UNIVERSITÀ DEGLI STUDI DI PADOVA

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CICLO XXII

# MODELS FOR EFFICIENCY OPTIMIZATION OF INDUSTRIAL PLANTS AND LOGISTICS

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*...dedicated to my beloved Rossana*

*and to our families with love.*

# INTRODUZIONE

Negli ultimi decenni, il mercato ha portato le aziende manifatturiere e di servizi ad essere più flessibili e efficienti nella produzione dei propri beni e servizi. Maggior attenzione è stata quindi posta nei confronti delle performance dei sistemi produttivi-logistici. É infatti noto lo stretto legame tra flessibilità e competitività delle aziende con l'efficienza degli impianti produttivi e della logistica caratterizzanti le stesse.

É importante quindi riuscire a definire, monitorare e migliorare l'efficienza degli impianti industriali e dei sistemi logistici. Sono stati introdotte numerose definizioni di efficienza, tra le quali la più importante è l'Overall Equipment Efficiency (OEE), introdotta alla fine degli anni '80 da Nakajima. Tale indice si è presto diffuso in numerosi settori e lega l'efficienza dei sistemi produttivi e logistici a tre fattori principali: la disponibilità dei sistemi, la qualità dei beni/servizi e le performance produttive e logistiche.

Tra questi fattori, maggior attenzione è stata posta al parametro relativo alla disponibilità dei sistemi. Inoltre è noto come la disponibilità dei sistemi sia strettamente legata al comportamento affidabilistico dei sistemi stessi.

Su tale fronte, questo elaborato introduce innovativi modelli teorici per l'analisi dell'affidabilità e disponibilità di componente e sistemi logistico-produttivi, con particolare attenzione ai sistemi soggetti a diverse condizioni operative.

La tesi si articola in tali principali parti:

1. Introduzione ai principali modelli di OEE e loro legame con la disponibilità dei sistemi logistico-produttivi;
2. Discussione del legame stretto tra comportamento affidabilistico e condizioni ambientali;
3. Sviluppo di modelli teorici innovativi per la modellazione delle caratteristiche affidabilistiche e loro validazione tramite applicazioni industriali;
4. Definizione e sviluppo di nuove politiche manutentive basate sull'estensione della letteratura scientifica grazie ai modelli teorici precedentemente verificati, per il miglioramento degli indici di efficienza;
5. Definizione di un nuovo modello per l'analisi delle performance di sistemi logistici quali gli Automated Storage/Retrieval Systems e comparazione con i modelli esistenti in letteratura.

Il lavoro di tesi è stato sviluppato in stretta collaborazione anche con il Prof. Hoang Pham, direttore del Department of Industrial and Systems Engineering, Rutgers University, Piscataway – New Jersey (USA), grazie ad un periodo di ricerca di alcuni mesi svolto dall'autore presso Quality and Reliability Engineering Laboratory di tale dipartimento.

Il presente lavoro ha portato anche alla pubblicazione di diversi contributi su importanti riviste e convegni internazionali, quali International Journal of Mathematics in Operational Research, International Journal of System Science e IEEE Transactions on Man, Cybernetics and Systems.

# ABSTRACT

In the last decades, the global markets have driven the manufacturing and service companies to be more flexible and efficient as for goods and services operations are concerned. More attention has also been paid to the performance of productive and logistic systems. In fact, the strict relation between flexibility and competitiveness of companies and efficiency of their productive plants and logistics is well known.

It is important to define, monitor and improve the efficiency of industrial and logistic systems. Many definitions of efficiency have been introduced and the most important is the Overall Equipment Efficiency (OEE), introduced by Nakajima at the end of the '80s. This index quickly spread in many industrial fields and it connects the efficiency of productive and logistic systems to three main factors: the availability of systems, the quality of produced goods/services and the productive and logistics performance.

Between these factors, more attention has been paid to the parameter related to the availability of systems. Moreover, it is known that the availability is strictly linked to the survival behavior of systems.

In this field, the manuscript introduces several innovative theoretical models for the survival analysis of components and productive-logistics complex systems, with particular attention to the systems which operate in different operative conditions.

This thesis is structured in the following main parts:

1. Introduction of most important models of OEE and their relationship with availability of productive and logistic systems;
2. Discussion about the relation between survival behavior and operative conditions;
3. Definition and development of innovative theoretical models for the system reliability modeling and their validation thanks to several industrial applications;
4. Definition and development of innovative maintenance policies for the efficiency improvement, based on the extension of scientific literature, thanks to the theoretical models introduced in previous parts;
5. Definition of innovative model for the performance analysis of logistic systems, in particular for Automated Storage/Retrieval Systems and comparison with the existing models.

The research has also been carried out in collaboration with Prof. Hoang Pham, director of Department of Industrial and Systems Engineering, Rutgers University, Piscataway – New Jersey (USA), during a period the author spent as *"visiting researcher"* in the Quality and Reliability Engineering Laboratory of that department.

The present work has carried out to the publishing of several scientific contributions in relevant International Journals and Conferences, like International Journal of Mathematics in Operational Research, International Journal of System Science and IEEE Transactions on Man, Cybernetics and Systems.

# ACKNOWLEDGEMENTS

First of all, I would really like to thank to my research supervisor, Professor Alessandro Persona. With his valuable professional and human suggestions, he was, is and will be a great example, who led me during this last three years which has driven me in this three years and has instilled me the passion for research.

A huge thank you goes to Professor Hoang Pham. He has given me the opportunity to live a beautiful period of my Ph.D. course in one of the most important universities, in the Quality and Reliability Engineering Laboratory, Department of Industrial and Systems Engineering, Rutgers University, Piscataway – New Jersey (USA). This thesis is also the result of our collaboration.

I cannot forget to thank all my research colleagues: Daria Battini, Maurizio Faccio, Anna Azzi, Chiara Ortolani and Pietro Vecchiato.

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I would like to greet all my colleagues in Ph.D. course, who believed in me, giving me an important charge, nominating me their Ph.D. student representative inside the Ph.D. course. I hope to have met all their expectations. We have spent nice time together.

A great thank goes to all my family, that has always been close to me in these last three years and above all to my beloved Rossana, who supported me in several difficult periods, we also shared many delighting times together and I hope we will keep on doing this forever.

Last but not least, I thank all my friends, in particular to the cool guys of B.B.L. band.

Thank you all,

Fabio Sgarbossa

31 January 2010

# TABLE OF CONTENTS

# 1. Introduction

## 1.1. Purpose of Research

Global market and customized demands are now satisfied by the quick and efficient responses of manufacturing and logistics systems.

The present thesis has been developed starting from the consideration that the industrial plants and logistic systems require high efficiency levels in order to guarantee operative flexibility and to achieve the competitiveness of the companies.

The relevant costs and investments of industrial plants and logistics have involved into the requirement of the measure of their manufacturing/logistics performances and the aim of introducting of these measures is to encourage their improvement.

There are many factors that influence the efficiency of a productive and logistic systems and many of these often depend on the operative and environmental conditions in which the systems operate.

The research carried out by the author starts from the analysis of the fundamentals theories concerning the definition of efficiency and performance of productive and logistics systems, such as the Overall Equipment Efficiency (OEE).

Then several different models, used to calculate this index, have been illustrated in order to demonstrate the relation between OEE and several factors, like availability of systems, quality of products/services and performances. Among these factors, it is well known that the availability is the most important one.

For this reason the main goal of this thesis is the System Reliability Modeling, deeply discussed in chapters 4, 5 and 6. In this research, the author has developed and introduced innovative theoretical models in order to define the availability and reliability of complex production and logistic systems.

These models have permitted to extend the literature contribution also to the systems that operate under random environmental conditions.

In fact, industrial applications often observe the difference between laboratory reliability test in standard conditions and component or system reliability when it is set in motion through different environments and real world conditions. As a matter of fact reliability variable is considerably influenced by environmental factors and environmental factors may change failure rate, reliability and availability of systems.

In particular the thesis have these main objectives:

- Introduction and analysis of principal efficiency indices and their relation with availability of systems;
- Introduction and analysis of the theory about reliability and availability fundamentals, presented in scientific literature;
- Definition and study of innovative theoretical models to estimate the survival parameters of complex productive and logistic systems when they operate in different environmental and operative conditions;
- Definition and development of innovative maintenance policies, based on new introduced models, in order to increase the efficiency of whole systems;
- Modeling of performance of innovative logistics systems, developed to improve the efficiency in logistics.

## 1.2. Structure of Thesis

The present thesis has been structured in the following parts, according the main aims of the research:

- ***Chapter 2***: in this chapter, the introduction of Overall Equipment Efficiency and the state of art of scientific contribution in this field are discussed. Then the definition of OEE and its relation with the six big losses in industrial and logistic systems are illustrated, reporting two important OEE calculations: the Nakajima and the SEMI models. At the end of this chapter, the most important strategies used to improve the efficiency OEE are discussed. These strategies are usually defined as parts of the Maintenance Engineering, due to the strict relationship between OEE and availability of systems.
- ***Chapter 3***: this chapter illustrates the fundamentals of reliability and maintenance theory, such as the definitions of failures, reliability function, probability density function and cumulative density function, hazard rate and its behavior for single component. Then, the system reliability calculations for different configurations of

systems are illustrated. The most important lifetime distribution are discussed in order to model the reliability data. At the end, several methods about the reliability estimation using data from the field are investigated.

- ***Chapter 4***: in this chapter, the impact of environmental conditions on reliability is studied. In particular, some models are just briefly mentioned in order to define the state of the art and mathematical models available: a literature analysis is summarized in order to show this innovative concept. Then, the mathematical function called systemability is introduced. Moreover, industrial applications of the systemability to several system configurations are illustrated.
  Systemability can also be used to model the cost of complex entire industrial system development life-cycle that perhaps reflects the perspectives from both developers and users and to determine the optimal release policies. An industrial application is discussed to illustrate the results of this study. At the end of the chapter, some careful considerations about the capability of the application and how to pursue with the research in that field are shown.
- ***Chapter 5***: this chapter introduces the principal maintenance policies in industrial and logistics systems, the Age Replacement Policy and the Periodic Replacement Policy. The literature review are reported in order to show the lack of policies that take into consideration the environmental factors. Starting from this consideration, the innovative application of systemability on these maintenance policies is investigated, developing two innovative preventive maintenance policies. An extensive sensitivity analysis is conducted in order to show the influence of environmental factors to maintenance results. Also in this chapter the innovative models are validated using several industrial applications.
- ***Chapter 6***: in this chapter, the performance of a logistic systems is investigated. Using FEM normative, the Overall Equipment Efficiency of Automated Storage/Retrieval Systems is modelled, related to the availability, reliability and cycle time estimation of these systems. Moreover an innovative travel time model, based on FEM normative, is introduced and compared with the models developed by the scientific literature, based on Bozer & White formulations.
- ***Chapter 7***: this chapter reports the conclusions about the researches and the future steps.

- *Chapter 8*: this chapter contains the Appendix I, related to section 4.
- *Chapter 9*: in this chapter, the references are listed, divided by chapter in which they are mentioned.

## 1.3. Main Scientific Contribution Developed

The presented research has permitted to write and publish several scientific contributions in many important international journals during the last three years, like International Journal of Mathematics in Operational Research, International Journal of System Science, IEEE Transactions on Man, Cybernetics and Systems and others.
Some of these have been also presented and discussed by the author in several international conferences, such as ISSAT and national conferences ANIMP.
Moreover, the research has also been carried out during a period of time in 2008 in the Quality and Reliability Engineering Laboratory, in collaboration with Prof. Hoang Pham, director of Department of Industrial and Systems Engineering, Rutgers University, Piscataway – New Jersey (USA).

Here below the list of scientific contributions developed from this research:

*INTERNATIONAL JOURNALS*

PERSONA A., PHAM H., SGARBOSSA F., (2009), Systemability Function to Optimisation Reliability in Random Environment. INTERNATIONAL JOURNAL OF MATHEMATICS IN OPERATIONAL RESEARCH. Vol. 1, No. 3, pp. 397–417. ISSN: 1757-5850.

SGARBOSSA, F., PERSONA A., PHAM H., (2009) Age Replacement Policy in Random Environment using Systemability. INTERNATIONAL JOURNAL OF SYSTEM SCIENCE. IN PROOFS. ISSN: 0020-7721. (Accepted 4 September 2009) (I.F. 0.634).

SGARBOSSA, F., PHAM H., (2008) A Weibull-based Cost Model with Considerations of Random Field Environments, IEEE TRANSACTION ON MAN, CYBERNETICS AND SYSTEMS. ISSN: 0018-9529. (I.F. 1.315) Under Review.

SGARBOSSA, F., PERSONA A., PHAM H., (2009) Periodic Replacement Policy in Random Environment using Systemability. QUALITY AND RELIABILITY ENGINEERING INTERNATIONAL. ISSN: 0748-8017. (I.F. 0.828) Under review.

AZZI A., BATTINI D., FACCIO M., PERSONA A., SGARBOSSA F., (2009) Innovative Travel Time Model For Dual-Shuttle Automated Storage/Retrieval Systems. European Journal of Operational Research. ISSN: 1545-8830 (I.F. 1.627) Under revies.

***INTERNATIONAL CONFERENCES***

BATTINI D., FACCIO M., PERSONA A., SGARBOSSA F.. (2007), Reliability In Random Environment: Systemability And Its Applications, In: PROCEEDING OF ISSAT 2007. 12th ISSAT CONFERENCE ON RELIABILITY AND QUALITY IN DESIGN. Seattle, USA. 2-4 August 2007. ISBN 978-0-9763486-2-7.

BATTINI D., FACCIO M., PERSONA A., SGARBOSSA F. (2008), Reliability of motorcycle components using systemability approach, 14th ISSAT INTERNATIONAL CONFERENCE ON RELIABILITY AND QUALITY IN DESIGN, Orlando, Florida USA. 7-9 August 2008. ISBN 978-0-9763486-4-1.

***NATIONAL CONFERENCES***

BATTINI D., FACCIO M., PERSONA A., SGARBOSSA F. (2008), L'innovativo concetto di Systemability per l'analisi affidabilistica: sviluppi e applicazioni. Summer School del gruppo ING-IND 17 "F. Turco", Gaeta, Italia. 9-12 Settembre 2008.

AZZI A., BATTINI D., PERSONA A., SGARBOSSA F. (2009), Approccio innovativo nella progettazione e gestione di AS/RS Multishuttle, In: ATTI DEL CONVEGNO ANIMP 2009. CONVEGNO NAZIONALE "INGEGNERIA E IMPIANTISTICA ITALIANA", Roma, Italia. 11-12 Giugno 2009.

# 2. Efficiency on Industrial Plants and Logistics

## 2.1. Introduction

Nowadays, quick responses of the companies are required in order to satisfy global markets and customized demands. To improve the competiveness and to guarantee the revenue of the companies, high levels of flexibility and efficiency are necessary. The relevant costs and investments of industrial plants and logistics have involved into the requirement of the measure of their manufacturing/logistics performances and the aim of the introduction of these measures is to encourage the improvement.

Most of these measures have been shown to be contradictory [2.11 – 2.13 – 2.17] and many of the measures used are considered obsolete and inconsistent for various reasons. The changing on performance measurements are well discussed by Schmenner and Vollmann [2.20], through an empirical study. They explained that most industrial plants and logistics were both using wrong measures and failing to use right measures in correct ways. Therefore, it seems important to identify the critical dimensions in a performance measurement systems and the optimum characteristics of the measures. Measurement systems could then be evaluated and improved with the dimensions and characteristics as comparative data.

Likewise, the usefulness of most cost accounting systems, individual measures as well as more comprehensive activity-based costing systems, are frequently questioned since they do not cover manufacturing performances relative to the competitive capabilities, e.g. [2.6 – 2.23]. Another serious problem with most performance measurement systems used in industrial plants and logistics is that they often include too many different measures, which makes it difficult to understand the “big picture” [2.14]. Integration between measures is often problematic, and many papers have emphasized that industrial systems have no effective system that covers all necessary performance dimensions, e.g. [2.2 – 2.8 – 2.16 – 2.20]. This is why it is not always obvious how companies should measure manufacturing performance.

Nakajima [2.17 – 2.18], the author of the total productive maintenance (TPM) philosophy, advocates the Overall Equipment Efficiency (OEE) as a metric for the evaluation of equipment effectiveness. OEE is often used as a driver for improving performance of the business by concentrating on quality, productivity and machine utilization issues and hence aimed at reducing non-value adding activities often inherent in manufacturing processes. Recent research [2.7] reports that accurate equipment performance data are essential to the success and long-term effectiveness of TPM activities, however individual equipment performance should never be considered in isolation because overall factory performance may or may not be dependent on individual machine performance.

The OEE measure can be applied at several different levels within a manufacturing environment. First, OEE can be used as a "benchmark" for measuring the initial performance of a manufacturing plant in its entirety. In this manner the initial OEE measure can be compared with future OEE values, thus quantifying the level of improvement made. Second, an OEE value, calculated for one manufacturing line can be used to compare line performance across the factory, thereby highlighting any poor line performance. Third, if the machine's processes work individually, an OEE measure can identify which machine performance is worst, and therefore indicate where to focus TPM resources [2.17]. In this respect the OEE measurement system within a company becomes the fundamental measure of TPM activities, and in fact the basis of improvements for the TPM system.

Dal et al. [2.3] said that by utilizing largely existing performance data, such as preventive maintenance, material utilization, absenteeism, accidents, labor recovery, conformance to schedule, set-up and changeover data, etc., the OEE measure could provide topical information for daily decision making. However, the role of OEE goes far beyond the task of just monitoring and controlling. It takes into account process improvement initiatives, prevents the sub-optimization of individual machines or product lines, provides a systematic method for establishing production targets, and incorporates practical management tools and techniques in order to achieve a balanced view of process availability, performance rate and quality.

## *2.2. Overall Equipment Efficiency (OEE): State of art*

A key performance measure in mass-production environments is the Overall Equipment Effectiveness (OEE). OEE was introduced by Nakajima [2.17 – 2.18] in the context of Total Productivity Maintenance (TPM) and is directed towards equipment/machines. OEE is a simple and clear overall metric, and managers appreciate such an aggregated metric instead of many detailed metrics. Huang et al. [2.9] report that the concept of OEE is becoming increasingly popular and that it has been widely used as a quantitative tool essential for the measurement of productivity in semiconductor manufacturing operations, because of extreme capacity constrained facility investment. They state that traditional metrics for measuring productivity, throughput and utilization, are insufficient for identifying the problems and underlying improvements needed to increase productivity.

Semiconductor Equipment and Materials International (SEMI) [2.21 – 2.22] has developed a standard for the definition and measurement of OEE as introduced by Nakajima [2.17 – 2.18]. The standard is directed towards measuring the effectiveness of equipment. SEMI renamed the metric Overall Equipment Efficiency, as it is expressed entirely in terms of time. The standard has been described by SEMI [2.21 – 2.22] and uses definitions as laid down by SEMI [2.21 – 2.22]. A guide for the application of OEE is described by Ames et al. [2.1].

Previous research has targeted various aspects of OEE. For example, Ljundberg [2.15] states that the definition of OEE does not take into account all factors that reduce the capacity utilization, e.g. planned downtime, lack of material input, lack of labor. In addition, the available time would be a more appropriate basis for time measurement than the loading time as it was originally used by Nakajima [2.17 – 2.18]. Similarly, De Groote [2.4] utilizes a fixed planned production time and calculates the difference between the actual and planned production time. The need for a more appropriate time basis is supported by Sattler and Schlueter [2.19], Jonsson and Lesshammar [2.12] and Jeong and Phillips [2.10]. It is generally observed that the accuracy of OEE is largely determined by the quality of the collected data. The description given by Nakajima [2.17] and SEMI [2.21] is directed towards equipment, but OEE is impacted greatly by factors beyond the equipment itself, including the operator, recipe, facilities, material (input items) availability,

scheduling requirements, etc. As this may result in OEE values influenced by factors beyond the equipment itself, a distinction can be made between stand-alone equipment and integrated equipment. OEE is directed towards equipment integrated in a manufacturing environment, so that OEE includes the influences of this

environment. As a metric was lacking directed towards stand-alone equipment, De Ron and Rooda [2.5] proposed the equipment effectiveness E. This performance measure was derived to monitor the effectiveness of stand-alone equipment, independent of the environment.

## *2.3. OEE Definition and the Six Big Losses*

Overall equipment effectiveness is a measure of total (complete, inclusive, whole) equipment performance–the degree to which the asset is doing what it is supposed to do. OEE is also a three-part analysis tool for equipment performance based on actual availability, performance efficiency, and quality of product or output. OEE can be applied to manufacturing, logistics, mobile, petrochemical processes, and environmental equipment.

Overall equipment effectiveness data is used to identify a single asset (machine or equipment) and/or single stream process (logistics) related losses for the purpose of improving total asset performance and reliability.

Overall equipment effectiveness data is used to identify and categorize major losses or reasons for poor performance. OEE provides the basis for setting improvement priorities and beginning root cause analysis.

OEE percentage is used to track and trend the improvement, or decline, in equipment effectiveness over a period of time. OEE percentages can point to hidden or untapped capacity in a manufacturing process and lead to balanced flow. The use of OEE is also intended to develop and improve collaboration between asset operations, maintenance, purchasing, and equipment engineering to jointly identify and eliminate (or reduce) the 2 major causes of poor performance since "maintenance" alone cannot improve OEE.

OEE is related to the Six Big Losses, that describes the most common causes for efficiency loss – almost always found in today's manufacturing environment.

One of the major goals of TPM and OEE programs is to reduce and/or eliminate what are called the Six Big Losses – the most common causes of efficiency loss in manufacturing.

In considering OEE, Nakajima (1988) defines six large equipment losses.

1. Equipment failure/breakdown losses are categorized as time losses when productivity is reduced, and quality losses caused by defective products.
2. Setup/adjustment time losses result from downtime and defective products that occur when the production of one item ends and the equipment is adjusted to meet the requirements of another item.
3. Idling and minor stop losses occur when production is interrupted by a temporary malfunction or when a machine is idling.

4. Reduced speed losses refer to the difference between the equipment design speed and the actual operating speed.
5. Reduced yield occurs during the early stage of production from machine startup stabilization.
6. Quality defects and rework are losses in quality caused by malfunctioning production equipment.

The following table lists the Six Big Losses, and shows how they relate to the OEE Loss categories.

| *Six Big Losses Category* | *OEE Loss Category* | *Event Examples* | *Comments* |
|---|---|---|---|
| Equipment failure/breakdown | Down Time Loss | • Tooling Failures<br>• Unplanned Maintenance<br>• General Breakdowns<br>• Equipment Failure | There is flexibility on where to set the threshold between a Breakdown (Down Time Loss) and a Small Stop (Speed Loss). |
| Setup/adjustment time | Down Time Loss | • Setup/Changeover<br>• Material Shortages<br>• Operator Shortages<br>• Major Adjustments<br>• Warm-Up Time | This loss is often addressed through setup time reduction programs. |
| Idling and minor stop | Speed Loss | • Obstructed Product Flow<br>• Component Jams<br>• Misfeeds<br>• Sensor Blocked<br>• Delivery Blocked<br>• Cleaning/Checking | Typically only includes stops that are under five minutes and that do not require maintenance personnel. |

| *Six Big Losses Category* | *OEE Loss Category* | *Event Examples* | *Comments* |
|---|---|---|---|
| Reduced yield | Quality Loss | • Scrap<br>• Rework<br>• In-Process Damage<br>• In-Process Expiration<br>• Incorrect Assembly | Rejects during warm-up, startup or other early production. May be due to improper setup, warm-up period, etc. |
| Quality defects and rework | Quality Loss | • Scrap<br>• Rework<br>• In-Process Damage<br>• In-Process Expiration<br>• Incorrect Assembly | Rejects during steady-state production. |

**Table 2.1: Six Big Losses vs. OEE Factor**

Now that it is known what the Six Big Losses are and some of the events that contribute to these losses, it can be focused on ways to monitor and correct them. Categorizing data makes loss analysis much easier, and a key goal should be fast and efficient data collection, with data put it to use throughout the day and in real-time.

- ***Breakdowns***

Eliminating unplanned Down Time is critical to improving OEE. Other OEE Factors cannot be addressed if the process is down. It is not only important to know how much Down Time your process is experiencing (and when) but also to be able to attribute the lost time to the specific source or reason for the loss (tabulated through Reason Codes). With Down Time and Reason Code data tabulated, Root Cause Analysis is applied starting with the most severe loss categories.

- ***Setup and Adjustments***

Setup and Adjustment time is generally measured as the time between the last good part produced before Setup to the first consistent good parts produced after Setup. This often includes substantial adjustment and/or warm-up time in order to consistently produce parts that meet quality standards.

Tracking Setup Time is critical to reducing this loss, together with an active program to reduce this time (such as an SMED – Single Minute Exchange of Dies program).

Many companies use creative methods of reducing Setup Time including assembling changeover carts with all tools and supplies necessary for the changeover in one place, pinned or marked settings so that coarse adjustments are no longer necessary, and use of prefabricated setup gauges.

- ***Small Stops and Reduced Speed***

Small Stops and Reduced Speed are the most difficult of the Six Big Losses to monitor and record. Cycle Time Analysis should be utilized to pinpoint these loss types. In most processes recording data for Cycle Time Analysis needs to be automated since cycles are quick and repetitive events that do not leave adequate time for manual data-logging.

By comparing all completed cycles to the Ideal Cycle Time and filtering the data through a Small Stop Threshold and Reduced Speed Threshold the errant cycles can be automatically categorized for analysis. The reason for analyzing Small Stops separately from Reduced Speed is that the root causes are typically very different, as can be seen from the Event Examples in the previous table.

- ***Startup Rejects and Production Rejects***

Startup Rejects and Production Rejects are differentiated, since often the root causes are different between startup and steady-state production. Parts that require rework of any kind should be considered rejects. Tracking when rejects occur during a shift and/or job run can help pinpoint potential causes, and in many cases patterns will be discovered.

### 2.3.1. OEE Calculation: Nakaijma model [2.17 – 2.18]

Nakajima [2.17 – 2.18] defined mathematically OEE as follows:

$$OEE = V * P * Q \qquad 2\text{-}1$$

with:

$$V = availability = \frac{actual\ operating\ time}{loading\ time} = \frac{loading\ time - downtime}{loading\ time}$$

$$P = performance = \frac{theoretical\ cycle\ time * processed\ amount}{actual\ operating\ time}$$

$$Q = quality = \frac{processed\ amount - defect\ amount}{processed\ amount}$$

where:

- Loading time is the total time that equipment is expected to produce, minus planned downtime, like annual leave.
- Downtime is the stoppage time loss due to breakdowns, setup and adjustments.
- Theoretical cycle time is the theoretical minimum time to produce one piece.
- Processed amount is the number of items produced.
- Operating time is the productive time available after downtime losses are subtracted.
- Defect amount is the total number of items that not meet quality standard.

***Availability***

The Availability is a percentage number showing how the machine was available when it was needed for production. It looks at the first two of the Six Big Losses, Breakdowns and Setup/Adjustment. That is the downtime that is measured at the equipment. Usually if the measurements at the equipment/machine are collected manually it is times longer than 5-10 minutes.

Availability is calculated by dividing the actual operating time by the loading time.

The loading time is given by subtracting the unscheduled time; e.g. no costumer demand, nonworking Sundays, during the day from the total available time or calendar time (24 hours a day).

The actual operating time is the loading time minus the sum of all downtime losses while operating, i.e. breakdowns and changeover.

***Performance***

The performance efficiency takes into account the unrecorded downtime. That is the third and fourth of the Six Big Losses, all unrecorded downtime, i.e. short stoppages, usually less than 5-10 minutes and losses due to the difference between ideal cycle time and actual cycle time.

To be able to calculate the performance efficiency an ideal cycle time for the job running at the machine is needed. If the ideal cycle time is multiplied with the total parts produced the outcome will be the time it should have taken to produce the parts. To calculate the performance efficiency the time it should have taken is divided by the actual operating time.

***Quality***

The quality rate concerns the rejected parts during production and the losses from initial startup to process stabilization. That is the last two of the Six Big Losses. The quality rate is calculated by dividing the good parts produced by the total number of parts produced. Good parts are all items that meet the quality standards. Items that have to be reworked are counted as scarp.

| **Availability** | |
|---|---|
| A. Total Available Time (3 shifts * 8 hours = 24 hours) | 1440 min |
| B. Planned Downtime (30 min lunch + 15 min breaks)*3 shifts | 135 min |
| C. Loading Time (A – B) | 1305 min |
| D. Unplanned Downtime | 250 min |
| E. Actual Operating Time (C – D) | 1055 min |
| ***F. Availability (E/C)*** | **80.8 %** |

| **Performance Efficiency** | |
|---|---|
| G. Total Parts Run | 2004 parts |
| H. Ideal Cycle Time (30 sec / 60) | 0.5 min/part |
| ***I. Performance Efficiency (G*H) / E*** | **95 %** |

| **Quality Rate** | |
|---|---|
| J. Total Defects (Scrap and Rework) | 78 parts |
| ***K. Quality Rate (G – J) / G*** | **96.1 %** |

| **Overall Equipment Efficiency ( F * I * K )** | **73.8 %** |
|---|---|

**Table 2.2: OEE Calculation**

### 2.3.2. OEE Calculation: SEMI model [2.21 – 2.22]

SEMI [2.21] expresses OEE entirely in terms of time. SEMI [2.22] defines six main states of manufacturing equipment (figure 2.1). Using these states, the following definition of the overall equipment efficiency can be given [2.21 – 2.22]:

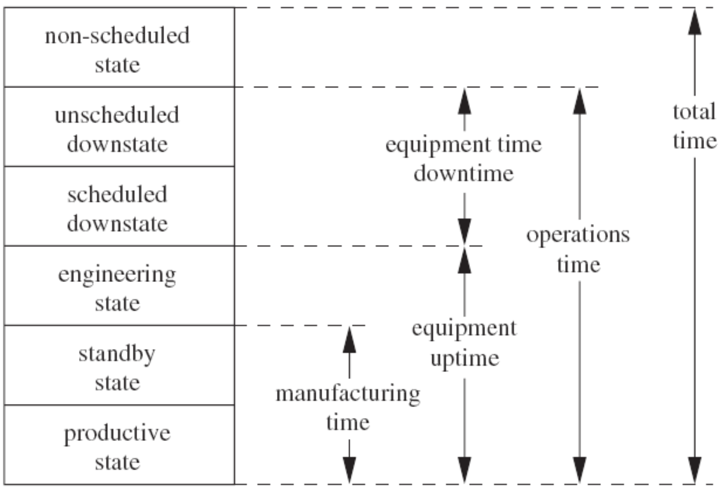



**Figure 2.1: OEE equipment states (from [2.5]).**

OEE consists of availability efficiency AE, operational efficiency OE, rate efficiency RE and quality efficiency QE:

$$OEE = AE * (OE * RE) * QE \qquad 2\text{-}2$$

with:

$$AE = \frac{equipment\ uptime}{total\ time}$$

$$OE = \frac{production\ time}{equipment\ uptime}$$

$$RE = \frac{theoretical\ production\ time\ for\ actual\ units}{production\ time}$$

$$QE = \frac{theoretical\ production\ time\ for\ effective\ units}{theoretical\ production\ time\ for\ actual\ units}$$

In these definitions, the theoretical production time is the production time at strictly theoretically efficient rates without efficiency losses.

The application of OEE is illustrated in the following examples. Table 2.3 shows the measurement data for the two situations. The difference between the situations is that, in case B, the throughput is larger than in case A so that the productive time is longer and more items are produced. Because of the longer production time, the downtime is increased and the standby time decreased as more capacity is required. The results are given in table 2.4.

| | A | B |
|---|---|---|
| Total time (h) | 168 | 168 |
| Non-scheduled time (h) | 0 | 0 |
| Standby time (h) | 72 | 48 |
| Production time (h) | 84 | 105 |
| Engineering time (h) | 0 | 0 |
| Total (un)scheduled downtime (h) | 12 | 15 |
| Theoretical production time per unit (h) | 0.044 | 0.044 |
| Maximum throughput (h_1) | 22.73 | 22.73 |
| Number of items | 1860 | 2324 |
| Number of qualified items | 1810 | 2261 |

**Table 2.3: Measurement data set (from [2.5])**

| | A | B |
|---|---|---|
| AE | (168 – 12)/168 = 0.929 | (168 – 15)/168 = 0.911 |
| OE | 84/(168 – 12) = 0.538 | 105/(168 – 15) = 0.686 |
| RE | 1860 x 0.044/84 = 0.974 | 2324 x 0.044/105 = 0.974 |
| QE | 1810/1860 = 0.973 | 2261/2324 = 0.973 |
| OEE | 0.929 x 0.538 x 0.974 x 0.973 = 0.474 | 0.911 x 0.686 x 0.974 x 0.973 = 0.592 |

**Table 2.4: Results of OEE for data set (from [2.5])**

The results show that RE and QE are the same for both situations. The differences are expressed by the AE and OE and, as a consequence, the OEE values. As the throughput is increased in case B, the differences between the measurements are not caused by the machine or workstation, but by their environment. The influence of the environment on OEE can be treated more generally by considering the utilization. This measure indicates the portion of the total time that the capacity of the machine is requested. Equation 2-2 shows that OEE contains the product of AE and OE.

## *2.4. Strategies to improve OEE: Maintenance Engineering*

As previously discussed, the introduction of Overall Equipment Efficiency index has been necessary to observe deeply the performance of the industrial and logistic systems.

The applications of OEE, reported in scientific contributions, and the real case studies have highlighted that the most important aspect which influences the OEE value is the Availability.

In details, the most relevant factors of the Six Big Losses are the first one: equipment failure/breakdown, for example tooling failures, unplanned maintenance, general breakdowns and equipment failures.

The reason why it is the most important loss is connected to two big issue:

- the high amount of time spent to repair the failure and to restart the systems;
- the unpredictable characteristic of breakdowns.

These two causes make the Availability the most significant factor of OEE index. The other factors can be reduced after a good cause-effect analysis, while the breakdowns and general failures are random features of the industrial and logistic systems.

Consequently, more relevance is given to a specific science: the Maintenance Engineering.

Maintenance Engineering is the combination of all technical and administrative actions, including supervision actions, intended to retain an item in (components, device, functional unit, equipment or system), or restore it to, a state in which it can perform a required function.

Maintenance is also defined as a set of organized activities that are carried out in order to keep an item in its best operational condition with minimum cost acquired.

Maintenance objectives should be consistent with and subordinate to production goals. The relation between maintenance objectives and production goals is reflected in the action of keeping production machines and facilities in the best possible condition.

Then, the main goal of Maintenance Engineering is to assure the higher level of Availability of industrial or logistic system at lower cost, in details, some sub-objective are:

- to minimize the breakdown, assuring the operation balance;
- to maintain operating structure and machine in several defined working conditions;
- to add and improve the global efficiency of production and logistic system;
- to make all the activities economically;
- to guarantee the personal safety and environment protection;

- to maximize production or increasing facilities availability at the lowest cost and at the highest quality and safety standards;
- to optimize resources utilization;
- to reduce downtime;
- to improve spares stock control,
- to improve equipment efficiency and reducing scrap rate;
- to minimize energy usage;
- to optimize the useful life of equipment;
- to provide reliable cost and budgetary control;
- to identify and implementing cost reductions;

Several different maintenance strategies and policies have been developed in order to achieve the main goal of Maintenance Engineering.
A very spread literature about different type of maintenance strategies and policies has been developed in the last decades.

The principal alternative approaches to support the Maintenance Engineering can be divided into:

- ***Breakdown/Corrective Maintenance:*** it is the oldest type of maintenance. The required repair, replacement, or restore action performed on a machine or a facility after the occurrence of a failure in order to bring this machine or facility to at least its minimum acceptable condition. It is subdivided into two types:
    - *Emergency maintenance:* it is carried out as fast as possible in order to bring a failed machine or facility to a safe and operationally efficient condition.
    - *Breakdown maintenance:* it is performed after the occurrence of an advanced considered failure for which advanced provision has been made in the form of repair method, spares, materials, labor and equipment.

  The principal disadvantages are:
    - its activities are expensive in terms of both direct and indirect cost;

- using this type of maintenance, the occurrence of a failure in a component can cause failures in other components in the same equipment, which leads to low production availability;
- its activities are very difficult to plan and schedule in advance.

This type of maintenance is useful in the following situations:

- the failure of a component in a system is unpredictable,
- the cost of performing run to failure maintenance activities is lower than performing other activities of other types of maintenance;
- the equipment failure priority is too low in order to include the activities of preventing it within the planned maintenance budget.

- ***Preventive Maintenance (PM):*** It is a set of activities that are performed on plant equipment, machinery, and systems before the occurrence of a failure in order to protect them and to prevent or eliminate any degradation in their operating conditions. It is also defined as the maintenance carried out at predetermined intervals or according to prescribed criteria and intended to reduce the probability of failure or the degradation of the functioning and the effects limited. The advantage of applying preventive maintenance activities is to satisfy most of maintenance objectives.

The factors that affect the efficiency of this type of maintenance:

- the need for an adequate number of staff in the maintenance department in order to perform this type of maintenance;
- the right choice of production equipment and machinery that is suitable for the working environment and that can tolerate the workload of this environment;
- the required staff qualifications and skills, which can be gained through training;
- the support and commitment from executive management to the PM program;
- the proper planning and scheduling of PM program;
- the ability to properly apply the PM program;

This type of maintenance is useful in the following situations:

- it is good for those machines and facilities which their failure would cause serious production losses;

- its aim is to maintain machines and facilities in such a condition that breakdowns and emergency repairs are minimised;
- its activities include replacements, adjustments, major overhauls, inspections and lubrications.

Researchers subdivided preventive maintenance into different kinds according to the nature of its activities:

- *Routine maintenance* which includes those maintenance activities that are repetitive and periodic in nature such as lubrication, cleaning, and small adjustment.
- *Running maintenance* which includes those maintenance activities that are carried out while the machine or equipment is running and they represent those activities that are performed before the actual preventive maintenance activities take place.
- *Opportunity maintenance* which is a set of maintenance activities that are performed on a machine or a facility when an unplanned opportunity exists during the period of performing planned maintenance activities to other machines or facilities.
- *Window maintenance* which is a set of activities that are carried out when a machine or equipment is not required for a definite period of time.
- *Shutdown preventive maintenance,* which is a set of preventive maintenance activities that are carried out when the production line is in total stoppage situation.

- ***Corrective Maintenance:*** In this type, actions such as repair, replacement, or restore will be carried out after the occurrence of a failure in order to eliminate the source of this failure or reduce the frequency of its occurrence. It is also defined as the maintenance carried out after recognition and intended to put an item into a state in which it can perform a required function.

  This type of maintenance is subdivided into three types:

  - *Remedial maintenance,* which is a set of activities that are performed to eliminate the source of failure without interrupting the continuity of the production process. The way to carry out this type of corrective maintenance is by taking the item to be corrected out of the production line and replacing it with reconditioned item or transferring its workload to its redundancy.

- *Deferred maintenance,* which is a set of corrective maintenance activities that are not immediately initiated after the occurrence of a failure but are delayed in such a way that will not affect the production process.
- *Shutdown corrective maintenance*, which is a set of corrective maintenance activities that are performed when the production line is in total stoppage situation.

The main objectives of corrective maintenance are the maximisation of the effectiveness of all critical plant systems, the elimination of breakdowns, the elimination of unnecessary repair, and the reduction of the deviations from optimum operating conditions.

The difference between corrective maintenance and preventive maintenance is that for the corrective maintenance, the failure should occur before any corrective action is taken.

Corrective maintenance is different from run to failure maintenance in that its activities are planned and regularly taken out to keep plant's machines and equipment in optimum operating condition.

- ***Improvement Maintenance:*** It aims at reducing or eliminating entirely the need for maintenance. This type of maintenance is subdivided into three types as follows:
    - *Design-out maintenance* which is a set of activities that are used to eliminate the cause of maintenance, simplify maintenance tasks, or raise machine performance from the maintenance point of view by redesigning those machines and facilities which are vulnerable to frequent occurrence of failure and their long term repair or replacement cost is very expensive.
    - *Engineering services* which includes construction and construction modification, removal and installation, and rearrangement of facilities.
    - *Shutdown improvement maintenance,* which is a set of improvement maintenance activities that are performed while the production line is in a complete stoppage situation.

- ***Predictive Maintenance:*** Predictive maintenance is a set of activities that detect changes in the physical condition of equipment (signs of failure) in order to carry out the appropriate maintenance work for maximising the service life of equipment without increasing the risk of failure.

  It is classified into two kinds according to the methods of detecting the signs of failure
  - *Condition-based predictive maintenance* depends on continuous or periodic condition monitoring equipment to detect the signs of failure.
  - *Statistical-based predictive maintenance* depends on statistical data from the meticulous recording of the stoppages of the in-plant items and components in order to develop models for predicting failures.

The drawback of predictive maintenance is that it depends heavily on information and the correct interpretation of the information. Some researchers classified predictive maintenance as a type of preventive maintenance. The main difference between preventive maintenance and predictive maintenance is that predictive maintenance uses monitoring the condition of machines or equipment to determine the actual mean time to failure whereas preventive maintenance depends on industrial average life statistics.

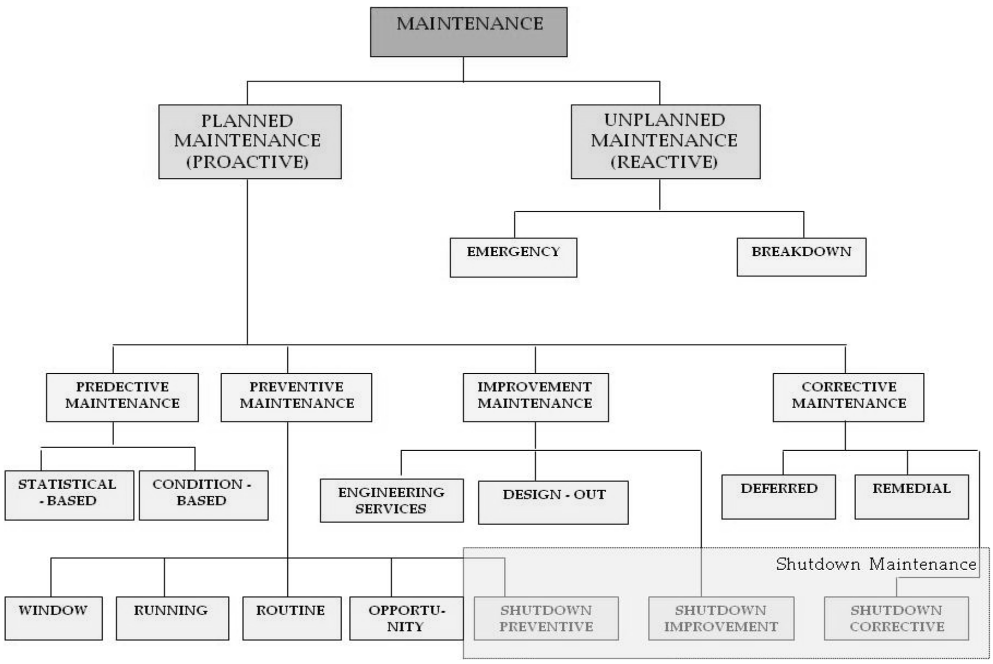



**Figure 2.2: Maintenance Policies Structure**

# 3. Reliability and Maintenance Engineering

## *3.1. Introduction*

The word "Maintenance" comes from Latin *manutere* (keep in hand) and meant the actions previewed to maintain in good conditions, the military units which were exhausted by battles.

Nowadays, in a similar way, maintenance is defined as "the combination of all technical and administrative actions, including supervision actions, intended to retain an item in (components, device, functional unit, equipment or system), or restore it to, a state in which it can perform a required function" [3.2].

More precisely, a component is a generic item (equipment, machine, plant) which survival behavior is known. On the contrary, a complex system is a set of components connected to satisfy a specific need. The reliability function of the system is not known, while that of the generic component is done. In fact, it can be calculated thanks to the survival analysis, applied to the components belonging to the system.

Several alternative approaches to the maintenance of a component or a system have been studied.

None of these can be defined as the best developed one, because the efficacy of a maintenance policy is strictly connected to the nature of the system to which it is applied.

It can be easily understood why maintenance represents a necessity for its function of assuring the availability of equipment and safety for personnel and goods. Unfortunately for many decades, and partly also today, maintenance is considered in a very reductive way as a "necessary evil", neglecting in this way the remarkable potential in terms of costs reduction and competitive improvement of companies.

In Italy, preventive maintenance policy is only used from 55% of firms, while the remaining 45% follows the traditional corrective maintenance; on the contrary, in the US, percentages are respectively 85% against 15% [3.4].

In the last few years, the concept of productive maintenance was introduced, that is the combination of actions executed for prevention and continuous improvement, using data collection and diagnostic on units to be maintained.

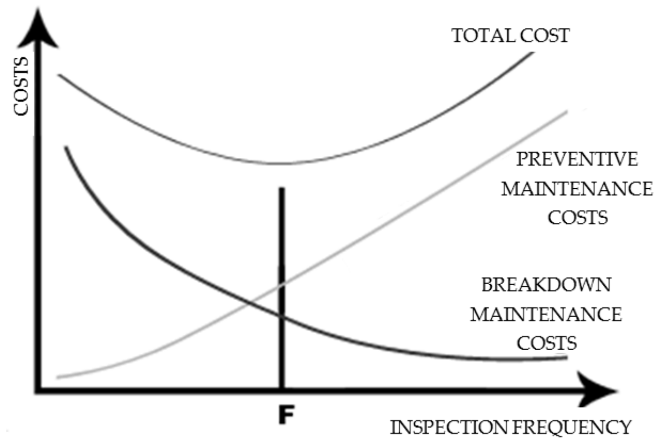



**Figure 3.1: Effects of productive maintenance on total costs.**

The aims of this philosophy concern the improvement of the OEE and the reduction of total costs (figure 3.1), through the use of innovative techniques and focused and complex interventions, in order to increment the firm profits.

Therefore it appears the importance of maintenance to guarantee safety, quality and reliability.

Later on, the fundamentals of reliability theory, starting from the failure analysis, will be illustrated [3.1-3.2-3.4-3.5-3.6].

## *3.2. Survival Fundamentals*

### 3.2.1. Failures Definition

The components and the systems that are currently designed, realized and developed, are tested during their operative life from a remarkable quantity and variety of stress (friction, abrasion, erosion, corrosion, thermal shocks, crashes... ) that bring to different types of fault that can be classified in the following structure:

- for their own <u>nature:</u>
  - Intrinsic: imputable to weaknesses belonging to the component;
  - External: imputable to the application of stresses, which are bigger than the specific possibilities of the component;
- for their <u>way of appearing</u>:
  - Progressive: those who could be prevented with a preliminary inspection;
  - Sudden: faults that cannot be prevented with a preliminary inspection;

- for their <u>magnitude</u>:
    - Partial: failures that alter the characteristics of the component, beyond the specified limits but without causing the total loss of performances;
    - Complete: those that cause the total loss of characteristics;
- for their <u>way of appearing and their magnitude</u>:
    - Catalectic: sudden and complete;
    - For deterioration: progressive and partial;
- for their <u>effects</u>:
    - Minor: not harming the good conclusion of the mission;
    - Major: harming the good conclusion of the mission;
    - Critical: harming the good conclusion of the mission and creating some serious risks;
- for their <u>appearing time</u>:
    - Premature: failures that occur at the beginning of the component life and whose appearing rate is quickly decreasing;
    - Random: failures that occur with a rate almost constant during the operational life (typical of electric components);
    - Wear-Out: failures that occur with a quickly increasing rate (typical of mechanical components).

### 3.2.2. Time to Failure

The under introduced reliability concept is strictly connected to the functioning and non functioning concept, and therefore to the failure process of a generic component or system. This process depends on several factors, many of which are unpredictable. For this reason, time to failure is not a deterministic variable but only random. To indicate this stochastic variable, the Greek $\tau$ letter was chosen, also called TTF (Time To Failure).

Not always the behavior of $\tau$ can be described through a known statistical distribution (normal, lognormal, exponential, etc.).

However, let $f(t)$ be the probability density function (pdf) of $\tau$ values, the following equation can be introduced:

$$P(\tau \le T) = \int_{-\infty}^{T} f(x) \cdot dx \qquad 3\text{-}1$$

This equation represents the probability that the stochastic variable is a value lower than T; while

$$\int_{-\infty}^{\infty} f(x) \cdot dx = 1 \qquad 3\text{-}2$$

represents the condition of normalization common to all statistical probability density functions. The function $f(t)$ is also called non conditional failure rate, because it is a punctual measure of speed with which a generic component breaks down in a time $t$, when $t$=0 is in operation.

### 3.2.3. Repairable and Non-Repairable Components

A distinction between repairable and non-repairable components is fundamental, since the analytical models that describe their survival behavior remarkably differ.

The acceptation "non-repairable" regards all the components that cannot be repaired after their failure and are replaced with other ones; while "reparable" is related to the components that can be repaired after their failure and therefore are subjects to several cycles of working- failure - repair. These definitions show that non-repairable components are some special cases of the repairable ones, therefore they can be treated with survival models, developed for repairable components. Consequently, models that describe the non-repairable components are used for repairable ones, when usually an industrial engineering wants to study their first failure.

The following concepts and the studied experimental analysis exclusively concern the non-repairable components.

### 3.2.4. Reliability Definition

Nowadays, the reliability of a product is design and assured with prevention methodologies. To prevent means above all to consider all the precautions in order not to let undesired phenomenon occur, such as failures.

The concept of failure does not only refer to considerable malfunctioning such as faults or blocks, but it also considers all the reactions of the products that are not included in the related "range of acceptability from the customer". This requires the identification of all the expected functions of the customer that the product must satisfy and, for each of them, the identification of a range of values for its reaction. Exceeding it the customer will be unsatisfied. Essentially, it means assuming some tolerances on the expected functions.

But it is not sufficient for a product to perform reactions within its related ranges of acceptability, only when it is new. It has to keep this characteristic in time, which means that the entity of the unavoidable operative decline has to result restrained, as required by the customers, at least during defined operation periods. This is called reliability in time, and corresponds to the classical idea of reliability.

More precisely, reliability is the attitude of a component to perform a required function under working conditions and for a specific period of time.

Studies on reliability of components and systems fulfilled important progresses as for the statistical issues are concerned.

Some functions have been developed to give analytical models to the problem.

### 3.2.5. Reliability Function *R(t)*

If the lifetime of a big number of identical devices under the same conditions is analyzed, and it is represented on a Cartesian diagram on x-axis the time value and on y-axis the rate of these components that still work properly in that given time, the reliability function $R(t)$ is obtained (figure 3.2).

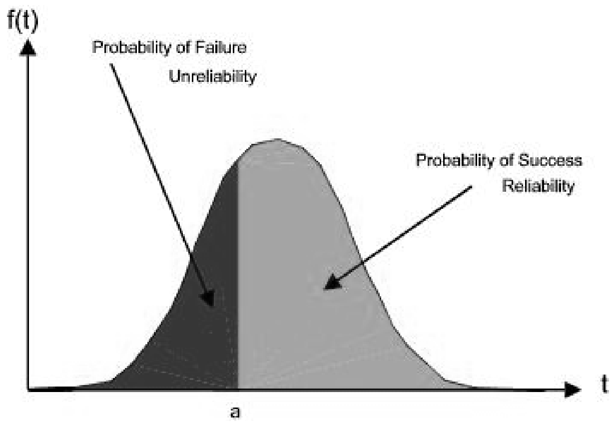



**Figure 3.2: Graphical interpretation of R(t).**

Therefore, reliability is not a punctual time-depended function; it is not defined on a singolar time t, but it is calculated on a specific time interval. In the formulas that are presented in this chapter, such as $r(t)$ or $R(t)$, the term $t$ identifies the range: $T = t - t_0$.

From the previous definitions of Time to Failure $\tau$ and its probability density function $f(t)$, reliability can be expressed:

$$R(t) = \int_T^{\infty} f(x)dx \qquad 3\text{-}3$$

If the following assumptions and definitions are considered:

- $N$ number of components working at time $t_0 = 0$;
- $N_G(t)$ number of components damaged at a generic time $t$;
- $N_S(t)$ number of components running at time $t$.

From the introduced definitions, the subsequent equations follow:

$$N_S(t) = N - N_G(t)$$

$$\lim_{t \to \infty}\left(\frac{N_G(t)}{N}\right) = 1$$

Next, the survival parameters $f(t)$ and $F(t)$ will be illustrated and defined in a more detailed way.

### 3.2.6. Probability Density Function of Failure *f(t)*

The probability density function $f(t)$ defines the instantaneous speed of failure of generic component, working at starting time $t_0 = 0$.

The following statistical equation is also useful to the reliability theory:

$$f(t) \cdot dt = P(t \le \tau \le dt) = \int_t^{t+dt} f(x)dx \qquad 3\text{-}4$$

### 3.2.7. Unreliability or Cumulative Density Function *F(t)*

The Cumulative Distribution Function $F(t)$ is a unique function which gives the probability that the component or the system will fail by time $T$.

So the following equations can be stated:

$$F(t) = P(-\infty \le \tau \le t) = \int_{-\infty}^{t} f(x) \cdot dx = P(0 \le \tau \le t) = \int_{0}^{t} f(x) \cdot dx \qquad 3\text{-}5$$

$$f(t) = \lim_{\Delta t \to 0} \frac{F(t+\Delta t) - F(t)}{\Delta t} = \frac{dF(t)}{dt} \qquad 3\text{-}6$$

Therefore $f(t)$ is the derivate of $F(t)$.

