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## Logically Unknowable Propositions: a criticism to Tennant's three-partition of Anti-Cartesian propositions.

## Abstract

The *Knowability Paradox* is a logical argument that, starting from the plainly innocent assumption that every true proposition is knowable, reaches the strong conclusion that every true proposition is known; i.e. if there are unknown truths, there are unknowable truths. The paradox has been considered a problem for every theory assuming the *Knowability Principle*, according to which *all truths are knowable* and, in particular, for semantic anti-realist theories.

A well known criticism to the *Knowability Paradox* is the so called restriction strategy. It bounds the scope of the universal quantification in (KP) to a set of formulas whose logical form avoids the paradoxical conclusion. Specifically, Tennant suggests to restrict the quantifier in (KP) to propositions whose knowledge is provably inconsistent. He calls them Anti-Cartesian propositions and distinguished them in three kinds. In this paper we will not be concerned with the soundness of the restriction proposal and the criticisms it has received. Rather, we are interested in analyzing the proposed distinction.

We argue that Tennant's distinction is problematic because it is not completely clear, it is not grounded on an adequate logical analysis, and it is incomplete. We suggest an alternative distinction, and we give some reasons for accepting it: it results logically grounded and more complete than Tennant's one, inclusive of it and independent from non-epistemic notions.

## Logically Unknowable Propositions: a criticism to Tennant's three-partition of Anti-Cartesian propositions.

## Introduction

The *Knowability Paradox* is a logical argument that, starting from the plainly innocent assumption that every true proposition is knowable, reaches the strong conclusion that every true proposition is known. The same conclusion could also be put as follows: if there are unknown truths, there are unknowable truths. The paradox has been considered a problem for every theory assuming the *Knowability Principle*, according to which *all truths are knowable* (formally: (KP)  $\forall q (q \rightarrow \Diamond Kq)$ ) and, in particular, for semantic antirealist theories.

A well known criticism to the *Knowability Paradox* is the so called restriction strategy. It bounds the scope of the universal quantification in (KP) to a set of formulas whose logical form avoids the paradoxical conclusion. Specifically, Tennant suggests to restrict the quantifier in (KP) to propositions whose knowledge is provably inconsistent. He calls them Anti-Cartesian propositions and distinguished them in three kinds. In this paper, we will not be concerned with the soundness of the restriction proposal, and the criticisms it has received. Rather, we are interested in analyzing the proposed distinction.

We argue that Tennant's distinction is problematic because it is not grounded on an adequate logical analysis, and it is incomplete. We suggest an alternative distinction, and we give some reasons for accepting it: it results logically grounded and more complete than Tennant's one, inclusive of it, and independent from non-epistemic notions.

## I. The Knowability Paradox

The *Knowability Paradox* is a logical argument published by F. Fitch in an article entitled *A Logical Analysis of Some Value Concepts* in 1963<sup>1</sup>.

It runs as follows. Adopt a standard system of modal logic with an epistemic operator K, where Kp stays for "someone knows that p"; "it is known that p" and 'p' is a proposition.

Assume the validity of two properties of knowledge: 1) distributivity over conjunction (if a conjunction is known, then also its conjuncts are), and 2) factivity (if a proposition is known, then it is true). Formally:

Dist) K(p & q) - Kp & Kq

<sup>&</sup>lt;sup>1</sup> Fitch (1963). For a detailed account of the argument and of the literature about it see Broogard & Salerno (2002) and Kvanvig (2006).

