Ph.D. thesis

The Goldstone Boson Higgs and
the effective Lagrangian(s)

Candidate:

Kirill Kanshin

Universita' degli Studi di Padova

Dipartimento di Fisica e Astronomia "G. Galilei"

Scuola di Dottorato di Ricerca in Fisica

Via Marzolo 8, I-35131 Padova, Italy

Ciclo XXVIII

Advisor: Prof. Stefano Rigolin

Director of Ph.D. school: Prof. Andrea Vitturi

January 2017

Abstract

The Goldstone boson nature of the observed Higgs scalar particle represents a tempting possible solution for the Standard Model hierarchy problem. We first discuss the essence of the problem in the context of low energy QCD and the Higgs sector of the Standard Model. As a step towards the solution we construct a UV complete model of the Goldstone Higgs based on the global $SO(5)/SO(4)$ symmetry breaking. The scalar sector of the theory is a linear sigma model extended by a scalar singlet σ, with mass $m_\sigma > 500$ GeV. In order to give mass to the SM fermions through the partial compositeness mechanism, the fermion sector is extended by heavy vectorlike fermions. We study in detail the possible direct detection of σ and the impact of the new scalar and fermion states on Electroweak Precision Tests. We conclude in particular that any reasonable contribution of the scalar sector can in principle be compensated by a fermionic one. At low energies any extension of the Standard Model results in a set of effective operators, describing the deviations of the couplings from their predicted values. Depending on how the electroweak symmetry is realised, two intrinsically different effective descriptions are possible: linear and non-linear one. Varying the σ mass allows to sweep from the regime of the perturbative linear UV completion to the non-linear one. The latter one is typically assumed in models in which the Higgs particle is a low-energy remnant of some strong dynamics at a higher scale. In the limit of large but finite masses of the new states we derive the benchmark non-linear effective Lagrangian. Furthermore the first order linear corrections originating from large, but finite mass of the additional scalar to the Higgs couplings have been derived and they are found to be suppressed by the scalar masses ratio. Finally, we consider the renormalization of the custodial preserving scalar sector of the non-linear effective Lagrangian in a general Goldstone bosons matrix parametrisation and identify the physical counterterms as well as the chiral-noninvariant divergences. The latter ones are shown to be unphysical as they can be removed by a field redefinition. The procedure allows to check the consistency of the non-linear effective Lagrangian at one loop. The results confirm the completeness of the scalar sector of NLO Lagrangian previously identified in the literature.

Abstract

Una particella di Higgs la cui natura sia di tipo Goldstone rappresenta una possibile soluzione al problema della gerarchia nel Modello Standard. Dapprima discutiamo l'essenza del problema nel contesto della QCD di bassa energia e del settore di Higgs del Modello Standard. Come passo successivo verso la soluzione, si costruisce un modello UV-completo del Goldstone Higgs, basato sulla rottura di simmetria globale $SO(5)/SO(4)$. Il settore scalare della teoria è un modello sigma lineare, esteso da un singoletto scalare σ, con massa $m_\sigma > 500$ GeV. Per dare massa ai fermioni del Modello Standard attraveso il meccanismo di compositezza parziale, il settore fermionico viene esteso da fermioni pesanti vectorlike. Si studia in dettaglio la possibile osservazione diretta di σ e l'impatto del nuovo scalare e dei nuovi stati fermionici sui test di precisione elettrodeboli. Si conclude, in particolare, che ogni ragionevole contributo dal settore scalare può , in linea di principio, essere compensato dal settore fermionico. A basse energie ogni estensione del Modello Standard risulta in un insieme di operatori effettivi che descrivono le deviazioni dei coupling dai loro valori predetti. A seconda di come la simmetria elettrodebole è realizzata, sono possibili due descrizioni effettive intrinsicamente differenti: lineare e non lineare. Variando la massa dell'σ passiamo con continuità dal regime in cui il completamento è perturbativo a quello non lineare. Quest'ultimo è in genere un presupposto di modelli nei quali la particella di Higgs è un residuo a bassa energia of dinamiche forti ad una scala superiore. Nel limite in cui le masse dei nuovi stati sono grandi, ma finite, deriviamo la Lagrangiana effettiva non lineare che può essere utilizzata come benchmark. Inoltre vengono derivate le correzioni lineari al primo ordine causate dalla massa – grande ma finita – degli scalari addizionali ai coupling dell'Higgs, dimostrando che sono soppresse di un fattore proporzionale al rapporto degli scalari della teoria. Infine, consideriamo la rinormalizzazione del settore scalare che preserva la simmetria custodial in una parametrizzazione matriciale dei bosoni di Goldstone e identifichiamo i controtermini fisici e le divergenze non-invarianti sotto trasformazioni chirali. Si dimostra che quest'ultime sono non fisiche, dal momento che possono essere rimosse da una ridefinizione dei campi. La procedura consente di controllare la consistenza della Lagrangiana effettiva non lineare a livello one-loop. I risultati confermano la completezza del settore scalare della Lagrangiana NLO precedentemente identificata in letteratura.

Contents

Preface

The electron is as inexhaustible as the atom, Nature is infinite, but it infinitely exists. And it is this sole categorical, this sole unconditional recognition of Nature's existence outside the mind and perception of man that distinguishes dialectical materialism from relativist agnosticism and idealism.

– V.I. Lenin, Materialism and empirio-criticism, 1908

As more than 100 years have passed since the times of Lenin many more elementary particles have been discovered with the famous Higgs boson being the last missing piece of the Standard Model (SM) – fundamental quantum field theory (QFT) of electromagnetic, weak and strong interactions, describing with unprecedented precision the wide range of phenomena observed. The discovery of the Higgs boson [1, 2] completes the struggle for experimental verification of this unifying description of the forces of Nature (all but gravity), yet it poses new theoretical challenges for the upcoming generations of physicists. In this sense, though we seem to understand the theory of the electron through Quantum Electrodynamics quite well, it is the physics of the Higgs boson what seems to be inexhaustible nowadays and so the pursuit for the knowledge of infinite Nature continues.

One of the main theoretical challenges motivating to look for BSM physics is the so called hierarchy problem. It can be stated as the failure of the SM to provide a theoretical explanation of the relatively light Higgs boson mass. From the QFT point of view the SM is a well defined, renormalisable theory. However, it can be thought as an effective field theory, correctly describing the fundamental interactions at the energies accessible by modern experiments, but has to be extended to include explanations of several theoretical puzzles, such as small neutrino masses, unobserved strong CP angle, dark matter and dark energy being some of them. In addition, it cannot be the ultimate theory since it does not include the description of gravity and is known to contain a Landau pole for the electromagnetic coupling at very high energies. If the SM is treated as a theory valid up to some cut-off scale Λ, any naive extension of the SM involving heavy states would introduce quantum corrections of order $\sim \Lambda^2$ to the Higgs mass parameter. From this point of view the observed value of the Higgs mass around the EW scale is unnaturally small and requires a screening mechanism, stabilizing the Higgs mass at its observed value. Typically this requires new heavy BSM states in the vicinity of the electroweak scale.

On the other hand, the persistent absence of evidence for new resonances calls for an in-depth exploration of BSM theories which may separate and isolate the Higgs mass from the putative

scale of exotic BSM resonances. Several solutions have been proposed over decades, such as supersymmetry, compact extra dimensions, the relaxion mechanism and composite Higgs. Here we will focus on the idea of the Higgs being a pseudo-Nambu-Goldstone boson (pNGB) of some global symmetry breaking, as was originally proposed decades ago in Refs. [3–8].

The first formulations of pNGB Higgs models were based on the assumption of a strong dynamics obeying an $SU(5)$ global group of symmetry broken down spontaneously to $SO(5)$. The Higgs was identified with one of the NGB states of this breakdown. Recent attempts tend to start from a global $SO(5)$ symmetry [9, 10] at a high scale Λ, spontaneously broken to $SO(4)$ and producing at this stage an ancestor of the Higgs particle. In [11, 12] the next-to-minimal coset $SO(6)/SO(5)$ with one more additional Goldstone boson (GB) been considered and studied as possible dark matter candidate in [13, 14].

In this thesis an explicit model of a pNGB Higgs based on $SO(5)/SO(4)$ coset will be presented. While most of the literature uses an effective non-linear formulation of the models [10, 15–21], we adopt the linear realization of the symmetry as suggested by [22]. See also [21] for the microscopic realisation of the linear scalar sector in terms of Nambu-Jona-Lasino four-fermion interactions. More recent developments based on the idea of the linear scalar sector can be found in [23–27]. The scalar sector of the theory is represented by a scalar multiplet in the fundamental representation of $SO(5)$ and contains the four SM scalar degrees of freedom plus an additional singlet σ. This will allow to gain intuition on the dependence on the ultraviolet (UV) completion scale of the model, by varying the σ mass: a light σ particle corresponds to a weakly coupled regime, while in the large mass limit the theory should fall back onto a usual effective non-linear construction. Our complete renormalizable model can thus be considered either as an ultimate model made out of elementary fields, or as a renormalizable version of a deeper dynamics, much as the linear σ-model [28] is to QCD.

The fermion sector of the theory is extended by heavy vectorlike fermions forming complete representations under the $SO(5)$ global group. The linear couplings between light and heavy fermions allow for a see-saw like mechanism for quarks, whose masses are inversely proportional to the heavy fermion mass scale. This construction is known in the literature as the partial compositeness mechanism [29]. Many choices of the heavy fermions representations are available in the literature, see Ref. [16] for a comparison between the possible options. The direction explored in this work employs heavy fermions in vectorial and singlet representations of $SO(5)$.

$SO(5)$ breaking couplings of the heavy fermions to the SM ones together with proto-Yukawa couplings between scalar multiplet and heavy fermions allow for the generation of the GB Higgs potential. Its minimum breaks the electroweak symmetry at scale v and gives mass to Higgs particle, massive gauge bosons and SM fermions.

Furthermore, we analyse the phenomenological implications of the modified scalar sector and the exotic fermions. In particular the contribution of the new sectors to the oblique S and T parameters will be computed. We also study the possible LHC reach for the direct detection of the σ scalar, set a bound $m_\sigma > 500$ GeV and analyse its possible relation with the diphoton anomaly observed in 2014.

Next, we consequently integrate out the heavy states and identify the leading order effective

operators. Firstly, as the heavy fermions removed from the spectrum , we obtain a linear effective Lagrangian composed of the Higgs, σ and SM fermions. Secondly, we integrate out the remaining scalar state, which results in the benchmark non-linear effective Lagrangian with the light Higgs, pointing out the leading contributions to the Higgs Effective Field Theory (HEFT) operators. In addition we compute first linear corrections to the deviations of the Higgs to gauge bosons and fermions (top and bottom) couplings and compare the results for different heavy fermions representations.

Finally, we perform a complete one-loop off-shell renormalization of the scalar sector of HEFT. While the previous literature has restricted the one-loop renormalization of this sector to on-shell analysis, the off-shell renormalization procedure guarantees that all the counterterms needed are identified. We have obtained the explicit expressions for the chiral noninvariant divergences obtained previously for the higgsless case and generalize it to the HEFT with the light Higgs. We also demonstrate that they vanish on-shell and have no impact on physical observables.

The structure of the thesis is as follows. Chapter 1 contains an introductory review of the topics covered in the following chapters. In Chapter 2 we construct the renormalizable theory of the pNGB Higgs. In Chapter 3 we continue the study of the previously introduced model, switching to the non-linear regime by integrating out the heavy scalar state. In Chapter 4 we study the one loop off-shell renormalization of the scalar sector of HEFT. We conclude the thesis with a Summary.

Chapters 2-4 are based on publications [30–32].

1

Introduction

1.1   The Standard Model

The Standard Model (SM) of particle physics [33–35] is a model within the framework of Quantum Field Theory (QFT), which has been proven to be a very precise description of the fundamental laws of Nature. The discovery of a Higgs(-like) boson at LHC [1,2], predicted in milestone papers [36–39] and awarded by a Nobel Prize in Physics in 2013, concludes the continuous effort in constructing the universal model describing the fundamental interactions of all known elementary particles and leaves us with a consistent framework allowing to reproduce the observed collider data.

The theory is based on the principles of gauge symmetry and renormalizability. In this section we will introduce the quantum states of the theory and the symmetries which those quantum states obey.

The postulated gauge symmetry imposed on the Lagrangian of the SM is

$$G_{SM} = SU(3)_c \ \times SU(2)_L \times U(1)_Y,$$

where $SU(3)_c$ is the color symmetry, governing strong interactions of quarks and gluons, while $SU(2)_L \times U(1)_Y$ is the electroweak (EW) symmetry group – direct product of the chiral $SU(2)$ group and the group of hypercharge, which is spontaneously broken at low energies and gives rise to weak and electromagnetic interactions. The gauge symmetry manifests itself in the invariance of the Lagrangian under local (space-time dependent) gauge transformations which is assured by the covariant derivatives of fields. Spin-1 gauge bosons form adjoint representations of the corresponding groups, while fermionic spin-1/2 fields are embedded into fundamental representations of the groups.

The SM is a chiral theory, meaning that left- and right-handed fermion fields have different transformation properties under the gauge group. This fact forbids the gauge non-invariant Dirac mass term for the SM fermions. The Higgs mechanism triggers the electroweak symmetry breaking (EWSB) and gives masses to quarks, charged leptons, W and Z bosons and the Higgs scalar itself.

The fermion sector consists of left-handed quarks q_L^i, transforming under the full G_{SM}, right-

7

handed quarks u^i and d^i transforming under $SU(3)_c \times U(1)_Y$, left-handed leptons l^i transforming under the electroweak symmetry group and right-handed leptons e^i charged only under $U(1)_Y$. Both quarks and leptons come in three copies or *generations*, index $i = 1, 2, 3$ indicating that. The charge assignment for the SM fields is summarized in Table 1.1.

	Fields	Representation
Gauge bosons	B_μ	(1, 1, 0)
	W^a_μ	(1, 3, 0)
	G^A_μ	(8, 1, 0)
Quarks	$q_L^i = \left(\begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix} \right)$	(3, 2, +1/6)
	$u^i = \begin{pmatrix} u, c, t \end{pmatrix}$	(3, 1, +2/3)
	$d^i = \begin{pmatrix} d, s, b \end{pmatrix}$	(3, 1, −1/3)
Leptons	$l_L^i = \left(\begin{pmatrix} \nu_L^e \\ e_L \end{pmatrix}, \begin{pmatrix} \nu_L^\mu \\ \mu_L \end{pmatrix}, \begin{pmatrix} \nu_L^\tau \\ \tau_L \end{pmatrix} \right)$	(1, 2, −1/2)
	$e^i = \begin{pmatrix} e, \mu, \tau \end{pmatrix}$	(1, 1, −1)
Higgs	H	(1, 2, +1/2)

Table 1.1: Representations of the SM field content. Numbers in the brackets denote the dimension of the representation of $SU(3)_c$, $SU(2)_L$ and charge under $U(1)_Y$ correspondingly.

The Lagrangian of the SM consists of the gauge, fermion and scalar sectors, and is written in terms of renormalizable operators up to dimension $d = 4$.

The gauge sector contains the invariant product of field strengths of the gauge bosons

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W_a^{\mu\nu} - \frac{1}{4} G^A_{\mu\nu} G_A^{\mu\nu}, \tag{1.1}$$

where the field strength is

$$V_A^{\mu\nu} = \partial^\mu V_A^\nu - \partial^\nu V_A^\mu - g f_{ABC} V_B^\mu V_C^\nu, \tag{1.2}$$

where g is the coupling constant and f_{ABC} are the group structure constants, defined by a commutator of generators Σ of the group

$$[\Sigma_A, \Sigma_B] = i f_{ABC} \Sigma_C.$$

The $SU(2)$ generators read $\Sigma^a = \tau^a/2$, where τ^a are Pauli matrices, and the $SU(3)$ generators are $\Sigma^A = \lambda^A/2$, with λ^A being Gell-Mann matrices. Sign conventions vary in literature, Ref. [40] considers all possible sign conventions at once.

The fermionic sector contains the kinetic term and the Yukawa term, describing the Gauge–fermion and Higgs–fermion interactions correspondingly

$$\mathcal{L}_f = \sum_{f=q,u,d} \bar{f} i D_\mu f - \left((q_L^i H) \, \mathbf{y}_d^{ij} \, d + (q_L^i \widetilde{H}) \, \mathbf{y}_u^{ij} \, u + + (l_L^i H) \, \mathbf{y}_e^{ij} \, e + h.c. \right), \tag{1.3}$$

8

where in order to give mass to the up type quarks, the conjugated Higgs doublet $\widetilde{H} \equiv i\tau^2 H^*$ with the hypercharge $-1/2$ is introduced.

In general the Yukawa matrices $\mathbf{y}^{ij}$ are complex and are not diagonal in the fermion generations space. Indeed, after the absorption of the complex phases by the redefinition of the fermionic fields and the diagonalisation of the fermion mass matrix, the flavour non-diagonal matrix known as Cabibbo-Kobayashi-Maskawa (CKM) matrix encodes the flavour changing interactions of the fermion doublets and W boson.

The Higgs Lagrangian contains the kinetic term, describing interactions with the gauge bosons, and a potential

$$\mathcal{L}_H = (D_\mu H)^\dagger D^\mu H - V(H), \tag{1.4}$$

with the covariant derivative defined according to the charge assignments for the Higgs doublet

$$D_\mu H = \partial_\mu H + \frac{ig}{2} W_\mu^a \tau^a H + \frac{ig'}{2} B_\mu H \tag{1.5}$$

The only two renormalizable and gauge invariant terms of the potential are weighed by the dimensionful parameter μ and the dimensionless self coupling λ. The potential therefore reads

$$V(H) = -\mu^2 |H|^2 + \lambda |H|^4 \tag{1.6}$$

The local gauge symmetries allow for the elimination of the three out of four scalar degrees of freedom of the Higgs doublet. This gauge choice corresponds to the unitary gauge (u.g.) and the remaining scalar is the physical Higgs boson. To minimize the potential, the Higgs field develops a non-zero vacuum expectation value (VEV)

$$H = \begin{pmatrix} H^u \\ H^d \end{pmatrix} \overset{\text{u.g.}}{\Rightarrow} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ h+v \end{pmatrix} \quad , \quad \widetilde{H} = \begin{pmatrix} \widetilde{H}^u \\ \widetilde{H}^d \end{pmatrix} \overset{\text{u.g.}}{\Rightarrow} \frac{1}{\sqrt{2}} \begin{pmatrix} h+v \\ 0 \end{pmatrix}, \tag{1.7}$$

where h is the fluctuation of the field around the minimum of the potential, $v^2 = \mu^2/\lambda$ is the VEV and $1/\sqrt{2}$ factor accounts for the proper normalization of the kinetic term of a real scalar field h. In this way the spontaneous electroweak symmetry breaking (EWSB) is realised. The unphysical gauge bosons mix and the physical combinations of them result in the massive W and Z bosons and massless photon. At the same time the Higgs field itself gets mass. The bosonic masses have the following expressions

$$M_W^2 = \frac{1}{4} g^2 v^2, \qquad M_Z^2 = \frac{1}{4}(g^2 + g'^2) v^2, \qquad M_H^2 = 2\lambda v^2, \tag{1.8}$$

where g' and g are the coupling constants of the gauge groups $SU(2)_L$ and $U(1)_Y$ correspondingly. Finally the fermions of the SM obtain masses proportional to the v and to the Yukawa coupling y.

Assuming that (1.6) is a correct description of the Higgs potential, the Higgs mass, measured to be 125 GeV [41], together with the known expression of Z boson mass and the gauge coupling constants, allows to reconstruct the values of the parameters of the scalar potential

$$v = 246 \text{ GeV}, \ \lambda = 0.13 \Rightarrow \mu \sim 90 \text{ GeV} \tag{1.9}$$

The Higgs sector of the SM can be rewritten in a different way, from which the global structure of the scalar sector can be seen more easily. Denoting the components of the doublet as $(\phi_0, \phi_1, \phi_2, \phi_3)$, the Higgs field and its conjugate before EWSB read

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_2 + i\phi_1 \\ \phi_0 - i\phi_3 \end{pmatrix}, \qquad \tilde{H} \equiv i\sigma_2 H^* = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_0 + i\phi_3 \\ -\phi_2 + i\phi_1 \end{pmatrix}. \tag{1.10}$$

The index assignment of the components of the doublet will become clear in the following. We can rewrite the Higgs sector of the SM, introducing a bidoublet notation

$$\mathbf{M}(x) = \phi_0(x)\,\mathbf{1} + i\boldsymbol{\phi}(x)\boldsymbol{\tau} = \begin{pmatrix} \phi_0 + i\phi_3 & \phi_2 + i\phi_1 \\ -\phi_2 + i\phi_1 & \phi_0 - i\phi_3 \end{pmatrix} \equiv \sqrt{2}\left(\tilde{H}\ H\right) \tag{1.11}$$

where the latter notation stands for the matrix where the columns coincide with the components of the corresponding doublets. The scalar invariant can be obtained by taking the trace $\langle\dots\rangle$ of the product of the $\mathbf{M}$ and its hermitian conjugate

$$\frac{1}{2}\langle\mathbf{M}^\dagger\mathbf{M}\rangle = \phi_0^2 + \boldsymbol{\phi}^2 \tag{1.12}$$

The scalar potential rewritten in terms of bidoublet $\mathbf{M}(x)$ is similar to the one of the linear sigma model [28], which is discussed in greater details further on

$$V(\mathbf{M}) = \lambda\left(\frac{1}{2}\langle\mathbf{M}^\dagger\mathbf{M}\rangle - v^2\right)^2. \tag{1.13}$$

It obeys global $SU(2)_L \times SU(2)_R$ symmetry with the following transformation property

$$\mathbf{M}(x) \to L\mathbf{M}(x)R^\dagger,$$
$$L = e^{il^a\Sigma^a}, \qquad R(x) = e^{ir^a\Sigma^a}, \tag{1.14}$$

where l^a and r^a are space-time independent parameters of the global transformations. The global symmetry of the scalar potential is violated explicitly by $U(1)_Y$ gauging and by the Yukawa couplings of the right-handed fermionic fields.

The local symmetry transformations can be embedded into the global $SU(2)_L \times SU(2)$. Indeed, taking into account that the hypercharges of H and $\widetilde{H}$ are opposite in sign one can write the transformation law under the local $SU(2)_L \times U(1)_Y$ as following

$$\mathbf{M}(x) \to L(x)\mathbf{M}(x)R_3^\dagger(x),$$
$$L(x) = e^{il^a(x)\Sigma^a}, \qquad R_3(x) = e^{ir^3(x)\Sigma^3}, \tag{1.15}$$

where $l^a(x)$ and $r^3(x)$ this time are space-time dependent parameters of the local transformations. The G_{SM} covariant derivative reads

$$D_\mu\mathbf{M}(x) = \partial_\mu\mathbf{M}(x) + igW_\mu^a\Sigma^a\mathbf{M}(x) - ig'B_\mu\mathbf{M}(x)\,\Sigma^3. \tag{1.16}$$

In the bidoublet notation the Higgs sector of the SM is

$$\mathcal{L} = \frac{1}{4}\langle D_\mu \mathbf{M}^\dagger D^\mu \mathbf{M}\rangle - \frac{1}{\sqrt{2}}\left(q_L \mathbf{M}\,\mathbf{Y_q}\,q_R + l_L \mathbf{M}\,\mathbf{Y_l}\,l_R + h.c.\right) - V(\mathbf{M}), \tag{1.17}$$

Where flavour indexes are suppressed and right-handed doublets $q_R = (u_R, d_R)$, $l_R = (0, e_R)$ with generalised Yukawa matrices have been introduced

$$\mathbf{Y_q} = \begin{pmatrix} \mathbf{y_u} & 0 \\ 0 & \mathbf{y_d} \end{pmatrix}, \qquad \mathbf{Y_l} = \begin{pmatrix} 0 & 0 \\ 0 & \mathbf{y_e} \end{pmatrix}. \tag{1.18}$$

In the limit of $g' \to 0$ and $\mathbf{y_u} = \mathbf{y_d}$ the global $SU(2)_L \times SU(2)_R$ is exact. The vacuum state

$$\langle \mathbf{M} \rangle = \begin{pmatrix} v & 0 \\ 0 & v \end{pmatrix}, \tag{1.19}$$

breaks the global symmetry down to vectorial subgroup $SU(2)_V$ which is also known as *custodial symmetry* [42].

If only $SU(2)_L$ is gauged, the Higgs mechanism provides an equal mass to W and Z bosons $M_W = M_Z = gv/2$, which reflects the fact that all the three gauge bosons belong to the triplet of the custodial symmetry group. The effect of the $U(1)_Y$ gauging results in the splitting between the gauge bosons masses and can be characterised by a dimensionless parameter

$$\rho = \frac{M_W^2}{M_Z^2 \cos \theta_W}, \tag{1.20}$$

where $\cos \theta_W = g/\sqrt{g^2 + g'^2}$. The custodial symmetry guarantees that at tree level the value of the parameter is $\rho = 1$. In the SM it receives loop corrections proportional to the custodial violating coupling g' and to the difference between the Yukawa couplings of fermion doublets.

It is possible to introduce one more parametrisation of the scalar sector, the so called non-linear one. Consider the non-linear coordinate redefinition $(\phi_0, \boldsymbol{\phi}) \to (\varphi, \boldsymbol{\pi})$

$$\begin{aligned} \phi_0(x) &\to \varphi(x)\cos\left(i\boldsymbol{\pi}(x)\boldsymbol{\tau}/v\right), \\ \boldsymbol{\phi}(x)\boldsymbol{\tau} &\to \varphi(x)\sin\left(i\boldsymbol{\pi}(x)\boldsymbol{\tau}/v\right), \end{aligned} \quad \Rightarrow \quad \mathbf{M}(x) \to \varphi(x)\,\mathbf{U}(x), \quad \mathbf{U}(x) = e^{i\boldsymbol{\pi}(x)\boldsymbol{\tau}/v}, \tag{1.21}$$

The dimensionless object $\mathbf{U}(x)$ represents a matrix of would-be-Goldstone bosons of the Standard model. The field $\varphi(x)$ is a singlet under G_{SM}, while all the transformation properties are carried out by the matrix $\mathbf{U}(x)$

$$\varphi(x) \to \varphi(x), \qquad \mathbf{U}(x) \to L(x)\mathbf{U}(x)R_3^\dagger(x). \tag{1.22}$$

Finally the Higgs potential $V(\varphi) = \lambda\left(\varphi^2 - v^2\right)^2$ triggers EWSB, and the field φ develops a VEV giving rise to the physical Higgs boson
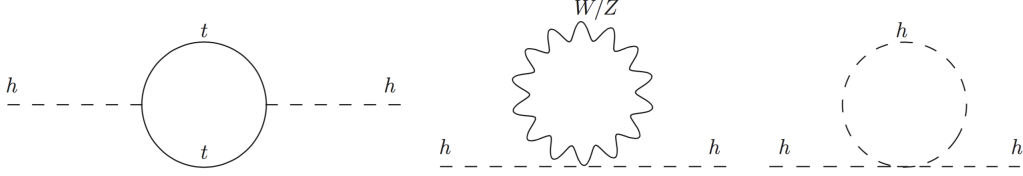
$$\varphi \to h + v. \tag{1.23}$$

Figure 1.1: Dominant SM contributions to Higgs mass parameter at one loop.

The Lagrangian after EWSB is rewritten accordingly

$$\mathcal{L} = \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{(h+v)^2}{4}\langle D_\mu \mathbf{U}^\dagger D^\mu \mathbf{U}\rangle - \frac{(h+v)}{\sqrt{2}}\left(q_L \mathbf{U}\,\mathbf{Y_q}\,q_R + l_L^i \mathbf{U}\,\mathbf{Y_l}\,l_R + h.c.\right) - V(h),$$
$$V(h) = \frac{1}{2}m_h^2 h^2 + \lambda v h^3 + \frac{\lambda}{4}h^4. \tag{1.24}$$

With the covariant derivative defined in the same way as for $\mathbf{M}(x)$

$$D_\mu \mathbf{U}(x) = \partial_\mu \mathbf{U}(x) + igW_\mu^a \Sigma^a \mathbf{U}(x) - ig'B_\mu \mathbf{U}(x)\,\Sigma^3. \tag{1.25}$$

The kinetic term of the would-be-Goldstone bosons accompanied by $(h+v)^2$ in the unitary gauge $\mathbf{U}_{\text{U.G.}} = \mathbf{1}$ provides the mass terms for the W and Z bosons, while the photon remains massless.

It is important to note that despite the fact that the scalar Lagrangian contains arbitrary high powers of the π fields, the SM remains renormalizable [43]: the non-linear field redefinition does not affect the property of renormalizability.

Hierarchy problem   The parameters of the scalar sector of the SM with the values given in Eq. (1.9) are the input parameters, reconstructed from the tree level expressions for the observables. At loop order they receive quantum corrections. The most relevant ones are of top quark (due to the big value of y_t), gauge bosons and Higgs boson loops, they all contain *quadratically divergent* contributions and are not suppressed by a small coefficient.

Assuming that the SM is an effective theory, which is valid only up to a cutoff energy scale Λ, the quantum corrections of the diagrams Fig 1.1 read [44]

$$\left(\delta\mu_{(t)}\right)^2 \sim -\frac{3}{8\pi^2}y_t^2\Lambda^2,$$
$$\left(\delta\mu_{(g)}\right)^2 \sim \frac{1}{16\pi^2}g^2\Lambda^2, \tag{1.26}$$
$$\left(\delta\mu_{(H)}\right)^2 \sim \frac{1}{16\pi^2}\lambda^2\Lambda^2.$$

The top quark loop numerically has the largest contribution and contributes negatively to the Higgs mass parameter, while the bosonic loops are smaller and contribute positively. The resulting mass parameter at one loop reads:

$$\mu^2 = \mu_{\text{tree}}^2 + \sum_{t,g,H}\delta\mu^2 \tag{1.27}$$

12

Thus, for example for $\Lambda = 10\text{TeV}$, in order to obtain $\mu \sim 100\text{GeV}$, we need to set $\mu_{\text{tree}} \sim 2\text{TeV}$ to cancel the contribution of the SM loops almost precisely. This fact is referred to as hierarchy problem, which manifests itself in the large amount of *fine-tuning* required to obtain the observed value of the Higgs mass parameter [45]. The observed value of it is a small part of the contribution generated by the SM loops

$$\mu^2 / \sum_{t,g,H} \delta\mu^2 \sim 1\% \quad \text{for} \quad \Lambda = 10 \text{ TeV}. \tag{1.28}$$

A smaller value of this ratio corresponds to a higher fine-tuning.

The clear way out of this problem is to lower the cutoff scale. Indeed for a lower cutoff the problem is not so dramatic and the fine-tuning is reasonable, for example

$$\mu^2 / \sum_{t,g,H} \delta\mu^2 \sim 25\% \quad \text{for} \quad \Lambda = 1 \text{ TeV}. \tag{1.29}$$

Naively, the SM is valid all the way up to the Plank scale $M_P \sim 10^{18}$ GeV . At this energy the effects of quantum gravity are expected to become important. If there is no new physics responsible for the Higgs mass generation and protection all the way up to M_P, the tree level input parameter and the quantum corrections would have to cancel with incredible precision of order $\mu^2/\delta\mu^2 \sim 10^{-30}$.

Even if one rejects the cutoff argument used above (for example by using the scaleless regularization scheme, such as dimensional regularization [46], see Ref. [47]) the hierarchy problem can be understood as the unclear mechanism separating M_P and EWSB scale v. There are strong theoretical evidences that new physics must be present at lower scales in order to explain the origin of neutrino masses, the strong CP problem and dark matter. Most of the solutions to the mentioned problems require introduction of new heavy quantum states. If this is the case the masses of those new states will play the role of the cutoff scale Λ and in turn will introduce fine-tuning in the theory, unless some mechanism protects the mass parameter from quantum corrections of all the states in the extended theory.

The Higgs parameter thus is seen as unnaturally small. Following t'Hooft [48] the concept of naturalness can be formulated as follows: *"A parameter can be naturally small only if setting it to zero increases the symmetry of the system"*. In this case one says that the parameter is *protected* by the symmetry. While a non-zero value of the parameter breaks the symmetry, quantum corrections will be suppressed due to fact that the system obeys the approximate symmetry. The SM does not have any symmetry protecting the Higgs mass parameter.

One possible solution is that the Higgs might be a pseudo-Nambu-Goldstone boson (pNGB) of the global symmetry breakdown. In the following we will first discuss the properties of the GB in low energy QCD and then focus on the pNGB Higgs.

1.2   Goldstone bosons

In this section we are to describe the notion of Goldstone boson and then apply it to the cases of pion physics and BSM physics. All the developed concepts will be then applied to construct the linear sigma model for the Goldstone Higgs described in Chapters 2 and 3.

A common feature of particle physics models is the presence of a continuous internal symmetry manifesting itself in the invariance of the Lagrangian of the theory under a group of transformations parametrised by a continuous parameter. The parameter may be space-time dependent, and then one talks about local symmetry, or constant, with the corresponding symmetry being global in this case. We will focus on the global case. If the potential of the theory is such that some of the scalar fields develop a non-zero vacuum expectation value then the ground state of the theory has less symmetry than the original Lagrangian. The symmetry is thus broken or hidden, since the resulting Lagrangian is not manifestly invariant under the original transformation group. This phenomenon is known as a spontaneous symmetry breakdown. The Nambu-Goldstone theorem states that **in a theory with spontaneous breakdown of a global symmetry, for every generator of the broken symmetry there is a correspondent massless spinless particle, representing the transformations of the vacuum state along the broken directions**. First discovered by Nambu [49] in the context of superconductivity, it has been subsequently generalized by Goldstone [50] for the case of QFT and summarised in Ref. [51].

Consider a global continuous symmetry $\mathcal{G}$, $\dim(\mathcal{G}) = N$, broken down spontaneously to the group $\mathcal{H}$, $\dim(\mathcal{H}) = K < N$. The vacuum state of the theory $|0\rangle$, which can be understood as vector of VEVs of the scalar fields, is invariant only under the action of a group element of $\mathcal{H}$, but not of an element from the coset $\mathcal{G}/\mathcal{H}$. Considering the infinitesimal transformations we can see that the generators V^a, $a = 1, \ldots, n$ corresponding to the unbroken symmetry annihilate the vacuum, while the broken ones $A^{\hat{a}}$, $\hat{a} = K + 1, \ldots, N$ do not

$$
\begin{aligned}
e^{i\alpha V^a}|0\rangle = |0\rangle &\quad \Rightarrow \quad V^a|0\rangle = 0, \\
e^{i\alpha A^{\hat{a}}}|0\rangle \neq |0\rangle &\quad \Rightarrow \quad A^{\hat{a}}|0\rangle \neq 0,
\end{aligned}
\tag{1.30}
$$

where α is the parameter of the transformation. By definition the vacuum state is the state with the minimum possible energy $E_{\min}$. Since $\mathcal{G}$ is the symmetry of the theory, all the group elements, including the ones with the broken generators, commute with the Hamiltonian and therefore

$$
H\left(e^{i\alpha A^{\hat{a}}}|0\rangle\right) = E_{\min}\left(e^{i\alpha A^{\hat{a}}}|0\rangle\right).
\tag{1.31}
$$

The states obtained by the action of a group element from the coset $\mathcal{G}/\mathcal{H}$ correspond to a degenerate set of states having the same energy as the ground state. Thus, we obtain a degenerate set of $N - K$ vacuum states connected to each other by transformations proportional to the generators of broken symmetries.

1.2.1   Pions as Pseudo-Nambu-Goldstone bosons in QCD

In this subsection we describe a QFT model of pion interactions: the linear sigma model with spontaneous chiral symmetry breaking. The non-linear sigma model will be obtained by integrating out the scalar state σ, which is not a part of the observed low energy QCD spectrum and therefore can be removed from, assuming that it is much heavier than pions. The dynamics of pions at low energies is described in terms of leading and next to leading order non-linear effective Lagrangians. We will mention the explicit symmetry breaking needed to provide non-zero mass to pions, making them pseudo-Goldstone bosons.

Quantum Chromo Dynamics (QCD) is the fundamental theory of the strong interaction of quarks and gluons. It is a gauge theory based on the local $SU(3)_c$ group. At low energy it has the property of confining, meaning that colored objects – quarks, gluons or their non-color-neutral combinations cannot be observed separately in an experiment. Instead they form composite objects consisting of multiple (anti-)quarks. Historically the first observed composite objects of this type were protons and neutrons – the constituents of the physical nuclei and therefore called nucleons.

Later on it was proposed by Yukawa [52] that nucleons stick together in the atom due to the strong nuclear force, with the force carrier being nearly degenerate in mass triplet of pions $(\pi^0, \pi^{\pm})$. From the typical size of the atom it was possible to estimate the mass of the pions being of order 100 MeV.

From the high energy perspective pions are understood as composite objects made of u and d (anti-)quarks, while from low energy they are seen as the pNGB of a global symmetry breaking. The pNGB nature explains why pions are significantly lighter than all the other known QCD states. The global symmetry $SU(2)_L \times SU(2)$, called chiral symmetry, is dictated by the underlying symmetry of quasi-degenerate quark (u, d) doublet. It is broken down to the vectorial subgroup by spinless quark condensate developing a non-zero vacuum expectation value, $\langle \bar{q}_L q \rangle \neq 0$, which is not invariant under the whole chiral group. Inclusion of the additional strange quark, being heavier than the other two, into a triplet of quarks (u, d, s) transforming under global flavour group $SU(3)_F$ accommodates for the additional pNGB states: 4 kaons and the η-meson, together with pions forming an adjoint **8** representation of $SU(3)_F$. In the following discussion we will restrict ourselves to the case of pions only.

Linear σ - model

The linear sigma model is a phenomenological model constructed by Levi and Gell-Mann [28] to describe the interaction of nucleons with pions.

Consider a doublet of nucleon fields consisting of proton and neutron fields

$$N = \begin{pmatrix} p \\ n \end{pmatrix}. \tag{1.32}$$

Even though those two states differ by a unit of electrical charge, from a purely hadronic perspective they are very similar: they have almost the same mass $m_p = 938$ MeV, $(m_n - m_p)/m_n = 0.13\%$ and are known to interact in a similar manner. Therefore in the limit of vanishing electroweak interactions, the description of the states as a doublet is a good approximation. The kinetic part of the nucleon Lagrangian, written in terms of chiral fields ,

$$\mathcal{L} = i\bar{N}_L \slashed{\partial} N_L + i\bar{N}_R \slashed{\partial} N_R, \tag{1.33}$$

obeys a global chrial symmetry $\mathcal{G} = SU(2)_L \times SU(2)$ under which the nucleon doublet transforms as

$$\begin{aligned} N_L &\to L N_L, \quad L = e^{il^a \Sigma^a}, \\ N_R &\to R N_R, \quad R = e^{ir^a \Sigma^a}, \end{aligned} \tag{1.34}$$

with l^a and r^a being parameters of a global $SU(2)$ transformations.

In infinitesimal form they can be rewritten as

$$\begin{array}{ll}
\delta N_L = il^a\Sigma^a N_L, \\
\delta N_R = ir^a\Sigma^a N_R,
\end{array} \quad \rightarrow \quad \delta N = i(v^a - a^a\gamma^5)\Sigma^a N, \quad \text{with} \quad \begin{array}{l} v^a = (l^a + r^a)/2, \\ a^a = (-l^a + r^a)/2. \end{array} \tag{1.35}$$

where v^a, a^a are the parameters of vectorial or isospin group transformation $SU(2)_V$ and axial or pure chiral transformation $SU(2)_A$ correspondingly.

If one naively attempts to introduce the mass term of the nucleon field this would break the axial symmetry explicitly, leaving only the vectorial one. Indeed, the mass term is a cross term between left and right chiral fields. Under $SU(2)_L \times SU(2)$ symmetry it transforms as

$$\bar{N}_L N \rightarrow \bar{N}_L e^{-il^a\Sigma^a} e^{ir^b\Sigma^b} N \tag{1.36}$$

For this to be invariant one has to put a constraint on the transformation parameters, $l^a = r^a$, which corresponds to a smaller symmetry $\mathcal{H} = SU(2)_V$. Not to break axial symmetry explicitly but instead spontaneously Gell-Mann and Levy introduced a bidoublet of scalar fields

$$\mathbf{M}(x) = \sigma(x)\,\mathbf{1} + i\boldsymbol{\pi}(x)\boldsymbol{\tau}, \tag{1.37}$$

containing four scalar fields: singlet σ and triplet of pions $\boldsymbol{\pi}$. The chiral symmetry transformation reads

$$\mathbf{M} \rightarrow L\mathbf{M}R^\dagger. \tag{1.38}$$

Using the expression for the product of Pauli matrices $\tau^a\tau^b = \delta^{ab}\mathbf{1} + i\varepsilon^{abc}\tau^C$ the explicit transformation properties of the field in infinitesimal limit are derived

$$\delta\sigma = a^a\pi^a, \qquad \delta\pi^a = -\varepsilon^{abc}v^b\pi^c - a^a\sigma. \tag{1.39}$$

The fields variations are linear functions of the non-transformed field. Therefore the chiral symmetry is realised *linearly* on the set of scalar fields.

The renormalizable chiral invariant Lagrangian describes the nucleon-scalar system with the interaction strength parametrised by coupling constant g

$$\mathcal{L} = i\bar{N}\slashed{\partial}N + \frac{1}{4}\langle\partial_\mu\mathbf{M}^\dagger\partial^\mu\mathbf{M}\rangle - V(\mathbf{M}) - g\bar{N}_L\mathbf{M}N - g\bar{N}\mathbf{M}^\dagger N_L, \tag{1.40}$$

where the second term gives rise to kinetic terms of the scalar fields, $V(\mathbf{M})$ is the scalar potential and the last two terms describe the interaction of nucleons with the scalars, which in terms of physical fields can be written as

$$-g\sigma\bar{N}N - ig\boldsymbol{\pi}\bar{N}\gamma^5\boldsymbol{\tau}N. \tag{1.41}$$

The second term of Eq. (1.41) describes the interaction of nucleons with the triplet of pseudoscalar pions which are indeed observed in QCD. Nevertheless this model does not meet yet the observations for the following reasons:

1. Nucleons are known to have mass, but for the moment they remain massless. In order to introduce nucleon masses spontaneous symmetry breaking has to occur.

2. The σ particle introduced previously is not exactly the state observed in QCD spectrum. Even though there is a state $f_0(500)$ with the appropriate quantum numbers (0^{++}) it is very broad and is barely seen in experiments. It is also about 5 times heavier than pions. We will integrate out the σ particle from the spectrum and concentrate on the physics of pions only. This will lead to the non-linear σ - model.

3. Even though pions are decoupled from the heavy hadrons spectrum $(m_\pi/m_p \sim 10\%)$, they do have a non-zero mass $m_\pi \sim 140$ MeV. In order to introduce the pion mass an explicit symmetry breaking is needed.

4. Multi-pion processes have been observed in the experiments, but the Lagrangian of the linear σ-model (1.40) does not contain any interactions between pions and has to be extended.

In the following we address each of these problems.

Spontaneous symmetry breaking: nucleon masses.

Lagrangian (1.40) does not contain a mass term for the nucleon field, but it does contain nucleon–scalar interactions and a scalar potential, which has not been specified yet. The mass can be obtained if one of the scalars obtains a non-zero VEV.

The potential is written in terms of scalar chiral invariants, such as

$$\frac{1}{2}\langle \mathbf{M}^\dagger \mathbf{M}\rangle = \sigma^2 + \boldsymbol{\pi}^2. \tag{1.42}$$

The trace of the product of higher number of $\mathbf{M}$ for the 2×2 case is proportional to the same combination $\sigma^2 + \boldsymbol{\pi}^2$ [53]. There are only two $d \leq 4$ terms without derivative

$$d = 2 \quad : \quad \langle \mathbf{M}^\dagger \mathbf{M}\rangle, \tag{1.43}$$
$$d = 4 \quad : \quad \langle \mathbf{M}^\dagger \mathbf{M}\rangle^2. \tag{1.44}$$

Therefore the potential is parametrised by two independent coefficients and can written as

$$V = \lambda \left(\sigma^2 + \boldsymbol{\pi}^2 - F^2\right)^2, \tag{1.45}$$

where F is a dimensionful parameter and the dimensionless λ parametrises self couplings among scalar fields. The parameter λ must be positive, such that the potential is bounded from below. To break the chiral symmetry F^2 must be positive as well, while a negative value corresponds to the situation with no spontaneous breaking and is disregarded. To minimize the potential, the σ scalar obtains a non-zero VEV. With a slight abuse of notation we will denote by the same letter both the unphysical scalar field and its physical fluctuation around vacua

$$\sigma(x) \to F + \sigma(x). \tag{1.46}$$

The Lagrangian after the shift reads

$$\mathcal{L} = i\bar{N}\slashed{\partial}N - m_N \bar{N}N + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi} - g\sigma\bar{N}N - ig\boldsymbol{\pi}\bar{N}\gamma^5\boldsymbol{\tau}N -$$
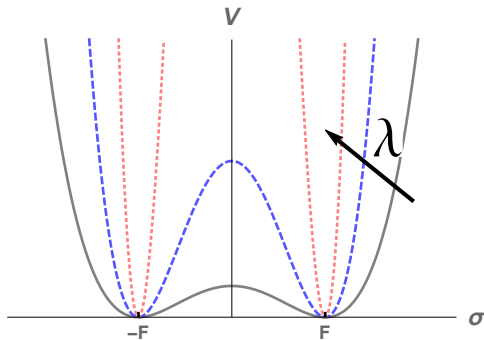
Figure 1.2: Growing λ shrinks the potential. The limit $\lambda \to \infty$ sets a restriction on the potential and eliminates one degree of freedom.

$$- \quad \lambda \left(\sigma^2 + \boldsymbol{\pi}^2\right)^2 - 4\sigma\lambda F \left(\sigma^2 + \boldsymbol{\pi}^2\right) - \frac{1}{2}m_\sigma^2 \sigma^2, \tag{1.47}$$

where fermion and scalar masses are

$$m_N = gF, \qquad m_\sigma^2 = 8\lambda F^2. \tag{1.48}$$

We can see that even though those masses are proportional to the same scale F they are controlled by two different dimensionless parameters g and λ. Since these parameters are independent in theory the hierarchy between fermionic and scalar mass could be arbitrary.

The VEV of σ breaks the chiral symmetry $SU(2)_L \times SU(2)$ down to $SU(2)_V$

$$\langle \mathbf{M} \rangle = F\,\mathbf{1} \to L\langle \mathbf{M}\rangle R^\dagger = F(LR^\dagger) \Rightarrow L = R \Rightarrow a^a = 0, \tag{1.49}$$

while pions remain as massless degrees of freedom being Goldstone bosons associated to the three broken generators of $SU(2)_A$.

Since we disregarded isospin breaking effects the nucleons as well as pions masses are equal. In the following the isospin group $SU(2)_V$ will be considered an exact symmetry.

Non-Linear σ-model.

The linear σ-model is a renormalizable theory, accounting for the spontaneous chiral symmetry breaking and describing pions as QCD Goldstone bosons (at the moment yet massless). We have assumed that the object $\mathbf{M}(x)$ is a simple linear function of the fields $\sigma(x)$ and $\boldsymbol{\pi}(x)$, such that they transform linearly under the chiral symmetry. As the σ particle has not been observed , [1] it can be seen as an artefact of the mathematical apparatus used to construct the theory.

The most straightforward way to remove the σ scalar from the spectrum is to integrate it out assuming it is heavy. According to Eq. (1.48) in order to obtain $m_\sigma \to \infty$ limit, while still keeping

---

[1]The observed 0^{++} state $f_0(500)$ has a very large width [54] and cannot be identified with the scalar of the linear sigma model.

masses of nucleon finite the limit $\lambda \to \infty$ has to be taken. Bigger λ corresponds to a sharper potential as illustrated by Fig.1.2. The limit $\lambda \to \infty$ effectively turns the potential into a delta function $V \sim \delta\left(\sigma^2 + \boldsymbol{\pi}^2 - F^2\right)$. In this case the energy required to excite a physical particle is infinite. This can be seen as a constraint on a field content of the theory

$$\sigma^2 + \boldsymbol{\pi}^2 - F^2 = 0 \qquad \Rightarrow \qquad \sigma = F\sqrt{1 - \boldsymbol{\pi}^2/F^2}. \tag{1.50}$$

It is convenient to introduce a dimensionless matrix of Goldstones $\mathbf{U}$ as a non-linear function of pion fields parametrising the coset $SU(2)_L \times SU(2)/SU(2)_V$ and containing the scale F explicitly .

$$\mathbf{M} \to \rho \, \mathbf{U}, \qquad \mathbf{U} = \sqrt{1 - \boldsymbol{\pi}^2/F^2} + i\boldsymbol{\pi}\boldsymbol{\tau}/F, \qquad \langle \mathbf{U}^\dagger \mathbf{U}\rangle = \mathbf{1}. \tag{1.51}$$

The potential is then written only in terms of the field ρ

$$V = \lambda \left(\frac{1}{2}\langle \mathbf{M}^\dagger \mathbf{M}\rangle - F^2\right)^2 \quad \Rightarrow \quad V = \lambda \left(\rho^2 - F^2\right)^2. \tag{1.52}$$

To minimize the potential the field ρ develops a VEV $\langle\rho\rangle = F$ and obtains a mass $m_\rho^2 = 8\lambda F^2$. In the infinite scalar mass limit ρ is simply replaced by its VEV and becomes non-dynamical.

An equivalent way to parametrise the scalar fields is

$$\begin{aligned} \sigma &\to \rho \, \cos\left(i\boldsymbol{\pi}\boldsymbol{\tau}/F\right), \\ \boldsymbol{\pi}\boldsymbol{\tau} &\to \rho \, \sin\left(i\boldsymbol{\pi}\boldsymbol{\tau}/F\right), \end{aligned} \quad \Rightarrow \quad \mathbf{M} \to \rho \, \mathbf{U}, \quad \mathbf{U} = e^{i\boldsymbol{\pi}\boldsymbol{\tau}/F} = e^{2i\pi^a T^a/F}, \tag{1.53}$$

where $\mathbf{U}$ is the Goldstone bosons matrix in exponential parametrisation. It has been shown [55] that the physical results are independent from the choice of coordinates [2]. For this reason we use the same letter $\boldsymbol{\pi}$ to denote pions before and after the non-linear field redefinition. In the following we will use the exponential form for $\mathbf{U}$.

The transformation properties of $\mathbf{U}$ are the same as of $\mathbf{M}$

$$\mathbf{U} \to L\mathbf{U}R^\dagger. \tag{1.54}$$

To derive the pion field transformation law we restrict the chiral transformation first to the vectorial and then to the axial subgroup. According to (1.35) the vectorial subgroup induces the following transformation

$$a^a = 0 \quad \Rightarrow \quad l^a = r^a = 2v^a, \tag{1.55}$$

$$\mathbf{U} \to e^{iv^a\tau^a}\mathbf{U}e^{-iv^a\tau^a}. \tag{1.56}$$

From the expansion of $\mathbf{U}$ as a power series of $\boldsymbol{\pi}$ it follows that the pion field transforms linearly under the unbroken group $SU(2)_V$

$$\boldsymbol{\pi}\boldsymbol{\tau} \to e^{iv^a\tau^a}\boldsymbol{\pi}\boldsymbol{\tau}e^{-iv^a\tau^a}, \tag{1.57}$$

---

[2]In Chapter 4 we will introduce a general parametrisation of the Goldstone bosons matrix and show that the expressions for the on-shell scattering amplitudes are parametrisation independent.

while under the spontaneously broken group $SU(2)_A$ it transforms *non-linearly*

$$v^a = 0 \quad \Rightarrow \quad -l^a = r^a = 2a^a, \tag{1.58}$$

$$\mathbf{U} \to e^{-ia^a\tau^a}\mathbf{U}e^{-ia^a\tau^a} \tag{1.59}$$

$$(1 + i\boldsymbol{\pi\tau}/F + \dots) \to (1 - ia^a\tau^a + O(a^2))(1 + i\boldsymbol{\pi\tau}/F + \dots)(1 - ia^a\tau^a + O(a^2)). \tag{1.60}$$

Perturbatively the latter transformation can be written as

$$\pi^a \to \pi^a - 2a^aF + O(\pi^2), \tag{1.61}$$

containing a constant shift of the pion field. This shift signals the spontaneous breaking of the correspondent symmetry and forbids the mass term for pions.

