Università degli Studi di Padova

Dipartimento di Fisica e Astronomia "Galileo Galilei"

Scuola di Dottorato di Ricerca in Fisica

Ciclo XXV

On the growth of structures in Galileon cosmologies

Coordinatore: Ch.mo Prof. Andrea Vitturi

Supervisori: Ch.mo Prof. Sabino Matarrese

Ch.mo Dr. Nicola Bartolo

Dottorando: Emilio Bellini

To Marianna

CONTENTS

INTRODUCTION

Several observations, such as those referring to the magnitude-redshift relation for type-Ia Supernovae (SNIa) [1–3], Cosmic Microwave Background (CMB) temperature anisotropies [4–6] and Baryonic Acoustic Oscillations (BAO) features in galaxy clustering [7, 8], suggest that the universe is currently undergoing an accelerated expansion phase, caused by the presence of a positive cosmological constant or a more general Dark Energy (DE) component or a suitable modified gravity model. Assuming that the matter distribution is dominated by Cold Dark Matter (CDM), the simplest model that reproduces this effect and fits present data is the so-called ΛCDM one, based on the existence of a cosmological constant term that fills the gap between the matter energy density and the critical one. Even though the presence of a cosmological constant term Λ is fully consistent with General Relativity, its value appears too small to be explained by fundamental physics [9]. Thus, cosmologists explored alternative theories by e.g., modifying the Einstein-Hilbert action:

$$S = \frac{M_{\rm pl}^2}{2} \int {\rm d}^4x \sqrt{-g}\, R + \int {\rm d}^4x\, \mathcal{L}_M\,, \tag{1}$$

where $M_{\rm pl}$ represents the reduced Planck mass. A representative list of the models investigated is quintessence [10, 11], $f(R)$ gravity (for a review see [12]), massive gravity [13], scalar-tensor theories [14], Brane-World models (e.g. [15]) and others (see [16] and references therein).

Few years ago a new class of scalar-tensor theories was introduced by *Nicolis et al.* [17], the so-called *Galileon.* This model was constructed as an effective field theory, which is based upon and aims at extending the decoupling limit of the Dvali-Gabadadze-Porratti model (DGP) [18]. Originally the Galileon was proposed as the most general theory containing second-order derivatives in the scalar field that preserves the Galilean shift symmetry ($\pi \rightarrow \pi + b_\mu x^\mu + c$, with b_μ and c constants) and avoids the Ostrogradski instabilities [19]. Unfortunately, in the original model, these properties were respected only in flat space-time. Then, the works by *Deffayet et al.* [20, 21] found a way to generalize Galileons to curved space-times. To do this, it is necessary to break the Galilean symmetry and to add certain extra terms which couple the scalar field with curvature terms. The result is a scalar-tensor theory, the Covariant Galileon, which keeps the equations of motion up to second-order in time-derivatives (i.e. it avoids Ostrogradski instabilities) and preserves the shift symmetry ($\pi \rightarrow \pi + c$) in a curved space-time. In addition, a fundamental property is that on non-linear scales the self-interactions of the Galileon field screen the fifth force through the Vainshtein mechanism [22, 23], see also [24] for a discussion in the most general second order scalar-tensor theory. The essence of this mechanism lies in the non-standard kinetic terms (i.e. $\Box\pi\partial_\mu\pi\partial^\mu\pi$), which decouple the scalar field from gravity at small scales ($r \ll r_V$, where r_V is a characteristic scale around a matter source, named "Vainshtein radius"). On the other hand, on linear scales ($r \gg r_V$) the Galileon is coupled with gravity causing observable modifications w.r.t. the standard gravity. DGP theory is one example that allows us to understand the magnitude of the Vainshtein radius. It possesses a Vainshtein radius defined by $r_V = (r_s r_c^2)^{1/3}$ (where r_s is the Schwarzschild radius of the source, r_c is a coupling constant which defines the crossover scale between a 5-dimensional Minkowsky space and the embedded 4-dimensional space-time).

Many literature has recently appeared on these models and their generalizations [25–40]. Galileon models have been extensively studied at late-times [41–57] and a subclass of these models has been already compared by observations [58–60]. Even though in this paper focus on the effects of the late-time cosmic acceleration produced by the scalar field, it is worth mentioning that the importance of the Galileon field also relies on the fact that it can inspire some "inflationary-like" model [61–68].

In this thesis we have analyzed, using Perturbation Theory (PT) and semi-analytical methods, some aspects of the growth of structures in Galileon cosmologies [51, 56]. Working at the late-times we consider the universe filled by Dark Matter (DM), radiation and a field responsible for the accelerated expansion of the universe (the Galileon). Baryonic matter is subdominant w.r.t. DM, thus it will be included in the total amount of DM.

This work is organized as follows.

In Chapter 1 we outline the main ideas to construct the lagrangian of the Covariant Uncoupled Galileon [17, 20, 21]. Then we add some explicit coupling between the Galileon and the matter fields, i.e. the *Coupled Galileon* [50]. Finally we calculate the equations of motion that will be used in the next Chapters.

In Chapter 2 we study the background evolution for the Galileon models in a Friedmann-Lemaître-Robertson-Walker (FLRW) universe. In Sec. 2.1 we use the tracker solution found in [46]. This solution is an attractor and ensures a stable de Sitter (dS) point after the radiation and the matter dominated epochs. Moreover, it is parameter independent, thus the cosmology is fixed and the parameters of the theory will be used only to control the results at the perturbation level. This approach will be used to study the spherical collapse model in Chapter 6. In Sec. 2.2 we study the background evolution of the *Cubic Galileon* using a general solution, to have some freedom in the the background evolution. The results of this section will be used in Chapter 4, when we will show the DM bispectrum.

In Chapter 3 we study the linear evolution of the DM density perturbations. In Sec. 3.1 we use the tracker solution to show the time evolution of the modified Newton's constant. Then, in Sec. 3.2 we analyze the linear perturbations with the background of Sec. 2.2. We derive the equations of motion of the *Cubic Galileon* in a general gauge. We are able to single out an equation for the DM density perturbations. For this equation we find two integral solutions for the growing and the decaying modes, and we study the growth rate.

In Chapter 4 we move to the weakly non-linear regime. We study the late-time non-Gaussianities (NG) of the matter distribution arising from gravitational instability in the cubic covariant Galileon theory. It is well known that NG can be classified in primordial and late-time. The primordial ones come from non-linearities encoded in the inflationary perturbations [69]; these are imprinted in the CMB and in the Large-Scale Structure (LSS) of the universe [70–76], and should be constrained by present and future surveys [77, 78]. The late-time non-Gaussianity in the LSS is generated classically by gravitational instability, when cosmological perturbations enter non-linear scales. While a Gaussian universe can be completely described by the power-spectrum, the deviations from Gaussianity are encoded in higher-order statistics, such as the bispectrum and the trispectrum [79, 80].

The interest in studying the dark matter bispectrum in the Galileon model comes from the possibility to measure the signature of modifications from standard gravity.[1] If this is the case, the bispectrum can be used to lift degeneracies among different models giving rise to the same observed power spectrum and the same background cosmology. We choose Gaussian initial conditions, in order to extract only the late-time non-Gaussianity. In particular, we will focus on the dark matter bispectrum calculated at tree-level (second-order perturbations), since it gives the leading contribution in the weakly non-linear regime. Even though we consider models with important modifications in the background and in the growth rate w.r.t. ΛCDM, we will show that the matter bispectrum deviations that we obtain are less than 5%. We think that this suppression is connected with a compensation effect when the equation of state is $w \lesssim -0.8$. Our results are obtained by using a semi-analytic technique both at first and second-order in perturbations.

In Chapter 5, using the same techniques of Chapter 4, we calculate the DM bispectrum of the *Coupled Galileon* theory. Using the conformal time an the Poisson gauge, we only provide the analytical results. Further investigation of this aspect is left for future work.

In Chapter 6 we focus on perturbations in a highly non-linear regime. This regime allows us to study the spherical collapse model (e.g. [85–87]), which analyzes the evolution of a spherical Dark Matter (DM) overdensity to explain the formation of cosmic structures. We will use the top-hat approximation, taking into account the energy non-conservation problem noted in [86]. This problem affects theories with a time-dependent dark energy component, and it can substantially modify the virialisation process. Our results include the calculation of the linearized density contrast and the virial overdensity, quantities that can be related with observables such as the halo mass function and bias.

In Conclusions we draw our conclusions and provide some comments. In

[1] For other works on the dark matter bispectrum within other modified gravity models see [81–84].

Appendix A we set out the components of the stress-energy tensor found in Chapter 1. In Appendix B we give some useful functions involved in the linear perturbation theory (Sec. 3.1) and in the highly non-linear regime (Sec. 6.1). In Appendix C we derive an equation for the first-order DM perturbations in different gauges (Sec. 3.2). In Appendix D we provide the source terms of the second-order field equations (Sec. 4.1). In Appendix E we show the coefficients of the kernel of the second-order DM fluctuations (Sec. 4.1.1).

Throughout the paper we adopt units $c = \hbar = G = 1$, except where explicitly indicated; our signature is $(-, +, +, +)$. Greek indices run over $\{0, 1, 2, 3\}$, denoting space-time coordinates, whereas Latin indices run over $\{1, 2, 3\}$, labelling spatial coordinates.

CHAPTER 1

LAGRANGIAN AND FIELD EQUATIONS

In this chapter we want to briefly outline the construction, the main ideas and the results of the most general action for the Galileon models [17]. As stated before, this action should preserve the galilean shift symmetry and should avoid Ostrogradski instabilities (i.e. no more than second-order time derivatives in the equations of motion) in a flat space-time. Following [17], the Galilean shift symmetry is respected if the equation of motion takes this form

$$\frac{\delta \mathcal{L}_\pi}{\delta \pi} = F(\partial_\mu \partial_\nu \pi)\,, \tag{1.1}$$

where F is a non-linear Lorentz invariant function. The generic term that satisfies this condition contains $n = 1 \ldots 5$ powers of the Galileon field π and $2n-2$ space-time derivatives. One peculiarity is the fact there are only five distinct operators in a 4-dimensional space-time of this kind. Then, each lagrangian term of order n in π can be schematically written as $\left(\partial^2\pi\right)^{n-2}\partial\pi\partial\pi$. Here, the central point to note is, because of the conserved current associated with the shift symmetry, the equation of motion can be written as a total derivative. Then, the authors proceed in demonstrating that this total derivative is unique and in finding the form of each lagrangian operator.

This construction is done in a flat-space time, and these properties hold exactly. However, if we consider a curved space-time it is necessary to introduce some coupling terms between gravity and the Galileon in order to prevent third-order derivatives in the equations of motion. In this case the shift symmetry is preserved, while the galilean symmetry is softly broken. The resulting theory is the covariant Galileon [20, 21, 46]

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R + \frac{1}{2}\sum_{i=1}^{5} c_i\mathcal{L}_i\right] + \int \mathrm{d}^4x\,\mathcal{L}_M\,, \tag{1.2}$$

where c_{1-5} are dimensionless constants. We consider $\mathcal{L}_M$ as the Lagrangian of a pressurless perfect fluid with density ρ and four-velocity u^μ. The five Lagrangian densities for the scalar field are

$$\mathcal{L}_1 = M^3\pi \tag{1.3}$$
$$\mathcal{L}_2 = (\nabla\pi)^2 \tag{1.4}$$
$$\mathcal{L}_3 = (\Box\pi)(\nabla\pi)^2/M^3 \tag{1.5}$$
$$\mathcal{L}_4 = (\nabla\pi)^2\left[2(\Box\pi)^2 - 2\pi_{;\mu\nu}\pi^{;\mu\nu} - R(\nabla\pi)^2/2\right]/M^6 \tag{1.6}$$
$$\begin{aligned}\mathcal{L}_5 = &(\nabla\pi)^2[(\Box\pi)^3 - 3(\Box\pi)\,\pi_{;\mu\nu}\pi^{;\mu\nu} + 2\pi_{;\mu}{}^{\nu}\pi_{;\nu}{}^{\rho}\pi_{;\rho}{}^{\mu} + \\ &- 6\pi_{;\mu}\pi^{;\mu\nu}\pi^{;\rho}G_{\nu\rho}]/M^9\,,\end{aligned} \tag{1.7}$$

where M is a constant with dimensions of mass. Here, $\mathcal{L}_1$ can be intended as a linear potential, while $\mathcal{L}_2$ is the standard kinetic term. $\mathcal{L}_3$ comes directly from the decoupling limit of DGP theory. $\mathcal{L}_4$ and $\mathcal{L}_5$ provide the full generalization of an action containing at most second derivatives w.r.t. Galilean shift symmetry in a flat space-time.

The galilean shift symmetry imposes severe constraints on the form of the action, however further freedom remains if we add a direct coupling with matter. It was shown in [88] that a linear coupling between π and the stress-energy tensor of matter enters the effective action in the decoupling limit of DGP. In [50] a linear and a derivative coupling were first introduced in the context of the covariant

Galileon. Thus the action, Eq. (1.2), becomes

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\rm pl}^2}{2} R + \frac{1}{2} \sum_{i=1}^{5} c_i \mathcal{L}_i - \mathcal{L}_{\rm m} - \frac{c_{\rm G}}{M_{\rm pl} M^3} T^{\mu\nu} \pi_{;\mu} \pi_{;\nu} - \frac{c_0}{M_{\rm pl}} \pi T \right] . \tag{1.8}$$

where the lagrangians $\mathcal{L}_i$ have the same form as in Eq. (1.2) and c_0 and c_G are two new coupling parameters. For our purposes it is convenient to write Eq. (1.8) in the Jordan frame, where the direct coupling between the Galileon and the matter is removed through a metric redefinition (see Appendix A of [50]) and the stress-energy tensor is covariantly conserved. The Jordan frame action reads

$$S = \int d^4x \sqrt{-g} \left[\left(1 - 2c_0 \frac{\pi}{M_{\rm pl}} \right) \frac{M_{\rm pl}^2}{2} R + \frac{1}{2} \sum_{i=1}^{5} c_i \mathcal{L}_i - \frac{M_{\rm pl}}{M^3} c_{\rm G} G^{\mu\nu} \pi_{;\mu} \pi_{;\nu} - \mathcal{L}_{\rm m} \right] . \tag{1.9}$$

It is important to note that, in a flat space-time, the new terms trivially preserve the galilean shift symmetry, since they vanish.

In the following we will refer to the action (1.2) as the *Uncoupled Galileon*, since it has no direct coupling with the matter fields. Instead we will call the *Cubic Galileon* Eq. (1.2), provided $c_{4,5} = 0$. Eq. (1.9) will be considered as the *Coupled Galileon*.

Varying the action Eq. (1.9) w.r.t. the metric $g_{\mu\nu}$ and the scalar field π we obtain the equations of motion. For the metric we obtain

$$G_{\mu\nu} = M_{\rm pl}^{-2} \left[T^{(m)}{}_{\mu\nu} + T^{(\pi)}{}_{\mu\nu} \right] , \tag{1.10}$$

where we have written the Galileon contribution in terms of a stress-energy tensor given by

$$T^{(\pi)}{}_{\mu\nu} = \sum_{i=1}^{5} c_i T^{(i)}{}_{\mu\nu} + c_0 T^{(0)}{}_{\mu\nu} + c_{\rm G} T^{(G)}{}_{\mu\nu} . \tag{1.11}$$

Here the terms $T^{(i,G,0)}_{\mu\nu}$ are listed in Appendix A. Instead, varying w.r.t. the scalar field, we obtain

$$\sum_{i=1}^{5} c_i \xi^{(i)} + c_0 \xi^{(0)} + c_{\rm G} \xi^{(\rm G)} = 0 , \tag{1.12}$$

where $\xi^{(i,G,0)}$ are also listed in Appendix A.

In the following our matter fluid will be a pressureless perfect fluid, which includes the Dark Matter and the Baryonic components

$$T^{(m)}{}_{\mu\nu} = \rho_m u_\mu u_\nu \,. \tag{1.13}$$

In the Jordan frame the stress-energy tensor continuity equation reads

$$\nabla_\mu T^{(m)\mu}{}_\nu = 0 \,. \tag{1.14}$$

CHAPTER 2

BACKGROUND EVOLUTION

In this chapter we study the background evolution of Eq. (1.2), the Uncoupled Galileon. We perform the analysis in two different contexts.

In Sec. 2.1 we use a flat Friedmann-Lemaître-Robertson-Walker (FLRW) metric with the physical time t

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j \, . \tag{2.1}$$

The Hubble parameter is defined, as usual, by $H(t) = a'(t)/a(t)$. In Eq. (1.2) we define the value of M as $M^3 \equiv M_{\rm pl}H_{\rm dS}^2$. $H_{\rm dS}$ is the value of the Hubble parameter $H(t)$ in a FLRW universe at the de Sitter fixed point. Indeed, as we will see, [46] found a tracker solution that ends at a stable point called “de Sitter point”, at which the energy density of the scalar field dominates. $\mathcal{L}_1$ can be understood as a potential term and for this reason we set $c_1 = 0$, since we are interested in analyzing the contribution of the new kinetic terms (the case in which a standard minimally coupled scalar field is introduced in the field equations was already studied in [89]). Moreover, with this choice we can employ the tracker solution given in [46], that is not admitted if $c_1 \neq 0$.

In Sec. 2.2 we use a FLRW background with the conformal time τ. In this case the metric reads

$$ds^2 = a^2(\tau)\left[-dt^2 + \delta_{ij}dx^i dx^j\right] , \tag{2.2}$$

and we use the the Hubble parameter defined by $\mathcal{H}(\tau) = a'(\tau)/a(\tau)$. We do not use the tracker solution in order to have more freedom in the evolution of the background. In Chapter 4 we shall use the solutions found in this section to study the matter bispectrum generated at late-times by gravitational instability. Our objective is to study how the non-linear Galileon terms in the lagrangian modify the bispectrum, thus, as a first step, we can set $c_{4-5} = 0$, i.e. the Cubic Galileon.

2.1 Tracker solution

Calling $\pi \equiv \pi(t)$ and $\rho \equiv \rho_m(t) + \rho_r(t)$, the background scalar field and background matter and radiation density respectively, the field equations, Eqs. (1.10) and (1.12), read

$$3M_{\rm pl}^2 H^2 = \rho_\pi + \rho_m + \rho_r \,, \tag{2.3}$$

$$3M_{\rm pl}^2 H^2 + 2M_{\rm pl}^2 \dot{H} = -P_\pi - \rho_r/3 \,, \tag{2.4}$$

and

$$\begin{aligned} &c_2\left[3H\dot{\pi} + \ddot{\pi}\right] - \frac{3c_3}{M^3}\dot{\pi}\left[3H^2\dot{\pi} + \dot{H}\dot{\pi} + 2H\ddot{\pi}\right] + \frac{18c_4}{M^6}H\dot{\pi}^2\left[3H^2\dot{\pi} + \right. \\ &\left. + 2\dot{H}\dot{\pi} + 3H\ddot{\pi}\right] - \frac{15c_5}{M^9}H^2\dot{\pi}^3\left[3H^2\dot{\pi} + 3\dot{H}\dot{\pi} + 4H\ddot{\pi}\right] = 0 \,, \end{aligned} \tag{2.5}$$

where

$$\rho_\pi \equiv -\frac{c_2}{2}\dot{\pi}^2 + \frac{3c_3}{M^3}H\dot{\pi}^3 - \frac{45c_4}{2M^6}H^2\dot{\pi}^4 + \frac{21c_5}{M^9}H^3\dot{\pi}^5 \,, \tag{2.6}$$

$$\begin{aligned} P_\pi \equiv &-\frac{c_2}{2}\dot{\pi}^2 - \frac{c_3}{M^3}\dot{\pi}^2\ddot{\pi} + \frac{3c_4}{2M^6}\dot{\pi}^3[8H\ddot{\pi} + (3H^2 + 2\dot{H})\dot{\pi}] + \\ &- \frac{3c_5}{M^9}H\dot{\pi}^4[5H\ddot{\pi} + 2(H^2 + \dot{H})\dot{\pi}] \,, \end{aligned} \tag{2.7}$$

are scalar field density and pressure, respectively.

As in [46], to study the background we work with the new variables

$$r_1 \equiv \dot{\pi}_{\rm dS}H_{\rm dS}/(\dot{\pi}H) \,, \quad r_2 \equiv (\dot{\pi}/\dot{\pi}_{\rm dS})^4/r_1 \,, \quad \Omega_r = \rho_r/(3M_{\rm pl}^2 H^2) \,, \tag{2.8}$$

where $\dot{\pi}_{\rm dS}$ is the time derivative of the scalar field at the dS point. At this point Eqs. (2.3) and (2.4) becomes

$$c_2 x_{\rm dS}^2 = 6 + 9\alpha - 12\beta \,, \tag{2.9}$$

$$c_3 x_{\rm dS}^3 = 2 + 9\alpha - 9\beta \,, \tag{2.10}$$

where $x_{\rm dS} \equiv \dot{\pi}_{\rm dS}/(H_{\rm dS} M_{\rm pl})$. These equations give two conditions for the coefficients c_2 and c_3. We also set $\alpha \equiv c_4 x_{\rm dS}^4$ and $\beta \equiv c_5 x_{\rm dS}^5$; therefore our free parameters become α, β and $x_{\rm dS}$. For simplicity, the assumption $x_{\rm dS} = 1$ will be often used in the next chapters. An approximation we have done is $H_{\rm dS} \simeq H_0$, where H_0 is the value of the Hubble parameter today.

As we already mentioned, [46] found a stable tracker solution ($r_1 = 1$), which drives the universe expansion from the radiation-dominated epoch ($r_2 \ll 1$, $\Omega_r = 1$), through the matter-dominated epoch ($r_2 = 1$, $\Omega_r \ll 1$), until the dS point ($r_2 = 1$, $\Omega_r = 0$). Note that along $r_1 = 1$, $\Omega_\pi \equiv \rho_\pi/(3M_{\rm pl}^2 H^2) = r_2$. Following this solution, Eqs. (2.4) and (1.12) with our new variables can be written as

$$r_2' = \frac{2r_2\,(3 - 3r_2 + \Omega_r)}{1 + r_2}, \qquad \Omega_r' = \frac{\Omega_r\,(\Omega_r - 1 - 7r_2)}{1 + r_2} \,, \tag{2.11}$$

where primes denote differentiation w.r.t. $N = \ln a$. In Fig. 2.1 we show the numerical solution of these equations with boundary conditions $\Omega_{r_0} = 4.8 \cdot 10^{-5}$ and $\Omega_{\Lambda_0} = 0.74$, where Ω_{r_0} and Ω_{Λ_0} are the density parameter values today, for the radiation and the dark energy component, respectively. These equations cannot be solved analytically; however we have found two analytic functions that approximate the numerical results with an accuracy better than 1.2% at redshift $z \lesssim 21$

$$r_2(N) \simeq 1 + \left[\frac{(1-\Omega_{\Lambda_0})^2}{2\Omega_{\Lambda_0}} - \frac{1-\Omega_{\Lambda_0}}{2\sqrt{\Omega_{\Lambda_0}}} \cdot \sqrt{4e^{6N} + \frac{(1-\Omega_{\Lambda_0})^2}{\Omega_{\Lambda_0}}} \right] \cdot e^{-6N} \,, \tag{2.12}$$

and

$$\Omega_r(N) \simeq 2\Omega_{r_0} e^{-N} \Big(1 - \Omega_{\Lambda_0} + \sqrt{4\Omega_{\Lambda_0} e^{6N} + (1-\Omega_{\Lambda_0})^2}\Big)^{-1} \,. \tag{2.13}$$

To study the stability of the solution $r_1(N) = 1$, Eqs. (2.3), (2.4) and (2.5) can be expanded at linear order in perturbations δr_1, δr_2 and $\delta\Omega_r$. Thus, it can be obtained

$$\delta {r_1}'(N) = -\frac{9 + \Omega_r(N) + 3r_2(N)}{2\,[1 + r_2(N)]} \delta r_1(N) \,, \tag{2.14}$$

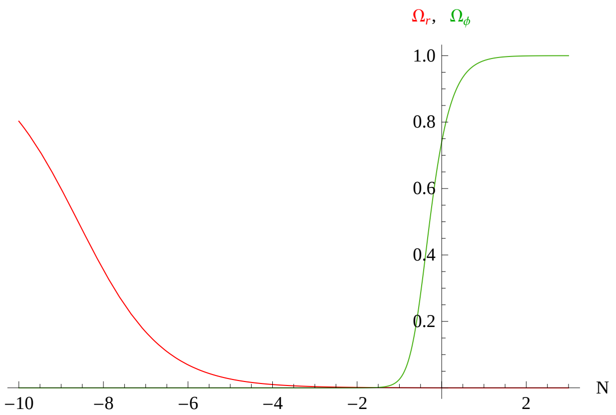

Figure 2.1: In the figure we show the evolution of Ω_r (red line) and Ω_π (green line), functions of $N = \ln a$.

which reads

$$\delta r_1(N) = \delta r_1(0) \exp\left[-\int_0^N dN' \frac{9 + \Omega_r(N') + 3r_2(N')}{2\,(1 + r_2(N'))}\right] \leq f_0\, e^{-\frac{9}{2}N}\,. \qquad (2.15)$$

f_0 is a finite integration constant, and this relation proves that any solution that approaches $r_1(N) = 1$, finally reaches it. Therefore we shall suppose that at least after the matter-dominated epoch the evolution of the universe can be described by $\delta r_1 \ll 1$.

In [46], the authors also find constraints on the parameters α and β (assuming $x_{\rm dS} = 1$). These constraints follow from the requirement of ghost avoidance. They study scalar (S) and tensor (T) perturbations, expanding the action Eq. (1.2) at second-order in perturbation theory (see [90, 91], for the complete procedure), finding conditions for the sign of the kinetic term (Q_S and Q_T) and the squared sound speed (${c_S}^2$ and ${c_T}^2$). Thus, in every epoch we have four conditions that must be satisfied. Reminding that α and β are constants, we can find a region of parameter space where no ghost modes exist. This area is bounded by the analytic functions

$$\begin{cases} \alpha > 2\,\beta \\ \alpha < 2\,\beta + 2/3 \\ \alpha < 12\,\sqrt{\beta} - 9\,\beta - 2 \\ \alpha > 12/13\,\beta + 10/13\,. \end{cases} \qquad (2.16)$$

2.2 General solution

In this section we study the non-tracker background evolution of Eqs. (1.10) and (1.12) in the context of the Cubic Galileon theory. Let $\pi \equiv \pi(\tau)$ be the Galileon field at the background level and $\rho_m(\tau)$ and $\rho_\pi(\tau)$ the background matter and the Galileon energy density respectively. The $(0,0)$ and the (i,i) components of the Einstein equations read

$$\frac{3M_{\rm pl}^2\mathcal{H}^2}{a^2} = \rho_m + \rho_\pi \,, \tag{2.17}$$

$$\frac{M_{\rm pl}^2}{a^2}\left(\mathcal{H}^2 + 2\mathcal{H}'\right) = -p_\pi \,, \tag{2.18}$$

where primes represent derivatives w.r.t. the conformal time τ and

$$\rho_\pi \equiv -\frac{c_1 M^3}{2}\pi - \frac{c_2}{2a^2}\pi'^2 + \frac{3c_3}{M^3a^4}\mathcal{H}\pi'^3 \,, \tag{2.19}$$

$$p_\pi \equiv \frac{c_1 M^3}{2}\pi - \frac{c_2}{2a^2}\pi'^2 - \frac{c_3}{M^3a^4}\pi'^2\left(\pi'' - \mathcal{H}\pi'\right) \,, \tag{2.20}$$

are the scalar field density and pressure, respectively. The equation of motion for the Galileon, Eq. (1.12), becomes

$$\frac{c_1M^3}{2} + \frac{c_2}{a^2}\left[\pi'' + 2\mathcal{H}\pi'\right] - \frac{3c_3}{M^3a^4}\pi'\left[2\mathcal{H}\pi'' + \mathcal{H}'\pi'\right] = 0 \,. \tag{2.21}$$

Here, without loss of generality, we have defined $M^3 \equiv M_{\rm pl}\mathcal{H}_0^2$. Here $\mathcal{H}_0$ is the value of the Hubble parameter $\mathcal{H}(\tau)$ in a FLRW universe today.

We have studied the background evolution solving Eqs. (2.18) and (2.21). The initial conditions are determined fixing an initial vacuum energy density $\rho_\pi(\tau_i)$ and using the background equations in the regime $\rho_\pi(\tau) \ll \rho_m(\tau)$. The DM and the DE energy densities today are $\Omega_m(\tau_0) \equiv \rho_m(\tau_0)/(3M_{\rm pl}^2\mathcal{H}_0^2) = 0.27$ and $\Omega_\pi(\tau_0) \equiv \rho_\pi(\tau_0)/(3M_{\rm pl}^2\mathcal{H}_0^2) = 0.73$ respectively [5]. We take into account the parameter $c_1 \neq 0$, which is the most general potential term preserving the Galilean shift symmetry. It acts as a cosmological constant in the case $\pi' \to 0$.

In Fig. 2.2 we show the evolution of $\mathcal{H}(a)$, $\Omega_\pi(a)$ and the equation of state $w_\pi(a) \equiv \rho_\pi(a)/p_\pi(a)$ for the models we are considering. In the limit $c_1 \to 0$ (green line) we have noted that the evolution of the background is c_2 and c_3 independent. This behavior is expected because if c_2 (or c_3) is absorbed through

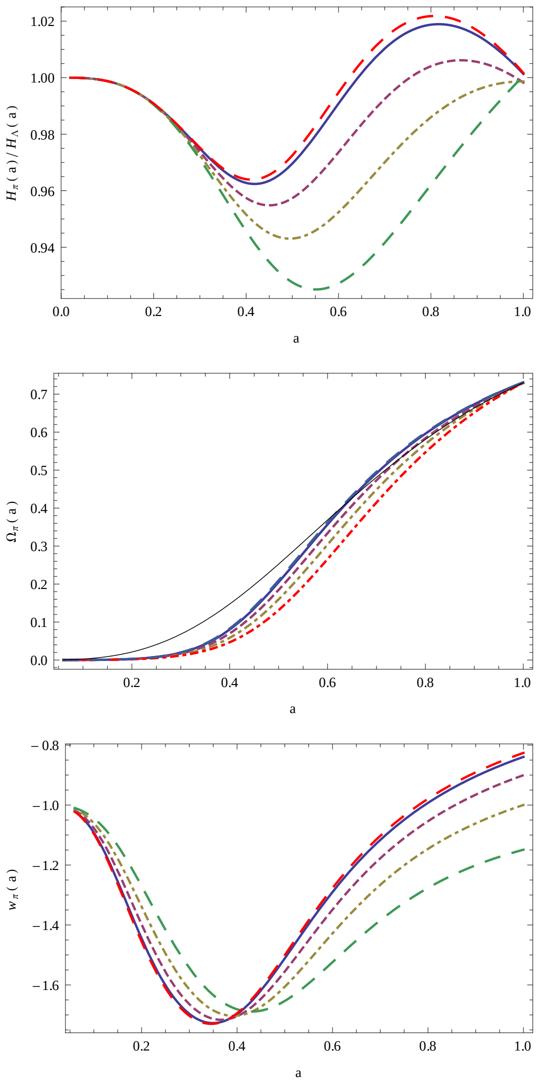

Figure 2.2: Background evolution of the Galileon model. In the top panels we plot the deviations of the Hubble parameter w.r.t. the Hubble parameter in ΛCDM and the evolution of the DE density. In the bottom panel we plot $w_\pi(a) \equiv \rho_\pi(a)/p_\pi(a)$. The parameter values are: $c_1 = 1.6$, $c_2 = 0.04$, $c_3 = 10^{-3}$ (red line); $c_1 = 1.5$, $c_2 = 0.04$, $c_3 = 10^{-3}$ (blue line); $c_1 = 11$, $c_2 = 3.8$, $c_3 = 1$ (purple line); $c_1 = 6$, $c_2 = 3.6$, $c_3 = 1$ (yellow line); $c_1 = 10^{-4}$, $c_2 = 3.3$, $c_3 = 1$ (green line); ΛCDM (black line).

a redefinition of the Galileon field, c_3 (or c_2) is constrained by the condition $\Omega_\pi(\tau_0) = 0.73$. In order to have a free parameter that allows to decrease the difference between ΛCDM and our Galileon models it is crucial to impose $c_3 \sim c_1 \neq 0$.

CHAPTER 3

LINEAR PERTURBATION THEORY

In this Chapter we focus on the linear perturbation theory. In Sec. 3.1 we solve the linear equations using the tracker solution and the notation given in Sec. 2.1. In Sec. 3.2 we analyze the linear growth rate using the general solution and the notation given in Sec. 2.2.

3.1 Tracker solution

In this section we study the evolution of the scalar perturbations on sub-horizon scales. Our work focuses on the dynamics of a spherically symmetric perturbed metric. Let us choose the Newtonian gauge,

$$ds^2 = -(1+2\Psi)dt^2 + a^2(t)(1+2\Phi)\delta_{ij}dx^i dx^j \,. \tag{3.1}$$

Perturbations of the energy density and the scalar field are given by

$$\rho(\vec{x},t) \equiv \rho_0(t) + \delta\rho(\vec{x},t) \qquad \pi(\vec{x},t) \equiv \pi_0(t) + \varphi(\vec{x},t)\,. \tag{3.2}$$

In the following we will drop the suffix "0". In this regime there are two valid approximations that simplify the field equations. The first one is the sub-horizon approximation $\mathcal{O}(\nabla^2\Phi/a^2) \gg \mathcal{O}(H^2\Phi)$. The second one is the quasi-static approximation, which allows us to neglect time derivatives of perturbations compared with space derivatives, assuming we are working with non-relativistic matter at short distances.

Replacing physical gradients with comoving gradients, at linear order Eqs. (1.10) and (1.12) become (∇ denotes a spatial gradient):

$$\left(2M_{\rm pl}^2 + \dot{\pi}^2\gamma_1(t)\right)\nabla^2\Phi = -\delta\rho + \gamma_2(t)\nabla^2\varphi\,, \tag{3.3}$$

$$\left(2M_{\rm pl}^2 + 3\gamma_3(t)\right)\nabla^2\Phi + \left(2M_{\rm pl}^2 + \dot{\pi}^2\gamma_1(t)\right)\nabla^2\Psi = 3\gamma_4(t)\nabla^2\varphi\,, \tag{3.4}$$

and

$$\gamma_5(t)\nabla^2\varphi + \gamma_2(t)\nabla^2\Psi + 3\gamma_4(t)\nabla^2\Phi = 0\ , \tag{3.5}$$

where $\gamma_i(t)$ are functions of the background, whose explicit form is given in Appendix B. It is important to note that one of the differences between these equations and those for the kinetic braiding model studied in [87] is the presence of an anisotropic stress in the RHS of Eq. (3.4).

Manipulating Eqs. (3.3) and (3.4), we obtain the modified Poisson equation

$$\frac{\left(2M_{\rm pl}^2 + \dot{\pi}^2\gamma_1\right)^2}{2M_{\rm pl}^2 + 3\gamma_3}\nabla^2\Psi = \delta\rho - \left[\gamma_2 - 3\gamma_5\frac{2M_{\rm pl}^2 + \dot{\pi}^2\gamma_1}{2M_{\rm pl}^2 + 3\gamma_3}\right]\nabla^2\varphi\,. \tag{3.6}$$

Using Eqs. (3.5), (3.3) and (3.6), the differential equation for the evolution of the scalar field takes the form

$$\nabla^2\varphi = A(t)\,\delta\rho(t,\vec{r}), \tag{3.7}$$

where

$$A(t) \equiv \frac{\gamma_2(t)\gamma_7(t) - 3\gamma_4(t)\gamma_6(t)}{\gamma_2(t)^2\gamma_7(t) - \gamma_5(t)\gamma_6(t)^2 - 6\gamma_2(t)\gamma_4(t)\gamma_6(t)}\ , \tag{3.8}$$

with

$$\gamma_6(t) \equiv \left[2M_{\rm pl}^2 + \dot{\pi}^2\gamma_1(t)\right] \tag{3.9}$$

$$\gamma_7(t) \equiv \left[2M_{\rm pl}^2 + 3\gamma_3(t)\right] . \tag{3.10}$$

Considering a spherically symmetric object of radius R_S, we can easily integrate Eq. (3.7) to obtain an analytic expression for the evolution of the scalar field. Defining $m(t,r) \equiv 4\pi \int_0^r \mathrm{d}r' r'^2 \delta\rho$, we obtain

$$\frac{\mathrm{d}\varphi}{\mathrm{dr}} = \frac{A(t)\, m(t,r)}{4\pi r^2} + \frac{C}{r^2} , \tag{3.11}$$

where C is an integration constant that, outside the source, can be viewed as an increase in $M_s \equiv m(t, R_S)$. While this term is present in φ', it does not enter in $\nabla^2\varphi$, so that the gravitational potential is not affected by our choice of C. Therefore, for our purposes we can set $C = 0$.

3.1.1 The Vainshtein mechanism and the linear regime

The Vainshtein mechanism works by screening the effects of the scalar field on the gravitational potential at small distances, so that one can satisfy the constraints coming from solar-system tests, while preserving the accelerated expansion of the universe on cosmological scales. The difference between this mechanism and the Chamaleon one is that the first also works by using non-linearities of the perturbations to this aim. At large distances ($r \gg r_V$, where r_V is the Vainshtein radius of the source) linear terms of the scalar field become dominant, while for $r \ll r_V$ non-linear terms become dominant (these terms will be shown in Eqs. (6.1), (6.2) and (6.3)). This is called "self-screening effect". A discussion about the magnitude of the Vainshtein radius (r_V) of a spherically symmetric source will be given later (Sec. 6.1.1).

A first approach is to study within the linear approximation the contribution of the scalar field to the gravitational potential. Recalling Eq. (3.6), to have a qualitative knowledge that outside the Vainshtein radius the scalar field drives the late time cosmic acceleration, we have to compare the contribution of the gravitational with the scalar field intensity [92]. Indeed, our request is that the two are comparable:

$$\frac{\varphi'(r)}{\Psi'(r)} \simeq 1 . \tag{3.12}$$

It can be shown that the above ratio is a monotone function, which starts from $\simeq 0$ during the radiation-matter-dominated epoch. At the dS point, recalling Eq. (3.7) with $x_{\rm dS} = 1$, we obtain

$$\left|\frac{\varphi'(r)}{\Psi'(r)}\right|_{\rm dS} = \left|\frac{A(t_{\rm dS})}{4\pi}\right| = \left|\frac{1}{24\pi M_{\rm pl}(2\beta - \alpha)}\right| . \tag{3.13}$$

Taking into account the region in the plane $(x_{\rm dS} = 1, \beta, \alpha)$ bounded by the no-ghost condition (2.16), it can be shown that the magnitude of the last ratio at the dS point is bounded by

$$\frac{1}{4\sqrt{2\pi}} < \left|\frac{\varphi'(r)}{\Psi'(r)}\right|_{\rm dS} < +\infty \tag{3.14}$$

This result means that the contribution of the scalar field at the dS point on scales $r \gg r_V$ is always important, and the importance can be set choosing proper values for α and β. In particular we can find a couple (α, β) which satisfies Eq. (3.12).

With Eq. (3.7) we can write the modified Poisson equation (3.6) in a more convenient form

$$\nabla^2\Psi = 4\pi G_\pi \delta\rho(t, \vec{r}) , \tag{3.15}$$

where

$$G_\pi(t) = \frac{\gamma_5(t)\gamma_7(t) + 9\gamma_4(t)^2}{4\pi\left[6\gamma_2(t)\gamma_4(t)\gamma_6(t) - \gamma_2(t)^2\gamma_7(t) + \gamma_5(t)\gamma_6(t)^2\right]} . \tag{3.16}$$

The modified gravitational constant assumes the value of the Newtonian one during the radiation-matter-dominated era, while it is

$$G_\pi(t_{\rm dS}) = \frac{G}{3(\alpha - 2\beta)} \tag{3.17}$$

at the dS point (when $x_{\rm dS} = 1$). The limit $x_{\rm dS} \to 0$ gives us the usual GR result $G_\pi(t_{\rm dS}) = G$. Instead, the limit $x_{\rm dS} \to \infty$ gives $G_\pi(t_{\rm dS}) \to 0$, which means, as expected, that the effective gravitational constant becomes small w.r.t. the Newtonian one ($G \propto M_{\rm pl}^{-2}$). The plots in Figs. 3.1, 3.2 and 3.3 show that we can vary the asymptotic value of G_π as we desire, to obtain, in principle, any reasonable model for the late time cosmic acceleration. The difference between the three graphs is the value of the parameter $x_{\rm dS}$, which sets the contribution of the Galileon field at the dS point. This result also agrees with the expectations of Eq. (3.14), quantifying the effective contribution of the scalar field at large distances on observables quantities. Of course, these results do not represent any realistic model, we are only interested here in investigating the range of possibilities offered by the Galileon theory. Moreover, astrophysical and cosmological constraints on the Galileon model have just started being considered [93–96].

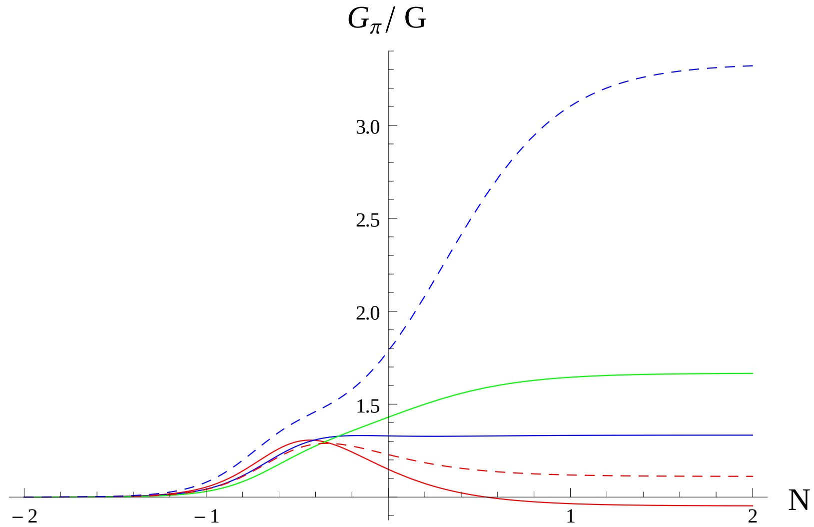

Figure 3.1: This plot shows the evolution of G_π, with $x_{\rm dS} = 1$, in different cases.The values for (α, β) are: $(-1, -0.55)$, blue dashed line; $(-0.45, -0.4)$, red line; $(-0.2, -0.2)$, green line; $(-0.55, -0.4)$, blue solid line; $(0.1, -0.1)$, red dashed line.