Fact) Kp - p

Assume the following common modal inferences:

Nec) if |-p|, then  $\Box p$ 

ER)  $\Box \neg p \models \neg \Diamond p$ 

Assume also the *Knowability Principle*:

 $\text{KP}) \forall q (q \rightarrow \Diamond Kq)$ 

Finally, assume that we are not omniscient, i.e. there is at least a truth that is not known, and instantiate an example of it; formally:

2) p & ¬Kp

Consider an example of (KP) resulting by the substitution of q in (KP) with (2):

3)  $p \& \neg Kp \rightarrow \Diamond K(p \& \neg Kp)$ 

By (2) and (3) we obtain:

4) ◊K(p & ¬Kp)

Consider the following argument *per absurdum* (independent from (2)-(4)):

assumption
by (5) and (Dist)
applying (Fact) to (6)
by (5)-(7), negating (5)
by (8) and (Nec)
by (9) and (ER)

(4) is inconsistent with (10). If so, (2) and (KP) are incompatible.

There are (at least) two strategies one might adopt in order to avoid the paradox: she could either negate (2) or (KP). If she negates (2), p &  $\neg$ Kp, that is equivalent to:

11)  $p \rightarrow Kp^2$ 

 $<sup>^{2}</sup>$  (11) is classically but not intuitionistically deducible by the negation of (2). Some criticisms against the paradox has been carried on this line. See, for example, Williamson (1982).

If every truth is knowable ((KP)) then every truth is also actually known (from (11) by generalization). But the last thesis is clearly indefensible. So, in order to avoid the contradiction, (KP) must be negated, i.e:

12) 
$$\neg \forall p (p \rightarrow \Diamond Kp)$$

informally: Not every true proposition is knowable.

## II. The restriction strategies and Tennant's Anti-Cartesian propositions

A seemingly weak demand of the so called semantic anti-realism is that every truth must be knowable, i.e. the *Knowability Principle* (KP) is true. But, as we have seen in the previous section, according to the *Knowability Paradox* if one accepts (KP), she must accept that all truths are known, a very demanding conclusion. So, the *Knowability Paradox* seems to be a hard challenge for semantic anti-realism.

One way to reply is to deny that the anti-realist commitment to the epistemic character of truth involves any commitment to the *Knowability Principle*. Many philosophers have followed this train of thought, suggesting a solution of the paradox called *restriction strategy*. It bounds the scope of the universal quantification in (KP) to a set of formulas whose logical form avoids the paradoxical conclusion.

One of the most famous and debated proposals is Neil Tennant's one in *The Taming of the True* (Tennant (1997), pp. 272 - 276).<sup>3</sup> His restriction is based on the distinction between Cartesian and Anti-Cartesian propositions. Propositions whose corresponding knowledge claims are consistent will be called Cartesian:

Kp | not-⊥

By contrast, every proposition whose knowledge is provably inconsistent is an Anti-Cartesian<sup>4</sup> one:

# Kp ⊢ ⊥

Tennant proposed the distinction between Cartesian and Anti-Cartesian propositions for formulating a restricted principle allowing to escape the latter ones from quantification. The restricted principle is the following one:

(KCP)  $q \rightarrow \Diamond Kq$ , where "q" is Cartesian

<sup>&</sup>lt;sup>3</sup> Other well known restrictions are those of M. Dummett (2001) and D. Edgington (1985). For a short list see Kvanvig (2008).

<sup>&</sup>lt;sup>4</sup> J. Hintikka's *epistemically indefensible* propositions (Hintikka (1962b)), Routley's *unknowable propositions* (Routley (1981)), Soerensen's *epistemic blindspots* (Soerensen (1988)) are characterizations similar to Tennant's ones.

His restriction avoids the paradox because the second assumption in the argument, (2) p &  $\neg$ Kp, is an Anti-Cartesian proposition, since its knowledge is provably inconsistent (from the *reductio* in the paradox, steps (5)-(7)). Then, we cannot substitute (2) for q in the restricted *Knowability Principle* (KCP). The first step in the argument:

3)  $(p \& \neg Kp) \rightarrow \Diamond K(p \& \neg Kp)$ 

is incorrect and the paradox is stopped at the beginning.