Reliability can be expressed by:

$$R(t) = \frac{N_S(t)}{N} = \frac{N - N_G(t)}{N} \qquad 3\text{-}7$$

while failure distribution function $F(t)$ can be explained with the following formula:

$$F(t) = \frac{N_G(t)}{N} = \frac{N - N_S(t)}{N} = 1 - R(t) \qquad 3\text{-}8$$

### 3.2.8. Hazard Rate Function *λ(t)*

Hazard Rate function $\lambda(t)$, is a very important survival function that represents the failure behavior of a component, and more precisely it expresses its failure rate. It is a instantaneous rate and is defined by:

$$\lambda(t) \cdot \Delta t = P(t \le \tau \le t + \Delta t) \qquad 3\text{-}9$$

Therefore, $\lambda(t)$ is defined as the probability that a failure per unit time occurs in the interval, given that a failure has not occurred prior to $t$, the beginning of the interval.

Known $\lambda(t)$, the survival functions $R(t)$ and $F(t)$ can be expressed by the following formulas:

$$R(t) = e^{(-\int_{0}^{t} \lambda(t) \cdot dt)} \qquad 3\text{-}10$$

$$F(t) = 1 - e^{(-\int_{0}^{t} \lambda(t) \cdot dt)} \qquad 3\text{-}11$$

### 3.2.9. Mean Time To Failure (MTTF)

Suppose that the reliability function for a system is given by R(t). the expected failure time during which a component is expected to perform successfully, or the system mean time to failure (MTTF), is given by:

$$MTTF = \int_0^t t \cdot f(t)dt = -\int_0^\infty t \cdot \frac{dR(t)}{dt} \cdot dt \qquad 3\text{-}12$$

Thus, MTTF is the definite integral evaluation of the reliability function. In general, if $\lambda(t)$ is defined as the failure rate function, then, by definition, MTTF is not equal to $1/\lambda(t)$.

### 3.2.10. Singular Case: Constant Hazard Rate

Assuming the hazard rate $\lambda(t)$ a constant value $\lambda$ , the following simplified equations can be traced:

$$R(t) = e^{-\lambda \cdot t} \qquad 3\text{-}13$$

$$F(t) = 1 - e^{-\lambda \cdot t} \qquad 3\text{-}14$$

$$f(t) = \frac{dF(t)}{dt} = \lambda e^{-\lambda \cdot t} \qquad 3\text{-}15$$

$$MTTF = \int_0^\infty R(t)dt = \frac{1}{\lambda} \qquad 3\text{-}16$$

These equations demonstrate that if a constant hazard rate is given, the time to failure $\tau$ follows an exponential distribution. As a result, the failure rate does not depend on time: failure is a random process and the component doesn't age.

Since $\lambda(t)$ is constant, furthermore, its opposite $1/\lambda(t)$ represents the Mean Time to Failure (MTTF).

### 3.2.11. Bathtub Curve: Description of Hazard Function *λ(t)*

In practice, the hazard rate is described by different trends, according to the component characteristics and the use which it is designed for.

For mechanical components subject to wear-out, the hazard rate starts to decrease from release time (where the so-called infant mortality is common to happen), and stabilize at a more or less constant level that usually last for most part of the component life. In this period the hazard rate $\lambda(t)$ can be seen as the addition of infant mortality hazard rate, which is slowly increasing, and hazard rate, slowly increasing, caused by ware-out and degradation phenomenon.

But from a certain time, this last component becomes preponderant and hazard rate $\lambda(t)$ trends to increase quickly as time goes by. Studying this last phase is very important because it would be a real advantage to replace a component in the moment when the hazard rate exceeds a fixed upper bound.

These three phases are in succession in a mechanical system and highlight a "bathtub" trend of hazard rate function $\lambda(t)$ (figure 3.3).

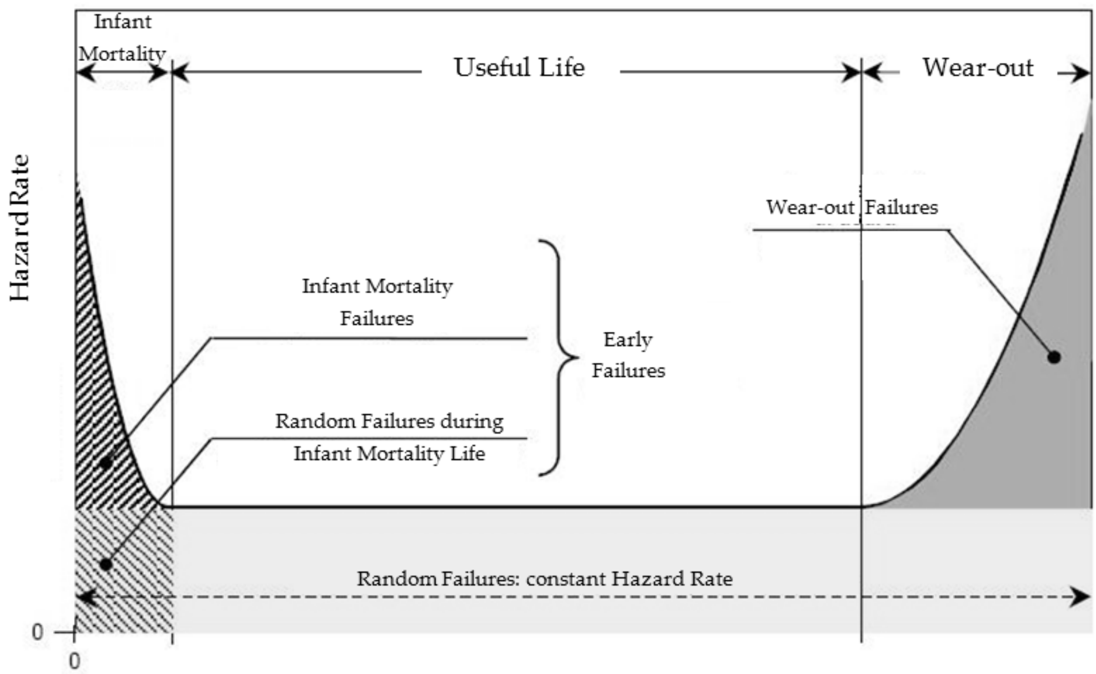



**Figure 3.3: Hazard rate function: Bathtub curve**

The main part of electrical and electronic components are characterized by a constant hazard rate $\lambda(t)$. The probability that a component fails in a generic time $t_1$ is the same that if it fails in a different time $t_2$, which means that its failure is random.

An increasing and constant hazard rate is typical of building components, such as protection and insulations, where wear-out process is slow but continue.

In industrial application, it is well seen that the survival functions *R(t)* and *f(t)* can be represented by:

- Exponential distribution for random failure process, with constant hazard rate;
- Normal distribution for wear-out process, with increasing hazard rate;
- Weibull distribution for both cases, using its flexibility thanks to its three parameters.

## *3.3. System Reliability Calculation*

The functional structures describe the physical connections of a system, for example a complex industrial plant, in order to understand the functioning.

Instead, the reliability structures define the functioning and non-functioning logics of the system, based on the physical connections and operational conditions.

### 3.3.1. Series Configuration

In this configuration, the components of generic system "S" are all essential for the correct functioning. A series system is a configuration such that, if any one of the system components fails, the entire system fails. Conceptually, a series system is one that is as weak as its weakest link.

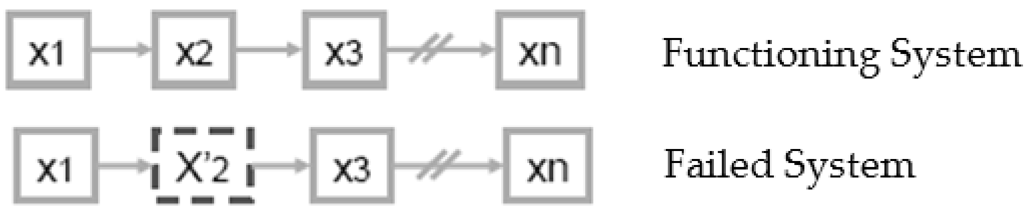



**Figure 3.4: Series Configuration Systems**

The reliability of system "S" is expressed by the following statistical equation:

$$R_S(T) = r_1(T) \cdot r_2(T) \cdot \ldots \cdot r_n(T) = \prod_{i=1}^{n} r_1(T) = e^{-\int_0^T \lambda(t)dt} = e^{-\int_0^T \sum_{i=1}^{n} \lambda_i(t)dt} \tag{3-17}$$

where:

$r_i(T)$: reliability of component i by time T;

$\lambda_i$: hazard rate of component i;

$n$: number of components in series.

From the previous formula, next general equation can be calculated:

$$\lambda_S(t) = \sum_{i=1}^{n} \lambda_i(t) \tag{3-18}$$

Consequently, if the generic hazard rate $\lambda_i$ is constant $\forall i = 1,\ldots,n,$ the previous equation gives a constant value: it means that the system has a random failure behavior, too. In other words, it does not exist any time interval where the system has a bigger probability to be damaged.
From the general equation of MTTF of a component, MTTF for series systems is explain below:

$$MTTF = \int_0^{\infty} R_S(t)dt = \int_0^{\infty} e^{-\int_0^t \lambda_S(x)dx} dt \tag{3-19}$$

Therefore, if all components have a constant hazard rate, it can be stated that:

$$MTTF = \frac{1}{\lambda_S} = \frac{1}{\sum_{i=1}^{n} \lambda_i(t)} = \frac{1}{\sum_{i=1}^{n} \frac{1}{MTTF_i}} \tag{3-20}$$

With the series configuration, the reliability of a system is lower than components' one. If it is important to increase the reliability of the system, using the same configuration, it is necessary to operate on components with lower value of reliability.

### 3.3.2. Parallel Configuration

A parallel system is a configuration such that, as long as not all of the system components fail, the entire system works. Conceptually, in a parallel configuration the total system reliability is higher than the reliability of any single system component.

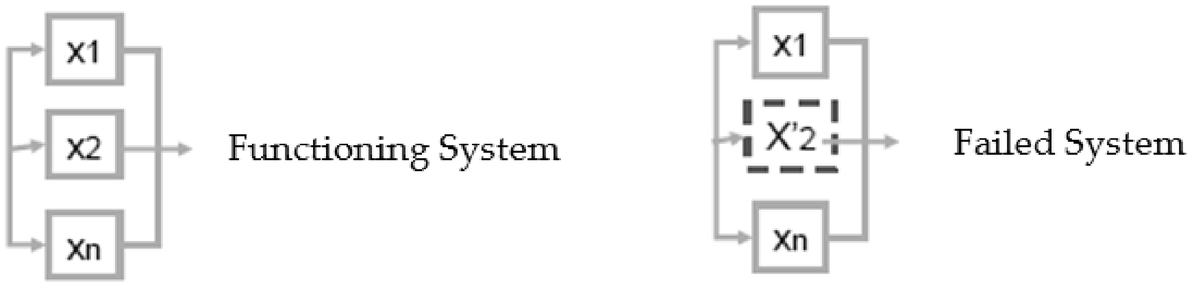



**Figure 3.5: Parallel Configuration Systems**

The reliability of system "S" is described by the following:

$$R_S(T) = 1 - F_S(T) = 1 - (1 - r_1(T)) \cdot \ldots \cdot (1 - r_n(T)) = 1 - \prod_{i=1}^{n} \left[1 - r_1(T)\right] = \coprod_{i=1}^{n} r_1(T) \qquad 3\text{-}21$$

where:

$r_i(T)$ : reliability of component i by time T;

$\lambda_i$ : hazard rate of component i;

$n$ : number of components in series.

$F_S(T)$ : unreliability of system "S".

Hence:

$$R_S(T) = 1 - \prod_{i=1}^{n} \left[ 1 - e^{-\int_0^T \lambda_i(t) \cdot dt} \right] = \coprod_{i=1}^{n} e^{-\int_0^T \lambda_i(t) \cdot dt} \qquad 3\text{-}22$$

If it assumes that the generic hazard rate $\lambda_i$ is constant and equal for all components of the system ($\lambda_i = \lambda \ \ \forall i = 1,..,n$), the previous equation is modified in this particular form:

$$R_S(T) = e^{-\int_0^T \lambda_S(t) \cdot dt} = 1 - \left[1 - e^{-\lambda T}\right]^n \qquad 3\text{-}23$$

where $\lambda_S(t)$: hazard rate of system "S".

The value of $\lambda_S(t)$ is not constant, as demonstrated by the following equation:

$$\lambda_S(t) = \frac{f(t)}{R(t)} = \frac{n\lambda e^{-\lambda t}(1-e^{-\lambda t})^{n-1}}{1-(1-e^{-\lambda t})^n} \qquad 3\text{-}24$$

Consequently the Mean Time to Failure of system S is:

$$MTTF_S \neq \frac{1}{\lambda_S} \qquad 3\text{-}25$$

### 3.3.3. Parallel Configuration with Stand-by Component

The following figure shows the reliability structure of the parallel configuration with stand-by component or sequential parallel. In this case, a switch modifies the connection in order to guarantee the functioning of the system.

For example, if component A fails during its functioning, component B starts to operate as a replacement of A, thanks to switch S'

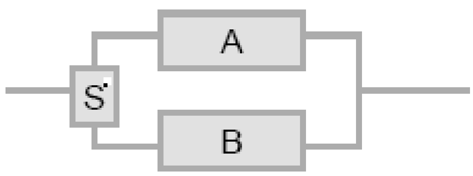



**Figure 3.6: Parallel Configuration with Stand-by component S'.**

Reliability of system "S" can be expressed by:

$$R_S(t) = R_I(t) + R_{II}(t) \qquad 3\text{-}26$$

where:

$R_I(t)$: reliability of component A;

$R_{II}(t)$: probability of situation where: A fails, B starts to operate and it is available.

The previous statistical equation can be introduced because the two terms are measure of probability of independent events, as shown in figure 3.7.

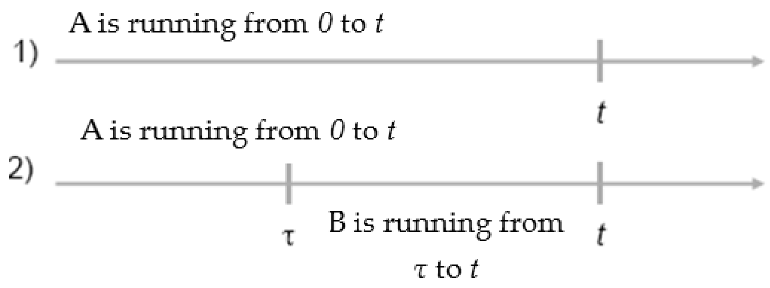



**Figure 3.7: Stand-by parallel functioning states**

Assuming that switch has reliability value equal to 1, the two equations related to $R_I(t)$ and $R_{II}(t)$ are defined as:

$$R_I(t) = R_A(t) \quad 3\text{-}27$$

$$R_{II}(t) = \int_0^t f_A(\tau) \cdot R_B(t-\tau) d\tau \quad 3\text{-}28$$

hence:

$$R_S(t) = R_I(t) + R_{II}(t) = \exp\left[-\int_0^t \lambda_A(x)dx\right] + \int_0^t \left\{ f_A(\tau) \cdot \left[\exp(-\int_0^{t-\tau} \lambda_B(x)dx\right]\right\} d\tau \quad 3\text{-}29$$

In a particular case, when $\lambda_A(t) = \lambda_B(t) = \lambda$, reliability of the system is given by:

$$R_S(t) = e^{-\lambda t}(1+\lambda t) \quad 3\text{-}30$$

Under this assumption the general equation of MTTF of system can be calculated using:

$$MTTF = \frac{2}{\lambda} \quad 3\text{-}31$$

### 3.3.4. *k*-out-of-*n* Configuration

A k-out-of-n configuration can have n components, and requires that k of those n components must work for the system to function.

A practical example is about a feeding system of a coke oven with 5 conveyors. In order to have a correct feeding of oven, it can be necessary that at least 3 of 5 conveyors work properly.

This configuration is called majority logic configuration or also partial redundant configuration, because it is a particular case of parallel structure where not all the components are replacement as in traditional parallel configuration is.

The number of situations where $k$ of $n$ components are working is:

$$\binom{n}{k} = \frac{n!}{k! \cdot (n-k)!} \qquad 3\text{-}32$$

In order to better understand the concept of partial redundancy configuration and how the reliability of system can be calculated, the following example is reported in table. It is related to a system 2/3 (where $k = 2$, $n = 3$). Each row of the table shows the reliability of functioning configuration of system. The case A has the assumption of functioning of all components and reliability can be calculated using the series configuration formulas. The cases B, C and D are related to the situation of 2 working components of 3 total components of system.

| FUNCTIONING COMBINATIONS | RELIABILITY $R_j(t)$ |
|---|---|
| A 1,2,3 | $r_1 \cdot r_2 \cdot r_3$ |
| B 1,2 | $r_1 \cdot r_2 \cdot (1 - r_3)$ |
| C 2,3 | $r_2 \cdot r_3 \cdot (1 - r_1)$ |
| D 1,3 | $r_1 \cdot r_3 \cdot (1 - r_2)$ |

**Table 3.1: Functioning combinations for 2-out-of-3**

The sets in table are independent and consequently, the reliability of system $R_{k/n}$ is equal to:

$$R_{2/3}(t)=\sum_{j=A,B,C,D} R_j \qquad 3\text{-}33$$

This equation is related to the assumption that at least 2 components work well in global system. If $r_i = R \ (\forall i = 1,..,n)$ the previous equation can be expressed as:

$$R_{2/3}(t)=R_A+\binom{3}{2}R_B=R^3+\binom{3}{2}R^2\cdot(1-R)=3R^2-2R^3 \qquad 3\text{-}34$$

Under this assumption, if all components have constant hazard rate $\lambda$, MTTF of system can be determined by:

$$MTTF_{2/3}=\int_0^\infty\left(3\cdot e^{-2\lambda t}-2\cdot e^{-3\lambda t}\right)\cdot dt=\frac{5}{6}MTTF \qquad 3\text{-}35$$

Generally, assuming that at least $k$ of $n$ identical components, the equation of reliability of a *k*-out-of-*n* system can be extended, as follows:

$$R_{k/n}(t)=\sum_{i=k}^{n}\binom{n}{i}[r(t)]^i[1-r(t)]^{n-1} \qquad 3\text{-}36$$

where: $r(t)$ is the reliability of each component.

The general equation of MTTF is calculated with following formula, assuming that all components are characterized by constant hazard rate:

$$MTTF=\sum_{i=k}^{n}\frac{1}{\lambda\cdot i} \qquad 3\text{-}37$$

## 3.4. Lifetime Distributions

In order to evaluate the reliability function and its parameters, it is necessary to use a continue statistical distribution, where the variable is expressed in a continue time values. Next sub-sections illustrate several statistical distributions, which are the most used to estimate the survival functions.

### 3.4.1. Exponential Distribution

The exponential distribution has a relevant role in reliability field because of its main characteristic is the constant hazard rate. Mainly it is used to represent the lifetime of electric and electronic components and systems. This distribution can be applied to components subjected to random failures. Consequently components has not memory about the previous faults and they are not subject to aging phenomenon.
This assumption is rather strictly and for this reason the exponential distribution has to be used with several attention because there are a lot of cases where it cannot be applied.
Time to Failure $\tau$ is described by the exponential probability density function as follows:

$$f(t) = \lambda e^{-\lambda \cdot t} \qquad 3\text{-}38$$

Then reliability function is given by:

$$R(t) = e^{-\lambda \cdot t} \qquad 3\text{-}39$$

where $t \geq 0$ and $\frac{1}{\lambda}$ represents MTTF.

The hazard rate is constant and it is equal to $\lambda$:

$$h(t) = \frac{f(t)}{R(t)} = \lambda \qquad 3\text{-}40$$

The main reason of diffusion of this distribution is related just to the constant hazard rate and the simplicity of formulas. The exponential distribution is a good model for the middle part of bathtub curve. The bigger number of components spent a lot of time in this part of curve and this justify the use of exponential distribution, but it does not concern the initial failures and wear-out cases.

### 3.4.2. Normal Distribution

The normal distribution is very important because of Middle Limit Theorem. This distribution has two parameters and it is used to described the components and systems characterized by wear-out failures. The normal distribution, also known as Guassian distribution, is symmetric in respect to the mean value and its extension is measured by variance.

The probability density function is given by:

$$f(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2} \; , \; -\infty \leq t \leq \infty \; , \qquad 3\text{-}41$$

where $\mu$ is the mean value and $\sigma$ is the standard deviation.

The cumulative density function is expressed by:

$$F(t) = \int_{-\infty}^{t} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{s-\mu}{\sigma}\right)^2} ds \qquad 3\text{-}42$$

And reliability is given by:

$$R(t) = \int_{t}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{s-\mu}{\sigma}\right)^2} ds \qquad 3\text{-}43$$

There is not an analytical solution for previous equation 3-43. However, several tables for normal density function can be used to determine the probability for different normal distributions.

If this variable:

$$Z = \frac{T-\mu}{\sigma} \qquad 3\text{-}44$$

is inserted in the normal probability density function, the following equation can be obtained:

$$f(z) = \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} \; , \; -\infty \leq Z \leq \infty \qquad 3\text{-}45$$

This is the standard normal probability density function and it is characterized by mean value equal to 0 and standard deviation equal to 1.

The standard cumulative density function is given by:

$$\Phi(t) = \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} \cdot s^2} ds \qquad 3\text{-}46$$

Therefore, for a random normal variable $T$, with mean value $\mu$ and standard deviation $\sigma$, it can be defined:

$$P(T \leq t) = P\left(Z \leq \frac{t-\mu}{\sigma}\right) = \Phi\left(\frac{t-\mu}{\sigma}\right) \qquad 3\text{-}47$$

The hazard rate function for a normal distribution is increasingly monotonic over $t$, and it is demonstrated by $h'(t) \geq 0$ for each value of $t$.

Since:

$$h(t) = \frac{f(t)}{R(t)} \qquad 3\text{-}48$$

then:

$$h'(t) = \frac{R(t)f'(t) + f^2(t)}{R^2(t)} \geq 0 \qquad 3\text{-}49$$

The normal distribution is enough flexible and it is very useful, in particular when the failures are caused by cracks, corrosion, chemical reactions and wear-out phenomenon.

### 3.4.3. Log-Normal Distribution

The Log-Normal distribution is a very flexible model which can be used to described a large number of failure modes.

This distribution, applied to mechanical reliability, is able to model the failure probability of repairable system, compression and tensile stress of components.

The probability density function is given by:

$$f(t) = \frac{1}{\sigma t\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\ln t - \mu}{\sigma}\right)^2}, \ t \geq 0 \qquad 3\text{-}50$$

Where $\mu$ and $\sigma$ are parameters and $-\infty \leq \mu \leq \infty$ e $\sigma \geq 0$. It is important to highlight that $\mu$ and $\sigma$ are not the mean value and standard deviation of the distribution.

Its relation with the normal distribution, given by the logarithms of data, make easy its use to elaborate data set.

The cumulative density function is defined by:

$$F(t) = \int_0^t \frac{1}{\sigma s\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\ln s - \mu}{\sigma}\right)^2} ds \qquad 3\text{-}51$$

and it is related with the standard normal distribution Z by:

$$F(t) = P(T \le t) = P(\ln T \le \ln t) = P\left( Z \le \frac{\ln t - \mu}{\sigma} \right) \quad 3\text{-}52$$

Therefore, reliability is expressed by:

$$R(t) = P\left( Z \ge \frac{\ln t - \mu}{\sigma} \right) \quad 3\text{-}53$$

and the hazard rate is given by:

$$h(t) = \frac{f(t)}{R(t)} = \frac{\Phi\left( \frac{\ln t - \mu}{\sigma} \right)}{\sigma t R(t)} \quad 3\text{-}54$$

where $\Phi$ is the cumulative density function of normal distribution.

The log-normal model is very flexible to represent the lifetime of component in a very useful way.

The application fields are very similar to those of normal distribution, that are when the failures are caused by cracks, corrosion, chemical reactions and wear-out phenomenon.

### 3.4.4. Gamma Distribution

The Gamma distribution can be used as survival density function for components and systems with asymmetric failure distribution.

The probability density function of Gamma distribution is given by:

$$f(t) = \frac{t^{\alpha-1}}{\beta^{\alpha}\Gamma(\alpha)} e^{-\frac{t}{\beta}}, \; t \ge 0, \alpha, \beta > 0 \quad 3\text{-}55$$

where $\alpha$ is the shape parameter and $\beta$ is the scale one.

In equation 3-55, $\Gamma(\alpha)$ is the Gamma function, defined by:

$$\Gamma(\alpha) = \int_0^{\infty} t^{\alpha-1} e^{-t} dt, \text{ con } \alpha > 0$$

3-56

Hence:

$$R(t) = \int_t^{\infty} \frac{1}{\beta^{\alpha}\Gamma(\alpha)} s^{\alpha-1} e^{-\frac{s}{\beta}} \quad 3\text{-}57$$

The gamma distribution is a good model for several failure modes. Although, it is not used commonly to model the survival behavior of mechanical components, this distribution is widely used in Bayesian application.

### 3.4.5. Weibull Distribution

The exponential distribution is not often applied because of its characteristic of no-memory of occurred failure.

The Weibull distribution is a generalization of exponential one and it is commonly used to represent wear-out failures. However, it can be applied to model all parts of bathtub curve of hazard rate.

For this reason, this distribution is very flexible and suitable to study the lifetime of components with different hazard rate functions.

The probability density function with 3 parameters is:

$$f(t) = \frac{\beta(t-\gamma)^{\beta-1}}{\alpha^{\beta}} e^{-\left(\frac{t-\gamma}{\alpha}\right)^{\beta}}, \quad t \le \gamma \le 0 \qquad 3\text{-}58$$

where $\alpha$ and $\beta$ are known respectively as shape and scale parameters, while $\lambda$ is the location one. These parameters are always positive. Using different values for these parameters, the weibull distribution can be adapt to exponential, normal and other distributions.

For $t \ge \gamma$, the reliability function is given by:

$$R(t) = e^{-\left(\frac{t-\gamma}{\alpha}\right)^{\beta}} \quad \text{with } t \ge \gamma > 0, \beta > 0, \alpha > 0 \qquad 3\text{-}59$$

Hence:

$$h(t) = \frac{\beta(t-\gamma)^{\beta-1}}{\alpha^{\beta}}, \quad t \ge \gamma > 0, \beta > 0, \alpha > 0 \qquad 3\text{-}60$$

The hazard rate function decreases for $\beta < 1$, increases for $\beta > 1$, and it is constant for $\beta = 1$ (figure 3.8)

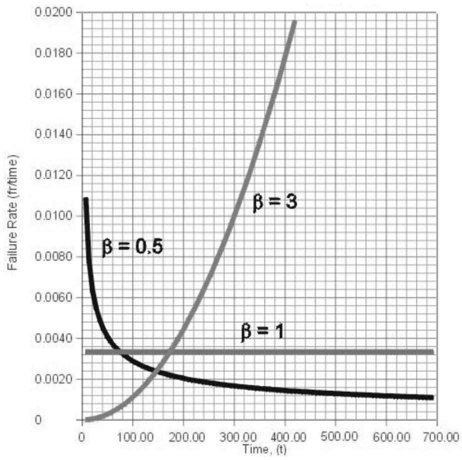



**Figure 3.8: Hazard rate function varying $\beta$ parameter.**

Therefore, the exponential distribution function is a particular case of Weibull distribution when $\beta = 1$ and $\gamma = 0$. Using this parameters values, the reliability function expressed by Weibull model, is given by:

$$R(t) = e^{-\frac{t}{\alpha}} \qquad \text{3-61}$$

And the hazard rate is equal to $1/\alpha$.

In figure 3.9 the probability density function $f(t)$ and reliability $R(t)$, varying $\beta$ parameters, are well shown.

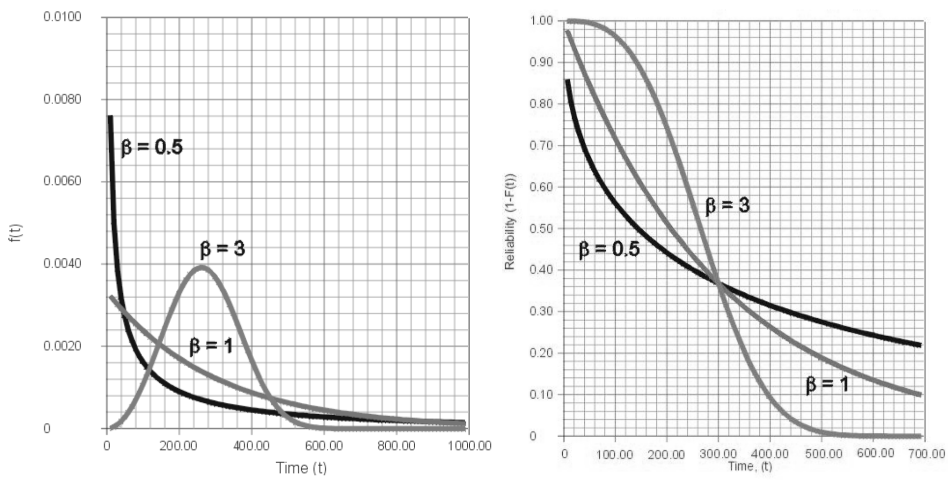



**Figure 3.9: $f(t)$ and $R(t)$ function varying $\beta$ parameter.**

The $\beta$ values, used in above inserted, represents the 3 parts of "bathtube curve", that are:

- $\beta = 0.5$: it represents the infant mortality characterized by decreasing hazard rate;
- $\beta = 1$: it models the middle part, that has a constant hazard rate and the failures are caused by random factors;
- $\beta = 3$: it defines the wear-out part, where hazard rate is increasing.

A simplified version of Weibull distribution is the two-parameters one: $\alpha, \beta$, assuming $\gamma = 0$. The fundamentals equation are modified as follows:

$$f(t) = \frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1} e^{-\left(\frac{t}{\alpha}\right)^{\beta}} \quad 3\text{-}62$$

$$R(t) = e^{-\left(\frac{t}{\alpha}\right)^{\beta}} \quad 3\text{-}63$$

$$h(t) = \frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1} \quad 3\text{-}64$$

## 3.5. *Reliability Estimation: Data from field*

The estimation of reliability functions from data collected on field is very important. Generally, several data of operating field are collected, in more details are related to failures, such as number of working cycle, travelled kilometres or time before the component fault. In the most part of cases they are time to failure and removed time (the components fail and test is finished, otherwise they are replaced during the test phase then their information are not available).
The goal is to determine an estimation of fundamental reliability functions, in particular the failure cumulative density function $\hat{F}(t)$, the survival functions, such as the reliability $\hat{R}(t)$ and hazard function $\hat{\lambda}(t)$.

### 3.5.1. Complete Data and Censored Data

Generally, starting from a population of $n$ components, Time To Failure can be estimated for each one. This condition is also called complete data and it means that all Times To Failure of $n$ components are available.

Often, this situation is difficult that it happens because it requires a lot of time and many information to be collected.

Therefore, in industrial field, failure test are concluded before all components have failed, or many other have reached their work objective, so the real Times To Failure are unknown.

In this situation, data collected are also called as censured data.

In other words, most types of failure data, as well as some life data, are as complete data. Complete data means that the value of each sample unit is observed or known. In many cases, life data contains uncertainty as to when exactly an event happened (i.e. when the unit failed). Data containing such uncertainty as to exactly when the event happened is termed as censored data.

***Complete Data***

Complete data means that the value of each sample unit is observed or known. For example, if it has had to compute the average test score for a sample of ten students, complete data would consist of the known score for each student. Likewise in the case of life data analysis, the data set (if complete) would be composed of the times-to-failure of all units in the sample. For example, if five units were tested and they all failed (and their times-to-failure were recorded), then complete information as to the time of each failure in the sample is given.

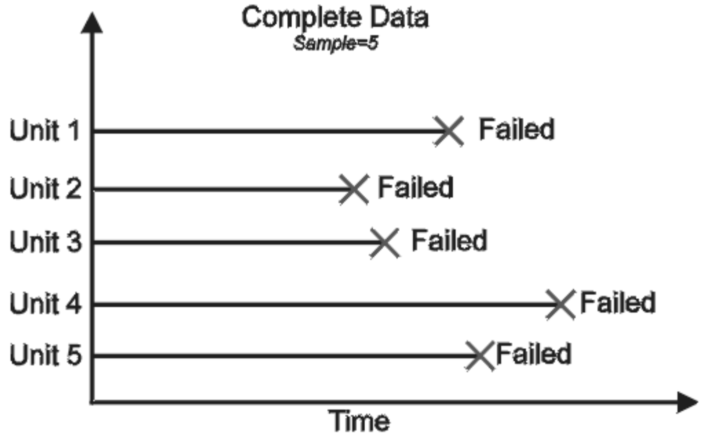



**Figure 3.10: Example of Complete Data Set**

Complete data is much easier to work with than censored data. For example, it would be much harder to compute the average test score of the students if the data set were not complete, i.e.

the average test score given scores of 30, 80, 60, 90, 95, three scores greater than 50, a score that is less than 70 and a score that is between 60 and 80.

*Censored Data*

In many cases when life data are analyzed, all of the units in the sample may not have failed (i.e. the event of interest was not observed) or the exact times-to-failure of all the units are not known. This type of data is commonly called censored data . There are three types of possible censoring schemes, right censored (also called suspended data), interval censored and left censored.

- *Right Censored (Suspended)*

  The most common case of censoring is what is referred to as right censored data, or suspended data. In the case of life data, these data sets are composed of units that did not fail. For example, if five units were tested and only three had failed by the end of the test, data (or right censored data) for the two unfailed units would be have suspended. The term "right censored" implies that the event of interest (i.e. the time-to-failure) is to the right of the data point. In other words, if the units were to keep on operating, the failure would occur at some time after the data point (or to the right on the time scale).

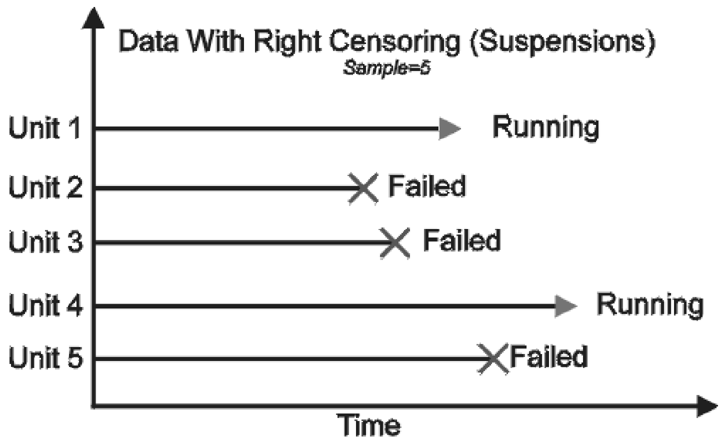



**Figure 3.11: Example of Right Censoring Data Set**

- *Interval Censored*

  The second type of censoring is commonly called interval censored data. Interval censored data reflects uncertainty as to the exact times the units failed within an interval. This type of data frequently comes from tests or situations where the objects of interest are not constantly monitored. If a test on five units is running and inspecting them every 100 hours, it is only known that a unit failed or did not fail between inspections. More specifically, if a certain unit at 100 hours is inspected and found it is operating and then perform another inspection at 200 hours to find that the unit is no longer operating, it is known that a failure occurred in the interval between 100 and 200 hours. In other words, the only available information is that it failed in a certain interval of time. This is also called inspection data by some authors.

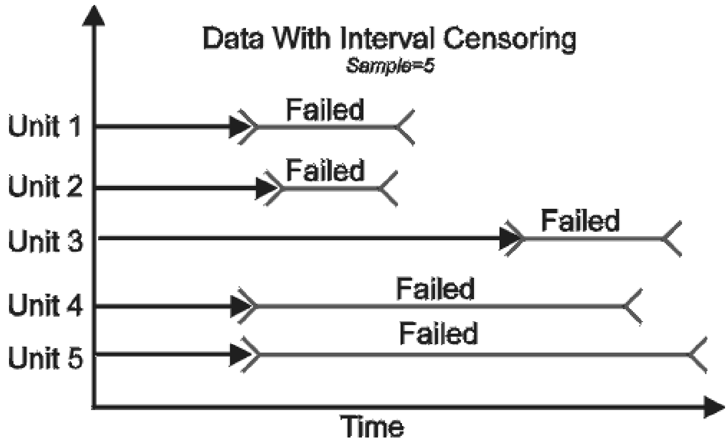



**Figure 3.12: Example of Interval Censoring Data Set**

- *Left Censored*

  The third type of censoring is similar to the interval censoring and is called left censored data. In left censored data, a failure time is only known to be before a certain time. For instance, it may be known that a certain unit failed sometime before 100 hours but not exactly when. In other words, it could have failed any time between 0 and 100 hours. This is identical to interval censored data in which the starting time for the interval is zero.

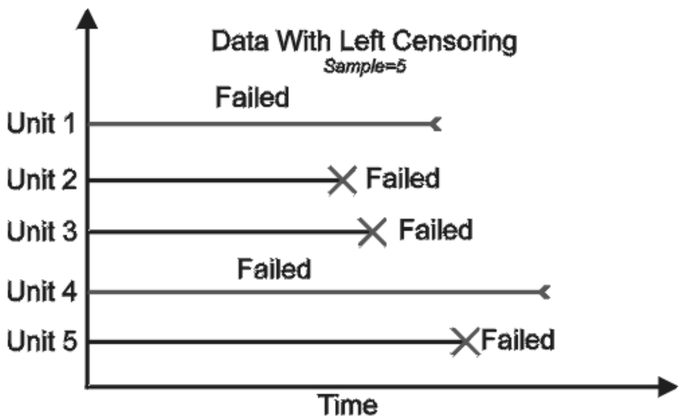



**Figure 3.13: Example of Left Censoring Data Set**

### 3.5.1. Evaluation of Reliability Functions

The main approaches to estimate the reliability functions from real collected data set are generally two, used both for complete and censored data.

The first approach derives directly empirical survival functions and it is called Empirical Function Direct to Data (EFDD).

The second one, called Theoretical Distribution Research (TDR), is more complicated and fit the theoretical distributions, such as Weibull, Exponential, Normal etc.

Generally, TDR is used after EFDD. For both presented approaches several methods have been developed and introduced in scientific literature.

Next sub-sections face in more details these two approaches.

### 3.5.2. Empirical Functions Direct to Data (EFDD)

Empirical methods, also called non parametric methods, have the goal to evaluate the main survival functions, such as $F(t)$, $R(t)$ e $\lambda(t)$, directly from collected data set from field. These data are related to Time To Failure of Removed Time. The functions calculated using these approaches are indicated as: $\hat{F}(t)$, $\hat{R}(t)$, $\hat{\lambda}(t)$.

The empirical method is extremely simple to use and very quick. Usually, the survival function curves are used to evaluate the estimation of empirical methods.

### 3.5.3. Complete Data – Direct Method

Given $t_1, t_2, t_3, ..., t_n$, where $t_i \leq t_{i+1}$, are $n$ ordered Times To Failures belonged to a random sample, and $i$ is the number of faults occur before time $t_i$. The estimation of reliability function $R(t)$ at time $t_i$ can be calculated using the quotient of running components at time $t_i$ over the total number of components $n$:

$$\hat{R}(t_i) = \frac{n-i}{n} = 1 - \frac{i}{n} \qquad \text{3-65}$$

This equation permits to evaluate the cumulative density function $\hat{F}(t)$:

$$\hat{F}(t_i) = 1 - \hat{R}(t_i) = 1 - \frac{n-i}{n} = \frac{i}{n} \qquad \text{3-66}$$

Using the definition of probability density function $f(t)$ and hazard function $\lambda(t)$, from the previous equation, it can be defined:

$$\hat{f}(t) = \frac{dF(t)}{dt} = \frac{-dR(t)}{dt} \cong -\frac{R_{(t_i+1)} - R_{(t_i)}}{t_{i+1} - t_i} = \frac{1}{n(t_{i+1} - t_i)} \quad \text{for } t_i < t < t_{i+1} \qquad 3\text{-}67$$

$$\hat{\lambda}(t) = \frac{\hat{f}(t)}{\hat{R}(t)} = \frac{1}{(t_{i+1} - t_i)(n-1)} \quad \text{for } t_i < t < t_{i+1} \qquad 3\text{-}68$$

If $\hat{F}(t_i)$ is calculated, where $i = n$: $F(t_n) = n/n = 1$, then the probability that a component runs at time $t_n$ is equal to zero.

Since it is unlikely that any analyzed sample contains the longest survival time. For this reason this method underestimate the reliability of components, $\hat{R}(t_i)$.

### 3.5.4. Complete Data – Improved Direct Method

The Improved Direct Method allows a better estimation of cumulative distribution function as follows:

$$\hat{F}(t_i) = \frac{i}{n+1} \qquad 3\text{-}69$$

Several applications on scientific literature of this method have obtained better results respect to Direct Method and for this reason it is widely known and used.

It is simply to estimate:

$$\hat{R}(t_i) = 1 - \hat{F}(t_i) = 1 - \frac{i}{n+1} = \frac{n+1-i}{n+1} \qquad 3\text{-}70$$

$$\hat{f}(t) = \frac{dF(t)}{dt} = \frac{-dR(t)}{dt} \cong -\frac{R_{(t_i+1)} - R_{(t_i)}}{t_{i+1} - t_i} = \frac{1}{(t_{i+1} - t_i)(n+1)} \quad \text{for } t_i < t < t_{i+1} \qquad 3\text{-}71$$

$$\hat{\lambda}(t) = \frac{\hat{f}(t)}{\hat{R}(t)} = \frac{1}{(t_{i+1} - t_i)(n+1-i)} \quad \text{for } t_i < t < t_{i+1} \qquad 3\text{-}72$$

### 3.5.5. Complete Data - Median Rank Method

The Improved Direct Method gives the mean plotting position for the i-th ordered failure. An alternative approaches is based on median. That method is better when the distribution of $F(t)$ is near $i$ values near zero and near to $n^3$.

The Median Rank MR depends on failure order $(i)$ and on the number of components $n$; it represents the value of $F(t)$ by which the probability to have $i$ or more faults is equal to 0,50.

Analytically MR is calculated with:

$$\sum_{k=i}^{n}\binom{n}{k}MR^k(1-MR)^{n-k}=0{,}50 \qquad 3\text{-}73$$

Median Rank can be easily derived from previous equation as follows:

$$MR \approx \hat{F}(t_i)=1-\frac{i-0{,}3}{n+0{,}4}=\frac{n+0{,}7-i}{n+0{,}4} \qquad 3\text{-}74$$

$$\hat{f}(t)=\frac{dF(t)}{dt}=\frac{-dR(t)}{dt}\cong-\frac{R_{(t_i+1)}-R_{(t_i)}}{t_{i+1}-t_i}=\frac{1}{(t_{i+1}-t_i)(n+0{,}4)} \quad \text{for } t_i<t<t_{i+1} \qquad 3\text{-}75$$

$$\hat{\lambda}(t)=\frac{\hat{f}(t)}{\hat{R}(t)}=\frac{1}{(t_{i+1}-t_i)(n+0{,}7-i)} \quad \text{for } t_i<t<t_{i+1} \qquad 3\text{-}76$$

### 3.5.6. Mean Time To Failure Calculation and its Variance

Others important survival parameters are the Mean Time To Failure and its Variance.

From the data set, MTTF and Variance can be directly calculated using:

$$MTTF^*=\sum_{i=1}^{n}\frac{t_i}{n} \qquad 3\text{-}77$$

and

$$s^2=\sum_{i=1}^{n}\frac{(t_i-MTTF^*)^2}{n-1}=\frac{\sum_{i=1}^{n}t_i^{\,2}-n(MTTF^*)^2}{n-1} \qquad 3\text{-}78$$

If the sample size is enough large to permit the use of Central Limit Theorem, the confidence interval for MTTF, based on Student distribution, can be calculated as follows:

$$\Pr\left\{MTTF^*-t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}\le MTTF\le MTTF^*+t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}\right\}=(1-\alpha) \qquad 3\text{-}79$$

where:

$\alpha$ : confidence level;

$t_{\alpha/2,n-1}$ : Student distribution parameter.

### 3.5.7. Censored Data – Product Limit Estimator

Considering $n$ tested components and $r < n$ are the failures occurred. Test is suspended before $n$ failures, then data are right censored.

The estimation of $\hat{F}(t)$, $\hat{R}(t)$, $\hat{\lambda}(t)$ can be obtained, thanks to the approaches used for complete data, until the suspension time of test. This estimation will be right truncated.

Usually, for multiple censored data, $t_i$ defines the Time to Failure while $t_i^+$ represents the censored time. The distribution of lifetime for censored components can be considered equal to that of those non censored.

The Product Limit Estimator method is based on the Improved Direct method used for complete data.

Hence, using IDM approach:

$$\hat{R}(t_i) = 1 - \hat{F}(t_i) = 1 - \frac{i}{n+1} = \frac{n+1-i}{n+1} \qquad \text{3-80}$$

and

$$\hat{R}(t_{i-1}) = 1 - \hat{F}(t_{i-1}) = 1 - \frac{i}{n+1} = \frac{n+2-i}{n+1} \qquad \text{3-81}$$

Then:

$$\frac{\hat{R}(t_i)}{\hat{R}(t_{i-1})} = \frac{n+1-i}{n+2-i}, \qquad \text{or: } \hat{R}(t_i) = \frac{n+1-i}{n+2-i}\hat{R}(t_{i-1}) \qquad \text{3-82}$$

If censoring occurs at time $t_i^+$, the reliability at that time is estimate equal to that at time $t_{i-1}$. While, if the failure occurs before time $t_i$, reliability is given by the previous equation:

$$\hat{R}(t_i) = \frac{n+1-i}{n+2-i}\hat{R}(t_{i-1}) \qquad \text{3-83}$$

So, using this formula, it can be stated:

$$\hat{R}(t_i) = \left(\frac{n+1-i}{n+2-i}\right)^{\delta_i}\hat{R}(t_{i-1}) \qquad \text{3-84}$$

where:

$\delta_i = (1,0)$ (if the failure occurs at time $t_i$, if censoring occurs at time $t_i$)

$\hat{R}(0) = 1$

Therefore, the estimation of $\hat{F}(t), \hat{R}(t), \hat{\lambda}(t)$ is done using the equation of Improved Direct Method, adapting the values of $\hat{R}(t)$ only with $t_i$ corresponding to the Time To Failure.

### 3.5.8. Censored Data – Kaplan Meier Approach

Kaplan & Meier [3.3] have introduced a variation in Product Limit Estimator. Considering the ordered Times To Failure $t_i$ and $n_i$, the number of components at risk before the i-th fault, the estimated reliability function is given by:

$$\hat{R}(t_i) = \left(1 - \frac{1}{n_i}\right)^{\delta_i} \hat{R}(t_{i-1}) \qquad \text{3-85}$$

where:

$\delta_i = (1,0)$ (if the failure occurs at time $t_i$, if censoring occurs at time $t_i$)

$\hat{R}(0) = 1$

Also in this case, the estimation of $\hat{F}(t), \hat{R}(t), \hat{\lambda}(t)$ is done using the equation of Improved Direct Method, adapting the values of $\hat{R}(t)$ only with $t_i$ corresponding to the Time To Failure.

### 3.5.9. Censored Data – Rank Adjustment Method

This method is based on the determination of failure rank, which is influenced by the position of censored data. The base equation is given by:

$$\hat{R}(t_i) = \left(1 - \frac{i_{t_i} - 0{,}3}{n + 0{,}4}\right) \qquad \text{3-86}$$

Where $n$ is the total number of components and $i_{t_i}$ is the rank order of failure at time $t_i$.

In particular:

$$i_t = i_{t_i} + RI \qquad \text{3-87}$$

Where $RI$ is the rank increment:

$$RI = \frac{(n+1) - i_{t_i}}{1 + n^{**}} \qquad 3\text{-}88$$

$n^{**}$ represents the number of component at risk (considering also the present component).

The Rank Increment is elaborated for the successive Time To Failure, which follows a censored component, then it remains the same until a new censoring occurs. At first Time To Failure $i_{t_i}$ and $RI$ is equal to 1.

### 3.5.10. Theoretical distribution research (TDR)

Previous sub-sections have dealt with the methods to calculate the empirical distribution of survival function, based on the estimation of working and failure information of components, directly collected from the fields.

An alternative approach permit to determine the theoretical distribution also using the collected data from field.

Generally speaking it is preferred to used the second approach because it provides more information, in particular it permits the evaluation of reliability also out of the range of collected data set. Moreover the theoretical distribution can be used in more complex analysis in reliability engineering and maintenance policy decisions.

The starting point, also in this case, is the collection of Times To Failure. The calculation of Empirical Function Direct to Data (EFDD) is useful to adapt the right distribution.

When the data set includes both complete and censored data, the fitting process is the same, while different adapting processes are necessary for the estimation of cumulative function, according with the approaches, discussed in previous sub-sections.

The fitting of a theoretical distribution can be viewed as a two step process: first it is necessary to identify a distribution and second a test is conducted to evaluate the goodness of fit.

Both these phases have been widely developed in scientific literature. In the next sub-sections one of the most used approach is introduced and applied to estimate different theoretical distributions.

### 3.5.11. Least Square Method for the curve fitting

This method uses the linear regression and least square in the $y = a + bx$ form and it adapts it to the transformed data set that depends on considered theoretical distribution.

If the index of fit, called $r$, is near to 1 then the fitting is good. The most used distributions with this approach are the Exponential, Weibull and Normal ones.

### 3.5.12. Application of Least Square Method to Exponential Distribution

Considering the Exponential distribution, the cumulative distribution function is given by:

$$F(t) = 1 - e^{-\lambda t} \qquad 3\text{-}89$$

Applying the natural logarithm to each terms:

$$-\ln(1 - F(t)) = \ln\left(\frac{1}{1 - F(t)}\right) = \lambda t \qquad 3\text{-}90$$

Considering $y_i = \ln\left(\frac{1}{1 - \hat{F}(t)}\right)$ and $x_i = t_i$, the slope of the line represents an estimation of hazard rate $\lambda$.

Using the Least Square method in $y = bx$ form, the value of $b$ term is:

$$b = \hat{\lambda} = \frac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2} \qquad 3\text{-}91$$

The index of fit, expressed by Correlation Index, using the Least Square method, is given by:

$$r = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}\frac{(x_i - \bar{x})^2}{n}}\sqrt{\sum_{i=1}^{n}\frac{(y_i - \bar{y})^2}{n}}} \qquad 3\text{-}92$$

where $\bar{y}$ and $\bar{x}$ are respectively the mean value of $y_i$ and $x_i$, and $n$ is the number of available couple $(x_i, y_i)$.

### 3.5.13. Application of Least Square Method to Weibull Distribution

The Weibull cumulative distribution function gives:

$$F(t) = 1 - e^{-\left(\frac{t}{\theta}\right)^{\beta}} \qquad 3\text{-}93$$

Applying twice in sequence the natural logarithms:

$$\ln\ln\left(\frac{1}{1-F(t)}\right) = \beta \ln t - \beta \ln \theta \qquad 3\text{-}94$$

Defining $y_i = \ln\ln\left(\frac{1}{1-\hat{F}(t)}\right)$ and $x_i = \ln t_i$, the linear regression form is determined, in particular:

$$y_i = a + bx_i \qquad 3\text{-}95$$

where:

$$b = \hat{\beta} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2} \text{ and } a = -\hat{\beta}\ln\hat{\theta} = \bar{y} - b\bar{x} \qquad 3\text{-}96$$

Calculated $b$, $\hat{\theta}$ is easily determined.

### 3.5.14. Application of Least Square Method to Normal Distribution

Considering the cumulative distribution function $F(t)$ as a Normal distribution, it is possible to calculate the normalized variable $z$.

In particular:

$$F(t) = \phi(z) = \phi\left(\frac{t-\mu}{\sigma}\right) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\tau}} \cdot e^{\frac{y^2}{2}} dy \qquad 3\text{-}97$$

where $\sigma$ is the standard deviation and $\mu$ is the mean value of normal distribution.

The relation between $z$ and $\phi(z)$ can be briefly obtained with the inverse function of standard Normal distribution, available in literature.

Using the inverse function:

$$\phi^{-1}[F(t_i)] = \phi^{-1}[\phi(z_i)] = z_i = \frac{t_i - \mu}{\sigma} = \frac{t_i}{\sigma} - \frac{\mu}{\sigma} \qquad \text{3-98}$$

The previous function is linear over $t$, and thanks to this it is possible to apply the fitting process through the Least Square method considering:

$$y_i = z_i = \phi^{-1}[F(t_i)] \qquad \text{3-99}$$

$$x_i = t_i . \qquad \text{3-100}$$

Applying the Least Square method:

$$\hat{\sigma} = \frac{1}{b} \qquad \text{3-101}$$

$$\hat{\mu} = -a\hat{\sigma} = -\frac{a}{b} \qquad \text{3-102}$$

# 4. Systemability: the impact of environmental factors on reliability estimation

## *4.1. Introduction*

As well discussed in the introductory chapters, the estimation of reliability and availability parameters is extremely important in order to guarantee the correct measure of Overall Equipment Efficiency (OEE) of production and logistic systems.

