})$	0.975	0.976	0.970	0.989
$corr(I_t, I_{t-2})$	0.963	0.965	0.958	0.975
$corr \left(\begin{array}{c} C_t/Y_t, \\ C_{t-1}/Y_{t-1} \end{array} \right)$	0.875	0.868	0.867	0.947
$corr \left(\begin{array}{c} C_t/Y_t, \\ C_{t-2}/Y_{t-2} \end{array} \right)$	0.800	0.796	0.805	0.901
J_T	0.037	0.039	0.042	
$\chi^2(2), 5\%$	2.062	2.194	2.310	9.21
$p - value$	0.356	0.333	0.315	

Table 2.8: Estimated Moments and Goodness of Fit Test, France

<i>Moments</i>	<i>Model 1</i>	<i>Model 2</i>	<i>Model 3</i>	<i>Empirical</i>
$\sigma_{\ln(Y_t)}$	0.452	0.564	0.454	0.565
$\sigma_{\ln(C_t)}$	0.541	0.915	0.543	0.543
$\sigma_{\ln(I_t)}$	0.666	1.050	0.668	0.634
$\sigma_{\ln(C_t/Y_t)}$	0.118	0.466	0.118	0.035
$corr(Y_t, C_t)$	0.979	0.964	0.978	0.998
$corr(Y_t, I_t)$	0.979	0.890	0.979	0.983
$corr(C_t, I_t)$	0.963	0.874	0.962	0.985
$corr(Y_t, Y_{t-1})$	0.995	0.977	0.995	0.998
$corr(Y_t, Y_{t-2})$	0.992	0.971	0.992	0.995
$corr(C_t, C_{t-1})$	0.999	0.998	0.999	0.999
$corr(C_t, C_{t-2})$	0.999	0.997	0.999	0.998
$corr(I_t, I_{t-1})$	0.989	0.863	0.990	0.991
$corr(I_t, I_{t-2})$	0.982	0.820	0.982	0.979
$corr \left(\begin{array}{c} C_t/Y_t, \\ C_{t-1}/Y_{t-1} \end{array} \right)$	0.892	0.879	0.894	0.908
$corr \left(\begin{array}{c} C_t/Y_t, \\ C_{t-2}/Y_{t-2} \end{array} \right)$	0.810	0.855	0.813	0.821
J_T	0.021	0.565	0.021	
$\chi^2(2), 5\%$	1.204	31.097	1.88	9.21
$p - value$	0.547	$0.176 * 10^{-6}$	0.552	

Table 2.9: Estimated Moments and Goodness of Fit Test, Italy

<i>Moments</i>	<i>Model 1</i>	<i>Model 2</i>	<i>Model 3</i>	<i>Empirical</i>
$\sigma_{\ln(Y_t)}$	0.441	0.269	0.438	0.580
$\sigma_{\ln(C_t)}$	0.571	0.459	0.566	0.619
$\sigma_{\ln(I_t)}$	0.635	0.621	0.635	0.502
$\sigma_{\ln(C_t/Y_t)}$	0.151	0.266	0.148	0.057
$corr(Y_t, C_t)$	0.981	0.902	0.983	0.998
$corr(Y_t, I_t)$	0.979	0.561	0.978	0.981
$corr(C_t, I_t)$	0.974	0.441	0.974	0.969
$corr(Y_t, Y_{t-1})$	0.994	0.922	0.995	0.998
$corr(Y_t, Y_{t-2})$	0.992	0.909	0.991	0.995
$corr(C_t, C_{t-1})$	0.999	0.994	0.999	0.999
$corr(C_t, C_{t-2})$	0.999	0.989	0.999	0.997
$corr(I_t, I_{t-1})$	0.990	0.463	0.989	0.985
$corr(I_t, I_{t-2})$	0.984	0.376	0.982	0.971
$corr \left(\begin{array}{c} C_t/Y_t, \\ C_{t-1}/Y_{t-1} \end{array} \right)$	0.920	0.870	0.932	0.945
$corr \left(\begin{array}{c} C_t/Y_t, \\ C_{t-2}/Y_{t-2} \end{array} \right)$	0.871	0.816	0.878	0.882
J_T	0.049	1.295	0.049	
$\chi^2(2), 5\%$	2.717	71.236	2.722	9.21
$p - value$	0.257	$0.333 * 10^{-15}$	0.256	

Table 2.10: Parameter Estimates, U.S.

<i>Parameter</i>	<i>Model 1</i>	<i>Model 2</i>	<i>Model 3</i>
γ	1.928	1.961	1.948
σ	5.474	5.787	5.479
β_0	0.983	0.983	—
$\bar{\beta}$	0.977	0.977	—
β	—	—	0.977
ρ_β	0.909	0.901	—
σ_{ϵ_t}	0.001	0.001	—
α	0.289	0.292	0.280
ρ_Z	1.267	1.250	1.198
σ_{ε_t}	0.033	0.035	0.033
μ	0.021	0.021	0.021
λ_0	0.519	0.518	0.517
λ_1	0.102	0.097	0.102
λ_2	-0.197	-0.194	-0.196
λ_3	0.100	0.098	0.100
τ_0	—	—	-0.088
τ_1	—	—	0.014
τ_2	—	—	-0.002

Table 2.11: Parameter Estimates, France

<i>Parameter</i>	<i>Model 1</i>	<i>Model 2</i>	<i>Model 3</i>
γ	2.588	1.960	2.572
σ	4.887	5.791	4.822
β_0	1.065	1.020	—
$\bar{\beta}$	1.059	1.014	—
β	—	—	1.057
ρ_β	0.639	0.942	—
σ_{ϵ_t}	0.001	0.001	—
α	0.327	0.292	0.332
ρ_Z	1.265	1.199	1.228
σ_{ε_t}	0.035	0.035	0.036
μ	0.021	0.021	0.020
λ_0	0.523	0.516	0.549
λ_1	0.100	0.096	0.105
λ_2	-0.146	-0.194	-0.166
λ_3	0.102	0.098	0.066
τ_0	—	—	-0.092
τ_1	—	—	0.011
τ_2	—	—	-0.002

Table 2.12: Parameter Estimates, Italy

<i>Parameter</i>	<i>Model 1</i>	<i>Model 2</i>	<i>Model 3</i>
γ	2.057	2.048	2.016
σ	5.050	5.766	5.148
β_0	1.066	0.983	—
$\bar{\beta}$	1.060	0.978	—
β	—	—	1.060
ρ_β	0.908	0.898	—
σ_{ϵ_t}	0.001	0.001	—
α	0.311	0.293	0.296
ρ_Z	1.214	1.201	1.125
σ_{ε_t}	0.037	0.035	0.033
μ	0.021	0.021	0.021
λ_0	0.518	0.517	0.517
λ_1	0.089	0.096	0.104
λ_2	-0.182	-0.195	-0.190
λ_3	0.097	0.098	0.101
τ_0	—	—	-0.089
τ_1	—	—	0.014
τ_2	—	—	-0.002

With a decreasing discount rate, in time, society will end up consuming more and progressively save less, fact that is also confirmed by the data. Turning finally to the age-specific utility weights profiles in models 1 and 2, for all three countries (see Figures 2-3 and 2-4), we notice that they register a bell-shape, confirming a shifting structure of the intratemporal preferences that favors the contemporaneous old, with respect to the young categories of population.

The process of shifting preferences influences the values of national saving rates. Tables 2.13 - 2.15 show actual saving rates as well as the simulated ones, for each country and for all three models. The baseline versions of models 1 and 2 refer to the scenario in which the stochastic variables (discount factor, multifactor productivity and government spending) do not display any shocks, maintaining only the AR(1) structure, while for model 3 it implies a time-variant structure of utility weights, but still no technology and government spending shocks; for models

1 and 2, the ' *β const*' scenario accounts for the fact that the discount factor levels were kept constant at their level in 1950, while for model 3, the '*time const*' label denotes no variation in the age-specific utility weights (constant at 1950 levels).

As it clearly results, saving rates in model 1 would have been substantially higher if the