A common criticism to Tennant's strategy is that it is  $ad-hoc^5$ . It seems to be correct because there is not a clear reason for excluding Anti-Cartesian propositions from the quantification inside the *Knowability Principle*, except that of escaping the paradox. This solution seems obviously unprincipled. Tennant has tried to answer to this criticism.<sup>6</sup> However, here we are not concerned with Tennant's restriction and its problems. We are more interested in the distinction given by Tennant of the Anti-Cartesian propositions into different kinds.

According to Tennant,

«[t]here are three broad kinds of Anti-Cartesian proposition  $\phi$ , corresponding to the kind of reason why knowledge that  $\phi$  is impossible:

First, the proposition  $\phi$  itself may be inconsistent; whence the proposition that  $\phi$  is known will be inconsistent. So, for example, any compound proposition of the form ( $\phi \& \neg \phi$ ) is Anti-Cartesian.

Secondly, knowledge of a (consistent) proposition  $\phi$  may be impossible because the very act of considering or judging (falsely) that  $\phi$ requires the falsity of (some consequence of)  $\phi$ . A fortiori the proposition that  $\phi$  is known is inconsistent. It is in this way that the proposition that no thinkers exist is Anti-Cartesian.

Thirdly, the proposition that  $\phi$  is known may be logically inconsistent because of its own overall logical structure, involving iterations of K (and perhaps of other attitudes). Thus for any  $\phi$  the proposition ( $\phi \& \neg K \phi$ ) is such that *that* ( $\phi \& \neg K \phi$ ) *is known* turns out to be logically inconsistent [(as the *Knowability Paradox* shows)] [...]. That is, ( $\phi \& \neg K \phi$ ) is Anti-Cartesian» (Tennant (1997), pp 272 – 273).

Unfortunately, Tennant does not explain why he brings in these and only these kinds of propositions: he does not argue for his distinction, but he simply lists it. We have found only some hints for justifying it.

About the first kind of propositions, their inclusion in the list seems quite clear: every factive operator – as it is the case of the epistemic one – applied to an inconsistency implies the truthfulness of such inconsistency. But inconsistencies are necessarily false.

<sup>&</sup>lt;sup>5</sup> For this criticism see, for instance, Hand-Kvanvig (1999).

<sup>&</sup>lt;sup>6</sup> See, for example, Tennant (2001).

The reason motivating Tennant to introduce the second kind of Anti-Cartesian propositions is particularly unclear. Some authors have argued that these propositions are the *existentially inconsistent* ones of J. Hintikka (1962a)<sup>7</sup>. Hintikka defines this concept as follows:

«Let p be a sentence and a a singular term (e.g. a name, a pronoun, or a definite description). We shall say that p is *existentially inconsistent for the person referred to by a to utter* if and only if the longer sentence "p; and a exists" is inconsistent. [...] Uttering such a sentence [...] means making a statement which, if true, entails that its maker does not exist» (Hintikka (1962b), p. 11).

Tennant example "no thinkers exist" could be considered a case of existential inconsistency and this seems the reason advanced for its unknowability. Tennant seems to move away by every particular interpretation of the origin of the term "Cartesian" (he writes: «"Cartesian" has been chosen for convenience» (Tennant (1997) p. 273, footnote 25). However, given that the title of Hintikka's article is just "Cogito, Ergo Sum: Inference or Performance?", the term "Cartesian" could be considered a clue of the above interpretation.

About the third kind of Anti-Cartesian propositions, the Anti-Cartesianity of the logical form  $p \& \neg Kp$  has been deduced by the *Knowability Paradox*; Tennant includes this form in a range of propositions whose knowledge is inconsistent as a consequence of some iteration of K, but he does not point out other clarifying examples.

Given the above considerations, it seems that Tennant's distinction is not particularly accurate and not grounded on any sort of analysis. Moreover, it can be the target of criticisms pointing out its incompleteness. Take for example the following case: imagine there is a machine such that if one pushes on a button, the machine affects his memory and she forgets to have pushed on the button. <sup>8</sup> The proposition "I pushed on the button" is Anti-Cartesian. In fact, if known, it is true (by (Fact)); but if true it implies that it is unknown, because it has been forgotten. Therefore, if known, it is unknown: an inconsistency results. But the proposition is not in itself inconsistent (kind 1), nor it is a case of knowledge iteration (2) or of existential inconsistency (3). The proposition seems not to be considered in Tennant's distinction.