The non-linear sigma model is defined by the chiral invariant Lagrangian (1.40) in the limit of infinite scalar mass

$$\mathcal{L} = i\bar{N}\slashed{\partial}N + \frac{F^2}{4}\langle\partial_\mu\mathbf{U}^\dagger\partial^\mu\mathbf{U}\rangle - gF\bar{N}_L\mathbf{U}N - gF\bar{N}\mathbf{U}^\dagger N_L. \tag{1.62}$$

The potential has been eliminated by the restriction (1.50), promoting the heavy scalar field to be non-dynamical. The kinetic term of pions dictates higher order interaction terms, which can be read out expanding $\mathbf{U}$ in terms of physical fields

$$\mathcal{L} \supset \frac{1}{2}\partial_\mu\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi} + \frac{1}{6F^2}(\boldsymbol{\pi}\partial_\mu\boldsymbol{\pi})(\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi}) - \frac{1}{6F^2}(\boldsymbol{\pi\pi})(\partial_\mu\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi}) + O(\pi^6) \tag{1.63}$$

Linear corrections to the non-linear σ-model

The restriction (1.50) corresponds to the exact $\lambda \to \infty$ limit. Keeping λ large but finite and solving perturbatively the equations of motion for the heavy scalar, one obtains the effective Lagrangian describing interactions of pions written as $1/\lambda$ expansion. In the following we demonstrate the effects of finite λ.

Disregarding the nucleon fields, the Lagrangian of the linear sigma model (1.40) in the exponential parametrisation is rewritten as

$$\mathcal{L} = \frac{1}{2}\partial_\mu\rho\partial^\mu\rho + \frac{\rho^2}{4}\langle\partial_\mu\mathbf{U}^\dagger\partial^\mu\mathbf{U}\rangle - \lambda(\rho^2 - F^2)^2. \tag{1.64}$$

The equation of motion for ρ reads

$$\Box\rho - \frac{\rho}{2}\langle\partial_\mu\mathbf{U}^\dagger\partial^\mu\mathbf{U}\rangle + 4\lambda\rho\left(\rho^2 - F^2\right) = 0. \tag{1.65}$$

The perturbative solution up to the next to leading order in $1/\lambda$ expansion

$$\rho = \rho_0 + \rho_1/\lambda + O(1/\lambda^2), \tag{1.66}$$

results in the corresponding effective Lagrangian

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1/\lambda + O(1/\lambda^2). \tag{1.67}$$

Solving the equation of motion for ρ order by order we obtain

$$\rho_0 = F, \tag{1.68}$$

$$\rho_1 = \frac{1}{16F}\langle \partial_\mu \mathbf{U}^\dagger \partial^\mu \mathbf{U} \rangle. \tag{1.69}$$

Plugging these expressions back to the Lagrangian in addition to the pion kinetic term we obtain the NLO correction [53]

$$\mathcal{L}_1 = -\lambda \left(\frac{2F\rho_1}{\lambda}\right)^2 = -\frac{F^2}{8m_\sigma^2}\langle \partial_\mu \mathbf{U}^\dagger \partial^\mu \mathbf{U} \rangle^2, \tag{1.70}$$

where we have traded λ for the mass of the heavy scalar.

Explicit symmetry breaking: pion masses

The theory we have constructed up to now describes the interactions of massless pions, as the group $\mathcal{G}$ remains exact symmetry of the Lagrangian. Pions do have mass and therefore the chiral symmetry has to be broken explicitly. In the underlying fundamental theory of pion interactions - QCD - it is the mass matrix of quarks what breaks the chiral symmetry. The rigorous way to introduce pion masses in the context of non-linear QCD is through *source* fields, which will allow to track the explicit breaking of the symmetry. The sources are auxiliary fields, allowing for a formally chiral invariant Lagrangian. Fixing the values of source fields to non-dynamical values we will obtain the physical Lagrangian with massive pions.

Consider the simplest scalar source χ of canonical dimension $d = 2$ and promote it to have the following transformation properties under the chiral symmetry

$$\chi \to R\chi L^\dagger \tag{1.71}$$

The only scalar invariant involving both GBs and source fields reads

$$d = 2 \ : \ \langle \chi \mathbf{U}^\dagger + \mathbf{U}\chi^\dagger \rangle. \tag{1.72}$$

The Lagrangian of the non-linear sigma model with a scalar source then reads

$$\mathcal{L}_2 = \frac{F^2}{4}\langle \partial_\mu \mathbf{U}^\dagger \partial^\mu \mathbf{U} \rangle + \frac{F^2}{4}\langle \chi \mathbf{U}^\dagger + \mathbf{U}\chi^\dagger \rangle, \tag{1.73}$$

where index 2 indicates the canonical dimension of the terms, disregarding the constant F.

Setting $\langle \chi \rangle = m_\pi^2$ results in the mass term for pion

$$-\frac{1}{2}m_\pi^2 \boldsymbol{\pi}\boldsymbol{\pi} + O(\boldsymbol{\pi}^4). \tag{1.74}$$

The pion mass term breaks the axial symmetry explicitly, while the vector one remains intact.

Effective Lagrangian

Lagrangian (1.73) represents the minimal set of terms required to give an approximate description of pion physics. This framework can be extended by the inclusion of higher dimensional terms $d > 2$. The general basis of $d = 4$ terms was first derived by Gasser and Leutwyler [56] and for the case of $SU(2) \times SU(2)$ chiral symmetry reads

$$
\begin{aligned}
\mathcal{L}_4 \ = \ & \alpha_1 \langle \partial_\mu \mathbf{U} \partial^\mu \mathbf{U}^\dagger \rangle^2 + \alpha_2 \langle \partial_\mu \mathbf{U} \partial_\nu \mathbf{U}^\dagger \rangle \langle \partial^\mu \mathbf{U} \partial^\nu \mathbf{U}^\dagger \rangle + \\
+ \ & \alpha_4 \langle \partial_\mu \mathbf{U} \partial^\mu \mathbf{U}^\dagger \rangle \langle \chi \mathbf{U}^\dagger + \mathbf{U} \chi^\dagger \rangle + \alpha_5 \langle \chi \mathbf{U}^\dagger + \mathbf{U} \chi^\dagger \rangle^2,
\end{aligned} \tag{1.75}
$$

where α_i are dimensionless coefficients with measured values of order 10^{-2} –10^{-3}.

To ensure the convergence of the expansion in canonical dimension the higher order terms have to be suppressed by a scale Λ. Any additional power of the derivative or pion mass will be then accompanied by an inverse of the sale. To reveal the general counting scheme we rewrite the kinetic term in a symbolic form, omitting the numerical coefficient and keeping track of scales

$$
F^2 \Lambda^2 \left(\frac{\partial_\mu}{\Lambda} \mathbf{U} \right)^2. \tag{1.76}
$$

Taking into account that each pion field is suppressed by F it is easy to estimate the contribution of any vertex with N_π pion fields originating from this term

$$
F^2 \Lambda^2 \left(\frac{\partial}{\Lambda} \right)^2 \left(\frac{\boldsymbol{\pi}}{f} \right)^{N_\pi}. \tag{1.77}
$$

Generalizing this result to N_p powers of derivatives and N_m mass insertions the counting formula reads [57, 58]

$$
F^2 \Lambda^2 \left(\frac{\partial}{\Lambda} \right)^{N_p} \left(\frac{m_\pi}{\Lambda} \right)^{N_m} \left(\frac{\boldsymbol{\pi}}{f} \right)^{N_\pi}. \tag{1.78}
$$

To estimate the relative weight between the Lagrangians with two and four derivatives we will focus on the $\pi\pi \to \pi\pi$ scattering. Each derivative corresponds to a characteristic energy of the process $\partial \sim E$. Using the counting formula Eq. (1.78) the contributions read

$$
\mathcal{L}_2 \quad : \quad (\partial^2 \pi^4) \ \to \ \frac{E^2}{F^2}, \tag{1.79}
$$

$$
\mathcal{L}_4 \quad : \quad (\partial^4 \pi^4) \ \to \ \frac{E^2}{\Lambda^2} \frac{E^2}{F^2}. \tag{1.80}
$$

The contribution of four-derivative term is suppressed by a factor $(E/\Lambda)^2$ and can be neglected at low energies $E \ll \Lambda$. Thus, the expansion in "derivatives" (both space-time derivative and mass are counted the same) allows for a more precise description, taking into account the (E/Λ) corrections to the leading order (LO) Lagrangian with two derivatives. This expansion is known as chiral perturbation theory and can be summarised as

$$
\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 + \dots. \tag{1.81}
$$

The expansion above is valid all the way up to the energies $E = \Lambda$, at which all the terms give contributions of the same order and the expansion breaks down.

Now consider loop corrections to the tree level 4π scattering amplitude. Computing the effects of virtual particles at one loop with the LO Lagrangian vertexes results in

$$\mathcal{L}_2^{\text{loop}} \quad \rightarrow \quad \left(\frac{E}{4\pi F}\right)^2 \frac{E^2}{F^2} \text{ (fin + div)}, \tag{1.82}$$

where "finite" is the dimensionless finite part of the amplitude, "div" denotes the infinite term extracted for example by dimensional regularisation and the 4π coefficient is the result of loop momentum integration. This divergence is proportional to four powers of energy and cannot be reabsorbed by the leading order terms, $\mathcal{L}_4$ counterterms are required instead. Thus, higher order terms in the chiral expansion are needed for the consistency of the chiral perturbation theory at loop level. The validity of the loop expansion requires the loop contribution to be subdominant to the tree level one, which is true for $E < 4\pi F$.

Assuming that the tree level NLO contribution is compatible with the loop contribution or larger

$$\frac{F^2}{\Lambda^2} \geq \frac{1}{16\pi^2} \tag{1.83}$$

we obtain the famous relation [58]

$$\Lambda \leq 4\pi F. \tag{1.84}$$

External gauging

Using the language of external sources it is possible to introduce also the SM gauge fields W and Z by gauging the electroweak group. Pions are composed of particles charged under $SU(2)_L \times U(1)_Y$ and therefore they participate in those interactions as well. Under $U(1)_{\text{em}}$, embedded into the unbroken $SU(2)_V$, pions form a triplet (π^-, π^0, π^+). We omit the details here and will mention several properties of the gauged non-linear sigma model.

- **Leptonic decays of pions.** The pion is the lightest hadron and remains stable if electroweak interactions are switched off. Turning on the EW interaction allows for leptonic decay of charged pion π^+ or π^- predominantly into $\mu^+\nu_\mu$ or $\mu^-\bar{\nu}_\mu$ correspondingly, with branching ratio $\sim 99\%$. The Goldstone boson scale enters in the expression for the matrix element of quark axial current $\langle 0|j_5^\mu|\pi^-(p)\rangle = iFp_\mu$ with $j_5^\mu = \frac{1}{\sqrt{2}}\bar{u}\gamma^\mu\gamma_5 d$, parametrizing the hadronic part of the decay amplitude and consequently in the pion lifetime [59]. It is identified with the pion decay constant $F = 93$ MeV.

- **Difference of masses of $\pi^\pm$ and π^0.** The quark composition of charged and neutral pions is different. Electromagnetic interactions violate the $SU(2)_V$ symmetry, generate a pion potential and introduce a small splitting between the masses of pions $(m_{\pi^+} - m_{\pi^0})/m_{\pi^+} \sim 3\%$.

Under the assumption that the dominant contribution to pion form factors comes from the first heavy meson resonances ρ and a_1 (so-called vector dominance) the splitting reads [60]

$$m_{\pi^\pm}^2 - m_{\pi^0}^2 \simeq \frac{3\alpha_{em}}{4\pi} \frac{m_\rho^2 m_{a_1}^2}{m_{a_1}^2 - m_\rho^2} \log\left(\frac{m_{a_1}^2}{m_\rho^2}\right) \Rightarrow m_{\pi^\pm} - m_{\pi^0} \sim 5.8 \text{MeV} \qquad (1.85)$$

which is in good agreement with experimental value $m_{\pi^\pm} - m_{\pi^0} \simeq 4.6$ MeV.

It is interesting too see that assuming that the splitting is controlled by a cutoff $\Lambda \sim 4\pi F$ we can estimate the mass splitting purely from the dimensional analysis as

$$m_{\pi^\pm}^2 - m_{\pi^0}^2 \simeq \frac{\alpha_{em}}{4\pi}(4\pi F)^2 \Rightarrow m_{\pi^\pm} - m_{\pi^0} \sim 8.8 \text{ MeV} \qquad (1.86)$$

- **Massive $W^\pm$ and Z.** QCD vacuum state breaks chrial invariance and therefore it breaks electroweak symmetry down to $U(1)_{em}$, a situation completely analogous to the SM itself, with the only difference that there is no physical scalar obtaining the VEV. The pions are eaten to form longitudinal polarisations of W and Z bosons and therefore the gauge bosons obtain masses [45]

$$m_W = \frac{gF}{2} \simeq 29 \text{ MeV}, \qquad m_Z = \frac{gF}{2\cos\theta_W} \simeq 33 \text{ MeV}. \qquad (1.87)$$

The values obtained are by 3 orders smaller than the real ones, though the important observation that the strong dynamics is able to provide a mass to the gauge bosons has led to the idea of technicolor and consequently to Composite Higgs models.

1.2.2 Higgs as Pseudo-Nambu-Goldstone boson in BSM

One possible solution to the hierarchy problem is to promote the physical Higgs boson to be a pseudo-Nambu-Goldstone boson of a global symmetry breaking. In this case, on dimensional analysis grounds the Higgs mass parameter does not receive corrections from arbitrary high scales of the BSM theory, but instead only from the global symmetry breaking scale multiplied by the explicit symmetry breaking parameters. While still some fine-tuning is typically required, the Goldstone Higgs option represents a tempting possibility for the new physics and will be discussed in details in what is following.

A peculiar feature of pNGB Higgs is the split spectrum – the Higgs, being the lightest state of the new sector, is significantly lighter than all the other new states and in the limit of the unbroken global symmetry remains precisely massless. Thus the non-observation of the new states at scale v is thus naturally explained, while the typical scale of new heavy states is lifted up to a few TeV.

Finally, all the known scalar particles, with the possible exception of the QCD $\sigma-$meson, are understood as (pseudo-)Goldstone bosons, such is the case for pions, discussed in the previous section or the longitudinal polarisations of W and Z.

Technicolor

The idea of technicolor models [45, 61, 62] being to a large extent the upscaled version of QCD was proposed long before the experimental discovery of the Higgs boson and exhibited no massive scalar in the spectrum, reproducing only the Goldstone bosons of the SM and predicting heavy resonances around TeV scale. As in QCD the Lagrangian of the model is invariant under chiral symmetry group with N_{TC}, flavours $SU(N_{\mathrm{TC}}) \times SU(N_{\mathrm{TC}})$, broken down spontaneously to $SU(N_{\mathrm{TC}})_V$ by condensate of (techi-)quark-antiquark pairs. The technipion decay constant $F_{\mathrm{TC}} = 246$ GeV , as dictated by the expression for the SM gauge bosons masses Eq. (1.87) , is analogous to the pion decay constant of QCD $F_{\mathrm{QCD}} = 93$ MeV. Scaling up the mass of the first resonance in QCD and assuming for simplicity that the number of colors is not so different from the three colors of QCD we can estimate the mass of techi-ρ meson as

$$\rho_{\mathrm{TC}} \sim \frac{F_{\mathrm{TC}}}{F_{QCD}} m_\rho \sim 2 \text{ TeV}. \tag{1.88}$$

The original technicolor models encountered serious phenomenological problems and were eventually ruled out. Even before the Higgs discovery severe constrains were posed on the parameters of the models.

- **Flavour changing neutral currents (FNCN)**. In the absence of the Higgs mechanism providing mass to the SM quarks, to generate mass term for the light quarks both QCD and TC are embedded in a larger Extended Technicolor (ETC) group [63, 64], broken down spontaneously at high scale Λ_{ETC}

$$SU(N_{\mathrm{ETC}}) \to SU(3)_c \times SU(N_{\mathrm{TC}}) \tag{1.89}$$

This in turn produces a set of heavy gauge bosons with the mass of order Λ_{ETC}. The exchange of the heavy bosons generate four-fermion operators, suppressed by the gauge boson mass. As techiquarks condensate, the mixed light-heavy operators lead to an effective mass term for the SM quarks

$$m_q \simeq \frac{g_{\mathrm{ETC}}^2}{\Lambda_{\mathrm{ETC}}^2} (\bar{q}q) \langle \bar{\psi}_{\mathrm{TC}} \psi_{\mathrm{TC}} \rangle \simeq \Lambda_{\mathrm{TC}} \left(\frac{\Lambda_{\mathrm{TC}}}{\Lambda_{\mathrm{ETC}}} \right)^2. \tag{1.90}$$

For $\Lambda_{\mathrm{TC}} = v = 245$ GeV the strange quark mass sets $\Lambda_{\mathrm{ETC}} \sim 10$ TeV. Moreover, to reproduce the hierarchy of the quark masses the $SU(N_{\mathrm{ETC}})$ breaking scale has to be different for each quark flavour. The group has to undergo series of hierarchical breakings, at different scales for each quark, inversely proportional to their masses.

On the other hand the SM four-fermion operators are suppressed by the same scale as mixed ones and in general violate both CP and flavour

$$\frac{g_{\mathrm{ETC}}^2}{\Lambda_{\mathrm{ETC}}^2} (\bar{q}q)(\bar{q}q) \tag{1.91}$$

The experimental bound $\Lambda_{\mathrm{ETC}} > 10^5$ TeV does not allow to reproduce the observed light quarks masses and therefore this set-up is ruled out.

25

- **Electroweak Precision Tests (EWPT)** It has been shown that in the Large N approximation that the contribution of the techicolor sector to Peskin–Takeuchi parameter S is approximately [65–67]

$$S \sim \frac{N_{TC} N_{TD}}{\pi}, \tag{1.92}$$

where N_{TD} is the number of technidoublets. Even for the small number of colors and flavours the contribution is $S \sim 1$, which is ruled out by EWPT performed at LEP.

The Composite Higgs as Nambu-Goldstone boson

Technicolor has been a motivation for a more elaborated idea of composite Higgs [3–8], being an interpolation between the strongly interacting non-linear picture and the weakly interacting SM Higgs. Complete reviews of the topic can be found in [68, 69]. As in technicolor or QCD, in the composite Higgs framework one assumes a strongly interacting sector obeying a global symmetry $\mathcal{G}$ at high energies of order $\Lambda \sim 1$ TeV, with a zoo of composite resonances forming complete representations of the group. At low energies it is broken spontaneously to a smaller group $\mathcal{H}$. The scale of the breaking f is analogous to the pion decay constant F at which the chiral symmetry of the low energy QCD is broken. The Goldstone bosons resulting from the breaking must include both the longitudinal degrees of freedom of the SM gauge bosons and the Higgs boson itself, produced firstly as an exact massless GB. The coset $\mathcal{G}/\mathcal{H}$ has to be large enough in order to include all the four degrees of freedom. For example for the most simple choices of special unitary and orthogonal groups this means

$$SU(N)/SU(N-1) \quad \Rightarrow \quad (2N-1) \text{ GB} \quad \Rightarrow \quad N \geq 3, \tag{1.93}$$
$$SO(N)/SO(N-1) \quad \Rightarrow \quad (N-1) \text{ GB} \quad \Rightarrow \quad N \geq 5. \tag{1.94}$$

The original Georgi-Kaplan model [5] relies on the $SU(5)/SO(5)$ coset. Later the minimal composite Higgs (MCHM) model was proposed [9, 10], based on the $SO(5)/SO(4)$ symmetry breaking and containing exactly 4 GBs. In [70] the effective operators produced from the cosets of the theories above, as well as the minimal intrinsically custodial violating coset $SU(3)/(SU(2) \times U(1))$, have been derived. In [11] the next-to-minimal coset $SO(6)/SO(5)$ with one more additional GB was considered. It has been studied as possible dark matter candidate in [13, 14].

The typical way to include the SM gauge group and its subsequent breaking can be described as a three step process

1. At scale Λ the theory is invariant under the global group $\mathcal{G}$, broken down spontaneously to group $\mathcal{H}$ at the scale f. This breaking produces $n = \dim(\mathcal{G}) - \dim(\mathcal{H})$ Goldstone bosons.

2. The Standard Model gauge group G_{SM} is embedded into the unbroken group $\mathcal{H}$. The Higgs boson is one of the Goldstone bosons originating from the $\mathcal{G}/\mathcal{H}$ breaking, transforming under the G_{SM}. Since the Higgs is an exact Golstone boson at tree level there are no potential terms and therefore G_{SM} remains unbroken.
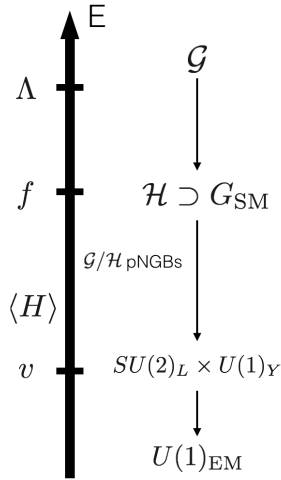
Figure 1.3: Scales and symmetry breaking patterns of a typical composite Higgs model.
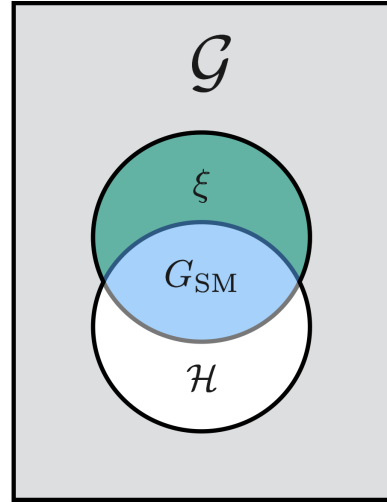

Figure 1.4: The group structure of a composite Higgs model.

3. EWSB is achieved due to a small explicit breaking of $\mathcal{G}$ in the fermion and gauge sectors. At the loop level this breaking propagates to the scalar sector and generates G_{SM}-invariant Coleman-Weinberg [71] effective potential for the Higgs, which in turn leads to the Higgs VEV, breaking spontaneously G_{SM} and setting the electroweak scale $v < f$, which in general differs from the Higgs VEV. Thus the Higgs boson acquires a mass term, making it a pNGB.

The misalignment between the G_{SM} preserving and G_{SM} breaking vacua is parametrised by a dimensionless parameter

$$\xi = \frac{v^2}{f^2}. \tag{1.95}$$

In the limit $f \to \infty$ or equivalently $\xi \to 0$ all the effects coming from the non-linear strongly interacting sector decouple and the SM is recovered. Thus, ξ parametrises the degree of non-linearity of the Higgs dynamics. The pictorial representation of the scales of the typical Composite Higgs model can be seen in Fig. 1.3.

The parameter ξ can also be seen as a measure of separation of the pseudo Goldstone Higgs mass of order v and the masses of heavy resonances with the masses $g^* f$, with the g^* being a (possibly strong) coupling of the new sector $1 < g^* < 4\pi$. The inclusion of those heavy states modifies the couplings of the Higgs boson to the SM particles which can be written as power series in ξ.

The explicit breaking of $\mathcal{G}$, required to misalign the vacuum state, is partially provided by the gauge group G_{SM} being a subgroup of $\mathcal{G}$, but typically this source is not enough. One way to achieve the necessary breaking is to introduce linear couplings of the SM fermion fields to a set of heavy fermionic resonances with appropriate quantum numbers. Since the SM fermions are considered to be elementary and not transforming under the global group of strongly interacting sector those couplings break $\mathcal{G}$ explicitly and allow for fermion mass generation. This mechanism is known as *partial compositeness* [29] and is to be discussed in more details in the following subsection.

The GB degrees of freedom can be represented by a matrix

$$\Omega(x) = e^{i\Pi(x)/f}, \tag{1.96}$$

where $\Pi = \Pi^{\hat{a}}(x)A^{\hat{a}}$, the vector $\Pi^{\hat{a}}(x)$ is the set of Goldstone bosons of $\mathcal{G}/\mathcal{H}$ breaking and $A^{\hat{a}}$ as before denotes the generators of broken symmetries, $\hat{a} = 1\ldots\dim(\mathcal{G}/\mathcal{H})$, V^a are the generators of the unbroken symmetry group $\mathcal{H}$, $a = 1\ldots\dim(\mathcal{H})$.

The matrix Ω is a group transformation corresponding to the $\mathcal{G}/\mathcal{H}$ coset. The action of an arbitrary group element $g \in \mathcal{G}$ on Ω parametrises another group element $g' \in \mathcal{G}$, as it follows from the closure property of the group product operation. Since every group element of $\mathcal{G}$ can be decomposed as a product of a broken and unbroken symmetry transformation, which we denote as Ω' and h correspondingly, the following decomposition is valid

$$g\,\Omega(x) = g' = \Omega'(x)h(g,\Pi) \tag{1.97}$$

Therefore the matrix Ω has the following transformation properties [72, 73]

$$\Omega(x) \to \Omega'(x) = g\,\Omega h^{-1}(g,\Pi). \tag{1.98}$$

If the global group $\mathcal{G}$ allows for an automorphism $g \to g = R(g)$, with

$$R : A^{\hat{a}} \to -A^{\hat{a}}, \tag{1.99}$$

the GB matrix transformation law can be recast as

$$\Omega(x) \to \Omega'(x) = h(g,\Pi)\,\Omega g^{-1}. \tag{1.100}$$

It is immediate to see that the squared GB matrix obeys a simple transformation law

$$\mathbf{U}(x) = \Omega(x)^2 = e^{2i\Pi(x)/f}, \qquad \mathbf{U}' = g\mathbf{U}g^{-1}. \tag{1.101}$$

This form has been used previously for the low energy QCD and will be used in the following discussion of the Electroweak Chiral Lagrangian in section 1.3.2.

Fermions and partial compositeness

The strongly coupled sector of the composite Higgs models typically includes a set of composite heavy fermionic resonances Ψ, forming complete representations under the global group of symmetry $\mathcal{G}$. The elementary fermions of the SM, to which we will refer as light ones, instead transform only under the gauge group G_{SM}. In the simplest case of vectorlike heavy fermions, the mass term $M\bar{\Psi}_L\Psi$ can be introduced straightforwardly and is invariant under $\mathcal{G}$. To provide mass for the SM quarks and leptons the couplings between the elementary and composite fermions have to be considered. In technicolor this has been achieved through the effective nonrenormalizable bilinear couplings (1.90), which led to severe phenomenological problems. The alternative idea proposed in Ref. [29] employs *linear* couplings of the light fermions q and operators originating from the

strongly coupled sector, such as Ψ itself. The couplings violate explicitly the global group $\mathcal{G}$ and can be formally written as

$$\mathcal{L} \supset \Lambda_L \left(\bar{q}_L \, \Delta_{q-\Psi} \right) \Psi + \Lambda \, \bar{\Psi}_L \left(\Delta_{\Psi-q} \, q \right) + h.c., \qquad (1.102)$$

where $\Delta_{q-\Psi}$ ($\Delta_{\Psi-q}$) are the spurion fields connecting G_{SM} and $\mathcal{G}$ representations (and vice versa), while $\Lambda_{L/R}$ are dimensionful couplings. The latter ones can be made much smaller than the scale of composite resonances by the RG evolution, thus reproducing the hierarchy for the SM quark masses.

The heavy fermions representations under $\mathcal{G}$ are not defined uniquely and introduce a strong model dependence in the phenomenological results. Multiple possible embeddings of the heavy fermions for the MCHM and their impact on phenomenology have been analysed in [16].

Provided that proto-Yukava coupling between the heavy fermions and composite Higgs state is allowed and parametrised by Y, the explicitly $\mathcal{G}$-breaking couplings $\Lambda_{L/R}$ propagate to the scalar sector at loop level and generate the CW potential for the Higgs, which eventually triggers EWSB.

The mixing between heavy and light fermions together with EWSB characterized by the scale v, result in the see-saw like expression for the light SM fermions, with the Yukawa coupling being a function of the parameters of the strongly interacting sector

$$m_q \sim y \, v, \qquad y = Y \frac{\Lambda_L}{M} \frac{\Lambda}{M}. \qquad (1.103)$$

The ratios $\Lambda_{L/R}/M$ characterize the degree of mixing between the fermions. Only the heaviest top quark has a significant admixture of the composite state, while the other light quarks are mostly elementary.

Other models of Goldstone Boson

Above we have briefly outlined the idea of the strong dynamics at high energies leading to the composite Higgs as a composite pseudo Goldstone boson.

The complementary concept of the pNGB Higgs is a Little Higgs model, originally proposed in [74] and further studied in [75], see [76] for a review. The Little Higgs construction does not necessarily rely on strong dynamics at high energies and instead assumes perturbative regime at scales around TeV. The additional 5th space dimension is deconstructed and represented by a discrete number of sites. The 4D symmetry is extended into the five dimensions, which in the deconstructed space results in a direct product of multiple copies of the gauge groups living in the different sites. The Higgs boson seen as a fifth component of the 5D gauge field becoming a GB of the collective breaking of the global symmetries. Due to the collective nature of the breaking the quadratic divergences are suppressed by the product of the breaking parameters, such that fine-tuning is reasonable.

The descriptions above implement the effective description of the Golstone boson degrees of freedom and therefore are written typically in terms of effective and therefore non-renormalizable

Lagrangian [72, 73]. While this does not pose a big problem for the calculability of observables, renormalizable, UV complete underlying theories are considered in the literature.

One possible direction to go is to consider a microscopic UV completion, the dynamics of particles forming the bound states at energies around TeV scale. Depending on the particular choice of the rank of the new gauge group and the number of colors, either through lattice simulations or from purely group theoretical considerations, it is possible to restrict the emerging global symmetry group and its breaking pattern. This eventually will allow for the determination of preferable cosets and define the number of emerging Goldstone bosons. Following this program [77, 78] analysed several possible gauge theories leading to $SU(5)/SO(5)$ and $SU(4)/Sp(4)$ cosets. The latter breaking pattern has been also analysed in Ref. [79], as the one emerging from the minimal strongly interacting theory consisting of and $SU(2)$ gauge group with two Dirac fermions in the fundamental representation.

Another way to construct a renormalizable theory is to promote the Higgs boson to be a part of a complete $\mathcal{G}$ representation, transforming linearly under the global group. The completion of the multiplet requires the existence of new scalar degrees of freedom. Ref. [12] considered an elementary realisation of pNGB Higgs for $SU(4)/Sp(4) \sim SO(6)/SO(5)$ coset. In the simplest case of $SO(5)/SO(4)$ linear model the spectrum is extended by one electroweak singlet scalar. In the context of the Goldstone Higgs this set up has been first mentioned first [22] and consequently [17]. More detailed phenomenological study of the additional scalar, perspectives for the direct detection, impact of the extended fermionic sector on EWPT and the effective field theory resulting from the integrating out the new degrees of freedom have been conducted in [31, 32] and will be discussed in details in Chapters 2,3. Furthermore, UV complete linear realisations of the global symmetry have been considered in [23–27].

1.3 Effective Field Theory

Though numerous theories have been proposed in the literature, dealing with the hierarchy problem and more generally with challenges of the current understanding of what the complete theory of particle physics should be, the parameter space of the possible extensions of the SM model is somewhat overwhelming. In order to confirm or disprove a theory in hand one can directly calculate the expressions for the possible observables and compare them with the available measurements of these observables provided by experiments. This procedure, known as the top-down approach, is a possible way, but due to the large number of the New Physics theories, if one is interested in considering various possible models it is not effective. The more effective way involves the use of Effective Field Theories as a tool making possible the study of all the theories at once or - if some additional assumptions are made - whole classes of BSM extensions. Once the observables are expressed in terms of the parameters of the EFT, a given UV theory can be matched into the EFT, resulting in relations among its (otherwise free) parameters and this way it can be either validated or disproved.

The construction of the EFT within QFT can be performed considering the following steps

1. **Physical scales.** Whether one studies the beta decay of neutron or performs EWPTs at LEP it is always possible to define the characteristic scale of the problem E, being for example the momentum transfer between the initial and final state.

2. **Particle content.** The next step requires identification of the relevant degrees of freedom in terms of quantum fields. Fortunately, to study the interactions at the given scale one does not need to know the complete theory of everything up to infinite energies [80]. This has always been the case and it is very hard to imagine any progress in physics otherwise. Therefore the heavy (much heavier than the typical scale of interactions under study) or simply unknown degrees of freedom can be neglected and not included explicitly in the Lagrangian.

3. **Symmetries**. The third step consists of imposing the physical symmetries. Thus, transformation properties under Lorentz group will define scalar, fermion, vectorial and possibly higher spin fields, while the internal symmetries with the proper charge assignments will further restrict the possible interaction terms.

4. **Counting.** Finally, in order to distinguish among more and less important effects, and possibly take into account the subleading contributions to the modifications of interactions , a small parameter has to be defined. The Lagrangian of the EFT can be expressed as a power series of such parameter. The famous decoupling theorem [80] states that in a local quantum theory with some light particles with mass $m \lesssim E$ interacting with a particle of mass $M \gg E$, the Green functions with only light particles as external legs can be obtained simply by omitting the heavy particles, up to the corrections of inverse powers of M. The dimensionless expansion parameter in this case is E/M. In the following we will actively exploit this fact, assuming that the new physics is represented by heavy states.

Probably the most prominent example of the effective theory in QFT is the Fermi theory of beta decay. Consider for example muon decay process $\mu \to e\,\nu_\mu\,\bar{\nu}_e$. It has been inferred from the experimental data that the interaction can be written as derivativeless product of two fermionic chiral $V - A$ currents

$$\mathcal{L}_{\text{Fermi}} = \frac{G_F}{\sqrt{2}} \, \bar{\nu}_\mu \gamma^\alpha (1 - \gamma^5)\mu \, \bar{e}\gamma_\alpha(1 - \gamma^5)\nu_e \tag{1.104}$$

with G_F being the dimensionful (d=2) Fermi constant parametrising the strength of the contact interaction. The calculated cross section grows with energy like $\sigma \sim \Gamma_F^2 E^2$, indicating that for the energies around $E \sim 100$ GeV it becomes bigger than one and therefore has to be replaced by a more fundamental theory smearing the energy dependence of the cross section.

The completion of the Fermi theory being the gauge theory of electroweak interactions is a part of the SM of particle physics. The interaction of weak currents is mediated by $W^\pm$ gauge boson, resulting in renormalizable interactions of spacial dimension $d = 4$ with a dimensionless coupling constant g. The low energy limit of the the propagator of the gauge boson reads

$$\frac{-g_{\mu\nu} + q^\mu q^\nu / M_W^2}{q^2 - M_W^2} \xrightarrow{q^2 \ll M_W^2} \frac{g_{\mu\nu}}{M_W^2} \tag{1.105}$$

The approximate momentum independent expression justifies the use of the Lagrangian (1.104) and allows to identify the expression of the Fermi constant in terms of the parameters of the SM

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} \qquad (1.106)$$

Further expansion of the bosonic propagator as powers of q^2/M_W^2 determines the corrections to the Fermi Lagrangian.

Another example of effective Lagrangian has been described before considering the low energy QCD Lagrangian – non linear sigma model of pion interactions. Theoretically it can be seen either as a power series of operators, sorted according to the small parameter E/Λ_{QCD}, or as a low energy limit of the UV complete model – linear sigma model. The first point of view on the chiral Lagrangian of QCD turned out to be correct, while the parameters of the effective Lagrangian obtained from the integrating out heavy σ fields do not match physically measured values.

Finally, there are good reasons to believe that the SM itself is an effective theory. Being the most successful theory of fundamental interactions it yet does not address some key issues, such as dark matter, neutrino masses, the strong θ angle and eventually it does not provide the quantum description of gravity. One typically assumes that the physics resolving and explaining all these issues will be accompanied by new heavy states which have not been yet observed, since the effect of their interaction is suppressed by some high scale Λ.

The use of the Effective Theories for BSM models have been summarised in [81] and can be seen as three step process (see also the original Fig.1.5)

1. Given the UV model at high scales Λ the parameters or the Wilson coefficients $c_i(\Lambda)$ of the EFT can be calculated. This step is called *matching* of UV completion into the EFT.

2. In general, the UV completion may be well separated in energy scale with the difference being of orders of magnitude. In this case the *running* effects are relevant and in order to match the low energy observables the running coefficients with γ_{ij} – the anomalous dimensions matrix have to be computed.

3. Finally, the coefficients of the EFT are run down to the EW scale $c_i(m_W)$. The observables are to be computed at this scale and compared to the predictions of the UV completions. This is *mapping*.

This is a general plan to proceed towards the construction of the EFT for the study of BSM effects. The particle content will include all the known elementary particles discovered, thus coinciding with the particle content of the SM.

Furthermore, we will rely on Lorentz symmetry and gauge symmetries of the SM, exploring only deviations from the SM preserving G_{SM}. Any BSM theory compatible with the symmetries assumed can be projected down into the set EFT operators. G_{SM} can be realised either linearly or non-linearly leading to two intrinsically different effective theories. The linear realisation as it is the case for the SM itself and generally for the weakly coupled BSM models such as supersymmetry corresponds to scenarios in which the Higgs belongs to an elementary $SU(2)_L$ doublet
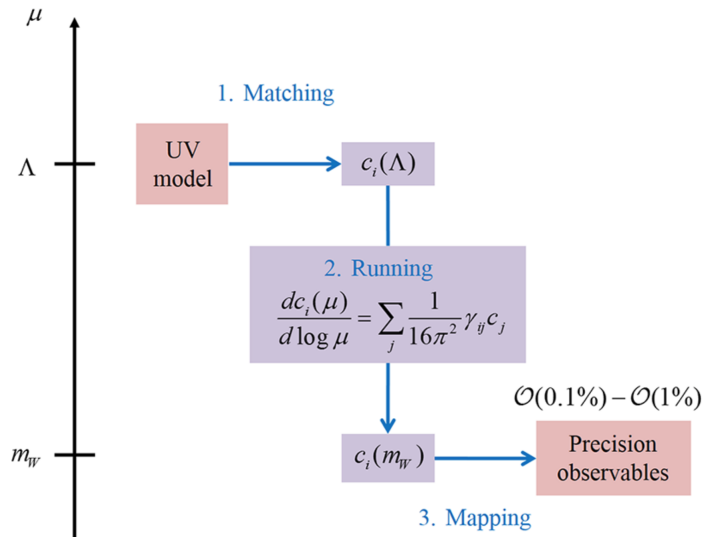
Figure 1.5: Connection between UV completion and the effective theory at EW scale [81].

and commonly known as Standard Model Effective Field Theory (SMEFT) or linear EFT. The non-linear case is preferable for strongly interacting theories or for the theories with Goldstone boson origin of the Higgs as is shown in Chapter 3. It corresponds to the case where all the gauge transformation properties are carried out by the would-be-Goldstones of the SM, while the physical higgs boson is a complete singlet under G_{SM} and therefore might enter through polynomial functions of the physical Higgs field. This scenario is known as Higgs Effective Field Theory (HEFT) or electroweak chrial Lagrangian (EWChL) resembling the chiral Lagrangian of the low energy QCD. Both approaches are perfectly viable and we will consider them separately in the following subsections, where we introduce the building blocks for both, discuss power counting for each case and demonstrate LO and NLO effective Lagrangians. For an extensive overview of the topic see also CERN "Handbook of LHC Higgs Cross Sections 4" [82].

1.3.1 SMEFT: Linear realisation of EW symmetry

Assuming that the BSM physics is weakly coupled and well separated from the SM, it can be represented by a cutoff scale Λ being for example the mass of the lightest BSM state. The Lagrangian of the linear EFT can be written simply as power series on Λ. Assuming in addition lepton and baryon number conservation, the lowest order correction is suppressed by two powers of cutoff scale

$$\mathcal{L}_{\text{linear}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{c_i}{\Lambda^2} O_i + O\left(\frac{1}{\Lambda^4}\right). \tag{1.107}$$

The set of operators O_i of spacial dimension 6 represent linear independent basis parametrising corrections to the SM Lagrangian originating from the BSM physics, preserving the symmetries assumed. Giving up the lepton number symmetry, there is only one $d = 5$ operator of the form $(lH)^2$ providing Majorana mass term for the neutrinos and thus violating the lepton number by

33

X^3		H^6 and H^4D^2		$\psi^2 H^3$	
Q_G	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	Q_H	$(H^\dagger H)^3$	Q_{eH}	$(H^\dagger H)(\bar{l}_p e H)$
$Q_{\widetilde{G}}$	$f^{ABC}\widetilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$Q_{H\square}$	$(H^\dagger H)\square(H^\dagger H)$	Q_{uH}	$(H^\dagger H)(\bar{q}_p u \widetilde{H})$
Q_W	$\varepsilon^{abc}W_\mu^{a\nu}W_\nu^{b\rho}W_\rho^{c\mu}$	Q_{HD}	$\left(H^\dagger D^\mu H\right)^*\left(H^\dagger D_\mu H\right)$	Q_{dH}	$(H^\dagger H)(\bar{q}_p d H)$
$Q_{\widetilde{W}}$	$\varepsilon^{abc}\widetilde{W}_\mu^{a\nu}W_\nu^{b\rho}W_\rho^{c\mu}$				

$X^2 H^2$		$\psi^2 X H$		$\psi^2 H^2 D$	
Q_{HG}	$H^\dagger H\, G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$(\bar{l}\sigma^{\mu\nu}e)\tau^a H W_{\mu\nu}^a$	$Q_{Hl}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{l}\gamma^\mu l)$
$Q_{H\widetilde{G}}$	$H^\dagger H\, \widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar{l}\sigma^{\mu\nu}e)H B_{\mu\nu}$	$Q_{Hl}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}_\mu^I H)(\bar{l}\tau^a\gamma^\mu l)$
Q_{HW}	$H^\dagger H\, W_{\mu\nu}^a W^{a\mu\nu}$	Q_{uG}	$(\bar{q}\sigma^{\mu\nu}\lambda^A u)\widetilde{H}\, G_{\mu\nu}^A$	Q_{He}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{e}\gamma^\mu e)$
$Q_{H\widetilde{W}}$	$H^\dagger H\, \widetilde{W}_{\mu\nu}^a W^{a\mu\nu}$	Q_{uW}	$(\bar{q}\sigma^{\mu\nu}u)\tau^a\widetilde{H}\, W_{\mu\nu}^a$	$Q_{Hq}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{q}\gamma^\mu q)$
Q_{HB}	$H^\dagger H\, B_{\mu\nu}B^{\mu\nu}$	Q_{uB}	$(\bar{q}\sigma^{\mu\nu}u)\widetilde{H}\, B_{\mu\nu}$	$Q_{Hq}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}_\mu^I H)(\bar{q}\tau^a\gamma^\mu q)$
$Q_{H\widetilde{B}}$	$H^\dagger H\, \widetilde{B}_{\mu\nu}B^{\mu\nu}$	Q_{dG}	$(\bar{q}\sigma^{\mu\nu}\lambda^A d)H\, G_{\mu\nu}^A$	Q_{Hu}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{u}\gamma^\mu u)$
Q_{HWB}	$H^\dagger\tau^a H\, W_{\mu\nu}^a B^{\mu\nu}$	Q_{dW}	$(\bar{q}\sigma^{\mu\nu}d)\tau^a H W_{\mu\nu}^a$	Q_{Hd}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{d}\gamma^\mu d)$
$Q_{H\widetilde{W}B}$	$H^\dagger\tau^a H\, \widetilde{W}_{\mu\nu}^a B^{\mu\nu}$	Q_{dB}	$(\bar{q}\sigma^{\mu\nu}d)H\, B_{\mu\nu}$	Q_{Hud}	$i(\widetilde{H}^\dagger D_\mu H)(\bar{u}\gamma^\mu d)$

Table 1.2: Dimension-six operators with up to two fermion fields in Warsaw basis.

two units, $\Delta L = 2$ [83]. The fact that this operator violates the accidental symmetry of the SM can be related to a bigger suppression scale Λ. Therefore operators of $d = 6$, preserving all the symmetries of the SM, are expected to give more significant contributions.

The building blocks for the effective operators are the SM fields and their functions, which are covariant under G_{SM} : H – Higgs field transforming linearly under G_{SM}, the covariant derivatives D_μ, the field strengths of the gauge bosons $V_{\mu\nu}$ and its dual $\tilde{V}_{\mu\nu} = \varepsilon^{\mu\nu\rho\sigma}V_{\rho\sigma}$ and finally the fermionic quark and lepton fields.

The exact form of the basis is not uniquely defined since it can be changed by applying the Equations of Motions (EOM) derived from the $d = 4$ Lagrangian to the $d = 6$ operators and by integration by parts. The choice of the basis in each particular study is motivated by the set of phenomenological processes one is planning to analyse, nonetheless since they all represent the basis of operators, different choices of it have the same phenomenological consequences [84].

The complete basis of $d = 6$ operators was obtained for the first time by Buchmuller et al. in Ref. [85]. Further on, some modifications of this basis have been proposed in [86]. Ref. [87] proposed a linear basis under assumption of the pNGB Higgs originating from strong dynamics and finally Grzadkowski et al [88] have removed the redundancies of Buchmuller et al. basis obtaining 59 independent baryon number conserving operators. The latter is also known as Warsaw basis. The list of the linear bosonic and two fermion operators in the Warsaw basis is given in Table 1.2, while

	$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$		$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$
Q_{ll}	$(\bar{l}\gamma_\mu l)(\bar{l}\gamma^\mu l)$	Q_{ee}	$(\bar{e}\gamma_\mu e)(\bar{e}\gamma^\mu e)$	Q_{le}	$(\bar{l}\gamma_\mu l)(\bar{e}\gamma^\mu e)$	Q_{ledq}	$(\bar{l}e)(\bar{d}q)$
$Q_{qq}^{(1)}$	$(\bar{q}\gamma_\mu q)(\bar{q}\gamma^\mu q)$	Q_{uu}	$(\bar{u}\gamma_\mu u)(\bar{u}\gamma^\mu u)$	Q_{lu}	$(\bar{l}\gamma_\mu l)(\bar{u}\gamma^\mu u)$	$Q_{quqd}^{(1)}$	$(\bar{q}u)\varepsilon_{jk}(\bar{q}d)$
$Q_{qq}^{(3)}$	$(\bar{q}\gamma_\mu \tau^a q)(\bar{q}\gamma^\mu \tau^a q)$	Q_{dd}	$(\bar{d}\gamma_\mu d)(\bar{d}\gamma^\mu d)$	Q_{ld}	$(\bar{l}\gamma_\mu l)(\bar{d}\gamma^\mu d)$	$Q_{quqd}^{(8)}$	$(\bar{q}\lambda^A u)\varepsilon_{jk}(\bar{q}\lambda^A d)$
$Q_{lq}^{(1)}$	$(\bar{l}\gamma_\mu l)(\bar{q}\gamma^\mu q)$	Q_{eu}	$(\bar{e}\gamma_\mu e)(\bar{u}\gamma^\mu u)$	Q_{qe}	$(\bar{q}\gamma_\mu q)(\bar{e}\gamma^\mu e)$	$Q_{lequ}^{(1)}$	$(\bar{l}e)\varepsilon_{jk}(\bar{q}u)$
$Q_{lq}^{(3)}$	$(\bar{l}\gamma_\mu \tau^a l)(\bar{q}\gamma^\mu \tau^a q)$	Q_{ed}	$(\bar{e}\gamma_\mu e)(\bar{d}\gamma^\mu d)$	$Q_{qu}^{(1)}$	$(\bar{q}\gamma_\mu q)(\bar{u}\gamma^\mu u)$	$Q_{lequ}^{(3)}$	$(\bar{l}\sigma_{\mu\nu}e)\varepsilon_{jk}(\bar{q}\sigma^{\mu\nu}u)$
		$Q_{ud}^{(1)}$	$(\bar{u}\gamma_\mu u)(\bar{d}\gamma^\mu d)$	$Q_{qu}^{(8)}$	$(\bar{q}\gamma_\mu \lambda^A q)(\bar{u}\gamma^\mu \lambda^A u)$		
		$Q_{ud}^{(8)}$	$(\bar{u}\gamma_\mu \lambda^A u)(\bar{d}\gamma^\mu \lambda^A d)$	$Q_{qd}^{(1)}$	$(\bar{q}\gamma_\mu q)(\bar{d}\gamma^\mu d)$		
				$Q_{qd}^{(8)}$	$(\bar{q}\gamma_\mu \lambda^A q)(\bar{d}\gamma^\mu \lambda^A d)$		

Table 1.3: Barion number preserving four-fermion operators in Warsaw basis.

the ones with four fermions can are shown in Table 1.3 with the chiral indexes L/R and family indexes of fermions omitted.

1.3.2 HEFT: Nonlinear realisation of EW symmetry

In section 1.1 we have established the non-linear formulation for the scalar sector of the SM. The difference with respect to the linear one is that all the transformation properties of scalars under G_{SM} are carried by the function of only three scalar fields $\mathbf{U}(x) = e^{i\tau\phi/v}$ – the matrix of would-be-Goldstone bosons, while the physical Higgs scalar h remains singlet. This situation is completely analogous to the case of non-linear sigma model in QCD, where the matrix $\mathbf{U}$ describes the dynamics of the physical particles – pion triplet. The effective Lagrangian can be written in terms of $\mathbf{U}$ as expansion in derivatives, corresponding to the expansion in energy. The same can be done to describe the electroweak effective theory.

Higgsless case

Historically, the EWChL was firstly derived for the case of the SM with a heavy Higgs, before its discovery as a rather light state with mass of order EW scale v. Indeed, in the heavy Higgs limit, the physical scalar excitation decouples from the spectrum, producing a tower of effective operators as functions of $\mathbf{U}$, gauge field strengths and fermions. The bosonic basis for this set-up has been derived in [89–92] and is known as Appelquist-Longhitano-Feruglio basis . The Lagrangian is written in terms of the $SU(2)_L$ covariant objects

$$\mathbf{V}_\mu = D_\mu\mathbf{U}\mathbf{U}^\dagger, \qquad \mathbf{T} = \mathbf{U}\tau^3\mathbf{U}^\dagger. \tag{1.108}$$

To form gauge invariant quantities the traces of the product of matrices have to be taken.

The leading order higgsless Lagrangian reads

$$
\mathcal{L}_{LO} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}W_{\mu\nu}^A W_A^{\mu\nu} - \frac{1}{4}G_{\mu\nu}^a G_a^{\mu\nu} - \frac{v^2}{4}\langle \mathbf{V}_\mu \mathbf{V}^\mu \rangle + \tag{1.109}
$$

$$
+ \sum_{\psi=q,u,d} \bar{\psi} i D_\mu \psi - \frac{v}{\sqrt{2}}\left(\bar{q}_L \mathbf{U}\, \mathbf{Y_q}\, q_R^j + \bar{l}_L^i \mathbf{U}\, \mathbf{Y_l}\, l_R^j + h.c. \right), \tag{1.110}
$$

which is simply the SM Lagrangian with the Higgs boson integrated out at leading order. It contains up to two space time derivatives. As before the kinetic term of $\langle \mathbf{V}_\mu \mathbf{V}^\mu \rangle$ in unitary gauge provides mass for the gauge bosons.

The NLO Lagrangian parametrising the deviations from the higgsless SM has been derived assuming two parameters of EFT expansion, the first being the derivative as in QCD case, and the second is the custodial breaking object $\mathbf{T}$. The latter one is motivated by the known phenomenologically small effects of custodial symmetry breaking. The ALF basis contains up to four powers of momenta and $\mathbf{T}$ and can be written as

$$
\mathcal{L}_{\mathrm{NLO}} = c_T \mathcal{P}_T + \sum c_i \mathcal{P}_i. \tag{1.111}
$$

where $\mathcal{P}_i$ are the bosonic operators to be defined later for the case of NLO effective Lagrangian with the light Higgs. The custodial breaking $\mathcal{P}_T$ operator reads

$$
\mathcal{P}_T = \frac{v^2}{4}\langle \mathbf{T} \mathbf{V}_\mu \rangle^2. \tag{1.112}
$$

It contains two powers of $\mathbf{T}$ but its coefficient is known to be parametrically small $c_T < 1.9 \times 10^{-3}$ [87] so it is included at NLO. It also provides a necessary counterterm to absorb divergences arising from the loops of $U(1)_Y$ gauge bosons obtained from the LO Lagrangian.

Non-linear EFT with the light Higgs

With the discovery of the Higgs boson the basis has to be extended to include a new light singlet degree of freedom h. While in the linear case the light physical state is embedded into the doublet H, thus restricting its possible couplings, in the non-linear case h is a singlet and therefore it can be included through arbitrary polynomials, with the coefficients being free parameters which we define as

$$
c_i \mathcal{F}_i(h) = c_i + 2a_i \frac{h}{v} + b_i \frac{h^2}{v^2} + \dots \tag{1.113}
$$

Note that in these parametrisations the natural dependence on h/f expected from the underlying pNGB Higgs models has been traded by h/v: the relative $v/f < 1$ normalization is thus implicitly reabsorbed in the definition of the constant coefficients, which are expected to be small parameters, justifying the expansion.

