Approximating Identity Criteria

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1. Introduction: Identity criteria and their logical adequacy

In a loose and philosophically popular view, derived from Quine, *identity criteria* are required for ontological respectability: Only entities with clearly determined identity criteria are ontologically acceptable. Think, for example, of the case of *propositions*: They would not be ontologically acceptable, because they do not have any identity criteria. The credit for introducing the notion of an *identity criterion* is usually attributed to Frege. He suggests that an identity criterion has the function of providing a general way of answering the following question:

> *Fregean Question*: How can we know whether *a* is identical to *b*? (Frege 1884, §62)

However, both Frege's examples (recall, for instance, the Fregean identity criterion for directions: If *a* and *b* are lines, then the direction of line *a* is identical to the direction of line *b* iff *a* is parallel to *b*) and later philosophical formulations assume that such a question is to be restricted to specific kinds of objects. In the philosophical literature, the *Fregean Question* has been reformulated in the following ways:

> *Epistemic Question* (EQ): If *a* and *b* are *K*s, how can we know that *a* is the same as *b*?

> *Ontological Question* (OQ): If *a* and *b* are *K*s, what is it for the object *a* to be identical to *b*?

> *Semantic Question* (SQ): If *a* and *b* are *K*s, when do '*a*' and '*b*' refer to the same object?

The difference between an answer to (EQ) and an answer to (OQ) is not purely formal. When answering (EQ), we think of conditions associated with a more or less general procedure for deciding the identity questions concerning objects of some kind *K*. In answering (OQ), we think of conditions which are meant to provide an ontological analysis of the identity between objects of kind *K*. Finally, an answer to (SQ) concerns sameness and difference of reference of simple or complex names. In the present paper we do not deal with (EQ), but we restrict our analysis to (OQ) and (SQ). Specifically, we think that an answer to (OQ) can shed some light on an answer to (SQ), too.

Each formulation of identity criteria contains an identity condition represented by a possibly complex formula $F(x, y)$ or a binary predicate *R*. In our paper, we want to focus on such a relation *R*. Among the possible formulations, we consider the following:

(IC) $\forall x \forall y \in D(f(x) = f(y) \leftrightarrow R(x, y))$

It is assumed that there is a domain of individuals *D* and a function *f* such that $f(D)$ constitutes a sort of individuals *K*. *R* represents the condition under which *x* and *y* are said to be identical. In the left side of the biconditional in (IC), there is an identity relation, which is an equivalence relation. Consequently, the relation *R* on the right side of the biconditional must be an equivalence relation, too. Unfortunately, as has been observed in the philosophical debate about identity criteria, some relations considered as candidates for *R* often fail to be transitive. The following are examples of transitivity failure of *R* (see (Williamson 1986)):

- Let *x, y,* and *z* range over colour samples and *f* be the function that maps colour samples to perceived colours. A plausible candidate for *R* might be the relation of perceptual indistinguishability. It is easy to verify, though, that such an *R* is not necessarily transitive: It might happen that *x* is indistinguishable in colour from *y* and *y* from *z*, but *x* and *z* can be perceived as different in colour.

- If $f(x)$ and $f(y)$ are physical magnitudes, to determine whether $f(x) = f(y)$, one could think to measure *x* and *y* with a measurement instrument. Instruments, though, are not infinitely precise. Suppose that *x* and *y* differ by very little and that our instrument does not detect such a difference. If we use the result of the measurement by the instrument as what provides the identity condition, it can happen that *x* turns out to be identical to *y* under the measurement, even if they actually differ. Roughly speaking, it is easy to see how in such a situation, transitivity of the identity condition can fail.

The examples above show how some relations that are intuitively plausible candidates to be identity conditions do not meet the logical constraint that (IC) demands. However, instead of refusing this kind of plausible but inadequate identity criteria, it has been suggested to approximate the relation *R* whenever it is not transitive. That means that, given a non-transitive *R*, we can obtain equivalence relations that approximate *R* by some operations. Some approaches have been suggested: Two of them are due to (Williamson 1986, 1990), while a third approach is due to (De Clercq and Horsten 2005). The aim of this paper is to present an improvement of De Clercq and Horsten's approach.