Industrial applications often observe the difference between laboratory reliability test in standard conditions and component or system reliability when it is set in motion through different environments and real world conditions. As a matter of fact reliability variable is considerably influenced by environmental factors. Environmental factors may change failure rate, reliability and availability of systems. When a component or a system works in an operative plant, it reflects a reliability function that is usually different from the theory reliability but also from all its similar applications in other industrial plants. This concept also concerns a service or a logistic system and all the other systems in which the reliability calculation is extremely necessary: it is confirmed in several remote maintenance applications [4.25]. Usually parameters of survival function of a component or system are calculated during the testing phases, but environmental factors (i.e. operating temperature, vibrations, possible shocks, moisture etc.) can change the hazard rate of the components/systems and consequently their reliability functions during their production time. For this reason, it's difficult to estimate the real lifetime distributions of the system products. The operating environment is often unknown and it is different from the laboratory or standard environment. Therefore, in reliability engineering, conversion problems and synthesis of test results are often faced from different environments.

Incorrect estimation of reliability function could lead to the wrong functional design of the system and an incorrect definition of the appropriate maintenance policies to improve the efficiency of industrial systems.

The aim of this chapter is to overlap this problem by operational study of the new parameter introduced by Pham [4.30 – 4.31 – 4.32] called systemability. This approach is very suitable for theoretical modelling of manufacturing systems and its application is convenient and simple particularly in presence of components with Weibull distribution lifetime [4.5 – 4.6].

In the next section, some models are just briefly mentioned in order to define the state of the art and mathematical models available: a literature analysis is summarized in order to show this innovative concept. Section 4.3 introduces the mathematical function called systemability. Moreover, theoretical application of the systemability to several system configurations are illustrated in section 4.4. Section 4.5 contains details about systemability approach applied to two different real case studies. Section 4.5.1 is related to automatic filler for packaging of bottles of beer, while section 4.5.2 presents the development of systemability on motorcycle components, demonstrating the goodness-of-fit of this approach. Section 4.6 discusses the application of systemability on reliability growth modeling of industrial systems. The literature review on this topic demonstrates that no one global approach exists to estimate the reliability growth considering also the important impacts of environmental and operating conditions.

At the end of the chapter, some careful consideration about the capability of the application and how to pursue with the research in that field are shown.

## *4.2. Scientific Contributions on Environmental Effects and Reliability Estimation in Random Environments*

Reliability is well known as the probability that a component (system) meets a determined mission, for a determined time and in determined environment [4.33]. The operating environment is often unknown and it is different from the laboratory or standard environments. Most used reliability approaches suppose the lifetime distributions to be depending on time only, so test reliability functions are used to describe also the operating lifetimes. The environmental factors may change failure rate, reliability and availability of components, so traditional approaches could cause the incorrect estimation of reliability. For this reason some researches create several models to estimate the reliability in operating environments, using the data collected during the test phases. Cox [4.10] firstly studied the relation between the environmental conditions and the hazard rate, introducing the proportional and additive hazard rate. These models [4.4 – 4.8 – 4.13 – 4.14] have been widely used in several experiments where the time to failure depends on a group of covariates. These covariates are usually used to define qualitative and quantitative variables, representing different operating conditions, different environments, different treatments and so on. Badia et al. [4.4] applied the proportional and additive hazard rate for modelling the change of lifetime distribution of components. Environmental Factors (EF) have been defined to assess covariates. In fact, an environmental factor converts reliability data in one environmental condition into equivalent information in other ones, so EF are defined as the quotient of the mean lives X1 and X2 in two different environments [4.12 – 4.52 – 4.53 – 4.54 – 4.55]. In scientific literature, the definition of EF for different distributions has been developed and accepted. Wang et al. [4.52 – 4.53 – 4.54 – 4.55] have defined EF for gamma, normal, log-normal, inverse Gaussian distribution and Elsayed & Wang [4.12] have studied environmental factors for the binomial distribution.

In the software reliability field, many studies have developed several models to estimate the reliability of the software in different working conditions [4.29 – 4.45 – 4.47 – 4.48 – 4.56 – 4.57 – 4.59]. In this field, Non Homogeneous Poisson Process (NHPP) has been successfully applied to model the software failure, and it is widely used to determine when stop testing and release the software. Generally, these models however assume that the field environments are the same as a testing environment. So, models with environmental factor consideration are developed: Zhang et al. [4.57] propose a general NHPP model, based on proportional hazard rate, considering constant η the environmental factor. Pham et al. [4.29] introduce a new model, called Random Field Environment Model (RFE), where they describe the η environmental factor by gamma and beta distribution.

In mechanical engineering, several contributions have been given and widely accepted. Oh and Bai [4.24] have proposed some models to study the lifetime distributions based on the test data, the warranty data and additional field data after the warranty expires. They have illustrated the methods to estimate the maximum likelihood and so defined specific formulas for Weibull distribution. Attardi et al. [4.3] have studied the survival characteristics of a component installed in two different cars with different working conditions. They have introduced a mixed Weibull distribution which depends on the covariates through the Weibull scale parameters.

Abbassi et al. [4.1] have introduced an approach based to simulated annealing algorithm to estimate the parameters of Weibull distribution. Sohn et al. [4.44] proposed a random effects Weibull regression model for forecasting the occupational lifetime of the employees who join another company, based on their characteristics. Advantage of using such a random effects model is the ability of accommodating not only the individual. Ram and Tiwari [4.40] have estimated the reliability of a component through a Monte-Carlo simulation, with the introduction of the factor η that is greater than 1 if the operating condition is more stressful than the testing ones, otherwise, it is less than 1.

Sun et al. have introduce a new model enables maintenance personnel to predict the reliability of pipelines with different Preventive Maintenance (PM) strategies and hence effectively assists them in making optimal PM decisions.

Pham [4.30 – 4.31 – 4.32] recently introduced an innovative approach, called systemability. It is very innovative and interesting because it's quite different from literature studies described before; in fact, it calculates the reliability in random environment using, as starting data, the reliability obtained during the test, and processing it using a gamma distribution in particular, or a distribution that represents operating environments in general, which takes into consideration the EF. This is a fundamental condition for applications in real contexts.

## *4.3. Systemability concept: new approach for the assessment of Reliability in random Environments*

The traditional reliability definitions and its calculations have commonly been carried out through the failure rate function within a controlled laboratory-test environment. In other words, such a reliability function is applied to the failure testing data and, with the help of parameter estimation approaches, it then can be used to make predictions on the reliability of the system used in the field. The underlying assumption for such calculation is that the field (or operating) environments and the testing environments are the same.

By definition, a mathematical reliability function is the probability that a system will be successful in the interval from time 0 to time t, given by:

$$R(t) = \int_t^\infty f(s)ds = e^{-\int_o^t h(s)ds} \qquad 4\text{-}1$$

where *f*(*s*) and *h*(*s*) are the failure time density and failure rate function, respectively.

The operating environments are often unknown and yet different due to the uncertainties of environments in the field. A new look at how reliability researchers can take account of the randomness of the field environments into mathematical reliability modelling covering system failure in the field is great interest.

Pham [4.30 – 4.31 – 4.32] recently developed a new mathematical function, called *systemability,* considering the uncertainty of the operating environments in the function for predicting the reliability of systems.

**Notation**

| | |
|---|---|
| $h(t_i)$ | i[th] component hazard rate function |
| $R(t_i)$ | $i^{th}$ component reliability function |
| $\lambda_i$ | Intensity parameter of Weibull distribution for $i^{th}$ component |
| $\underline{\lambda}$ | $\underline{\lambda} = (\lambda_1, \lambda_2, \lambda_3 ..., \lambda_n)$ |
| $\gamma_i$ | Shape parameter of Weibull distribution for $i^{th}$ component |
| $\underline{\gamma}$ | $\underline{\gamma} = (\gamma_1, \gamma_2, \gamma_3 ..., \gamma_n)$ |
| $\eta$ | A common environment factor |
| $G(\eta)$ | Cumulative distribution function of $\eta$ |
| $\alpha$ | Shape parameter of Gamma distribution |
| $\beta$ | Scale parameter of Gamma distribution |

**Definition [4.14]:** *systemability* is defined as the probability that the system will perform its intended function for a specified mission time under the random operating environments.

In a mathematical form, the *systemabililty* function is given by:

$$R_s(t) = \int_{\eta} e^{-\eta \int_0^t h(s)ds} dG(\eta) \qquad 4\text{-}2$$

where $\eta$ is a random variable that represents the system operating environments with a distribution function *G*.

This new function captures the uncertainty of complex operating environments of systems in terms of the system failure rate. It also would reflect the reliability estimation of the system in the field.

If it assumes that $\eta$ has a gamma distribution with parameters $\alpha$ and $\beta$, *i.e.* $\eta \sim gamma(\alpha, \beta)$ where the pdf of $\eta$ is given by:

$$f_\eta(x) = \frac{\beta^\alpha x^{\alpha-1} e^{-\beta x}}{\Gamma(\alpha)} \quad \text{for} \quad \alpha, \beta > 0; \quad x \geq 0 \qquad \text{4-3}$$

then the systemability function of the system in eq. 4-2 using the Laplace transform is given by:

$$R_s(t) = \left[ \frac{\beta}{\beta + \int_0^t h(s)ds} \right]^\alpha \qquad \text{4-4}$$

## 4.4. Systemability Calculations

This section presents several systemability results and its variances of some system configurations such as series, parallel, and k-out-of-n systems. Considering the following assumptions:

1. A system consists of $n$ independent components where the system is subject to a random operational environment $\eta$.
2. $i^{th}$ component lifetime is assumed to follow the Weibull density function, *i.e.*
   - Component hazard rate $h_i(t) = \lambda_i \gamma_i t^{\gamma_i - 1}$ 4-5
   - Component reliability $R_i(t) = e^{-\lambda_i t^{\gamma_i}}$ for $t > 0$ 4-6

Given common environment factor $\eta \sim gamma(\alpha, \beta)$, the series systemability function can be calculated as follows.

### 4.4.1. Systemability Calculations: Series system configuration

Now a specific systemability calculation for a series system configuration is presented. In a series system, all components must operate successfully if the system is to function. The conditional reliability function of series systems subject to an actual operating environment $\eta$ is given by:

$$R_{Series} = (t \mid \eta, \underline{\lambda}, \underline{\gamma}) = \exp\left(-\eta \sum_{i=1}^{n} \lambda_i t^{\gamma_i}\right) \quad 4\text{-}7$$

Therefore, from eq. 4-2, the series systemability is given as follows:

$$R_{Series} = (t \mid \eta, \underline{\lambda}, \underline{\gamma}) = \int_{\eta} \exp\left(-\eta \sum_{i=1}^{n} \lambda_i t^{\gamma_i}\right) dG(\eta) = \left[\frac{\beta}{\beta + \sum_{i=1}^{n} \lambda_i t^{\gamma_i}}\right]^{\alpha} \quad 4\text{-}8$$

The variance of a function R(t) is given by:

$$Var[R(t)] = E[R^2(t)] - (E[R(t)])^2 \quad 4\text{-}9$$

Given $\eta \sim gamma(\alpha, \beta)$, the variance of systemability for any system structure can be easily obtained. Therefore, the variance of series systemability can be obtained:

$$Var[R_{Series} = (t \mid \eta, \underline{\lambda}, \underline{\gamma})] = \left[\frac{\beta}{\beta + 2\sum_{i=1}^{n} \lambda_i t^{\gamma_i}}\right]^{\alpha} - \left[\frac{\beta}{\beta + \sum_{i=1}^{n} \lambda_i t^{\gamma_i}}\right]^{2\alpha} \quad 4\text{-}10$$

Figures 4.1 and 4.2 show the reliability and systemability functions of a series system (here k=5) for α = 2, β = 3 and for α = 2, β =1, respectively.

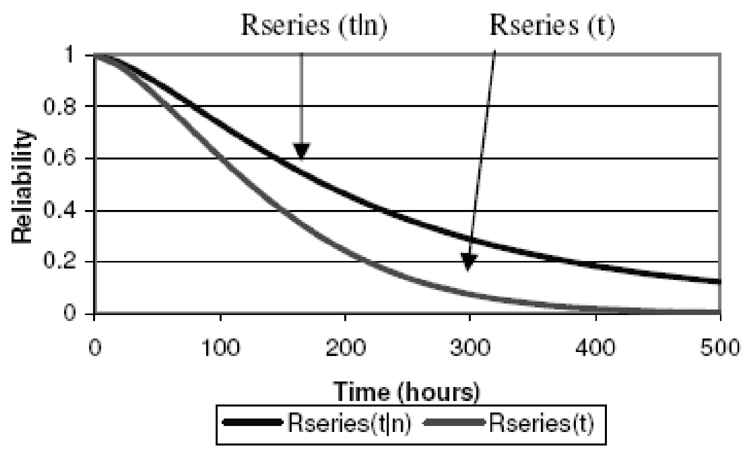



**Figure 4.1: Comparisons of Series System Reliability vs. Systemability Functions for α = 2 and β = 3 [4.14]**

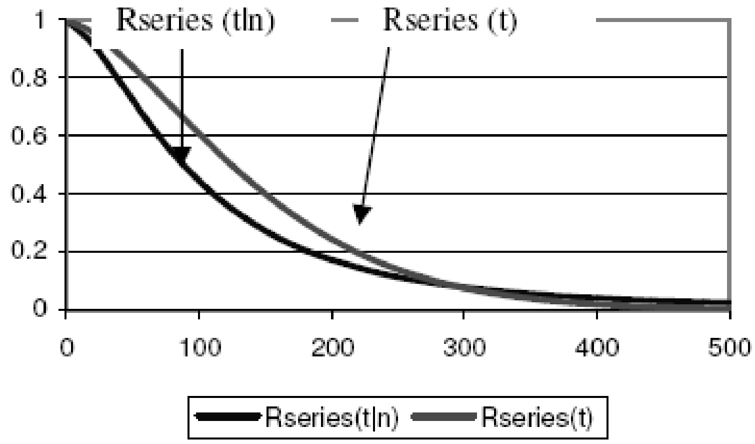



**Figure 4.2: Comparisons of Series System Reliability vs. Systemability Functions for α = 2 and β = 1 [4.14]**

### 4.4.2. Systematibility Calculations: Parallel system configuration

Similarly, the systemability of parallel systems (conditional reliability function subject to a randomly operating environment) is given by:

$$R_{Parallel} = (t \mid \eta, \underline{\lambda}, \underline{\gamma}) = \exp\left(-\eta \lambda_i t^{\gamma_i}\right) - \sum_{i_1, i_2 = 1; i_1 \neq i_2}^{n} \exp\left(-\eta\left(\lambda_{i1} t^{\gamma_{i1}} + \lambda_{i2} t^{\gamma_{i2}}\right)\right) + \sum_{i_1, i_2, i_3 = 1; i_1 \neq i_2 \neq i_3}^{n} \exp\left(-\eta\left(\lambda_{i1} t^{\gamma_{i1}} + \lambda_{i2} t^{\gamma_{i2}} + +\lambda_{i3} t^{\gamma_{i3}}\right)\right) - \ldots + (-1)^{n-1} \exp\left(-\eta \sum_{i=1}^{n} \lambda_i t^{\gamma_i}\right) \quad \text{4-11}$$

Hence, the parallel systemability is given by:

$$R_{Parallel} = (t \mid \eta, \underline{\lambda}, \underline{\gamma}) = \sum_{k=1}^{n} (-1)^{k-1} \sum_{i_1, i_2, \ldots i_k = 1; i_1 \neq i_2 \ldots \neq i_k}^{n} \left[ \frac{\beta}{\beta + \sum_{j = i_1, \ldots i_k} \lambda_j t^{\gamma_j}} \right]^{\alpha} \quad \text{4-12}$$

To simplify the calculation of a general n-component parallel system, here a parallel system consisting of two components is considered. The variance of series systemability of a two-component parallel system is given by:

$$Var\left[R_{Parallelo}(t\mid\eta,\underline{\lambda},\underline{\gamma})\right]=\left[\frac{\beta}{\beta+2\lambda_1 t^{\gamma_1}}\right]^{\alpha}+\left[\frac{\beta}{\beta+2\lambda_2 t^{\gamma_2}}\right]^{\alpha}+\left[\frac{\beta}{\beta+2\lambda_1 t^{\gamma_1}+2\lambda_2 t^{\gamma_2}}\right]^{\alpha}+\left[\frac{\beta}{\beta+\lambda_1 t^{\gamma_1}+\lambda_2 t^{\gamma_2}}\right]^{\alpha}-$$

$$\left[\frac{\beta}{\beta+2\lambda_1 t^{\gamma_1}+\lambda_2 t^{\gamma_2}}\right]^{\alpha}-\left[\frac{\beta}{\beta+\lambda_1 t^{\gamma_1}+2\lambda_2 t^{\gamma_2}}\right]^{\alpha}-$$

$$\left[\left[\frac{\beta}{\beta+\lambda_1 t^{\gamma_1}}\right]^{\alpha}+\left[\frac{\beta}{\beta+\lambda_2 t^{\gamma_2}}\right]^{\alpha}-\left[\frac{\beta}{\beta+\lambda_1 t^{\gamma_1}+\lambda_2 t^{\gamma_2}}\right]^{\alpha}\right]^{2} \qquad 4\text{-}13$$

Figures 4.3 and 4.4 show the reliability and systemability functions of a parallel system (here k=1) for α = 2, β = 3 and for α = 2, β =1, respectively.

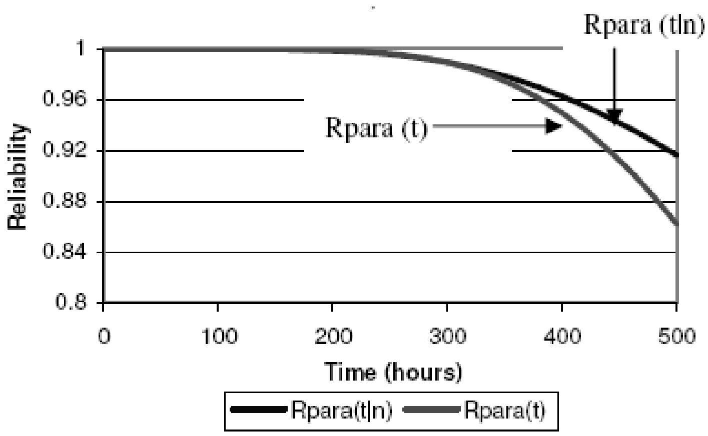



**Figure 4.3: Comparisons of Parallel System Reliability vs. Systemability Functions for α = 2 and β = 3 [4.14]**

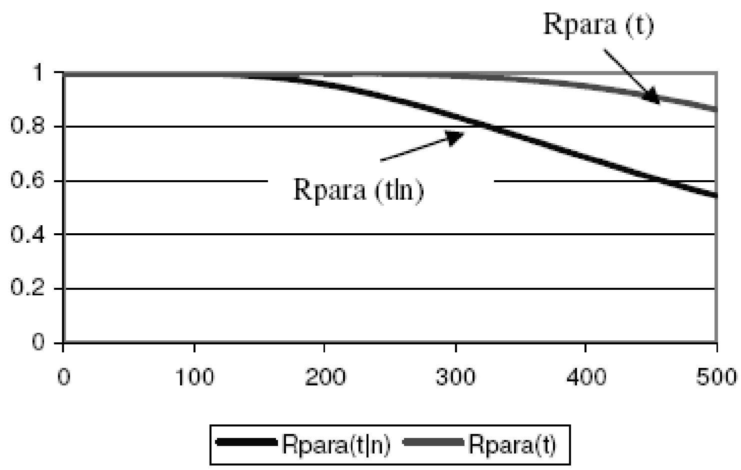



**Figure 4.4: Comparisons of Parallel System Reliability vs. Systemability Functions for α = 2 and β =1 [4.14]**

### 4.4.3. Systemability Calculations: K-out-of-n system configuration

For k-out-of-n system configuration, in order to simplify the complexity of the systemability function [4.30], it assumes that all the components in the k-out-of-n systems are identical. Therefore, the conditional reliability function of a component subject to a randomly operating environment can be written as:

$$R(t \mid \eta, \lambda, \gamma) = e^{-\eta \lambda t^{\gamma}} \qquad 4\text{-}14$$

The systemability of k-out-of-n systems is given by

$$R_{K-out-of-N}(t \mid \mu, \lambda, \gamma) = \sum_{j=k}^{n} \binom{n}{j} \sum_{l=0}^{n-j} \binom{n-j}{l} (-1)^{l} e^{-\eta (j+l) \lambda t^{\gamma}} \qquad 4\text{-}15$$

Note that

$$\left(1-e^{-\eta \lambda t^{\gamma}}\right)^{(n-j)} = \sum_{l=0}^{n-j} \binom{n-j}{l} \left(-e^{-\eta \lambda t^{\gamma}}\right)^{l} \qquad 4\text{-}16$$

The conditional reliability function of k-out-of-n systems, from eq. 4-15, can be rewritten as:

$$R_{k/n}(t \mid \eta, \lambda, \gamma) = \sum_{j=k}^{n} \binom{n}{j} \sum_{l=0}^{n-j} \binom{n-j}{l} (-1)^{l} e^{-\eta (j+l) \lambda t^{\gamma}} \qquad 4\text{-}17$$

Then if $\eta \sim gamma(\alpha, \beta)$ then the k-out-of-n systemability is given by:

$$R_{(T_1,\ldots,T_n)}(t \mid \eta, \lambda, \gamma) = \sum_{j=k}^{n} \binom{n}{j} \sum_{l=0}^{n-j} \binom{n-j}{l} (-1)^{l} \left[ \frac{\beta}{\beta + \lambda (j+l) t^{\gamma}} \right]^{\alpha} \qquad 4\text{-}18$$

It can be easily shown that:

$$R^{2}{}_{k/n}(t \mid \eta, \lambda, \gamma) = \sum_{i=k}^{n} \binom{n}{i} \sum_{j=k}^{n} \binom{n}{j} e^{-\eta (i+j) \lambda t^{\gamma}} \left(1-e^{-\eta \lambda t^{\gamma}}\right)^{(2n-i-j)} \qquad 4\text{-}19$$

Since

$$\left(1-e^{-\eta \lambda t^{\gamma}}\right)^{(2n-i-j)} = \sum_{l=0}^{2n-i-j} \binom{2n-i-j}{l} \left(-e^{-\eta \lambda t^{\gamma}}\right)^{l} \qquad 4\text{-}20$$

Equation 4-19 can be rewritten, after several simplifications, as follows:

$$R^{2}{}_{k/n}(t \mid \eta, \lambda, \gamma) = \sum_{i=k}^{n} \binom{n}{i} \sum_{j=k}^{n} \binom{n}{j} (-1)^{l} \sum_{l=0}^{2n-i-j} \binom{2n-i-j}{l} e^{-\eta (i+j+l) \lambda t^{\gamma}} \qquad 4\text{-}21$$

Therefore, the variance of *k*-out-of-*n* systemability function is given by

$$Var(R_{k/n}(t\mid\lambda,\gamma)=\int_{\eta} R^2{}_{k/n}(t\mid\eta,\lambda,\gamma)dG(\eta)-\left[\int_{\eta} R^2{}_{k/n}(t\mid\eta,\lambda,\gamma)dG(\eta)\right]^2$$

$$=\sum_{i=k}^{n}\binom{n}{i}\sum_{j=k}^{n}\binom{n}{j}\sum_{l=0}^{2n-i-j}\binom{2n-i-j}{l}(-1)^l\left(\frac{\beta}{\beta+(i+j+l)\lambda t^{\gamma}}\right)^2-$$

$$\left(\sum_{j=k}^{n}\binom{n}{j}\sum_{l=0}^{n-j}\binom{n-j}{l}(-1)^l\left(\frac{\beta}{\beta+(j+l)\lambda t^{\gamma}}\right)^2\right)^2 \qquad 4\text{-}22$$

Figures 4.5 and 4.6 show the reliability and systemability functions of a 3-out-of-5 system for α = 2, β = 3 and for α = 2, β =1, respectively.

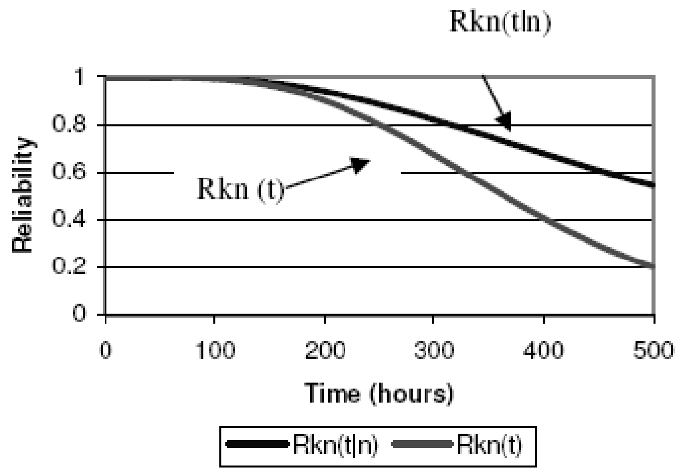



**Figure 4.5: Comparisons of k-out-of-*n* System Reliability vs. Systemability Functions for α = 2 and β =3 [4.14]**

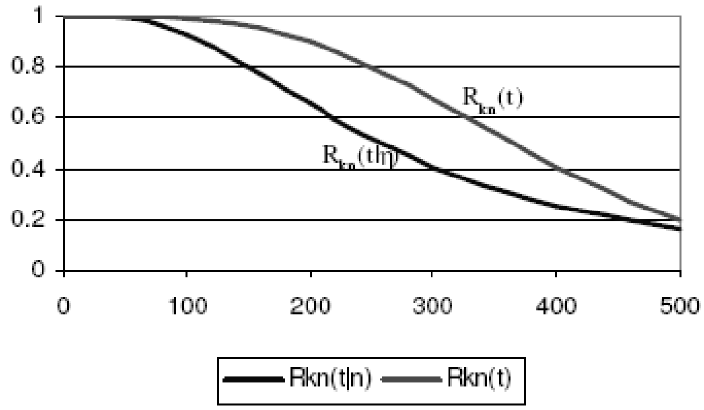



**Figure 4.6: Comparisons of k-out-of-*n* System Reliability vs. Systemability Functions for α = 2 and β = 1 [4.14]**

Assume λ = 0.00001, γ_ = 1.5, *n* = 3, k = 2, and η ~ *gamma* (α ,β), the systemability and its confidence intervals of a 2-out-of-3 system for α = 2, β =1 and α = 2, β = 2 , are shown in figures 4.7 and 4.8, respectively. Figure 4.9 is the same calculations for α = 3 and β = 2.

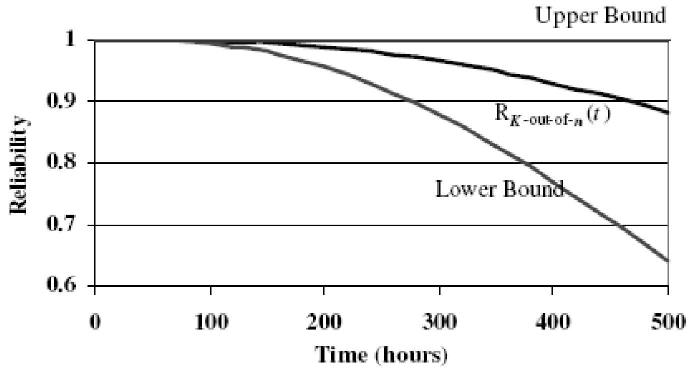



**Figure 4.7: A 2-out-of-3 Systemability and its 95% Confidence Interval (α = 2, β = 1) [4.14]**

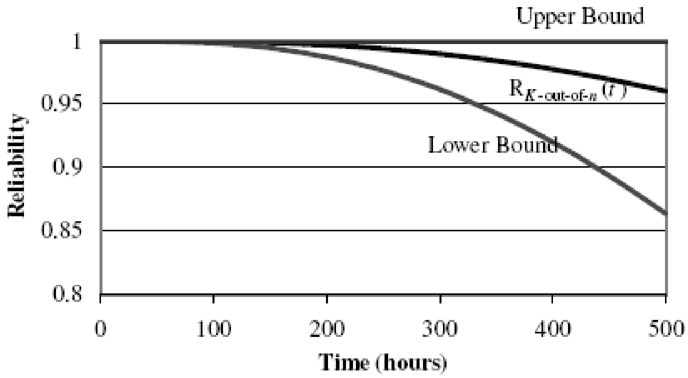



**Figure 4.8: A 2-out-of-3 Systemability and its 95% Confidence Interval (α = 2, β = 2) [4.14]**

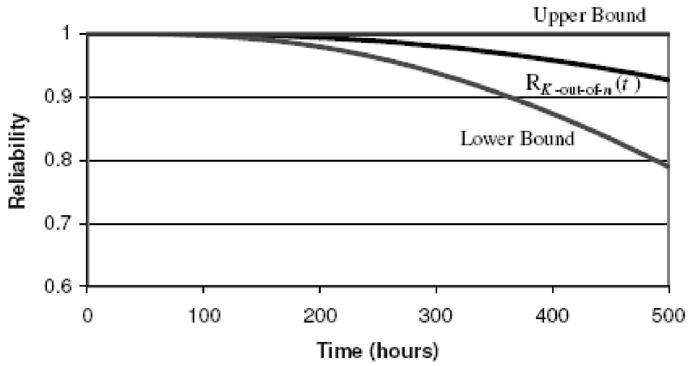



**Figure 4.9: A 2-out-of-3 Systemability and its 95% Confidence Interval (α = 3, β = 2) [4.14]**

## 4.5. Systemability Application

### 4.5.1. Systemability Application 1: Automatic Packaging Machines

In this section, the systemability function is validated using two different data sets of the same component but in different operating conditions.

An interesting case study in the automatic packaging machines for beer production is discussed. A manufacturer of these expensive systems (throughput at least 46.000 bottles per hour of production) has given us the data for analysis related to different applications of the same model of machine in several customer plants located in different countries. The two plants (Peroni Beer Plants) have been visited and analyzed in details, and they present the same machine subject to different environmental conditions for location, personnel education and maintenance policies.

Figure 4.10 shows the layout of a typical plant (a) and the automatic bottle filler (b). This study gives special attentions to this machine because usually it is the bottleneck of the whole production system and it is more influenced by the environmental factors than the other machines.

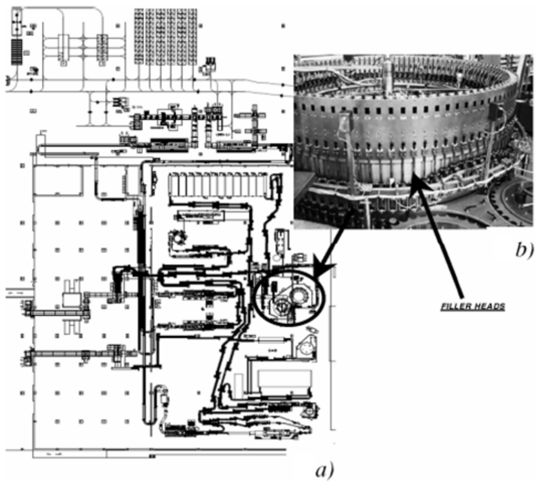



**Figure 4.10: (a) Equipments layout of a beverage _ oduction line (b) Automatic bottle filler**

The main causes of failure are shown in figure 4.11. As it can be observed, the principal downtimes are caused by failures of the filler heads upon which this study has been oriented.

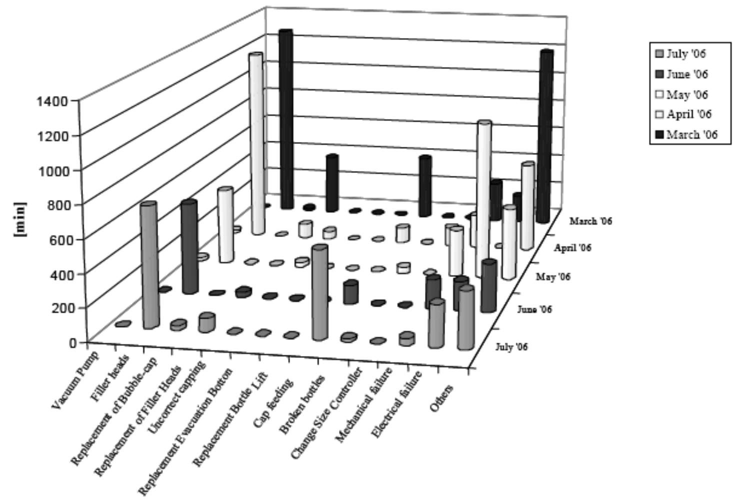



**Figure 4.11: Principal causes of failure**

Table 4.1 shows the data set that has taken from this component where the time values *t* has been normalized.

*$R_a(t)$* represents reliability function values collected from the test environment. The first plant presents the same data of the test environment while the second one is described by *$R_b(t)$*.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $t$ | 0.00 | 0.03 | 0.05 | 0.08 | 0.10 | 0.13 | 0.15 | 0.18 | 0.20 | 0.23 | 0.25 |
| $R_a(t)$ | 100.00 | 98.46 | 97.92 | 96.58 | 96.10 | 94.20 | 93.60 | 91.82 | 89.88 | 87.34 | 85.21 |
| $R_b(t)$ | 100.00 | 94.78 | 93.21 | 89.56 | 88.23 | 85.41 | 82.63 | 78.99 | 75.09 | 71.02 | 66.38 |
| $t$ | – | 0.28 | 0.30 | 0.33 | 0.35 | 0.38 | 0.40 | 0.43 | 0.45 | 0.48 | 0.50 |
| $R_a(t)$ | – | 82.39 | 79.30 | 74.70 | 72.21 | 68.26 | 65.40 | 59.72 | 55.21 | 52.00 | 45.10 |
| $R_b(t)$ | – | 60.52 | 56.74 | 51.73 | 45.23 | 41.68 | 35.98 | 32.08 | 27.65 | 23.47 | 19.66 |
| $t$ | – | 0.53 | 0.55 | 0.58 | 0.60 | 0.63 | 0.65 | 0.68 | 0.70 | 0.73 | 0.75 |
| $R_a(t)$ | – | 41.33 | 37.60 | 32.38 | 26.60 | 24.21 | 19.00 | 17.70 | 13.50 | 11.30 | 9.00 |
| $R_b(t)$ | – | 16.32 | 13.19 | 10.55 | 8.34 | 6.41 | 4.86 | 3.78 | 2.64 | 1.92 | 1.32 |
| $t$ | – | 0.78 | 0.80 | 0.83 | 0.85 | 0.88 | 0.90 | 0.93 | 0.95 | 0.98 | 1.00 |
| $R_a(t)$ | – | 6.80 | 5.30 | 3.40 | 2.60 | 2.30 | 1.63 | 1.14 | 0.77 | 0.52 | 0.33 |
| $R_b(t)$ | – | 0.94 | 0.59 | 0.41 | 0.26 | 0.16 | 0.09 | 0.04 | 0.03 | 0.02 | 0.01 |

**Table 4.1: Testing and operating data set**

The new approach is applied to the second plant where there are differences between testing values and operational ones.

As it can be noticed from the data set in table 4.1, the reliability of the second working system field is very different from the one that was revealed in the initial test. What interesting us is that it is able to apply the systemability concept by using Weibull parameters found in the test and to fit the new curve of reliability with respect to the systemability parameters (fig. 4.12).

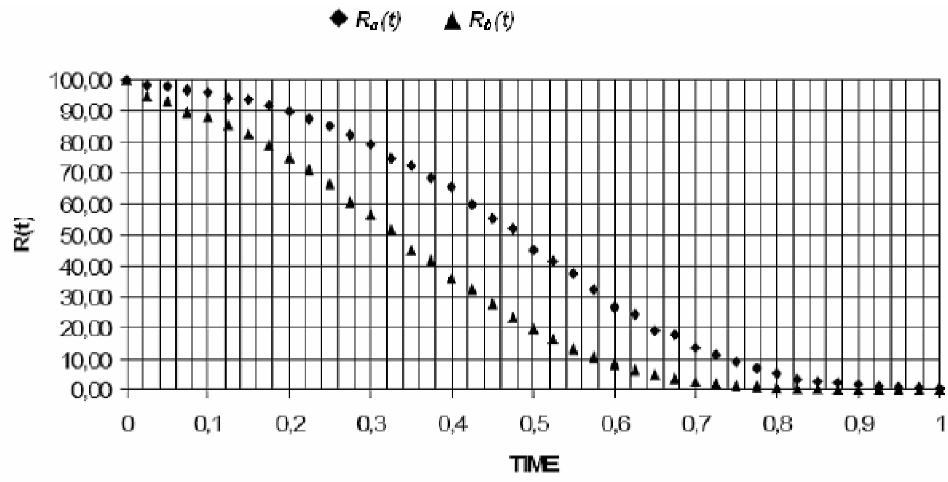



**Figure 4.12: Reliability during testing ($R_a(t)$) and reliability during operation ($R_b(t)$)**

The Weibull model has been used to fit the reliability data set in the testing environment, while systemability has been used to fit the reliability data in operational environment. The Weibull model, that has been used to estimate the reliability in the testing environment, is the following:

$$R_t(t) = e^{-\int_0^t h(s)ds} \qquad 4\text{-}23$$

where $h(s)$ is the Weibull baseline hazard rate:

$$h(t) = \lambda \gamma t^{\gamma - 1} \qquad 4\text{-}24$$

where $\gamma$ is the shape parameter and $\lambda$ is the intensity one.

So Weibull reliability function is:

$$R_t(t) = e^{-\lambda t^{\gamma}} \qquad 4\text{-}25$$

The Weibull parameters that best fit the reliability data are given by: $\gamma$ = 8,1 and $\lambda$ = 0,0000915.

In this case the systemability function will assume the following mathematical form:

$$R_s(t) = \left[ \frac{\beta}{\beta + \lambda t^{\gamma}} \right]^{\alpha} \qquad 4\text{-}26$$

where $\lambda$ and $\gamma$ are Weibull parameters calculated with the data set related to the environment test.

Figure 4.13 illustrates the used methodology. Research step were the following:

- Weibull parameters ($\lambda$ and $\gamma$) calculus with data set collected from the laboratory environment;
- Systemability application, considering based parameters as the Weibull previously calculated;
- The goodness-of-fit estimation of the function, as explained in figure 4.13.

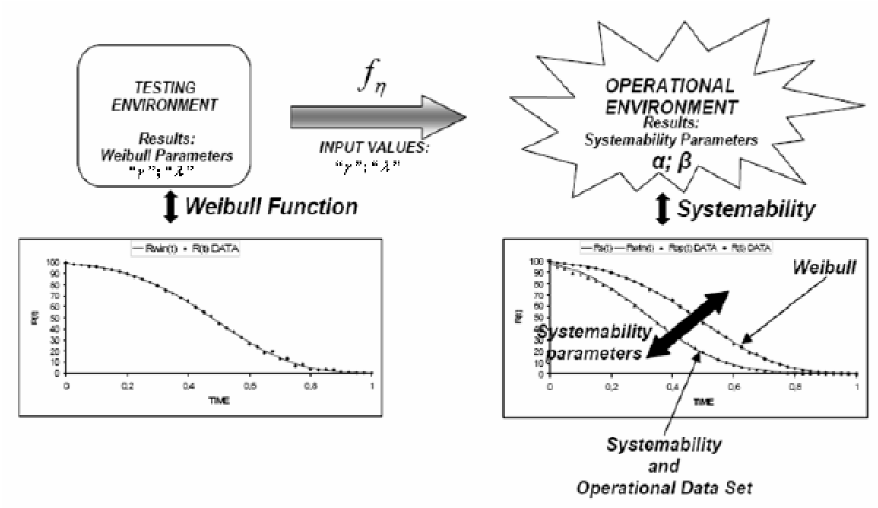



**Figure 4.13: Steps of the application of systemability model**

The parameter's value of systemability can be estimated using the least squares estimate method. In this analysis, the goodness-of-fit of the curve is measured by the sum of squares of errors, *SSE*. The SSE is defined as follows:

$$SSE = \sum_{i=1}^{n} (y_i - R_s(t_i))^2 \qquad 4\text{-}27$$

where $n$ represents the size of data set, $R_s(t_i)$ represents the estimated value of systemability function to the time $t_i$, and $y_i$ the observed value of reliability in the operational environment to the time $t_i$. The smaller the value of *SSE* is, the better the curve fits.

Systemability parameters have been calculated based on the *SSE*. The underlying assumption is to use Weibull testing reliability to calculate the systemability values, using 4-26

These results are shown in table 4.2. Figure 4.14 shows the goodness-of-fit of systemability function.

| *Weibull* | | *Systemability* | | *Goodness-of-fit* |
|---|---|---|---|---|
| $\lambda$ | $\gamma$ | $\alpha$ | $\beta$ | *SSE* |
| 0.0000915 | 8.10 | 3.10 | 1.42 | 33.82 |

**Table 4.2: Parameters of Weibull testing function; parameters of systemability function and goodness-of-fit of systemability model**

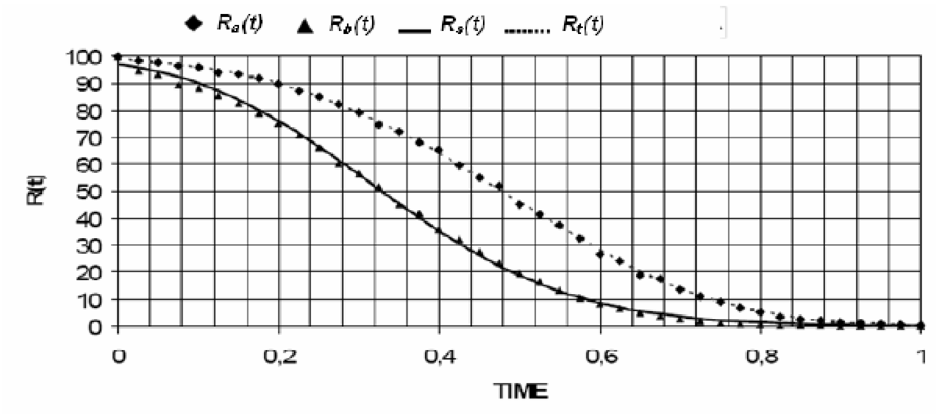



**Figure 4.14: Reliability during testing ($R_a(t)$), during operation ($R_b(t)$), Weibull function ($R_t(t)$) and systemability function ($R_s(t)$)**

## 4.5.2. Systemability Application 2: Motorcycle Drive-System

Motorcycle components present different lifetime distributions between the testing environment and the operating one. This aspect makes difficult the prediction of the reliability of these components in the working conditions, therefore motorcycle manufacturers meet troubles to estimate the correct warranty policies. After the validation of systemability function, explained in the previous section, this innovative concept is applied in order to predict and estimate the lifetime distribution of several components and systems.

*Examined Components and System*

The reliability analysis has been carried out on drive-systems (fig. 4.15) of motorcycles belonging to one of the most important world motorcycle manufacturer. The lifetime data set of the drive chain (indicated with number 1 in the figure 4.15), relative gear (indicated with number 2) and entire drive-system have been examined.

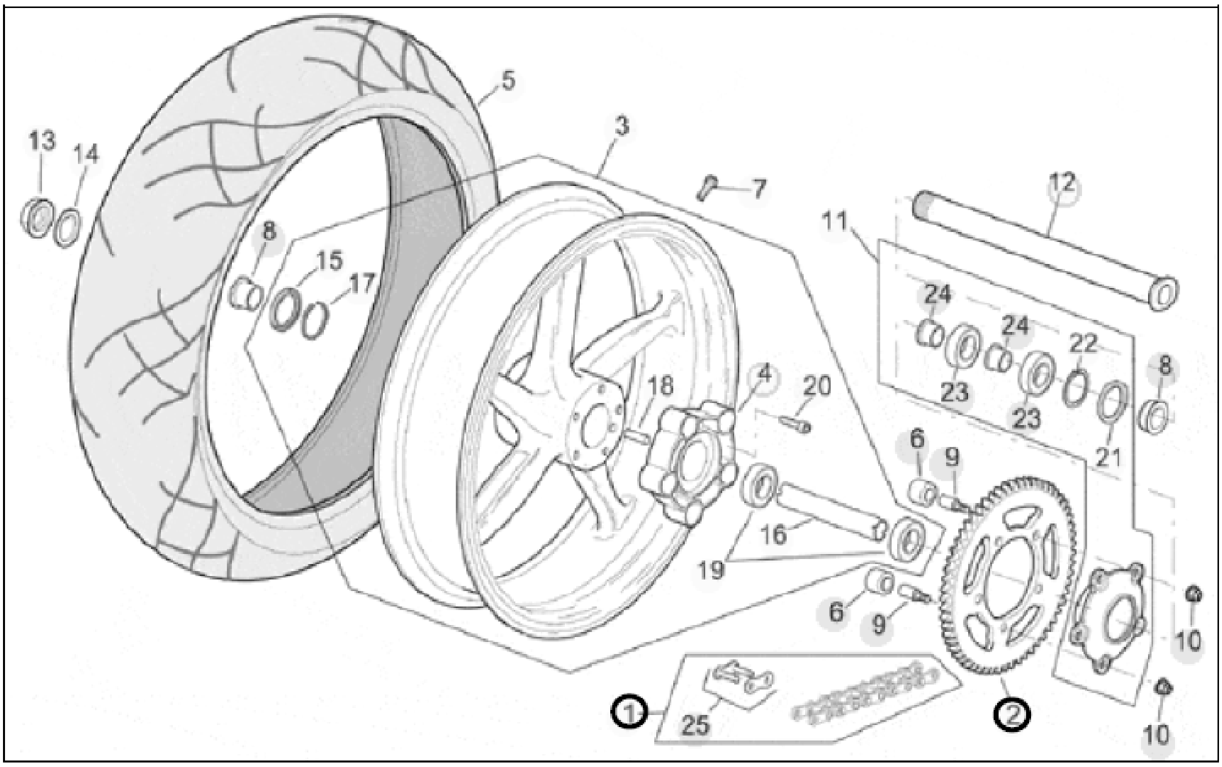



**Figure 4.15: Motorcycle Drive System**

The experimental analysis has studied the testing and operating data set to define the systemability parameters in order to calculate the goodness-of-fit of this new model to estimate the working lifetime distributions.

*Collected Data Set*

The reliability data set have been divided, on which the study has been developed, in these two different macro-sets:

- *Test Data Set (TDS):* collected during the testing phases, in well known conditions. This set contains the series of time to failure of the components, censured (1) and not (0).

- *Work Data Set (WDS):* related to the lifetime data during the operating conditions. This set contains the reliability of the components and system calculated using the sold spare parts in specific year and the number of motorcycles on field at the beginning of each year. It's important to notice that the operating conditions are the same for the two studied components, because they belong to the same system.

In the next page, the collected data set for each component is illustrated (table 4.3). The data related to the time have been normalized in order to guarantee the privacy policy of the manufacturer.

***Application of Systemability Concept***

With the collected data set, the application of systemability have been analysed through the following steps:

modelling of all testing data set by Weibull distribution;

estimation of the systemability parameters using the testing and operating data set of one component (drive-chain);

estimation of the operating lifetime distribution of the other component (gear) and calculation of the goodness-of-fit of these predicted data, using the systemability parameters, previously calculated;

estimation of the operating reliability of the entire drive-system using the systemability series calculations and evaluation of its goodness-of-fit.

| Test data set (TDS) | | | |
|---|---|---|---|
| Drive-chain | | Gear | |
| ttf | Censured | ttf | Censured |
| 0.363 | 1 | 0.151 | 1 |
| 0.418 | 1 | 0.363 | 1 |
| 0.351 | 1 | 0.418 | 1 |
| 0.289 | 1 | 0.351 | 1 |
| 0.355 | 1 | 0.289 | 1 |
| 0.320 | 1 | 0.355 | 1 |
| 0.352 | 1 | 0.352 | 1 |
| 0.283 | 1 | 0.305 | 1 |
| 0.334 | 1 | 0.283 | 1 |
| 0.212 | 1 | 0.295 | 1 |
| 0.190 | 1 | 0.334 | 1 |
| 0.226 | 1 | 0.212 | 1 |
| 0.349 | 1 | 0.190 | 1 |
| 0.400 | 1 | 0.226 | 1 |
| 0.270 | 0 | 0.349 | 1 |
| 0.398 | 0 | 0.400 | 1 |
| 0.295 | 0 | 0.398 | 0 |
| 0.391 | 0 | 0.664 | 0 |
| 0.257 | 0 | 0.263 | 0 |
| 0.350 | 0 | 0.823 | 0 |
| 0.472 | 0 | 0.221 | 0 |
| 0.221 | 0 | 0.321 | 0 |
| 0.051 | 0 | 0.418 | 0 |
| 0.204 | 0 | 0.218 | 0 |
| 0.214 | 0 | 0.320 | 0 |
| 0.218 | 0 | 0.435 | 0 |
| 0.242 | 0 | 0.370 | 0 |
| 0.151 | 0 | – | – |
| 0.236 | 0 | – | – |
| 0.435 | 0 | – | – |
| 0.370 | 0 | – | – |

| Work data set (WDS) | | | | | |
|---|---|---|---|---|---|
| Drive-chain | | Gear | | Drive-system | |
| t | R(t) | t | R(t) | t | R(t) |
| 0.173 | 0.992 | 0.173 | 0.981 | 0.173 | 0.973 |
| 0.212 | 0.980 | 0.212 | 0.977 | 0.212 | 0.958 |
| 0.280 | 0.971 | 0.280 | 0.969 | 0.280 | 0.940 |
| 0.345 | 0.960 | 0.345 | 0.955 | 0.345 | 0.915 |
| 0.407 | 0.941 | 0.407 | 0.934 | 0.407 | 0.874 |
| 0.471 | 0.910 | 0.471 | 0.899 | 0.471 | 0.810 |

**Table 4.3: Collected Data Set**

### *Modelling of the testing data*

Using the reliability and survival tool of Minitab® software, the parameters of the Weibull distribution have been calculated to model the testing data of each component where the Weibull distribution is given in 4-25.

| *Components* | *TDS modelling* | | |
|---|---|---|---|
| | *Weibull parameters* | | *Correlation index* |
| Drive-chain | $\lambda' = 8.71$ | $\gamma' = 2.428$ | 0.941 |
| Gear | $\lambda'' = 8.42$ | $\gamma'' = 3.288$ | 0.951 |

**Table 4.4: Reliability results: Weibull parameters**

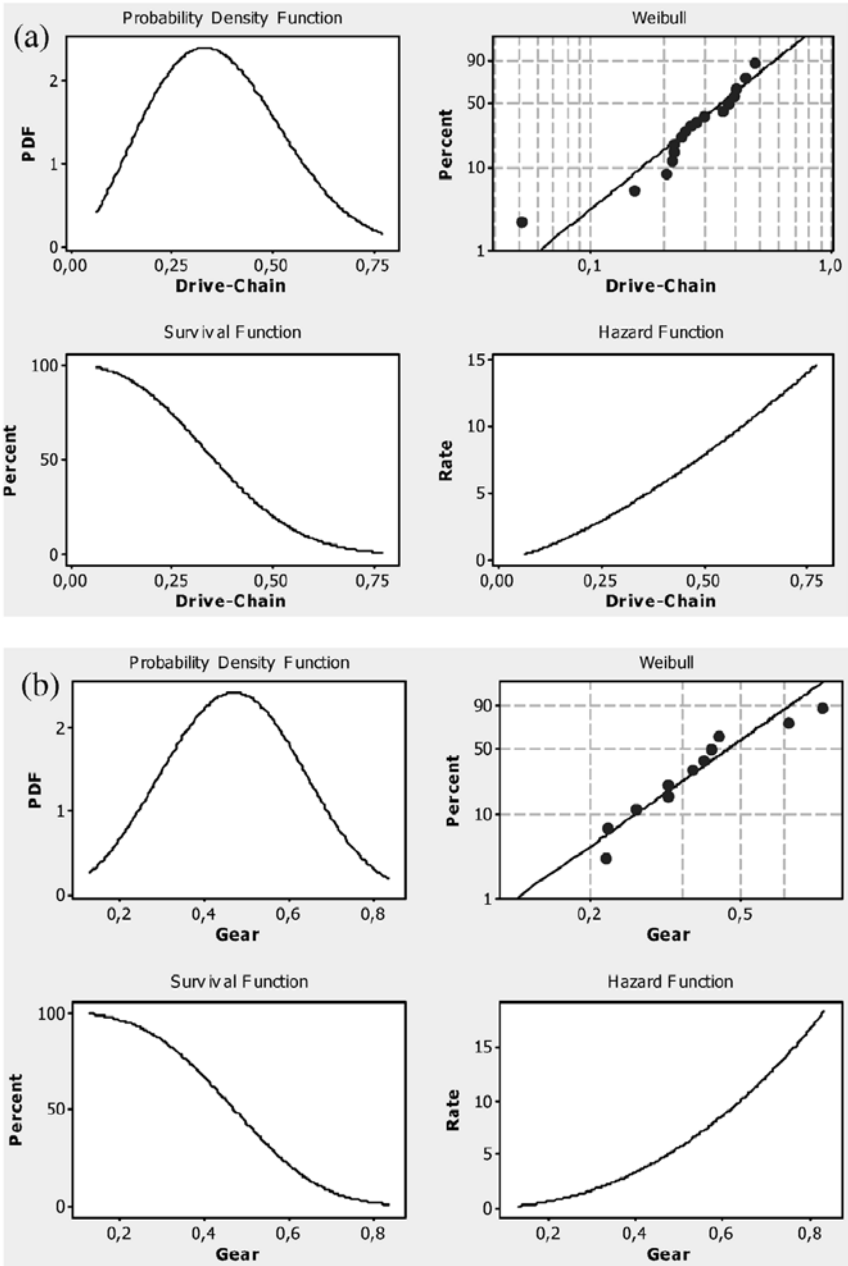



**Figure 4.16: Reliability analysis and functions for drive-chain (a) and gear (b)**

Figures 4.16a and 4.16b show the results of the Minitab® elaborations while table 4.4 reports the values of the Weibull parameters for the different components.

