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On product cannibalization

A new Lotka-Volterra model for asymmetric competition in the ICTs

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Keywords: product cannibalization, asymmetric competition, Lotka-Volterra model with churn, nonlinear regression

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1 Introduction

One of the key points to consider when studying the diffusion of a new product in a market is the presence of competitors. In fact competition may alter the dynamics of diffusion both in terms of penetration speed, time to maximum peak, and size of market potential. Concurrent products may act as a barrier to a product's success, but at the same time, their presence may enlarge the size of demand and thus imply a benefit for all market players. Although markets are increasingly becoming complex and in many commercial sectors the competitive environment counts several actors, the models so far produced in the field of new product diffusion, accounting for competition, have limited their attention to duopolistic conditions [15]. This is certainly due to the inherent difficulty of managing systems of differential equations, simultaneously describing the growth dynamics of each product. Diffusion models for competition have often focused on modelling the interaction between two products by splitting the word-of-mouth in two parts: the *within product* word-of-mouth, which is due to product's specific sales, and the *cross product* word-of-mouth, which is due to competition and may imply either a negative or a positive effect. Also,

competition has been considered to be both *synchronic*, when two products enter the market at the same time, or *diachronic*, when a product acts as a monopolist in a first phase and the second enters the market later. Among the papers dealing with diachronic competition, we recall Krishnan et al. (2000) [12], Savin and Terwiesh (2005) [16], Guseo and Mortarino (2010) [8], Guseo and Mortarino (2012) [9], Guseo and Mortarino (2014) [10], Guidolin and Guseo (2015) [6] and Guseo and Mortarino (2015) [11], where competition is modeled as a duopoly after a monopolistic period for the first entrant. In particular, Guseo and Mortarino (2014) in [10], have proposed a model called unrestricted UCRCD where both products share the same market potential and are influenced by within product and cross product word-of-mouth. In this sense the residual market is a common target too, which allows a totally free competition. This model has been generalized by Guseo and Mortarino (2015) in [11] by accounting for a time dependent market potential, $m(t)$. A different approach for the definition of the residual market has been proposed by Guidolin and Guseo (2015) [6] within a Lotka-Volterra model (Abramson and Zanette (1998) [1], Morris and Pratt (2003) [14], and Baláž and Williams (2012) [2], among others). This is characterized by an independent modulation of the residual market of each competitor, taking into account 'churn' effects.
Conventional wisdom suggests that competition deals with different products commercialized by different brands, struggling for market share. However, a special case of competition may also occur between products pertaining to the same brand: in this situation we talk about *product cannibalization*. Copulsky (1976) [5] defined product cannibalization as "the extent to which one product's sales are at the expense of other products offered by the same firm". In fact, in marketing strategy, cannibalization refers to a reduction in sales of one product due to the introduction of a new product by the same producer. Although the effect of cannibalization may appear negative, it may eventually result beneficial by growing the overall market. A particular case of cannibalization has been considered by Guidolin et al. (2016) in [7] by studying the situation when a retailer discounts a product: in this case consumers will tend to buy the discounted product rather than that with higher price. This change in consumer behaviour due to promotional activities gives rise to a special intra-brand competition that has been modelled with a LVch model. In this paper we consider a typical case of cannibalization in which competition has an asymmetric nature, so that the cannibalizing product is able to steal market to the other, while obviously the cannibalized cannot do the reverse. As will be illustrated in Section 2, this phenomenon may be described with a special Lotka-Volterra model with asymmetric competition, which derives as a special case of the LVch model by Guidolin and Guseo (2015) [6]. This model is applied to a well known case occurred in the ICTs where Apple iPhone was able to steal market and determine the crisis of the more recent Apple iPad. Interestingly, this is a very particular example of cannibalization, where it is the first entrant to cannibalize the second, while marketing theory suggests that the reverse typically occurs.
Stimulated by this applied case, we studied a different representation of the proposed LV model, which allows for a reduction of the involved parameters. This reduced representation proves very useful to understand whether and to what extent cannibalization modifies the temporal dynamics of a product's life cycle: as highlighted

in [18], the ability to evaluate the impact of cannibalization on a product's life cycle is of great importance for strategic and operational decision-making on marketing. In particular, the question we are able to answer through this different model representation is: does the entrance of a second competitor alter the timing of maximum peak of the first?

