Perfected science and the knowability paradox

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1 Introduction

In *The Limits of Science* [5], N. Rescher embraces a logical argument known as the *Knowability Paradox*, according to which, if every true proposition is knowable, then every true proposition is known, i.e., if there are unknown truths, there are unknowable truths. Rescher argues that the paradox, providing evidence of a limit of our knowledge (the existence of unknowable truths), could be used for arguing against perfected science.

In this article, we present two criticisms of Rescher's argument. The first one points out that Rescher is ambiguous on the meaning of "impossibility of a perfected science": it could be interpreted in at least two different ways, one of which is plainly unproblematic compared with the *Knowability Paradox*. In the second criticism, we argue that the kind of unknowability involved in the paradox is semantic, rather than epistemic. Therefore, it is not a real problem for science. The final conclusion of the paper is, if our criticisms are correct, that the paradox leaves open the possibility of a perfected science.

The paper is divided into three parts. In the first one, we give an account of the paradox and our reading of Rescher's argument. In the second and third parts, we point out our criticisms. If our arguments are correct, Rescher's conclusion, according to which the *Knowability Paradox* constitutes a problem for perfected science, is mistaken.

2 The *Knowability Paradox* and Rescher's argument for the imperfectibility of science

N. Rescher, in *The Limits of Science*, argues that "perfected science is a mirage; complete knowledge a chimera" [5, p. 150]. The above thesis is a consequence of the *Knowability Paradox*, a logical argument published by F. Fitch in an article entitled *A Logical Analysis of Some Value Concepts.*[1] Fitch's argument, starting from the assumption that every true proposition is knowable, reaches the strong conclusion that every true proposition is known or, in different terms: if there are unknown truths, there are unknowable truths. *Prima facie*, this argument seems to seriously narrow our epistemic possibilities and to constitute a limit for knowledge in general, for scientific knowledge in particular. The argument runs as follows: take the epistemic

[1] [2]. For an introduction to the literature about the argument, see [1] and [3].

operator K, where Kp stands for "someone knows that p" or "it is known that p",[2] and "p" is a proposition in a formal language.

Assume the following two properties of knowledge:

1. the distributive property over conjunction (Dist), i.e., if a conjunction is known, then also its conjuncts are, and

2. the factivity of knowledge (Fact), i.e., if a proposition is known, then it is true.

Formally:

$$K(p \wedge q) \vdash Kp \wedge Kq \tag{Dist}$$

$$Kp \vdash p \tag{Fact}$$

Assume the following two unremarkable modal claims, which can be formulated using the usual modal operators $\Diamond$ (which is read "it is possible that") and $\Box$ (which is read "it is necessary that"). The first is the *Rule of Necessitation*:

$$if \ \vdash p,\ then\ \Box p \tag{Nec}$$

The second rule establishes the interdefinability of the modal concepts of necessity and possibility:

$$\Box\neg p \dashv\vdash \neg\Diamond p \tag{ER}$$

Assume also the *Knowability Principle*, according to which every true proposition is knowable, formally:

$$\forall q(q \rightarrow \Diamond Kq) \tag{KP}$$

Finally, assume that we are not omniscient, i.e., there is at least a truth that is not known:

$$\exists r(r \wedge \neg Kr) \tag{NO}$$

an instantiation of (NO) is:

$$(p \wedge \neg Kp) \tag{2}$$

Consider an example of (KP) resulting by the substitution of q with (2):

$$((p \wedge \neg Kp) \rightarrow \Diamond K(p \wedge \neg Kp)) \tag{3}$$

By (2) and (3), we obtain:

$$\Diamond K(p \wedge \neg Kp) \tag{4}$$

Consider the following argument "per absurdum" (independent from (2)–(4)):

(5)	$K(p \wedge \neg Kp)$	[assumption]
(6)	$Kp \wedge K\neg Kp$	[by (5) and (Dist)]
(7)	$Kp \wedge \neg Kp$	[applying (Fact) to (6)]
(8)	$\neg K(p \wedge \neg Kp)$	[by (5)–(7), refusing (5) for the inconsistency of (7)]
(9)	$\Box\neg K(p \wedge \neg Kp)$	[by (8) and (Nec)]
(10)	$\neg\Diamond K(p \wedge \neg Kp)$	[by (9) and (ER)]

[2] Kp is commonly generalized at every subject and time: "someone knows at some time that p". For the purposes of our paper, the chosen reading of Kp is irrelevant.