*Systemability Parameters Estimation*

Using the data set of the drive-chain components, the systemability parameters are calculated in order to model the data set related to the operating conditions (WDS) of this component.

In this case, the systemability function assumes the following math form:

$$R_s(t) = \left[ \frac{\beta}{\beta + \lambda' t^{\gamma'}} \right]^{\alpha} \qquad 4\text{-}28$$

where $\lambda'$ and $\gamma'$ are Weibull parameters calculated with the data set related to the test of drive-chain.

Figure 4.17 and table 4.5 show the results of this first step.

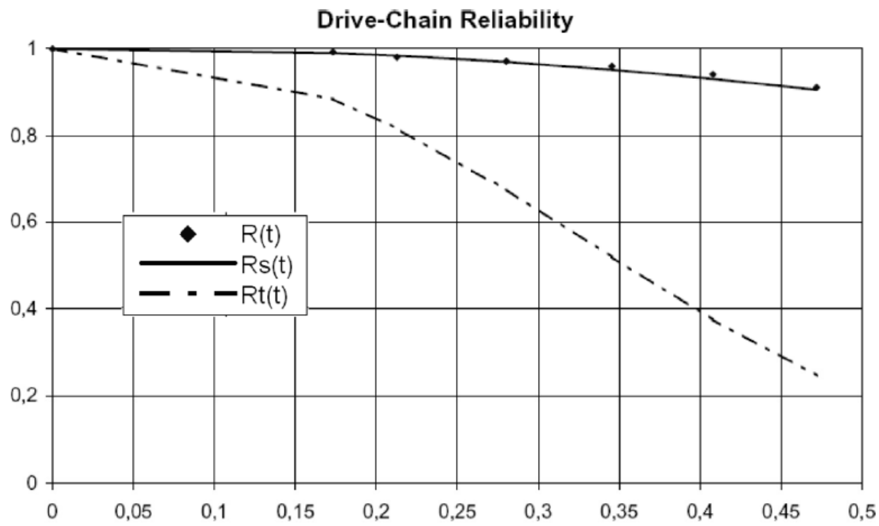



**Figure 4.17: Drive-chain reliability: systemability vs. work data set**

| *Drive-chain: reliability values* | | | | | |
|---|---|---|---|---|---|
| $t_i$ | $R(t)$ | | $Rs(t)$ | | $e_i$ % |
| 0.173 | 0.992 | | 0.990 | | 0.23 |
| 0.213 | 0.980 | | 0.984 | | 0.36 |
| 0.280 | 0.971 | | 0.970 | | 0.16 |
| 0.345 | 0.960 | | 0.951 | | 0.88 |
| 0.408 | 0.941 | | 0.930 | | 1.11 |
| 0.472 | 0.910 | | 0.905 | | 0.55 |
| | *Weibull* | | *Systemability* | | |
| | $\lambda'$ | $\gamma'$ | $\alpha$ | $\beta$ | $e$% |
| Drive-chain | 8.71 | 2.428 | 0.4 | 5 | 0.55 |

**Table 4.5: Reliability and systemability values of drive-chain component**

The error, in the last column, is given by:

$$e = \frac{\sum_{i=1}^{n} \frac{|R(t_i) - R_s(t_i)|}{R(t_i)}}{n} \qquad \text{4-29}$$

and

$$e_i = \frac{|R(t_i) - R_s(t_i)|}{R(t_i)} \qquad \text{4-30}$$

where $R_s(t_i)$ is the value of systemability in time $t_i$, using the systemability formula 4-28.

Moreover, in formula 4-30, $R(t_i)$ is the value of the Work Data Set *(WDS)* collected in the operating conditions. Finally, $n$ is the number of values.

***Application of systemability to Gear component***

Using the data related to the gear component and the parameters of systemability calculated here, the goodness-of-fit has been estimated using the absolute mean error defined before 4-30. The same values of α, β have been used, because drive-chain and gear belong to the same systems, so they work in the same operating conditions.

Table 4.6 illustrates the results of this step, also shown in figure 4.18.

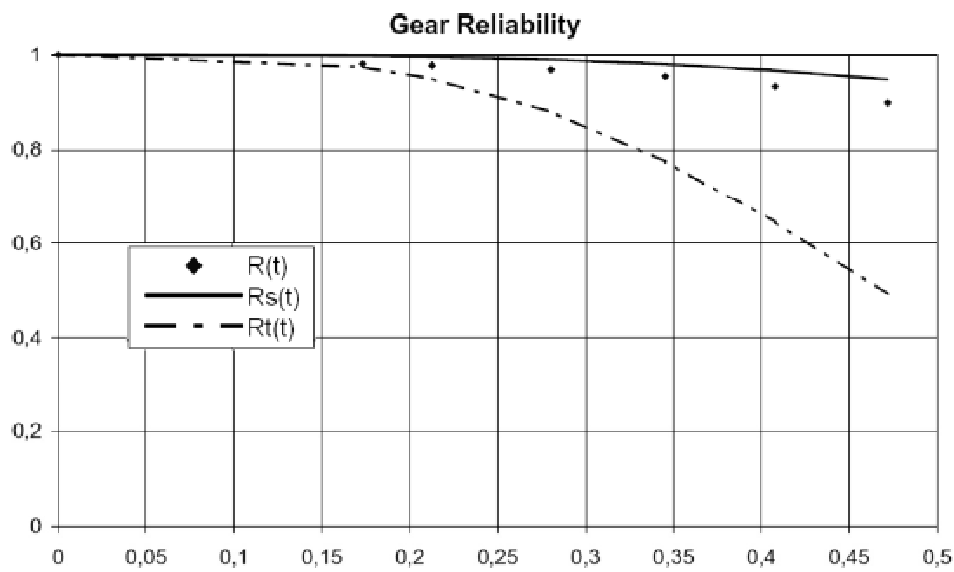



**Figure 4.18: Gear reliability: systemability vs. work data set**

| Gear: reliability values | | | | | |
|---|---|---|---|---|---|
| $t_i$ | $R(t)$ | | $Rs(t)$ | | $e_i$ % |
| 0.173 | 0.981 | | 0.998 | | 1.73 |
| 0.213 | 0.977 | | 0.996 | | 1.90 |
| 0.280 | 0.969 | | 0.990 | | 2.17 |
| 0.345 | 0.955 | | 0.980 | | 2.69 |
| 0.408 | 0.934 | | 0.967 | | 3.55 |
| 0.472 | 0.899 | | 0.948 | | 5.43 |
| | Weibull | | Systemability | | |
| | $\lambda''$ | $\gamma''$ | $\alpha$ | $\beta$ | $e\%$ |
| Gear | 8.42 | 3.288 | 0.4 | 5 | 2.91 |

**Table 4.6: Reliability and systemability values of gear component**

In this case, the systemability values have been calculated with:

$$R_s(t) = \left[ \frac{\beta}{\beta + \lambda'' t^{\gamma''}} \right]^{\alpha} \qquad \text{4-31}$$

where $\lambda''$ and $\gamma''$ are Weibull parameters calculated with the data set related to the test of gear. The errors have been given using formulas 4-29 and 4-30.

### *Application of systemability to Drive-System*

Using the data collected about Drive-System, shown in table 4.3, and using the Weibull parameters calculated before, the systemability function is applied in order to predict the system lifetime.

The Drive-System can be studied as a series-system configuration, composed by the gear and drive-chain components.

Therefore, the systemability of Drive-System , consisting of the gear and drive-chain components, is given as follows:

$$R_S(t \mid \underline{\lambda}, \underline{\gamma}) = \left[ \frac{\beta}{\beta + (\lambda' t^{\gamma'} + \lambda'' t^{\gamma''})} \right]^{\alpha} \qquad \text{4-32}$$

where $\lambda'$, $\lambda''$ and $\gamma'$, $\gamma''$ are the intensity and the shape parameters of Weibull distribution of the components.

As in previous application, the mean absolute error is calculated in order to estimate the goodness-of-fit of the systemability function using formulas 4-29 and 4-30.

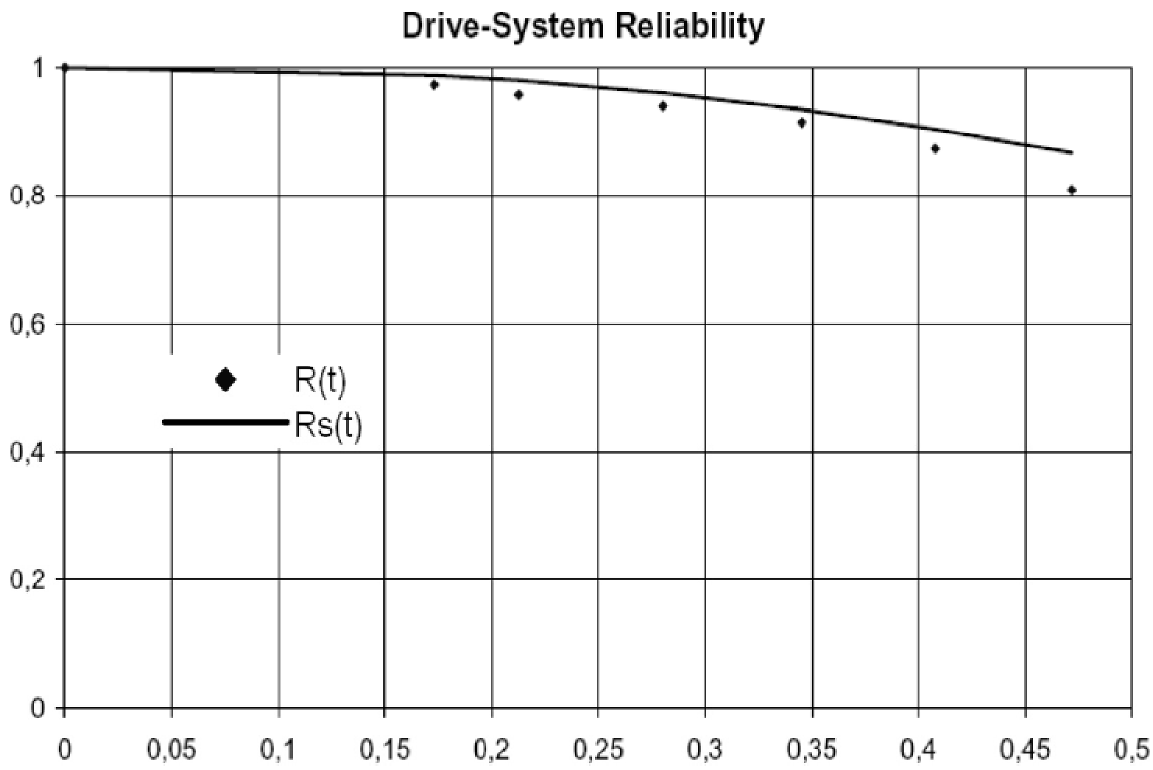



**Figure 4.19: Drive-system reliability: systemability vs. work data set**

| *Drive-system: reliability values* | | | |
|---|---|---|---|
| $t_i$ | *R(t)* | *Rs(t)* | $e_i$ % |
| 0.173 | 0.973 | 0.988 | 1.51 |
| 0.213 | 0.958 | 0.980 | 2.33 |
| 0.280 | 0.940 | 0.961 | 2.16 |
| 0.345 | 0.915 | 0.935 | 2.21 |
| 0.408 | 0.874 | 0.905 | 3.39 |
| 0.472 | 0.810 | 0.869 | 7.17 |

| | *Weibull* | | | | *Systemability* | | |
|---|---|---|---|---|---|---|---|
| | *Drive-chain* | | *Gear* | | | | |
| | $\lambda$ | $\gamma$ | $\lambda''$ | $\gamma''$ | $\alpha$ | $\beta$ | $e\%$ |
| Drive-system | 8.71 | 2.428 | 8.42 | 3.288 | 0.4 | 5 | 3.13 |

**Table 4.7: Reliability and systemability values of drive-system**

### 4.5.3. Several Considerations about Systemability Approach

In this Chapter, the reliability modeling of practical application-systems with respect to random field environments is discussed, addressing the fact that the difference between the lifetime distributions collected during the test and the distributions related to the operating environment is well known. For this reason, it is difficult to estimate the reliability of a component when it works in different environments.

Starting from Pham's paper [3.30] the concept of systemability, then, in this chapter, this approach is utilized to examine real world application case studies in automatic manufacturer packaging machines as well as motorcycle systems in determining the reliability of customer products. The first one shows the most important features of the Systemability approach, as a function enables to separate the intrinsic performance of the components or the systems from the random environmental factors. The second case study has been carried out in order to estimate the reliability and systemability of several motorcycle components and drive system in operative conditions, starting from the data collected during the tests.

The chapter demonstrates that this new concept is suitable to predict the reliability of system products in operating environments and it is easy to use also for predicting the lifetime distribution of complex system configuration, starting from the data related to single components.

My research addresses three important aspects:

a) the study of the performance of the systemability when its parameters change, especially in the field environments;
b) the definition of quantitative relationships between systemability parameters and different environment conditions; and
c) the definition of a general model for the estimation of reliability function in order to design and produce components or system products and reflect ways to define the best maintenance policies.

## 4.6. Reliability Growth Modeling using Systemability

Generally, the development of a new complex manufacturing or industrial system produces a series of prototypes that may contain faults during the processes including development, design and production. So during the early stages of prototyping complex systems, reliability often faces a major challenge in meeting the desired requirements. For these reasons, a typical reliability improvement process is carried out in order to increase a specified system reliability. Moreover, the penalty costs due to system failures in the field are much more significant than the system developmental costs. Consequently, the total system costs should consist of not only the developmental costs but also considers the penalty costs after the system release to the end-users. Usually, the testing developmental phase has been carried out in a given condition or control environment, while after the release of the system end-user environment factors can affect in an unpredictable way to the failure detection rate.

Many reliability cost models have been proposed in the areas of software reliability engineering [4.7, 4.9, 4.25, 4.21, 4.22, 4.27, 4.28, 4.34, 4.38, 4.46, 4.49, 4.50, 4.60], while, to knowledge, yet in the field environments of complex engineering systems, a generalized cost model have not been discussed [4.16, 4.51]. Most of the software cost models have been developed based on the non-homogeneous Poisson process (NHPP) Goel-Okumoto model to estimate the software reliability and the number of errors remaining in the software [4.38]. Most of them have assumed that the fault rate function and the mean value function are not affected by the environment factors. The considerations of the environmental effects have been studied by [4.7, 4.9, 4.16, 4.21, 4.22, 4.51, 4.59]. Huang, et al. [4.17] proposed a method for predicting the fault detection rate to reflect changes in the user's operational environments. Schneidewind [4.41] proposed an approach to relate fault correction to failure prediction by introducing a time delay between failure detection and fault correction time, where the rate of fault correction was proportional to the rate of failure detection.

Hwang and Pham [4.18] recently develops a generalized NHPP software reliability model considering quasi-renewal time-delay fault removal. Jeske et al. [4.20] recently extended an empirical calibration factor for adjusting the failure rate estimate obtained from a test dataset. The calibration factor studied in [4.20] is calculated from the averaged failure intensity during the testing and operation phases. It however does not utilize most of the software failure information in the operational phase. As a result, the method may not provide a good assessment on soft-ware reliability. Differing from the other NHPP models, the model proposed by Teng and Pham [4.47] assumes that the field environment affects the unit failure detection rate by incorporating a random factor. Assuming that the random effects of the field environments can be captured by a unit-free environmental factor, which is modeled as a random-distributed variable, they establish a generalized random field environment (RFE) software reliability model that covers both the testing phase and the operating phase in the software development cycle.

The motivation of this research is based on an industrial application (discussed in section 4) where the Weibull distribution would be a better fit instead of the NHPP Goel-Okumoto model. Manufacturing systems reliability is completely different from the software one. In fact the sample of faults, occurred during the development, design, release and production phases of a manufacturing system (like a CNC work station), is very small and the number of failures is not high as in software reliability growth process. Moreover each failure in manufacturing system development is very expensive in term of spent money and spent time to repair the system, in comparison to software field. Therefore the variance of the data is ignored and the model is based on the mean value function [4.7]. Because of this observation and motivation, in this chapter a cost model using Weibull distribution with considerations of the effects of random field environment is developed. The adopted modelling techniques are based on an innovative concept introduced recently by the authors, called systemability [4.30]. Systemability permits to estimate the reliability of a component/system in operating conditions, that are different from the testing ones [4.26, 4.43]. Industrial applications often observe the difference between laboratory reliability test in standard conditions and component or system reliability when it is set in motion through different environments and real-world conditions. As a matter of fact, reliability variable is considerably influenced by environmental factors. Environmental factors may change failure rate, reliability and availability of systems. When a component or a system works in an operative plant, it reflects a reliability function that is usually different from the theory reliability, but also from all its similar applications in other industrial plants. Incorrect estimation of reliability function could lead to the wrong functional design of the system and an incorrect definition of the appropriate maintenance policies. In general, this model can be used to model the cost of complex entire industrial system development life-cycle that perhaps reflects the perspectives from both developers and users and to determine the optimal release policies. An industrial application is discussed to illustrate the results of this study.

***Notation***

| | |
|---|---|
| $R(x \mid T)$ | reliability function of system by time $T$ for a mission time $x$ |
| $T$ | time to stop testing and release the system for field operations |
| $G(\eta)$ | Cumulative distribution function of random environmental factor $\eta$ |
| $\gamma$ | Shape parameter of field environmental factor |
| $\theta$ | Scale parameter of field environmental factor |
| $N(t)$ | The number of system failures discovered by time $t$ |
| $m(t)$ | Expected number of system failures detected by time $t$, $m(t) = E\left[N(T)\right]$ |
| $m_1(t)$ | Expected number of system failures detected during testing phase, by time $t$, $t < T$ |
| $m_2(t)$ | Expected number of system failures detected, since the beginning of the warranty period, by time $t$, $t \geq T$ |
| $m_F(t \mid \eta)$ | Expected number of system failures detected in field by time t |
| $C_0$ | Set-up cost for system testing |
| $C_1$ | System testing cost per unit time |
| $C_2$ | Cost of removing a fault per unit time during testing phase |
| $C_3$ | Cost of removing a fault per unit time during warranty period |
| $C_4$ | Penalty cost due to system failure |
| $\mu_y$ $(\mu_w)$ | Expected time to remove a fault during testing phase (warranty period) |
| $a$ | Number of initial system faults at the beginning of testing |
| $b$ | Scale parameter of Weibull distribution |
| $c$ | Shape parameter of Weibull distribution |
| $T_w$ | Time length of the warranty period |
| $x$ | Time length the system is going to be used in the field. |

### 4.6.1. Cost Model: Weibull Reliability Model with Environmental Factors

System reliability $R(x|t)$ is defined as the probability that a failure does not occur in the time interval $[t, t+x]$, where $t \geq 0$ and $x \geq 0$. Based on the non-homogeneus Poisson process (NHPP) [4.47]:

$$R(x|T) = e^{-[m(t+x)-m(t)]} \qquad \text{4-33}$$

In this chapter, the Weibull NHPP model is used, as a system basic reliability function [4.48]:

$$m(t) = a(1-\exp(-bt^c)) \qquad \text{4-34}$$

Note that this model (equation 4-34) becomes the Goel-Okumoto model when $c=1$, studied by Teng and Pham [4.47]. Incorporating the environmental field factors, the system reliability in the operational environments is obtained.

In this section, the generalized NHPP model with random field environments proposed by Teng and Pham [4.47] is briefly illustrated. Most existing NHPP models [4.15], [4.34], [4.36], do not consider the affects of the environmental factors on the unit failure detection rate, $b$. The contribution of Teng and Pham [4.47] assume that the field environment affects $b$ by multiplying a random factor $\eta$. In this chapter, it assumes that the probability that a fault is successful removed is equal to 1 and the error introduction rate in the testing phase is equal to 0. Based on these assumptions, the mean value $m_F(t|\eta)$ in operating environment introduced by [4.47] can be obtain as follows:

$$m_F(t|\eta) = a(1-\exp(-\eta \cdot bt^c)) \qquad \text{4-35}$$

The factor $\eta$ captures the effects of the field environmental factors upon the system failure rate. The system testing is commonly carried out in a control-known environment where a constant factor $\eta$ equal to 1 can be used. That environment is usually called testing environment. As for field environments, the factor $\eta$ can represent for different values that depend on the effects of the operational factors. For example, if the value of $\eta$ is greater than 1, the field conditions are more favorable to detect the failures than in the testing environment. Likewise, if the $\eta$ is less than 1, the environmental conditions are less favorable to fault detection than in the testing environment. Any suitable nonnegative distribution may be used to describe the uncertainty factor $\eta$. As discussed and supported in [4.48], also in this chapter a Gamma distribution is used to evaluate and predict the system reliability in the operating environments, and so to describe the field environmental factor $\eta$. If this factor $\eta$ is modeled by a Gamma distribution with the following probability density function (pdf):

$$f_\gamma(\eta)=\frac{\theta^\gamma \cdot \eta^{\gamma-1} \cdot e^{-\theta \cdot \eta}}{\Gamma(\gamma)}, \quad \gamma, \theta > 0; \eta \geq 0 \qquad \text{4-36}$$

then the mean value function of the Weibull generalized NHPP model is given by:

$$m(t)=\begin{cases} m_1(t)=a\left(1-e^{-bt^c}\right) & t \leq T \\ m_2(t)=a\left(1-e^{-bT^c}\left(\dfrac{\theta}{\theta+b\left(t-T\right)^c}\right)^\gamma\right) & t \geq T \end{cases} \qquad \text{4-37}$$

Usually, the prediction of system reliability is done after the release of system for the used of operating environments, i.e. $t \geq T$. Consequently, the reliability in the field can be obtained:

$$R(x \mid t)=\exp\left(ae^{-bT^c}\left(\left(\frac{\theta}{\theta+b(t+x-T)^c}\right)^\gamma-\left(\frac{\theta}{\theta+b(t-T)^c}\right)^\gamma\right)\right) \qquad \text{4-38}$$

Then, it is interesting to calculate the value of equation 4-38 for $t=T$, that is the reliability of the system immediately after the release. In this case, $R(x \mid T)$ is given by:

$$R(x \mid T)=\exp\left(-ae^{-bT^c}\left(1-\left(\frac{\theta}{\theta+b(x)^c}\right)^\gamma\right)\right) \qquad \text{4-39}$$

### 4.6.2. Model Development

Obviously the quality of the system will normally depend on the testing efforts such as testing times, testing methodologies. On the one hand, if the testing is too short, the cost of the system testing is lower, but the consumers may take higher risk of buying an unreliable system. It involves also in higher cost during the operating environments since it is much more expensive to detect a failure during operational phase than testing phase. On the other hand, the longer testing time the more faults can be removed, which leads to a reliable system; however, the testing cost of the system will also increase. Therefore, it is important to determine when to release the system. Figure 4.20 shows the entire system development life cycle that considers in the following cost model: testing phase before a release time $T$, in testing environment; warranty period and operational life in the field environment, which is usually quite different from the testing one.

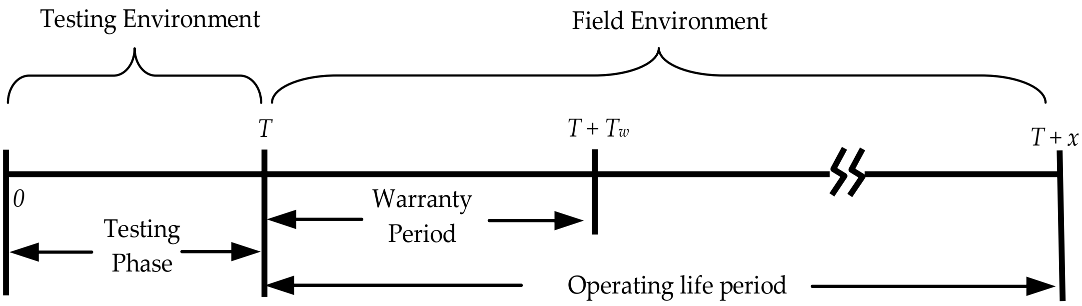



**Figure 4.20: System cost model infrastructure.**

In this study, it considers:

1. There is a set-up cost at the beginning of the system development process.
2. The cost to do testing is a linear function of testing time.
3. The cost to remove faults during the testing period is proportional to the total time of removing all faults detected during this phase.
4. The cost to remove the errors during warranty periods is proportional to the total time of removing all errors detected during the period $[T, T+T_w]$.
5. There is a risk due to the system failure after release the system.
6. Removing failures takes time and it is assumed that the time to remove each error follows a truncated exponential distribution.

From the last assumption, the expected time to remove each error during testing phase is [4-38]:

$$\mu_y = \frac{1-\left[\lambda_y T_0 + 1\right] e^{-\lambda_y T_0}}{\lambda_y \left[1 - e^{-\lambda_y T_0}\right]} \qquad 4\text{-}40$$

$\lambda_y$ a constant parameter associated with truncated exponential density function for the time to remove a fault during testing phase.

$T_0$ maximum time to remove any fault during testing phase.

Similarly, the expected time to remove each fault during the warranty period is given by [4-38]:

$$\mu_w = \frac{1-\left[\lambda_w T_1 + 1\right] e^{-\lambda_w T_1}}{\lambda_w \left[1 - e^{-\lambda_w T_1}\right]} \qquad 4\text{-}41$$

$\lambda_w$ a constant parameter associated with truncated exponential density function for the time to remove a fault during warranty period.

$T_1$ maximum time to remove any fault during warranty period.

The expected total cost $E(T)$ is given by the sum of the following parts [4.38]:

1. set-up cost for starting the system development process, and it is assumed as a constant $C_0$;
2. cost to do testing $E_1(T)$;
3. error removal cost during the testing period $E_2(T)$;
4. error removal cost during the warranty period $E_3(T)$;
5. risk cost due to system failure $E_4(T)$.

These addendums can be determined as follows:

1. Cost to do testing $E_1(T)$ is a linear function of $T$ :
   a. $E_1(T) = C_1 T$ .
2. The expected cost to remove all errors detected by time $T$ during the testing phase is given by:
   a. $E_2(T) = C_2 \mu_y m_1(T)$ .
3. The expected cost to remove all errors detected during the warranty period $[T, T+T_w]$ is given by:

a. $E_3(T)=C_3\mu_w\left(m_2(T+T_w)-m_2(T)\right)$.

4. The penalty cost $E_4(T)$ due to the failures occur after the system release time $T$, is given by:

a. $E_4(T)=C_4\left(1-R(x\mid T)\right)$.

Therefore, the total cost function $E(T)$ can be expressed as follows:

$$\begin{aligned}E(T)&=C_0+C_1T+C_2\mu_y m_1(T)\\&\quad+C_3\mu_w\left(m_2(T+T_w)-m_2(T)\right)+C_4\left(1-R(x\mid T)\right)\end{aligned} \tag{4-42}$$

where

$$m_1(T)=m_2(T)=a\left(1-e^{-bT^c}\right)$$

$$m_2(T+T_w)=a\left(1-e^{-bT^c}\left(\frac{\theta}{\theta+b\left(T+T_w-T\right)^c}\right)^{\gamma}\right)$$

## 4.6.3. Optimal Industrial System Release Policies

In this section, the behavior of the expected total cost function , $E(T)$, is studied and it is determined the optimal system release time $T^*$ which minimizes the expected total system cost. Taking the first and second derivatives of the cost function $E(T)$, respectively, it can be obtained:

$$\begin{aligned}\frac{\partial E(T)}{\partial T}&=C_1+C_2*\mu_y*a*e^{-f(T)}*f'(T)\\&\quad+C_3*\mu_w*\left(S_w-1\right)*a*e^{-f(T)}*f'(T)-\\&\quad-C_4*R(x\mid T)*\left(1-S_x\right)*\left[a*e^{-f(T)}*f'(T)\right]\\&=F(T)\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 E(T)}{\partial T^2}&=a*e^{-f(T)}*\left[f'(T)^2-f''(T)\right]\\&*\left\{C_4*R(x\mid T)*\left(1-S_x\right)*\left[1-\frac{a*\left(1-S_x\right)*e^{-f(T)}*\left(f'(T)\right)^2}{\left[f'(T)^2-f''(T)\right]}\right]\right.\\&\quad\left.-\left(C_2*\mu_y*+C_3*\mu_w*\left(S_w-1\right)\right)\right\}\\&=H(T)\left[G(T)-C\right]\end{aligned}$$

where $f(T)$, $f'(T)$, $f''(T)$, $S_w$, $S_x$, $F(T)$, $H(T)$, $G(T)$ and $C$ are defined as follows:

- $f(T) = bT^c$
- $f'(T) = \frac{\partial f(T)}{\partial T} = bcT^{c-1}$
- $f''(T) = \frac{\partial^2 f(T)}{\partial T^2} = bc(c-1)T^{c-2}$
- $S_w = \left(\frac{\theta}{\theta + bT_w^{\,c}}\right)^{\gamma}$
- $S_x = \left(\frac{\theta}{\theta + bx^c}\right)^{\gamma}$
- $F(T) = C_1 - a * e^{-f(T)} * f'(T) * \left[C_4 * R_s(x \mid T) * (1 - S_x) - C\right]$
- $H(T) = a * e^{-f(T)} * \left[f'(T)^2 - f''(T)\right]$
- $G(T) = C_4 * R(x \mid T) * (1 - S_x) * \left\{1 - \frac{a * (1 - S_x) * e^{-f(T)} * f'(T)}{\left[f'(T)^2 - f''(T)\right]}\right\}$
- $C = \left(C_2 * \mu_y * + C_3 * \mu_w * (S_w - 1)\right)$

Next the following results are presented. It is important to notice that each function used to define the different cases has not any engineering mean, but they are only mathematical function, derived by the analytical formulation. The general practitioners, which would like to apply this model, has to calculate only the functions F(t), G(t) and C as indicated in each case. Following these guide-lines, the solution could be easily calculated.

**Theorem T.** *Given* $C_0$, $C_1$, $C_2$, $C_3$, $C_4$, $x$, $T_w$, $\mu_y$, $\mu_w$, $b$, $\theta$ *and* $\gamma$; the optimal solutions of release time can be obtained for various values of Weibull shape parameter $c$, $c > 0$, as follows:

(a) For $c = 1$*: go to Theorem T1 (typical of intrinsic failure period also called useful life, with low constant failure rate);*

(b) For $0 < c < 1$*: go to Theorem T2 (typical of early failure period also called infant mortality life, with decreasing failure rate);*

(c) For $c > 1$*: go to Theorem T3 (typical of wear-out failure period also called end-of-life, with increasing failure rate);*

**Theorem T1.** (*For* $c=1$) *The optimal value of* $T$, *say* $T^*$, *which minimizes the expected total cost of system* $E(T)$ *is as follows:*

*Case 1.1: if* $G(0)>C$, *then*

(a) *If* $F(0)\geq 0$ *then* $T^*=0$ *minimizes* $E(T)$;

(b) *If* $F(\infty)<0$ *then* $T^*=\infty$ *minimizes* $E(T)$;

(c) *if* $F(0)<0$, $F(T)<0$ *for any* $T\in(0,T_1']$ *and* $F(T)>0$ *for any* $T\in(T_1',\infty)$ *then* $T^*=T_1'$, *where* $T_1'=F^{-1}(0)$.

*Case 1.2:if* $G(\infty)>C$, *then*

(a) *If* $F(0)\leq 0$ *then* $T^*=\infty$ *minimizes* $E(T)$

(b) *If* $F(\infty)>0$ *then* $T^*=0$ *minimizes* $E(T)$

(c) *if* $F(0)>0$, $F(T)>0$ *for any* $T\in(0,T_1'']$ *and* $F(T)<0$ *for any* $T\in(T_1'',\infty)$ *then:*

a. $T^*=0$, *if* $E(0)\leq E(\infty)$,

b. $T^*=\infty$, *if* $E(0)>E(\infty)$.

*Case 1.3: if* $G(0)<C$, $G(T)\leq C$ *for* $T\in(0,T_1^0]$ *and* $G(T)>C$ *for* $T\in(T_1^0,\infty)$, *where* $T_1^0=G^{-1}(C)$, *then:*

(a) *If* $F(0)\geq 0$ *then:*

a. $T^*=0$, *if* $E(0)\leq E(T_{1-b})$,

b. $T^*=T_b$, *if* $E(0)>E(T_{1-b})$, *where* $T_{1-b}=\inf\{T>T_{1-a}:F(T)>0\}$;

(b) *If* $F(0)<0$ *then* $T^*=T_{1-b}''$ *where* $T_{1-b}''=F^{-1}(0)$.

PROOF: Can be easily obtained from [4.47].

**Theorem T2.** (*For* $0<c<1$) *The optimal value of* $T$, *say* $T^*$, *which minimizes the expected total cost of system* $E(T)$ *is as follows:*

*Case 2.1: If* $E(T_2^*)\leq E(T)$ *where* $T\in(0,T_2^G]$, *then* $T^*=T_2^*$ *minimizes* $E(T)$;

*Case 2.2: If* $E(T_2^*) > E(T)$ *where* $T \in \left(0, T_2^G\right]$ *then:* $T^* = \left\{T \in \left(0, T_2^G\right] : \min\left[E(T)\right]\right\}$ *minimizes* $E(T)$*; where* $T_2^*$ *is as follows:*

*Case 2.0.1: if* $G(T_2^G) > C$ *then*

(a) *If* $F(T_2^G) \geq 0$ *then* $T_2^* = T_2^G$*;*

(b) *If* $F(\infty) < 0$ *then* $T_2^* = \infty$*;*

(c) *if* $F(T_2^G) < 0$, $F(T) < 0$ *for any* $T \in \left(T_2^G, T'\right]$ *and* $F(T) > 0$ *for any* $T \in \left(T', \infty\right)$ *then* $T_2^* = T'$, *where* $T_2^* = F^{-1}(0)$.

*Case 2.0.2: if* $G(\infty) > C$ *then*

(a) *If* $F(T_2^G) \leq 0$ *then* $T_2^* = \infty$

(b) *If* $F(\infty) > 0$ *then* $T_2^* = T_2^G$

(c) *If* $F(T_2^G) > 0$, $F(T) > 0$ *for any* $T \in \left(T_2^G, T''\right]$ *and* $F(T) < 0$ *for any* $T \in \left(T'', \infty\right)$ *then:*

a. $T_2^* = T_2^0$, *if* $E(T_2^G) \leq E(\infty)$,

b. $T_2^* = \infty$, *if* $E(T_2^G) > E(\infty)$.

*Case 2.0.3: if* $G(T_2^G) < C$, $G(T) \leq C$ *for* $T \in \left(T_2^G, T_2^0\right]$ *and* $G(T) > C$ *for* $T \in \left(T_2^0, \infty\right)$ *where* $T_2^0 = G^{-1}(C)$, *then*

(a) *If* $F(T_2^G) \geq 0$, *then:*

a. $T_2^* = T_2^G$, *if* $E(T_2^G) \leq E(T_b)$,

b. $T_2^* = T_{2-b}$, *if* $E(T_2^G) > E(T_{2-b})$, *where* $T_{2-b} = \inf\left\{T > T_{2-a} : F(T) > 0\right\}$*;*

(b) *If* $F(T_2^G) < 0$, *then* $T_2^* = T_{2-b}''$ *where* $T_{2-b}'' = F^{-1}(0)$.

PROOF: See the Appendix I. □

**Theorem T3.** **(***For* $c > 1$*) The optimal value of* $T$, *say* $T^*$, *which minimizes the expected total cost of system* $E(T)$ *is as follows:*

*Case 3.1: if* $G(\infty) < C$ *then* $T^* = 0$ *minimizes* $E(T)$.

*Case 3.2: if* $G(0) < C < G(\infty)$ *then:*

(a) *If* $F(T_3^2) \geq 0$ *then:* $T^* = 0$ *minimizes* $E(T)$, *where* $T_3^2 = \left\{T : \left[\frac{\partial^2 E^{-1}(0)}{\partial T^2}\right] and \left[\frac{\partial E^{-1}(T)}{\partial T} < C_1\right]\right\}$*;*

(b) *If* $F(T_3^2) < 0$ *then:*

a. $T^* = 0$, *if* $E(0) \le E(T_{3-b})$,

b. $T^* = T_{3-b}$, *if* $E(0) > E(T_{3-b})$,

$$where\, T_3^2 = \left\{ T : \left[ \frac{\partial^2 E^{-1}(0)}{\partial T^2} \right] and \left[ \frac{\partial E^{-1}(T)}{\partial T} < C_1 \right] \right\} \quad T_{3-b} = \inf\left\{ T > T_{3-a} : F(T) > 0 \right\};$$

*Case 3.3: if* $0 < C < G(0)$ *then:*

*(a) If* $F(T_3^3) \ge 0$ *then:* $T^* = 0$ *minimizes* $E(T)$, *where* $T_3^3 = \frac{\partial^2 E^{-1}(0)}{\partial T^2}$;

*(b) If* $F(T_3^3) < 0$ *then:*

a. $T^* = 0$, *if* $E(0) \le E(T_{3-d})$

b. $T^* = T_b$, *if* $E(0) > E(T_{3-d})$, *where* $T_3^3 = \frac{\partial^2 E^{-1}(0)}{\partial T^2}$, $T_{3-d} = \inf\left\{ T > T_{3-c} : F(T) > 0 \right\}$;

*Case 3.4: if* $C < 0$ *then* $T^* = \left\{ T \in (0, \infty) : \min\left[ E(T) \right] \right\}$ *minimizes* $E(T)$;

PROOF: See the Appendix I. □

### 4.6.4. Applications to a complex manufacturing system

In this section the application of the proposed methodology to a real case - a developmental process and release of a complex industrial system – is discussed. The analyzed case study deals with a complex flexible manufacturing system for production of several mechanical components for driver systems. The system is a CNC machine for the metal working of different kind of driver gears. The data, in term of the number of failures per week, was given by a development team that related to the testing phase and collected in a fixed interval. After the release of the system, the data set has been collected by the customer service team where the intervals are not necessary the same because it also depends on the user environment team. The data are summarized in table 4.8 and 4.9.

Note that the data have been normalized. The normalized time values have been obtained dividing the actual time by the total time. In the same way the normalized failure data have been calculated. Table 1 is related to the testing-development process and the last value of time is the actual time to release of the system. Table 2 shows the data about the system failures in the real environment.

| *Testing Environment* | |
|---|---|
| **T** | **Failures** |
| 0.000 | 0.0000 |
| 0.050 | 0.0296 |
| 0.100 | 0.0296 |
| 0.150 | 0.0888 |
| 0.200 | 0.3258 |
| 0.250 | 0.4739 |
| 0.300 | 0.5628 |
| 0.350 | 0.7997 |
| 0.400 | 0.8886 |
| 0.450 | 0.9182 |

**Table 4.8: Normalized cumulative Failures and Times during Testing Environments**

| *Field Environment* | |
|---|---|
| **T** | **Failures** |
| 0.623 | 0.9250 |
| 0.662 | 0.9359 |
| 0.730 | 0.9443 |
| 0.795 | 0.9546 |
| 0.857 | 0.9722 |
| 0.921 | 1.0000 |

**Table 4.9: Normalized cumulative Failures and Times during Field Environments**

It can be seen that the system appears to be more reliable in real environment than in the testing one. This means that the testing-developmental phase is more liable for the system to fail than in the operating environment. In fact one of the aims of the testing phases is to stress in various hard ways the entire system in order to find as many failures in less time as possible.

The model parameters in 4-37 have been obtained using the least squares estimated (LSE) method:

$$LSE = \sum_{i=1}^{n} (y_i - \tilde{y}_i)^2 \qquad \text{4-43}$$

where $y_i$ are the observed values and $\tilde{y}_i$ are the expected values, obtained using 4-37.

The parameters of the Weibull model are as follows:

$$\tilde{a} = 143.53\,,\ \tilde{b} = 0.000104\,,\ \tilde{c} = 2.685\,,\ \tilde{\theta} = 3.49\,,\ \tilde{\gamma} = 1.04\,.$$

It is also obtained the estimated parameters of the Goel-Okumoto model using LSE method. In this case, the parameters are as follows:

$$\hat{a} = 150.46\,,\ \hat{b} = 0.02718\,,\ \hat{\theta} = 1.00\,,\ \hat{\gamma} = 1.64\,.$$

Note that in the NHPP Goel-Okumoto model, the parameter $c$ is equal to 1. Figure 4.21 shows the mean value function fitting curve of the proposed model during testing, where the environmental factors do not affect the mean value function. The mean value function fitting curve for field environment in Figure 4.21 is also plotted. The Goel-Okumoto model is also included in Figure 4.21. The comparison of the values of Least Squares Function, given by formula 4-43, calculated for new model and Goel-Okumoto model, permits to validate the more accuracy of the here introduced model. In fact, from Figure 4.21, the Weibull model provides a better fit for the collected data than the Goel-Okumoto model based on LSE. In other words, $LSE = 270.28$ for Weibull model while $LSE = 4969.61$ is for Goel-Okumoto model.

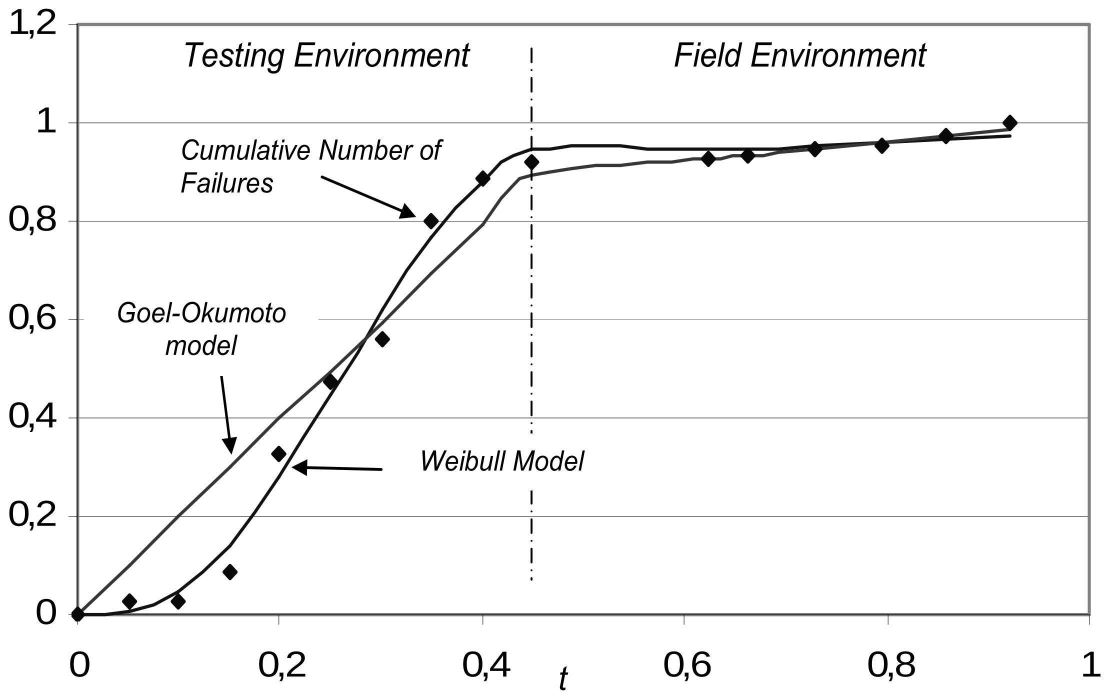



**Figure 4.21: Mean value function fitting curve of the proposed model.**

With a consultation and a discussion with the engineering team that developed the system, the cost coefficients and parameters values are given at the beginning of the applicaiton as follows: $C_0 = 100$, $C_1 = 400$, $C_2 = 1{,}000$, $C_3 = 4{,}000$, $C_4 = 200{,}000$, $x = 100{,}000$, $T_w = 10{,}000$, $\mu_y = 0{,}2$, $\mu_w = 0{,}5$.

$C_0$ is the set-up cost for system testing, paid at the beginning of the testing phase. It consists in the cost of initial activities to prepare the system to be tested. $C_1$ is the time-cost of testing phase, while $C_2$ is the cost to remove the failure occurred during the testing. After release, the company will pay a cost for each failure occurred during warranty period, $C_3$, and a penalty cost due to system failures, $C_4$. The total time of system operation in work condition is assumed a priori and it is given by $x$, while the warranty period is long $T_w$. When a failure occurs, the expected time to remove the fault is given by $\mu_y$ and $\mu_w$, respectively for testing and warranty phase.

Using the proposed theorems, the normalized optimal value of the time to release is calculated, that is equal to 0.717. This optimal value of time minimizes the total expected system cost to 58,459 €.

If this value is compared with the total cost calculated using the actual time to release 0.450, that is 245,170 €, it may be noticed the difference on the results using the proposed model cost model. Figure 4.22 shows the curve of the total expected cost using the parameters introduced before. This application is a specific example of the set of cases introduced before. Exactly, it is related to Theorem T3 and Case 4.2, in sub-case b.

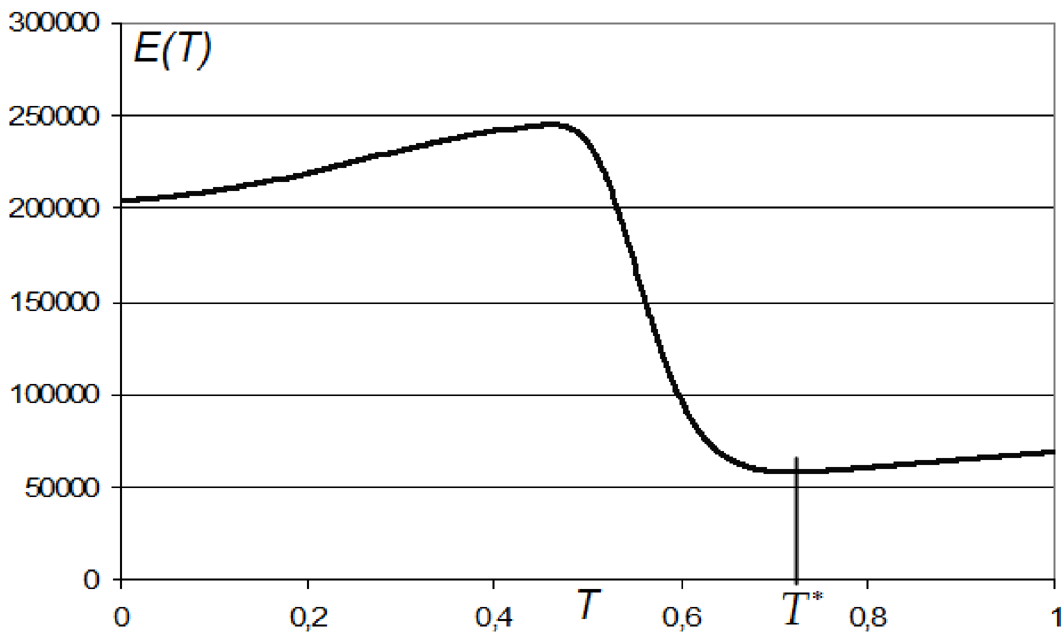



**Figure 4.22:** ***E(T)*** **vs** ***T*** **in a real application.**

### 4.6.5. Discussion about Systemability Application on Reliability Growth Modeling

A new generalized cost model to estimate the total expected cost of a general industrial system is discussed by considering the cost to do testing, to remove faults during testing phase, the cost of the warranty, and the penalty cost due to system failure after the release. This is the first study that generalized the NHPP reliability model using a Weibull distribution and also incorporates the random field environmental factors into the cost model. Then the optimal release policy is determined which minimizes the total expected cost and also illustrate the application of this methodology to a real application in order to validate the cost model. This model can help the development and testing engineering team to decide when to release the system to the end-user. The proposed model also can be easily applied to various applications in industries such as communications and manufacturing products.

# 5. Maintenance Policies on Industrial Plants

## 5.1. Introduction

Many modern companies invest relevant capital in the production systems, in order to have high level of flexibility and efficiency, necessary to guarantee the variety and the volume of different products to satisfy the various consumer demands. Moreover higher levels of efficiency of the production systems are required to make convenient the production systems. In this context the reliability and availability of the production systems become very important to satisfy the final production rate. Hence, the importance of maintenance of these systems has grown up, because their reliability can improve thanks to the different applied maintenance policies.

Starting from the collection of data, related to the systems lifetime and then estimating their different survival functions, like reliability, hazard rate and so on, several preventive maintenance policies have been planned [5.18, 5.26]. Thanks to these information, the service engineer can design the best maintenance strategies for improving system reliability, preventing the occurrence of system failures, and reducing maintenance costs of systems.

When a component or a system works in an operative plant, it reflects a reliability function that is usually different from the theory reliability but also from all its similar applications in other industrial plants. Environmental factors may change failure rate, reliability and availability of systems. Incorrect estimation of reliability function could lead to the wrong functional design of the system and an incorrect definition of the appropriate maintenance policies.

In the next section, a literature review is illustrated in order to discuss the importance of this work. In the section 5.3, the Age Replacement Policy (ARP), introduced by Barlow and Hunter [5.4] is discussed. the ARP using weibull reliability model is investigated in details in subsection 5.3.1. The environmental factors and their effects on ARP are investigated in subsection 5.3.2. In this part of chapter 4, the new concept systemability, introduced by Pham [5.19, 5.20, 5.21] is used to model the Age Replacement Policy. It allows to consider the environmental effects in the reliability estimation, separating their from the intrinsic survival performances of the component.

The subsection 5.3.3 includes several numerical examples of the Age Replacement Policy using Weibull and systemability reliability modeling. A sensitivity analysis is conducted in order to study the cost curves in function of change on systemability parameters.
The subsection 5.3.4 shows several graphics in order to analyze the changing of the optimal time of preventive maintenance and to estimate the difference between the UEC considering the environmental factors or not.
The proposed methodology is applied, in subsection 5.3.5, to an interesting case study in the automatic packaging machines for beer production and its performance is discussed in subsection 5.3.6.
The same section-structure is used for Periodic Replacement Policy (PRP), starting from a literature review, illustrating the Classical approach versus the systemability one, making a sensitive analysis of cost curves in function of systemability parameters and applying the new approach to an interesting real case study, finally discussing the results.

## *5.2. Maintenance Policies Types: State of Art and Future Researches*

In the last decades, most scientific contributions on maintenance policies have been developed and widely used. The first study of preventive maintenance was carried out by Barlow and Hunter [5.4]. Their research introduced two different maintenance policies to maximize 'limiting efficiency' i.e., fractional amount of up-time over long intervals. The first policy is called policy *type I*, or also age replacement policy, while the second one is called policy *type II* or periodic replacement. The optimum policies are determined, in each case, as unique solutions of certain integral equations depending on the failure distribution.
In the past several decades, maintenance and replacement problems of deteriorating systems have been extensively studied in the literature. A survey of all maintenance policies has been developed by Wang [5.31]. The author has summarized, classified, and compared various existing maintenance policies in several group, like: age replacement policy, random age replacement policy, block replacement policy, periodic preventive maintenance policy, failure limit policy, sequential preventive maintenance policy, repair cost limit policy, repair time limit policy, repair number counting policy, reference time policy, mixed age policy, preparedness

maintenance policy, group maintenance policy, opportunistic maintenance policy, etc. [5.16 - 5.22 – 5.23 - 5.32].

In some early works, the age replacement policy was extensively studied. Under this policy, a unit is always replaced at its age *T* or failure, whichever occurs first, where *T* is a constant [5.5]. Later, as the concepts of minimal repair and especially imperfect maintenance became more and more established, various extensions and modifications of the age replacement policy were proposed. For this class of policy, various maintenance models can be constructed according to different types of preventive maintenances (minimal, imperfect, perfect), corrective maintenances (minimal, imperfect, perfect), cost structures, etc. [5.8 – 5.16 – 5.23 – 5.32]. Recently other studies have been developed about the replacement policies and spare parts management under different conditions [5.9 – 5.10 – 5.15 – 5.33].

In the periodic preventive maintenance policy, a unit is preventively maintained at fixed time intervals kT (T = 1; 2; . . .) independent of the failure history of the unit, and repaired at intervening failures where T is a constant. The basic periodic preventive maintenance policy is ‘’periodic replacement with minimal repair at failures’’ policy under which a unit is replaced at predetermined times kT (T = 1; 2; . . .) and failures are removed by minimal repair ([5.4], *Type II*). As the concepts of minimal repair and especially imperfect maintenance [5.22] became more and more established, various extensions and variations of these two policies were proposed.

The effects of the environmental factors in the development of preventive maintenance models are not considered in the previous scientific contributions, as shown in the analysis of this survey.

It’s very important to design an accurate maintenance plan, to start from the collection of data on component lifetimes and then to estimate statistically the reliability of the components using quantities like the failure rate, hazard rate and so on [5.18 – 5.26] taking also in consideration the environment where the examined components operate.

In fact, the environmental factors may change failure rate, reliability and availability of components, so traditional approaches could cause the incorrect estimation of reliability. For this reason some researches create several models to estimate the reliability in operating environments, using the data collected during the test phases. The relation between the environmental conditions and the hazard rate was studied firstly by Cox [5.11]. The authors introduced the concept of proportional and additive hazard rate. These models have been widely used in several experiments where the time to failure depends on a group of covariates [5.3 – 5.8 – 5.13 – 5.14].

Badia et al. [5.3] study the aging characteristics in two mixing models having additive and proportional hazards; they consider a baseline hazard function and an additive or multiply functions to represent the changes in the operating conditions. Moreover, they consider three different types of the group of covariates for this function: Poisson, Bernoulli and Gamma distribution.

