

3.

Modelling a dynamic market potential as function of knowledge in innovation diffusion

3.1 Introduction

In a recent agenda for marketing science, Hauser, Tellis and Griffin (2006) have stated that the success of innovations ultimately depends on consumers accepting them. Mahajan, Muller and Wind (2000) have noticed that this success may depend on several factors, so that the rate of failure of innovations has been documented to vary in the range of 40 to 90 percent. Foster and Potts (2006) have reminded that history is full of examples of innovations that did not have success because of lacking connections with consumers: indeed, resistance of consumers to innovation is a well known concern, as highlighted by Moldovan and Goldenberg (2003). Since there is no guaranty that a new product or a new technology will succeed, the markets for innovations may appear quite unstable and uncertain at very early stages of their diffusion: the so-called phase of incubation, preceding the “take-off”, when a dramatic increase in sales occurs (Golder and Tellis, 1997), has been proven to have a chilling effect on sales (Kohli, Lehman and Pae, 1999; Goldenberg, Libai, Muller, 2005).

Rogers (2003) has suggested that adoption of an innovation may be explained by five attributes related to it, namely relative advantage, compatibility, complexity, trialability, observability. Indeed, the decision to adopt may require considerable time and cognitive effort: however, most of the times, people do not have sufficient knowledge to evaluate the advantage implied by a new product or technology compared to investment’s risks

and it is hard to acquire such a knowledge in the short term. In addition, the complexity of decision may be due to the fact that full benefits of adoption typically are delayed with respect to the time of purchase, when negative outcomes are more likely to occur.

A noteworthy situation is represented by interactive innovations, like telephones, fax machines, electronic mail, that present strong network externalities. In this case, the benefit of the innovation for a single user depends on the number of others who have already adopted it, thus on the existence of a physical network of interacting devices. A specific treatment of network externalities, that exert a considerable chilling effect on new product growth, depressing sales even for several years, will be provided in Chapter 4.

In this chapter it is argued that most innovations experience some difficulty in the first part of their life cycle, because a limited knowledge about their features, or even about their existence, prevents consumers to adopt them. In this sense, institutional communication efforts, like promotional activities, advertising, informative campaigns and experts' support play a crucial role in stimulating people awareness and in reducing the complexity associated to adoption decision. In addition, word-of-mouth communication has proven to be an increasingly powerful mean for the spread of information about an innovation, given the ease with which people can communicate, for example through the Internet, exchanging opinions, experiences and advices on new products or services. The rise of forms of organization like virtual communities of consumers well represent this natural tendency of individuals to interact with others and share both positive and negative information about a product. Recognizing the relevance of word-of-mouth justifies the increasing attempt from firms to manage it: in what is sometimes called *viral-marketing*, companies are currently investing much

effort to take advantage of the relationships between consumers, since a recommendation from a friend or other trusted source has the credibility that advertisements sometimes lack (Jurvetson, 2000).

Although communication, both institutional actions and word-of-mouth effects, has a conclusive part in innovation diffusion, which actually has been defined a *theory of communication* (Mahajan, Muller and Bass, 1990), the standard approach to diffusion modelling, essentially represented by the Bass model (equation (1)), does not provide an explicit formalization of it, synthesizing its effect within the adoption process parameters,

$$z(t) = m \frac{1 - e^{-(p+q)t}}{1 + \frac{q}{p} e^{-(p+q)t}} . \quad (1)$$

Equation (1) represents the closed-form solution of the Bass model: cumulative adoptions, $z(t)$, depend on adoption dynamics $\frac{1 - e^{-(p+q)t}}{1 + \frac{q}{p} e^{-(p+q)t}}$, defined in terms of innovative and imitative behaviour (parameters p and q) and multiplied by the market potential, m , that gives the absolute scale to the process. The diffusion process described by the Bass model has therefore a binary nature: potential adopters either adopt or do not adopt and consequently stages of knowledge or awareness are not taken into account. In the past, attempts to develop multistage models were made by Sharif and Ramanathan (1981), Mahajan, Muller and Kerin (1984), Kalish (1985), providing structures of difficult implementation.

This chapter presents an innovation diffusion model in which adoption and communication dynamics are considered simultaneously, but modelled in a separate

way. A separate modelling of communication dynamics allows to isolate them and measure to what extent these have an impact on diffusion: this might represent an efficient method for evaluating the effectiveness of informative actions and marketing strategies focused on communication. In particular, this model assumes that communication exerts its effect in generating the market potential for an innovation, since a person becomes a potential adopter after being informed on it. This point is particularly crucial, because the market potential (or carrying capacity) is no longer constant, as in the Bass model, but has a variable structure depending on the communication process underlying adoptions. Indeed, one of the characterising assumptions of the Bass model relates to the size of the market potential, m , whose value is determined at the time of introducing the new product and remains constant along the whole diffusion process. However, Mahajan, Muller and Bass (1990) have noticed that theoretically there is no rationale for a static adopter population and on the contrary a dynamic adopters population seems a reasonable choice.

The issue of a dynamic potential has been treated in literature since the 1970s: some works introduce a variable structure, modifying the residual market, $(m(t) - z(t))$, see for instance Mahajan and Peterson (1978), Horsky (1990), Kamakura and Balasubramanian (1988) and Mesak and Darat (2002). Other contributions also consider a modification of the word-of-mouth ratio, $\frac{z(t)}{m(t)}$, for example Sharif and Ramanathan (1981), Jain and Rao (1990), Parker (1992), Parker (1993), Goldenberg, Libai and Muller (2005), Centrone, Goia and Salinelli (2007). However, none of these works consider the variable structure of the market potential as dependent of a related communication process.

The rest of the chapter is structured as follows. Section 3.2 introduces a rationale for modelling a dynamic potential based on the concept of absorptive capacity: the communication process underlying adoptions is considered as an evolutionary network of interacting agents and modelled accordingly in section 3.3 with a class of Cellular Automata models, Network Automata models. In section 3.4 the aggregate structure of this network generating the market potential is then incorporated into the diffusion model, whose final structure jointly considers communication and adoption dynamics as separate but co-evolving processes. Section 3.5 deals with aspects of statistical implementation and empirical applications. Section 3.6 presents two case studies to test the performance of the proposed model. Section 3.7 is left for concluding remarks.

3.2 Absorptive capacity: a rationale for a dynamic market potential

The representation of a dynamic market potential generated by a communication process may ground on a theoretical contribution offered by Cohen and Levinthal (1990) with the concept of *absorptive capacity*. Though the authors' intended focus was the firm, this concept seems reasonably applicable also in a consumption perspective. Both at individual and organisational levels, the term absorptive capacity indicates the "ability to recognise the value of new information, assimilate it and apply it" (Cohen and Levinthal, 1990). It is argued that this ability is positively associated to the accumulation of *prior related knowledge*, meaning that the *receptiveness* to innovations depends on a background of relevant knowledge about it.

Applying this concept both to individuals and to organisations, Cohen and Levinthal (1990) maintain that individual absorptive capacity is related to cognitive functions of the single person, while to understand an organization's absorptive capacity it is

necessary to focus on its communication structure, since this ability for organizations is not the simple sum of those of its components, but has to do with *information transfers*.

