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1   Introduction

Volatility of high frequency financial time series shows long range dependence. In particular, when dealing with intradaily data one observes long range dependence merged with periodic behavior due to some operating features of financial markets as open, close and lunch time effects. Periodic components are represented as marked peaks at some frequencies of the periodogram of the time series. If this empirical evidence is neglected modelling and prediction of the volatility dynamics may be seriously affected. To face this problem two approaches have been considered in the literature. The first one uses suitable models that can explain non seasonal and seasonal long memory behavior and applies them directly to the squared returns. Examples of this approach are k-factor GARMA modelling (Woodward, Cheng and Gray, 1998; Bisaglia, Bordignon and Lisi, 2003) and SARFIMA modelling (Porter-Hudak, 1990). The second way consists of generalizing the basic GARCH model introducing seasonal effects in the volatility equation. Seasonal GARCH and Periodic GARCH (Bollerslev and Ghysel, 1996) belong to this class of models. Here we follow this approach and propose a model, which is an extension of the classical FI-GARCH model, to account for long memory also in the periodic component. Since it is a Seasonal Fractionally Integrated GARCH, we call this model SFIGARCH.

We show the effectiveness in capturing both seasonality and long memory through the hourly series of the FIB30, the future on the italian stock market index MIB30.

2 Seasonal FIGARCH

The main feature of the SFIGARCH model is to allow for both periodic patterns and long memory behavior in the conditional variance. In particular, it can also merge these two aspects allowing to model periodic *and* long memory components. Formally, in the SFIGARCH(p, d, q, S) model the conditional mean equation is defined as $y_t = \mu_t + \varepsilon_t$, where μ_t is the (possibly time varying) conditional mean of y_t that can be described, for example, by an ARMA model. The error term ε_t is such that $E\left(\varepsilon_t | I^{t-1}\right) = 0$ and $E\left(\varepsilon_t^2 | I^{t-1}\right) = \sigma_t^2$, $E\left(\cdot | I^{t-1}\right)$ denoting the conditional expectation with respect to the history of the process until time $t-1$.
The model assumes then that the conditional variance of ε_t has the following dynamic evolution

$$\sigma_t^2 = \omega + \sum_{j=1}^{p} \alpha_j \varepsilon_{t-j}^2 + \sum_{i=1}^{q} \beta_i \sigma_{t-i}^2 + \left[1 - \left(1 - L^S\right)^d\right] \varepsilon_t^2. \tag{1}$$

where L is the lag operator. The first three terms in the conditional variance reproduce the traditional GARCH(p,q) structure while the fourth term introduces a long memory component which operates at zero and at seasonal frequencies. The parameter S represents the length of the cycle and d modulates the (long) memory degree. As in the standard FIGARCH model, it must be $0 < d < 1$ to ensure positivity of the conditional variance and strictly stationarity. The additonal GARCH parameters must satisfy the standard restrictions for positivity ($\omega > 0$, $\alpha_j \geq 0$ for $j = 1, 2, ...p$, and $\beta_i \geq 0$ for $i = 1, 2, ...q$) and stationarity of the conditional variance ($\sum_{j=1}^{p} \alpha_j + \sum_{i=1}^{q} \beta_i < 1$). The specific assumptions required by the SFIGARCH for strictly stationarity can be derived adapting some results reported in Zaffaroni (2004).
Using the standard relation $\varepsilon_t^2 = \sigma_t^2 + \nu_t$, where ν_t is a martingale difference sequence, it is not difficult to show that if ε_t follows a SFIGARCH process the evolution of ε_t^2 can be described by the following Seasonal ARFIMA:

$$\left(1 - L^S\right)^d \varepsilon_t^2 = \omega + \sum_{j=1}^{k} \left(\alpha_j + \beta_j\right) \varepsilon_{t-j}^2 + \left(1 - \sum_{i=1}^{q} \beta_i L^i\right) \nu_t \tag{2}$$

where $k = \max(p, q)$. The representation (2) permits to derive the theoretical spectrum of the ε_t^2. For example, under the assumption $p = q = 0$ we have

$$f\left(\omega\right) = \frac{\sigma_z^2}{2\pi} \left(2 - 2\cos S\omega\right)^{-d} \tag{3}$$