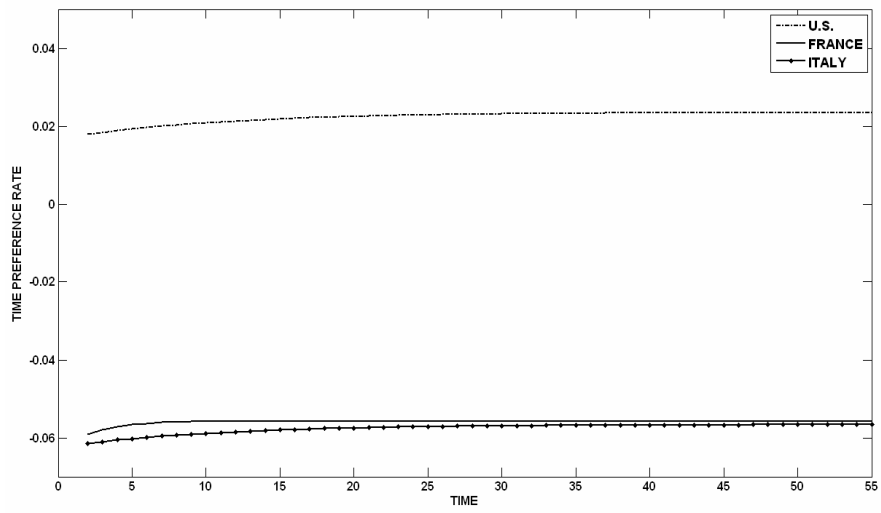


Figure 2-1: Time Preference Rate, model 1

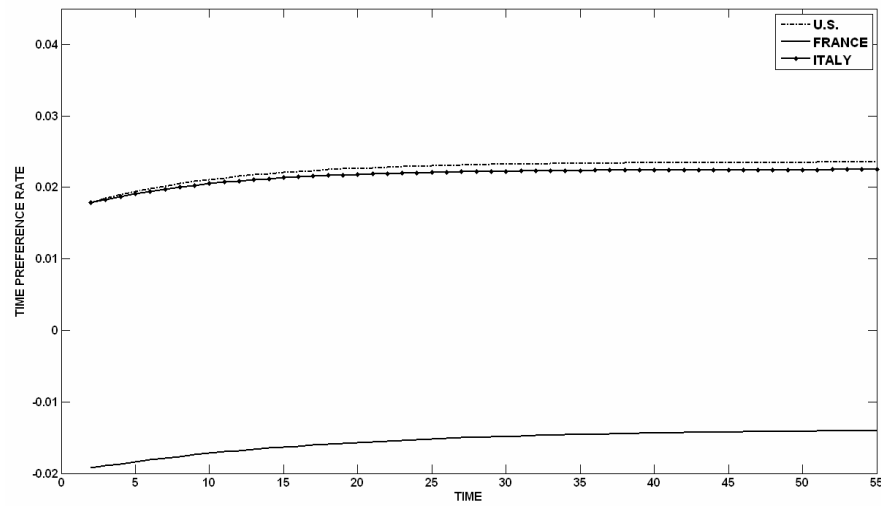


Figure 2-2: Time Preference Rate, model 2

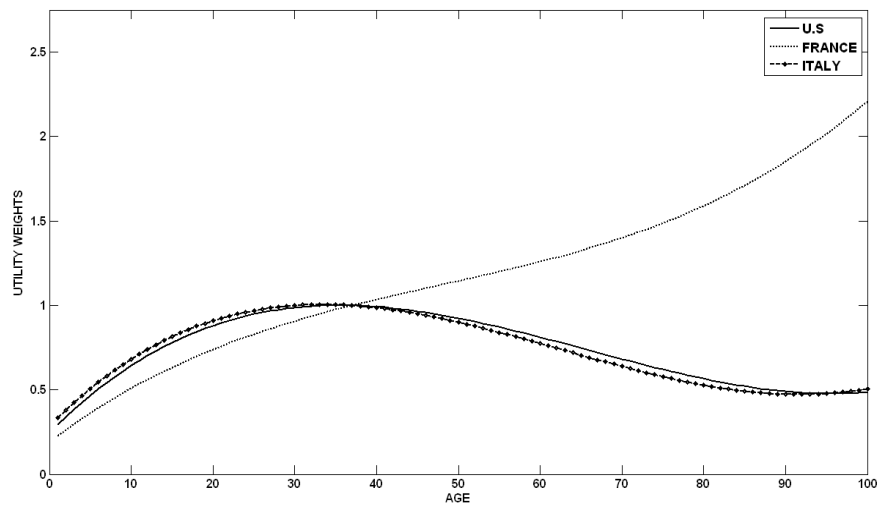


Figure 2-3: Age-Specific Utility Weights by Age Group, model 1

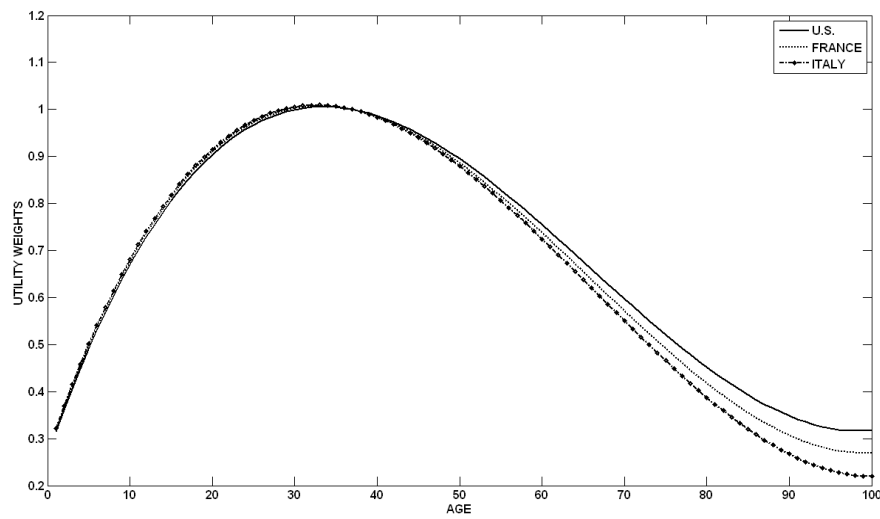


Figure 2-4: Age-Specific Utility Weights by Age Group, model 2

Table 2.13: Saving Rates, U.S.

<i>Saving Rate (%)</i>	1950	1960	1970	1980	1990	2000
<i>Actual sav. rate</i>	—	—	9.53	8.58	4.81	6.81
<i>Model 1 baseline</i>	24.79	7.36	8.38	2.59	4.01	5.02
<i>Model 1 β const</i>	25.26	16.55	9.34	9.15	5.98	11.28
<i>Model 2 baseline</i>	25.22	7.63	8.68	2.95	4.47	5.88
<i>Model 2 β const</i>	25.44	16.68	9.43	9.46	6.47	8.73
<i>Model 3 baseline</i>	25.16	7.63	8.44	2.59	3.85	4.76
<i>Model 3 time const</i>	26.49	7.59	8.50	2.53	3.47	4.23

Table 2.14: Saving Rates, France

<i>Saving Rate (%)</i>	1950	1960	1970	1980	1990	2000
<i>Actual sav. rate</i>	—	—	17.31	11.16	9.67	10.46
<i>Model 1 baseline</i>	18.57	22.17	21.40	16.65	12.69	3.60
<i>Model 1 β const</i>	29.54	34.79	40.22	36.09	27.83	13.29
<i>Model 2 baseline</i>	30.00	23.55	17.54	14.01	11.82	10.27
<i>Model 2 β const</i>	29.78	23.98	18.84	15.98	13.49	11.49
<i>Model 3 baseline</i>	19.91	22.55	21.20	16.11	12.66	4.00
<i>Model 3 time const</i>	20.12	21.38	19.38	14.20	11.30	2.63

Table 2.15: Saving Rates, Italy

<i>Saving Rate (%)</i>	1950	1960	1970	1980	1990	2000
<i>Actual sav. rate</i>	—	—	17.43	12.65	8.27	7.02
<i>Model 1 baseline</i>	9.77	18.26	19.68	14.82	11.71	2.96
<i>Model 1 β const</i>	19.41	30.78	33.42	33.88	26.49	17.69
<i>Model 2 baseline</i>	34.36	35.21	24.38	15.62	8.12	4.78
<i>Model 2 β const</i>	33.31	34.13	10.01	15.36	8.70	5.12
<i>Model 3 baseline</i>	28.41	27.82	17.71	18.04	13.42	3.60
<i>Model 3 time const</i>	27.46	25.54	14.58	15.01	10.55	0.76

discount factors were to be kept at the 1950 level: U.S., France and Italy would have respectively saved 1%, 18.8% and 13.7% more in 1970, and up to 6.3%, 9.7% and 14.7% in 2000. On the other hand, model 2 displays similar results to model 1, including how the discount factor evolves with respect to its long-run value. All three countries' discount factor declines in time, converging to its mean level while utility weights maintain the bell-shape profiles. Accordingly, saving rates for U.S., France and Italy would have respectively registered an increase of 6.5%, 1.9% and -0.3% in 1980 for instance, had the discount factor remained constant at its initial level. These differences would have amounted to 2.8%, 1.2% and 0.3% in 2000.

For model 3, our main interest is on the estimates for τ_0 , τ_1 , and τ_2 , that suggest a remarkable intratemporal preferences shift over time toward older generations. This holds true for all countries. Figures 2-5 - 2-7 show the normalized utility coefficients for different age groups in selected years. The continuous (respectively dot and diamond) line represents the utility coefficients function of age groups in 2004 (respectively 1950 and 1980).

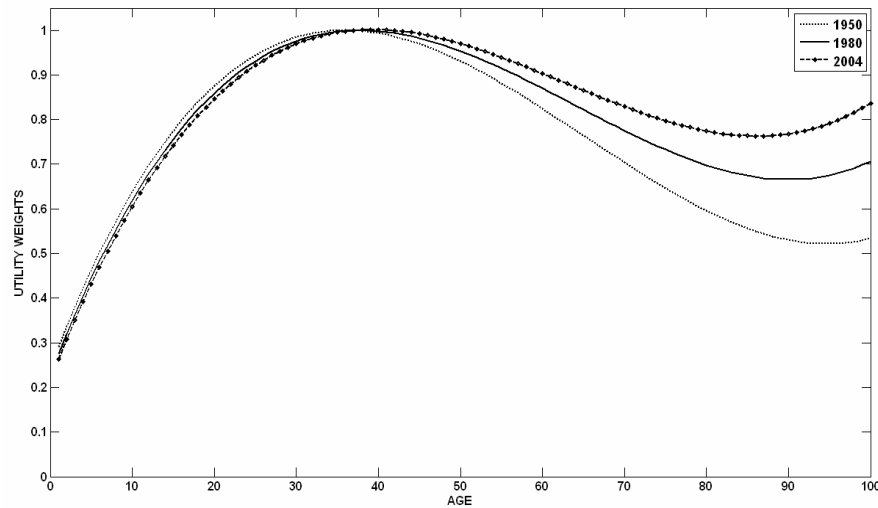


Figure 2-5: Age-Specific Utility Weights by Age Group for Selected Years, U.S.

Society in 2000 appears to care less about the young and more about the old, with respect to both years 1950 and 1980. In time, for model 3, the intratemporal preferences display a constant shift toward older generations at the detriment of the younger ones. This change in intratemporal preferences nevertheless seems not to affect drastically saving rates. Analyzing

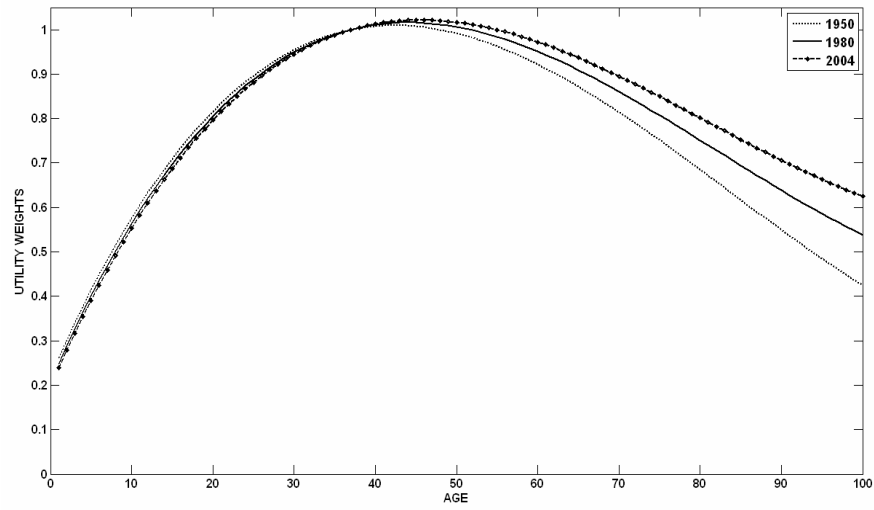


Figure 2-6: Age-Specific Utility Weights by Age Group for Selected Years, France

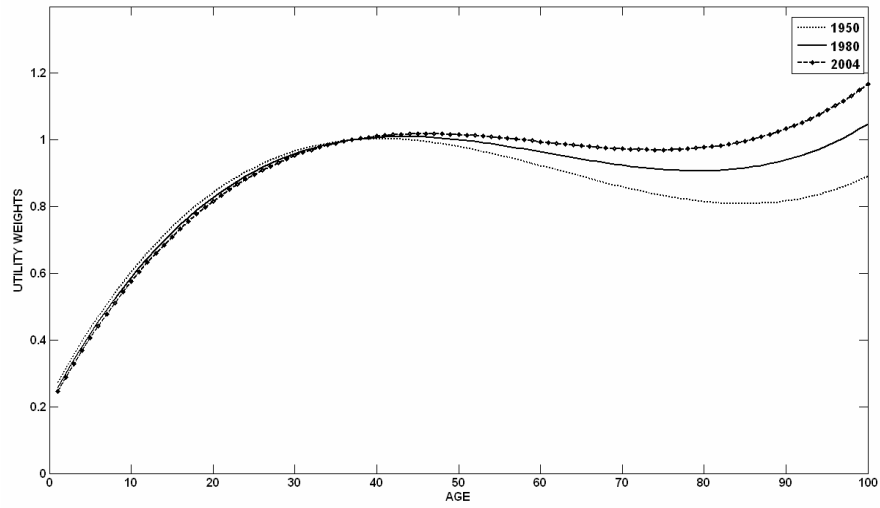


Figure 2-7: Age-Specific Utility Weights by Age Group for Selected Years, Italy

the difference between the simulated saving rate and the saving rate were the utility weights kept constant at 1950 for the three countries, it seems that all three countries register very small difference in terms of the two savings series. Moreover, most of the values for the change in saving rates are negative: it follows that the saving rate predicted by our model always overcome the saving rate with constant utility weights structure. In this case, differences for U.S., France and Italy evolve from 0.06%, -1.8% and -3.1% in 1970 to -0.5%, -1.4% and -3.2% in 2000 respectively. Consequently, it seems that model 3 is not succeeding in explaining the drop in the saving rates for none of the countries analyzed. A reasonable explanation would account for the following scenario: considering that from 1950 there were more young than old people in the society (as it has been registered due to the baby boom phenomenon, which lasted until 1964), it is normal that a society which expects to assign progressively more utility weight to the elderly in time, would anticipate the large amount of consumption to be attributed to the baby boomers when they become old. Consequently, in order to be able to afford this future expenses, society must save in the current period accordingly.

2.6 Conclusions

The estimation exercises in this chapter emphasize the aim of the present work, on proving the downward sloping path of the preference structure towards the future and the quasi bell-shaped form of the age-specific utility weights. The main results, based on modelling and estimating the intertemporal and intratemporal preferences, confirm the theory that in time society changed its preference structure towards assigning progressively more weight to the present generations with respect to the future ones. More than that, it also shows that in time, as far as the preferences within the present generations are concerned, society evolved more and more towards a preference structure which assigned higher level of importance to the old population categories with respect to the young ones. Indeed, our findings show that utility weights follow a bell-shape pattern, quickly increasing for the first half of the existing age groups, while for the second ones the preferences structure is more flat, declining at a slower rate than it raised in the first half. In other words, society tends to allocate less importance to the young generations than to the old ones, and this pattern is accentuated in time.