2. Closer approximations to identity conditions

(Williamson 1986, 1990) suggests giving up the requirement for the identity condition to be both necessary and sufficient. Given a non-transitive *R*, let $R_1, R_2, \ldots R_n$ be equivalence relations that approximate *R*. Among them, we want to find the relation R_i that best approximates *R*. Williamson's proposal is to apply one of the following approaches:

> *Approach from above*: Consider the smallest (unique) equivalence relation R^+ such that $R \subseteq R^+$.

> *Approach from below*: Consider the largest (not unique) equivalence relation R^- such that $R^- \subseteq R$.

Adopting the approach from above, you get a relation R^+ that is a sufficient identity condition. On the contrary, if you adopt the approach from below, you obtain a relation R^- that is a necessary identity condition. Consider the following example. Let *D* be a domain of objects:

D = (a, b, c, d, e).

Assume there is a candidate relation *R*, reflexive and symmetric, for the identity condition for the individual of *D*.

When R holds between two objects x and y, we denote this as $\overline{xy}$. Let R on D be the following:

$R = (\overline{ac}, \overline{ad}, \overline{bc}, \overline{bd}, \overline{cd}, \overline{de})$.

R is not an equivalence relation. In fact, it fails to be transitive. For instance, R holds between a and d and between d and e, but it does not hold between a and e.

Now, apply, firstly, Williamson's approach from above. We obtain the smallest equivalence relation R^{+} such that it is a superset of R, i.e.:

$R^{+} = (\overline{ab}, \overline{ac}, \overline{ad}, \overline{ae}, \overline{bc}, \overline{bd}, \overline{be}, \overline{cd}, \overline{ce}, \overline{de})$.

Consider, instead, the approach from below. We get a relation R^{-} that is not unique. For instance, one of the largest equivalence relations that are subsets of R is the following:

$R^{-} = (\overline{bc}, \overline{bd}, \overline{cd})$.

Now, we have at least two approximations, one from above and the other from below. Which is the best one? Following De Clercq and Horsten's suggestion, you first measure the degree of unfaithfulness of R^{+} and R^{-} with respect to R. Such a degree is the number of revisions you must make to get R^{+} or R^{-} from R. A revision is any adding or removing of an ordered pair to or from R. In the example considered above, R^{+} is obtained by adding four ordered pairs to R and R^{-} by removing three ordered pairs. The degree of unfaithfulness of R^{+} is 4 and the degree of R^{-} is 3. Thus, R^{-} is closer to R than R^{+}. That means that with R^{-}, you stay closer to your intuitive identity condition R, because R^{-} modifies R less than R^{+}.

De Clercq and Horsten claim that, given a kind of objects K, there are not always good reasons to decide whether you must take a necessary or a sufficient identity condition R. They consider a third option: to give up both the necessity and the sufficiency of the identity condition. They search for an overlapping relation $R^{\pm}$ that is neither a super- nor a sub-relation of R. Such an overlapping relation has the advantage of being closer to R than either R^{+} or R^{-}. With respect to the example given above, an overlapping relation that approximates the given R can be the following:

$R^{\pm} = (\overline{ab}, \overline{ac}, \overline{ad}, \overline{bc}, \overline{bd}, \overline{cd})$.

$R^{\pm}$ adds one ordered pair and removes another one. So the degree of unfaithfulness of $R^{\pm}$ is 2; that is, less than both R^{+} and R^{-}. It is, then, the best approximation to R. An overlapping relation can be closer to R than the relations obtained with the approaches from below and from above.

3. Refinement of the overlapping approach

Consider now the following variants of Williamson's example concerning perceived colours:

Example a: You see just two monochromatic spots, A and B, and you do not detect any difference with respect to their colour. Following Williamson, you claim that they have the same colour, because they are perceptually indistinguishable (the identity condition R is perceptual indistinguishability). Now, suppose you add two further monochromatic spots, C and D, such that they are perceptually distinguishable. However, A is indistinguishable from C and B from D. In such a scenario, you can accept to revise your previous judgement and say that A and B are distinct.

Example b: You see two colour samples A and B from a distant point of view such that you are not able to distinguish A-colour from B-colour. You say that A and B have the same colour (the identity condition is, again, perceptual indistinguishability in colour). Now, you get closer to them and detect a difference between them. So, you revise your previous judgement and say that A and B are distinct.