where σ_z^2 is the variance of the standardised residuals. In this case the spectrum shows peaks at the frequencies $\omega \in \left\{0, \frac{2\pi}{S}, \frac{4\pi}{S}, \frac{6\pi}{S}...\frac{2k\pi}{S}\right\}$ with $\frac{2k\pi}{S} < \pi$.
   From (1) it is trivial to see that the SFIGARCH model contains, as particular cases, several other well known GARCH models: the standard GARCH (for $d = 0$

and $S = 1$), and thus the IGARCH, the FIGARCH model (for $0 < d < 1$ and $S = 1$) and the SGARCH (for $d = 0$, $p = kS$, $q = rS$). The PGARCH model is not nested in the SFIGARCH. However the latter generalizes the PGARCH because it includes long memory at the non-seasonal and seasonal frequencies. Note that the long memory periodic modelling is entrusted with just a single parameter.

3   A case study: the FIB30 returns

In this section we perform an empirical comparison between the SFIGARCH and four alternative models on the hourly log-returns, r_t, of the FIB30 contract, the Future on the MIB30, the Italian stock market index of most highly capitalised firms. Our data cover the period 13/03/2000 - 20/03/2001 for a total of 260 market days. Since in that period the italian market was open from 9.30 to 17.30, we have 8 observations for every day, for a total of 1960 data. The raw data have been subjected to a correction for abnormal trading days and outliers (see Caporin, 2004).

In order to filter out the linear dependence on the conditonal mean we fitted an ARMA model to the log-returns. The selected model, chosen by considering the significance of the parameters and the ACF of the residuals, was a Seasonal AR $(1 - \phi_1 L)\left(1 - \phi_8 L^8\right) r_t = \varepsilon_t$, with estimated parameters $\hat{\phi}_1 = 0.1483$ (s.e. 0.0222) and $\hat{\phi}_8 = 0.0788$ (s.e. 0.0223). The residuals of the linear model, e_t, resulted to be incorrelated in the level, but they showed a clear linear dependence in the squares, as testified by the ACF of e_t^2. This latter, moreover, showed a seasonal pattern of period 8. The periodic beavhior is also confirmed by the periodogram of e_t^2 (Fig.2) which shows four peaks. The first one, at the zero frequency and of magnitude 158, is associated to a long memory component. The other three peaks are at frequencies associated with long memory seasonal components of periods 8, 4 and 2.6 hours. The first periodic component, corresponding to a daily cycle, is due to the operating features of the market. The second cycle probably reflects a different dynamics in the first and in the second half of a trading day. Finally the third cycle might be connected with the opening of the US market. The magnitudes of the three cycles are, respectively, 400, 140 and 60. Given these characteristics, we fittted to the series e_t our SFIGARCH model. Furthermore, for the same data, we estimated also other four models: a misspecified GARCH(1,1), a short memory Seasonal GARCH model (SGARCH), a short memory Periodic GARCH (PGARCH) and a misspecified long memory FIGARCH.

The models identification was carried out by choosing the models which, on the whole, gave the best performance in term of significance of the parameters and in term of periodogram and autocorrelation function of the squared standardized residuals, z_t^2. The expressions for the conditional variance of the identified models are given in Tab.1.

In the estimation step quasi maximum likelihood estimates have been considered. These estimates are reported in Tab.2. In the SFIGARCH model the parameter α_3 is significant only at a 6.7% level, but we chose to retain it because it improved the behavior of the ACF.

The comparison among the models is based on three criteria: the value of the

SFIGARCH	$\sigma_t^2 = \omega + \alpha_1 e_{t-1}^2 + \alpha_3 e_{t-3}^2 + \alpha_7 e_{t-7}^2 + \beta_1 \sigma_{t-1}^2 + \left[1 - \left(1 - L^8\right)^d\right] e_t^2$
GARCH	$\sigma_t^2 = \omega + \alpha_1 e_{t-1}^2 + \beta_1 \sigma_{t-1}^2$
SGARCH	$\sigma_t^2 = \omega + \alpha_1 e_{t-1}^2 + \alpha_7 e_{t-7}^2 + \alpha_8 e_{t-8}^2 + \alpha_{16} e_{t-16}^2 + \beta_1 \sigma_{t-1}^2$
PGARCH	$\sigma_t^2 = \omega + \left(\alpha_1 + \gamma_2 * I_2 + \gamma_3 * I_3 + \gamma_4 * I_4\right) e_{t-1}^2 + \beta_1 \sigma_{t-1}^2$
FIGARCH	$\sigma_t^2 = \omega + \alpha_1 e_{t-1}^2 + \alpha_8 e_{t-8}^2 + \left[1 - \left(1 - L\right)^d\right] e_t^2$