The adoption of an innovation in a social context may be considered as a direct evidence of an existing absorptive capacity, as the ability to assimilate and accept a novelty may be simply inferred from the observed adoption process. More in detail, the market potential or carrying capacity m , which is the number of adopters that will buy an innovation during its life cycle, may be considered a direct measure of the absorptive capacity of a system. Since the ability to accept an innovation depends on the accumulation of a prior knowledge, this provides an interesting rationale for defining the market potential accordingly. A process of accumulation of knowledge in a social system requires the transfer of information among its components. Thus, Cohen and Levinthal (1990) point out the importance of designing the communication structure of an organization to understand its absorptive capacity. Accumulating knowledge involves some learning dynamics, whose description may be made with an evolutionary perspective, rather than with a cross-sectional model as proposed in Cohen and Levinthal (1990). In fact, an evolutionary model seems the most suitable choice to represent the communication structure as a set of informational linkages between the units of the system. As individual knowledge is created connecting ideas and concepts between them but also destroying some existing connections (see Potts, 2001), the development of a collective knowledge may be considered as a set of connections between persons, an *evolving network*, in which some linkages already exist, some rise and some others die. Considering the market potential m as a function of this knowledge process, implies to make it *dependent* on a network of connections that evolves over time.

3.3 Collective knowledge as an evolutionary network

A growing body of research in economics and social sciences is currently applying the concept of network to various systems of connections. As Foster and Potts (2006) and Foster (2005) have pointed out, it seems that the most natural representation of systems of connections at all levels is the network. Recent trends of research aim to study the formation of networks with respect to dynamics of learning, adaptation and innovation using complex systems models and network models. Although the use of such models in this respect is relatively recent, the number of issues addressed in literature is expanding rapidly and it is almost impossible to make justice to all the research efforts made. However, it is possible to recognise some streams of research providing new insights in economics using this approach.

The so-called Complex Networks Analysis is an emerging scientific branch finding a wide range of applications, from physical sciences to social sciences. The most important result achieved by this field of research is the existence of scale-free networks (see Barabasi, 2004). Scale-free networks are essentially threshold-free networks, that pervade technology: Internet, power-grids and transportation are just few examples (Devezas, 2005).

Agent-based Computational Economics, ACE, uses the formal device of small-world Networks, proposed by Watts and Strogatz (1998) for analysing trade networks or representing specific markets, such as labour markets (see Tesfatsion, 2001).

In marketing research, Shaikh, Rangaswamy and Balakrishnan (2005) have tried to model the diffusion of innovations with small-world networks, while Goldenberg, Libai and Muller (2001b) have proposed to use Cellular Automata Models for describing the effect of strong and weak ties on innovation diffusion, suggesting that word-of-mouth

phenomenon may be considered a complex adaptive system. Hauser, Tellis and Griffin (2006) have identified social networks as one of the most relevant opportunities of research on innovation and more recently, Muller, Peres and Mahajan (2007) have stated that diffusion of innovation is strongly related to network formation, so that it can be argued that there is no diffusion without network effects and the reason why literature on networks and diffusion is sparse in this respect is mainly because it is hard to map networks and have diffusion over it at the same time.

Similarly to Goldenberg, Libai and Muller (2001b) the model presented in this chapter aims to analyse communication dynamics underlying the diffusion process, using Cellular Automata Models as an adequate tool of analysis. However, this model *will not* focus on the *strenght* of relationships between individuals to connote the network: in this sense it will adopt a different approach from that inspired by the theory of strong and weak ties proposed by Granovetter (1973), where network dynamics depend on the nature of interpersonal relationships.

Not focusing on the strenght of relationships, the proposed model is characterised by a high level of generality, since it does not require any specific information about the spatial connectivity of agents. A network of communication is designed using a special kind of Cellular Automata models, namely Network Automata models: the rise and evolution of connections between the components of a system are analysed, just focusing on the source of information, institutional communication or word-of-mouth, that allows the formation of such connections. Such a description of how a collective knowledge is created, just focuses on information channels and prescinds from other specifications like strong or weak ties. The structure of the network, in aggregate terms,

is finally incorporated into the definition of the market potential m , obtaining the result of making it dependent on a communication process.

3.3.1 A Network Automata model

Network Automata Models, NA, represent an evolution of Cellular Automata models (see Boccara, 2004). They consist of a graph, described in terms of *vertices*, individuals, and *edges*, connections between vertices. The model proposed in this section takes as its unit of analysis the *state of edges*, that may be *standard edges*, when a connection occurs between two different vertices, or *reflexive edges*, when a vertex is in relationship with itself.

Let $G=(V, E)$ be a finite graph, where $V=(1,2,...i,...,N)$ is a set of vertices whose cardinality is $N=c(V)$. The set of ordered pairs (i, j) called *directed edges* (for simplicity edges) or arcs, $E \subset V \times V$, represents a subset of all the possible binary relationships -also reflexive- between vertices V . Assuming possible limitations on connectivity between vertices the cardinality of E is $U = c(E) \leq N^2$.

Each edge in E may assume, at time t , a specific state within a set of possible states, $Q = \{0,1,2,...,K\}$. Here it is assumed a simple binary version, $Q = \{0,1\}$, so that an edge may be *active*, 1, when an information about an innovation is transmitted between its vertices, otherwise is *inactive*, 0. The state of an edge (i, j) at time t is denoted as $s(i, j; t)$. This indicator function is 1, $s(i, j; t) = 1$ if and only if edge (i, j) is active, otherwise is zero, $s(i, j; t) = 0$. Besides, we may consider the possibility of edges inactivation (state reversibility).

Following the standard CA approach, the change of state of edges is governed by a *transition rule* $g(\cdot)$. Transition rule $g(\cdot)$ is function of a neighbourhood $A_{(i,j)}$. Since the unit of analysis is the edge (i, j) , a suitable representation of $A_{(i,j)}$ jointly considers the local neighbourhoods of i and j with rays of interactions e_i and $e_j \in N$ (the set of natural numbers), so that $A_{(i,j)} = \{(r, s) \mid i - e_i \leq r \leq i + e_i, j - e_j \leq s \leq j + e_j\}$, where (r, s) is a generic edge within $A_{(i,j)}$.

Consistently with the standard CA approach, in equation (2) the state of the edge (i, j) at time $t+1$ depends on a transition rule $g(\cdot)$, defined in terms of the state of all the edges within a specified neighbourhood $A_{(i,j)}$.

$$s(i, j; t + 1) = g(s(i - e_i, j - e_j; t), \dots, s(i + e_i, j + e_j; t)) . \quad (2)$$

A precise definition of $g(\cdot)$ relies on the specification of the sources of information allowing the activation of an edge (i, j) : the two sources reasonably taken into account are institutional communication, namely advertising and mass media, and word-of-mouth. In this way it is possible to consider both external (institutional communication) and internal (word-of-mouth) influences.

At a first stage it is necessary to provide a specification of the neighbourhood of (i, j) . This local neighbourhood, named $\sigma(i, j; t)$, depends on a probability measure, $p_{n,m} \geq 0$ and presents the following structure

$$\sigma(i, j; t) = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} s(i + n, j + m; t) p_{n,m} \quad \sum_{n,m} p_{n,m} = 1. \quad (3)$$

Function $\sigma(i, j; t)$ acts as a *local pressure* on edge (i, j) and is a prerequisite for the existence of a word-of-mouth effect.

