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Luisa Bisaglia
Department of Statistical Sciences
University of Padua
Italy

Margherita Gerolimetto
Department of Statistics, Ca' Foscari University of Venice

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Keywords: Long memory, occasional structural breaks, Box-Pierce test.

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Department of Statistical Sciences
Via Cesare Battisti, 241
35121 Padova
Italy

tel: +39 049 8274168
fax: +39 049 8274170
http://www.stat.unipd.it

Corresponding author:
Luisa Bisaglia
tel: +39 049 827 4168
bisaglia@stat.unipd.it
http://www.stat.unipd.it/~bisaglia

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1 Introduction

Recently there has been an increasing interest in the possibility of confusing long memory and occasional-break processes. If a stationary short memory process is affected by occasional breaks, its realizations give the impression of persistence and the estimate of the long memory parameter is biased away from zero. In such a case, the appearance of long memory is not a genuine feature of the time series, but it occurs because occasional structural breaks are present (Granger and Hyung, 2004, Diebold and Inoue, 2001). Formal statistical tests to decide if a real time series is better described as a realization of a long memory process or a process with structural breaks are therefore of value. Notwithstanding, to our knowledge, in literature, there are not so many proposals to distinguish the long memory from the occasional breaks effects. Ohanissian et al. (2004) propose a test based on the invariance of the long memory parameter for temporal aggregates of the process under the null hypothesis of true long memory. Dolado *et al.* (2004) develop a test, conceived as an extension of the well-known Dickey and Fuller test, for the null hypothesis of long memory versus the alternative of short memory with deterministic components, subject to structural breaks. Perron and Qu (2004) propose a test for long memory, based on the difference in the log-periodogram estimate, computed using different numbers of frequencies. Hsu (2005) presents a modified local Whittle method for testing long memory when the data have a single change in mean at a know date. Shimotsu (2005) proposes a pair of tests based, the first, on the comparison among the estimates of the long memory parameter d, computed

for b subsamples and the second on the KPSS test after taking the d-th difference of the sample. Berkes et al. (2006) develop a testing procedure to discriminate between a long memory dependent process and a weakly dependent process with one change in mean at an unknown point.
In this paper we propose to use the well-known Box-Pierce (Box and Pierce, 1970) and Ljiung-Box (Ljiung and Box, 1979) statistics to distinguish between long memory and occasional-breaks data generating process (DGP) when more than one break in mean is present at unknown dates. The BP and LB statistics have the big advantage of being very easy to compute and they are also implemented in several softwares. To describe the idea, suppose that a time series is generated from a process with occasional breaks and its autocorrelation function (ACF) decreases very slowly to zero, giving the impression of persistence. Then we could estimate the date of breaks and filter the series to obtain a break-free series which should not present spurious dependence anymore. This can be tested using the Box-Pierce (BP) and Ljiung-Box (LB) statistics.
Via an extensive Monte Carlo experiment we investigate the finite sample performance of the BP and LB test and it with Hsu's modified local Whittle test (hereafter mLW) opportunely generalized to multiple breaks at unknown dates. We do not consider in the Monte Carlo experiment all the tests previously mentioned because either their null hypothesis is long memory (whereas we are focussed on the null hypothesis of occasional breaks) or they assume the occurrance of only one change possibly at known date (whereas we assume multiple breaks at unknown dates).
Moreover we present an application to the Italian inflation rate monthly series.
The paper is organized as follows. Section 2 gives some basics on the long memory and occasional-break models considered in the paper. Section 3 describes Hsu's modified local Whittle test and the BP and LB test. Section 4 is devoted to the Monte Carlo experiment. Section 5 presents an empirical example using the Italian inflation rate monthly data. Section 6 concludes.