Table 1: Identified models for the conditional variance. In the PGARCH model the I_j are dummy variables which take value 1 if e_{t-1}^2 refers to the j-th hour of a trading day.

loglikelihood, the autocorrelation function of the z_t^2 and the periodogram of the same quantities. Concerning the loglikelihood, the model with the best performance is the SFIGARCH, followed, respectively, by the SGARCH, PGARCH, FIGARCH and GARCH models (see Tab.2). We first note that the models have a different number of parameters. However, given the sample size, the consideration of this aspect, for example by using standard information criteria, is not rilevant. Furthermore, all periodic models have the same number of parameters. Since the models are not nested, standard LR tests can not be used to compare them.

Looking at Fig.2 it is possible to compare the periodogram of the z_t^2 for the five models. Not surprisingly the standard GARCH does not account for long memory or seasonal pattern and infact the periodogram of its squared residuals shows both these components. At the same way, it is not strange that the FIGARCH model explains the non periodic long memory behavior, but clearly fails to describe the periodic components. Among the periodic models, the SGARCH model seems to be able to capture short memory seasonal behavior but not the long memory at zero and seasonal frequencies. The PGARCH model sensibly reduces the peak at the zero frequency and partially explains the periodic component but the peak at the frequency $f = 0.125$ is again significant. Finally, the periodogram of z_t^2 for the SFIGARCH model does not show any dominant peak suggesting that it adequately accounts for both long memory and periodic behavior (see Fig.2).

Similar conclusions can be reached by analysing the autocorrelation functions of the z_t^2 (Fig. 1). To this purpose, we considered the Ljung-Box statistics at lags 5 and 16 to test for short time and seasonal residual correlation. For non periodic models, namely GARCH and FIGARCH, the ACF of z_t^2 is not significant at the first lags but it is clearly significant at the periodic lags (see Tab.3). For the PGARCH and SGARCH models the Ljung-Box test rejects the null both at the lag 5 and at the lag 16. On the contrary, for the SFIGARCH model both Q(5) and Q(16) lead to accept the hypothesis of incorrelation.

par.	est.	std.err.	t-stat	est.	std.err.	t-stat
	GARCH			SGARCH		
ω	0.0097	0.0075	1.290	0.0371	0.0063	5.884
α_1	0.0862	0.0275	3.140	0.0925	0.0396	2.336
α_7	-	-	-	0.2357	0.0533	4.418
α_8	-	-	-	0.0803	0.0237	3.386
α_{16}	-	-	-	0.1596	0.0323	4.937
β_1	0.8523	0.0742	11.489	0.1867	0.0543	3.434
$Logl$	982.5	-	-	1149.4	-	-
	FIGARCH			SFIGARCH		
ω_1	0.0165	0.0087	1.894	0.0000	0.0046	0.000
d	0.1156	0.0379	3.050	0.1815	0.0286	6.335
α_1	0.2454	0.0456	5.377	0.1107	0.0351	3.151
α_3	-	-	-	0.0192	0.0105	1.830
α_7	-	-	-	0.0643	0.0211	3.057
α_8	0.1375	0.0619	2.221	-	-	-
β_1	-	-	-	0.1354	0.0485	2.787
$Logl$	1079.3	-	-	1248.4	-	-
	PGARCH					
ω	0.0298	0.0062	4.841			
α_1	0.8625	0.2042	4.223			
γ_2	-0.8312	0.1931	-4.303			
γ_3	-0.8054	0.1891	-4.259			
γ_4	-0.8451	0.2073	-4.077			
β_1	0.4735	0.0691	6.851			
$Logl$	1082.3	-	-	-	-	-

Table 2: Estimated parameters of the conditional variance of the models.

In the whole, thus, with respect all the three criteria the best model is the SFI-GARCH which seems to adequately describe the dynamics of the empirical volatility.