In conclusion, Tennant's distinction seems to be not well grounded and incomplete. In the next parts of the paper we give an analysis of the Anti-Cartesian propositions, and we use the results of the analysis for giving a new foundation of Anti-Cartesian propositions' distinction, able to settle the faults of Tennant's one.

<sup>&</sup>lt;sup>7</sup> For M. Hand and J. Kvanvig «[Anti-Cartesian proposition] might be the sort of proposition Descartes considered, such as *no thinking thing exists*. These propositions are consistent but existentially inconsistent [...] The idea of existential inconsistency is due to Jaakko Hintikka (1962a)» (Hand & Kvanvig (1999), p. 423).

<sup>&</sup>lt;sup>8</sup> The example is freely adapted from Egrè (undated).

We recall the definition of Anti-Cartesian proposition:  $Kp \models \bot$ . Our analysis will proceed as follows: 1) we take an operator K, where the properties of K are just distributivity and factivity.<sup>9</sup> 2) We assume Kp and one of the properties (for example, (Fact)). 3) We list all the propositions deducible in propositional logic by those assumptions. 4) We ask: how should the proposition p be for being Anti-Cartesian? And we give an answer: if p denies at least a proposition deduced from the assumptions, then from its knowledge an inconsistency is deducible, and so it is Anti-Cartesian. On the contrary, if from p is not deducible a negation of a proposition deduced by the assumptions, p is not Anti-Cartesian, because no inconsistency can be derived. 5) We repeat the procedure with the other property of K and with the two properties taken together. 6) We give recursively an analysis of complex propositions including the ones obtained in the previous analysis. Given this analysis, we obtain the propositions that should be derivable from p, for p being Anti-Cartesian.

Case 1. Consider just (Fact); propositions p such that  $p \models \neg Kp$  and  $p \models \neg p$  are Anti-Cartesian.

*Proof.* Assume Kp. From (Fact),  $Kp \rightarrow p$ , in propositional logic, by the common rules of inference, we can only deduce p, Kp (the assumption) or a disjunction including those propositions. From those premises the sub-cases to consider are: 1.1) Kp, 1.2) p, 1.3) p & Kp (the conjunction of case 1.1 and 1.2), 1.4) every disjunction including one of the above cases. We have that p is Anti-Cartesian,

- 1.1) if  $\neg$ Kp is derivable from p. Because, given the assumption Kp, we derive (Kp &  $\neg$ Kp), an inconsistency;
- 1.2) if ¬p is derivable from p. Because, deduced p from (Fact) and the assumed Kp, we derive (p & ¬p), an inconsistency;
- 1.3) if  $\neg(p \& Kp)$  is derivable from p. Because it would contradict the conjunction of the two cases listed above (1 and 2)
- 1.4) if the negation of a disjunction including one of the above propositions is derivable from p. In fact, the negation of a disjunction is the negation of each disjunct. But if from p is deducible the negation of one of the above propositions in conjunction with another proposition, from p is

<sup>&</sup>lt;sup>9</sup> A complete analysis of Anti-Cartesian propositions should consider all the logical properties owned by K, but here we will be concerned only with the above properties because otherwise the analysis would require much more space. However we think that our analysis could be considered, if not complete, at least very reliable, because Anti-Cartesianity seems to grow just from no more than these two properties. So we could suspect that these ones and not others are the responsible of such phenomenon. Our intention is to extend in the future the analysis to the complete axiomatization of some specific epistemic logic.

deducible the negation of one of the above propositions alone, for A & B  $\mid$  A.