3.2 General solution

In this section we give some definitions needed to analyze the evolution of the DM perturbations on sub-horizon scales [97]. Without choosing any gauge the metric can be written as

$$ds^2 = a(\tau)^2 \left[-(1+2\psi)d\tau^2 + 2\hat{\omega}_i dx^i d\tau + [(1-2\phi)\delta_{ij} + \hat{\chi}_{ij}]\, dx^i dx^j\right] . \quad (3.18)$$

Here the dependence of all the perturbations on both the conformal time τ and the spatial coordinates $\vec{x}$ is implicit. The symmetric trace-free perturbation $\hat{\chi}_{ij}$ and $\hat{\omega}_i$ can be decomposed as

$$\hat{\omega}_i \equiv \omega_i + \partial_i \omega\,, \quad (3.19)$$

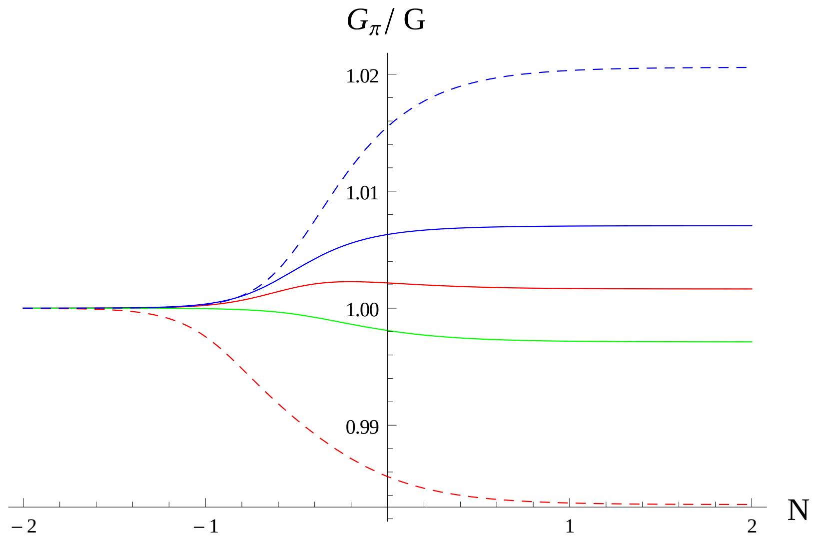

Figure 3.2: The same as in Fig. 3.1, but with $x_{\rm dS} = 0.3$.

$$\hat{\chi}_{ij} \equiv \chi_{ij} + \partial_i \chi_j + \partial_j \chi_i + D_{ij}\chi\,, \tag{3.20}$$

where ω_i and χ_i are transverse vectors (i.e. $\delta^{ij}\partial_i\omega_j = 0$), χ_{ij} is a trace-free transverse symmetric tensor ($\delta^{ij}\chi_{ij} = \delta^{ij}\partial_i\chi_{jk} = 0$) and D_{ij} is a trace-free operator defined by $D_{ij} \equiv \partial_i\partial_j - (1/3)\delta_{ij}\nabla^2$. Perturbations of the energy-density and the four-velocity of the DM fluid can be written as

$$\rho(\vec{x},\tau) \equiv \rho^{(0)}(\tau)\left[1 + \delta(\vec{x},\tau)\right]\,, \tag{3.21}$$

$$u^\mu(\vec{x},\tau) \equiv \frac{1}{a}\left[\delta^\mu{}_0 + v^\mu(\vec{x},\tau)\right]\,. \tag{3.22}$$

We can expand any perturbation up to the desired order in this way

$$\pi \simeq \pi^{(0)} + \pi^{(1)} + \frac{1}{2}\pi^{(2)} + \ldots + \frac{1}{n!}\pi^{(n)}\,. \tag{3.23}$$

In the following we will drop the suffix “0”. At first-order we can safely neglect vector and tensor perturbations. In fact the first-order vector perturbations have decreasing amplitudes and are not generated by the presence of a scalar field. Moreover, the first-order tensor perturbations give a negligible contribution to second-order perturbations. This result cannot be generalized to second-order perturbations, since second-order vector and tensor perturbations are generated by products of first-order scalars.

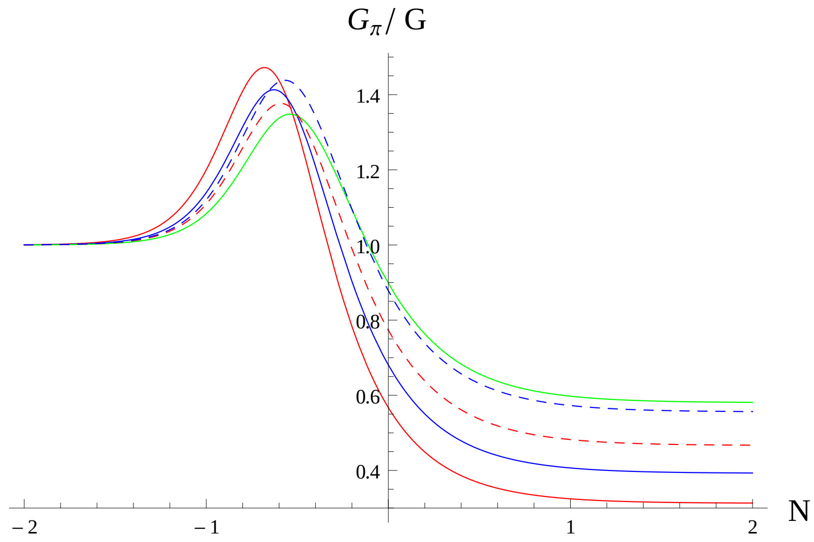

Figure 3.3: The same as in Fig. 3.1, but with $x_{\rm dS} = 1.2$.

Perturbing the well-known relation $u^\mu u_\mu = -1$, these useful equations can be obtained

$$v^{(1)0} = -\psi^{(1)}\,, \tag{3.24}$$

$$v^{(2)0} = -\psi^{(2)} + 3{\psi^{(1)}}^2 + 2\partial_i\omega^{(1)}\partial^i v^{(1)} + \partial_i v^{(1)}\partial^i v^{(1)}\,. \tag{3.25}$$

At the linear level from Eq. (1.10) we obtain four independent equations, the $(0,0)$, the $(0,i)$, the trace and the traceless of (i,j) parts. These are, respectively,

$$\begin{aligned}
&2{M_{\rm pl}}^2\nabla^2\phi^{(1)} + \frac{1}{3}{M_{\rm pl}}^2\nabla^2\nabla^2\chi^{(1)} - \left(6{M_{\rm pl}}^2\mathcal{H} - \frac{3c_3\pi'^3}{M^3a^2}\right)\phi^{(1)\prime} \\
&-\left(2{M_{\rm pl}}^2\mathcal{H} - \frac{c_3\pi'^3}{M^3a^2}\right)\nabla^2\omega^{(1)} = a^2\rho_m\delta^{(1)} + \left(c_2\pi'^2 + 6{M_{\rm pl}}^2\mathcal{H}^2 - \frac{12c_3\pi'^3\mathcal{H}}{M^3a^2}\right)\psi^{(1)} \\
&-\frac{c_3\pi'^2\nabla^2\pi^{(1)}}{M^3a^2} - \frac{1}{2}c_1M^3a^2\pi^{(1)} + \left(-c_2\pi' + \frac{9c_3\pi'^2\mathcal{H}}{M^3a^2}\right)\pi^{(1)\prime}\,,
\end{aligned} \tag{3.26}$$

$$\begin{aligned}
&-2{M_{\rm pl}}^2\phi^{(1)\prime} - \left(2{M_{\rm pl}}^2\mathcal{H} - \frac{c_3\pi'^3}{M^3a^2}\right)\psi^{(1)} - \frac{1}{3}{M_{\rm pl}}^2\nabla^2\chi^{(1)\prime} = a^2\rho_m v^{(1)} \\
&+a^2\rho_m\omega^{(1)} + \left(c_2\pi' - \frac{3c_3\pi'^2\mathcal{H}}{M^3a^2}\right)\pi^{(1)} + \frac{c_3\pi'^2\pi^{(1)\prime}}{M^3a^2}\,,
\end{aligned} \tag{3.27}$$

$$\frac{2}{3}{M_{\rm pl}}^2\nabla^2\left(\psi^{(1)}-\phi^{(1)}+\omega^{(1)\prime}+2\mathcal{H}\omega^{(1)}-\frac{1}{6}\nabla^2\chi^{(1)}\right)+2{M_{\rm pl}}^2\phi^{(1)\prime\prime}$$
$$+\left(2{M_{\rm pl}}^2\mathcal{H}^2+4{M_{\rm pl}}^2\mathcal{H}'-c_2\pi'^2-\frac{4c_3\pi''\pi'^2}{M^3a^2}+\frac{4c_3\pi'^3\mathcal{H}}{M^3a^2}\right)\psi^{(1)}$$
$$+\left(2{M_{\rm pl}}^2\mathcal{H}-\frac{c_3\pi'^3}{M^3a^2}\right)\psi^{(1)\prime}=-\left(c_2\pi'+\frac{2c_3\pi''\pi'}{M^3a^2}-\frac{3c_3\pi'^2\mathcal{H}}{M^3a^2}\right)\pi^{(1)\prime}$$
$$+4{M_{\rm pl}}^2\mathcal{H}\phi^{(1)\prime}+\frac{1}{2}c_1M^3a^2\pi^{(1)}-\frac{c_3\pi'^2\pi^{(1)\prime\prime}}{M^3a^2}\,, \tag{3.28}$$

$$\chi^{(1)\prime\prime}+2\mathcal{H}\chi^{(1)\prime}+\frac{1}{3}\nabla^2\chi^{(1)}-2\omega^{(1)\prime}-4\mathcal{H}\omega^{(1)}+2\phi^{(1)}-2\psi^{(1)}=0\,. \tag{3.29}$$

The equation of motion for the linear perturbation of the Galileon field, Eq. (1.12), reads

$$\left(c_2-\frac{2c_3\left(\pi''+\mathcal{H}\pi'\right)}{M^3a^2}\right)\nabla^2\pi^{(1)}=\left(2c_2\mathcal{H}-\frac{6c_3\pi'\mathcal{H}'}{M^3a^2}-\frac{6c_3\pi''\mathcal{H}}{M^3a^2}\right)\pi^{(1)\prime}$$
$$+\left(c_2-\frac{6c_3\pi'\mathcal{H}}{M^3a^2}\right)\pi^{(1)\prime\prime}+\left(-2c_2\pi''-4c_2\pi'\mathcal{H}+\frac{12c_3\pi'^2\mathcal{H}'}{M^3a^2}+\frac{24c_3\pi''\pi'\mathcal{H}}{M^3a^2}\right)\psi^{(1)}$$
$$+\left(-c_2\pi'+\frac{9c_3\pi'^2\mathcal{H}}{M^3a^2}\right)\psi^{(1)\prime}+\frac{c_3\pi'^2\nabla^2\psi^{(1)}}{M^3a^2}+\left(-3c_2\pi'+\frac{6c_3\pi''\pi'}{M^3a^2}+\frac{9c_3\pi'^2\mathcal{H}}{M^3a^2}\right)\phi^{(1)\prime}$$
$$+\frac{3c_3\pi'^2\phi^{(1)\prime\prime}}{M^3a^2}+\left(-c_2\pi'+\frac{2c_3\pi''\pi'}{M^3a^2}+\frac{3c_3\pi'^2\mathcal{H}}{M^3a^2}\right)\nabla^2\omega^{(1)}+\frac{c_3\pi'^2\nabla^2\omega^{(1)\prime}}{M^3a^2}\,. \tag{3.30}$$

From the time and the space components of the stress-energy tensor continuity equation, Eq. (1.14), we obtain

$$\delta^{(1)\prime}=3\phi^{(1)\prime}-\nabla^2v^{(1)}\,, \tag{3.31}$$

$$\omega^{(1)\prime}+v^{(1)\prime}+\mathcal{H}\omega^{(1)}+\psi^{(1)}+\mathcal{H}v^{(1)}=0\,. \tag{3.32}$$

There are many ways to decouple these equations . First of all, it is convenient to work in Fourier space. From Eqs. (3.29) we can immediately obtain $\psi^{(1)}$. In the sub-horizon ($k^2\gg\mathcal{H}^2$) and quasi-static ($|\phi''|\lesssim\mathcal{H}\,|\phi'|\ll k^2\,|\phi|$) approximation, the relevant equations we need are (3.26), (3.30) and the derivative of (3.31)

$$2{M_{\rm pl}}^2k^2\left(\phi^{(1)}-\frac{1}{6}k^2\chi^{(1)}\right)-\left(2{M_{\rm pl}}^2\mathcal{H}-\frac{c_3\pi'^3}{M^3a^2}\right)k^2\omega^{(1)}$$

$$= -\rho_m \delta^{(1)} a^2 - \frac{c_3 \pi'^2 k^2 \pi^{(1)}}{M^3 a^2}\,, \tag{3.33}$$

$$\left(c_2 - \frac{2c_3\pi''}{M^3a^2} - \frac{2c_3\pi'\mathcal{H}}{M^3a^2}\right)\pi^{(1)} + \left(c_2\pi' - \frac{2c_3\pi''\pi'}{M^3a^2} - \frac{c_3\pi'^2\mathcal{H}}{M^3a^2}\right)\omega^{(1)}$$
$$= \frac{c_3\pi'^2}{M^3a^2}\left(\phi^{(1)} - \frac{1}{6}k^2\chi^{(1)}\right)\,, \tag{3.34}$$

$$\delta^{(1)''} + \delta^{(1)'}\mathcal{H} + k^2\left(\phi^{(1)} - \frac{1}{6}k^2\chi^{(1)}\right) - \mathcal{H}k^2\omega^{(1)} = 0\,. \tag{3.35}$$

Combining Eqs. (3.33) and (3.34) to eliminate $\phi^{(1)}$ and $\chi^{(1)}$ it is straightforward to obtain

$$\left(c_2 {M_{\rm pl}}^2 + \frac{{c_3}^2\pi'^4}{2M^6a^4} - \frac{2c_3{M_{\rm pl}}^2\left(\pi'' + \mathcal{H}\pi'\right)}{M^3a^2}\right)k^2\left[\pi^{(1)} + \pi'\omega^{(1)}\right] = -\frac{c_3\rho_m\pi'^2\delta^{(1)}}{2M^3}\,. \tag{3.36}$$

Finally, using Eqs. (3.33), (3.35) and (3.36) we are able to single out an equation for the DM perturbation $\delta^{(1)}$

$$\delta^{(1)''} + \mathcal{H}\delta^{(1)'} = 4\pi G\left(1 - \frac{{c_3}^2\pi'^4}{2c_2M^6M_{\rm pl}^2a^4\alpha}\right)a^2\rho_m\delta^{(1)}\,, \tag{3.37}$$

where $G \equiv \left(8\pi M_{\rm pl}^2\right)^{-1}$ is Newton's constant and we have defined

$$\alpha(\tau) \equiv 1 - \frac{2c_3}{c_2M^3a^2}\left(\pi'' + \mathcal{H}\pi'\right) + \frac{{c_3}^2\pi'^4}{2c_2M^6M_{\rm pl}^2a^4}\,. \tag{3.38}$$

The crucial difference between Eq. (3.37) and the one obtained in the ΛCDM model is that the Galileon acts modifying the Newton's constant at late-times. To recover the standard Newton's constant it is sufficient to set $c_3 = 0$. On the left hand side of Eq. (3.37) the other modification lies inside the friction term ($\mathcal{H}\delta^{(1)'}$) due to the evolution of the Hubble parameter. As shown in Fig. 2.2 these differences cannot be neglected and should play an important role in the growth of structures.

Eq. (3.37), which describes the dynamics of DM perturbations on sub-horizon scales, together with Eqs. (3.31), (3.32), (3.34) and (3.36) forms our complete set of equations that allow to solve the dynamics of the fluctuations at first-order. In Appendix C we show how to obtain the same result in the Poisson, spatially flat and synchronous gauges. In particular it is important to pay attention doing the sub-horizon approximation in the synchronous gauge, due to the residual gauge freedom.

Eq. (3.37) can be divided in the linear combination of two independent solutions

$$\delta^{(1)}(\vec{k},\tau) = c_+ D_+(\tau)\delta^{(1)}(\vec{k}) + c_- D_-(\tau)\delta^{(1)}(\vec{k}) \,, \tag{3.39}$$

where $\delta^{(1)}(\vec{k})$ is the primordial amplitude of the density contrast perturbation. We have also added explicitly two integration constants, c_+ and c_-. $D_+(\tau)$ and $D_-(\tau)$ are the growing and the decaying modes and they depend on the coefficients c_i. In the next subsection we will find an integral solution for these modes.

3.2.1 Integral solutions for the growing and the decaying modes of DM perturbations

To solve Eq. (3.37) it is convenient to redefine

$$A(\tau) \equiv \frac{4\pi G}{\mathcal{H}^2}\left(1 - \frac{{c_3}^2 \pi'^4}{2c_2 M^6 M_{\rm pl}^2 a^4 \alpha}\right)\rho_m \,. \tag{3.40}$$

We can also use the scale factor as the new time variable

$$\frac{d^2\delta(a)}{da^2} + \left(\frac{2}{a} + \frac{1}{\mathcal{H}(a)}\frac{d\mathcal{H}(a)}{da}\right)\frac{d\delta(a)}{da} = A(a)\delta(a) \,. \tag{3.41}$$

After that we can perform the change of the variable

$$\delta(a) = u(a)\sqrt{\frac{\mathcal{H}_0}{a^2\mathcal{H}(a)}} \,. \tag{3.42}$$

After a straightforward calculation we shall obtain Eq. (3.37) in its normal form

$$\frac{d^2u(a)}{da^2} - I(a)u(a) = 0 \,, \tag{3.43}$$

where $(-I(a))$ is often called the *invariant* of the equation

$$I(a) = A(a) + \frac{1}{a\mathcal{H}(a)}\frac{d\mathcal{H}(a)}{da} - \frac{1}{4\mathcal{H}(a)^2}\left(\frac{d\mathcal{H}(a)}{da}\right)^2 + \frac{1}{2\mathcal{H}(a)}\frac{d^2\mathcal{H}(a)}{da^2} \,. \tag{3.44}$$

Now, suppose we have to solve

$$\frac{d^2y(a)}{da^2} + g(a)\frac{dy(a)}{da} + \frac{dg(a)}{da}y(a) = 0\,. \tag{3.45}$$

After the substitution

$$Y(a) = y(a)e^{+\frac{1}{2}\int_{a_m}^{a} da' g(a')}\,, \tag{3.46}$$

where a_m is some initial time deep inside the matter dominated era, we obtain

$$\frac{d^2Y(a)}{da^2} + \frac{1}{2}\left[\frac{dg(a)}{da} - \frac{1}{2}g^2(a)\right]Y(a) = 0\,. \tag{3.47}$$

We can choose $g(a)$ to be a solution of

$$\frac{dg(a)}{da} - \frac{1}{2}g^2(a) + 2I(a) = 0\,, \tag{3.48}$$

which is a particular Riccati equation. In this case Eqs. (3.43) and (3.47) become equals. Thus, we can relate the solutions of Eq. (3.45) with the ones of Eq. (3.37) through

$$\delta(a) = \frac{y(a)}{a}\sqrt{\frac{\mathcal{H}_0}{\mathcal{H}(a)}}e^{+\frac{1}{2}\int_{a_m}^{a} da' g(a')}\,. \tag{3.49}$$

It is straightforward to integrate Eq. (3.45) the first time

$$\frac{dy(a)}{da} + g(a)y(a) = a_2, \tag{3.50}$$

where a_2 is the first integration constant. A second integration is also possible, giving us the solutions for $y(\tau)$ in their integral form

$$y(\tau) = \kappa_1\gamma^2(a) + \kappa_2\gamma^2(a)\int_{a_m}^{a}\frac{da'}{\gamma^2(a')}\,, \tag{3.51}$$

where

$$\gamma^2(a) = e^{-\int_{a_m}^{a} da' g(a')}\,. \tag{3.52}$$

From Eq. (3.51) we have two independent solutions of Eq. (3.37) in their integral form

$$D_1(a) = \frac{\gamma(a)}{a}\sqrt{\frac{\mathcal{H}_0}{\mathcal{H}(a)}}$$

$$D_2(a) = \frac{\gamma(a)}{a}\sqrt{\frac{\mathcal{H}_0}{\mathcal{H}(a)}}\int_{a_m}^{a}\frac{da'}{\gamma^2(a')}\,. \tag{3.53}$$

To determine the growing and the decaying modes it is important to note that there is an additional degree of freedom due to the boundary condition in Eq. (3.48). If we want to separate them we have to choose carefully the behavior of $g(a)$ at early times. As shown in Fig. 2.2, during the matter dominated epoch the contribution of the Galileon field can be neglected, leading to an Einstein-de Sitter (EdS) universe. Indeed, during this epoch we expect that ${D_{EdS}}^{+}(a) \propto a$, and ${D_{EdS}}^{-}(a) \propto \mathcal{H}(a)/a \propto a^{-3/2}$. We can impose $D_1(a) = {D_{EdS}}^{+}(a) = a$, obtaining $g(a) = -7/(2a)$. Taking into account the right coefficients, we can extend this result to the general solution, i.e. valid also after the matter-dominated epoch

$$\begin{aligned} D_+(a) &= {a_m}^{7/4}D_1(a) = \frac{{a_m}^{7/4}\gamma(a)}{a}\sqrt{\frac{\mathcal{H}_0}{\mathcal{H}(a)}} \\ D_-(a) &= {a_m}^{-3/4}D_1(a) - \frac{5}{2{a_m}^{7/4}}D_2(a) \\ &= \frac{\gamma(a)}{{a_m}^{3/4}a}\sqrt{\frac{\mathcal{H}_0}{\mathcal{H}(a)}}\left(1 - \frac{5}{2a_m}\int_{a_m}^{a}\frac{da'}{\gamma^2(a')}\right)\,. \end{aligned} \tag{3.54}$$

These solutions are important because they are valid in every modified gravity theory in which the evolution of first-order DM perturbations, Eq. (3.37), is scale-independent. In Fig. 3.4 we show the evolution of our integral solution, Eq. (3.54), vs. the numerical solution of Eq. (3.37) for various and arbitrary initial conditions. It is important to note that every numerical solution approaches $D_+(a)$, this proves that the first line of Eq. (3.54) is the pure growing mode of Eq. (3.37). In Fig. 3.5 we plot the deviations of the Galileon growth rate, $f(a) \equiv d\ln D/d\ln a$, w.r.t. the growth rate of the ΛCDM model. For models in which the value of c_3 is negligible w.r.t. the value c_1 the deviations are large (up to about 100%), while, increasing c_3 the deviations decrease reaching $\simeq 10\%$.

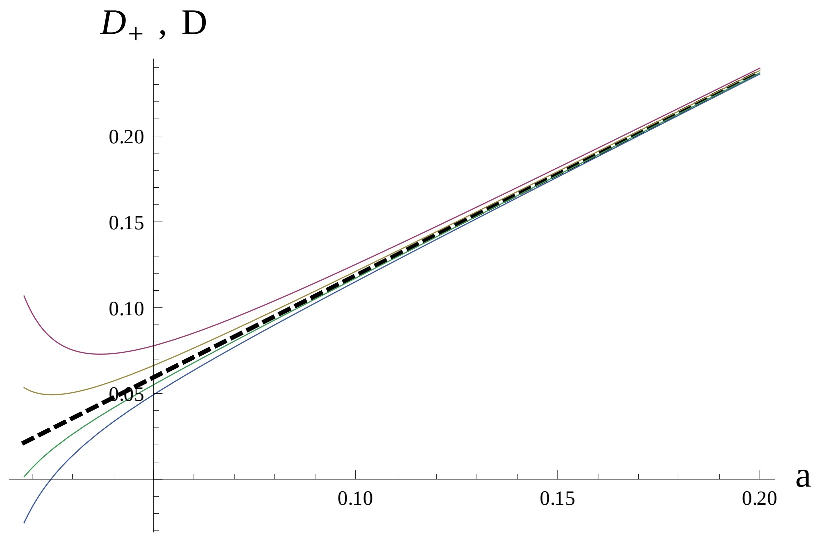

Figure 3.4: Evolution of $D_+(a)$ (black dashed line), Eq. (3.54), and $D(a)$ (the other lines), solutions of Eq. (3.37) with different initial conditions (for a fixed background corresponding to $c_1 = 1.5$, $c_2 = 0.04$ and $c_3 = 10^{-3}$).

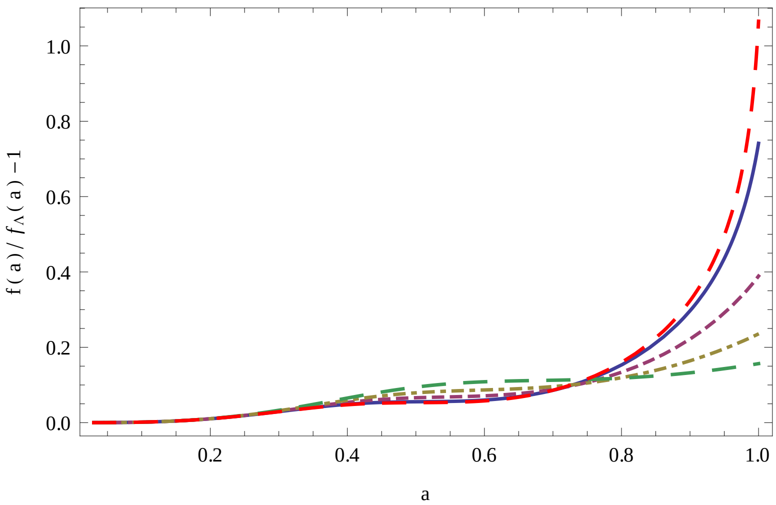

Figure 3.5: Growth rate $f(a)$ of the Galileon compared with the growth rate of ΛCDM. The values for the parameters c_1, c_2 and c_3 are the same as in Fig. 2.2.

CHAPTER 4

THE WEAKLY NON-LINEAR REGIME (CUBIC GALILEON)

After the analysis of the linear perturbation theory, it would be interesting to go beyond, in order to investigate some aspects on the weakly non-linear regime, i.e. the non-Gaussianities. In particular, our purpose is to calculcate the DM bispectrum at tree-level generated at late-times by gravitational instability. We use the notation and the results given in Sec. 2.2 and Sec. 3.2.

4.1 Second-order equations

By perturbing the Einstein and the Galileon field equations, Eqs. (1.10) and (1.12), at second order we can study the dynamics of the DM fluctuations in the weakly non-linear regime. The structure of these equations is the same as in the linear case, up to additional source terms formed by product of first-order scalar quantities that we will indicate with $S^{(n)}$ (their explicit expression in a general gauge can be found in Appendix D). From the Einstein equations we obtain, respectively, the $(0,0)$, $(0,i)$ the trace and the traceless part of (i,j)

$$
\begin{aligned}
&2{M_{\rm pl}}^2\nabla^2\phi^{(2)} - \left(6{M_{\rm pl}}^2\mathcal{H} - \frac{3c_3\pi'^3}{M^3a^2}\right)\phi^{(2)\prime} - \left(2{M_{\rm pl}}^2\mathcal{H} - \frac{c_3\pi'^3}{M^3a^2}\right)\nabla^2\omega^{(2)} \\
&+ \frac{1}{3}{M_{\rm pl}}^2\nabla^2\nabla^2\chi^{(2)} = a^2\rho_m\delta^{(2)} + \left(c_2\pi'^2 + 6{M_{\rm pl}}^2\mathcal{H}^2 - \frac{12c_3\pi'^3\mathcal{H}}{M^3a^2}\right)\psi^{(2)} \\
&- \frac{c_3\pi'^2\nabla^2\pi^{(2)}}{M^3a^2} - \frac{1}{2}c_1M^3a^2\pi^{(2)} + \left(-c_2\pi' + \frac{9c_3\pi'^2\mathcal{H}}{M^3a^2}\right)\pi^{(2)\prime} - S^{(1)}\,, \qquad (4.1)
\end{aligned}
$$

$$
\begin{aligned}
&- 2{M_{\rm pl}}^2\nabla^2\phi^{(2)\prime} - \left(2{M_{\rm pl}}^2\mathcal{H} - \frac{c_3\pi'^3}{M^3a^2}\right)\nabla^2\psi^{(2)} = a^2\rho_m\nabla^2v^{(2)} + a^2\rho_m\nabla^2\omega^{(2)} \\
&+ \frac{1}{3}{M_{\rm pl}}^2\nabla^2\nabla^2\chi^{(2)\prime} + \left(c_2\pi' - \frac{3c_3\pi'^2\mathcal{H}}{M^3a^2}\right)\nabla^2\pi^{(2)} + \frac{c_3\pi'^2}{M^3a^2}\nabla^2\pi^{(2)\prime} + S^{(2)}\,, \quad (4.2)
\end{aligned}
$$

$$
\begin{aligned}
&\frac{2}{3}{M_{\rm pl}}^2\nabla^2\left(\psi^{(2)} - \phi^{(2)} + \omega^{(2)\prime} + 2\mathcal{H}\omega^{(2)} - \frac{1}{6}\nabla^2\chi^{(2)}\right) + 2{M_{\rm pl}}^2\phi^{(2)\prime\prime} \\
&+ 4{M_{\rm pl}}^2\mathcal{H}\phi^{(2)\prime} + \left(2{M_{\rm pl}}^2\mathcal{H}^2 + 4{M_{\rm pl}}^2\mathcal{H}' - c_2\pi'^2 - \frac{4c_3\pi''\pi'^2}{M^3a^2} + \frac{4c_3\pi'^3\mathcal{H}}{M^3a^2}\right)\psi^{(2)} \\
&+ \left(2{M_{\rm pl}}^2\mathcal{H} - \frac{c_3\pi'^3}{M^3a^2}\right)\psi^{(2)\prime} = -\left(c_2\pi' + \frac{2c_3\pi''\pi'}{M^3a^2} - \frac{3c_3\pi'^2\mathcal{H}}{M^3a^2}\right)\pi^{(2)\prime} \\
&+ \frac{1}{2}c_1M^3a^2\pi^{(2)} - \frac{c_3\pi'^2\pi^{(2)\prime\prime}}{M^3a^2} + S^{(3)}\,, \qquad (4.3)
\end{aligned}
$$

$$
\nabla^4\left(\frac{1}{2}\chi^{(2)\prime\prime} + \mathcal{H}\chi^{(2)\prime} + \frac{1}{6}\nabla^2\chi^{(2)} - \omega^{(2)\prime} - 2\mathcal{H}\omega^{(2)} + \phi^{(2)} - \psi^{(2)}\right) = S^{(4)}\,. \quad (4.4)
$$

Eq. (1.12), for the Galileon field fluctuations, becomes

$$
\begin{aligned}
&\left(c_2 - \frac{6c_3\pi'\mathcal{H}}{M^3a^2}\right)\pi^{(2)\prime\prime} + \left(2c_2\mathcal{H} - \frac{6c_3\pi'\mathcal{H}'}{M^3a^2} - \frac{6c_3\pi''\mathcal{H}}{M^3a^2}\right)\pi^{(2)\prime} \\
&- \left(c_2 - \frac{2c_3\left(\pi'' + \mathcal{H}\pi'\right)}{M^3a^2}\right)\nabla^2\pi^{(2)} + \left(-c_2\pi' + \frac{9c_3\pi'^2\mathcal{H}}{M^3a^2}\right)\psi^{(2)\prime} \\
&+ \left(-2c_2\pi'' - 4c_2\pi'\mathcal{H} + \frac{12c_3\pi'^2\mathcal{H}'}{M^3a^2} + \frac{24c_3\pi''\pi'\mathcal{H}}{M^3a^2}\right)\psi^{(2)} \\
&+ \frac{c_3\pi'^2\nabla^2\psi^{(2)}}{M^3a^2} + \left(-3c_2\pi' + \frac{6c_3\pi''\pi'}{M^3a^2} + \frac{9c_3\pi'^2\mathcal{H}}{M^3a^2}\right)\phi^{(2)\prime} + \frac{3c_3\pi'^2\phi^{(2)\prime\prime}}{M^3a^2} \\
&+ \left(-c_2\pi' + \frac{2c_3\pi''\pi'}{M^3a^2} + \frac{3c_3\pi'^2\mathcal{H}}{M^3a^2}\right)\nabla^2\omega^{(2)} + \frac{c_3\pi'^2\nabla^2\omega^{(2)\prime}}{M^3a^2} = S^{(5)}\,. \qquad (4.5)
\end{aligned}
$$

The stress-energy tensor continuity equation reads

$$\delta^{(2)'} = 3\phi^{(2)'} - \nabla^2 v^{(2)} + S^{(6)}\,, \tag{4.6}$$

$$\nabla^2\left(\omega^{(2)'} + v^{(2)'} + \mathcal{H}\omega^{(2)} + \psi^{(2)} + \mathcal{H}v^{(2)}\right) = S^{(7)}\,. \tag{4.7}$$

In Eqs. (4.2), (4.4) and (4.7) second-order vector and tensor perturbations were present. In order to decouple scalar from vector and tensor perturbations we have used the operator ∂_i in Eqs. (4.2) and (4.7), while we have used $\partial_i\partial_j$ in Eq. (4.4). Once the equations of motion for the scalar perturbations are obtained the steps to obtain the evolution for δ are the same as in the linear case. The result is

$$\delta^{(2)\prime\prime} + \mathcal{H}\delta^{(2)\prime} - 4\pi G\left(1 - \frac{{c_3}^2\pi'^4}{2c_2M^6M_{\rm pl}^2a^4\alpha}\right)a^2\rho_m\delta^{(2)} = S^{(\delta)}\,, \tag{4.8}$$

where

$$S^{(\delta)} = -\left(1 - \frac{{c_3}^2\pi'^4}{2c_2M^6{M_{\rm pl}}^2a^4\alpha}\right)\left[\frac{S^{(1)}}{2{M_{\rm pl}}^2} - \frac{S^{(4)}}{k^2}\right] + \frac{c_3\pi'^2S^{(5)}}{2c_2M^3{M_{\rm pl}}^2a^2\alpha} + S^{(6)'} + \mathcal{H}S^{(6)} - S^{(7)} \tag{4.9}$$

4.1.1 Solution of the evolution equation for the second-order DM density contrast

In this section we study the behavior of Eq. (4.8). It is clear that the homogeneous part of this equation is equal to Eq. (3.37). Thus, using Green's method, and Eqs. (3.54), we can find an analytical (in its integral form) solution for the evolution of the second-order DM density perturbations. Using Eqs. (C.3), (C.4), (C.5), (3.31) and (3.37), in the Poisson gauge the Fourier transform of the source term Eq. (4.9) becomes

$$S^{(\delta)}(a,\vec{k}) = \int d^3k_1d^3k_2\delta^{(3)}(\vec{k}-\vec{k}_1-\vec{k}_2)\mathcal{K}(a,\vec{k}_1,\vec{k}_2)\delta^{(1)}(a,\vec{k}_1)\delta^{(1)}(a,\vec{k}_2)\,. \tag{4.10}$$

Here, the symmetrized kernel $\mathcal{K}(a,\vec{k}_1,\vec{k}_2)$ reads

$$\mathcal{K}(a,\vec{k}_1,\vec{k}_2) \equiv \gamma_1(a) + \gamma_2(a)\,\frac{\left(\vec{k}_1\cdot\vec{k}_2\right)\left({k_1}^2+{k_2}^2\right)}{{k_1}^2{k_2}^2} + \gamma_3(a)\frac{\left(\vec{k}_1\cdot\vec{k}_2\right)^2}{{k_1}^2{k_2}^2}$$

$$
+ \gamma_4(a)\frac{a^2\mathcal{H}^2\left(\vec{k}_1\cdot\vec{k}_2\right)}{{k_1}^2{k_2}^2} + \gamma_5(a)\frac{a^2\mathcal{H}^2}{k^2} + \gamma_6(a)\left(\frac{a^2\mathcal{H}^2}{{k_1}^2} + \frac{a^2\mathcal{H}^2}{{k_2}^2}\right)
$$

$$
+ \gamma_7(a)\frac{a^2\mathcal{H}^2\left({k_1}^4 + {k_2}^4\right)}{k^2{k_1}^2{k_2}^2} + \gamma_8(a)\frac{a^4\mathcal{H}^4}{{k_1}^2{k_2}^2}\,, \tag{4.11}
$$

where the background functions $\gamma_1(a)$, $\gamma_2(a)$ and $\gamma_3(a)$ are shown in the next section, while the other $\gamma_i(a)$ are listed in Appendix E. Finally, using Green's method with the homogeneous solutions, Eq. (3.54), we can find the evolution of the second-order density fluctuations

$$
\delta^{(2)}(a,\vec{k}) = D_+(a)\delta^{(2)}(\vec{k}) - D_+(a)\int_{a_m}^{a} da'\frac{D_-(a')S^{(\delta)}(a',\vec{k})}{a'^2\mathcal{H}^2(a')W(a')}
$$

$$
+ D_-(a)\int_{a_m}^{a} da'\frac{D_+(a')S^{(\delta)}(a',\vec{k})}{a'^2\mathcal{H}^2(a')W(a')}\,, \tag{4.12}
$$

where W is the Wronskian

$$
W(a) \equiv D_+(a){D_-}'(a) - D_-(a){D_+}'(a) = -\frac{5\mathcal{H}_0}{2a^2\mathcal{H}(a)}\,, \tag{4.13}
$$

a_m is some initial time deep inside the matter dominated era and $\delta^{(2)}(\vec{k})$ is the initial second-order DM perturbation. It is interesting to see that in this relation there is no explicit dependence on the coefficients c_i.

4.2 Dark Matter Bispectrum

To describe the DM distribution of the universe the first statistical interesting quantity is the power-spectrum

$$
\left\langle\delta(a,\vec{k}_1)\delta(a,\vec{k}_2)\right\rangle \equiv (2\pi)^3\delta^{(3)}(\vec{k}_1+\vec{k}_2)P(a,k_1)\,, \tag{4.14}
$$

where $\delta^{(3)}(\vec{k}_1+\vec{k}_2)$ is the three dimensional Dirac delta function and $\langle\ldots\rangle$ indicates ensemble averaging. Note that, under the assumption of spatial isotropy, the power-spectrum depends only on the absolute value of $\vec{k}_1$. By the Wick theorem, for Gaussian distributed fluctuations the power-spectrum contains all the information about the DM distribution. The linear power-spectrum, calculated using first-order equations, reads

$$
P(a,k) \propto |D_+(a)|^2T^2(k)\left(\frac{k}{\mathcal{H}_0}\right)^{n_s}\,, \tag{4.15}
$$

where n_s is the scalar spectral index of primordial fluctuations and $T(k)$ is the transfer function (for which we use for simplicity the fit provided in [98]). In the following computations we will take $n_s = 0.96$ [6]. The second statistic of interest is the bispectrum, defined by

$$\left\langle \delta(a,\vec{k}_1)\delta(a,\vec{k}_2)\delta(a,\vec{k}_3)\right\rangle \equiv (2\pi)^3\delta^{(3)}(\vec{k}_1+\vec{k}_2+\vec{k}_3)B(a,k_1,k_2,k_3)\,, \tag{4.16}$$

where the Dirac delta function imposes that only closed triangle configurations are to be considered. Since we are interested in studying the contribution generated by gravitational instability at late times in the Galileon theory, we impose Gaussian initial conditions. It is convenient to use the reduced bispectrum [99], defined by

$$Q(a,k_1,k_2,k_3) \equiv \frac{B(a,k_1,k_2,k_3)}{P(a,k_1)P(a,k_2)+cyc.}\,, \tag{4.17}$$

which has the property that it remove most of the scale dependence to lowest-order (tree-level) in non-linear perturbation theory. Using the results of the previous sections we can write the density contrast perturbation as [1]

$$\begin{aligned}\delta(a,\vec{k}) &\equiv \delta^{(1)}(a,\vec{k}) + \frac{1}{2}\delta^{(2)}(a,\vec{k}) = D_+(a)\delta^{(1)}(\vec{k})\\ &+ \int d^3q_1 \int d^3q_2 \delta^{(3)}(\vec{k}-\vec{q}_1-\vec{q}_2)F(a,\vec{q}_1,\vec{q}_2)\delta^{(1)}(a,\vec{q}_1)\delta^{(1)}(a,\vec{q}_2)\,,\end{aligned} \tag{4.18}$$

where

$$\begin{aligned}F(a,\vec{q}_1,\vec{q}_2) &= \int_{a_m}^{a} da' \frac{{D_+}^2(a')\left[D_-(a)D_+(a')-D_+(a)D_-(a')\right]}{2a'^2\mathcal{H}^2(a')W(a'){D_+}^2(a)}\mathcal{K}_{SH}(a',\vec{q}_1,\vec{q}_2)\\ &= \frac{a\sqrt{\mathcal{H}(a)}}{\gamma(a)}\int_{a_m}^{a} da' \left(\int_{a'}^{a}\frac{da''}{\gamma^2(a'')}\right)\frac{\gamma^3(a')}{a'^3\sqrt{\mathcal{H}(a')}}\frac{\mathcal{K}_{SH}(a',\vec{q}_1,\vec{q}_2)}{2\mathcal{H}^2(a')}\,.\end{aligned} \tag{4.19}$$

The kernel $\mathcal{K}_{SH}(a',\vec{q}_1,\vec{q}_2)$ is the leading order of Eq. (4.11) taking into account that we are working on scales much smaller than the horizon (${k_i}^2 \gg a^2\mathcal{H}$). This kernel can be recast in a more convenient form as

$$\frac{\mathcal{K}_{SH}(a,\vec{q}_1,\vec{q}_2)}{2\mathcal{H}^2(a)} = \gamma_1(a) + \gamma_2(a)\frac{\left(\vec{k}_1\cdot\vec{k}_2\right)\left({k_1}^2+{k_2}^2\right)}{{k_1}^2{k_2}^2} + \gamma_3(a)\frac{\left(\vec{k}_1\cdot\vec{k}_2\right)^2}{{k_1}^2{k_2}^2}\,, \tag{4.20}$$

[1] Notice that in the following we neglect the contribution proportional to the initial second-order DM perturbation $\delta^{(2)}(\vec{k})$ in Eq. (4.12). $\delta^{(2)}(\vec{k})$ contains both a possible primordial NG, and a non-primordial contribution, see, e.g. [70, 100]. However the non-primordial term gives a negligible contribution to our final results on the scales of the quasi-static regime.

where

$$\gamma_1(a) \equiv f^2(a) + \frac{\rho_m a^2}{2M_{\rm pl}^2\mathcal{H}^2} - \frac{c_3^2 a^2 \mathcal{H}^2 \pi'^4 \rho_m}{4c_2 M^6 M_{\rm pl}^4 \alpha} + \frac{c_3^4 a^2 \mathcal{H}^4 \pi'^6 \rho_m^2}{8c_2^3 M^{12} M_{\rm pl}^6 \alpha^3}$$

$$\gamma_2(a) \equiv f^2(a) + \frac{\rho_m a^2}{4M_{\rm pl}^2\mathcal{H}^2} - \frac{c_3^2 a^2 \mathcal{H}^2 \pi'^4 \rho_m}{8c_2 M^6 M_{\rm pl}^4 \alpha}$$

$$\gamma_3(a) \equiv f^2(a) - \frac{c_3^4 a^2 \mathcal{H}^4 \pi'^6 \rho_m^2}{8c_2^3 M^{12} M_{\rm pl}^6 \alpha^3}$$

$$f(a) = 1 + \frac{3p_\pi}{4\left(\rho_m + \rho_\pi\right)} + \frac{1}{2} a h(a)\,. \tag{4.21}$$

Here we introduce $h(a) = g(a) + 7/(2a)$, where $g(a)$ is the solution of Eq. (3.50), to parametrize the contribution of the accelerated expansion on the growth rate. Eq. (4.20) is one of the main results of our paper. It reduces to the usual form of the Newtonian kernel in the limit of an EdS universe [79, 80]. It shows that the different contributions to the bispectrum have the same scale dependence as in EdS and ΛCDM, while they are modulated by time dependent coefficients that depend on the particular Galileon model. Looking at Eq. (4.20) we can recognize three kind of modifications w.r.t. the ΛCDM kernel. The first is due to the different evolution of the growth rate w.r.t. ΛCDM and, as stated before, should produce deviations in the bispectrum that can reach $\simeq 100\%$. The second comes from the different evolution of the background, while the third is related to the parameters c_2 and c_3.