Example a shows that our judgements about colours depend on how we compare colour samples. It seems that R can vary across contexts: Two objects that are indistinguishable in one context, and therefore judged as identical, can turn out to be distinct in another context. Example b presents a different issue from example a. In b, a context is fixed and R varies among different levels of observation. Suppose that from a distant and coarse point of view, you make an identity statement about some objects x and y in a context o via the relation R: for instance, $x = y$. From a more precise, fine-grained point of view, you can make a different identity statement about the same objects x and y in the same context o via R: for instance, $x \neq y$. That means you can look at the elements of a context under different standards of precision, which we call *granular levels*. The finer the level is, the more differences between the individuals can be detected.

Our proposal is to integrate the notions of contexts and granular levels with De Clercq and Horsten's formal treatment of approximating relations. Informally, our suggestion is as follows: Given a fixed context, each granular level provides a relation R for that context; however, if we fix a granular level of observation, R can hold between two objects in a context and not hold between the same objects in a different context. In the following section, we sketch a formalisation of the above suggestion.

4. Granular models

Let L be a formal language through which we can represent English expressions. L consists of the following:

- individual constant symbols: $\bar{a}$, $\bar{b}$, … (there is a constant symbol for each element of the domain);
- individual variables: x_0, x_1, x_2, ... (countably many);
- two-arity predicate symbols P_1, P_2, …; and
- usual logical connectives with identity, quantifiers.

The set of terms consists of individual constants and individual variable symbols. The formulas can be defined in the usual way.

Consider now an interpretation of L. Let D be a fixed, non-empty domain of objects. A context o is defined as a subset of the domain D. So, the set of all contexts O in D is the powerset of D:

$O = \wp(D)$.

Consider now a binary relation R (a two-arity predicate). Assume that R is reflexive and symmetric, but not necessarily transitive. R pairs the elements in each context $o \in O$ that are indistinguishable in some respect. For instance, in the case of colour samples, R gives rise to a set of ordered pairs, each of them consisting of elements that are indistinguishable with regard to their (perceived) colour. We want R to vary across contexts as well as across granular levels. Consider, firstly, granular levels. R behaves in a specific way in each context $o \in O$ in each granular level. Take, for the sake of simplicity, the following context with three elements: $o = (a, b, c)$. One of the following scenarios can occur:

1. R gives rise to three ordered pairs.
2. R gives rise to two ordered pairs.
3. R gives rise to one ordered pair.
4. R does not give rise to any ordered pair.

We can understand the different behaviour of R in the scenarios 1–4 if we think of each scenario as a description of the context o given in a specific level of observation. For example, in 1, we are in a coarse-grained level; in 4, in a very fine-grained level; and in 2 and 3, in some intermediate granular level. The same can be done for all contexts $o \in O$. Now, call *context structure* a structure M consisting of the domain D, all the contexts in D, and a binary relation R (a two-arity predicate); formally, $M = \langle D, O, R \rangle$.

We have seen that, in a fixed domain and set of contexts, R can vary across granular levels. More precisely, we have more than one context structure: There is at least one context structure for each granular level. Consider again the scenarios 1–4. We have some very coarse context structures with an R that behaves as in 1, some refined context structures with an R that behaves as in 4, and other context structures with an R that behaves as in 2 or 3.

Now, consider the behaviour of R across contexts. Fix a context structure, say M_1. Consider two contexts: $o = (a, b, c)$, $o' = (a, b, c, d)$. Suppose that M_1 has a relation R such that $R_o = (\overline{ab}, \overline{bc})$ and $R_{o'} = (\overline{ab})$. You can observe that R holds between b and c in o, but it does not hold between them in o'. So, fixed a context structure, a relation R can vary across contexts.

If, according to some context structure, the relation R fails to be transitive with respect to some context $o \in O$, then the formal framework given by De Clercq and Horsten is applied. For instance, consider again M_1. Its relation R is not transitive in context o. Thus, an equivalence overlapping relation $R^{\pm}$ can be defined for R relatively to o. In contexts where R is not transitive, $R^{\pm}$ denotes a relation that differs from R in that it adds and/or removes some ordered pairs to or from R.

5. Conclusion

In this paper we have tried to show how the overlapping approach proposed by De Clercq and Horsten can be improved. Before determining the closest approximation to R, we suggest fixing a context and a granular level of observation, since R can vary along those two variables. If, according to a context structure M_i belonging to some granular level, R fails to be transitive in a context, you can build the closest approximation to R for that context in M_i.

Literature

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