Some remarks. In the first case we have a proposition p from which is deducible its own ignorance; p is such that:  $p \models \neg Kp$ . In the second case p implies its own negation,  $p \models \neg p$ , i.e. it is self-contradictory. Notice that this second kind of propositions includes inconsistencies (necessarily false propositions).

Case 2. From (Dist) alone, by the application of K to an arbitrary conjunction (say (p & q)) an inconsistency cannot be deduced<sup>10</sup>.

*Proof.* Assume K(p & q), where p & q is whatever conjunction (the same proof could be repeated with more than two conjuncts). From those premises we can only deduce Kp, Kq, Kp & Kq, or whatever disjunction including one of those propositions. So, from those premises the possible cases are: 2.1) Kp, 2.2) Kq, 2.3) Kp & Kq and 2.4) every disjunction including one of the above propositions.

We have that, assuming (Dist) alone, p or q (or both) cannot be Anti-Cartesian. In fact, note that whichever is the logical form of propositions p and q, an inconsistency cannot be deduced by (Dist) alone, because distributing K over propositions, whatsoever negation in p and q remains under the scope of K and we cannot have  $\neg$ Kp or  $\neg$ Kq. But, for contradicting Kp, Kq or their conjunction (cases 2.1-2.3), we request  $\neg$ (Kp & Kq), that is  $\neg$ Kp v  $\neg$ Kq, where the negation is out of the scope of the operator. So, by (Dist) alone, and cases 2.1-2.3, there are not Anti-Cartesian propositions.

Case 3. From (Dist) and (Fact) together, propositions with logical forms p & q, where  $q \models \neg p$ , or  $p \models \neg q$ , or  $q \models \neg Kp$ , or  $p \models \neg Kq$  are Anti-Cartesian.

*Proof.* Assume K(p & q), where p & q is whatever conjunction (the same proof could be repeated with more than two conjuncts). From those we can only deduce p, q by (Fact), Kp, Kq by (Dist), a conjunction of two or more of those propositions, and whatever disjunction including one of those ones. From those premises, the possible cases are: 3.1) p, 3.2) q, 3.3) Kp, 3.4) Kq, 3.5) p & q, 3.6) Kp & Kq, 3.7) Kp & p (Kq & q), 3.8) Kp & q (Kq & p), 3.9) longer conjunctions (cases explainable appealing to 3.5-3.8), 3.10) every disjunction including one of the above propositions (3.1-3.9).

We have that, p or q (or both) are Anti-Cartesian,

3.1) a) if  $\neg p$  is derivable from p. See 1.2.

b) if  $\neg p$  is derivable from q. In this case we have that p & q is inconsistent, entailing p &  $\neg p$ .

<sup>&</sup>lt;sup>10</sup> Notice that here we assume just (Dist). The point here is that Anti-Cartesianity doesn't emerge by (Dist) alone.

- 3.2) see 3.1 inverting the cases ((a) for (b) and *vice versa*).
- 3.3) a) if ¬Kp is derivable from p. See case 1.1.
  b) if ¬Kp is derivable from q. Because in this case we would have Kp (by dist) and ¬Kp (from q, by Fact).
- 3.4) see 3.3 inverting the cases.
- 3.5) a) if ¬(p & q) is derivable from p. That is ¬p or ¬q. The reason is shown taking together points 3.1 and 3.2.
  b) if ¬(p & q) is derivable from q. See 3.5a.
- 3.6) a) if ¬(Kp & Kq) is derivable from p. That is ¬Kp or ¬Kq. See points 3.3 and 3.4 together.
  b) if ¬(Kp & Kq) is derivable from q. See 3.6a.
- 3.7 a) if ¬(Kp & p) (¬(Kq & q)) is derivable from p. That is ¬Kp or ¬p (¬Kq or ¬q). See points 3.1a and 3.3a (3.1b, 3.3b) together.
  b) if ¬(Kp & p) (¬(Kq & q)) is derivable from q. See 3.7a.
- 3.8 a) if ¬(Kp & q) is derivable from p. That is ¬Kp or ¬q. See points 3.1 3.4.
  b) See 3.8a.
- 3.9) Conjunction of cases 3.1-3.8.
- 3.10) if the negation of a disjunction including one of the above propositions is derivable from p. See case 1.4.