The reduced bispectrum, Eq. (4.17), assumes the standard form

$$Q(a, k_1, k_2, k_3) = \frac{2F(a, \vec{k}_1, \vec{k}_2)P(a, k_1)P(a, k_2) + cyc.}{P(a, k_1)P(a, k_2) + cyc.}\,. \tag{4.22}$$

The scales at which our approximations can give valid results are $10^{-4}\mathrm{h\,Mpc}^{-1} \ll \mathrm{k} \lesssim 10^{-1}\mathrm{h\,Mpc}^{-1}$. The first inequality follows from the sub-horizon approximation, while the second excludes the scales at which highly non-linear effects become non-negligible. In Figs. 4.1 and 4.3 we show the angular dependence of the reduced bispectrum for different Galileon models, at $a = 1$ and at $a = 0.6$ respectively, fixing $k_1 = k_2$, θ being the angle between $\vec{k}_1$ and $\vec{k}_2$ ($\vec{k}_1 \cdot \vec{k}_1 = k_1 k_2 \cos\theta$). In Fig. 4.2 and 4.4 we show the angular dependence of the reduced bispectrum, at $a = 1$ and at $a = 0.6$ respectively, fixing $k_1 = const. \times k_2$ and $k_2 = 10^{-3}\,\mathrm{h\,Mpc}^{-1}$.

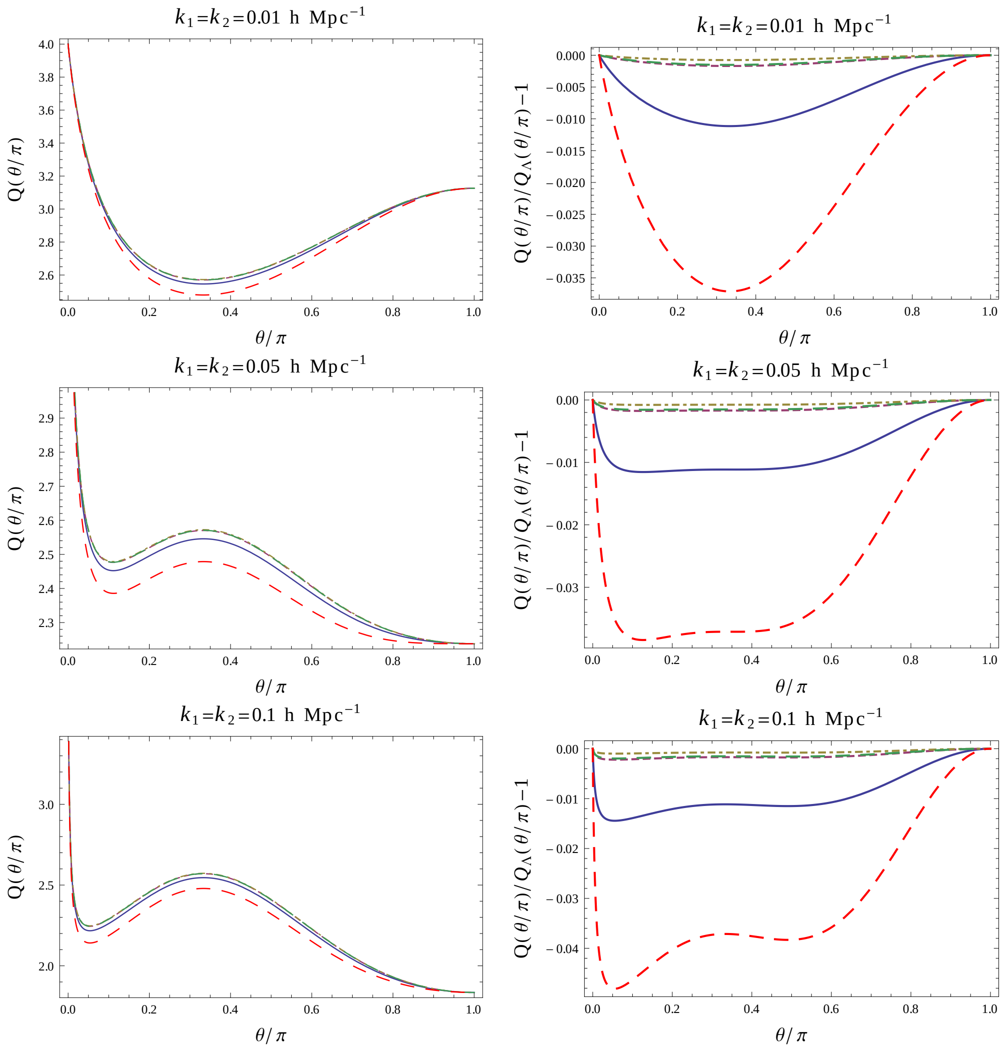

Figure 4.1: In the left panels we plot the reduced bispectrum for some Galileon models as a function of the angle θ, fixing $k_1 = k_2$ at $a = 1$. In the right panels we plot the relative deviations of the bispectrum of the Galileon w.r.t. the one of ΛCDM. The parameter values are: $c_1 = 1.6$, $c_2 = 0.04$, $c_3 = 10^{-3}$ (red line); $c_1 = 1.5$, $c_2 = 0.4$, $c_3 = 10^{-3}$ (blue line); $c_1 = 11$, $c_2 = 3.8$, $c_3 = 1$ (purple line); $c_1 = 6$, $c_2 = 3.6$, $c_3 = 1$ (yellow line); $c_1 = 10^{-4}$, $c_2 = 3.3$, $c_3 = 1$ (green line).

In Fig. 4.5 we show the evolution of

$$G(a', a) \equiv \frac{{D_+}^2(a') \left[D_-(a)D_+(a') - D_+(a)D_-(a')\right]}{a'^2 W(a') {D_+}^2(a)} \frac{\mathcal{K}_{SH}(a', \vec{q}_1, \vec{q}_2)}{2\mathcal{H}^2(a')} \tag{4.23}$$

for an equilateral configuration at $a = 1$ (left panel) and $a = 0.6$ (right panel).

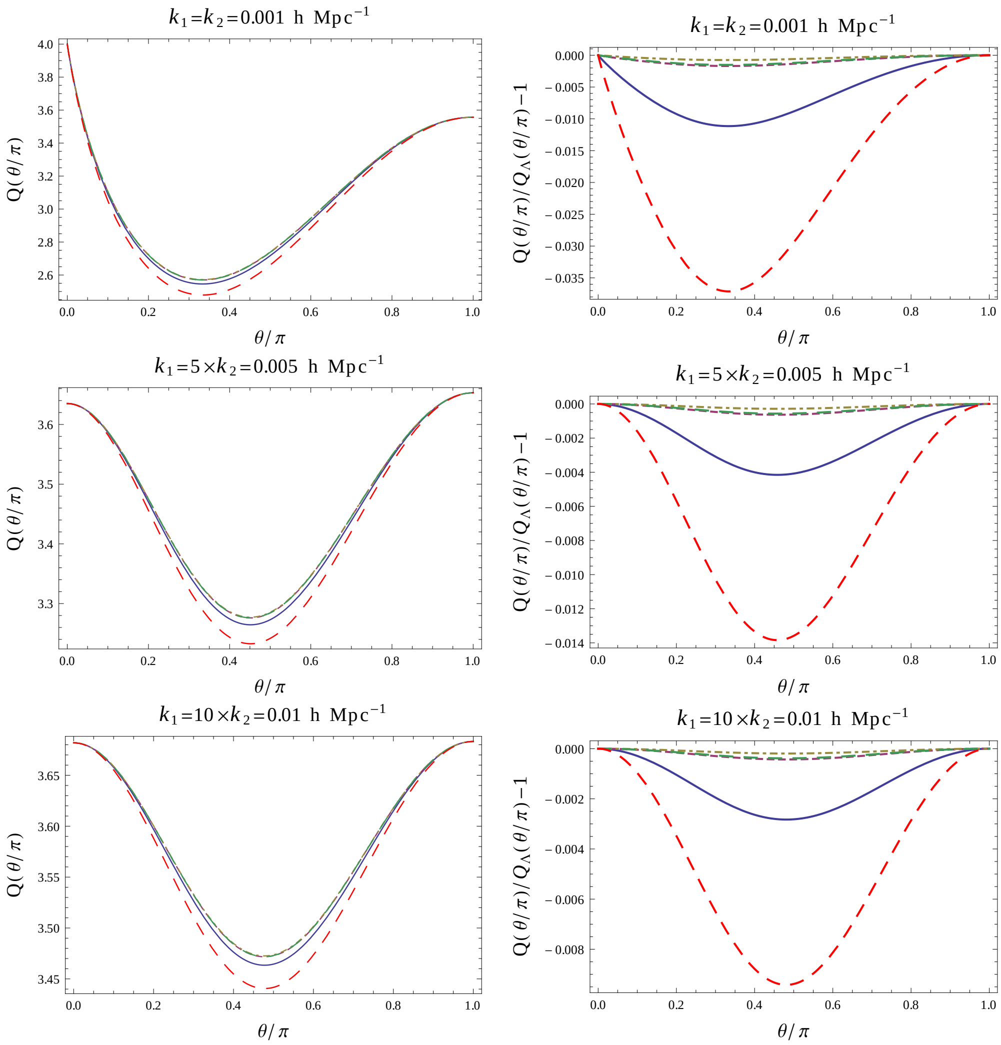

Figure 4.2: The same as Fig. 4.1 fixing $k_1 = const. \times k_2$ and $k_2 = 0.001\,\mathrm{h\,Mpc^{-1}}$.

This configuration is useful to understand the behavior of the reduced bispectrum, Eq. 4.22, because it is totally independent of the power-spectrum, in fact $Q(a, k_1, k_1, k_1) = 2F(a, \vec{k}_1, \vec{k}_1)$. As one can see in the left panel of Fig. 4.5 the function $G(a = 1, a')$ contains a compensation effect that reduces the deviations w.r.t. the ΛCDM model in the bispectrum, as shown in Figs. 4.1, 4.2, 4.3 and 4.4. Let us notice that, for $a' \lesssim 0.4$, the line of every Galileon model we consider lies below the ΛCDM line; viceversa, for $a' \gtrsim 0.4$, except for the red and blue lines, for which we find the strongest deviations, the Galileon lines lie above

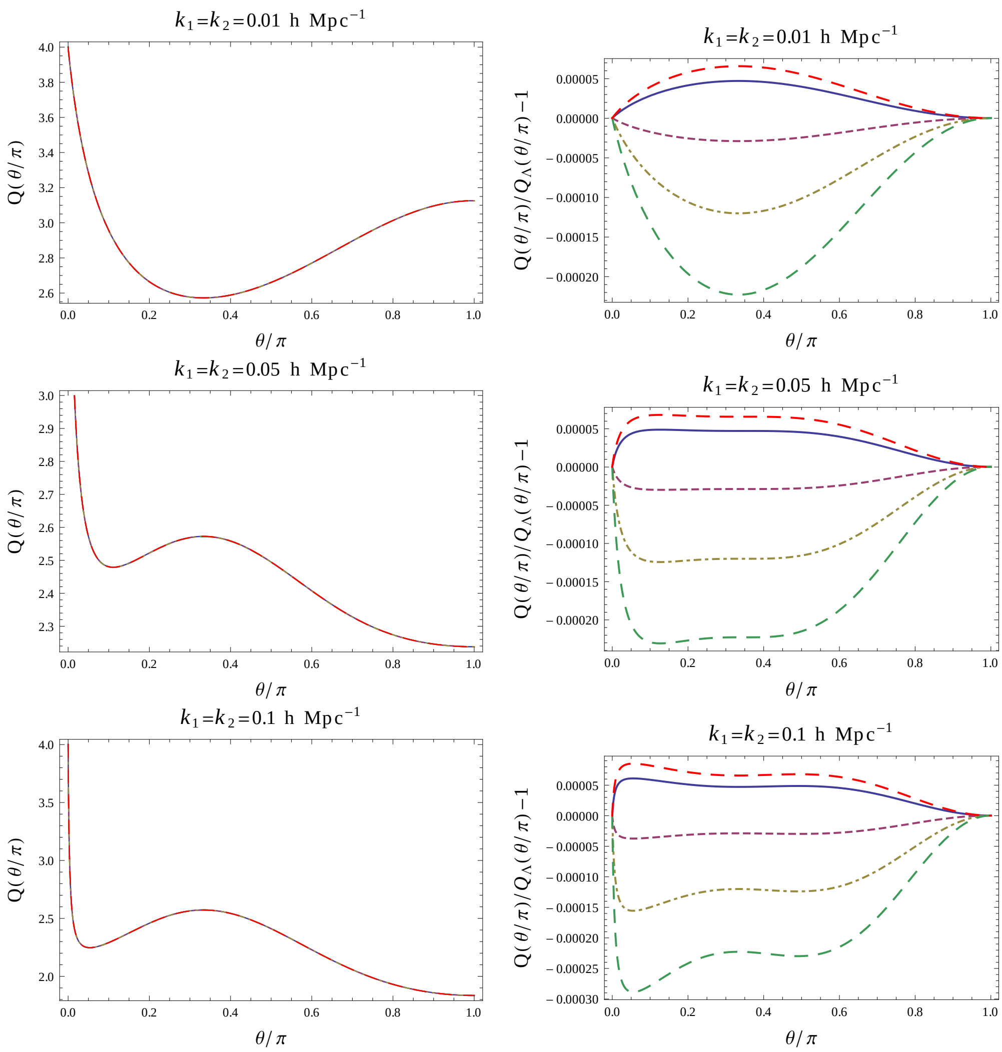

Figure 4.3: In the left panels we plot the reduced bispectrum for some Galileon models as a function of the angle θ, fixing $k_1 = k_2$ at $a = 0.6$. In the right panels we plot the relative deviations of the bispectrum of the Galileon w.r.t. the one of ΛCDM. The parameter values are: $c_1 = 1.6$, $c_2 = 0.04$, $c_3 = 10^{-3}$ (red line); $c_1 = 1.5$, $c_2 = 0.4$, $c_3 = 10^{-3}$ (blue line); $c_1 = 11$, $c_2 = 3.8$, $c_3 = 1$ (purple line); $c_1 = 6$, $c_2 = 3.6$, $c_3 = 1$ (yellow line); $c_1 = 10^{-4}$, $c_2 = 3.3$, $c_3 = 1$ (green line).

the ΛCDM line (up to the present epoch). Consequently, when we integrate $G(a = 1, a')$, the deviations that we have obtained studying the background and the power-spectrum are attenuated considerably. Instead, when $w \gtrsim -0.85$ – corresponding to the red and blue lines, see Fig. 2.2 – we see a minimum be-

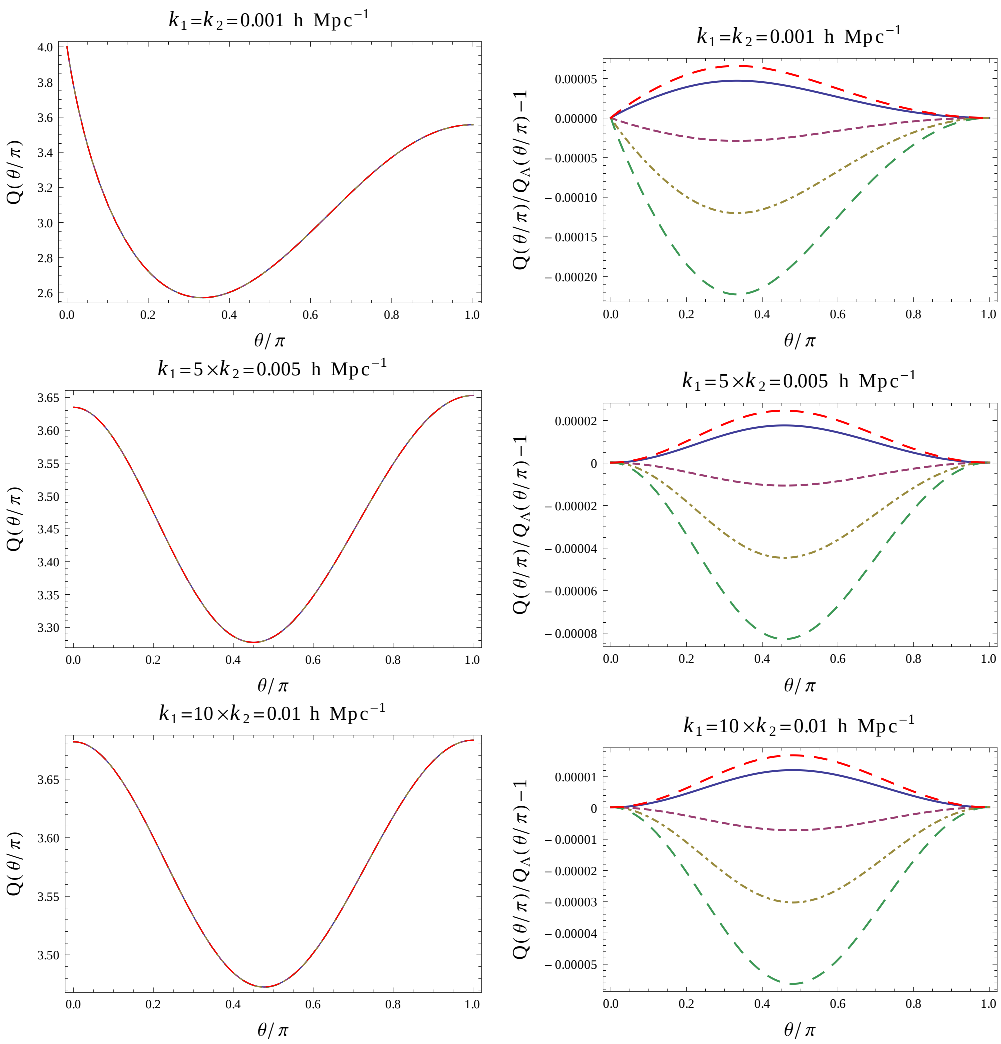

Figure 4.4: The same as Fig. 4.3 fixing $k_1 = const. \times k_2$ and $k_2 = 0.001\,\mathrm{h\,Mpc^{-1}}$.

low the ΛCDM around $a' \simeq 1$. This feature decreases the compensation effect and produces larger deviations in the dark matter bispectrum. This could be explained by the fact that the universe is not accelerating enough today and the evolution of the growth rate is strongly modified (see Fig. 3.5). For these cases the deviations we find in the bispectrum are about $\simeq 5\%$. Instead, computing $B(a, k_1, k_2, k_3)$ before the acceleration of the universe, the compensation effect is conserved because the contribution of the Galileon is negligible and all models are indistinguishable (see for example the right panel of Fig. 4.5 and the tiny

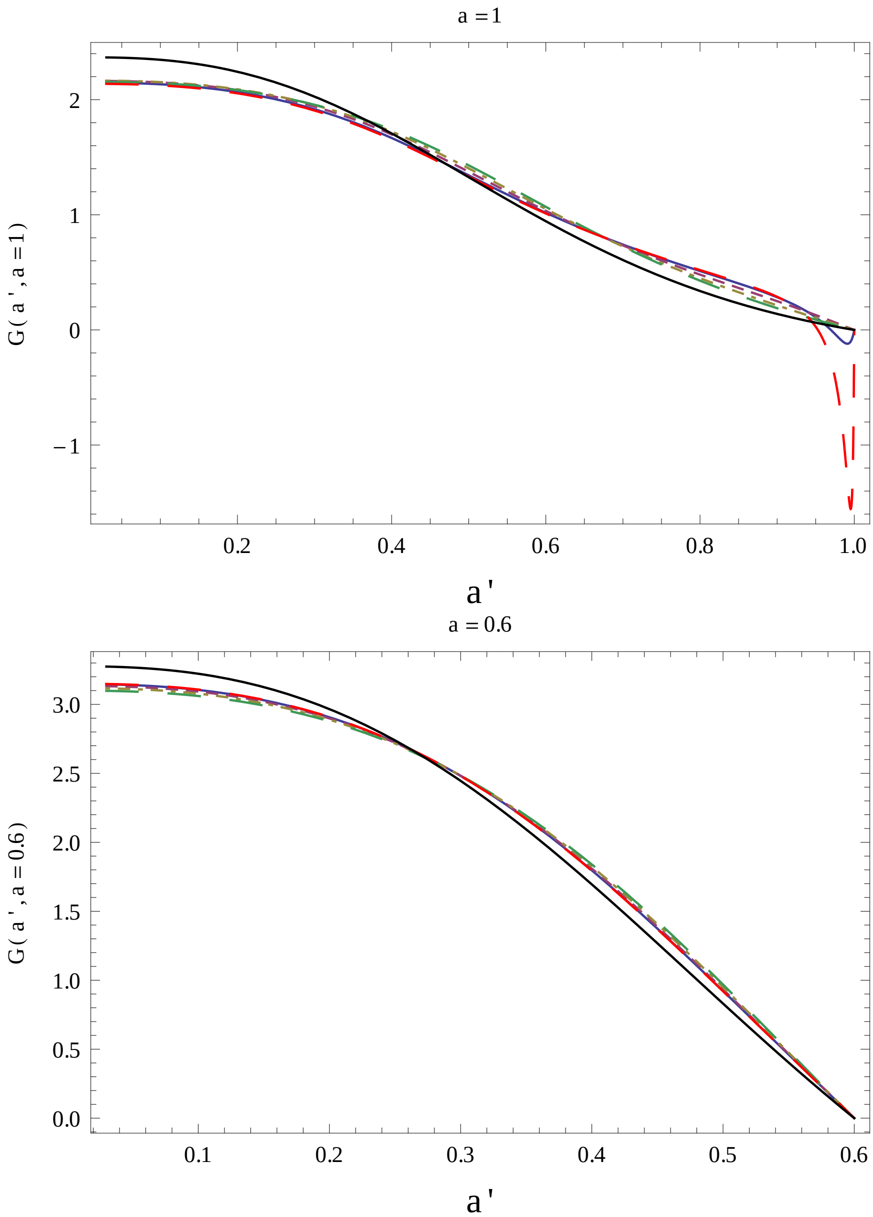

Figure 4.5: The integrand of Eq. 4.19 for an equilateral configuration at $a = 1$ (left panel) and $a = 0.6$ (right panel). The models we plot are the same as in Fig. 2.2, while the black line represents the ΛCDM

model.

deviations seen in Figs. 4.3 and 4.4).

CHAPTER 5

THE WEAKLY NON-LINEAR REGIME (COUPLED GALILEON)

In the previous Chapter we have found interesting results, as the suppression effect of the bispectrum w.r.t. the power spectrum. This property can be a particular behavior of the Galileon models or a more universal effect. Therefore, to conlcude this thesis, it is interesting to study the DM bispectrum of the Coupled Galileon theory, Eq. (1.9), i.e. the most general Galileon theory in literature. In this Chapter we shall first review the background and the linear perturbation theory (a detailed analysis can be found in [50, 52]). Here we only provide the analytical results and a brief discussion on the form of the equations. The last section is devoted to the second-order equations and the analytic form of the DM bispectrum calculated at tree-level.

5.1 Background evolution

Using the conformal time and the metric given in Eq. (3.18), the Friedmann equations for Eq. (1.9) read

$$\begin{aligned}\frac{3{M_{\rm pl}}^2\mathcal{H}^2}{a^2} =&\rho_m + \frac{1}{2}c_1M^3\pi + \frac{c_2\pi'^2}{2a^2} - \frac{3c_3\pi'^3\mathcal{H}}{M^3a^4} + \frac{45c_4\pi'^4\mathcal{H}^2}{2M^6a^6} \\ &- \frac{21c_5\pi'^5\mathcal{H}^3}{M^9a^8} - \frac{9c_GM_{\rm pl}\pi'^2\mathcal{H}^2}{M^3a^4} + \frac{6c_0M_{\rm pl}\mathcal{H}\left(\pi'+\mathcal{H}\pi\right)}{a^2}\,, \end{aligned}\tag{5.1}$$

$$\begin{aligned}&\frac{{M_{\rm pl}}^2}{a^2}\left(\mathcal{H}^2+2\mathcal{H}'\right) = \frac{c_1M^3}{2}\pi - \frac{c_2\pi'^2}{2a^2} - \frac{c_3\pi'^2\left(\pi''-\mathcal{H}\pi'\right)}{M^3a^4} \\ &\quad + \frac{3c_4\pi'^3}{2M^6a^6}\left(8\mathcal{H}\pi''-7\mathcal{H}^2\pi'+2\mathcal{H}'\pi'\right) - \frac{3c_5\mathcal{H}\pi'^4}{M^9a^8}\left(5\mathcal{H}\pi''-5\mathcal{H}^2\pi'\right. \\ &\quad \left.+2\mathcal{H}'\pi'\right) - \frac{c_GM_{\rm pl}\pi'}{M^3a^4}\left(2\mathcal{H}'\pi'+4\mathcal{H}\pi''-3\mathcal{H}^2\pi'\right) \\ &\quad + \frac{2c_0M_{\rm pl}}{a^2}\left(\pi''+\mathcal{H}^2\pi+\mathcal{H}\pi'+2\mathcal{H}'\pi\right)\,. \end{aligned}\tag{5.2}$$

The background Galileon field equation reads

$$\begin{aligned}&-\frac{c_1M^3}{2} - \frac{c_2}{a^2}\left(\pi''+2\mathcal{H}\pi'\right) + \frac{3c_3\pi'}{M^3a^4}\left(2\mathcal{H}\pi''+\mathcal{H}'\pi'\right) - \frac{18c_4\mathcal{H}\pi'^2}{M^6a^6}\left(3\mathcal{H}\pi''\right. \\ &\left.-2\mathcal{H}^2\pi'+2\mathcal{H}'\pi'\right) + \frac{15c_5\mathcal{H}^2\pi'^3}{M^9a^8}\left(4\mathcal{H}\pi''+3\mathcal{H}'\pi'-4\mathcal{H}^2\pi'\right) \\ &+\frac{6c_GM_{\rm pl}\mathcal{H}}{M^3a^4}\left(\mathcal{H}\pi''+2\mathcal{H}'\pi'\right) = \frac{6c_0M_{\rm pl}}{a^2}\left(\mathcal{H}^2+\mathcal{H}'\right)\,. \end{aligned}\tag{5.3}$$

5.2 Linear perturbation theory

At the linear level the first interesting equations in order to study the evolution of the DM perturbation in the Poisson gauge and on sub-horizon scales are the $(0,0)$ and the traceless part of the Einstein equations

$$\alpha_1\phi^{(1)} = -\frac{\rho_m\delta^{(1)}a^2}{2{M_{\rm pl}}^2k^2} - \alpha_2\frac{\pi^{(1)}}{M_{\rm pl}}\,,\tag{5.4}$$

$$\alpha_1\psi^{(1)} = \alpha_4\phi^{(1)} + 2\alpha_3\frac{\pi^{(1)}}{M_{\rm pl}}\,.\tag{5.5}$$

The equation for the Galileon takes this form

$$\alpha_5 \frac{\pi^{(1)}}{M_{\rm pl}} + 4\alpha_3 \phi^{(1)} - 2\alpha_2 \psi^{(1)} = 0\,, \tag{5.6}$$

where $\alpha_i \equiv \alpha_i(\tau)$ are dimensionless background functions

$$\alpha_1 \equiv 1 - \frac{3c_4 \pi'^4}{2M^6 {M_{\rm pl}}^2 a^4} + \frac{3c_5 \mathcal{H} \pi'^5}{M^9 {M_{\rm pl}}^2 a^6} + \frac{c_G \pi'^2}{M^3 M_{\rm pl} a^2} - \frac{2c_0 \pi}{M_{\rm pl}}\,, \tag{5.7}$$

$$\alpha_2 \equiv c_0 - \frac{c_3 \pi'^2}{2M^3 M_{\rm pl} a^2} + \frac{6c_4 \mathcal{H} \pi'^3}{M^6 M_{\rm pl} a^4} - \frac{15 c_5 \mathcal{H}^2 \pi'^4}{2M^9 M_{\rm pl} a^6} - \frac{2c_G \mathcal{H} \pi'}{M^3 a^2}\,, \tag{5.8}$$

$$\alpha_3 \equiv c_0 + \frac{c_4 \pi'^2}{M^6 M_{\rm pl} a^4} \left(3\pi'' - 2\mathcal{H}\pi'\right) - \frac{3c_5 \pi'^3}{2M^9 M_{\rm pl} a^6} \left(4\mathcal{H}\pi'' + \mathcal{H}'\pi' - 4\mathcal{H}^2 \pi'\right) - \frac{c_G \pi''}{M^3 a^2}\,, \tag{5.9}$$

$$\alpha_4 \equiv 1 + \frac{c_4 \pi'^4}{2M^6 {M_{\rm pl}}^2 a^4} + \frac{3c_5 \pi'^4}{M^9 {M_{\rm pl}}^2 a^6} \left(\pi'' - \mathcal{H}\pi'\right) - \frac{c_G \pi'^2}{M^3 M_{\rm pl} a^2} - \frac{2c_0 \pi}{M_{\rm pl}}\,, \tag{5.10}$$

$$\alpha_5 \equiv -c_2 + \frac{2c_3}{M^3 a^2} \left(\pi'' + \mathcal{H}\pi'\right) - \frac{2c_4 \pi'}{M^6 a^4} \left(12\mathcal{H}\pi'' + 6\mathcal{H}'\pi' - 5\mathcal{H}^2 \pi'\right) + \frac{12 c_5 \mathcal{H} \pi'^2}{M^9 a^6} \left(3\mathcal{H}\pi'' + 2\mathcal{H}'\pi' - 3\mathcal{H}^2 \pi'\right) + \frac{2c_G M_{\rm pl} \left(\mathcal{H}^2 + 2\mathcal{H}'\right)}{M^3 a^2}\,. \tag{5.11}$$

In the Jordan frame the stress-energy tensor of the DM component is decoupled from the Galileon field, therefore it is covariantly conserved and the results given in Eqs. (3.31) and (3.32) also hold in this case

$$\delta^{(1)''} + \mathcal{H}\delta^{(1)'} + k^2 \psi^{(1)} = 0\,. \tag{5.12}$$

It is important to note these equations have some structural differences w.r.t. the corresponding equations in Sec. 3.2. In particular, on sub-horizon scales and using the Poisson gauge, each new term (c_4, c_5, c_0 and c_G) in Eq. (1.9) contributes to produce an anysotropic stress in Eq. (5.5) w.r.t. Eq. (3.29). The other difference is the presence of the gravitational potential $\phi^{(1)}$ in the Galileon field equation, Eq. (5.6).

From Eqs. (5.4), (5.5), (5.6) and (5.12), it is straightforward to single out an equation for the evolution of the DM perturbations at the linear level. The result reads

$$\delta^{(1)''}(\tau) + \mathcal{H}\delta^{(1)'}(\tau) - 4\pi G_\pi(\tau) a^2 \rho_m \delta^{(1)}(\tau) = 0\,, \tag{5.13}$$

where we have absorbed the modifications due to the Galileon field into a modified Newton's constant

$$G_\pi(\tau) = \frac{8{\alpha_3}^2 - \alpha_4\alpha_5}{8\alpha_1\alpha_2\alpha_3 - 2{\alpha_2}^2\alpha_4 - {\alpha_1}^2\alpha_5} \,. \tag{5.14}$$

5.3 Second-order equations and Dark Matter bispectrum

As explained in the previous Chapter, in order to calculate the leading contribute to the DM bispectrum (tree-level) on weakly non-linear scales we also need the second-order field equations. The procedure to decouple the equations is the same as in Sec. 5.2, but the result should contain a source term

$$\delta^{(2)''}(\tau) + \mathcal{H}\delta^{(2)'}(\tau) - 4\pi G_\pi(\tau) a^2 \rho_m \delta^{(2)}(\tau) = S^{(\delta)} \,. \tag{5.15}$$

After a Fourier transform the source term can be expressed as

$$S^{(\delta)}(\tau,\vec{k}) = \int d^3k_1 d^3k_2 \delta^{(3)}(\vec{k}-\vec{k}_1-\vec{k}_2)\mathcal{K}(\tau,\vec{k}_1,\vec{k}_2)\delta^{(1)}(\tau,\vec{k}_1)\delta^{(1)}(\tau,\vec{k}_2) \,. \tag{5.16}$$

Even though in principle the source contains a huge quantity of terms, it is possible to simplify the calculation by expanding in $(k_{1,2}\tau)^{-1} \ll 1$ up to the leading order. This is allowed despite the fact that in Eq. (5.16) $k_{1,2}$ cover the full momenta-space (not only the sub-horizon scales). In fact, we know that, when calculating the bispectrum, the integrals in Eq. (5.16) are solved. Then the kernel $\mathcal{K}(\tau,\vec{k}_1,\vec{k}_2)$ reads

$$\begin{aligned}
\mathcal{K}(\tau,\vec{k}_1,\vec{k}_2) \equiv& \left[2\mathcal{H}^2 f^2 + 8\pi G_\pi a^2 \rho_m - (8\pi)^3 {G_\pi}^3 a^2 {\rho_m}^2 \gamma(\tau)\right] \\
&+ \left[2\mathcal{H}^2 f^2 + 4\pi G_\pi a^2 \rho_m\right] \frac{\left(\vec{k}_1\cdot\vec{k}_2\right)\left({k_1}^2+{k_2}^2\right)}{{k_1}^2{k_2}^2} \\
&+ \left[2\mathcal{H}^2 f^2 + (8\pi)^3 {G_\pi}^3 a^2 {\rho_m}^2 \gamma(\tau)\right] \frac{\left(\vec{k}_1\cdot\vec{k}_2\right)^2}{{k_1}^2{k_2}^2} \,,
\end{aligned} \tag{5.17}$$

where $\gamma(\tau)$ is a function defined by

$$\begin{aligned}
\gamma(\tau) \equiv& \frac{(2\alpha_1\alpha_3 - \alpha_2\alpha_4)}{\left(8{\alpha_3}^2 - \alpha_4\alpha_5\right)^3} \left[3\left(2\alpha_1\alpha_3 - \alpha_2\alpha_4\right)\left(8{\alpha_3}^2 - \alpha_4\alpha_5\right)\eta_1\right. \\
&- 3\left(2\alpha_1\alpha_3 - \alpha_2\alpha_4\right)\left(-4\alpha_2\alpha_3 + \alpha_1\alpha_5\right)\eta_2 + 2\left(2\alpha_1\alpha_3 - \alpha_2\alpha_4\right)^2\eta_3
\end{aligned}$$

$$+3\left(-4\alpha_2\alpha_3+\alpha_1\alpha_5\right)\left(8{\alpha_3}^2-\alpha_4\alpha_5\right)\eta_4\Big]\,, \tag{5.18}$$

and η_{1-4} read

$$\eta_1(\tau)\equiv\frac{c_G{M_{\rm pl}}^3}{M^3}-\frac{3c_4{M_{\rm pl}}^2{\pi'}^2}{M^6a^2}+\frac{6c_5{M_{\rm pl}}^2{\pi'}^3\mathcal{H}}{M^9a^4} \tag{5.19}$$

$$\eta_2(\tau)\equiv-\frac{c_G{M_{\rm pl}}^3}{M^3}+\frac{c_4{M_{\rm pl}}^2{\pi'}^2}{M^6a^2}-\frac{6c_5{M_{\rm pl}}^2{\pi'}^2}{M^9a^4}\left(\pi''-\mathcal{H}\pi'\right) \tag{5.20}$$

$$\eta_3(\tau)\equiv\frac{c_3{M_{\rm pl}}^3}{M^3}-\frac{6c_4{M_{\rm pl}}^3\pi''}{M^6a^2}+\frac{6{M_{\rm pl}}^3c_5\pi'}{M^9a^4}\left(2\mathcal{H}\pi''+\mathcal{H}'\pi'-2\mathcal{H}^2\pi'\right) \tag{5.21}$$

$$\eta_4(\tau)\equiv\frac{3c_5M_{\rm pl}{\pi'}^4}{2M^9a^4} \tag{5.22}$$

We are now ready to calculate the reduced DM bispectrum generated at late-times by gravitational instability. We use the well known definitions, Eqs. (4.16) and (4.17), to obtain

$$Q(\tau,k_1,k_2,k_3)=\frac{2F(\tau,\vec{k}_1,\vec{k}_2)P(\tau,k_1)P(\tau,k_2)+cyc.}{P(\tau,k_1)P(\tau,k_2)+cyc.}\,, \tag{5.23}$$

where $P(\tau,k_i)$ is the power spectrum and the time dependence part of $F(\tau,\vec{k}_1,\vec{k}_2)$ read

$$F(\tau)\equiv\int_{\tau_m}^{\tau}d\tau'\frac{\left[D_-(\tau)D_+(\tau')-D_+(\tau)D_-(\tau')\right]{D_+}^2(\tau')}{\left[D_+(\tau')D_-{}'(\tau')-D_-(\tau')D_+{}'(\tau')\right]{D_+}^2(\tau)}\mathcal{K}(\tau')\,. \tag{5.24}$$

Here we have neglected, as in Eq. 4.12, the contribution proportional to the initial second-order DM perturbations, since it gives a negligible contribution to the bispectrum on sub-horizon scales. The kernel shown in Eq. (5.17) agrees with the result obtained in [56] and Eq. (4.20) for the Cubic Galileon and also with the standard results for an EdS universe and the ΛCDM model [79, 80]. Comparing Eq. (5.17) with Eq. (4.20) we can recognize the same form, where the interesting time-dependent functions involved in the bispectrum are three. The first is the usual contribution coming from the growth rate, the second is linearly dependent on the modified Newton's constant ($\propto G_\pi a^2\rho_m$), while the third involves more non-linearities ($\propto {G_\pi}^3a^2{\rho_m}^2\gamma$). In the Cubic Galileon models we noted that the three terms give a comparable contribution on the DM bispectrum today. The next step will be to resolve numerically the bispectrum in order to understand if the new parameters introduced here can modify this behavior.

CHAPTER 6

THE HIGHLY NON-LINEAR REGIME

This Chapter is focused on the analysis of the collapse of a spherical DM density perturbation. This analysis involves highly non-linear effects, thus we have to take into account every non-linear term in the equations of motion. We use the notation and the results given in Sec. 2.1 and Sec. 3.1.

6.1 Non-Linear regime

When perturbations grow, Eqs. (3.3), (3.4) and (3.5) must be rewritten taking into account every perturbation term. Neglecting time-derivatives of perturbations and assuming that the characteristic scale of the perturbation is well within the Hubble radius, the fully non-linear equations are

$$\left(2M_{\rm pl}^2 + \dot{\pi}^2\gamma_1(t)\right)\nabla^2\Phi = -\delta\rho + \gamma_2(t)\nabla^2\varphi + \gamma_1(t)\left[\left(\nabla^2\varphi\right)^2 + \right.$$
$$\left. - \nabla^i{}_j\varphi\nabla^j{}_i\varphi\right] + \eta_1(t)\left[\left(\nabla^2\varphi\right)^3 + 2\nabla^i{}_j\varphi\nabla^j{}_k\varphi\nabla^k{}_i\varphi + \right.$$

$$-3\nabla^2\varphi\nabla^i{}_j\varphi\nabla^j{}_i\varphi\Big] - \frac{3}{2}\dot{\pi}^2\eta_1(t)\left[\nabla^2\varphi\nabla^2\Phi - \nabla^i{}_j\Phi\nabla^j{}_i\varphi\right], \tag{6.1}$$

$$\left(2M_{\rm pl}^2 + 3\gamma_3(t)\right)\nabla^2\Phi + \left(2M_{\rm pl}^2 + \dot{\pi}^2\gamma_1(t)\right)\nabla^2\Psi = 3\gamma_4(t)\nabla^2\varphi +$$
$$+ 3\eta_2(t)\left[\left(\nabla^2\varphi\right)^2 - \nabla^i{}_j\varphi\nabla^j{}_i\varphi\right] - \frac{3}{2}\dot{\pi}^2\eta_1(t)\left[\nabla^2\varphi\nabla^2\Psi - \nabla^i{}_j\Psi\nabla^j{}_i\varphi\right]. \tag{6.2}$$

Eq. (3.5), instead, takes the form

$$\gamma_5(t)\nabla^2\varphi + \gamma_2(t)\nabla^2\Psi + 3\gamma_4(t)\nabla^2\Phi + \eta_3(t)\left[\left(\nabla^2\varphi\right)^2 - \nabla^i{}_j\varphi\nabla^j{}_i\varphi\right] -$$
$$- \eta_4(t)\left[\left(\nabla^2\varphi\right)^3 + 2\nabla^i{}_j\varphi\nabla^j{}_k\varphi\nabla^k{}_i\varphi - 3\nabla^2\varphi\nabla^i{}_j\varphi\nabla^j{}_i\varphi\right] +$$
$$+ 2\gamma_1(t)\left[\nabla^2\varphi\nabla^2\Psi - \nabla^i{}_j\Psi\nabla^j{}_i\varphi\right] + 6\eta_2(t)\left[\nabla^2\varphi\nabla^2\Phi - \nabla^i{}_j\Phi\nabla^j{}_i\varphi\right] -$$
$$- \frac{3}{2}\dot{\pi}^2\eta_1(t)\left[\nabla^2\Psi\nabla^2\Phi - \nabla^i{}_j\Phi\nabla^j{}_i\Psi\right] + 3\eta_1(t)\Big[\left(\nabla^2\varphi\right)^2\nabla^2\Psi -$$
$$-2\nabla^2\varphi\nabla^i{}_j\varphi\nabla^j{}_i\Psi - \nabla^2\Psi\nabla^i{}_j\varphi\nabla^j{}_i\varphi + 2\nabla^i{}_j\varphi\nabla^j{}_k\varphi\nabla^k{}_i\Psi\Big] = 0\,, \tag{6.3}$$

where the $\eta_i(t)$ functions are listed in B.