The leading order Lagrangian with up to two derivatives reads

$$
\mathcal{L}_{LO}^{(h)} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}W_{\mu\nu}^A W_A^{\mu\nu} - \frac{1}{4}G_{\mu\nu}^a G_a^{\mu\nu} + \frac{1}{2}\partial_\mu h \partial^\mu h \mathcal{F}_H(h) - \frac{v^2}{4}\langle \mathbf{V}_\mu \mathbf{V}^\mu \rangle \mathcal{F}_C(h) - V(h),
$$
$$
+ \sum_{\psi=q,u,d} \bar{\psi} i D_\mu \psi - \frac{v}{\sqrt{2}}\left(\bar{q}_L \mathbf{U} \begin{pmatrix} \mathbf{y_u}\mathcal{F}_Y^u(h) & 0 \\ 0 & \mathbf{y_d}\mathcal{F}_Y^d(h) \end{pmatrix} q_R^j + \bar{l}_L^i \mathbf{U}\, \mathbf{Y_l}\mathcal{F}_Y^l(h)\, l_R^j + h.c. \right), \tag{1.114}
$$

$\mathcal{P}_1 = gg'B_{\mu\nu}\langle\mathbf{TW}^{\mu\nu}\rangle$	$\mathcal{P}_{14} = g\epsilon_{\mu\nu\rho\lambda}\langle\mathbf{TV}^{\mu}\rangle\langle\mathbf{V}^{\nu}\mathbf{W}^{\rho\lambda}\rangle$
$\mathcal{P}_2 = ig'B_{\mu\nu}\langle\mathbf{T}[\mathbf{V}^{\mu},\mathbf{V}^{\nu}]\rangle$	$\mathcal{P}_{15} = \langle\mathbf{T}\mathcal{D}_{\mu}\mathbf{V}^{\mu}\rangle\langle\mathbf{T}\mathcal{D}_{\nu}\mathbf{V}^{\nu}\rangle$
$\mathcal{P}_3 = ig\langle\mathbf{W}_{\mu\nu}[\mathbf{V}^{\mu},\mathbf{V}^{\nu}]\rangle$	$\mathcal{P}_{16} = \langle[\mathbf{T},\mathbf{V}_{\nu}]\mathcal{D}_{\mu}\mathbf{V}^{\mu}\rangle\langle\mathbf{TV}^{\nu}\rangle$
$\mathcal{P}_4 = ig'B_{\mu\nu}\langle\mathbf{TV}^{\mu}\rangle\partial^{\nu}(h/v)$	$\mathcal{P}_{17} = ig\langle\mathbf{TW}_{\mu\nu}\rangle\langle\mathbf{TV}^{\mu}\rangle\partial^{\nu}(h/v)$
$\mathcal{P}_5 = ig\langle\mathbf{W}_{\mu\nu}\mathbf{V}^{\mu}\rangle\partial^{\nu}(h/v)$	$\mathcal{P}_{18} = \langle\mathbf{T}[\mathbf{V}_{\mu},\mathbf{V}_{\nu}]\rangle\langle\mathbf{TV}^{\mu}\rangle\partial^{\nu}(h/v)$
$\mathcal{P}_6 = \langle\mathbf{V}_{\mu}\mathbf{V}^{\mu}\rangle^2$	$\mathcal{P}_{19} = \langle\mathbf{T}\mathcal{D}_{\mu}\mathbf{V}^{\mu}\rangle\langle\mathbf{TV}_{\nu}\rangle\partial^{\nu}(h/v)$
$\mathcal{P}_7 = \langle\mathbf{V}_{\mu}\mathbf{V}^{\mu}\rangle\partial_{\nu}\partial^{\nu}(h/v)$	$\mathcal{P}_{20} = \langle\mathbf{V}_{\mu}\mathbf{V}^{\mu}\rangle\partial_{\nu}(h/v)\partial^{\nu}(h/v)$
$\mathcal{P}_8 = \langle\mathbf{V}_{\mu}\mathbf{V}_{\nu}\rangle\partial^{\mu}(h/v)\partial^{\nu}(h/v)$	$\mathcal{P}_{21} = \langle\mathbf{TV}_{\mu}\rangle^2\partial_{\nu}(h/v)\partial^{\nu}(h/v)$
$\mathcal{P}_9 = \langle(\mathcal{D}_{\mu}\mathbf{V}^{\mu})^2\rangle$	$\mathcal{P}_{22} = \langle\mathbf{TV}_{\mu}\rangle\langle\mathbf{TV}_{\nu}\rangle\partial^{\mu}(h/v)\partial^{\nu}(h/v)$
$\mathcal{P}_{10} = \langle\mathbf{V}_{\nu}\mathcal{D}_{\mu}\mathbf{V}^{\mu}\rangle\partial^{\nu}(h/v)$	$\mathcal{P}_{23} = \langle\mathbf{V}_{\mu}\mathbf{V}^{\mu}\rangle\langle\mathbf{TV}_{\nu}\rangle^2$
$\mathcal{P}_{11} = \langle\mathbf{V}_{\mu}\mathbf{V}_{\nu}\rangle^2$	$\mathcal{P}_{24} = \langle\mathbf{V}_{\mu}\mathbf{V}_{\nu}\rangle\langle\mathbf{TV}^{\mu}\rangle\langle\mathbf{TV}^{\nu}\rangle$
$\mathcal{P}_{12} = g^2\langle\mathbf{TW}_{\mu\nu}\rangle^2$	$\mathcal{P}_{25} = \langle\mathbf{TV}_{\mu}\rangle^2\partial_{\nu}\partial^{\nu}(h/v)$
$\mathcal{P}_{13} = ig\langle\mathbf{TW}_{\mu\nu}\rangle\langle\mathbf{T}[\mathbf{V}^{\mu},\mathbf{V}^{\nu}]\rangle$	$\mathcal{P}_{26} = (\langle\mathbf{TV}_{\mu}\rangle\langle\mathbf{TV}_{\nu}\rangle)^2$

$$\mathcal{P}_{\Box H} = \frac{1}{v^2}\Box h\Box h\mathcal{F}_{\Box H}(h) \qquad \mathcal{P}_{\Delta H} = \frac{1}{v^3}\partial_{\mu}h\partial^{\mu}h\Box h\mathcal{F}_{\Delta H}(h) \qquad \mathcal{P}_{DH} = \frac{1}{v^3}(\partial_{\mu}h\partial^{\mu}h)^2\mathcal{F}_{DH}(h)$$

Table 1.4: Full bosonic NLO basis of non-linear CP-even operators of [93] with the pure higgs operators added.

The difference with respect to the higgsless Lagrangian is the kinetic term for the Higgs, extended by $\mathcal{F}_H(h)$, the Higgs potential and the polynomial functions $\mathcal{F}_i(h)$, appended to the kinetic term of would-be-Goldstones and fermionic Yukawa couplings. This Lagrangian reproduces the SM Lagrangian for the following choice of functions $\mathcal{F}(h)$

$$\mathcal{F}_C = \left(1 + \frac{h}{v}\right)^2, \qquad \mathcal{F}_Y^{u,d,l} = 1 + \frac{h}{v}, \qquad \mathcal{F}_H(h) = 1. \tag{1.115}$$

Present data set strong constraints on departures from SM expectations for the a_C and b_C, while a_H and b_H still can be large.

Note that the Lagrangian is written for the physical particle after EWSB, therefore it is assumed that $V(h)$ must provide no VEV to the scalar: $\langle h \rangle = 0$.

The full bosonic basis with the light Higgs at NLO reads

$$\mathcal{L}_{\text{NLO}}^{(h)} = c_T\mathcal{P}_T\mathcal{F}_T(h) + \sum c_i\mathcal{P}_i\mathcal{F}_i(h). \tag{1.116}$$

Some of the relevant operators have been mentioned in [94–96], while the full basis of bosonic CP-even operators has been derived in [93]. Later, Ref. [97] considered the full basis including fermionic operators, pure higgs operators and trading some of the bosonic operators for the fermionic ones using equations of motion. In Ref. [98] the full bosonic basis of CP-odd operators has been derived. In Ref. [99] B-number violating operators have been added.

The basis is presented in the Table 1.4 with the notations consistent with Ref. [32, 70]. It contains

1. The operators equivalent to ones in ALF basis, namely $\mathcal{P}_{1-3}, \mathcal{P}_6, \mathcal{P}_{11-14}\mathcal{P}_{23-24}$ and $\mathcal{P}_{26}$.

37

2. New operators with the derivative acting on the Higgs field, those are $\mathcal{P}_{4-5}, \mathcal{P}_{7-8}, \mathcal{P}_{10}, \mathcal{P}_{17-22}$ and $\mathcal{P}_{25}$

3. Operators $\mathcal{P}_{9-10}$ and $\mathcal{P}_{15}$ have no derivatives on h, but are rewritten in different form in comparison to the usual ALF basis.

4. Pure Higgs operators $\mathcal{P}_{\square H}, \mathcal{P}_{\Delta H}$ and $\mathcal{P}_{DH}$.

Counting and renormalization

There is a striking difference between the loop structures of the linear and non-linear Lagrangians. Indeed, the LO SMEFT is the SM, it is a renormalizable theory , which in principle can be used to calculate the observables at any precision, involving as many loop orders as needed. The diagrams with arbitrary number of loops and vertexes belonging to $\mathcal{L}_{LO}^{Lin}$ will generate divergent contributions only to $\mathcal{L}_{LO}^{Lin}$ and therefore $\mathcal{L}_{NLO}^{Lin}$ is not required for the theoretical consistency. Loops with one insertion of NLO vertexes will generate only divergences that are to be absorbed by the parameters of $\mathcal{L}_{NLO}^{Lin}$. The insertion of two vertexes from $\mathcal{L}_{NLO}^{Lin}$ or one from $\mathcal{L}_{NNLO}^{Lin}$ will renormalize $\mathcal{L}_{NNLO}^{Lin}$ terms and so on. Note that this rule does not depend on the number of loops contained in a diagram.

The situation is not the same for non-linear Lagrangian. Indeed, unless one restricts the parameters of $\mathcal{F}_i(h)$ to the SM case, even the LO HEFT Lagrangian is not renormalizable and the counterterms needed to render the theory finite do not have the structure of $\mathcal{L}_{LO}^{NL}$. *At one loop* they will generate divergences proportional to the structures of to $\mathcal{L}_{NLO}^{NL}$ as it is demonstrated for example in original papers [90, 91] and in Chapter 4. Beyond one loop NNLO Lagrangian is required for renormalization. The non-linear expansion thus is an example of non-decoupling EFT, a theory where higher order terms of EFT expansion have to be included to perform renormalization. Strictly speaking the number of counterterms for arbitrary number of loops is infinite. For one loop though the counterterms needed are the ones presented in Table 1.4.

A more rigorous way to define the order at which a given operator should appear and estimate its coefficient suppression relies on the use of power counting formula. Naive dimensional analysis formula [58], modified in order to include the weight of the gauge couplings by [100, 101], and applied for the HEFT case [102] reads

$$\frac{\Lambda^4}{16\pi^2} \left[\frac{\partial}{\Lambda}\right]^{N_P} \left[\frac{4\pi\,\phi}{\Lambda}\right]^{N_\phi} \left[\frac{4\pi\,A}{\Lambda}\right]^{N_A} \left[\frac{4\pi\,\psi}{\Lambda^{3/2}}\right]^{N_\psi} \left[\frac{g}{4\pi}\right]^{N_g} \left[\frac{y}{4\pi}\right]^{N_y} \left[\frac{\lambda}{16\pi^2}\right]^{N_\lambda}. \tag{1.117}$$

This formula reproduces the correct order 1 coefficient for the leading order terms, including the canonically normalized gauge kinetic terms, as well as reveals the suppression of beyond LO terms.
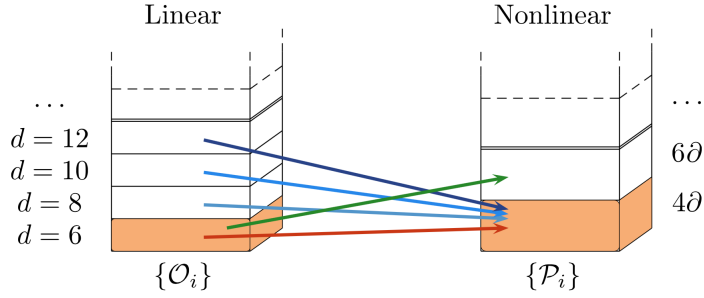
Figure 1.6: A representation of the correspondence between SMEFT and HEFT operators. The first order deviations from the SM are highlighted in color [103].

1.3.3 SMEFT vs. HEFT

The correspondence between the two approaches for BSM EFT can be summarised as follows [24, 70, 82, 103–108]

- **Higgs couplings.** Since in HEFT the Higgs is a singlet under G_{SM} its couplings are completely free parameters, while in the SMEFT they are restricted by gauge symmetry, due to the fact that the Higgs belongs to a doublet, and therefore might be correlated to the pure gauge couplings.

- **Power counting.** SMEFT is organised as an expansion in terms of canonical mass dimension with higher order (d>4) operators suppressed by corresponding powers of the cutoff scale Λ. In HEFT, with the matrix of Goldstones **U** being dimensionless, the counting is according to number of derivatives, similar to the counting of QCD. Moreover Higgs insertions are are considered all at once as arbitrary functions $\mathcal{F}(h)$, with all the powers of Higgs field h belonging to the same order of expansion.

- **Triple and quartic gauge couplings.** Furthermore, triple and quartic gauge and Higgs-gauge couplings are correlated in the SMEFT, while in HEFT they remain decorrelated. The observation of such decorrelated couplings would point to the validity of HEFT.

- **Couplings strength.** Some couplings that are expected to be strongly suppressed in the SMEFT, are instead predicted with higher strength in the HEFT and are potentially visible in the present LHC run.

The correspondence between SMEFT and HEFT can be identified simply by setting

$$H = \frac{v + h}{\sqrt{2}} \mathbf{U} \begin{pmatrix} 0 \\ 1 \end{pmatrix} . \tag{1.118}$$

Each SMEFT operator then corresponds to a combination of *sibling* operators of HEFT.

The two EFTs do not match each other at a given order. Instead, in general the correspondence connects operators of different orders in both expansions. This situation is depicted in Fig. 1.6. Moreover, the HEFT expansion contains more free parameters at any given order.

In the Chapter 3 we derive the effective non-linear Lagrangian of the linear Goldstone Higgs model developed before that in Chapter 2. The resulting Lagrangian will contain only a subset of the parameters of the general HEFT.

2

The minimal linear sigma model for the Goldstone Higgs

In this chapter we construct a UV complete model of the Goldstone Higgs and study its possible experimental validation at LHC as well as possible low energy effects. Out of many possible options for the symmetry breaking pattern, we will focus on the minimal case of $SO(5)/SO(4)$ coset. Most of the literature based on this coset assumes from the start a strong dynamics and uses an effective non-linear formulation of the model(s) [10, 15–21]. This approach has the advantage of being quite general, offering a parametrisation of all possible ultraviolet completions for the symmetry group chosen. At the same time one of its limitations is that it is applicable only in a finite domain of energies. Instead, we consider a renormalizable model which in its scalar part is a linear sigma model including a new heavy scalar particle σ, singlet under the gauge group. Our model can be considered either as an ultimate model made out of elementary fields, or as a renormalizable version of a deeper dynamics, much as the linear σ model is to QCD. One former attempt in this direction [22] did not fully take into account and computed the impact of the fermionic sector on the main phenomenological observables. Later, linearly completed models have been considered in Refs. [23–26, 31, 32] and extended the previous analyses.

While the choice of the minimal bosonic sector is clear, there is a number of possible choices for the fermionic sector. The option explored in this section assumes heavy fermions in vectorial representations of $SO(5)$, in contrast to models where the SM left doublets are embedded in $SO(5)$ multiplets [22]. Direct couplings between SM fermions and the heavy fermions will be the source of the soft $SO(5)$ breaking, while the Higgs particle has tree-level couplings only with the exotic fermionic sector, via $SO(5)$-invariant Yukawa couplings. It will be discussed how the induced Coleman-Weinberg potential requires soft breaking terms to be included in the scalar potential.

The usual SM Higgs sector is now substituted by a Higgs-σ sector, correcting the strength of the SM Higgs-gauge boson couplings and opening new interaction channels. The phenomenology of the σ production and decay will be also studied, including fermionic and bosonic tree-level and one loop decays (e.g. gluon-gluon and photon-photon). Analysis of present Higgs data will be used to set a constraint on the fine-tuning ratio v/f. The contribution of the Higgs, σ and the exotic fermions to the oblique S and T parameters will be computed. Particular emphasis will be

dedicated to the impact of the σ particle on present and future LHC data, produced either via gluon fusion or vector-boson fusion and decaying into a plethora of channels including diphoton final state.

Furthermore, we identify some of the leading low-energy bosonic operators stemming from the new physics when the exotic heavy fermion sector is integrated out. We determine the dominant effective operators made out of the σ field and SM fields, as a first step towards the identification of a "benchmark" electroweak effective Lagrangian, considered in Chapter 3.

2.1 The $SO(5)/SO(4)$ scalar sector

The complete Lagrangian can be written as the sum of three terms describing respectively the pure gauge, scalar and fermionic sectors,

$$\mathcal{L} = \mathcal{L}_\mathrm{g} + \mathcal{L}_\mathrm{s} + \mathcal{L}_\mathrm{f}, \tag{2.1}$$

where $\mathcal{L}_\mathrm{gauge}$ reduces to the SM gauge kinetic terms. This section discusses in detail the scalar sector and its interactions, while the study of the fermionic sector is deferred to the next section.

In order to define the linear σ model corresponding to an $SO(5)$ symmetry spontaneously broken to $SO(4)$, let us consider a real scalar field ϕ in the fundamental representation of $SO(5)$. Three among its five components will be ultimately associated with the longitudinal components of the SM gauge bosons ϕ_i, $i = 1, 2, 3$, while the other two will correspond to the Higgs particle h and to an additional scalar σ, respectively. For simplicity the results will be often presented in the unitary gauge $(u.g.)$, in which $\phi_i = 0$:

$$\phi = (\phi_1, \phi_2, \phi_3, h, \sigma)^T \quad \overset{u.g.}{\Rightarrow} \quad (0, 0, 0, h, \sigma)^T \ . \tag{2.2}$$

The scalar Lagrangian describing the scalar-gauge and the scalar-scalar interactions reads

$$\mathcal{L}_\mathrm{s} = \frac{1}{2}(D_\mu \phi)^T (D^\mu \phi) - V(\phi) \,, \tag{2.3}$$

where the $SU(2)_L \times U(1)_Y$ covariant derivative is given by

$$D_\mu \phi = \left(\partial_\mu + ig\Sigma_L^i W_\mu^i + ig'\Sigma_R^3 B_\mu \right) \phi \tag{2.4}$$

and Σ_L^i and Σ_R^i as before are the generators of the $SU(2)_L$ and $SU(2)_R$ subgroups of the custodial $SO(4)$ group contained in $SO(5)$. The embedding of the gauge group $SU(2)_L \times U(1)_Y$ inside $SO(5)$, implicitly assumed in Eqs. (2.2-2.4), is purely conventional. As we will see in section 2.1, both h and σ acquire a vacuum expectation value (vev), leaving unbroken an $SO(4)'$ subgroup which is rotated with respect to the group $SO(4) \approx SU(2)_L \times SU(2)_R$ containing $SU(2)_L \times U(1)_Y$.

For later convenience it is pertinent to introduce the complex notation for the scalar field ϕ in the fundamental representation of $SO(5)$

$$\hat{\phi} = \left(H^T, \widetilde{H}^T, \sigma \right)^T \ . \tag{2.5}$$

The relation between the real and the complex notation is given by

$$\phi = \frac{1}{\sqrt{2}} \left(-i(H^u + \tilde{H}^d) \, , \, H^u - \tilde{H}^d \, , \, i(H^d - \tilde{H}^u) \, , \, H^d + \tilde{H}^u \, , \, \sqrt{2}\sigma \right)^T .$$

(2.6)

2.1.1   The scalar potential

The most general $SO(4)$ preserving while $SO(5)$ breaking renormalizable potential depends a priori on ten parameters. Two of them can be reabsorbed via a redefinition of parameters [1], resulting on a Lagrangian dependent on one $SO(5)$ preserving coupling, λ, one scale f heralding spontaneous $SO(5)/SO(4)$ breaking, and six $SO(5)$ soft-breaking terms (denoted below $\alpha, \beta, a_{1,2,3,4}$). The Lagrangian in the unitary gauge reads:

$$V(h, \sigma) = \lambda \left(\sigma^2 + h^2 - f^2 \right)^2 + \alpha f^3 \sigma - f^2 \beta h^2 + a_1 \, f \, \sigma h^2 + a_2 \, \sigma^2 h^2 + a_3 \, f \, \sigma^3 + a_4 \, h^4 .$$

(2.7)

In order to retrieve the formulae in a general gauge it suffices to replace h^2 by the $SO(4)$ invariant combination $h^2 + \boldsymbol{\phi}^2$.

The only strictly necessary soft breaking terms are α and β as they need to be present to absorb divergences generated by one-loop Coleman-Weinberg contributions to the Lagrangian, as shown in Appendix A; only those terms will be considered in what follows [2], a procedure already previously adopted in Ref. [22]. The potential then reads

$$V(h, \sigma) = \lambda \left(h^2 + \sigma^2 - f^2 \right)^2 + \alpha f^3 \sigma - \beta f^2 h^2 ,$$

(2.8)

resulting on a system depending on four parameters. The scalar quartic coupling λ can be conventionally traded by the σ mass, given by $m_\sigma^2 \simeq 8\lambda f^2$ for negligible α and β; the non-linear model would be recovered in the limit $m_\sigma \gg f$, that is $\lambda \gg 1$.

A consistent electroweak (EW) symmetry breaking requires both scalars h, σ to acquire a non-vanishing vev, respectively dubbed as v and v_σ below, as for $v \neq 0$ the $SO(4)$ global group and the EW group are spontaneously broken. Note that the vev of h is identified with the electroweak scale since it can be related to the Fermi constant precisely as in the SM, see Sect. 2.1.2 below. For $\alpha, \beta \neq 0$ and assuming $v \neq 0$, it results

$$v_\sigma^2 = f^2 \frac{\alpha^2}{4\beta^2} \quad , \qquad v^2 = f^2 \left(1 - \frac{\alpha^2}{4\beta^2} + \frac{\beta}{2\lambda} \right) ,$$

(2.9)

satisfying the condition

$$v^2 + v_\sigma^2 = f^2 \left(1 + \beta/2\lambda \right) ,$$

(2.10)

---

[1] Here we choose to get rid of the σ^2 and σ^4 terms.

[2] Full renormalizability of the theory requires, in general, the presence of all gauge invariant operators of dimension equal to or smaller than four. At two or more loops, the renormalization procedure may thus require to include further symmetry breaking terms beyond those considered; we will assume that their finite contributions will be weighted by comparatively negligible coefficients and can be safely omitted in our analysys.

which indicates that the $SO(5)$ vev is "renormalized" by the β term in the potential. From Eqs. (2.9) and (2.10) it follows that both $f^2 > 0$ and $f^2 < 0$ are in principle allowed [3], in appropriate regions of the parameters (α, β, λ). However, in the $SO(5)$-invariant limit, for negative f^2 the minimum of the potential is at the origin and in consequence the symmetry is unbroken and there are no Goldstone bosons. The focus of this section is instead set on the interpretation of the Higgs particle as a PNGB, which requires $f^2 > 0$ as well as $|v| < |v_\sigma|$, the latter condition defining the region in parameter space continuously connected with the limiting case $v = 0$ in which the Higgs particle becomes a true Goldstone boson. For $f^2 > 0$, the positivity of v^2 in Eq. (2.9) and the $|v| < |v_\sigma|$ constraint lead respectively to the conditions [4]

$$\alpha^2 < 4\beta^2 \left(1 + \frac{\beta}{2\lambda}\right), \tag{2.11}$$

$$2\beta^2 \left(1 + \frac{\beta}{2\lambda}\right) < \alpha^2, \tag{2.12}$$

which for $|\beta| \ll \lambda$ would indicate $2\beta^2 \lesssim \alpha^2 \lesssim 4\beta^2$. Moreover, in order to get $v^2 \ll f^2$, Eq. (2.9) requires a fine-tuning such that $\alpha/2\beta$ is very close to unity.

Expanding the σ and h fields around their minima, $h \equiv \hat{h} + v$ and $\sigma \equiv \hat{\sigma} + v_\sigma$, and diagonalizing the scalar mass matrix, the mass eigenstates are given by

$$h_{\text{phys}} = \hat{h} \cos\gamma - \hat{\sigma} \sin\gamma \quad , \quad \sigma_{\text{phys}} = \hat{\sigma} \cos\gamma + \hat{h} \sin\gamma. \tag{2.13}$$

For simplicity, from now on the notation h_{phys} and σ_{phys} will be traded by h and σ, respectively. The mixing angle in Eq. (2.13) is given by

$$\tan 2\gamma = \frac{4vv_\sigma}{3v_\sigma^2 - v^2 - f^2} \tag{2.14}$$

and should remain in the interval $\gamma \in [-\pi/4, \pi/4]$ in order not to interchange the roles of the heavy and light mass eigenstates. The mass eigenvalues are given by

$$m^2_{\text{heavy, light}} = 4\lambda f^2 \left\{ \left(1 + \frac{3}{4}\frac{\beta}{\lambda}\right) \pm \left[1 + \frac{\beta}{2\lambda}\left(1 + \frac{\alpha^2}{2\beta^2} + \frac{\beta}{8\lambda}\right)\right]^{1/2} \right\}, \tag{2.15}$$

where the plus sign refers to the heavier eigenstate. For $f^2 > 0$, the squared masses are positive if the following two conditions are satisfied [5]

$$3\beta + 4\lambda > 0 \quad , \qquad 2\beta^2 + 4\beta\lambda - \alpha^2\lambda/\beta > 0, \tag{2.16}$$

with the second constraint coinciding with that in Eq. (2.11) multiplying it by $1/(4\beta\lambda)$; it follows that $\beta > 0$. If the soft mass term proportional to β in the scalar potential Eq. (2.8) would be overall positive (as for instance for $f^2 < 0$ and $\beta > 0$), the minimum would always correspond to

---

[3]For $f^2 < 0$, α would have to be purely imaginary because of hermiticity.

[4]For $f^2 < 0$, the inequality Eq. (2.11) is reverted.

[5]For $f^2 < 0$, both inequalities in Eq. (2.16) are reverted and as a consequence $\beta < 0$.

an undesired symmetric EW vacuum $v = 0$. Assuming the $SO(5)$ explicit breaking to be small, $|\beta|/4\lambda \ll 1$ which may only happen for positive f^2, the masses of the heavy and light eigenstates read

$$
\begin{aligned}
m^2_{\text{heavy}} &= 8\lambda f^2 + 2\beta(3f^2 - v^2) + O\left(\frac{\beta}{4\lambda}\right), \\
m^2_{\text{light}} &= 2\beta v^2 + O\left(\frac{\beta}{4\lambda}\right).
\end{aligned}
\tag{2.17}
$$

The physical scalars thus correspond to a "light" state with mass $O(\sqrt{\beta}v)$ and a "heavy" state with mass $O(\sqrt{\lambda}f)$. It will be later shown that, for a PNGB Higgs particle (that is $v < v_\sigma$ and $f^2 > 0$), the less fine-tuned regions in parameter space correspond to the case $m_{\text{light}} = m_h$ and $m_{\text{heavy}} = m_\sigma$ in the equations above. In fact, would the σ particle be lighter than the Higgs, the roles of the lighter and heavier eigenstates would be flipped and the mixing angle γ will be necessarily outside the region quoted above. The lighter σ scenario is quite different from the typical Higgs PNGB scenarios considered in the literature.

Notice that for $m_h < m_\sigma$ and at variance with the SM case, in the regime of small soft $SO(5)$ breaking the mass of the Higgs and its quartic self-coupling are controlled by two different parameters, β and λ, respectively. This is consistent with the PNGB nature of the Higgs boson whose mass should now appear protected from growing in the strong interacting regime of the theory –corresponding to large λ– in which instead the σ mass would increase. In other words, we have replaced the hierarchy problem for the Higgs particle mass by a sensitivity of the σ particle to heavier scales: the σ mass represents generically the heavy UV completion. The expression for m_h shows that the value of the β parameter for small $\beta/4\lambda$ is expected to be $\beta \sim m_h^2/2v^2 \sim 0.13$.

2.1.2 Scalar-gauge boson couplings

In the unitary gauge, the kinetic scalar Lagrangian written in terms or the unrotated fields reads

$$
\mathcal{L}_{s,kin} = \frac{1}{2}(\partial_\mu \hat{\sigma})^2 + \frac{1}{2}(\partial_\mu \hat{h})^2 + \frac{g^2}{4}\left(\hat{h} + v\right)^2 W_\mu^+ W^{\mu-} + \frac{(g^2 + g'^2)}{8}\left(\hat{h} + v\right)^2 Z_\mu Z^\mu,
$$

and justifies the previous identification of the Higgs vev v with the electroweak scale defined from the W mass.

The rotation to the physical h, σ fields results in the following Lagrangian for the scalar and scalar-gauge interactions, for $m_h < m_\sigma$,

$$
\begin{aligned}
\mathcal{L}_s &= \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu h)^2 - \frac{1}{2}m_\sigma^2 \sigma^2 - \frac{1}{2}m_h^2 h^2 - \lambda\left(h^2 + 2h\sigma + \sigma^2\right)^2 - \\
&\quad - 4\lambda(v\cos\gamma - v_\sigma \sin\gamma)\left(h^3 + h\sigma^2\right) - 4\lambda(v\sin\gamma + v_\sigma \cos\gamma)\left(\sigma^3 + h^2\sigma\right) + \\
&\quad + (1 + \frac{h}{v}\cos\gamma + \frac{\sigma}{v}\sin\gamma)^2\left(M_W^2 W_\mu^+ W^{\mu-} + \frac{1}{2}M_Z^2 Z_\mu Z^\mu\right).
\end{aligned}
\tag{2.18}
$$

The physical Higgs couplings are thus seen to be weighted down by a $\cos\gamma$ factor with respect to the SM Higgs ones, while the σ field acquires the same interactions than h albeit weighted down

by $\sin \gamma$, a fact rich in phenomenological consequences to be discussed further below. The SM limit is recovered when the σ field is decoupled from the spectrum and $\cos \gamma = 1$ follows from Eqs. (2.9) and (2.14). Conversely, for $m_h > m_\sigma$ the mixing dependence would correspond to the interchange $\cos \gamma \leftrightarrow \sin \gamma$ in Eq. (2.18).

2.1.3 Renormalization and scalar tree-level decays

The four independent parameters of the scalar Lagrangian can be expressed in terms of four observables, which we choose to be:

$$G_F \equiv (\sqrt{2}v^2)^{-1}, \qquad m_h, \qquad m_\sigma, \qquad \sin \gamma, \qquad (2.19)$$

with the Fermi constant G_F as measured from muon decay and m_h from the Higgs pole mass, while m_σ could be determined from future measurements of the σ mass, and $\sin \gamma$ from either deviations of the Higgs couplings or from the σ line shape obtained from its decay into four leptons for $m_\sigma \geq 300$ GeV, analogous to the case of a heavy SM Higgs boson [6].

Using Eqs. (2.9), (2.14), and (2.15), the exact expressions for the h and σ vevs in terms of those physical parameters can be obtained

$$v = \left(\sqrt{2}G_F \right)^{-1/2} ,$$
$$v_\sigma = \frac{v \sin(2\gamma)(m_\sigma^2 - m_h^2)}{m_\sigma^2 + m_h^2 - (m_\sigma^2 - m_h^2)\cos(2\gamma)} . \qquad (2.20)$$

These expressions in turn allow to express in terms of measured quantities the four independent parameters of the scalar potential Eq. (2.8), which can be written as

$$\lambda = \frac{\sin^2 \gamma m_\sigma^2}{8v^2} \left(1 + \cot^2 \gamma \frac{m_h^2}{m_\sigma^2} \right) ,$$

$$\frac{\beta}{4\lambda} = \frac{m_h^2 m_\sigma^2}{\sin^2 \gamma m_\sigma^4 + \cos^2 \gamma m_h^4 - 2m_h^2 m_\sigma^2} ,$$

$$\frac{\alpha^2}{4\beta^2} = \frac{\sin^2(2\gamma)(m_\sigma^2 - m_h^2)^2}{4(\sin^2 \gamma m_\sigma^4 + \cos^2 \gamma m_h^4 - 2m_h^2 m_\sigma^2)} , \qquad (2.21)$$

$$f^2 = \frac{v^2(\sin^2 \gamma m_\sigma^4 + \cos^2 \gamma m_h^4 - 2m_h^2 m_\sigma^2)}{\left(\sin^2 \gamma m_\sigma^2 + \cos^2 \gamma m_h^2 \right)^2} .$$

The above exact formulae show that the mixing angle γ does not coincide with the parameter $\sqrt{\xi}$, except in the limit $m_\sigma \gg m_h$ (or more precisely $\beta/4\lambda \ll 1$ and $v^2 \ll f^2$), where for sizeable $\sin \gamma$

---

[6]For a lighter σ, the decay width becomes too narrow –possibly even below the experimental resolution– and more ingenious procedures would be required to determine the scalar mixing strength, such as for instance on-shell to off-shell cross section measurements [109].
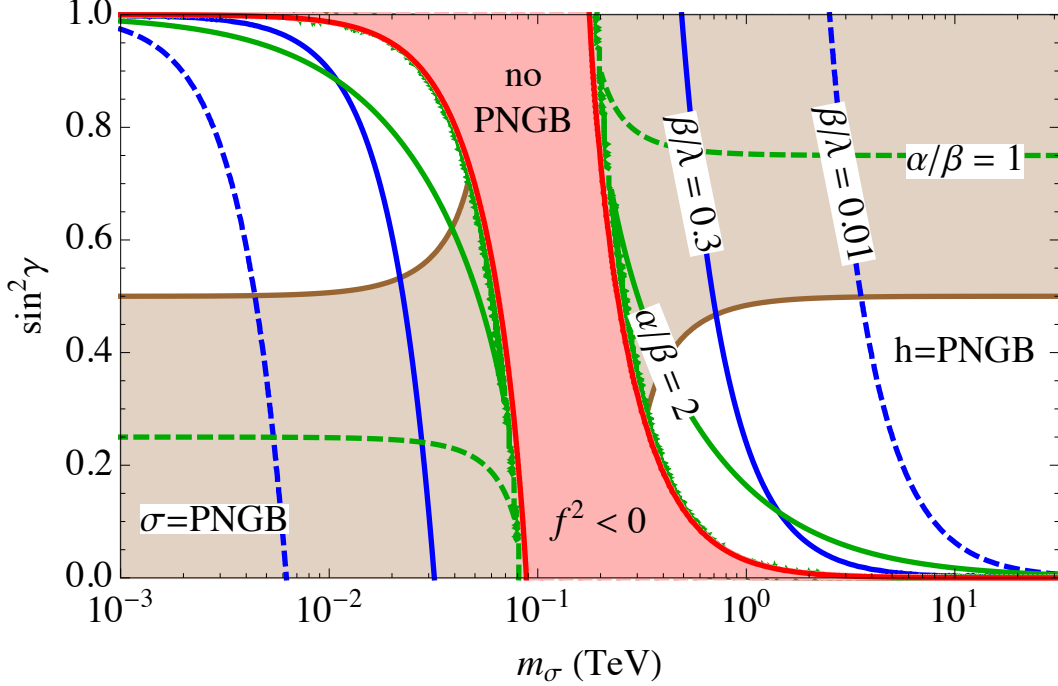
Figure 2.1: m_σ versus $\sin^2\gamma$ parameter space of the scalar sector. The Higgs mass m_h and the Higgs vev v have been fixed to their physical values. The red region corresponds to $f^2 < 0$, for which the $SO(5)$-invariant part of the potential is unbroken and there are no Goldstone bosons in the symmetric limit. The region where $|v| > |v_\sigma|$ is shown in brown (these regions are excluded by Higgs data, see text). The Higgs is a pseudo-Goldstone boson within the white regions at the bottom-right and the top-left part of the plane.

the last equation above leads to

$$\sin^2\gamma \xrightarrow[m_\sigma/m_h \gg 1]{} \frac{v^2}{f^2} + 4\frac{m_h^2}{m_\sigma^2} \,. \tag{2.22}$$

A few comments regarding the parameter space and the scalar spectrum are in order, as arbitrary values of m_σ and $\sin^2\gamma$ are not allowed if we insist on interpreting the Higgs boson as the pseudogoldstone boson of a spontaneous $SO(5)$ breaking. Fig. 2.1 displays the $(m_\sigma, \sin^2\gamma)$ plane: at each point the scalar sector is completely defined as m_h and v are fixed to their physical values. The differently colored regions correspond to

- No $SO(5)$ breaking in the light red region, where $f^2 < 0$; its red borders depict the $f^2 = 0$ frontier;

- The σ particle being the PNGB of the spontaneous breaking of $SO(5)$ in the light brown region, where $v_\sigma < v$;

- The Higgs as the PNGB of the $SO(5) \to SO(4)$ breaking in the white areas, where $v < v_\sigma$ and the Higgs would became a true goldstone boson in the absence of EW breaking ($v \to 0$).

47

A complementary divide is provided by the value of the Higgs mass:

- On the $m_h < m_\sigma$ region to the right of the figure, the physical Higgs couplings to SM particles are weighted down by $\cos\gamma$ with respect to SM values, see for instance Eq. (2.18). It will be shown in the next sections that present LHC Higgs data only allow for values $\sin^2\gamma < 0.18$ at 2σ CL, though, leaving as allowed parameter space a fraction of the lower white (Higgs PNGB) section of the figure. The analysis in the next sections will thus focus in this regime, for which Fig. 2.1 already suggests a lower bound on m_σ of a few hundreds of GeV. The relative importance of the soft breaking terms is also illustrated through the curves depicted for fixed α/β and β/λ;

- On the $m_\sigma < m_h$ area to the right of the figure, the physical Higgs couplings to SM particles are instead weighted down by $\sin\gamma$, whose value will thus be bounded by $\sin^2\gamma > 0.82$ at 2σ CL. It thus remains as available zone the upper part of the upper white (Higgs PNGB) region. Nevertheless, the quartic coupling λ is there very small, typically $\lambda < 10^{-3}$, making the $SO(5)$ invariant potential very flat and potentially unstable against radiative corrections; furthermore, if the soft breaking parameters are required to be small compared to the symmetric term, $\alpha, \beta < \lambda$, their values may require extra fine-tuning with respect to radiative corrections from the fermionic sector to be discussed further below. For these reasons we will not dwell further below on the case $m_\sigma < m_h$ even if phenomenologically of some interest.

Extending the renormalization scheme to the gauge sector, we choose the two extra observables needed to be the mass of the Z boson and the fine structure constant,

$$M_Z, \qquad \alpha_{em} = \frac{e^2}{4\pi}, \tag{2.23}$$

with M_Z and α_{em} as determined from Z-pole mass measurements and from Thompson scattering, respectively [110]. In our model, the relation between the gauge boson masses is the same than that for the SM,

$$M_W = \cos\theta_W M_Z, \tag{2.24}$$

where the weak angle is given at tree-level by

$$\sin^2\theta_W = \frac{1}{2}\left(1 - \sqrt{1 - \frac{4\pi\alpha_{em}}{\sqrt{2}G_F M_Z^2}}\right). \tag{2.25}$$

Using all the above, it is straightforward to compute the relevant tree-level branching ratios for the heavy and light scalar boson decays into SM bosons:

$$
\begin{aligned}
\Gamma(h \to WW^*) &= \Gamma_{\text{SM}}(h \to WW^*)\cos^2\gamma, \\
\Gamma(h \to ZZ^*) &= \Gamma_{\text{SM}}(h \to ZZ^*)\cos^2\gamma, \\
\Gamma(\sigma \to W^+W^-) &= \frac{\sqrt{2}G_F}{16\pi}m_\sigma^3\sin^2\gamma\left[1 + \mathcal{O}\left(\frac{M_W^2}{m_\sigma^2}\right)\right], \\
\Gamma(\sigma \to ZZ) &= \frac{\sqrt{2}G_F}{32\pi}m_\sigma^3\sin^2\gamma\left[1 + \mathcal{O}\left(\frac{M_Z^2}{m_\sigma^2}\right)\right],
\end{aligned}
$$

$$\Gamma(\sigma \to h\,h) \quad = \quad \frac{\sqrt{2}G_F}{32\pi} m_\sigma^3 \sin^2\gamma \left[1 + \mathcal{O}\left(\frac{m_h^2}{m_\sigma^2}\right)\right], \tag{2.26}$$

where the SM widths can be found for instance in Ref. [111]. The σ partial widths above will dominate the total σ width unless the mixing is unnaturally tiny, and thus measuring the branching ratios is not enough to infer the value of the scalar mixing. It is easy to see that

$$\frac{\Gamma_\sigma}{m_\sigma} \simeq \frac{m_\sigma^2 \sin^2\gamma}{8v^2}, \tag{2.27}$$

and thus the measurement of the line shape of the σ seems feasible only for m_σ above the EW breaking scale (assuming non-negligible mixing). In that regime, the value of $\sin\gamma$ can be inferred from the line shape and all other observables in Eq. (2.26) can then be predicted in terms of the physical parameters defining our renormalization scheme. Other bosonic decay channels requiring one-loop amplitudes will be discussed later on.

2.2 Fermionic sector

The fermionic sector is unavoidably an important source of model dependency as diverse choices of $SO(5)$ fermionic multiplets are possible. Moreover the achievement of the desired symmetry breaking pattern in "composite Higgs" models relies on the fermionic sector. A schematic picture is given in Fig. (2.2) considering a high energy global symmetry group - SO(5) in the case under discussion:

- Heavy scalar and fermion representations of the high-energy global symmetry are considered. $SO(5)$ breaks spontaneously to $SO(4)$ at a scale f, resulting in four massless Goldstone bosons: the three longitudinal components of the electroweak gauge bosons and a "Higgs Goldstone boson". The fifth component σ remains massive.

- Furthermore, $SO(5)$ is explicitly broken by the coupling of the exotic heavy representations to the SM fermions (soft breaking) and to the gauge bosons (hard breaking). This induces at one-loop a potential for the h field with a non-trivial minimum, providing a mass for h and breaking the SM electroweak symmetry at a scale $v \neq f$.

Several "minimal" possibilities have been explored in the literature for the exotic fermionic representations (see for instance Refs. [16, 22]). The setup considered here contains:

1. Heavy (exotic) vector-like fermions in complete representations of $SO(5)$, either in the fundamental representation, denoted below by ψ - or singlets denoted by χ.

2. A scalar field ϕ in the fundamental representation of $SO(5)$, which contains the h and σ particles. Its vev breaks $SO(5)$ spontaneously to $SO(4)'$. By construction *only* the heavy exotic fermions couple directly to the scalar ϕ.
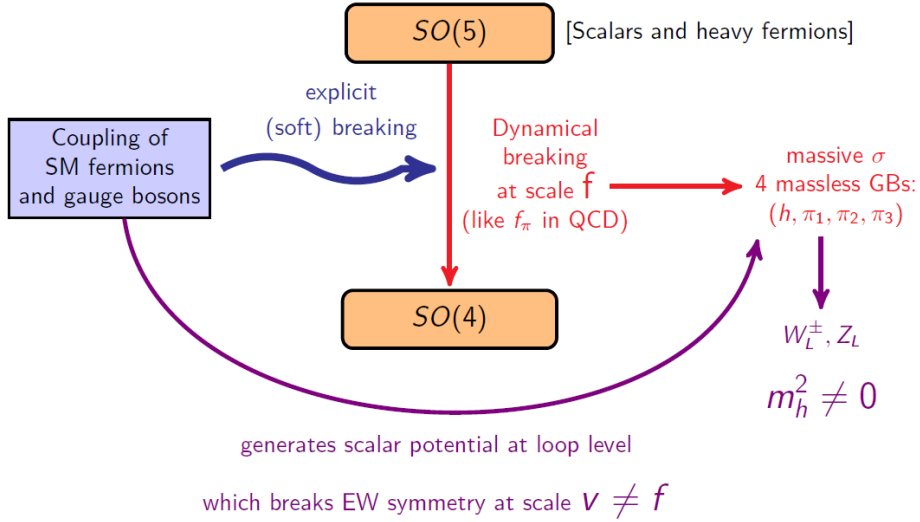
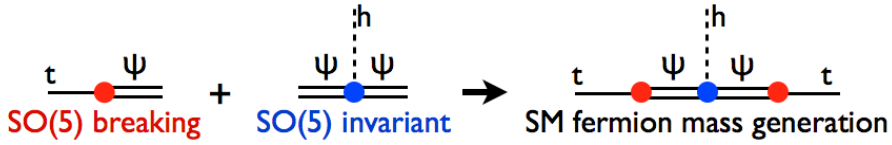**Figure 2.2:** Schematics of the $SO(5) \to SO(4)$ model

Figure 2.3: Schematics of light fermion mass generation. The light SM fermions -here the top quark- couple to the heavy partners breaking explicitly $SO(5)$. The middle image depicts the $SO(5)$ invariant Yukawa interactions between the Higgs and the heavy partners. The combination of both couplings induces and effective top Yukawa coupling and thus a massive top quark.

3. The Higgs field couples to the exotic fermions only via $SO(5)$ invariant Yukawa couplings. The sources of $SO(5)$ breaking lie instead in the electroweak gauge interactions and in mixing terms between the heavy exotic fermions and the SM fermions. Such a breaking is fed via loop corrections to the scalar potential, where it is modeled by two $SO(5)$ soft breaking terms which are custodial preserving.

This choice of fermionic representation respects an approximate custodial symmetry which protects the *Zbb* coupling [112]. Fig. (2.3) illustrates a characteristic of the partially composite fermionic sector in this class of models [29]: *a seesaw-like mechanism is at work in the generation of all low-energy fermion masses*. The heavier the exotic fermions the lighter the light fermions.

To ensure correct hypercharge assignments for the SM fermions coupled directly to heavy exotic fields, the global symmetry is customarily enlarged by (at least) an extra $U(1)_X$ sector, leading finally to a pattern of spontaneous global symmetry breaking given by

$$SO(5) \times U(1)_X \to SO(4) \times U(1)_X \approx SU(2)_L \times SU(2)_R \times U(1)_X \,, \qquad (2.28)$$

with the hypercharge corresponding now to a combination of the new generator and that of $SU(2)_R$

generator, see Eq. (2.4),

$$Y = \Sigma_R^{(3)} + X \, . \tag{2.29}$$

As the global $U(1)_X$ symmetry remains unbroken, no additional Goldstone bosons are generated. Two different $U(1)_X$ charges are compatible with SM hypercharge assignments: 2/3 and $-1/3$. We will indeed consider two different copies of heavy fermions for each representation, differentiated by the $U(1)_X$, as they are necessary to induce mass terms for both the SM up and the down quark sectors. Schematically, the fundamental and singlet representations can be decomposed under $SU(2)_L$ quantum numbers as follows,

$$\begin{aligned}
\psi^{(2/3)} &\sim (X, Q, T^{(5)}) &,& \quad \psi^{(-1/3)} \sim (Q', X', B^{(5)}) \, , \\
\chi^{(2/3)} &\sim T^{(1)} &,& \quad \chi^{(-1/3)} \sim B^{(1)} \, ,
\end{aligned}$$

where $X^{(')}, Q^{(')}$ denote the two different $SU(2)_L$ doublets contained in the fundamental representation of $SO(5)$. In each multiplet, the first doublet has $\Sigma_R^{(3)} = 1/2$ while the second one has $\Sigma_R^{(3)} = -1/2$. $T_{(1,5)}, B_{(1,5)}$ denote instead $SU(2)_L \times SU(2)_R$ singlets, respectively in the **5** and **1** representation of $SO(5)$. Table 2.1 summarizes the relevant quantum numbers for all heavy fermions.

Charge/Field	X	Q	$T_{(1,5)}$	Q'	X'	$B_{(1,5)}$
$\Sigma_R^{(3)}$	$+1/2$	$-1/2$	0	$+1/2$	$-1/2$	0
$SU(2)_L \times U(1)_Y$	$(2, +7/6)$	$(2, +1/6)$	$(1, +2/3)$	$(2, +1/6)$	$(2, -5/6)$	$(1, -1/3)$
x	$+2/3$	$+2/3$	$+2/3$	$-1/3$	$-1/3$	$-1/3$
q_{EM}	$X^u = +5/3$ $X^d = +2/3$	$Q^u = +2/3$ $Q^d = -1/3$	$+2/3$	$Q'^u = +2/3$ $Q'^d = -1/3$	$X'^u = -1/3$ $X'^d = -4/3$	$-1/3$

Table 2.1: Heavy fermion charges assignments.

The fermionic Lagrangian

For the SM fermions, the analysis below will be restricted to the third generation of SM quarks for simplicity, denoting by q_L and t_R and b_R the doublet and singlets, respectively. It would be straightforward to extend the results to the other two generations, for instance introducing heavier replica of the exotic sector, leading to very minor additional phenomenological impact.

Assuming the "minimal" content specified in the previous sections, the fermionic Lagrangian is given by

$$\begin{aligned}
\mathcal{L}_F &= \bar{q}_L i \slashed{D} q_L + \bar{t}_R i \slashed{D} t_R + \bar{b}_R i \slashed{D} b_R \\
&+ \bar{\psi}^{(2/3)} \left(i\slashed{D} - M_5 \right) \psi^{(2/3)} + \bar{\psi}^{(-1/3)} \left(i\slashed{D} - M_5' \right) \psi^{(-1/3)} \\
&+ \bar{\chi}^{(2/3)} \left(i\slashed{D} - M_1 \right) \chi^{(2/3)} + \bar{\chi}^{(-1/3)} \left(i\slashed{D} - M_1' \right) \chi^{(-1/3)}
\end{aligned}$$

$$-\left[y_1\, \bar{\psi}_L^{(2/3)} \phi\, \chi_R^{(2/3)} + y_2\, \bar{\psi}_R^{(2/3)} \phi\, \chi_L^{(2/3)} + y_1'\, \bar{\psi}_L^{(-1/3)} \phi\, \chi_R^{(-1/3)} + y_2'\, \bar{\psi}_R^{(-1/3)} \phi\, \chi_L^{(-1/3)} \right.$$
$$+ \ \Lambda_1 \left(\bar{q}_L \Delta_{2\times5}^{(2/3)} \right) \psi_R^{(2/3)} + \Lambda_2\, \bar{\psi}_L^{(2/3)} \left(\Delta_{5\times1}^{(2/3)} t_R \right) + \Lambda_3\, \bar{\chi}_L^{(2/3)} t_R$$
$$\left. + \ \Lambda_1' \left(\bar{q}_L \Delta_{2\times5}^{(-1/3)} \right) \psi_R^{(-1/3)} + \Lambda_2'\, \bar{\psi}_L^{(-1/3)} \left(\Delta_{5\times1}^{(-1/3)} b_R \right) + \Lambda_3'\, \bar{\chi}_L^{(-1/3)} b_R + h.c. \right]. \qquad (2.30)$$

The first lines contain the kinetic terms for the SM fermions. The second and third lines include the kinetic and mass terms for the exotic fermions. The kinetic terms become $SO(5)$-invariant in the gaugeless limit. The fourth line contains the $SO(5)$ invariant Yukawa couplings of the exotic sector to the Higgs field. Finally, the last two lines of the Lagrangian contain the $SO(5)$ soft-breaking interactions of SM fermions with exotic fermions. $\Delta_{2\times5}$ and $\Delta_{5\times1}$ denote suitable spurions connecting $SO(5)$ and $SU(2) \times U(1)$ representations. If the primed parameters were set to zero no bottom mass would be generated through this mechanism. All parameters in Eq. (2.30) are assumed real for simplicity, that is, we will assume CP invariance in what follows.