Notice that introducing a simplifying assumption, that excludes a differential effect of individual edges and neglects spatial correlations, we may consider its *mean field approximation*, as proposed in Chapter 2

$$\sigma(i, j; t) \equiv v(t) = \sum_{i,j} \frac{s(i, j; t)}{U} \quad (4)$$

where $v(t)$ is the proportion of active edges in E , that is the sum of active edges over the total number of potential edges U .

The transition rule $g(\cdot)$ is defined as follows

$$s(i, j; t + 1) = s(i, j; t) + Bi(1, p_c)I_{(s(i, j; t)=0)} + Bi(1, q_c \sigma_c(i, j; t))I_{(s(i, j; t)=0)} + \\ - Bi(1, e_c)I_{(s(i, j; t)=1)} - Bi(1, w_c \sigma_c(i, j; t))I_{(s(i, j; t)=1)} \quad (5)$$

where $I_{(\cdot)}$ is an indicator function and $Bi(1, \cdot)$ is a binomial random variable.

In equation (5) the state of edge (i, j) at time $t + 1$, $s(i, j; t + 1)$, depends on its state at time t , $s(i, j; t)$ and on the realization of a binomial experiment, represented by the other components of (5).

The second component of (5), $Bi(1, p_c)I_{(s(i, j; t)=0)}$ is a binomial experiment, realised once and with probability of success p_c . In this case, the event “success” is the activation of edge (i, j) . Parameter p_c defines the probability of edge activation through an external source of information, like mass media and advertising. Notice that this experiment may be realised only if the indicator function $I_{(s(i, j; t)=0)}$ is set to one, i.e.,

if $s(i, j; t) = 0$. In other words the binomial experiment is possible *only if* the state of the edge is still *inactive*.

With a similar reasoning, the third component of (5), $Bi(1, q_c \sigma_c) I_{(s(i, j; t)=0)}$ refers to edge activation through a word-of-mouth effect, expressed with the joint probability $q_c \sigma_c(i, j; t)$, where q_c represents, as usual, a specific aptitude towards imitation, combined with the local pressure $\sigma_c(i, j; t)$: this expresses the fact that word-of-mouth is the combined result of a relational characteristic, parameter q_c , with a local evidence (pressure), $\sigma_c(i, j; t)$.

It is important to stress that these two experiments represent *strictly alternative possibilities*, so that if an edge is activated through an external source of information, its status becomes active and the binomial experiment based on a word-of-mouth effect is no longer possible and vice versa, if an edge becomes active through a word-of-mouth effect the experiment based on the external source of information can not be realised. Moreover, observing rule (5), we may see that the activation of an edge occurs if *both* vertices *either* receive the information from an external source (institutional communication) *or* from an internal one (word-of-mouth). In other words, the possibility of a *mixed connection*, when a vertex is informed by an external source and the other by an internal one is not considered in the model. Although a mixed connection is possible, the focus is just on *pure cases*, i.e. individuals (vertices) that are influenced by the same source of information: this is certainly a simplification, though probably reasonable. In fact, the rise of a collective knowledge may be seen as a process of *convergence*, in which individuals exchange information and establish connections

with persons with *similar references*, in order to share the meaning they give to phenomena and reach a *mutual understanding* (see Rogers, 2003).

The fourth component of equation (5), $Bi(1, e_c)$, represents a *decay effect* and describes the possible inactivation of an edge, occurring with probability e_c , due to a normal loss of information. Notice that this binomial experiment may be realised only if the edge is already active, that is if $s(i, j; t) = 1$.

Finally, the fifth component, $Bi(1, w_c \sigma_c)$, describes the possibility of edge inactivation due to negative word-of-mouth, expressed through parameter $w_c \sigma_c$, representing a phenomenon of resistance to innovation. Also in this case the binomial experiment is possible when $s(i, j; t) = 1$. Again, the two binomial experiments are strictly alternative, so that an edge may become inactive either for a loss of information or for a negative word-of-mouth.

Given transition rule (5), that describes how the status of a single edge may change in time, it is now interesting to analyse the collective behaviour emerging from many individual changes governed by (5). The *bottom-up* construction of the collective behaviour of the set of edges E may be obtained in a simplified way, through the *mean field approximation* of (5), namely

$$v(t+1) = v(t) + p_c(1-v(t)) + q_c v(t)(1-v(t)) - e_c v(t) - w_c v^2(t) \quad (6)$$

that is

$$v(t+1) - v(t) = p_c(1-v(t)) + q_c v(t)(1-v(t)) - e_c v(t) - w_c v^2(t) \quad (6a)$$

and approximating the finite difference with the first derivative

$$v'(t) = p_c(1-v(t)) + q_c v(t)(1-v(t)) - e_c v(t) - w_c v^2(t). \quad (6b)$$

Equation (6b) describes the average evolution of the network: the variation over time of the rate of active edges, $v'(t)$, is proportional to the rate of edges still inactive, $(1 - v(t))$, and is defined by two positive flows of information, respectively due to institutional communication, $p_c(1 - v(t))$, and positive word-of-mouth, $q_c v(t)(1 - v(t))$, and by two negative flows, due to normal losses of information, $e_c v(t)$, and negative word-of-mouth, $w_c v^2(t)$.

We may observe that excluding the effect of parameters e_c and w_c , so that just a positive flow of information is considered, yields the structure of a standard Bass model,

$$v'(t) = (p_c + q_c v(t))(1 - v(t)). \quad (7)$$

The closed form solution of (6b) (provided in analytic form in Guseo and Guidolin, 2007b) is

$$v(t) = \frac{1 - e^{-D_c t}}{\frac{1}{r_2} - \frac{1}{r_1} e^{-D_c t}}, \quad D_c = \sqrt{(q_c - p_c - e_c)^2 + 4(q_c + w_c)p_c} > 0 \quad (8)$$

where $r_i = -(q_c - p_c - e_c) \pm D_c / (-2(q_c + w_c))$, $i = 1, 2$, $r_2 > r_1$

or its reduced version, if the negative flow of information is not considered

$$v(t) = \frac{1 - e^{-(p_c + q_c)t}}{1 + \frac{q_c}{p_c} e^{-(p_c + q_c)t}}. \quad (9)$$

that is the solution of a standard Bass model.

Equations (8) and (9) describe the dynamics of information diffusion in a social system, through the formation of a network of connections (edges) between agents and

function $v(t)$ is the *rate of active edges* at time t . If we want to analyse this diffusion in *absolute terms*, we just need to multiply $v(t)$ by the total number of edges U within the set E , obtaining $Uv(t)$ that represents the *number of active edges* at time t .

3.3.2 Defining a variable market potential (or carrying capacity)

$Uv(t)$ describes an aggregate evolution of the knowledge about an innovation within the network: as discussed in previous sections, this knowledge is a prerequisite for the concrete adoption process, so that $Uv(t)$ should be used for defining the market potential. Using the terminology proposed by Cohen and Levinthal we may think of $Uv(t)$ as the prior related knowledge that allows the development of an absorptive capacity here represented by the market potential m . To formalise the fact that absorptive capacity is a function of prior knowledge, it is necessary to provide a definition of m as function of $Uv(t)$.