Techniquely, the work illustrates an equilibrium model where expectations and realizations of technological shock, government expenditure shock, intertemporal preferences (rate of time preference) and current intratemporal preferences (relative weighting of different age groups' utilities from consumption and leisure) were qualitatively and quantitatively able to explain several patterns associated with the saving rates the models generated and could yield a good fit of these models with the true time-series of economic indicators. Consequently, the models confirmed the intuition of our results which rest with the process of decreasing discount factor - society is progressively becoming more present - oriented, and increasing utility weights for the old demographic categories, starting from 1950 to the present. On the contrary, our findings are not equally strong in the case of a model that considers society to have stable intertemporal preferences and time-varying intratemporal preferences, but the explanation lies within the demographic structure that changes over time.

Chapter 3

Stochastic preferences: lessons on government inconsistency

3.1 Introduction

The present work is intended to extend the answer to the question raised in Chapter 2, relative to which are the main factors that determine the national saving rate of a country. It does so by estimating a finite-horizon overlapping generational model of consumption choice where, in each and every moment of time, government and households decide together what is to be consumed and saved. As in the previous chapter, the collective-choice setup of this paper is based on the work of Green and Kotlikoff [2006], which underlined the fact that, in terms of public and private property, economics makes actually no difference between them. On the contrary, the government and household sector effectively play the role of two partners whose assets are community property. Consequently, each party can argue that it owns entirely or partially the household's collective resources and that it makes a specific decision, but without an economic basis whatsoever.

However, the parties come to reach a consensus on who consumes particular amounts if their bargaining is efficient, and their decision can be modeled as maximizing a weighted sum of the parties' utilities. This maximization is, however, complicated in a dynamic setting by the potential for changes over time in preferences, bargaining power, economic conditions, and, indeed, the number and identities of future agents. Given these factors, current parties realize

that they can't dictate future decisions. But they also realize that they can influence how such future decisions will be made by bringing more or less resources into the next period.

To keep the model tractable, I made the simplifying assumption that society attempts at each point in time to maximize an expected social welfare function, whose arguments are the annual utility levels of all current and future generations. The function weighs these annual utility levels based on the ages of the agents in question. It also discounts future annual utility levels of current and future generations at a fixed discount rate. I assumed that the age-specific weights are time invariant, i.e., all future societies share the age weights. On the other hand, I allowed the society's preference rate for one aggregate age group with respect to the other (young versus old generations) for successive societies to vary through time.

An efficient bargaining, in this dynamic context, thus entails, using backwards induction, to determine how current choices can indirectly influence future choices. This eventuates in a time-consistent solution. In order to solve for such a solution, if today's society has preferences about how to discount the utility of young/old/unborn generations that differ from those of future society, I would have to determine the consumption rules, conditional on state variables, of future societies and plug them into a direct expected utility function that incorporates current society's aggregate age-preference rates at all future dates.

These social aggregate age-preference rates are the sole preference parameter subject to change in the model. They rise or fall from period to period as successive societies place less or respectively more weight on the future wellbeing of those now alive and young (defined as being between 0 and 64 years old), of those now alive and old (defined as being between 65 and 100 years old) and of those to be born in the future. Furthermore, the model also incorporates time-varying stochastic shocks to demographics, multifactor productivity and the government's own consumption. We solve for the time-consistent consumption decisions using dynamic programming and estimate the model using the method of moments.

The main result confirms the theory that, in time, successive societies translated their preference structure for alive and unborn generations progressively towards the old generations. The procedure chosen in order to illustrate this idea was estimating the two stochastic processes that describe the structure of the preferences that a society has towards the young and old age groups. The idea is also confirmed by the age-specific utility weights that display a bell-shape

which raise sharper for the first half of the population's age structure than it decrease in the second half. This finding is also validating the basic fact that, within the same time period, young people benefitted of less weight, while the emphasis increased on the old categories of individuals.

The chapter proceeds in Section 2 with a detailed presentation of the model and a solution for optimal consumption using backward induction. Further, Section 3 describes the U.S. data used, while Section 4 analyses the estimation procedure, including the simulated method of moments (SMM) and its application to the specific problem. I present the results in Section 5 and conclude in Section 6.

3.2 The model

Consider a finite-horizon overlapping generation model with periods t , $t \in \{-\infty, T\}$, each one governed by a different social planner. The model refers to a centralized economy, in which society is choosing how much agents will consume, in a stochastic framework that refers to four types of shocks. On one hand, I considered the typical shocks in productivity and government expenditure, while on the other hand, the model was further extended to account for shocks in society preferences for one aggregate age group (young/old) with respect to another. More specifically, the scope of this chapter is to model society's preferences towards present young, present old and future generations, in the context of an existing structure of age-specific utility weights.

Consider that the maximum longevity is 100 years old and further and that the world ends in the following way. Given that I modelled the economy as ending at T period, at time T just the generation of age 100 is alive, while at time $T - 1$ the generations of 99 and 100 years old are alive and so on; consequently, there are a total of 100 alive generations at time $T - 100$ aged from 0 to 100 years old inclusive. The last generation alive, age 100 years at time T , is characterized by a lifetime utility at time T , $V_{T,-100+T}$ which means that this generation was born 100 periods before time T . The last but one generation is 99 years old in the $T - 1$ period and 100 year old in the T period, so its lifetime utility $V_{T-1,-99+T-1}$, will account for the fact that this generation will be alive for two periods, through the discounted factor β , and so on.

The finite horizon is implied by the scenario that from time $T - 100$ onward, the birth rate is constant at zero level.

For completeness, I assumed that each society from time $T - 100$ backward, cares not only about the lifetime utility of agents that are alive during its mandate, but also for the unborn generations in an infinite horizon; moreover, each society has the same preferences, $\theta(t)$ over the utilities of young generations (defined as the age groups between 0 and 64 age old) and $\delta(t)$ over the utilities of old generations (defined as age groups 65+). Please note that although these preference parameters are the same for all the young categories and old respectively, they vary in time and change from one period to another, capturing the issue of society inconsistency. By working backwards the best lifetime allocation that it can achieve given the last preference structure, society needs to implement a time-consistent policy in the current period. Obviously, the economy can be decentralized in numerous ways (with agent-specific lump-sum payments, plus agent-specific payments per unit of expenditure on old-age consumption, or with a capital-income or consumption taxation system)¹. For the unborn generations, all societies attach a preference parameter of zero until the time they are born, and unit after they are born; also, society will apply a discount factor of β from the moment they are considered alive in the utility function.

Under these circumstances, the issue is choosing consumption in each periods so as to maximize

$$W_t = \sum_{\tau=-\infty}^{t+1} E_t(V_{t,\tau}^f) + \sum_{\tau=t}^{-64+t} \tilde{\theta}(t) E_t(V_{t,\tau}) + \sum_{\tau=-65+t}^{-100+t} \tilde{\delta}(t) E_t(V_{t,\tau}), \quad (3.1)$$

with

$$V_{t,\tau}^{y,o} = \sum_{a=t-\tau}^{100} P_{a,\tau} \theta_a \beta^{a-(t-\tau)} \frac{C_{a,\tau}^{1-\gamma} - 1}{1-\gamma}, \quad (3.2)$$

and

$$V_{t,\tau}^f = \sum_{a=0}^{100} P_{a,\tau} \theta_a \beta^{a-(t-\tau)} \frac{C_{a,\tau}^{1-\gamma} - 1}{1-\gamma}. \quad (3.3)$$

The function W_t is the total welfare function of society at time t , and indicates the fact that society takes into consideration the utility functions of all 100 generations alive at time t

¹Fischer (1980).

(separated in two groups - young and old), but also the utility functions of the infinite number of generations that are yet to be born. Within each age group, utility is multiplied by an age-specific set of utility weights for each age a , θ_a , determining the shape of the cross-sectional age-consumption profile,

$$\theta_a = \lambda_0 + \lambda_1 * age + \lambda_2 * age^2 + \lambda_3 * age^3. \quad (3.4)$$

The function $V(\cdot)$ multiplied by the correspondent preference parameter gives society's welfare at time t associated with the consumption of a member age a , born at time τ , $C_{a,\tau}$. It is further assumed that utility is of constant-elastic form, i.e.,

$$U(C) = \frac{C^{1-\gamma} - 1}{1 - \gamma},$$

with γ relative risk aversion coefficient for consumption.

$P_{t,\tau}$ represents the number of individuals alive at time t that have been born at time τ (with maximum longevity of 100) and $\beta > 0$ is the discount factor. $\theta(t)$ and $\delta(t)$ are the preference parameters, for young and old respectively, of current society that exercises its mandate at time t , both exogenous, following AR(1) processes with mutually-correlated shocks,

$$\ln \tilde{\theta}(t) = (1 - \rho_\theta) \ln \bar{\theta} + \rho_\theta \ln \tilde{\theta}(t-1) + \epsilon_t \text{ with } \epsilon_t \sim N(0, \sigma_{\epsilon_t}^2), \quad (3.5)$$

and

$$\ln \tilde{\delta}(t) = (1 - \rho_\delta) \ln \bar{\delta} + \rho_\delta \ln \tilde{\delta}(t-1) + \varphi_t \text{ with } \varphi_t \sim N(0, \sigma_{\varphi_t}^2) \text{ and } \sigma_{\epsilon_t, \varphi_t} = cov(\epsilon_t, \varphi_t). \quad (3.6)$$

Each agent has an endowment of time for each period normalized to one and for simplicity, I assumed that agents are working full time. Consequently, the only difference between them will be given by their age-specific earning ability.

The single, perishable good in this economy is produced by perfectly competitive firms using a constant returns to scale technology. Since in this setup the number of firms in equilibrium is

indeterminate, it is convenient to focus on the economy's production function. In the economy wide production function, firms rent labor and capital from the agents and combines them according to

$$Y_t = \widetilde{Z}_t A_t K_t^\alpha \left(\sum_{\substack{\tau=\infty \\ a=t-\tau}}^{-100+t} P_{a,\tau} e_a \right)^{1-\alpha}, \quad (3.7)$$

where K_t is the capital in the economy at time t , considered state variable in the model, $\alpha \in (0, 1)$ is the capital share in production, $A_t = (1 + \mu)A_{t-1}$ is the deterministic technical progress that grows at exponential rate μ , and \widetilde{Z}_t is the economy's productivity shock, which is considered to be exogenous, stochastic following a normal distribution with an AR(1) process of the form:

$$\ln \widetilde{Z}_t = \rho \ln \widetilde{Z}_{t-1} + \varepsilon_t \text{ with } \varepsilon_t \sim N(0, \sigma_{\varepsilon_t}^2), \quad (3.8)$$

where $\rho > 0$ and ε_t is the innovation assumed to be independently, identically, and normally distributed with zero mean and variance $\sigma_{\varepsilon_t}^2$. In every period, the firm chooses input levels to maximize profits and equates the marginal product of labor (capital) to the real wage (rental rate). Due to the assumptions of perfect competition and constant returns to scale, firms make zero profits in equilibrium.

We note by e_a the relative earning ability of each age group. This parameter varies with age from 15 years old to 75 years old², and it is defined as follows:

$$e_a = e^{4.47+0.0033*(a-15)-0.000067*(a-15)^2}. \quad (3.9)$$

Considering all these elements, the economy's capital accumulation process will be:

$$K_{t+1} = (1 - d)K_t + \widetilde{Z}_t A_t K_t^\alpha \left(\sum_{\substack{\tau=\infty \\ a=t-\tau}}^{-100+t} P_{a,\tau} e_a \right)^{1-\alpha} - \sum_{\substack{\tau=\infty \\ a=t-\tau}}^{-100+t} P_{a,\tau} C_{a,\tau} - \widetilde{G}_t. \quad (3.10)$$

²For a complete formulation of this matter see Fehr, H., Jokisch, S., Kotlikoff, L.J., (2007).