It is useful to rewrite the Lagrangian in Eq.(2.30) in terms of $SU(2)_L$ components. For this purpose, from this point and until Eq. (2.39) below, h and σ will denote again the unshifted and unrotated original scalar fields in Eq. (2.8):

$$\begin{aligned}
\mathcal{L}_F &= \ \bar{q}_L i\slashed{D} q_L + \bar{t}_R i\slashed{D} t_R + \bar{b}_R i\slashed{D} b_R + \bar{Q} \left(i\slashed{D} - M_5 \right) Q + \bar{X} \left(i\slashed{D} - M_5 \right) X \\
&+ \ \bar{T}^{(5)} \left(i\slashed{D} - M_5 \right) T^{(5)} + \bar{T}^{(1)} \left(i\slashed{D} - M_1 \right) T^{(1)} + \bar{Q}' \left(i\slashed{D} - M_5' \right) Q' + \bar{X}' \left(i\slashed{D} - M_5' \right) X' \\
&+ \ \bar{B}^{(5)} \left(i\slashed{D} - M_5' \right) B^{(5)} + \bar{B}^{(1)} \left(i\slashed{D} - M_1' \right) B^{(1)} \\
&- \left[y_1 \left(\bar{X}_L H T_R^{(1)} + \bar{Q}_L \widetilde{H} T_R^{(1)} + \bar{T}_L^{(5)} \sigma T_R^{(1)} \right) + y_2 \left(\bar{T}_L^{(1)} H^\dagger X_R + \bar{T}_L^{(1)} \widetilde{H}^\dagger Q_R + \bar{T}_L^{(1)} \sigma T_R^{(5)} \right) \right. \\
&+ \ y_1' \left(\bar{X}'_L \widetilde{H} B_R^{(1)} + \bar{Q}'_L H B_R^{(1)} + \bar{B}^{(5)}_L \sigma B_R^{(1)} \right) + y_2' \left(\bar{B}_L^{(1)} \widetilde{H}^\dagger X'_R + \bar{B}_L^{(1)} H^\dagger Q'_R + \bar{B}_L^{(1)} \sigma B_R^{(5)} \right) \\
&+ \left. \Lambda_1 \bar{q}_L Q_R + \Lambda_1' \bar{q}_L Q'_R + \Lambda_2 \bar{T}_L^{(5)} t_R + \Lambda_3 \bar{T}_L^{(1)} t_R + \Lambda_2' \bar{B}_L^{(5)} b_R + \Lambda_3' \bar{B}_L^{(1)} b_R + h.c. \right]. \qquad (2.31)
\end{aligned}$$

Eq. (2.31) shows that the light fermion masses must be proportional to the $SO(5)$ invariant Yukawa couplings of heavy fermions and to the explicitly $SO(5)$ breaking light-heavy fermionic interactions. The generation of light quark masses requires a vev for the scalar doublet H. For instance, a $\bar{t}_L t_R$ mass term is seen to result from the following chain of couplings,

$$q_l \ \xrightarrow{\Lambda_1}\ Q_R \ \xrightarrow{M_5}\ Q_L \ \xrightarrow{y_1 \langle \widetilde{H} \rangle}\ T_R^{(1)} \ \xrightarrow{M_1}\ T_L^{(1)} \ \xrightarrow{\Lambda_3}\ t_R \,, \qquad (2.32)$$

suggesting

$$m_t \propto y_1 \frac{\Lambda_1 \, \Lambda_3}{M_1 \, M_5}\, v \,, \qquad (2.33)$$

see also Fig. 2.3 and Sect. 2.5. Furthermore, both the $+2/3$ and $-1/3$ electrically charged sectors acquire off-diagonal mixing terms.

The expression for the fermionic Lagrangian Eq. (2.31) can be rewritten in a compact form defining a fermionic vector whose components are ordered by their electrical charges $q_{EM} = (+5/3, +2/3, -1/3, -4/3)$,

$$\Psi = \left(X^u, \ \mathcal{T}, \ \mathcal{B}, \ X'^d \right), \qquad (2.34)$$

where $\mathcal{T}$ and $\mathcal{B}$ include the top and bottom quarks together with their heavy fermionic partners

$$\mathcal{T} = \left(t, Q^u, X^d, T^{(5)}, T^{(1)}, Q'^u\right), \quad \mathcal{B} = \left(b, Q'^d, X'^u, B^{(5)}, B^{(1)}, Q^d\right). \tag{2.35}$$

The fermion mass terms in the weak basis can then be written as

$$\mathcal{L}_{\mathcal{M}} = -\bar{\Psi}_L \, \mathcal{M}(h, \sigma) \, \Psi_R, \tag{2.36}$$

where here and in what follows the sum over all components of the fermionic vector is left implicit and the block diagonal 14×14 fermion mass matrix $\mathcal{M}$ reads

$$\mathcal{M}(h, \sigma) \;=\; \mathrm{diag}\left(M_5, \mathcal{M}^{\mathcal{T}}(h, \sigma), \mathcal{M}^{\mathcal{B}}(h, \sigma), M_5'\right), \tag{2.37}$$

$$\mathcal{M}^{\mathcal{T}}(h, \sigma) \;=\; \begin{pmatrix} 0 & \Lambda_1 & 0 & 0 & 0 & \Lambda_1' \\ 0 & M_5 & 0 & 0 & y_1\frac{h}{\sqrt{2}} & 0 \\ 0 & 0 & M_5 & 0 & y_1\frac{h}{\sqrt{2}} & 0 \\ \Lambda_2 & 0 & 0 & M_5 & y_1\sigma & 0 \\ \Lambda_3 & y_2\frac{h}{\sqrt{2}} & y_2\frac{h}{\sqrt{2}} & y_2\sigma & M_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & M_5' \end{pmatrix}, \tag{2.38}$$

$$\mathcal{M}^{\mathcal{B}}(h, \sigma) \;=\; \mathcal{M}^{\mathcal{T}}(h, \sigma) \text{ with } \{y_i, \Lambda_i, M_i\} \leftrightarrow \{y_i', \Lambda_i', M_i'\}. \tag{2.39}$$

The mass matrices can be diagonalised by bi-unitary (or for the case of the real parameters bi-orthogonal) transformations,

$$\Psi_L^{\mathrm{phys}} \;=\; L\Psi_L, \quad \Psi_R^{\mathrm{phys}} = R\Psi_R, \quad \mathcal{M}^{\mathrm{diag}} = L^\dagger \mathcal{M} R. \tag{2.40}$$

These matrices can be diagonalized analytically in some interesting limits; in general they will be diagonalized numerically.

The physical light eigenstates are admixtures of the light and heavy fermion fields appearing in the Lagrangian. The scalar fields vevs induce in addition heavy fermion mass splittings. Notice however that, even in the limit of vanishing Yukava couplings, the exotic fermions get mixed via the $SO(5)$ breaking couplings. Moreover, although the various dimensional couplings Λ_i and Λ_i' in Eqs. (2.30) and (2.31) may be of the same order, the top and bottom components of the heavy doublets are splitted by $SO(4)$ breaking terms, generically of $O(y_i v)$.

2.3   Phenomenology

In this section, bounds are derived first on the model parameters resulting from present LHC Higgs data and from electroweak precision tests - namely S,T and g_L^b. Future signals are discussed next, focusing in particular on σ physics.

2.3.1 Bounds from Higgs measurements

The tree-level mixing of the scalar singlet σ with the Higgs resonance h can be strongly bounded from present data and in particular from h to ZZ and W^+W^- decays, and from h-gluon-gluon transitions: the Higgs coupling strength to SM fields is weighted down simply by a $\cos\gamma$ factor with respect the SM value, as previously explained and shown in Eq. (2.18). We use ATLAS and CMS combined results for the gluon-gluon and vector boson mediated Higgs production processes [113].

A χ^2 fit taking into account the correlation between the corresponding coupling modifiers in the combined fit of the 7 and 8 TeV LHC data -given by figure 23.B of Ref. [113]- constrains directly $\cos\gamma$, translating into the following bound

$$\sin^2\gamma \lesssim 0.18 \text{ (at } 2\sigma\text{)}, \tag{2.41}$$

which in the $m_\sigma \gg m_h$ limit would point to a value for the non-linearity parameter of composite Higgs models, $\xi \equiv v^2/f^2 \sim \sin^2\gamma$, consistent with the limits found in the literature [69], see Eqs. (2.21) and (2.22) and the discussion below.

Comparison with literature on non-linear realizations

Ref. [69] shows that in non-linear realizations of the composite Higgs scenario the behaviour of the Higgs couplings modifications varies depending on the $SO(5)$ fermionic representations chosen. In particular they compare the so called $MCHM_4$ and $MCHM_5$ scenarios:

- In $MCHM_4$, the fermions (both the embedded light ones and the heavy partners) are in the **4** (spinorial) representation of $SO(5)$; the coupling modifiers then obey $\kappa_V^{(4)} = \kappa_f^{(4)} = (1-\xi)^{1/2}$, leading to a bound from Higgs data $\xi^{(4)} < 0.18$ at 2σ CL. $MCHM_4$ is actually ruled out by its impact on the Zbb coupling and thus for instance disregarded in Ref. [16].

- In $MCHM_5$, the fermions are instead in the **5** (fundamental) of $SO(5)$, and in this case $\kappa_V^{(5)} = (1-\xi)^{1/2}$ differs from $\kappa_f^{(5)} = (1-2\xi)(1-\xi)^{-1/2}$, that is, $\kappa_f^{(5)}/\kappa_V^{(5)} \approx 1-\xi$ for small ξ values. LHC Higgs data set then a bound $\xi^{(5)} < 0.12$ at 2σ CL.

Now, the heavy fermion configuration of our model seems alike to that in $MCHM_5$ if the σ particle was disregarded (σ does not intervene in the tree-level Higgs data analysis of SM couplings), and in spite of including one heavy fermion in a **1** (singlet) of SO(5) as the latter could be integrated out to mimic the $MCHM_5$ spectrum discussed for instance in Refs. [114] or [16]. Indeed, according to the notation in Ref. [16], the fermion representation in our model would be given by $MCHM_{Q-T-B} \rightarrow MCHM_{5-5,1-5,1}$; nevertheless, the behaviour for the modifiers –and thus the resulting ξ bound– is the same than in $MCHM_4$, see Eq. (2.41). Plausibly, this apparent paradox may be resolved by taking into account that the limit of very heavy σ corresponds to the λ coupling reaching the non-perturbative regime, and a resumation of the strong interacting effects may be needed to fully reach the non-linear regime.

2.3.2 Precision electroweak constraints

Analyses available on precision tests for composite Higgs models, such as that in Ref. [115], tend to consider non-linear versions of the theory where the only scalar present is the Higgs particle, but for Ref. [22], which discusses qualitatively the interplay of scalar and exotic fermion contributions. see also Ref. [12]. We present here an explicit computation of the scalar (h and σ) and exotic fermion contributions, discussing the impact of varying the σ mass. S, T and U parameters are considered together with g_L^b and parameter correlations.

S, T and g_L^b

Consider the parameter definitions in Ref. Ata ,

$$
\begin{aligned}
\alpha\, S &= 4 s_W c_W \frac{d\Pi_{30}(q^2)}{dq^2}\Big|_{q^2=0} = 4 s_W c_W\, F_{30}\,, \\
\alpha\, T &= \frac{1}{M_W^2}[\Pi_{11}(0) - \Pi_{33}(0)] = \frac{1}{M_W^2}[A_{11} - A_{33}]\,, \\
\alpha\, U &= -4 s_W^2 \frac{d}{dq^2}[\Pi_{33}(q^2) - \Pi_{11}(q^2)]|_{q^2=0} = 4 s_W^2 (F_{11} - F_{33})\,,
\end{aligned}
\tag{2.42}
$$

where c_W (s_W) denotes the cosinus (sinus) of the Weinberg angle $c_W = M_W/M_Z$, and the electroweak vacuum polarization functions are given by

$$
\Pi_{ij}^{\mu\nu}(q) = -i[\Pi_{ij}(q^2)g^{\mu\nu} + (q^\mu q^\nu - terms)]\,; \qquad \Pi_{ij}(q^2) \equiv A_{ij}(0) + q^2 F_{ij} + ...
\tag{2.43}
$$

with $i, j = W, Z$ or $i, j = 0, 3$ for the B or the W_3 bosons, respectively, and the dots indicating an expansion in powers of q^2. We will not consider further U as it typically corresponds to higher order (mass dimension eight) couplings while only low-energy data (e.g. LEP) will be used here [7]. On the contrary, relevant constraints could stem from deviations induced in the $Z\bar{b}_L b_L$ coupling, parametrised by g_L^b in the decay amplitude

$$
\mathcal{M}_{Z\to\bar{b}_L b_L} = -\frac{e\, g_L^b}{s_W c_W}\bar{b}(p_2)\,\not{\epsilon}(q)\frac{1-\gamma_5}{2}\,b(p_1),
\tag{2.44}
$$

where $\epsilon(q)$ denotes the Z boson polarization and p_i the b quark and antiquark momenta.

The values of S, T and g_L^b are allowed to deviate from the SM prediction within the constraints [115, 117]

$$
\begin{aligned}
\Delta S &\equiv S - S_{SM} = 0.0079 \pm 0.095\,, \\
\Delta T &\equiv T - T_{SM} = 0.084 \pm 0.062\,, \\
\Delta g_L^b &\equiv g_L^b - g_{LSM}^b = (-0.13 \pm 0.61)\times 10^{-3}\,,
\end{aligned}
\tag{2.45}
$$

with the (S, T, g_L^b) correlation matrix given by

$$
\begin{pmatrix}
1 & 0.864 & 0.06 \\
0.864 & 1 & 0.123 \\
0.06 & 0.123 & 1
\end{pmatrix}.
\tag{2.46}
$$

---

[7]Some other handful parameter definitions are: $\epsilon_1 \equiv \alpha T$; $\epsilon_3 = \alpha S/(4 s_W^2)$, see Ref. [116].

Scalar contributions in the linear $SO(5)$ model: h and σ

Given the scalar couplings in Eq. (2.18), their contributions to S and T can be formulated as

$$\Delta T^{(h \,\text{and}\, \sigma)} = -\Delta T^h_{SM}(m_h) + c_\gamma^2 \Delta T^h_{SM}(m_h) + \Delta T^{(\sigma)} = s_\gamma^2 \left[-\Delta T^h_{SM}(m_h) + \Delta T^h_{SM}(m_\sigma) \right], \quad (2.47)$$

$$\Delta S^{(h \,\text{and}\, \sigma)} = -\Delta S^h_{SM}(m_h) + c_\gamma^2 \Delta S^h_{SM}(m_h) + \Delta S^{(\sigma)} = s_\gamma^2 \left[-\Delta S^h_{SM}(m_h) + \Delta S^h_{SM}(m_\sigma) \right], \quad (2.48)$$

where the σ contributions $\Delta T^{(\sigma)}$ and $\Delta S^{(\sigma)}$ have been simply written in terms of the usual SM formulae for the Higgs contribution ΔT^h_{SM} and ΔS^h_{SM} with the replacement $m_h \to m_\sigma$ and using the formulae valid for masses much above the electroweak scale. The scalar contribution to ΔT is then given by

$$\begin{aligned} \Delta T^{(h \,\text{and}\, \sigma)} =& \frac{3 G_F M_W^2}{8\pi^2 \sqrt{2}} s_\gamma^2 \left(-m_h^2 \frac{\log(m_h^2/M_W^2)}{M_W^2 - m_h^2} + m_\sigma^2 \frac{\log(m_\sigma^2/M_W^2)}{M_W^2 - m_\sigma^2} \right. \\ &\left. + \frac{M_Z^2}{M_W^2} \left\{ m_h^2 \frac{\log(m_h^2/M_Z^2)}{M_Z^2 - m_h^2} - m_\sigma^2 \frac{\log(m_\sigma^2/M_Z^2)}{M_Z^2 - m_\sigma^2} \right\} \right), \end{aligned} \quad (2.49)$$

which in the limit $m_\sigma \gg m_h, M_W, M_Z$ reduces to

$$\Delta T^{(h \,\text{and}\, \sigma)} \sim s_\gamma^2 \frac{3 G_F M_W^2}{8\pi^2 \sqrt{2}} \frac{s_W^2}{c_W^2} \log(m_\sigma^2/M_W^2). \quad (2.50)$$

For the ΔS corrections, the formulation in Refs. [118, 119] is used, leading to

$$\alpha \Delta S^h_{SM}(m) = s_W^2 \frac{2 G_F}{\sqrt{2}\pi^2} M_W^2 \left(\frac{x}{12(x-1)} \log(x) + \left(-\frac{x}{6} + \frac{x^2}{12} \right) F(x) - \left(1 - \frac{x}{3} + \frac{x^2}{12} \right) F'(x) \right), \quad (2.51)$$

where $x \equiv m^2/M_Z^2$ and for $x < 4$:

$$\begin{aligned} F(x) &= 1 + \left(\frac{x}{x-1} - \frac{1}{2}x \right) \log x - x \sqrt{\frac{4}{x} - 1} \arctan \sqrt{\frac{4}{x} - 1}, \\ F'(x) &= -1 + \frac{x-1}{2} \log x + (3-x) \sqrt{\frac{x}{4-x}} \arctan \sqrt{\frac{4}{x} - 1}, \end{aligned} \quad (2.52)$$

while for $x > 4$:

$$\begin{aligned} F(x) &= 1 + \left(\frac{x}{x-1} - \frac{1}{2}x \right) \log x - x \sqrt{1 - \frac{4}{x}} \log \left(\sqrt{\frac{x}{4} - 1} + \sqrt{\frac{x}{4}} \right), \\ F'(x) &= -1 + \frac{x-1}{2} \log x + (3-x) \sqrt{\frac{x}{x-4}} \log \left(\sqrt{\frac{x}{4} - 1} + \sqrt{\frac{x}{4}} \right). \end{aligned} \quad (2.53)$$

In the limit of very large m_σ, the σ contribution to S can be approximated by:

$$\alpha \, \Delta S^{(\sigma)} \xrightarrow{\sigma \to \infty} s_\gamma^2 s_W^2 \frac{2 G_F}{\sqrt{2}\pi^2} M_W^2 \left[\frac{1}{12} \log \left(\frac{m_\sigma^2}{M_W^2} \right) \right], \quad (2.54)$$
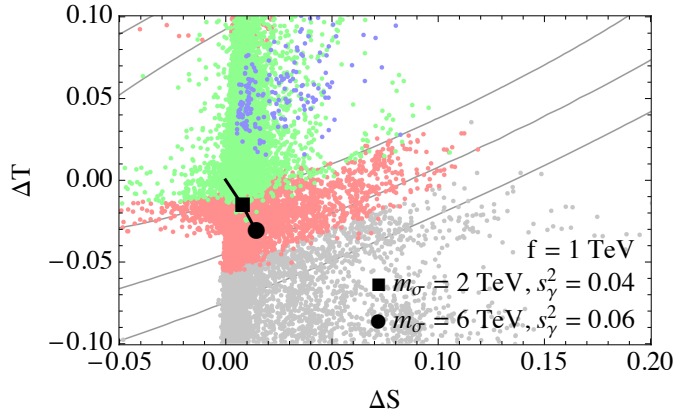
Figure 2.4: Uncombined contributions of the scalar sector (black curve) and the exotic fermionic sector to the parameters S and T.

consistent with the statements in the literature for a very heavy Higgs particle [65].

The black curve in Fig. 2.4 displays examples of the ΔS and ΔT corrections induced by the σ scalar as it follows from the formulae shown above. As earlier explained, the set of parameters in the scalar potential $(f, \lambda, \alpha, \beta)$ has been traded by four observables: G_F, m_h, m_σ and the scalar mixing γ (with the latter two yet to be experimentally measured). It is nevertheless theoretically illuminating to indicate the corresponding values for f and the scalar quartic self-coupling λ for each example analyzed, and their values are shown in all figures to follow. We present numerical results for two typical parameter regimes:

- $m_\sigma = 2$ TeV, $s_\gamma^2 = 0.04$, which corresponds to $f = 1$ TeV and to scalar potential couplings $\lambda = 0.38$, $\alpha = 0.35$ and $\beta = 0.16$, which clearly lie within the perturbative regime of the linear $SO(5)$ sigma model.

- $m_\sigma = 6$ TeV, $s_\gamma^2 = 0.06$, which also correspond to $f = 1$ TeV, while $\lambda = 4.3$ -closer to the limit of validity of the perturbative expansion- and $\alpha = 0.25$, $\beta = 0.13$; this pattern corresponds then to a mainly $SO(5)$ symmetric scenario with small soft symmetry breaking.

Fig. 2.4 shows a sizeable negative contribution of the σ particle to ΔT which increases with m_σ, and positive contribution to ΔS; the result is consistent with the pattern expected in Ref. [22], and similar to that for the heavy Higgs case (see e.g. Ref. [120]). In the limit $m_\sigma \to m_h$ the total scalar contribution matches that in the SM due to the Higgs particle. It is easy to extrapolate the S and T scalar contributions to other mixing regimes as they scale with s_γ^2: for instance the effect would be amplified by a factor of ~ 3 when raising the mixing towards the maximal value allowed, see Eq (2.41).

For g_L^b instead we will not analyze the one-loop σ contributions, as they would be proportional to the bottom Yukawa couplings and thus negligible compared to the top and top-partner contributions to be discussed next.
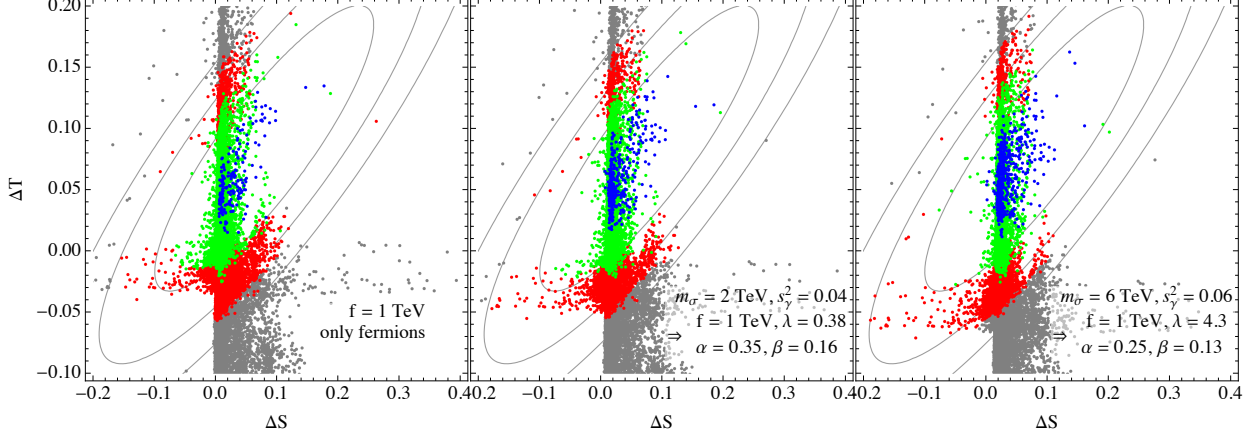
Figure 2.5: Combined contributions to S and T from the scalar sector and the exotic fermionic sector. The blue, green and red points are allowed at $1, 2, 3\sigma$ by the combined (S, T, g_L^b) fit, while gray points are outside the 3σ region.

Fermionic contributions

The heavy fermion sector may have an impact on the oblique parameters and on g_L^b. This sector adds additional parameter dependence on top of the four renormalization parameters already discussed for the scalar sector of the linear $SO(5)$ sigma model. The fermionic parameter space is quite large and adjustable, and thus in practice m_σ and γ will be treated here as independent from them. It will also be assumed that the inclusion of quarks and leptons from the first two generations does not alter significantly the analysis of electroweak precision tests, as lighter fermions tend to have very small mixing with their heavy partners.

The gauge boson couplings to neutral (NC) and charged (CC) fermionic currents in the weak basis can be read from Table 2.1. After rotation to the mass basis, the corresponding Lagrangians can be written as [114]:

$$
\begin{aligned}
\mathcal{L}_{\text{NC}} &= \bar{\Psi}^{\text{phys}} \gamma^\mu \left[\frac{g}{2} \left(C_L P_L + C_R P_R \right) W_\mu^3 - g'(Y_L P_L + Y_R P_R) B_\mu \right] \Psi^{\text{phys}} \\
&= \bar{\Psi}^{\text{phys}} \gamma^\mu \left[\frac{g}{2 c_W} \left(C_L P_L + C_R P_R - 2 s_W^2 \mathcal{Q} \right) Z_\mu - e \mathcal{Q} A_\mu \right] \Psi^{\text{phys}} , \\
\mathcal{L}_{CC} &= \bar{\Psi}^{\text{phys}} \gamma^\mu \left[\frac{g}{\sqrt{2}} \left(V_L P_L + V_R P_R \right) W_\mu^+ \right] \Psi^{\text{phys}} + h.c. ,
\end{aligned}
\tag{2.55}
$$

where P_L and P_R are chirality projectors, Ψ^{phys} denotes the generic fermionic vector in the physical mass basis and e is the absolute value of the electric charge unit. In the model under discussion, the matrices C and Y are related via the electric charge matrix –see also Eq. (2.29):

$$
Y_\alpha = \mathcal{Q} - \frac{1}{2} C_\alpha \quad \alpha = L \text{ or } R ,
\tag{2.56}
$$

with

$$
\mathcal{Q} = \left(+\frac{5}{3}, +\frac{2}{3} \, \mathbf{1}_{6\times 6}, -\frac{1}{3} \, \mathbf{1}_{6\times 6}, -\frac{4}{3} \right) .
\tag{2.57}
$$

58

The relation between the NC coupling matrices in the mass basis, $C_{L,R}$ and $Y_{L,R}$, and their counterparts in the interaction basis (same symbols in curly characters below) is given by

$$C_L = L\mathcal{C}_L L^\dagger, \ C_R = R\mathcal{C}_R R^\dagger, \quad \mathcal{C}_{L;R} = \mathrm{diag}(+1, \mathcal{C}_{L;R}^\mathcal{T}, \mathcal{C}_{L;R}^\mathcal{B}, -1), \tag{2.58}$$

$$\mathcal{C}_{L;R}^\mathcal{T} = -\mathcal{C}_{L;R}^\mathcal{B} = \mathrm{diag}(+1; 0, +1, -1, 0, 0, +1); \tag{2.59}$$

$$Y_L = L\mathcal{Y}_L L^\dagger, \ Y_R = R\mathcal{Y}_R R^\dagger, \quad \mathcal{Y}_{L;R} = \mathrm{diag}\left(+\frac{7}{6}, \mathcal{Y}_{L;R}^\mathcal{T}, \mathcal{Y}_{L;R}^\mathcal{B}, -\frac{5}{6}\right), \tag{2.60}$$

$$\mathcal{Y}_{L;R}^\mathcal{T} = \mathrm{diag}\left(\frac{1}{6}; \frac{2}{3}, \frac{1}{6}, \frac{7}{6}, \frac{2}{3}, \frac{2}{3}, \frac{1}{6}\right), \ \mathcal{Y}_{L;R}^\mathcal{B} = \mathrm{diag}\left(\frac{1}{6}; -\frac{1}{3}, \frac{1}{6}, -\frac{5}{6}, -\frac{1}{3}, -\frac{1}{3}, \frac{1}{6}\right). \tag{2.61}$$

Analogously, for the CC coupling matrices $V_{L,R}$:

$$V_L = L\mathcal{V}L^\dagger, V_R = R\mathcal{V}R^\dagger; \mathcal{V}_{L;R} = \begin{pmatrix} 0 & \mathcal{V}^{X^u\mathcal{T}} & \mathbf{0}_{1\times6} & 0 \\ \mathbf{0}_{6\times1} & \mathbf{0}_{6\times6} & \mathcal{V}_{L;R}^{\mathcal{T}\mathcal{B}} & \mathbf{0}_{6\times1} \\ \mathbf{0}_{6\times1} & \mathbf{0}_{6\times6} & \mathbf{0}_{6\times6} & \mathcal{V}^{\mathcal{B}X'^d} \\ 0 & \mathbf{0}_{1\times6} & \mathbf{0}_{1\times6} & 0 \end{pmatrix}, \tag{2.62}$$

$$\mathcal{V}^{X^u\mathcal{T}} = \left(\mathcal{V}^{\mathcal{B}X'^d}\right)^\dagger = (0, 0, 1, 0, 0, 0), \tag{2.63}$$

while $\mathcal{V}_L^{\mathcal{T}\mathcal{B}}$ is a 6×6 matrix whose elements are null but for its $(1,1)$, $(2,6)$ and $(6,2)$ entries with value 1, and $\mathcal{V}_R^{\mathcal{T}\mathcal{B}}$ is a 6×6 matrix with null elements but for its $(2,6)$ and $(6,2)$ entries with value 1.

T parameter. The contribution of the fermionic sector to the T parameter, ΔT^f, is given by [121]

$$\Delta T^f = \frac{3}{16\pi s_W^2 c_W^2} \left\{ \sum_{ij} \left[\left((V_L^{ij})^2 + (V_R^{ij})^2\right)\theta_+(\eta_i, \eta_j) + 2V_L^{ij}V_R^{ij}\theta_-(\eta_i, \eta_j)\right] - \right.$$

$$\left. -\frac{1}{2}\sum_{ij}\left[\left((C_L^{ij})^2 + (C_R^{ij})^2\right)\theta_+(\eta_i, \eta_j) + 2C_L^{ij}C_R^{ij}\theta_-(\eta_i, \eta_j)\right]\right\}$$

$$-\frac{3}{16\pi s_W^2 c_W^2 M_Z^2}\left(m_t^2 + m_b^2 - 2\frac{m_t^2 m_b^2}{m_t^2 - m_b^2}\ln\frac{m_t^2}{m_b^2}\right), \tag{2.64}$$

where m_i denotes the fermion masses, $m_i \equiv \mathcal{M}_{ii}^{\mathrm{diag}}$, and $\eta_i \equiv m_i^2/M_Z^2$. The last line in this equation corresponds to the substraction of the SM contribution from the light fermions (top and bottom). The $\theta_\pm$ functions are defined as [121]:

$$\theta_+(\eta_1, \eta_2) = \eta_1 + \eta_2 - \frac{2\eta_1\eta_2}{\eta_1 - \eta_2}\ln\frac{\eta_1}{\eta_2} - 2(\eta_1\ln\eta_1 + \eta_2\ln\eta_2) + \mathrm{div}\frac{\eta_1 + \eta_2}{2}, \tag{2.65}$$

$$\theta_-(\eta_1, \eta_2) = 2\sqrt{\eta_1\eta_2}\left(\frac{\eta_1 + \eta_2}{\eta_1 - \eta_2}\ln\frac{\eta_1}{\eta_2} - 2 + \ln(\eta_1\eta_2) - \frac{\mathrm{div}}{2}\right). \tag{2.66}$$

S parameter. The fermionic contribution to S, ΔS^f, can be computed following Ref. [121],

$$\Delta S^f = -\frac{1}{\pi} \sum_{ij} \left\{ (C_L^{ij} Y_L^{ij} + C_R^{ij} Y_R^{ij}) \left[-\frac{\text{div}}{12} - \frac{5}{9} + \frac{\eta_i + \eta_j}{3} + \frac{\ln(\eta_i \eta_j)}{6} \right. \right.$$

$$\left. + \frac{\eta_i - 1}{12} f(\eta_i, \eta_i) + \frac{\eta_j - 1}{12} f(\eta_j, \eta_j) - \frac{\chi_+(\eta_i, \eta_j)}{2} \right]$$

$$- (C_L^{ij} Y_R^{ij} + C_R^{ij} Y_L^{ij}) \left[2\sqrt{\eta_i \eta_j} + \sqrt{\eta_i \eta_j} \, \frac{f(\eta_i, \eta_i) + f(\eta_j, \eta_j)}{4} + \frac{\chi_-(\eta_i, \eta_j)}{2} \right] \right\} - \Delta S_{SM}^f, \quad (2.67)$$

with the functions $f(\eta_1, \eta_2)$ and $\chi_\pm(\eta_i, \eta_j)$ as defined in Ref. [121], "div" standing for the divergent contributions typically appearing in dimensional regularisation, and the last term corresponding to the substraction of the SM light (top and bottom) fermionic contributions. [8]

Anomalous Zbb coupling. We follow Ref. [114] for the computation of the corrections to the g_L^b parameter defined in Eq. (2.44), δg_L^b. Only the top and bottom sectors will be taken into account as the mass generation mechanism for the lighter fermions are expected to have a lesser impact on EW precision tests since either the exotic fermions involved are much heavier or the Yukawa couplings connecting them to the SM fermions are much smaller. Moreover, the bottom quark mass will be neglected ($y_1' = y_2' = 0$) [9]. The fermion-gauge couplings relevant to this case are the Z couplings for the charge $2/3$ and $-1/3$ sectors which can be read from Eqs. (2.57) and (2.59), and the couplings to the $W^\pm$ boson between the $(2/3, R)$ and the $(-1/3, L)$ sectors (see the matrix $\mathcal{V}_L^{\mathcal{TB}}$ defined after Eq. (2.63)). In addition to the NC and CC couplings in Eq. (2.55), the interactions of the charged longitudinal gauge boson components "π_i" are needed,

$$\mathcal{L}_{\pi^\pm} = \bar{\Psi}^{\text{phys}} \frac{g}{\sqrt{2}} \left(W_L P_L + W_R P_R \right) \Psi^{\text{phys}} \pi^+ + h.c. \quad (2.68)$$

where

$$W_L = R \mathcal{W}_L L^\dagger, \qquad W_R = L \mathcal{W}_R R^\dagger, \quad (2.69)$$

with $\mathcal{W}_L$ and $\mathcal{W}_R$ being the mixing matrices in the interaction basis, given in the present model by

$$\mathcal{W}_{L;R} = \begin{pmatrix} 0 & \mathcal{W}_{L;R}^{X^u \mathcal{T}} & \mathbf{0}_{1\times 6} & 0 \\ \mathbf{0}_{6\times 1} & \mathbf{0}_{6\times 6} & \mathcal{W}_{L;R}^{\mathcal{TB}} & \mathbf{0}_{6\times 1} \\ \mathbf{0}_{6\times 1} & \mathbf{0}_{6\times 6} & \mathbf{0}_{6\times 6} & \mathcal{W}_{L;R}^{\mathcal{B}X'^d} \\ 0 & \mathbf{0}_{1\times 6} & \mathbf{0}_{1\times 6} & 0 \end{pmatrix}, \quad (2.70)$$

$$\mathcal{W}_{L;R}^{X^u \mathcal{T}} = \frac{\sqrt{2}}{g}(0,0,0,0,-y_2; -y_1, 0), \quad \left(\mathcal{W}_{L;R}^{\mathcal{B}X'^d}\right)^\dagger = \frac{\sqrt{2}}{g}(0,0,0,0,y_1'; y_2', 0).$$

---

[8] The SM fermionic contributions to S and T with only one generation of quarks follow from Eq. (2.55) considering a two-component fermion field $\Psi^{SM} = (t,b)$, with $\mathcal{M}^{SM} = \text{diag}(m_t, m_b)$ and coupling matrices $\mathcal{Q}^{SM} = Y_R^{SM} = \text{diag}(+2/3, -1/3)$, $C_L^{SM} = \text{diag}(+1, -1)$, $Y_L^{SM} = +\frac{1}{6}\mathbb{1}_{2\times 2}$, $V_L^{SM} = \text{antidiag}(0,1)$, $C_R^{SM} = V_R^{SM} = \mathbb{0}_{2\times 2}$.

[9] The cancellation of divergences in the computation of δg_L^b has been verified in this approximation.
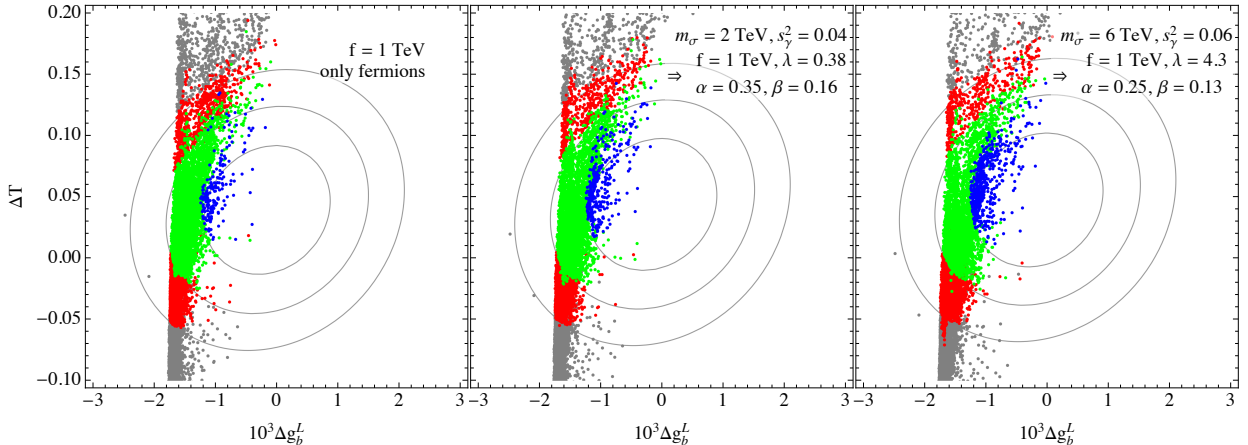
Figure 2.6: Scalar and fermionic impact on the T parameter and on the Z-b_L-b_L coupling g_b^L.

The 6×6 matrix $\mathcal{W}_L^{\mathcal{TB}}$ in this equation has all elements null but for its $(5,6)$ and $(6,5)$ entries which take values y_1 and $-y_2'$, respectively, while $\mathcal{W}_R^{\mathcal{TB}}$ is a 6×6 matrix of null elements but for its $(5,6)$ and $(6,5)$ entries which take values y_2 and $-y_1'$, respectively. In practice, only the entries connecting –after rotation– the charge $2/3$ fermions to b_L enter the computation.

In the numerical analysis, the two sets of values considered earlier on for the numerical analysis of the pure scalar contributions will be retained: $(m_\sigma = 2 \text{ TeV}, \sin_\gamma^2 = 0.04)$ and $(m_\sigma = 6 \text{ TeV}, \sin_\gamma^2 = 0.06)$, both corresponding to $f = 1$ TeV and within the soft breaking regime $\alpha, \beta < \lambda$, with the latter being kept within its perturbative range [10]. Note that, for a σ particle much heavier than the Higgs, values of f below 700 GeV would be difficult to accommodate experimentally as $\sin \gamma^2 \simeq v^2/f^2$, see Eq. (2.41). The exotic fermionic masses will be allowed to vary randomly between 800 GeV and $\mathcal{O}(10 \text{ TeV})$, as the heavy top partners with electric charges $+5/3$ and $+2/3$ are bounded to be above $800 - 1000$ GeV [122, 123], depending on the dominant decay mode. In the light fermion sector, the top and bottom masses will be allowed to vary within the intervals $m_t = 173 \pm 5$ GeV and $m_b = 4.6 \pm 2$ GeV, respectively, for illustrative purposes.

Figs. 2.4 to 6 depict the points that satisfy a χ^2 global fit to the precision pseudo-observables S, T and δg_L^b, where the blue, green, and red points are the allowed $1\sigma, 2\sigma$ and 3σ regions, respectively, while gray points lie above the 3σ limit. The central values, uncertainties and correlation matrix are taken from Ref. [115]. The ellipses drawn in the $\Delta S - \Delta T$ plane in Figs. 2.4 and 2.5 are the projection for $\Delta g_b^L = 0$, while those in the $\Delta T - \Delta g_b^L$ plane in Fig. 2.6 use the ΔS value coming from the scalar sector. The latter is a good approximation since S gets in practice a very small correction from the heavy fermions, as seen in Fig. (2.5).

S versus T. The fermion sector can lead to large deviations in the value of the T parameter. In Fig. (2.4) and in the first panel of Fig. (2.5) only the fermionic contributions are depicted. The last two panels in Fig (2.5) show the fermion plus scalar combined results: the lighter the σ particle, the less tension follows with respect to electroweak precision data, in particular due to the impact

---

[10]$m_\sigma = 4\pi f$ is roughly where perturbativity is lost in chiral perturbation theory [58].

on ΔT, although it is to be noted that even for large m_σ fermionic contributions can bring (S, T) within the experimentally allowed region.

The sign of the fermionic contributions to S and T can be largely understood in terms of the light-heavy fermion mixings and the mass hierarchy between the heavy eigenstates. For instance, large mixing values with a heavy singlet are known to induce large positive contributions to ΔT, as pointed out in Ref. [114], as a result from the custodial symmetry being broken by the singlet-doublet mixing. It is possible to illustrate the analysis more in detail following Ref. [124], which uses a different fermionic but nonetheless illuminating embedding. They consider heavy vector fermions which couple directly to both the light doublet q and the light singlet q_R. When only a heavy singlet is present, the expected contributions to ΔT and ΔS are both significant (though the first are more important) and positive for the regime we consider, see their Eq. (32). Instead, when only a heavy vector doublet was taken into account, the sign of the correction to the oblique parameters was proportional to the sign of the mass splitting between the heavy eigenstates with charge $2/3$ and $-1/3$, resulting in sizeable contributions to ΔT and very small to ΔS.

It is not possible to apply those conclusions in Ref. [124] directly here, though, as in our setup the light fermion mass generation involves necessarily and simultaneously *both* a heavy doublet and a heavy singlet, see Eq. (2.33): the light doublet q mixes directly only with the heavy doublet Q, while q_R mixes with T_1. Nevertheless, the mainly positive fermionic corrections to ΔS found are consistent with being dominated by the participation of a heavy singlet. The results, in Fig. 2.7 show indeed that a large mixing between t_L and the singlet $T^{(1)}$ leads to a positive ΔT (left panel) while the negative corrections to ΔT obtained are consistent instead with a large mixing between t_R and the doublet component X^d (middle panel).

T versus g_L^b. The deviations induced in the *Zbb* coupling provide additional bounds: even if the model parameters do not impact on Δg_b^L at tree level, the top partners may induce at loop level deviations from the SM value. Fig. 2.6 depicts the purely fermionic and the scalar plus fermion combined contributions in the $T - g_b^b$ parameter space. Finally, the right panel of Fig. 2.7 shows a sizeable and positive impact on g_L^b of the mixing between t_L and the charge $2/3$ heavy singlets $T^{(1)}$ and $T^{(5)}$.

As a final remark, there are considerable mixings in the fermion sector for which the dominant effects go schematically as $\tan\theta_{ij} \sim \Lambda_i/M_j$. It could be thus suspected that large deviations in the Wtb coupling should occur. However, these rotations are mainly driven by the $SO(5)$ breaking couplings Λ_i and Λ_i', which are custodial symmetry preserving. Therefore, a large rotation in the top sector is mostly compensated by a corresponding one in the bottom sector, leading to practically no deviation in V_{tb}.

2.3.3 Higgs and σ coupling to gluons

This section and the next one deal with the scalar to photons and to gluons effective couplings, arising at one-loop level. Define the scalar-gluon-gluon amplitudes hgg (σgg) as

$$\mathcal{A}_{h(\sigma)} \equiv \mathcal{A}_{h(\sigma) \leftrightarrow gg}(m_{h(\sigma)}^2) = -i\frac{\alpha_s}{\pi}\, g_{h(\sigma)}\, (p \cdot k\ g^{\mu\nu} - p^\mu k^\nu)\delta^{ab}, \tag{2.71}$$
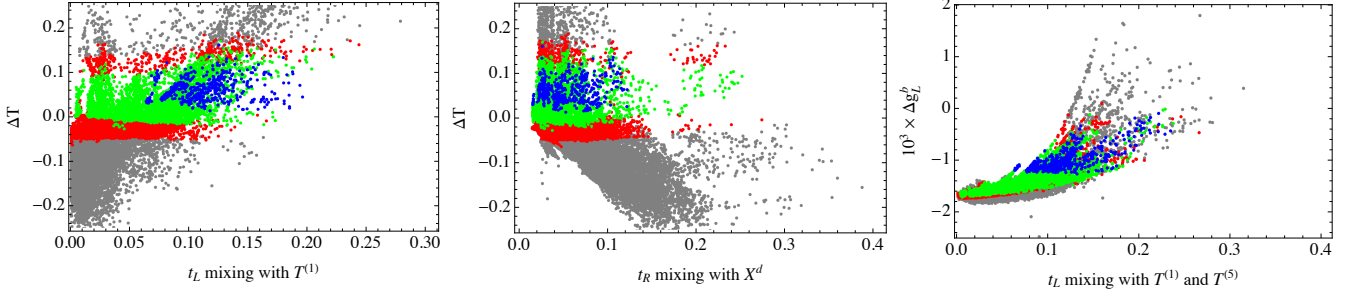
Figure 2.7: Examples of correlations between the fermion mixing strength and electroweak precision measurements. The label "$t_{L,R}$ mixing with Ψ_1, Ψ_2, ..." indicates $\left(|U_1^{L,R}|^2 + |U_2^{L,R}|^2 + \dots\right)^{1/2}$, where $U^{L,R}$ indicates the left or right rotation that diagonalizes the mass matrix.

where g_h and g_σ are scale dependent functions that parametrise the amplitude strength, $\alpha_s = g_s^2/4\pi$ with g_s denoting the QCD coupling constant, p and k stand for the gluon four-momenta, and a, b are color indices. In the case of the SM, the hgg coupling is induced only at one loop level and the amplitude is dominated by the top quark,

$$g_h^{SM} = \left(\frac{y_t}{\sqrt{2}}\right) \frac{1}{m_t} I\left(\frac{m_h^2}{m_t^2}\right) , \qquad (2.72)$$

where y_t is the top Yukawa coupling ($m_t \equiv y_t\, v/\sqrt{2}$) and $I(m_h^2/m_t^2)/m_t$ is the loop factor with

$$I\left(\frac{q^2}{m^2}\right) = \int_0^1 dx \int_0^{1-x} dz \, \frac{1 - 4xz}{1 - xz\frac{q^2}{m^2}} \quad \approx \quad \left\{ \begin{array}{lll} 1/3 & \text{for} & m^2 \gg q^2 \\[2mm] 0 & \text{for} & m^2 \ll q^2 \end{array} \right\} .$$

The SM bottom contribution corresponds to $I(m_h^2/m_b^2) \approx 10^{-2}$ and is thus usually neglected [11].

There are no direct hgg or σgg couplings in the Lagrangian discussed here, but effective hgg and σgg interactions arise via fermion loops. Expanding the global field-dependent mass matrix $\mathcal{M}(h,\sigma)$ in Eqs. (2.36)-(2.39) around the scalar field vevs, v and v_σ, and defining the following constant matrices

$$\overline{\mathcal{M}} \equiv \mathcal{M}(v, v_\sigma) \quad , \quad \frac{\partial\overline{\mathcal{M}}}{\partial h} \equiv \left.\frac{\partial\mathcal{M}(h,\sigma)}{\partial h}\right|_{\substack{h = v \\ \sigma = v_\sigma}} \quad , \quad \frac{\partial\overline{\mathcal{M}}}{\partial \sigma} \equiv \left.\frac{\partial\mathcal{M}(h,\sigma)}{\partial \sigma}\right|_{\substack{h = v \\ \sigma = v_\sigma}} \quad ,$$

the fermionic mass Lagrangian Eq. (2.36) can be written as

$$-\mathcal{L}_Y \;\; = \;\; \bar{\Psi}_L \,\overline{\mathcal{M}}\, \Psi_R + \hat{h}\, \bar{\Psi}_L \frac{\partial\overline{\mathcal{M}}}{\partial h}\, \Psi_R + \hat{\sigma}\, \bar{\Psi}_L \frac{\partial\overline{\mathcal{M}}}{\partial \sigma}\, \Psi_R + h.c. \qquad (2.73)$$

where $\hat{h}$ and $\hat{\sigma}$ are the unrotated scalar fluctuations, see Eq. (2.13). Performing the rotation to the fermionic mass eigenstate basis $\{\Psi_i \to \Psi_i^{phys}\}$,

$$\left(\overline{\mathcal{M}}\right)_{ij} \to m_i\, \delta_{ij} \quad , \quad \left(\frac{\partial\overline{\mathcal{M}}}{\partial h}\right)_{ij} \to (Y_h)_{ij} \quad , \quad \left(\frac{\partial\overline{\mathcal{M}}}{\partial \sigma}\right)_{ij} \to (Y_\sigma)_{ij} , \qquad (2.74)$$

---

[11]The large mass limit in the integral is customarily applied for $m_h < 2m_i$, which includes the top case.

where m_i, Y_h and Y_σ are respectively the masses and the couplings to the unrotated scalars fields $\hat{h}, \hat{\sigma}$ of the physical fermionic states [12]. For simplicity, CP invariance will be assumed in what follows.

It is straightforward to obtain the physical $h \leftrightarrow gg$ and $\sigma \leftrightarrow gg$ amplitudes combining those involving the unrotated $\hat{h}$ and $\hat{\sigma}$ fields. The latter will require the substitution of the SM loop factor in Eq. (2.73) as follows,

$$\frac{y_t}{\sqrt{2}} \frac{1}{m_t} I\left(\frac{m_h^2}{m_t^2}\right) \qquad \rightarrow \qquad \left\{ \begin{array}{ll} \sum_i (Y_h)_{ii} \dfrac{1}{m_i}\, I\left(\dfrac{q^2}{m_i^2}\right) & \text{for } \hat{h} \leftrightarrow gg \\[3mm] \sum_i (Y_\sigma)_{ii} \dfrac{1}{m_i}\, I\left(\dfrac{q^2}{m_i^2}\right) & \text{for } \hat{\sigma} \leftrightarrow gg \end{array} \right\}, \qquad (2.75)$$

where $q^2 = m_h^2$ for $h \leftrightarrow gg$ on-shell transitions, while $q^2 = m_\sigma^2$ for $\sigma \leftrightarrow gg$ on-shell transitions, and where the sum runs over all colored fermion species present in the model.

$h \leftrightarrow gg$ transitions

If all fermion masses were much larger than m_h, it would be possible to simply factorize the constant integral outside the sum as follows:

$$\sum_i \frac{(Y_h)_{ii}}{m_i}\, I\left(\frac{m_h^2}{m_i^2}\right) \quad \approx \quad \frac{1}{3} \sum_i \frac{(Y_h)_{ii}}{m_i} = \frac{1}{6} \frac{d}{dh} \log \det(\mathcal{M}\,\mathcal{M}^\dagger), \qquad (2.76)$$

where the last term is written in the original unrotated fermionic basis since trace and determinant are invariant under a change of basis. All fermions in the model under consideration are indeed much heavier than the Higgs particle but for the bottom, whose loop contribution $I(m_h^2/m_b^2)$ is negligible. Therefore the false "heavy" bottom contribution included in Eq. (2.76) should be removed at energies $q^2 \approx m_h^2$, resulting in the following effective couplings at the scale m_h:

$$\begin{aligned} g_{\hat{h}}(m_h^2) &\approx \frac{1}{6}\frac{d}{dh}\left(\log\det(\mathcal{M}\,\mathcal{M}^\dagger) - \log(m_b(h,\sigma)\, m_b^*(h,\sigma))\right)\Big|_{\substack{h=v \\ \sigma=v_\sigma}} \\ &= \frac{1}{3v} + O\left(\frac{v}{M_1' M_5'}\right), \qquad\qquad\qquad\qquad (2.77) \\ g_{\hat{\sigma}}(m_h^2) &\approx \frac{1}{6}\frac{d}{d\sigma}\left(\log\det(\mathcal{M}\,\mathcal{M}^\dagger) - \log(m_b(h,\sigma)\, m_b^*(h,\sigma))\right)\Big|_{\substack{h=v \\ \sigma=v_\sigma}} \\ &= -\frac{y_2}{3M_5}\frac{\Lambda_2}{\Lambda_3} + O\left(\frac{v_\sigma}{M_1' M_5'}, \frac{v_\sigma}{M_5^2}\right), \qquad\quad (2.78) \end{aligned}$$

where the eigenvalue of the field dependent mass matrix corresponding to the bottom quark reads:

$$m_b(h,\sigma) = \frac{y_1'\Lambda_1'\Lambda_3' - y_1'y_2'\Lambda_1'\Lambda_2'\, \sigma/M_5'}{M_1' M_5' - y_1'y_2'\, (h^2 + \sigma^2)}\, \frac{h}{\sqrt{2}}. \qquad (2.79)$$

---

[12]For instance, in this notation $(Y_h)_{tt} = y_t/\sqrt{2}$.

The $h \leftrightarrow gg$ amplitude is then given by Eq. (2.71), with

$$g_h \equiv g_{\hat{h}}(m_h^2) \cos\gamma - g_{\hat{\sigma}}(m_h^2) \sin\gamma \,.$$

In the limit $m_t \gg m_h$, the hgg effective coupling is exactly as in the SM. The contribution from the heavy vector-like quarks tends to cancel out for bare vector-like masses substantially larger than v, a result well-known in the literature.