Eqs. (6.1), (6.2) and (6.3) are more complicated than in the linear case, however, assuming spherical symmetry, they are in fact integrable. The boundary values of the perturbations can be determined by resorting to the physical meaning to these fields. For example, from GR we know that the physical solution of the Poisson equation is

$${\Psi_{GR}}'(t,r) = \frac{Gm(t,r)}{r^2}. \tag{6.4}$$

Recalling the definition of the mass function, $m(t,r) \equiv 4\pi \int_0^r \mathrm{d}r' r'^2 \delta\rho(t,r)$, if there are no singularities at $r = 0$ for the density perturbation, this relation tells us that ${\Psi_{GR}}'(t,0) = 0$ (to violate this limit we have to choose $\delta\rho(r) \propto r^{-n}$, with $n \geq 3$). At small scales we want to recover GR, so the physical meaning of $\Psi(t,r)$ should be that of gravitational potential ($\Psi'(t,r) \simeq {\Psi_{GR}}'(t,r)$). Indeed, the natural assignment is $\Psi'(t,0) = 0$. The same argument applies to $\Phi'(t, r \to 0) \simeq -{\Psi_{GR}}'(t, r \to 0)$. Instead, the scalar field and its perturbations are not directly observable quantities, so we have to choose the correct boundary value by mathematical arguments or by its effect on measurable physical quantities. As in Eq. (3.11), at $r \to 0$ there should be some divergent term. However, the same reasoning used in the linear case allows us to consider $\varphi'(r \to 0)$ finite.

Integrating Eqs. (6.1), (6.2) and (6.3) for a spherically symmetric object, we obtain

$$\gamma_6 \frac{\Phi'}{r} = -\frac{m(t,r)}{4\pi r^3} + \gamma_2 \frac{\varphi'}{r} + 2\gamma_1 \frac{\varphi'^2}{r^2} + 2\eta_1 \frac{\varphi'^3}{r^3} - 2\eta_1 \frac{\varphi'(0)^3}{r^3} - 3\dot{\pi}^2 \eta_1 \frac{\varphi' \Phi'}{r^2} \tag{6.5}$$

$$\gamma_7 \frac{\Phi'}{r} + \gamma_6 \frac{\Psi'}{r} = 3\gamma_4 \frac{\varphi'}{r} + 6\eta_2 \frac{\varphi'^2}{r^2} - 3\dot{\pi}^2 \eta_1 \frac{\varphi' \Psi'}{r^2} \tag{6.6}$$

$$\begin{aligned} &\gamma_5 \frac{\varphi'}{r} + \gamma_2 \frac{\Psi'}{r} + 3\gamma_4 \frac{\Phi'}{r} + 2\eta_3 \frac{\varphi'^2}{r^2} - 2\eta_4 \frac{\varphi'^3}{r^3} \\ &+ 2\eta_4 \frac{\varphi'(0)^3}{r^3} + 4\gamma_1 \frac{\varphi' \Psi'}{r^2} + 6\eta_1 \frac{\varphi'^2 \Psi'}{r^3} + 12\eta_2 \frac{\varphi' \Phi'}{r^2} - 3\dot{\pi}^2 \eta_1 \frac{\Phi' \Psi'}{r^2} = 0\,, \end{aligned} \tag{6.7}$$

Note that we have not yet analyzed the case in which the scalar field has a boundary value $\varphi'(r=0)$ finite, but different from zero; to do this we have to impose a physical condition. From Eq. (6.6), we can write

$$\begin{aligned} \frac{\varphi'(r)}{r} = &-\frac{\gamma_4}{4\eta_2} + \frac{\dot{\pi}^2 \eta_1}{4\eta_2} \cdot \frac{\Psi'(r)}{r} + \frac{\mathrm{Sgn}(\gamma_4)}{4\eta_2} \left[\left(-\gamma_4 + \dot{\pi}^2 \eta_1 \frac{\Psi'(r)}{r} \right)^2 + \right. \\ &\left. + \frac{8}{3}\eta_2 \left(2M_{\rm pl}^2 + 3\gamma_3 \right) \frac{\Phi'(r)}{r} + \frac{8}{3}\eta_2 \left(2M_{\rm pl}^2 + \dot{\pi}^2 \gamma_1 \right) \frac{\Psi'(r)}{r} \right]^{1/2} ; \end{aligned} \tag{6.8}$$

here we have chosen the solution which matches the linear one (3.11) when $r \to \infty$. Without any loss of generality, the metric perturbations can be written as

$$\Psi'(r) = \Psi'_{GR}(r) \left[1 + \delta_\Psi(r)\right] = \frac{G\, m(t,r)}{r^2} \left[1 + \delta_\Psi(r)\right] \tag{6.9}$$

$$\Phi'(r) = \Phi'_{GR}(r) \left[1 + \delta_\Phi(r)\right] = -\frac{G\, m(t,r)}{r^2} \left[1 + \delta_\Phi(r)\right] . \tag{6.10}$$

In this case, $\Psi_{GR}(r)$ can be understood as the gravitational potential generated by a perturbation in the ΛCDM model. When $r \ll r_V$, δ_Ψ and δ_Φ have to be small by solar-system constraints ($\delta_\Psi\,, \delta_\Phi \lesssim 10^{-3}$), so we can treat them as small perturbations. In this limit, at first-order, Eq. (6.8) becomes

$$\begin{aligned} \frac{\varphi'(r)}{r} \simeq &- \frac{\gamma_4}{4\eta_2} + \frac{\dot{\pi}^2 \eta_1 \Psi'_{GR}(r)}{4\eta_2 r} + \frac{\mathrm{Sgn}(\gamma_4) f(t,r)}{4\eta_2} + \frac{\dot{\pi}^2 \eta_1 \Psi'_{GR}(r)}{4\eta_2 r} \delta_\Psi(t,r) + \\ &+ \frac{\mathrm{Sgn}(\gamma_4) \Psi'_{GR}(r)}{12\eta_2 f(t,r) r} \left[3\dot{\pi}^4 {\eta_1}^2 \frac{\Psi'_{GR}(r)}{r} - 3\dot{\pi}^2 \gamma_4 \eta_1 + 4\gamma_6 \eta_2 \right] \delta_\Psi(t,r) + \\ &- \frac{\mathrm{Sgn}(\gamma_4) \gamma_7 \Psi'_{GR}(r)}{f(t,r) r} \delta_\Phi(t,r)\,, \end{aligned} \tag{6.11}$$

where

$$f(t,r) \equiv \left[{\gamma_4}^2 + \dot{\pi}^4 {\eta_1}^2 \frac{{\Psi'_{GR}(r)}^2}{r^2} - 8\gamma_3\eta_2 \frac{\Psi'_{GR}(r)}{r} - 2\dot{\pi}^2\gamma_4\eta_1 \frac{\Psi'_{GR}(r)}{r} + \right.$$
$$\left. + \frac{8}{3}\dot{\pi}^2\gamma_1\eta_2 \frac{\Psi'_{GR}(r)}{r} \right]^{1/2} . \tag{6.12}$$

From Eq. (6.11), we are now ready to choose a reasonable boundary value for $\varphi'(r)$. It is sufficient to suppose that neither δ_Ψ nor δ_Φ diverge in the limit $r \to 0$, to show that $\varphi'(r \to 0) = 0$.

6.1.1 Vainshtein radius

Having obtained the non-linear equations of motion, we are now ready to investigate the radius at which non-linearities become important. The simplest way to estimate r_V is to plug-in the linear solutions into the non-linear equations, and estimate when the non-linear terms become comparable with the linear ones. First, considering Eq. (6.5), from the quadratic term we obtain

$$2\frac{\gamma_1}{\gamma_2} \cdot \left. \frac{\varphi'}{r} \right|_{r=r_{V_1}} \simeq 1 \, . \tag{6.13}$$

To solve this equation, we need to know the matter density profile. However, using Eq. (3.11), in the general case we find

$${r_{V_1}}^3 = \frac{\gamma_1(t)A(t)}{2\pi\gamma_2(t)} \left[m(t,r) + \Delta m(t,r,r_{V_1}) \right] , \tag{6.14}$$

where $\Delta m(t,r,r_{V_1}) = 4\pi \int_r^{r_{V_1}} dr' {r'}^2 \delta\rho$. The interior solution for a top-hat profile leads to an r-invariant equation. The simple consideration is that, depending on the epoch and on the choice of the background parameters, we can have this region all inside or all outside the Vainshtein region. Instead, outside a source of mass M_s we find (defining $R_V \equiv r_V(R)$)

$$\left(\frac{R_{V_1}}{R} \right)^3 = \left| \frac{2\gamma_1}{\gamma_2} \cdot \frac{A(t)M_s}{4\pi R^3} \right| . \tag{6.15}$$

The same procedure for the cubic term leads to

$$\left(\frac{R_{V_2}}{R} \right)^3 = \left| \sqrt{\frac{2\eta_1}{\gamma_2}} \cdot \frac{A(t)M_s}{4\pi R^3} \right| . \tag{6.16}$$

Comparing the two Vainshtein radii we see that they are comparable. This means that we have an exterior linear region, but, when we enter the non-linear one, quadratic and cubic terms can both dominate. Indeed, the contribution derived from the terms c_4 and c_5 influences in a non-negligible way the scalar field profile. This also proves that at sufficiently large distances we recover the predictions of the linear theory, discussed in Sec. 3.1.

Other three important Vainshtein radii, coming from Eqs. (6.6) and (6.7), are

$$\left(\frac{R_{V_3}}{R}\right)^3 = \frac{2\eta_2}{\gamma_4}\cdot\frac{A(t)M_s}{4\pi R^3}\,, \tag{6.17}$$

$$\left(\frac{R_{V_4}}{R}\right)^3 = \frac{2\eta_3}{\gamma_5}\cdot\frac{A(t)M_s}{4\pi R^3}\,. \tag{6.18}$$

and

$$\left(\frac{R_{V_5}}{R}\right)^3 = \sqrt{\frac{2\eta_4}{\gamma_5}}\cdot\frac{A(t)M_s}{4\pi R^3}\,. \tag{6.19}$$

Here we have neglected non-linear interactions which couple φ with Φ and Ψ, because they produce results analogous to the previous ones. The Vainshtein radius can be set as $R_V \equiv \text{Max}(R_{V_i})$, where $i = 1, .., 11$. It is straightforward to prove that $R_V(t \to -\infty) \to +\infty$, while $R_V(t \to +\infty) = f(\alpha, \beta, x_{\text{dS}})M_s/(4\pi M_{\text{pl}}H^2_{\text{dS}})$, where f is a generic function of the background parameters. This result agrees with the predictions of [86] and [87].

6.1.2 Galileon field evolution

In this section we study the Galileon field evolution, starting from Eqs. (6.5), (6.6) and (6.7). These are three algebraic equations in $\Psi'(r)$, $\Phi'(r)$ and $\varphi'(r)$, so it is straightforward to obtain a sixth-order polynomial equation in $\varphi'(r)$ (to simplify the problem we will work under the assumption that $x_{\text{ds}} = 1$)

$$\begin{aligned}&\frac{\varphi'^6}{r^6} + \lambda_1(t)\frac{\varphi'^5}{r^5} + \lambda_2(t)\frac{\varphi'^4}{r^4} + \left[\lambda_3(t)\delta_m + \lambda_4(t)\right]\frac{\varphi'^3}{r^3} + \left[\lambda_5(t)\delta_m + \right.\\ &\left.+\lambda_6(t)\right]\frac{\varphi'^2}{r^2} + \left[\lambda_7(t)\delta_m + \lambda_8(t)\right]\frac{\varphi'}{r} + \lambda_9(t)\delta_m + \lambda_{10}(t){\delta_m}^2 = 0\,,\end{aligned} \tag{6.20}$$

where λ_i are background functions, combinations of γ_i and η_i. From Eq. (6.20) it follows that $\varphi'(r)$ has six branches of solutions. What is the correct one? Remembering the Vainshtein effect, we want that the physical solution reduces to Eq. (3.11) at large distances. Of course, this condition cannot be verified analytically, but it is sufficient to choose between the real solutions of Eq. (6.20).

Are we sure that, for a given couple (α, β), Eq. (6.20) has at least a couple of solutions during the whole evolution of the universe? Obviously this condition is not sufficient to ensure the existence of the physical solution, but it is a necessary condition. In the linear regime the existence of a physical solution was proved in Sec. 3.1, thus the problems can be inside the Vainshtein radius. As proved in Sec. 6.1.1, at small distances non-linear terms become dominant for the evolution of the scalar field. In particular, instead of Eq. (6.20), we can work with the equation

$$\frac{\varphi'^6}{r^6} + \lambda_1(t)\frac{\varphi'^5}{r^5} + \lambda_2(t)\frac{\varphi'^4}{r^4} + \lambda_{10}(t){\delta_m}^2 = 0. \tag{6.21}$$

Also in this case we do not have an analytic solution for the scalar field; however Eq. (6.21) gives new constraints on the allowed region in the parameter space (α, β).

Consider a function as

$$f(x) = x^6 + A\,x^5 + B\,x^4 + C\,, \tag{6.22}$$

where A, B, $C \neq 0$ are real coefficients. The RHS of this equation has the same form as Eq. (6.21), after the substitution $\varphi'(t,r)/r \to x$. It was demonstrated that there is no analytic method to find a solution for $f(x) = 0$, when $f(x)$ is a fifth or higher degree polynomial. However, since

$$\lim_{x\to\pm\infty} f(x) = +\infty\,, \tag{6.23}$$

it is sufficient to require that a minimum of this function is < 0, to be sure to have at least a couple of real solutions. The points which satisfy $f'(x) = 0$ are

$$x_{1,2,3} = 0 \qquad x_{4,5} = -\frac{5}{12}A \pm \sqrt{\frac{25}{144}A^2 - \frac{2}{3}B}\,. \tag{6.24}$$

The zeros of Eq. (6.20) can be understood as six perturbative terms around x_i. Let us assume that, for the purpose of this section, these perturbations are small.

The set of parameters for which $f(x) = 0$ has at least a couple of solutions, which are given by

$$f(x_1) < 0 \quad \vee \quad f(x_4) < 0 \quad \vee \quad f(x_5) < 0\,. \tag{6.25}$$

Substituting our background functions into the parameters A, B, C, we must pay attention to the dependence on t, because the previous inequalities have to be hold true $\forall\, t$. The first one becomes

$$f(x_1) = \frac{H_{\rm ds}^{12}\, M_{\rm Pl}^{6}}{144\, \dot{\pi}^4\, \beta^2} \cdot \frac{\left(\frac{\dot{\pi}}{H_{\rm ds} M_{\rm Pl}}\right)^4 \left[\alpha + 6\beta \left(\frac{\ddot{\pi}}{H_{\rm ds}^2 M_{\rm Pl}}\right)\right] - 2}{4 + \left(\frac{\dot{\pi}}{H_{\rm ds} M_{\rm Pl}}\right)^4 \left[-5\alpha + 42\beta \left(\frac{\ddot{\pi}}{H_{\rm ds}^2 M_{\rm Pl}}\right)\right]} \delta_m(t)^2 < 0\,. \tag{6.26}$$

It can be proved that this condition is verified $\forall\, t$, when

$$\begin{cases} \alpha < 4/5 \\ \alpha \lesssim 5.22\beta + 1.93 \\ \alpha \lesssim -3.73\beta + 4.83\,. \end{cases} \tag{6.27}$$

These relations were obtained evaluating the above expression at some critical times, when $f(x_1)$ results maximized/minimized. We were able to do this because $f(0)$ takes a simple form, but this is not the case for $f(x_4)$ and $f(x_5)$. In fact, the form of these functions at the points $x_{4,5}$ is

$$\begin{aligned} f(x_{4,5}) =& C - \frac{2}{12^6}\left(\pm 5A + \sqrt{25A^2 - 96B}\right)^4 \left(5A^2 - 24B + \right. \\ & \left. \pm A\sqrt{25A^2 - 96B}\right)\,. \end{aligned} \tag{6.28}$$

In our case, the parameter C depends on the matter-density perturbation, so the inequalities which follow from the above expression have to be evaluated in two distinct cases. The first one is when the density term dominates on the other terms (the analysis is the same as in $f(0) < 0$ case), the second one when it is subdominant. The latter case involves more complicated expressions for the parameters α and β, so we were only able to solve it numerically. Combining these results with the no-ghost condition given in [46], the constraints on the parameters α and β become

$$\begin{cases} \alpha > 2\,\beta \\ \alpha < 2\,\beta + 2/3 \\ \alpha < 4/5 \\ \alpha \lesssim 5.7\,\beta + 2.62\,, \end{cases} \tag{6.29}$$

and are represented in Fig. 6.1.

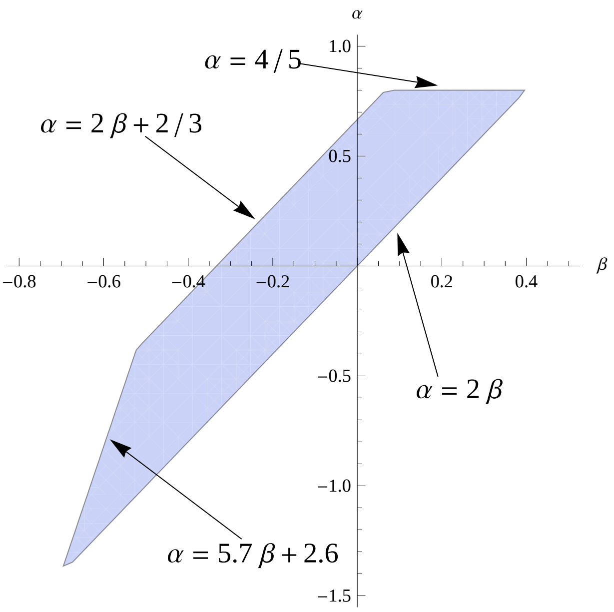

Figure 6.1: In this figure we show the allowed region in the plane (β, α) obtained by mixing the no-ghost conditions and the conditions for the existence of the scalar field in the non-linear regime.

6.2 Spherical collapse

In this section we will restrict our analysis to a top-hat matter configuration

$$\rho(r) = \begin{cases} \rho_0 + \delta\rho & r \leq R \\ \rho_0 & r > R \end{cases} , \; m(r) = \begin{cases} \delta M \, (r/R)^3 & r \leq R \\ \delta M & r > R \end{cases} . \tag{6.30}$$

The mass δM is the total mass of the density perturbation $\delta\rho$, while $M \equiv 4/3\,\pi\,(\rho_0 + \delta\rho)\,R^3$. The two masses are related by

$$\delta M = \frac{\delta}{1+\delta}\, M \,, \tag{6.31}$$

where $\delta \equiv \delta\rho/\rho_0$ is the density contrast.

To study the dynamics of a spherical matter perturbation we need the well known equation

$$\ddot{\delta} - \frac{4}{3}\frac{\dot{\delta}^2}{1+\delta} + 2H\dot{\delta} = (1+\delta)\,\nabla^2\Psi \,, \tag{6.32}$$

which follows from the non-linear continuity and the Euler equation for a pressureless fluid of non-relativistic matter in a top-hat configuration [85]. Eqs. (6.1), (6.2) and (6.3) tell us that, inside a top-hat density perturbation, $\Psi'(r) \propto r$, which means that $\nabla^2\Psi$ will be r-independent. Indeed, a top-hat profile, remains a top-hat profile during its whole evolution despite the non-validity of Birkhoff's theorem.

To solve Eq. (6.32), we have followed [85]; here we briefly summarize the main steps. Assuming the total mass conservation, $R^3\,\rho_0\,(1+\delta) = \text{const.}$, Eq. (6.32) can be rewritten in terms of R

$$\frac{\ddot{R}}{R} = H^2 + \dot{H} - \frac{1}{3}\nabla^2\Psi\,. \tag{6.33}$$

From this equation we can distinguish all the sources that affect the collapse dynamics: $H^2 + \dot{H}$ contains the contribution of the background (matter and dark energy), while $\nabla^2\Psi$ contains the contribution of matter and scalar field perturbations. Using $N = \ln a$ as a time variable and defining

$$y \equiv \frac{R}{R_i} - \frac{a}{a_i}\,, \tag{6.34}$$

where R_i and a_i are the initial radius of the perturbation and the initial scale factor, Eq. (6.33) becomes

$$y'' + \frac{H'}{H}\,y' - \left(1 + \frac{H'}{H}\right)\,y = -\frac{1}{3}\,\left(y + e^{N-N_i}\right)\,\nabla^2\Psi\,, \tag{6.35}$$

where a prime denotes differentiation w.r.t. N. The density contrast is

$$\delta = (1+\delta_i)\cdot\left(e^{N_i-N}y + 1\right)^{-3} - 1\,. \tag{6.36}$$

Eq. (6.35) can be solved numerically setting the initial conditions. From Eq. (6.34) we know that $y_i = 0$ and $y'_i = -\delta'_i/(3(1+\delta_i))$. Supposing that the perturbations start growing linearly during matter-dominance, the linearization of Eq. (6.32) can be solved analytically. The growing mode is $\delta \propto a$, so $\delta' = \delta$, thus the second initial condition becomes $y'_i = -\delta_i/3$. We also set $a_i = 10^{-5}$, while the initial density perturbation is set to collapse exactly at $a_0 = 1$.

6.2.1 Virialisation

The Virial Theorem states that a stable system must satisfy the relation

$$W + 2T = 0\,, \tag{6.37}$$

where

$$T \equiv \frac{1}{2}\int d^3x\rho v^2 = \frac{3}{10}M\dot{R}^2 \tag{6.38}$$

is the kinetic energy (the last equality holds true for a top-hat profile), while

$$W \equiv -\int d^3x\rho_m(\vec{x})\vec{x}\cdot\nabla\Psi = -\frac{3M}{R^3}\sum_i\int_0^R \mathrm{d}r\cdot r^3\frac{\mathrm{d}\Psi_i(r)}{\mathrm{d}r} \tag{6.39}$$

is the trace of the potential energy tensor. As in the previous equation the last equality holds true only for a top-hat profile. $\Psi_i(r)$ denotes each component that contributes to the total gravitational potential.

Usually energy conservation is used, but, as noted in [86], for a time-dependent dark energy model, energy is not strictly conserved. So, during the collapse phase, the virial radius can be estimated as the radius at which the virial condition (6.37) is satisfied.

Important quantities that can be extrapolated from the dynamics of the collapse are the linearized density contrast δ_c, and the virial overdensity

$$\Delta_{vir} \equiv \frac{\rho_{vir}}{\rho_{collapse}} = [1+\delta(R_{vir})]\cdot\left(\frac{a_{collapse}}{a_{vir}}\right)^3. \tag{6.40}$$

6.2.2 Numerical Results

Case $\beta = 0$**,** $x_{\mathrm{dS}} = 1$**.**

This is the case in which the fifth term of Eq. (1.2) gives no contribution. Eqs. (6.5), (6.6) and (6.7) become simpler. In particular, the modified Poisson equation reads

$$\nabla^2\Psi = 3\Omega_m H_{\mathrm{dS}}^2 a^{-3} x^4 \frac{2x^4-\alpha}{2x^4+3\alpha}\,\delta +$$

$$
-\frac{3H_{\rm dS}^2x^2\left[2x^4(2+\alpha)+\alpha(-2+15\alpha)\right]-36\alpha(2x^4+3\alpha)\dot{H}}{H_{\rm dS}^2M_{\rm pl}(2x^4+3\alpha)^2}\cdot\frac{\varphi'}{r}+
$$
$$
-\frac{12\alpha x^2(2x^4-3\alpha)}{H_{\rm dS}^2M_{\rm pl}^2(2x^4+3\alpha)^2}\cdot\frac{\varphi'^2}{r^2}\,, \tag{6.41}
$$

with $x \equiv H/H_{\rm dS}$, and φ' is a solution of

$$
\alpha_1\cdot\frac{\varphi'^3}{r^3}+\alpha_2\cdot\frac{\varphi'^2}{r^2}+(\alpha_3+\alpha_4\delta)\cdot\frac{\varphi'}{r}+\alpha_5\delta=0\,, \tag{6.42}
$$

with

$$
\alpha_1 = 4\alpha x^2\left(4x^8+24x^4\alpha-45\alpha^2\right) \tag{6.43}
$$
$$
\alpha_2 = 2M_{\rm pl}\left[H_{\rm dS}^2x^2\left(4x^4(2+3\alpha)(x^4+6\alpha)-9\alpha^2(2-21\alpha)\right)+\right. \tag{6.44}
$$
$$
\left.+6\alpha\left(4x^8-24\alpha x^4-45\alpha^2\right)\dot{H}\right]
$$
$$
\alpha_3 = -2H_{\rm dS}^2M_{\rm pl}^2\left[H_{\rm dS}^2x^2\left[2x^8(2+\alpha)+x^4\left(-4+8\alpha+21\alpha^2\right)+\right.\right.
$$
$$
\left.+\alpha\left(2-21\alpha+45\alpha^2\right)\right]-\left[4x^8(2+3\alpha)+27\alpha^2(-2+5\alpha)+\right.
$$
$$
\left.\left.+12x^4\alpha(-2+9\alpha)\right]\dot{H}\right] \tag{6.45}
$$
$$
\alpha_4 = -8e^{-3n}H_{\rm dS}^4M_{\rm pl}^2\Omega_m x^4\left(2x^4-3\alpha\right)\alpha \tag{6.46}
$$
$$
\alpha_5 = -e^{-3n}H_{\rm dS}^4M_{\rm pl}^3\Omega_m x^2\left[H_{\rm dS}^2x^2\left(2x^4(2+\alpha)+\alpha(-2+15\alpha)\right)+\right.
$$
$$
\left.-12\alpha\left(2x^4+3\alpha\right)\dot{H}\right]\,. \tag{6.47}
$$

Of course, among the solutions we want the one that reduces to Eq. (3.11) when $r \gg r_V$.

Although this is a particular case, it is interesting to show the role of $\mathcal{L}_4$ in Eq. (1.2). In Fig. 6.2 we have plotted the solution of Eq. (6.35) for various α. It should be noted that modifications w.r.t. the ΛCDM model are present during the collapse phase. This is, as expected, an effect of the increasing contribution from the scalar field. In Tab. (6.1) we show the values assumed by the linearized density contrast and the virial overdensity.

Case $\alpha = 0$, $x_{\rm dS} = 1$.

In this paragraph we analyze another particular case, the one which shows the role of $\mathcal{L}_5$, Eq. (1.2), in the dynamics of the collapse. Compared to the previous

Model	δ_i (10^{-5})	δ_c	a_{tur}	R_{tur}/R_i	Δ_{tur}	a_{vir}	R_{vir}/R_i	Δ_{vir}
ΛCDM	2.220	1.674	0.553	28840	42	0.919	13910	371
$\alpha = 0$	2.205	1.689	0.551	28990	41	0.914	14170	351
$\alpha = 1/10$	2.243	1.723	0.537	28380	44	0.899	14500	328
$\alpha = 1/5$	2.272	1.757	0.527	27930	46	0.884	14850	305
$\alpha = 1/3$	2.300	1.801	0.515	27470	48	0.863	15430	272
$\alpha = 1/2$	2.327	1.847	0.504	27020	51	0.836	16150	238
$\alpha = 2/3$	2.345	1.882	0.495	26680	53	0.812	16710	215

Table 6.1: Here we show numerical results of physical interesting quantities in the case $\beta = 0$, $x_{\rm dS} = 1$ *for various* α

paragraph, when $\beta \neq 0$ Eq. (6.20) cannot have an analytic solution. By the parameter conditions, Eqs. (2.16) and (6.29), $-1/3 \leq \beta \leq 0$, so, to investigate the parameter region in which $\beta > 0$ we need to set $\alpha > 0$.

The dynamics of the collapse is shown in Fig. 6.3, while the linearized density contrast and the virial overdensity for various β can be found in Table (6.2). It is important to note that the onset of the fifth term in Eq. (1.2) plays a crucial role in the virialisation process. In fact we can see that varying the parameter β there is a substantial modification of Δ_{vir} w.r.t. the ΛCDM model.

Case $\alpha \neq 0$, $\beta \neq 0$, $x_{\rm dS} = 1$.

This is the most general case, despite the assumption $x_{\rm dS} = 1$. Here we can evaluate the sum of the contribution of the terms $\mathcal{L}_4$ and $\mathcal{L}_5$, Eqs. (1.6) and (1.7), in the whole parameter region defined by Eq. (6.29). The dynamics of the collapse is shown in Fig. 6.4. In Table (6.3) it can be noted that as α and β grow we obtain a larger δ_c, thus it should be easy to remove a large piece of parameter space from the allowed region.

Model	δ_i (10^{-5})	δ_c	a_{tur}	R_{tur}/R_i	Δ_{tur}	a_{vir}	R_{vir}/R_i	Δ_{vir}
ΛCDM	2.220	1.674	0.553	28840	42	0.919	13910	371
$\beta = 0$	2.205	1.689	0.551	28990	41	0.914	14170	351
$\beta = -0.005$	2.219	1.700	0.547	28800	42	0.907	14410	334
$\beta = -0.01$	2.227	1.707	0.544	28680	42	0.911	13810	380
$\beta = -0.02$	2.238	1.717	0.540	28500	43	0.910	13600	398
$\beta = -0.05$	2.263	1.742	0.531	28120	45	0.895	14050	361
$\beta = -0.07$	2.277	1.757	0.527	27910	46	0.883	14470	330
$\beta = -0.1$	2.296	1.780	0.520	27620	47	0.866	15060	293
$\beta = -0.2$	2.356	1.857	0.501	26740	52	0.813	16440	225
$\beta = -0.3$	2.412	1.928	0.484	25980	57	0.769	17150	198

Table 6.2: Here we show numerical results of physically interesting quantities, in the case $\alpha = 0$, $x_{\rm dS} = 1$ for various β

Model		δ_i (10^{-5})	δ_c	a_{tur}	R_{tur}/R_i	Δ_{tur}	a_{vir}	R_{vir}/R_i	Δ_{vir}
ΛCDM		2.220	1.674	0.553	28840	42	0.919	13910	371
α	β								
0.1	-0.1	2.323	1.815	0.511	27220	50	0.862	14760	311
-0.2	-0.2	2.356	1.831	0.499	26710	52	0.791	17230	195
-0.45	-0.4	2.383	1.875	0.492	26400	54	0.763	17820	177
-0.55	-0.4	2.362	1.851	0.496	26660	53	0.773	17710	180

Table 6.3: Here we show numerical results of physically interesting quantities in the case $\alpha = 0$, $x_{\rm dS} = 1$ for various β

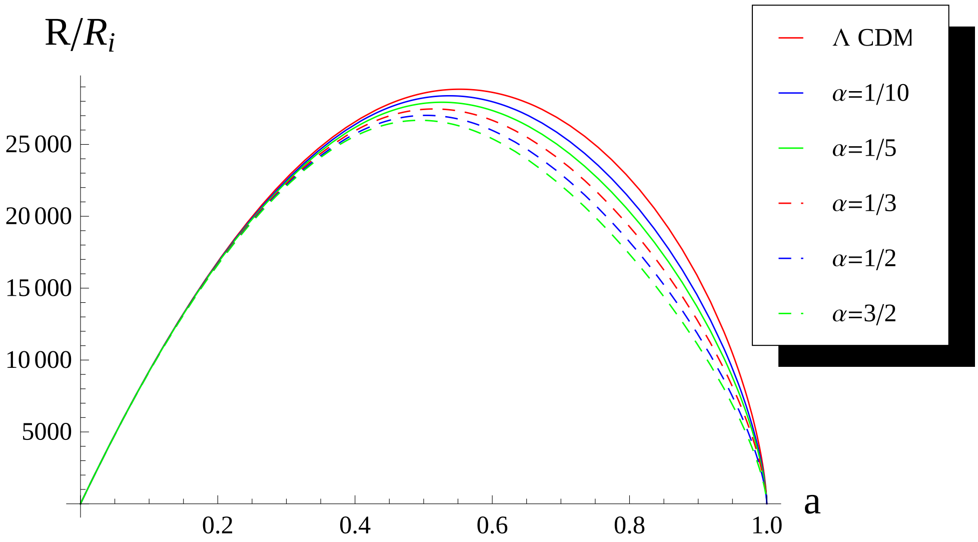

Figure 6.2: In the figure we plot the solution of Eq. (6.35), in terms of the normalized radius R/R_i of the top-hat perturbation, when $\beta = 0$ and $x_{\rm dS} = 1$. The initial density for each model is shown in Tab. (6.1).

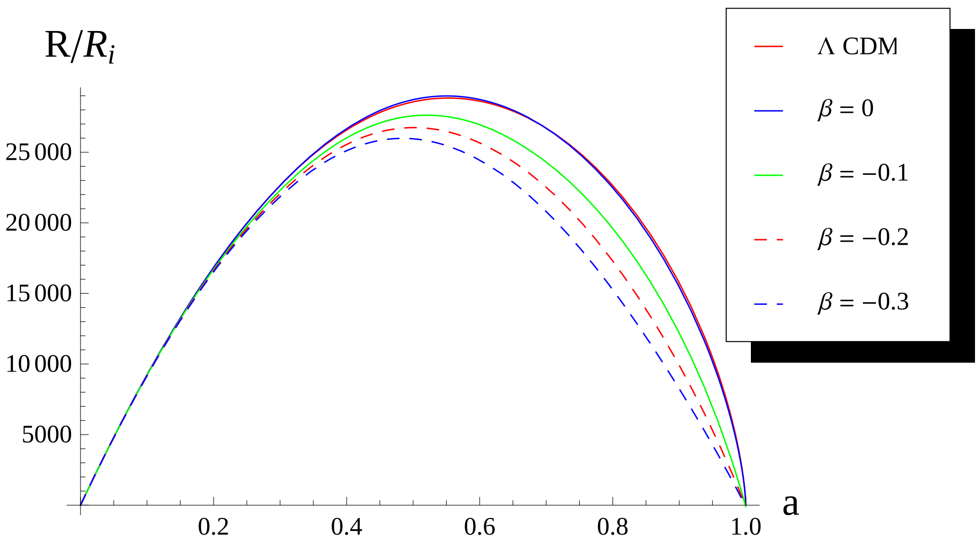

Figure 6.3: In the figure we plot the solution of Eq. (6.35), in terms of the normalized radius R/R_i of the top-hat perturbation, when $\alpha = 0$ and $x_{\rm dS} = 1$. The initial density for each model is shown in Tab. (6.2).

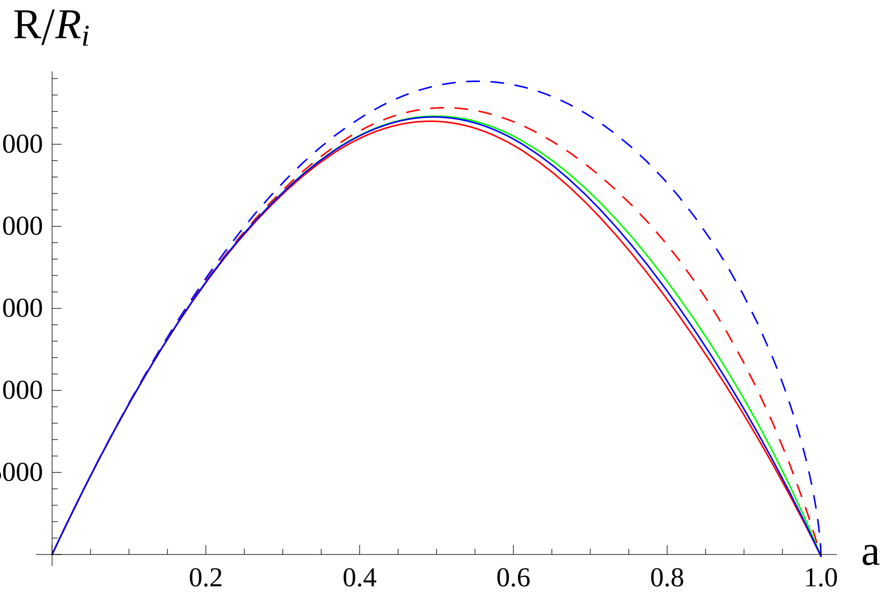

Figure 6.4: In the figure we plot the solution of Eq. (6.35), in terms of the normalized radius R/R_i *of the top-hat perturbation, when* $\alpha \neq 0$, $\beta \neq 0$ *and* $x_{\mathrm{dS}} = 1$*. The initial density for each model is shown in Tab. (6.3). The values for* (α, β) *are:* ΛCDM *blue dashed line;* $(-0.45, -0.4)$ *red line;* $(-0.2, -0.2)$ *green line;* $(-0.55, -0.4)$ *blue thick line;* $(0.1, -0.1)$ *red dashed line.*

CONCLUSIONS

In this thesis we have investigated certain aspects of the formation and the growth of cosmic structures within the context of the Galileon field theory [51, 56].

A central topic of this thesis is the study of the collapse of a spherical top-hat DM density perturbation in the highly non-linear regime. With this objective we have first reviewed the background evolution of the Galileon model, following the tracker solution found in [46]. The peculiarity of this tracker solution is that it ensures a dS stable point independent of the c_i parameters of Eq. (1.2). This assumption simplifies our equations, but it should also be easy to generalize our work to a more general background evolution. Once c_1 is set to zero, in Eq. (1.2) should remain only kinetic terms for the scalar field, thus the Galileon cannot be considered as a deviation from the ΛCDM model. It should work as a substitute of the cosmological constant, mimicking the effects of Λ on cosmological scales. Later, we have found two analytic functions that describe the evolution of the components of the universe at late-times (Sec. 2.1).

Then, in Sec. 3.1 we have shown that, in the linear approximation the scalar perturbations of a FLRW universe lead to a time-dependent gravitational constant $G_\pi(t)$, that modifies the gravitational potential generated by a distant or, equivalently, small source. The results we give do not represent a realistic model, i.e. they are not required to satisfy the observational bounds, rather they are chosen in order to display what one can generally expect from this theory.

The purpose of this work was the study of the role the new terms $\mathcal{L}_4$ and $\mathcal{L}_5$ have on the spherical collapse. It would be interesting to relate the linearized density contrast and the virial overdensity with observable quantities such as the halo mass function and bias. However, a better understanding of how Galileon field cosmologies confront observation is needed before proceeding to these detailed predictions. Our results indeed represent the first mandatory step in this direction.

The Galileon model can be successful because it possesses a Vainshtein mechanism, by which we can consider two distinct regions; the first one at large scales, where the linear approximation applies and the Galileon drives the cosmic acceleration, the second one where non-linearities are dominant. We have also shown how to recover a Vainshtein radius in agreement with the one of DGP and other simpler models (Sec. 6.1.1). Even though the study of the perturbations in a highly non-linear regime can not be completely analytic, we found some constraints, whose fulfillment allows Eq. (6.21) to have at least a couple of real solutions (Sec. 6.1.2).

Chapter 6 is devoted to the study of the collapse of a spherical top-hat matter perturbation. We have shown that the new terms $\mathcal{L}_4$ and $\mathcal{L}_5$ affect in a non-negligible way the dynamics of the collapse and the value of δ_c and Δ_{vir}. To study the virialisation process we paid attention to the energy non-conservation problem, calculating point by point the virial condition Eq. (6.37).

Another important aspect of the growth of structures we discussed in this thesis is the DM modified bispectrum within the context of the cubic covariant Galileon theory [56]. We worked on sub-horizon scales at second-order in the perturbations, to show the leading contribution in the weakly non-linear regime.

We have first studied the background with the most general potential that preserves the Galilean shift symmetry in a flat space-time. The contribution of c_3 is crucial to drive the late-time cosmic acceleration, however the deviations w.r.t. the ΛCDM model are smaller if $c_1 \sim c_3$ (Sec. 2.2).

At the linear level we have studied the evolution of the DM perturbations finding semi-analytical expressions for the growing and the decaying modes. In Fig. 3.5 we plot the deviations of the Galileon growth rate w.r.t. the growth rate of the ΛCDM model. For models in which the value of c_3 is negligible w.r.t. the value of c_1 the deviations are large (until about 100%), while, increasing c_3 the deviations decrease reaching $\simeq 10\%$ (Sec. 3.2).

Then, in Chapter 4 we have extended our analysis to second-order perturbations in order to calculate the DM bispectrum. Eq. (4.20), is one of our main results. It shows that the overall $\vec{k}$-dependence of the bispectrum is the same as in the ΛCDM model, with time dependent coefficients which depend on the particular Galileon model. We noted that, in general, there is a compensation effect (see Fig. 4.5) in the integrand of Eq. (4.20), $G(a, a')$, that reduces the deviations w.r.t. the ΛCDM model in the bispectrum. This effect is conserved if we compute $B(a, k_1, k_2, k_3)$ before the accelerated phase of the universe (see the right panel of Fig. 4.5), because the contribution of the Galileon is negligible and all models are indistinguishable. If the bispectrum is evaluated today and the model has $w \lesssim -0.85$, the compensation effect is preserved, giving deviations up to $\simeq 1\%$. Instead, we noted that this effect is less strong for those models which have $w \gtrsim -0.85$, allowing for larger deviations in the bispectrum up to $\simeq 5\%$. We argue that the Vainshtein mechanism can be a possible explanation for the overall suppression of the deviations w.r.t. the ΛCDM model in the DM bispectrum and we leave for future work further investigation of this aspect.

In Chapter 5 we have calculated the DM bispectrum of the Coupled Galileon to extend the analysis of Chapter 4. Even though we only give the analytical result, it is important to recognize in Eq. 5.17 the same form of Eq. 4.20. In fact it has the same overall $\vec{k}$-dependence and the same structure of the time-dependent functions in the kernel of the *Cubic Galileon*.

There are two main reasons to extend our analysis to a more general framework, i.e. the *Coupled Galileon*. The first is that comparisons with observational data prefer the *Coupled Galileon*, imposing severe constraints on the *Uncoupled Galileon* [59]. The second reason is that it is fundamental to answer the following question: is the suppression effect found in [56] a particular property of the Galileon models or a more universal effect? Note that a similar result was found for $f(R)$ theories in [84].

In order to explain this suppression effect, there are alternative directions that can be explored. First one can investigate the DM bispectrum using improvements as the renormalized perturbation theory [101], resummed perturbation theory or time-RG [102], to take into account more non-linear effects. In this regime the Vainshtein mechanism should play a key role, increasing the suppression effect we see on weakly non-linear scales. Another interesting test would be to extend our analysis using a General Relativistic approach, i.e. relaxing the sub-horizon approximation, in order to get closer to the horizon. In this regime the Vainshtein mechanism should lower its effect, increasing the deviations of the modified DM bispectrum w.r.t. the DM bispectrum of the ΛCDM model.