	FIGARCH	PGARCH	GARCH	SGARCH	SFIGARCH
$Q(5)$	0.274	0.000	0.149	0.001	0.173
$Q(16)$	0.000	0.000	0.000	0.009	0.123

Table 3: p-values of the Ljung-Box test for the squared standardized residuals z_t^2 of the models.

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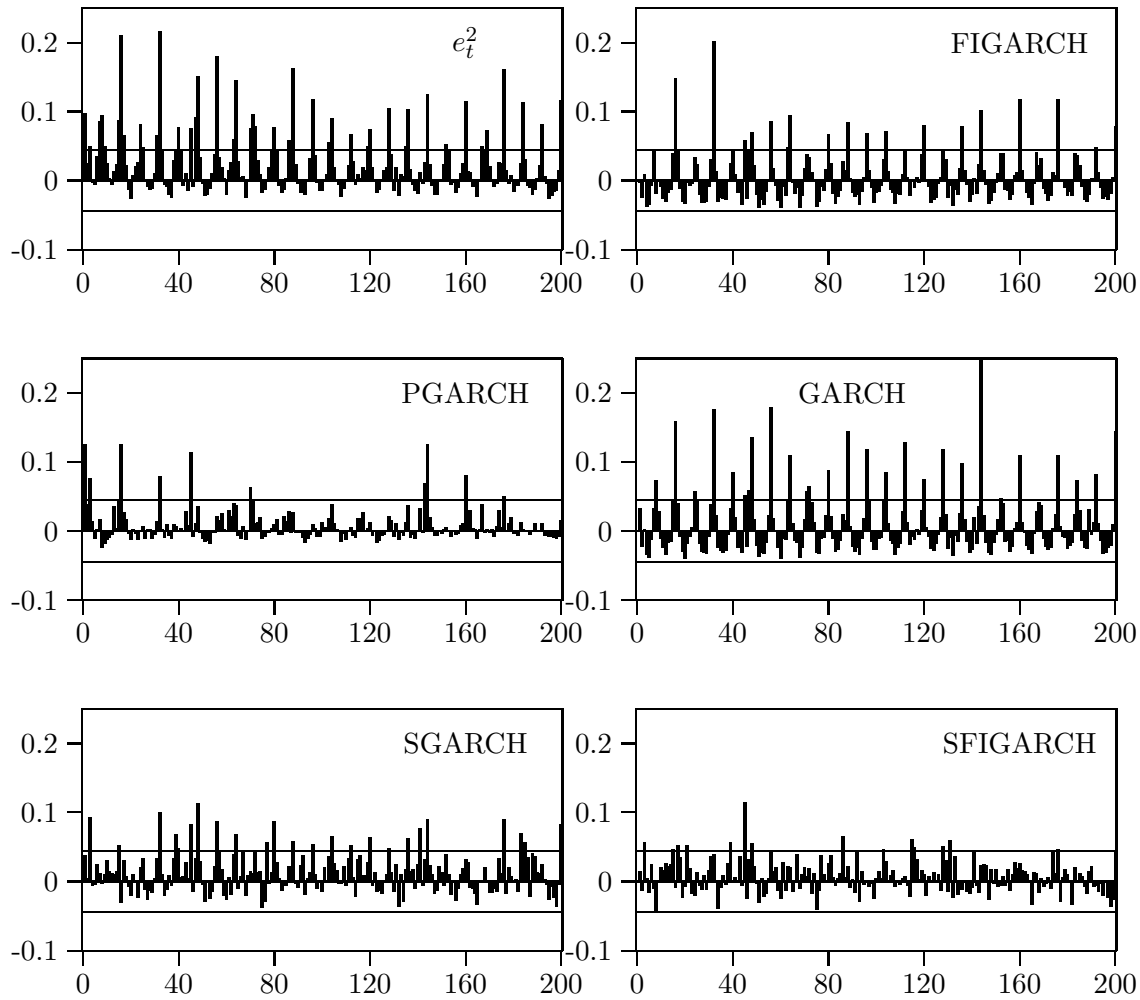