Recall that m is the number of individuals that might adopt within the social system considered and $Uv(t)$ is the number of active connections between individuals. Reasonably, the number of potential adopters may be viewed as a *subset* of the number of individuals informed about the innovation, because we could not expect that all those that are informed will eventually become potential adopters. To obtain the number of informed individuals, it is sufficient to remind that in the network the cardinality of the set of edges E has been defined as $U = c(E) \leq N^2$, where N is the cardinality of vertices of G . Introducing a simplification and considering E as an approximate squared subset of $V \times V$, the positive squared root of $Uv(t)$

$$k(t) = \sqrt{U} \sqrt{v(t)} \tag{10}$$

gives the number of all the *active vertices* of G , i.e. persons that are aware of the innovation at time t .

All these persons may be potential adopters, since they are informed about the innovation, thus $k(t)$ may represent the *upper bound*, i.e. a limit condition of the potential market $m(t)$, reached when all those that are informed will adopt. But because we have told that the number of potential adopters may be viewed as a subset of the number of informed persons, the *actual* potential market will be defined as

$$m(t) = K\sqrt{v(t)} \quad (11)$$

where $K \leq \sqrt{U}$.

Equation (11) can be rewritten in explicit form

$$m(t) = K \sqrt{\frac{1 - e^{-D_c t}}{\frac{1}{c r_2} - \frac{1}{c r_1} e^{-D_c t}}} \quad (12)$$

or in its reduced version, when exit parameters are excluded

$$m(t) = K \sqrt{\frac{1 - e^{-(p_c + q_c)t}}{1 + \frac{q_c}{p_c} e^{-(p_c + q_c)t}}} \quad (13)$$

Equation (12) and (13) show that the market potential may be conceived as function of a knowledge process, expressed in terms of the proportion of active (i.e. informed) vertices at time t , multiplied by the scale parameter K , describing the phenomenon in absolute terms. The definition of the market potential as function of the informed vertices within an evolving network implies, as crucial consequence, its time dependence, $m(t)$.

At this stage, one might wonder why focusing on the state of *edges*, when the relevant measure for defining the market potential is the *number of active vertices*, obtained through equation (10). Should not be fair to consider the evolution of vertices and design a transition rule for representing their changing state?

Indeed, it is easy to prove that modelling the evolution of vertices or that of edges within the same graph, generates pretty much equivalent models. However, the model focuses on edges and on the formation of a network, in order to represent the development of a collective knowledge described as an evolutionary network in which some linkages already exist, some rise, some others die.

A graphical representation (see Figure 1) of equation (13), i.e. without exit rates, may help to appreciate the role of a good or bad communication process in the development of the market potential: a very good communication process, with high parameters of institutional communication, p_c , and word-of-mouth, q_c implies a fast development of the market potential, that soon reaches its asymptotic level and then may be considered constant (as assumed in the Bass model). On the contrary, bad communication, that is low values of p_c and q_c implies a great delay in the development of the market potential, which obviously will exert a negative impact on adoptions (as will be explained in section 3.4).

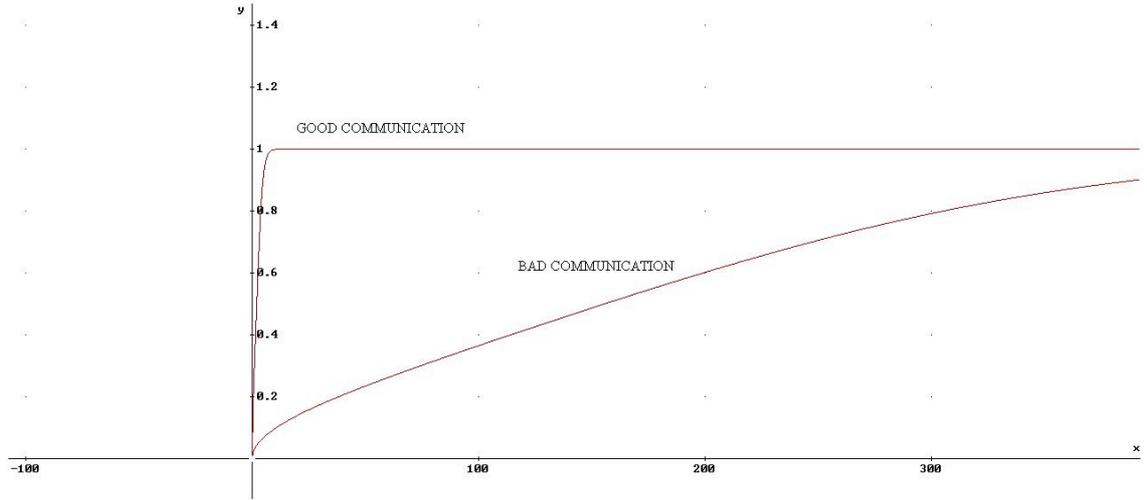


Figure 1: The development of a market potential, y , as function of time, x , under good and bad communication processes. The value of parameters is $K=1$ for both cases. Bad communication: $p_c=0,0009$ and $q_c=0,009$; Good communication $p_g=0,06$ and $q_g=0,8$.

3.4 An innovation diffusion model with a variable market potential

Once defined the variable market potential $m(t)$ its structure has to be incorporated into the model describing the adoption process, in our case the Bass model.

The modified structure of the Bass model results as

$$z'(t) = m(t) \left\{ \left(p + q \frac{z(t)}{m(t)} \right) \left(1 - \frac{z(t)}{m(t)} \right) \right\} + z(t) \frac{m'(t)}{m(t)}. \quad (14)$$

where the last term $z(t) \frac{m'(t)}{m(t)}$ represents a self-reinforcing effect to the adoption

process given by the variable structure of the market potential. Equation (14) may be rewritten as

$$\frac{z'(t)m(t) - z(t)m'(t)}{m^2(t)} = \left(\frac{z(t)}{m(t)} \right)' = \left(p + q \frac{z(t)}{m(t)} \right) \left(1 - \frac{z(t)}{m(t)} \right) \quad (15)$$

and assuming $y(t) = \frac{z(t)}{m(t)}$ yields the well known structure of the standard Bass model

$$y'(t) = (p + qy(t))(1 - y(t)). \quad (16)$$

The closed-form solution of equation (15) is consequently

$$z(t) = m(t) \frac{1 - e^{-(p+q)t}}{1 + \frac{q}{p} e^{-(p+q)t}}. \quad (17)$$

Notice that for a constant market potential $m(t) = m$, equation (17) is completely equivalent to the solution of the standard Bass model.