2 Long memory and occasional-break processes

In the time domain, a stationary discrete time series is said to be long memory if its autocorrelation function decays to zero like a power function, that is

$$\rho(k) \approx Ck^{2d-1} \quad as \quad k \to \infty, \tag{1}$$

where $C \neq 0$ and $d < 0.5$. This definition implies that the dependence between successive observations decays slowly as the number of lags tends to infinity. Alternatively, in the frequency domain, a stationary discrete time series is said to be long memory if its spectral density is unbounded at low frequencies, that is

$$f(\lambda) \approx \frac{\sigma^2}{2\pi}\lambda^{-2d} \quad as \quad \lambda \to 0.$$

In the following we will concentrate on the fractionally integrated, or $I(d)$ processes, with $d \in (0, 1/2)$: for this range of values the process is stationary, invertible and possesses long-range dependence (see Beran (1994) for more details on long memory processes).

In this paper we will also consider occasional-break models, i.e. such that the number of breaks that can occur in a specific period of time is somehow bounded. More formally, for these models the probability of breaks, p, is assumed to converge to zero slowly as the sample size increases, i.e. $p \to 0$ as $T \to \infty$, yet $\lim_{T\to\infty} Tp$ is a non-zero finite constant.

The first model we consider is the mean plus noise or occasional-break model (Chen and Tiao, 1990, Engle and Smith, 1999)

$$\begin{aligned} y_t &= m_t + \epsilon_t, \qquad t = 1, ..., T, \\ m_t &= m_{t-1} + q_t \eta_t \end{aligned} \tag{2}$$

where ϵ_t is a noise variable and occasional level shifts, m_t, are controlled by two variables q_t (date of breaks) and η_t (size of jump). η_t is an i.i.d. $N(0, \sigma^2_\eta)$. In the following we assume that η_t is $N(0, 1)$ and q_t follows an i.i.d. binomial distribution, such that $q_t = 0$ with probability $1 - p$ and 1 with probability p.

The binomial model (2) is characterized by sudden changes only. When the structural changes are gradual the Markov switching model (Hamilton, 1989) is more suitable. Suppose s_t is a latent random variable that can assume only values 0 or 1. s_t is assumed to be a Markov chain with transition probability $p_{ij} = Pr\,(s_t = j/s_{t-1} = i)$. Then, in model (2), it is possible to use a switching model for q_t such that $q_t = 0$ when $s_t = 0$ and $q_t = 1$ when $s_t = 1$. Hence $s_t = 1$ indicates a break date and this is independent of the value taken by s_{t-1} that might be also equal to 1. This allows graduality in the structural changes.

3 Alternative approaches to structural change testing

In this Section we consider three tests to verify the occasional breaks hypothesis vs. long range dependence. The first test we discuss is a generalization of Hsu's (2005) test to take into account the instance of multiple breaks at unknown dates. Then we present two tests based on the Box-Pierce and Ljung-Box statistics.

3.1 A generalization of the modified local Whittle test

When $y_t, t = 1, \ldots, T$ is a realization of an ARFIMA (p, d, q) process, Sowell (1992) suggested to estimate the parameters by the maximum likelihood method, hereafter MLE. On the ground of the approximation proposed in Whittle (1953), maximizing the log-likelihood function is equivalent to minimizing the spectral likelihood function:

$$L^W_T(\beta) = \sum_{j=1}^{[T/2]} \left\{ \frac{I(v_j)}{f(v_j, \beta)} + log f(v_j, \beta) \right\}$$

where $f(v_j, \beta)$ is the spectral density of y_t and

$$I(v_j) = \frac{1}{2\pi T} \left| \sum_{t=1}^{T} (y_t - \mu) e^{itv_j} \right|^2$$

is the periodogram. β is a vector including the short and long memory parameters and $v_j = 2\pi j/T$, for $j = 1, \ldots, [T^c] < [T/2]$, are the spectral ordinates. The resulting minimizer is an approximation of the MLE and it is known as the Whittle estimator. The Whittle