$\sigma \leftrightarrow gg$ transitions

With analogous procedure, the σgg amplitude can be obtained using Eq. (2.75) for $q^2 = m_\sigma^2$. The difference with the previous case is that now the top quark is lighter or comparable in mass to σ and it cannot be integrated out, that is $m_b \ll m_t, m_\sigma \ll m_i$, where here m_i denotes the heavy fermion masses, and in consequence it is necessary to subtract the bottom contribution and to take into account the q^2 dependence in the top loop. In the approximation $I(m_\sigma^2/m_b^2) \approx 0$, it results

$$
\begin{aligned}
g_{\hat{h}}(m_\sigma^2) &\approx \frac{1}{6}\frac{d}{dh}\left(\log\det(\mathcal{M}\mathcal{M}^\dagger) - \log(m_t(h,\sigma)\,m_t^*(h,\sigma)) - \log(m_b(h,\sigma)\,m_b^*(h,\sigma))\right)\Big|_{\substack{h=v\\\sigma=v_\sigma}} \\
&\quad + \frac{1}{v}I\left(\frac{m_\sigma^2}{m_t^2}\right) \\
&= \frac{1}{v}I\left(\frac{m_\sigma^2}{m_t^2}\right) - \frac{2}{3}v\left(\frac{y_1 y_2}{M_1 M_5} + \frac{y_1' y_2'}{M_1' M_5'}\right) + O\left(\frac{v v_\sigma^2}{M_1^2 M_5^2}, \frac{v v_\sigma^2}{M_1'^2 M_5'^2}\right)\,,
\end{aligned} \tag{2.80}
$$

$$
\begin{aligned}
g_{\hat{\sigma}}(m_\sigma^2) &\approx \frac{1}{6}\frac{d}{d\sigma}\left(\log\det(\mathcal{M}\mathcal{M}^\dagger) - \log(m_t(h,\sigma)\,m_t^*(h,\sigma)) - \log(m_b(h,\sigma)\,m_b^*(h,\sigma))\right)\Big|_{\substack{h=v\\\sigma=v_\sigma}} \\
&\quad - \frac{y_2}{M_5}\frac{\Lambda_2}{\Lambda_3}I\left(\frac{m_\sigma^2}{m_t^2}\right) \\
&= -\frac{y_2}{M_5}\frac{\Lambda_2}{\Lambda_3}I\left(\frac{m_\sigma^2}{m_t^2}\right) - \frac{2}{3}v_\sigma\left(\frac{y_1 y_2}{M_1 M_5} + \frac{y_1' y_2'}{M_1' M_5'}\right) + O\left(\frac{v_\sigma^3}{M_1^2 M_5^2}, \frac{v_\sigma^3}{M_1'^2 M_5'^2}\right)\,,
\end{aligned} \tag{2.81}
$$

where the eigenvalue of the field dependent mass matrix corresponding to the top quark reads

$$m_t(h,\sigma) = \frac{y_1\Lambda_1\Lambda_3 - y_1 y_2\Lambda_1\Lambda_2\,\sigma/M_5}{M_1 M_5 - y_1 y_2\,(h^2+\sigma^2)}\frac{h}{\sqrt{2}}\,. \tag{2.82}$$

Note that the dominant contribution to the σgg effective coupling requires both y_1 and y_2 to be non-vanishing. In contrast, the dominant contribution to the top quark mass is proportional to y_1 but independent of y_2.

The $\sigma \leftrightarrow gg$ amplitude is finally given by Eqs. (2.71), (2.80) and (2.81), with

$$g_\sigma \equiv g_{\hat{h}}(m_\sigma^2) \sin\gamma + g_{\hat{\sigma}}(m_\sigma^2) \cos\gamma \,.$$

The matrix elements modulus square for $gg \to h$ and $gg \to \sigma$, averaged over the polarisations of the initial state, are then given by

$$\overline{|\mathcal{A}_h|^2} = \frac{\alpha_S^2 m_h^4}{64\pi^2}\,g_h^2\,,$$

$$\overline{|\mathcal{A}_\sigma|^2} \;\; = \;\; \frac{\alpha_S^2 m_\sigma^4}{64\pi^2}\, g_\sigma^2 \,. \tag{2.83}$$

In terms of those amplitudes, the cross section at the parton level can be expressed as

$$\sigma_{part}(gg \to h) \;\; = \;\; \overline{|\mathcal{A}_h|^2}\, \frac{\pi}{s_{part}}\, \delta(s_{part} - m_h^2)$$

$$= \;\; \overline{|\mathcal{A}_h|^2}\, \frac{\pi}{\tau s^2}\, \delta(\tau - \frac{m_h^2}{s})\,, \tag{2.84}$$

where as usual s_{part} denotes the center-of-mass energy at the parton level $s_{part} = \tau s$. A similar expression holds for $\sigma_{part}(gg \to \sigma)$. By convoluting the cross-section with the gluon densities $G(x)$ we finally obtain

$$\sigma(pp \to h) \;\; = \;\; \overline{|\mathcal{A}_h|^2}\, \frac{\pi}{m_h^2 s} \times \int_{m_h^2/s}^1 \frac{dx}{x}\, G(x)\, G\left(\frac{m_h^2}{sx}\right),$$

$$\sigma(pp \to \sigma) \;\; = \;\; \overline{|\mathcal{A}_\sigma|^2}\, \frac{\pi}{m_\sigma^2 s} \times \int_{m_\sigma^2/s}^1 \frac{dx}{x}\, G(x)\, G\left(\frac{m_\sigma^2}{sx}\right). \tag{2.85}$$

In resume, for very heavy fermion partners the h-gluon-gluon transitions are dominated by the top quark contribution, while they have a more significant impact on σ-gluon-gluon transitions.

2.3.4   Higgs and σ decay into $\gamma\gamma$

There are no direct $h\gamma\gamma$ or $\sigma\gamma\gamma$ couplings in our Lagrangian. They arise instead as effective interactions from loops of fermions and, in the case of $h\gamma\gamma$, also of massive vector bosons. As in the case of h and σ production via gg fusion, we distinguish between mass eigenstates h, σ and the unrotated (interaction) eigenstates $\hat{h}, \hat{\sigma}$.

Let the scalar-photon-photon amplitudes $h\gamma\gamma$ and $\sigma\gamma\gamma$ be defined as

$$\mathcal{A}_{h(\sigma)\leftrightarrow\gamma\gamma}(m_{h(\sigma)}^2) = i\frac{\alpha}{\pi}\, \Omega_{h(\sigma)}\, (p\cdot k\; g^{\mu\nu} - p^\mu k^\nu)\delta^{ab}\,, \tag{2.86}$$

where again Ω_h and Ω_σ are scale dependent functions. The decay amplitudes are then given by

$$\Gamma(h \to \gamma\gamma) = \frac{\alpha^2 m_h^3}{64\pi^3}|\Omega_h|^2\,, \qquad \Gamma(\sigma \to \gamma\gamma) = \frac{\alpha^2 m_\sigma^3}{64\pi^3}|\Omega_\sigma|^2\,. \tag{2.87}$$

In the model under study, the contributions can again be decomposed as

$$\Omega_h \;\; = \;\; \cos\gamma\, \Omega_{\hat{h}}(m_h^2) - \sin\gamma\, \Omega_{\hat{\sigma}}(m_h^2)\,,$$

$$\Omega_\sigma \;\; = \;\; \sin\gamma\, \Omega_{\hat{h}}(m_\sigma^2) + \cos\gamma\, \Omega_{\hat{\sigma}}(m_\sigma^2)\,.$$

While both unrotated scalar fields $\hat{h}$ and $\hat{\sigma}$ couple to fermions, only $\hat{h}$ couples to the W boson,

$$\Omega_{\hat{h}}(q^2) = \Omega_{\hat{h}}^F(q^2) + \Omega_{\hat{h}}^W(q^2) \qquad , \qquad\qquad \Omega_{\hat{\sigma}}(q^2) = \Omega_{\hat{\sigma}}^F(q^2)\,,$$

where the superscripts F and W stand for fermionic and gauge contributions, respectively. The latter is akin to the SM one, that is,

$$\Omega_{\hat{h}}^W(q^2) = \frac{g^2 v}{8 m_W^2} \, I_W\left(\frac{4\, m_W^2}{q^2}\right),$$ (2.88)

where the factor $g^2 v$ results from the Higgs$-WW$ vertex, and the remaining part $I_W/8 m_W^2$ results from the kinematics of the loop integral

$$I_W(x) = 2 + 3x + 3x(2-x)f(x), \qquad f(x) = \begin{cases} \arcsin^2(1/\sqrt{x}) & x \geq 1 \\ -\frac{1}{4}\left[\log\frac{1+\sqrt{1-x}}{1-\sqrt{1-x}} - i\pi\right]^2 & x < 1 \end{cases}.$$ (2.89)

$h \leftrightarrow \gamma\gamma$ transitions

At the Higgs mass scale, the SM $\hat{h}WW$ coupling in Eq. (2.88) is given by

$$\Omega_{\hat{h}}^W(m_h^2) \approx \frac{4.2}{v}.$$ (2.90)

The SM quark contributions are in turn given by

$$\Omega_h^{SM,F} = -6\sum_f Q_f^2 \left(\frac{y_f}{\sqrt{2}}\right) \frac{1}{m_f} I\left(\frac{m_h^2}{m_f^2}\right) \approx -\frac{8}{9}\frac{1}{v},$$ (2.91)

where y_f is the fermion Yukawa coupling, $m_f = y_f\, v/\sqrt{2}$, and the remaining factor I/m_f results from the loop integral. The last expression in Eq. (2.91) corresponds to the top contribution, which dominates the SM fermionic contribution. The SM decay $h \to \gamma\gamma$ decay rate is as given in Eq. (2.86) with $\Omega_h = \Omega_h^{SM,W} + \Omega_h^{SM,F}$. In the model under consideration these expressions for the quark contributions to the $h \to \gamma\gamma$ transitions are generalized as follows, in analogy with the gg fusion analysis above,

$$
\begin{aligned}
\Omega_{\hat{h}}^F(m_h^2) &= -2\sum_f N_C^f Q_f^2\, \omega_f^h(m_h^2) = -2\left[3\left(\frac{2}{3}\right)^2 \omega_{2/3}^h(m_h^2) + 3\left(-\frac{1}{3}\right)^2 \omega_{-1/3}^h(m_h^2)\right], \\
\Omega_{\hat{\sigma}}^F(m_h^2) &= -2\sum_f N_C^f Q_f^2\, \omega_f^\sigma(m_h^2) = -2\left[3\left(\frac{2}{3}\right)^2 \omega_{2/3}^\sigma(m_h^2) + 3\left(-\frac{1}{3}\right)^2 \omega_{-1/3}^\sigma(m_h^2)\right],
\end{aligned}
$$ (2.92)

where N_C^f is the number of colors of a given quark species f, and ω_f^h are scale-dependent functions, which for charge $2/3$ and $-1/3$ fermions read

$$
\begin{aligned}
\omega_{2/3}^h(m_h^2) &\equiv \left.\frac{1}{6}\frac{d}{dh}\left(\log\det(\mathcal{M}_\mathcal{T}\,\mathcal{M}_\mathcal{T}^\dagger)\right)\right|_{\substack{h=v \\ \sigma=v_\sigma}} = \frac{1}{3v}, \\
\omega_{-1/3}^h(m_h^2) &\equiv \left.\frac{1}{6}\frac{d}{dh}\left(\log\det(\mathcal{M}_\mathcal{B}\,\mathcal{M}_\mathcal{B}^\dagger) - \log(m_b(h,\sigma)\, m_b^*(h,\sigma))\right)\right|_{\substack{h=v \\ \sigma=v_\sigma}}
\end{aligned}
$$ (2.93)

$$= -\frac{2}{3}v\frac{y'_1 y'_2}{M'_1 M'_5} + \mathcal{O}\left(\frac{v\,v_\sigma^2}{M'^2_1 M'^2_5}\right).$$

For the ω^σ functions, it holds instead

$$\omega^\sigma_{2/3}(m_h^2) \equiv \frac{1}{6}\frac{d}{d\sigma}\left(\log\det(\mathcal{M_T}\,\mathcal{M_T^\dagger})\right)\Big|_{\substack{h=v\\\sigma=v_\sigma}} = -\frac{y_2}{3M_5}\frac{\Lambda_2}{\Lambda_3} + \mathcal{O}\left(\frac{v_\sigma}{M_5^2}\right),$$

$$\omega^\sigma_{-1/3}(m_h^2) \equiv \frac{1}{6}\frac{d}{d\sigma}\left(\log\det(\mathcal{M_B}\,\mathcal{M_B^\dagger}) - \log(m_b(h,\sigma)\,m_b^*(h,\sigma))\right)\Big|_{\substack{h=v\\\sigma=v_\sigma}} \tag{2.94}$$

$$= -\frac{2}{3}\frac{v_\sigma y'_1 y'_2}{M'_1 M'_5} + \mathcal{O}\left(\frac{v_\sigma^3}{M'^2_1 M'^2_5}\right).$$

In these expressions the bottom contribution was neglected, while it has been assumed $m_h \ll m_i$ for the top mass and all other exotic fermion masses m_i.

$\sigma \leftrightarrow \gamma\gamma$ transitions

Similarly, for σ decaying into two photons the contributions for the unrotated field $\hat\sigma$ are given by

$$\Omega^F_{\hat\sigma}(m_\sigma^2) = -2\sum_f N_C^f Q_f^2\,\omega^\sigma_f(m_\sigma^2) = -2\left[3\left(\frac{2}{3}\right)^2\omega^\sigma_{2/3}(m_\sigma^2) + 3\left(-\frac{1}{3}\right)^2\omega^\sigma_{-1/3}(m_\sigma^2)\right],$$

$$\Omega^F_{\hat h}(m_\sigma^2) = -2\sum_f N_C^f Q_f^2\,\omega^h_f(m_\sigma^2) = -2\left[3\left(\frac{2}{3}\right)^2\omega^h_{2/3}(m_\sigma^2) + 3\left(-\frac{1}{3}\right)^2\omega^h_{-1/3}(m_\sigma^2)\right],$$

where

$$\omega^\sigma_{2/3}(m_\sigma^2) \equiv \frac{1}{6}\frac{d}{d\sigma}\left(\log\det(\mathcal{M_T}\,\mathcal{M_T^\dagger}) - \log(m_t(h,\sigma)\,m_t^*(h,\sigma))\right)\Big|_{\substack{h=v\\\sigma=v_\sigma}} - \frac{y_2}{M_5}\frac{\Lambda_2}{\Lambda_3}I\left(\frac{m_\sigma^2}{m_t^2}\right)$$

$$= -\frac{y_2}{M_5}\frac{\Lambda_2}{\Lambda_3}I\left(\frac{m_\sigma^2}{m_t^2}\right) - \frac{2}{3}v_\sigma\frac{y_1 y_2}{M_1 M_5} + \mathcal{O}\left(\frac{v_\sigma^3}{M_1^2 M_5^2}\right),$$

$$\omega^\sigma_{-1/3}(m_\sigma^2) \equiv \frac{1}{6}\frac{d}{d\sigma}\left(\log\det(\mathcal{M_B}\,\mathcal{M_B^\dagger}) - \log(m_b(h,\sigma)\,m_b^*(h,\sigma))\right)\Big|_{\substack{h=v\\\sigma=v_\sigma}} \tag{2.95}$$

$$= -\frac{2}{3}v_\sigma\frac{y'_1 y'_2}{M'_1 M'_5} + \mathcal{O}\left(\frac{v_\sigma^3}{M'^2_1 M'^2_5}\right),$$

while for $\omega^h(m_\sigma^2)$ it results

$$\omega^h_{2/3}(m_\sigma^2) \equiv \frac{1}{6}\frac{d}{dh}\left(\log\det(\mathcal{M_T}\,\mathcal{M_T^\dagger}) - \log(m_t(h,\sigma)\,m_t^*(h,\sigma))\right)\Big|_{\substack{h=v\\\sigma=v_\sigma}} + \frac{1}{v}I\left(\frac{m_\sigma^2}{m_t^2}\right)$$

$$= \frac{1}{v}I\left(\frac{m_\sigma^2}{m_t^2}\right) - \frac{2}{3}v\frac{y_1 y_2}{M_1 M_5} + \mathcal{O}\left(\frac{v v_\sigma^2}{M_1^2 M_5^2}\right),$$

$$\omega^h_{-1/3}(m^2_\sigma) \equiv \frac{1}{6}\frac{d}{dh}\left(\log\det(\mathcal{M}_\mathcal{B}\,\mathcal{M}^\dagger_\mathcal{B}) - \log(m_b(h,\sigma)\,m^*_b(h,\sigma))\right)\Big|_{\substack{h=v\\\sigma=v_\sigma}}$$

$$= -\frac{2}{3}v\frac{y'_1 y'_2}{M'_1 M'_5} + \mathcal{O}\left(\frac{vv^2_\sigma}{M'^2_1 M'^2_5}\right), \tag{2.96}$$

where it has been assumed that $m_b \ll m_t, m_\sigma$ while $m_\sigma \ll m_i$ for all the other heavy quarks.

Finally, the physical h and σ decay widths into two photons are given by

$$\Gamma(h \to \gamma\gamma) = \frac{\alpha^2 m^3_h}{64\pi^3}\left|\cos\gamma\left[\Omega^W_{\hat{h}}(m^2_h) + \Omega^F_{\hat{h}}(m^2_h)\right] - \sin\gamma\,\Omega^F_{\hat{\sigma}}(m^2_h)\right|^2,$$

$$\Gamma(\sigma \to \gamma\gamma) = \frac{\alpha^2 m^3_\sigma}{64\pi^3}\left|\sin\gamma\left[\Omega^W_{\hat{h}}(m^2_\sigma) + \Omega^F_{\hat{h}}(m^2_\sigma)\right] + \cos\gamma\,\Omega^F_{\hat{\sigma}}(m^2_\sigma)\right|^2. \tag{2.97}$$

Quantitatively, $\sigma \to \gamma\gamma$ transitions are dominated by the $W^\pm$ loop contributions unless the scalar mixing is small enough for the heavy partner loop contribution to be significant.

2.4 The σ resonance at the LHC

The constraints from electroweak precision tests explored in Sect. 2.3.2 showed that a scenario with a light σ particle tends to diminish the tension with data. On the other side, from the theoretical viewpoint the assumption of a PNGB nature for the Higgs boson within an approximate global $SO(5)$ symmetry mildly broken by soft terms prefers a sizeable mass for the σ particle, see Fig. (2.1). The PNGB interpretation implies the existence of a non-zero mixing between σ and h, specially when considering naturalness as a guideline since $\sin^2\gamma \sim \xi \ll 1$ would require a strong fine-tuning of the theory -see the discussion after Eq. (2.12) and Eqs. (2.21), (2.22) and (2.41).

As argued in Sect. 2.1.3, the scalar potential is completely determined by the masses m_h and m_σ, the constant G_F, and the scalar mixing $\sin\gamma$. The conclusions obtained for the linear σ model together with generic soft breaking terms are of general validity. The extra ingredients needed to determine the phenomenology of the σ particle are its couplings to the vector-like fermions of the theory, which introduce instead significant model-dependence and may have important consequences particularly in the production of this scalar.

In order to estimate the LHC constraints on the model, we recast many LHC searches for scalar resonances into the σ parameter space, calculating the production cross section and decays of the σ particle. The production of the σ particle at the LHC may proceed mainly via two processes, gluon fusion and vector boson fusion (VBF). Gluon fusion usually dominates the production due to the large gluon pdfs. Nevertheless, this conclusion is somewhat model-dependent as the heavy fermion couplings to σ may a priori enhance or diminish the cross section. VBF depends essentially on the mixing angle γ, but it typically yields a lower production cross section than gluon fusion for $m_\sigma < \mathcal{O}(1\text{ TeV})$, for which it will have unnoticeable impact in what follows.

Consider then the cross section for σ production via gluon fusion $\sigma(gg \to \sigma)$. To account for higher order corrections to $\Gamma(\sigma \to gg)$, we will profit from the results in the literature for a heavy

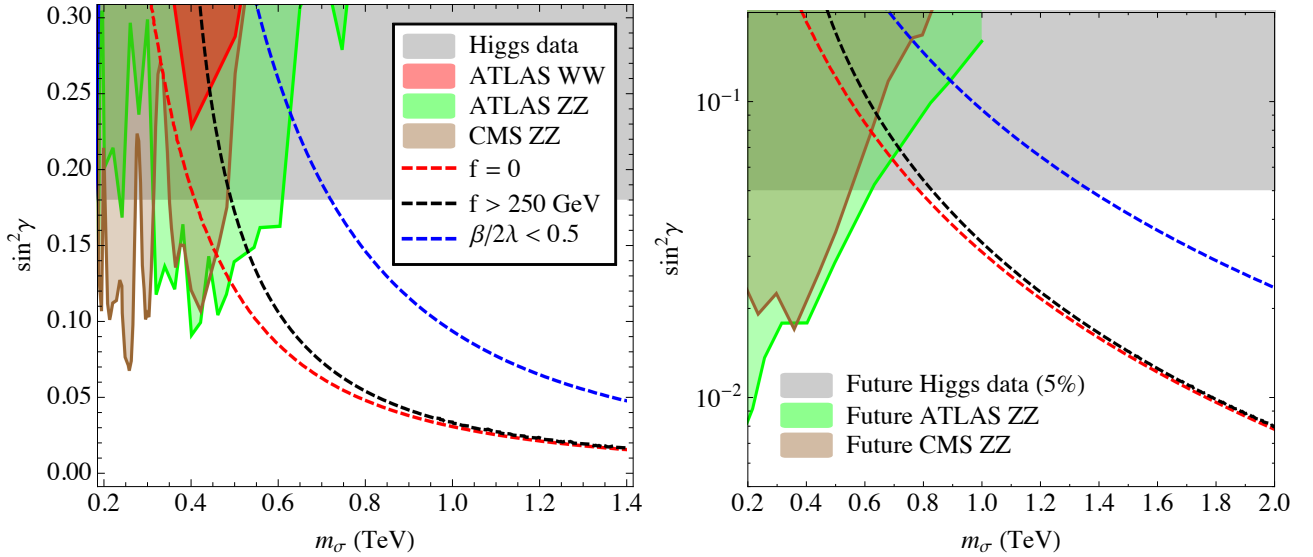**Figure 2.8:** Present LHC (left panel) and future LHC run-2 (right panel) constraints on $\sin^2 \gamma$ versus σ mass parameter space in the case where gluon fusion is dominated by the top loop. For the latter, a total luminosity of 3ab^{-1} was assumed.

SM-like Higgs boson H', using the following approximation

$$\sigma(gg \to \sigma) \simeq \frac{|A(\sigma \to gg)|^2}{|A_{SM}(H' \to gg)|^2} \, \sigma_{SM}(gg \to H') \,, \tag{2.98}$$

where $A(\sigma, H' \to gg)$ refers to leading order (one loop) amplitudes and $\sigma_{SM}(gg \to H')$ is the NNLO standard gluon fusion production cross section given in Ref. [125]. For illustrative purposes we discuss next the LHC impact of the σ particle in two steps: first an "only scalars" analysis will be considered, to add next to it the effect of the rather model-dependent fermionic sector.

In the only scalars scenario, that is, a case in which the impact of the heavy fermions on gluon fusion is negligible compared to the top contribution, the production amplitude can be approximated by the top loop contribution for a heavy SM Higgs weighted down by $\sin \gamma$. Under this assumption, we have recasted the LHC searches for a heavy Higgs-like particle into constraints in the $\{m_\sigma, \sin^2 \gamma\}$ plane, and the results are shown on the left panel of Fig. 2.8. The searches taken into account here include diphoton [126, 127], diboson [128–131] and dihiggs [132, 133] decays. The figure shows that LHC data are sensitive to $\sin^2 \gamma \simeq 0.1$ for $m_\sigma < 600$ GeV, otherwise Higgs measurements put a bound on $\sin^2 \gamma < 0.18$ independently of m_σ. It is worth noting that these bounds apply well beyond the model discussed here: they are valid for any physics scenario in which the role of the Higgs particle is substituted by a Higgs-scalar system with a generic mixing angle γ, independently of the details of the theory. In addition, by combining the LHC data with theoretically motivated constraints as those mentioned above, interesting bounds can be derived: a PNGB nature for the Higgs boson corresponds to the area to the right of the red curve depicted, see also Fig. 2.1, corresponding to the minimal theoretical requirement $f^2 > 0$ for $SO(5)$ to be spontaneously broken, resulting in the bound $m_\sigma > 500$ GeV in particular from the impact of ATLAS $H_{\text{heavy}} \to ZZ$ searches. If f^2 values above the electroweak scale are instead required (black curve) $m_\sigma > 550$ GeV
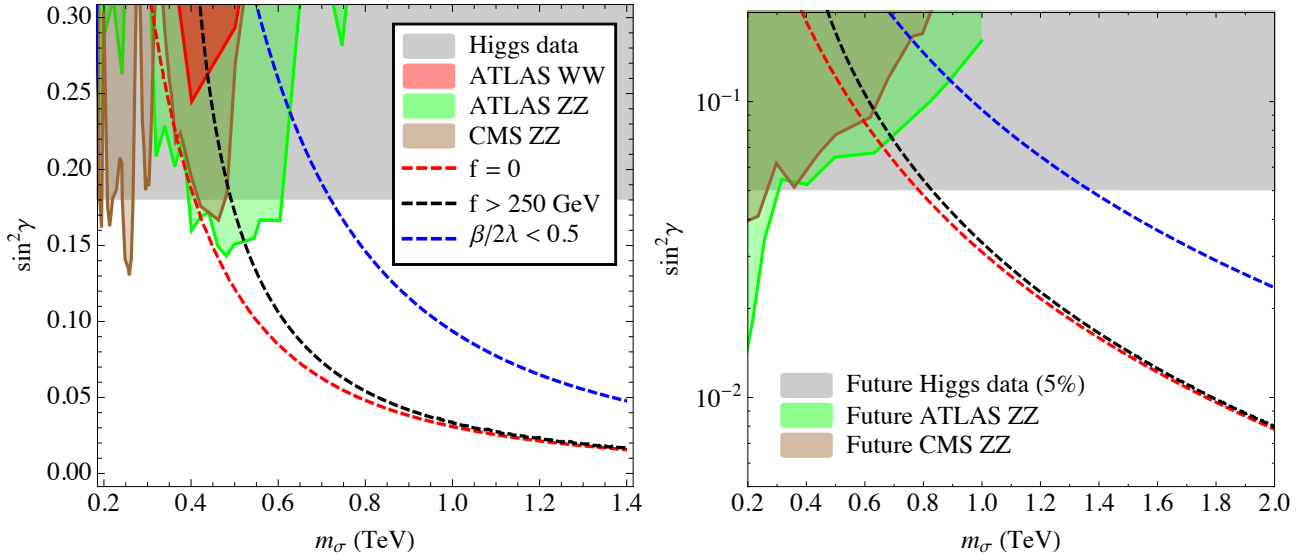
Figure 2.9: Same as fig. (2.8), but considering a sizeable contribution of the heavy fermion sector to gluon fusion. See text for details.

follows. The future prospects for this "only scalars" scenario are depicted on the right panel of Fig. 2.8. It shows the future LHC sensitivity in the ZZ decay channel of the 14 TeV LHC run with an integrated luminosity of 3 ab^{-1}, for both ATLAS and CMS [134], as well as the mixing disfavoured by Higgs data assuming a 5% precision on the Higgs couplings to SM particles. In the absence of any beyond the SM signal, future LHC data together with the aforementioned theory constraints could push the limit on the σ mass above 900 GeV–1.4 TeV.

The difference between the LHC predictions of the model discussed and those stemming from extending the SM by a generic scalar singlet (see e.g. Ref. [135]) is the underlying $SO(5)$ structure of the former, which prescribes a specific relation between the quartic terms in the potential as well as specific soft breaking terms. In the generic extra singlet scenarios, the allowed parameter space is given by the entire white area in Fig. 2.8, while a PNGB nature for the Higgs restricts the allowed region to the area to the right of the curves depicted in the figure.

Finally, the impact of the heavy fermions of the model on the gluon fusion cross section may be significant. Using the approximate expressions in Eqs. (2.83) and (2.85), assuming that the factor $y_1 y_2 / M_1 M_5$ in Eqs. (2.80) and (2.81) is the largest contribution between $1/(4\pi f)^2$ and 1 TeV^{-2} (the latter will also be assumed when $f^2 < 0$), the results obtained are depicted in Fig. 2.9. It shows that the present LHC bounds on $\sin^2 \gamma$ can be weakened by $\mathcal{O}(30 - 50\%)$ with respect to the "only scalars" bounds in Fig. 2.8. This is due to a destructive interference between the heavy fermions and the top loops, for the set of parameters considered. Moreover, future searches will be much more sensitive to the heavy fermion sector as they probe smaller mixing angles, and therefore they enter regions in parameter space where the top quark is relatively less important to the σ phenomenology.

Coefficient	Leading Order in f/M
$\mathcal{Z}_{q_L}$	$\left(1 + \frac{\Lambda_1^2}{M_5^2} + \frac{\Lambda_1'^2}{M_5'^2}\right)$
$\mathcal{Z}_{t_R}$	$\left(1 + \frac{\Lambda_2^2}{M_5^2} + \frac{\Lambda_3^2}{M_1^2}\right)$
$\mathcal{Z}_{b_R}$	$\left(1 + \frac{\Lambda_2'^2}{M_5'^2} + \frac{\Lambda_3'^2}{M_1'^2}\right)$

Table 2.2: Table with the definitions for the renormalization factors.

The 750 GeV di-photon excess

In 2015 both ATLAS [136] and CMS [137] reported the mild 750 GeV di-photon excess. Even though it disappeared in the updated results we have analysed weather the σ resonance could have been responsible for generating this excess.

It turned out to be highly unlikely because of the constraints imposed on the scalar couplings by the approximate $SO(5)$ symmetry of the scalar potential, as well as the uniqueness of the signal, as explained next.

Since the decay $\sigma \to \gamma\gamma$ is loop induced, the corresponding branching ratio tends to be very small. In order to be able to account for the excess observed, the $h-\sigma$ mixing needs to be tiny, so that for instance the WW and ZZ channels are suppressed and the loop-induced processes may dominate the decay. This requires $\sin^2\gamma \ll (m_h/m_\sigma)^2 \simeq 0.03$, which Eq. (2.21) shows to require on one side $f^2 < 0$ –for which the Higgs cannot be interpreted as a PNGB, and on the other a very small and fine-tuned α value as $\alpha^2 \propto \beta^2 \sin^2(2\gamma)$; overall a very unnatural scenario.

Furthermore, since the mixing is so small, the top loop essentially does not contribute to the production and decay anymore. Hence, to obtain a large enough $\Gamma(\sigma \to \gamma\gamma)$ and $\Gamma(\sigma \to gg)$ the fermion content needs a higher multiplicity than what is assumed in this analysis, as well as large Yukawa couplings, since σ mainly couples to T_1 and T_5 with electric charges 2/3. Even assuming such an extreme and extended configuration, the stability of the potential could become a very serious issue as the fermion contribution to the beta function of the quartic couplings is negative. Additional field content would then be possibly required to compensate for this effect, making the model extremely *ad hoc*. Therefore, we found no compelling argument to interpret the 750 GeV excess as corresponding to the σ scalar studied here.

2.5 $d \leq 6$ Fermionic Effective Lagrangian

The linear model described is renormalizable and valid for any mass range of the fermionic and/or scalar exotic fields. Two simplifying limits are specially interesting: i) the heavy fermion regime, $M \gg m_\sigma \gg v$, where M generically represents the exotic fermionic scales M_i in Eq. (2.30); ii) the heavy singlet regime, $m_\sigma \gg M \gg v$. We have concentrated on the first scenario, considering

	Operator	c_i	Leading Order in f/M
dim 4	$\bar{q}_L \, \widetilde{H} \, t_R$	$-y_t$	$-\left(\frac{y_1 \Lambda_1 \Lambda_3}{M_1 M_5}\right) \mathcal{Z}_{q_L}^{-1/2} \mathcal{Z}_{t_R}^{-1/2}$
	$\bar{q}_L \, H \, b_R$	$-y_b$	$-\left(\frac{y_1' \Lambda_1' \Lambda_3'}{M_1' M_5'}\right) \mathcal{Z}_{q_L}^{-1/2} \mathcal{Z}_{b_R}^{-1/2}$
dim 5	$\sigma \, (\bar{q}_L \widetilde{H} t_R)$	$c_{\sigma 1}^t$	$\frac{y_t}{M_5}\left(y_2 \frac{\Lambda_2}{\Lambda_3} - \left(y_1 \frac{\Lambda_2 \Lambda_3}{M_1 M_5} + y_2 \frac{\Lambda_2 \Lambda_3}{M_1^2}\right) \mathcal{Z}_{t_R}^{-1}\right)$
	$\sigma \, (\bar{q}_L H b_R)$	$c_{\sigma 1}^b$	$\frac{y_b}{M_5'}\left(y_2' \frac{\Lambda_2'}{\Lambda_3'} - \left(y_1' \frac{\Lambda_2' \Lambda_3'}{M_1' M_5'} + y_2' \frac{\Lambda_2' \Lambda_3'}{M_1'^2}\right) \mathcal{Z}_{b_R}^{-1}\right)$
dim 6	$\sigma^2 \, (\bar{q}_L \widetilde{H} t_R)$	$c_{\sigma 2}^t$	$-\frac{y_t}{M_1 M_5}\left(y_1 y_2 - \left(y_1 y_2 \left(2\frac{\Lambda_2^2}{M_5^2} + \frac{\Lambda_3^2}{M_1^2}\right) + \frac{3 y_2^2 \Lambda_2^2 + y_1^2 \Lambda_3^2}{2 M_1 M_5}\right) \mathcal{Z}_{t_R}^{-1} \right.$ $\left. + 2\frac{\Lambda_2^2 \Lambda_3^2}{M_1 M_5}\left(\frac{y_1^2}{M_5^2} + \frac{2 y_1 y_2}{M_5 M_1} + \frac{y_2^2}{M_1^2}\right) \mathcal{Z}_{t_R}^{-2}\right)$
	$\sigma^2 \, (\bar{q}_L H b_R)$	$c_{\sigma 2}^b$	$-\frac{y_b}{M_1' M_5'}\left(y_1' y_2' - \left(y_1' y_2' \left(2\frac{\Lambda_2'^2}{M_5'^2} + \frac{\Lambda_3'^2}{M_1'^2}\right) + \frac{3 y_2'^2 \Lambda_2'^2 + y_1'^2 \Lambda_3'^2}{2 M_1' M_5'}\right) \mathcal{Z}_{b_R}^{-1} \right.$ $\left. + 2\frac{\Lambda_2'^2 \Lambda_3'^2}{M_1' M_5'}\left(\frac{y_1'^2}{M_5'^2} + \frac{2 y_1' y_2'}{M_5' M_1'} + \frac{y_2'^2}{M_1'^2}\right) \mathcal{Z}_{b_R}^{-2}\right)$
	$|H|^2 \, (\bar{q}_L \widetilde{H} t_R)$	c_{H2}^t	$-\frac{y_t}{M_1 M_5}\left(2 y_1 y_2 - \left(2 y_1 y_2 \frac{\Lambda_3^2}{M_1^2} + y_1^2 \frac{\Lambda_3^2}{M_1 M_5}\right) \mathcal{Z}_{t_R}^{-1} \right.$ $\left. - \left(y_1 y_2 \frac{\Lambda_1^2}{M_5^2} + \frac{y_1^2}{2} \frac{\Lambda_1^2}{M_1 M_5}\right) \mathcal{Z}_{q_L}^{-1}\right)$
	$|H|^2 \, (\bar{q}_L H b_R)$	c_{H2}^b	$-\frac{y_b}{M_1' M_5'}\left(2 y_1' y_2' - \left(2 y_1' y_2' \frac{\Lambda_3'^2}{M_1'^2} + y_1'^2 \frac{\Lambda_3'^2}{M_1' M_5'}\right) \mathcal{Z}_{b_R}^{-1} \right.$ $\left. - \left(y_1' y_2' \frac{\Lambda_1'^2}{M_5'^2} + \frac{y_1'^2}{2} \frac{\Lambda_1'^2}{M_1' M_5'}\right) \mathcal{Z}_{q_L}^{-1}\right)$
	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{q}_L \gamma^\mu q_L)$	$c_L^{(1)}$	$\frac{1}{4}\left(\frac{y_1^2 \Lambda_1^2}{M_1^2 M_5^2} - \frac{y_1'^2 \Lambda_1'^2}{M_1'^2 M_5'^2}\right) \mathcal{Z}_{q_L}^{-1}$
	$(H^\dagger i \overleftrightarrow{D}_\mu^i H)(\bar{q}_L \tau^i \gamma^\mu q_L)$	$c_L^{(3)}$	$-\frac{1}{4}\left(\frac{y_1^2 \Lambda_1^2}{M_1^2 M_5^2} + \frac{y_1'^2 \Lambda_1'^2}{M_1'^2 M_5'^2}\right) \mathcal{Z}_{q_L}^{-1}$
	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{t}_R \gamma^\mu t_R)$	c_R^t	0
	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{b}_R \gamma^\mu b_R)$	c_R^b	0
	$i(\widetilde{H}^\dagger D_\mu H)(\bar{t}_R \gamma^\mu b_R)$	c_R^{tb}	0

Table 2.3: Leading order low-energy effective operators induced and their coefficients. Note that the Yukawa couplings defined in the two first rows appear as well in coefficients of some higher order operators. The renormalization factors present have been defined in Table 2.2. Those operators made out exclusively of SM fields have been written in the Warsaw basis [88].

a not-so-heavy extra singlet in the spectrum and its phenomenological consequences. The second limit is instead interesting to elucidate the connection between the linear (weak) and non-linear (strong) BSM physics scenarios: the $m_\sigma \to \infty$ regime should lead to the non-linear scenarios usually explored in the literature about composite Higgs; it will be discussed in detail in Chapter 3.

When condition i) is satisfied, some important low-energy effects (and model dependencies) induced by the exotic fermions are easily inferred by integrating them out. The procedure is quite lengthy; here only the resulting mass-dimension (d) 4, 5 and 6 effective operators and their coefficients are summarized. For energies $E < M$, the effective Lagrangian describing the $d \leq 6$ interactions of fermions with gauge and scalar fields can be written as

$$\mathcal{L}_{eff} = \bar{q}_L i \slashed{D} \, q_L + \bar{t}_R i \slashed{D} \, t_R + \bar{b}_R i \slashed{D} \, b_R + \sum_i c_i \mathcal{O}_i \,, \tag{2.99}$$

where the set $\{\mathcal{O}_i\}$ includes operators of dimension four (for which the induced coefficients are the leading contributions to the top and bottom Yukawa couplings), five and six. We will use the "Warsaw basis" [88] below.

In most models (for instance in composite Higgs ones) it is reasonable to assume that the goldstone boson scale f and the scalar vevs all satisfy $f \,, v \,, v_\sigma \ll M$. In what follows we will thus assume $f/M \ll 1$ for simplicity, while $\Lambda \approx M$ will be considered with Λ denoting generically the composite Λ_i scales in Eq. (2.30). The light field kinetic energies get contributions which require wave function renormalization in order to recover canonically normalized kinetic energies,

$$q_L \quad \to \quad \mathcal{Z}_{q_L}^{-1/2} q_L \,, \tag{2.100}$$

$$q_R \quad \to \quad \begin{pmatrix} \mathcal{Z}_{t_R}^{-1/2} & 0 \\ 0 & \mathcal{Z}_{b_R}^{-1/2} \end{pmatrix} \begin{pmatrix} t_R \\ b_R \end{pmatrix} \,, \tag{2.101}$$

where $\mathcal{Z}_{t_R}^{-1/2}$ and $\mathcal{Z}_{b_R}^{-1/2}$ are given in Table 2.2. The operators obtained and their coefficients resulting after those redefinitions, at leading order in f/M, are shown in Table 2.3, where the following definitions have been used,

$$(H^\dagger i \overleftrightarrow{D}_\mu H) \equiv i \left(H^\dagger (\overrightarrow{D}_\mu H) - (H^\dagger \overleftarrow{D}_\mu) H \right) \,,$$

$$(H^\dagger i \overleftrightarrow{D}^i_\mu H) \equiv i \left(H^\dagger \tau^i (\overrightarrow{D}_\mu H) - (H^\dagger \overleftarrow{D}_\mu) \tau^i H \right) \,.$$

In writing Eq. (2.99) and Table 2.3 the unshifted scalar fields have been assumed. This fact introduces a potential subtlety that we discuss next. Consider for instance the top and bottom quark masses corresponding to the first two operators in the table, which are their respective Yukawa couplings: when the Higgs gets a vev, mass terms for the light quarks are generated. Additional contributions to the light quark masses stem however from the next six operators in the list, for $\sigma = \langle \sigma \rangle$ and $H = \langle H \rangle$. The corrections induced in the top and bottom mass are of higher order in f/M, though, and do not need to be retained when working at leading order. Finally,

$$m_t = \frac{v}{\sqrt{2}} \left(\frac{y_1 \Lambda_1 \Lambda_3}{M_1 M_5} \right) \frac{1}{\sqrt{\left(1 + \frac{\Lambda_1^2}{M_5^2} + \frac{\Lambda_1'^2}{M_5'^2}\right) \left(1 + \frac{\Lambda_2^2}{M_5^2} + \frac{\Lambda_3^2}{M_1^2}\right)}} \left(1 + \mathcal{O}\left(\frac{f}{M}\right)\right) \,,$$

$$m_b = \frac{v}{\sqrt{2}} \left(\frac{y_1' \Lambda_1' \Lambda_3'}{M_1' M_5'} \right) \frac{1}{\sqrt{\left(1 + \frac{\Lambda_1^2}{M_5^2} + \frac{\Lambda_1'^2}{M_5'^2}\right)\left(1 + \frac{\Lambda_2'^2}{M_5'^2} + \frac{\Lambda_3'^2}{M_1'^2}\right)}} \left(1 + \mathcal{O}\left(\frac{f}{M}\right)\right).$$

The same reasoning applies to other couplings. For example the fermion-σ coupling via the $\mathcal{O}_{\sigma 1}^t$ operator [88] would get corrections proportional to $c_{\sigma 2}^t \langle \sigma \rangle$ –see Table (2.3)– which are of higher order in the f/M expansion, and can thus be disregarded when restraining to the leading contributions.

3

The linear-non-linear frontier for the Goldstone Higgs

A complete renormalizable model of Goldstone Higgs was constructed in Chapter 2. Its scalar part is a linear sigma model including a new singlet scalar σ. The procedure and first steps to obtain the non-linear effective Lagrangian were developed. In particular we have derived the leading order low-energy effective operators composed of the SM fields and scalar σ. In this section we explore the divide between linear and non-linear regimes by varying the mass of the extra scalar: a light σ particle corresponds to a weakly coupled regime, while in the high mass limit the theory will fall back onto the effective non-linear construction.

The effective low-energy effective Lagrangian in a general case contains a large number of couplings. For the generic $SO(5)/SO(4)$ construction, a very reduced subset of those couplings, constituting a complete basis of bosonic operators, was established in Ref. [70]. Here we will focus on the particular case of the minimal $SO(5)$ sigma model, leading to an even more reduced subset of operators –the benchmark low-energy effective Lagrangian. We consider here both bosonic and fermionic operators. The leading linear corrections and the leading dependence on the explicit $SO(5)$-breaking mechanism will be determined as well.

While the bosonic couplings are general, the fermionic part may instead be quite model-dependent. We will address the problem in two approaches:

- A rather model independent one in which the field content of the SM is augmented exclusively by a singlet scalar within the minimal $SO(5)$ setup, while the leading phenomenological impact of heavy fermions is encoded in an effective Yukawa coupling of the SM fields that we will define. This effective operator will serve to parametrise and disentangle among different choices of BSM fermion embeddings.

- In a second step, we consider the heavy fermion representations of the complete model described in the previous chapter. This sector will be integrated out explicitly.

By furthermore keeping track of the linear and heavy-fermion corrections, the analysis will provide
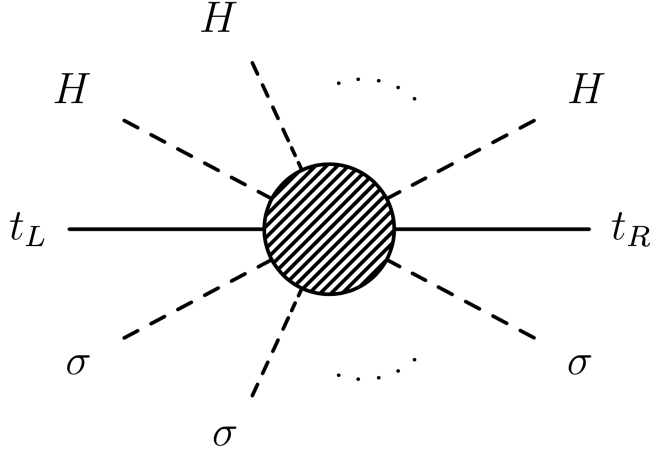
Figure 3.1: Schematic fermion mass operator at low scales with arbitrary insertions of the scalar fields.

candles to identify whether a renormalizable ultraviolet completion exists in nature or alternatively an underlying composite mechanism is at work at high-energy, analogous to QCD for the chiral dynamics involving pions.

3.1 Model independent analysis

The scalar sector has been fully defined in the previous chapter by Eqs. (2.3, 2.8) and remains universal.

Consider now the fermion sector. A generic feature is that the phenomenological constraints on partial compositeness require additional vector-like fermions, which couple and act as mediators among the SM fields. The exact form of the effective coupling is model-dependent and varies according to how the SM fermions are embedded in $SO(5)$. We will obviate until Sect. 3.2 the details of the heavy fermion spectrum, and use instead in this section a simplified –effective– approach to the dominant fermion-induced effects.

The fermionic part of the Lagrangian in Eq. (2.1) will be written as the sum of two terms,

$$\mathcal{L}_{\text{f}} = \mathcal{L}_{\text{f,SM}}^{\text{kin}} + \mathcal{L}_{\text{f}}^{\text{Yuk}}, \tag{3.1}$$

where $\mathcal{L}_{\text{f,SM}}^{\text{kin}}$ comprises the kinetic terms for only SM fermions.

The schematic effective Yukawa coupling in the presence of the σ particle is presented in figure 3.1. It follows that at low energies it is possible to write an effective Yukawa Lagrangian in terms of only the SM fermions, plus h and σ, which respects electroweak gauge invariance but not $SO(5)$ invariance,

$$\mathcal{L}_{\text{f}}^{\text{Yuk}} \equiv y_{\text{f}}^{0} \, \mathcal{O}_{\text{Yuk,f}}^{(n,m)} + \cdots + \text{h.c.}, \tag{3.2}$$

Fermion representation (q_L-q_R)	Yukawa interactions $y_{\mathrm{f}}^0 \mathcal{O}_{\mathrm{Yuk}}^{(n,m)}$
5-1, 5-10, 10-5	$y\mathcal{O}_{\mathrm{Yuk}}^{(0,0)}$
5-5, 10-10, 14-10, 10-14, 14-1	$y\mathcal{O}_{\mathrm{Yuk}}^{(1,0)}$
14-14	$3y\mathcal{O}_{\mathrm{Yuk}}^{(1,0)} - 2y'\mathcal{O}_{\mathrm{Yuk}}^{(1,1)} + 8y'\mathcal{O}_{\mathrm{Yuk}}^{(3,0)}$
14-5	$y\mathcal{O}_{\mathrm{Yuk}}^{(0,0)} + y'\mathcal{O}_{\mathrm{Yuk}}^{(2,0)}$
5-14	$y\mathcal{O}_{\mathrm{Yuk}}^{(0,0)} + y'\mathcal{O}_{\mathrm{Yuk}}^{(0,1)} - 4y'\mathcal{O}_{\mathrm{Yuk}}^{(2,0)}$

Table 3.1: Yukawa operators corresponding to particular embeddings (see e.g. Ref [16]) of a SM quark doublet q_L and right-handed q_R fermion (either up-type or down-type right-handed quark) into $SO(5)$. The coefficients y, y' refer to distinct possible relative weights of $SO(5)$ invariant operators allowed by the models

where the constant y_{f}^0 is a model-dependent coefficient [1] and we define the effective Yukawa operator for a given fermion f as

$$\mathcal{O}_{\mathrm{Yuk,f}}^{(n,m)} \equiv \bar{q}_L \widetilde{H} \, \mathrm{f}_R \left(\frac{\sigma}{f}\right)^n \left(\frac{2H^\dagger H}{f^2}\right)^m . \tag{3.3}$$

The ellipses in Eq. (3.2) refer to other SM fermion operators and possibly extra model-dependent terms coming from the heavy fermion sector.

In the literature of composite Higgs models the notation $\mathrm{MCHM}_{\mathrm{A-B-C}}$ is often used to indicate their fermion composition, with $\mathrm{A, B, C}$ indicating the $SO(5)$ representation in which the SM doublet q_L, up-type right-handed and down-type right-handed fermions are embedded, respectively; else, when only one subindex appears as in $\mathrm{MCHM_A}$ it is understood to be of the type $\mathrm{MCHM_{A-A-A}}$. Table 3.1 summarizes the $\{n, m\}$ parameter values for different models. We do not take into account the models with spinorial $SO(5)$ embeddings [9] as they are phenomenologically excluded [10].

Eq. (3.3) assumes that a given fermion mass corresponds to a single set of $\{n, m\}$ values. This is often the case; for instance, the top and bottom Yukawa couplings in the $\mathrm{MCHM_{5-1-1}}$ model of the previous chapter correspond to $\mathcal{O}_{\mathrm{Yuk}}^{(0,0)}$, while in the $\mathrm{MCHM_5}$ scenario they both correspond to $\mathcal{O}_{\mathrm{Yuk}}^{(1,0)}$ (see e.g. Ref. [69]). Notice that, for these cases with a single Yukawa operator, the global coefficients and suppression scales in Eqs. (3.2)-(3.3) are constrained by the fermion masses and therefore do not constitute any additional model dependence.

Nevertheless, in some scenarios a given fermion mass results instead from combining several operators of the type in Eq. (3.3) with different $\{n, m\}$ values. The procedure derived can be easily

---

[1]The superscript 0 indicates that y_{f}^0 only encodes the leading contributions induced by the heavy fermionic sector.

extended to encompass it. A model-dependence remains then in the relative size of the y and y' weights in Table 3.1. An example is the MCHM$_{14-14-10}$ scenario [16] in which different sets of $\{n, m\}$ values are involved in generating the top mass, while the bottom mass only requires set $\{n, m\} = \{1, 0\}$. The cases of single and of multiple Yukawa operators contributing to a given mass will be further considered explicitly below. We focus in what follows on the top Yukawa coupling unless otherwise explicitly stated, while the conclusions to be obtained are easily generalized to all light fermions.

3.1.1 Polar coordinates

Armed with the tools described, it is quite straightforward to derive the benchmark bosonic Lagrangian as well as the leading couplings involving fermions. To this aim, it is convenient to rewrite the scalar degrees of freedom in polar coordinates,

$$\sigma \;\equiv\; \rho\, c_\varphi\,, \tag{3.4}$$

$$H \;\equiv\; \frac{1}{\sqrt{2}}\, \rho\, \mathbf{U}\, s_\varphi\,, \tag{3.5}$$

with $c_\varphi \equiv \cos \varphi/f$, $s_\varphi \equiv \sin \varphi/f$, and $\mathbf{U}(x) \equiv \exp\{i\boldsymbol{\pi}(x)\boldsymbol{\tau}/f\}$ as usual being the Goldstone matrix corresponding to the longitudinal components of the electroweak gauge bosons. This notation differs from the one used before, as the f scale instead of v divides the field π. In order to obtain the usual v scale suppression and the canonical kinetic term for the would-be-Goldstones after the EWSB the π field has to be rescaled by f/v. Having this in mind, we will keep the f scale suppression in what is following for simplicity. In this notation the scalar Lagrangian in Eq. (2.3) with the potential defined in Eq. (2.8) reads

$$\mathcal{L}_s = \frac{1}{2}\partial_\mu\rho\,\partial^\mu\rho + \frac{\rho^2}{2f^2}\left[\partial_\mu\varphi\,\partial^\mu\varphi - \frac{f^2}{2}s_\varphi^2\,\langle\mathbf{V}_\mu\mathbf{V}^\mu\rangle\right] - \lambda(\rho^2 - f^2)^2 - \alpha f^3 \rho\, c_\varphi + \beta f^2 \rho^2 s_\varphi^2. \tag{3.6}$$

The effective top Yukawa operator in Eqs. (3.2) and (3.3) is then given by

$$y_t^0\, \mathcal{O}_{\text{Yuk},t}^{(n,m)} = \frac{y_t^0}{\sqrt{2}}(\bar{q}_L \mathbf{U} P_+ \mathbf{q}_R)\rho \left(\frac{\rho}{f}\right)^{n+2m} c_\varphi^n s_\varphi^{2m+1}\,, \tag{3.7}$$

where the right-handed SM fermions have been gathered in a formal doublet $\mathbf{q}_R \equiv (t_R,\ b_R)$, with $P_+ \equiv \text{diag}(1,\ 0)$ ($P_- = \text{diag}(0,\ 1)$) being a projector onto the up-type (down-type) right-handed SM fermions.