Other researches have carried out to define the Environmental Factor (EF); an environmental factor converts reliability test result at one environmental condition into equivalent information at other ones [5.12 – 5.27 – 5.28 – 5. 29 – 5.30]. It is defined as the quotient of the mean life in two different environments. The definition of EF for different distribution has been developed and accepted. Wang et al. [5.27 – 5.28 – 5. 29 – 5.30] have defined EF for gamma, normal, log-normal, inverse Gaussian distribution and Elsayed & Wang [5.12] have studied environmental factors for the binomial distribution.

In mechanical engineering, several contributions have been given and widely accepted. Ram and Tiwari [5.24] have estimated the reliability of a component through a Monte-Carlo simulation, with the introduction of the factor $\eta$ that is greater than 1 if the operating condition is more stressful than the testing ones, otherwise, it is less than 1. Other models to study the lifetime distributions based on the test data, the warranty data and additional field data after the warranty expires have been developd by Oh and Bai [5.17] illustrating the methods to estimate the maximum likelihood and so defined specific formulas for Weibull distribution. The reliability characteristics of a component installed in two different cars with different working conditions have been studied by Attardi et al. [5.2]. In their work, a mixed Weibull distribution have been used, depending on the covariates through the Weibull scale parameters.

Abbassi et al. [5.1] have introduced an approach based to simulated annealing algorithm to estimate the parameters of Weibull distribution. Sohn et al. [5.25] proposed a random effects Weibull regression model for forecasting the occupational lifetime of the employees who join another company, based on their characteristics. Advantage of using such a random effects model is the ability of accommodating not only the individual.

An innovative and important contribution different from the previous ones, has been given by Pham [5.19 – 5.20 – 5.21]. Pham recently introduced an innovative approach, called systemability. It calculates the reliability in random environment using, as starting data, the reliability obtained during the test, and processing it using a gamma distribution in particular, or a distribution that represents operating environments in general, which takes into consideration the EF. This is a fundamental condition for applications in real contexts [5.6 – 5.7].

## 5.3. Theory on Age Replacement Policy (ARP)

This policy is one of the most used time-based preventive maintenance policies. It is also called policy *type I*. It involves the change of the component after $t_p$ time of work. If a failure occurs before age $t_p$, the component is replaced at failure, and the next planned replacement operation is schedule after $t_p$ time.

After each maintenance replacement action, the component becomes as good as new; in this case the age replacement policy is also called perfect preventive maintenance.

The optimum policy for minimizing the total expected rate and for maximizing the availability of the component is shown before.

***Notation***

$f(t)$ probability density function of failure

$h(t)$ hazard rate function

$R(t)$ reliability function

$F(t)$ failure function

$\lambda$ intensity parameter of Weibull distribution

$\gamma$ shape parameter of Weibull distribution

$\eta$ a common environment factor

$G(\eta)$ cumulative distribution function of $\eta$

$\alpha$ shape parameter of Gamma distribution

$\beta$ scale parameter of Gamma distribution

$c_p$ cost of a preventive maintenance operation

$c_f$ cost of a breakdown maintenance operation

$UEC$ Unit Expected Cost of the replacement policy

$t_p$ planned time of replacement

Barlow et al. defined the Unit Excepted Cost as following:

$$UEC(t_p) = \frac{c_p * R(t_p) + c_f * F(t_p)}{\int_0^{t_p} R(t)dt} \qquad 5\text{-}1$$

where $c_p$ and $c_f$ are the cost of preventive and breakdown replacement, with $c_p < c_f$.

Suppose that a component is replaced at a planned time $t_p$ or at failure, whichever occurs first, it is obviously to claim that if $t_p = \infty$, the policy aims to replacement the unit upon failure and the Unit Excepted Cost is:

$$UEC(\infty) = \frac{c_p * R(\infty) + c_f * F(\infty)}{\int_0^{\infty} R(t)dt} \qquad 5\text{-}2$$

Note that $R(\infty) = 0$, $F(\infty) = 1$ and $\int_0^{\infty} R(t)dt = MTTF$, where MTTF is the Mean Time to Failure, then the $UEC(t_p)$ is: $UEC(\infty) = \frac{c_f}{MTTF}$.

Differentiating the $UEC(t_p)$ with respect to time $t_p$ and setting it equal to zero:

$$\frac{\partial UEC(t_p)}{\partial t_p} = \frac{\partial}{\partial t_p}\left( \frac{c_p * R(t_p) + c_f * F(t_p)}{\int_0^{t_p} R(t)dt} \right) = 0 \qquad 5\text{-}3$$

The following equation is given:

$$h(t_p) * \int_0^{t_p} R(t)dt - F(t_p) = \frac{c_p}{c_f - c_p} \qquad 5\text{-}4$$

As well shown in [5.18] it can be found an optimum time $t_p^*$ which minimizes the function $UEC(t_p)$, provided the hazard rate function $h(t_p)$ is strictly increasing. Since $\lim_{t_p \to 0} UEC(t_p) = \infty$, a positive $t_p^*$ must exist.

To find a unique optimum solution, the function $h(t_p)$ has to be increasing. If this hypothesis is not guaranteed, the optimal solution could not be the unique one [5.18]. The left-hand side of equation 5-4 strictly increases from *0* to $h(\infty) * MTTF - 1$, because $h(t_p)$ is strictly increasing.

Hence, if $h(\infty) \leq \frac{c_f}{MTTF*(c_f - c_p)}$ then $t_p^* = \infty$, so the component is replaced at failure. On the other hand, if $h(\infty) > \frac{c_f}{MTTF*(c_f - c_p)}$, then there is a unique and finite $t_p^*$ that satisfies the equation 5-4. In this case, the Unit Expected Cost is:

$$UEC(t_p^*) = (c_f - c_p) * h(t_p^*) \text{ .} \qquad 5\text{-}5$$

### 5.3.1. Classical Approach: Weibull Age Replacement Policy (W-ARP)

Considering Weibull distribution for the lifetime data of replaced components, the age replacement policy, its minimal Unit Expected Cost $UEC(t_p^*)$ and optimal preventive maintenance time $t_p^*$ could be analyzed using the following formulas.

In this case, the survival functions are:

$$R(t) = e^{-\lambda t^{\gamma}} \qquad 5\text{-}6$$

$h(t) = \lambda \gamma t^{\gamma - 1}$ (strictly increasing with $\gamma > 1$, see [5.18]) 5-7

then the Unit Expected Cost function is:

$$UEC(t_p) = \frac{c_p * e^{-\lambda t_p^{\gamma}} + c_f * (1 - e^{-\lambda t_p^{\gamma}})}{\int_0^{t_p} e^{-\lambda t^{\gamma}} dt} \text{ .} \qquad 5\text{-}8$$

**Algorithm A1:** *Given $c_p$, $c_f$, $\lambda$, $\gamma$ , the optimal value of $t_p$, say $t_p^*$, which minimizes the Unit Expected Cost for age replacement policy is as follows:*

- Step 1: Set-up Phase

  *Set $c_p$, $c_f$, $\lambda$, $\gamma$, real positive constants*

  *Set $t_p$, $t$ real positive variables.*

- Step 2: Calculation Phase

  *Calculate* $h(t) = \lambda \gamma t^{\gamma - 1}$

  *Calculate* $R(t) = e^{-\lambda t^{\gamma}}$

  *Calculate* $MTTF = \int_0^{\infty} e^{-\lambda t^{\gamma}} dt$

*Calculate* $UEC(t_p) = \dfrac{c_p * e^{-\lambda t_p{}^{\gamma}} + c_f * (1 - e^{-\lambda t_p{}^{\gamma}})}{\int_0^{t_p} e^{-\lambda t^{\gamma}} dt}$

- Step 3: Decision-Making Phase

Condition A1_C1: *If* $h(\infty) \le \dfrac{c_f}{MTTF * (c_f - c_p)}$ *then* $t_p{}^* = \infty$;

Condition A1_C2: *If* $h(\infty) > \dfrac{c_f}{MTTF * (c_f - c_p)}$ *for any* $t_p \in (0, t_p^*]$ *and* $t_p \in (t_p^*, \infty)$, *then* $UEC(t_p) > UEC(t_p{}^*)$, *then* $t_p{}^*$ *minimizes* $UEC(t_p)$, *where* $UEC(t_p{}^*) = (c_f - c_p) * \lambda\gamma(t_p{}^*)^{\gamma-1}$.

### 5.3.2. New Approach: Systemability Age Replacement Policy (S-ARP)

As described in Chapter 3, the mathematical form of systemability function is given by:

$$R_s(t) = \int_{\eta} e^{-\eta \int_0^t h(s)ds} dG(\eta) \qquad \text{5-9}$$

where $\eta$ is a random variable that represents the system operating environments with a distribution function G.

If it assumes that $\eta$ has a gamma distribution with parameters $\alpha$ and $\beta$, *i.e.*, $\eta \sim gamma\ (\alpha, \beta)$ where the pdf of $\eta$ is given by:

$$f_{\eta}(x) = \frac{\beta^{\alpha} x^{\alpha-1} e^{-\beta x}}{\Gamma(\alpha)} \quad \text{for} \quad \alpha, \beta > 0; \quad x \ge 0 \qquad \text{5-10}$$

where $\Gamma(\alpha) = (\alpha - 1)!$, then the systemability function of the system in equation 5-9 using the Laplace transform is given by:

$$R_s(t) = \left[ \frac{\beta}{\beta + \int_0^t h(s)ds} \right]^{\alpha} \qquad \text{5-11}$$

Using formulas 5-1 and 5-12, the $UEC(t_p)$ assumes this mathematical expression:

$$UEC_{\alpha\beta}(t_p) = \frac{c_p * \left[ \frac{\beta}{\beta + \lambda t^{\gamma}} \right]^{\alpha} + c_f * \left( 1 - \left[ \frac{\beta}{\beta + \lambda t^{\gamma}} \right]^{\alpha} \right)}{\int_0^{t_p} \left[ \frac{\beta}{\beta + \lambda t^{\gamma}} \right]^{\alpha} dt} \qquad \text{5-12}$$

Also in this case the ${t_{sp}}^*$ is a finite and unique solution if the hazard rate function $h_s(t)$ of systemability is strictly increasing.

If the systemability formula 5-9 is considered with Weibull distribution of testing data, with hazard rate *h(t)* as shown in formula 5-7, the systemabilty function is as follows:

$$R_s(t) = \left[ \frac{\beta}{\beta + \lambda t^\gamma} \right]^\alpha \qquad 5\text{-}13$$

and known that $h_s(t) = \frac{1}{R_s(t)} * \left( -\frac{\partial R_s(t)}{\partial t} \right)$,

where $\frac{\partial R_s(t)}{\partial t} = -\frac{\alpha\lambda\gamma * \left( \frac{\beta}{\beta + \lambda t^\gamma} \right)^\alpha}{\beta + \lambda t^\gamma} * t^{\gamma - 1}$

so the related hazard rate function $h_s(t)$ becomes:

$$h_s(t) = \frac{\alpha\lambda\gamma * t^{\gamma-1}}{\beta + \lambda t^\gamma} \qquad 5\text{-}14$$

To find a unique optimum solution, the function 5-14 has to be increasing. If this function is studied, it can be found that the $\lim_{t\to 0} h_s(t) = 0$ and also $\lim_{t\to\infty} h_s(t) = 0$. Moreover, the function $h_s(t)$ is major than 0 for all values of variable *t*.

Differentiating the $h_s(t)$ with respect of time $t$ and setting it equal to zero:

$$h_s'(t) = \frac{\partial h_s(t)}{\partial t} = \frac{\partial}{\partial t} \left( \frac{\alpha\lambda\gamma * t^{\gamma-1}}{\beta + \lambda t^\gamma} \right) = 0 \qquad 5\text{-}15$$

The following solution is given:

$$t^* = \sqrt[\gamma]{\frac{\beta(\gamma - 1)}{\lambda}} \qquad 5\text{-}16$$

Considering the $h_s'(t)$, it can be noticed that $h_s'(t=0) = 0$ and $h_s'(t=\infty) = 0$. Hence, if $0 \le t \le t^*$ the hazard rate function $h_s(t)$ is strictly increasing, while on the other hand, if $t \ge t^*$ the hazard rate function $h_s(t)$ is strictly decreasing, due also to $h_s'(t=0) \ge 0$ and $h_s'(t=\infty) \ge 0$.

It's interesting to notice that $t^*$ depends only on a parameter of systemability, that is $\beta$ one. Moreover, increasing $\beta$ value, the value of $t^*$ increases, as shown in the following figure 5.1.

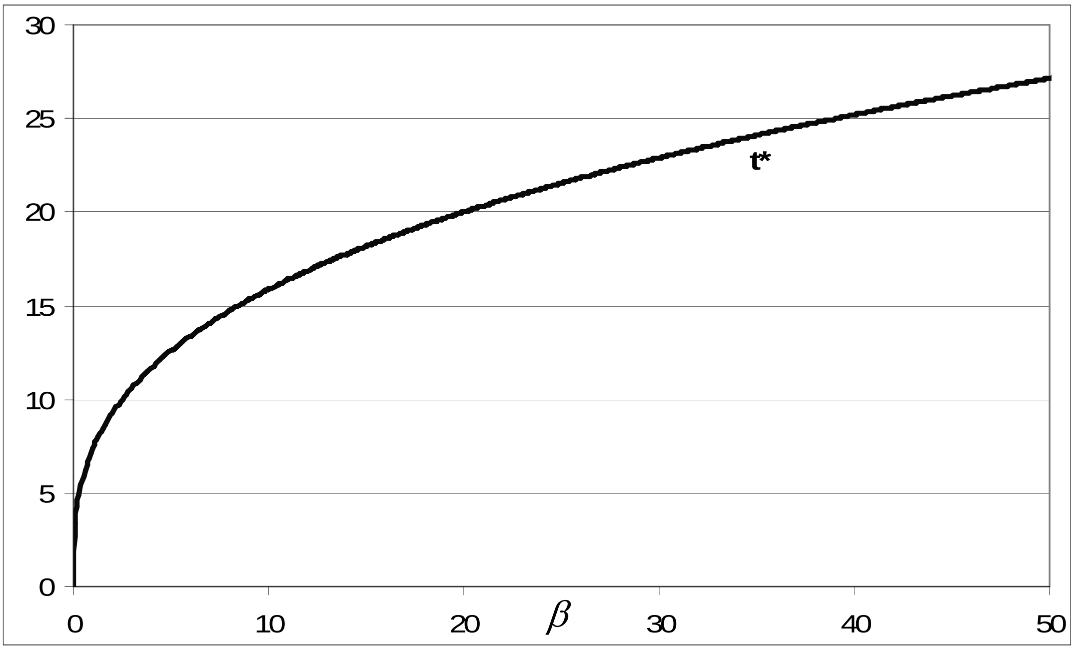



**Figure 5.1:** $t^*(\beta)$ **versus** $\beta$

To find a unique and finite solution $t_{s_p}{}^*$, which minimizes $UEC_{\alpha\beta}(t_p)$, the value of the unique optimal solution calculated in $t_p \le t^*$, based on the hazard rate function $h_s(t)$ of systemability is strictly increasing, has to be compared with each values of $UEC_{\alpha\beta}(t_p)$ for $t_p \ge t^*$ as follows:

**Algorithm A2:** *Given* $c_p$, $c_f$, $\lambda$, $\gamma$, $\alpha$, $\beta$ the *optimal value of* $t_p$, *say* $t_{s_p}{}^*$, *which minimizes the Unit Expected Cost* $UEC_{\alpha\beta}(t_p)$ *for age replacement policy using systemabilty Function is as follows:*

- Step 1: Set-up Phase

  *Set* $c_p$, $c_f$, $\lambda$, $\gamma$, $\alpha$, $\beta$ *real positive constants*

  *Set* $t_p$, $t$ *real positive variables.*

- Step 2: Calculation Phase

*Calculate* $h_s(t) = \frac{\alpha\lambda\gamma * t^{\gamma-1}}{\beta + \lambda t^{\gamma}}$

*Calculate* $R_s(t) = \left[\frac{\beta}{\beta + \lambda t^{\gamma}}\right]^{\alpha}$

*Calculate* $UEC_{\alpha\beta}(t_p) = \frac{c_p * \left[\frac{\beta}{\beta+\lambda t^{\gamma}}\right]^{\alpha} + c_f * \left(1 - \left[\frac{\beta}{\beta+\lambda t^{\gamma}}\right]^{\alpha}\right)}{\int_0^{t_p} \left[\frac{\beta}{\beta+\lambda t^{\gamma}}\right]^{\alpha} dt}$

*Calculate* $t^* = \sqrt[\gamma]{\frac{\beta(\gamma-1)}{\lambda}}$

- Step 3: Decision-Making Phase

  *For* $t_p \leq t^*$, *for any* $t_p \in \left(0, t_p^*\right]$ *and* $t_p \in \left(t_p^*, t^*\right)$, *then* $UEC_{\alpha\beta}(t_p) > UEC_{\alpha\beta}(t_p^*)$, *then* $t_p^*$ *minimizes* $UEC_{\alpha\beta}(t_p^*)$.

  Condition A2_C1: *If* $UEC_{\alpha\beta}(t_p^*) < UEC_{\alpha\beta}(t_p)$ *for any* $t_p \geq t^*$, $t_{sp}^* = t_p^*$ *minimizes* $UEC_{\alpha\beta}(t_p)$, *then* $t_{sp}^*$ *is the unique optimal solution;*

  Condition A2_C2: *If* $UEC_{\alpha\beta}(t_p^*) \geq UEC_{\alpha\beta}(t_p^i)$, *where for any* $t_p \in \left(t^*, t_p^i\right]$ *and* $t_p \in \left(t_p^i, \infty\right)$, *then* $UEC(t_p) > UEC(t_p^i)$, *then* $t_{sp}^* = t_p^i$ *minimizes* $UEC_{\alpha\beta}(t_p)$, *then* $t_{sp}^*$ *is the unique optimal solution.*

### 5.3.3. W-ARP vs S-ARP: Numerical Examples

***Age replacement policy of component with Weibull distribution***

Let us consider a component with Weibull distribution of failure rate, where intensity parameter $\lambda = 0.005$, shape parameter $\gamma = 3$, so with increasing hazard rate *h(t)*. it is also supposed that the cost of a preventive maintenance operation $c_p = 5k€/action$ the cost of a breakdown maintenance operation $c_f = 50k€/action$.

Using *algorithm A1*, proposed in the previous section, let us see how the steps follow:

- Step 1: Set-up Phase

  *Set* $c_p = 5; c_f = 50,\ \lambda = 0.005,\ \gamma = 3$

  *Set* $t_p,\ t$ *real positive variables.*

- Step 2: Calculation Phase

  *Calculate* $h(t) = \lambda\gamma t^{\gamma-1} = 0.005*3*t^{3-1} = 0.015*t^2$

  *Calculate* $R(t) = e^{-\lambda t^{\gamma}} = e^{-0.005*t^3}$

  *Calculate* $MTTF = \int_0^{\infty} e^{-\lambda t^{\gamma}} dt = \int_0^{\infty} e^{-0.005*t^3} dt$

  *Calculate* $UEC(t_p) = \dfrac{c_p * e^{-\lambda t_p^{\gamma}} + c_f * (1 - e^{-\lambda t_p^{\gamma}})}{\int_0^{t_p} e^{-\lambda t^{\gamma}} dt} = \dfrac{5 * e^{-0.005 t_p^3} + 50 * (1 - e^{-0.005 t_p^3})}{\int_0^{t_p} e^{-0.005 t^3} dt}$

- Step 3: Decision-Making Phase

  Condition A1_C1: *If* $h(\infty) \le \dfrac{c_f}{MTTF * (c_f - c_p)}$ *then* $t_p^* = \infty$;

  Condition A1_C2: *If* $h(\infty) > \dfrac{c_f}{MTTF * (c_f - c_p)}$ *for any* $t_p \in (0, t_p^*]$ *and* $t_p \in (t_p^*, \infty)$, *then* $UEC(t_p) > UEC(t_p^*)$, *then* $t_p^*$ *minimizes* $UEC(t_p^*)$, *where* $UEC(t_p^*) = (c_f - c_p) * \lambda\gamma(t_p^*)^{\gamma-1}$.

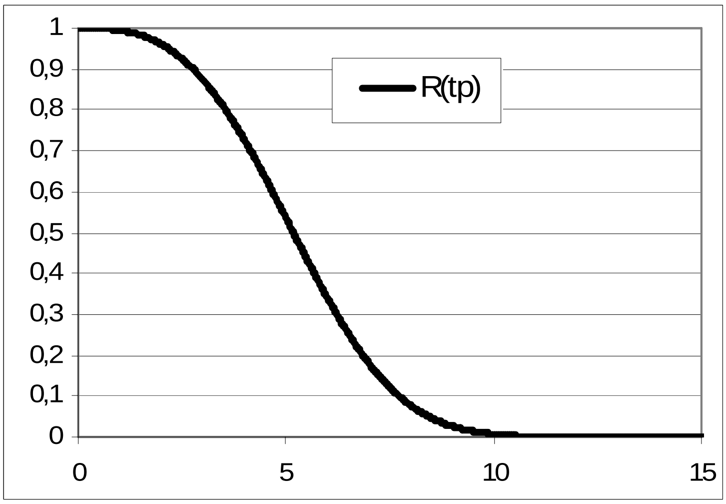
  RESULTS:

  $h(\infty) = \infty$, *so* Condition A1_C1 = FALSE *and* Condition A1_C2: TRUE. *Then:*

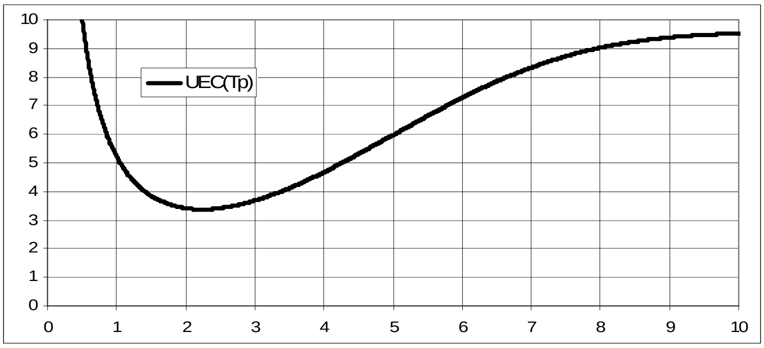
  $t_p^* = 2.24 months$;

  $UEC(t_p^*) = (c_f - c_p) * \lambda\gamma(t_p *)^{\gamma-1} = (50 - 5) * 0.015 * (2.24)^2 = 3.386 k€ / month$

With the parameters, introduced before, and using algorithm A1, the values of optimum time $t_p^*$ is equal to 2.24 months, where the $UEC(t_p^*)$ assumes value 3.386 k€/months (see figure 5.2).




*a) Reliability Function*




*b) Cost Function*

**Figure 5.2: a) Reliability for** $\lambda = 0,005\ \gamma = 3$; **b) Unit Expected Cost for** $\lambda = 0,005\ \gamma = 3$.

### *Age replacement policy using systemability function: UEC curves in function of systemability parameters*

Using the same Weibull parameters and cost values, defined before, in this section the function of $UEC(t_p)$ are tested with the introduction of systemability formula in order to study the effect of environmental factors, using algorithm A2.

Here below, several graphics show the function $UEC_{\alpha\beta}(t_p)$ for different values of the systemability parameters $\alpha$ and $\beta$, fixing the values of the costs parameters, $c_p$ and $c_f$.

This series of graphics illustrates systemability, $R_s(t_p)$, and $UEC_{\alpha\beta}(t_p)$ functions, fixing one of the systemability parameters to a defined value and changing the other one in a series of values between 1, 10, 50.

Let us consider the following values of the parameters in the cost function to carry out this study: $c_p = 5$;

$c_f = 50$; $\lambda = 0.005$ and $\gamma = 3$.

Using algorithm A2, proposed in the previous section, let us see how the following steps as the example with $\alpha = 10$ and $\beta = 50$:

- Step 1: Set-up Phase

  *Set* $c_p = 5$; $c_f = 50$; $\lambda = 0.005$, $\gamma = 3$, $\alpha = 10$, $\beta = 50$

  *Set* $t_p$, $t$ *real positive variables.*

- Step 2: Calculation Phase

  *Calculate* $h_s(t) = \dfrac{\alpha\lambda\gamma * t^{\gamma-1}}{\beta + \lambda t^{\gamma}} = \dfrac{10 * 0.005 * 3 * t^{3-1}}{50 + 0.005t^3} = \dfrac{0.15 * t^2}{50 + 0.005t^3}$

  *Calculate* $R_s(t) = \left[\dfrac{\beta}{\beta + \lambda t^{\gamma}}\right]^{\alpha} = \left[\dfrac{50}{50 + 0.005t^3}\right]^{10}$

  *Calculate*

$$UEC_{\alpha\beta}(t_p) = \frac{c_p * \left[\dfrac{\beta}{\beta + \lambda t^{\gamma}}\right]^{\alpha} + c_f * \left(1 - \left[\dfrac{\beta}{\beta + \lambda t^{\gamma}}\right]^{\alpha}\right)}{\displaystyle\int_0^{t_p} \left[\dfrac{\beta}{\beta + \lambda t^{\gamma}}\right]^{\alpha} dt} = \frac{5 * \left[\dfrac{50}{50 + 0.005t^3}\right]^{10} + 50 * \left(1 - \left[\dfrac{50}{50 + 0.005t^3}\right]^{10}\right)}{\displaystyle\int_0^{t_p} \left[\dfrac{50}{50 + 0.005t^3}\right]^{10} dt}$$

  *Calculate* $t^* = \sqrt[\gamma]{\dfrac{\beta(\gamma - 1)}{\lambda}} = \sqrt[3]{\dfrac{50(3-1)}{0.005}} = 27.144\,months$

- Step 3: Decision-Making Phase

  *For* $t_p \le t^*$, *for any* $t_p \in \left(0, t_p^*\right]$ *and* $t_p \in \left(t_p^*, t^*\right)$, *then* $UEC_{\alpha\beta}(t_p) > UEC_{\alpha\beta}(t_p^*)$, *then* $t_p^*$ *minimizes* $UEC_{\alpha\beta}(t_p^*)$.

  Condition A2_C1: *If* $UEC_{\alpha\beta}(t_p^*) < UEC_{\alpha\beta}(t_p)$ *for any* $t_p \ge t^*$, $t_{s_p}^* = t_p^*$ *minimizes* $UEC_{\alpha\beta}(t_p)$, *then* $t_{s_p}^*$ *is the unique optimal solution;*

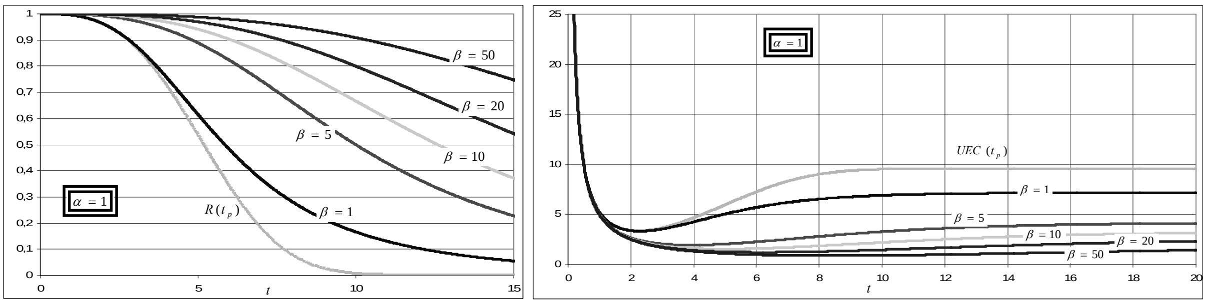
Condition A2_C2: *If* $UEC_{\alpha\beta}(t_p^{\ *}) \geq UEC_{\alpha\beta}(t_p^{\ i})$, *where for any* $t_p \in \left(t^*, t_p^{\ i}\right]$ *and* $t_p \in \left(t_p^{\ i}, \infty\right)$, *then* $UEC(t_p) > UEC(t_p^{\ i})$, *then* $t_{s\,p}^{\ *} = t_p^{\ i}$ *minimizes* $UEC_{\alpha\beta}(t_p)$, *then* $t_{s\,p}^{\ *}$ *is the unique optimal solution.*

RESULT:

$t_p^{\ *} = 3.83 months$

$UEC_{\alpha\beta}(t_p^{\ *}) = 1.973 < UEC_{\alpha\beta}(t_p)$: Condition A2_C1=TRUE → $t_{s\,p}^{\ *} = t_p^{\ *}$ *minimizes* $UEC_{\alpha\beta}(t_p)$, *then* $t_{s\,p}^{\ *}$ *is the unique optimal solution.*

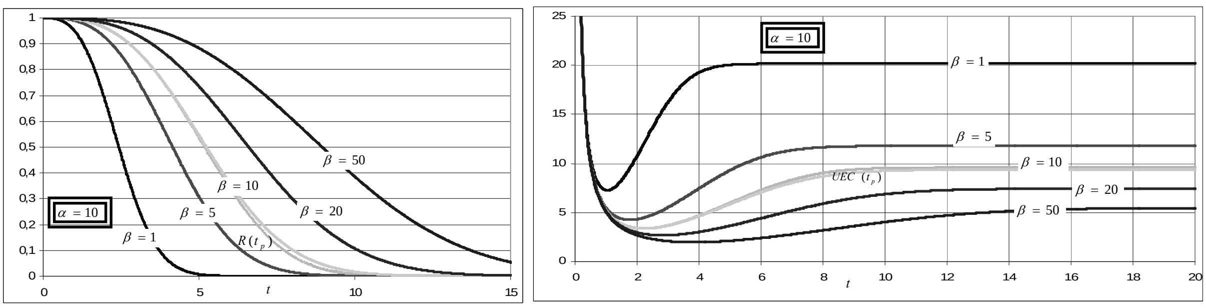
Figures 5.3, 5.4, 5.5 show the systemability function $R_s(t_p)$ and the cost function $UEC_{\alpha\beta}(t_p)$ for different values of $\alpha$, varying the $\beta$ parameter, in comparison with $R(t_p)$ and $UEC(t_p)$.




*a) Reliability Functions* *b) Cost Functions*

**Figure 5.3: a) Comparison of reliability vs. systemability function for α = 1 and β = 1, 5, 10, 20 and 50.**

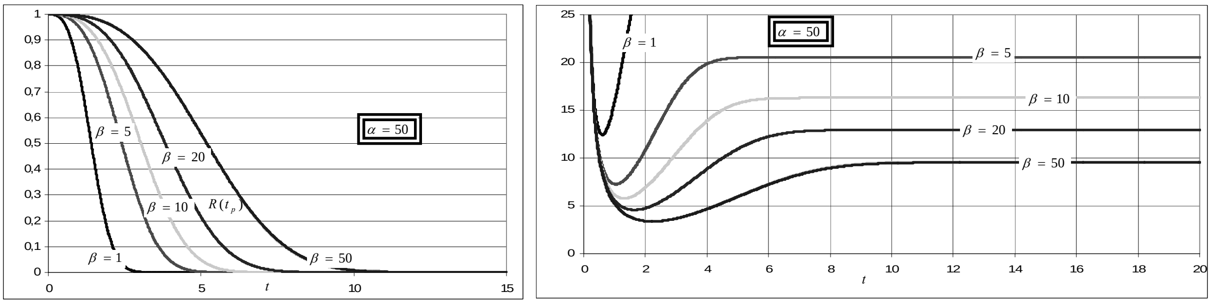
**b) Comparison of cost function $UEC(t_p)$ vs. $UEC_{\alpha\beta}(t_p)$ for α = 1 and β = 1, 5, 10, 20 and 50.**




*a) Reliability Functions* *b) Cost Functions*

**Figure 5.4: a) Comparison of reliability vs. systemability function for α = 10 and β = 1, 5, 10, 20 and 50.**

**b) Comparison of cost function $UEC(t_p)$ vs. $UEC_{\alpha\beta}(t_p)$ for α = 10 and β = 1, 5, 10, 20 and 50.**

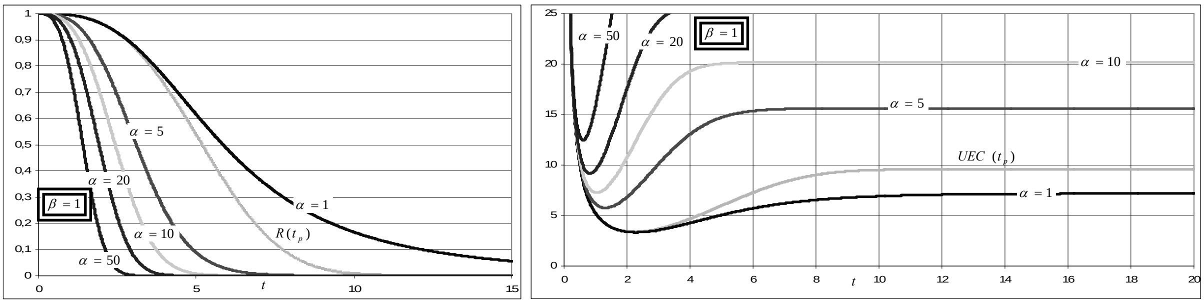



a) Reliability Functions    b) Cost Functions

**Figure 5.5: a) Comparison of reliability vs. systemability function for α = 50 and β = 1, 5, 10, 20 and 50.**

**b) Comparison of cost function $UEC(t_p)$ vs. $UEC_{\alpha\beta}(t_p)$ for α = 50 and β = 1, 5, 10, 20 and 50.**

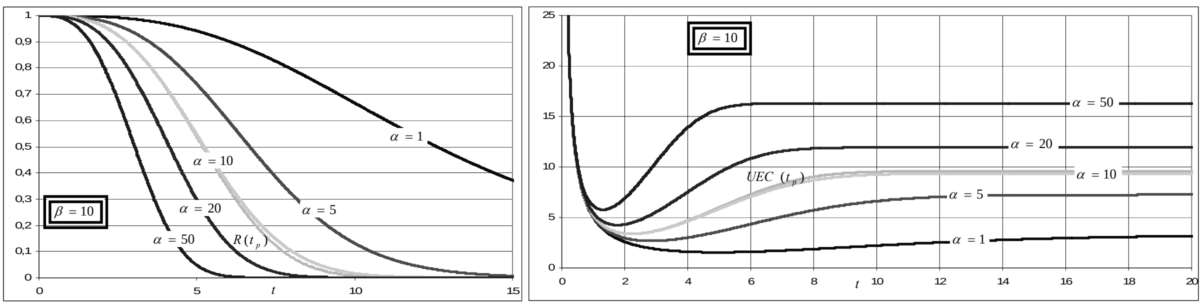
Furthermore, figures 5.6, 5.7 and 5.8 show the systemability function $R_s(t_p)$ and the cost function $UEC_{\alpha\beta}(t_p)$ for different values of $\beta$, varying the $\alpha$ parameter, in comparison with $R(t_p)$ and $UEC(t_p)$.




a) Reliability Functions    b) Cost Functions

**Figure 5.6: a) Comparison of reliability vs. systemability function for β = 1 and α = 1, 5, 10, 20 and 50.**

**b) Comparison of cost function $UEC(t_p)$ vs. $UEC_{\alpha\beta}(t_p)$ for β = 1 and α = 1, 5, 10, 20 and 50.**




a) Reliability Functions    b) Cost Functions

**Figure 5.7: a) Comparison of reliability vs. systemability Function for β = 10 and α = 1, 5, 10, 20 and 50.**

**b) Comparison of cost function $UEC(t_p)$ vs. $UEC_{\alpha\beta}(t_p)$ for β = 10 and α = 1, 5, 10, 20 and 50.**

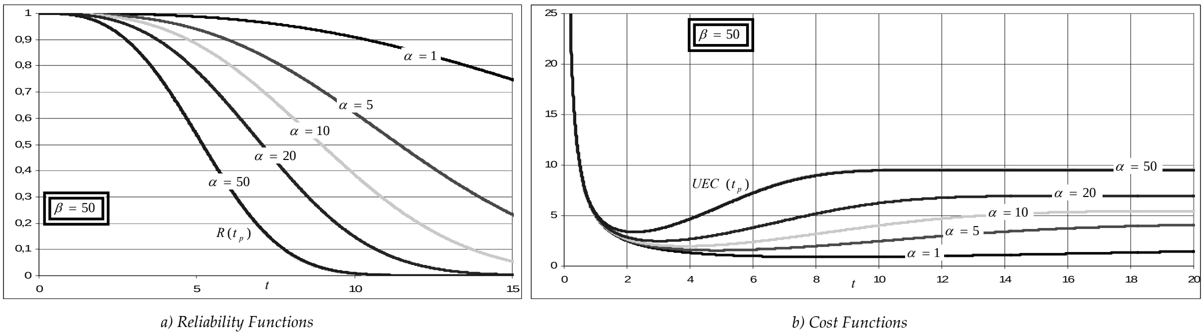



**Figure 5.8: a) Comparison of reliability vs. systemability function for β = 50 and α = 1, 5, 10, 20 and 50.**

**b) Comparison of cost function $UEC(t_p)$ vs. $UEC_{\alpha\beta}(t_p)$ for β = 50 and α = 1, 5, 10, 20 and 50.**

As well shown in these graphics, two scenarios can be defined:

- Scenario 1:

*hard environment effects:* when $\alpha > \beta$, the $UEC_{\alpha\beta}(t_p)$ is higher than $UEC(t_p)$ because $R_s(t)$ is worse than $R(t)$.

- Scenario 2:

*soft environment effects:* when $\alpha < \beta$, the $UEC_{\alpha\beta}(t_p)$ is lower than $UEC(t_p)$ because $R_s(t)$ is better than $R(t)$.

The previous figures show the effects of the environmental factors, represented with the systemability parameters $\alpha$ and $\beta$, on reliability and on Expected Unit Cost functions. In fact, as explained before, if the operating environment is different from the testing one, the reliability will be different from the one calculated during the test phases. As a consequence, also the Expected Unit Cost will be different and it could be higher or lower, depending on the effects of environmental factors, described by $\alpha$ and $\beta$ parameters, as shown before.

### 5.3.4. Further analysis about S-ARP: $t_{sp}^*$ and $\%UEC(t_p^*)$ curves in function of systemability parameters

The second group shows several graphics in order to analyze the changing of the optimal time of preventive maintenance and to estimate the difference between the UEC considering the environmental factors or not.

Let us consider the following values of the parameters: $\lambda$ = 0.005 and $\gamma$ = 3, for different values of costs parameters, $c_p$ and $c_f$, and it has been illustrated:

- a) $t_{sp}^*$ curve for $0<\alpha<\infty$ and $\beta$ = 1, 5, 10, 20, 50;
- b) $t_{sp}^*$ curve for $0<\beta<\infty$ and $\alpha$ = 0.5, 1, 5, 10, 20, 50;
- c) $\%UEC(t_p^*)$ curve for $0<\alpha<\infty$ and $\beta$ = 1, 5, 10, 20, 50;
- d) $\%UEC(t_p^*)$ curve for $0<\beta<\infty$ and $\alpha$ = 0.5, 1, 5, 10, 20, 50;
- e) $t_{sp}^*$ surface for $0<\alpha<\infty$ and $0<\beta<\infty$;
- f) $\%UEC(t_p^*)$ surface for $0<\alpha<\infty$ and $0<\beta<\infty$.

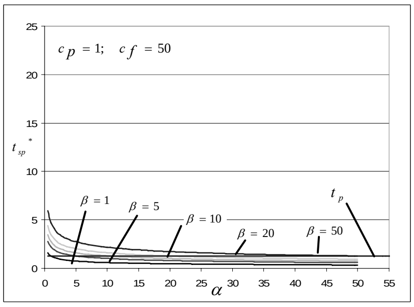
where $t_{sp}^*$ is the value which minimized the $UEC_{\alpha\beta}(t_p)$, calculated with equation 5-12

Then $\%UEC(t_p^*)$ is calculated as:

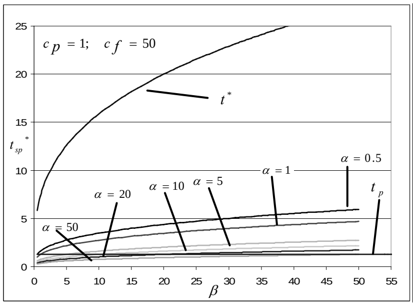
$$\%UEC(t_p^*) = 100 * \left( \frac{UEC_{\alpha\beta}(t_p^*)}{UEC_{\alpha\beta}(t_{sp}^*)} - 1 \right) \qquad \text{5-17}$$

It indicates the percent difference between the real Unit Expected Cost $UEC_{\alpha\beta}(t_p^*)$, using a planned time $t_p^*$ calculated with only Weibull distribution, without consider the environmental effects, and the optimum value of Unit Expected Cost $UEC_{\alpha\beta}(t_{sp}^*)$ estimated considering also the environmental effects with systemability parameters, calculated in the real optimum time value $t_{sp}^*$.

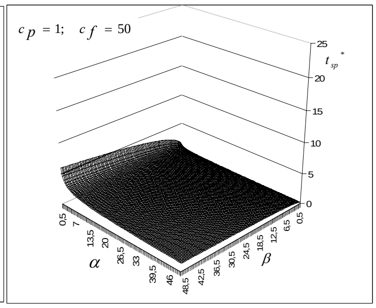
All calculated $t_{sp}^*$ are minor than $t^*$, given by equation 5-16, and satisfy condition (a) of algorithm A2 hence $t_{sp}^*$ can be considered the unique and finite solution of $UEC_{\alpha\beta}(t_{sp}^*)$.
Figures 5.9, 5.10 and 5.11 show the curve of $t_{sp}^*$ and $\%UEC(t_p^*)$ for different values of $c_p$, fixed $c_f$ = 50, varying $\alpha$ and $\beta$ values. For each figure, the optimal times $t_p^*$ has been shown which values have been calculated with the formula related to the test reliability using algorithm A1.



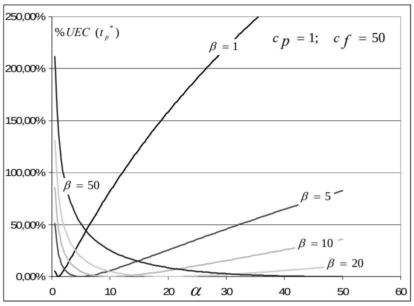

a) Optimal Time vs. parameter "α"



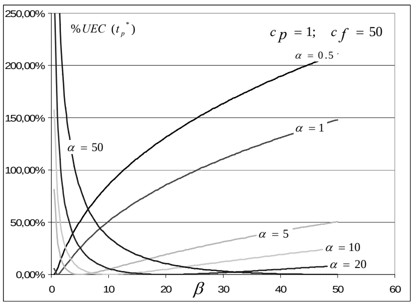

b) Optimal Time vs. parameter "β"



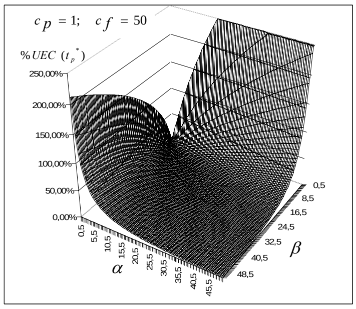

c) Optimal Time vs. parameters "α" and "β"




d) $\%UEC(t_p^*)$ vs. parameter "α"




e) $\%UEC(t_p^*)$ vs. parameter "β"




f) $\%UEC(t_p^*)$ vs. parameters "α" and "β"

**Figure 5.9: a) Curve of $t_{sp}^*$ in function of α and β = 1, 5, 10, 20 and 50;**

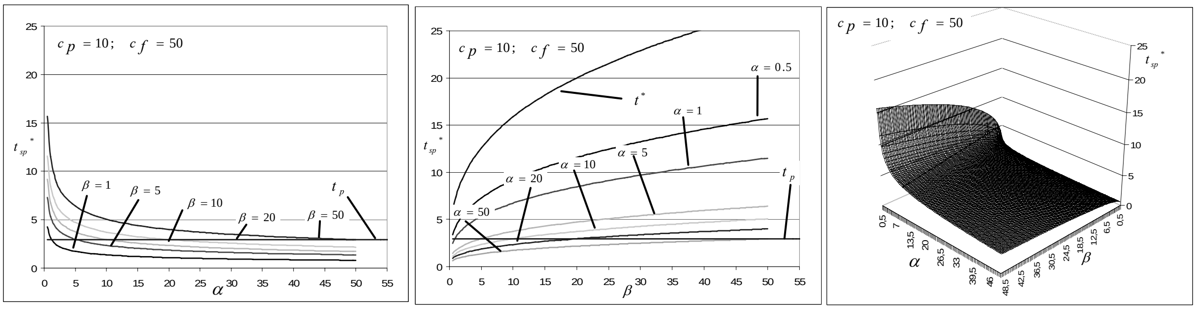
**b) Curve of $t_{sp}^*$ in function of β and α = 0.5, 1, 5, 10, 20 and 50;**

**c) Curve of $t_{sp}^*$ in function of α and β. All these for $c_p$ = 1 and $c_f$ = 50;**

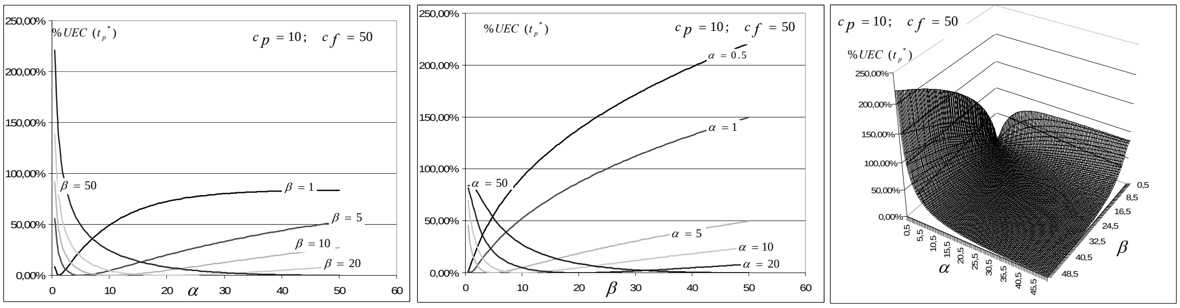
**d) Curve of $\%UEC(t_p^*)$ in function of α and β = 1, 5, 10, 20 and 50;**

**e) Curve of $\%UEC(t_p^*)$ in function of β and α = 0.5, 1, 5, 10, 20 and 50;**

**f) Curve of $\%UEC(t_p^*)$ in function of α and β. All these for $c_p$ = 1 and $c_f$ = 50.**




a) Optimal Time vs. parameter "α" b) Optimal Time vs. parameter "β" c) Optimal Time vs. parameters "α" and "β"




d) %UEC($t_p^*$) vs. parameter "α" e) %UEC($t_p^*$) vs. parameter "β" f) %UEC($t_p^*$) vs. parameters "α" and "β"

**Figure 5.10: a) Curve of $t_{sp}^*$ in function of α and β = 1, 5, 10, 20 and 50;**

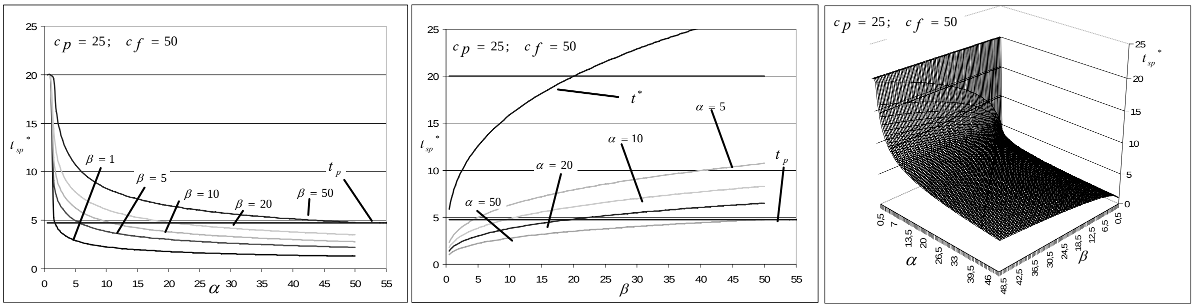
**b) Curve of $t_{sp}^*$ in function of β and α = 0.5, 1, 5, 10, 20 and 50;**

**c) Curve of $t_{sp}^*$ in function of α and β. All these for $c_p$ = 10 and $c_f$ = 50;**

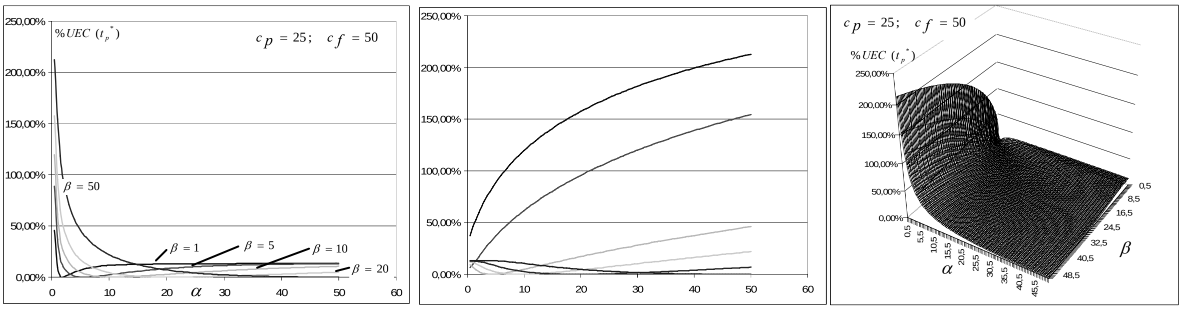
**d) Curve of *%UEC($t_p^*$)* in function of α and β = 1, 5, 10, 20 and 50;**

**e) Curve of *%UEC($t_p^*$)* in function of β and α = 0.5, 1, 5, 10, 20 and 50;**

**f) Curve of *%UEC($t_p^*$)* in function of α and β. All these for $c_p$ = 10 and $c_f$ = 50.**




a) Optimal Time vs. parameter "α" b) Optimal Time vs. parameter "β" c) Optimal Time vs. parameters "α" and "β"




d) %UEC($t_p^*$) vs. parameter "α" e) %UEC($t_p^*$) vs. parameter "β" f) %UEC($t_p^*$) vs. parameters "α" and "β"

**Figure 5.11: a) Curve of $t_{sp}^*$ in function of $\alpha$ and β = 1, 5, 10, 20 and 50;**

**b) Curve of $t_{sp}^*$ in function of β and $\alpha$ = 0.5, 1, 5, 10, 20 and 50;**

**c) Curve of $t_{sp}^*$ in function of $\alpha$ and β. All these for $c_p$ = 25 and $c_f$ = 50;**

**d) Curve of %UEC($t_p^*$) in function of $\alpha$ and β = 1, 5, 10, 20 and 50;**

**e) Curve of %UEC($t_p^*$) in function of β and $\alpha$ = 0.5, 1, 5, 10, 20 and 50;**

**f) Curve of %UEC($t_p^*$) in function of $\alpha$ and β. All these for $c_p$ = 25 and $c_f$ = 50.**

From the analysis of the previous graphics some considerations could be made:

- fixed parameter $\beta$, $t_{sp}^*$ is a decreasing function of parameter $\alpha$;
- fixed parameter $\alpha$, $t_{sp}^*$ is an increasing function of parameter $\beta$;
- overall, a general increasing of $t_{sp}^*$ is well shown for increasing values of couple ($\alpha$; $\beta$), from (0,0) to (50,50).
- increasing the cost parameter $c_p$, the $\%UEC(t_p^*)$ is less influenced by parameter $\alpha$ and more influenced by the other systemability parameter $\beta$;

Furthermore, the $\%UEC(t_p^*)$ values have been collected in different intervals, every five points percent, and for each interval, the probability distribution $Pc$ of $\%UEC(t_p^*)$ has been calculated in order to define the mean value of $\%UEC(t_p^*)$ and its distribution.

Here below, several graphics (figure 5.12) show the probability distribution $Pc$ of $\%UEC(t_p^*)$ for different values of the cost parameter $c_p$, respectively 1, 5, 10, 25, while the cost parameter $c_f$ is equal to 50.

The mean value for each case is about 10%, so if a preventive maintenance time is design using the test data set, the mean major cost, due to the environmental effects, is about 10 %.