The paper is structured as follows. In Section 2 we present the LVch model and some special cases derived by the modulation of churn parameters. In Section 3 we illustrate some aspects concerning statistical inference and model selection, while in Section 4 we analyze the case of intra-brand competition between Apple iPhone and iPad, which gave rise to a case of product cannibalization, which is well described by the Lotka-Volterra model with asymmetric competition, a special case of the LVch model. In Section 5 we propose a nondimensional representation of the model employed in Section 4, through which we are able to better understand whether the temporal dynamics of the first product are influenced by the entrance of the second. Section 6 contains some final remarks.

2 Lotka-Volterra model with churn and possible reductions for asymmetric competition

The Lotka-Volterra with churn model, LVch, by Guidolin and Guseo [6] is described by a system of differential equations, namely,

$$\begin{aligned}
z_1'(t) &= \left[p_{1a} + q_{1a}\frac{z_1(t)}{m_a}\right][m_a - z_1(t)], \quad t \le c_2 \\
z_1'(t) &= \left[p_1 + \frac{a_1 z_1(t) + \alpha_2 b_1 z_2(t)}{m_1 + \alpha_2 m_2}\right][(m_1 - z_1(t)) + \alpha_2(m_2 - z_2(t))] \\
z_2'(t) &= \left[p_2 + \frac{a_2 z_2(t) + \alpha_1 b_2 z_1(t)}{m_2 + \alpha_1 m_1}\right][(m_2 - z_2(t)) + \alpha_1(m_1 - z_1(t))].
\end{aligned} \tag{1}$$

In this model the first equation describes the stand-alone phase ($t \le c_2$), when the first product acts as a monopolist in the marekt. We may see that the product is assumed to behave according to a standard Bass model (Bass (1969), [3]) with parameters m_a, p_{1a} and q_{1a}. The second and third equations are defined for $t > c_2$, when the second product has entered the market, and describe competition dynamics. Each product's rate sales, $z_i'(t), i = 1, 2$, for $t > c_2$, are proportional to the corresponding residual $[(m_i - z_i(t)) + \alpha_j(m_j - z_j(t))], i = 1, 2, j = 1, 2, i \neq j$, where m_i is the product's specific market potential under competition and $z_i(t), i = 1, 2$, represent the cumulative sales at time t. As we may see, the residual is the sum of the product specific residual $m_i - z_i(t)$ plus a fraction of the other's, $\alpha_j(m_j - z_j)$. Parameter $\alpha_j, j = 1, 2$, modulates the size of this second element. Parameter $p_i, i = 1, 2$, defines innovative behavior in adoption, while the WOM components have a more complex structure made of a within-product element $[a_1 z_1(t)/(m_1 + \alpha_2 m_2)]$ and of a cross-product one, $[\alpha_2 b_1 z_2(t)/(m_1 + \alpha_2 m_2)]$, for the first competitor and, similarly, $[a_2 z_2(t)/(m_2 + \alpha_1 m_1)]$ and $[\alpha_1 b_2 z_1(t)/(m_2 + \alpha_1 m_1)]$ for the second. Notice that α_1 and α_2 operate on both the WOM and the residual market potentials. The

presence of variable parameters α_1 and α_2 that control a sort of "churn" effect between the two competitors has suggested the name *Lotka-Volterra with churn* model.
Interestingly, the modulation of parameters α_1 and α_2 allows to distinguish some specific cases, useful to represent different market environments:

1. if $0 < \alpha_1 < 1$ and $0 < \alpha_2 < 1$ we have the Lotka-Volterra with churn model, where both products are affected by within and cross WOM and each one may have access to a portion of the other's residual market;
2. if $\alpha_1 = \alpha_2 = 1$ the LVch model reduces to the UCRCD by Guseo and Mortarino (2014) [10];
3. if $\alpha_1 = \alpha_2 = 0$ there is no competition between the two products, which are in fact described through simple independent Bass models;
4. if $\alpha_1 = 1$ and $\alpha_2 = 0$ the life cycle of the first product is described with a standard independent Bass model. The second product's residual market is made by the sum of both residual markets, $(m_2 - z_2(t)) - (m_1 - z_1(t))$ since $\alpha_1 = 1$. In this sense we may observe the total asymmetry of competition, where the second product has a complete access to the residual market of the first. At the same time we may see that the first product may still have an impact on sales of the second, by means of the cross product WOM, $[\alpha_1 b_2 z_1(t)/(m_2 + \alpha_1 m_1)]$, which may be either positive or negative depending on the sign of parameter b_2;
5. if $\alpha_1 = 0$ and $\alpha_2 = 1$ similar considerations hold. In this case the first product acts as a competitor, cannibalizing the market of the second, while the second is described with a standard independent Bass model.

3 Statistical inference and estimation

The statistical implementation of the models presented in previous section is based on nonlinear least squares (NLS), (see [17]), under a convenient stacking of the two submodels; the stacking procedure is necessary in order to obtain a unidimensional nonlinear model estimated with standard nonlinear least squares (NLS) methodology, under Levemberg–Marquardt algorithm.
In particular, we may consider the structure of a nonlinear regression model

$$w(t) = \eta(\beta, t) + \varepsilon(t), \tag{2}$$

where $w(t)$ is the observed response, $\eta(\beta, t)$ is the deterministic component describing instantaneous or cumulative processes, depending on parameter set β and time t, and $\varepsilon(t)$ is a zero mean residual term, not necessarily independent identically distributed (i.i.d.).
The performance of an extended model, m_2, compared with a nested one, m_1, may be evaluated through a squared multiple partial correlation coefficient $\tilde{R}^2$ in the interval $[0; 1]$, namely,

$$\tilde{R}^2 = (R^2_{m_2} - R^2_{m_1})/(1 - R^2_{m_1}), \tag{3}$$

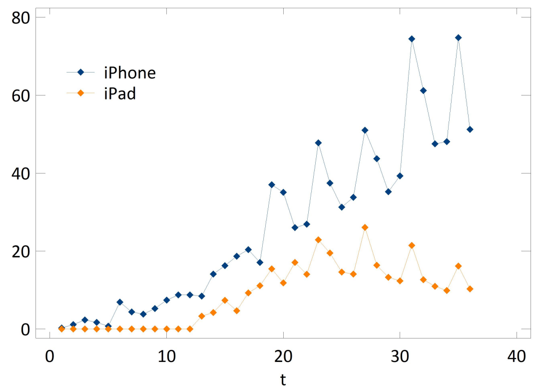

Figure 1: Quarterly unit sold of iPhone and iPad (data source: Apple Inc).

where $R^2_{m_i}$, $i = 1, 2$ is the standard determination index of model m_i.
The $\tilde{R}^2$ coefficient has a monotone correspondence with the F-ratio, i.e.,

$$F = [\tilde{R}^2(n - v)]/[(1 - \tilde{R}^2)u], \tag{4}$$

where n is the number of observations, v the number of parameters of the extended model m_2, and u the incremental number of parameters from m_1 to m_2. Under strong conditions on the distributional shape of the error term $\varepsilon(t)$, particularly independence, identical distribution, and normality, the F-ratio statistic, for the null hypothesis of equivalence of the two models, is a central Snedecor's F with u degrees of freedom for numerator and $n - v$ degrees of freedom for denominator, $F \sim F_{u,n-v}$, [6].