The ρ and φ fields will develop vevs,

$$\rho \to \rho + \langle\rho\rangle\,, \qquad \varphi \to h + \langle\varphi\rangle\,, \tag{3.8}$$

where at the minimum of the potential the φ field corresponds to

$$\cos\left(\frac{\langle\varphi\rangle}{f}\right) = -\frac{\alpha}{2\beta}\left(1 + \frac{\beta}{2\lambda}\right)^{-1/2}. \tag{3.9}$$

The connection between the vevs of the fields in the linear and polar parametrisations is

$$\langle\rho\rangle = \sqrt{\langle\sigma\rangle^2 + 2\langle H\rangle^2}, \qquad \langle\varphi\rangle = f\,\tan^{-1}\left(\frac{\sqrt{2}\,\langle H\rangle}{\langle\sigma\rangle}\right). \tag{3.10}$$

The scalar resonance, which in the linear parametrisation is customarily denoted σ, is traded by ρ in the polar parametrisation, with $m_\rho = m_\sigma$ exactly as expected for a physical observable, while the Higgs resonance h corresponds now to the excitation of the φ field, see Eq. 3.8.

Finally, as the pure gauge Lagrangian $\mathcal{L}_g$ and the weak coupling to fermions are not modified, the coefficient of the W_μ mass term in Eq. (3.6) allows to identify the electroweak scale v in terms of the Lagrangian parameters:

$$v^2 = \langle\rho\rangle^2\,\sin^2\left(\frac{\langle\varphi\rangle}{f}\right). \tag{3.11}$$

3.1.2   Expansion in $1/\lambda$

The scalar quartic coupling λ can be conventionally traded by the ρ mass, given by $m_\rho^2 \simeq 8\lambda f^2$ for negligible α and β, the non-linear model would be recovered in the limit $m_\rho \gg f$, that is $\lambda \to \infty$. Varying the ρ mass (that is, λ) allows to switch from the regime of perturbative ultraviolet completion to the non-linear one assumed in models in which the Higgs particle is a low-energy remnant of some strong dynamics. We will explore this limit next.

The exact equation of motion for ρ reads

$$\Box\rho - \frac{\rho}{f^2}\left[\partial_\mu\varphi\,\partial^\mu\varphi - \frac{f^2}{2}s_\varphi^2\,\langle\mathbf{V}_\mu\mathbf{V}^\mu\rangle\right] + 4\lambda\rho(\rho^2 - f^2) + \alpha f^3 c_\varphi - 2\beta\rho f^2 s_\varphi^2$$
$$+ (n + 2m + 1)\left(\frac{y_t^0}{\sqrt{2}}\bar{q}_L\mathbf{U}P_+\mathbf{q}_R + \text{h.c.}\right)c_\varphi^n s_\varphi^{2m+1}\left(\frac{\rho}{f}\right)^{n+2m} = 0, \tag{3.12}$$

where $\Box \equiv \partial_\mu\partial^\mu$. In a $1/\lambda$ expansion, the ρ field can be expressed as

$$\rho \equiv \rho_0 + \rho_1/\lambda + \rho_2/\lambda^2 + \dots$$

where the leading terms are given by

$$\rho_0 = f, \tag{3.13}$$

$$\rho_1 = \frac{f}{4}\left[\frac{1}{2f^4}\partial_\mu\varphi\,\partial^\mu\varphi - \frac{1}{4f^2}\langle\mathbf{V}_\mu\mathbf{V}^\mu\rangle\,s_\varphi^2 - \frac{1}{2}\alpha c_\varphi + \beta s_\varphi^2\right.$$
$$\left. - \frac{(n+2m+1)}{2f^3}\left(\frac{y_t^0}{\sqrt{2}}\bar{q}_L\mathbf{U}P_+\mathbf{q}_R + \text{h.c.}\right)c_\varphi^n s_\varphi^{2m+1}\right], \tag{3.14}$$

and subsequent ones can be written as polynomial functions of ρ_1. Substituting these in Eq. (3.6) yields the $1/\lambda^n$ Lagrangian corrections,

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1/\lambda + \mathcal{L}_2/\lambda^2 + \dots, \tag{3.15}$$

80

where the different terms in this equation are given by

$$\mathcal{L}_0 = \frac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi - \frac{f^2}{4}\,\langle V_\mu V^\mu\rangle\,s_\varphi^2 - \alpha f^4 c_\varphi + \beta f^4 s_\varphi^2 - f\,\frac{y_t^0}{\sqrt{2}}\bar{q}_L \mathbf{U} P_+ \mathbf{q}_R c_\varphi^n s_\varphi^{2m+1}, \qquad (3.16)$$

$$\mathcal{L}_1 = 4f^2\rho_1^2, \qquad (3.17)$$

$$\mathcal{L}_2 = \frac{1}{2}\left\{ (\partial_\mu\rho_1)^2 + \left[\alpha f^2 c_\varphi \right.\right. \qquad (3.18)$$

$$\left.\left. + \left(1 - (n+2m)^2\right)\frac{1}{f}\left(\frac{y_t^0}{\sqrt{2}}\bar{q}_L \mathbf{U} P_+ \mathbf{q}_R + \text{h.c.}\right)c_\varphi^n s_\varphi^{2m+1})\right]\rho_1^2\right\}.$$

$\mathcal{L}_0$ coincides with the leading-order Lagrangian for the scalar sector of the minimal composite Higgs model [9], as expected. The expressions obtained for $\mathcal{L}_1$ and $\mathcal{L}_2$ are remarkably compact and a similar pattern holds for higher orders in $1/\lambda$.

The maximum number of derivatives of $\mathcal{L}_n$ is $2 + 2n$, although not all $2 + 2n$ derivative operators are generated at order n. This is as foreseen, as for large λ the non-linear regime is approached and ordering the operators by their $1/\lambda$ dependence does not coincide with the ordering given by mass dimensions. The ordering in which the operators appear is akin to the power counting of non-linear Higgs effective theory [58, 100–102].

Eqs. (3.14) and (3.17) suggest interesting correlations between operators involving the Higgs boson, gauge bosons and fermions. In particular, operators such as $(\partial_\mu h)^2\bar{\psi}\psi$ or $\langle \mathbf{V}_\mu \mathbf{V}^\mu\rangle\bar{\psi}\psi$, where ψ denotes a generic fermion, are weighted by the fermion mass and also bear a dependence on the SM embedding into $SO(5)$, parametrised by the set $\{n, m\}$ in Eq. (3.7). From those equations emerges the low-energy effective Lagrangian in terms of SM fields at a given order in $1/\lambda$,

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_g + \mathcal{L}_{\text{f,SM}}^{\text{kin}} + \sum_i \mathcal{P}_i\,\mathcal{F}_i(\varphi), \qquad (3.19)$$

where the first two terms in the right-hand side contain respectively the kinetic terms for gauge bosons and fermions and the index i runs over all operator labels and coefficient functions $\mathcal{F}_i(\varphi)$ in Table 3.2. The table collects all couplings corresponding to two and four "derivatives", where plain derivatives and gauge boson insertions are counted with equal weight, as they come together in the covariant derivative. The notation/basis for the purely bosonic operators was chosen according to Ref. [70] to facilitate the comparison with a model-independent approach. From this tree-level analysis we draw the following conclusions:

- Up to first order in the linear corrections, the benchmark effective Lagrangian is determined to be composed of ten operators, five of them bosonic and the rest fermionic [2] including that responsible for Yukawa couplings. The coefficients of those operators are not free but intimately correlated by the coefficient functions explicitly determined here, and shown in the Table 3.2.

---

[2] For fermionic operators only the generic Lorentz and flavor structure are explicited, being trivial their decomposition in terms of different flavors.

- Among the couplings which first appear at $\mathcal{O}(1/\lambda)$, three bosonic operators are singled out in the SO(5)-invariant limit ($\alpha = \beta = 0$, massless SM fermions): $\mathcal{P}_6$, $\mathcal{P}_{20}$ and $\mathcal{P}_{DH}$. The two latter ones involve multiple Higgs insertions and are out of present experimental reach while the strength of $\mathcal{P}_6$, which involves vertices with four gauge bosons, is already tested directly by data, although the present sensitivity is very weak [138, 139].[3]

- Operators involving SM fermions have an implicit dependence on the symmetry-breaking terms in the Lagrangian – they are weighted by the fermion masses in a pattern alike to that of the Minimal Flavour Violation setup [140–142]. Most interestingly, the corresponding $\mathcal{F}_i(\varphi)$ coefficients, written as a function of the $\{n, m\}$ parameters, allow to differentiate the expected impact of different fermionic ultraviolet completions in the literature.

- All operators derived from Eqs. (3.16)-(3.18) are at most four-derivative ones, and they are all shown in table 3.2, including the only one appearing at $\mathcal{O}(1/\lambda^3)$.

- The gauge field dependence is present only through powers of $\langle \mathbf{V}_\mu \mathbf{V}^\mu \rangle$, consistent with its exclusively scalar covariant derivative origin, see Eq. (3.6). Other Lorentz contractions such as $\langle \mathbf{V}_\mu \mathbf{V}^\mu \rangle^2$ would be loop-induced, and thus expected to be subleading.

- The scalar functions $\mathcal{F}_i(\varphi)$ obtained as operator weights of bosonic couplings are in agreement with those derived in Ref. [70] in the $SO(5)$ invariant limit, for the subset of operators identified here as benchmarks, see their Eqs. (2.5)-(2.8). Table 3.2 provides in addition the leading deviations due to the presence of explicit $SO(5)$-breaking parameters α and β.

3.1.3   Impact on Higgs observables

Bosonic sector

From Eqs. (3.16) and (3.17), the potential at order $1/\lambda$ reads

$$\frac{V}{f^4} = \alpha c_\varphi - \beta s_\varphi^2 - \frac{1}{16\lambda}\left(\alpha c_\varphi - 2\beta s_\varphi^2\right)^2 + \mathcal{O}\left(\frac{1}{\lambda^2}\right), \tag{3.20}$$

with minimum at $\cos\left(\frac{\langle\varphi\rangle}{f}\right) \simeq -\frac{\alpha}{2\beta}\left(1 - \frac{\beta}{4\lambda}\right)$, see Eq. (3.9). The kinetic energy of the physical Higgs excitation h (see Eq. (3.8)) gets then a correction given by

$$\frac{1}{2}\left(1 + \frac{\beta}{2\lambda}\right)\partial_\mu h\, \partial^\mu h, \tag{3.21}$$

which is reabsorbed by a field redefinition

$$h \to (1 + Z_h)\, h, \qquad \text{with} \qquad Z_h = -\frac{\beta}{4\lambda}. \tag{3.22}$$

---

[3]These bounds can be translated for instance in $m_\rho \gtrsim 70$ GeV for $f \approx 600$ GeV.

	Operator	$\mathcal{F}_k(\varphi)$	$1/\lambda^n$
$\mathcal{P}_H$	$\dfrac{1}{2}(\partial_\mu h)^2$	$1 - \dfrac{1}{4\lambda}(\alpha cos\tfrac{\hat{h}}{f} - 2\beta sin^2\tfrac{\hat{h}}{f})$	0
$\mathcal{P}_C$	$-\dfrac{v^2}{4}\langle \mathbf{V}_\mu \mathbf{V}^\mu \rangle$	$\dfrac{1}{\xi}\left[1 - \dfrac{1}{4\lambda}\left(\alpha cos\tfrac{\hat{h}}{f} - 2\beta sin^2\tfrac{\hat{h}}{f}\right)\right]s_\varphi^2$	0
$\mathcal{P}_{\text{Yuk}}$	$v\,\bar{q}_{iL}\mathbf{U}P_\pm \mathbf{q}_{iR} + \text{h.c.}$	$-\dfrac{y_i^0}{\sqrt{2\xi}}c_\varphi^n s_\varphi^{2m+1}\left(1 - \dfrac{n+2m+1}{8\lambda}(\alpha c_\varphi - 2\beta s_\varphi^2)\right)$	0
$\mathcal{P}_{DH}$	$\dfrac{1}{v^4}(\partial_\mu h)^4$	$\dfrac{\xi^2}{16\lambda}$	1
$\mathcal{P}_6$	$\langle \mathbf{V}_\mu \mathbf{V}^\mu \rangle^2$	$\dfrac{s_\varphi^4}{64\lambda}$	1
$\mathcal{P}_{20}$	$\dfrac{1}{v^2}\langle \mathbf{V}_\mu \mathbf{V}^\mu \rangle(\partial_\nu h)^2$	$-\dfrac{\xi}{16\lambda}s_\varphi^2$	1
$\mathcal{P}_{qH}$	$\dfrac{1}{v^3}(\partial_\mu h)^2\bar{q}_{iL}\mathbf{U}P_\pm \mathbf{q}_{iR} + \text{h.c.}$	$-\dfrac{y_i^0}{\sqrt{2}}\xi^{3/2}\left(\dfrac{n+2m+1}{8\lambda}\right)c_\varphi^n s_\varphi^{2m+1}$	1
$\mathcal{P}_{qV}$	$\dfrac{1}{v}\langle \mathbf{V}_\mu \mathbf{V}^\mu \rangle\bar{q}_{iL}\mathbf{U}P_\pm \mathbf{q}_{iR} + \text{h.c.}$	$\dfrac{y_i^0}{\sqrt{2}}\sqrt{\xi}\left(\dfrac{n+2m+1}{16\lambda}\right)c_\varphi^n s_\varphi^{2m+3}$	1
$\mathcal{P}_{4q}$	$\dfrac{1}{v^2}(\bar{q}_{iL}\mathbf{U}P_\pm \mathbf{q}_{iR})(\bar{q}_{jL}\mathbf{U}P_\pm \mathbf{q}_{jR}) + \text{h.c.}$	$(2-\delta_{ij})y_i^0 y_j^0 \xi\dfrac{(n+2m+1)^2}{32\lambda}c_\varphi^{2n} s_\varphi^{4m+2}$	1
$\mathcal{P}_{4q'}$	$\dfrac{1}{v^2}(\bar{q}_{iL}\mathbf{U}P_\pm \mathbf{q}_{iR})(\bar{\mathbf{q}}_{jR}P_\pm \mathbf{U}^\dagger q_{jL}) + \text{h.c.}$	$(2-\delta_{ij})y_i^0 y_j^0 \xi\dfrac{(n+2m+1)^2}{32\lambda}c_\varphi^{2n} s_\varphi^{4m+2}$	1
$\mathcal{P}_7$	$\dfrac{1}{v}\langle \mathbf{V}_\mu \mathbf{V}^\mu \rangle(\Box h)$	$\sqrt{\xi}\left[\dfrac{1}{128\lambda^2}\left(\alpha + 4\beta cos\tfrac{\hat{h}}{f}\right)s_\varphi^3\right]$	2
$\mathcal{P}_{\Delta H}$	$\dfrac{1}{v^3}(\partial_\mu h)^2(\Box h)$	$-\xi^{3/2}\left[\dfrac{1}{64f^3\lambda^2}\left(\alpha + 4\beta cos\tfrac{\hat{h}}{f}\right)sin\tfrac{\hat{h}}{f}\right]$	2
$\mathcal{P}_{\Box H}$	$\dfrac{1}{v^2}\left(\Box h\right)^2$	$\mathcal{O}\left(\dfrac{1}{\lambda^3}\right)$	3

Table 3.2: Effective operators before electroweak symmetry breaking, including two and four derivative couplings, together with their coefficients up to their first corrections in the $1/\lambda$ expansion. The bosonic contributions from $SO(5)$ breaking contributions ($\alpha \neq 0$ and/or $\beta \neq 0$) are also shown. The right-hand column indicates the order in $1/\lambda$ at which a given couplings first appears. The Higgs field h is defined as the excitation of the field φ, see Eq. (3.8).

The scalar sector depends on four independent parameters explicitated in Eq. (2.19), with the only difference that the mass of the heavy scalar is labelled m_ρ.

In the non-linear limit $\lambda \to \infty$ the ρ field decouples from the spectrum and the scalar sector would depend on just three renormalized parameters. It is now possible to foresee the impact of the linear corrections in terms of mass dependence. Precisely because the large λ and m_ρ limits are in correspondence, dimensional arguments suggest the equivalence

$$\frac{1}{\lambda} \qquad \Rightarrow \qquad \frac{m_h^2}{m_\rho^2} \simeq \frac{\beta \xi}{4\lambda} \, , \tag{3.23}$$

as the expansion parameter, as it follows from the expressions for the scalar masses in the small explicit breaking limit Eq. (2.17). In other words, the linear corrections are expected to be proportional to the two small parameters β and ξ and thus doubly suppressed.

The modification of the Higgs – gauge boson coupling can be parametrised by κ_V and can be extracted for instance from the expression for Higgs to WW decay width as

$$\Gamma(h \to WW^*) \equiv \Gamma_{\mathrm{SM}}(h \to WW^*) \, \kappa_V^2 \, .$$

A generic expectation is the departure of κ_V from 1. Indeed, the coupling between the Higgs and the gauge bosons which stems from the Lagrangian Eqs. (3.15)-(3.18) at order $1/\lambda$ is that encoded in the operator $\mathcal{P}_C$ in Table 3.2 and reads

$$\mathcal{L}_{hVV} = -\left(\frac{1}{2}\sqrt{1-\xi} + \frac{\beta}{8\lambda}\frac{(2-\xi)\xi}{\sqrt{1-\xi}} \right) \langle V_\mu V^\mu \rangle \, vh \, , \tag{3.24}$$

or in other words

$$\kappa_V = \sqrt{1-\xi} + \frac{\beta\xi}{2\lambda}\frac{(1-\xi/2)}{\sqrt{1-\xi}} \, . \tag{3.25}$$

Assuming for illustrative purposes $\mathcal{O}(\xi) \sim \mathcal{O}(1/\lambda)$ and expanding up to second order in these parameters, the result simplifies to

$$\kappa_V \simeq \sqrt{1-\xi} + \frac{\beta\xi}{2\lambda}. \tag{3.26}$$

The first term on the right-hand side of this equation is the well-known correction present in non-linear scenarios [9], while the second term encodes the linear correction linked to the scale of ultraviolet completion, which in terms of physical parameters we predict to be given by

$$\kappa_V^2 \simeq 1 - \xi + 4\frac{m_h^2}{m_\rho^2} \, , \tag{3.27}$$

where Eq. (3.23) has been used. Higher order corrections are expected to be very small, as they will stem from operators with at least 4 derivatives. For instance, the first extra tree-level contribution to κ_V is the $1/\lambda^2$ weight of the operator $\mathcal{P}_7$ in Table 3.2,

$$\delta\kappa_V^2 \simeq \frac{1}{2\sqrt{2}G_F}\frac{m_h^2}{m_\rho^4} \, .$$

Fermionic sector

Consider first the case in which the fermion mass is generated by a single Yukawa operator $\mathcal{O}_{\text{Yuk},f}^{(n,m)}$, see Eqs. (3.2) and (3.3). From the Lagrangian in Eqs. (3.15)-(3.18), and more specifically from the Yukawa operator in the third line of Table 3.2, an expression for the fermion mass follows after applying Eqs. (3.8) and Eq. (3.22),

$$\mathcal{L}_{\text{f}}^{\text{Yuk}} \supset -m_{\text{f}}\,\bar{\text{f}}_L \text{f}_R + \text{h.c.}\,, \qquad m_{\text{f}} \simeq \frac{y_{\text{f}}^0}{\sqrt{2}}\,f\,\sqrt{\xi}\,(1-\xi)^{n/2}\xi^m\left(1 + n\,\frac{1}{\xi(1-\xi)}\,\frac{m_h^2}{m_\rho^2}\right)\,. \tag{3.28}$$

The renormalization scheme is now enlarged to the fermion sector choosing as observable precisely the fermion masses. The prediction that follows for the Higgs coupling to a given fermion f,

$$\mathcal{L}_{h\text{ff}} \equiv -g_{h\text{ff}}\,h\,\bar{\text{f}}_L \text{f}_R + \text{h.c.}\,, \tag{3.29}$$

takes then the form

$$g_{h\text{ff}} \simeq \frac{y_{\text{f}}^0}{\sqrt{2}}(1-\xi)^{\frac{n-1}{2}}\xi^m\Bigg\{(1+2m)(1-\xi) - n\xi + \frac{\beta}{\xi(1-\xi)}\,\frac{m_h^2}{m_\rho^2}\times \tag{3.30}$$

$$\times\Big[(1+2m+n)\xi(1-\xi)(2-\xi) + n\left(1+2m(1-\xi) - n\xi\right)\Big]\Bigg\}.$$

Encoding the deviations with respect to the SM expectations through the conventional κ_{f} parameter,

$$\kappa_{\text{f}} \equiv g_{h\text{ff}}/g_{h\text{ff}}^{\text{SM}}\,, \tag{3.31}$$

where $g_{h\text{ff}}^{\text{SM}} = m_{\text{f}}/v$, the exact and somewhat lengthy expression for κ_{f} up to order $1/\lambda$ follows. The latter can be simply recast assuming again $\mathcal{O}(\xi) \sim \mathcal{O}(1/\lambda)$, leading to

$$\kappa_{\text{f}} \simeq \frac{(1+2m)(1-\xi) - n\xi}{\sqrt{1-\xi}} + (2+4m+3n)\,\frac{m_h^2}{m_\rho^2}, \tag{3.32}$$

where once again Eq. (3.23) has been used. It is straightforward to check that the first term on the right-hand side of this equation reproduces well-known κ_{f} results for different models in the literature, which assume a non-linear realization. The second term gives instead the leading linear corrections. For instance, this equation leads to the following results for the MCHM$_{5-1-1}$ (corresponding to $n = m = 0$ in our parametrisation) and MCHM$_5$ (corresponding to $n = 1$, $m = 0$):

$$\kappa_{\text{f}}^{\text{MCHM}_{5-1-1}} \simeq \sqrt{1-\xi} + 2\frac{m_h^2}{m_\rho^2}, \qquad \kappa_{\text{f}}^{\text{MCHM}_5} \simeq \frac{1-2\xi}{\sqrt{1-\xi}} + 5\frac{m_h^2}{m_\rho^2}\,. \tag{3.33}$$

obtaining again at order $1/\lambda$ a correction doubly suppressed as proportional to both β and ξ, see Eq. (3.23).

Consider next the case in which a given fermion mass corresponds to the combination of several $SO(5)$ invariant Yukawa operators, instead of just one as developed above,

$$\mathcal{L}_{\text{f}}^{\text{Yuk}} = -\,c_{(n,m)}\,\mathcal{O}_{\text{Yuk},\text{f}}^{(n,m)} + \cdots + \text{h.c.}, \tag{3.34}$$

where $c_{(n,m)}$ are related to the generators of $SO(5)$ and the fermion embedding in a given model. The procedure is still quite straightforward. The fermion mass will be a sum of contributions similar to that in Eq. (3.28) weighted by the coefficients $c_{(n,m)}$, and a similar combination protocol will apply to the obtention of the fermion-Higgs coupling g_{hff} and κ_f. As an example, consider the MCHM$_{14-14-10}$ scenario [16], in which the third family quark doublet and the right-handed top are embedded each in a **14**-plet of $SO(5)$, denoted Q_L and U_R respectively, while the right-handed bottom is included in a **10**-plet representation denoted D_R. Two $SO(5)$ invariant operators [16] contribute in this case to the top quark mass,

$$y_u \phi^\dagger \bar{Q}_L U_R \phi - \tilde{y}_u (\phi^\dagger \bar{Q}_L \phi)(\phi^\dagger U_R \phi) \rightarrow 3 y_u \mathcal{O}_{\text{Yuk}}^{(1,0)} - \tilde{y}_u \left(2\mathcal{O}_{\text{Yuk}}^{(1,1)} - 8\mathcal{O}_{\text{Yuk}}^{(3,0)} \right) , \tag{3.35}$$

leading to

$$\kappa_{\text{t}}^{\text{MCHM}_{14-14-10}} \simeq \frac{y_u(3 - 6\xi) + 2\tilde{y}_u(4 - 23\xi + 20\xi^2)}{\sqrt{1 - \xi}\,(3y_u + 2\tilde{y}_u(4 - 5\xi))} + \frac{15 y_u^2 + 32 \tilde{y}_u(8\tilde{y}_u - 3y_u)}{(8\tilde{y}_u - 3y_u)^2} \frac{3m_h^2}{m_\rho^2} . \tag{3.36}$$

In contrast, in this same scenario only one effective Yukawa operator contributes to the bottom quark mass,

$$y_d \phi^\dagger \bar{Q}_L D_R \phi \rightarrow y_d \mathcal{O}_{\text{Yuk}}^{(1,0)} , \tag{3.37}$$

and consequently

$$\kappa_{\text{b}}^{\text{MCHM}_{14-14-10}} \simeq \frac{1 - 2\xi}{\sqrt{1 - \xi}} + \frac{5\beta\xi}{4\lambda} \simeq \frac{1 - 2\xi}{\sqrt{1 - \xi}} + \frac{5m_h^2}{m_\rho^2} . \tag{3.38}$$

All $\mathcal{O}(1/\lambda)$ corrections considered above show again the double suppression in ξ and β, which after Eq. (3.23) is tantamount to a m_h^2/m_ρ^2 suppression factor, as expected.

3.2 Explicit fermion sector

In the previous section, the infinite mass limit for the heavy fermion sector was assumed from the start, while the corrections due to the heavy scalar singlets were explored. In this section we start instead of a complete (bosons plus fermions) renormalizable model, so as to estimate the impact of a fermionic ultraviolet completion beyond that related to the Yukawa couplings discussed earlier. The low-energy effective Lagrangian made out of SM fields will be then explicitly determined up to the leading corrections stemming from the heavy scalar and fermion sectors: respectively up to $\mathcal{O}(1/\lambda) \sim \mathcal{O}(m_h^2/m_\rho^2)$ and $\mathcal{O}(f/\mathcal{M}_i)$, where $\mathcal{M}_i$ denotes generically the heavy fermion masses.

The fermionic Lagrangian $\mathcal{L}_{\text{f}}$ needs to be redefined,

$$\mathcal{L}_{\text{f}} = \mathcal{L}_{\text{f}}^{\text{kin}} + \mathcal{L}_{\text{f}}^{\text{Yuk}}, \tag{3.39}$$

where $\mathcal{L}_{\text{f}}^{\text{kin}}$ contains now kinetic terms for all fermions, light and heavy, and the fermion mass Lagrangian denoted by $\mathcal{L}_{\text{f}}^{\text{Yuk}}$ needs to be specified for a particular ultraviolet fermion completion. The model developed in Chapter 2 will be analyzed as illustration.

The heavy fermion content is given in Eq. (2.30) and $\mathcal{L}_{\text{f}}^{\text{Yuk}}$ reads

$$\mathcal{L}_{\text{f}}^{\text{Yuk}} = - \left[y_1 \left(\bar{X}_L H T_R^{(1)} + \bar{Q}_L \widetilde{H} T_R^{(1)} + \bar{T}_L^{(5)} \sigma T_R^{(1)} \right) \right. \tag{3.40}$$

$$+ y_2 \left(\bar{T}_L^{(1)} H^\dagger X_R + \bar{T}_L^{(1)} \widetilde{H}^\dagger Q_R + \bar{T}_L^{(1)} \sigma T_R^{(5)} \right)$$

$$+ y_1' \left(\bar{X}'_L \widetilde{H} B_R^{(1)} + \bar{Q}'_L H B_R^{(1)} + \bar{B}_L^{(5)} \sigma B_R^{(1)} \right)$$

$$+ y_2' \left(\bar{B}_L^{(1)} \widetilde{H}^\dagger X'_R + \bar{B}_L^{(1)} H^\dagger Q'_R + \bar{B}_L^{(1)} \sigma B_R^{(5)} \right)$$

$$+ \Lambda_1 \bar{q}_L Q_R + \Lambda_1' \bar{q}_L Q'_R + \Lambda_2 \bar{T}_L^{(5)} t_R + \Lambda_3 \bar{T}_L^{(1)} t_R + \Lambda_2' \bar{B}_L^{(5)} b_R + \Lambda_3' \bar{B}_L^{(1)} b_R + h.c. \Big].$$

In Sec. 2.5 we had first integrated out the heavy fermions of this Lagrangian, determining then the effective Lagrangian made out of SM fields plus the singlet scalar present in the minimal $SO(5)$ sigma model. Here we reverse the order of integration of the heavy fields, taking first the limit of heavy ρ and then that of heavy BSM fermions. We have explicitly checked that the final low-energy effective Lagrangian made out only of SM fields is independent of the order in which those limits are taken.

Using polar coordinates and integrating out the radial mode ρ does not bring any novel complication with respect to the procedure carried out in the previous section, except for lengthier expressions. Nevertheless, $\mathcal{L}_F^{\text{Yuk}}$ can be compactly written prior to any integration procedure as

$$\mathcal{L}_F^{\text{Yuk}} = - \Big[\rho \left(s_\varphi \mathcal{O}_s^F + c_\varphi \mathcal{O}_c^F \right) + \Lambda_1 \bar{q}_L Q_R + \Lambda_1' \bar{q}_L Q'_R$$
$$+ \Lambda_2 \bar{T}_L^{(5)} t_R + \Lambda_3 \bar{T}_L^{(1)} t_R + \Lambda_2' \bar{B}_L^{(5)} b_R + \Lambda_3' \bar{B}_L^{(1)} b_R + h.c. \Big], \tag{3.41}$$

where $\mathcal{O}_s^F$ and $\mathcal{O}_c^F$ are heavy fermion bilinears corresponding to the first four lines in Eq. (3.40):

$$\mathcal{O}_s^F \equiv -\frac{1}{\sqrt{2}} \Big[y_1 \left(\bar{X}_L U e_- T_R^{(1)} + \bar{Q}_L U e_+ T_R^{(1)} \right) + y_2 \left(\bar{T}_L^{(1)} U e_- X_R + \bar{T}_L^{(1)} U e_+ Q_R \right)$$
$$+ y_1' \left(\bar{X}'_L U e_+ B_R^{(1)} + \bar{Q}'_L U e_- B_R^{(1)} \right) + y_2' \left(\bar{B}_L^{(1)} U e_+ X'_R + \bar{B}_L^{(1)} U e_- Q'_R \right) \Big], \tag{3.42}$$

$$\mathcal{O}_c^F \equiv -\frac{1}{\sqrt{2}} \Big[y_1 \bar{T}_L^{(5)} T_R^{(1)} + y_2 \bar{T}_L^{(1)} T_R^{(5)} + y_1' \bar{B}_L^{(5)} B_R^{(1)} + y_2' \bar{B}_L^{(1)} B_R^{(5)} \Big], \tag{3.43}$$

where $e_+ = (1,0)$ and $e_- = (0,1)$.

Consider next the limit of very large scalar mass m_ρ (that is $\lambda \to \infty$) and very heavy fermions. Implementing first the $1/\lambda$ corrections, the effective Lagrangian at this order takes exactly the form in Eq. (3.17), although ρ_1 shows now an explicit dependence on the heavy fermion spectrum,

$$\rho_1 = \frac{f}{4} \left[\frac{1}{2f^4} \partial_\mu \varphi \, \partial^\mu \varphi - \frac{1}{4f^2} \langle V_\mu V^\mu \rangle s_\varphi^2 - \frac{1}{2} \alpha c_\varphi + \beta s_\varphi^2 - \left\{ \frac{1}{2f^3} \mathcal{O}_c^F c_\varphi + \frac{1}{2f^3} \mathcal{O}_s^F s_\varphi + h.c. \right\} \right],$$

instead of the effective dependence in Eq. (3.14). New operators beyond those previously considered appear, such as

$$- \frac{1}{8\lambda f^3} \partial_\mu \varphi \, \partial^\mu \varphi \left(\mathcal{O}_c^F c_\varphi + \mathcal{O}_s^F s_\varphi \right), \tag{3.44}$$

$$\frac{1}{16\lambda f} \langle V_\mu V^\mu \rangle s_\varphi^2 \left(\mathcal{O}_c^F c_\varphi + \mathcal{O}_s^F s_\varphi \right), \tag{3.45}$$

	Operator	$\mathcal{F}_i(\varphi)$	$1/\lambda^n$
$\mathcal{P}_{\text{Yuk}}$	$v(\bar{q}_{iL} U P_\pm \mathbf{q}_{iR})$	$-\dfrac{y_t^0}{\sqrt{2\xi}} s_\varphi \left[1 - \dfrac{1}{8\lambda}(\alpha c_\varphi - 2\beta s_\varphi^2) - 2\dfrac{f}{\mathcal{M}_i} a_{\sigma 1}^i c_\varphi\right]$	0
$\mathcal{P}_{qh}$	$(\partial_\mu h)^2 (\bar{q}_{iL} U P_\pm \mathbf{q}_{iR})$	$-\dfrac{y_i^0}{8\sqrt{2}\lambda f^3} sin\dfrac{\hat{h}}{f} \left(1 - 2\dfrac{f}{\mathcal{M}_i} a_{\sigma 1}^i cos\dfrac{\hat{h}}{f}\right)$	1
$\mathcal{P}_{qV}$	$\langle \mathbf{V}_\mu \mathbf{V}^\mu \rangle (\bar{q}_{iL} U P_\pm \mathbf{q}_{iR})$	$\dfrac{y_i^0}{16\sqrt{2}\lambda f} sin\dfrac{\hat{h}}{f} \left(1 - 2\dfrac{f}{\mathcal{M}_i} a_{\sigma 1}^i cos\dfrac{\hat{h}}{f}\right)$	1
$\mathcal{P}_{4q}$	$(\bar{q}_{iL} U P_\pm \mathbf{q}_{iR})(\bar{q}_{jL} U P_\pm \mathbf{q}_{jR})$	$(2-\delta_{ij})\dfrac{y_i^0 y_j^0}{32\lambda f^2} sin\dfrac{\hat{h}}{f}^2 \left[1 - 2\left(a_{\sigma 1}^i \dfrac{f}{\mathcal{M}_i} + a_{\sigma 1}^j \dfrac{f}{\mathcal{M}_j}\right) cos\dfrac{\hat{h}}{f}\right]$	1
$\mathcal{P}_{4q'}$	$(\bar{q}_{iL} U P_\pm \mathbf{q}_{iR})\left(\bar{\mathbf{q}}_{jR} P_\pm U^\dagger q_{jL}\right)$	$(2-\delta_{ij})\dfrac{y_i^0 y_j^0}{32\lambda f^2} sin\dfrac{\hat{h}}{f}^2 \left[1 - 2\left(a_{\sigma 1}^i \dfrac{f}{\mathcal{M}_i} + a_{\sigma 1}^j \dfrac{f}{\mathcal{M}_j}\right) cos\dfrac{\hat{h}}{f}\right]$	1

Table 3.3: Effective operators, up to order $f/\mathcal{M}_i$ and $1/\lambda$, after integrating out the radial mode ρ and the heavy fermions in a UV realisation of partial compositeness. The Hermitian conjugate should be included for all operators here. The Higgs field h is defined as the excitation of the field φ, see Eq. (3.8).

$$\frac{1}{16\lambda f^2}\left(\mathcal{O}_c^F c_\varphi + \mathcal{O}_s^F s_\varphi\right)^2 . \tag{3.46}$$

They are higher-order operators made out of both SM and heavy BSM fermions and related to the explicit fermionic ultraviolet completion. Furthermore, it is again easy to verify that the counting rule matches the NDA rule [58, 102] by identifying $\lambda f \sim \Lambda$.

Consider next the integration of the heavy fermion sector in the results just obtained. To estimate the corrections, we adopt here a universal heavy fermion mass scale $\mathcal{M}_i$ associated with the mass generation mechanism of a given SM fermion, so that $M_1 \sim M_5 \sim \Lambda_1 \sim \cdots \sim \mathcal{M}_t$. Assuming this scale to be larger than f, $f/\mathcal{M}_i$ is a good expansion parameter. The final set of five effective operators resulting up to first order in the $1/\lambda$ and $f/\mathcal{M}_i$ expansions is shown in Table 3.3, where the $a_{\sigma 1}^i$ operator coefficients weighting the $f/\mathcal{M}_i$ corrections are expected to be $\mathcal{O}(1)$. [4] Noteworthy consequences include:

- At tree level, the heavy fermions have no impact on the gauge-Higgs coupling and κ_V is still given by Eq. (3.25). The coupling to top quarks, on the contrary, will receive fermionic contributions from the first operator in Table 3.3,

$$\kappa_t = \sqrt{1-\xi} + 2\frac{m_h^2}{m_\rho^2} + a_{\sigma 1}^t \frac{f}{\mathcal{M}_t}\xi + \dots \tag{3.47}$$

Again, a double suppression acts on the leading heavy fermion corrections $\sim \xi f/\mathcal{M}_t$, alike to the case for the bosonic ones in $\sim \beta\xi/(2\lambda)$. It is important to note, though, that the tree-level fermionic contributions found may be larger than those induced by the scalar sector

---

[4]The $a_{\sigma 1}^i$ coefficients are a redefinition of the c_σ^i operator coefficients used before in Chapter 2, so as to extract explicitly the $f/\mathcal{M}_i$ dependence: $c_{\sigma 1}^i \to y_t^0 a_{\sigma 1}^i/\mathcal{M}_t$; the exact expressions for c_σ^i can be found in Tab. 2.3.

if $f/\mathcal{M}_t > \beta/\lambda$; this may occur specially for the top quark since the top partners, with characteristic mass scale $\mathcal{M}_t$, should be light enough in order not to generate a hierarchy problem.

- On top of the above, higher order effective operators involving SM fields are singled out at low scales: the dominant ones are the last three presented in Table 3.3. For these operators, the inclusion of an explicit heavy fermion sector does not change much the conclusions obtained previously by using an effective Yukawa coupling as defined in Eq. (3.3).

- In the limit $f/\mathcal{M}_i \to 0$, the operators in Table 3.3 coincide as expected with the fermion-Higgs and four-fermion operators given previously in Table 3.2 using the effective Yukawa operator $\mathcal{O}_{\text{Yuk}}^{(0,0)}$.

4

Non-linear EFT at one loop

In the previous chapter we have identified the benchmark effective non-linear Lagrangian originating from the UV complete model for the Goldstone Higgs. It contains a subset of operators of the general non-linear Lagrangian. While that study has been restrained only to the tree level computations – the loop corrections might contribute significantly and have to be considered as well.

In this chapter we will focus on the loop structure of the scalar sector of the general non-linear Lagrangian (i.e. longitudinal components of the W and Z bosons plus h), up to four derivatives in the chiral expansion. While most of the literature [1], [143–146] has restrained the one-loop renormalization of this sector to on-shell analysis, we implement the complete off-shell renormalization procedure, by considering the corrections to the leading - up to two-derivatives - scalar Lagrangian, and furthermore taking into account the finite Higgs mass. The off-shell procedure will allow:

- To guarantee that all counterterms required for consistency are identified, and that the corresponding basis of chirally invariant scalar operators is thus complete. It will follow that some operators often disregarded previously are mandatory when analysing the bosonic sector by itself.

- To shed light on the expected size of the counterterm coefficients, in relation with NDA [58, 147] for light h.

- To identify the renormalization group equations (RGE) for the bosonic sector of the chiral Lagrangian.

A complete one-loop off-shell renormalization of the electroweak chiral Lagrangian with a decoupled Higgs particle was performed in Ref. [148]. Using the non-linear sigma model and a perturbative analysis, apparently chiral non-invariant divergences (NID) were shown to appear as counterterms of four-point functions for the "pion" fields, in other words, for the longitudinal components of the W and Z bosons. Physical consistency was guaranteed as those NID were shown to vanish on-shell

---

[1]The on-shell precursory study in Ref. [91] assumed no Higgs in the low-energy spectrum.

and thus did not contribute to physical amplitudes. They were an artefact of the perturbative procedure – which is not explicitly chiral invariant – and a redefinition of the pion fields leading to their reabsortion was identified, see also Refs. [55, 149, 150]. In the present work, additional new NID in three and four-point functions involving the Higgs field will be shown to be present, and their reabsortion explored. Furthermore, a general parametrisation of the pseudo-goldstone boson matrix will be formulated, defining a parameter η which reduces to the various usual pion parametrisations for different values of η, and the non-physical character of all NID will be analysed.

An alternative approach based on the path integral formulation of QFT has been developed in Refs. [151–154] for the case of pions only interactions. It has been demonstrated that the appearance of the NIDs can be avoided if the derivative, covariant with respect to chiral symmetry, is implemented. In Refs. [155, 156] the same method has been applied to the case of the non-linear Lagrangian with the light Higgs, with the similar conclusions. The results presented there confirm our expressions for the counterterms and NIDs.

The resulting RGE restricted to the bosonic sector may eventually illuminate future experimental searches when comparing data to be obtained at different energy scales.

4.1   The Lagrangian

The focus of the present analysis will be set on the physics of the longitudinal components of the gauge bosons π, to which we will refer sometimes as "pions" for shortness and of the h scalar, and only these degrees of freedom will be explicited below. The non-linear effective Lagrangian with the light Higgs can be ordered as

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_2 + \mathcal{L}_4 \,, \tag{4.1}$$

where the $\mathcal{L}_i$ subindex indicates number of derivatives:

$$\mathcal{L}_0 = - V(h) \,, \tag{4.2}$$

$$\mathcal{L}_2 = \frac{1}{2}\partial_\mu h \partial^\mu h \; \mathcal{F}_H(h) - \frac{v^2}{4}\mathrm{Tr}[\mathbf{V}_\mu \mathbf{V}^\mu] \; \mathcal{F}_C(h) \,, \tag{4.3}$$

$$\mathcal{L}_4 = \sum_i c_i \mathcal{P}_i \,. \tag{4.4}$$

Here we have omitted the two-derivative custodial breaking operator, because the size of its coefficient is phenomenologically very strongly constrained. In consequence, and as neither gauge nor Yukawa interactions are considered in this chapter, no custodial-breaking countertem will be required by the renormalization procedure to be present among the four-derivative operators in $\mathcal{L}_4$. Our analysis is thus restricted to the custodial-preserving sector.

The $\mathcal{P}_i$ operators in Eq. (4.4) being a custodial preserving subset of the general non-linear effective Lagrangian given in Eq. (1.4) with only scalar degrees of freedom. They are shown explicitly in Table 4.1. The potential $V(h)$ in Eq. (4.2) denotes a general potential for the h field, for which

only up to terms quartic in h will be made explicit, with arbitrary coefficients μ_i and λ,

$$V \equiv \mu_1^3 h + \frac{1}{2}m_h^2 h^2 + \frac{\mu_3}{3!}h^3 + \frac{\lambda}{4!}h^4 \, . \tag{4.5}$$

The first term in $V(h)$ is provisionally kept in order to cancel the tadpole amplitude at one loop; we will clarify this point in Sect. 4.2.1. As before, the Higgs field is assumed to be physical one with $\langle h \rangle = 0$.

The present analysis will only require up to four-field vertices, for which it suffices to explicit the h dependence of the functions $\mathcal{F}_i(h)$ up to quadratic terms. $\mathcal{F}_{H,C}(h)$ will be thus parametrised as

$$\mathcal{F}_{H,C}(h) \equiv 1 + 2a_{H,C}h/v + b_{H,C}h^2/v^2 \, , \tag{4.6}$$

while for all $\mathcal{P}_i(h)$ operators in Table 4.1 the corresponding functions will be defined as

$$c_i\mathcal{F}_i(h) \equiv c_i + 2a_i h/v + b_i h^2/v^2 \, . \tag{4.7}$$

A further comment on $\mathcal{F}_H(h)$ may be useful: through a redefinition of the h field [87] it would be possible to absorb it completely. Nevertheless, this redefinition would affect all other couplings in which h participates and induce for instance corrections on fermionic couplings which are weighted by SM Yukawa couplings; it is thus pertinent not to disregard $\mathcal{F}_H(h)$ here, as otherwise consistency would require to include in the analysis the corresponding $\mathcal{F}_i(h)$ fermionic and gauge functions. If a complete basis including all SM fields is considered assigning individual arbitrary functions $\mathcal{F}_i(h)$ to all operators, it would then be possible to redefine away completely one $\mathcal{F}_i(h)$ without loss of generality: it is up to the practitioner to decide which set of independent operators he/she may prefer, and to redefine away one of the functions, for instance $\mathcal{F}_H(h)$. For the time being, we keep explicit $\mathcal{F}_H(h)$ all through, for the sake of generality [2].

We analyse next the freedom in defining the $\mathbf{U}$ matrix and work with a general parametrization truncated up to some order in π/v. On-shell quantities must be independent of the choice of parametrisation for the $\mathbf{U}$ matrix they thh, while it will be shown below that all NID depend instead on the specific parametrisation chosen. The NID in which the h particle participates will turn out to offer a larger freedom to be redefined away than the pure pionic ones.

4.1.1 The Lagrangian in a general U parametrisation

The non-linear σ model can be written as [55]

$$\mathcal{L}_{\mathrm{NL}} = \frac{1}{2}D_\mu\boldsymbol{\pi}D^\mu\boldsymbol{\pi} = \frac{v^2}{4}\mathrm{Tr}[\partial_\mu\mathbf{U}\partial^\mu\mathbf{U}^\dagger] = \frac{1}{2}G_{ij}(\boldsymbol{\pi}^2)\partial_\mu\pi_i\partial^\mu\pi_j \, , \tag{4.8}$$

where D_μ is a derivative "covariant" under the non-linear chiral symmetry. In geometric language, $G_{ij}(\boldsymbol{\pi}^2)$ can be interpreted as the metric of a 3-sphere in which the pions live, and the freedom

---

[2]Note that $\mathcal{F}_H(h)$ is not expected to be generated from the most popular composite Higgs models, as the latter break explicitly the chiral symmetry only via a potential for h externally generated, while $\mathcal{F}_H(h)$ would require derivative sources of explicit breaking of the chiral symmetry. A similar comment could be applied to $\mathcal{P}_{\Delta H}$.

of parametrisation is just a coordinate transformation (see Ref. [148] and references therein). Indeed, Weinberg has shown [55] that different linear realizations of the chiral symmetry would lead to different metrics, which turn out to correspond to different $\mathbf{U}$ parametrisations; they are all equivalent with respect to the dynamics of the pion fields as the non-linear transformation induced on them is unique, and they are connected via redefinitions of the pion fields. In order to illustrate this correspondence explicitly, let us define general X and Y functions as follows:

$$\mathbf{U} \equiv X(z) + \frac{i\boldsymbol{\tau}\cdot\boldsymbol{\pi}}{v}Y(z), \qquad z = \boldsymbol{\pi}^2/v^2\,. \tag{4.9}$$

$X(z)$ and $Y(z)$ are related via the unitarity condition $\mathbf{U}\mathbf{U}^\dagger = \mathbf{1}$,

$$X(z) = \sqrt{1 - zY(z)^2}\,. \tag{4.10}$$

The G_{ij} metric can now be rewritten as

$$G_{ij}(\boldsymbol{\pi}^2) = Y(z)^2\delta_{ij} + 4\left(X'(z)^2 + zY'(z)^2 + Y(z)Y'(z)\right)\frac{\pi_i\pi_j}{v^2}, \tag{4.11}$$

where the primes indicate derivatives with respect the the z variable, and $Y(0) = \pm 1$ is required for canonically normalized pion kinetic terms.

The Lagrangian in Eq. (4.8) is invariant under the transformation $Y \to -Y$, or equivalently $\boldsymbol{\pi} \to -\boldsymbol{\pi}$. It is easy to relate X and Y to the functions in Weinberg's analysis of chiral symmetry [3]. A Taylor expansion of $\mathbf{U}$ up to order $\boldsymbol{\pi}^{2N+2}$ bears N free parameters. *A priori* the present analysis requires to consider in $\mathcal{L}_2$ terms up to $O(\boldsymbol{\pi}^6)$, as the latter may contribute to 4-point functions joining two of its pion legs into a loop. Nevertheless, the latter results in null contributions for massless pions, and in practice it will suffice to consider inside $\mathbf{U}$ up to terms cubic on the pion fields. We thus define a single parameter η which encodes all the parametrisation dependence,

$$Y(z) \equiv 1 + \eta\,z + O(z^2)\,, \tag{4.12}$$

resulting in

$$\mathbf{U} = 1 - \frac{\boldsymbol{\pi}^2}{2v^2} - \left(\eta + \frac{1}{8}\right)\frac{\boldsymbol{\pi}^4}{v^4} + \frac{i(\boldsymbol{\pi}\boldsymbol{\tau})}{v}\left(1 + \eta\frac{\boldsymbol{\pi}^2}{v^2}\right) + \ldots \tag{4.13}$$

Specific values of η can be shown to correspond to different parametrisations up to terms with four pions, for instance:

- $\eta = 0$ yields the square root parametrisation: $\mathbf{U} = \sqrt{1 - \boldsymbol{\pi}^2/v^2} + i(\boldsymbol{\pi}\boldsymbol{\tau})/v\,,$

- $\eta = -1/6$ yields the exponential one: $\mathbf{U} = \exp(i\boldsymbol{\pi}\cdot\tau/v)\,.$

The $\mathcal{L}_2$ Lagrangian can now be written in terms of pion fields. Using the $\mathcal{F}_i(h)$ expansions in Eqs. (4.6) and (4.7) it results

$$\mathcal{L}_2 = \frac{1}{2}\partial_\mu h\partial^\mu h\left(1 + 2a_H\frac{h}{v} + b_H\frac{h^2}{v^2}\right) \tag{4.14}$$

---

[3] The $f(\boldsymbol{\pi}^2)$ function defined in Ref. [55] is related to X and Y simply by $f(x) = X(x)/Y(x)$

$$+\left\{\frac{1}{2}\partial_\mu\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi}+\frac{(\boldsymbol{\pi}\partial_\mu\boldsymbol{\pi})^2}{2v^2}+\eta\left[\frac{\boldsymbol{\pi}^2(\partial_\mu\boldsymbol{\pi})^2}{v^2}+2\frac{(\boldsymbol{\pi}\partial_\mu\boldsymbol{\pi})^2}{v^2}\right]\right\}\left(1+2a_C\frac{h}{v}+b_C\frac{h^2}{v^2}\right),$$

where terms containing more than four fields are to be disregarded. The operators required by the renormalization procedure to be present in $\mathcal{L}_4$ as counterterms will be shown below to correspond to those on the left-hand side of Table 4.1. The expansion up to four fields of the terms in $\mathcal{L}_4$ is shown on the right column of Table 4.1.

Counterterm Lagrangian

It is straightforward to obtain the counterterm Lagrangian via the usual procedure of writing the bare parameters and field wave functions in terms of the renormalized ones (details in Appendix B),

$$\begin{aligned}
\delta\mathcal{L}_0+\delta\mathcal{L}_2 =&\frac{1}{2}\partial_\mu h\partial^\mu h\left(\delta_h+2\delta a_H\frac{h}{v}+\delta b_H\frac{h^2}{v^2}\right)-\frac{1}{2}\delta m_h^2 h^2-\delta\mu_1^3 h-\frac{\delta\mu_3}{3!}h^3-\frac{\delta\lambda}{4!}h^4\\
&+\frac{1}{2}\partial_\mu\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi}\left(\delta_\pi+2\delta a_C\frac{h}{v}+\delta b_C\frac{h^2}{v^2}\right)\\
&+\left(\delta_\pi-\frac{\delta v^2}{v^2}\right)\frac{1}{2v^2}\left((\boldsymbol{\pi}\partial_\mu\boldsymbol{\pi})(\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi})+2\eta\left(\boldsymbol{\pi}^2(\partial_\mu\boldsymbol{\pi})^2+2(\boldsymbol{\pi}\partial_\mu\boldsymbol{\pi})^2\right)\right). \quad (4.15)
\end{aligned}$$

$\delta\mathcal{L}_4$ is simply given by $\mathcal{L}_4$ with the replacement $c_i, a_i, b_i\to\delta c_i,\delta a_i,\delta b_i$, apart from operator $\mathcal{P}_9$, for which

$$\begin{aligned}
\delta(c_9\mathcal{P}_9)\to&-\frac{2\delta c_9}{v^4}\Big[(1+4\eta)(\boldsymbol{\pi}\Box\boldsymbol{\pi})^2+2(1+2\eta)(\boldsymbol{\pi}\Box\boldsymbol{\pi})(\partial_\mu\boldsymbol{\pi})^2\\
&\qquad\qquad+2\eta\boldsymbol{\pi}^2(\Box\boldsymbol{\pi})^2+8\eta(\Box\boldsymbol{\pi}\partial_\mu\boldsymbol{\pi})(\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi})\Big]\\
&-\frac{2}{v^2}\Box\boldsymbol{\pi}\Box\boldsymbol{\pi}\left[\left(\delta c_9-\frac{\delta v^2}{v^2}\right)+\frac{2\delta a_9 h}{v}+\frac{\delta b_9 h^2}{v^2}\right].
\end{aligned}$$

It turns out that the counterterm coefficient $\delta v^2=0$ as shown below. We have left explicit the δv^2 dependence, though, in case it may be interesting to apply our results to some scenario which includes sources of explicit chiral symmetry breaking in a context different than the SM one; it also serves as a check-point of our computations.