ACKNOWLEDGEMENTS

A heartfelt thanks to my supervisors, Nicola Bartolo and Sabino Matarrese. It is beautiful to see their passion for physics and the love for their students. It is a pleasure to work with them and, most important, to learn from them. They gave me a constant help and the encouragement to take the opportunities as conferences, workshops, schools. In particular, thanks to them I had the privilege to join the *CosmoClassic* collaboration. Then, a sincere thanks to the *CosmoClassic* members, a collaboration formed by really nice people and excellent physicists.

I also would like to thanks Daniele Bertacca with whom I shared my last work, always helpful for suggestions and discussions.

For the wonderful experience in Padova I am also honoured to thanks the other members of its cosmology group. In particular Angelo Ricciardone and Bin Hu, with whom I shared many useful discussions and experiences.

There are no words to express the gratitude I feel for my family: Anna, Franco, Giovanna, Maria and Massimo. I am delighted to thanks them for the freedom and support (financial and affective) they always gave me. A special thanks to my future wife, Marianna, to whom this thesis and my life are devoted.

APPENDIX A

STRESS-ENERGY TENSOR AND GALILEON FIELD EQUATION COMPONENTS

The terms of the stress-energy tensor of the scalar field read

$$T^{(0)}_{\mu\nu} = 2M_{\rm pl}\left[\pi G_{\mu\nu} + g_{\mu\nu}\Box\pi - \pi_{;\mu\nu}\right] \tag{A.1}$$

$$\begin{aligned} T^{(G)}_{\mu\nu} =& \frac{M_{\rm pl}}{M^3}\big[g_{\mu\nu}(\Box\pi)^2 - 2\Box\pi\pi_{;\mu\nu} - g_{\mu\nu}\pi_{;\alpha\beta}\pi^{;\alpha\beta} + 2\pi_{;\mu\alpha}\pi^{;\alpha}{}_{\nu} \\ & - 2g_{\mu\nu}R^{\alpha\beta}\pi_{;\alpha}\pi_{;\beta} + 2R_{\mu\alpha\nu\beta}\pi^{;\alpha}\pi^{;\beta} - G_{\mu\nu}\pi_{;\alpha}\pi^{;\alpha} \\ & + 2R_{\nu\alpha}\pi^{;\alpha}\pi_{;\mu} + 2R_{\mu\alpha}\pi^{;\alpha}\pi_{;\nu} - R\pi_{;\mu}\pi_{;\nu}\big] \end{aligned} \tag{A.2}$$

$$T^{(1)}_{\mu\nu} = \frac{1}{2}M^3 g_{\mu\nu}\,\pi \tag{A.3}$$

$$T^{(2)}_{\mu\nu} = -\,\pi_{;\mu}\,\pi_{;\nu} + \frac{1}{2}g_{\mu\nu}\,(\nabla\pi)^2 \tag{A.4}$$

$$T^{(3)}_{\mu\nu} = -\,\frac{1}{M^3}\left[\pi_{;\mu}\,\pi_{;\nu}\,\Box\pi - \pi_{;\{\mu}\,\pi_{;\nu\}\alpha}\,\pi^{;\alpha} + g_{\mu\nu}\,\pi^{;\alpha}\,\pi_{;\alpha\beta}\,\pi^{;\beta}\right] \tag{A.5}$$

$$T^{(4)}_{\mu\nu} = -\,\frac{2}{M^6}\Big[-\frac{1}{2}R\,\pi_{;\mu}\,\pi_{;\nu}\,(\nabla\pi)^2 + 2\pi_{;\mu\alpha}\,\pi^{;\alpha}\,\pi_{;\nu\beta}\,\pi^{;\beta}$$

$$-2\pi_{;\mu\nu}\,\pi^{;\alpha}\,\pi_{;\alpha\beta}\,\pi^{;\beta}+2\pi_{;\{\mu}\,\pi_{;\nu\}\alpha}\,\pi_{;\beta}\,\pi^{;\beta\alpha}-\pi_{;\mu}\,\pi_{;\nu}\,\pi_{;\alpha\beta}\,\pi^{;\alpha\beta}$$
$$-g_{\mu\nu}\,R_{\alpha\beta}\,\pi^{;\alpha}\,\pi^{;\beta}\,(\nabla\pi)^2+R_{\mu\alpha\nu\beta}\,\pi^{;\alpha}\,\pi^{;\beta}\,(\nabla\pi)^2+\frac{1}{2}g_{\mu\nu}\,(\Box\pi)^2\,(\nabla\pi)^2$$
$$-2g_{\mu\nu}\,\pi_{;\alpha}\,\pi^{;\beta}\,\pi_{;\beta\gamma}\,\pi^{;\alpha\gamma}-\pi_{;\mu\nu}\,\Box\pi\,(\nabla\pi)^2-2\pi_{;\{\mu}\,\pi_{;\nu\}\alpha}\,\pi^{;\alpha}\,\Box\pi$$
$$+\pi_{;\mu}\,\pi_{;\nu}\,(\Box\pi)^2-\frac{1}{2}g_{\mu\nu}\,\pi_{;\alpha\beta}\,\pi^{;\alpha\beta}\,(\nabla\pi)^2+2g_{\mu\nu}\,\pi^{;\alpha}\,\pi_{;\alpha\beta}\,\pi^{;\beta}\,\Box\pi$$
$$+R_{\alpha\{\mu}\,\pi_{;\nu\}}\,\pi^{;\alpha}\,(\nabla\pi)^2-\frac{1}{4}G_{\mu\nu}(\nabla\pi)^4+\pi_{;\mu\alpha}\,\pi_{;\nu}{}^{\alpha}\,(\nabla\pi)^2\Big] \tag{A.6}$$

$$T^{(5)}{}_{\mu\nu}=-\frac{2}{M^9}\Big[3\pi_{;\mu\gamma}\,\pi_{;\nu}{}^{\gamma}\,\pi^{;\alpha}\,\pi_{;\alpha\beta}\,\pi^{;\beta}-3\pi_{;\mu\alpha}\,\pi_{;\nu\gamma}\,\pi^{;\gamma}\,\pi_{;\beta}\,\pi^{;\beta\alpha}$$
$$+\frac{3}{2}\pi_{;\mu\nu}\,R_{\alpha\beta}\,\pi^{;\alpha}\,\pi^{;\beta}\,(\nabla\pi)^2-\frac{3}{2}R_{\alpha\{\mu}\,\pi_{;\nu\}\beta}\,\pi^{;\alpha}\,\pi^{;\beta}\,(\nabla\pi)^2$$
$$+\frac{3}{4}R\,\pi_{;\{\mu}\,\pi_{;\nu\}\alpha}\,\pi^{;\alpha}\,(\nabla\pi)^2+\frac{3}{2}G_{\mu\nu}\,\pi^{;\alpha}\,\pi_{;\alpha\beta}\,\pi^{;\beta}\,(\nabla\pi)^2$$
$$-\frac{3}{2}R_{\beta}{}^{\alpha}\,\pi_{;\{\mu}\,\pi_{;\nu\}\alpha}\,\pi^{;\beta}\,(\nabla\pi)^2-3\pi_{;\mu\alpha}\,\pi_{;\nu\beta}\,\pi^{;\alpha}\,\pi_{;\gamma}\,\pi^{;\beta\gamma}$$
$$-3\pi_{;\mu\alpha}\,\pi^{;\alpha\beta}\,\pi_{;\nu\beta}\,(\nabla\pi)^2+3\pi_{;\mu\nu}\,\pi_{;\alpha}\,\pi^{;\alpha\beta}\,\pi_{;\gamma\beta}\,\pi^{;\gamma}$$
$$-3\pi_{;\{\mu}\,\pi_{;\nu\}\alpha}\,\pi^{;\alpha\beta}\,\pi_{;\gamma\beta}\,\pi^{;\gamma}+\frac{3}{2}R_{\alpha\beta}\,\pi_{;\mu}\,\pi_{;\nu}\,\pi^{;\alpha\beta}\,(\nabla\pi)^2$$
$$+\pi_{;\mu}\,\pi_{;\nu}\,\pi_{;\alpha}{}^{\beta}\,\pi_{;\beta\gamma}\,\pi^{;\alpha\gamma}+\frac{3}{4}R\,\pi_{;\mu}\,\pi_{;\nu}\,\Box\pi\,(\nabla\pi)^2$$
$$+\frac{3}{2}\pi_{;\mu\nu}\,\pi_{;\alpha\beta}\,\pi^{;\alpha\beta}\,(\nabla\pi)^2+\frac{3}{2}\pi_{;\{\mu}\,\pi_{;\nu\}\alpha}\,\pi^{;\alpha}\,\pi_{;\beta\gamma}\,\pi^{;\beta\gamma}$$
$$-\frac{3}{2}\pi_{;\mu}\,\pi_{;\nu}\,\pi_{;\alpha\beta}\,\pi^{;\alpha\beta}\,\Box\pi+\frac{3}{2}R_{\alpha\gamma\beta\{\mu}\,\pi_{;\nu\}}{}^{\alpha}\,\pi^{;\beta}\,\pi^{;\gamma}\,(\nabla\pi)^2$$
$$-\frac{3}{2}R_{\alpha\{\mu}\,\pi_{;\nu\}}\,\pi_{;\beta}\,\pi^{;\beta\alpha}\,(\nabla\pi)^2-\frac{3}{2}R_{\alpha\gamma\beta\{\mu}\,\pi_{;\nu\}}\,\pi^{;\gamma}\,\pi^{;\alpha\beta}\,(\nabla\pi)^2$$
$$-\frac{3}{2}R\,\pi_{;\mu}\,\pi_{;\nu}\,\Box\pi\,(\nabla\pi)^2+\frac{3}{2}R_{\alpha\{\mu}\,\pi_{;\nu\}}\,\pi^{;\alpha}\,\Box\pi\,(\nabla\pi)^2$$
$$+3g_{\mu\nu}\,\pi_{;\alpha}\,\pi_{;\beta}\,\pi^{;\alpha\gamma}\,\pi_{;\gamma\tau}\,\pi^{;\beta\tau}+3g_{\mu\nu}\,R_{\gamma\beta}\,\pi_{;\alpha}\,\pi^{;\gamma}\,\pi^{;\alpha\beta}\,(\nabla\pi)^2$$
$$-\frac{3}{2}R_{\mu\nu\alpha\beta}\,\pi_{;\gamma}\,\pi^{;\alpha}\,\pi^{;\beta\gamma}\,(\nabla\pi)^2-3R_{\mu\beta\nu\gamma}\,\pi_{;\alpha}\,\pi^{;\gamma}\,\pi^{;\alpha\beta}\,(\nabla\pi)^2$$
$$+\frac{3}{2}g_{\mu\nu}\,R_{\alpha\gamma\beta\tau}\,\pi^{;\alpha}\,\pi^{;\beta}\,\pi^{;\gamma\tau}\,(\nabla\pi)^2+g_{\mu\nu}\,\pi^{;\alpha}{}_{\beta}\,\pi_{;\alpha\gamma}\,\pi^{;\beta\gamma}\,(\nabla\pi)^2$$
$$+3\pi_{;\mu\alpha}\,\pi^{;\alpha}\,\pi_{;\nu\beta}\,\pi^{;\beta}\,\Box\pi-3\pi_{;\mu\nu}\,\pi^{;\alpha}\,\pi_{;\alpha\beta}\,\pi^{;\beta}\,\Box\pi+\frac{1}{2}g_{\mu\nu}(\Box\pi)^3(\nabla\pi)^2$$
$$+3\pi_{;\{\mu}\,\pi_{;\nu\}\alpha}\,\pi^{;\beta\alpha}\,\pi_{;\beta}\,\Box\pi+3\pi_{;\mu\alpha}\,\pi_{;\nu}{}^{\alpha}\,\Box\pi\,(\nabla\pi)^2$$
$$-\frac{3}{2}g_{\mu\nu}\,R_{\alpha\beta}\,\pi^{;\alpha}\,\pi^{;\beta}\,\Box\pi\,(\nabla\pi)^2+\frac{3}{2}R_{\mu\alpha\nu\beta}\,\pi^{;\alpha}\,\pi^{;\beta}\,\Box\pi\,(\nabla\pi)^2$$
$$-\frac{3}{2}\pi_{;\mu\nu}\,(\Box\pi)^2\,(\nabla\pi)^2-\frac{3}{2}\pi_{;\{\mu}\,\pi_{;\nu\}\alpha}\,\pi^{;\alpha}\,(\Box\pi)^2+\frac{1}{2}\pi_{;\mu}\,\pi_{;\nu}\,(\Box\pi)^3$$
$$-\frac{3}{2}g_{\mu\nu}\,\pi^{;\alpha}\,\pi_{;\alpha\beta}\,\pi^{;\beta}\,\pi_{;\gamma\tau}\,\pi^{;\gamma\tau}-\frac{3}{2}g_{\mu\nu}\,\pi_{;\alpha\beta}\,\pi^{;\alpha\beta}\,\Box\pi\,(\nabla\pi)^2$$

$$-3g_{\mu\nu}\,\pi_{;\alpha}\,\pi^{;\gamma}\,\pi_{;\gamma\beta}\,\pi^{;\alpha\beta}\,\Box\pi+\frac{3}{2}g_{\mu\nu}\,\pi^{;\alpha}\,\pi_{;\alpha\beta}\,\pi^{;\beta}\,(\Box\pi)^2\Big]\,. \tag{A.7}$$

The terms appearing in the equation of motion for the scalar field read

$$\xi^{(0)}=-\,M_{\rm pl}R \tag{A.8}$$

$$\xi^{(G)}=\frac{2M_{\rm pl}}{M^3}G_{\mu\nu}\pi^{;\mu\nu} \tag{A.9}$$

$$\xi^{(1)}=\frac{M^3}{2} \tag{A.10}$$

$$\xi^{(2)}=-\,\Box\pi \tag{A.11}$$

$$\xi^{(3)}=\frac{1}{M^3}\left[-(\Box\pi)^2+R_{\mu\nu}\,\pi^{;\mu}\,\pi^{;\nu}+\pi_{;\mu\nu}\,\pi^{;\mu\nu}\right] \tag{A.12}$$

$$\begin{aligned}\xi^{(4)}=&\frac{1}{M^6}\Big[2R\,\pi^{;\mu}\,\pi_{;\mu\nu}\,\pi^{;\nu}-8R_{\nu\alpha}\,\pi^{;\mu}\,\pi^{;\nu}\,\pi_{;\mu}{}^{\alpha}-2R_{\mu\nu}\,\pi^{;\mu\nu}\,(\nabla\pi)^2\\&-4R_{\mu\alpha\nu\beta}\,\pi^{;\mu}\,\pi^{;\nu}\,\pi^{;\alpha\beta}-4\pi_{;\mu}{}^{\nu}\,\pi_{;\nu}{}^{\alpha}\,\pi^{;\mu}{}_{\alpha}+R\,(\nabla\pi)^2\,\Box\pi\\&+4R_{\mu\nu}\,\pi^{;\mu}\,\pi^{;\nu}\,\Box\pi+6\pi_{;\mu\nu}\,\pi^{;\mu\nu}\,\Box\pi-2(\Box\pi)^3\Big]\end{aligned} \tag{A.13}$$

$$\begin{aligned}\xi^{(5)}=&\frac{1}{M^9}\Big[\frac{3}{2}R\,(\nabla\pi)^2\,(\Box\pi)^2+3R\,\pi^{;\mu}\,\pi_{;\mu\nu}\,\pi^{;\nu}\,\Box\pi\\&+3R_{\mu}{}^{\alpha}\,R_{\nu\alpha}\,\pi^{;\mu}\,\pi^{;\nu}\,(\nabla\pi)^2-\frac{3}{2}R\,R_{\mu\nu}\,\pi^{;\mu}\,\pi^{;\nu}\,(\nabla\pi)^2\\&+3R^{\mu\nu}\,R_{\alpha\mu\beta\nu}\,\pi^{;\alpha}\,\pi^{;\beta}\,(\nabla\pi)^2-\frac{3}{2}R_{\mu}{}^{\alpha\beta\gamma}\,R_{\nu\alpha\beta\gamma}\,\pi^{;\mu}\,\pi^{;\nu}\,(\nabla\pi)^2\\&-3R\,\pi^{;\mu}\,\pi^{;\nu}\,\pi_{;\mu\alpha}\,\pi_{;\mu}{}^{\alpha}-\frac{3}{2}R\,\pi_{;\mu\nu}\,\pi^{;\mu\nu}\,(\nabla\pi)^2-(\Box\pi)^4\\&+3R_{\mu\nu}\,\pi^{;\mu}\,\pi^{;\nu}\,(\Box\pi)^2-12R_{\mu\alpha}\,\pi^{;\mu}\,\pi^{;\nu}\,\pi_{;\nu}{}^{\alpha}\,\Box\pi\\&+6R_{\alpha\beta}\,\pi^{;\mu}\,\pi^{;\nu}\,\pi_{;\mu}{}^{\alpha}\,\pi_{;\nu}{}^{\beta}+6R_{\mu\nu}\,\pi^{;\mu\alpha}\,\pi_{;\alpha}{}^{\nu}\,(\nabla\pi)^2\\&+6\pi^{;\mu\nu}\,\pi_{;\mu\alpha}\,\pi^{;\alpha\beta}\,\pi_{;\nu\beta}-8\pi^{;\mu\nu}\,\pi_{;\nu\alpha}\,\pi_{;\mu}{}^{\alpha}\,\Box\pi\\&+12R_{\nu\beta}\,\pi^{;\mu}\pi^{;\nu}\,\pi_{;\mu\alpha}\,\pi^{;\alpha\beta}-6R_{\mu\nu}\,\pi^{;\mu\nu}\,(\nabla\pi)^2\,\Box\pi\\&-6R_{\mu\nu}\,\pi^{;\mu\nu}\,\pi^{;\alpha}\,\pi_{;\alpha\beta}\,\pi^{;\beta}+6\pi_{;\mu\nu}\,\pi^{;\mu\nu}\,(\Box\pi)^2-3R_{\alpha\beta}\,\pi^{;\alpha}\,\pi^{;\beta}\,\pi_{;\mu\nu}\,\pi^{;\mu\nu}\\&-3(\pi_{;\mu\nu}\,\pi^{;\mu\nu})^2+6R_{\mu\alpha\nu\beta}\,\pi^{;\mu}\,\pi^{;\nu}\,\pi^{;\gamma\alpha}\,\pi_{;\gamma}{}^{\beta}-6R_{\mu\alpha\nu\beta}\,\pi^{;\mu}\,\pi^{;\nu}\,\pi^{;\alpha\beta}\,\Box\pi\\&+12R_{\mu\alpha\nu\beta}\,\pi^{;\gamma}\,\pi^{;\mu}\,\pi_{;\gamma}{}^{\nu}\,\pi^{;\alpha\beta}+3R_{\mu\alpha\nu\beta}\,\pi^{;\mu\nu}\,\pi^{;\alpha\beta}\,(\nabla\pi)^2\Big]\end{aligned} \tag{A.14}$$

APPENDIX B

BACKGROUND FUNCTIONS

Background functions involved in the linear perturbation theory (Sec. 3.1)

$$\gamma_1(t) \equiv 3\left(\alpha - 2x_{\rm dS}\beta\right)\frac{\dot{\pi}^2}{H_{\rm dS}^4 M_{\rm pl}^2} \tag{B.1}$$

$$\gamma_2(t) \equiv \left(2 + 9\alpha - 9\beta - 12x_{\rm dS}\alpha + 15x_{\rm dS}^2\beta\right)\frac{\dot{\pi}^2}{H_{\rm dS}^2 M_{\rm pl}} \tag{B.2}$$

$$\gamma_3(t) \equiv -\frac{\dot{\pi}^4}{3H_{\rm dS}^4 M_{\rm pl}^2}\left(\alpha + 6\beta\frac{\ddot{\pi}}{H_{\rm dS}^2 M_{\rm pl}}\right) \tag{B.3}$$

$$\gamma_4(t) \equiv -\frac{2\dot{\pi}^2}{3H_{\rm dS}^2 M_{\rm pl}}\left(2\alpha - 3x_{\rm dS}\beta\right)\left(x_{\rm dS} + \frac{3\ddot{\pi}}{H_{\rm dS}^2 M_{\rm pl}}\right) \tag{B.4}$$

$$\begin{aligned}\gamma_5(t) \equiv & -6 - 9\alpha + 12\beta - 26x_{\rm dS}^2\alpha + 4x_{\rm dS}\left(2 + 9\alpha - 9\beta\right) + 24x_{\rm dS}^3\beta \\ & + 2\left[2 + 9\alpha - 9\beta - 6x_{\rm dS}\left(\alpha - x_{\rm dS}\beta\right)\right]\frac{\ddot{\pi}}{H_{\rm dS}^2 M_{\rm pl}}\end{aligned} \tag{B.5}$$

Background functions involved in the non-linear dynamics (Sec. 6.1)

$$\eta_1(t) \equiv \frac{2\beta}{H_{\rm dS}^6 M_{\rm pl}^3}\dot{\pi}^2 \tag{B.6}$$

$$\eta_2(t) \equiv \frac{\dot{\pi}^2}{3H_{\rm dS}^4 M_{\rm pl}^2}\left(\alpha - 6\beta\frac{\ddot{\pi}}{H_{\rm dS}^2 M_{\rm pl}}\right) \tag{B.7}$$

$$\eta_3(t) \equiv \frac{1}{H_{\mathrm{dS}}^2 M_{\mathrm{pl}}} \left[2 + 9\alpha - 9\beta - 6\left(\alpha - x_{\mathrm{dS}}\beta\right) \left(x_{\mathrm{dS}} + \frac{\ddot{\pi}}{H_{\mathrm{dS}}^2 M_{\mathrm{pl}}} \right) \right] \tag{B.8}$$

$$\eta_4(t) \equiv \frac{2}{H_{\mathrm{dS}}^4 M_{\mathrm{pl}}^2} \left(\alpha - 2\beta \frac{\ddot{\pi}}{H_{\mathrm{dS}}^2 M_{\mathrm{pl}}} \right) \tag{B.9}$$

APPENDIX C

GAUGES IN LINEAR APPROXIMATION

Poisson Gauge

This gauge is very useful because in many cases the scalar metric perturbation ψ can be interpreted as the Newtonian potential. It can be obtained suppressing the off-diagonal terms of the metric

$$\begin{aligned} \chi^{(1)} &= 0 \\ \omega^{(1)} &= 0\,. \end{aligned} \tag{C.1}$$

From Eq. (3.29), we obtain the standard result

$$\psi^{(1)} = \phi^{(1)}\,, \tag{C.2}$$

while Eq. (3.32) reads

$$\psi^{(1)} + v^{(1)\prime} = 0\,. \tag{C.3}$$

In sub-horizon approximation the field equation for the galileon, Eq. (3.30) reads

$$\left(c_2 - \frac{2c_3\pi''}{M^3a^2} - 2\frac{c_3\mathcal{H}\pi'}{M^3a^2}\right)k^2\pi^{(1)} - \frac{c_3\pi'^2}{M^3a^2}k^2\psi^{(1)} = 0\,. \tag{C.4}$$

Using also the time-time component of the Einstein equations, Eq. (3.26), we obtain

$$\begin{aligned}&\left(1 + \frac{c_3^2\pi'^4}{2c_2M^6M_{\rm pl}^2a^4} - \frac{2c_3\pi''}{c_2M^3a^2} - \frac{2c_3\mathcal{H}\pi'}{c_2M^3a^2}\right)k^2\psi^{(1)}\\ &+\left(\frac{1}{2M_{\rm pl}^2} - \frac{c_3\pi''}{c_2M^3M_{\rm pl}^2a^2} - \frac{c_3\mathcal{H}\pi'}{c_2M^3M_{\rm pl}^2a^2}\right)a^2\rho_m\delta^{(1)} = 0\,.\end{aligned} \tag{C.5}$$

Substituting Eqs. (C.3) and then (C.2) into the derivative of Eq. (3.31), in sub-horizon approximation we obtain

$$\delta^{(1)''} + \mathcal{H}\delta^{(1)'} = -k^2\psi^{(1)}\,. \tag{C.6}$$

Using Eq. (C.5) to eliminate the metric perturbation ϕ in Eq. (C.6), the result is

$$\delta^{(1)''} + \mathcal{H}\delta^{(1)'} = 4\pi G\left(1 - \frac{{c_3}^2\pi'^4}{2c_2M^6M_{\rm pl}^2a^4\alpha}\right)a^2\rho_m\delta^{(1)}\,. \tag{C.7}$$

This equations studies the dynamics of the DM perturbation $\delta^{(1)}$, and it is the same equation obtained without choosing a gauge, Eq. (3.37).

Spatially Flat Gauge

The spatially flat gauge can be obtained by considering the spatial scalar fluctuations equal to zero

$$\begin{aligned}\phi^{(1)} &= 0\\ \chi^{(1)} &= 0\,.\end{aligned} \tag{C.8}$$

In this gauge Eq. (3.32) remains the same, while from Eq. (3.29) and its derivative we can solve for $\psi^{(1)}$

$$\psi^{(1)} = -\omega^{(1)'} - 2\mathcal{H}\omega^{(1)}\,. \tag{C.9}$$

To separate the galileon perturbation we use Eq. (3.30) in sub-horizon approximation

$$\left(c_2 - \frac{2c_3\pi''}{M^3a^2} - \frac{2c_3\mathcal{H}\pi'}{M^3a^2}\right)\pi^{(1)} + \left(c_2\pi' - \frac{2c_3\pi'\pi''}{M^3a^2} - \frac{c_3\mathcal{H}\pi'^2}{M^3a^2}\right)\omega^{(1)} = 0\,. \quad \text{(C.10)}$$

Using the last equation in Eq. (3.26), after a sub-horizon approximation, to eliminate the galileon field $\pi^{(1)}$ we obtain

$$\begin{aligned}&\left(c_2 - \frac{2c_3\pi''}{M^3a^2} - \frac{2c_3\mathcal{H}\pi'}{M^3a^2}\right)a^2\rho_m\delta^{(1)} \\ &\quad = \left(2c_2M_{\rm pl}^2\mathcal{H} + \frac{c_3^2\mathcal{H}\pi'^4}{M^6a^4} - \frac{4c_3M_{\rm pl}^2\mathcal{H}\pi''}{M^3a^2} - \frac{4c_3M_{\rm pl}^2\mathcal{H}^2\pi'}{M^3a^2}\right)k^2\omega^{(1)}\,. \quad \text{(C.11)}\end{aligned}$$

Finally, to find the dynamics of $\delta^{(1)}$ we have to substitute $\omega^{(1)}$ from the last equation in the derivative of Eq. (3.31). It is straightforward to show that also in this gauge the result is identical w.r.t. Eq. (3.37).

Synchronous Gauge

The synchronous gauge is a gauge that, at first-order, leaves only the spatial scalar perturbations

$$\begin{aligned}\psi^{(1)} &= 0 \\ \omega^{(1)} &= 0\,. \end{aligned} \quad \text{(C.12)}$$

It is slightly different from the other gauges described, because it has a residual gauge freedom. From Eq. (3.32) we find that the velocity $v^{(1)}$ must satisfy

$$v^{(1)\prime} + \mathcal{H}v^{(1)} = 0\,. \quad \text{(C.13)}$$

One can fix the residual gauge freedom imposing the additional condition $v^{(1)} = 0$. However we do not need to fix it to decouple on sub-horizon scales the DM density fluctuation $\delta^{(1)}$. Taking the difference between Eq. (3.28) and Eq. (3.26), and performing a sub-horizon approximation the result is

$$6M_{\rm pl}^2\phi^{(1)\prime\prime} + \left(6M_{\rm pl}^2\mathcal{H} + \frac{3c_3\pi'^3}{M^3a^2}\right)\phi^{(1)\prime} = a^2\rho_m\delta^{(1)} + \frac{c_3\pi'^2}{M^3a^2}k^2\pi^{(1)}\,. \quad \text{(C.14)}$$

In this gauge Eq. (3.30) reads

$$\frac{3c_3\pi'^2}{M^3a^2}\phi^{(1)\prime\prime} + \left(\frac{6c_3\pi'\pi''}{M^3a^2} + \frac{9c_3\mathcal{H}\pi'^2}{M^3a^2} - 3c_2\pi'\right)\phi^{(1)\prime} = -\left(c_2 - \frac{2c_3\pi''}{M^3a^2} - \frac{2c_3\mathcal{H}\pi'}{M^3a^2}\right)k^2\pi^{(1)}\,. \tag{C.15}$$

Combining Eqs. (C.14) and (C.15) to eliminate the galileon field $\pi^{(1)}$ we obtain

$$\left(6c_2M_{\rm pl}^2 + \frac{3c_3^2\pi'^4}{M^6a^4} - \frac{12c_3M_{\rm pl}^2\left(\pi'' + \mathcal{H}\pi'\right)}{M^3a^2}\right)\left[\phi^{(1)\prime\prime} + \mathcal{H}\phi^{(1)\prime}\right] = \left(c_2 + \frac{2c_3\left(\pi'' + \mathcal{H}\pi'\right)}{M^3}\right)a^2\rho_m\delta^{(1)}\,. \tag{C.16}$$

It is now straightforward to use this equation, Eq. (C.13) and the derivative of Eq. (3.31),

$$\delta^{(1)\prime\prime} + \mathcal{H}\delta^{(1)\prime} = 3\left(\phi^{(1)\prime\prime} + \mathcal{H}\phi^{(1)\prime}\right)\,, \tag{C.17}$$

to obtain Eq. (3.37).

APPENDIX $\mathcal{D}$

SOURCE TERMS FOR THE SECOND-ORDER EQUATIONS OF MOTION

In the following we give the explicit expression in a general gauge of the source terms found in Sec. 4. They reads

$$
\begin{aligned}
S^{(1)} \equiv\; & c_2 {\pi^{(1)}}'^2 + 6{M_{\rm pl}}^2 {\phi^{(1)}}'^2 + \frac{1}{12}{M_{\rm pl}}^2\nabla^2{\chi^{(1)}}'^2 + \frac{2}{3}{M_{\rm pl}}^2\mathcal{H}\nabla^2{\chi^{(1)}}'\nabla^2\chi^{(1)} \\
& + \frac{1}{9}{M_{\rm pl}}^2\nabla^2\nabla^2\chi^{(1)}\nabla^2\chi^{(1)} + 4{M_{\rm pl}}^2{\phi^{(1)}}'\nabla^2\omega^{(1)} - \frac{1}{3}{M_{\rm pl}}^2\nabla^2{\chi^{(1)}}'\nabla^2\omega^{(1)} \\
& - \frac{4}{3}{M_{\rm pl}}^2\mathcal{H}\nabla^2\chi^{(1)}\nabla^2\omega^{(1)} + \frac{2}{3}{M_{\rm pl}}^2\nabla^2\chi^{(1)}\nabla^2\phi^{(1)} + 8{M_{\rm pl}}^2\mathcal{H}\nabla^2\omega^{(1)}\psi^{(1)} \\
& + 16{M_{\rm pl}}^2\nabla^2\phi^{(1)}\phi^{(1)} + \frac{8}{3}{M_{\rm pl}}^2\nabla^2\nabla^2\chi^{(1)}\phi^{(1)} - 8{M_{\rm pl}}^2\mathcal{H}\nabla^2\omega^{(1)}\phi^{(1)} \\
& - 24{M_{\rm pl}}^2{\phi^{(1)}}'\mathcal{H}\phi^{(1)} - 4c_2\pi'{\pi^{(1)}}'\psi^{(1)} + 24{M_{\rm pl}}^2{\phi^{(1)}}'\mathcal{H}\psi^{(1)} + {M_{\rm pl}}^2\nabla^2{\omega^{(1)}}^2 \\
& + 4c_2\pi'^2{\psi^{(1)}}^2 + 24{M_{\rm pl}}^2\mathcal{H}^2{\psi^{(1)}}^2 + \frac{18c_3\pi'^2{\pi^{(1)}}'{\phi^{(1)}}'}{M^3a^2} - \frac{18c_3\pi'{\pi^{(1)}}'^2\mathcal{H}}{M^3a^2}
\end{aligned}
$$

$$
\begin{aligned}
&- \frac{c_3\pi'^3\nabla^2\chi^{(1)\prime}\nabla^2\chi^{(1)}}{3M^3a^2} + \frac{4c_3\pi'\pi^{(1)\prime}\nabla^2\pi^{(1)}}{M^3a^2} + \frac{2c_3\pi'^2\nabla^2\chi^{(1)}\nabla^2\pi^{(1)}}{3M^3a^2} \\
&+ \frac{6c_3\pi'^2\pi^{(1)\prime}\nabla^2\omega^{(1)}}{M^3a^2} + \frac{2c_3\pi'^3\nabla^2\chi^{(1)}\nabla^2\omega^{(1)}}{3M^3a^2} + \frac{12c_3\pi'^3\phi^{(1)\prime}\phi^{(1)}}{M^3a^2} \\
&+ \frac{4c_3\pi'^2\nabla^2\pi^{(1)}\phi^{(1)}}{M^3a^2} + \frac{4c_3\pi'^3\nabla^2\omega^{(1)}\phi^{(1)}}{M^3a^2} - \frac{24c_3\pi'^3\phi^{(1)\prime}\psi^{(1)}}{M^3a^2} \\
&+ \frac{72c_3\pi'^2\pi^{(1)\prime}\mathcal{H}\psi^{(1)}}{M^3a^2} - \frac{4c_3\pi'^2\nabla^2\pi^{(1)}\psi^{(1)}}{M^3a^2} - \frac{8c_3\pi'^3\nabla^2\omega^{(1)}\psi^{(1)}}{M^3a^2} \\
&- \frac{72c_3\pi'^3\mathcal{H}\psi^{(1)2}}{M^3a^2} + \frac{2c_3\pi'^2\partial_i\omega^{(1)}\partial^i\pi^{(1)\prime}}{M^3a^2} + \frac{2}{3}{M_{\rm pl}}^2\partial_i\omega^{(1)}\partial^i\nabla^2\chi^{(1)\prime} \\
&+ 4{M_{\rm pl}}^2\partial_i\omega^{(1)}\partial^i\phi^{(1)\prime} - \frac{5}{12}{M_{\rm pl}}^2\partial_i\nabla^2\chi^{(1)}\partial^i\nabla^2\chi^{(1)} - \frac{4c_3\pi'^2\partial_i\pi^{(1)}\partial^i\nabla^2\chi^{(1)}}{3M^3a^2} \\
&+ \frac{8}{3}{M_{\rm pl}}^2\mathcal{H}\partial_i\omega^{(1)}\partial^i\nabla^2\chi^{(1)} - \frac{4c_3\pi'^3\partial_i\omega^{(1)}\partial^i\nabla^2\chi^{(1)}}{3M^3a^2} + c_2\partial_i\pi^{(1)}\partial^i\pi^{(1)} \\
&+ \frac{2c_3\pi'\mathcal{H}\partial_i\pi^{(1)}\partial^i\pi^{(1)}}{M^3a^2} + \frac{12c_3\pi'^2\mathcal{H}\partial_i\pi^{(1)}\partial^i\omega^{(1)}}{M^3a^2} - c_2\pi'^2\partial_i\omega^{(1)}\partial^i\omega^{(1)} \\
&- 6{M_{\rm pl}}^2\mathcal{H}^2\partial_i\omega^{(1)}\partial^i\omega^{(1)} + \frac{12c_3\pi'^3\mathcal{H}\partial_i\omega^{(1)}\partial^i\omega^{(1)}}{M^3a^2} - \frac{2c_3\pi'^2\partial_i\pi^{(1)}\partial^i\phi^{(1)}}{M^3a^2} \\
&+ 4{M_{\rm pl}}^2\mathcal{H}\partial_i\omega^{(1)}\partial^i\phi^{(1)} - \frac{2c_3\pi'^3\partial_i\omega^{(1)}\partial^i\phi^{(1)}}{M^3a^2} + 6{M_{\rm pl}}^2\partial_i\phi^{(1)}\partial^i\phi^{(1)} \\
&+ 4{M_{\rm pl}}^2\mathcal{H}\partial_i\omega^{(1)}\partial^i\psi^{(1)} - \frac{2c_3\pi'^3\partial_i\omega^{(1)}\partial^i\psi^{(1)}}{M^3a^2} + 4{M_{\rm pl}}^2\mathcal{H}\partial_j\partial^i\chi^{(1)}\partial^j\partial_i\omega^{(1)} \\
&- 2\rho_m a^2\partial_i\omega^{(1)}\partial^i v^{(1)} - 2\rho_m a^2\partial_i v^{(1)}\partial^i v^{(1)} - \frac{1}{4}{M_{\rm pl}}^2\partial_j\partial^i\chi^{(1)\prime}\partial^j\partial_i\chi^{(1)\prime} \\
&- 2{M_{\rm pl}}^2\mathcal{H}\partial_j\partial^i\chi^{(1)}\partial^j\partial_i\chi^{(1)\prime} + \frac{c_3\pi'^3\partial_j\partial^i\chi^{(1)}\partial^j\partial_i\chi^{(1)\prime}}{M^3a^2} \\
&- \frac{1}{3}{M_{\rm pl}}^2\partial_j\partial^i\chi^{(1)}\partial^j\partial_i\nabla^2\chi^{(1)} - \frac{2c_3\pi'^2\partial_j\partial^i\chi^{(1)}\partial^j\partial_i\pi^{(1)}}{M^3a^2} \\
&- \frac{2c_3\pi'^3\partial_j\partial^i\chi^{(1)}\partial^j\partial_i\omega^{(1)}}{M^3a^2} - {M_{\rm pl}}^2\partial_j\partial^i\omega^{(1)}\partial^j\partial_i\omega^{(1)} - 2{M_{\rm pl}}^2\partial_j\partial^i\chi^{(1)}\partial^j\partial_i\phi^{(1)} \\
&+ \frac{1}{4}{M_{\rm pl}}^2\partial_k\partial^j\partial^i\chi^{(1)}\partial^k\partial_j\partial_i\chi^{(1)} + {M_{\rm pl}}^2\partial_j\partial^i\omega^{(1)}\partial^j\partial_i\chi^{(1)\prime} \qquad \text{(D.1)}
\end{aligned}
$$

$$
\begin{aligned}
S^{(2)} \equiv\ & \frac{4}{3}{M_{\rm pl}}^2\phi^{(1)\prime}\nabla^2\nabla^2\chi^{(1)} + \frac{1}{18}{M_{\rm pl}}^2\nabla^2\chi^{(1)\prime}\nabla^2\nabla^2\chi^{(1)} - \frac{2}{3}{M_{\rm pl}}^2\nabla^2\phi^{(1)\prime}\nabla^2\chi^{(1)} \\
&- \frac{1}{9}{M_{\rm pl}}^2\nabla^2\nabla^2\chi^{(1)\prime}\nabla^2\chi^{(1)} + 2c_2\pi^{(1)\prime}\nabla^2\pi^{(1)} - 4{M_{\rm pl}}^2\phi^{(1)\prime}\nabla^2\psi^{(1)} \\
&- \frac{1}{3}{M_{\rm pl}}^2\nabla^2\nabla^2\chi^{(1)}\nabla^2\omega^{(1)} + 8{M_{\rm pl}}^2\phi^{(1)\prime}\nabla^2\phi^{(1)} + \frac{1}{3}{M_{\rm pl}}^2\nabla^2\chi^{(1)\prime}\nabla^2\phi^{(1)} \\
&- 2{M_{\rm pl}}^2\nabla^2\omega^{(1)}\nabla^2\phi^{(1)} + \frac{1}{3}{M_{\rm pl}}^2\nabla^2\chi^{(1)\prime}\nabla^2\psi^{(1)} - 2{M_{\rm pl}}^2\nabla^2\omega^{(1)}\nabla^2\psi^{(1)}
\end{aligned}
$$