Some remarks. Observe that, in 3., the cases of Anti-Cartesianity not reducible to a sub-case of case 1 or to cases 3.1 - 3.4, are only the cases 3.1b) q  $\mid \neg p$ , 3.2b) p  $\mid \neg q$ , 3.3b) q  $\mid \neg Kp$ , and 3.4b) p  $\mid \neg Kq$ . The first two cases are Anti-Cartesian because from a conjunct of p & q is deducible the contradictory of the other conjunct.

Notice that, if p & q is contradictory, (Fact) is sufficient to show its Anti-Cartesianity. In fact, as said in the explanation of case 1, Case 1.2) p  $\mid \neg p$ includes all self-contradictory propositions, including inconsistencies (necessarily false propositions). On the contrary, cases 3.3b) and 3.4b) are not deducible by (Fact) or (Dist) separately, but only if taken together. The logical form of the propositions responsible of the *Knowability Paradox*, p & ¬Kp, is an example of those cases: it is obtained substituting ¬Kp for q in the conjunction p & q (and, therefore, q  $\mid \neg$ Kp, case 3.3b).

At this point we have exhausted all the possible cases where K is a distributive and factive operator and Kp  $\vdash \perp$ . We can consider propositions

included in those cases as "basic" Anti-Cartesian ones, but for the list being complete, we need to show how those propositions behave if included in complex propositions. This job can be achieved recursively, showing if, how and in what cases the composition of "basic" Anti-Cartesian propositions results in "complex" Anti-Cartesian propositions.

#### Case 4. Complex cases.

4.K) If p is Anti-Cartesian also Kp is Anti-Cartesian. In fact, given (Fact), if Kp is Anti-Cartesian, it results in an inconsistency (by the definition of Anti-Cartesian). But an inconsistency is itself an Anti-Cartesian proposition (see 3.1b, 3.2b, or simply 1.2).

4.&) If p is Anti-Cartesian also p & q, for every q, is Anti-Cartesian. In fact, if p & q is known, from (Dist) we have that each conjunct is known (p included). But if p is Anti-Cartesian, from his knowledge an inconsistency follows. If in a conjunction one of the conjuncts is inconsistent, all the conjunction is inconsistent. So, knowing p & q implies inconsistency, i.e. p & q is Anti-Cartesian.

4.v) The disjunction's case is a bit more complex. In particular, we have examples of known disjunctions of Anti-Cartesian propositions. Take the example of a proposition that we know that is true or false, but we do not known whether it is true or false. In this case we have a proposition like: K(p &  $\neg$ Kp v  $\neg$ p &  $\neg$ K $\neg$ p). As the *Knowability Paradox* shows, knowledge of both the disjuncts of the above proposition gives inconsistency, but their disjunction is knowable. The same discourse is valid for other Anti-Cartesian propositions.

The only exception are propositions like 1.2)  $p \mid \neg p$ . Those propositions are necessarily false because self-contradictory; and a disjunction of inconsistencies is itself an inconsistency; but, by (Fact), we have that if a disjunction including inconsistencies is known, it is true, and this is contradictory. The same if we have a disjunction of Anti-Cartesian propositions including propositions like (1.2) and only one Anti-Cartesian proposition deriving from a different case. In fact, knowing a disjunction of *n* disjuncts where *n*-1 disjuncts are inconsistencies is the same as knowing the only possibly true disjunct; but we supposed that it is Anti-Cartesian. So knowledge of the whole disjunction gives an inconsistency, and, therefore, it is Anti-Cartesian.