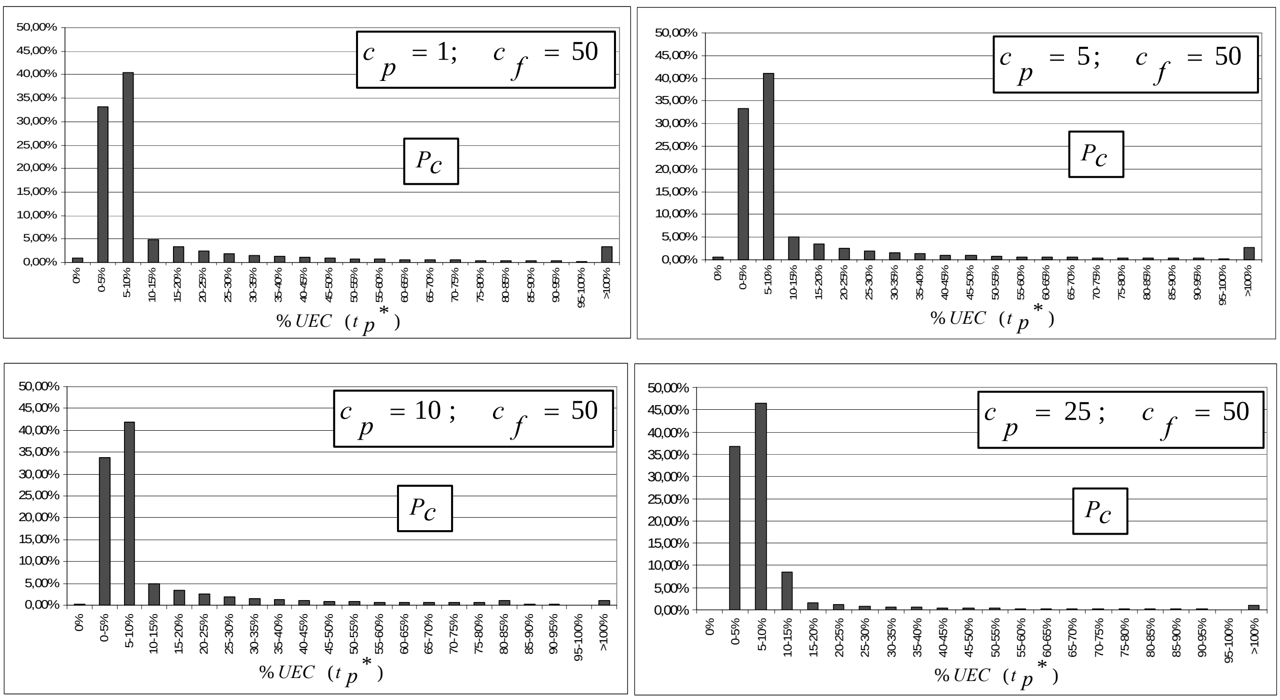



**Figure 5.12: Probability distribution of $\%UEC(t_p^*)$ for different values of $c_p$**

### 5.3.5. Real Application: ARP on Automatic Packaging Machines.

In this section, the proposed methodology applied to an interesting case study in the automatic packaging machines for beer production is illustrated. Some data related to different applications of the same model of machine, in several customer plants located in different countries, have been collected. The two plants have been visited and analyzed in detail, that present the same machine subject to different environmental conditions.

The study gives special attention to the automatic bottle filler because usually it is the bottleneck of the whole production system and it is more influenced by the environmental factors than the other machines.

The main causes of failure are investigated. As it is observed, the principal downtimes are caused by failures of the filler heads upon which the study has been oriented.

Table 5.1 shows the data set that has taken from this component where the time values $t$ has been normalized. $R_a(t)$ represents reliability function values collected from the test environment. The first plant presents the same data of the test environment while the second one is described by $R_b(t)$.

| T | 0.00 | 0.03 | 0.07 | 0.10 | 0.13 | 0.17 | 0.20 | 0.23 | 0.27 | 0.30 | 0.33 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $R_a(T)$ | 100.00 | 99.98 | 99.95 | 99.92 | 99.86 | 99.76 | 99.59 | 99.33 | 98.94 | 98.36 | 97.52 |
| $R_b(T)$ | 100.00 | 99.88 | 99.79 | 99.64 | 99.40 | 99.03 | 98.50 | 97.73 | 96.66 | 95.20 | 93.25 |

| T | 0.37 | 0.40 | 0.43 | 0.47 | 0.50 | 0.53 | 0.57 | 0.60 | 0.63 | 0.67 |
|---|---|---|---|---|---|---|---|---|---|---|
| $R_a(T)$ | 96.34 | 94.71 | 93.21 | 89.59 | 85.81 | 82.30 | 75.17 | 66.79 | 60.10 | 52.21 |
| $R_b(T)$ | 90.70 | 87.44 | 83.36 | 78.37 | 72.44 | 65.59 | 57.91 | 49.61 | 41.00 | 32.47 |

| T | 0.70 | 0.73 | 0.77 | 0.80 | 0.83 | 0.87 | 0.90 | 0.93 | 0.97 | 1.00 |
|---|---|---|---|---|---|---|---|---|---|---|
| $R_a(T)$ | 42.21 | 33.20 | 23.45 | 14.88 | 9.68 | 4.50 | 2.56 | 1.06 | 0.37 | 0.10 |
| $R_b(T)$ | 24.46 | 17.37 | 11.52 | 7.05 | 3.94 | 1.98 | 0.88 | 0.34 | 0.11 | 0.03 |

**Table 5.1: Testing and operating data set**

Fitting the data using Weibull distribution and systemability function, the parameters illustrated in table 5.2 and figure 5.13 have been obtained. Also the cost parameters given from the maintenance service of the company are included.

| Weibull parameters | | Systemability parameters | | Cost parameters | |
|---|---|---|---|---|---|
| $\lambda$ | $\gamma$ | $\alpha$ | $\beta$ | $c_p$ | $c_f$ |
| 0.0000915 | 8.10 | 3.10 | 1.42 | 3 | 30 |

**Table 5.2: Parameters of Weibull and systemability function and cost parameters**

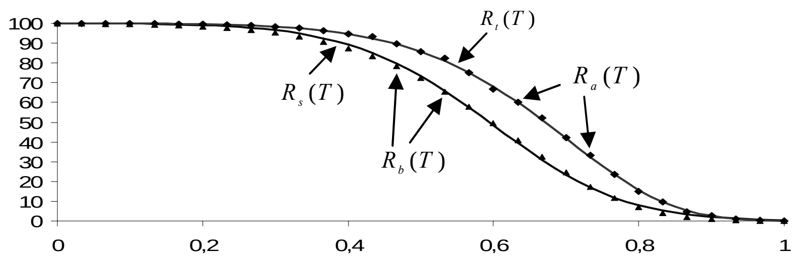



**Figure 5.13: Reliability during Testing ($R_a(t)$), during Operation ($R_b(t)$), Weibull Function ($R_t(t)$) and Systemability Function ($R_s(t)$).**

Considering $c_p = 3$, $c_f = 30$, $\lambda = 0.0000915$, $\gamma = 8.1$, $\alpha = 3.10$, $\beta = 1.42$, where the parameters values of systemability have been estimated using the least squares estimate method [5.6], the age-replacement policy of the second plant has been optimized thanks to the consideration of the environmental factors with systemability function.

Considering only the Weibull parameters, the application of algorithm A1 have given us the replacement time related to the testing data, $t_p{*} = 2.24 months$, and the Unit Expected Cost is $UEC(t_p{*}) = 1.184 k€ / month$.

Using algorithm A2, the optimal time to replacement has been calculated considering also the environmental effects to the survival function, applying the systemability function. In this case, the optimal solution is $t_{sp}{*} = 1.71 months$ and the Unit Expected Cost is $UEC_{\alpha\beta}(t_{sp}{*}) = 1.998 k€ / month$.

If it is calculated the Unit Expected Cost in the optimal solution of testing data, but taking into consideration the environmental factors, its value will be $UEC_{\alpha\beta}(t_p*) = 2.392k€/month$ (figure 5.14). As well demonstrated, the Unit Expected Costs are different between these cases and the optimal solution, using algorithm A2 and considering the environmental effects, allows to saving about the 20%.

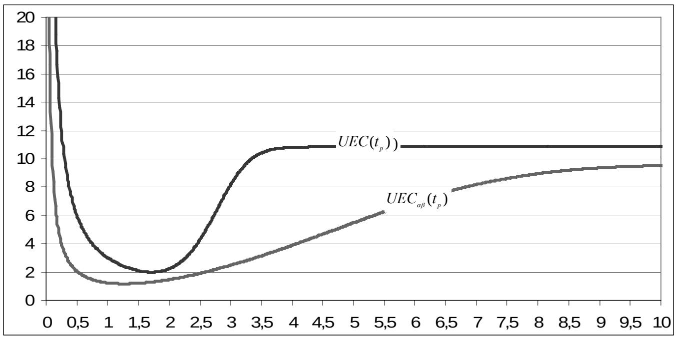



**Figure 5.14: Real application of the methodology to the age replacement policy on the automatic machine component**

### 5.3.6. Some considerations about Systemability application on ARP

In the last decades, maintenance policies design have become necessary to guarantee high level of efficiency of the complex industrial systems. Each maintenance plan has been defined starting with the collection of data about the lifetime of components in order to use it to estimate the different survival functions. Thanks to these ones, several maintenance policies can be applied for improving system reliability, preventing the occurrence of system failures, and reducing maintenance costs of systems.

Many maintenance models have been developed and widely used in scientific literature. The age replacement policy is one of the most important.

Incorrect estimation of reliability function could lead to the wrong functional design of the system and an incorrect definition of the appropriate maintenance policies.

In this chapter, the effects of the different operating environments in the age replacement policy have been demonstrate using a new concept, called systemability [5.19 – 5.20 – 5.21].

The aim of this work is to introduce the systemability concept in the maintenance policies design in order to show its benefits, because it considers also the environmental conditions in which the systems operate.

With its parameters, systemability calculates the reliability in random environments, using as starting data, the reliability obtained during the test, and processing it using a gamma distribution in particular, or a distribution that represents operating environments in general, which takes into consideration the environmental factors. In other words, each operating environment is characterized by related systemability parameters calculated before with several failure data. These parameters allow to estimate the reliability behaviour of each system operating in such environment conditions, using systemability function.

Hence, the purpose of this chapter is to evaluate the application of systemability function to the age replacement policy and highlight the benefits arisen with the use of this concept in comparison with the classical methodology.

The analytical behaviour of the total cost function and a useful algorithm have been introduced in order to help the practitioners to apply this innovative model. Furthermore, a sensitivity analysis of these benefits is conducted in order to show how the outcomes vary in function of the different environmental conditions, represented by systemability parameters $\alpha$ and $\beta$. In conclusions the application of systemability permits the correct estimation of time-replacement and consequently the minimization of global maintenance cost UEC. The results of this application have been illustrated with a series of graphics and summarized at the end. It's important to highlight that the mean value of $\%UEC(t_p^*)$ is about 10%, that is the percent difference between the real Unit Expected Cost $UEC_{\alpha\beta}(t_p^*)$, using a planned time $t_p^*$ calculated with only Weibull distribution, without consider the environmental effects, and the optimum value of Unit Expected Cost $UEC_{\alpha\beta}(t_{s\,p}^*)$ estimated considering also the environmental effects with systemability parameters, calculated in the real optimum time value $t_{s\,p}^*$. A real industrial application has demonstrated the importance of this study, and shown about a 20% saving (about 5,000 €/years) using this methodology in comparison with the application of traditional age replacement policy.

In few words, if a preventive maintenance time is design using the test data set, the mean major cost, due to the environmental effects, will be about 10% in comparison with the use of systemability on estimating of the time to replacement.

## *5.4. Theory on Periodic Replacement Policy (PRP)*

This policy is one of the most used time-based preventive maintenance policies. It is also called policy *type II*. It involves a preventive maintenance action on the system after a operative time $t_p$, independent from the number of replacements at failure, occur in time $t_p$.

After each maintenance replacement action, the component becomes as good as new, hence the number of preventive maintenance actions do not affect to the failure rate of the system.
In other words the system failure rate remains undisturbed by the minimal repair.
The optimum policy for minimizing the total expected rate and for maximizing the availability of the component is shown before.

***Notation***

$f(t)$ probability density function of failure

$h(t)$ hazard rate function

$R(t)$ reliability function

$F(t)$ failure function

$\lambda$ intensity parameter of Weibull distribution

$\gamma$ shape parameter of Weibull distribution

$\eta$ a common environment factor

$G(\eta)$ cumulative distribution function of $\eta$

$\alpha$ shape parameter of Gamma distribution

$\beta$ scale parameter of Gamma distribution

$c_p$ cost of a preventive maintenance operation

$c_f$ cost of a breakdown maintenance operation

$UEC$ Unit Expected Cost of the replacement policy

$t_p$ planned time of replacement

Barlow et al. defined the Unit Excepted Cost as following:

$$UEC(t_p) = \frac{c_p + c_f * \int_0^{t_p} h(t)dt}{t_p} \quad \text{5-18}$$

where $c_p$ and $c_f$ are the cost of preventive and breakdown replacement, with $c_p < c_f$.

Differentiating the $UEC(t_p)$ with respect of time $t_p$ and setting it equal to zero:

$$\frac{\partial UEC(t_p)}{\partial t_p}=\frac{\partial}{\partial t_p}\left(\frac{c_p+c_f*\int_0^{t_p}h(t)dt}{t_p}\right)=0 \qquad 5\text{-}19$$

The followed equation has given:

$$t_p*h(t_p)-\int_0^{t_p}h(t)dt=\frac{c_p}{c_p} \qquad 5\text{-}20$$

As well shown in [5.18] it can be found an optimum time $t_p^*$ which minimized the function $UEC(t_p)$, provided the hazard rate function $h(t_p)$ is strictly increasing. Since $\lim_{t_p\to 0}UEC(t_p)=\infty$, a positive $t_p^*$ must exist.

In this case, the Unit Expected Cost is:

$$UEC(t_p)=c_p*h(t_p) \qquad 5\text{-}21$$

### 5.4.1. Classical Approach: Weibull Periodic Replacement Policy (W-PRP)

Considering Weibull distribution for the lifetime data of replaced components, the periodic replacement policy, its minimal Unit Expected Cost $UEC(t_p^*)$ and optimal preventive maintenance time $t_p^*$ could be analyzed using the following formulas.

In this case, the different survival functions are:

$$R(t)=e^{-\lambda t^{\gamma}} \qquad 5\text{-}22$$

$h(t)=\lambda\gamma t^{\gamma-1}$ (strictly increasing with $\gamma>1$, see (Pham, [1])) 5-23

then the Unit Expected Cost function is:

$$UEC(t_p)=\frac{c_p*e^{-\lambda t_p^{\gamma}}+c_f*(1-e^{-\lambda t_p^{\gamma}})}{\int_0^{t_p}e^{-\lambda t^{\gamma}}dt}=\frac{c_p+c_f*\lambda t_p^{\gamma}}{t_p}. \qquad 5\text{-}24$$

If the hazard rate follows a Weibull distribution, that is $h(t)=\lambda\gamma t^{\gamma-1}$, the optimal time is given solving equation 5-20:

$$t_p=\left(\frac{c_p}{(\gamma-1)*c_f*\lambda}\right)^{1/\gamma}.$$

**Algorithm A3:** *Given $c_p$, $c_f$, $\lambda$, $\gamma$, the optimal value of $t_p$, say $t_p^*$, which minimized the Unit Expected Cost for age replacement policy is as follows:*

- Step 1: Set-up Phase

  *Set $c_p$, $c_f$, $\lambda$, $\gamma$, real positive constants*

  *Set $t_p$, $t$ real positive variables.*

- Step 2: Optimal-Solution Calculation Phase

  *Calculate* $t_p{*} = \left( \dfrac{c_p}{(\gamma - 1) * c_f * \lambda} \right)^{1/\gamma}$.

  *Calculate* $UEC\,(t_p{*}) = \dfrac{c_p * e^{-\lambda t_p{*}^{\gamma}} + c_f * (1 - e^{-\lambda t_p{*}^{\gamma}})}{\int_0^{t_p{*}} e^{-\lambda t^{\gamma}}\, dt} = \dfrac{c_p + c_f * \lambda (t_p{*})^{\gamma}}{t_p{*}}$

### 5.4.2. New Approach: Systemability Periodic Replacement Policy (S-PRP)

The $UEC(t_p)$ assumes this mathematical expression in the operating environments:

$$UEC_{\alpha\beta}(t_p) = \frac{c_p + c_f * \int_0^{t_p} h_s(t)dt}{t_p} \qquad \text{5-25}$$

Also in this case the $t_{s\,p}^{\,*}$ is a finite and unique solution if the hazard rate function $h_s(t)$ of systemability is strictly increasing.

If the systemability formula is considered:

$$R_s(t) = \left[ \frac{\beta}{\beta + \lambda t^{\gamma}} \right]^{\alpha}$$

and kwon that $h_s(t) = \dfrac{1}{R_s(t)} * \left( -\dfrac{\partial R_s(t)}{\partial t} \right)$, where $\dfrac{\partial R_s(t)}{\partial t} = -\dfrac{\alpha\lambda\gamma * \left( \dfrac{\beta}{\beta + \lambda t^{\gamma}} \right)^{\alpha}}{\beta + \lambda t^{\gamma}} * t^{\gamma - 1}$

so the hazard rate function $h_s(t)$ becomes:

$$h_s(t) = \frac{\alpha\lambda\gamma * t^{\gamma-1}}{\beta + \lambda t^{\gamma}} \qquad \text{5-26}$$

Using formulas 5-18 and 5-26, the $UEC(t_p)$ assumes this mathematical expression:

$$UEC_{\alpha\beta}(t_p) = \frac{c_p + c_f * \int_0^{t_p} \frac{\alpha\lambda\gamma * t^{\gamma-1}}{\beta + \lambda t^{\gamma}} dt}{t_p} = \frac{c_p + c_f * \left[\alpha * \ln\left(1 + \frac{\lambda}{\beta} t_p^{\gamma}\right)\right]}{t_p} \qquad \text{5-27}$$

Also in this case the $t_{sp}^*$ is a finite and unique solution if the hazard rate function $h_s(t)$ of systemability is strictly increasing. If this function is studied, it can be found that the $\lim_{t\to 0} h_s(t) = 0$ and also $\lim_{t\to\infty} h_s(t) = 0$. Moreover, the function $h_s(t)$ is major than 0 for all values of variable *t*.

Differentiating the $h_s(t)$ with respect of time $t$ and setting it equal to zero:

$$\frac{\partial h_s(t)}{\partial t} = \frac{\partial}{\partial t}\left(\frac{\alpha\lambda\gamma * t^{\gamma-1}}{\beta + \lambda t^{\gamma}}\right) = 0 \qquad \text{5-28}$$

The followed solution is given:

$$t^* = \sqrt[\gamma]{\frac{\beta(\gamma-1)}{\lambda}} \qquad \text{5-29}$$

Considering the $h_s'(t)$, it can be noticed that $h_s'(t=0) = 0$ and $h_s'(t=\infty) = 0$. Hence, if $0 \le t \le t^*$ the hazard rate function $h_s(t)$ is strictly increasing, while on the other hand, if $t \ge t^*$ the hazard rate function $h_s(t)$ is strictly decreasing, due also to $h_s'(t=0) \ge 0$ and $h_s'(t=\infty) \ge 0$.

It's interesting to notice that the $t^*$ depends only on a parameter of systemability, that is $\beta$ one. Moreover, increasing $\beta$ value the value of $t^*$ increases, as shown in the following figure 5.15.

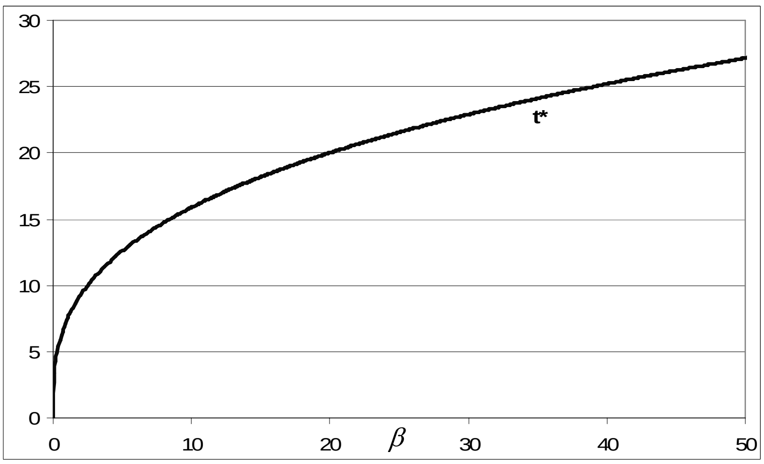



**Figure 5.15: $t^*(\beta)$ versus $\beta$**

To find a unique and finite solution $t_{sp}^{*}$, which minimizes $UEC_{\alpha\beta}(t_p)$, the value of the unique optimal solution calculated in $t_p \le t^{*}$, based on the hazard rate function $h_s(t)$ of systematability is strictly increasing, with each values of $UEC_{\alpha\beta}(t_p)$ for $t_p \ge t^{*}$ has have to compare as follows:

**Algorithm A4:** *Given $c_p$, $c_f$, $\lambda$, $\gamma$, $\alpha$, $\beta$, the optimal value of $t_p$, say $t_{sp}^{*}$, which minimized the Unit Expected Cost $UEC_{\alpha\beta}(t_p)$ for periodic replacement policy using systemabilty function is as follows:*

- Step 1: Set-up Phase

  *Set $c_p$, $c_f$, $\lambda$, $\gamma, \alpha$, $\beta$, real positive constants*

  *Set $t_p$, $t$ real positive variables.*

- Step 2: Calculation Phase

  *Calculate* $h_s(t) = \dfrac{\alpha\lambda\gamma * t^{\gamma-1}}{\beta + \lambda t^{\gamma}}$

  *Calculate* $R_s(t) = \left[\dfrac{\beta}{\beta + \lambda t^{\gamma}}\right]^{\alpha}$

  *Calculate* $UEC_{\alpha\beta}(t_p) = \dfrac{c_p + c_f * \int_0^{t_p} \frac{\alpha\lambda\gamma * t^{\gamma-1}}{\beta + \lambda t^{\gamma}} dt}{t_p} = \dfrac{c_p + c_f * \left[\alpha * \ln\left(1 + \frac{\lambda}{\beta} t_p^{\gamma}\right)\right]}{t_p}$

  *Calculate* $t^{*} = \sqrt[\gamma]{\dfrac{\beta(\gamma-1)}{\lambda}}$

- Step 3: Decision-Making Phase

  *For $t_p \le t^{*}$, for any $t_p \in \left(0, t_p^{*}\right]$ and $t_p \in \left(t_p^{*}, t^{*}\right)$, then $UEC_{\alpha\beta}(t_p) > UEC_{\alpha\beta}(t_p^{*})$, then $t_p^{*}$ minimizes $UEC_{\alpha\beta}(t_p^{*})$.*

  Condition A4_C1: *If $UEC_{\alpha\beta}(t_p^{*}) < UEC_{\alpha\beta}(t_p)$ for any $t_p \ge t^{*}$, $t_{sp}^{*} = t_p^{*}$ minimizes $UEC_{\alpha\beta}(t_p)$, then $t_{sp}^{*}$ is the unique optimal solution;*

  Condition A4_C2: *If $UEC_{\alpha\beta}(t_p^{*}) \ge UEC_{\alpha\beta}(t_p^{i})$, where for any $t_p \in \left(t^{*}, t_p^{i}\right]$ and $t_p \in \left(t_p^{i}, \infty\right)$, then $UEC(t_p) > UEC(t_p^{i})$, then $t_{sp}^{*} = t_p^{i}$ minimizes $UEC_{\alpha\beta}(t_p)$, then $t_{sp}^{*}$ is the unique optimal solution.*

### 5.4.3. W-PRP vs S-PRP: Numerical Examples

***Periodic replacement policy of component with Weibull distribution***

Let us consider a component with Weibull distribution of failure rate, where intensity parameter $\lambda = 0.005$, shape parameter $\gamma = 3$, so with increasing failure intensity rate *h(t)*. The cost of a preventive maintenance operation $c_p = 5k€/action$ and the cost of a breakdown maintenance operation $c_f = 50k€/action$ are also supposed.

Using algorithm A3, proposed in the previous section, let us how follow the steps as:

- Step 1: Set-up Phase

  *Set* $c_p = 5;\ c_f = 50;\ \lambda = 0.005, \gamma = 3,$

  *Set* $t_p$, $t$ *real positive variables.*

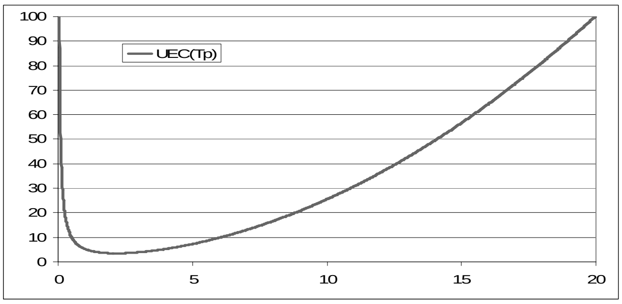
- Step 2: Optimal-Solution Calculation Phase

  *Calculate* $t_p{*} = \left(\frac{c_p}{(\gamma-1)*c_f*\lambda}\right)^{1/\gamma} = \left(\frac{5}{2*50*0.005}\right)^{1/3} = 2.154 months$.

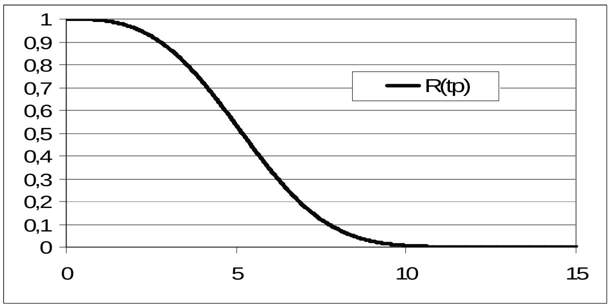
  *Calculate*

$$UEC(t_p{*}) = \frac{c_p * e^{-\lambda t_p{*}^{\gamma}} + c_f * (1 - e^{-\lambda t_p{*}^{\gamma}})}{\int_0^{t_p{*}} e^{-\lambda t^{\gamma}} dt} = \frac{c_p + c_f * \lambda t_p{*}^{\gamma}}{t_p{*}} = \frac{5 + 50 * 0.005 * 2.154^3}{2.154} = 3.481 k€/month$$

With the parameters, introduced before, and using algorithm A1, the values of optimum time $t_p^*$ is equal to 2.154 months, where the $UEC(t_p^*)$ assumes value 3.481 k€/month, the time could be intended as days, or weeks, or months and so on, and the $UEC(t_p^*)$ is related to the cost of chosen time unit (see figure 5.16).




*b) Cost Function*




*a) Reliability Function*

**Figure 5.16: a) Reliability for** $\lambda = 0{,}005\ \gamma = 3$**; b) Unit Expected Cost for** $\lambda = 0{,}005\ \gamma = 3$.

### *Periodic replacement policy using systemability function: UEC curves in function of systemability parameters*

Using the same Weibull parameters and cost values, defined before, in this section the function of $UEC(t_p)$ are tested with the introduction of systemability formula in order to study the effect of environmental factors, using algorithm A4.

Here below, several graphics show the function $UEC_{\alpha\beta}(t_p)$ for different values of the systemability parameters $\alpha$ and $\beta$ and fixed the values of the costs parameters, $c_p$ and $c_f$.

This series of graphics illustrates systemability, $R_s(t_p)$, and $UEC_{\alpha\beta}(t_p)$ functions fixed one of the systemability parameters to a defined value and changing the other one in a series of values between 1, 5, 10, 20, 50.

Let us consider the following values of the parameters in the cost function to carry out this study: $c_p = 5$; $c_f = 50$; $\lambda = 0{,}005$ and $\gamma = 3$.

Using algorithm A4, proposed in the previous section, let us how follow the steps as the example with $\alpha = 10$ and $\beta = 50$ :

- Step 1: Set-up Phase

*Set* $c_p = 5;\ c_f = 50;\ \lambda = 0.005,\ \gamma = 3,\ \alpha = 10,\ \beta = 50$

*Set* $t_p$, $t$ *real positive variables.*

- Step 2: Calculation Phase

*Calculate* $h_s(t) = \dfrac{\alpha\lambda\gamma * t^{\gamma-1}}{\beta + \lambda t^{\gamma}} = \dfrac{10 * 0.005 * 3 * t^{3-1}}{50 + 0.005t^3} = \dfrac{0.15 * t^2}{50 + 0.005t^3}$

*Calculate* $R_s(t) = \left[\dfrac{\beta}{\beta + \lambda t^{\gamma}}\right]^{\alpha} = \left[\dfrac{50}{50 + 0.005t^3}\right]^{10}$

*Calculate* $UEC_{\alpha\beta}(t_p) = \dfrac{c_p + c_f * \left[\alpha * \ln\left(1 + \dfrac{\lambda}{\beta} t_p^{\ \gamma}\right)\right]}{t_p} = \dfrac{5 + 50 * \left[10 * \ln\left(1 + \dfrac{0.005}{50} t_p^{\ 3}\right)\right]}{t_p}$

*Calculate* $t^* = \sqrt[\gamma]{\dfrac{\beta(\gamma - 1)}{\lambda}} = \sqrt[3]{\dfrac{50(3-1)}{0.005}} = 27.144 months$

- Step 3: Decision-Making Phase

*For* $t_p \le t^*$, *for any* $t_p \in \left(0, t_p^*\right]$ *and* $t_p \in \left(t_p^*, t^*\right)$, *then* $UEC_{\alpha\beta}(t_p) > UEC_{\alpha\beta}(t_p^{\ *})$, *then* $t_p^{\ *}$ *minimizes* $UEC_{\alpha\beta}(t_p^{\ *})$.

Condition A4_C1: *If* $UEC_{\alpha\beta}(t_p^{\ *}) < UEC_{\alpha\beta}(t_p)$ *for any* $t_p \ge t^*$, $t_{sp}^{\ *} = t_p^{\ *}$ *minimizes* $UEC_{\alpha\beta}(t_p)$, *then* $t_{sp}^{\ *}$ *is the unique optimal solution;*

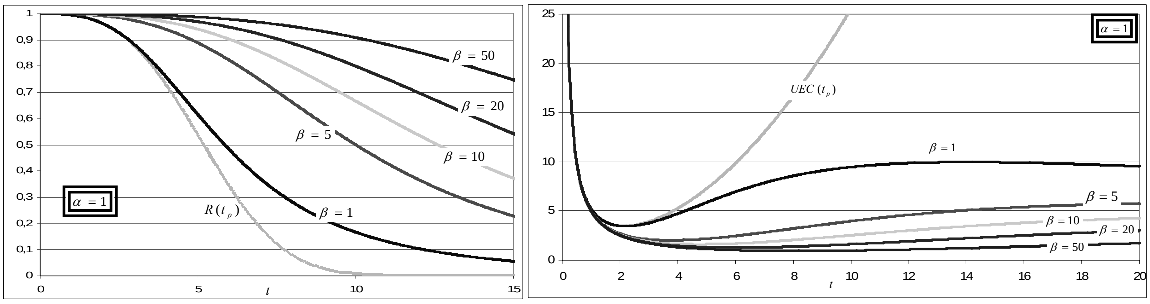
Condition A4_C2: *If* $UEC_{\alpha\beta}(t_p^{\ *}) \ge UEC_{\alpha\beta}(t_p^{\ i})$, *where for any* $t_p \in \left(t^*, t_p^{\ i}\right]$ *and* $t_p \in \left(t_p^{\ i}, \infty\right)$, *then* $UEC(t_p) > UEC(t_p^{\ i})$, *then* $t_{sp}^{\ *} = t_p^{\ i}$ *minimizes* $UEC_{\alpha\beta}(t_p)$, *then* $t_{sp}^{\ *}$ *is the unique optimal solution.*

RESULT:

$t_p^{\ *} = 3.69 months$

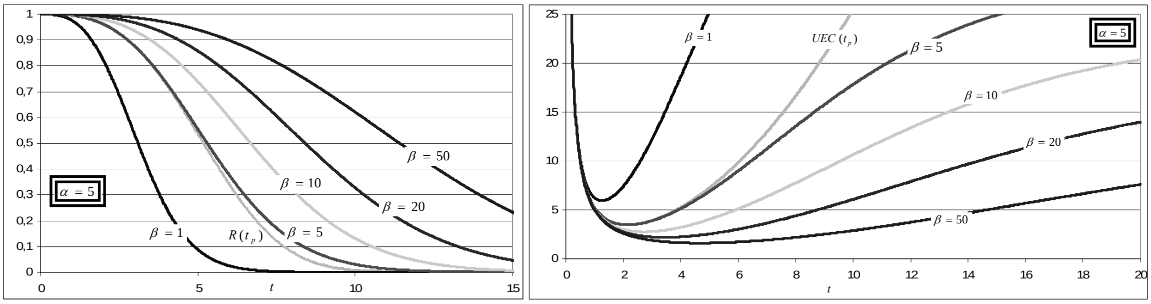
$UEC_{\alpha\beta}(t_p*) = 2.034 < UEC_{\alpha\beta}(t_p)$: Condition A4_C1=TRUE → $t_{sp}^{\ *} = t_p^{\ *}$ *minimizes* $UEC_{\alpha\beta}(t_p)$, *then* $t_{sp}^{\ *}$ *is the unique optimal solution.*

Figures 5.17a, 5.18a, 5.19a, 5.20a and 5.21a show the systemability function $R_s(t_p)$ for different values of $\alpha$, respectively 1, 5, 10, 20, 50 and varying the $\beta$ parameter, where $R(t_p)$ is the reliability function related to the test environment calculated with formula 5-13 and these weibull parameters $\lambda$ = 0,005 and $\gamma$ = 3. Figures 5.17b, 5.18b, 5.19b, 5.20b and 5.21b show the cost function $UEC_{\alpha\beta}(t_p)$ for different values of $\alpha$ and varying the $\beta$ parameter.



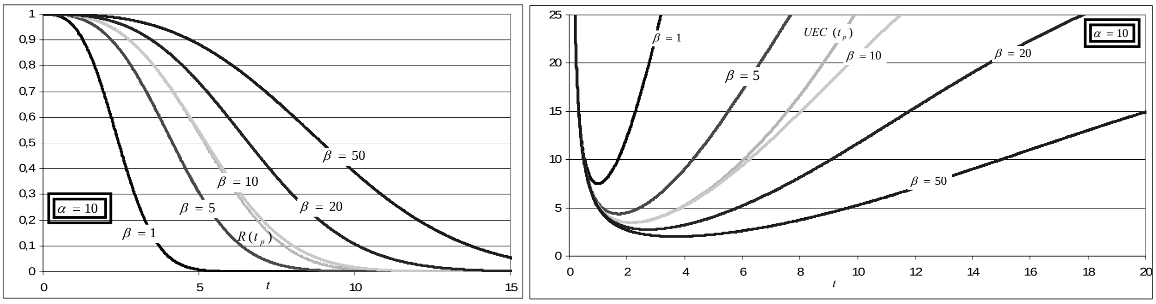

*a) Reliability Functions* *b) Cost Functions*

**Figure 5.17: a) Comparison of reliability vs. systemability function for α = 1 and β = 1, 5, 10, 20 and 50. b) Comparison of cost function $UEC(t_p)$ vs. $UEC_{\alpha\beta}(t_p)$ for α = 1 and β = 1, 5, 10, 20 and 50.**




*a) Reliability Functions* *b) Cost Functions*

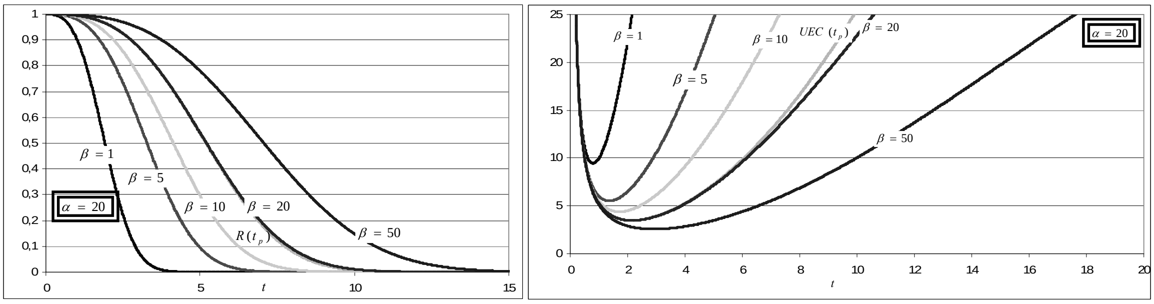
**Figure 5.18: a) Comparison of reliability vs. systemability function for α = 5 and β = 1, 5, 10, 20 and 50. b) Comparison of cost function $UEC(t_p)$ vs. $UEC_{\alpha\beta}(t_p)$ for α = 5 and β = 1, 5, 10, 20 and 50.**




a) Reliability Functions b) Cost Functions

**Figure 5.19: a) Comparison of reliability vs. systemability function for α = 10 and β = 1, 5, 10, 20 and 50.**

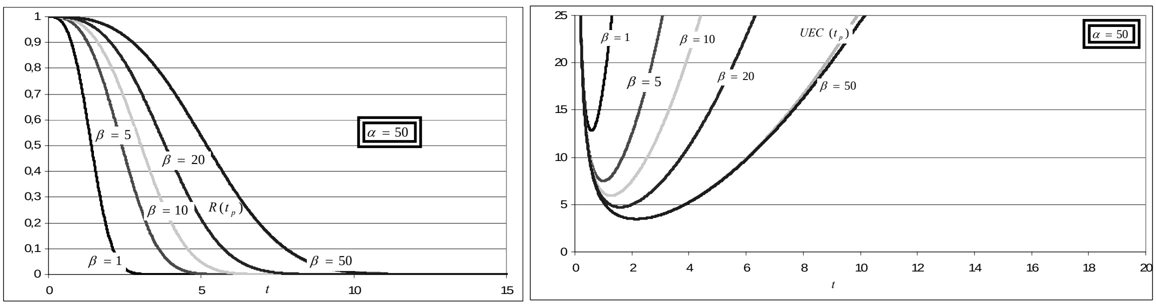
**b) Comparison of cost function $UEC(t_p)$ vs. $UEC_{\alpha\beta}(t_p)$ for α = 10 and β = 1, 5, 10, 20 and 50.**




a) Reliability Functions b) Cost Functions

**Figure 5.20: a) Comparison of reliability vs. systemability function for α = 20 and β = 1, 5, 10, 20 and 50.**

**b) Comparison of cost function $UEC(t_p)$ vs. $UEC_{\alpha\beta}(t_p)$ for α = 20 and β = 1, 5, 10, 20 and 50.**

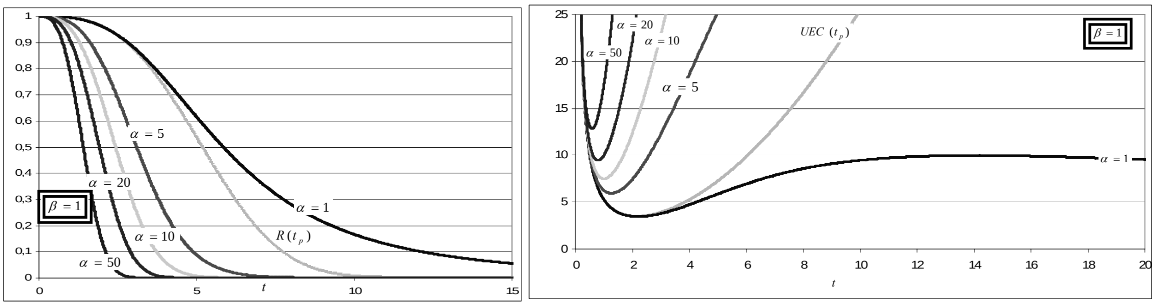



a) Reliability Functions b) Cost Functions

**Figure 5.21: a) Comparison of reliability vs. systemability function for α = 50 and β = 1, 5, 10, 20 and 50.**

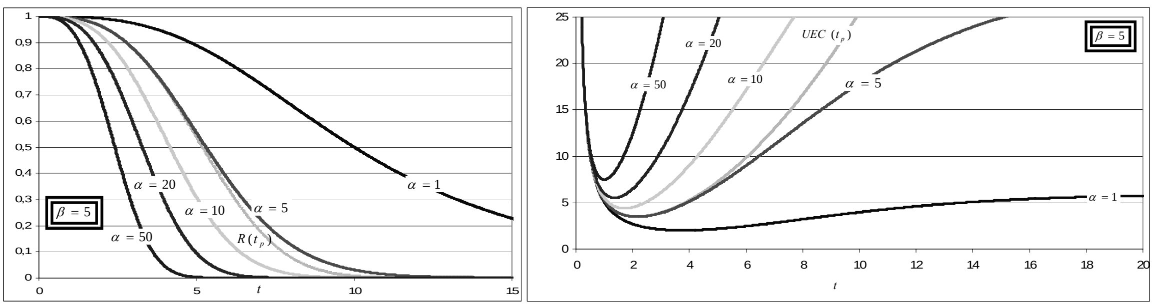
**b) Comparison of cost function $UEC(t_p)$ vs. $UEC_{\alpha\beta}(t_p)$ for α = 50 and β = 1, 5, 10, 20 and 50.**

Furthermore, figures 5.22a, 5.23a, 5.24a, 5.25a and 5.26a show the systemability function $R_S(t_p)$ for different values of $\beta$, respectively 1, 5, 10, 20, 50 and varying the $\alpha$ parameter, where $R(t_p)$ is the reliability function related to the test environment calculated with formula (11) and these Weibull parameters $\lambda$ = 0.005 and $\gamma$ = 3. Figures 5.22b, 5.23b, 5.24b, 5.25b and 5.26b show the cost function $UEC_{\alpha\beta}(t_p)$ for different values of $\beta$ and varying the $\alpha$ parameter.



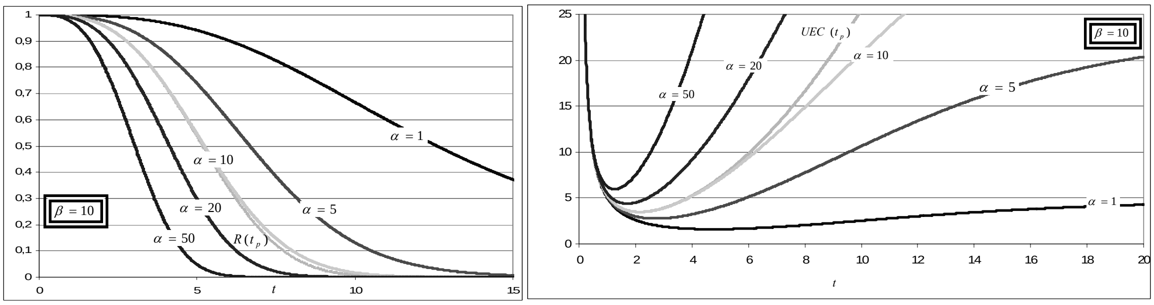

*a) Reliability Functions* *b) Cost Functions*

**Figure 5.22: a) Comparison of reliability vs. systemability function for β = 1 and α = 1, 5, 10, 20 and 50.**
**b) Comparison of cost function $UEC(t_p)$ vs. $UEC_{\alpha\beta}(t_p)$ for β = 1 and α = 1, 5, 10, 20 and 50.**




*a) Reliability Functions* *b) Cost Functions*

**Figure 5.23: a) Comparison of reliability vs. systemability Function for β = 5 and α = 1, 5, 10, 20 and 50.**
**b) Comparison of cost function $UEC(t_p)$ vs. $UEC_{\alpha\beta}(t_p)$ for β = 5 and α = 1, 5, 10, 20 and 50.**

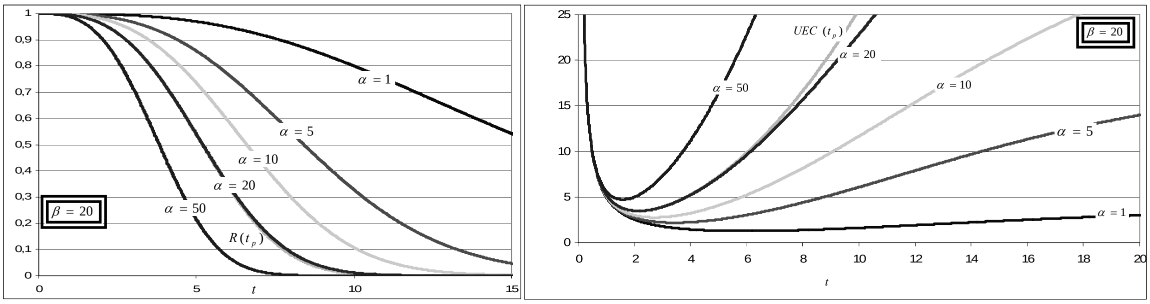



a) Reliability Functions

b) Cost Functions

**Figure 5.24: a) Comparison of reliability vs. systemability funct. for β = 10 and α = 1, 5, 10, 20 and 50.**

**b) Comparison of cost function $UEC(t_p)$ vs. $UEC_{\alpha\beta}(t_p)$ for β = 10 and α = 1, 5, 10, 20 and 50.**

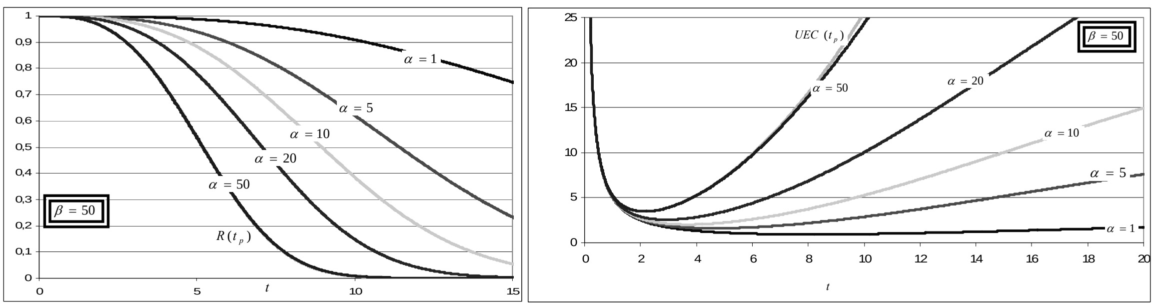



a) Reliability Functions

b) Cost Functions

**Figure 5.25: a) Comparison of reliability vs. systemability function for β = 20 and α = 1, 5, 10, 20 and 50.**

**b) Comparison of cost function $UEC(t_p)$ vs. $UEC_{\alpha\beta}(t_p)$ for β = 20 and α = 1, 5, 10, 20 and 50.**




a) Reliability Functions

b) Cost Functions

**Figure 5.26: a) Comparison of reliability vs. systemability function for β = 50 and α = 1, 5, 10, 20 and 50.**

**b) Comparison of cost function $UEC(t_p)$ vs. $UEC_{\alpha\beta}(t_p)$ for β = 50 and α = 1, 5, 10, 20 and 50.**

As well shown in these graphics, two scenario can be defined:

- Scenario 1:

  *hard environment effects:* when $\alpha > \beta$, the $UEC_{\alpha\beta}(t_p)$ is higher than $UEC(t_p)$ because the $R_s(t)$ is worse than $R(t)$ .

- Scenario 2:

  *soft environment effects:* when $\alpha < \beta$, the $UEC_{\alpha\beta}(t_p)$ is lower than $UEC(t_p)$ because the $R_s(t)$ is better than $R(t)$ .

The previous figures show the effects of the environmental factor, represented with the systemability parameters $\alpha$ and $\beta$, to the reliability and to the Expected Unit Cost functions. In fact, as explained before, if the operating environment is different from the testing one, the reliability will be different from the one calculated during the test phases. As a consequence, also the Expected Unit Cost will be different and it could be higher or lower, depends on the effects of environmental factors, described by $\alpha$ and $\beta$ parameters, as shown before.

### 5.4.4. Further analysis about S-PRP: ${t_s}_p^*$ and $\%UEC(t_p^*)$ curves in function of systemability parameters

The second group shows several graphics in order to analyze the changing of the optimal time of preventive maintenance and to estimate the difference between the UEC considering the environment factors or not.

Let us consider the following values of the parameters: $\lambda$ = 0.005 and $\gamma$ = 3, for different values of the costs parameters, $c_p$ and $c_f$, and it has been illustrated:

- a) ${t_s}_p^*$ curve for $0<\alpha<\infty$ and $\beta$ = 1, 5, 10, 20, 50;
- b) ${t_s}_p^*$ curve for $0<\beta<\infty$ and $\alpha$ = 0.5, 1, 5, 10, 20, 50;
- c) $\%UEC(t_p^*)$ curve for $0<\alpha<\infty$ and $\beta$ = 1, 5, 10, 20, 50;
- d) $\%UEC(t_p^*)$ curve for $0<\beta<\infty$ and $\alpha$ = 0.5, 1, 5, 10, 20, 50;
- e) ${t_s}_p^*$ surface for $0<\alpha<\infty$ and $0<\beta<\infty$ ;
- f) $\%UEC(t_p^*)$ surface for $0<\alpha<\infty$ and $0<\beta<\infty$ .

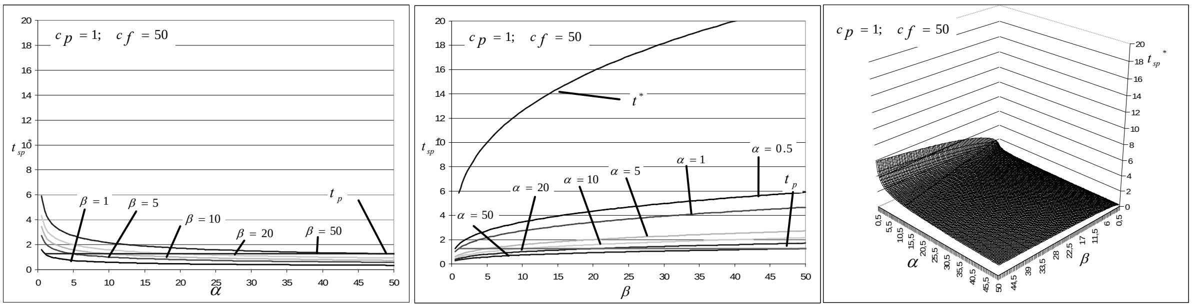
where ${t_{s}}_{p}^{*}$ is the value which minimized the $UEC_{\alpha\beta}(t_p)$, calculated with formula 5-27.

Then $\%UEC(t_p^{*})$ is calculated as:

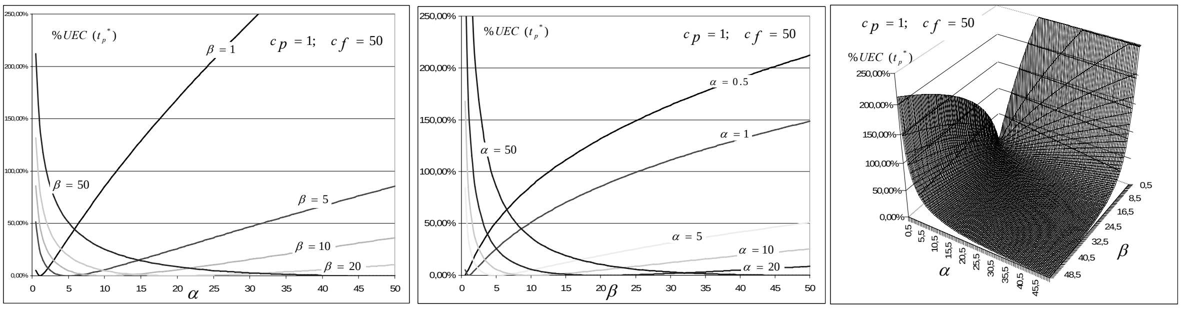
$$\%UEC(t_p^{*}) = 100 * \left( \frac{UEC_{\alpha\beta}(t_p^{*})}{UEC_{\alpha\beta}({t_s}_p^{*})} - 1 \right) \qquad 5\text{-}30$$

It indicates the percent difference between the real Unit Expected Cost $UEC_{\alpha\beta}(t_p^{*})$, using a planned time $t_p^{*}$ calculated with only Weibull distribution, without consider the environmental effects, and the optimum value of Unit Expected Cost $UEC_{\alpha\beta}({t_s}_p^{*})$ estimated considering also the environmental effects with systemability parameters, calculated in the real optimum time value ${t_s}_p^{*}$. All calculated ${t_s}_p^{*}$ are minor than $t^{*}$, calculate with 5-29, and satisfy condition (a) of algorithm A4 hence ${t_s}_p^{*}$ can be considered the unique and finite solution of $UEC_{\alpha\beta}({t_s}_p^{*})$.

Figures 5.27a, 5.28a, 5.29a and 5.30a show the curve of $t_{sp}^{*}$ for different values of $c_p$, respectively 1, 5, 10, 25, fixed $c_f$ = 50, and for each case, different values of $\beta$ (1, 5, 10, 20, 50) varying continuously $\alpha$ from 0 to 50. Figures 5.27b, 5.28b, 5.29b and 5.30b show the curve of $t_{sp}^{*}$ for different values of $c_p$, respectively 1, 5, 10, 25, fixed $c_f$ = 50, and for each case, different values of $\alpha$ (1, 5, 10, 20, 50) varying continuously $\beta$ from 0 to 50. For each figure, the optimal times $t_p^{*}$ are also shown, which values have been calculated with the formula related to the test reliability using algorithm A4. Moreover figures 5.27c, 5.28c, 5.29c and 5.30c show the curve of $t_{sp}^{*}$ for different values of $c_p$, respectively 1, 5, 10, 25, fixed $c_f$ = 50, and for each case, varying continuously $\alpha$ and $\beta$ from 0 to 50. Instead, figures 5.27d, 5.28d, 5.29d and 5.30d illustrate the curve of $\%UEC(t_p^{*})$ for different values of $c_p$, respectively 1, 5, 10, 25, fixed $c_f$ = 50, and for each case, different values of $\beta$ (1, 5, 10, 20, 50) varying continuously $\alpha$ from 0 to 50. Figures 5.27e, 5.28e, 5.29e and 5.30e illustrate the curve of $\%UEC(t_p^{*})$ for different values of $c_p$, respectively 1, 5, 10, 25, fixed $c_f$ = 50, and for each case, different values of $\alpha$ (1, 5, 10, 20, 50) varying continuously $\beta$ from 0 to 50. Last figures 5.27f, 5.28f, 5.29f and 5.30f show the curve of $\%UEC(t_p^{*})$ for different values of $c_p$, respectively 1, 5, 10, 25, fixed $c_f$ = 50, and for each case, varying continuously $\alpha$ and $\beta$ from 0 to 50.