$$
\begin{aligned}
&+ 8{M_{\rm pl}}^2\nabla^2\phi^{(1)\prime}\phi^{(1)} + \frac{4}{3}{M_{\rm pl}}^2\nabla^2\nabla^2\chi^{(1)\prime}\phi^{(1)} - 8{M_{\rm pl}}^2\nabla^2\phi^{(1)\prime}\psi^{(1)} \\
&- \frac{4}{3}{M_{\rm pl}}^2\nabla^2\nabla^2\chi^{(1)\prime}\psi^{(1)} - 4c_2\pi'\nabla^2\pi^{(1)}\psi^{(1)} - \frac{12c_3\pi'\pi^{(1)\prime}\mathcal{H}\nabla^2\pi^{(1)}}{M^3a^2} \\
&- 16{M_{\rm pl}}^2\mathcal{H}\nabla^2\psi^{(1)}\psi^{(1)} + \frac{4c_3\pi'\pi^{(1)\prime}\nabla^2\pi^{(1)\prime}}{M^3a^2} + \frac{6c_3\pi'^2\phi^{(1)\prime}\nabla^2\pi^{(1)}}{M^3a^2} \\
&+ \frac{2c_3\pi'\nabla^2{\pi^{(1)}}^2}{M^3a^2} + \frac{2c_3\pi'^2\nabla^2\pi^{(1)}\nabla^2\omega^{(1)}}{M^3a^2} - \frac{6c_3\pi'^2\pi^{(1)\prime}\nabla^2\psi^{(1)}}{M^3a^2} \\
&- \frac{8c_3\pi'^2\nabla^2\pi^{(1)\prime}\psi^{(1)}}{M^3a^2} + \frac{24c_3\pi'^2\mathcal{H}\nabla^2\pi^{(1)}\psi^{(1)}}{M^3a^2} + \frac{12c_3\pi'^3\nabla^2\psi^{(1)}\psi^{(1)}}{M^3a^2} \\
&- \frac{2}{3}\rho_m\nabla^2\chi^{(1)}\nabla^2v^{(1)}a^2 + 2\rho_m\nabla^2\omega^{(1)}\delta^{(1)}a^2 - 2\rho_m\nabla^2v^{(1)}\psi^{(1)}a^2 \\
&+ \frac{2}{3}{M_{\rm pl}}^2\partial_i\phi^{(1)}\partial^i\nabla^2\chi^{(1)\prime} - 4\rho_m\nabla^2v^{(1)}\phi^{(1)}a^2 - 4\rho_m\nabla^2\omega^{(1)}\psi^{(1)}a^2 \\
&+ \frac{4c_3\pi'\partial_i\pi^{(1)\prime}\partial^i\pi^{(1)\prime}}{M^3a^2} + 2c_2\partial_i\pi^{(1)}\partial^i\pi^{(1)\prime} - \frac{12c_3\pi'\mathcal{H}\partial_i\pi^{(1)}\partial^i\pi^{(1)\prime}}{M^3a^2} \\
&+ \frac{6c_3\pi'^2\partial_i\pi^{(1)}\partial^i\phi^{(1)\prime}}{M^3a^2} + 16{M_{\rm pl}}^2\partial_i\phi^{(1)}\partial^i\phi^{(1)\prime} - 12{M_{\rm pl}}^2\partial_i\psi^{(1)}\partial^i\phi^{(1)\prime} \\
&- \frac{7}{18}{M_{\rm pl}}^2\partial_i\nabla^2\chi^{(1)}\partial^i\nabla^2\chi^{(1)\prime} + 2\rho_m a^2\partial_i\delta^{(1)}\partial^i\omega^{(1)} - 4c_2\pi'\partial_i\pi^{(1)}\partial^i\psi^{(1)} \\
&+ 2\rho_m\nabla^2v^{(1)}\delta^{(1)}a^2 - 2{M_{\rm pl}}^2\partial_i\psi^{(1)}\partial^i\nabla^2\chi^{(1)\prime} - \frac{2}{3}{M_{\rm pl}}^2\partial_i\omega^{(1)}\partial^i\nabla^2\nabla^2\chi^{(1)} \\
&+ \frac{8}{3}{M_{\rm pl}}^2\partial_i\phi^{(1)\prime}\partial^i\nabla^2\chi^{(1)} + \frac{4}{3}\rho_m a^2\partial_i v^{(1)}\partial^i\nabla^2\chi^{(1)} - \frac{2c_3\pi'^2\partial_i\omega^{(1)}\partial^i\nabla^2\pi^{(1)}}{M^3a^2} \\
&+ 4{M_{\rm pl}}^2\mathcal{H}\partial_i\omega^{(1)}\partial^i\nabla^2\omega^{(1)} - \frac{2c_3\pi'^3\partial_i\omega^{(1)}\partial^i\nabla^2\omega^{(1)}}{M^3a^2} - 4{M_{\rm pl}}^2\partial_i\omega^{(1)}\partial^i\nabla^2\phi^{(1)} \\
&+ \frac{24c_3\pi'^2\mathcal{H}\partial_i\pi^{(1)}\partial^i\psi^{(1)}}{M^3a^2} - 4\rho_m a^2\partial_i\omega^{(1)}\partial^i\psi^{(1)} - 16{M_{\rm pl}}^2\mathcal{H}\partial_i\psi^{(1)}\partial^i\psi^{(1)} \\
&+ \frac{12c_3\pi'^3\partial_i\psi^{(1)}\partial^i\psi^{(1)}}{M^3a^2} - 4\rho_m a^2\partial_i\phi^{(1)}\partial^i v^{(1)} + 4{M_{\rm pl}}^2\mathcal{H}\partial_j\partial^i\omega^{(1)}\partial^j\partial_i\omega^{(1)} \\
&- 2\rho_m a^2\partial_i\psi^{(1)}\partial^i v^{(1)} - {M_{\rm pl}}^2\partial_j\partial^i\phi^{(1)}\partial^j\partial_i\chi^{(1)\prime} - {M_{\rm pl}}^2\partial_j\partial^i\psi^{(1)}\partial^j\partial_i\chi^{(1)\prime} \\
&+ 2{M_{\rm pl}}^2\partial_j\partial^i\chi^{(1)}\partial^j\partial_i\phi^{(1)\prime} + \frac{1}{3}{M_{\rm pl}}^2\partial_j\partial^i\chi^{(1)}\partial^j\partial_i\nabla^2\chi^{(1)\prime} \\
&- \frac{1}{6}{M_{\rm pl}}^2\partial_j\partial^i\chi^{(1)\prime}\partial^j\partial_i\nabla^2\chi^{(1)} - \frac{1}{3}{M_{\rm pl}}^2\partial_j\partial^i\omega^{(1)}\partial^j\partial_i\nabla^2\chi^{(1)} \\
&- \frac{2c_3\pi'\partial_j\partial^i\pi^{(1)}\partial^j\partial_i\pi^{(1)}}{M^3a^2} - \frac{4c_3\pi'^2\partial_j\partial^i\pi^{(1)}\partial^j\partial_i\omega^{(1)}}{M^3a^2} + 2\rho_m a^2\partial_i\delta^{(1)}\partial^i v^{(1)} \\
&- \frac{2c_3\pi'^3\partial_j\partial^i\omega^{(1)}\partial^j\partial_i\omega^{(1)}}{M^3a^2} - 2{M_{\rm pl}}^2\partial_j\partial^i\omega^{(1)}\partial^j\partial_i\phi^{(1)} \\
&+ 2\rho_m a^2\partial_j\partial^i\chi^{(1)}\partial^j\partial_i v^{(1)} + \frac{1}{2}{M_{\rm pl}}^2\partial_k\partial^j\partial^i\chi^{(1)}\partial^k\partial_j\partial_i\chi^{(1)\prime}
\end{aligned}
$$

$$- \frac{14c_3\pi'^2\partial_i\psi^{(1)}\partial^i\pi^{(1)\prime}}{M^3a^2} + 2{M_{\rm pl}}^2\partial_j\partial^i\omega^{(1)}\partial^j\partial_i\psi^{(1)} \tag{D.2}$$

$$\begin{aligned}
S^{(3)} \equiv &- \frac{12c_3\pi^{(1)\prime\prime}\pi'\pi^{(1)\prime}}{M^3a^4} - \frac{6c_3\pi''{\pi^{(1)\prime}}^2}{M^3a^4} + \frac{18c_3\pi'^2\pi^{(1)\prime}\psi^{(1)\prime}}{M^3a^4} + \frac{18c_3\pi'{\pi^{(1)\prime}}^2\mathcal{H}}{M^3a^4} \\
&+ \frac{24c_3\pi^{(1)\prime\prime}\pi'^2\psi^{(1)}}{M^3a^4} + \frac{48c_3\pi''\pi'\pi^{(1)\prime}\psi^{(1)}}{M^3a^4} + \frac{{M_{\rm pl}}^2\nabla^2\nabla^2\chi^{(1)}\nabla^2\chi^{(1)}}{9a^2} \\
&- \frac{36c_3\pi'^3\psi^{(1)\prime}\psi^{(1)}}{M^3a^4} - \frac{72c_3\pi'^2\pi^{(1)\prime}\mathcal{H}\psi^{(1)}}{M^3a^4} - \frac{72c_3\pi''\pi'^2{\psi^{(1)}}^2}{M^3a^4} - \frac{3c_2{\pi^{(1)\prime}}^2}{a^2} \\
&+ \frac{72c_3\pi'^3\mathcal{H}{\psi^{(1)}}^2}{M^3a^4} - \frac{6{M_{\rm pl}}^2{\phi^{(1)\prime}}^2}{a^2} + \frac{12{M_{\rm pl}}^2\phi^{(1)\prime}\psi^{(1)\prime}}{a^2} + \frac{5{M_{\rm pl}}^2{\nabla^2\chi^{(1)\prime}}^2}{12a^2} \\
&+ \frac{2{M_{\rm pl}}^2\nabla^2\chi^{(1)\prime\prime}\nabla^2\chi^{(1)}}{3a^2} + \frac{4{M_{\rm pl}}^2\mathcal{H}\nabla^2\chi^{(1)\prime}\nabla^2\chi^{(1)}}{3a^2} - \frac{4{M_{\rm pl}}^2\nabla^2\omega^{(1)\prime}\nabla^2\chi^{(1)}}{3a^2} \\
&+ \frac{4{M_{\rm pl}}^2\phi^{(1)\prime}\nabla^2\omega^{(1)}}{a^2} + \frac{4{M_{\rm pl}}^2\psi^{(1)\prime}\nabla^2\omega^{(1)}}{a^2} - \frac{{M_{\rm pl}}^2\nabla^2\chi^{(1)\prime}\nabla^2\omega^{(1)}}{3a^2} \\
&- \frac{8{M_{\rm pl}}^2\mathcal{H}\nabla^2\chi^{(1)}\nabla^2\omega^{(1)}}{3a^2} + \frac{{M_{\rm pl}}^2{\nabla^2\omega^{(1)}}^2}{a^2} + \frac{2{M_{\rm pl}}^2\nabla^2\chi^{(1)}\nabla^2\phi^{(1)}}{3a^2} \\
&- \frac{4{M_{\rm pl}}^2\nabla^2\chi^{(1)}\nabla^2\psi^{(1)}}{3a^2} - \frac{24{M_{\rm pl}}^2\phi^{(1)\prime\prime}\phi^{(1)}}{a^2} - \frac{8{M_{\rm pl}}^2\nabla^2\psi^{(1)}\phi^{(1)}}{a^2} \\
&- \frac{48{M_{\rm pl}}^2\phi^{(1)\prime}\mathcal{H}\phi^{(1)}}{a^2} - \frac{8{M_{\rm pl}}^2\nabla^2\omega^{(1)\prime}\phi^{(1)}}{a^2} + \frac{8{M_{\rm pl}}^2\nabla^2\nabla^2\chi^{(1)}\phi^{(1)}}{3a^2} \\
&- \frac{16{M_{\rm pl}}^2\mathcal{H}\nabla^2\omega^{(1)}\phi^{(1)}}{a^2} + \frac{16{M_{\rm pl}}^2\nabla^2\phi^{(1)}\phi^{(1)}}{a^2} + \frac{24{M_{\rm pl}}^2\phi^{(1)\prime\prime}\psi^{(1)}}{a^2} \\
&+ \frac{12c_2\pi'\pi^{(1)\prime}\psi^{(1)}}{a^2} + \frac{48{M_{\rm pl}}^2\phi^{(1)\prime}\mathcal{H}\psi^{(1)}}{a^2} + \frac{48{M_{\rm pl}}^2\psi^{(1)\prime}\mathcal{H}\psi^{(1)}}{a^2} \\
&+ \frac{8{M_{\rm pl}}^2\nabla^2\omega^{(1)\prime}\psi^{(1)}}{a^2} - \frac{12c_2\pi'^2{\psi^{(1)}}^2}{a^2} + \frac{16{M_{\rm pl}}^2\mathcal{H}\partial_i\omega^{(1)}\partial^i\nabla^2\chi^{(1)}}{3a^2} \\
&+ \frac{16{M_{\rm pl}}^2\mathcal{H}\nabla^2\omega^{(1)}\psi^{(1)}}{a^2} + \frac{8{M_{\rm pl}}^2\nabla^2\psi^{(1)}\psi^{(1)}}{a^2} + \frac{48{M_{\rm pl}}^2\mathcal{H}'{\psi^{(1)}}^2}{a^2} \\
&+ \frac{24{M_{\rm pl}}^2\mathcal{H}^2{\psi^{(1)}}^2}{a^2} + \frac{8c_3\pi'\partial_i\pi^{(1)}\partial^i\pi^{(1)\prime}}{M^3a^4} + \frac{10c_3\pi'^2\partial_i\omega^{(1)}\partial^i\pi^{(1)\prime}}{M^3a^4} \\
&+ \frac{6c_3\pi'^2\partial_i\pi^{(1)}\partial^i\omega^{(1)\prime}}{M^3a^4} + \frac{6c_3\pi'^3\partial_i\omega^{(1)}\partial^i\omega^{(1)\prime}}{M^3a^4} - \frac{12{M_{\rm pl}}^2\mathcal{H}\partial_i\omega^{(1)}\partial^i\omega^{(1)\prime}}{a^2} \\
&+ \frac{4{M_{\rm pl}}^2\partial_i\phi^{(1)}\partial^i\omega^{(1)\prime}}{a^2} + \frac{12{M_{\rm pl}}^2\partial_i\omega^{(1)}\partial^i\phi^{(1)\prime}}{a^2} + \frac{2{M_{\rm pl}}^2\partial_i\omega^{(1)}\partial^i\nabla^2\chi^{(1)\prime}}{a^2} \\
&+ \frac{8{M_{\rm pl}}^2\partial_i\omega^{(1)\prime}\partial^i\nabla^2\chi^{(1)}}{3a^2} - \frac{5{M_{\rm pl}}^2\partial_i\nabla^2\chi^{(1)}\partial^i\nabla^2\chi^{(1)}}{12a^2} + \frac{c_2\partial_i\pi^{(1)}\partial^i\pi^{(1)}}{a^2} \\
&+ \frac{8{M_{\rm pl}}^2\partial_i\psi^{(1)}\partial^i\nabla^2\chi^{(1)}}{3a^2} + \frac{2c_3\pi''\partial_i\pi^{(1)}\partial^i\pi^{(1)}}{M^3a^4} - \frac{2c_3\pi'\mathcal{H}\partial_i\pi^{(1)}\partial^i\pi^{(1)}}{M^3a^4}
\end{aligned}$$

$$
\begin{aligned}
&+\frac{12c_3\pi''\pi'\partial_i\pi^{(1)}\partial^i\omega^{(1)}}{M^3a^4}-\frac{12c_3\pi'^2\mathcal{H}\partial_i\pi^{(1)}\partial^i\omega^{(1)}}{M^3a^4}+\frac{4c_2\pi'\partial_i\pi^{(1)}\partial^i\omega^{(1)}}{a^2}\\
&+\frac{12c_3\pi''\pi'^2\partial_i\omega^{(1)}\partial^i\omega^{(1)}}{M^3a^4}-\frac{12c_3\pi'^3\mathcal{H}\partial_i\omega^{(1)}\partial^i\omega^{(1)}}{M^3a^4}+\frac{3c_2\pi'^2\partial_i\omega^{(1)}\partial^i\omega^{(1)}}{a^2}\\
&-\frac{12{M_{\rm pl}}^2\mathcal{H}'\partial_i\omega^{(1)}\partial^i\omega^{(1)}}{a^2}-\frac{6{M_{\rm pl}}^2\mathcal{H}^2\partial_i\omega^{(1)}\partial^i\omega^{(1)}}{a^2}-\frac{2c_3\pi'^2\partial_i\pi^{(1)}\partial^i\psi^{(1)}}{M^3a^4}\\
&+\frac{8{M_{\rm pl}}^2\mathcal{H}\partial_i\omega^{(1)}\partial^i\phi^{(1)}}{a^2}+\frac{6{M_{\rm pl}}^2\partial_i\phi^{(1)}\partial^i\phi^{(1)}}{a^2}-\frac{2{M_{\rm pl}}^2\partial_j\partial^i\chi^{(1)}\partial^j\partial_i\chi^{(1)''}}{a^2}\\
&-\frac{4c_3\pi'^3\partial_i\omega^{(1)}\partial^i\psi^{(1)}}{M^3a^4}+\frac{8{M_{\rm pl}}^2\mathcal{H}\partial_i\omega^{(1)}\partial^i\psi^{(1)}}{a^2}+\frac{4{M_{\rm pl}}^2\partial_i\phi^{(1)}\partial^i\psi^{(1)}}{a^2}\\
&-\frac{5{M_{\rm pl}}^2\partial_j\partial^i\chi^{(1)'}\partial^j\partial_i\chi^{(1)'}}{4a^2}-\frac{4{M_{\rm pl}}^2\mathcal{H}\partial_j\partial^i\chi^{(1)}\partial^j\partial_i\chi^{(1)'}}{a^2}\\
&+\frac{{M_{\rm pl}}^2\partial_j\partial^i\omega^{(1)}\partial^j\partial_i\chi^{(1)'}}{a^2}+\frac{4{M_{\rm pl}}^2\partial_j\partial^i\chi^{(1)}\partial^j\partial_i\omega^{(1)'}}{a^2}+\frac{4{M_{\rm pl}}^2\partial_i\psi^{(1)}\partial^i\psi^{(1)}}{a^2}\\
&-\frac{{M_{\rm pl}}^2\partial_j\partial^i\chi^{(1)}\partial^j\partial_i\nabla^2\chi^{(1)}}{3a^2}-\frac{2{M_{\rm pl}}^2\partial_j\partial^i\chi^{(1)}\partial^j\partial_i\phi^{(1)}}{a^2}+2\rho_m\partial_i\omega^{(1)}\partial^iv^{(1)}\\
&+\frac{8{M_{\rm pl}}^2\mathcal{H}\partial_j\partial^i\chi^{(1)}\partial^j\partial_i\omega^{(1)}}{a^2}-\frac{{M_{\rm pl}}^2\partial_j\partial^i\omega^{(1)}\partial^j\partial_i\omega^{(1)}}{a^2}+2\rho_m\partial_iv^{(1)}\partial^iv^{(1)}\\
&+\frac{4{M_{\rm pl}}^2\partial_j\partial^i\chi^{(1)}\partial^j\partial_i\psi^{(1)}}{a^2}+\frac{{M_{\rm pl}}^2\partial_k\partial^j\partial^i\chi^{(1)}\partial^k\partial_j\partial_i\chi^{(1)}}{4a^2}
\end{aligned}\tag{D.3}
$$

$$
\begin{aligned}
S^{(4)}\equiv\;&\frac{1}{2}\nabla^2\chi^{(1)'}\nabla^2\phi^{(1)'}-\frac{1}{2}\nabla^2\chi^{(1)'}\nabla^2\psi^{(1)'}-\phi^{(1)'}\nabla^2\nabla^2\chi^{(1)'}+\psi^{(1)'}\nabla^2\nabla^2\chi^{(1)'}\\
&-\frac{1}{3}\nabla^2\chi^{(1)'}\nabla^2\nabla^2\chi^{(1)'}-4\phi^{(1)'}\mathcal{H}\nabla^2\nabla^2\chi^{(1)}-\frac{1}{6}\nabla^2\chi^{(1)''}\nabla^2\nabla^2\chi^{(1)}\\
&-2\phi^{(1)''}\nabla^2\nabla^2\chi^{(1)}-\frac{1}{3}\mathcal{H}\nabla^2\chi^{(1)'}\nabla^2\nabla^2\chi^{(1)}+\frac{4}{3}\mathcal{H}\nabla^2\nabla^2\omega^{(1)}\nabla^2\chi^{(1)}\\
&+\frac{2}{3}\nabla^2\omega^{(1)'}\nabla^2\nabla^2\chi^{(1)}+\frac{1}{36}\nabla^2\nabla^2\chi^{(1)^2}-2\phi^{(1)'}\nabla^2\nabla^2\omega^{(1)}-2\psi^{(1)'}\nabla^2\nabla^2\omega^{(1)}\\
&+\frac{1}{6}\nabla^2\chi^{(1)'}\nabla^2\nabla^2\omega^{(1)}+2\mathcal{H}\nabla^2\phi^{(1)'}\nabla^2\chi^{(1)}-\frac{1}{6}\nabla^2\nabla^2\chi^{(1)''}\nabla^2\chi^{(1)}\\
&-\frac{1}{3}\mathcal{H}\nabla^2\nabla^2\chi^{(1)'}\nabla^2\chi^{(1)}+\frac{2}{3}\nabla^2\nabla^2\omega^{(1)'}\nabla^2\chi^{(1)}-\frac{1}{18}\nabla^2\nabla^2\nabla^2\chi^{(1)}\nabla^2\chi^{(1)}\\
&-\frac{1}{3}\nabla^2\nabla^2\phi^{(1)}\nabla^2\chi^{(1)}+\frac{2}{3}\nabla^2\nabla^2\psi^{(1)}\nabla^2\chi^{(1)}-\frac{3c_2\nabla^2\pi^{(1)^2}}{{M_{\rm pl}}^2}+\nabla^2\phi^{(1)'}\nabla^2\omega^{(1)}\\
&+\frac{4}{3}\mathcal{H}\nabla^2\nabla^2\chi^{(1)}\nabla^2\omega^{(1)}-\frac{3c_2\pi'\nabla^2\pi^{(1)}\nabla^2\omega^{(1)}}{{M_{\rm pl}}^2}+8\mathcal{H}\nabla^2\omega^{(1)}\nabla^2\phi^{(1)}\\
&+\nabla^2\phi^{(1)''}\nabla^2\chi^{(1)}+\nabla^2\psi^{(1)'}\nabla^2\omega^{(1)}+\frac{2}{3}\nabla^2\nabla^2\chi^{(1)'}\nabla^2\omega^{(1)}-\nabla^2\nabla^2\omega^{(1)}\nabla^2\omega^{(1)}\\
&+\nabla^2\chi^{(1)''}\nabla^2\phi^{(1)}+2\mathcal{H}\nabla^2\chi^{(1)'}\nabla^2\phi^{(1)}+4\nabla^2\omega^{(1)'}\nabla^2\phi^{(1)}-\frac{2}{3}\nabla^2\nabla^2\chi^{(1)}\nabla^2\phi^{(1)}
\end{aligned}
$$

$$
\begin{aligned}
&- 5\nabla^2\phi^{(1)^2} - \nabla^2\chi^{(1)''}\nabla^2\psi^{(1)} - 2\mathcal{H}\nabla^2\chi^{(1)'}\nabla^2\psi^{(1)} + 2\nabla^2\omega^{(1)'}\nabla^2\psi^{(1)} \\
&+ \frac{2}{3}\nabla^2\nabla^2\chi^{(1)}\nabla^2\psi^{(1)} - 2\mathcal{H}\nabla^2\omega^{(1)}\nabla^2\psi^{(1)} + 4\nabla^2\nabla^2\omega^{(1)'}\phi^{(1)} + 2\nabla^2\nabla^2\chi^{(1)''}\psi^{(1)} \\
&+ 4\nabla^2\phi^{(1)}\nabla^2\psi^{(1)} - \nabla^2\psi^{(1)^2} - 2\nabla^2\nabla^2\chi^{(1)''}\phi^{(1)} - \frac{3c_3\pi'^2\nabla^2\pi^{(1)'}\nabla^2\omega^{(1)}}{M^3{M_{\rm pl}}^2a^2} \\
&- \frac{4}{3}\nabla^2\nabla^2\nabla^2\chi^{(1)}\phi^{(1)} + 8\mathcal{H}\nabla^2\nabla^2\omega^{(1)}\phi^{(1)} - 8\nabla^2\nabla^2\phi^{(1)}\phi^{(1)} + 4\nabla^2\nabla^2\psi^{(1)}\phi^{(1)} \\
&+ 4\mathcal{H}\nabla^2\nabla^2\chi^{(1)'}\psi^{(1)} - 4\nabla^2\nabla^2\omega^{(1)'}\psi^{(1)} - 8\mathcal{H}\nabla^2\nabla^2\omega^{(1)}\psi^{(1)} - 4\nabla^2\nabla^2\psi^{(1)}\psi^{(1)} \\
&- 4\mathcal{H}\nabla^2\nabla^2\chi^{(1)'}\phi^{(1)} - \frac{6c_3\pi'\nabla^2\pi^{(1)'}\nabla^2\pi^{(1)}}{M^3{M_{\rm pl}}^2a^2} + \frac{3c_3\pi''\nabla^2\pi^{(1)^2}}{M^3{M_{\rm pl}}^2a^2} + \frac{6c_3\pi'\mathcal{H}\nabla^2\pi^{(1)^2}}{M^3{M_{\rm pl}}^2a^2} \\
&+ \frac{9c_3\pi'^2\mathcal{H}\nabla^2\pi^{(1)}\nabla^2\omega^{(1)}}{M^3{M_{\rm pl}}^2a^2} + \frac{6c_3\pi'^2\nabla^2\pi^{(1)}\nabla^2\psi^{(1)}}{M^3{M_{\rm pl}}^2a^2} + \frac{3c_3\pi'^3\nabla^2\omega^{(1)}\nabla^2\psi^{(1)}}{M^3{M_{\rm pl}}^2a^2} \\
&- 4\partial_i\phi^{(1)}\partial^i\nabla^2\chi^{(1)''} + \frac{3\rho_m\nabla^2v^{(1)^2}a^2}{{M_{\rm pl}}^2} - 4\partial_i\nabla^2\chi^{(1)}\partial^i\phi^{(1)''} + \frac{1}{6}\partial_i\nabla^2\chi^{(1)}\partial^i\nabla^2\chi^{(1)''} \\
&+ \frac{3\rho_m\nabla^2\omega^{(1)}\nabla^2v^{(1)}a^2}{{M_{\rm pl}}^2} + 2\partial_i\psi^{(1)'}\partial^i\nabla^2\chi^{(1)'} - \frac{4c_3\pi'\partial_i\pi^{(1)}\partial^i\nabla^2\pi^{(1)'}}{M^3{M_{\rm pl}}^2a^2} \\
&+ \frac{1}{6}\partial_i\nabla^2\chi^{(1)'}\partial^i\nabla^2\chi^{(1)'} + \frac{1}{3}\mathcal{H}\partial_i\nabla^2\chi^{(1)}\partial^i\nabla^2\chi^{(1)'} + \frac{1}{3}\partial_i\nabla^2\omega^{(1)}\partial^i\nabla^2\chi^{(1)'} \\
&+ 4\partial_i\psi^{(1)}\partial^i\nabla^2\chi^{(1)''} - 2\partial_i\phi^{(1)'}\partial^i\nabla^2\chi^{(1)'} - 8\mathcal{H}\partial_i\phi^{(1)}\partial^i\nabla^2\chi^{(1)'} + 8\mathcal{H}\partial_i\psi^{(1)}\partial^i\nabla^2\chi^{(1)'} \\
&- \frac{2c_3\pi'^2\partial_i\omega^{(1)}\partial^i\nabla^2\pi^{(1)'}}{M^3{M_{\rm pl}}^2a^2} - \frac{1}{2}\partial_i\nabla^2\chi^{(1)}\partial^i\nabla^2\omega^{(1)'} + 12\partial_i\phi^{(1)}\partial^i\nabla^2\omega^{(1)'} \\
&- 8\partial_i\psi^{(1)}\partial^i\nabla^2\omega^{(1)'} - \frac{1}{3}\partial_i\omega^{(1)'}\partial^i\nabla^2\nabla^2\chi^{(1)} + \frac{7}{18}\partial_i\nabla^2\chi^{(1)}\partial^i\nabla^2\nabla^2\chi^{(1)} \\
&- \frac{11}{3}\partial_i\phi^{(1)}\partial^i\nabla^2\nabla^2\chi^{(1)} - \frac{1}{3}\partial_i\psi^{(1)}\partial^i\nabla^2\nabla^2\chi^{(1)} - 8\mathcal{H}\partial_i\phi^{(1)'}\partial^i\nabla^2\chi^{(1)} \\
&- \frac{4c_3\pi'\partial_i\pi^{(1)'}\partial^i\nabla^2\pi^{(1)}}{M^3{M_{\rm pl}}^2a^2} - \frac{4c_2\partial_i\pi^{(1)}\partial^i\nabla^2\pi^{(1)}}{{M_{\rm pl}}^2} + \frac{4c_3\pi''\partial_i\pi^{(1)}\partial^i\nabla^2\pi^{(1)}}{M^3{M_{\rm pl}}^2a^2} \\
&+ \frac{8c_3\pi'\mathcal{H}\partial_i\pi^{(1)}\partial^i\nabla^2\pi^{(1)}}{M^3{M_{\rm pl}}^2a^2} - \frac{2c_2\pi'\partial_i\omega^{(1)}\partial^i\nabla^2\pi^{(1)}}{{M_{\rm pl}}^2} + \frac{6c_3\pi'^2\mathcal{H}\partial_i\omega^{(1)}\partial^i\nabla^2\pi^{(1)}}{M^3{M_{\rm pl}}^2a^2} \\
&+ \frac{4c_3\pi'^2\partial_i\psi^{(1)}\partial^i\nabla^2\pi^{(1)}}{M^3{M_{\rm pl}}^2a^2} - \frac{2c_3\pi'^2\partial_i\pi^{(1)'}\partial^i\nabla^2\omega^{(1)}}{M^3{M_{\rm pl}}^2a^2} - 4\partial_i\phi^{(1)'}\partial^i\nabla^2\omega^{(1)} \\
&- \mathcal{H}\partial_i\nabla^2\chi^{(1)}\partial^i\nabla^2\omega^{(1)} - \partial_i\nabla^2\omega^{(1)}\partial^i\nabla^2\omega^{(1)} - \frac{2c_2\pi'\partial_i\pi^{(1)}\partial^i\nabla^2\omega^{(1)}}{{M_{\rm pl}}^2} \\
&- 4\partial_i\psi^{(1)'}\partial^i\nabla^2\omega^{(1)} - \frac{2}{3}\mathcal{H}\partial_i\omega^{(1)}\partial^i\nabla^2\nabla^2\chi^{(1)} + \frac{2\rho_ma^2\partial_i\omega^{(1)}\partial^i\nabla^2v^{(1)}}{{M_{\rm pl}}^2} \\
&+ \frac{6c_3\pi'^2\mathcal{H}\partial_i\pi^{(1)}\partial^i\nabla^2\omega^{(1)}}{M^3{M_{\rm pl}}^2a^2} + 24\mathcal{H}\partial_i\phi^{(1)}\partial^i\nabla^2\omega^{(1)} - 20\mathcal{H}\partial_i\psi^{(1)}\partial^i\nabla^2\omega^{(1)}
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{2c_3\pi'^3\partial_i\psi^{(1)}\partial^i\nabla^2\omega^{(1)}}{M^3{M_{\rm pl}}^2a^2}+\frac{2\rho_m a^2\partial_i v^{(1)}\partial^i\nabla^2\omega^{(1)}}{{M_{\rm pl}}^2}+4\partial_i\omega^{(1)\prime}\partial^i\nabla^2\phi^{(1)}\\
&-\frac{1}{6}\partial_i\nabla^2\chi^{(1)}\partial^i\nabla^2\phi^{(1)}+8\mathcal{H}\partial_i\omega^{(1)}\partial^i\nabla^2\phi^{(1)}-28\partial_i\phi^{(1)}\partial^i\nabla^2\phi^{(1)}\\
&-\frac{1}{2}\partial_i\nabla^2\chi^{(1)}\partial^i\nabla^2\psi^{(1)}+\frac{4c_3\pi'^2\partial_i\pi^{(1)}\partial^i\nabla^2\psi^{(1)}}{M^3{M_{\rm pl}}^2a^2}-4\mathcal{H}\partial_i\omega^{(1)}\partial^i\nabla^2\psi^{(1)}\\
&+\frac{2c_3\pi'^3\partial_i\omega^{(1)}\partial^i\nabla^2\psi^{(1)}}{M^3{M_{\rm pl}}^2a^2}+12\partial_i\phi^{(1)}\partial^i\nabla^2\psi^{(1)}-12\partial_i\psi^{(1)}\partial^i\nabla^2\psi^{(1)}\\
&+\frac{4\rho_m a^2\partial_i v^{(1)}\partial^i\nabla^2 v^{(1)}}{{M_{\rm pl}}^2}-3\partial_j\partial^i\phi^{(1)}\partial^j\partial_i\chi^{(1)\prime\prime}+3\partial_j\partial^i\psi^{(1)}\partial^j\partial_i\chi^{(1)\prime\prime}\\
&-3\partial_j\partial^i\chi^{(1)}\partial^j\partial_i\phi^{(1)\prime\prime}-6\mathcal{H}\partial_j\partial^i\phi^{(1)}\partial^j\partial_i\chi^{(1)\prime}+6\mathcal{H}\partial_j\partial^i\psi^{(1)}\partial^j\partial_i\chi^{(1)\prime}\\
&-\frac{2c_3\pi'\partial_j\partial^i\pi^{(1)}\partial^j\partial_i\pi^{(1)\prime}}{M^3{M_{\rm pl}}^2a^2}-\frac{c_3\pi'^2\partial_j\partial^i\omega^{(1)}\partial^j\partial_i\pi^{(1)\prime}}{M^3{M_{\rm pl}}^2a^2}+4\partial_i\psi^{(1)}\partial^i\nabla^2\phi^{(1)}\\
&+8\partial_j\partial^i\phi^{(1)}\partial^j\partial_i\omega^{(1)\prime}-6\partial_j\partial^i\psi^{(1)}\partial^j\partial_i\omega^{(1)\prime}-\frac{3}{2}\partial_j\partial^i\chi^{(1)\prime}\partial^j\partial_i\phi^{(1)\prime}\\
&-6\mathcal{H}\partial_j\partial^i\chi^{(1)}\partial^j\partial_i\phi^{(1)\prime}-3\partial_j\partial^i\omega^{(1)}\partial^j\partial_i\phi^{(1)\prime}+\frac{3}{2}\partial_j\partial^i\chi^{(1)\prime}\partial^j\partial_i\psi^{(1)\prime}\\
&-3\partial_j\partial^i\omega^{(1)}\partial^j\partial_i\psi^{(1)\prime}+\frac{1}{2}\partial_j\partial^i\chi^{(1)}\partial^j\partial_i\nabla^2\chi^{(1)\prime\prime}+\partial_j\partial^i\chi^{(1)\prime}\partial^j\partial_i\nabla^2\chi^{(1)\prime}\\
&+\mathcal{H}\partial_j\partial^i\chi^{(1)}\partial^j\partial_i\nabla^2\chi^{(1)\prime}-\partial_j\partial^i\omega^{(1)}\partial^j\partial_i\nabla^2\chi^{(1)\prime}-2\partial_j\partial^i\chi^{(1)}\partial^j\partial_i\nabla^2\omega^{(1)\prime}\\
&+\frac{1}{6}\partial_j\partial^i\chi^{(1)}\partial^j\partial_i\nabla^2\nabla^2\chi^{(1)}+\frac{1}{2}\partial_j\partial^i\chi^{(1)\prime\prime}\partial^j\partial_i\nabla^2\chi^{(1)}+\mathcal{H}\partial_j\partial^i\chi^{(1)\prime}\partial^j\partial_i\nabla^2\chi^{(1)}\\
&-\frac{8}{3}\partial_j\partial^i\omega^{(1)\prime}\partial^j\partial_i\nabla^2\chi^{(1)}+\frac{1}{3}\partial_j\partial^i\nabla^2\chi^{(1)}\partial^j\partial_i\nabla^2\chi^{(1)}-\frac{16}{3}\mathcal{H}\partial_j\partial^i\omega^{(1)}\partial^j\partial_i\nabla^2\chi^{(1)}\\
&-2\partial_j\partial^i\phi^{(1)}\partial^j\partial_i\nabla^2\chi^{(1)}-\frac{8}{3}\partial_j\partial^i\psi^{(1)}\partial^j\partial_i\nabla^2\chi^{(1)}-\frac{1}{2}\partial_j\partial^i\chi^{(1)\prime}\partial^j\partial_i\nabla^2\omega^{(1)}\\
&-4\mathcal{H}\partial_j\partial^i\chi^{(1)}\partial^j\partial_i\nabla^2\omega^{(1)}+\partial_j\partial^i\omega^{(1)}\partial^j\partial_i\nabla^2\omega^{(1)}+\partial_j\partial^i\chi^{(1)}\partial^j\partial_i\nabla^2\phi^{(1)}\\
&-2\partial_j\partial^i\chi^{(1)}\partial^j\partial_i\nabla^2\psi^{(1)}-\frac{c_2\partial_j\partial^i\pi^{(1)}\partial^j\partial_i\pi^{(1)}}{{M_{\rm pl}}^2}+\frac{c_3\pi''\partial_j\partial^i\pi^{(1)}\partial^j\partial_i\pi^{(1)}}{M^3{M_{\rm pl}}^2a^2}\\
&+\frac{2c_3\pi'\mathcal{H}\partial_j\partial^i\pi^{(1)}\partial^j\partial_i\pi^{(1)}}{M^3{M_{\rm pl}}^2a^2}-\frac{c_2\pi'\partial_j\partial^i\pi^{(1)}\partial^j\partial_i\omega^{(1)}}{{M_{\rm pl}}^2}+\frac{3c_3\pi'^2\mathcal{H}\partial_j\partial^i\pi^{(1)}\partial^j\partial_i\omega^{(1)}}{M^3{M_{\rm pl}}^2a^2}\\
&+16\mathcal{H}\partial_j\partial^i\omega^{(1)}\partial^j\partial_i\phi^{(1)}-15\partial_j\partial^i\phi^{(1)}\partial^j\partial_i\phi^{(1)}+\frac{2c_3\pi'^2\partial_j\partial^i\pi^{(1)}\partial^j\partial_i\psi^{(1)}}{M^3{M_{\rm pl}}^2a^2}\\
&+\frac{\rho_m a^2\partial_j\partial^i\omega^{(1)}\partial^j\partial_i v^{(1)}}{{M_{\rm pl}}^2}-14\mathcal{H}\partial_j\partial^i\omega^{(1)}\partial^j\partial_i\psi^{(1)}+\frac{c_3\pi'^3\partial_j\partial^i\omega^{(1)}\partial^j\partial_i\psi^{(1)}}{M^3{M_{\rm pl}}^2a^2}\\
&+8\partial_j\partial^i\phi^{(1)}\partial^j\partial_i\psi^{(1)}-7\partial_j\partial^i\psi^{(1)}\partial^j\partial_i\psi^{(1)}+\frac{\rho_m a^2\partial_j\partial^i v^{(1)}\partial^j\partial_i v^{(1)}}{{M_{\rm pl}}^2}\\
&+\frac{1}{2}\partial_k\partial^j\partial^i\chi^{(1)}\partial^k\partial_j\partial_i\chi^{(1)\prime\prime}+\frac{1}{2}\partial_k\partial^j\partial^i\chi^{(1)\prime}\partial^k\partial_j\partial_i\chi^{(1)\prime}+\mathcal{H}\partial_k\partial^j\partial^i\chi^{(1)}\partial^k\partial_j\partial_i\chi^{(1)\prime}
\end{aligned}
$$

$$
\begin{aligned}
&-\partial_k\partial^j\partial^i\omega^{(1)}\partial^k\partial_j\partial_i\chi^{(1)\prime}-\frac{5}{2}\partial_k\partial^j\partial^i\chi^{(1)}\partial^k\partial_j\partial_i\omega^{(1)\prime}-\frac{1}{6}\partial_k\partial^j\partial^i\chi^{(1)}\partial^k\partial_j\partial_i\nabla^2\chi^{(1)}\\
&-5\mathcal{H}\partial_k\partial^j\partial^i\chi^{(1)}\partial^k\partial_j\partial_i\omega^{(1)}+\partial_k\partial^j\partial^i\omega^{(1)}\partial^k\partial_j\partial_i\omega^{(1)}+\frac{1}{2}\partial_k\partial^j\partial^i\chi^{(1)}\partial^k\partial_j\partial_i\phi^{(1)}\\
&-\frac{5}{2}\partial_k\partial^j\partial^i\chi^{(1)}\partial^k\partial_j\partial_i\psi^{(1)}-\frac{1}{4}\partial_l\partial^k\partial^j\partial^i\chi^{(1)}\partial^l\partial_k\partial_j\partial_i\chi^{(1)} \qquad \text{(D.4)}
\end{aligned}
$$