3.5 Statistical implementation and applications

Equation (17) may be rewritten in explicit form, so that the final model to be implemented is

$$z(t) = K \sqrt{\frac{1 - e^{-D_c t}}{\frac{1}{r_2} - \frac{1}{r_1} e^{-D_c t}}} \frac{1 - e^{-(p_s + q_s)t}}{1 + \frac{q}{p} e^{-(p_s + q_s)t}} \quad (18)$$

or its reduced version

$$z(t) = K \sqrt{\frac{1 - e^{-(p_c + q_c)t}}{1 + \frac{q_c}{p_c} e^{-(p_c + q_c)t}}} \frac{1 - e^{-(p_s + q_s)t}}{1 + \frac{q}{p} e^{-(p_s + q_s)t}} \quad (19)$$

if we do not consider exit parameters in information diffusion. Equations (18) and (19) highlight that communication and adoption dynamics are *separate* but *co-evolutionary* processes.

Figure 2 presents the structure of two market potentials and corresponding adoption dynamics, to show how communication can affect the final diffusion process. For appreciating the net effect of communication, adoption parameters p_s and q_s have been set at the same level in both cases. As is evident in Figure 2, ineffective communication

has negative consequences on the final diffusion process, that develops with a considerable delay.

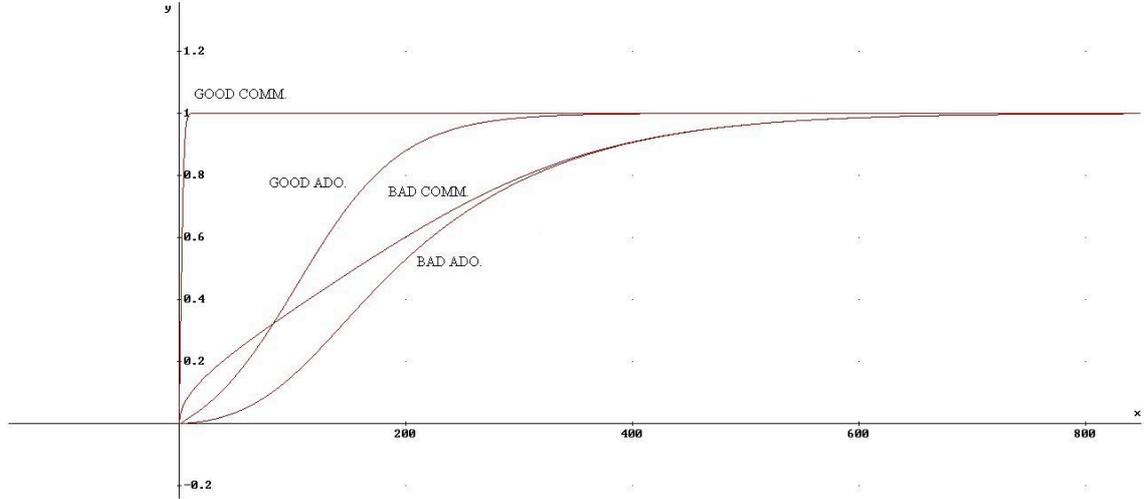


Figure 2: The impact of different communication processes on adoption dynamics: adoption parameters have constant values, $p_s=0,002$ and $q_s=0,02$. The value of communication parameters is : $p_c=0,0009$ and $q_c=0,009$ (bad communication) and $p_c=0,06$ and $q_c=0,8$ (good communication), $K=1$. Exit parameters are not considered in this figure.

One of the most interesting aspects of this model is that its statistical implementation just requires *aggregate adoption data*, $z(t)$: there is no need of observed data on the network of information, whose *latent* structure may be inferred in parametric terms just using data of adoptions. Using a NLS procedure (e.g. Levenberg-Marquardt, see Seber and Wild, 1989) the estimated parameters are: K, p_c, q_c, p_s, q_s . Parameter K represents the upper bound of $m(t)$ and acts as a scale parameter, defining the absolute size of the process. Observe that in the case without exit parameters, here represented by equation (19) the asymptotic behaviour of $m(t)$ is controlled by K , i.e. $\lim_{t \rightarrow +\infty} m(t) = Kr_2$ where $r_2 = 1$ as in the Bass model. Parameters p_c and q_c define the dynamics of information diffusion within the network, respectively due to institutional communication and word-of-mouth, while p_s and q_s are the usual Bass model parameters, describing innovative and imitative behaviour in adoptions.

The advantage of proposing a co-evolutive model in which communication dynamics act as a shadow process, is not only related to the statistical parsimony of the model, but also has a substantial meaning: adoption data testify the concrete decision of consumers, that may result from a good or bad communication effort. In this way, the model offers a useful benchmark for evaluating the effectiveness of various communication strategies. Indeed, a typical problem for firms investing much effort on marketing and promotional activities is the difficulty to test the impact of these actions on the final purchase decision.

The central result of the model is a new connotation of the market potential m , whose structure is no longer constant, but is *generated through an evolutionary process* of information diffusion that creates a collective knowledge. The crucial consequence of this choice is the possibility to isolate the effect of communication dynamics on adoptions. In particular, as the structure and size of the market potential is made depend on the development of a collective knowledge, it is essential to concentrate communication efforts in early stages of diffusion and generate the market potential as soon as possible. On the contrary, a limited awareness due to insufficient communication hinders the formation of it, resulting in an incubation phase that penalizes sales. Interestingly, the standard Bass model is characterised by a structural tendency to overestimate the first part of data series, not recognising the presence of an incubation phase and counterbalances this fact with an underestimation of the saturation phase. A comparison between the standard Bass model and that proposed here within empirical applications, will confirm the superiority of the latter in terms of parameter estimates, global fitting and forecasts.

3.6 Empirical applications

The use of this model appears a suitable diagnostic in all those diffusion cases, in which firms have adopted marketing strategies that try to exploit the rise of networks between consumers, like *viral marketing* techniques. While viral marketing techniques are typically implemented to trigger the communication process among consumers, such a communication is perceived as a “black box” by managers and academic marketing research has so far offered little to mitigate managers’ sense of inefficacy (Goldenberg, Libai and Muller, 2001b). In this sense, the model proposed here might offer a new insight on the communication process underlying adoptions.

In addition, this model may serve as a managerial instrument for a general evaluation of the impact of communications efforts on adoptions, since it is always a difficult task for firms to understand if their investments in communication have been effective, and for comparing different markets in their receptiveness to the same innovative product. It is well known that the attractiveness of a market is function of the eventual market potential: however, as highlighted by Talukdar, Sudhir and Ainslie (2002) cross-country studies have generally tended to focus on coefficients of internal and external influence, rather than on the determinants of the market potential, pointing out a theoretical lack in this sense.

The first application is therefore dedicated to test the performance of the model in a case where viral marketing has been crucial for innovation success, while the second one proposes a comparison between two regions in their receptiveness to a new pharmaceutical drug. In this second application the focus will be in particular on physicians’ acceptance of medical innovation, a quite investigated theme in sociological and economic literature.

3.6.1 Successful viral marketing: the case of Hotmail

The electronic mail service Hotmail represents one of the most notable cases of how a firm has been able to exploit the network effect created by consumers. This case is described also in Barabasi (2004) as an illuminating example of the power of consumption networks.