*a) Optimal Time vs. parameter "α"* *b) Optimal Time vs. parameter "β"* *c) Optimal Time vs. parameters "α" and "β"*




*a) %UEC($t_p^*$) vs. parameter "α"* *b) %UEC($t_p^*$) vs. parameter "β"* *c) %UEC($t_p^*$) vs. parameters "α" and "β"*

**Figure 5.27: a) Curve of $t_{sp}^*$ in function of α and β = 1, 5, 10, 20 and 50;**

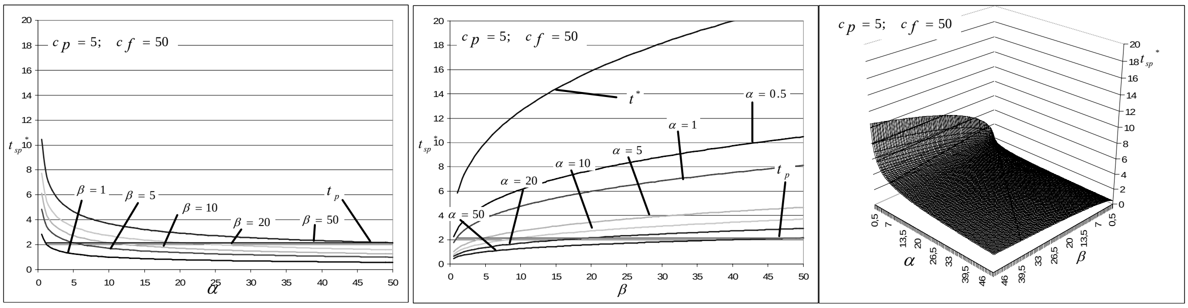
**b) Curve of $t_{sp}^*$ in function of β and α = 0.5, 1, 5, 10, 20 and 50;**

**c) Curve of $t_{sp}^*$ in function of α and β. All these for $c_p$ = 1 and $c_f$ = 50;**

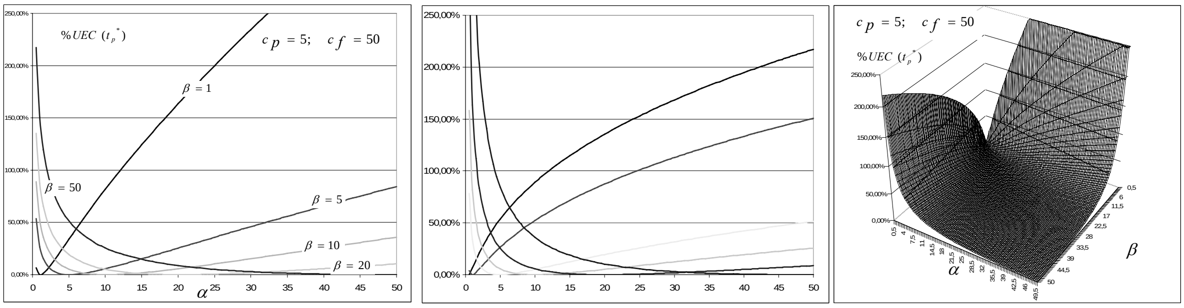
**d) Curve of %*UEC*($t_p^*$) in function of α and β = 1, 5, 10, 20 and 50;**

**e) Curve of %*UEC*($t_p^*$) in function of β and α = 0.5, 1, 5, 10, 20 and 50;**

**f) Curve of %*UEC*($t_p^*$) in function of α and β. All these for $c_p$ = 1 and $c_f$ = 50.**




a) Optimal Time vs. parameter "α" b) Optimal Time vs. parameter "β" c) Optimal Time vs. parameters "α" and "β"




a) %UEC($t_p^*$) vs. parameter "α" b) %UEC($t_p^*$) vs. parameter "β" c) %UEC($t_p^*$) vs. parameters "α" and "β"

**Figure 5.28: a) Curve of $t_{sp}^*$ in function of $\alpha$ and β = 1, 5, 10, 20 and 50;**

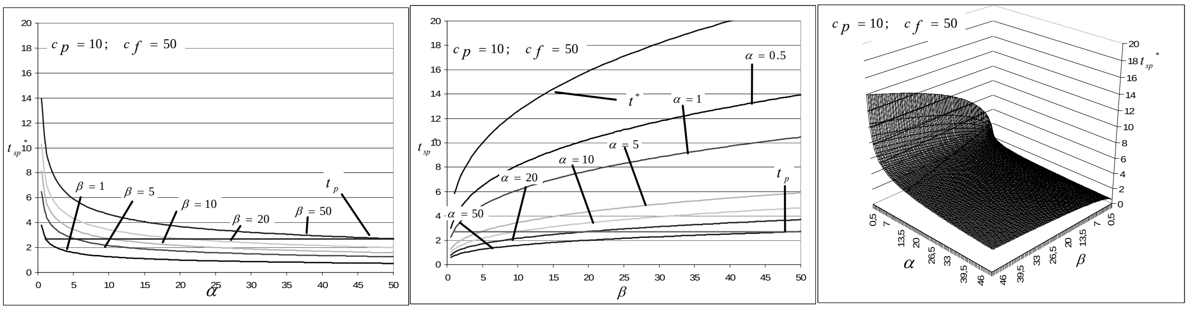
**b) Curve of $t_{sp}^*$ in function of β and $\alpha$ = 0.5, 1, 5, 10, 20 and 50;**

**c) Curve of $t_{sp}^*$ in function of $\alpha$ and β. All these for $c_p$ = 5 and $c_f$ = 50;**

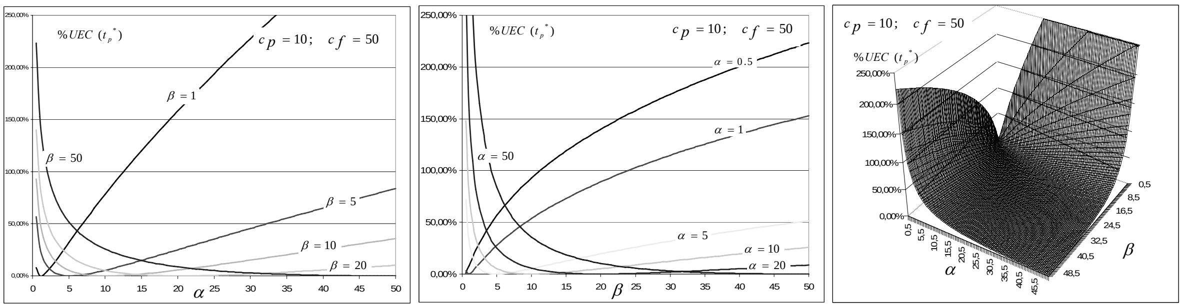
**d) Curve of *%UEC($t_p^*$)* in function of $\alpha$ and β = 1, 5, 10, 20 and 50;**

**e) Curve of *%UEC($t_p^*$)* in function of β and $\alpha$ = 0.5, 1, 5, 10, 20 and 50;**

**f) Curve of *%UEC($t_p^*$)* in function of $\alpha$ and β. All these for $c_p$ = 5 and $c_f$ = 50.**




*a) Optimal Time vs. parameter "α"* *b) Optimal Time vs. parameter "β"* *c) Optimal Time vs. parameters "α" and "β"*




*a) %UEC(tp*) vs. parameter "α"* *b) %UEC(tp*) vs. parameter "β"* *c) %UEC(tp*) vs. parameters "α" and "β"*

**Figure 5.29: a) Curve of $t_{sp}^*$ in function of α and β = 1, 5, 10, 20 and 50;**

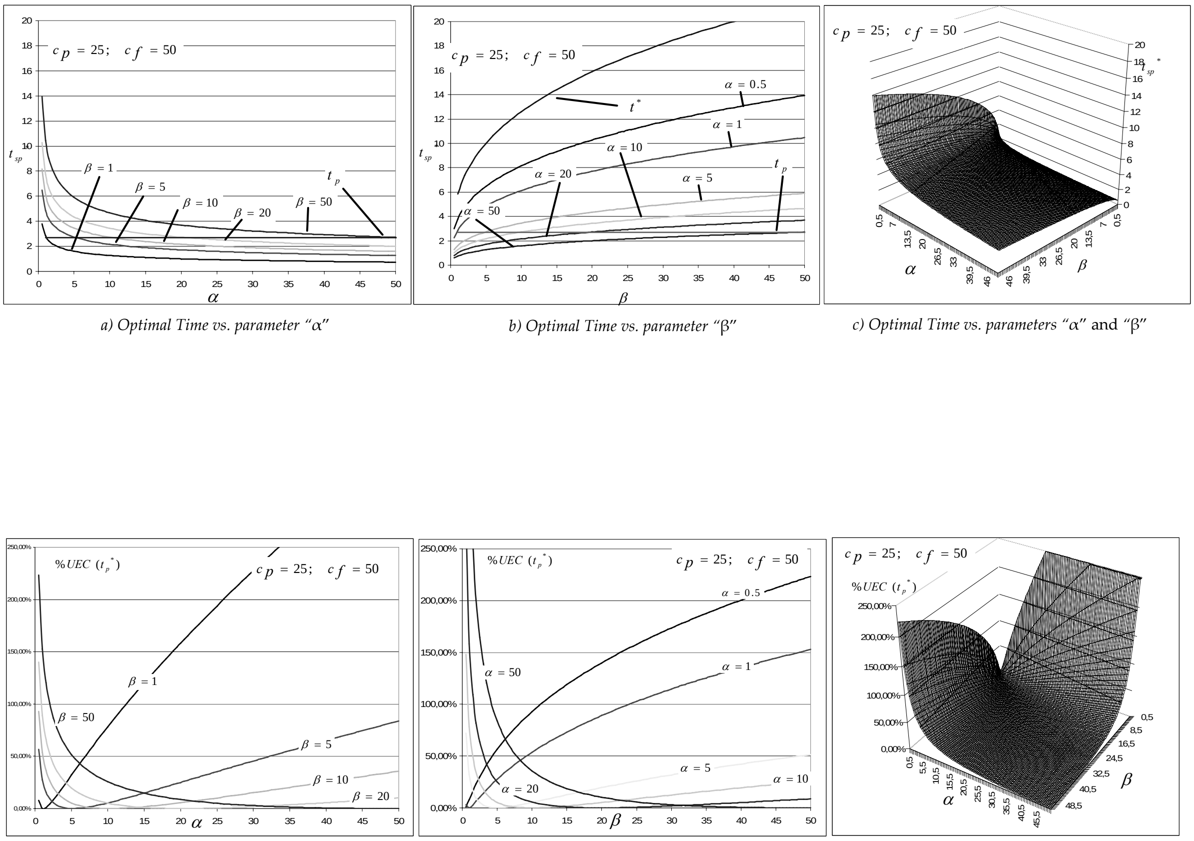
**b) Curve of $t_{sp}^*$ in function of β and α = 0.5, 1, 5, 10, 20 and 50;**

**c) Curve of $t_{sp}^*$ in function of α and β. All these for $c_p$ = 10 and $c_f$ = 50;**

**d) Curve of *%UEC($t_p^*$)* in function of α and β = 1, 5, 10, 20 and 50;**

**e) Curve of *%UEC($t_p^*$)* in function of β and α = 0.5, 1, 5, 10, 20 and 50;**

**f) Curve of *%UEC($t_p^*$)* in function of α and β. All these for $c_p$ = 10 and $c_f$ = 50.**




a) Optimal Time *vs.* parameter "α"
b) Optimal Time *vs.* parameter "β"
c) Optimal Time *vs.* parameters "α" and "β"

a) %UEC($t_p^*$) *vs.* parameter "α"
b) %UEC($t_p^*$) *vs.* parameter "β"
c) %UEC($t_p^*$) *vs.* parameters "α" and "β"

**Figure 5.30: a) Curve of $t_{sp}^*$ in function of α and β = 1, 5, 10, 20 and 50;**

**b) Curve of $t_{sp}^*$ in function of β and α = 0.5, 1, 5, 10, 20 and 50;**

**c) Curve of $t_{sp}^*$ in function of α and β. All these for $c_p$ = 25 and $c_f$ = 50;**

**d) Curve of *%UEC($t_p^*$)* in function of α and β = 1, 5, 10, 20 and 50;**

**e) Curve of *%UEC($t_p^*$)* in function of β and α = 0.5, 1, 5, 10, 20 and 50;**

**f) Curve of *%UEC($t_p^*$)* in function of α and β. All these for $c_p$ = 25 and $c_f$ = 50.**

From the analysis of the previous graphics some considerations could be made:

- fixed parameter $\beta$, $t_{sp}^*$ is a decreasing function of parameter $\alpha$;
- fixed parameter $\alpha$, $t_{sp}^*$ is a increasing function of parameter $\beta$;
- overall, a general increasing of $t_{sp}^*$ is well shown.
- the $\%UEC(t_p^*)$ is not influenced by the increasing the cost parameter $c_p$;
- parameter $\alpha$ and $\beta$ affect in the same mode the $\%UEC(t_p^*)$;

Furthermore, the $\%UEC(t_p^*)$ values have been collected in different interval, every five points percent, and for each interval the probability distribution $Pc$ of $\%UEC(t_p^*)$ has been calculated in order to define the mean value of $\%UEC(t_p^*)$ and its distribution.

Here below, several graphics (figure 5.31) show the probability distribution $Pc$ of $\%UEC(t_p^*)$ for different values of the cost parameter $c_p$, respectively 1, 5, 10, 25, while the cost parameter $c_f$ equal to 50.

The mean value for each cases is about 5%, so if a preventive maintenance time is design using the test data set, the mean major cost, due to the environmental effects, is about 5 %.

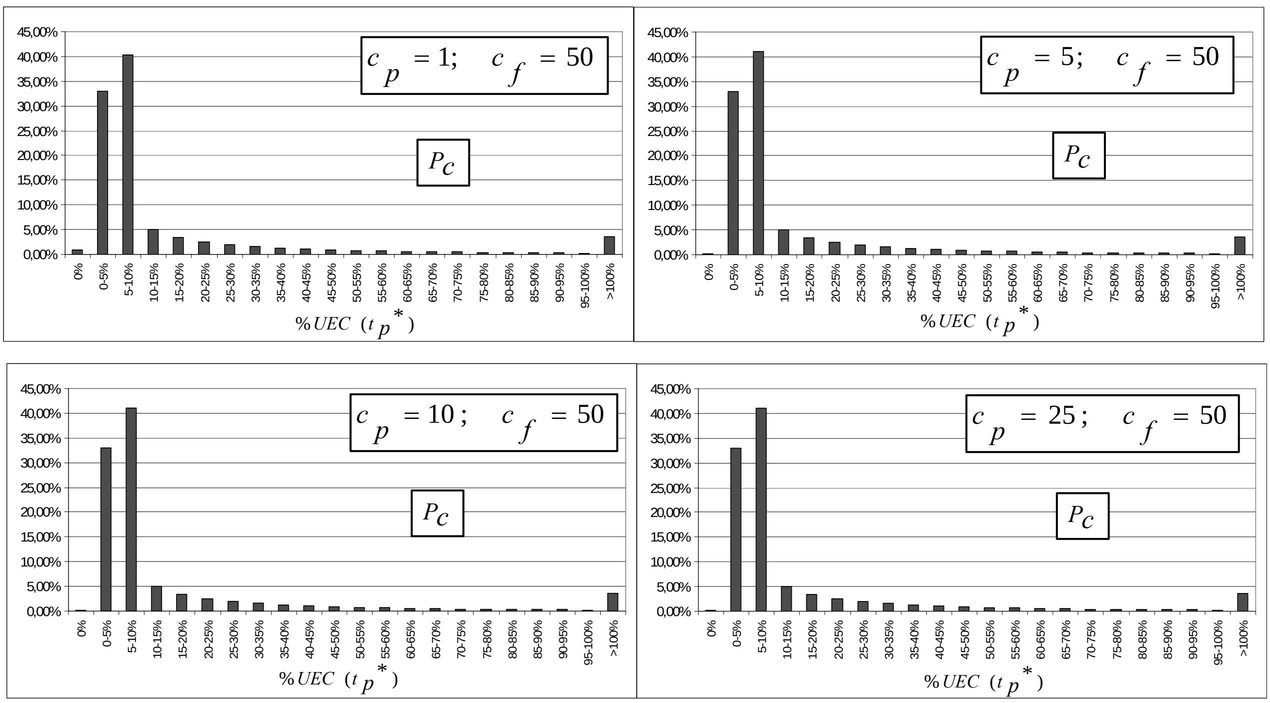



**Figure 5.31: Probability distribution of $\%UEC(t_p^*)$ for different values of $c_p$**

### 5.4.5. Real Application: PRP on Automatic Packaging Machines.

In this section, the proposed methodology is illustrated, applied to an interesting case study in the automatic packaging machines for beer production. Data related to different applications of the same model of machine in several customer plants located in different countries has been collected. The two plants have been visited and analyzed in details, that present the same machine subject to different environmental conditions.

This study gives special attentions to the automatic bottle filler because usually it is the bottleneck of the whole production system and it is more influenced by the environmental factors than the other machines. The main causes of failure are investigated. As it is observed, the principal downtimes are caused by failures of the filler heads upon which the study has been oriented.

Table 5.3 shows the data set that has taken from this component where the time values *t* has been normalized. $R_a(t)$ represents reliability function values collected from the test environment. The first plant presents the same data of the test environment while the second one is described by $R_b(t)$.

| T | 0.00 | 0.03 | 0.07 | 0.10 | 0.13 | 0.17 | 0.20 | 0.23 | 0.27 | 0.30 | 0.33 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $R_a(T)$ | 100.00 | 99.98 | 99.95 | 99.92 | 99.86 | 99.76 | 99.59 | 99.33 | 98.94 | 98.36 | 97.52 |
| $R_b(T)$ | 100.00 | 99.88 | 99.79 | 99.64 | 99.40 | 99.03 | 98.50 | 97.73 | 96.66 | 95.20 | 93.25 |

| T | 0.37 | 0.40 | 0.43 | 0.47 | 0.50 | 0.53 | 0.57 | 0.60 | 0.63 | 0.67 |
|---|---|---|---|---|---|---|---|---|---|---|
| $R_a(T)$ | 96.34 | 94.71 | 93.21 | 89.59 | 85.81 | 82.30 | 75.17 | 66.79 | 60.10 | 52.21 |
| $R_b(T)$ | 90.70 | 87.44 | 83.36 | 78.37 | 72.44 | 65.59 | 57.91 | 49.61 | 41.00 | 32.47 |

| T | 0.70 | 0.73 | 0.77 | 0.80 | 0.83 | 0.87 | 0.90 | 0.93 | 0.97 | 1.00 |
|---|---|---|---|---|---|---|---|---|---|---|
| $R_a(T)$ | 42.21 | 33.20 | 23.45 | 14.88 | 9.68 | 4.50 | 2.56 | 1.06 | 0.37 | 0.10 |
| $R_b(T)$ | 24.46 | 17.37 | 11.52 | 7.05 | 3.94 | 1.98 | 0.88 | 0.34 | 0.11 | 0.03 |

**Table 5.3: Testing and operating data set**

Fitting the data using Weibull distribution and systemability function, the parameters illustrated in table 5.4 and figure 5.32 have been obtained. The cost parameters given us from the maintenance service of the company are also included.

| Weibull parameters | | Systemability parameters | | Cost parameters | |
|---|---|---|---|---|---|
| $\lambda$ | $\gamma$ | $\alpha$ | $\beta$ | $c_p$ | $c_f$ |
| 0.0000915 | 8.10 | 3.10 | 1.42 | 3 | 30 |

**Table 5.4: Parameters of Weibull and systemability function and cost parameters**

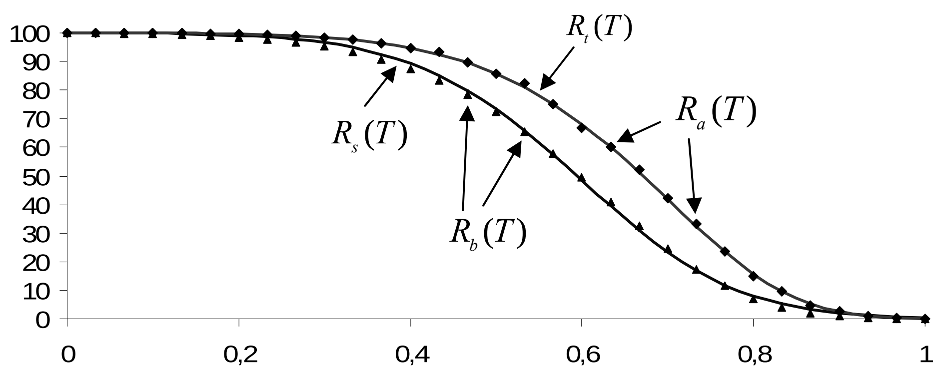



**Figure 5.32: Reliability during Testing ($R_a$(t)), during Operation ($R_b$(t)), Weibull Function ($R_t$(t)) and Systemability Function ($R_s$(t)).**

Considering $c_p = 3$, $c_f = 30$, $\lambda = 0.0000915$, $\gamma = 8.1$, $\alpha = 3.10$, $\beta = 1.42$, the periodic replacement policy of the second plant has been optimized thanks to the consideration of the environmental factors with systemability function.

Considering only the Weibull parameters, the application of algorithm A1 have given us the replacement time related to the testing data, $t_p* = 1.91 months$, and the Unit Expected Cost is $UEC(t_p*) = 2.086 k€ / month$.

Using with algorithm A4, the optimal time to replacement has been calculated considering also the environmental effects to the survival function, applying the systemability function. In this case, the optimal solution is $t_{sp}* = 1.65 months$ and the Unit Expected Cost is

$UEC_{\alpha\beta}(t_{sp}*) = 2.414k€/month$ . If the Unit Expected Cost is calculated in the optimal solution of testing data, but taking into consideration the environmental factors, its value will be $UEC_{\alpha\beta}(t_p*) = 2.657k€/month$ (figure 5.33). As well demonstrated, the Unit Expected Costs are different between these cases and the optimal solution, using algorithm A4 and considering the environmental effects, allows to saving about the 10%.

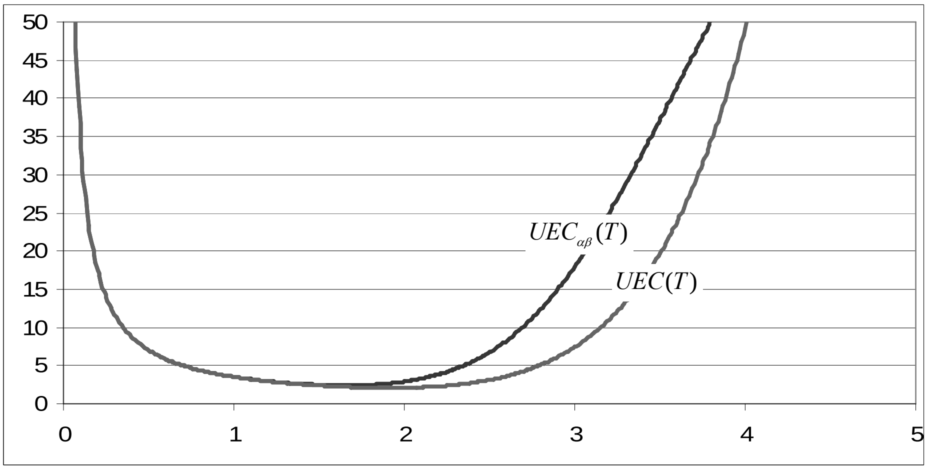



**Figure 5.33: Real application of the methodology to the periodic replacement policy on the automatic machine component**

### 5.4.6. Some considerations about Systemability application on PRP

In the modern industrial sectors, many companies are characterized by expensive industrial systems in order to guarantee high level of flexibility and efficiency to better serve the different final costumers. In this context the reliability and availability of the production systems become very important to satisfy the final production rate. Hence, the importance of maintenance of these systems has grown up, because their reliability can improve thanks to the different applied maintenance policies. When a component or a system works in an operative plant, it reflects a reliability function that is usually different from the theory reliability but also from all its similar applications in other industrial plants.

Many maintenance model have been developed and widely used in scientific literature. The Periodic Replacement Policy is one of the most important.

Incorrect estimation of reliability function could lead to the wrong functional design of the system and an incorrect definition of the appropriate maintenance policies.

In this chapter, the effects of the different operating environments in the Periodic Replacement Policy have been demonstrate using a new concept, called systemability [5.19]. Systemability, with its parameters, allows to consider the environmental effects in the reliability estimation, separating their from the intrinsic survival performances of the component..

Several numerical examples have been carry out in order to validate the initial assumption. The results of this application have been illustrated with a series of graphics and summarized at the end.

It's important to highlight that the mean value of $\%UEC(t_p^*)$ is about 5%. An real industrial application have demonstrated the importance of this study, shown a saving about 10% using this methodology.

In few word, if a preventive maintenance time is design using the test data set, the mean major cost, due to the environmental effects, will be about 5%.

# 6. Performance Improvement in Logistics: the Automated Storage/Retrieval Systems (AS/RSs)

## *6.1. Introduction*

Recently, many companies have invested in intensive automatic storage systems to keep their logistics activities more flexible and efficient. Automatic Storage and Retrieval Systems, also called AS/RSs, have been widely used in different logistics applications, from manufacturing to warehousing.

An AS/RS is defined as a storage system where a machine performs material handling activities, on fixed paths in the aisles inside the storage racks. An AS/RS carries out its activities without the interference of an operator, thus it is completely automated. Typically, an AS/RS is composed by these mainly components: racks, cranes, aisles, input-output locations and picking positions.

Racks are typically metal structures with locations that can accommodate loads (e.g., pallets) that need to be stored. Cranes are the fully automated storage and retrieval machines that can autonomously move, pick up and drop off loads. Aisles are formed by the empty spaces between the racks, where the cranes can move. An input/output point (I/O-point) is a location where retrieved loads are dropped off, and where incoming loads are picked up for storage. Pick positions (if any) are places where people are working to remove individual items from a retrieved load before the load is sent back into the system.

The application of AS/RSs has had a relevant development in the last decades. The main savings are the reduction of direct labor cost, the decrease of handling errors, the improvement of logistic flows and better inventory control results. On the other hand, these systems require an high investment mainly due to the high level of automation.

One possible variation of the basic AS/RS is when cranes are capable of changing aisles. In this case, it is possible to have fewer cranes than aisles in the system. This may be useful if the amount of requests does not justify the purchase of a crane for each aisle. To overcome the restriction of the crane's unit-load capacity, multi-shuttle cranes exist. Such a crane can transport two or more loads at a time. Cranes which can transport two loads are also referred to as dual-shuttle cranes; cranes capable of transporting more than two loads are still rarely seen.

The increased transport capacity enables a crane, for example, to first retrieve one load and then store another load in the same location without having to go to the I/O-point in between.

Often an AS/RS is installed for handling unit-loads only (typically, pallets). Unit-loads arrive at the I/O-point of the AS/RS from other parts of the warehouse by means of, for example, automated guided vehicles, conveyors, or forklift trucks. The unit-loads are stored in the AS/RS and after a period of time they are retrieved again, for example, to be shipped to a customer. In many cases, however, only part of the unit-load may be needed to fulfill a customer's order. This can be resolved by having a separate picking area in the warehouse; in which case the AS/RS serves to replenish the picking area. Alternatively, the picking operation can be integrated with the AS/RS. One option is to design the crane such that a person can ride along (person-onboard). Instead of retrieving a full pallet automatically from the location, the person can pick one item from the location. A more common option to integrate item picking is when the AS/RS drops off the retrieved unit loads at a workstation. A picker at this workstation takes the required amount of products from the unit-load after which the AS/RS moves the remainder of the load back into the storage rack. This system is often referred to as an end-of-aisle system. If the unit-loads are bins, then the system is generally called a miniload AS/RS. Storage in the racks may occur single or double deep. In a double-deep rack, each rack location has space for two unit-loads; one load is stored in front of the other load. A load can only be put into or retrieved from the second position if there is no load in the first position. Double-deep storage might be beneficial if the variety of loads is relatively low and the turnover rate of these loads is high [6-15]. Modifications to the crane may be required to be able to store and retrieve loads from both positions. Carousel systems (horizontal or vertical, single or double) are suitable for storing small and medium-sized products at different levels. A crane is used to store and retrieve items from the rotating carousel. The lower and upper part of a double carousel can rotate independently of each other. Finally, worthy of mentioning is a special type of AS/RSs called autonomous vehicle storage and retrieval systems. This system separates horizontal and vertical travel. Vehicles travel horizontally over rails through aisles, while lifts are used to transfer loads vertically.

To keep the investment more convenient and to increase the performances of the AS/RSs, several structural solutions have been developed, like AS/RSs with double deep racks, to contain more loads, or systems with cranes that can change the aisle to serve.

In the last years, a new kind of AS/RS has been introduced, where the cranes have multi-shuttles. Usually a normal crane with a single-shuttle moves one load at the time, while a multi-shuttle crane can transport more than one load, typically two or three (fig. 6.1).

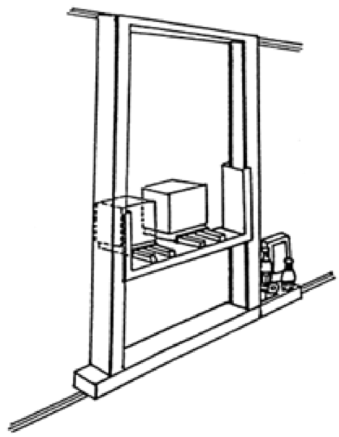


**Figure 6.1: Automated Storage and Retrieval System with Dual-Shuttles**

***(Meller and Mungtwattana [6.9])***

Due to the different AS/RS configurations and the high cost of these systems, particular attention has to be paid during the design process of AS/RS. It is crucial to design AS/RS based on the current and future demand requirements. An important system performance measure is the throughput capacity of the systems, calculated as the inverse of the mean time to complete a storage/retrieval command.

Several different models can help the practitioners to estimate the performance of the system during the feasibility design process. The mainly used models are based on Bozer and White formulations [6.1] or on the F.E.M. 9851 (*Federation Europeenne de la Manutention* normative) [6.5]. This last solution is often required by the specifications and terms of design contracts. In scientific literature, many models have been developed to modify the Bozer and White one to adapt it in order to calculate the multi-shuttle AS/RSs performances, but increasing the complexity. On the other hand, F.E.M. 9851 normative gives simple models but none of these consider the case of multi-shuttle AS/RSs.

The purpose of this chapter is to extend the F.E.M. 9851 normative to calculate the throughput of dual-shuttle AS/RS and compare the results with the applications of the models presented in literature.

In the next section and sub-sections the Overall Equipment Efficiency of AS/RSs is investigated, using the guidelines indicated by the normative F.E.M. 9221 [6.4]. In section 6.3 an innovative AS/RS is illustrated: the AS/RS multi-shuttle. This kind of system permits an improvement of efficiency because it eliminates the spare time during the command cycle, moving more than one load in a cycle. In that section a rigorous review of the scientific contributions concerning this kind of AS/RS is reported [6.11]. In section 6.4, the mainly travel-time models, presented in scientific literature, are discussed. At the end of this section, the new travel-time model is introduced based on the extension of F.E.M. 9851 normative and compared to the other models. In section 6.5, a Monte Carlo simulation is conducted to evaluate the goodness of fit of the new model. Finally, in section 6.6, several discussions and further possible researches are introduced.

## 6.2. Overall Equipment Efficiency of AS/RSs: Reliability, Availability and Cycle Time Estimations

The performance of Automated Storage and Retrieval Systems is strictly connected to their reliability and availability. The *Federation Europeenne de la Manutention* has discussed the standard method to determine the survival parameters for this kind of machine. The rule F.E.M. 9221 [6.4] and F.E.M. 9851 [6.5] permit to draw conclusion about the throughput which the user of AS/RSs requires from the manufacturer.

As previously described, throughput is defined as the number of storage and retrieval operations per unit of time, then it is a measure of Overall Equipment Efficiency.

Consequently the Overall Equipment Efficiency depends on [6.4]:

- Reliability and Availability of the AS/RSs;
- Cycle Time of the AS/RSs;

In the next sub-section 6.2.1., the theoretical principles, reported in rule F.E.M. 9221 on Reliability and Availability of AS/RSs will be discussed, while in sub-section 6.2.2. the guidelines to estimate the Cycle Time will be illustrated, as described in rule F.E.M. 9851.

### 6.2.1. Theoretical Principles of Reliability and Availability of AS/RSs

The reliability of a discontinuously loaded system unit is equal to the probability of this unit carrying out a particular operation correctly and without malfunctions. In tests, reliability is determined by the quotient:

$$\eta = \frac{n_r}{n_r + n_f} \qquad 5\text{-}1$$

where:

$n_r$ = number of correctly carried out operations;

$n_f$ = number of operations carried out incorrectly or with faults;

The particular operation should be tested with an adequate statistical frequency.

The availability of a system unit for a particular operation is equal to the probability of finding that unit, at any given time during the period of operation, in a state which will allow the operation concerned to be carried out correctly and without malfunctions. In order to determine the availability of individual system units in tests, the unit concerned is considered under clearly defined operating conditions with the planned average loading for a statistically adequate period of time T.

Availability is determined by the quotient:

$$\eta_T = \frac{T - T_{aus}}{T} \qquad 5\text{-}2$$

where:

$T$ = total operating time

$T_{aus}$ = sum of individual periods of downtime

This gives the net operational time:

$$T_{net} = T - T_{aus} \qquad 5\text{-}3$$

The mean downtime is the total downtime $T_{aus}$ divided by the number of failures, $n_{aus}$

$$\frac{T_{aus}}{n_{aus}} = MTTR \qquad 5\text{-}4$$

Similarly the mean time without failures is

$$\frac{T - T_{aus}}{n_{aus}} = MTBF \qquad 5\text{-}5$$

And, therefore, availability can also be expressed by the quotient:

$$n_T = \frac{MTBF}{MTBF + MTTR} \qquad 5\text{-}6$$

Each period of downtime can be divided into the following sub-period:

- Period between stoppage of machine and start of search for fault by appropriate personnel;
- Time needed to identify cause of failure;
- Time needed to correct fault and restore serviceability.

### 6.2.2. Cycle Time Estimation of AS/RSs

The cycle time of an AS/RS is the total sum of constant time periods, consisting in positioning, location check, switching and checking operations and fork cycle, and travel periods. These periods depend on the specific technical AS/RS data and on the travel path in the x, y and z-directions. The mean cycle time is the average value, assuming random storage of unit load. An exact determination of the mean cycle time for calculation and analysis purposes is highly complex.

The mean cycle time depends on the parameter:

$$a = \frac{H}{L} * \frac{v_x}{v_y} \qquad 5\text{-}7$$

where:

$H$ = Maximum lifting path;

$L$ = Maximum travel path;

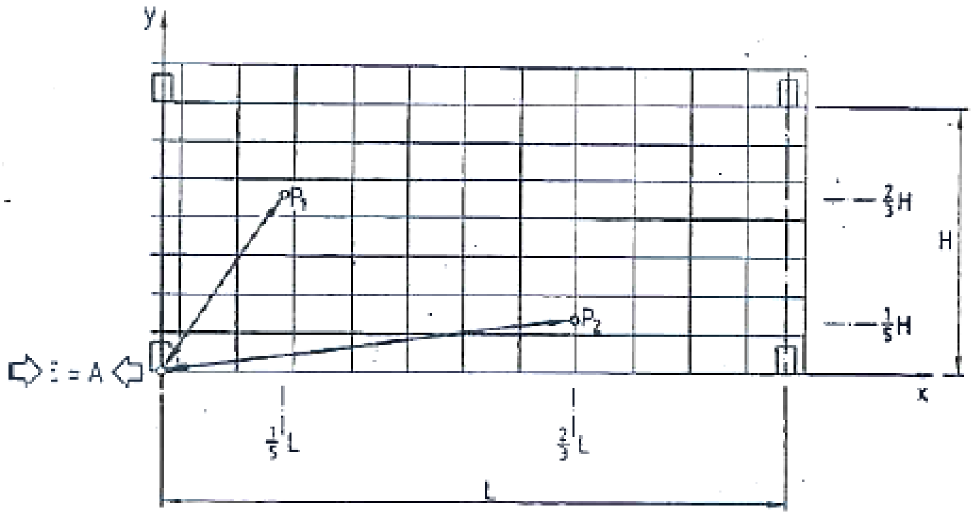
$v_x$ = Maximum travel speed;

$v_y$ = Maximum lifting speed.

The velocity vector corresponds with the diagonal of the vertical rack plane, if a = 1. The test cycle estimation differ from the theoretical time values if a is not equal to 1. Therefore the limits $0.5 < a < 2$ should not be exceeded in order to obtain informative values. The normative F.E.M. 9851 [6.5] are developed for a typical AS/RS with single-shuttle, where usually two locations are interested by the storage and retrieval process. This normative gives two theoretical reference points P1 and P2 to be determined according to requirements and warehouse layout. Here below, several examples of the most frequently cases, from F.E.M. 9851.

*CASE 1: Pick-up and transfer at lower corner point*

| Point | Coordinates | |
|---|---|---|
| | x | y |
| E = A | 0 | 0 |
| P1 | 1/5 L | 2/3 H |
| P2 | 2/3 L | 1/5 H |



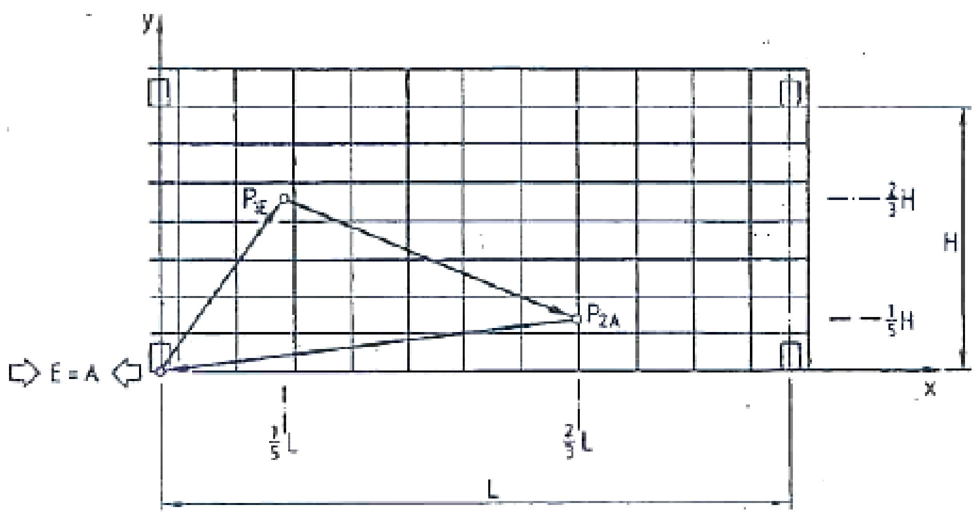




**Figure 6.2: Rack plane, single and combined cycle for case 1** ***(from F.E.M. 9851 [6.5]).***

*CASE 2: Pick-up at corner point E and transfer at corner point A*

| Point | Coordinates | |
|---|---|---|
| | x | y |
| E | 0 | 0 |
| A | L | 0 |
| P1 E | 1/5 L | 2/3 H |
| P2 E | 2/3 L | 1/5 H |
| P'1A | 1/5 L | 2/3 H |
| P'2A | 2/3 L | 1/5 H |

measured beginning at A

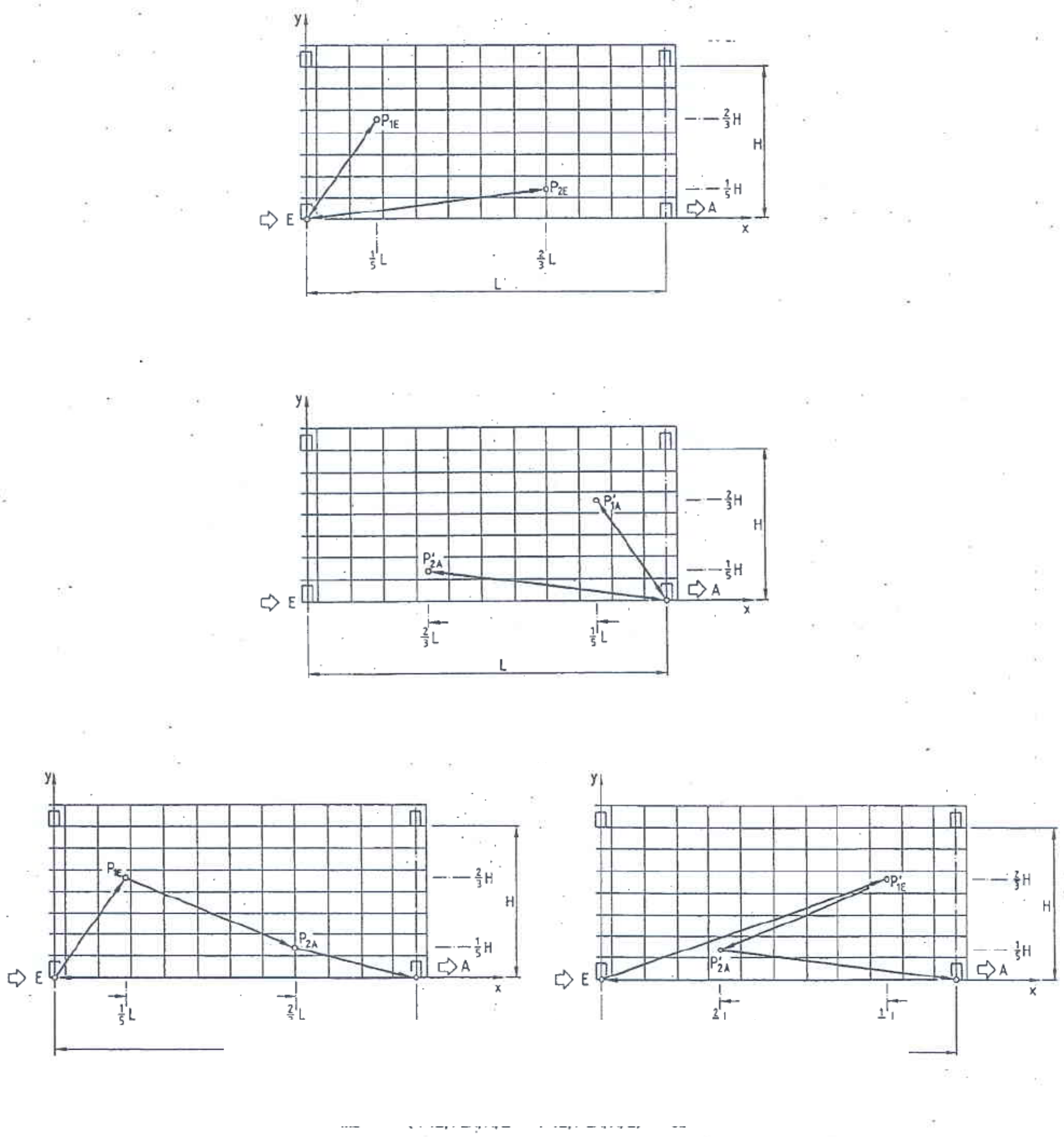
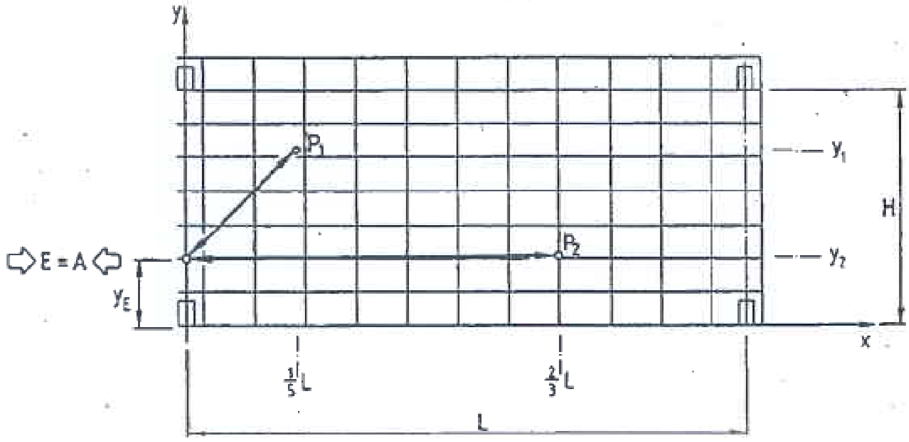



**Figure 6.3: Rack plane, single and combined cycle for case 2 *(from F.E.M. 9851 [6.5])*.**

*CASE 3: Pick-up and transfer at a location above the corner point*

| Point | | Coordinates | |
|---|---|---|---|
| | | x | y |
| E = A | | 0 | yE |
| P1 | 0 < yE ≤ H/2 | 1/5 L | y1 = 2/3 H + 1/3 yE |
| P2 | 0 < yE ≤ H/2 | 2/3 L | y2 = 1/5 H + 1/3 yE |
| P1 | H/2 < yE ≤ H | 1/5 L | y1 = 2/3 H - 1/3 yE |
| P2 | H/2 < yE ≤ H | 2/3 L | y2 = 1/5 H + 3/5 yE |



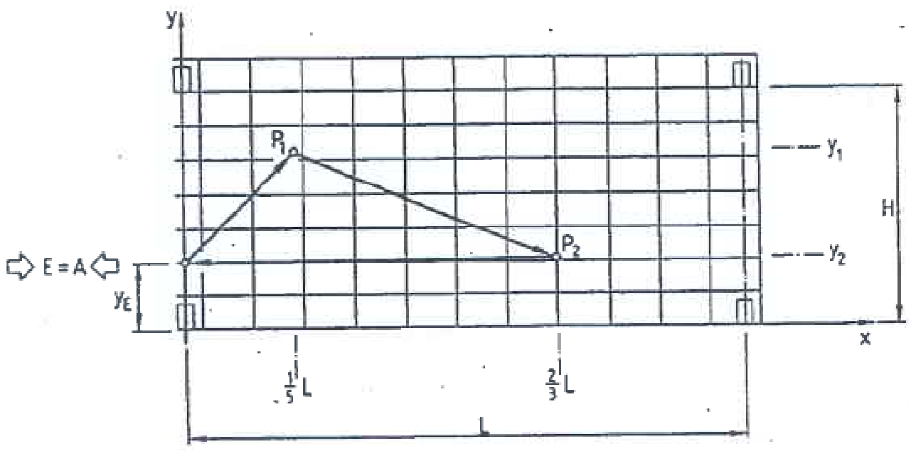




**Figure 6.4: Rack plane, single and combined cycle for case 3** ***(from F.E.M. 9851 [6.5])*.**

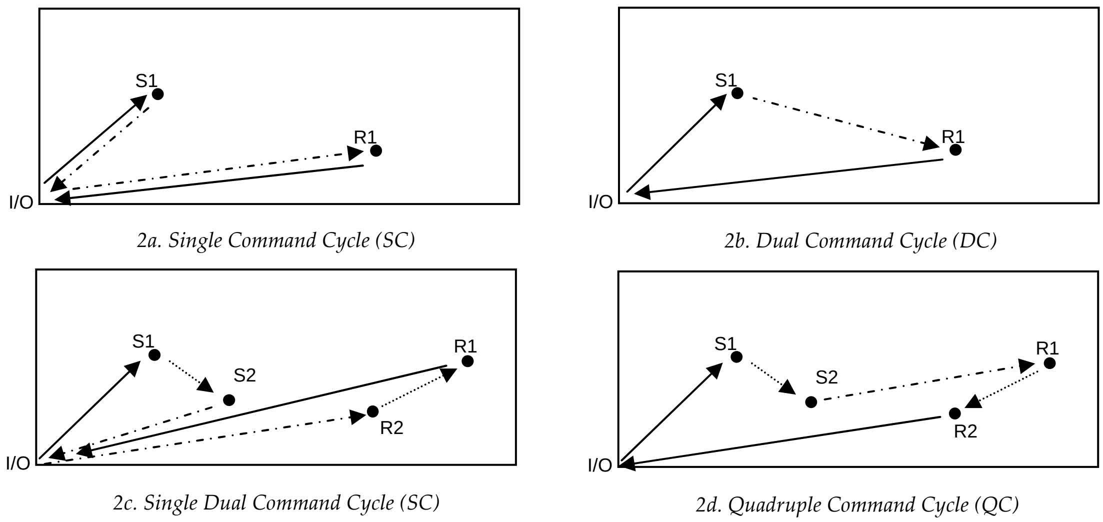
## 6.3. Development of Innovative AS/RSs with multi-shuttles

Typically, AS/RSs are characterized by single-shuttle cranes that permit to move one load at the time. Fixed the cinematic performances of the cranes, multi-shuttle AS/RS have been developed to increase the throughput, where each crane can transport more than one load at the time. Generally, increasing the number of shuttles, the performances of the system increase because of the related decrease of empty trips. On the other hand, increasing the number of shuttles gives an higher investment cost.

For these reasons, the applications of dual-shuttle AS/RSs are more used than other kinds of multi-shuttle AS/RSs.

Recent surveys of scientific contributions on warehouse design and control [6.12] and one in particular on AS/RS [6.11] show the relevant aspects of design process of warehouse and AS/RS to take into consideration to define the right configuration of the system in order to satisfy the project requirements. The authors have underlined the importance of the multi-shuttle AS/RSs in the next future and the related necessity to estimate correctly the performance of such systems in order to design them properly.

The performance of AS/RS mainly depends on the type of command cycle [6.13]. Figure 6.5 shows the most used command cycles to storage, in locations S1 and S2, and retrieve, from locations R1 and R2, the loads on racks. For single-shuttle AS/RSs, it can be given the Single Command Cycle and the Dual Command Cycle, while for dual-shuttle AS/RSs, the possible cycles are the Single Dual Command Cycle (SDC) and the Quadruple Command Cycle (QC).




*2a. Single Command Cycle (SC)* *2b. Dual Command Cycle (DC)*

*2c. Single Dual Command Cycle (SC)* *2d. Quadruple Command Cycle (QC)*

*S1 – S2: Storage Locations* *R1 – R2: Retrieval Locations*

**Figure 6.5: Travel cycle for Single Shuttle and Dual Shuttle AS/RSs**

Roodbergen and Vis [6.11] have underlined the requirement to introduced new analytical models to calculate the performances of non standard AS/RSs, for example in case of dual-shuttle AS/RSs.

In fact, few contributions have been published on the methodology of estimation of travel-time for SDC and QC [6.1-6.2-6.3-6.6-6.7-6.8-6.9-6.13-6.14]. Sarker et al [6.13] firstly developed a model to calculate the performance of these systems using the formulations introduced before by Bozer and White [6.1]. They introduced also an improving heuristic for the sequencing of retrievals. Keserla and Peters [6.7] studied and compared the performance of AS/RS with single and dual shuttles and they also introduced a new heuristic to sequence the retrievals in order to minimize the travel time in QC mode.

Another analytical model, also based on Bozer and White formulations, has been developed by Meller and Mungtwattana [6.8-6.9]. They estimated the travel-time of both dual shuttle and triple shuttle, varying the different storage and retrieval policies. First-come-first-served (FCFS) policy is the most used in the feasibility design phase, while nearest neighbor (NN) one is related to the management phase of systems and permits to reduce the total travel-time.

These scientific contributions show the close link to the Bozer and White formulations [6.1] and all these models do not consider the acceleration and deceleration into the travel-time calculation.

For this reason, other models have been developed in order to consider also these cinematic aspects. De Puy [6.3] extended the model of Meller and Mungtwattana for each policy, applying the research of Chang et al [6.2] about the extension of Bozer and White model, with acceleration and deceleration consideration. This model better fits the real travel-time values but it is more complex to apply.

Other contributions have used a simulation approach to evaluate and estimate the different travel-time for multi-shuttle AS/RSs [6.10] and to validate the increasing of efficiency and flexibility in comparison to single-shuttle AS/RSs, as demonstrated in [6.11].

It is important to highlight that in many industrial applications the feasibility study of AS/RS required to follow several normatives, recognized by the international industries.

The *Federation Européenne de la Manutention* gives the guide-lines to estimate the travel-time for different configurations of AS/RS, in section IX of the F.E.M. 9851 normative [6.5], which is related to the storage plants. In more details, this normative introduces several coefficients to be used during the design process, but it does not consider the case of multi-shuttle AS/RSs.

From the literature review, it points out the lack of an analytical model that has to be used during the feasibility study of the system and as consequence it should be simple and quick to use but with high accuracy in travel-time estimation.

The purpose of this chapter is to introduce an analytical model, extending the F.E.M. 9851 normative, giving the right coefficients useful to estimate the travel-time in case of dual-shuttle AS/RSs. The results here obtained are validated using a Monte Carlo simulation and the better fitting is compared to the other models.

## 6.4. Estimation of Performance of Dual-Shuttle AS/RSs

Bozer and White [6.1] were the first ones to introduce an analytical travel time model to estimate the transaction time of an AS/RS. Their developed model has been widely used in the design of traditional automated storage systems.

Meller and Mungtwattana [6.8-6.9] extended this formulation to multi-shuttle AS/RSs in order to evaluate the time saving in comparison to traditional systems.

They assumed a randomized storage and a FCFS processing of both storage and retrieval. Generally, other assumptions regard the dimension of rack, length L and height H, the input/output station, that is located at the lower left-hand corner, and the cranes and shuttles, that travel simultaneously in the horizontal and vertical directions.

Here below, the main two travel time models for dual-shuttle AS/RSs, based on Bozer and White formulations, are discussed. The first model was introduced by Meller and Mungtwattana [6.8-6.9] and the second one is an extension developed by De Puy [6.3]. The latter model is based on studies conducted by Chang et al. [6.2] in order to consider also the acceleration and deceleration on classical models.