4.2 Renormalization of off-shell Green functions

We present in this section the results for the renormalization of the 1- 2-, 3-, and 4-point functions involving h and/or $\boldsymbol{\pi}$ in a general $\mathbf{U}$ parametrisation, specified by the η parameter in Eq. (4.13). Dimensional regularization is a convenient regularization scheme as it avoids quadratic divergencies, some of which would appear to be chiral noninvariant, leading to further technical complications [149, 150]. Dimensional regularization is thus used below, as well as minimal subtraction scheme as renormalization procedure. The notation

$$\Delta=+\frac{1}{16\pi^2}\frac{2}{\varepsilon}$$

	$\mathcal{L}_4$ operators	Expansion in π fields
$c_6\mathcal{P}_6$	$c_6\left[\text{Tr}(V_\mu V^\mu)\right]^2 \mathcal{F}_6(h)$	$\dfrac{4c_6}{v^4}(\partial_\mu\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi})^2$
$c_7\mathcal{P}_7$	$c_7\text{Tr}(V_\mu V^\mu)\frac{1}{v}\Box h\,\mathcal{F}_7(h)$	$-\dfrac{2c_7}{v^3}\Box h(\partial_\nu\boldsymbol{\pi}\partial^\nu\boldsymbol{\pi}) - \dfrac{4a_7}{v^4}(h\Box h)(\partial_\nu\boldsymbol{\pi}\partial^\nu\boldsymbol{\pi})$
$c_8\mathcal{P}_8$	$c_8\text{Tr}(V_\mu V_\nu)\frac{1}{v^2}\partial^\mu h\partial^\nu h\,F_8(h)$	$-2\dfrac{c_8}{v^4}(\partial_\mu h\partial_\nu h)(\partial^\mu\boldsymbol{\pi}\partial^\nu\boldsymbol{\pi})$
$c_9\mathcal{P}_9$	$c_9\text{Tr}[(\mathcal{D}_\mu V^\mu)^2]\,\mathcal{F}_9(h)$	$-\dfrac{2c_9}{v^4}\Big[v^2(\Box\boldsymbol{\pi}\Box\boldsymbol{\pi})+2\eta\boldsymbol{\pi}^2(\Box\boldsymbol{\pi})^2+(1+4\eta)\,(\boldsymbol{\pi}\Box\boldsymbol{\pi})^2$
		$+8\eta(\boldsymbol{\pi}\partial_\mu\boldsymbol{\pi})(\partial^\mu\boldsymbol{\pi}\Box\boldsymbol{\pi}) + (2+4\eta)\,(\partial_\mu\boldsymbol{\pi})^2(\boldsymbol{\pi}\Box\boldsymbol{\pi})\Big]$
		$-\dfrac{4a_9}{v^3}h(\Box\boldsymbol{\pi}\Box\boldsymbol{\pi}) - \dfrac{2b_9}{v^4}h^2(\Box\boldsymbol{\pi}\Box\boldsymbol{\pi})$
$c_{10}\mathcal{P}_{10}$	$c_{10}\text{Tr}(V_\nu\mathcal{D}_\mu V^\mu)\frac{1}{v}\partial^\nu h\,\mathcal{F}_{10}(h)$	$\dfrac{-2c_{10}}{v^3}\partial^\nu h(\partial_\nu\boldsymbol{\pi}\Box\boldsymbol{\pi}) + \dfrac{-4a_{10}}{v^4}h\partial^\nu h(\partial_\nu\boldsymbol{\pi}\Box\boldsymbol{\pi})$
$c_{11}\mathcal{P}_{11}$	$c_{11}\left[\text{Tr}(V_\mu V_\nu)\right]^2 \mathcal{F}_{11}(h)$	$\dfrac{4c_{11}}{v^4}(\partial_\mu\boldsymbol{\pi}\partial_\nu\boldsymbol{\pi})^2$
$c_{20}\mathcal{P}_{20}$	$c_{20}\text{Tr}(V_\mu V^\mu)\frac{1}{v^2}\partial_\nu h\partial^\nu h\,\mathcal{F}_{20}(h)$	$-2\dfrac{c_{20}}{v^4}(\partial_\mu h\partial^\mu h)(\partial_\nu\boldsymbol{\pi}\partial^\nu\boldsymbol{\pi})$
$c_{\Box H}\mathcal{P}_{\Box H}$	$\dfrac{1}{v^2}(\Box h\Box h)\,\mathcal{F}_{\Box H}(h)$	$\dfrac{c_{\Box H}}{v^2}(\Box h\Box h) + \dfrac{2a_{\Box H}}{v^3}h\,(\Box h\Box h) + \dfrac{b_{\Box H}}{v^4}h^2\,(\Box h\Box h)$
$c_{\Delta H}\mathcal{P}_{\Delta H}$	$\dfrac{1}{v^3}(\partial_\mu h\partial^\mu h)\Box h\,\mathcal{F}_{\Delta H}(h)$	$\dfrac{c_{\Delta H}}{v^3}(\partial_\mu h\partial^\mu h)\Box h + \dfrac{2a_{\Delta H}}{v^4}(\partial_\mu h\partial^\mu h)h\Box h$
$c_{DH}\mathcal{P}_{DH}$	$\dfrac{1}{v^4}(\partial_\mu h\partial^\mu h)^2\,\mathcal{F}_{DH}(h)$	$\dfrac{c_{DH}}{v^4}(\partial_\mu h\partial^\mu h)^2$

Table 4.1: The two columns on the left show the operators required to be in $\mathcal{L}_4$, Eq. (4.4), by the renormalization procedure. The right hand side gives the corresponding explicit expansion in terms of pion and h fields (up to four fields), following the **U** expansion in Eq. (4.13) and the $\mathcal{F}_i(h)$ parametrisation in Eq. (4.7).

95

	Amplitudes						
	$2h$	$3h$	$4h$	2π	$2\pi h$	$2\pi 2h$	4π
$\mathcal{P}_6$							c_6
$\mathcal{P}_7$					c_7	a_7	
$\mathcal{P}_8$						c_8	
$\mathcal{P}_9$				c_9	a_9	b_9	c_9
$\mathcal{P}_{10}$					c_{10}	a_{10}	
$\mathcal{P}_{11}$							c_{11}
$\mathcal{P}_{20}$						c_{20}	
$\mathcal{P}_{\square H}$	$c_{\square H}$	$a_{\square H}$	$b_{\square H}$				
$\mathcal{P}_{\Delta H}$		$c_{\Delta H}$	$a_{\Delta H}$				
$\mathcal{P}_{DH}$			c_{DH}				

Table 4.2: Illustration of which operators in $\mathcal{L}_4$ (see Eq. (4.4) and Table 4.1) contribute to 2-, 3-, and 4-point amplitudes involving pions and/or h fields. The specific operator coefficients contributing to each amplitude are indicated, following the $c_i\mathcal{F}_i$ expansion in Eq. (4.7).

will be adopted, while FeynRules, FeynArts, and FormCalc [157–161] will be used to compute one-loop amplitudes. Diagrams with closed pion loops give zero contribution for the case of massless pions under study, and any reference to them will be omitted below.

Table 4.2 provides and overview of which $\mathcal{L}_4$ operator coefficients contribute to amplitudes involving pions and/or h, up to 4-point vertices. It also serves as an advance over the results: all operators in (4.4) will be shown to be required by the renormalization procedure. Furthermore we have checked that they are all independent and thus their ensemble, when considered by itself, constitutes a complete and independent basis of scalar operators, up to four-derivatives in the non-linear expansion. None of them should be disregarded on arguments of their tradability for other operators, for instance for fermionic ones via the application of the equations of motion (EOM), unless the latter operators are explicitly included as part of the analysis, or without further assumptions (i.e. to neglect all fermion masses). See Sect. 4.4 for comparison with previous literature.

4.2.1 1-point functions

Because of chiral symmetry pions always come in even numbers in any vertex, unlike Higgs particles, thus tadpole contributions may be generated only for the latter. At tree-level it would suffice to set $\mu_1 = 0$ in $V(h)$ (Eq. (4.2)) in order to insure $\langle h \rangle = 0$. At one-loop, a tadpole term is induced from the triple Higgs couplings μ_3 and a_H, though, via the Feynman diagram in Fig. 4.1. The counterterm required to cancel this contribution reads

$$\delta\mu_1^3 = m_h^2 \left(\mu_3 - 4a_H \frac{m_h^2}{v} \right) \Delta_\varepsilon \,, \tag{4.16}$$
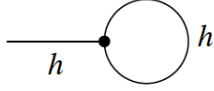
and has no impact on the rest of the Lagrangian.

Figure 4.1: Diagram contributing to the Higgs 1-point function.
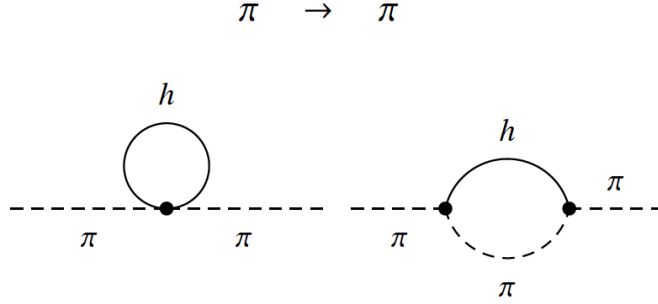
$$\pi \quad \to \quad \pi$$

Figure 4.2: Diagrams contributing to the π self-energy.

4.2.2 2-point functions

Consider mass and wave function renormalization for the pion and h fields. Because of chiral symmetry no pion mass will be induced by loop corrections at any order, unlike for the h field, whose mass is not protected by that symmetry. The diagrams contributing to the pion self-energy are shown in Fig. 4.2. The divergent part of the amplitudes, $\Pi_{\text{div}}^{ij}(p^2)\Delta_\varepsilon$, and the counterterm structure are given by

$$\Pi_{\text{div}}^{ij}(p^2) = \left[p^2 \left(a_C^2 - b_C \right) \frac{m_h^2}{v^2} + p^4 \frac{a_C^2}{v^2} \right] \delta_{ij} , \tag{4.17}$$

$$\Pi_{ctr}^{ij}(p^2) = \left[p^2 \delta_\pi - p^4 \frac{4}{v^2} \left(\delta c_9 - \frac{\delta v^2}{v^2} \right) \right] \delta_{ij} . \tag{4.18}$$

In an off-shell renormalization scheme, it is necessary to match all the momenta structure of the divergent amplitude with that of the counterterms, which leads to the following determination

$$\delta_\pi = - \left(a_C^2 - b_C \right) \frac{m_h^2}{v^2} \Delta_\varepsilon ,$$
$$\delta c_9 - \frac{\delta v^2}{v^2} = \frac{a_C^2}{4} \Delta_\varepsilon . \tag{4.19}$$

It follows that the π wave function renormalization has no divergent part whenever $a_C^2 = b_C$, which happens for instance in the case of the SM ($a_C = b_C = 1$). Note as well that the absence of a
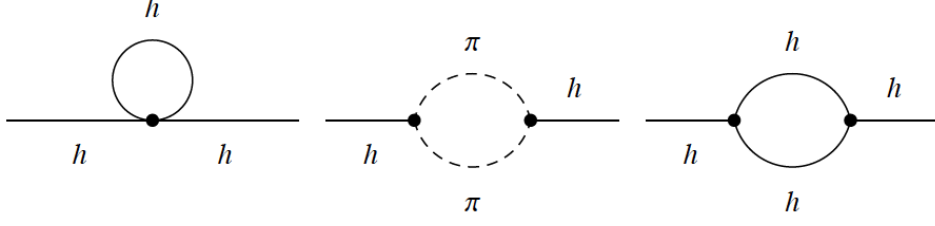
Figure 4.3: Diagrams contributing to the Higgs self-energy.

constant term in eq. (4.17) translates into massless pions at 1-loop level, as mandated by chiral symmetry at any loop order. Furthermore, the p^4 term stems from the $h - \pi$ coupling a_C, which is an entire new feature compared to the non-linear σ model renormalization. This term demands the presence of a $\Box\pi\Box\pi$ counterterm in the $\mathcal{L}_4$ Lagrangian, as expected by naive dimensional analysis.

Turning to the Higgs particle, the diagrams contributing to its self-energy are shown in Fig. 4.3, with the divergent part and the required counterterm structure given by

$$
\Pi_{\text{div}}(p^2) = p^4 \frac{(3a_C^2 + a_H^2)}{2v^2} + p^2 \left(-\frac{\mu_3}{v} a_H + \frac{m_h^2 (5a_H^2 - b_H)}{v^2} \right)
$$
$$
+ \left(\frac{1}{2}\mu_3^2 + \frac{1}{2}m_h^2 \left(\lambda - 8\frac{\mu_3}{v} a_H \right) + \frac{m_h^4 (6a_H^2 - b_H)}{v^2} \right), \tag{4.20}
$$

$$
\Pi_{ctr}(p^2) = p^4 \frac{2\delta c_{\Box H}}{v^2} + p^2 \delta_h - \delta m_h^2. \tag{4.21}
$$

It follows that the required counterterms are given by

$$
\delta_h = \left[\frac{\mu_3}{v} a_H + \frac{m_h^2 (b_H - 5a_H^2)}{v^2} \right] \Delta_\varepsilon,
$$
$$
\delta m_h^2 = \left[\frac{1}{2}\mu_3^2 + \frac{1}{2}m_h^2 \left(\lambda - 8\frac{\mu_3}{v} a_H \right) + \frac{m_h^4 (6a_H^2 - b_H)}{v^2} \right] \Delta_\varepsilon, \tag{4.22}
$$
$$
\delta c_{\Box H} = -\frac{1}{4} \left(3a_C^2 + a_H^2 \right) \Delta_\varepsilon.
$$

This result implies that a non-vanishing a_C (as in the SM limit) and/or a_H leads to a p^4 term in the counterterm Lagrangian, requiring a $\Box h\Box h$ term in $\mathcal{L}_4$. In this scheme, a Higgs wave function renormalization is operative only in deviations from the SM with non-vanishing a_H and/or b_H.
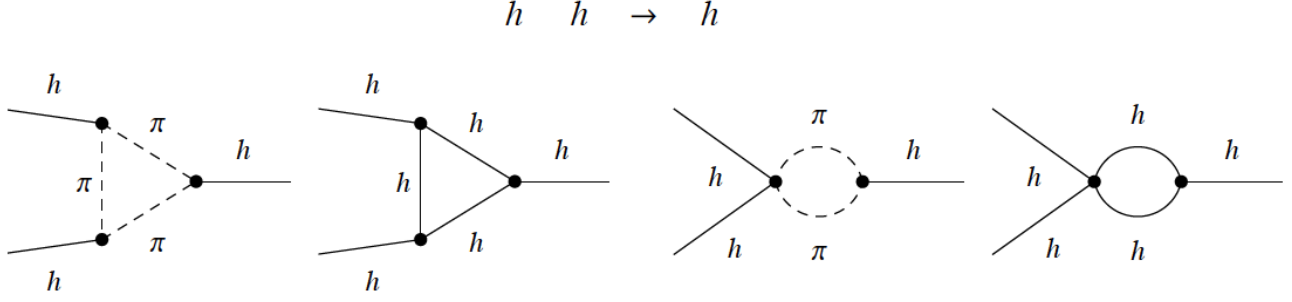
Figure 4.4: Diagrams contributing to the $hh \to h$ amplitude, not including diagrams obtained by crossing.

4.2.3   3-point functions

The calculational details for the 3- and 4-point functions will not be explicitly shown as they are not particularly illuminating. See Appendix B for more details. Vertices with an odd number of legs necessarily involve at least one Higgs particle.

hhh

Let us consider first the *hhh* amplitude at one loop. The relevant diagrams to be computed are displayed in Fig. 4.4. As h behaves as a generic singlet, the vertices involving uniquely external h legs which appear in the Lagrangian Eq. (4.1) will span all possible momentum structures that can result from one-loop amplitudes. Hence any divergence emerging on amplitudes involving only external h particles will be easily absorbable. The specific results for the counterterms emerging from $\mathcal{L}_0$ and $\mathcal{L}_2$ can be found in Appendix B.

ππh

The diagrams for $\pi\pi h$ amplitudes are shown in Fig. 4.5. The one-loop divergences are studied in detail in Appendix B; for instance, it turns out that neither δa_C nor δa_9 are induced in the SM limit. Chiral symmetry restricts the possible structures spanned by the pure π and $h - \pi$ operators. Because of this, it turns out that part of the divergent amplitude induced by the last diagram in Fig. 4.5 cannot be cast as a function of the $\mathcal{L}_2$ and $\mathcal{L}_4$ operators, that is, it cannot be reabsorbed by chiral-invariant counterterms, and furthermore its coefficient depends on the pion parametrisation used: an apparent non chiral-invariant divergence (NID) has been identified. NIDs are an artefact of the apparent breaking of chiral symmetry when the one-loop analysis is treated in perturbation theory [55] and have no physical impact as they vanish for on-shell amplitudes. While long ago NIDs had been isolated in perturbative analysis of four-pion vertices in the non-linear sigma model [148], the result obtained here is a new type of NID: a three-point function
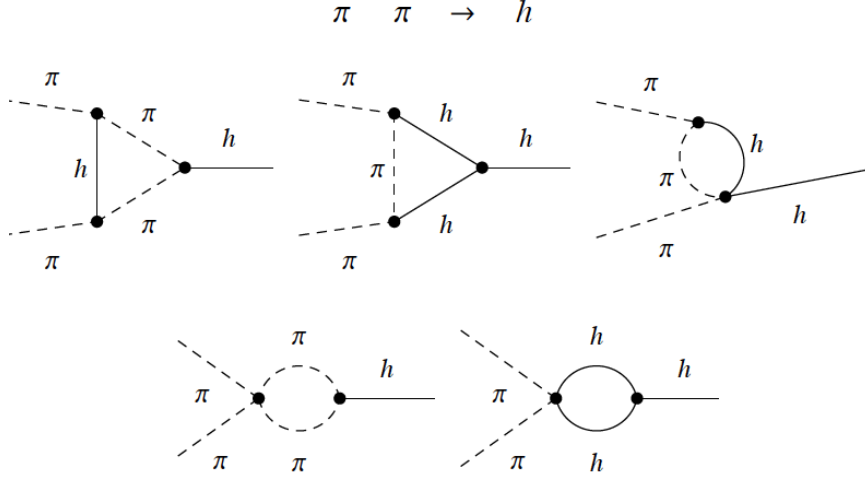
Figure 4.5: Diagrams contributing to the $\pi\pi \to h$ scattering amplitude, not including diagrams obtained by crossing.

involving the Higgs particle, corresponding to the chiral non-invariant operator

$$O_1^{\text{NID}} = -a_C \left(\frac{3}{2} + 5\eta \right) \frac{\Delta_\varepsilon}{v^3} \, \boldsymbol{\pi} \Box \boldsymbol{\pi} \Box h \,. \tag{4.23}$$

This coupling cannot be reabsorbed as part of a chiral invariant counterterm, but its contribution to on-shell amplitudes indeed vanishes. It is interesting to note that while the renormalization conditions of all physical parameters turn out to be independent of the choice of $\mathbf{U}$ parametrisation, as they should, NIDs exhibit instead an explicit η dependence, as illustrated by Eq. (4.23). This pattern will be also present in the renormalization of 4-point functions, developed next.

4.2.4   4-point functions

The analysis of this set of correlation functions turns out to be tantalizing when comparing the results for mixed $\pi - h$ vertices with those for pure pionic ones [4].

$\pi\pi hh$

The computation of the $\pi\pi \to hh$ one-loop amplitude shows that the renormalization procedure requires the presence of all possible chiral invariant $hh\pi\pi$ counterterms in the Lagrangian, in the most general case.

---

[4]It provides in addition nice checks of the computations; for instance we checked explicitly in the present context that the consistency of the renormalization results for four-point functions requires $\delta_v^2 = 0$.

Furthermore, we have identified new NIDs in $hh\pi\pi$ amplitudes:

$$O_2^{\text{NID}} = +(2a_C^2 - b_C)\left(\frac{3}{2} + 5\eta\right)\frac{\Delta_\varepsilon}{v^4}\,\boldsymbol{\pi}\Box\boldsymbol{\pi}\,h\Box h\,,$$

$$O_3^{\text{NID}} = +(a_C^2 - b_C)\left(\frac{3}{2} + 5\eta\right)\frac{\Delta_\varepsilon}{v^4}\,\boldsymbol{\pi}\Box\boldsymbol{\pi}\,\partial_\mu h\partial^\mu h\,, \qquad (4.24)$$

$$O_4^{\text{NID}} = -2a_C^2\left(\frac{3}{2} + 5\eta\right)\frac{\Delta_\varepsilon}{v^4}\,\boldsymbol{\pi}\partial_\mu\boldsymbol{\pi}\,\partial^\mu h\Box h\,.$$

While these NIDs differ from that for the three-point function in Eq. (4.23) in their counterterm structure, they all share an intriguing fact: to be proportional to the factor $(3/2 + 5\eta)$. Therefore a proper choice of parametrisation, i.e. $\eta = -3/10$, removes all mixed $h - \pi$ NIDs. That value of η is of no special significance as fas as we know, and in fact there is no choice of parametrisation that can avoid *all* noninvariant divergencies, as proved next.

$\pi\pi\pi\pi$

Consider now $\pi\pi \to \pi\pi$ amplitudes. Only two counterterms are necessary to reabsorb chiral-invariant divergencies, namely δc_6 and δc_{11}. In this case, we find no other NIDs than those already present in the non-linear σ model [148], which stemmed from the insertion in the loop of the four-pion vertex (whose coupling depends on η). Our analysis shows that the four-π NIDs read:

$$O_5^{\text{NID}} = +\left(9\eta^2 + 5\eta + \frac{3}{4}\right)\frac{\Delta_\varepsilon}{v^4}\,(\boldsymbol{\pi}\Box\boldsymbol{\pi})^2,$$

$$O_6^{\text{NID}} = +\left[1 + 4\eta + \left(\frac{1}{2} + \eta\right)a_C^2\right]\frac{\Delta_\varepsilon}{v^4}\,(\boldsymbol{\pi}\Box\boldsymbol{\pi})(\partial_\mu\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi}),$$

$$O_7^{\text{NID}} = +2\eta^2\frac{\Delta_\varepsilon}{v^4}\,\boldsymbol{\pi}^2(\Box\boldsymbol{\pi})^2, \qquad (4.25)$$

$$O_8^{\text{NID}} = +2\eta\left(a_C^2 - 1\right)\frac{\Delta_\varepsilon}{v^4}\,(\Box\boldsymbol{\pi}\partial_\mu\boldsymbol{\pi})(\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi}).$$

As expected, the parametrisation freedom – the dependence on the η parameter – appears only in NIDs, and never on chiral-invariant counterterms, as the latter describe physical processes. Furthermore, the contribution of all NIDs to on-shell amplitudes vanishes as expected [5]. Finally, the consideration of the ensemble of three and four-point NIDs in Eqs. (4.23), (4.24) and (4.25) shows inmediately that no parametrisation can remove *all* NIDs: it is possible to eliminate those involving h [6], but no value of η would remove all pure pionic ones.

hhhh

The renormalization procedure for $hh \to hh$ amplitudes is straightforward. It results in contributions to $\delta a_{\Delta H}$, δc_{DH}, and δb_H. Interestingly, Appendix C illustrates that large coefficients are

---

[5]This is not always seen when taken individually. For instance, the contribution of O_4^{NID} to the $hh\pi\pi$ amplitude is cancelled by that of O_1^{NID}, which corrects the $h\pi\pi$ vertex.

[6]This may be linked to the larger freedom of redefinition for fields not subject to chiral invariance.

present in some terms of the RGE for b_H and λ; this might *a priori* translate into measurable effects when comparing data at different scales, if ever deviations from the SM predictions are detected, see Sect. (4.3).

————

There is a particularity of the off-shell renormalization scheme which deserves to be pointed out. A closer look at the counterterms reveals that, in the SM case, that is

$$a_C = b_C = 1, \quad a_H = b_H = 0, \quad \mu_3 = 3\frac{m_h^2}{v} \quad and \quad \lambda = 3\frac{m_h^2}{v^2}, \tag{4.26}$$

several BSM operator coefficients do not vanish. Although at first this might look counterintuitive, when calculating physical amplitudes the contribution of these non-vanishing operator coefficients all combine in such a way that the overall BSM contribution indeed cancels. The same pattern propagates to the renormalization group equations discussed in Sect. 4.

4.2.5 Dealing with the apparent non-invariant divergencies

For the non-linear σ model the issue of NIDs was analyzed long ago [148, 150–154]). In that case, it was finally proven that a non-linear redefinition of the pion field which includes space-time derivatives could reabsorb them [148]. This method reveals a deeper rationale in understanding the issue, as Lagrangians related by a local field redefinition are equivalent, even when it involves derivatives [162–165]. Consequently, if via a general pion field redefinition

$$\boldsymbol{\pi} \to \boldsymbol{\pi}\, f(\boldsymbol{\pi}, h, \partial_\mu \boldsymbol{\pi}, \partial_\mu h, \dots)\,,$$

with $f(0) = 1$, the Lagrangian is shifted

$$\mathcal{L} \to \mathcal{L}' = \mathcal{L} + \delta\mathcal{L}\,,$$

from the equivalence between $\mathcal{L}$ and $\mathcal{L}'$ it follows that $\delta\mathcal{L}$ must be unphysical. Thus, if an appropriate pion field redefinition is found which is able to absorb all NDIs, it automatically implies that NDIs do not contribute to the S-matrix, and therefore that chiral symmetry remains unbroken. *In other words, the non-invariant operators can be identifed with quantities in the functional generator that vanish upon performing the path integral.*

Let us consider the following pion redefinition, in which we propose new terms not considered previously and which contain the h field:

$$\begin{aligned}
\pi_i \to \pi_i &\left(1 + \frac{\alpha_1}{2v^4}\boldsymbol{\pi}\Box\boldsymbol{\pi} + \frac{\alpha_2}{2v^4}\partial_\mu\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi} + \frac{\beta}{2v^3}\Box h + \frac{\tilde{\gamma}_1}{2v^4}h\Box h + \frac{\gamma_2}{2v^4}\partial_\mu h\partial^\mu h\right) \\
&+ \frac{\alpha_3}{2v^4}\Box\pi_i(\boldsymbol{\pi}\boldsymbol{\pi}) + \frac{\alpha_4}{2v^4}\partial_\mu\pi_i(\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi}).
\end{aligned} \tag{4.27}$$

The application of this redefinition to $\mathcal{L}_4$ is immaterial, as it would only induce couplings of higher order. As all terms in the shift contain two derivatives, when applied to $\mathcal{L}_2$ contributions to $\mathcal{L}_4$

and NID operator coefficients do follow. Indeed, the action of Eq. (4.27) on $\mathcal{L}_2$ reduces to that on the term

$$\frac{1}{4}\text{Tr}(\partial_\mu \mathbf{U}\partial^\mu \mathbf{U})\mathcal{F}_C(h)\,, \tag{4.28}$$

which produces the additional contribution to NDI vertices given by

$$\Delta\mathcal{L}^{\text{NID}} = -\boldsymbol{\pi}\Box\boldsymbol{\pi}\left(\frac{\alpha_1}{v^4}\boldsymbol{\pi}\Box\boldsymbol{\pi} + \frac{\alpha_2}{v^4}\partial_\mu\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi} + \frac{\beta}{v^3}\Box h + \frac{\gamma_1}{v^4}h\Box h + \frac{\gamma_2}{v^4}\partial_\mu h\partial^\mu h\right)$$
$$- \frac{\alpha_3}{v^4}(\Box\boldsymbol{\pi}\Box\boldsymbol{\pi})(\boldsymbol{\pi}\boldsymbol{\pi}) - \frac{\alpha_4}{v^4}(\Box\boldsymbol{\pi}\partial_\mu\boldsymbol{\pi})(\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi}) - \frac{2a_C\beta}{v^4}\boldsymbol{\pi}\partial_\mu\boldsymbol{\pi}\partial^\mu h\Box h + ... \tag{4.29}$$

where $\gamma_1 = 2a_C\beta + \tilde{\gamma}_1$, and where the dots indicate other operators with either six derivatives or that have more than four fields and are beyond the scope of this work. Comparing the terms in $\Delta\mathcal{L}^{\text{NID}}$ with the NID operators found, Eqs. (4.23), (4.24) and (4.25), it follows that by choosing

$$\alpha_1 = \left(9\eta^2 + 5\eta + \frac{3}{4}\right)\Delta_\varepsilon, \qquad\qquad \beta = -\left(\frac{3}{2} + 5\eta\right)a_C\Delta_\varepsilon,$$

$$\alpha_2 = \left[1 + 4\eta + \left(\frac{1}{2} + \eta\right)a_C^2\right]\Delta_\varepsilon, \qquad \gamma_1 = \left(\frac{3}{2} + 5\eta\right)\left(2a_C^2 - b_C\right)\Delta_\varepsilon,$$

$$\alpha_3 = 2\eta^2\Delta_\varepsilon,$$

$$\alpha_4 = 2\eta\left(a_C^2 - 1\right)\Delta_\varepsilon, \qquad\qquad \gamma_2 = \left(\frac{3}{2} + 5\eta\right)\left(a_C^2 - b_C\right)\Delta_\varepsilon.$$

all 1-loop NID are removed away.

A few comments are in order. The off-shell renormalization of one loop amplitudes is delicate and physically interesting. Because of chiral symmetry, the pure pionic or mixed pion-h operators do not encode all possible momentum structures, even after pion field redefinitions. Hence, the appearance of divergent structures that can be absorbed by $\delta\mathcal{L}_0$, $\delta\mathcal{L}_2$, $\delta\mathcal{L}_4$ and $\Delta\mathcal{L}^{\text{NI}}$ is a manisfestation of chiral invariance and of the field redefinition equivalence discussed above. We have shown consistently that NIDs appearing in the one-loop renormalization of the electroweak chiral Lagrangian do not contribute to on-shell quantities. In fact, a closer examination has revealed that the apparent chiral non-invariant divergencies emerge from loop diagrams which have at least one four-pion vertex in it, and this is why all of them depend on η. We have also shown that the presence of a light Higgs boson modifies the coefficients of the unphysical counterterms made out purely of pions, but not their structure, neither -of course- breaks chiral symmetry.

4.3 Renormalization Group Equations

It is straightforward to derive the RGE from the δc_i divergent contributions determined in the previous section. The complete RGE set can be found in Appendix C. As illustration, the evolution of those Lagrangian coefficients which do not vanish in the SM limit is given by:

$$16\pi^2\frac{d}{d\ln\mu}a_C = a_C\left[\left(5a_H^2 - 3b_C - b_H\right)\frac{m_h^2}{v^2} - a_H\frac{\mu_3}{v}\right]$$

$$+ a_C^2 \left(4a_H \frac{m_h^2}{v^2} - \frac{\mu_3}{v} \right) + 3a_C^3 \frac{m_h^2}{v^2} + b_C \frac{\mu_3}{v} - 4a_H b_C \frac{m_h^2}{v^2} \ ,$$

$$16\pi^2 \frac{d}{d\ln\mu} b_C = b_C \left[2\left(5a_C^2 + 8a_C a_H + 17a_H^2 - 3b_H \right) \frac{m_h^2}{v^2} + \lambda - 2(2a_C + 5a_H)\frac{\mu_3}{v} \right]$$

$$- 2b_C^2 \frac{m_h^2}{v^2} + a_C^2 \left(-\lambda + 4a_C \frac{\mu_3}{v} + 8a_H \frac{\mu_3}{v} \right) - 4a_C^2 \left(2a_C^2 + 4a_C a_H + 6a_H^2 - b_H \right) \frac{m_h^2}{v^2} \ ,$$

$$16\pi^2 \frac{d}{d\ln\mu} m_h^2 = m_h^2 \left(\lambda - 10a_H \frac{\mu_3}{v} \right) + \left(22a_H^2 - 4b_H \right) \frac{m_h^4}{v^2} + \mu_3^2 \ ,$$

$$16\pi^2 \frac{d}{d\ln\mu} \mu_3 = \mu_3 \left[\left(87a_H^2 - 15b_H \right) \frac{m_h^2}{v^2} + 3\lambda \right] - 15a_H \frac{\mu_3^2}{v} - 12\lambda a_H \frac{m_h^2}{v} - \left(96a_H^3 - 36a_H b_H \right) \frac{m_h^4}{v^3} \ ,$$

$$16\pi^2 \frac{d}{d\ln\mu} \lambda = \lambda \left[4\left(41a_H^2 - 7b_H \right) \frac{m_h^2}{v^2} - 52a_H \frac{\mu_3}{v} \right] + 3\lambda^2 + 24\left(6a_H^2 - b_H \right) \frac{\mu_3^2}{v^2}$$

$$- 96a_H \left(8a_H^2 - 3b_H \right) \frac{\mu_3 m_h^2}{v^3} + 12\left(80a_H^4 - 48a_H^2 b_H + 3b_H^2 \right) \frac{m_h^4}{v^4} \ ,$$

These and the rest of the RGE in Appendix C show as well that the running of the parameters a_C, b_C, a_H, b_H, and v^2 is only induced by the couplings entering the Higgs potential, Eq. (4.5).

Note that in the RGE for the Higgs quartic self-coupling λ, some terms are weighted by numerical factors of $\mathcal{O}(100)$. This suggests that if a BSM theory results in small couplings for a_H and b_H, those terms could still induce measurable phenomenological consequences. Nevertheless, physical amplitudes will depend on a large combination of parameters, which might yield cancellations or enhancements as pointed out earlier, and only a more thorough study can lead to firm conclusions. Such large coefficients turn out to be also present in the evolution of some BSM couplings, such as the four-Higgs coupling b_H for which

$$16\pi^2 \frac{d}{d\ln\mu} b_H = b_H \left[2\left(-a_C^2 + 97a_H^2 + b_C \right) \frac{m_h^2}{v^2} - 44a_H \frac{\mu_3}{v} + 3\lambda \right] - 18b_H^2 \frac{m_h^2}{v^2}$$

$$+ a_H^2 \left(-13\lambda + 84a_H \frac{\mu_3}{v} \right) - 240a_H^4 \frac{m_h^2}{v^2} \ , \tag{4.30}$$

On general grounds a_H is expected to be small, and for instance the a_H^4 dependence in Eq. (4.30) is not expected to be relevant in spite of the numerical prefactor. On the other side, present data set basically no bound on the couplings involving three or more external Higgs particles, and thus the future putative impact of this evolution should not be dismissed yet.

4.4 Comparison with the literature

The works on the one-loop renormalization of the scalar sector of the non-linear Lagrangian with a light Higgs typically use either the square root parametrisation ($\eta = 0$ in our parametrisation) or the exponential one ($\eta = -1/6$), and have

- concentrated on on-shell analyses,

- disregarded the impact of $\mathcal{F}_H(h)$,

- disregarded fermionic operators; in practice this means to neglect all fermion masses.

This last point is not uncorrelated with the fact that the basis of independent four-derivative operators determined here has a larger number of elements than previous works about the scalar sector. Those extra bosonic operators have been shown here to be required by the counterterm procedure. It is possible to demonstrate, though, that they can be traded via EOM by other type of operators including gauge corrections and Yukawa-like operators. In a complete basis of all possible operators it is up to the practitioner to decide which set is kept, as long as it is complete and independent. When restricting instead to a given subsector, the complete and consistent treatment requires to consider all independent operators of the kind selected (anyway the renormalization procedure will indicate their need), or to state explicitly any extra assumptions to eliminate them. Some further specific comments:

Ref. [143] considers, under the first two itemized conditions above plus disregarding the impact of $V(h)$ (and in particular neglecting the Higgs mass), the scattering processes $hh \to hh$, $\pi\pi \to hh$ and $\pi\pi \to \pi\pi$. With the off-shell treatment, five additional operators result in this case with respect to those obtained in that reference (assuming the rest of their assumptions), $\mathcal{P}_7$, $\mathcal{P}_9$, $\mathcal{P}_{10}$, $\mathcal{P}_{\square H}$ and $\mathcal{P}_{\Delta H}$ in Table 4.1. Note that all these operators contain either $\square h$ or $\square \pi$ inside; they may thus be implicitly traded by fermionic operators via EOM, and can only be disregarded if all fermion masses are neglected. Assuming this extra condition, we could reproduce their results using the EOMs. For instance, the RGEs derived here for c_6, c_8, c_{20}, a_C and b_C differ from the corresponding ones in that reference: an off-shell renormalization analysis entails the larger number of operators mentioned. In any case, we stress again that the results of both approaches coincide when calculating physical amplitudes. Another contrast appears in the running of a_C, b_C, a_H, b_H, as well as the mass, the triple, and the quartic coupling of the Higgs, for which the running is induced by the Higgs potential parameters, disregarded in that reference.

In Ref. [144] the on-shell scattering process of the longitudinal modes of the process $W^+W^- \to ZZ$ is considered, disregarding $\mathcal{F}_H(h)$ but including the impact of $V(h)$. Our off-shell treatment results in this case in one additional operator (assuming the rest of their assumptions) with respect to those in that reference, $\mathcal{P}_9$, as only processes involving four goldstone bosons were considered there. Again this extra operator contains $\square \pi$ in all its terms and it could be neglected in practice if disregarding all fermion masses. With this extra assumption, our results reduce to theirs in the limit indicated.

Finally, in Refs. [155, 156] the geometrical formulation of HEFT has been adopted. It has been demonstrated that quantum corrections to the theory are given in terms of the curvature and connection of the scalar manifold. In particular it has been shown that the NIDs appear due to noncovariant term in the second variation of action which is explicitly proportional to EOM and can be eliminated if the variation is promoted to be covariant with respect to the chiral symmetry of the Lagrangian (see Eq.7 in Ref. [156]). The expressions for NID match exactly the results presented in this Chapter.

Summary

Completely clarifying the mechanism of the electroweak breaking is one of the main goals of particle physics today and an important role, on the theory side, is played by simple and motivated extensions of the SM which could provide guidance in the experimental search.

To this purpose we have formulated a model where the scalar sector of the SM is minimally extended to include an additional scalar particle σ, singlet under the SM gauge group. To a large extent our set up is motivated by the QCD linear sigma model. Similarly, we consider a linear realization of symmetry, where the Higgs fourplet together the σ scalar forms a fundamental representation of $SO(5)$ global symmetry, broken down spontaneously to $SO(4)$. This minimal $SO(5)/SO(4)$ coset includes four GB states, three of which are to become longitudinal polarisations of W and Z bosons as in the SM itself and one additional state is the observed Higgs particle. In order to reproduce the observed Higgs mass an explicit breaking of the global $SO(5)$ symmetry is needed. To introduce it we consider the extended fermionic sector in complete representations of $SO(5)$ group, linearly coupled to the SM fermions through $SO(5)$ violating couplings. Proto-Yukawa interactions between the scalar 5plet and the new fermions allow for the explicit breaking to propagate to the scalar potential due to Coleman-Weinberg mechanism and to generate a potential for the Higgs fourplet, which eventually triggers the EWSB.

The previous attempt in this direction [22] did not fully take into account the impact of the extended fermionic sector on the main phenomenological observables. Instead, we considered the restrictions set on the parameter space of the fermionic sector by EWPT parameters, namely S, T and R_b. Concerning the scalar contribution, from an explicit one-loop computation we recover a well-known result: a positive contribution to the S parameter and a negative contribution to the T parameter. In our model both extra contributions ΔS and ΔT are finite, so there is no cut-off ambiguity. The main impact of the heavy fermion contribution is on the T parameter. Letting the heavy fermions to be as light as 800 GeV, the lower limit from direct searches, contributions to T of both signs as large as 0.2. Even the largest scalar contributions to ΔS and ΔT, obtained when the scalars mixing angle γ saturates its experimental bound and σ is very heavy, can always be compensated by the fermionic ones for an appropriate choice of the parameters, keeping S and T within the experimentally allowed region. Thus, we conclude that large part of the parameter space is allowed by EWPT and therefore the theory is fully viable.

The direct LHC searches for a scalar particle together with the theoretical requirement that the Higgs behaves as a pseudo-Goldstone boson, allow to exclude σ masses below about 500 GeV.

Furthermore, we have integrated out the BSM states and obtained the effective Lagrangian. First,

linear effective Lagrangian has been derived by integration out of the heavy fermionic states. This Lagrangian describes interactions of the Higgs and σ scalars with the SM fermionic matter. As a second step we introduce the effective Yukawa operator, characterized by two parameters, which depend only on the embeddings of the fermions into $SO(5)$ representations . Having this, we have derived the benchmark non-linear effective Lagrangian for the large mass limit of the heavy scalar σ. Up to the first order in the linear corrections it contains ten operators, five bosonic and five fermionic. The coefficients of the latter ones are given as functions of the two parameters of Yukawa interaction, thus allowing for a simple comparison between the models with various embeddings adopted.

We have derived for the first time σ induced modifications of the Higgs to gauge bosons and fermions couplings κ_V and κ_f and shown that they are quite universal. While the leading order corrections, obtained in the limit of the infinite scalar mass coincide with the ones previously obtained in the literature, the new next to leading corrections are doubly suppressed by small explicit breaking parameters and large scalar self coupling, a combination corresponding to a m_h^2/m_s^2 suppression. We have repeated the integration out procedure for the UV complete model considered previously and identified the coefficients of the effective operators in terms of the parameters of extended fermionic sector as well as scalar sector with the σ boson. The resulting operators represent a subset of the most general non-linear Lagrangian and are consistent with the results for the general $SO(5)/SO(4)$ constructions given in [70].

Next, we have performed the one-loop off-shell renormalization of the custodial-preserving sector of effective non-linear Lagrangian with the finite Higgs mass, focusing on its scalar sector composed of longitudinal components of the SM gauge bosons and the Higgs boson h. We have identified the NLO counterterms required to absorb the divergences originating from the diagrams with up to 4 external legs, in the agreement with the NDA the general non-linear Lagrangian.

As a useful tool we introduce a novel general parametrisation of the Goldstone boson matrix, at the order considered depending on single parameter η. All the counterterms induced are parametrisation independent. We have also found chrial noninvariant divergences, which depend explicitly on η and generalize the result of [148] to the case of the light Higgs present in the spectrum. We demonstrated how they can be removed by the pion field redefinition, thus being unphysical and vanishing on-shell. Our findings have been later confirmed by an independent group [155, 156], where the same non-invariant divergences have been found, it has also been shown that they can be avoided by employing of covariant variation of the action in the path integral approach to the renormalization.

To sum up the above we have constructed the viable UV complete model of Goldstone Higgs which is possible to validate by the ongoing LHC run either through direct observation of the heavy scalar σ and/or heavy fermionic partners of the SM fermions. At low energies, below the masses of BSM particles the model results in the set of non-linear operators parametrizing the deviations from the SM predictions with the coefficients being the explicit functions of the parameters of the UV completion. We have analysed the consistency of the subset of non-linear Lagrangian by analysing its one loop structure in general **U** parametrisation and confirmed the previous results based on NDA.

Acknowledgements

Appendix A

Coleman–Weinberg Potential

In Sect. 2.1 it was assumed a specific form for the $SO(5)$ scalar potential broken to $SO(4)$, introducing two additional $SO(5)$ breaking parameters α and β. Here we will further motivate this assumption. Even assuming that the tree level scalar potential would preserve the global $SO(5)$ symmetry, the presence of a $SO(5)$ breaking couplings in the fermionic sector will generate at one-loop level $SO(5)$ breaking terms through the Coleman-Weinberg mechanism [71]. The one-loop fermionic contribution can be obtained from the field dependent mass matrix $\mathcal{M}$ as

$$
\begin{aligned}
V_{\text{loop}} &= -\frac{i}{2} \int \frac{d^4 k}{(2\pi)^4} \sum_{n=1}^{\infty} \frac{1}{n} \text{Tr} \left(\frac{\mathcal{M}\mathcal{M}^\dagger}{k^2} \right)^n = \frac{i}{2} \int \frac{d^4 k}{(2\pi)^4} \text{Tr} \log \left(1 - \frac{\mathcal{M}\mathcal{M}^\dagger}{k^2} \right) \\
&= -\frac{1}{64\pi^2} \left(\Lambda^2 \text{Tr} \left[\mathcal{M}\mathcal{M}^\dagger \right] - \text{Tr} \left[(\mathcal{M}\mathcal{M}^\dagger)^2 \right] \log \left(\frac{\Lambda^2}{\mu^2} \right) + \right. \\
&\quad \left. + \text{Tr} \left[(\mathcal{M}\mathcal{M}^\dagger)^2 \log \left(\frac{\mathcal{M}\mathcal{M}^\dagger}{\mu^2} \right) \right] - \frac{1}{2} \text{Tr} \left[(\mathcal{M}\mathcal{M}^\dagger)^2 \right] \right) ,
\end{aligned} \tag{A.1}
$$

where Λ is the UV cutoff scale while μ is a generic renormalization scale. The first two terms on the right-hand side of this equation are divergent, respectively quadratically and logarithmically, while the last two terms are finite. For the model under discussion it results:

$$
\begin{aligned}
\text{Tr}[\mathcal{M}\mathcal{M}^\dagger] &= c_1 + c_2 \left(\phi^T \phi \right) , \tag{A.2} \\
\text{Tr}[(\mathcal{M}\mathcal{M}^\dagger)^2] &= d_1 + d_2 \, \sigma + d_3 \, \sigma^2 + d_4 \left(\phi^T \phi \right) + d_5 \left(\phi^T \phi \right)^2 , \tag{A.3}
\end{aligned}
$$

where

$$
\begin{aligned}
c_1 &= 2\Lambda_1^2 + \Lambda_2^2 + \Lambda_3^2 + M_1^2 + 5\, M_5^2 + (\{\} \leftrightarrow \{\}') \\
c_2 &= y_1^2 + y_2^2 + (\{\} \leftrightarrow \{\}') ,
\end{aligned} \qquad ,
$$

and

$$
\begin{aligned}
d_1 &= M_1^4 + 5M_5^4 + 2M_5^2 \left(2\Lambda_1^2 + \Lambda_2^2 \right) + 2M_1^2\Lambda_3^2 + 2\Lambda_1^4 + \left(\Lambda_2^2 + \Lambda_3^2 \right)^2 + (\{\} \leftrightarrow \{\}') , \\
d_2 &= 4 \left(y_1 M_1 + y_2 M_5 \right) \Lambda_2 \Lambda_3 + (\{\} \leftrightarrow \{\}') ,
\end{aligned}
$$

$$d_3 = 2\,y_1^2\Lambda_2^2 - y_2^2\Lambda_1^2 + (\{\} \leftrightarrow \{\}')\,,$$

$$d_4 = 4\,y_1 y_2 M_1 M_5 + 2\left(y_1^2 + y_2^2\right)\left(M_1^2 + M_5^2\right) + y_2^2\left(\Lambda_1^2 + 2\Lambda_3^2\right) + (\{\} \leftrightarrow \{\}')\,,$$

$$d_5 = y_1^4 + y_2^4 + (\{\} \leftrightarrow \{\}')\,.$$

In consequence, only the quadratically divergent piece is seen to remain $SO(5)$ invariant, while the rest of the potential introduces an explicit breaking of the $SO(5)$ symmetry to $SO(4)$, see Eq. (A.3). The quadratic divergence can be thus absorbed in the parameters of the tree-level Lagrangian, and the same holds for the $SO(5)$ invariant component of the logarithmically divergent terms (d_1, d_4 and d_5). However, the presence of the d_2 and d_3 divergent $SO(5)$-breaking terms require to add the two corresponding counterterms in the potential, so as to obtain a renormalizable theory. These two necessary terms are those defined with coefficients α and β in the potential definition Eq. (2.8). The gauge couplings appearing in the covariant derivatives also break explicitly the $SO(5)$ symmetry, but they do not induce extra one-loop divergent contributions to the effective potential, and in consequence only α and β are required for consistency.

The computation of the finite part of V_{loop} should provide the dependence of the parameters on the renormalization scale [1] and should thus be equivalent to the computation of their renormalization group equations; this task is beyond the scope of the present analysis.

---

[1]The gauge bosons induce finite contributions which are usually neglected with respect to the fermionic ones, as their relative ratio is proportional to the ratio between the mass scale for the heavy fermions and that for the gauge bosons.

Appendix B

The counterterms

Details about the computation of the counterterms and the renormalization of the chiral Lagrangian are given in this Appendix, including the derivation of the RGEs.