First, notice that the model economy I considered is closed³. Since there is just one good, there is no international trade, while international borrowing is not allowed. Second, in the above formulation, \tilde{G}_t represents government spending, assumed to evolve partially exogenously according to a serially correlated process,

$$\ln \tilde{G}_t = (1 - \rho_G) \ln \bar{G} + \rho_G \ln \tilde{G}_{t-1} + \eta_t \text{ with } \eta_t \sim N(0, \sigma_{\eta_t}^2), \quad (3.11)$$

and partially according to a trend, $AG_t = (1 + mg)AG_{t-1}$. I further considered that d is the depreciation rate, and $(K_0, \tilde{Z}_0, \tilde{G}_0, \tilde{\delta}_0, \tilde{\theta}_0)$ is given. To clarify the time indexes, t is the current calendaristic year, τ is the year of birth, and consequently, $a = t - \tau$ represents current age. Given that I assumed that the problem has a recursive formulation, its solution satisfies the equation:

$$\begin{aligned} & W_t(\tilde{Z}_t, \tilde{G}_t, \tilde{\delta}_t, \tilde{\theta}_t, K_t) = \quad (3.12) \\ = & \max_{C_{a,\tau}} \left\{ \sum_{\tau=-\infty}^{t+1} \sum_{a=0}^{100} P_{a,\tau} \theta_a \frac{C_{a,\tau}^{1-\gamma} - 1}{1-\gamma} + \sum_{\tau=t}^{-64+t} \tilde{\theta}(t) \sum_{a=t-\tau}^{100} P_{a,\tau} \theta_a \frac{C_{a,\tau}^{1-\gamma} - 1}{1-\gamma} + \right. \\ & \left. + \sum_{\tau=-65+t}^{-100+t} \tilde{\delta}(t) \sum_{a=t-\tau}^{100} P_{a,\tau} \theta_a \frac{C_{a,\tau}^{1-\gamma} - 1}{1-\gamma} \right\} + \beta E_t \left[V_{t+1}(K_{t+1}, \tilde{Z}_{t+1}, \tilde{G}_{t+1}, \tilde{\delta}_{t+1}, \tilde{\theta}_{t+1}) \right] \\ & \text{s.t.} \quad \text{equation (3.10)}. \end{aligned}$$

The last system of equations can be used to solve for the optimal choice of consumption at any point in time. Equilibrium paths can be calculated by backward recursion, at each and every period adjusting the welfare function backwards for all the previous periods preference structure. Consequently, time- t society can solve for the choices of society at time- $(t + 1)$, which will depend on the amount of capital transferred from period t and then it can use these partial outcomes (because they depend on t -period variables) to solve for its t -period choices of consumption. By reacting at time- $(t + 1)$ society' choices and adjust its own policy to this

³For the case of an open economy, the interest rate would have to be considered as exogeneous, with the capital-labour ratio being determined by this exogeneous interest rate and the draw of the productivity shock, \tilde{Z}_t . The wage would be based on the previously formulated capital-labour ratio and the draw of the productivity shock, \tilde{Z}_t .

conditions, the time- t society is formulating a time-consistent policy.

At any point in time the relationship between consumption of different age groups is given as follows:

For the last 35 periods ($T \rightarrow T - 35$) (only the old generations are alive),

$$\frac{C_{a,\tau}}{C_{a^*,\tau}} = \left[\frac{\theta_a}{\theta_{a^*}} \right]^{\frac{1}{\gamma}}. \quad (3.13)$$

For all the previous periods ($T - 36 \rightarrow T - \infty$) (all the 101 generations are alive plus the ∞ number of unborn generations),

$$\frac{C_{a,\tau}}{C_{a^*,\tau}} = \begin{cases} \left[\frac{\theta_a}{\theta_{a^*}} \right]^{\frac{1}{\gamma}} & \text{if } a, a^* \geq 65 \\ \left[\frac{\theta_a}{\theta_{a^*}} \frac{\theta(t)}{\delta(t)} \right]^{\frac{1}{\gamma}} & \text{if } a = 64 \text{ and } a^* = 65 \\ \left[\frac{\theta_a}{\theta_{a^*}} \right]^{\frac{1}{\gamma}} & \text{if } a, a^* \leq 64, a \geq 0 \\ 0 & \text{if } a < 0 \end{cases}. \quad (3.14)$$

This model takes as input the predetermined level of capital and four exogenous shocks, and generates predictions about three observable endogenous variables, namely output, consumption and investment.

3.2.1 Solving for optimal consumption through backward recursion

The strategy of solving for finite horizon models in the context of extrapolating the results to infinite time models implies solving for a sufficiently large number of periods such that any further extension would not introduce any changes in the results. The idea is simulating the economy for a number of periods large enough such that it permits the model to converge for the first 55 periods, in this case. Data cover the period from 1950 - 2004, and I chose as optimal year 2100, which is the last year covered by Census Population Projections. The optimal consumption values for the years 1950 and 2004 based on terminal year 2099 were less than half percent different with respect to the ones obtained by considering as terminal year 2100.

The dynamic programming problem is solved backwards, but with certain thresholds where

the problem changes due to changes in the structure of the population and in society preferences. Consider period t in which the structure of the population is complete (demographically, I considered that there are old, young and future generations). From period t onwards, the economy is still populated by different age groups and, for simplicity, I considered the representative agent as, say, the forty years old individual. The strategy is obtaining consumption from the budget constraint as a function of the state variables (next period capital included):

$$\begin{aligned}
C_{a^*,t}(\tilde{Z}_t, \tilde{G}_t, \tilde{\delta}_t, \tilde{\theta}_t, K_t, K_T) &= \\
&= \left((1-d)K_t + \tilde{Z}_t A_t K_t^\alpha \left(\sum_{\substack{\tau=\infty \\ a=t-\tau}}^{-100+t} P_{t,\tau} e_a \right)^{1-\alpha} - \sum_{\substack{\tau=\infty \\ a=t-\tau}}^{-100+t} P_{t,\tau} C_{a,\tau} - G_t - K_{t+1} \right). \tag{3.15}
\end{aligned}$$

At time t , society in charge displays an utility function as described in equation (3.1), where the first term in the equation concerns the unborn generations; moreover, since the actual society has no control over the final period, $V_{t,\tau}^f$ is given by

$$V_{t,\tau}^f = E_t \left[\sum_{a=1}^{100} P_{a,\tau} \theta_a \beta \frac{\left(\frac{\theta_a}{\theta_{a^*}} \right)^{\frac{1-\gamma}{\gamma}} C_{a^*,t+1}(\tilde{Z}_{t+1}, \tilde{G}_{t+1}, \tilde{\delta}_{t+1}, \tilde{\theta}_{t+1}, K_{t+1}^*)^{1-\gamma} - 1}{1-\gamma} \right]. \tag{3.16}$$

The utility function for the current young is given by

$$\begin{aligned}
V_{t,\tau}^y &= \frac{\left(\frac{\theta_a}{\theta_{a^*}} \right)^{\frac{1-\gamma}{\gamma}} C_{a^*,t}(\tilde{Z}_t, \tilde{G}_t, \tilde{\delta}_t, \tilde{\theta}_t, K_t, K_{t+1})^{1-\gamma} - 1}{1-\gamma} + \\
&+ E_t \left[\sum_{a=T-\tau}^{T-\tau} P_{a,\tau} \theta_a \beta^{a-(T-1-\tau)} \frac{\left(\frac{\theta_a}{\theta_{a^*}} \right)^{\frac{1-\gamma}{\gamma}} C_{a^*,t+1}(\tilde{Z}_{t+1}, \tilde{G}_{t+1}, \tilde{\delta}_{t+1}, \tilde{\theta}_{t+1}, K_{t+1}^*) - 1}{1-\gamma} \right]. \tag{3.17}
\end{aligned}$$

The utility function for the current old is given by

$$\begin{aligned}
V_{t,\tau}^o &= \frac{\left(\frac{\theta_a}{\theta_{a^*}} \frac{\theta(t+1)}{\delta(t+1)}\right)^{\frac{1-\gamma}{\gamma}} C_{a^*,t}(\tilde{Z}_t, \tilde{G}_t, \tilde{\delta}_t, \tilde{\theta}_t, K_t, K_{t+1})^{1-\gamma} - 1}{1-\gamma} + \\
&+ E_t \left[\sum_{a=T-\tau}^{t+1-\tau} P_{a,\tau} \theta_a \beta^{a-(t-\tau)} \frac{\left(\frac{\theta_a}{\theta_{a^*}} \frac{\theta(t+1)}{\delta(t+1)}\right)^{\frac{1-\gamma}{\gamma}} C_{a^*,t+1}(\tilde{Z}_{t+1}, \tilde{G}_{t+1}, \tilde{\delta}_{t+1}, \tilde{\theta}_{t+1}, K_{t+1}^*) - 1}{1-\gamma} \right].
\end{aligned} \tag{3.18}$$

The solutions for the consumption choice were found numerically; using Gaussian quadrature procedure, described in Appendix A, to deal with the stochastic part allowed to rewrite the above expressions in the following way:

- for the unborn future generations:

$$\begin{aligned}
V_{t,t+1}^f &= \sum_{i=1}^m \pi_{qi} \sum_{j=1}^n \pi_{rj} \sum_{k=1}^{mm} \pi_{ok} \sum_{l=1}^{nn} \pi_{sl} \\
&\left[\sum_{a=1}^{100} P_{a,\tau} \theta_a \beta \left(\frac{\theta_a}{\theta_{a^*}}\right)^{\frac{1-\gamma}{\gamma}} \frac{C_{a^*,t+1}(\tilde{Z}_{lt+1}, \tilde{G}_{kt+1}, \tilde{\theta}_{jt+1}, \tilde{\delta}_{it+1}, K_{t+1}^*)^{1-\gamma} - 1}{1-\gamma} \right],
\end{aligned} \tag{3.19}$$

- for the alive young generations:

$$\begin{aligned}
V_{t,\tau}^y &= \frac{\left(\frac{\theta_a}{\theta_{a^*}}\right)^{\frac{1-\gamma}{\gamma}} C_{a^*,t}(\tilde{Z}_{st}, \tilde{G}_{ot}, \tilde{\delta}_{qt}, \tilde{\theta}_{rt}, K_t, K_{t+1})^{1-\gamma} - 1}{1-\gamma} + \sum_{i=1}^m \pi_{qi} \sum_{j=1}^n \pi_{rj} \sum_{k=1}^{mm} \pi_{ok} \sum_{l=1}^{nn} \pi_{sl} \\
&\left[\sum_{a=T-\tau}^{100} P_{a,\tau} \theta_a \beta^{a-(t+1-\tau)} \left(\frac{\theta_a}{\theta_{a^*}}\right)^{\frac{1-\gamma}{\gamma}} \frac{C_{a^*,T}(\tilde{Z}_{lt+1}, \tilde{G}_{kt+1}, \tilde{\theta}_{jt+1}, \tilde{\delta}_{it+1}, K_{t+1}^*)^{1-\gamma} - 1}{1-\gamma} \right],
\end{aligned} \tag{3.20}$$

- for the alive old generations:

$$\begin{aligned}
V_{t,\tau}^o &= \left(\frac{\theta_a}{\theta_{a^*}} \frac{\theta(t+1)}{\delta(t+1)}\right)^{\frac{1-\gamma}{\gamma}} \frac{C_{a^*,t}(\tilde{Z}_{st}, \tilde{G}_{ot}, \tilde{\delta}_{qt}, \tilde{\theta}_{rt}, K_t, K_{t+1})^{1-\gamma} - 1}{1-\gamma} + \\
&+ \sum_{q,i=1}^{mm} \pi_{qi} \sum_{r,j=1}^{nn} \pi_{rj} \sum_{o,k=1}^m \pi_{ok} \sum_{s,l=1}^n \pi_{sl} \left[\frac{\sum_{a=T-\tau}^{100} P_{a,\tau} \theta_a \beta^{a-(T-\tau)} \left(\frac{\theta_a}{\theta_{a^*}} \frac{\theta(t+1)}{\delta(t+1)}\right)^{\frac{1-\gamma}{\gamma}}}{\frac{C_{a^*,t+1}(\tilde{Z}_{lt+1}, \tilde{G}_{kt+1}, \tilde{\theta}_{jt+1}, \tilde{\delta}_{it+1}, K_{t+1}^*) - 1}{1-\gamma}} \right],
\end{aligned} \tag{3.21}$$

where π_{sl} represents the transition probability from state s to state l , for the productivity shock with n total possible states,

$$\pi_{sl} = Prob(Z_{t+1} = Z_l | Z_t = Z_s),$$

and π_{ok} represents the transition probability from state o to state k , for the government expenditure shock with m total possible states,

$$\pi_{ok} = Prob(G_{t+1} = G_k | G_t = G_o).$$

Similarly, π_{rj} is the transition probability from state r to state j , for the preference parameter for young shock with nn total possible states,

$$\pi_{rj} = Prob(\theta_{t+1} = \theta_j | \theta_t = \theta_r),$$

while π_{qi} is the transition probability from state q to state i , for the preference parameter for old shock with mm total possible states,

$$\pi_{qi} = Prob(\delta_{t+1} = \delta_i | \delta_t = \delta_q).$$

Let's now concentrate on the decision variables. At time t , note that society's utility function is both depending on K_{t+1} and K_t . Given the grid for capital, I could compute the optimal value, K_{t+1}^* , as a function of each and every possible value of K_t ,

$$K_{t+1}^* = h(\tilde{Z}_t, \tilde{G}_t, \tilde{\delta}_t, \tilde{\theta}_t, K_t).$$