The introduction of Hotmail is a classic Internet start-up success story. Launched in July 1996, Hotmail was a simple email service, but free and available all around the world. Thanks to its amazing diffusion, today almost 25% of email users uses Hotmail. In Sweden and India, where there was no advertising of the service, it is the major provider of email service (Barabasi, 2004). Indeed, a compelling aspect of the Hotmail story is its phenomenal growth rate. What was the secret of such a success? An email service for free with a very simplified registration procedure made things easier. However, this is not enough: a traditional answer for Hotmail growth would be lots of advertising, yet practically all the initial seed money from venture capitalists was spent on hard-ware and personnel, leaving very little for traditional promotional activities.

Tim Draper, partner of the investment company Draper, Fisher and Jurvetson convinced the inventors of Hotmail, Saber Bhatia and Jack Smith, to add at the bottom of a message sent with Hotmail the sentence: “get your private, free email at <http://www.hotmail.com>”. In this way, every time a message was sent from a Hotmail account, the service was known by receivers. In other words, the knowledge of the service increases with sent messages. Every recipient of a message from a Hotmail account, not yet registered, automatically becomes a *potential user just for knowing the service*. Thus, the creation of a potential market as function of knowledge is particularly evident in this case. Initially, the founders were against this type of

promotion, thinking that their users would be repelled by any advertising: however, the Hotmail's subscriber base grew faster than any other online company.

The key to Hotmail's success was *viral marketing*, a term coined by Steve Jurvetson and Tim Draper, to denote a type of marketing that infects its customers with an advertising message, which passes from one customer to another. Viral marketing seeks to increase awareness or adoption of a product by taking advantage of the relationship network among consumers (Hill, Provost and Volinsky, 2006).

A literature review on viral marketing and, in particular, on statistical analysis for it proposed in Hill, Provost and Volinsky (2006) highlights a clear deficiency in current research to answer the question, "does viral marketing improve over traditional marketing techniques?". The authors argue that this question remains open because such an evaluation requires both data on communication and on adoptions. In particular, they observe that the traditional approach to diffusion, namely the Bass model does not focus specifically on this aspect: models of product diffusion assume and firms hope that viral marketing is effective. However, the understanding of when it occurs and to what extent it is effective, is important for the firm.

Montgomery (2001) has maintained that the case of Hotmail could be analysed with a standard Bass model, having the *weekly cumulative data* of registration between June 1996 and June 1997.

Using this data series, the case of Hotmail is analysed in this section with a standard Bass model, yielding the results summarised in Table 1.

A standard Bass model for the diffusion of Hotmail (data source: Montgomery, 2001)			
m	p	q	R ²
6,66948	0,0012	0,0968	99,9634%
(6,40728)	(0,00122)	(0,0940)	
(6,93169)	(0,00130)	(0,09968)	

Table 1. Bass model parameters' estimates, R² index, asymptotic 95% confidence intervals into parentheses.

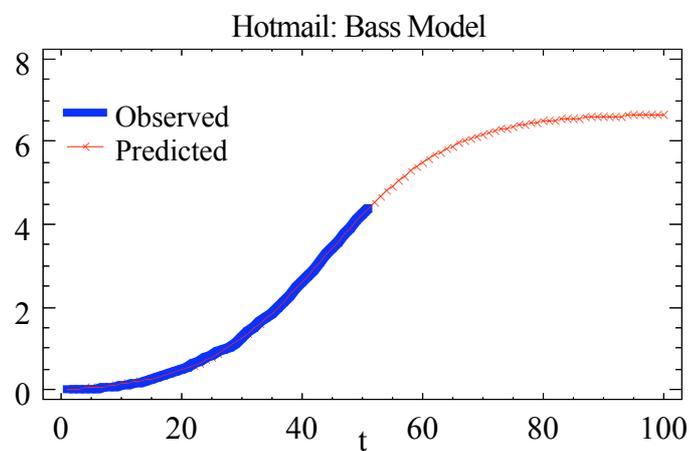


Figure 3. Bass model: observed weekly data and predicted values (time origin: June 1996). The Bass model predicts an early saturation of the market, denied by facts.

Apart from an unrealistic estimate of the market potential m -about 6 millions of subscribers- that has been denied by facts, the proposed analysis does not allow conclusions about the effectiveness of the viral marketing strategy, which was apparently crucial for the success of the service. So, we may expect to improve the analysis using the model proposed in this chapter, in which it is possible to determine simultaneously adoption and communication dynamics. The applied model is therefore

$$z(t) = K \sqrt{\frac{1 - e^{-(p_c + q_c)t}}{1 + \frac{q_c}{p_c} e^{-(p_c + q_c)t}} \frac{1 - e^{-(p_s + q_s)t}}{1 + \frac{q_s}{p_s} e^{-(p_s + q_s)t}}} + \varepsilon(t)$$

where $z(t)$ are observed data, in this case cumulative data of registration. Table 2 presents the results of this application.

K	q _c	p _c	q _s	p _s	R ²
69,3037	0,1120	0,0010	0,0089	0,0011	99,9836%
(43,644)	(0,1055)	(0,00042)	(-0,0037)	(-0,0159)	
(94,9635)	(0,1185)	(0,0016)	(0,0216)	(0,0183)	

Table 2: NA model parameters' estimates, R², asymptotic 95% confidence intervals into parentheses.

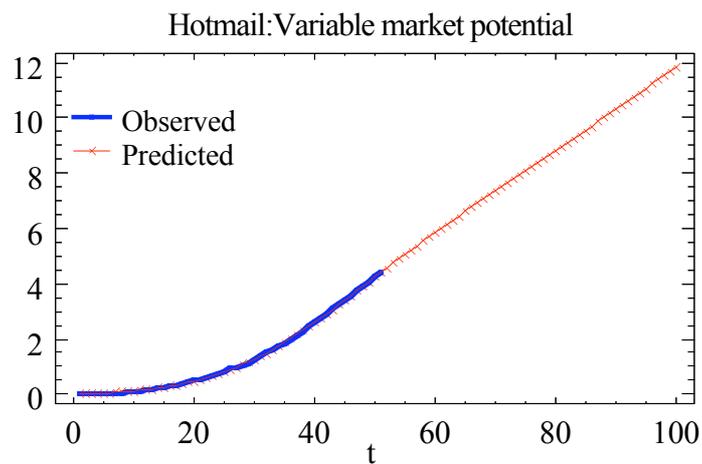


Figure 4. Observed and predicted weekly cumulative values: the model with a variable market potential departs from the Bass model.

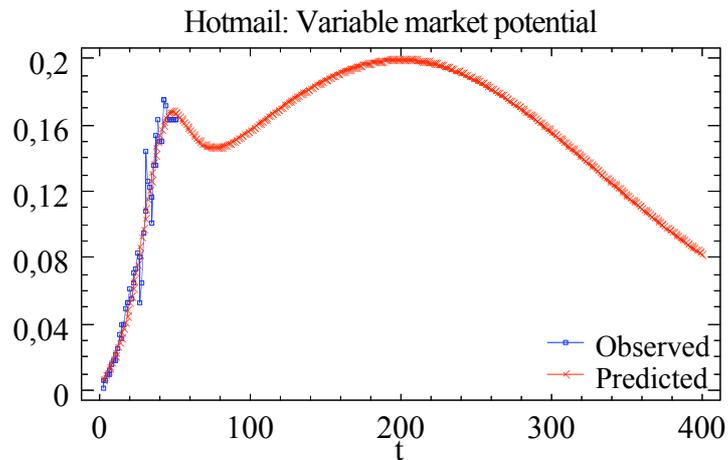


Figure 5. Instantaneous observed data and predicted values: after a fast growth in the first part of the life cycle the model highlights the possible presence of a “slowdown”.