(10) is inconsistent with (4).[3] If so, (NO) and (KP) are incompatible. One of the two assumptions must be abandoned. The advocate of the view that all truths are knowable must negate (NO):

$$\neg\exists r(r \wedge \neg Kr) \qquad \text{(Not-NO)}$$

according to (Not-NO), there are not unknown truths, i.e., every truth is known:

$$\forall r(r \rightarrow Kr) \qquad \text{(Not-NO*)}$$

Otherwise, one must negate (KP):

$$\neg\forall q(q \rightarrow \Diamond Kq) \qquad \text{(Not-KP)}$$

obtaining that there are unknowable truths:

$$\exists q(q \wedge \neg\Diamond Kq) \qquad \text{(Not-KP*)}$$

"This argumentation shows that in the presence of (relatively unproblematic) principles [(Dist)–(Fact)], the thesis that all truths are knowable [(KP)] entails that all truths are known, that is, [(Not-NO*)]. Since the latter thesis is clearly unacceptable, the former must be rejected. We must concede that some truths are unknowable: $\exists q(q \wedge \neg\Diamond Kq)$" [5, p. 150].[4]

Rescher points out that "No doubt this sort of argumentation for the incompleteness of knowledge is too abstract [...] to carry much conviction in itself. But it does provide some suggestive stage setting for the more concrete rationale of the imperfectibility of science" [5, p.150].[5]

Rescher's argument for the imperfectibility of science could be analyzed in the following way:

if the *Knowability Paradox* holds, then there are unknowable truths (Ass.) (R1)

the *Knowability Paradox* holds (Ass.) (R2)

there are unknowable truths (Conclusion I) (R3)

if there are unknowable truths, then perfected science is impossible (Ass.) (R4)

perfected science is impossible (Conclusion II) (R5)

Rescher's argument (as it is here reformulated) is based on three different premises, (R1), (R2), and (R4). Here we are not interested in how correct (R1)–(R3) is: we just assume that Fitch's argument is sound.[6] Is Rescher's second part of the argument (R3)–(R5) correct?

[3]Here we have take the freedom of substituting the argument as it was originally proposed by Rescher with the equivalent, clearer, and commonly used formulation. See, for instance, [1].

[4]For a formal proof of $\exists q(q \wedge \neg\Diamond Kq)$, see [6].

[5]Routley in [6] considers the paradox in a more serious way, as an authentic limitation for knowledge in general. For articles related to Rescher's, see [7]and [10].

[6]There is a long list of criticisms of the paradox. For an introduction to the main literature, see [1].

3 First criticism: (R4) is ambiguous

The first problem of Rescher's argument is that (R4):

$$\text{If there are unknowable truths, then perfected science is impossible} \tag{R4}$$

is ambiguous. In particular, there are at least two meanings of "imperfectibility of science" — where the expression is here considered equivalent to "impossibility of a perfected science" — and one of them is plainly unproblematic compared with the paradoxical conclusion.

Consider (R4): if the existence of unknowable truths is a problem for the perfectibility of science, it seems reasonable to think that a perfected science is equivalent or at least implies an omniscient science.[7] The following is Rescher's train of thought: if there are unknowable truths, scientific omniscience is impossible, and a perfected science is impossible, too.

Here an ambiguity rises. What does it mean that "omniscience is impossible"? We could read it as:

$$\text{It is impossible that every true proposition is known} \tag{IO1}$$

Formally:

$$\neg\Diamond\forall q(q \to Kq) \tag{IO1}$$

But we could also read "omniscience is impossible" as:

$$\text{not every true proposition is knowable.} \tag{IO2}$$

Formally:

$$\neg\forall q(q \to \Diamond Kq) \tag{IO2}$$

Are (IO1) and (IO2) both implied by the paradox? If there are unknown truths, the result of the paradox (according to Rescher) is the negation of the Knowability Principle: (Not-KP) $\neg\forall q(q \to \Diamond Kq))$ and (Not-KP) is (IO2). So, (IO2) is the proper conclusion of the paradox.