In this way, I obtained the decision rule that allowed the writing of the utility function of society at time t as function of only K_t (and no more of K_{t+1}),

$$W_t(\tilde{Z}_t, \tilde{G}_t, \tilde{\delta}_t, \tilde{\theta}_t, K_t).$$

This operation can be recursively applied to obtain the complete decision rule, for any period,

$$K_{t+1}^* = h(\tilde{Z}_t, \tilde{G}_t, \tilde{\delta}_t, \tilde{\theta}_t, K_t).$$

3.3 Description of data

The dataset used to estimate the model consists of a sample of 55 annual observation starting from 1950 to 2004. The interest series were considered to be GDP, private consumption, government expenditures and investment, all in real terms and in billions of 2000 chained dollars. These economic macro-indicators together with real government expenditures were provided by the Bureau of Economic Affairs office. Total consumption is considered to include the NIPA personal consumer expenditure on nondurable goods and services as well as the government consumption expenditures. Government spending was obtained by applying to the existing NIPA data which cover the period 1990-2004, the percent change from preceding period in real government consumption expenditures. Consumption as share of the output was obtained by simply calculating the ratio between the two variables.

The sample on population was obtained based on the US Census national estimates for the period 1950-2004 and the projection on the evolution of population structure for the period 2005-2100, and were further detailed by 1 year age groups.

3.4 Estimation procedure

This section refers to the estimation procedure. In order to be able to compute the variety of descriptive statistics which will summarize the behavior of the individuals in the economic model and will be further used in the simulated method of moments (SMM) estimation, I generated the artificial time series taking into consideration the set of initial values of the state variables, the vector of stochastic unobserved shocks and the set of parameters. Based on these elements, one can compute N simulated paths for the variables and use them to estimate the parameters of interest.

3.4.1 Data simulation

To compute the numerical simulations I used the economic model which is adjusted for the population growth. For the purpose of analyzing the qualitative implications of the model, I needed to assign certain initial values for the model parameters. As a result, I calibrated the initial model to fit the standard values used in the literature. Under these circumstances, I estimated all the parameters of the distribution of the two stochastic processes for the society preferences towards the young generations and towards the old generations, as well as for technological shock, and I calibrated the depreciation at 5 percent, the initial level of labour-augmenting technical progress at unit and the parameters for the distribution of the government shock with their values from the data. The government consumption process is modelled as a VAR(1) process and its parameters are estimated using least-squares. I obtained the 1950 value of K from data on fixed reproducible tangible wealth⁴. Using the parameter estimates, I then proceeded and estimated all the other parameters using SMM. The values obtained were further used in the simulation of the artificial time-series. The parameters definition is summarized in Table 3.1.

One issue encountered is that I would have liked to be able to evaluate the expectation operator in the above equations. For all the shocks, I evaluated the model using the Gauss-Hermite quadrature approach to discretization; I modelled the shock in technology and the one in government expenditure as independent, while I considered the two shocks regarding society preferences for young and old generation as correlated⁵ throughout the work. I therefore had to discretize the shocks and transform the continuous problem into a discrete one with the constraint that the asymptotic properties of the continuous and the discrete processes should be the same. In this context, I used Markov chains to represent each of the stochastic processes.

Using these parameters, I computed life-cycle histories in a series of 20 different scenarios corresponding to 20 different random draws, each of them simulating a higher number of periods (172 periods) than SMM actually uses for model fit (55 periods). Each scenario is based on a certain range of values for the state variables drawn from a distribution which had been further

⁴The net foreign asset position was obtained by subtracting the foreign investments in the country from the investments abroad. For the U.S., data was obtain from BEA.

⁵For a more detailed approach to Gauss-Hermite quadrature procedure of discretization, see Appendix A.

Table 3.1: Parameters Estimated - Definitions

<i>Parameter</i>	<i>Definition</i>
γ	utility function parameter
β	discount factor
d	capital depreciation rate
ρ_δ	delta process persistence coefficient
ρ_θ	theta process persistence coefficient
$\bar{\delta}$	mean of delta stochastic process
$\bar{\theta}$	mean of theta stochastic process
σ_{φ_t}	std. dev. of the delta stochastic process
$\sigma_{\epsilon_t, \varphi_t}$	cov between delta and theta processes
σ_{ϵ_t}	std. dev. of theta stochastic process
λ_0	intercept for the age-specific utility weights
λ_1	age param. for the age-specific U weights
λ_2	age ² param. for the age-specific U weights
λ_3	age ³ param. for the age-specific U weights
α	share of capital in the production function
ρ	technology shock persistence coefficient
σ_{ϵ_t}	std. dev. of technology shock
ρ_G	government exp. shock persistence coefficient
σ_{η_t}	std. dev. of government exp. shock

estimated. Solving numerically the model yields a set of decision rules obtained by grid search with respect to the state variable of capital and four categories of shocks.

3.4.2 SMM

Using the simulated series generated for different realizations of the stochastic processes, I computed the moments of each artificial economy and tried to match them with the corresponding moments registered by the true economy, based on the data described above. I used the SMM procedure which consists in choosing the parameters values that minimize the difference between the true and the artificial data moments. Since the number of moments is higher than the number of parameters to estimate, and so since I adopted the hypothesis of an overidentified model, I used a two stage SMM: in the first stage, I minimized the difference between the true and the artificial data moments and obtained consistent estimates for the parameters, which will be consistent; in the second stage, starting from the parameters values obtained at the previous stage, I minimized a weighted difference that will account for a distance matrix, and

obtained also the efficiency of the estimated parameters. More precise, multiplying the vector of the differences between the historic data moments and the simulated data moments by the inverse of a consistent estimate of its asymptotic variance matrix will yield a test statistic that is distributed as a χ^2 , with degrees of freedom equals to the number of moments minus the number of parameters to be estimated.

The procedure is as follows. Once the N ($=20$ in the simulation exercise) paths of simulated $T_N \simeq 3 \times T$ ($T_N = 172, T = 55$) periods for each and every variable are generated, I computed the set moments conditional on the initial values of the state variables x_0 and of the parameters ϕ_0 . As in the previous two chapters, given that I had already calculated the set of real data moments, what SMM did was searching for the value of the following objective function:

$$J_T = \arg \min_{\tilde{\phi}} [m_T - \frac{1}{N} m_N(x_0, \phi_0)]' W_T [m_T - \frac{1}{N} m_N(x_0, \phi_0)], \quad (3.22)$$

where m_T represents the set of the true data moments and $m_N(x_0, \phi_0)$ is the set of moments of each of the N simulated paths of the artificial economy. W_T is the weighting or distance matrix which almost surely converges to $W_0 = S_0^{-1}$, where S_0 is the limit constant full rank matrix of the covariance of the estimations errors. Using the method of Andrews [1991], I obtained a positive definite matrix. This matrix is consistently estimated using the estimator proposed by Newey and West [1994]. Heuristically, it gives more weight to moments that are precisely estimated in the data.

What I will interpret as the result from this χ^2 -test statistic or associated p -value is whether the true data moments are equal to the realized data moments of the stochastic processes for which the true time series is just one realization. Consequently, this hypothesis fits perfectly the sets of parameters to be estimated, which included also the means, standard deviations and covariance for the distributions of the stochastic processes.

As mentioned, the actual choice of moments for the SMM is still an open issue in the literature. For the model, in order to ease the interpretation and restrain the set of moments that would potentially be too large, I considered a measure of variability (standard deviations of output, consumption, investment and share of consumption in output), a measure of instantaneous output-consumption, output-investment, output-consumption to output ratio and consumption-investment, consumption-consumption to output ratio and investment-consumption to output

Table 3.2: Parameters Estimated

$$\{ \gamma, \beta, \rho_\delta, \rho_\theta, \bar{\delta}, \bar{\theta}, \sigma_{\varphi_t}, \sigma_{\epsilon_t, \varphi_t}, \sigma_{\epsilon_t}, \alpha, \rho, \sigma_{\epsilon_t}, \lambda_0, \lambda_1, \lambda_2, \lambda_3 \}.$$

Table 3.3: Choice of Moments

$$\left\{ \begin{array}{lll} \sigma_{\ln(Y_t)}, & \sigma_{\ln(C_t)}, & \sigma_{\ln(X_t)}, \\ \sigma_{\ln(C_t/Y_t)}, & \text{corr}(Y_t, C_t), & \text{corr}(Y_t, X_t), \\ \text{corr}(Y_t, C_t/Y_t), & \text{corr}(C_t, X_t), & \text{corr}(C_t, C_t/Y_t), \\ \text{corr}(Y_t, Y_{t-1}), & \text{corr}(Y_t, Y_{t-2}), & \text{corr}(X_t, C_t/Y_t), \\ \text{corr}(C_t, C_{t-1}), & \text{corr}(X_t, X_{t-1}), & \text{corr}(C_t/Y_t, C_{t-1}/Y_{t-1}), \\ \text{corr}(C_t, C_{t-2}), & \text{corr}(X_t, X_{t-2}), & \text{corr}(C_t/Y_t, C_{t-2}/Y_{t-2}) \end{array} \right\}.$$

ratio correlation coefficients, and a measure of persistence (first and second-order autocorrelation coefficients for every variable).

In particular, using SMM, I restricted the estimation to a set of moments involving three variables, namely output, consumption, and investment. In practice, in order to be able to estimate the vector of sixteen parameters in Table 3.2, I estimated the model using a set of eighteen moments m_T/m_N , showed in Table 3.3. The parameters ρ_G and σ_{η_t} , as well as the mean of the process are calibrated such that to match the time series data from NIPA table for the U.S. in terms of government expenditure.

3.5 Results

Based on initial values of parameters, on range and formulation of states variables and on the simulation procedure previously mentioned, I reported here the parameter estimates and the goodness-of-fit measures from the SMM method. Notice in Table 3.4 that the parameters yielded by the estimation are very close to what it would have been obtained via conventional calibration procedure.

As it can be seen, all general estimates are in line with the findings of the previous chapter. The only difference is in the discount factor which in this model is registering a lower value with respect to the one displayed by the second model in Chapter 2. However, we need to compare the right elements: model 2 in the previous chapter embedded a stochastic discount factor. In the case of the present model, the correspondent discount factor should be calculated

Table 3.4: Parameter Estimates

γ	β	ρ_δ	ρ_θ	$\bar{\delta}$	$\bar{\theta}$	σ_{φ_t}	$\sigma_{\epsilon_t, \varphi_t}$
2.152	0.865	0.960	0.907	1.020	1.004	0.032	0.015
σ_{ϵ_t}	α	ρ	σ_{ϵ_t}	λ_0	λ_1	λ_2	λ_3
0.040	0.300	0.954	0.040	0.488	0.107	-0.204	0.108

by multiplying the actual discount factor with the stochastic preference processes for old and young. These two 'aggregated' discount factors give relative measures that can be compared to the discount factor in the previous chapter; by performing some simple comparative analysis, it can be seen that actually, although the exact variation interval is not maintained, the scale and trend displayed by the two models' discount factor are the same.