$$
\begin{aligned}
S^{(5)}\equiv\ &\pi^{(1)\prime}\psi^{(1)}(8c_2\mathcal{H}-\frac{48c_3\pi'\mathcal{H}'}{M^3a^2}-\frac{48c_3\pi''\mathcal{H}}{M^3a^2})+\pi^{(1)\prime\prime}\psi^{(1)}(4c_2-\frac{48c_3\pi'\mathcal{H}}{M^3a^2})\\
&+\pi^{(1)\prime}\psi^{(1)\prime}(2c_2-\frac{36c_3\pi'\mathcal{H}}{M^3a^2})+\pi^{(1)\prime}\phi^{(1)\prime}(6c_2-\frac{12c_3\pi''}{M^3a^2}-\frac{36c_3\pi'\mathcal{H}}{M^3a^2})\\
&+\pi^{(1)\prime}\nabla^2\omega^{(1)}(2c_2-\frac{4c_3\pi''}{M^3a^2}-\frac{12c_3\pi'\mathcal{H}}{M^3a^2})+\nabla^2\pi^{(1)}\phi^{(1)}(4c_2-\frac{8c_3\pi''}{M^3a^2}-\frac{8c_3\pi'\mathcal{H}}{M^3a^2})\\
&+\nabla^2\chi^{(1)}\nabla^2\pi^{(1)}(\frac{2}{3}c_2-\frac{4c_3\pi''}{3M^3a^2}-\frac{4c_3\pi'\mathcal{H}}{3M^3a^2})+\nabla^2\pi^{(1)}\psi^{(1)}(\frac{8c_3\pi''}{M^3a^2}+\frac{8c_3\pi'\mathcal{H}}{M^3a^2})\\
&+\psi^{(1)2}(-8c_2\pi''-16c_2\pi'\mathcal{H}+\frac{72c_3\pi'^2\mathcal{H}'}{M^3a^2}+\frac{144c_3\pi''\pi'\mathcal{H}}{M^3a^2})+\phi^{(1)\prime}\phi^{(1)}(12c_2\pi'\\
&-\frac{24c_3\pi''\pi'}{M^3a^2}-\frac{36c_3\pi'^2\mathcal{H}}{M^3a^2})+\nabla^2\omega^{(1)}\phi^{(1)}(4c_2\pi'-\frac{8c_3\pi''\pi'}{M^3a^2}-\frac{12c_3\pi'^2\mathcal{H}}{M^3a^2})\\
&+\nabla^2\chi^{(1)}\nabla^2\omega^{(1)}(\frac{2}{3}c_2\pi'-\frac{4c_3\pi''\pi'}{3M^3a^2}-\frac{2c_3\pi'^2\mathcal{H}}{M^3a^2})+\nabla^2\chi^{(1)\prime}\nabla^2\chi^{(1)}(-\frac{1}{3}c_2\pi'\\
&+\frac{2c_3\pi''\pi'}{3M^3a^2}+\frac{c_3\pi'^2\mathcal{H}}{M^3a^2})+\nabla^2\omega^{(1)}\psi^{(1)}(-4c_2\pi'+\frac{16c_3\pi''\pi'}{M^3a^2}+\frac{24c_3\pi'^2\mathcal{H}}{M^3a^2})\\
&+\phi^{(1)\prime}\psi^{(1)}(-12c_2\pi'+\frac{48c_3\pi''\pi'}{M^3a^2}+\frac{72c_3\pi'^2\mathcal{H}}{M^3a^2})+\psi^{(1)\prime}\psi^{(1)}(-8c_2\pi'+\frac{108c_3\pi'^2\mathcal{H}}{M^3a^2})\\
&-\frac{12c_3\phi^{(1)\prime\prime}\pi'\pi^{(1)\prime}}{M^3a^2}+\frac{6c_3\mathcal{H}'\pi^{(1)\prime 2}}{M^3a^2}-\frac{12c_3\pi^{(1)\prime\prime}\pi'\phi^{(1)\prime}}{M^3a^2}-\frac{4c_3\pi'\partial_j\partial^i\pi^{(1)}\partial^j\partial_i\omega^{(1)}}{M^3a^2}\\
&-\frac{2c_3\pi'^2\partial_j\partial^i\omega^{(1)}\partial^j\partial_i\omega^{(1)}}{M^3a^2}+\frac{6c_3\pi'^2\phi^{(1)\prime 2}}{M^3a^2}+\frac{18c_3\pi'^2\phi^{(1)\prime}\psi^{(1)\prime}}{M^3a^2}+\frac{12c_3\pi^{(1)\prime\prime}\pi^{(1)\prime}\mathcal{H}}{M^3a^2}\\
&+\frac{c_3\pi'^2\nabla^2\chi^{(1)\prime 2}}{3M^3a^2}-\frac{4c_3\pi'\pi^{(1)\prime}\nabla^2\omega^{(1)\prime}}{M^3a^2}+\frac{c_3\pi'^2\nabla^2\chi^{(1)\prime\prime}\nabla^2\chi^{(1)}}{3M^3a^2}\\
&-\frac{2c_3\pi'^2\nabla^2\omega^{(1)\prime}\nabla^2\chi^{(1)}}{3M^3a^2}-\frac{4c_3\pi^{(1)\prime\prime}\nabla^2\pi^{(1)}}{M^3a^2}+\frac{8c_3\pi'\phi^{(1)\prime}\nabla^2\pi^{(1)}}{M^3a^2}\\
&+\frac{4c_3\pi'\psi^{(1)\prime}\nabla^2\pi^{(1)}}{M^3a^2}-\frac{4c_3\pi^{(1)\prime}\mathcal{H}\nabla^2\pi^{(1)}}{M^3a^2}-\frac{2c_3\pi'\nabla^2\chi^{(1)\prime}\nabla^2\pi^{(1)}}{3M^3a^2}+\frac{2c_3\nabla^2\pi^{(1)2}}{M^3a^2}\\
&-\frac{4c_3\pi^{(1)\prime\prime}\pi'\nabla^2\omega^{(1)}}{M^3a^2}+\frac{8c_3\pi'^2\phi^{(1)\prime}\nabla^2\omega^{(1)}}{M^3a^2}+\frac{6c_3\pi'^2\psi^{(1)\prime}\nabla^2\omega^{(1)}}{M^3a^2}\\
&-\frac{2c_3\pi'^2\nabla^2\chi^{(1)\prime}\nabla^2\omega^{(1)}}{3M^3a^2}+\frac{4c_3\pi'\nabla^2\pi^{(1)}\nabla^2\omega^{(1)}}{M^3a^2}+\frac{2c_3\pi'^2\nabla^2\omega^{(1)2}}{M^3a^2}\\
&-\frac{4c_3\pi'\pi^{(1)\prime}\nabla^2\psi^{(1)}}{M^3a^2}-\frac{2c_3\pi'^2\nabla^2\chi^{(1)}\nabla^2\psi^{(1)}}{3M^3a^2}-\frac{12c_3\phi^{(1)\prime\prime}\pi'^2\phi^{(1)}}{M^3a^2}
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{4c_3\pi'^2\nabla^2\omega^{(1)\prime}\phi^{(1)}}{M^3a^2}-\frac{4c_3\pi'^2\nabla^2\psi^{(1)}\phi^{(1)}}{M^3a^2}+\frac{24c_3\phi^{(1)\prime\prime}\pi'^2\psi^{(1)}}{M^3a^2}\\
&+\frac{8c_3\pi'^2\nabla^2\omega^{(1)\prime}\psi^{(1)}}{M^3a^2}+\frac{8c_3\pi'^2\nabla^2\psi^{(1)}\psi^{(1)}}{M^3a^2}+\frac{4c_3\partial_i\pi^{(1)\prime}\partial^i\pi^{(1)\prime}}{M^3a^2}\\
&-\frac{8c_3\mathcal{H}\partial_i\pi^{(1)}\partial^i\pi^{(1)\prime}}{M^3a^2}+4c_2\partial_i\omega^{(1)}\partial^i\pi^{(1)\prime}-\frac{24c_3\pi'\mathcal{H}\partial_i\omega^{(1)}\partial^i\pi^{(1)\prime}}{M^3a^2}\\
&-\frac{8c_3\pi'\partial_i\psi^{(1)}\partial^i\pi^{(1)\prime}}{M^3a^2}+2c_2\partial_i\pi^{(1)}\partial^i\omega^{(1)\prime}-\frac{12c_3\pi'\mathcal{H}\partial_i\pi^{(1)}\partial^i\omega^{(1)\prime}}{M^3a^2}\\
&+2c_2\pi'\partial_i\omega^{(1)}\partial^i\omega^{(1)\prime}-\frac{18c_3\pi'^2\mathcal{H}\partial_i\omega^{(1)}\partial^i\omega^{(1)\prime}}{M^3a^2}+\frac{2c_3\pi'^2\partial_i\phi^{(1)}\partial^i\omega^{(1)\prime}}{M^3a^2}\\
&+\frac{8c_3\pi'\partial_i\pi^{(1)}\partial^i\phi^{(1)\prime}}{M^3a^2}+\frac{8c_3\pi'^2\partial_i\omega^{(1)}\partial^i\phi^{(1)\prime}}{M^3a^2}+\frac{4c_3\pi'\partial_i\pi^{(1)}\partial^i\nabla^2\chi^{(1)\prime}}{3M^3a^2}\\
&+\frac{4c_3\pi'^2\partial_i\omega^{(1)}\partial^i\nabla^2\chi^{(1)\prime}}{3M^3a^2}+\frac{4c_3\pi'^2\partial_i\omega^{(1)\prime}\partial^i\nabla^2\chi^{(1)}}{3M^3a^2}-\frac{4}{3}c_2\partial_i\pi^{(1)}\partial^i\nabla^2\chi^{(1)}\\
&+\frac{8c_3\pi''\partial_i\pi^{(1)}\partial^i\nabla^2\chi^{(1)}}{3M^3a^2}+\frac{8c_3\pi'\mathcal{H}\partial_i\pi^{(1)}\partial^i\nabla^2\chi^{(1)}}{3M^3a^2}-\frac{4}{3}c_2\pi'\partial_i\omega^{(1)}\partial^i\nabla^2\chi^{(1)}\\
&+\frac{8c_3\pi''\pi'\partial_i\omega^{(1)}\partial^i\nabla^2\chi^{(1)}}{3M^3a^2}+\frac{4c_3\pi'^2\mathcal{H}\partial_i\omega^{(1)}\partial^i\nabla^2\chi^{(1)}}{M^3a^2}\\
&+\frac{4c_3\pi'^2\partial_i\psi^{(1)}\partial^i\nabla^2\chi^{(1)}}{3M^3a^2}-\frac{2c_3\mathcal{H}'\partial_i\pi^{(1)}\partial^i\pi^{(1)}}{M^3a^2}+4c_2\mathcal{H}\partial_i\pi^{(1)}\partial^i\omega^{(1)}\\
&-\frac{12c_3\pi'\mathcal{H}'\partial_i\pi^{(1)}\partial^i\omega^{(1)}}{M^3a^2}-\frac{12c_3\pi''\mathcal{H}\partial_i\pi^{(1)}\partial^i\omega^{(1)}}{M^3a^2}+2c_2\pi''\partial_i\omega^{(1)}\partial^i\omega^{(1)}\\
&+4c_2\pi'\mathcal{H}\partial_i\omega^{(1)}\partial^i\omega^{(1)}-\frac{12c_3\pi'^2\mathcal{H}'\partial_i\omega^{(1)}\partial^i\omega^{(1)}}{M^3a^2}-\frac{24c_3\pi''\pi'\mathcal{H}\partial_i\omega^{(1)}\partial^i\omega^{(1)}}{M^3a^2}\\
&-2c_2\partial_i\pi^{(1)}\partial^i\phi^{(1)}+\frac{4c_3\pi''\partial_i\pi^{(1)}\partial^i\phi^{(1)}}{M^3a^2}+\frac{4c_3\pi'\mathcal{H}\partial_i\pi^{(1)}\partial^i\phi^{(1)}}{M^3a^2}-2c_2\pi'\partial_i\omega^{(1)}\partial^i\phi^{(1)}\\
&+\frac{4c_3\pi''\pi'\partial_i\omega^{(1)}\partial^i\phi^{(1)}}{M^3a^2}+\frac{6c_3\pi'^2\mathcal{H}\partial_i\omega^{(1)}\partial^i\phi^{(1)}}{M^3a^2}+2c_2\partial_i\pi^{(1)}\partial^i\psi^{(1)}\\
&+\frac{4c_3\pi'\mathcal{H}\partial_i\pi^{(1)}\partial^i\psi^{(1)}}{M^3a^2}-2c_2\pi'\partial_i\omega^{(1)}\partial^i\psi^{(1)}+\frac{18c_3\pi'^2\mathcal{H}\partial_i\omega^{(1)}\partial^i\psi^{(1)}}{M^3a^2}\\
&+\frac{2c_3\pi'^2\partial_i\phi^{(1)}\partial^i\psi^{(1)}}{M^3a^2}+\frac{6c_3\pi'^2\partial_i\psi^{(1)}\partial^i\psi^{(1)}}{M^3a^2}-\frac{c_3\pi'^2\partial_j\partial^i\chi^{(1)}\partial^j\partial_i\chi^{(1)\prime\prime}}{M^3a^2}\\
&-\frac{c_3\pi'^2\partial_j\partial^i\chi^{(1)\prime}\partial^j\partial_i\chi^{(1)\prime}}{M^3a^2}+c_2\pi'\partial_j\partial^i\chi^{(1)}\partial^j\partial_i\chi^{(1)\prime}-\frac{2c_3\pi''\pi'\partial_j\partial^i\chi^{(1)}\partial^j\partial_i\chi^{(1)\prime}}{M^3a^2}\\
&-\frac{3c_3\pi'^2\mathcal{H}\partial_j\partial^i\chi^{(1)}\partial^j\partial_i\chi^{(1)\prime}}{M^3a^2}+\frac{2c_3\pi'\partial_j\partial^i\pi^{(1)}\partial^j\partial_i\chi^{(1)\prime}}{M^3a^2}+\frac{2c_3\pi'^2\partial_j\partial^i\omega^{(1)}\partial^j\partial_i\chi^{(1)\prime}}{M^3a^2}\\
&+\frac{2c_3\pi'^2\partial_j\partial^i\chi^{(1)}\partial^j\partial_i\omega^{(1)\prime}}{M^3a^2}-2c_2\partial_j\partial^i\chi^{(1)}\partial^j\partial_i\pi^{(1)}+\frac{4c_3\pi''\partial_j\partial^i\chi^{(1)}\partial^j\partial_i\pi^{(1)}}{M^3a^2}\\
&+\frac{4c_3\pi'\mathcal{H}\partial_j\partial^i\chi^{(1)}\partial^j\partial_i\pi^{(1)}}{M^3a^2}-\frac{2c_3\partial_j\partial^i\pi^{(1)}\partial^j\partial_i\pi^{(1)}}{M^3a^2}-2c_2\pi'\partial_j\partial^i\chi^{(1)}\partial^j\partial_i\omega^{(1)}
\end{aligned}
$$

$$+\frac{4c_3\pi''\pi'\partial_j\partial^i\chi^{(1)}\partial^j\partial_i\omega^{(1)}}{M^3a^2}+\frac{6c_3\pi'^2\mathcal{H}\partial_j\partial^i\chi^{(1)}\partial^j\partial_i\omega^{(1)}}{M^3a^2}+\frac{2c_3\pi'^2\partial_j\partial^i\chi^{(1)}\partial^j\partial_i\psi^{(1)}}{M^3a^2} \tag{D.5}$$

$$\begin{aligned}S^{(6)}\equiv&-\frac{1}{3}\nabla^2\chi^{(1)\prime}\nabla^2\chi^{(1)}+6\phi^{(1)\prime}\delta^{(1)}-2\nabla^2v^{(1)}\delta^{(1)}+12\phi^{(1)\prime}\phi^{(1)}-2\nabla^2v^{(1)}\psi^{(1)}\\&-2\partial_i\omega^{(1)}\partial^i\omega^{(1)\prime}-2\partial_i\omega^{(1)}\partial^iv^{(1)\prime}-4\partial_iv^{(1)}\partial^iv^{(1)\prime}-2\partial_i\delta^{(1)}\partial^iv^{(1)}\\&-2\mathcal{H}\partial_i\omega^{(1)}\partial^iv^{(1)}+6\partial_i\phi^{(1)}\partial^iv^{(1)}-2\mathcal{H}\partial_iv^{(1)}\partial^iv^{(1)}+\partial_j\partial^i\chi^{(1)}\partial^j\partial_i\chi^{(1)\prime}\\&-4\partial_iv^{(1)}\partial^i\omega^{(1)\prime}-4\partial_i\psi^{(1)}\partial^iv^{(1)}\end{aligned} \tag{D.6}$$

$$\begin{aligned}S^{(7)}\equiv&\frac{2}{3}\nabla^2v^{(1)\prime}\nabla^2\chi^{(1)}-2\delta^{(1)\prime}\nabla^2\omega^{(1)}+6\phi^{(1)\prime}\nabla^2\omega^{(1)}+2\psi^{(1)\prime}\nabla^2\omega^{(1)}-2\delta^{(1)\prime}\nabla^2v^{(1)}\\&+10\phi^{(1)\prime}\nabla^2v^{(1)}+\frac{2}{3}\nabla^2\chi^{(1)\prime}\nabla^2v^{(1)}+\frac{2}{3}\mathcal{H}\nabla^2\chi^{(1)}\nabla^2v^{(1)}-2\nabla^2\omega^{(1)}\nabla^2v^{(1)}\\&-2\nabla^2{v^{(1)}}^2-2\nabla^2\omega^{(1)\prime}\delta^{(1)}-2\nabla^2v^{(1)\prime}\delta^{(1)}-2\mathcal{H}\nabla^2\omega^{(1)}\delta^{(1)}+4\mathcal{H}\nabla^2\omega^{(1)}\psi^{(1)}\\&-2\nabla^2\psi^{(1)}\delta^{(1)}-2\mathcal{H}\nabla^2v^{(1)}\delta^{(1)}+4\nabla^2v^{(1)\prime}\phi^{(1)}+4\mathcal{H}\nabla^2v^{(1)}\phi^{(1)}+4\nabla^2\omega^{(1)\prime}\psi^{(1)}\\&+4\nabla^2\psi^{(1)}\psi^{(1)}+2\mathcal{H}\nabla^2v^{(1)}\psi^{(1)}-2\partial_i\omega^{(1)}\partial^i\delta^{(1)\prime}-2\partial_iv^{(1)}\partial^i\delta^{(1)\prime}-2\partial_i\delta^{(1)}\partial^i\omega^{(1)\prime}\\&+4\partial_i\psi^{(1)}\partial^i\omega^{(1)\prime}+6\partial_i\omega^{(1)}\partial^i\phi^{(1)\prime}+10\partial_iv^{(1)}\partial^i\phi^{(1)\prime}+2\partial_i\omega^{(1)}\partial^i\psi^{(1)\prime}\\&+2\partial_i\psi^{(1)}\partial^iv^{(1)\prime}-\frac{4}{3}\partial_iv^{(1)}\partial^i\nabla^2\chi^{(1)\prime}-\frac{4}{3}\partial_iv^{(1)\prime}\partial^i\nabla^2\chi^{(1)}-2\partial_j\partial^iv^{(1)}\partial^j\partial_iv^{(1)}\\&+2\nabla^2v^{(1)\prime}\psi^{(1)}+4\partial_i\phi^{(1)}\partial^iv^{(1)\prime}-2\partial_i\delta^{(1)}\partial^i\psi^{(1)}+2\mathcal{H}\partial_i\psi^{(1)}\partial^iv^{(1)}-2\partial_i\delta^{(1)}\partial^iv^{(1)\prime}\\&-\frac{4}{3}\mathcal{H}\partial_iv^{(1)}\partial^i\nabla^2\chi^{(1)}-2\partial_i\omega^{(1)}\partial^i\nabla^2v^{(1)}-4\partial_iv^{(1)}\partial^i\nabla^2v^{(1)}-2\mathcal{H}\partial_i\delta^{(1)}\partial^i\omega^{(1)}\\&+4\mathcal{H}\partial_i\omega^{(1)}\partial^i\psi^{(1)}+4\partial_i\psi^{(1)}\partial^i\psi^{(1)}-2\mathcal{H}\partial_i\delta^{(1)}\partial^iv^{(1)}+4\mathcal{H}\partial_i\phi^{(1)}\partial^iv^{(1)}\\&-2\partial_j\partial^iv^{(1)}\partial^j\partial_i\chi^{(1)\prime}-2\partial_j\partial^i\chi^{(1)}\partial^j\partial_iv^{(1)\prime}-2\mathcal{H}\partial_j\partial^i\chi^{(1)}\partial^j\partial_iv^{(1)}\end{aligned} \tag{D.7}$$

APPENDIX E

BACKGROUND QUANTITIES FOR THE SECOND-ORDER DM KERNEL

In the following we give the explicit expression for the background functions $\gamma_i(a)$ found in the kernel (4.11). They reads

$$\begin{aligned}\gamma_4(\tau) \equiv &-\frac{5\rho_m f^2}{2{M_{\rm pl}}^2} + \frac{7{c_3}^6{\rho_m}^2\pi'^{12}}{64{c_2}^3M^{18}{M_{\rm pl}}^{10}\alpha^3\mathcal{H}^2a^{10}} + \frac{3{c_3}^5{\rho_m}^2\alpha'\pi'^9}{8{c_2}^3M^{15}{M_{\rm pl}}^8\alpha^4\mathcal{H}^2a^8}\\ &+ \frac{7{c_3}^5{\rho_m}^2\pi'^9}{16{c_2}^3M^{15}{M_{\rm pl}}^8\alpha^3\mathcal{H}a^8} - \frac{3{c_3}^5{\rho_m}^2\pi'^9 f}{8{c_2}^3M^{15}{M_{\rm pl}}^8\alpha^3\mathcal{H}a^8} - \frac{35{c_3}^4{\rho_m}^2\pi'^6}{2{c_2}^3M^{12}{M_{\rm pl}}^6\alpha^3a^6}\\ &+ \frac{6{c_3}^4{\rho_m}^2\pi'^6 f}{{c_2}^3M^{12}{M_{\rm pl}}^6\alpha^3a^6} - \frac{{c_3}^4{\rho_m}^2\pi'^6 f^2}{2{c_2}^3M^{12}{M_{\rm pl}}^6\alpha^3a^6} - \frac{{c_3}^4{\rho_m}^2\alpha'^2\pi'^6}{2{c_2}^3M^{12}{M_{\rm pl}}^6\alpha^5\mathcal{H}^2a^6}\\ &+ \frac{7{c_3}^4{\rho_m}^2\pi'^8}{32{c_2}^2M^{12}{M_{\rm pl}}^8\alpha^3\mathcal{H}^2a^6} + \frac{9{c_3}^4{\rho_m}^2\pi'^8}{16{c_2}^2M^{12}{M_{\rm pl}}^8\alpha^2\mathcal{H}^2a^6} + \frac{2f^2\mathcal{H}^2}{a^2}\end{aligned}$$

$$
\begin{aligned}
&+\frac{c_3{}^4\rho_m{}^2\pi'^6\mathcal{H}'}{4c_2{}^3M^{12}M_{\rm pl}{}^6\alpha^3\mathcal{H}^2a^6}-\frac{6c_3{}^4\rho_m{}^2\alpha'\pi'^6}{c_2{}^3M^{12}M_{\rm pl}{}^6\alpha^4\mathcal{H}a^6}+\frac{2c_3\rho_m\pi'^3f}{M^3M_{\rm pl}{}^4\alpha\mathcal{H}a^2}\\
&+\frac{c_3{}^4\rho_m{}^2\alpha'\pi'^6f}{c_2{}^3M^{12}M_{\rm pl}{}^6\alpha^4\mathcal{H}a^6}+\frac{c_3{}^3\rho_m\pi'^7f}{c_2M^9M_{\rm pl}{}^6\alpha\mathcal{H}a^6}-\frac{8c_3{}^2\rho_m\pi'^4f}{c_2M^6M_{\rm pl}{}^4\alpha a^4}\\
&+\frac{5c_3{}^2\rho_m\pi'^4f^2}{4c_2M^6M_{\rm pl}{}^4\alpha a^4}+\frac{c_3{}^3\rho_m{}^2\alpha'\pi'^5}{c_2{}^2M^9M_{\rm pl}{}^6\alpha^4\mathcal{H}^2a^4}-\frac{7c_3{}^3\rho_m{}^2\alpha'\pi'^5}{4c_2{}^2M^9M_{\rm pl}{}^6\alpha^3\mathcal{H}^2a^4}\\
&+\frac{6c_3{}^3\rho_m{}^2\pi'^5}{c_2{}^2M^9M_{\rm pl}{}^6\alpha^3\mathcal{H}a^4}-\frac{71c_3{}^3\rho_m{}^2\pi'^5}{8c_2{}^2M^9M_{\rm pl}{}^6\alpha^2\mathcal{H}a^4}-\frac{c_3{}^2\rho_m\alpha'\pi'^4f}{c_2M^6M_{\rm pl}{}^4\alpha^2\mathcal{H}a^4}\\
&-\frac{c_3{}^3\rho_m{}^2\pi'^5f}{c_2{}^2M^9M_{\rm pl}{}^6\alpha^3\mathcal{H}a^4}+\frac{3c_3{}^3\rho_m{}^2\pi'^5f}{4c_2{}^2M^9M_{\rm pl}{}^6\alpha^2\mathcal{H}a^4}-\frac{2\mathcal{H}'f^2}{a^2}-\frac{2c_3\rho_m\pi'^3f}{M^3M_{\rm pl}{}^4\mathcal{H}a^2}\\
&-\frac{c_3{}^2\rho_m{}^2\pi'^4}{2c_2M^6M_{\rm pl}{}^6\alpha^3\mathcal{H}^2a^2}+\frac{25c_3{}^2\rho_m{}^2\pi'^4}{16c_2M^6M_{\rm pl}{}^6\alpha^2\mathcal{H}^2a^2}-\frac{25c_3{}^2\rho_m{}^2\pi'^4}{16c_2M^6M_{\rm pl}{}^6\alpha\mathcal{H}^2a^2}
\end{aligned}
\tag{E.1}
$$

$$
\begin{aligned}
\gamma_5(\tau)\equiv&\frac{3\rho_mf^2}{2M_{\rm pl}{}^2}+\frac{15c_3{}^6\rho_m{}^2\pi'^{12}}{64c_2{}^3M^{18}M_{\rm pl}{}^{10}\alpha^3\mathcal{H}^2a^{10}}-\frac{3c_3{}^5\rho_m{}^2\alpha'\pi'^9}{8c_2{}^3M^{15}M_{\rm pl}{}^8\alpha^4\mathcal{H}^2a^8}\\
&-\frac{33c_3{}^5\rho_m{}^2\pi'^9}{16c_2{}^3M^{15}M_{\rm pl}{}^8\alpha^3\mathcal{H}a^8}+\frac{3c_3{}^5\rho_m{}^2\pi'^9f}{8c_2{}^3M^{15}M_{\rm pl}{}^8\alpha^3\mathcal{H}a^8}+\frac{15c_3{}^4\rho_m{}^2\pi'^8}{32c_2{}^2M^{12}M_{\rm pl}{}^8\alpha^3\mathcal{H}^2a^6}\\
&-\frac{3c_3{}^4\rho_m{}^2\pi'^8}{4c_2{}^2M^{12}M_{\rm pl}{}^8\alpha^2\mathcal{H}^2a^6}-\frac{3c_3{}^2\rho_m\pi'^4f^2}{4c_2M^6M_{\rm pl}{}^4\alpha a^4}+\frac{3c_3{}^3\rho_m{}^2\alpha'\pi'^5}{4c_2{}^2M^9M_{\rm pl}{}^6\alpha^3\mathcal{H}^2a^4}\\
&+\frac{33c_3{}^3\rho_m{}^2\pi'^5}{8c_2{}^2M^9M_{\rm pl}{}^6\alpha^2\mathcal{H}a^4}-\frac{3c_3{}^3\rho_m{}^2\pi'^5f}{4c_2{}^2M^9M_{\rm pl}{}^6\alpha^2\mathcal{H}a^4}-\frac{15c_3{}^2\rho_m{}^2\pi'^4}{16c_2M^6M_{\rm pl}{}^6\alpha^2\mathcal{H}^2a^2}\\
&+\frac{3c_3{}^2\rho_m{}^2\pi'^4}{16c_2M^6M_{\rm pl}{}^6\alpha\mathcal{H}^2a^2}+\frac{3\rho_m{}^2a^2}{4M_{\rm pl}{}^4\mathcal{H}^2}
\end{aligned}
\tag{E.2}
$$

$$
\begin{aligned}
\gamma_6(\tau)\equiv&\frac{15\rho_mf}{2M_{\rm pl}{}^2}-\frac{15\rho_mf^2}{4M_{\rm pl}{}^2}-\frac{9c_2\rho_m\pi'^2}{2M_{\rm pl}{}^4\mathcal{H}^2}+\frac{9c_2\rho_m\pi'^2}{4M_{\rm pl}{}^4\alpha\mathcal{H}^2}+\frac{9c_2\alpha\rho_m\pi'^2}{4M_{\rm pl}{}^4\mathcal{H}^2}\\
&+\frac{3\rho_m\mathcal{H}'}{2M_{\rm pl}{}^2\mathcal{H}^2}+\frac{59c_3{}^6\rho_m{}^2\pi'^{12}}{128c_2{}^3M^{18}M_{\rm pl}{}^{10}\alpha^3\mathcal{H}^2a^{10}}-\frac{9c_3{}^5\rho_m{}^2\alpha'\pi'^9}{16c_2{}^3M^{15}M_{\rm pl}{}^8\alpha^4\mathcal{H}^2a^8}\\
&+\frac{21c_3{}^4\rho_m\pi'^{10}}{16c_2M^{12}M_{\rm pl}{}^8\alpha\mathcal{H}^2a^8}-\frac{c_3{}^6\rho_m{}^3\pi'^{10}}{16c_2{}^4M^{18}M_{\rm pl}{}^{10}\alpha^4\mathcal{H}^2a^8}-\frac{157c_3{}^5\rho_m{}^2\pi'^9}{32c_2{}^3M^{15}M_{\rm pl}{}^8\alpha^3\mathcal{H}a^8}\\
&+\frac{9c_3{}^5\rho_m{}^2\pi'^9f}{16c_2{}^3M^{15}M_{\rm pl}{}^8\alpha^3\mathcal{H}a^8}+\frac{5c_3{}^4\rho_m{}^2\pi'^6}{c_2{}^3M^{12}M_{\rm pl}{}^6\alpha^3a^6}-\frac{5c_3{}^4\rho_m{}^2\pi'^6f}{2c_2{}^3M^{12}M_{\rm pl}{}^6\alpha^3a^6}\\
&-\frac{c_3{}^4\alpha''\rho_m{}^2\pi'^6}{4c_2{}^3M^{12}M_{\rm pl}{}^6\alpha^4\mathcal{H}^2a^6}-\frac{3c_3{}^3\rho_m\alpha'\pi'^7}{2c_2M^9M_{\rm pl}{}^6\alpha^2\mathcal{H}^2a^6}+\frac{79c_3{}^4\rho_m{}^2\pi'^8}{64c_2{}^2M^{12}M_{\rm pl}{}^8\alpha^3\mathcal{H}^2a^6}\\
&-\frac{17c_3{}^4\rho_m{}^2\pi'^8}{16c_2{}^2M^{12}M_{\rm pl}{}^8\alpha^2\mathcal{H}^2a^6}-\frac{5c_3{}^4\rho_m{}^2\pi'^6\mathcal{H}'}{4c_2{}^3M^{12}M_{\rm pl}{}^6\alpha^3\mathcal{H}^2a^6}-\frac{7\rho_m{}^2a^2}{8M_{\rm pl}{}^4\mathcal{H}^2}
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{9{c_3}^4{\rho_m}^2\alpha'\pi'^6}{4{c_2}^3M^{12}{M_{\rm pl}}^6\alpha^4\mathcal{H}a^6}-\frac{33{c_3}^3\rho_m\pi'^7}{2c_2M^9{M_{\rm pl}}^6\alpha\mathcal{H}a^6}-\frac{{c_3}^4{\rho_m}^2\alpha'\pi'^6f}{2{c_2}^3M^{12}{M_{\rm pl}}^6\alpha^4\mathcal{H}a^6}\\
&+\frac{15{c_3}^3\rho_m\pi'^7f}{4c_2M^9{M_{\rm pl}}^6\alpha\mathcal{H}a^6}+\frac{54{c_3}^2\rho_m\pi'^4}{c_2M^6{M_{\rm pl}}^4\alpha a^4}-\frac{135{c_3}^2\rho_m\pi'^4f}{4c_2M^6{M_{\rm pl}}^4\alpha a^4}\\
&+\frac{15{c_3}^2\rho_m\pi'^4f^2}{8c_2M^6{M_{\rm pl}}^4\alpha a^4}-\frac{3{c_3}^2\alpha''\rho_m\pi'^4}{4c_2M^6{M_{\rm pl}}^4\alpha^2\mathcal{H}^2a^4}+\frac{3{c_3}^2\rho_m\alpha'^2\pi'^4}{2c_2M^6{M_{\rm pl}}^4\alpha^3\mathcal{H}^2a^4}\\
&-\frac{{c_3}^3{\rho_m}^2\alpha'\pi'^5}{2{c_2}^2M^9{M_{\rm pl}}^6\alpha^4\mathcal{H}^2a^4}+\frac{{c_3}^3{\rho_m}^2\alpha'\pi'^5}{8{c_2}^2M^9{M_{\rm pl}}^6\alpha^3\mathcal{H}^2a^4}+\frac{15c_3\rho_m\pi'^3f}{2M^3{M_{\rm pl}}^4\alpha\mathcal{H}a^2}\\
&-\frac{15{c_3}^2\rho_m\pi'^6}{4M^6{M_{\rm pl}}^6\mathcal{H}^2a^4}+\frac{15{c_3}^2\rho_m\pi'^6}{4M^6{M_{\rm pl}}^6\alpha\mathcal{H}^2a^4}+\frac{{c_3}^4{\rho_m}^3\pi'^6}{8{c_2}^3M^{12}{M_{\rm pl}}^8\alpha^3\mathcal{H}^2a^4}\\
&-\frac{27{c_3}^2\rho_m\pi'^4\mathcal{H}'}{4c_2M^6{M_{\rm pl}}^4\alpha\mathcal{H}^2a^4}+\frac{51{c_3}^2\rho_m\alpha'\pi'^4}{4c_2M^6{M_{\rm pl}}^4\alpha^2\mathcal{H}a^4}-\frac{15c_3\rho_m\pi'^3f}{2M^3{M_{\rm pl}}^4\mathcal{H}a^2}\\
&-\frac{3{c_3}^3{\rho_m}^2\pi'^5}{2{c_2}^2M^9{M_{\rm pl}}^6\alpha^3\mathcal{H}a^4}+\frac{29{c_3}^3{\rho_m}^2\pi'^5}{16{c_2}^2M^9{M_{\rm pl}}^6\alpha^2\mathcal{H}a^4}-\frac{15{c_3}^2\rho_m\alpha'\pi'^4f}{4c_2M^6{M_{\rm pl}}^4\alpha^2\mathcal{H}a^4}\\
&+\frac{{c_3}^3{\rho_m}^2\pi'^5f}{2{c_2}^2M^9{M_{\rm pl}}^6\alpha^3\mathcal{H}a^4}-\frac{9{c_3}^3{\rho_m}^2\pi'^5f}{8{c_2}^2M^9{M_{\rm pl}}^6\alpha^2\mathcal{H}a^4}-\frac{3c_3\rho_m\alpha'\pi'^3}{M^3{M_{\rm pl}}^4\alpha^2\mathcal{H}^2a^2}\\
&+\frac{3c_3\rho_m\alpha'\pi'^3}{2M^3{M_{\rm pl}}^4\alpha\mathcal{H}^2a^2}+\frac{{c_3}^2{\rho_m}^2\pi'^4}{8c_2M^6{M_{\rm pl}}^6\alpha^3\mathcal{H}^2a^2}-\frac{15{c_3}^2{\rho_m}^2\pi'^4}{32c_2M^6{M_{\rm pl}}^6\alpha^2\mathcal{H}^2a^2}\\
&+\frac{{c_3}^4{\rho_m}^2\alpha'^2\pi'^6}{2{c_2}^3M^{12}{M_{\rm pl}}^6\alpha^5\mathcal{H}^2a^6}+\frac{23{c_3}^2{\rho_m}^2\pi'^4}{32c_2M^6{M_{\rm pl}}^6\alpha\mathcal{H}^2a^2}-\frac{21c_3\rho_m\pi'^3}{M^3{M_{\rm pl}}^4\alpha\mathcal{H}a^2}\\
&+\frac{21c_3\rho_m\pi'^3}{M^3{M_{\rm pl}}^4\mathcal{H}a^2}
\end{aligned}
\tag{E.3}
$$

$$
\begin{aligned}
\gamma_7(\tau)\equiv&-\frac{3\rho_mf^2}{4{M_{\rm pl}}^2}-\frac{15{c_3}^6\pi'^{12}{\rho_m}^2}{128{c_2}^3M^{18}{M_{\rm pl}}^{10}\alpha^3\mathcal{H}^2a^{10}}+\frac{3{c_3}^5\pi'^9{\rho_m}^2\alpha'}{16{c_2}^3M^{15}{M_{\rm pl}}^8\alpha^4\mathcal{H}^2a^8}\\
&+\frac{33{c_3}^5\pi'^9{\rho_m}^2}{32{c_2}^3M^{15}{M_{\rm pl}}^8\alpha^3\mathcal{H}a^8}-\frac{3{c_3}^5\pi'^9{\rho_m}^2f}{16{c_2}^3M^{15}{M_{\rm pl}}^8\alpha^3\mathcal{H}a^8}-\frac{15{c_3}^4\pi'^8{\rho_m}^2}{64{c_2}^2M^{12}{M_{\rm pl}}^8\alpha^3\mathcal{H}^2a^6}\\
&+\frac{3{c_3}^4\pi'^8{\rho_m}^2}{8{c_2}^2M^{12}{M_{\rm pl}}^8\alpha^2\mathcal{H}^2a^6}+\frac{3{c_3}^2\pi'^4\rho_mf^2}{8c_2M^6{M_{\rm pl}}^4\alpha a^4}-\frac{3{c_3}^3\pi'^5{\rho_m}^2\alpha'}{8{c_2}^2M^9{M_{\rm pl}}^6\alpha^3\mathcal{H}^2a^4}\\
&-\frac{33{c_3}^3\pi'^5{\rho_m}^2}{16{c_2}^2M^9{M_{\rm pl}}^6\alpha^2\mathcal{H}a^4}+\frac{3{c_3}^3\pi'^5{\rho_m}^2f}{8{c_2}^2M^9{M_{\rm pl}}^6\alpha^2\mathcal{H}a^4}+\frac{15{c_3}^2\pi'^4{\rho_m}^2}{32c_2M^6{M_{\rm pl}}^6\alpha^2\mathcal{H}^2a^2}\\
&-\frac{3{c_3}^2\pi'^4{\rho_m}^2}{32c_2M^6{M_{\rm pl}}^6\alpha\mathcal{H}^2a^2}-\frac{3{\rho_m}^2a^2}{8{M_{\rm pl}}^4\mathcal{H}^2}
\end{aligned}
\tag{E.4}
$$

$$
\gamma_8(\tau)\equiv\frac{69c_2\pi'^2{\rho_m}^2}{8{M_{\rm pl}}^6\mathcal{H}^4}+\frac{c_2\pi'^2{\rho_m}^2}{8{M_{\rm pl}}^6\alpha^2\mathcal{H}^4}-\frac{19c_2\pi'^2{\rho_m}^2}{4{M_{\rm pl}}^6\alpha\mathcal{H}^4}-\frac{9c_2\alpha\pi'^2{\rho_m}^2}{2{M_{\rm pl}}^6\mathcal{H}^4}
$$

$$
\begin{aligned}
&-\frac{3\rho_m{}^2\mathcal{H}'}{M_{\rm pl}{}^4\mathcal{H}^4}-\frac{3\rho_m{}^2}{4M_{\rm pl}{}^4\mathcal{H}^2}-\frac{21\rho_m{}^2f}{2M_{\rm pl}{}^4\mathcal{H}^2}+\frac{9\rho_m{}^2f^2}{4M_{\rm pl}{}^4\mathcal{H}^2}+\frac{39c_3{}^8\pi'^{18}\rho_m{}^2}{16c_2{}^3M^{24}M_{\rm pl}{}^{14}\alpha^3\mathcal{H}^4a^{16}}\\
&-\frac{3c_3{}^8\pi'^{16}\rho_m{}^3}{8c_2{}^4M^{24}M_{\rm pl}{}^{14}\alpha^4\mathcal{H}^4a^{14}}-\frac{165c_3{}^7\pi'^{15}\rho_m{}^2\alpha'}{32c_2{}^3M^{21}M_{\rm pl}{}^{12}\alpha^4\mathcal{H}^4a^{14}}-\frac{141c_3{}^6\pi'^{14}\rho_m{}^2}{16c_2{}^2M^{18}M_{\rm pl}{}^{12}\alpha^2\mathcal{H}^4a^{12}}\\
&-\frac{1227c_3{}^7\pi'^{15}\rho_m{}^2}{32c_2{}^3M^{21}M_{\rm pl}{}^{12}\alpha^3\mathcal{H}^3a^{14}}+\frac{165c_3{}^7\pi'^{15}\rho_m{}^2f}{32c_2{}^3M^{21}M_{\rm pl}{}^{12}\alpha^3\mathcal{H}^3a^{14}}+\frac{623c_3{}^6\pi'^{14}\rho_m{}^2}{64c_2{}^2M^{18}M_{\rm pl}{}^{12}\alpha^3\mathcal{H}^4a^{12}}\\
&-\frac{3c_3{}^6\pi'^{12}\alpha''\rho_m{}^2}{2c_2{}^3M^{18}M_{\rm pl}{}^{10}\alpha^4\mathcal{H}^4a^{12}}+\frac{3c_3{}^7\pi'^{13}\rho_m{}^3\alpha'}{8c_2{}^4M^{21}M_{\rm pl}{}^{12}\alpha^5\mathcal{H}^4a^{12}}+\frac{183c_3{}^6\pi'^{12}\rho_m{}^2\alpha'^2}{32c_2{}^3M^{18}M_{\rm pl}{}^{10}\alpha^5\mathcal{H}^4a^{12}}\\
&-\frac{105c_3{}^6\pi'^{12}\rho_m{}^2\mathcal{H}'}{16c_2{}^3M^{18}M_{\rm pl}{}^{10}\alpha^3\mathcal{H}^4a^{12}}+\frac{27c_3{}^7\pi'^{13}\rho_m{}^3}{16c_2{}^4M^{21}M_{\rm pl}{}^{12}\alpha^4\mathcal{H}^3a^{12}}+\frac{453c_3{}^6\pi'^{12}\rho_m{}^2\alpha'}{8c_2{}^3M^{18}M_{\rm pl}{}^{10}\alpha^4\mathcal{H}^3a^{12}}\\
&-\frac{3c_3{}^7\pi'^{13}\rho_m{}^3f}{8c_2{}^4M^{21}M_{\rm pl}{}^{12}\alpha^4\mathcal{H}^3a^{12}}-\frac{135c_3{}^6\pi'^{12}\rho_m{}^2\alpha'f}{16c_2{}^3M^{18}M_{\rm pl}{}^{10}\alpha^4\mathcal{H}^3a^{12}}+\frac{5541c_3{}^6\pi'^{12}\rho_m{}^2}{32c_2{}^3M^{18}M_{\rm pl}{}^{10}\alpha^3\mathcal{H}^2a^{12}}\\
&-\frac{465c_3{}^6\pi'^{12}\rho_m{}^2f}{8c_2{}^3M^{18}M_{\rm pl}{}^{10}\alpha^3\mathcal{H}^2a^{12}}+\frac{87c_3{}^6\pi'^{12}\rho_m{}^2f^2}{32c_2{}^3M^{18}M_{\rm pl}{}^{10}\alpha^3\mathcal{H}^2a^{12}}-\frac{c_3{}^6\pi'^{12}\rho_m{}^3}{2c_2{}^3M^{18}M_{\rm pl}{}^{12}\alpha^4\mathcal{H}^4a^{10}}\\
&+\frac{27c_3{}^6\pi'^{12}\rho_m{}^3}{16c_2{}^3M^{18}M_{\rm pl}{}^{12}\alpha^3\mathcal{H}^4a^{10}}-\frac{29c_3{}^5\pi'^{11}\rho_m{}^2\alpha'}{2c_2{}^2M^{15}M_{\rm pl}{}^{10}\alpha^4\mathcal{H}^4a^{10}}+\frac{81c_3{}^5\pi'^{11}\rho_m{}^2\alpha'}{8c_2{}^2M^{15}M_{\rm pl}{}^{10}\alpha^3\mathcal{H}^4a^{10}}\\
&+\frac{3c_3{}^5\pi'^9\alpha''\rho_m{}^2\alpha'}{2c_2{}^3M^{15}M_{\rm pl}{}^8\alpha^5\mathcal{H}^4a^{10}}-\frac{3c_3{}^5\pi'^9\rho_m{}^2\alpha'^3}{c_2{}^3M^{15}M_{\rm pl}{}^8\alpha^6\mathcal{H}^4a^{10}}+\frac{27c_3{}^5\pi'^9\rho_m{}^2\alpha'\mathcal{H}'}{4c_2{}^3M^{15}M_{\rm pl}{}^8\alpha^4\mathcal{H}^4a^{10}}\\
&-\frac{1375c_3{}^5\pi'^{11}\rho_m{}^2}{16c_2{}^2M^{15}M_{\rm pl}{}^{10}\alpha^3\mathcal{H}^3a^{10}}+\frac{1107c_3{}^5\pi'^{11}\rho_m{}^2}{16c_2{}^2M^{15}M_{\rm pl}{}^{10}\alpha^2\mathcal{H}^3a^{10}}+\frac{27c_3{}^5\pi'^9\alpha''\rho_m{}^2}{4c_2{}^3M^{15}M_{\rm pl}{}^8\alpha^4\mathcal{H}^3a^{10}}\\
&+\frac{3c_3{}^6\pi'^{10}\rho_m{}^3\alpha'}{8c_2{}^4M^{18}M_{\rm pl}{}^{10}\alpha^5\mathcal{H}^3a^{10}}-\frac{243c_3{}^5\pi'^9\rho_m{}^2\alpha'^2}{8c_2{}^3M^{15}M_{\rm pl}{}^8\alpha^5\mathcal{H}^3a^{10}}+\frac{30c_3{}^5\pi'^9\rho_m{}^2\mathcal{H}'}{c_2{}^3M^{15}M_{\rm pl}{}^8\alpha^3\mathcal{H}^3a^{10}}\\
&+\frac{29c_3{}^5\pi'^{11}\rho_m{}^2f}{2c_2{}^2M^{15}M_{\rm pl}{}^{10}\alpha^3\mathcal{H}^3a^{10}}-\frac{261c_3{}^5\pi'^{11}\rho_m{}^2f}{16c_2{}^2M^{15}M_{\rm pl}{}^{10}\alpha^2\mathcal{H}^3a^{10}}-\frac{3c_3{}^5\pi'^9\alpha''\rho_m{}^2f}{2c_2{}^3M^{15}M_{\rm pl}{}^8\alpha^4\mathcal{H}^3a^{10}}\\
&+\frac{6c_3{}^5\pi'^9\rho_m{}^2\alpha'^2f}{c_2{}^3M^{15}M_{\rm pl}{}^8\alpha^5\mathcal{H}^3a^{10}}-\frac{27c_3{}^5\pi'^9\rho_m{}^2\mathcal{H}'f}{4c_2{}^3M^{15}M_{\rm pl}{}^8\alpha^3\mathcal{H}^3a^{10}}+\frac{15c_3{}^6\pi'^{10}\rho_m{}^3}{8c_2{}^4M^{18}M_{\rm pl}{}^{10}\alpha^4\mathcal{H}^2a^{10}}\\
&-\frac{114c_3{}^5\pi'^9\rho_m{}^2\alpha'}{c_2{}^3M^{15}M_{\rm pl}{}^8\alpha^4\mathcal{H}^2a^{10}}-\frac{3c_3{}^6\pi'^{10}\rho_m{}^3f}{8c_2{}^4M^{18}M_{\rm pl}{}^{10}\alpha^4\mathcal{H}^2a^{10}}+\frac{195c_3{}^5\pi'^9\rho_m{}^2\alpha'f}{4c_2{}^3M^{15}M_{\rm pl}{}^8\alpha^4\mathcal{H}^2a^{10}}\\
&-\frac{1335c_3{}^5\pi'^9\rho_m{}^2}{8c_2{}^3M^{15}M_{\rm pl}{}^8\alpha^3\mathcal{H}a^{10}}+\frac{483c_3{}^5\pi'^9\rho_m{}^2f}{4c_2{}^3M^{15}M_{\rm pl}{}^8\alpha^3\mathcal{H}a^{10}}-\frac{147c_3{}^5\pi'^9\rho_m{}^2f^2}{8c_2{}^3M^{15}M_{\rm pl}{}^8\alpha^3\mathcal{H}a^{10}}\\
&-\frac{375c_3{}^4\pi'^6\rho_m{}^2}{2c_2{}^3M^{12}M_{\rm pl}{}^6\alpha^3a^8}+\frac{120c_3{}^4\pi'^6\rho_m{}^2f}{c_2{}^3M^{12}M_{\rm pl}{}^6\alpha^3a^8}-\frac{33c_3{}^4\pi'^6\rho_m{}^2f^2}{2c_2{}^3M^{12}M_{\rm pl}{}^6\alpha^3a^8}\\
&+\frac{175c_3{}^4\pi'^{10}\rho_m{}^2}{16c_2M^{12}M_{\rm pl}{}^{10}\alpha^3\mathcal{H}^4a^8}-\frac{601c_3{}^4\pi'^{10}\rho_m{}^2}{32c_2M^{12}M_{\rm pl}{}^{10}\alpha^2\mathcal{H}^4a^8}+\frac{39c_3{}^4\pi'^{10}\rho_m{}^2}{8c_2M^{12}M_{\rm pl}{}^{10}\alpha\mathcal{H}^4a^8}\\
&-\frac{2c_3{}^4\pi'^8\alpha''\rho_m{}^2}{c_2{}^2M^{12}M_{\rm pl}{}^8\alpha^4\mathcal{H}^4a^8}+\frac{9c_3{}^4\pi'^8\alpha''\rho_m{}^2}{4c_2{}^2M^{12}M_{\rm pl}{}^8\alpha^3\mathcal{H}^4a^8}-\frac{3c_3{}^5\pi'^9\rho_m{}^2\alpha'f^2}{c_2{}^3M^{15}M_{\rm pl}{}^8\alpha^4\mathcal{H}^2a^{10}}
\end{aligned}
$$