At this point we have shown every case where a proposition is a "basic" or a "complex" Anti-Cartesian one. So, every Anti-Cartesian proposition could be reduced to one of the shown cases. In fact, we have considered for each property and for their conjunction all the possible cases in which any proposition, if known, is inconsistent. By (Dist) and (Fact) we could deduce only the basic cases 1-3, and recursively only the complex sub-cases in case 4. So, our analysis can be considered complete.

A noteworthy thing is that the complex cases obtained recursively in 4 and the other sub-cases in 1-3 are all reducible to three basic sub-cases: 1.1) p  $\mid \neg \text{Kp}$ , 1.2) p  $\mid \neg \text{p}$ , and 3.3b) q  $\mid \neg \text{Kp}$  (or equivalently 3.4b). Whatever Anti-Cartesianity is reducible to one of those basic cases. For example, every inconsistency can be reduced to 1.2, sentences like "no thinkers exist"<sup>11</sup> (assuming that thinking is a necessary condition for knowing) to case 1.1, and Moorean sentences ("p and it is not believed that p") to case 3.3b.<sup>12</sup> So, it seems that every case of logical unknowable proposition is (or is reducible to) a self-contradiction (1.2), or a proposition from which is derivable its own ignorance (1.1), or a conjunct from which is derivable the ignorance of the other conjunct (3.3b).

# *IV. A new distinction of the Anti-Cartesian propositions and a comparison with Tennant's distinction.*

In section II, we have introduced Tennant's distinction of Anti-Cartesian propositions. In section III we proposed our analysis of Anti Cartesian propositions. in this section we are going to introduce a new distinction based on such analysis. Then we suggest some reasons for preferring our distinction, by showing some advantages deriving by it.

Our distinction is a bipartition between propositions derivable by (Fact) alone (case 1), call them *kind I*, and propositions derivable by (Dist) and (Fact) together (case 3); let us call them *kind II*. Given this distinction, we can bring back each Anti-Cartesian proposition to one of these cases.<sup>13</sup>

In our opinion, there are at least three reasons for preferring it:

(\*) "there are not subjects"

<sup>&</sup>lt;sup>11</sup> As Hand & Kvanvig (1999) noted, the unknowability of those propositions faces semantic Anti-Realism with a further issue known as "Idealism Problem": if propositions like "no thinkers exist" are unknowable, and if knowability is a necessary condition of truth, then those propositions are necessarily false. The consequence is that it is impossible that no thinkers exist, that is it is necessarily true that there are thinking creatures.

<sup>&</sup>lt;sup>12</sup> Notice that whatever necessary condition for knowledge can be substituted with the K operator in 1.1 and 3.3b cases preserving Anti-Cartesianity. For example, admitting that the existence of subjects is a necessary condition for knowing, the proposition

is Anti-Cartesian. In fact, if there are no subjects there are not known propositions, and the same proposition (\*) is one of them. So, (\*) is a proposition included in the case 1.1, from which it is deducible its own ignorance. On the contrary, if we do not admit that the existence of subjects is a necessary condition for knowing, (\*) is perfectly knowable and not Anti-Cartesian. An analogous reasoning is valid with every other necessary property of knowledge. "no propositions are believed" and "p and it is not believed that p" are other examples of this kind.

<sup>&</sup>lt;sup>13</sup> Note that Kind I might be reduced to Kind II, because the analysis of (Fact) could be included inside that of both the properties, in fact in case 3 we have found repeated all the sub-cases of 1. But the distinction of the two kinds is not endangered by this: in fact our distinction is based on the sufficiency or insufficiency of (Fact) for having Anti-Cartesianity.

1) It is more grounded. Our distinction is based on an analysis that, although only sketched (and probably still incomplete), is grounded on theoretical bases (specifically, on the rule plaied by each property of knowledge in generating Anti-Cartesianity), whereas Tennant doesn't justify his distinction in any way.