The dynamics of all variables seems to be well reproduced. The lead and lag correlations are positive and close to the real historical paths. These results come both from the chosen formulation and from the informational structure of the model. Since information on shocks values is imperfect at the current period, and since it is expected that the technical progress affect future technology, it increases agents anticipated capital. Consequently, consumption increases. On the other hand, output will also increase given the higher capital. Depending on the ratio between the growth in output and the growth in consumption, a corresponding path will be registered by investments.

One remark. The information is imperfect in the sense that the current society does not know what will be the preferences towards the young and old generations in the next period, which will be governed by the next society, but it values the present and future young and old, under the same preference scheme. In other words, it values the whole lifetime utility of the young (which will subsequently become old after the age of 64) according to the same stochastic preference process, with the only difference given by the age-specific utility weights. The same holds for the old.

Results on moments simulation and tests of goodness of fit are reported in Table 3.5.

Analyzing this table, it is obvious that all the eighteen moments are close to their empirical counterparts showed by the true data. The SMM allows to test if all the restrictions used to estimate the structural parameters are verified. Indeed, the overidentifying χ^2 -test shows that

Table 3.5: Estimated Moments and Goodness of Fit Test

<i>Moments</i>	<i>Artificial</i>	<i>Empirical</i>
$\sigma_{\ln(Y_t)}$	0.544	0.535
$\sigma_{\ln(C_t)}$	0.662	0.571
$\sigma_{\ln(X_t)}$	0.575	0.612
$\sigma_{\ln(C_t/Y_t)}$	0.144	0.040
$\text{corr}(Y_t, C_t)$	0.974	0.999
$\text{corr}(Y_t, X_t)$	0.926	0.986
$\text{corr}(Y_t, C_t/Y_t)$	0.975	0.925
$\text{corr}(C_t, X_t)$	0.940	0.980
$\text{corr}(C_t, C_t/Y_t)$	0.955	0.932
$\text{corr}(X_t, C_t/Y_t)$	0.742	0.877
$\text{corr}(Y_t, Y_{t-1})$	0.996	0.999
$\text{corr}(Y_t, Y_{t-2})$	0.993	0.998
$\text{corr}(C_t, C_{t-1})$	0.937	0.999
$\text{corr}(C_t, C_{t-2})$	0.945	0.998
$\text{corr}(X_t, X_{t-1})$	0.987	0.989
$\text{corr}(X_t, X_{t-2})$	0.943	0.975
$\text{corr}(C_t/Y_t, C_{t-1}/Y_{t-1})$	0.965	0.947
$\text{corr}(C_t/Y_t, C_{t-2}/Y_{t-2})$	0.949	0.901
J_T	0.058	<i>na</i>
$\chi^2(2)$	3.19	<i>na</i>
<i>p</i> - value	0.104	<i>na</i>

model cannot be rejected by the data at standard significance levels.

The core and the final consideration of this work are both sustained by the results. Figure 3-1 and 3-2 below plot the age-specific utility weights structure by age groups and the two stochastic preference processes.

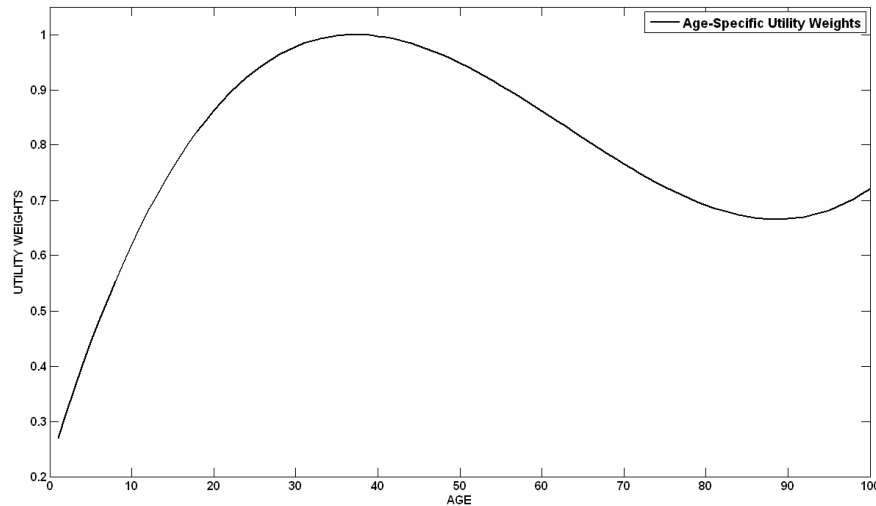


Figure 3-1: Age-Specific Utility Weights by Age Group

The idea is that, in time, societies preferences evolved towards the old categories of population, in the detriment of the young generations (see Figure 3-1) and also in the detriment of the future ones. In other words, it means that from 1950 to present more importance was given to present with respect to future and that the preference structure moved its weight toward the old categories of individuals, which confirms the findings of Chapter 2.

As it can be seen from Figure 3-2, the long-term trend of social aggregated age-preference rates in both cases (old and young) is declining. This general path combined with a faster rate of decrease registered for young leads in the end to the situation in which societies preferences shifted towards the old categories of population. Indeed, the preference stochastic process for the elderly register a slowly decreasing path, while young age groups fall sharply and future unborn remain at the lowest level.

Finally consider a comparison between the second model in Chapter 2 and this model: notice that by aggregating the discount factor to the two preference processes in the present

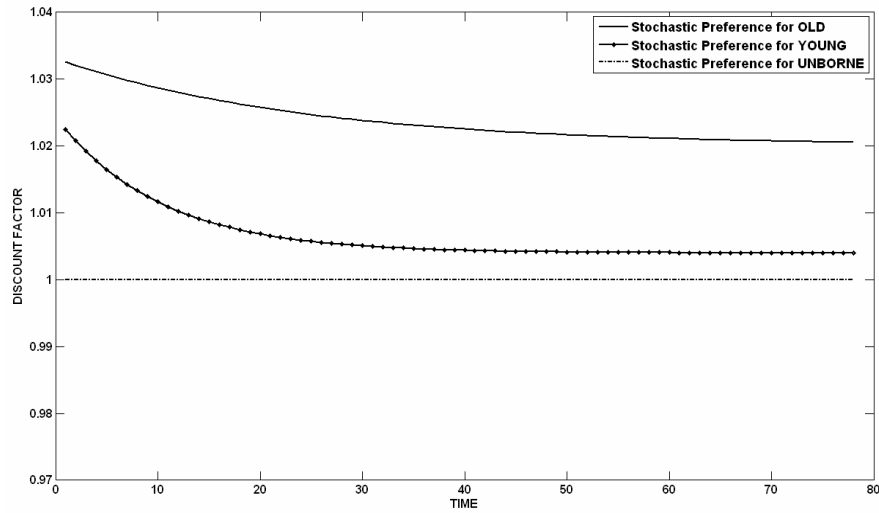


Figure 3-2: Stochastic Preference Paths in Time.

model, we are able to get an approximation at the limit of the discount factor estimated in the inconsistency model of the previous chapter. Consequently, the general decreasing path registered by both preference processes, young and old, multiplied by the discount factor of 0.865 is actually in line with the findings in Chapter 2, which stated that in time, progressively less importance was assigned to the future as registered by the declining path of the discount factor (from 0.983 to 0.977).

3.6 Conclusions

The focus of this chapter was on re-confirming the idea that, in time, successive societies translated their preference structure for alive and unborn generations progressively towards the old generations. The procedure chosen in order to illustrate this idea was estimating the two stochastic processes that described the structure of the preferences that a society has towards the young and old age groups. The main result confirmed the theory that in time, societies changed their preference structure towards assigning progressively more importance to the old age groups with respect to the young or future ones.

The idea is also confirmed by the path of the age-specific utility weights that displays a bell-

shape, that rises sharper for the first half of the population age structure than it decreases in the second half. This shape is also re-validating the basic fact of this chapter, that in time, to young people it was assign less and less weight while the emphasize increase on the old categories of individuals.

The work showed that representing the different preference structures of societies for the different groups of population through correlated stochastic processes can generate a good fit of the model with the true time-series of economic indicators. Consequently, the model predicted a positive correlation between the two processes that generated the preferences for young and old, confirming the expectations. The estimation was improved by simultaneously considering the traditional technological shock as well as a shock in government expenditure. More than that, it was registered a considerable variability in both consumption and output, as data confirm.

One last question may arise on whether the tests for accepting or rejecting the model have any reliability, given that I was considering only 55 data points. The answer can be provided by arguing that the work actually concentrated on eighteen moments. One other aspect is that I was estimating the parameters simultaneously. However, future extensions are considering a dataset with a larger number of observations, based on quarterly or perhaps monthly data.

Furthermore, some sources of shock that I do not consider in the present work are linked to variables with international character. One further extension that I intend to approach in future work would be to consider an open economy model.

Bibliography

- [1] Ameriks, J., Caplin, A., Laufer, S., Van Nieuwerburgh, S., 2005, Strategic Surveys and the Bequest Motive, working paper
- [2] Ando, A., Modigliani, F., 1963, The Life Cycle Hypothesis of Savings: Aggregate Implications and Tests, *American Economic Review* 53: 55-84
- [3] Andrews, D.,(1991), *Heteroskedasticity and autocorrelation consistent covariance matrix estimation*, *Econometrica*, 59 (3), 817-858
- [4] Bajtelsmit, V. L., Bernasek, A., 2001, Risk Preferences and the Investment Decisions of Older Americans, AARP Working Paper 2001-11
- [5] Bernheim, B. D., Shleifer, A., Summers, L. H., 1985, The strategic bequest motive, *Journal of Political Economy* 93, 1045-1076
- [6] Brown, J. R., Finkelstein, A., 2004, The Interaction of Public and Private Insurance, Medicaid and the Long Term Care Insurance Market, Working Paper, Downloadable from Amy Finkelstein's homepage, National Bureau of Economic Research, <http://www.nber.org/~afinkels/>
- [7] Bjornerud, S., Oliveira Martins, J., 2005, Disentangling demographic and nondemographic drivers of health spending: a possible methodology and data requirements, Economics Department, OECD, Joint EC/OECD Workshop February 2005
- [8] Cagetti, M., 2003, Wealth accumulation over the life cycle and precautionary savings, *Journal of Business and Economic Statistics*, 21(3):339–353

- [9] Chiuri, M. C., Jappelli, T., 2006, Do the elderly reduce housing equity? An international comparison, CSEF Working Paper No. 158
- [10] Cooley, T. F., Prescott, E., (1995), *Economic Growth and Business Cycle*, in T. Cooley, ed., Frontiers of business cycle research, Princeton University Press, Princeton, NJ, pp. 1-38
- [11] Declercq, A., Van Audenhove, C., 2004, National Background report for Belgium, Services for supporting family carers of elderly people in Europe: Characteristics, Coverage and Usage - EUROFAMCARE, LUCAS-KU Leuven
- [12] Dynan, K., Skinner, J., Zeldes, S. P., 2002, The Importance of Bequests and Life-Cycle Saving in Capital Accumulation: A New Answer, AER Papers and Proceedings, 274-278
- [13] Eckstein, Z., Eichenbaum, M., Peled, D., 1985, Uncertain Lifetimes and the Welfare Enhancing Properties of Annuity Markets and Social Security, Journal of Public Economics, vol. 26, issue 3, pg. 303-326
- [14] Fehr, H., Jokisch, S., Kotlikoff, L.J., (2007), *Will China Eat Our Lunch or Take Us to Dinner? – Simulating the Transition Paths of the U.S., EU, Japan, and China*, Boston University Working Paper
- [15] Fischer, S., (1980), *Dynamic inconsistency, cooperation and the benevolent dissembling government*, Journal of Economic Dynamics and Control, 2:93-107
- [16] French, E., Jones, J. B., 2004, The effects of health insurance and self-insurance on retirement behavior. Center for Retirement Research Working Paper WP 2004-12
- [17] Gourinchas, P. O., Parker, J. A., 2002, Consumption over the life cycle. Econometrica, 70(1):47–89
- [18] Hubbard, R.G., Skinner, J., Zeldes, S.P., 1994, “The importance of precautionary motives in explaining individual and aggregate saving, ” Carnegie-Rochester Conference Series on Public Policy, 40, 59-125
- [19] Hubbard, R.G., Skinner, J., Zeldes, S.P., 1995, “Precautionary Saving and Social Insurance, ” Journal of Political Economy, 1995, vol. 103, no.2