4 On cannibalization: competition between Apple iPhone and iPad

In this section we analyze a recent case of product cannibalization occurred between Apple iPhone and iPad. Figure 1 shows the time series of quarterly sales of both products (data source: Apple Inc).
We may observe some preliminary aspects:

1. the iPhone entered the market in Q3/2007 and is still experiencing an increasing trend;

2. the iPad entered the market in Q3/2010 and is characterized by an evident declining trend, having already undertaken the life cycle peak;

Table 1: Parameter estimates of a standard Bass model for Apple iPhone before t=13; Marginal linearized asymptotic 95% confidence limits into brackets. Estimates performed on instantaneous data.

m_a	p_a	q_a	R^2
145.809	0.005	0.265	0.8083
(-163.199)	(-0.001)	(-0.001)	
(454.817)	(0.011)	(0.531)	

Table 2: Parameter estimates of LVch model. Marginal linearized asymptotic 95% confidence limits into brackets. Estimates performed on instantaneous data.

p_1	a_1	b_1	α_2	m_1	R^2
-0.010	0.526	-0.840	0.998	1347.57	0.8766
(-0.037)	(0.002)	(-8.050)	(-7.668)	(661.55)	
(0.015)	(1.049)	(6.370)	(9.665)	(2033.60)	
p_2	a_2	b_2	α_1	m_2	DW
0.011	0.167	1.058	0.001	378.76	2.073
(-0.096)	(-1.081)	(-395.802)	(-0.989)	(-21.38)	
(0.118)	(1.417)	(397.918)	(0.993)	(778.91)	

3. both products are characterized by an evident seasonal component;

4. Apple reports sales data of all its products without making a distinction between product generations.

In order to analyze the presence and the nature of competition between the two products, we estimated the LVch model presented in Equation (1) and the reduced models deriving by the modulation of parameters α_1 and α_2 as illustrated in Section 2. Estimates of the LVch model -an independent Bass model- are presented in Table 1 for the stand-alone component, before the entry of the iPad. Parameters of the competition phase are outlined in Table 2. In particular, we may see that $\hat{\alpha}_2 = 0.998$ and $\hat{\alpha}_1 = 0.001$, which suggests a polarization of the two parameters, according to case 5) in Section 2. Following this observation we estimated this reduced version of the model by setting $\alpha_2 = 1$ and $\alpha_1 = 0$. Also, we interpreted the negative estimate of parameter p_1, $\hat{p}_1 = -0.010$, (which is incoherent with the theory of diffusion models), as a signal of the absence of an innovative component for the iPhone within the competition phase: in fact, it is well known that the success of the iPhone heavily relied on word-of-mouth, as highlighted in [15]. This appears reasonable since the innovation component for the iPhone is already described by parameter p_{1a} in the stand-alone phase. We therefore estimated a reduced version of LVch model, a LV model with asymmetric competition ($\alpha_2 = 1$ and $\alpha_1 = 0$) and $p_1 = 0$. The estimated parameters of the model have been outlined in Table 3. In order to understand if this model reduction is effective, we compared its performance with that of the general LVch model through

Table 3: Parameter estimates of LV model with asymmetric competition and $p_{1c} = 0$. Marginal linearized asymptotic 95% confidence limits into brackets. Estimates performed on instantaneous data.

a_1	b_1	m_1	R^2
0.238	-0.260	1798.18	0.8733
(0.146)	(-0.493)	(1415.45)	
(0.328)	(-0.028)	(2180.90)	
p_2	a_2	m_2	DW
0.011	0.172	379.71	2.077
(-0.004)	(0.077)	(299.54)	
(0.027)	(0.266)	(459.89)	