What about (IO1)? Let us first note that the result of the paradox is that (NO) and (KP) are incompatible. The result of the paradox can be summarized in the following theorem:

$$\vdash \exists q(q \wedge \neg Kq) \to \neg\forall q(q \to \Diamond Kq) \tag{T1}$$

Furthermore, notice also that the converse of (T1) can be easily demonstrated; in fact, by the principle that what is actual is possible, we obtain:

$$\vdash \forall q(q \to Kq) \to \forall q(q \to \Diamond Kq). \tag{T2}$$

which is provably equivalent to:

$$\vdash \neg\forall q(q \to \Diamond Kq) \to \exists q(q \wedge \neg Kq) \tag{T3}$$

[7] Here with *omniscience* we do not mean a property of a subject, i.e. the property of possessing an effective knowledge of every truth. Rather, *omniscience* is specifically referred to science: an omniscient science is a science having the means to acquire the knowledge of every truth.

(T1) and (T3) validate the following theorem:

$$\vdash \exists q(q \wedge \neg Kq) \leftrightarrow \neg\forall q(q \rightarrow \Diamond Kq)) \tag{T}$$

If (T) is a theorem, by applying the *Rule of Necessitation* to (T), we obtain:

$$\vdash \Box(\exists q(q \wedge \neg Kq) \leftrightarrow \neg\forall q(q \rightarrow \Diamond Kq)) \tag{TN}$$

Now, notice that (NO) $\exists r(r \wedge \neg Kr)$ — the non-omniscience thesis — is the result of a commonsensical observation according to which, *de facto*, actually there are true propositions that we do not know. It is not a logical principle of the paradox, nor it is introduced through a logical argument.[8] If it is so, (NO) is contingently true, i.e., it is possibly false:

$$\Diamond\neg\exists r(r \wedge \neg Kr) \tag{CNO}$$

But, by (CNO), (TN), and the modal rule ($\Diamond A$, $\Box(A \leftrightarrow B) \vdash \Diamond B$), we obtain the following:

$$\Diamond\forall q(q \rightarrow \Diamond Kq) \tag{CIO2}$$

(IO2) is contingent, that is, it is possibly false. By (CIO2), (TN), and the modal rule ($\Diamond A$, $\Box(A \leftrightarrow B) \vdash \Diamond B$) it is easy to derive (Not-IO1):

$$\Diamond\forall q(q \rightarrow Kq) \tag{Not-IO1}$$

Summarizing: we assumed that (NO) is only contingently true (i.e., it is possible that it is false). If (NO) is only contingently true, (IO2) is contingently true, too. But if (IO2) is only contingently true, then (IO1) is false. So (IO1) is false. Accepting the contingency of (NO), Fitch's paradox does not imply (IO1), Fitch's paradox implies the negation of (IO1). With this result at hand, let us return to our considerations concerning (R4); given the two readings of "omniscience is impossible", there are two corresponding readings of "imperfectibility of science":

Assuming (IO1) $\neg\Diamond\forall q(q \rightarrow Kq)$ (it is logically impossible that every truth is known), the imperfectibility of science is equivalent to the logical impossibility of a perfected science.

Assuming (IO2) $\neg\forall q(q \rightarrow \Diamond Kq)$ (Not every truth is knowable), and the actual, contingent existence of unknown truths, the imperfectibility of science is equivalent to the actual unrealizability of a perfected science.

The *Knowability Paradox* is an argument only for the second reading. It is not an argument for the first one. If it is so, Rescher's premise (R4):

if there are unknowable truths, then perfected science is impossible (R4)

is ambiguous.

[8]As C. Wright writes in [9], (NO) says just that p is true and not actually known. Furthermore, if (NO) were necessarily true, (Not-KP) would be easily proved by it, without the necessity of introducing Fitch's argument.

Rescher does not seem to be aware of the above specified distinction, and for this reason he falls in the mentioned ambiguity: the *Knowability Paradox* is an argument for (IO2) and the actual unrealizability of a perfected science, but it is not an argument for (IO1) and the logical impossibility of a perfected science.

4 A second criticism: an incorrectness in Rescher's argument

The second problem of Rescher's argument is that, given

there are unknowable truths (Conclusion I) (R3)

and

if there are unknowable truths, then perfected science is impossible (Ass.) (R4)

the conclusion

perfected science is impossible (Conclusion II) (R5)

is misleading.