The bare parameters (denoted by b) written in terms of the renormalized ones and the counterterms for the $\mathcal{L}_2$ and $\mathcal{L}_4$ Lagrangians are given by

$$h_b = \sqrt{Z_h}\,h, \qquad \delta_h \equiv Z_h - 1,$$

$$\boldsymbol{\pi}_b = \sqrt{Z_\pi}\,\boldsymbol{\pi}, \qquad \delta_\pi \equiv Z_\pi - 1,$$

$$v_b^2 = Z_\pi(v^2 + \delta v^2)\mu^{-\varepsilon}, \qquad a_C^b = \frac{1}{Z_\pi^{1/2} Z_h^{1/2}}\left(a_C + \delta a_C + \frac{a_C}{2}\frac{\delta v^2}{v^2}\right),$$

$$(m_h^2)^b = \frac{1}{Z_h}(m_h^2 + \delta m_h^2), \qquad b_C^b = \frac{1}{Z_h}\left(b_C + \delta b_C + b_C\frac{\delta v^2}{v^2}\right),$$

$$(\mu_1^3)^b = \frac{1}{Z_h^{1/2}}\left(\mu_1^3 + \delta\mu_1^3\right)\mu^{3\varepsilon/2},$$

$$\mu_3^b = \frac{1}{Z_h^{3/2}}\left(\mu_3 + \delta\mu_3\right)\mu^{\varepsilon/2}, \qquad a_H^b = \frac{Z_\pi^{1/2}}{Z_h^{3/2}}\left(a_H + \delta a_H + \frac{a_H}{2}\frac{\delta v^2}{v^2}\right),$$

$$\lambda^b = \frac{1}{Z_h^2}\left(\lambda + \delta\lambda\right)\mu^\varepsilon, \qquad b_H^b = \frac{Z_\pi}{Z_h^2}\left(b_H + \delta b_H + b_H\frac{\delta v^2}{v^2}\right),$$

(B.1)

where

$$X_i^b = \left(X_i + \delta X_i + 2X_i \frac{\delta v^2}{v^2} \right) \mu^{-\varepsilon}, \qquad\qquad X_i = c_6, c_9, c_{11},$$

$$X_i^b = \frac{Z_\pi^{1/2}}{Z_h^{1/2}} \left(X_i + \delta X_i + \frac{3}{2}X_i \frac{\delta v^2}{v^2} \right) \mu^{-\varepsilon}, \qquad\qquad X_i = c_7, a_9, c_{10},$$

$$X_i^b = \frac{Z_\pi}{Z_h} \left(X_i + \delta X_i + 2X_i \frac{\delta v^2}{v^2} \right) \mu^{-\varepsilon}, \qquad X_i = a_7, c_8, b_9, a_{10}, c_{20},$$

$$X_i^b = \frac{Z_\pi}{Z_h} \left(X_i + \delta X_i + X_i \frac{\delta v^2}{v^2} \right) \mu^{-\varepsilon}, \qquad\qquad X_i = c_{\square H}, \tag{B.2}$$

$$X_i^b = \frac{Z_\pi^{3/2}}{Z_h^{3/2}} \left(X_i + \delta X_i + \frac{3}{2}X_i \frac{\delta v^2}{v^2} \right) \mu^{-\varepsilon}, \qquad\qquad X_i = a_{\square H}, c_{\Delta H},$$

$$X_i^b = \frac{Z_\pi^2}{Z_h^2} \left(X_i + \delta X_i + 2X_i \frac{\delta v^2}{v^2} \right) \mu^{-\varepsilon}, \qquad X_i = b_{\square H}, a_{\Delta H}, c_{DH}.$$

The counterterms required to absorb the divergencies of the hhh 3-point function are

$$\delta a_{\square H} = \frac{1}{2} \left(-\frac{3a_C b_C}{2} - \frac{a_H b_H}{2} + 3a_C^3 + a_H^3 \right) \Delta_\varepsilon,$$

$$\delta c_{\Delta H} = \frac{1}{2} \left(-3a_C b_C + 3a_C^3 - a_H^3 \right) \Delta_\varepsilon,$$

$$\delta a_H = \left[\frac{1}{2} \left(-9\frac{\mu_3}{v}a_H^2 + \lambda a_H + 2\frac{\mu_3}{v}b_H \right) + a_H \left(15a_H^2 - 7b_H \right) \frac{m_h^2}{v^2} \right] \Delta_\varepsilon, \tag{B.3}$$

$$\delta \mu_3 = \left[\frac{3}{2}\mu_3 \left(\lambda - 4\frac{\mu_3}{v}a_H \right) + 6 \left(6\mu_3 a_H^2 - \lambda v a_H - \mu_3 b_H \right) \frac{m_h^2}{v^2} \right.$$

$$\left. + 6a_H \left(3b_H - 8a_H^2 \right) \frac{m_h^4}{v^3} \right] \Delta_\varepsilon,$$

while those for $\pi\pi \to h$ read

$$\delta a_C = \frac{1}{2} \left(a_C^2 - b_C \right) \left[2(a_C + 2a_H)\frac{m_h^2}{v^2} - \frac{\mu_3}{v} \right] \Delta_\varepsilon,$$

$$\delta c_7 = \frac{1}{4} \left(-a_H b_C + a_C^2 a_H - a_C^3 - 2a_C \right) \Delta_\varepsilon, \tag{B.4}$$

$$\delta a_9 = -\frac{1}{8}a_C \left(a_C a_H + a_C^2 - b_C \right) \Delta_\varepsilon,$$

$$\delta c_{10} = \frac{1}{2}a_C \left(-a_C a_H + a_C^2 + b_C \right) \Delta_\varepsilon.$$

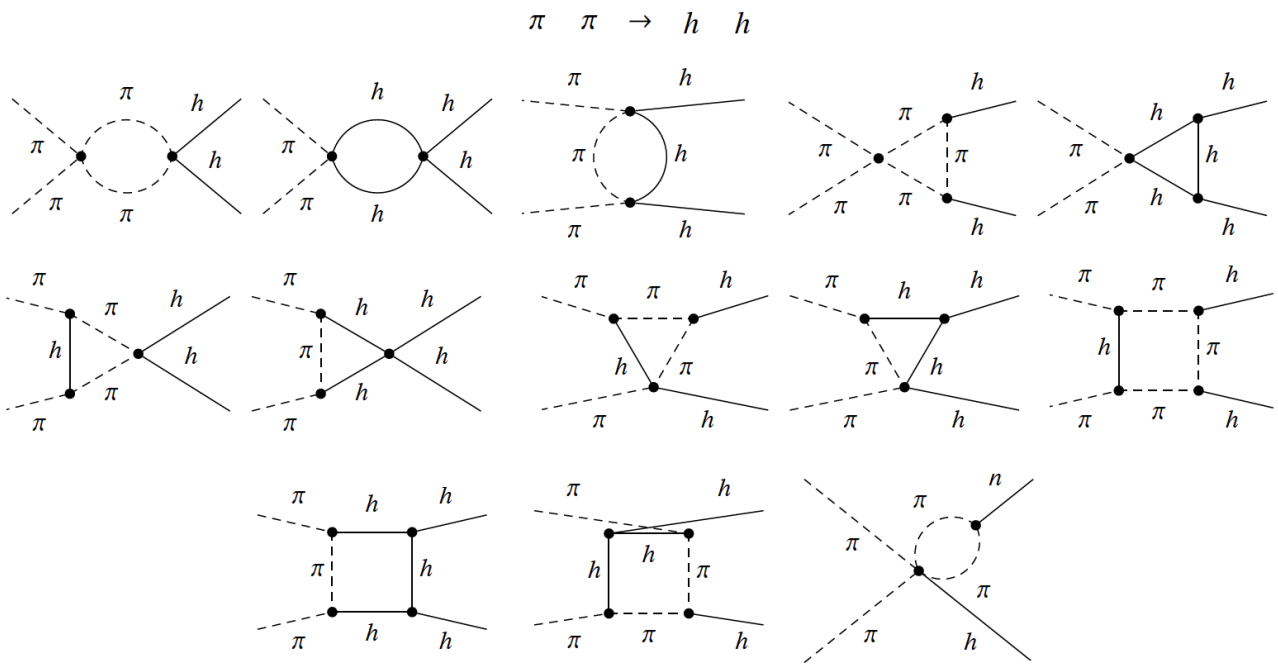In the case of the $\pi\pi \to hh$ amplitudes, the relevant diagrams are displayed in Fig. B.1, and the

112

Figure B.1: Diagrams contributing to the $\pi\pi \to hh$ amplitude, not including diagrams obtained by crossing.
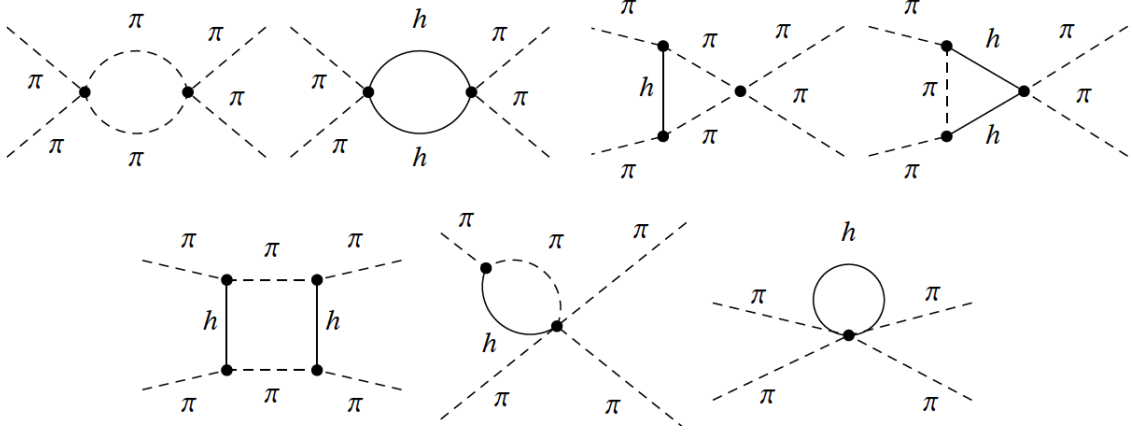
Figure B.2: Diagrams contributing to the $\pi\pi \rightarrow \pi\pi$ amplitude, not including diagrams obtained by crossing.

counterterms correspond to

$$\delta b_C = \frac{1}{2}\left(a_C^2 - b_C\right)\left[\left(4a_C + 8a_H\right)\frac{\mu_3}{v} - \lambda\right.$$
$$\left. - 2\left(8a_C a_H + 4a_C^2 + 12a_H^2 - b_C - 2b_H\right)\frac{m_h^2}{v^2}\right]\Delta_\varepsilon,$$
$$\delta a_7 = \frac{1}{8}\left[a_C^2\left(-4a_H^2 - 3b_C + b_H + 4\right) + 2a_C a_H b_C + b_C\left(4a_H^2 - b_H - 2\right) + 4a_C^4\right]\Delta_\varepsilon,$$
$$\delta c_8 = \frac{1}{3}\left[a_C^2\left(a_H^2 + b_C\right) - 2a_C a_H b_C - a_C^3 a_H + a_C^4 + b_C^2\right]\Delta_\varepsilon,$$
$$\delta b_9 = \frac{1}{4}\left[-a_C^2\left(-4a_H^2 + 5b_C + b_H\right) - 4a_C a_H b_C + 4a_C^3 a_H + 4a_C^4 + b_C^2\right]\Delta_\varepsilon,$$
$$\delta a_{10} = \frac{1}{4}\left[a_C^2\left(4a_H^2 + b_C - b_H\right) - 4a_C a_H b_C - 4a_C^4 + b_C^2\right]\Delta_\varepsilon,$$
$$\delta c_{20} = \frac{1}{12}\left[a_C^2\left(2a_H^2 - b_C + 6\right) + 2a_C a_H b_C\right.$$
$$\left. - b_C\left(3a_H^2 + b_C + 6\right) - 2a_C^3 a_H + 2a_C^4\right]\Delta_\varepsilon.$$

(B.5)

For $\pi\pi \rightarrow \pi\pi$ amplitudes, the relevant diagrams are displayed in Fig. B.2, and the required counterterms are given by

$$\delta c_6 = \frac{1}{48}\left[a_C^2\left(6b_C - 8\right) - 2a_C^4 - 3b_C^2 - 2\right]\Delta_\varepsilon,$$
$$\delta c_{11} = -\frac{1}{12}\left(a_C^2 - 1\right)^2\Delta_\varepsilon.$$
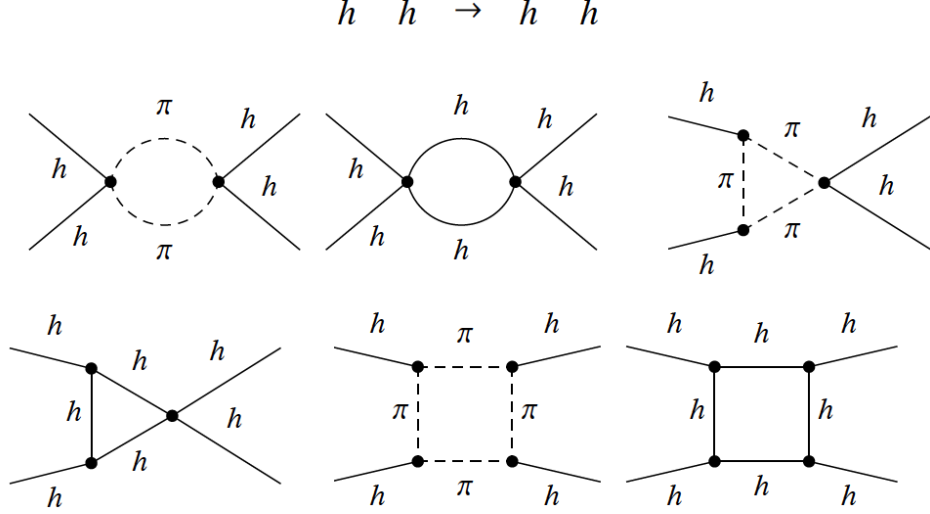
(B.6)

$$h \quad h \quad \rightarrow \quad h \quad h$$

Figure B.3: Diagrams contributing to the $hh \to hh$ amplitude, not including diagrams obtained by crossing.

Finally, the relevant diagrams for $hh \to hh$ amplitudes are shown in Fig. B.3, and the renormalization conditions read

$$\delta b_H = \left[\frac{1}{2} \left(\frac{\mu_3}{v} \left(-40 a_H b_H + 84 a_H^3 \right) - 13 \lambda a_H^2 + 3 \lambda b_H \right) \right.$$
$$\left. + \left(87 a_H^2 b_H - 120 a_H^4 - 7 b_H^2 \right) \frac{m_h^2}{v^2} \right] \Delta_\varepsilon,$$

$$\delta b_{\square H} = \frac{1}{4} \left[-3 \left(4 a_H^4 + b_C^2 \right) + 30 a_C^2 b_C + 10 a_H^2 b_H - 36 a_C^4 - b_H^2 \right] \Delta_\varepsilon,$$

$$\delta a_{\Delta H} = -\frac{3}{4} \left(-7 a_C^2 b_C + a_H^2 b_H + 6 a_C^4 - 2 a_H^4 + b_C^2 \right) \Delta_\varepsilon,$$

$$\delta c_{DH} = \left[-\frac{3}{4} \left(a_C^2 - b_C \right)^2 - \frac{a_H^4}{4} \right] \Delta_\varepsilon,$$

$$\delta \lambda = \left\{ \frac{3}{2 v^2} \left[8 \mu_3^2 \left(6 a_H^2 - b_H \right) - 16 \lambda \mu_3 v a_H + \lambda^2 v^2 \right] \right.$$
$$- 12 \left(-12 \mu_3 a_H b_H + 32 \mu_3 a_H^3 - 6 \lambda v a_H^2 + \lambda v b_H \right) \frac{m_h^2}{v^3}$$
$$\left. + 6 \left(-48 a_H^2 b_H + 80 a_H^4 + 3 b_H^2 \right) \frac{m_h^4}{v^4} \right\} \Delta_\varepsilon.$$

(B.7)

Appendix C

The Renormalization Group Equations

This Appendix provides the expressions for the RGE of all couplings discussed above, at the order considered.

$$16\pi^2 \frac{d}{d\ln\mu} a_C = a_C \left[\left(5a_H^2 - 3b_C - b_H\right) \frac{m_h^2}{v^2} - a_H \frac{\mu_3}{v} \right]$$

$$+ a_C^2 \left(4a_H \frac{m_h^2}{v^2} - \frac{\mu_3}{v}\right) + 3a_C^3 \frac{m_h^2}{v^2} + b_C \frac{\mu_3}{v} - 4a_H b_C \frac{m_h^2}{v^2} \ , \tag{C.1}$$

$$16\pi^2 \frac{d}{d\ln\mu} b_C = b_C \left[2 \left(5a_C^2 + 8a_C a_H + 17a_H^2 - 3b_H\right) \frac{m_h^2}{v^2} + \lambda - 2(2a_C + 5a_H)\frac{\mu_3}{v} \right]$$

$$- 2b_C^2 \frac{m_h^2}{v^2} + a_C^2 \left(-\lambda + 4a_C \frac{\mu_3}{v} + 8a_H \frac{\mu_3}{v}\right)$$

$$- 4a_C^2 \left(2a_C^2 + 4a_C a_H + 6a_H^2 - b_H\right) \frac{m_h^2}{v^2} \ , \tag{C.2}$$

$$16\pi^2 \frac{d}{d\ln\mu} a_H = a_H \left[\lambda - \left(a_C^2 - b_C + 17b_H\right) \frac{m_h^2}{v^2} \right] - 12a_H^2 \frac{\mu_3}{v} + 45a_H^3 \frac{m_h^2}{v^2} + 2b_H \frac{\mu_3}{v} \ , \tag{C.3}$$

$$16\pi^2 \frac{d}{d\ln\mu} b_H = b_H \left[2 \left(-a_C^2 + 97a_H^2 + b_C\right) \frac{m_h^2}{v^2} - 44a_H \frac{\mu_3}{v} + 3\lambda \right] - 18b_H^2 \frac{m_h^2}{v^2}$$

$$+ a_H^2 \left(-13\lambda + 84a_H \frac{\mu_3}{v}\right) - 240a_H^4 \frac{m_h^2}{v^2} \ , \tag{C.4}$$

$$16\pi^2 \frac{d}{d\ln\mu} m_h^2 = m_h^2 \left(\lambda - 10a_H \frac{\mu_3}{v}\right) + \left(22a_H^2 - 4b_H\right) \frac{m_h^4}{v^2} + \mu_3^2 \ , \tag{C.5}$$

$$16\pi^2 \frac{d}{d\ln\mu} \mu_3 = \mu_3 \left[\left(87a_H^2 - 15b_H\right) \frac{m_h^2}{v^2} + 3\lambda \right] - 15a_H \frac{\mu_3^2}{v}$$

$$- 12\lambda a_H \frac{m_h^2}{v} - \left(96a_H^3 - 36a_H b_H\right) \frac{m_h^4}{v^3} \ , \tag{C.6}$$

$$16\pi^2 \frac{d}{d\ln\mu} \lambda = \lambda \left[4 \left(41a_H^2 - 7b_H\right) \frac{m_h^2}{v^2} - 52a_H \frac{\mu_3}{v} \right] + 3\lambda^2 + 24 \left(6a_H^2 - b_H\right) \frac{\mu_3^2}{v^2}$$

$$-96a_H\left(8a_H^2-3b_H\right)\frac{\mu_3 m_h^2}{v^3}+12\left(80a_H^4-48a_H^2 b_H+3b_H^2\right)\frac{m_h^4}{v^4}\ , \tag{C.7}$$

$$16\pi^2\frac{d}{d\ln\mu}v^2=-2\left(a_C^2-b_C\right)m_h^2\ , \tag{C.8}$$

$$16\pi^2\frac{d}{d\ln\mu}c_6=-\frac{1}{24}\left[2+2a_C^4+3b_C^2-a_C^2\left(-8+6b_C\right)\right]\ , \tag{C.9}$$

$$16\pi^2\frac{d}{d\ln\mu}c_7=-c_7\left[\left(a_C^2-5a_H^2-b_C+b_H\right)\frac{m_h^2}{v^2}+a_H\frac{\mu_3}{v}\right]$$
$$+\frac{1}{2}\left(-2a_C-a_C^3+a_C^2 a_H-a_H b_C\right)\ , \tag{C.10}$$

$$16\pi^2\frac{d}{d\ln\mu}a_7=-a_7\left[2\left(a_C^2-5a_H^2-b_C+b_H\right)\frac{m_h^2}{v^2}+2a_H\frac{\mu_3}{v}\right]$$
$$+\frac{1}{4}\left[4a_C^4+2a_C a_H b_C+b_C\left(-2+4a_H^2-b_H\right)+a_C^2\left(4-4a_H^2-3b_C+b_H\right)\right]\ , \tag{C.11}$$

$$16\pi^2\frac{d}{d\ln\mu}c_8=-c_8\left[2\left(a_C^2-5a_H^2-b_C+b_H\right)\frac{m_h^2}{v^2}+2a_H\frac{\mu_3}{v}\right]$$
$$+\frac{2}{3}\left[a_C^4-a_C^3 a_H-2a_C a_H b_C+b_C^2+a_C^2\left(a_H^2+b_C\right)\right]\ , \tag{C.12}$$

$$16\pi^2\frac{d}{d\ln\mu}c_9=\frac{a_C^2}{2}\ , \tag{C.13}$$

$$16\pi^2\frac{d}{d\ln\mu}a_9=-a_9\left[\left(a_C^2-5a_H^2-b_C+b_H\right)\frac{m_h^2}{v^2}+a_H\frac{\mu_3}{v}\right]$$
$$-\frac{1}{2}a_C\left(a_C^2+a_C a_H-b_C\right)\ , \tag{C.14}$$

$$16\pi^2\frac{d}{d\ln\mu}b_9=-b_9\left[2\left(a_C^2-5a_H^2-b_C+b_H\right)\frac{m_h^2}{v^2}+2a_H\frac{\mu_3}{v}\right]$$
$$+\frac{1}{2}\left[4a_C^4+4a_C^3 a_H-4a_C a_H b_C+b_C^2+a_C^2\left(4a_H^2-5b_C-b_H\right)\right]\ , \tag{C.15}$$

$$16\pi^2\frac{d}{d\ln\mu}c_{10}=-c_{10}\left[\left(a_C^2-5a_H^2-b_C+b_H\right)\frac{m_h^2}{v^2}+a_H\frac{\mu_3}{v}\right]+a_C\left(a_C^2-a_C a_H+b_C\right)\ , \tag{C.16}$$

$$16\pi^2\frac{d}{d\ln\mu}a_{10}=-a_{10}\left[2\left(a_C^2-5a_H^2-b_C+b_H\right)\frac{m_h^2}{v^2}+2a_H\frac{\mu_3}{v}\right]$$
$$+\frac{1}{2}\left(-4a_C^4-4a_C a_H b_C+b_C^2+a_C^2\left(4a_H^2+b_C-b_H\right)\right)\ , \tag{C.17}$$

$$16\pi^2\frac{d}{d\ln\mu}c_{11}=-\frac{1}{6}\left(a_C^2-1\right)^2\ , \tag{C.18}$$

$$16\pi^2\frac{d}{d\ln\mu}c_{20}=-c_{20}\left[2\left(a_C^2-5a_H^2-b_C+b_H\right)\frac{m_h^2}{v^2}+2a_H\frac{\mu_3}{v}\right] \tag{C.19}$$
$$+\frac{1}{6}\left[2a_C^4-2a_C^3 a_H+a_C^2\left(6+2a_H^2-b_C\right)+2a_C a_H b_C-b_C\left(6+3a_H^2+b_C\right)\right]\ ,$$

$$16\pi^2 \frac{d}{d\ln\mu}c_{\square H} = -c_{\square H}\left[2\left(a_C^2 - 5a_H^2 - b_C + b_H\right)\frac{m_h^2}{v^2} + 2a_H\frac{\mu_3}{v}\right] + \frac{1}{2}\left(-3a_C^2 - a_H^2\right) \,, \qquad \text{(C.20)}$$

$$16\pi^2 \frac{d}{d\ln\mu}a_{\square H} = -a_{\square H}\left[3\left(a_C^2 - 5a_H^2 - b_C + b_H\right)\frac{m_h^2}{v^2} + 3a_H\frac{\mu_3}{v}\right]$$
$$+ 3a_C^3 + a_H^3 - \frac{3a_C b_C}{2} - \frac{a_H b_H}{2} \,, \qquad \text{(C.21)}$$

$$16\pi^2 \frac{d}{d\ln\mu}b_{\square H} = -b_{\square H}\left[4\left(a_C^2 - 5a_H^2 - b_C + b_H\right)\frac{m_h^2}{v^2} + 4a_H\frac{\mu_3}{v}\right]$$
$$- 18a_C^4 - 6a_H^4 + 15a_C^2 b_C - \frac{3b_C^2}{2} + 5a_H^2 b_H - \frac{b_H^2}{2} \,, \qquad \text{(C.22)}$$

$$16\pi^2 \frac{d}{d\ln\mu}c_{\Delta H} = -c_{\Delta H}\left[3\left(a_C^2 - 5a_H^2 - b_C + b_H\right)\frac{m_h^2}{v^2} + 3a_H\frac{\mu_3}{v}\right] + 3a_C^3 - a_H^3 - 3a_C b_C \,, \quad \text{(C.23)}$$

$$16\pi^2 \frac{d}{d\ln\mu}a_{\Delta H} = -a_{\Delta H}\left[4\left(a_C^2 - 5a_H^2 - b_C + b_H\right)\frac{m_h^2}{v^2} + 4a_H\frac{\mu_3}{v}\right]$$
$$- \frac{3}{2}\left(6a_C^4 - 2a_H^4 - 7a_C^2 b_C + b_C^2 + a_H^2 b_H\right) \,, \qquad \text{(C.24)}$$

$$16\pi^2 \frac{d}{d\ln\mu}c_{DH} = -c_{DH}\left[4\left(a_C^2 - 5a_H^2 - b_C + b_H\right)\frac{m_h^2}{v^2} + 4a_H\frac{\mu_3}{v}\right]$$
$$- \frac{1}{2}\left[a_H^4 + 3\left(a_C^2 - b_C\right)^2\right] \,. \qquad \text{(C.25)}$$

Bibliography

[1] ATLAS collaboration, G. Aad et al., *Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC*, *Phys. Lett.* **B716** (2012) 1–29, [`1207.7214`].

[2] CMS collaboration, S. Chatrchyan et al., *Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC*, *Phys. Lett.* **B716** (2012) 30–61, [`1207.7235`].

[3] D. B. Kaplan, H. Georgi and S. Dimopoulos, *Composite Higgs Scalars*, *Phys. Lett.* **B136** (1984) 187–190.

[4] D. B. Kaplan and H. Georgi, *SU(2) x U(1) Breaking by Vacuum Misalignment*, *Phys. Lett.* **B136** (1984) 183–186.

[5] H. Georgi and D. B. Kaplan, *Composite Higgs and Custodial SU(2)*, *Phys. Lett.* **B145** (1984) 216–220.

[6] H. Georgi, D. B. Kaplan and P. Galison, *Calculation of the Composite Higgs Mass*, *Phys. Lett.* **B143** (1984) 152–154.

[7] T. Banks, *Constraints on SU(2) x U(1) breaking by vacuum misalignment*, *Nucl. Phys.* **B243** (1984) 125–130.

[8] M. J. Dugan, H. Georgi and D. B. Kaplan, *Anatomy of a Composite Higgs Model*, *Nucl. Phys.* **B254** (1985) 299–326.

[9] K. Agashe, R. Contino and A. Pomarol, *The Minimal composite Higgs model*, *Nucl. Phys.* **B719** (2005) 165–187, [`hep-ph/0412089`].

[10] R. Contino, L. Da Rold and A. Pomarol, *Light custodians in natural composite Higgs models*, *Phys. Rev.* **D75** (2007) 055014, [`hep-ph/0612048`].

[11] B. Gripaios, A. Pomarol, F. Riva and J. Serra, *Beyond the Minimal Composite Higgs Model*, *JHEP* **04** (2009) 070, [`0902.1483`].

[12] H. Gertov, A. Meroni, E. Molinaro and F. Sannino, *Theory and phenomenology of the elementary Goldstone Higgs boson*, *Phys. Rev.* **D92** (2015) 095003, [`1507.06666`].

[13] M. Frigerio, A. Pomarol, F. Riva and A. Urbano, *Composite Scalar Dark Matter*, *JHEP* **07** (2012) 015, [`1204.2808`].

119

[14] D. Marzocca and A. Urbano, *Composite Dark Matter and LHC Interplay*, *JHEP* **07** (2014) 107, [1404.7419].

[15] G. Panico, M. Redi, A. Tesi and A. Wulzer, *On the Tuning and the Mass of the Composite Higgs*, *JHEP* **03** (2013) 051, [1210.7114].

[16] M. Carena, L. Da Rold and E. Pontón, *Minimal Composite Higgs Models at the LHC*, *JHEP* **06** (2014) 159, [1402.2987].

[17] R. Contino, D. Marzocca, D. Pappadopulo and R. Rattazzi, *On the effect of resonances in composite Higgs phenomenology*, *JHEP* **10** (2011) 081, [1109.1570].

[18] D. Marzocca, M. Serone and J. Shu, *General Composite Higgs Models*, *JHEP* **08** (2012) 013, [1205.0770].

[19] M. Redi and A. Tesi, *Implications of a Light Higgs in Composite Models*, *JHEP* **10** (2012) 166, [1205.0232].

[20] A. Carmona and F. Goertz, *A naturally light Higgs without light Top Partners*, *JHEP* **05** (2015) 002, [1410.8555].

[21] G. von Gersdorff, E. Pontón and R. Rosenfeld, *The Dynamical Composite Higgs*, *JHEP* **06** (2015) 119, [1502.07340].

[22] R. Barbieri, B. Bellazzini, V. S. Rychkov and A. Varagnolo, *The Higgs boson from an extended symmetry*, *Phys. Rev.* **D76** (2007) 115008, [0706.0432].

[23] T. Alanne, H. Gertov, A. Meroni and F. Sannino, *Vacuum alignment with and without elementary scalars*, *Phys. Rev.* **D94** (2016) 075015, [1608.07442].

[24] G. Buchalla, O. Cata, A. Celis and C. Krause, *Standard Model Extended by a Heavy Singlet: Linear vs. Nonlinear EFT*, 1608.03564.

[25] S. Fichet, G. von Gersdorff, E. Pontón and R. Rosenfeld, *The Global Higgs as a Signal for Compositeness at the LHC*, *JHEP* **01** (2017) 012, [1608.01995].

[26] S. Fichet, G. von Gersdorff, E. Pontón and R. Rosenfeld, *The Excitation of the Global Symmetry-Breaking Vacuum in Composite Higgs Models*, *JHEP* **09** (2016) 158, [1607.03125].

[27] K. Kannike, G. M. Pelaggi, A. Salvio and A. Strumia, *The Higgs of the Higgs and the diphoton channel*, *JHEP* **07** (2016) 101, [1605.08681].

[28] M. Gell-Mann and M. Levy, *The axial vector current in beta decay*, *Nuovo Cim.* **16** (1960) 705.

[29] D. B. Kaplan, *Flavor at SSC energies: A New mechanism for dynamically generated fermion masses*, *Nucl. Phys.* **B365** (1991) 259–278.

[30] M. B. Gavela, K. Kanshin, P. A. N. Machado and S. Saa, *On the renormalization of the electroweak chiral Lagrangian with a Higgs*, *JHEP* **03** (2015) 043, [`1409.1571`].

[31] F. Feruglio, B. Gavela, K. Kanshin, P. A. N. Machado, S. Rigolin and S. Saa, *The minimal linear sigma model for the Goldstone Higgs*, *JHEP* **06** (2016) 038, [`1603.05668`].

[32] M. B. Gavela, K. Kanshin, P. A. N. Machado and S. Saa, *The linear–non-linear frontier for the Goldstone Higgs*, *Eur. Phys. J.* **C76** (2016) 690, [`1610.08083`].

[33] S. L. Glashow, *Partial Symmetries of Weak Interactions*, *Nucl. Phys.* **22** (1961) 579–588.

[34] S. Weinberg, *A Model of Leptons*, *Phys. Rev. Lett.* **19** (1967) 1264–1266.

[35] A. Salam, *Weak and Electromagnetic Interactions*, *Conf. Proc.* **C680519** (1968) 367–377.

[36] P. W. Higgs, *Broken symmetries, massless particles and gauge fields*, *Phys. Lett.* **12** (1964) 132–133.

[37] P. W. Higgs, *Broken Symmetries and the Masses of Gauge Bosons*, *Phys. Rev. Lett.* **13** (1964) 508–509.

[38] F. Englert and R. Brout, *Broken Symmetry and the Mass of Gauge Vector Mesons*, *Phys. Rev. Lett.* **13** (1964) 321–323.

[39] G. S. Guralnik, C. R. Hagen and T. W. B. Kibble, *Global Conservation Laws and Massless Particles*, *Phys. Rev. Lett.* **13** (1964) 585–587.

[40] J. C. Romao and J. P. Silva, *A resource for signs and Feynman diagrams of the Standard Model*, *Int. J. Mod. Phys.* **A27** (2012) 1230025, [`1209.6213`].

[41] ATLAS, CMS collaboration, G. Aad et al., *Combined Measurement of the Higgs Boson Mass in pp Collisions at $\sqrt{s} = 7$ and 8 TeV with the ATLAS and CMS Experiments*, *Phys. Rev. Lett.* **114** (2015) 191803, [`1503.07589`].

[42] P. Sikivie, L. Susskind, M. B. Voloshin and V. I. Zakharov, *Isospin Breaking in Technicolor Models*, *Nucl. Phys.* **B173** (1980) 189–207.

[43] G. 't Hooft, *Renormalizable Lagrangians for Massive Yang-Mills Fields*, *Nucl. Phys.* **B35** (1971) 167–188.

[44] M. Schmaltz, *Physics beyond the standard model (theory): Introducing the little Higgs*, *Nucl. Phys. Proc. Suppl.* **117** (2003) 40–49, [`hep-ph/0210415`].

[45] L. Susskind, *Dynamics of Spontaneous Symmetry Breaking in the Weinberg-Salam Theory*, *Phys. Rev.* **D20** (1979) 2619–2625.

[46] G. 't Hooft and M. J. G. Veltman, *Regularization and Renormalization of Gauge Fields*, *Nucl. Phys.* **B44** (1972) 189–213.

[47] A. Salvio and A. Strumia, *Agravity*, *JHEP* **06** (2014) 080, [`1403.4226`].

[48] G. 't Hooft, *Naturalness, chiral symmetry, and spontaneous chiral symmetry breaking*, *NATO Sci. Ser. B* **59** (1980) 135–157.

[49] Y. Nambu, *Quasiparticles and Gauge Invariance in the Theory of Superconductivity*, *Phys. Rev.* **117** (1960) 648–663.

[50] J. Goldstone, *Field Theories with Superconductor Solutions*, *Nuovo Cim.* **19** (1961) 154–164.

[51] J. Goldstone, A. Salam and S. Weinberg, *Broken Symmetries*, *Phys. Rev.* **127** (1962) 965–970.

[52] H. Yukawa, *On the Interaction of Elementary Particles I*, *Proc. Phys. Math. Soc. Jap.* **17** (1935) 48–57.

[53] J. F. Donoghue, E. Golowich and B. R. Holstein, *Dynamics of the standard model*, *Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol.* **2** (1992) 1–540.

[54] Particle Data Group collaboration, C. Patrignani et al., *Review of Particle Physics*, *Chin. Phys.* **C40** (2016) 100001.

[55] S. Weinberg, *Nonlinear realizations of chiral symmetry*, *Phys. Rev.* **166** (1968) 1568–1577.

[56] J. Gasser and H. Leutwyler, *Chiral Perturbation Theory to One Loop*, *Annals Phys.* **158** (1984) 142.

[57] S. Weinberg, *Phenomenological Lagrangians*, *Physica* **A96** (1979) 327–340.

[58] A. Manohar and H. Georgi, *Chiral Quarks and the Nonrelativistic Quark Model*, *Nucl. Phys.* **B234** (1984) 189–212.

[59] H. Georgi, *Weak Interactions and Modern Particle Theory*. 1984.

[60] T. Das, G. S. Guralnik, V. S. Mathur, F. E. Low and J. E. Young, *Electromagnetic mass difference of pions*, *Phys. Rev. Lett.* **18** (1967) 759–761.

[61] S. Weinberg, *Implications of Dynamical Symmetry Breaking*, *Phys. Rev.* **D13** (1976) 974–996.

[62] S. Weinberg, *Implications of Dynamical Symmetry Breaking: An Addendum*, *Phys. Rev.* **D19** (1979) 1277–1280.

[63] E. Eichten and K. D. Lane, *Dynamical Breaking of Weak Interaction Symmetries*, *Phys. Lett.* **B90** (1980) 125–130.

[64] S. Dimopoulos and L. Susskind, *Mass Without Scalars*, *Nucl. Phys.* **B155** (1979) 237–252.

[65] M. E. Peskin and T. Takeuchi, *Estimation of oblique electroweak corrections*, *Phys. Rev.* **D46** (1992) 381–409.

[66] B. Holdom and J. Terning, *Large corrections to electroweak parameters in technicolor theories*, *Phys. Lett.* **B247** (1990) 88–92.

[67] M. Golden and L. Randall, *Radiative Corrections to Electroweak Parameters in Technicolor Theories*, *Nucl. Phys.* **B361** (1991) 3–23.

[68] R. Contino, *The Higgs as a Composite Nambu-Goldstone Boson*, in *Physics of the large and the small, TASI 09, proceedings of the Theoretical Advanced Study Institute in Elementary Particle Physics, Boulder, Colorado, USA, 1-26 June 2009*, pp. 235–306, 2011. `1005.4269`. DOI.

[69] G. Panico and A. Wulzer, *The Composite Nambu-Goldstone Higgs*, *Lect. Notes Phys.* **913** (2016) pp.1–316, [`1506.01961`].

[70] R. Alonso, I. Brivio, B. Gavela, L. Merlo and S. Rigolin, *Sigma Decomposition*, *JHEP* **12** (2014) 034, [`1409.1589`].

[71] S. R. Coleman and E. J. Weinberg, *Radiative Corrections as the Origin of Spontaneous Symmetry Breaking*, *Phys. Rev.* **D7** (1973) 1888–1910.

[72] S. R. Coleman, J. Wess and B. Zumino, *Structure of phenomenological Lagrangians. 1.*, *Phys. Rev.* **177** (1969) 2239–2247.

[73] C. G. Callan, Jr., S. R. Coleman, J. Wess and B. Zumino, *Structure of phenomenological Lagrangians. 2.*, *Phys. Rev.* **177** (1969) 2247–2250.

[74] N. Arkani-Hamed, A. G. Cohen and H. Georgi, *Electroweak symmetry breaking from dimensional deconstruction*, *Phys. Lett.* **B513** (2001) 232–240, [`hep-ph/0105239`].

[75] N. Arkani-Hamed, A. G. Cohen, T. Gregoire and J. G. Wacker, *Phenomenology of electroweak symmetry breaking from theory space*, *JHEP* **08** (2002) 020, [`hep-ph/0202089`].

[76] M. Schmaltz and D. Tucker-Smith, *Little Higgs review*, *Ann. Rev. Nucl. Part. Sci.* **55** (2005) 229–270, [`hep-ph/0502182`].

[77] G. Ferretti and D. Karateev, *Fermionic UV completions of Composite Higgs models*, *JHEP* **03** (2014) 077, [`1312.5330`].

[78] G. Ferretti, *UV Completions of Partial Compositeness: The Case for a SU(4) Gauge Group*, *JHEP* **06** (2014) 142, [`1404.7137`].

[79] G. Cacciapaglia and F. Sannino, *Fundamental Composite (Goldstone) Higgs Dynamics*, *JHEP* **04** (2014) 111, [`1402.0233`].

[80] T. Appelquist and J. Carazzone, *Infrared Singularities and Massive Fields*, *Phys. Rev.* **D11** (1975) 2856.

[81] B. Henning, X. Lu and H. Murayama, *How to use the Standard Model effective field theory*, *JHEP* **01** (2016) 023, [`1412.1837`].

[82] LHC Higgs Cross Section Working Group collaboration, D. de Florian et al., *Handbook of LHC Higgs Cross Sections: 4. Deciphering the Nature of the Higgs Sector*, `1610.07922`.

[83] S. Weinberg, *Baryon and Lepton Nonconserving Processes, Phys. Rev. Lett.* **43** (1979) 1566–1570.

[84] G. Passarino, *Field reparametrization in effective field theories, Eur. Phys. J. Plus* **132** (2017) 16, [`1610.09618`].

[85] W. Buchmuller and D. Wyler, *Effective Lagrangian Analysis of New Interactions and Flavor Conservation, Nucl. Phys.* **B268** (1986) 621–653.

[86] K. Hagiwara, S. Ishihara, R. Szalapski and D. Zeppenfeld, *Low-energy effects of new interactions in the electroweak boson sector, Phys. Rev.* **D48** (1993) 2182–2203.

[87] G. F. Giudice, C. Grojean, A. Pomarol and R. Rattazzi, *The Strongly-Interacting Light Higgs, JHEP* **06** (2007) 045, [`hep-ph/0703164`].

[88] B. Grzadkowski, M. Iskrzynski, M. Misiak and J. Rosiek, *Dimension-Six Terms in the Standard Model Lagrangian, JHEP* **10** (2010) 085, [`1008.4884`].

[89] T. Appelquist and C. W. Bernard, *Strongly Interacting Higgs Bosons, Phys. Rev.* **D22** (1980) 200.

[90] A. C. Longhitano, *Heavy Higgs Bosons in the Weinberg-Salam Model, Phys. Rev.* **D22** (1980) 1166.

[91] A. C. Longhitano, *Low-Energy Impact of a Heavy Higgs Boson Sector, Nucl. Phys.* **B188** (1981) 118–154.

[92] F. Feruglio, *The Chiral approach to the electroweak interactions, Int. J. Mod. Phys.* **A8** (1993) 4937–4972, [`hep-ph/9301281`].

[93] R. Alonso, M. B. Gavela, L. Merlo, S. Rigolin and J. Yepes, *The Effective Chiral Lagrangian for a Light Dynamical "Higgs Particle", Phys. Lett.* **B722** (2013) 330–335, [`1212.3305`].

[94] B. Grinstein and M. Trott, *A Higgs-Higgs bound state due to new physics at a TeV, Phys. Rev.* **D76** (2007) 073002, [`0704.1505`].

[95] R. Contino, C. Grojean, M. Moretti, F. Piccinini and R. Rattazzi, *Strong Double Higgs Production at the LHC, JHEP* **05** (2010) 089, [`1002.1011`].

[96] A. Azatov, R. Contino and J. Galloway, *Model-Independent Bounds on a Light Higgs, JHEP* **04** (2012) 127, [`1202.3415`].

[97] G. Buchalla, O. Catà and C. Krause, *Complete Electroweak Chiral Lagrangian with a Light Higgs at NLO, Nucl. Phys.* **B880** (2014) 552–573, [`1307.5017`].

[98] M. B. Gavela, J. Gonzalez-Fraile, M. C. Gonzalez-Garcia, L. Merlo, S. Rigolin and J. Yepes, *CP violation with a dynamical Higgs*, *JHEP* **10** (2014) 044, [`1406.6367`].

[99] L. Merlo, S. Saa and M. S. Barbero, *Baryon Non-Invariant Couplings in Higgs Effective Field Theory*, `1612.04832`.

[100] A. G. Cohen, D. B. Kaplan and A. E. Nelson, *Counting 4 pis in strongly coupled supersymmetry*, *Phys. Lett.* **B412** (1997) 301–308, [`hep-ph/9706275`].

[101] M. A. Luty, *Naive dimensional analysis and supersymmetry*, *Phys. Rev.* **D57** (1998) 1531–1538, [`hep-ph/9706235`].

[102] B. M. Gavela, E. E. Jenkins, A. V. Manohar and L. Merlo, *Analysis of General Power Counting Rules in Effective Field Theory*, *Eur. Phys. J.* **C76** (2016) 485, [`1601.07551`].

[103] I. Brivio, T. Corbett, O. J. P. Éboli, M. B. Gavela, J. Gonzalez-Fraile, M. C. Gonzalez-Garcia et al., *Disentangling a dynamical Higgs*, *JHEP* **03** (2014) 024, [`1311.1823`].

[104] R. Alonso, M. B. Gavela, L. Merlo, S. Rigolin and J. Yepes, *Minimal Flavour Violation with Strong Higgs Dynamics*, *JHEP* **06** (2012) 076, [`1201.1511`].

[105] G. Buchalla, O. Cata and C. Krause, *A Systematic Approach to the SILH Lagrangian*, *Nucl. Phys.* **B894** (2015) 602–620, [`1412.6356`].

[106] I. M. Hierro, L. Merlo and S. Rigolin, *Sigma Decomposition: The CP-Odd Lagrangian*, *JHEP* **04** (2016) 016, [`1510.07899`].

[107] I. Brivio, J. Gonzalez-Fraile, M. C. Gonzalez-Garcia and L. Merlo, *The complete HEFT Lagrangian after the LHC Run I*, *Eur. Phys. J.* **C76** (2016) 416, [`1604.06801`].

[108] O. J. P. Éboli and M. C. Gonzalez–Garcia, *Classifying the bosonic quartic couplings*, *Phys. Rev.* **D93** (2016) 093013, [`1604.03555`].

[109] L. J. Dixon and Y. Li, *Bounding the Higgs Boson Width Through Interferometry*, *Phys. Rev. Lett.* **111** (2013) 111802, [`1305.3854`].

[110] Particle Data Group collaboration, K. A. Olive et al., *Review of Particle Physics*, *Chin. Phys.* **C38** (2014) 090001.

[111] J. F. Gunion, H. E. Haber, G. L. Kane and S. Dawson, *The Higgs Hunter's Guide*, *Front. Phys.* **80** (2000) 1–404.

[112] K. Agashe, R. Contino, L. Da Rold and A. Pomarol, *A Custodial symmetry for $Z b \bar{b}$*, *Phys. Lett.* **B641** (2006) 62–66, [`hep-ph/0605341`].

[113] ATLAS, CMS collaboration, *Measurements of the Higgs boson production and decay rates and constraints on its couplings from a combined ATLAS and CMS analysis of the LHC pp collision data at $\sqrt{s} = 7$ and 8 TeV,* .

[114] C. Anastasiou, E. Furlan and J. Santiago, *Realistic Composite Higgs Models*, *Phys. Rev.* **D79** (2009) 075003, [0901.2117].

[115] D. Ghosh, M. Salvarezza and F. Senia, *Extending the Analysis of Electroweak Precision Constraints in Composite Higgs Models*, *Nucl. Phys.* **B914** (2017) 346–387, [1511.08235].

[116] G. Altarelli and R. Barbieri, *Vacuum polarization effects of new physics on electroweak processes*, *Phys. Lett.* **B253** (1991) 161–167.

[117] M. Ciuchini, E. Franco, S. Mishima, M. Pierini, L. Reina and L. Silvestrini, *Update of the electroweak precision fit, interplay with Higgs-boson signal strengths and model-independent constraints on new physics*, *Nucl. Part. Phys. Proc.* **273-275** (2016) 2219–2225, [1410.6940].

[118] V. A. Novikov, L. B. Okun and M. I. Vysotsky, *On the Electroweak one loop corrections*, *Nucl. Phys.* **B397** (1993) 35–83.

[119] A. Orgogozo and S. Rychkov, *The S parameter for a Light Composite Higgs: a Dispersion Relation Approach*, *JHEP* **06** (2013) 014, [1211.5543].

[120] H. E. Haber and D. O'Neil, *Basis-independent methods for the two-Higgs-doublet model III: The CP-conserving limit, custodial symmetry, and the oblique parameters S, T, U*, *Phys. Rev.* **D83** (2011) 055017, [1011.6188].

[121] L. Lavoura and J. P. Silva, *The Oblique corrections from vector - like singlet and doublet quarks*, *Phys. Rev.* **D47** (1993) 2046–2057.

[122] ATLAS collaboration, G. Aad et al., *Search for production of vector-like quark pairs and of four top quarks in the lepton-plus-jets final state in pp collisions at $\sqrt{s} = 8$ TeV with the ATLAS detector*, *JHEP* **08** (2015) 105, [1505.04306].

[123] CMS collaboration, C. Collaboration, *Search for top quark partners with charge 5/3 at $\sqrt{s} = 13$ TeV*, .

[124] S. Dawson and E. Furlan, *A Higgs Conundrum with Vector Fermions*, *Phys. Rev.* **D86** (2012) 015021, [1205.4733].

[125] LHC Higgs Cross Section Working Group collaboration, J. R. Andersen et al., *Handbook of LHC Higgs Cross Sections: 3. Higgs Properties*, 1307.1347.

[126] ATLAS collaboration, G. Aad et al., *Search for Scalar Diphoton Resonances in the Mass Range $65 - 600$ GeV with the ATLAS Detector in pp Collision Data at $\sqrt{s} = 8\,TeV$*, *Phys. Rev. Lett.* **113** (2014) 171801, [1407.6583].

[127] CMS collaboration, V. Khachatryan et al., *Search for diphoton resonances in the mass range from 150 to 850 GeV in pp collisions at $\sqrt{s} = 8$ TeV*, *Phys. Lett.* **B750** (2015) 494–519, [1506.02301].

126

[128] ATLAS collaboration, G. Aad et al., *Search for a high-mass Higgs boson decaying to a W boson pair in pp collisions at $\sqrt{s} = 8$ TeV with the ATLAS detector*, JHEP **01** (2016) 032, [1509.00389].

[129] ATLAS collaboration, G. Aad et al., *Search for an additional, heavy Higgs boson in the $H \to ZZ$ decay channel at $\sqrt{s} = 8$ TeV in pp collision data with the ATLAS detector*, Eur. Phys. J. **C76** (2016) 45, [1507.05930].

[130] CMS collaboration, *Properties of the Higgs-like boson in the decay H to ZZ to 4l in pp collisions at sqrt s =7 and 8 TeV*, .

[131] CMS collaboration, *Update on the search for the standard model Higgs boson in pp collisions at the LHC decaying to W + W in the fully leptonic final state*, .

[132] CMS collaboration, V. Khachatryan et al., *Search for resonant pair production of Higgs bosons decaying to two bottom quark–antiquark pairs in proton–proton collisions at 8 TeV*, Phys. Lett. **B749** (2015) 560–582, [1503.04114].

[133] ATLAS collaboration, G. Aad et al., *Searches for Higgs boson pair production in the $hh \to bb\tau\tau, \gamma\gamma WW^*, \gamma\gamma bb, bbbb$ channels with the ATLAS detector*, Phys. Rev. **D92** (2015) 092004, [1509.04670].

[134] ATLAS, CMS collaboration, A. Holzner, *Beyond standard model Higgs physics: prospects for the High Luminosity LHC*, 1411.0322.

[135] V. Martín Lozano, J. M. Moreno and C. B. Park, *Resonant Higgs boson pair production in the $hh \to b\bar{b}\,WW \to b\bar{b}\ell^+\nu\ell^-\bar{\nu}$ decay channel*, JHEP **08** (2015) 004, [1501.03799].

[136] ATLAS collaboration, *Search for resonances decaying to photon pairs in 3.2 fb^{-1} of pp collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector*, .

[137] CMS collaboration, C. Collaboration, *Search for new physics in high mass diphoton events in proton-proton collisions at 13TeV*, .

[138] ATLAS collaboration, G. Aad et al., *Evidence for Electroweak Production of $W^\pm W^\pm jj$ in pp Collisions at $\sqrt{s} = 8$ TeV with the ATLAS Detector*, Phys. Rev. Lett. **113** (2014) 141803, [1405.6241].

[139] ATLAS collaboration, G. Aad et al., *Measurements of $W^\pm Z$ production cross sections in pp collisions at $\sqrt{s} = 8$ TeV with the ATLAS detector and limits on anomalous gauge boson self-couplings*, Phys. Rev. **D93** (2016) 092004, [1603.02151].

[140] R. S. Chivukula and H. Georgi, *Composite Technicolor Standard Model*, Phys. Lett. **B188** (1987) 99–104.

[141] G. D'Ambrosio, G. F. Giudice, G. Isidori and A. Strumia, *Minimal flavor violation: An Effective field theory approach*, Nucl. Phys. **B645** (2002) 155–187, [hep-ph/0207036].

[142] B. Grinstein, M. Redi and G. Villadoro, *Low Scale Flavor Gauge Symmetries*, JHEP **11** (2010) 067, [1009.2049].

[143] R. L. Delgado, A. Dobado and F. J. Llanes-Estrada, *One-loop $W_L W_L$ and $Z_L Z_L$ scattering from the electroweak Chiral Lagrangian with a light Higgs-like scalar*, *JHEP* **02** (2014) 121, [1311.5993].

[144] D. Espriu, F. Mescia and B. Yencho, *Radiative corrections to WL WL scattering in composite Higgs models*, *Phys. Rev.* **D88** (2013) 055002, [1307.2400].

[145] R. L. Delgado, A. Dobado, M. J. Herrero and J. J. Sanz-Cillero, *One-loop $\gamma\gamma \to W_L^+ W_L^-$ and $\gamma\gamma \to Z_L Z_L$ from the Electroweak Chiral Lagrangian with a light Higgs-like scalar*, *JHEP* **07** (2014) 149, [1404.2866].

[146] F.-K. Guo, P. Ruiz-Femenía and J. J. Sanz-Cillero, *One loop renormalization of the electroweak chiral Lagrangian with a light Higgs boson*, *Phys. Rev.* **D92** (2015) 074005, [1506.04204].

[147] E. E. Jenkins, A. V. Manohar and M. Trott, *Naive Dimensional Analysis Counting of Gauge Theory Amplitudes and Anomalous Dimensions*, *Phys. Lett.* **B726** (2013) 697–702, [1309.0819].

[148] T. Appelquist and C. W. Bernard, *The Nonlinear σ Model in the Loop Expansion*, *Phys. Rev.* **D23** (1981) 425.

[149] I. S. Gerstein, R. Jackiw, S. Weinberg and B. W. Lee, *Chiral loops*, *Phys. Rev.* **D3** (1971) 2486–2492.

[150] J. M. Charap, *Closed-loop calculations using a chiral-invariant lagrangian*, *Phys. Rev.* **D2** (1970) 1554–1561.

[151] D. I. Kazakov, V. N. Pervushin and S. V. Pushkin, *Invariant Renormalization for the Field Theories with Nonlinear Symmetry*, *Teor. Mat. Fiz.* **31** (1977) 169–176.

[152] D. I. Kazakov, V. N. Pervushin and S. V. Pushkin, *An Invariant Renormalization Method for Nonlinear Realizations of the Dynamical Symmetries*, *Theor. Math. Phys.* **31** (1977) 389.

[153] B. de Wit and M. T. Grisaru, *On-shell Counterterms and Nonlinear Invariances*, *Phys. Rev.* **D20** (1979) 2082.

[154] J. Honerkamp, *Chiral multiloops*, *Nucl. Phys.* **B36** (1972) 130–140.

[155] R. Alonso, E. E. Jenkins and A. V. Manohar, *A Geometric Formulation of Higgs Effective Field Theory: Measuring the Curvature of Scalar Field Space*, *Phys. Lett.* **B754** (2016) 335–342, [1511.00724].

[156] R. Alonso, E. E. Jenkins and A. V. Manohar, *Geometry of the Scalar Sector*, *JHEP* **08** (2016) 101, [1605.03602].

[157] R. Mertig, M. Bohm and A. Denner, *FEYN CALC: Computer algebraic calculation of Feynman amplitudes*, *Comput. Phys. Commun.* **64** (1991) 345–359.

[158] A. Alloul, N. D. Christensen, C. Degrande, C. Duhr and B. Fuks, *FeynRules 2.0 - A complete toolbox for tree-level phenomenology, Comput. Phys. Commun.* **185** (2014) 2250–2300, [`1310.1921`].

[159] J. Kublbeck, M. Bohm and A. Denner, *Feyn Arts: Computer Algebraic Generation of Feynman Graphs and Amplitudes, Comput. Phys. Commun.* **60** (1990) 165–180.

[160] T. Hahn, *Generating Feynman diagrams and amplitudes with FeynArts 3, Comput. Phys. Commun.* **140** (2001) 418–431, [`hep-ph/0012260`].

[161] T. Hahn and M. Perez-Victoria, *Automatized one loop calculations in four-dimensions and D-dimensions, Comput. Phys. Commun.* **118** (1999) 153–165, [`hep-ph/9807565`].

[162] M. Ostrogradsky, *Mémoire sur les équations différentielles relatives an probléme des isopérimétres.* 1850.

[163] C. Grosse-Knetter, *Effective Lagrangians with higher derivatives and equations of motion, Phys. Rev.* **D49** (1994) 6709–6719, [`hep-ph/9306321`].

[164] S. Scherer and H. W. Fearing, *Field transformations and the classical equation of motion in chiral perturbation theory, Phys. Rev.* **D52** (1995) 6445–6450, [`hep-ph/9408298`].

[165] C. Arzt, *Reduced effective Lagrangians, Phys. Lett.* **B342** (1995) 189–195, [`hep-ph/9304230`].