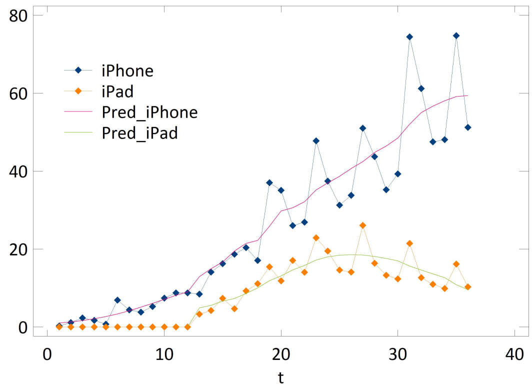

Figure 2: Lotka-Volterra model with asymmetric competition for iPhone and iPad (data source: Apple Inc).

the tests for nested models presented in Section 3. On this basis, we accepted the reduced model since $\tilde{R}^2 = (0.87662 - 0.873347)/(1 - 0.873347) = 0.0258$ and $F = 0.0258(60 - 10)/(1 - 0.0258)4 = 0.33$.
The results show that the residual market for the iPhone is given by $(m_1 - z_1(t)) + (m_2 - z_2(t))$, that is the residual market of the iPad results to be completely available to the iPhone. Conversely, by setting $\alpha_1 = 0$, the residual market for the iPad is just given by $m_2 - z_2(t)$, and the cross-product WOM vanishes, $[\alpha_1 b_2 z_1(t)/(m_2 + \alpha_1 m_1)] = 0$. Interestingly, we may see that the iPad is described by an independent standard Bass model and is therefore not influenced by the iPhone, while the iPhone has been affected by the iPad both in negative and positive terms. In fact, the iPad implied an extension of the iPhone's residual market but also a negative cross product WOM,

since parameter b_1 is negative, $b_1 = 0.26$.

5 Lotka-Volterra model with asymmetric competition: a nondimensional representation

The model selected in Section 4 to treat the case of Apple iPhone and iPad has the following reduced form (for $t > c_2$)

$$\begin{aligned} z_1'(t) &= \left[\frac{a_1 z_1(t) + b_1 z_2(t)}{m_1 + m_2}\right] [(m_1 - z_1(t)) + (m_2 - z_2(t))] \\ z_2'(t) &= \left[p_2 + \frac{a_2 z_2(t)}{m_2}\right] [(m_2 - z_2(t))]\,. \end{aligned} \tag{5}$$

As we may see, this system depends upon 6 parameters, namely $a_1, b_1, m_1, p_2, a_2, m_2$. This number may be reduced by expressing the system of equations in non-dimensional terms, following for instance Boccara (2004) [4]. Nondimensionalization may recover characteristic properties of a system of equations through a convenient scaling of involved variables. To this end, let us consider the following rescaled variables $x_1 = z_1/z_{10}$, $x_2 = z_2/z_{20}$, $\tau = t/t_0$. By setting $z_{20} = m_2$ and $z_{10} = m_1$, Equation (5) may be rewritten as

$$\begin{aligned} x_1' &= \left[\frac{a_1 z_{10} x_1 + b_1 z_{20} x_2}{m_1 + m_2}\right] \frac{t_0}{z_{10}} [(m_1 - z_{10} x_1) + (m_2 - z_{20} x_2)] \\ x_2' &= \left[p_2 \frac{t_0}{z_{20}} + a_2 x_2 \frac{t_0}{z_{20}}\right] [(m_2 - z_{20} x_2)]\,. \end{aligned} \tag{6}$$

A further reduction is obtained through $s = m_2/m_1$,

$$\begin{aligned} x_1' &= \left[\frac{a_1 x_1 t_0 + b_1 s x_2 t_0}{1 + s}\right] [(1 - x_1) + s(1 - x_2)] \\ x_2' &= [p_2 t_0 + a_2 x_2 t_0] [(1 - x_2)] \end{aligned} \tag{7}$$

and setting $t_0 = 1/a_2$ and $r = p_2/a_2$ we obtain

$$\begin{aligned} x_1' &= \left[\frac{a_1 x_1 + b_1 s x_2}{a_2(1 + s)}\right] [(1 - x_1) + s(1 - x_2)] \\ x_2' &= (r + x_2) [(1 - x_2)]\,. \end{aligned} \tag{8}$$