- [20] Green, J., Kotlikoff, L.J., (2006), *On the General Relativity of Fiscal Language*, <http://people.bu.edu/kotlikoff/General%20Relativity%206-10-061.pdf>
- [21] Gunnar J., Klein, P., 1996, Stochastic fiscal policy and the Swedish business cycle, *Journal of Monetary Economics* 38 245-268
- [22] Hansen, G., (1985), *Indivisible labour and the business cycle*, *Journal of Monetary Economics*, 16, 309-328
- [23] Hansen, G., Prescott, E., (1995), *Recursive methods for computing equilibria of business cycle models*, in T. Cooley, ed., *Frontiers of business cycle research*, Princeton University Press, Princeton, NJ
- [24] Heston, A., Summers, R., Aten, B., Penn World Table Version 6.2, Center for International Comparisons of Production, Income and Prices at the University of Pennsylvania, September 2006
- [25] Jonsson, G., Klein, P., (1996), *Stochastic fiscal policy and the Swedish business cycle*, *Journal of Monetary Economics*, vol. 38, pp. 245-268
- [26] Kotlikoff, L. J., Summers, L. H., 1981, The Role of Intergenerational Transfers in Aggregate Capital Accumulation, *Journal of Political Economy*, August 1981, 59:4, 706-732
- [27] Kotlikoff, L. J., Spivak, A., 1981, The Family as an Incomplete Annuities Market, *Journal of Political Economy*, April 1981, 89.2, 372-91
- [28] Kotlikoff, L. J., Shoven, J., Spivak, A., 1987, Annuity Market's Savings, and the Capital Stock, in Rodie, Z., Shoven, J. B. and Wise, D. A. eds., *Issues in Pension Economics*, Chicago: University of Chicago Press, 21 1-254
- [29] Kotlikoff, L. J., Shoven, J., Spivak, A., 1986, The Impact of Annuity Insurance on Savings and Inequality, *Journal of Labor Economics*, July 1986, 4: 3, part 2
- [30] Lee, B., Ingram, B., 1991, Simulation estimation of time-series models, *Journal of Econometrics* 47, 197-205

- [31] Modigliani, F., 1986, Life Cycle, Individual Thrift, and the Wealth of Nations, *American Economic Review*, 76: 297-313
- [32] Morin, R.A., Suarez A.F., 1983, Risk Aversion Revisited, *Journal of Finance*, Vol. 38, No. 4, pp. 1201-1216
- [33] Newey, W.K., West, K.D. (1994), *Automatic Lag Selection in Covariance Matrix Estimation*, *Review of Economic Studies*, Blackwell Publishing, vol. 61(4), pages 631-53
- [34] Newey, W.K., West, K.D., (1987), *A Simple, Positive Semi-definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix*, *Econometrica*, Econometric Society, vol. 55(3), pages 703-08
- [35] OECD in Figures 2006-2007, Demography and health - Health spending and resources, OECD © 2006 - ISBN 9264022635
- [36] OECD, 2006, Projecting OECD health and long-term expenditures: what are the main drivers?, OECD Economics Department Working Papers No. 477
- [37] Paccagnella, O., Rebba, V., Weber, G., 2006, Voluntary private health care insurance among the over fifties in Europe: a comparative analysis of SHARE data, mimeo University of Padua, Department of Economics
- [38] Palumbo, M.G., 1999, Uncertain Medical Expenses and Precautionary Saving Near the End of the Life Cycle, *Review of Economic Studies*, 66, 395-421
- [39] Reher, D.S., 1998, Family ties in western Europe: persistent contrasts, *Population and Development Review*. 24(2), pp. 203 - 234
- [40] United Nations, Department of Economic and Social Affairs, Population Division (2007). *World Population Prospects: The 2006 Revision. CD-ROM Edition - Extended Dataset* (United Nations publications)
- [41] Venti, S. F., Wise, D. A., 2004, Aging and housing equity: Another look, in D. A. Wise (Ed.), *Perspectives on the Economics of Aging*, pp. 127-175, Chicago: The University of Chicago Press

- [42] Wang, H., Hanna, S., 1997, Does Risk Tolerance Decrease With Age?, Association for Financial Counseling and Planning Education, Vol. 8, No. 2

Appendix A: Gauss-Hermite quadrature approach to discretization

This section clarifies the concept used in order to be able to discretize the shocks, in terms of alternative values for the states and gives some details on the construction of the method to discretize uncorrelated shocks.

Taking as base the processes adequate to implement the Gaussian quadrature, Tauchen and Hussey [1991] provided a simple way to discretize stochastic processes. Given the formulation for a stochastic process:

$$s_{t+1} = (1 - \rho)\bar{s} + \rho s_t + \varepsilon_{t+1},$$

where $\varepsilon_{t+1} \sim N(0, \sigma^2)$, it implies that:

$$\frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp \left\{ -\frac{1}{2} \left(\frac{s_{t+1} - (1 - \rho)\bar{s} - \rho s_t}{\sigma} \right)^2 \right\} ds_{t+1} = \int \mathbf{f}(s_{t+1} | \mathbf{s}_t) d\mathbf{s}_{t+1} = 1,$$

or, in Tauchen and Hussey [1991] terms:

$$\int \Phi(s_{t+1}; s_t, \bar{s}) f(s_{t+1} | \bar{s}) ds_{t+1} \equiv 1,$$

where $f(s_{t+1} | \bar{s})$ defines the density of s_{t+1} given that $s_t = \bar{s}$. This expression yields that

$$\Phi(s_{t+1}; s_t, \bar{s}) \equiv \frac{f(s_{t+1}; s_t)}{f(s_{t+1} | \bar{s})} = \exp \left\{ -\frac{1}{2} \left[\left(\frac{s_{t+1} - (1 - \rho)\bar{s} - \rho s_t}{\sigma} \right)^2 - \left(\frac{s_{t+1} - \bar{s}}{\sigma} \right)^2 \right] \right\},$$

and by noting $z_t = \frac{(s_t - \bar{s})}{\sigma\sqrt{2}}$ I obtain

$$\frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \exp \left\{ -(z_{t+1} - \rho z_t)^2 - z_{t+1}^2 \right\} \exp(-z_{t+1}^2) dz_{t+1},$$

to which it can be applied Gauss - Hermite quadrature. If z_i are the quadrature nodes, and ω_i , $i = 1, \dots, n$ the weights, I get that

$$\frac{1}{\sqrt{\pi}} \sum_{j=1}^n \omega_j \Phi(z_j; z_i, \bar{x}) \simeq 1,$$

with $\omega_j \Phi(z_j; z_i, \bar{x})$ "estimating"

$$\hat{\pi}_{ij} = Prob(s_{t+1} = s_j | s_t = s_i),$$

the transition probability from state i to state j .

Since using Gaussian quadrature is just a way of approximating the continuous distribution of a stochastic process by its discrete counterpart, it will usually be the case that $\sum_{j=1}^n \hat{\pi}_{ij} = 1$ will not hold exactly.

The solution of Tauchen and Hussey was defining the following expression for achieving the discretization of a continuous stochastic process:

$$\hat{\pi}_{ij} = \frac{\omega_j \Phi(z_j; z_i, \bar{x})}{\sum_{j=1}^n \omega_j \Phi(z_j; z_i, \bar{x})}.$$

For the cases of uncorrelated shocks, we will use the same technique for discretization as above in order to be able to construct different states and their transition probabilities, with the only difference on the parameters of the respective distributions. For the health and health spending shocks considered in Chapter 1, I considered 4 different states for each stochastic process. In Chapter 2, for the stochastic process of the discount factor I considered the scenario with 10 different states, while both in Chapter 2 and 3, for the technological shock and for government expenditure shock, I considered 4 states and 3 states respectively. For the society's preference process for young and for old in Chapter 3, I considered again 4 possible stochastic states.

Moreover, for the particular case of the correlated shocks in Chapter 3, I used the same technique as in the case of the independent shocks in order to construct the different states and their transition probabilities, with the only difference that I determined the states not just function of the variance across the computed nodes of the Gaussian quadrature of the process but also as function of the covariance with respect to the nodes of the process with which there exists the correlation.

Appendix B: Moment Conditions

The following summary is based on the work of Jonsson and Klein [1996] that presents and extends the work of McFadden [1989] and Lee and Ingram [1991]. The idea behind SMM is very similar as a concept to the one of GMM. Suppose there is a model which is fully specified except for an unknown parameter vector $\beta \in R^k$, and which can be used to simulate K independent l -dimensional sequences $\left(y_{t,i}(\tilde{\beta})\right)_{t=1}^N$, $i = 1, 2, \dots, K$, and have an observed l -dimensional sequence $(x_t)_{t=1}^T$. Suppose $n = NK/T > 1$ ⁶. Without loss of generality, let the mean of these series be 0. I now want to (i) estimate the model's parameters, and (ii) test the model.

I begin by specifying a 'raw' moment function $v : R^l \rightarrow R^{l(l+1)/2}$ defined by

$$v(x_t) = \text{vech}(x_t x_t').$$

Thus, $v(x_t)$ is just the vector of all possible products between the elements of x_t arranged in suitable order. Define

$$v_T = \frac{1}{T} \sum_{t=1}^T v(x_t),$$

and

$$v_{N,i}(\tilde{\beta}) = \frac{1}{N} \sum_{t=1}^N v(y_{t,i}(\tilde{\beta})).$$

If I wanted only variances and covariances as moments, the above would suffice. But since it is easier to interpret standard deviations, relative standard deviations and correlation coefficients, I go one step further and define a function

$$m : R^{l(l+1)/2} \rightarrow R^j.$$

Since sample standard deviations and correlation coefficients are functions of the raw moments in v_T and $v_{N,i}(\tilde{\beta})$, this is all I need. Finally, define

⁶The reason for simulating K independent sequences of length N rather than a single sequence of length NK is the gain in efficiency. This gain is large because of the high degree of persistence in our simulated series.

$$m_T = m(v_T),$$

and

$$m_{N,i}(\tilde{\beta}) = m(v_{N,i}(\tilde{\beta})),$$

with

$$m_N(\tilde{\beta}) = \frac{1}{K} \sum_{t=1}^K m_{N,i}(\tilde{\beta}).$$

If the model is true, then, for some $\beta \in R^k$, I should presumably have

$$p \lim_{T \rightarrow \infty} [m_T] = p \lim_{N, K \rightarrow \infty} [m_N(\tilde{\beta})] = \mathbf{m},$$

where $\mathbf{m} \in R^j$ is a nonrandom vector of population moments. This motivates the following estimator of $\tilde{\beta}$ (the SMM estimator):

$$\hat{\beta} = \arg \min_{\tilde{\beta}} [m_T - m_N(\tilde{\beta})]' \widehat{W} [m_T - m_N(\tilde{\beta})],$$

where \widehat{W} is some, possibly random, positive definite matrix such that $p \lim \widehat{W} = W$, where W is nonrandom. Then, under various technical assumptions, $\hat{\beta}$ is consistent and asymptotically normal, with

$$\sqrt{T} (\hat{\beta} - \tilde{\beta}) \rightarrow N(0, Q_n(W)),$$

where $Q_n(W) = (1 + \frac{1}{n}) \left[E_0 \frac{\partial m(v_T)'}{\partial v_T} W_T^{-1} \frac{\partial m(v_T)}{\partial v_T'} \right]^{-1} E_0 \frac{\partial m(v_T)'}{\partial v_T} W_T^{-1} S(\tilde{\beta}) W_T^{-1} \frac{\partial m(v_T)}{\partial v_T'}$
 $\times \left[E_0 \frac{\partial m(v_T)'}{\partial v_T} W_T^{-1} \frac{\partial m(v_T)}{\partial v_T'} \right]^{-1}$ and $S(\tilde{\beta})$ is the covariance matrix of $\frac{1}{\sqrt{T}} \left(\frac{1}{T} \sum_{t=1}^T [m_T - m_N(\tilde{\beta})] \right)$.