The obtained results seem particularly satisfactory. The value of index $R^2= 0,999836$ denotes a global good fitting and parameter estimates, with the exception of some instability in confidence intervals for parameters p_s and q_s , likely to be due to the limited dataset available, seem good and reliable. Notice the quite high value of q_c describing the word-of-mouth effect especially with respect to p_c , the institutional communication parameter: indeed this confirms that word-of-mouth was mostly responsible for the fast growth of the service and that viral marketing was effective. Besides, it is interesting to observe that adoptions are characterised by a similar structure in parameter terms.

The value of parameter K , depicting the upper bound of the market potential $m(t)$ - about 70 millions of subscribers- assumes a notably higher value than that of m estimated with a simple Bass model, highlighting the superiority in forecasting terms of this new model with respect to the standard Bass model. Although the analysis applied to the case of Hotmail has *no predictive* value and serves more as a post-hoc

explanation, it has appeared particularly appropriate for testing and showing the performance of the proposed model, both for forecasting and for evaluating the impact of communication strategies on adoptions.

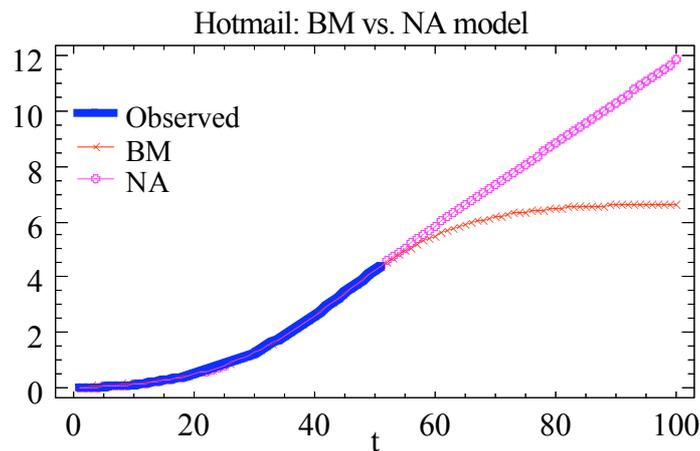


Figure 6. The two models compared (BM vs. NA): there is an evident difference in forecasts.

In any case, it should be recognised that the example of Hotmail is good for representing the formation of a market potential as a function of connections between consumers, because the email service is interactive and typically exploits the existence of relationships between persons: the strategy for promoting the service has been chosen coherently with this aspect. This strategy has been also defined *frictionless*, as the message is diffused without an explicit will of the sender.

Nevertheless, Hotmail had success because potential adopters received and maintained the information, subsequently adopting the service. In other words, *viral marketing was succesful because this message was retained by persons*. The success of a marketing strategy is not an obvious result: many others firms that tried to replicate the experience of Hotmail, like Epidemic Marketing, failed because of a negative or

lacking response by consumers. In this sense, the case of Hotmail is an illustrative example to show that the success of an innovation really depends on the response of consumers to it (see Foster and Potts, 2006).

3.6.2 The diffusion of a pharmaceutical drug in Italy

The second application proposed in this chapter analyses the regional markets for a new pharmaceutical drug in Italy. Several studies were dedicated to pharmaceuticals in the 1980s, mainly due to the complex distribution chain involved in new drugs' marketing. Recently, less attention has been paid to the diffusion of drugs: however, the complexity of these markets, the high level of competition between firms, the shortening of product life cycles and the consequent search for successive generations of product or completely new generations through intensive research activities, are all elements indicating that new drugs' sector is particularly fit for diffusion research. Desiraju, Naik and Chintagunta (2004) have examined differences in diffusion parameters of drugs between developed and developing countries, comparing the relative receptiveness of markets in terms of speed of adoption. This certainly represents a stream of research to be investigated, since the acceptance of a new drug by a community relies on a widespread knowledge about the benefits implied by its assumption. It has been demonstrated that various forces contribute to the construction of this knowledge, like scientific research, the medical community, institutions, profit and no-profit organisations. In addition, pharmaceutical industries make strong investments on communication efforts, through the direct action of detailmen, that promote new products to physicians or through the sponsorization of conventions and conferences with the purpose of stimulating awareness and word-of-mouth effects. Indeed, a considerable body of research has been produced in order to understand if

physicians' acceptance of medical innovation is due to word-of-mouth effects (see for instance the pioneering work of Coleman, Katz and Menzel, 1966) or to marketing efforts exerted by detailmen (see Van den Bulte and Lilien, 2001). In particular, Van den Bulte and Lilien (2001) propose a model for evaluating the impact of marketing efforts on physicians' acceptance of a new drug assuming that physicians are aware of the existence of the new drug. They specify that such awareness can be driven by marketing efforts, free publicity in medical journals and exposure to peers who have adopted previously and with whom one shares information on medical practice. Interestingly, they recognize that in their model the stage of awareness is not isolated within the adoption process, though this separation may help gain a better understanding of the differential effects of advertising and social contagion, as the former is believed to operate mainly early in the decision process and the latter mainly in later stages (see Van den Bulte and Lilien, 2001).

The model developed in this chapter suggests that both social contagion and marketing efforts have a specific role in creating physicians' awareness and provides measures of their relative effects, in terms of communication parameters.

The case study proposed in this section deals with the diffusion of a new calcium antagonist drug for treating hypertension problems. The innovativeness of this product would rely on the active principle based on a new molecule, whose effect lasts for 24 hours. Several studies have documented that high blood pressure must be controlled both with life style modifications, like weight reduction, dietary sodium reduction, physical activity, moderation of alcohol consumption, and with an appropriate pharmacological treatment, that helps to prevent risks of infarction and stroke.

This product, here denoted by “Libr”, was introduced in the Italian market in 2005. The data of diffusion refer to the *number of weekly sold packages* and are provided by IMS health. They cover the period between 2005 and 2007 with a spatial disaggregation by geographic areas: this fact may allow interesting comparisons on the response of different markets to the same product, promoted with equivalent communication strategies.

As in the case of Hotmail, the model is applied in its reduced version (i.e. without exit rates in communication dynamics) in two areas of Italy, “Nord-Est” (North-East) and “Nord-Ovest” (North-West), using a standard NLS procedure. The results of this application are summarised in Table 3.

The diffusion of “Libr” in Nord-Est of Italy					
K	q _c	p _c	q _s	p _s	R ²
647.061	0,0811	0,0038	0,0185	0,0010	99,9939%
(533821)	(0,0748)	(0,0034)	(0,0169)	(0,0008)	
(760300)	(0,0874)	(0,0042)	(0,0201)	(0,0011)	
The diffusion of “Libr” in Nord-Ovest of Italy					
K	q _c	p _c	q _s	p _s	R ²
1.543.200	0,0767	0,0039	0,0156	0,00063	99,9901%
(650413)	(0,0693)	(0,0034)	(0,0132)	(0,0003)	
(2436100)	(0,0840)	(0,0044)	(0,0180)	(0,0009)	

Table 3. Parameters’ estimates for the two areas, R², asymptotic 95% confidence intervals into parentheses.