The mistake is due to the fact that Rescher does not take into consideration the special *status* of the propositions that lead to the paradox. The propositions resulting unknowable by the paradox, as the *reductio* (5)–(8) in the paradox showed, are instances of (NO): e.g., (2) $(p \wedge \neg Kp)$. But why are those propositions unknowable?

First of all, let us distinguish between two different kinds of unknowability. A true proposition could be unknowable because of some epistemic limits. Take, for example, Heisenberg's indetermination principle: according to a certain interpretation, the principle seems to give an ineliminable epistemic limit to human knowledge. On the other hand, a different kind of unknowability is just based on semantic considerations: the unknowability of a proposition could result just from its meaning. In the last case, there are no effective limits to our (scientific) knowledge.

Consider the proposition:

perfected science is unrealized (S)

(S) jeopardizes the realization of perfected science: if it is true, then it is false that perfected science is realized. But the reason for such unrealizability is not ascribable to an epistemic limit. It is simply a semantic consequence of the logical law that a proposition is incompatible with its negation (the *law of non-contradiction*): (S) is incompatible with:

perfected science is realized (S*)

(S*) is false simply because (S) is true. Propositions that lead the paradox emerge, and have the logical form (2), present the same sort of problem. This semantic phenomenon has been studied in the literature, and some authors have called this kind of propositions "blindspots":[9] $(p \wedge \neg Kp)$ is unknowable just because it is a conjunction of two propositions, p and $\neg Kp$, and the knowledge of the first conjunct implies the falsity of the second one just for semantic reasons: "it is known that p" is trivially incompatible with "it is not known that p". Notice that the paradox does not concern the knowability of each conjunct in (2). Each one is independently knowable, whereas their contemporary knowledge is impossible, for the demonstrated semantic reason. If it is so, the problem of the paradox is strictly semantic, not epistemic: it does not concern any specific area of science or, more generally, human epistemic skills.

In light of the above explanation, Rescher's unproblematic acceptance of (R4):

if there are unknowable truths, then perfected science is impossible (R4)

is mistaken. The unknowability problematic for science, referred to by Rescher in the antecedent of (R4), is an epistemic one: given our epistemic limits perfected science is impossible. On the other hand, the unknowability resulting in the paradox, assumed in (R3), is of the semantic kind. So, the conclusion of the paradox is not the intended premise of (R4), and from (R3) and (R4) we cannot infer (R5):

perfected science is impossible (Conclusion II) (R5)

5 Conclusion

To conclude: if our criticisms are satisfactory, Rescher's argument according to which the *Knowability Paradox* constitutes a limit for perfected science, is ambiguous and mistaken. Specifically, the *Knowability Paradox* cannot be used — as Rescher has — as an argument for the imperfectibility of science. The final conclusion of our paper is that the paradox leaves open the possibility of a perfected science.

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BIBLIOGRAPHY

[1] B. Brogaard and J. Salerno. Fitch's paradox of knowability. In E. Zalta (ed.), *The Stanford Encyclopedia of philosophy* (Summer 2004 edition), 2008.
[2] F. Fitch. A logical analysis of some value concepts. *The Journal of Symbolic Logic*, 28: 135–142, 1963.
[3] J. Kvanvig. *The Knowability Paradox*. Oxford University Press, Oxford 2006.
[4] B. Linsky. Factives, Blindspots and Some Paradoxes. *Analysis*, 46: 10–15, 1986.
[5] N. Rescher. *The Limits of Science*. University of California Press, Berkeley 1984.

[9] See, for instance, [8] and [4].

[6] R. Routley. Necessary limits to knowledge: unknown truths. In E. Morscher *et al.* (eds.), *Essays in Scientific Philosophy*, Bad Reichental, 1981, pp. 93–113 (reprinted in a special issue of *Synthese* edited by J. Salerno (forthcoming)).
[7] N.G. Schlesinger. On the limits of science. *Analysis*, 46: 24–26, 1986.
[8] R.A. Sorensen. *Blindspots.* Clarendon-Press, New York 1988.
[9] C. Wright. Truth as sort of epistemic: Putnam's peregrinations. *Journal of Philosophy*, 97 (6): 335–364, 2000.
[10] E.M. Zemach. Are there logical limits for science? *The British Journal for the Philosophy of Science*, 38 (4): 527–532, 1987.