***Notation***

$L$ = Length of rack [m]

$H$ = Height of rack [m]

$s_h$ = horizontal speed [ $m/s$ ]

$a_h$ = horizontal acceleration [ $m/s^2$ ]

$s_v$ = vertical speed [ $m/s$ ]

$a_v$ = vertical acceleration [ $m/s^2$ ]

$t_h$ = horizontal dimension of rack in time [s]

$t_v$ = vertical dimension of rack in time [s]

$T$ = scaling factor

$b$ = shape factor

$E(SC)_{BW}$ = expected single-command round-trip travel time calculated with classical Bozer and White [6.1] formulations [s].

$E(TB)_{BW}$ = expected travel time between two locations calculated with classical Bozer and White [6.1] formulations [s].

$E(SC)_{BW}'$ = expected single-command round-trip travel time calculated with extended Bozer and White [6.1] formulations [s].

$E(TB)_{BW}'$ = expected travel time between two locations calculated with extended Bozer and White [6.1] formulations [s].

$E(SDC)_{BW}$ = expected single-dual-command round-trip time calculated with classical Bozer and White [6.1] formulations [s].

$E(QC)_{BW}$ = expected quadruple-command round-trip time calculated with classical Bozer and White [6.1] formulations [s].

$E(SDC)_{BW}'$ = expected single-dual-command round-trip time calculated with extended Bozer and White [6.1] formulations [s].

$E(QC)_{BW}'$ = expected quadruple-command round-trip time calculated with extended Bozer and White [6.1] formulations [s].

$E(SDC)_{FEM}$ = expected single-dual-command round-trip time calculated with extended F.E.M. formulations [s].

$E(QC)_{FEM}$ = expected quadruple-command round-trip time calculated with extended F.E.M. formulations [s].

$E(SDC)_{MC}$ = expected single-dual-command round-trip time resulted from Monte Carlo simulation [s].

$E(QC)_{MC}$ = expected quadruple-command round-trip time resulted from Monte Carlo simulation [s].

$MAD$ = Mean Absolute Deviation.

$MRD$ = Mean Relative Deviation.

$D$ = Dispersion.

### 6.4.1. Bozer & White Based Formulation without acceleration impact

The first model introduced in international literature was developed by Meller and Mungtwattana [6.8-6.9] starting from the Bozer and White formulations, that do not consider the impact of acceleration in travel time estimation.

Here below the algorithm, studied by Meller and Mungtwattana [6.8-6.9], to calculated the expected travel times:

***Algorithm of Meller and Mungtwattana* [6.8-6.9]:**

Defined the dimensions of the rack, $L$ and $H$, and the cinematic parameter of crane, $s_h$ and $s_v$:

Calculate the dimension of rack in time as follows:

$t_h = \frac{L}{s_h}$ and $t_v = \frac{H}{s_v}$;

Determine the *factor scale* $T$ and the *shape scale* $b$:

$T = \max(t_h, t_v)$ and $b = \min\left(\frac{t_h}{T}, \frac{t_v}{T}\right)$;

Compute the normalized travel times:

$E(SC)_{BW} = \left(1 + \frac{b^2}{3}\right)$ and $E(TB)_{BW} = \left(\frac{1}{3} + \frac{b^2}{6} - \frac{b^3}{30}\right)$;

Calculate the expected times for dual-shuttle AS/RSs cycles:

$$E(SDC)_{BW} = 2T\left[E(SC)_{BW} + E(TB)_{BW}\right]$$

$$E(QC)_{BW} = T\left[E(SC)_{BW} + 3E(TB)_{BW}\right].$$

### 6.4.2. Bozer & White Based Formulation within acceleration impact

The impact of acceleration and deceleration on expected travel time was studied by Chang et al. [6.2] and extended by De Puy [6.3].

The author developed an extended formulation to estimate the expected travel times for dual-shuttle AS/RSs based on Bozer and White paper, considering the acceleration and deceleration of the system, too.

***Algorithm of De Puy [6.3] using the formulations of Chang et al [6.2]:***

Defined the dimensions of the rack, $L$ and $H$, and the cinematic parameter of crane, $s_h$, $a_h$ and $s_v, a_v$:

Compute the travel time for half rack length $t_x$ as follows:

$$if\ \frac{L}{2} \geq \frac{s_h^2}{a_h}, \qquad t_x = \frac{L}{2s_h} + \frac{s_h}{a_h};$$

$$if\ \frac{L}{2} < \frac{s_h^2}{a_h}, \qquad t_x = \sqrt{\frac{4L}{2a_h}};$$

Compute the travel time for half rack height $t_y$ as follows:

$$if\ \frac{H}{2} \geq \frac{s_v^2}{a_v}, \qquad t_y = \frac{H}{2s_v} + \frac{s_v}{a_v};$$

$$if\ \frac{H}{2} < \frac{s_v^2}{a_v}, \qquad t_y = \sqrt{\frac{4H}{2a_v}};$$

Calculate the acceleration/deceleration parameters $\beta_1, \beta_2, \lambda_1, \lambda_2$:

$$if\ t_h \geq t_v, \qquad \beta_1 = \frac{\ln t_x - \ln t_h}{\ln 0.5} \quad \beta_2 = \frac{\ln t_y - \ln t_v}{\ln 0.5}$$

$$if\ t_h < t_v, \qquad \beta_1 = \frac{\ln t_y - \ln t_v}{\ln 0.5} \quad \beta_2 = \frac{\ln t_x - \ln t_h}{\ln 0.5}$$

$$\lambda_1 = \frac{1}{\beta_1} \quad \lambda_2 = \frac{1}{\beta_2}$$

Determine the normalized travel times:

$$E(SC)_{BW}' = 2\frac{\lambda_1}{\lambda_1 + 1} + 2\left(\frac{1}{\lambda_1 + 1} - \frac{1}{\lambda_1 + \lambda_2 + 1}\right)b^{\lambda_1 + 1}$$

$$E(TB)_{BW}' = \left(1 - \frac{2}{\lambda_1 + 1} + \frac{1}{2\lambda_1 + 1}\right) + b^{\lambda_1 + 1}\left(\frac{2}{\lambda_1 + 1} - \frac{4}{\lambda_1 + \lambda_2 + 1} + \frac{2}{\lambda_1 + 2\lambda_2 + 1}\right) + $$
$$+ b^{2\lambda_1 + 1}\left(\frac{2}{2\lambda_1 + \lambda_2 + 1} - \frac{1}{2\lambda_1 + 2\lambda_2 + 1} - \frac{1}{2\lambda_1 + 1}\right);$$

Calculate the expected times for dual-shuttle AS/RSs cycles:

$$E(SDC)_{BW}' = 2T\left[E(SC)_{BW}' + E(TB)_{BW}'\right]$$

$$E(QC)_{BW}' = T\left[E(SC)_{BW}' + 3E(TB)_{BW}'\right]$$

### 6.4.3. Innovative Travel Time Model based on F.E.M. rules

The model developed and introduced in this chapter follows the guidelines given by the *Federation Européenne de la Manutention*, in section IX, Series Lifting Equipment, Rules F.E.M. 9851: *"Performance Data of S/R Machines Cycle Times"* [6.5].

The F.E.M. 9851 rules are widely used by practitioners and industrial engineers to conduct the feasibility studies in future AS/RS applications because they are quick and simple to use and the estimated travel times are close to real values.

Such rules permit to calculate the travel-times for traditional single-shuttle AS/RSs, introducing two virtual locations, characterized by parametric coordinates of length and height of rack.

The F.E.M. 9851 normative mentions the possibility to extend the traditional formulations also to dual-shuttle AS/RSs but the rules do not suggest any parametric coordinates for the other two locations, served by the system during single-dual command or quadruple command.

For this reason, the aim of this chapter is to cover this lack, introducing the parametric coordinates also for the two new locations.

A Monte Carlo simulation has been conducted varying the cinematic parameters and the dimension of rack. In total, 1512 scenarios have been created to collected a wide set of values, composed by 54 different cinematic profiles and 28 different rack configurations (Table 6.1).

| Factor | Levels |
|---|---|
| $s_h$ $[m/s]$ | *2; 3; 4;* |
| $a_h$ $[m/s^2]$ | *0.8; 1.2; 1.6;* |
| $s_v$ $[m/s]$ | *0.4; 0.8; 1.2;* |
| $a_v$ $[m/s^2]$ | *0.4; 0.8;* |
| $L$ $[m]$ | *80; 120; 160; 200;* |
| $H$ $[m]$ | $k * L \frac{s_v}{s_h}$ *where* $k = [0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1]$ |

**Table 6.1: Levels of factors for scenarios simulation**

The Monte Carlo simulation has been carried out with 10,000 runs for each scenario, generating for each run four different locations to be served and calculating the travel time to make the Single Dual Command and the Quadruple Command. The main assumptions regard: the randomized distribution of the unit load inside the rack, the First Come First Served (FCFS) policy used to complete the travel and the input/output station located at the lower left-hand corner. This last assumption could be relaxed, dividing the rack into two independent ones and calculating the travel times as weighted average of travel times in the two independent areas of rack.

Table 6.2 shows the parametric coordinates of modified F.E.M. 9851 for dual-shuttle AS/RSs.

| Point | Coordinates | |
|---|---|---|
| | X | Y |
| I/O | 0 | 0 |
| P1 *(from F.E.M. 9851 rules)* | 1/5 L | 2/3 H |
| P2 | 0.38 L | 0.49 H |
| P3 | L | 0.80 H |
| P4 *(from F.E.M. 9851 rules)* | 2/3 L | 1/5 H |

**Table 6.2: Estimated modified F.E.M. coefficients**

The method of least squares has been used to calculate the values of parametric coordinates, comparing the estimated travel time of the extended F.E.M. model with Monte Carlo values. Noted the four locations P1, P2, P3 and P4, the estimation of travel-times is simple and quick, following the present algorithm:

***Algorithm of Extended F.E.M.***

Defined the dimensions of the rack, $L$ and $H$, and the cinematic parameter of crane, $s_h$, $a_h$ and $s_v, a_v$:

Calculate the coordinates $(X_i, Y_i)$ of locations P1, P2, P3, P4 using the coefficients defined in table 2.

Calculate the expected times for AS/RS dual-shuttle cycles:

$$E(SDC)_{FEM} = T(I/O-P1)+T(P1-P2)+T(P2-I/O)+ \\ +T(I/O-P3)+T(P3-P4)+T(P4-I/O)$$

$$E(QC)_{FEM} = T(I/O-P1)+T(P1-P2)+T(P2-P3)+ \\ +T(P3-P4)+T(P4-I/O)$$

The travel-time between two locations is calculated by:

$$T(P_i-P_j)=\max\left(T_x(P_i-P_j);T_y(P_i-P_j)\right)$$

where each time is the following:

$T_x(P_i-P_j)$:

$$if\ \frac{s_h^{\ 2}}{a_h}<|X_i-X_j| \qquad T_x(P_i-P_j)=\frac{|X_i-X_j|}{s_h}+\frac{s_h}{a_h};$$

$$if\ \frac{s_h^{\ 2}}{a_h}\geq|X_i-X_j| \qquad T_x(P_i-P_j)=2\sqrt{\frac{|X_i-X_j|}{a_h}};$$

$T_y(P_i-P_j)$:

$$if\ \frac{s_v^{\ 2}}{a_v}<|Y_i-Y_j| \qquad T_y(P_i-P_j)=\frac{|Y_i-Y_j|}{s_v}+\frac{s_v}{a_v};$$

$$if\ \frac{s_v^{\ 2}}{a_v}\geq|Y_i-Y_j| \qquad T_y(P_i-P_j)=2\sqrt{\frac{|Y_i-Y_j|}{a_v}};$$

## *6.5. Evaluation of Models*

In this section, the introduced models are compared with the travel-time values obtained by the Monte Carlo simulation.

A series of evaluation functions is introduced to measure the goodness of fit of each model. The Mean Absolute Deviation (MAD) is defined as the average of the absolute differences between the values estimated by the model and the Monte Carlo ones. The Mean Relative Deviation (MRD) compares the difference between the estimated value and Monte Carlo one with the Monte Carlo value. It represents the percentage error of the model. The last evaluation function is the Deviation (D) that is the measure of the variability or dispersion.

The following equations have been used to calculate the evaluation functions:

$$MAD_j = \sum_i \frac{\left|E(c)_{i,j} - E(c)_{i,MC}\right|}{n}$$

$$MRD_j = \frac{1}{n} * \sum_i \frac{\left|E(c)_{i,j} - E(c)_{i,MC}\right|}{T_{i,MC}}$$

$$D_j = \sum_i \frac{\left[E(c)_{i,j} - E(c)_{i,MC}\right]^2}{n}$$

where i = scenarios, j = BW, BW', F.E.M.' and c = SDC, QC.

The following tables reports the values of these functions based on type of command: Single Dual Command or Quadruple Command.

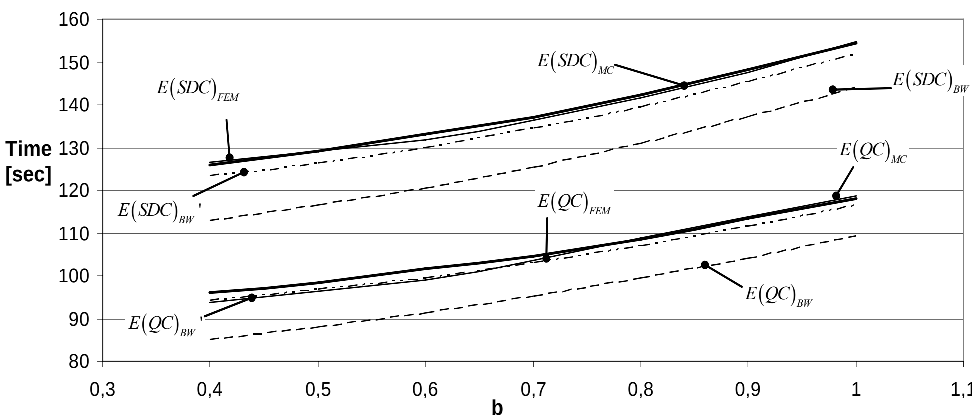
| SINGLE DUAL COMMAND | | | |
|---|---|---|---|
| | MAD | MRD | D |
| BOZER & WHITE | 13.62 | 9.54% | 139.99 |
| BOZER & WHITE MODIFIED | 3.04 | 2.00% | 3.83 |
| F.E.M. MODIFIED | 0.87 | 0.58% | 4.68 |

**Table 6.3: Comparison between models on Single Dual Command travel time estimation.**

| QUADRUPLE COMMAND | | | |
|---|---|---|---|
| | MAD | MRD | D |
| BOZER & WHITE | 11.24 | 10.27% | 206.33 |
| BOZER & WHITE MODIFIED | 1.84 | 1.56% | 10.21 |
| F.E.M. MODIFIED | 1.56 | 1.24% | 1.44 |

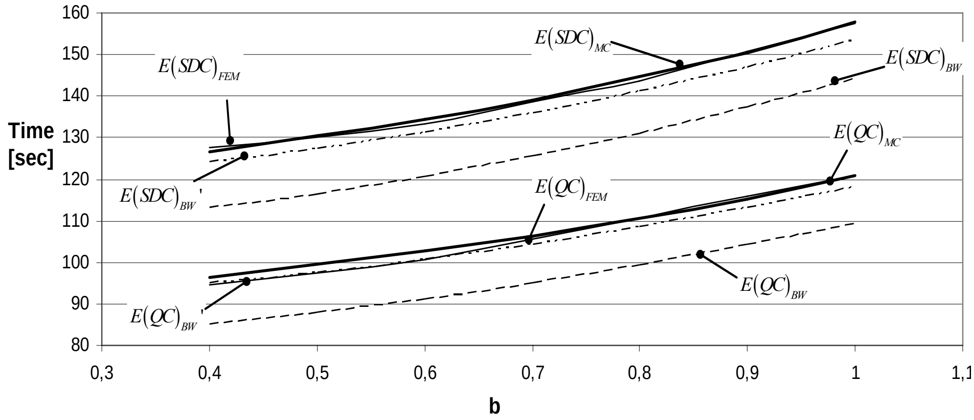
**Table 6.4: Comparison between models on Quadruple Command travel time estimation.**

As well shown in these tables and in the following figures, the modified F.E.M. model developed in this chapter has a better accuracy than the previous models. In fact, each evaluation function has lower value than the other ones.



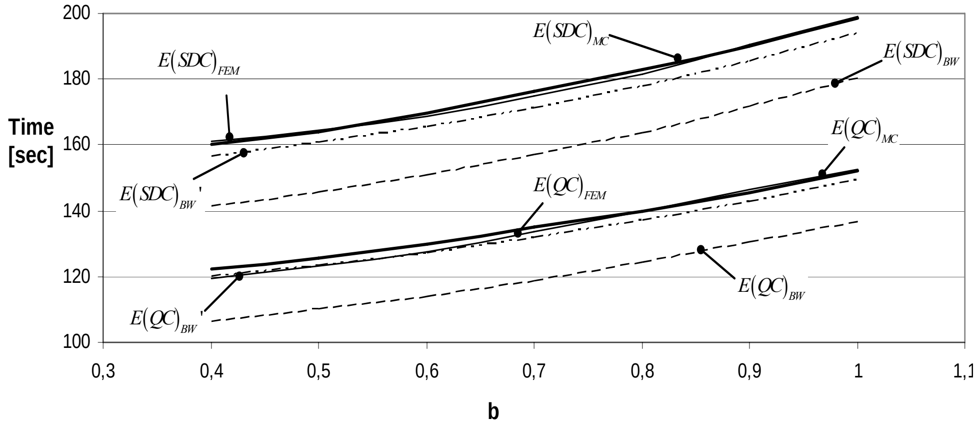

| Factor | Levels |
|---|---|
| $s_h$ | 2 |
| $a_h$ | 0.8 |
| $s_v$ | 0.4 |
| $a_v$ | 0.4 |
| $L$ | 80 |

**Figure 6.6: Estimated Travel Times for scenario** $s_h = 2\ m/s$; $a_h = 0.8\ m/s^2$; $s_v = 0.4\ m/s$; $a_v = 0.4\ m/s^2$; $L = 80\ m$




| Factor | Levels |
|---|---|
| $s_h$ | *3* |
| $a_h$ | *1.2* |
| $s_v$ | *0.8* |
| $a_v$ | *0.4* |
| $L$ | *120* |

**Figure 6.7: Estimated Travel Times for scenario** $s_h = 3\ m/s;\ a_h = 1.2\ m/s^2;\ s_v = 0.8\ m/s;\ a_v = 0.4\ m/s^2;\ L = 120\ m$




| Factor | Levels |
|---|---|
| $s_h$ | *4* |
| $a_h$ | *0.8* |
| $s_v$ | *1.2* |
| $a_v$ | *0.8* |
| $L$ | *200* |

**Figure 6.8: Estimated Travel Times for scenario** $s_h = 4\ m/s;\ a_h = 0.8\ m/s^2;\ s_v = 1.2\ m/s;\ a_v = 0.8\ m/s^2;\ L = 200\ m$

## 6.6. Considerations about the models comparison

The present work focuses on designing of automated storage/retrieval systems, called AS/RSs. Such a type of plants is widely used in different businesses, from the manufacturing to warehousing applications. They permit to achieve several benefits like reduction of errors, reduction of direct labor costs, better logistics performances, but they require an elevate capital investment. To increase their performances and reduce the cycle times, several different configurations have been developed and introduced in the last decades.

One of the most relevant structures of AS/RS is a system that uses a multi-shuttle cranes to move more than one load during each cycle, typically two or three loads.

In scientific literature, several models have been introduced to calculate the performance of such AS/RS in order to help the practitioners during the feasibility study and first phase of design process.

In this chapter, a model based on F.E.M. 9851 rules has been developed to estimate the travel-time for dual-shuttle AS/RSs. The results of this application have been compared with the previous models presented in literature. Meller and Mungtwattana model [6.8-6.9] is based on Bozer and White formulations, not considering the impact of acceleration/deceleration while De Puy model [6.3] extends it using the formulas by Chang et al. [6.2] in order to consider the acceleration and deceleration. Each model has been evaluated comparing it to the real values resulting from a Monte Carlo Simulation, over about 1500 different scenarios.

Generally, the Mean Relative Deviation (MRD) is about 1% for the model here introduced. This result is more convenient compared to the other methods. Meller and Mungtwattana model [6.8-6.9] fits the data with a MRD of about 11%, while De Puy one, considering also the acceleration/deceleration, is characterized by a MRD of about 2%.

The future researches will consider several aspects related to different movement policies and to the percentage of Single Dual Command and Quadruple Command to be used during the preliminary design of the system. Also the class based storage policy will be studied in the next model development.

# 7. Conclusions and Future Researches

In the last years, the efficiency of industrial plants and logistic systems, defined by the Overall Equipment Efficiency (OEE), have been considered with more relevance by the researchers and industrial managers, in order to improve the flexibility and to quicken the response to the market of productive and logistics systems.

About this, the estimation of reliability and availability of complex industrial systems is very important to calculate the OEE indices in the right way. Consequently, it is relevant to evaluate the correct survival functions, in order to monitory and improve the efficiency of productive and logistics systems. Wrong values of OEE indices lead to wrong actions of improvement.

In the first three chapters of this thesis, an introduction to the problems of efficiency and performance evaluation has been treated.

In particular, chapter 1 has illustrated the goals of this research, the structure of the thesis and the scientific contributions published by the author in the last three years about these topics.

Chapter 2 has introduced the definition of efficiency on industrial and logistic systems. Two Overall Equipment Efficiency indices have been investigated: the Nakajima and the SEMI model. At the end of this chapter, the relation between efficiency, defined by OEE, and the maintenance of productive and logistic systems has been well studied. The scientific literature and the practice have shown the relevance of developing of several different maintenance policies in order to improve the overall efficiency.

Then in chapter 3, the survival fundamentals have been introduced in order to specify the models for the estimation of reliability and availability starting, from the data collected from the industrial fields.

The main part of the thesis is included in chapters 4, 5 and 6, where:

- an innovative model for the estimation of reliability of systems operating in different productive environments has been defined and introduced;
- new maintenance policies have been developed and studied, in order to improve the efficiency of productive and logistic systems;
- the efficiency of a particular case of logistic system has been investigated. The efficiency and performance models have been introduced and deeply studied.

Chapter 4 discussed the reliability modeling of industrial systems with respect to random field environments has been discussed, addressing the fact that it is difficult to estimate the reliability of a component when it works in different environments.
The concept of systemability has been introduced and used to examine real world application case studies in automatic manufacturer packaging machines as well as motorcycle systems in determining the reliability of customer products. The first one shows the most important features of the systemability approach, as a function that enables to separate the intrinsic performance of the components or the systems from the random environmental factors. The second case study has been carried out in order to estimate the reliability and systemability of several motorcycle components and drive system in operative conditions, starting from the data collected during the tests.
The chapter has demonstrated that this new concept is suitable to predict the reliability of system products in operating environments and it is easy to use also for predicting the lifetime distribution of complex system configuration, starting from the data related to single components.
At the end of this chapter, this innovative concept has been applied to model the cost of complex entire industrial system development life-cycle that perhaps reflects the perspectives from both developers and users and to determine the optimal release policies. An industrial application has been discussed to illustrate the results of this study. This is the first study that generalized the NHPP reliability model using a Weibull distribution and also incorporates the random field environmental factors into the cost model.

Several future researches about reliability modeling concern the study of the performance of the systemability when its parameters change; the definition of quantitative relationships between systemability parameters and different environment conditions and the definition of a general model for the estimation of reliability function in order to design and produce components or system products and reflect ways to define the best maintenance policies.

Chapter 5 highlighted the necessary of maintenance policies design to guarantee high level of efficiency of the complex industrial systems, improving system reliability, preventing the occurrence of system failures, and reducing maintenance costs of systems.

Many maintenance models have been developed and widely used in scientific literature. The Age Replacement Policy (ARP) and the Periodic Replacement Policy (PRP) are some of the most important policies. Incorrect estimation of reliability function could lead to the wrong functional design of the system and to an incorrect definition of the appropriate maintenance policies, and as a consequence the wrong design of actions to improve the efficiency of systems.

In this chapter, the effects of the different operating environments in the Age Replacement Policy have been demonstrated using the innovative concept, introduced before, called systemability. The aim of this chapter is introducing the systemability concept in the maintenance policies design, in order to show its benefits, because it also considers the environmental conditions in which the systems operate.

The purpose of this chapter is evaluating the application of systemability function to the Age Replacement Policy and Periodic Replacement Policy and highlighting the benefits arisen with the use of this concept in comparison with the classical methodology.

The analytical behaviour of the total cost function and a useful algorithm have been introduced in order to help the practitioners to apply this innovative model. Furthermore, a sensitivity analysis of these benefits is conducted in order to show how the outcomes vary in function of the different environmental conditions, represented by systemability parameters $\alpha$ and $\beta$.

In conclusions the application of systemability permits the correct estimation of time-replacement and consequently the minimization of global maintenance cost UEC.

The results of this application have been illustrated with a series of graphics and summarized at the end.

Real industrial applications have demonstrated the relevance of this study, and shown about a 10-20% saving (about 5,000 €/years) using this methodology in comparison with the application of traditional replacement policy. In few words, if a preventive maintenance time is designed using the test data set, the mean major cost, due to the environmental effects, will be about 10-20% in comparison with the use of systemability on estimating the time to replacement.

The future research in this field will extend other maintenance policies using the systemability function in order to give a general overview. It will also investigated the behavior of the most used maintenance policies, varying the parameters of systemability function, which model the environmental effects.

Chapter 6 has introduced the study of the efficiency of a particular logistic system: the Automated Storage and Retrieval System (AS/RS). In this industrial application, the efficiency is strictly related to the availability and performance of system. In this chapter, the Overall Equipment Efficiency of AS/RSs has been investigated, using the guidelines indicated by the normative F.E.M. 9221.

In the last years, a new kind of AS/RS has been introduced in order to improve the efficiency of the whole system. In these kind of industrial application, the cranes have multishuttles. Usually a normal crane with a single-shuttle moves one load at the time, while a multishuttles crane can transport more than one load, typically two or three. In few words, this kind of system permits an improvement of efficiency because it eliminates the spare time during the command cycle, moving more than one load in a cycle.

It is very important to estimate also the performance of those systems, defined by the travel time. Then the mainly travel-time models, presented in scientific literature, are discussed and an innovative travel-time model is introduced based on the extension of F.E.M. 9851 normative and compared to other models.

The results of this application have been compared with the previous models presented in literature. Meller and Mungtwattana model is based on Bozer and White formulations, not considering the impact of acceleration/deceleration while De Puy model extends it using the formulas by Chang et al. in order to consider the acceleration and deceleration. Each model has been evaluated comparing it to the real values resulting from a Monte Carlo Simulation, over about 1500 different scenarios.

Generally, the Mean Relative Deviation (MRD) is about 1% for the model here introduced. This result is more conveniently compared to other methods. Meller and Mungtwattana model fits the data with a MRD of about 11%, while De Puy one, considering also the acceleration/deceleration, is characterized by a MRD of about 2%.

The future researches about innovative logistic systems will consider several aspects related to different movement policies and to the percentage of Single Dual Command and Quadruple Command to be used during the preliminary design of the system. Also the class based storage policy will be studied in the next model development.

In conclusion, this thesis has investigated the importance of correct estimation of reliability and performance, in different operating conditions, in order to define, monitory and improve the Overall Equipment Efficiency (OEE) of industrial and logistics plants.

Several innovative models have been introduced and their goodness has been evaluated, comparing to models presented in scientific literature.

The innovative characteristic of this research has been well demonstrated and its applications on industrial systems have been deeply investigated, well highlighting the great benefits of the models introduced in this thesis. Several future steps have been introduced in order to continue the research activity in this important field.

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# 9. Appendix I

## 9.1. Proof of Theorem T2 (Chapter 4).

Taking the second derivate of $E(T)$, given by (10):

- $\frac{\partial^2 E(T)}{\partial T^2} = H(T)\left[G(T) - C\right]$
- $H(T) = a * e^{-f(T)} * \left[f'(T)^2 - f''(T)\right]$
  $= a * e^{-bT^c} * \left[-bcT^{c-2} * (c - 1 - bcT^{c-1})\right]$
- $G(T) = C_4 * R(x \mid T) * (1 - S_x) * \left\{1 - \frac{a * (1 - S_x) * e^{-f(T)} * f'(T)}{\left[f'(T)^2 - f''(T)\right]}\right\}$
  $= C_4 * R(x \mid T) * (1 - S_x) * L(T)$

where $L(T)$ is as follows:

- $L(T) = \left\{1 - \frac{a * (1 - S_x) * e^{-f(T)} * f'(T)}{\left[f'(T)^2 - f''(T)\right]}\right\}$
- $H(T) \geq 0$

Then $\frac{\partial^2 E(T)}{\partial T^2} \geq 0$ where $\left[G(T) - C\right] \geq 0$.

$R(x \mid T) \geq 0$ and $\frac{\partial R(x \mid T)}{\partial T} \geq 0$, so $R(x \mid T)$ is strictly increasing, then $G(T)$ is strictly increasing if

- $\frac{\partial G(T)}{\partial T} = C_4 * \left(1 - S_x\right)$
  $* \left[\frac{\partial R(x \mid T)}{\partial T} * L(T) + R(X \mid T) * \frac{\partial L(x \mid T)}{\partial T}\right] \geq 0$

so if $L(T)$ is strictly increasing and $L(T) > 0$.

Studying $L(T)$, then $L(0) = 1$ and $L(\infty) = 1$.

Taking the first derivate of $L(T)$:

- $\frac{\partial L(T)}{\partial T} = a * (1 - S_x) * e^{-f(T)} * f'(T) * \frac{f(T)^2 - \frac{c-1}{c} * f(T) + \frac{c-1}{c}}{\left[f(T) - \frac{c-1}{c}\right]^2}$
- $\frac{\partial L(T)}{\partial T} \geq 0$ if $L_1(T) = f(T)^2 - \frac{c-1}{c} * f(T) + \frac{c-1}{c} \geq 0$.

The equation $L_1(T)$ has two roots but only one, called $T_{L1}$. is major than 0, then:

- $\frac{\partial L(T)}{\partial T} < 0$ for $0 < T < T_{L1}$

- $\frac{\partial L(T)}{\partial T} = 0$ for $T = T_{L1}$
- $\frac{\partial L(T)}{\partial T} > 0$ for $T > T_{L1}$

Then $L(T)$ is decreasing for $0 < T < T_{L1}$ and increasing for $T > T_{L1}$, and the behavior of $L(T)$ is as follows (see Figures 10.1 and 10.2):

- If $L(T_{L1}) < 0$, $\frac{\partial L(T)}{\partial T} \geq 0$ and $L(T) > 0$ for $T > T_2^G$, where $T_2^G : \inf\{T > T_{L1} : L(T) \geq 0\}$;
- If $L(T_{L1}) > 0$, $\frac{\partial L(T)}{\partial T} \geq 0$ and $L(T) > 0$ for $T > T_2^G$, where $T_2^G = T_{L1}$;

Then:

- if $T > T_2^G$, $G(T)$ is strictly increasing;
- if $T < T_2^G$, $G(T)$ could have any behavior.

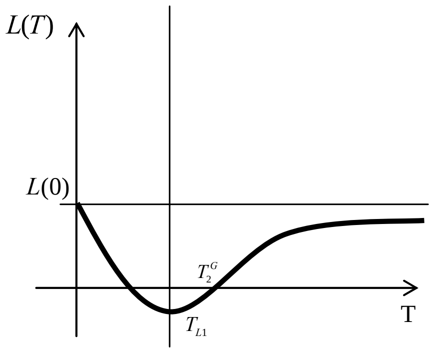


**Figure 9.1:** $L(T)$ **vs.** $T$ **for** $0 < c < 1$ **and** $L(T_{L1}) < 0$

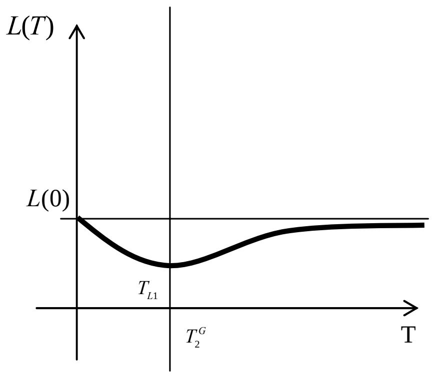


**Figure 9.2:** $L(T)$ **vs.** $T$ **for** $0 < c < 1$ **and** $L(T_{L1}) > 0$

So all the optimal solutions of the previous cases, have to be compared to any value of $E(T)$ for $T < T_2^G$. □

## 9.2. Proof of Theorem T3 (Chapter 4)

Taking the second derivate of $E(T)$, given by (10), it can be obtained:

- $$\frac{\partial^2 E(T)}{\partial T^2} = H(T)\left[G(T) - C\right]$$

- $$H(T) = a * e^{-f(T)} * \left[f'(T)^2 - f''(T)\right]$$
  $$= a * e^{-bT^c} * \left[-bcT^{c-2} * (c - 1 - bcT^{c-1})\right]$$

where:

- $H(T) < 0 \qquad 0 < T < T_3^H$, for $T_3^H = \left(\frac{c-1}{bc}\right)^{1/c}$

- $H(T) \geq 0 \qquad T \geq T_3^H$, for $T_3^H = \left(\frac{c-1}{bc}\right)^{1/c}$

Studying function $G(T)$:

- $$G(T) = C_4 * R(x \mid T) * (1 - S_x) * \left\{1 - \frac{a * (1 - S_x) * e^{-f(T)} * f'(T)}{\left[f'(T)^2 - f''(T)\right]}\right\}$$
  $$= C_4 * R(x \mid T) * (1 - S_x) * L(T)$$

and:

- $G(0) = C_4 * (1 - S_x) * e^{-a(1-S_x)}$
- $G(\infty) = C_4 * (1 - S_x)$
- $G(\infty) = G(0)$

Then $L(T)$ is as follows:

- $$L(T) = \left\{1 - \frac{a * (1 - S_x) * e^{-f(T)} * f'(T)}{\left[f'(T)^2 - f''(T)\right]}\right\}$$

Note that $L(0) = 1$ and $L(\infty) = 1$. Taking the first derivative of $L(T)$, it is given:

- $$\frac{\partial L(T)}{\partial T} = a * (1 - S_x) * e^{-f(T)} * f'(T) * \frac{f(T)^2 - \frac{c-1}{c} * f(T) + \frac{c-1}{c}}{\left[f(T) - \frac{c-1}{c}\right]^2}$$

- $\frac{\partial L(T)}{\partial T} \geq 0$ if $L_1(T) = f(T)^2 - \frac{c-1}{c} * f(T) + \frac{c-1}{c} \geq 0$.

The equation $L_1(T)$ has negative discriminant, then $\frac{\partial L(T)}{\partial T} \geq 0 \quad for \quad \forall T$.

Studying function $L(T)$, the singular point $T_3^H = \left(\frac{c-1}{bc}\right)^{1/c}$ is well noted. In this point, $L(T) = \pm\infty$, then the behavior of $L(T)$ is shown in Figure 10.3.

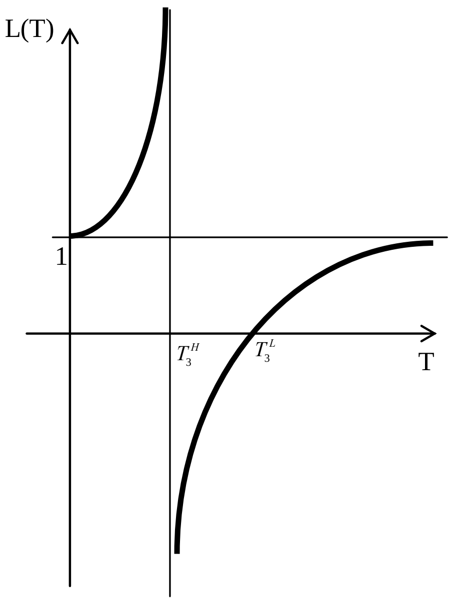


**Figure 9.3:** $L(T)$ **vs.** $T$ **for** $c > 1$.

Note that $R(x \mid T) \geq 0$ and $\frac{\partial R(x \mid T)}{\partial T} \geq 0$, so $R(x \mid T)$ is strictly increasing, and the formula of $G(T)$, the behavior of function $G(T)$ is as shown in Figure 10.4. For $T_3^H < T < T_3^G$, $G(T) < 0$, and it could have any behavior.

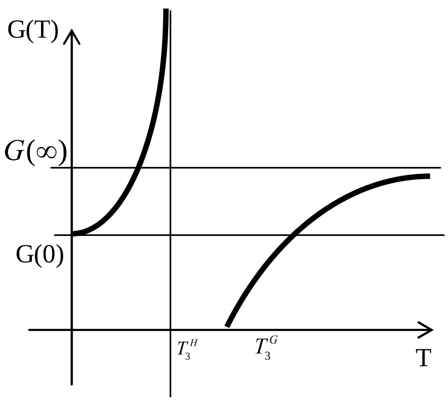


**Figure 9.4:** $G(T)$ **vs.** $T$ **for** $c > 1$.

Since $\frac{\partial^2 E(T_3^H)}{\partial T^2} < 0$, the following cases are given about the study of $\frac{\partial^2 E(T)}{\partial T^2} = H(T)\left[G(T) - C\right]$:

Case 3.1: if $G(\infty) < C$, it is given:

(a) $G(T) - C \le 0 \qquad 0 < T \le T_{G-1}$;

(b) $G(T) - C > 0 \qquad T_{G-1} < T < T_3^H$;

(c) $G(T) - C < 0 \qquad T > T_3^H$.

Considering the behavior of $H(T)$, $\frac{\partial^2 E(T)}{\partial T^2}$ is as follows:

- $\frac{\partial^2 E(T)}{\partial T^2} \ge 0 \qquad 0 < T \le T_{G-1}$;
- $\frac{\partial^2 E(T)}{\partial T^2} < 0 \qquad T > T_{G-1}$.

So, $\frac{\partial E(T)}{\partial T} = F(T)$ is increasing for $0 < T \le T_{G-1}$ and decreasing for $T > T_{G-1}$. Given $\frac{\partial E(0)}{\partial T} = \frac{\partial E(\infty)}{\partial T} = C_1$, then $\frac{\partial E(T)}{\partial T} > 0 \quad \forall T$. So $E(T)$ is strictly increasing, then $T^* = 0$ minimizes $E(T)$ (see Figures 10.5 – 10.7).

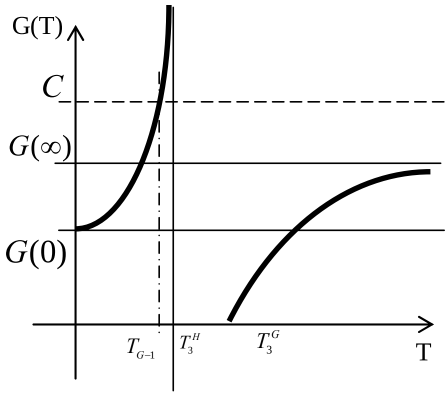


**Figure 9.5:** $G(T)$ **vs.** $T$ **for** $c > 1$ **and** $G(\infty) < C$

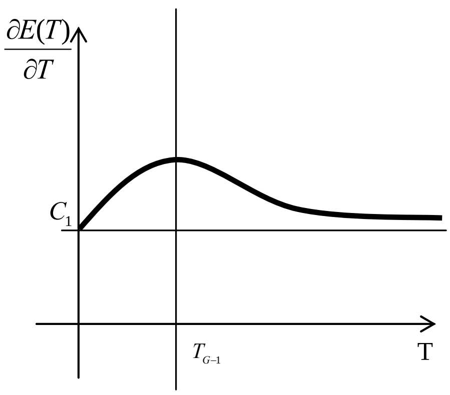


**Figure 9.6:** $\frac{\partial E(T)}{\partial T} = F(T)$ **vs.** $T$ **for** $c > 1$ **and** $G(\infty) < C$

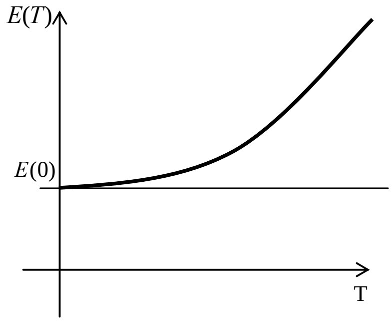


**Figure 9.7:** $E(T)$ **vs.** $T$ **for** $c > 1$ **and** $G(\infty) < C$

**Case 3.2:** if $G(0) < C < G(\infty)$, it is given:

(a) $G(T) - C \leq 0 \qquad 0 < T \leq T^1_{G-2}$;

(b) $G(T) - C > 0 \qquad T^1_{G-2} < T < T^H_3$;

(c) $G(T) - C \leq 0 \qquad T^H_3 < T \leq T^2_{G-2}$

(d) $G(T) - C > 0 \qquad T > T^2_{G-2}$.

Considering the behavior of $H(T)$, $\frac{\partial^2 E(T)}{\partial T^2}$ is as follows:

- $\frac{\partial^2 E(T)}{\partial T^2} \geq 0 \qquad 0 < T \leq T^1_{G-2}$;
- $\frac{\partial^2 E(T)}{\partial T^2} < 0 \qquad T^1_{G-2} < T < T^2_{G-2}$;
- $\frac{\partial^2 E(T)}{\partial T^2} \geq 0 \qquad T \geq T^2_{G-2}$.

So, $\frac{\partial E(T)}{\partial T} = F(T)$ is increasing for $0 < T \leq T^1_{G-2}$, decreasing for $T^1_{G-2} < T < T^2_{G-2}$ and increasing for $T \geq T^2_{G-2}$.

Given $\frac{\partial E(0)}{\partial T} = \frac{\partial E(\infty)}{\partial T} = C_1$, two subcases are defined:

*3.2.a)* if $\frac{\partial E(T^2_{G-2})}{\partial T} > 0$, then $\frac{\partial E(T)}{\partial T} > 0 \quad \forall T$. So $E(T)$ is strictly increasing, then $T^* = 0$ minimizes $E(T)$;

*3.2.b)* if $\frac{\partial E(T^2_{G-2})}{\partial T} < 0$, then:

- $\frac{\partial E(T)}{\partial T} \geq 0 \qquad 0 < T \leq T_{3-a}$;
- $\frac{\partial E(T)}{\partial T} < 0 \qquad T_{3-a} < T < T_{3-b}$;
- $\frac{\partial E(T)}{\partial T} \geq 0 \qquad T \geq T_{3-b}$.

So, $E(T)$ has a minimum at $T = T_{3-b}$. If $E(0) > E(T_{3-b})$, $T^* = T_{3-b}$ minimizes $E(T)$, otherwise $T^* = 0$ minimizes $E(T)$ (see Figures 10.8 – 10.13).

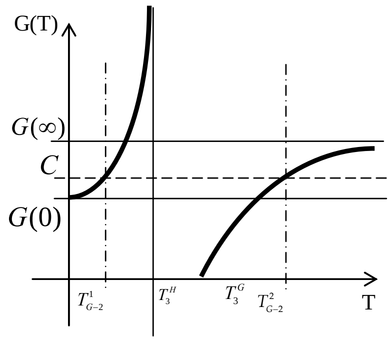


**Figure 9.8:** $G(T)$ **vs.** $T$ **for** $c > 1$ **and** $G(0) < C < G(\infty)$

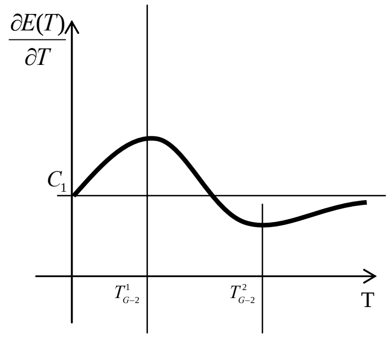


**Figure 9.9:** $\frac{\partial E(T)}{\partial T} = F(T)$ **vs.** $T$ **for** $c > 1$ **and** $G(0) < C < G(\infty)$ **and** $\frac{\partial E(T_{G-2}^2)}{\partial T} > 0$ **.**

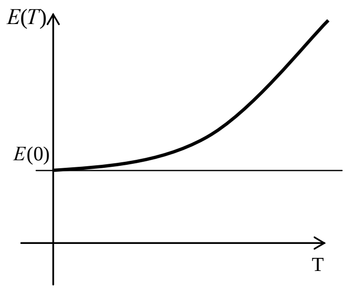


**Figure 9.10:** $E(T)$ **vs.** $T$ **for** $c > 1$ **and** $G(0) < C < G(\infty)$ **and** $\frac{\partial E(T_{G-2}^2)}{\partial T} > 0$ **.**

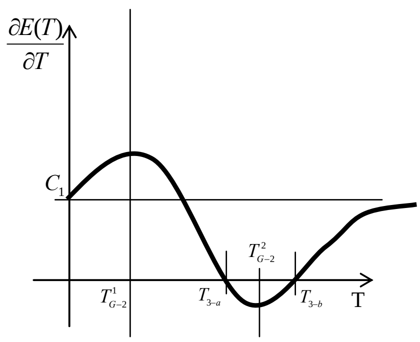


**Figure 9.11:** $\frac{\partial E(T)}{\partial T} = F(T)$ **vs.** $T$ **for** $c > 1$ **and** $G(0) < C < G(\infty)$ **.**

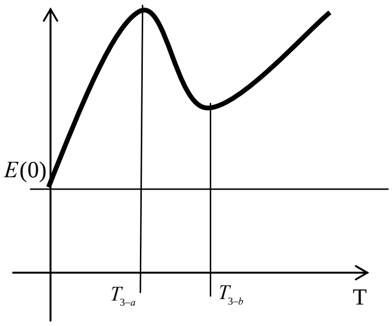


**Figure 9.12:** $E(T)$ **vs.** $T$ **for** $c>1$ **and** $G(0)<C<G(\infty)$ **and** $\frac{\partial E(T_{G-2}^{2})}{\partial T}>0$ **and** $E(0)>E(T_{3-b})$ **.**

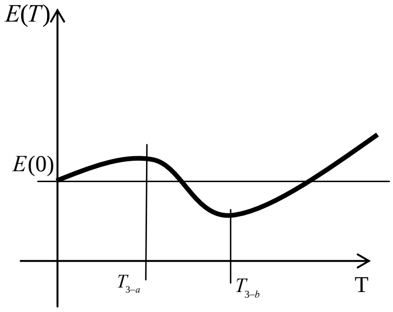


**Figure 9.13:** $E(T)$ **vs.** $T$ **for** $c>1$ **and** $G(0)<C<G(\infty)$ **and** $\frac{\partial E(T_{G-2}^{2})}{\partial T}>0$ **and** $E(0)<E(T_{3-b})$

**Case 3.3:** if $0<C<G(0)$, it is given:

(a) $G(T)-C>0 \quad 0<T<T_3^H$ ;

(b) $G(T)-C\le 0 \quad T_3^H<T\le T_{G-3}$

(c) $G(T)-C>0 \quad T>T_{G-3}$ .

Considering the behavior of $H(T)$, $\frac{\partial^2 E(T)}{\partial T^2}$ is as follows:

- $\frac{\partial^2 E(T)}{\partial T^2}<0 \quad 0<T<T_{G-3}$ ;
- $\frac{\partial^2 E(T)}{\partial T^2}\ge 0 \quad T\ge T_{G-3}$ .

So, $\frac{\partial E(T)}{\partial T}=F(T)$ is decreasing for $0<T<T_{G-3}$ and increasing for $T\ge T_{G-3}$. Given $\frac{\partial E(0)}{\partial T}=\frac{\partial E(\infty)}{\partial T}=C_1$, two subcases are defined:

*3.3.a)* if $\frac{\partial E(T_{G-3})}{\partial T}>0$, then $\frac{\partial E(T)}{\partial T}>0 \quad \forall T$ . So $E(T)$ is strictly increasing, then $T^*=0$ minimizes $E(T)$;

*3.2.b)* if $\frac{\partial E(T_{G-3})}{\partial T}<0$, then:

- $\frac{\partial E(T)}{\partial T} \geq 0 \qquad 0 < T \leq T_{3-c}$;
- $\frac{\partial E(T)}{\partial T} < 0 \qquad T_{3-c} < T < T_{3-d}$;
- $\frac{\partial E(T)}{\partial T} \geq 0 \qquad T \geq T_{3-d}$.

So, $E(T)$ has a minimum at $T = T_{3-d}$. If $E(0) > E(T_{3-d})$, $T^* = T_{3-d}$ minimizes $E(T)$, otherwise $T^* = 0$ minimizes $E(T)$ (see Figures 10.14 – 10.19).

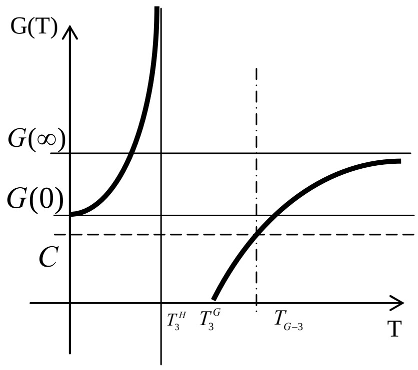


**Figure 9.14:** $G(T)$ **vs.** $T$ **for** $c > 1$ **and** $0 < C < G(0)$.

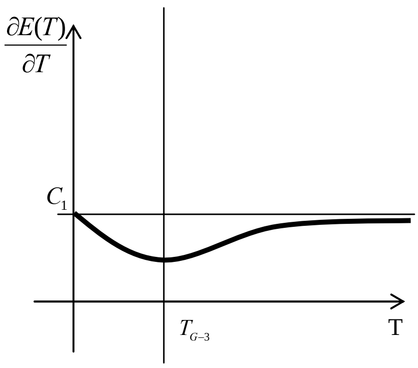


**Figure 9.15:** $\frac{\partial E(T)}{\partial T} = F(T)$ **vs.** $T$ **for** $c > 1$ **and** $0 < C < G(0)$ **and** $\frac{\partial E(T_{G-3})}{\partial T} > 0$.

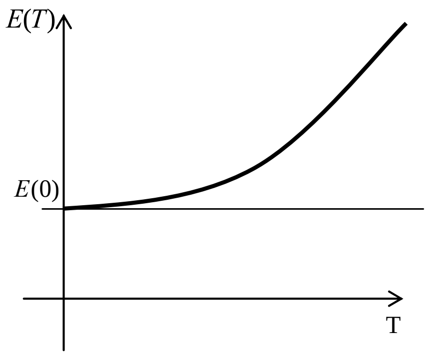


**Figure 9.16:** $E(T)$ **vs.** $T$ **for** $c > 1$ **and** $0 < C < G(0)$ **and** $\frac{\partial E(T_{G-3})}{\partial T} > 0$.

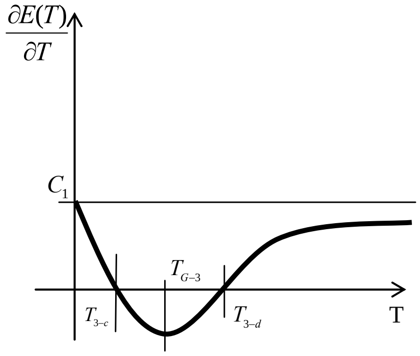


**Figure 9.17:** $\frac{\partial E(T)}{\partial T} = F(T)$ **vs.** $T$ **for** $c > 1$ **and** $0 < C < G(0)$ **and** $\frac{\partial E(T_{G-3})}{\partial T} < 0$.

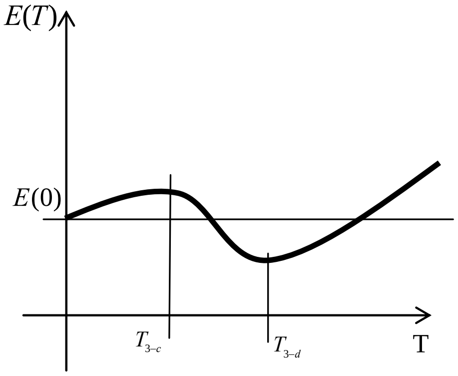


**Figure 9.18:** $E(T)$ **vs.** $T$ **for** $c > 1$ **and** $0 < C < G(0)$ **and** $\frac{\partial E(T_{G-3})}{\partial T} < 0$ **and** $E(0) > E(T_{3-d})$

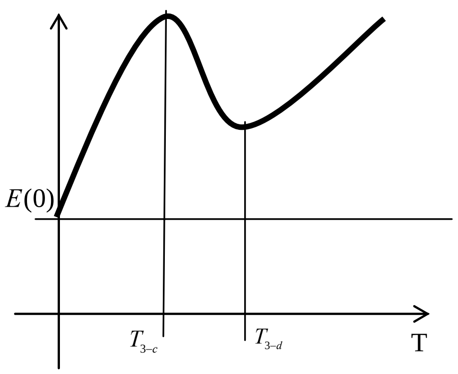


**Figure 9.19:** $E(T)$ **vs.** $T$ **for** $c > 1$ **and** $0 < C < G(0)$ **and** $\frac{\partial E(T_{G-3})}{\partial T} < 0$ **and** $E(0) < E(T_{3-d})$

**Case 3.4:** if $C < 0$, the behavior of $\frac{\partial^2 E(T)}{\partial T^2}$, for $T_3^H < T < T_3^G$, where $G(T) < 0$, is unknown, then $T^* = \{T \in (0, \infty) : \min[E(T)]\}$ minimizes $E(T)$. □