$$
\begin{aligned}
&- \frac{9{c_3}^5\pi'^9{\rho_m}^3\alpha'}{8{c_2}^3M^{15}{M_{\rm pl}}^{10}\alpha^4\mathcal{H}^4a^8} + \frac{135{c_3}^4\pi'^8{\rho_m}^2\alpha'^2}{16{c_2}^2M^{12}{M_{\rm pl}}^8\alpha^5\mathcal{H}^4a^8} - \frac{87{c_3}^4\pi'^8{\rho_m}^2\alpha'^2}{16{c_2}^2M^{12}{M_{\rm pl}}^8\alpha^4\mathcal{H}^4a^8} \\
&- \frac{37{c_3}^4\pi'^8{\rho_m}^2\mathcal{H}'}{4{c_2}^2M^{12}{M_{\rm pl}}^8\alpha^3\mathcal{H}^4a^8} + \frac{3{c_3}^4\pi'^6{\rho_m}^2\alpha'^2\mathcal{H}'}{4{c_2}^3M^{12}{M_{\rm pl}}^6\alpha^5\mathcal{H}^4a^8} - \frac{3{c_3}^5\pi'^9{\rho_m}^3}{8{c_2}^3M^{15}{M_{\rm pl}}^{10}\alpha^4\mathcal{H}^3a^8} \\
&- \frac{15{c_3}^5\pi'^9{\rho_m}^3}{4{c_2}^3M^{15}{M_{\rm pl}}^{10}\alpha^3\mathcal{H}^3a^8} + \frac{479{c_3}^4\pi'^8{\rho_m}^2\alpha'}{8{c_2}^2M^{12}{M_{\rm pl}}^8\alpha^4\mathcal{H}^3a^8} - \frac{33{c_3}^4\pi'^8{\rho_m}^2\alpha'}{{c_2}^2M^{12}{M_{\rm pl}}^8\alpha^3\mathcal{H}^3a^8} \\
&+ \frac{3{c_3}^4\pi'^6\alpha''{\rho_m}^2\alpha'}{2{c_2}^3M^{12}{M_{\rm pl}}^6\alpha^5\mathcal{H}^3a^8} - \frac{3{c_3}^4\pi'^6{\rho_m}^2\alpha'^3}{{c_2}^3M^{12}{M_{\rm pl}}^6\alpha^6\mathcal{H}^3a^8} + \frac{15{c_3}^4\pi'^6{\rho_m}^2\alpha'\mathcal{H}'}{{c_2}^3M^{12}{M_{\rm pl}}^6\alpha^4\mathcal{H}^3a^8} \\
&+ \frac{3{c_3}^5\pi'^9{\rho_m}^3f}{2{c_2}^3M^{15}{M_{\rm pl}}^{10}\alpha^3\mathcal{H}^3a^8} + \frac{51{c_3}^4\pi'^8{\rho_m}^2\alpha'f}{4{c_2}^2M^{12}{M_{\rm pl}}^8\alpha^3\mathcal{H}^3a^8} - \frac{3{c_3}^4\pi'^6{\rho_m}^2\alpha'\mathcal{H}'f}{2{c_2}^3M^{12}{M_{\rm pl}}^6\alpha^4\mathcal{H}^3a^8} \\
&+ \frac{2095{c_3}^4\pi'^8{\rho_m}^2}{16{c_2}^2M^{12}{M_{\rm pl}}^8\alpha^3\mathcal{H}^2a^8} - \frac{189{c_3}^4\pi'^8{\rho_m}^2}{16{c_2}^2M^{12}{M_{\rm pl}}^8\alpha^2\mathcal{H}^2a^8} + \frac{15{c_3}^4\pi'^6\alpha''{\rho_m}^2}{2{c_2}^3M^{12}{M_{\rm pl}}^6\alpha^4\mathcal{H}^2a^8} \\
&- \frac{30{c_3}^4\pi'^6{\rho_m}^2\alpha'^2}{{c_2}^3M^{12}{M_{\rm pl}}^6\alpha^5\mathcal{H}^2a^8} + \frac{225{c_3}^4\pi'^6{\rho_m}^2\mathcal{H}'}{4{c_2}^3M^{12}{M_{\rm pl}}^6\alpha^3\mathcal{H}^2a^8} - \frac{103{c_3}^4\pi'^8{\rho_m}^2\alpha'f}{8{c_2}^2M^{12}{M_{\rm pl}}^8\alpha^4\mathcal{H}^3a^8} \\
&- \frac{495{c_3}^4\pi'^8{\rho_m}^2f}{8{c_2}^2M^{12}{M_{\rm pl}}^8\alpha^3\mathcal{H}^2a^8} + \frac{615{c_3}^4\pi'^8{\rho_m}^2f}{8{c_2}^2M^{12}{M_{\rm pl}}^8\alpha^2\mathcal{H}^2a^8} - \frac{3{c_3}^4\pi'^6\alpha''{\rho_m}^2f}{2{c_2}^3M^{12}{M_{\rm pl}}^6\alpha^4\mathcal{H}^2a^8} \\
&+ \frac{6{c_3}^4\pi'^6{\rho_m}^2\alpha'^2f}{{c_2}^3M^{12}{M_{\rm pl}}^6\alpha^5\mathcal{H}^2a^8} - \frac{15{c_3}^4\pi'^6{\rho_m}^2\mathcal{H}'f}{{c_2}^3M^{12}{M_{\rm pl}}^6\alpha^3\mathcal{H}^2a^8} + \frac{71{c_3}^4\pi'^8{\rho_m}^2f^2}{16{c_2}^2M^{12}{M_{\rm pl}}^8\alpha^3\mathcal{H}^2a^8} \\
&- \frac{117{c_3}^4\pi'^8{\rho_m}^2f^2}{16{c_2}^2M^{12}{M_{\rm pl}}^8\alpha^2\mathcal{H}^2a^8} + \frac{3{c_3}^4\pi'^6{\rho_m}^2\mathcal{H}'f^2}{4{c_2}^3M^{12}{M_{\rm pl}}^6\alpha^3\mathcal{H}^2a^8} - \frac{225{c_3}^4\pi'^6{\rho_m}^2\alpha'}{2{c_2}^3M^{12}{M_{\rm pl}}^6\alpha^4\mathcal{H}a^8} \\
&+ \frac{5{c_3}^4\pi'^8{\rho_m}^3}{4{c_2}^2M^{12}{M_{\rm pl}}^{10}\alpha^3\mathcal{H}^4a^6} - \frac{15{c_3}^4\pi'^8{\rho_m}^3}{8{c_2}^2M^{12}{M_{\rm pl}}^{10}\alpha^2\mathcal{H}^4a^6} - \frac{63{c_3}^3\pi'^7{\rho_m}^2\alpha'}{8c_2M^9{M_{\rm pl}}^8\alpha^4\mathcal{H}^4a^6} \\
&+ \frac{93{c_3}^4\pi'^6{\rho_m}^2\alpha'f}{2{c_2}^3M^{12}{M_{\rm pl}}^6\alpha^4\mathcal{H}a^8} + \frac{77{c_3}^3\pi'^7{\rho_m}^2\alpha'}{8c_2M^9{M_{\rm pl}}^8\alpha^3\mathcal{H}^4a^6} + \frac{15{c_3}^3\pi'^7{\rho_m}^2\alpha'}{8c_2M^9{M_{\rm pl}}^8\alpha^2\mathcal{H}^4a^6} \\
&- \frac{3{c_3}^4\pi'^6{\rho_m}^2\alpha'f^2}{{c_2}^3M^{12}{M_{\rm pl}}^6\alpha^4\mathcal{H}a^8} - \frac{3{c_3}^3\pi'^5{\rho_m}^2\alpha'\mathcal{H}'}{2{c_2}^2M^9{M_{\rm pl}}^6\alpha^4\mathcal{H}^4a^6} + \frac{6{c_3}^3\pi'^5{\rho_m}^2\alpha'\mathcal{H}'}{{c_2}^2M^9{M_{\rm pl}}^6\alpha^3\mathcal{H}^4a^6} \\
&- \frac{261{c_3}^3\pi'^7{\rho_m}^2}{8c_2M^9{M_{\rm pl}}^8\alpha^3\mathcal{H}^3a^6} + \frac{22{c_3}^3\pi'^7{\rho_m}^2}{c_2M^9{M_{\rm pl}}^8\alpha^2\mathcal{H}^3a^6} + \frac{291{c_3}^3\pi'^7{\rho_m}^2}{8c_2M^9{M_{\rm pl}}^8\alpha\mathcal{H}^3a^6} \\
&- \frac{3{c_3}^3\pi'^5\alpha''{\rho_m}^2}{2{c_2}^2M^9{M_{\rm pl}}^6\alpha^4\mathcal{H}^3a^6} - \frac{3{c_3}^4\pi'^6{\rho_m}^3\alpha'}{4{c_2}^3M^{12}{M_{\rm pl}}^8\alpha^4\mathcal{H}^3a^6} + \frac{6{c_3}^3\pi'^5{\rho_m}^2\alpha'^2}{{c_2}^2M^9{M_{\rm pl}}^6\alpha^5\mathcal{H}^3a^6} \\
&- \frac{45{c_3}^3\pi'^5{\rho_m}^2\alpha'^2}{4{c_2}^2M^9{M_{\rm pl}}^6\alpha^4\mathcal{H}^3a^6} - \frac{15{c_3}^3\pi'^5{\rho_m}^2\mathcal{H}'}{{c_2}^2M^9{M_{\rm pl}}^6\alpha^3\mathcal{H}^3a^6} + \frac{6{c_3}^3\pi'^5\alpha''{\rho_m}^2}{{c_2}^2M^9{M_{\rm pl}}^6\alpha^3\mathcal{H}^3a^6} \\
&+ \frac{60{c_3}^3\pi'^5{\rho_m}^2\mathcal{H}'}{{c_2}^2M^9{M_{\rm pl}}^6\alpha^2\mathcal{H}^3a^6} + \frac{63{c_3}^3\pi'^7{\rho_m}^2f}{8c_2M^9{M_{\rm pl}}^8\alpha^3\mathcal{H}^3a^6} - \frac{83{c_3}^3\pi'^7{\rho_m}^2f}{4c_2M^9{M_{\rm pl}}^8\alpha^2\mathcal{H}^3a^6} \\
&+ \frac{51{c_3}^3\pi'^7{\rho_m}^2f}{8c_2M^9{M_{\rm pl}}^8\alpha\mathcal{H}^3a^6} + \frac{3{c_3}^3\pi'^5\alpha''{\rho_m}^2f}{2{c_2}^2M^9{M_{\rm pl}}^6\alpha^3\mathcal{H}^3a^6} - \frac{3{c_3}^3\pi'^5{\rho_m}^2\alpha'^2f}{{c_2}^2M^9{M_{\rm pl}}^6\alpha^4\mathcal{H}^3a^6}
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{3{c_3}^3{\pi'}^5{\rho_m}^2\mathcal{H}'f}{2{c_2}^2M^9{M_{\rm pl}}^6\alpha^3\mathcal{H}^3a^6}+\frac{3{c_3}^3{\pi'}^5{\rho_m}^2\mathcal{H}'f}{2{c_2}^2M^9{M_{\rm pl}}^6\alpha^2\mathcal{H}^3a^6}-\frac{15{c_3}^4{\pi'}^6{\rho_m}^3}{4{c_2}^3M^{12}{M_{\rm pl}}^8\alpha^3\mathcal{H}^2a^6}\\
&+\frac{81{c_3}^3{\pi'}^5{\rho_m}^2\alpha'}{2{c_2}^2M^9{M_{\rm pl}}^6\alpha^4\mathcal{H}^2a^6}-\frac{135{c_3}^3{\pi'}^5{\rho_m}^2\alpha'}{2{c_2}^2M^9{M_{\rm pl}}^6\alpha^3\mathcal{H}^2a^6}+\frac{3{c_3}^4{\pi'}^6{\rho_m}^3f}{4{c_2}^3M^{12}{M_{\rm pl}}^8\alpha^3\mathcal{H}^2a^6}\\
&+\frac{3{c_3}^3{\pi'}^5{\rho_m}^2\alpha'f^2}{{c_2}^2M^9{M_{\rm pl}}^6\alpha^3\mathcal{H}^2a^6}+\frac{90{c_3}^3{\pi'}^5{\rho_m}^2}{{c_2}^2M^9{M_{\rm pl}}^6\alpha^3\mathcal{H}a^6}-\frac{825{c_3}^3{\pi'}^5{\rho_m}^2}{4{c_2}^2M^9{M_{\rm pl}}^6\alpha^2\mathcal{H}a^6}\\
&-\frac{42{c_3}^3{\pi'}^5{\rho_m}^2f}{{c_2}^2M^9{M_{\rm pl}}^6\alpha^3\mathcal{H}a^6}+\frac{117{c_3}^3{\pi'}^5{\rho_m}^2f}{2{c_2}^2M^9{M_{\rm pl}}^6\alpha^2\mathcal{H}a^6}+\frac{3{c_3}^3{\pi'}^5{\rho_m}^2f^2}{{c_2}^2M^9{M_{\rm pl}}^6\alpha^3\mathcal{H}a^6}\\
&+\frac{51{c_3}^3{\pi'}^5{\rho_m}^2f^2}{4{c_2}^2M^9{M_{\rm pl}}^6\alpha^2\mathcal{H}a^6}+\frac{33{c_3}^2{\pi'}^6{\rho_m}^2}{4M^6{M_{\rm pl}}^8\mathcal{H}^4a^4}+\frac{43{c_3}^2{\pi'}^6{\rho_m}^2}{16M^6{M_{\rm pl}}^8\alpha^3\mathcal{H}^4a^4}\\
&-\frac{9{c_3}^2{\pi'}^6{\rho_m}^2}{2M^6{M_{\rm pl}}^8\alpha^2\mathcal{H}^4a^4}-\frac{91{c_3}^2{\pi'}^6{\rho_m}^2}{16M^6{M_{\rm pl}}^8\alpha\mathcal{H}^4a^4}-\frac{{c_3}^2{\pi'}^4\alpha''{\rho_m}^2}{2c_2M^6{M_{\rm pl}}^6\alpha^3\mathcal{H}^4a^4}\\
&+\frac{3{c_3}^2{\pi'}^4\alpha''{\rho_m}^2}{2c_2M^6{M_{\rm pl}}^6\alpha^2\mathcal{H}^4a^4}+\frac{3{c_3}^3{\pi'}^5{\rho_m}^3\alpha'}{4{c_2}^2M^9{M_{\rm pl}}^8\alpha^3\mathcal{H}^4a^4}-\frac{9{c_3}^3{\pi'}^5{\rho_m}^2\alpha'f}{{c_2}^2M^9{M_{\rm pl}}^6\alpha^4\mathcal{H}^2a^6}\\
&+\frac{7{c_3}^2{\pi'}^4{\rho_m}^2{\alpha'}^2}{8c_2M^6{M_{\rm pl}}^6\alpha^4\mathcal{H}^4a^4}-\frac{3{c_3}^2{\pi'}^4{\rho_m}^2{\alpha'}^2}{c_2M^6{M_{\rm pl}}^6\alpha^3\mathcal{H}^4a^4}-\frac{21c_3{\pi'}^3{\rho_m}^2f}{2M^3{M_{\rm pl}}^6\alpha\mathcal{H}^3a^2}\\
&+\frac{3{c_3}^2{\pi'}^4{\rho_m}^2\mathcal{H}'}{4c_2M^6{M_{\rm pl}}^6\alpha^3\mathcal{H}^4a^4}-\frac{17{c_3}^2{\pi'}^4{\rho_m}^2\mathcal{H}'}{2c_2M^6{M_{\rm pl}}^6\alpha^2\mathcal{H}^4a^4}+\frac{c_3{\pi'}^3{\rho_m}^2f}{4M^3{M_{\rm pl}}^6\alpha^2\mathcal{H}^3a^2}\\
&+\frac{111{c_3}^2{\pi'}^4{\rho_m}^2\mathcal{H}'}{4c_2M^6{M_{\rm pl}}^6\alpha\mathcal{H}^4a^4}+\frac{3{c_3}^3{\pi'}^5{\rho_m}^3}{4{c_2}^2M^9{M_{\rm pl}}^8\alpha^3\mathcal{H}^3a^4}+\frac{3{c_3}^3{\pi'}^5{\rho_m}^3}{4{c_2}^2M^9{M_{\rm pl}}^8\alpha^2\mathcal{H}^3a^4}\\
&-\frac{15{c_3}^2{\pi'}^4{\rho_m}^2\alpha'}{4c_2M^6{M_{\rm pl}}^6\alpha^4\mathcal{H}^3a^4}+\frac{33{c_3}^2{\pi'}^4{\rho_m}^2\alpha'}{2c_2M^6{M_{\rm pl}}^6\alpha^3\mathcal{H}^3a^4}+\frac{45c_3{\pi'}^3{\rho_m}^2f}{4M^3{M_{\rm pl}}^6\mathcal{H}^3a^2}\\
&-\frac{51{c_3}^2{\pi'}^4{\rho_m}^2\alpha'}{2c_2M^6{M_{\rm pl}}^6\alpha^2\mathcal{H}^3a^4}-\frac{3{c_3}^3{\pi'}^5{\rho_m}^3f}{2{c_2}^2M^9{M_{\rm pl}}^8\alpha^2\mathcal{H}^3a^4}+\frac{2{c_3}^2{\pi'}^4{\rho_m}^2\alpha'f}{c_2M^6{M_{\rm pl}}^6\alpha^3\mathcal{H}^3a^4}\\
&+\frac{21{c_3}^2{\pi'}^4{\rho_m}^2\alpha'f}{4c_2M^6{M_{\rm pl}}^6\alpha^2\mathcal{H}^3a^4}-\frac{57{c_3}^2{\pi'}^4{\rho_m}^2}{4c_2M^6{M_{\rm pl}}^6\alpha^3\mathcal{H}^2a^4}+\frac{561{c_3}^2{\pi'}^4{\rho_m}^2}{8c_2M^6{M_{\rm pl}}^6\alpha^2\mathcal{H}^2a^4}\\
&-\frac{1317{c_3}^2{\pi'}^4{\rho_m}^2}{8c_2M^6{M_{\rm pl}}^6\alpha\mathcal{H}^2a^4}+\frac{15{c_3}^2{\pi'}^4{\rho_m}^2f}{4c_2M^6{M_{\rm pl}}^6\alpha^3\mathcal{H}^2a^4}-\frac{49{c_3}^2{\pi'}^4{\rho_m}^2f}{4c_2M^6{M_{\rm pl}}^6\alpha^2\mathcal{H}^2a^4}\\
&+\frac{60{c_3}^2{\pi'}^4{\rho_m}^2f}{c_2M^6{M_{\rm pl}}^6\alpha\mathcal{H}^2a^4}-\frac{23{c_3}^2{\pi'}^4{\rho_m}^2f^2}{8c_2M^6{M_{\rm pl}}^6\alpha^2\mathcal{H}^2a^4}+\frac{21{c_3}^2{\pi'}^4{\rho_m}^2f^2}{8c_2M^6{M_{\rm pl}}^6\alpha\mathcal{H}^2a^4}\\
&-\frac{{c_3}^2{\pi'}^4{\rho_m}^3}{2c_2M^6{M_{\rm pl}}^8\alpha^2\mathcal{H}^4a^2}-\frac{3{c_3}^2{\pi'}^4{\rho_m}^3}{4c_2M^6{M_{\rm pl}}^8\alpha\mathcal{H}^4a^2}-\frac{3c_3{\pi'}^3{\rho_m}^2\alpha'}{4M^3{M_{\rm pl}}^6\alpha^3\mathcal{H}^4a^2}\\
&+\frac{23c_3{\pi'}^3{\rho_m}^2\alpha'}{4M^3{M_{\rm pl}}^6\alpha^2\mathcal{H}^4a^2}-\frac{3c_3{\pi'}^3{\rho_m}^2\alpha'}{M^3{M_{\rm pl}}^6\alpha\mathcal{H}^4a^2}-\frac{171c_3{\pi'}^3{\rho_m}^2}{4M^3{M_{\rm pl}}^6\mathcal{H}^3a^2}+\frac{3{\rho_m}^3a^2}{2{M_{\rm pl}}^6\mathcal{H}^4}\\
&+\frac{3c_3{\pi'}^3{\rho_m}^2}{4M^3{M_{\rm pl}}^6\alpha^3\mathcal{H}^3a^2}-\frac{27c_3{\pi'}^3{\rho_m}^2}{4M^3{M_{\rm pl}}^6\alpha^2\mathcal{H}^3a^2}+\frac{215c_3{\pi'}^3{\rho_m}^2}{4M^3{M_{\rm pl}}^6\alpha\mathcal{H}^3a^2}\,.
\end{aligned}
\tag{E.5}
$$

REFERENCES

[1] S. Perlmutter et al. Measurements of Omega and Lambda from 42 high redshift supernovae. *Astrophys.J.*, 517:565–586, 1999, astro-ph/9812133.

[2] Adam G. Riess et al. Observational evidence from supernovae for an accelerating universe and a cosmological constant. *Astron.J.*, 116:1009–1038, 1998, astro-ph/9805201.

[3] Adam G. Riess, Robert P. Kirshner, Brian P. Schmidt, Saurabh Jha, Peter Challis, et al. BV RI light curves for 22 type Ia supernovae. *Astron.J.*, 117:707–724, 1999, astro-ph/9810291.

[4] D.N. Spergel et al. First year Wilkinson Microwave Anisotropy Probe (WMAP) observations: Determination of cosmological parameters. *Astrophys.J.Suppl.*, 148:175–194, 2003, astro-ph/0302209.

[5] E. Komatsu et al. Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation. *Astrophys.J.Suppl.*, 192:18, 2011, arXiv:1001.4538.

[6] G. Hinshaw, D. Larson, E. Komatsu, D.N. Spergel, C.L. Bennett, et al. Nine-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Parameter Results. 2012, astro-ph/1212.5226.

[7] Daniel J. Eisenstein et al. Detection of the baryon acoustic peak in the large-scale correlation function of SDSS luminous red galaxies. *Astrophys.J.*, 633:560–574, 2005, astro-ph/0501171.

[8] Will J. Percival et al. Baryon Acoustic Oscillations in the Sloan Digital Sky Survey Data Release 7 Galaxy Sample. *Mon.Not.Roy.Astron.Soc.*, 401:2148–2168, 2010, 0907.1660.

[9] Steven Weinberg. The Cosmological constant problems. pages 18–26, 2000, astro-ph/0005265.

[10] R.R. Caldwell, Rahul Dave, and Paul J. Steinhardt. Cosmological imprint of an energy component with general equation of state. *Phys.Rev.Lett.*, 80:1582–1585, 1998, astro-ph/9708069.

[11] Ivaylo Zlatev, Li-Min Wang, and Paul J. Steinhardt. Quintessence, cosmic coincidence, and the cosmological constant. *Phys.Rev.Lett.*, 82:896–899, 1999, astro-ph/9807002.

[12] Antonio De Felice and Shinji Tsujikawa. f(R) theories. *Living Rev.Rel.*, 13:3, 2010, 1002.4928.

[13] Kurt Hinterbichler. Theoretical Aspects of Massive Gravity. *Rev.Mod.Phys.*, 84:671–710, 2012, 1105.3735.

[14] C. Brans and R.H. Dicke. Mach's principle and a relativistic theory of gravitation. *Phys.Rev.*, 124:925–935, 1961.

[15] Roy Maartens. Brane world cosmology. *AIP Conf.Proc.*, 736:21–34, 2005.

[16] Shinji Tsujikawa. Dark energy: investigation and modeling. 2010, 1004.1493.

[17] Alberto Nicolis, Riccardo Rattazzi, and Enrico Trincherini. The Galileon as a local modification of gravity. *Phys.Rev.*, D79:064036, 2009, 0811.2197.

[18] G.R. Dvali, Gregory Gabadadze, and Massimo Porrati. 4-D gravity on a brane in 5-D Minkowski space. *Phys.Lett.*, B485:208–214, 2000, hep-th/0005016.

[19] M. Ostrogradski. Member academic science st. *Petersbourg*, 1:385, 18502.

[20] C. Deffayet, Gilles Esposito-Farese, and A. Vikman. Covariant Galileon. *Phys.Rev.*, D79:084003, 2009, 0901.1314.

[21] C. Deffayet, S. Deser, and G. Esposito-Farese. Generalized Galileons: All scalar models whose curved background extensions maintain second-order field equations and stress-tensors. *Phys.Rev.*, D80:064015, 2009, 0906.1967.

[22] A.I. Vainshtein. To the problem of nonvanishing gravitation mass. *Physics Letters B*, 39(3):393–394, 1972.

[23] Nemanja Kaloper, Antonio Padilla, and Norihiro Tanahashi. Galileon Hairs of Dyson Spheres, Vainshtein's Coiffure and Hirsute Bubbles. *JHEP*, 1110:148, 2011, 1106.4827.

[24] Rampei Kimura, Tsutomu Kobayashi, and Kazuhiro Yamamoto. Vainshtein screening in a cosmological background in the most general second-order scalar-tensor theory. *Phys.Rev.*, D85:024023, 2012, 1111.6749.

[25] Claudia de Rham and Andrew J. Tolley. DBI and the Galileon reunited. *JCAP*, 1005:015, 2010, 1003.5917.

[26] C. Deffayet, S. Deser, and G. Esposito-Farese. Arbitrary p-form Galileons. *Phys.Rev.*, D82:061501, 2010, 1007.5278.

[27] Antonio Padilla, Paul M. Saffin, and Shuang-Yong Zhou. Bi-galileon theory I: Motivation and formulation. *JHEP*, 1012:031, 2010, 1007.5424.

[28] Karel Van Acoleyen and Jos Van Doorsselaere. Galileons from Lovelock actions. *Phys.Rev.*, D83:084025, 2011, 1102.0487.

[29] Justin Khoury, Jean-Luc Lehners, and Burt A. Ovrut. Supersymmetric Galileons. *Phys.Rev.*, D84:043521, 2011, 1103.0003.

[30] C. Deffayet, Xian Gao, D.A. Steer, and G. Zahariade. From k-essence to generalised Galileons. *Phys.Rev.*, D84:064039, 2011, 1103.3260.

[31] Clare Burrage, Claudia de Rham, and Lavinia Heisenberg. de Sitter Galileon. *JCAP*, 1105:025, 2011, 1104.0155.

[32] Zhi-Guo Liu, Jun Zhang, and Yun-Song Piao. A Galileon Design of Slow Expansion. *Phys.Rev.*, D84:063508, 2011, 1105.5713.

[33] Claudia de Rham and Lavinia Heisenberg. Cosmology of the Galileon from Massive Gravity. *Phys.Rev.*, D84:043503, 2011, 1106.3312.

[34] Cristiano Germani, Luca Martucci, and Parvin Moyassari. Introducing the Slotheon: a slow Galileon scalar field in curved space-time. *Phys.Rev.*, D85:103501, 2012, 1108.1406.

[35] Claudia de Rham. Galileons in the Sky. *Comptes Rendus Physique*, 13:666–681, 2012, 1204.5492.

[36] Taotao Qiu, Jarah Evslin, Yi-Fu Cai, Mingzhe Li, and Xinmin Zhang. Bouncing Galileon Cosmologies. *JCAP*, 1110:036, 2011, 1108.0593.

[37] Xian Gao. Conserved cosmological perturbation in Galileon models. *JCAP*, 1110:021, 2011, 1106.0292.

[38] Mark Trodden and Kurt Hinterbichler. Generalizing Galileons. *Class.Quant.Grav.*, 28:204003, 2011, 1104.2088.

[39] E. Babichev. Galileon accretion. *Phys.Rev.*, D83:024008, 2011, 1009.2921.

[40] Damien A. Easson, Ignacy Sawicki, and Alexander Vikman. G-Bounce. *JCAP*, 1111:021, 2011, 1109.1047.

[41] Nathan Chow and Justin Khoury. Galileon Cosmology. *Phys.Rev.*, D80:024037, 2009, 0905.1325.

[42] Fabio P Silva and Kazuya Koyama. Self-Accelerating Universe in Galileon Cosmology. *Phys.Rev.*, D80:121301, 2009, 0909.4538.

[43] Tsutomu Kobayashi, Hiroyuki Tashiro, and Daichi Suzuki. Evolution of linear cosmological perturbations and its observational implications in Galileon-type modified gravity. *Phys.Rev.*, D81:063513, 2010, 0912.4641.

[44] Tsutomu Kobayashi. Cosmic expansion and growth histories in Galileon scalar-tensor models of dark energy. *Phys.Rev.*, D81:103533, 2010, 1003.3281.

[45] Radouane Gannouji and M. Sami. Galileon gravity and its relevance to late time cosmic acceleration. *Phys.Rev.*, D82:024011, 2010, 1004.2808.

[46] Antonio De Felice and Shinji Tsujikawa. Cosmology of a covariant Galileon field. *Phys.Rev.Lett.*, 105:111301, 2010, 1007.2700.

[47] Amna Ali, Radouane Gannouji, and M. Sami. Modified gravity a la Galileon: Late time cosmic acceleration and observational constraints. *Phys.Rev.*, D82:103015, 2010, 1008.1588.

[48] Antonio De Felice and Shinji Tsujikawa. Generalized Galileon cosmology. *Phys.Rev.*, D84:124029, 2011, 1008.4236.

[49] Antonio De Felice, Ryotaro Kase, and Shinji Tsujikawa. Matter perturbations in Galileon cosmology. *Phys.Rev.*, D83:043515, 2011, 1011.6132.

[50] Stephen Appleby and Eric V. Linder. The Paths of Gravity in Galileon Cosmology. *JCAP*, 1203:043, 2012, 1112.1981.

[51] Emilio Bellini, Nicola Bartolo, and Sabino Matarrese. Spherical Collapse in covariant Galileon theory. *JCAP*, 1206:019, 2012, 1202.2712.

[52] Alexandre Barreira, Baojiu Li, Carlton M. Baugh, and Silvia Pascoli. Linear perturbations in Galileon gravity models. *Phys.Rev.*, D86:124016, 2012, 1208.0600.

[53] Thomas L. Curtright and David B. Fairlie. Geons of Galileons. *Phys.Lett.*, B716:356–360, 2012, 1206.3616.

[54] Genly Leon and Emmanuel N. Saridakis. Dynamical analysis of generalized Galileon cosmology. 2012, 1211.3088.

[55] Eugeny Babichev and Gilles Esposito-Farese. Time-dependent spherically symmetric covariant Galileons. 2012, 1212.1394.

[56] Nicola Bartolo, Emilio Bellini, Daniele Bertacca, and Sabino Matarrese. Matter bispectrum in cubic Galileon cosmologies. 2013, 1301.4831.

[57] Garrett Goon, Kurt Hinterbichler, and Mark Trodden. Galileons on Cosmological Backgrounds. *JCAP*, 1112:004, 2011, 1109.3450.

[58] Savvas Nesseris, Antonio De Felice, and Shinji Tsujikawa. Observational constraints on Galileon cosmology. *Phys.Rev.*, D82:124054, 2010, 1010.0407.

[59] Stephen A. Appleby and Eric V. Linder. Trial of Galileon gravity by cosmological expansion and growth observations. *JCAP*, 1208:026, 2012, 1204.4314.

[60] Hiroyuki Okada, Tomonori Totani, and Shinji Tsujikawa. Constraints on f(R) theory and Galileons from the latest data of galaxy redshift surveys. 2012, 1208.4681.

[61] Paolo Creminelli, Alberto Nicolis, and Enrico Trincherini. Galilean Genesis: An Alternative to inflation. *JCAP*, 1011:021, 2010, 1007.0027.

[62] Tsutomu Kobayashi, Masahide Yamaguchi, and Jun'ichi Yokoyama. G-inflation: Inflation driven by the Galileon field. *Phys.Rev.Lett.*, 105:231302, 2010, 1008.0603.

[63] Shuntaro Mizuno and Kazuya Koyama. Primordial non-Gaussianity from the DBI Galileons. *Phys.Rev.*, D82:103518, 2010, 1009.0677.

[64] Clare Burrage, Claudia de Rham, David Seery, and Andrew J. Tolley. Galileon inflation. *JCAP*, 1101:014, 2011, 1009.2497.

[65] Xian Gao and Daniele A. Steer. Inflation and primordial non-Gaussianities of 'generalized Galileons'. *JCAP*, 1112:019, 2011, 1107.2642.

[66] Yi Wang and Robert Brandenberger. Scale-Invariant Fluctuations from Galilean Genesis. *JCAP*, 1210:021, 2012, 1206.4309.

[67] Junko Ohashi and Shinji Tsujikawa. Potential-driven Galileon inflation. *JCAP*, 1210:035, 2012, 1207.4879.

[68] Paolo Creminelli, Kurt Hinterbichler, Justin Khoury, Alberto Nicolis, and Enrico Trincherini. Subluminal Galilean Genesis. 2012, 1209.3768.

[69] N. Bartolo, E. Komatsu, Sabino Matarrese, and A. Riotto. Non-Gaussianity from inflation: Theory and observations. *Phys.Rept.*, 402:103–266, 2004, astro-ph/0406398.

[70] Nicola Bartolo, Sabino Matarrese, and Antonio Riotto. Signatures of primordial non-Gaussianity in the large-scale structure of the Universe. *JCAP*, 0510:010, 2005, astro-ph/0501614.

[71] Emiliano Sefusatti, Martin Crocce, Sebastian Pueblas, and Roman Scoccimarro. Cosmology and the Bispectrum. *Phys.Rev.*, D74:023522, 2006, astro-ph/0604505.

[72] Nicola Bartolo, J.P. Beltran Almeida, Sabino Matarrese, Massimo Pietroni, and Antonio Riotto. Signatures of Primordial non-Gaussianities in the Matter Power-Spectrum and Bispectrum: the Time-RG Approach. *JCAP*, 1003:011, 2010, 0912.4276.

[73] Emiliano Sefusatti, Martin Crocce, and Vincent Desjacques. The Matter Bispectrum in N-body Simulations with non-Gaussian Initial Conditions. *Mon.Not.Roy.Astron.Soc.*, 406:1014–1028, 2010, 1003.0007.

[74] M. Liguori, E. Sefusatti, J.R. Fergusson, and E.P.S. Shellard. Primordial non-Gaussianity and Bispectrum Measurements in the Cosmic Microwave Background and Large-Scale Structure. *Adv.Astron.*, 2010:980523, 2010, 1001.4707.

[75] Licia Verde. Non-Gaussianity from Large-Scale Structure Surveys. *Adv.Astron.*, 2010:768675, 2010, 1001.5217.

[76] D.G. Figueroa, E. Sefusatti, A. Riotto, and F. Vernizzi. The Effect of Local non-Gaussianity on the Matter Bispectrum at Small Scales. *JCAP*, 1208:036, 2012, 1205.2015.

[77] See `http://planck.esa.it`.

[78] Luca Amendola et al. Cosmology and fundamental physics with the Euclid satellite. 2012, 1206.1225.

[79] P.J.E. Peebles. *Large-scale structure of the Universe*. Princeton university press, 1980.

[80] F. Bernardeau, S. Colombi, E. Gaztanaga, and R. Scoccimarro. Large scale structure of the universe and cosmological perturbation theory. *Phys.Rept.*, 367:1–248, 2002, astro-ph/0112551.

[81] Alex Borisov and Bhuvnesh Jain. Three-Point Correlations in f(R) Models of Gravity. *Phys.Rev.*, D79:103506, 2009, 0812.0013.

[82] Takayuki Tatekawa and Shinji Tsujikawa. Second-order matter density perturbations and skewness in scalar-tensor modified gravity models. *JCAP*, 0809:009, 2008, 0807.2017.

[83] Francis Bernardeau and Philippe Brax. Cosmological Large-scale Structures beyond Linear Theory in Modified Gravity. *JCAP*, 1106:019, 2011, 1102.1907.

[84] Hector Gil-Marin, Fabian Schmidt, Wayne Hu, Raul Jimenez, and Licia Verde. The Bispectrum of f(R) Cosmologies. *JCAP*, 1111:019, 2011, 1109.2115.

[85] Fabian Schmidt, Marcos Vinicius Lima, Hiroaki Oyaizu, and Wayne Hu. Non-linear Evolution of f(R) Cosmologies III: Halo Statistics. *Phys.Rev.*, D79:083518, 2009, 0812.0545.

[86] Fabian Schmidt, Wayne Hu, and Marcos Lima. Spherical Collapse and the Halo Model in Braneworld Gravity. *Phys.Rev.*, D81:063005, 2010, 0911.5178.

[87] Rampei Kimura and Kazuhiro Yamamoto. Large Scale Structures in Kinetic Gravity Braiding Model That Can Be Unbraided. *JCAP*, 1104:025, 2011, 1011.2006.

[88] Gregory Gabadadze and Alberto Iglesias. (De)coupling limit of DGP. *Phys.Lett.*, B639:88–94, 2006, hep-th/0603199.

[89] Paolo Creminelli, Guido D'Amico, Jorge Norena, Leonardo Senatore, and Filippo Vernizzi. Spherical collapse in quintessence models with zero speed of sound. *JCAP*, 1003:027, 2010, 0911.2701.

[90] Antonio De Felice and Teruaki Suyama. Vacuum structure for scalar cosmological perturbations in Modified Gravity Models. *JCAP*, 0906:034, 2009, 0904.2092.

[91] Antonio De Felice, Shinji Mukohyama, and Shinji Tsujikawa. Density perturbations in general modified gravitational theories. *Phys.Rev.*, D82:023524, 2010, 1006.0281.

[92] Clare Burrage and David Seery. Revisiting fifth forces in the Galileon model. *JCAP*, 1008:011, 2010, 1005.1927.

[93] Philippe Brax, Clare Burrage, and Anne-Christine Davis. Laboratory Tests of the Galileon. *JCAP*, 1109:020, 2011, 1106.1573.

[94] Antonio De Felice and Shinji Tsujikawa. Cosmological constraints on extended Galileon models. *JCAP*, 1203:025, 2012, 1112.1774.

[95] Koichi Hirano and Zen Komiya. Observational tests of Galileon gravity with growth rate. 2010, 1012.5451.

[96] Koichi Hirano, Zen Komiya, and Hisato Shirai. Constraining Galileon gravity from observational data with growth rate. *Prog.Theor.Phys.*, 127:1041–1056, 2012, 1103.6133.

[97] Sabino Matarrese, Silvia Mollerach, and Marco Bruni. Second order perturbations of the Einstein-de Sitter universe. *Phys.Rev.*, D58:043504, 1998, astro-ph/9707278.

[98] JM Bardeen, JR Bond, N. Kaiser, and AS Szalay. The statistics of peaks of gaussian random fields, 1986. *Astrophys. J*, 304:15.

[99] EJ Groth and PJE Peebles. Statistical analysis of catalogs of extragalactic objects. vii-two-and three-point correlation functions for the high-resolution shane-wirtanen catalog of galaxies. *The Astrophysical Journal*, 217:385–405, 1977.

[100] Nicola Bartolo, Sabino Matarrese, and Antonio Riotto. CMB Anisotropies at Second-Order. 2. Analytical Approach. *JCAP*, 0701:019, 2007, astro-ph/0610110.

[101] Martin Crocce and Roman Scoccimarro. Renormalized cosmological perturbation theory. *Phys.Rev.*, D73:063519, 2006, astro-ph/0509418.

[102] Massimo Pietroni. Flowing with Time: a New Approach to Nonlinear Cosmological Perturbations. *JCAP*, 0810:036, 2008, 0806.0971.