2) It is inclusive of Tennant's one. We could settle Tennant's examples inside our frame and give an explanation of their unknowability:

- About inconsistencies. Even though they could be derived in two different ways (1.2 and 3.1b, 3.2b), we include them into *kind I*. Our distinction is able to show that (Fact) is sufficient for explaining their Anti-Cartesianity.

- Propositions whose form is  $(p \& \neg Kp)$  are included into *kind II*. Their form is a particular case of (p & q), where  $q \models \neg Kp$ .

- About "No thinkers exist", the advantage of our distinction over Tennant's one is clear. We should not recall Hintikka's notion of existential inconsistency for considering the above proposition unknowable. Existential inconsistency is not the reason of logical unknowability (Anti-Cartesianity) of the above proposition, but *vice versa* the existentially inconsistent proposition is unknowable because, on condition that the existence of thinkers is necessary for knowing, it is a proposition p such that  $p \models \neg Kp$  (sub-case 1.1). A check of the validity of this explanation is the fact that if we imagine (per absurdum) that knowing is possible without existing, then existential inconsistent propositions are dead knowable.

Given our analysis, the references to concepts like Hintikka's existential indefensibility in explaining Anti-Cartesianity seems to be misleading. Anti-Cartesianity grows up just by knowledge and its properties, and other propositions like the existential indefensible ones are indirectly unknowable, only in so far as they are logically unknowable (i.e. reducible to the exploited cases). In this sense, our analysis is independent from non-epistemic notions.

3) It is more complete. Our distinction can include examples of propositions that in Tennant's distinction wouldn't find a place or would stay in the middle between two kinds: the above reported problematic "button-sentence" is an example of the former case. The proposition "I pushed on the button", given that if one pushes on it, she forgets it, is a proposition with this property:  $p \rightarrow \neg Kp$ ; and so, it is a proposition p such that  $p \models \neg Kp$  (sub-case 1.1). The proposition, not considered in Tennant's distinction, is here of kind 1.

An example of a proposition in the middle between two Tennant's kinds is "there are not known propositions". In Tennant's distinction this proposition could be included in the third kind: "proposition that  $\phi$  is known may be logically inconsistent because of its own overall logical structure, involving

iterations of K"<sup>14</sup>, or in the second kind: "knowledge of a (consistent) proposition  $\phi$  may be impossible because the very act of considering or judging (falsely) that  $\phi$  requires the falsity of (some consequence of)  $\phi$ ". In our distinction the above proposition is an Anti-Cartesian one of the first kind because it is a proposition p such that  $p \models \neg Kp$ , deducible with (Fact) alone.

### V. Conclusion

Some final comments on our analysis of Anti-Cartesian propositions: as shown in the last section, such analysis can be taken as the basis for a new distinction of Anti-Cartesian propositions, better motivated and complete than Tennant's one. But the analysis could also be interesting for other reasons, independent by the purpose of classifying such kind of propositions. For example, if complete and correct, the analysis gives a list of propositions, problematic for the supporters of an epistemic notion of truth.

In our opinion, our analysis should be useful to an anti-realist arguing for the Knowability Principle, at least for understanding more deeply the reasons of some of its problems. However, we don't want to consider our results as a criticism to the anti-realist position (at least, no more than how the Knowability Paradox yet does). On the contrary, they could also be used for analyzing some features of the Anti-Cartesian propositions and for suggesting some other restrictive solutions to the paradox better motivated and more complete than Tennant's one.<sup>15</sup>

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<sup>&</sup>lt;sup>14</sup> By "there are not known propositions" is derived an inconsistency for the iteration of K as follows:

1) K(∀p (¬Kp))	(Ass.)
2) ∀p (¬Kp)	by (1) and (Fact.)
3) $\neg K(\forall p (\neg Kp))$	by (2), substituting (2) in (2)

(1) and (3) are inconsistent between them.

<sup>&</sup>lt;sup>15</sup> Rosenkranz (2008) gives an example of a more complete restriction, able to solve some further problems of Tennant. Our results could be useful for a further improvement of the restriction.

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