The form $(a_1 x_1 + b_1 s x_2)/(a_2(1 + s))$ may be multiplied by $a_2(1 + s)/b_1$, obtaining $(a_1/b_1)x_1 + s x_2$. Moreover, if we set $v = a_1/b_1$ we obtain a *nondimensional representation* of Equation (5), which is based on only 3 parameters

$$\begin{aligned} x_1' &= (v x_1 + s x_2) [(1 - x_1) + s(1 - x_2)] \\ x_2' &= (r + x_2) [(1 - x_2)]\,. \end{aligned} \tag{9}$$

5.1 Peak conditions for x_1'

Let us indicate with $F_2 = (1 - e^{-(r+1)\tau})/(1 + 1/re^{(r+1)\tau})$ the solution of the second equation in (9) and rewrite the first equation accordingly,

$$x_1' = (vx_1 + sF_2)(-x_1 + 1 + s(1 - F_2)). \tag{10}$$

Taking the first derivative of x_1' with respect to x_1 and setting it equal to zero we obtain the maximum density condition, $\hat{x}_1$

$$\hat{x}_1 = \frac{1}{2} + \frac{s}{2}(1 - F_2) - \frac{s}{2v}F_2 = \frac{1}{2} + \frac{s}{2}\left(1 - F_2 - \frac{F_2}{v}\right). \tag{11}$$

Since $v = a_1/b_1$ is typically negative because b_1, expressing the cross WOM effect, is negative, $(1 - F_2 - F_2/v)$ will be positive.
Reminding that $s = m_2/m_1$ we may rewrite $\hat{z}_1 = m_1\hat{x}_1$ in a more interesting form

$$\hat{z}_1 = m_1 x_1 = \frac{m_1}{2} + \frac{m_2}{2}\left(1 - F_2 - \frac{F_2}{v}\right). \tag{12}$$

Equation (12) highlights that as long as the market potential of the second entrant m_2 increases, the maximum peak for z_1', $\hat{z}_1$, is delayed and reached beyond $m_1/2$. Competition for the specific case at hand, following Equation (11), gives rise to

$$\hat{x}_1 = \frac{1}{2} + 0.08327477(1 + 0.04669954\ F_2(\tau)). \tag{13}$$

Reminding that $F_2(\tau)$ is a cumulative distribution function that may take values between 0 and 1 we will have that $0.58327377 \le \hat{x}_1 \le 0.58716$ or approximately $\hat{x} \simeq 0.585$.

6 Discussion and concluding remarks

In this paper we have proposed a competition model for describing the well known phenomenon of cannibalization and applied it to the case of Apple Inc, where the iPhone was able to cannibalize the more recent iPad. This is a very peculiar case of cannibalization, because it is generally the second entrant that is able to steal sales and market to the first one, while in the Apple case the reverse occurred. Through our proposed model, LV with asymmetric competition, we described the interaction between the two technologies, showing that the life cycle of the iPad is described by a simple Bass model, while the iPhone life cycle has a more complex structure, which accounts for the presence of the iPad. In particular, the competing role of the iPad had both a negative and positive role: on the one hand the iPad has exerted competition on the iPhone through a negative WOM, but its presence has also been beneficial since its residual market potential is completely available to the iPhone. Moreover, through a non dimensional representation of the proposed LV model we were able to show that competition has implied a delay in the peak time of the iPhone: so the entrance of the iPad has been strongly beneficial for the iPhone, in terms of market potential definition and length of life cycle. It is worth noting that in our model we just considered the mean trajectory of sales, without taking into account the evident seasonal pattern characterizing both products. Seasonality may be adequately modelled with SARMAx models once the mean behaviour of the series is adequately described, see for instance [7].

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