The optimal choice of W is the inverse of the asymptotic variance matrix of $[m_T - m_N(\tilde{\beta})]$,
i.e. $\widehat{W}_T = \widehat{S}_T$,

$$W = \left[\frac{\partial m(v_T)}{\partial v_T'} \left(1 + \frac{1}{n} \right) S \frac{\partial m(v_T)}{\partial v_T} \right]^{-1},$$

where S is the asymptotic variance matrix of v_T . A suitable estimator of S is given by

Newey and West [1987] as

$$\widehat{S} = \widehat{R}_0 + \sum_{i=1}^p \left(1 - \frac{i}{p+1}\right) (\widehat{R}_i - \widehat{R}'_i),$$

where p is some suitable integer-valued strictly increasing function of T and

$$\widehat{R}_i = \frac{1}{T} \sum_{t=i+1}^T (v(x_t) - v_T)(v(x_{t-i}) - v_T)'$$

To test the model, I use the fact that if the weighting matrix is chosen optimally, then, as $T \rightarrow \infty$ (keeping n fixed), I have

$$T[m_T - m_N(\widetilde{\beta})]' \widehat{W} [m_T - m_N(\widetilde{\beta})] \rightarrow \chi^2(j - k),$$

where j is the number of moments and k is the number of estimated parameters.

In each estimation, the N random sequences are fixed. As a reasonable compromise between speed and efficiency, I set $N = 36$ and $K = 100$ in Chapter 1 and $N = 172$ and $K = 20$ in Chapter 2 and 3. In practice, minimization of SMM estimator is done by a grid search where each parameter takes on different values. Note that the SMM requires a large number of simulations to compute the standard errors of the estimator, even if the estimator is consistent for a fixed number of simulations.

Appendix C: Numerical simulation, Chapter 1

As mentioned in the first chapter, for the simulation I used the prospective-retrospective method of the dynamic programming and, more precisely, I will use backward induction to compute value functions and policy functions. Given that the model lacks a closed form solution, these decision rules are found numerically. The optimization problem is solved by grid search, and the state-space for "wealth" is made discrete. In the last period, the decision is trivial, with the agent consuming and leaving bequest all residual available wealth. Here and throughout the chapter, I normalize utility after death at zero.

At time T , the individual do not formally insure for the next period, and so the issue is

choosing consumption

$$\begin{aligned} \underset{C_T, f_T}{Max} V_T(m_T, C_T, I_T) &= \left[\begin{array}{c} (1 + \delta m_T) \frac{C_T^{1-\gamma} - 1}{1-\gamma} + (1 - m_T) \\ \frac{[\alpha(\omega f_{T-1} + \bar{f})^\theta + (1-\alpha)(\eta_T(1-s_T)a_{T+1})^\theta]^{\frac{1-\sigma}{\theta}} - 1}{1-\sigma} \end{array} \right] \\ s.t. a_{T+1} &= a_T + y - C_T - (h_T - (\omega f_{T-1} + \bar{f}) - \eta_T(1-s_T)a_{T+1} + \sigma_{\varepsilon T} * \psi_T), \\ &\text{under the condition } s_T = 0 \text{ and } f_T = 0. \end{aligned}$$

Once the policy function is solved, I can obtain the corresponding value function and policy rule in the last period and that can be used in computing policy rules for the previous period. I find the decision rules at time $T-1$ by solving the previous equation with V_T and the first order conditions for consumption and formal insurance. This iteration is continued backward using Euler equations until $t=1$. For all the shocks, I evaluated the model using the Gauss-Hermite quadrature approach to discretization, and transform the continuous problem into a discrete one with the constraint that the asymptotic properties of the continuous and of the discrete processes should be the same⁷. In this case, as showed in the model, I used Markov chains to represent each of the stochastic processes.

The value function is directly computed at a finite number of points within the wealth grid, $\{A_T\}_{ia=1}^{40}$, the consumption grid $\{C_T\}_{ic=1}^{1000}$, and within the formal insurance grid, $\{F_{T-1}\}_{f=1}^5$. On the other hand, at time $t+1$, medical expenses (partially) and health status (totally) will be random variables. To capture uncertainty over the stochastic components of medical expenses and health status I convert m_t and ψ_t into discrete Markov chains, and calculate the conditional expectation of V_{t+1} accordingly. I integrate the value function with respect to the stochastic component of medical expenses, ψ_t , using 4-node Gauss-Hermite quadrature, while for health status m_t I will use also 4 as the number of nodes. In order to be able to find the solution, our approach is to discretize the consumption and formal insurance decision space and to search over this grid. Experiments with the fineness of the grids suggested that the grids I used (with 40 points for wealth and 5 points for the health insurance) gave reasonable approximations. In particular, I increased the number of grid points until the stage at which a further increase seemed to have a small effect on the results. Based on these grids, I use the decision rules to

⁷Please refer to Appendix A for further details.

generate simulated time series.

Appendix D: Data, Chapter 1

The data are drawn from SHARE, a cross-national database on the microeconomics level, regarding health, socioeconomic status and social and family networks of individuals aged 50 or over living in the representative regions of Europe: Scandinavia (Denmark and Sweden), Central Europe (Austria, France, Germany, Switzerland, Belgium, and the Netherlands) and the Mediterranean (Spain, Italy and Greece). Data are measured at the household level and refers to the first wave corresponding to year 2004 (the second data collection wave was conducted in 2006-2007).

The true variables chosen as benchmark for the simulated ones are referring to the amount of formal health insurance that individuals are contracting, their consumption of goods and services and finally to their comprehensive wealth, in which I also included pension earnings and benefits (since I am considering just the individuals aged 65 and over) as part of their income flow, financial assets, as well as the real assets.

For the formal health insurance that individuals acquired during the last year it was considered the amount spent on all voluntary and supplementary health insurance contracts. The rationale behind this choice is based on the fact that the compulsory insurance in Europe is usually covered partially by the government, and constitute its public spending related to health care, and partially by the employers/employees through contributions to the health system. Consequently, the amount of formal health insurance that individuals actually buy is identified by the supplementary, voluntary formal insurance contracts that they acquire. In order to eliminate the missing values in the formal insurance variable for those individuals that have reported wealth and consumption, I predicted formal insurance using a linear model that related the interest variable to the non-durable expenditures, wealth and individual observable characteristic (age, proxy for health status, number of children).

The value of total consumption of goods and services was obtained by aggregating the monthly data on consumption on all goods and services at annual level, while the latter was imputed using the amount spent on food at home, food outside home and telephone bills,

each weighted with certain coefficients. The coefficients used to obtain the monthly non-durable expenditure of an individual were obtained using an OLS procedure on the same variables but from additional datasets, that are specific to the countries that I analyzed. For Italy, Spain and Greece, the dataset used to obtain the weights of the different components in the total non-durable expenditure was ISTAT, while for all the other countries I used the Dutch Consumption Dataset.

The pension earnings and benefits were considered to their extensive definition, meaning I considered all the types of pensions and associated benefits: old age/early retirement/pre-retirement pension, public disability/unemployment /survivor/invalidity or incapacity/war pension and private (occupational) old age/early retirement/disability or invalidity/survivor pension.

The other categories of income flows and financial assets include the level and the interest on the amount in bank accounts, government and corporate bonds, amount in stocks and correspondent dividends, amount in mutual funds and correspondent interest or dividend, amount in individual retirement accounts and contractual savings and face value of life policies, net of total amount of money owed to other parties. One last item that it was included in the category of income flows is the amount of income from renting other real estates owned, while the market values of the main property and of other real estates and car(s) (if owned any), net of mortgage on main residence were considered to represent the real assets that individual possesses. I do not include bequests in the wealth measure due to the fact that, the age for the individuals considered in the analysis being 65 and over, very few of them actually receive any bequests.

One last remark on the data. The levels of minimum formal insurance were considered to be approximated by the public expenditure with health per capita in all the countries analyzed. Data were provided by the OECD database⁸.

⁸OECD in Figures 2006-2007, Demography and health - Health spending and resources.

Appendix E: The Health Cost Model, Chapter 1

The issue of medical costs is central to the analysis presented in Chapter 1, especially since the aim is to account properly also for the possibility of high costs associated with long-term care and invalidity, case in which the informal arrangement is highly important. The distribution of these costs is controlled by the medical spending associated to each health state and by the one-period 4×4 health state transition matrix $P(t)$.

The transition matrix for health status is parameterized by twelve elements, nine probabilities that determine the value of $P(1)$ (of the sixteen elements, four are fixed by the death state being un-reversible and there are three further restrictions so that each row sums to one) and three parameters that control the row of probability from greater health to poorer health as t increases. I selected values for these parameters to match the values computed by Ameriks et al. [2005] as starting points and then I estimated them through SMM.

Some remarks on the construction of the health expenditure function. First of all, let us explain the significance of the terms that I will attach to each health status. The curative and rehabilitation expenditures comprises medical and paramedical services delivered during an episode of curative and/or rehabilitative care. An episode of curative care is one in which the principal medical intent is to relieve symptoms of illness or injury, to reduce the severity of an illness or injury or to protect against exacerbation and/or complication of an illness and/or injury which could threaten life or normal function. Rehabilitative care comprises services where the emphasis lies on improving the functional levels of the persons served and where the functional limitations are either due to a recent event of illness or injury or of a recurrent nature (regression or progression). Included are services delivered to persons where the onset of disease or impairment to be treated occurred further in the past or has not been subject to prior rehabilitation services. (Note: This item corresponds to HC.1+HC.2 in the ICHA-HC classification of health care functions.). These expenditures will be adequate to be considered in the case that the fair health status (3) is verified.

On the other hand, long-term health care comprises ongoing health and nursing care given to in-patients who need assistance on a continued basis due to chronic impairments and a reduced degree of independence and activities of daily living. In-patient long-term care is provided in institutions or community facilities. Long-term care is typically a mix of medical

(including nursing care) and social services. Only the former is recorded in the SHA under health expenditure. (Note: This item corresponds to HC.3 in the ICHA-HC classification of health care functions.). These expenditures will be adequate to be considered in the case that the poor health status (2) is verified.

In the spirit of the Ameriks et al. [2005] paper, I consider the OECD Health Data October 2006 Statistics reports in each country, namely 2004 average medical expenses for non-institutionalized individuals and for assisted ones. According to their study, I find that among the periods our simulated retirees spend out of invalidity and death status (health states 3 and 4), a certain average amount specific to each country, in state 3 (fair health) so that $h(m_t(4)) = 0$ and $h(m_t(3)) > h(m_t(4))$ will reproduce these averages.

For the invalidity state, I use Brown and Finkelstein's approach that consider the cost of long term care facility. This leaves an annual expense for a full year of long term care at a higher amount than the costs of fair health. Consequently, I take $h(m_t(2)) > h(m_t(3))$. I also consider the costs associated with death to be the highest ones, after the annual costs of long-term care, according to the formula used in the OECD calculations, and set $h(m_t(1)) < h(m_t(2))$. For each and every country analyzed, I will first determine these costs based on the OECD Health Data October 2006 Statistics reports, and further use these information in simulating the data.

In practice, the primary data for the OECD countries analyzed are drawn from the AGIR data set (Westerhout and Pelikaan, 2005, based on EPC, 2001) for EU-15 countries, and OECD calculations.

The cost of death for the oldest group (95+) is assumed to be the lowest and was proxied by their observed health expenditure per person when available. For France, Germany, Italy, Spain, and Netherlands for which the expenditure for the oldest group were not available, the cost of people aged 75-79 was taken as a proxy. In fact, when available, expenditure at age 95+ is roughly equal to the level of expenditure at age 75-79. For the countries where no cost expenditures were available, the cost of death for the oldest group was estimated by taking 3 times the average health expenditure per capita, adjusted by the country-specific residual. The total long-term care expenditure in percentage of GDP in 2005 was calibrated to fit the estimates of the OECD Long-term Care study (OECD, 2005b), when available. Data for the countries not available in this study were obtained by applying the ratios of long-term care to

Table 3.6: Benchmark countries in OECD studies

Country estimated	Benchmark countries
Belgium	Netherlands
Denmark	average (Norway, Sweden)
France	Germany
Greece	Spain
Italy	average (Germany, Spain)
Switzerland	Germany

GDP observed in ‘similar’ benchmark countries, as indicated in Table 3.6.