Figure 1: Autocorrelation function of squared residuals e_t^2 and of squared standardized residuals of the FIGARCH, PGARCH, GARCH, SGARCH and SFIGARCH models. Horizontal lines are 95% confidence intervals.
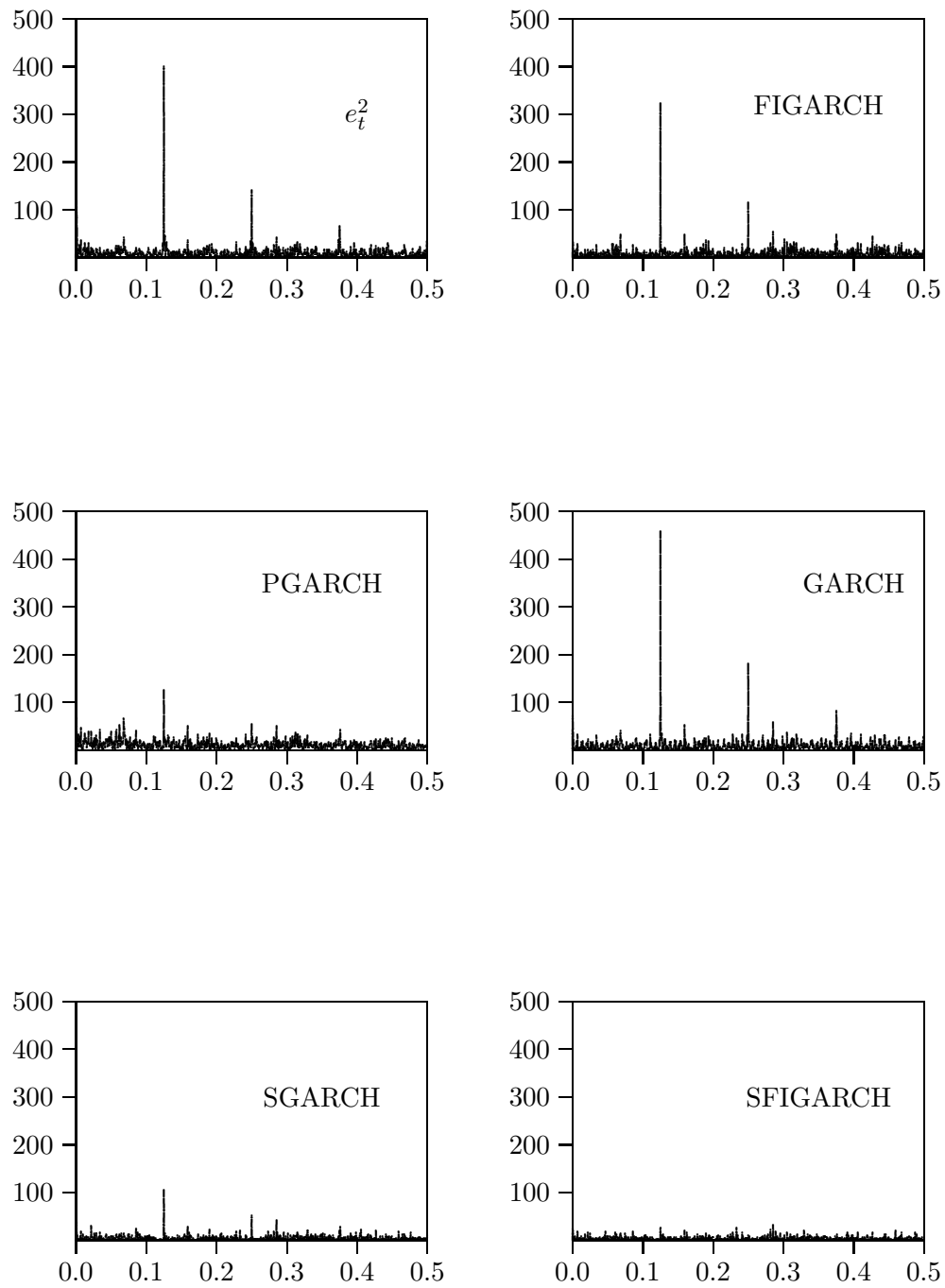
Figure 2: Periodograms of squared residuals e_t^2 of the ARMA model and of squared standardized residuals z_t^2 of the FIGARCH, PGARCH, GARCH, SGARCH and SFIGARCH models.

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