Observing the value of the determination index R², the model presents very good levels of global fitting in both cases and all the involved parameters are significant.

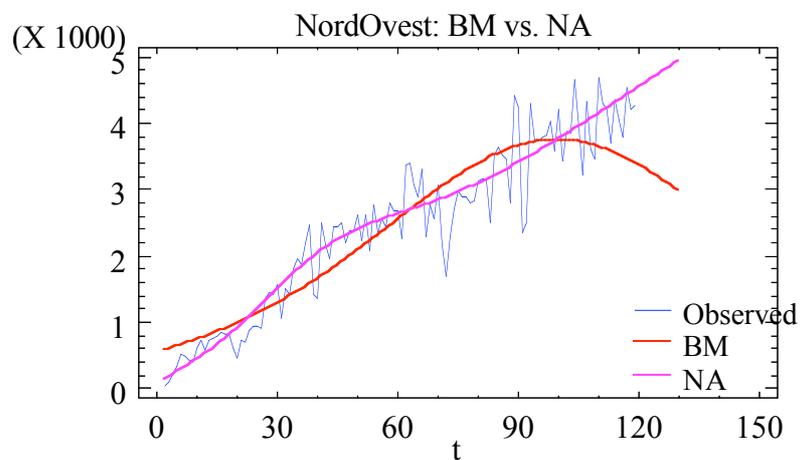
Apart from differences in the estimate of K , that are due to the relative size of the two markets, the interesting result of this application refers to the value of communication parameters p_c and q_c , that are quite similar in the two areas. This would suggest that the two markets are not characterised by evident differences in terms of receptiveness to a given new drug and that physicians' acceptance has been substantially the same. In another case, analysed by Guseo and Guidolin (2007b) the application of the model to the diffusion of a new product in two areas of Italy has highlighted structural differences in markets' response, suggesting a change in marketing choices, able to improve not enough effective communication efforts.

Also in this case a comparison with the standard Bass model may be useful for appreciating the improvements obtained in modelling terms. The application of a BM yields the results summarised in Table 4.

BM: the diffusion of "Libr" in Nord-Est			
m	p	q	R ²
305.253	0,00124	0,0323	99,8584%
(289917)	(0,00120)	(0,0308)	
(320588)	(0,00128)	(0,0338)	
BM: the diffusion of "Libr" in Nord-Ovest			
m	p	q	R ²
443.815	0,00124	0,0312	99,8508%
(418344)	(0,00120)	(0,0296)	
(469287)	(0,00128)	(0,0327)	

Table 4. Standard Bass model parameters' estimates, R², asymptotic 95% confidence limits into parentheses: a considerable underestimation of m is evident.

The application of a standard Bass model to our data series has produced a weaker performance, as shown by the values of the determination indexes R^2 , that in these cases are much lower. However, the most surprising results are the estimated values of the market potential m (or K), that are significantly different from those obtained with the co-evolutive model. This confirms that not taking into account the presence of an incubation period in which the market potential is still under development, may seriously affect forecasting of market potentials and related strategic measures, like the peak of sales (which is reached at about $\frac{m}{2}$). A graphical comparison of the performances of the models for both data series may help to clarify the obtained improvements with respect to a standard Bass model (see Figure 7).



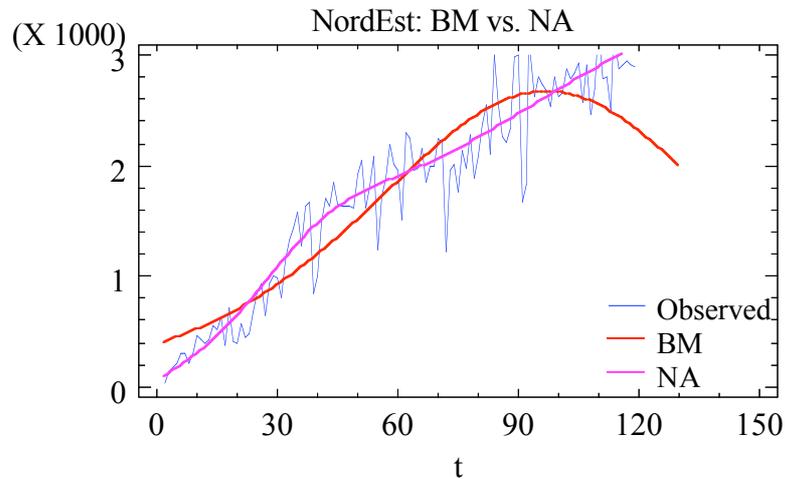


Figure 7. Actual non cumulative data, predicted values with a standard Bass model (BM), predicted values with the model with the variable market potential (NA): in both regions (NordOvest, NordEst) the improvement in fitting terms is evident especially in the first part of data series.

3.7 Concluding remarks and further research

This chapter has presented a new innovation diffusion model in which the market potential, or carrying capacity, is no longer constant, but is generated through the construction of a collective knowledge from agents. This collective knowledge, considered an essential step for diffusion, has been represented as an evolving network, using a class of Complex systems models, namely network Automata models. The structure of this network in aggregate terms, obtained with a *mean field approximation*, has been incorporated into the final diffusion model, showing that information dynamics and adoptions are separate but co-evolving phases.

The model may be a useful managerial tool for analysing the impact of marketing strategies focused on communication. In particular, the example of Hotmail has proven the good performance of the model in testing if viral marketing techniques, that seek to take advantage of the existence of consumption networks, are really effective or not.

Moreover, it may be an efficient diagnostic for evaluating different levels of receptiveness to innovation of different social contexts or geographic areas: in this sense it may be used for cross-country analyses that try to understand how cultural, social and informational aspects may influence the speed of an innovation diffusion process.

The predictive performance of the model has been compared with the standard Bass one, showing that the hypothesis of a variable market potential allows to recognise the incubation phase, typical of early stages of diffusion, when a limited knowledge about the innovation penalizes adoptions, and, as a counterbalancing effect, there is no underestimation of the size of the market potential.

There is a fundamental difference between the generalization provided by this model and that of the Generalized Bass model (presented in Chapter 1). While the GBM allows a modification of the timing of diffusion but not of the size of it through an external function $x(t)$, considering a dynamic market potential permits to deal with a variable size of the process, that depends on internal diffusion dynamics.

Further research should of course test the model in many other situations, included the case of successive generations of product, that are characterised by an intriguing paradox: while several contributions in literature have documented that adoption parameters across generations essentially do not change, another branch of literature has proven that there the speed of products life cycles accelerates over time. A possible explanation of this fact may be given using this model and showing that, if adoption parameters are the same in two diffusion processes, that just differ for the values of information parameters, indeed life cycles will have different speeds (see Figure 2). In other words, the acceleration over time of life cycles may be explained in terms of market potential formation: while the first generation of an innovation may reasonably

experience some difficulty in taking off because people are not sufficiently aware of it, this is not the case of a successive generation, when people are ready to accept it, so that the corresponding market potential is soon developed.