estimator is easy to compute since $L_T^W(\beta)$ does not depend on the unknown mean μ and is a simple function of β. However the Whittle estimator has the disadvantage of requiring the *a priori* specification of a parametric form of $f(v_j, \beta)$.
With specific regards to the estimation of parameter d, Kunsch (1987) and Robinson (1995b) suggested a local Whittle estimator that neither imposes the Gaussian assumption nor requires the specification of the spectral density. Indeed in the neighbourhood of frequency zero the spectral density of a long memory process is

$$f(v, d) = G|v|^{-2d}$$

as $v \to 0$, where $G \in (0, \infty)$ and $d \in (-0.5, 0.5)$. Therefore replacing $f(v, \beta)$ with $G|v|^{-2d}$ it is possible to obain a local version of $L_T^W(\beta)$ that depends only on G and d, but not on the ARMA coefficients. The local analogue of $L_T^W(\beta)$ is then

$$R(G, d) = \frac{1}{m} \sum_{j=1}^{m} \left(\frac{I(v_j) v_j^{2d}}{G} + logGv_j - 2d \right) \tag{3}$$

where m represents the number of frequencies included in the neighbourhood of frequency zero and is an integer smaller than $[T/2]$, such that $1/m + m/T \to 0$ as $T \to 0$. The local Whittle estimator $\hat{d}_{mLW}$ is obtained by minimazing equation (3) and, when there is no change in mean, Robinson (1995b) proves that

$$m^{1/2}(\hat{d}_{mLW} - d) \xrightarrow{d} N(0, 1/4).$$

Hsu (2005) suggests the use of the local Whittle estimator and includes potential breaks in the models in order to discriminate between long memory and occasional-break DGP. In particular, Hsu considers the case of one single break at an unknown date. The idea is, firstly, to obtain the break-free series by subtracting the sample mean from each regime, then compute the local Whittle estimator $\hat{d}(\tilde{\tau})$ of each subseries, conditionally to the date of the break $[\tilde{\tau}T]$. The estimated break point is $\tilde{\tau} = n/T$, where $1 < n \leq T$. Under these circumstances the local Whittle estimator (hereafter called modified Local Whittle estimator, mLW) maintains the asymptotic normality.
In our context, Hsu's settings of a single structural break are too restrictive. Hence it is opportune to generalize the mLW estimator to the instance of multiple breaks at unknown dates and employ the Bai and Perron test (Bai and Perron, 1998) to detect the break points in the series.[1] On the ground of the T-consistence of the estimated break points, the test statistics still has the asymptotic normality. Therefore a test statistics, mLW, for detecting long memory is

$$mLW = 2m^{1/2}(\hat{d}(\hat{\tau}) - d) \to N(0, 1)$$

where $\hat{\tau} = (\hat{\tau}_{\mathbf{1}}, \hat{\tau}_{\mathbf{2}}, \ldots, \hat{\tau}_{\mathbf{k}})$ is the vector of the estimated break points. Under the null hypothesis of structural breaks we have $d = 0$.

[1] Actually, in his paper, Hsu estimates jointly the long memory parameter and change points, but he supposes the number of breaks is known.

3.2 A pair of tests based on the Box-Pierce statistics

In this Section we propose the use of the BP and LB statistics to discriminate between long memory and occasional-break processes. As in Hsu (2005), we work with the break free series. More precisely, using the BP or LB statistics, we investigate whether there is correlation in the break free series. Under the null hypothesis of occasional-break DGP, the BP and LB tests should not find correlation in the break free series and the appearance of persistence of the original series can be interpreted as a spurious feature, induced by the presence of the (unaccounted) structural breaks. On the contrary, under the alternative of long memory DGP, the BP and LB tests should find correlation in the break free series, as filtering the original series from the (spurious) structural breaks can not destroy its (genuine) persistence.
Given the time series y_t, $t = 1, ..., T$ and its break free version y'_t, defined in the previous section, the test statistics we consider is:

$$BP_K = T \sum_{k=1}^{K} (\hat{\rho}(k))^2 \tag{4}$$

where

$$\hat{\rho}(k) = \frac{\sum_{j=1}^{T-k}(y'_t - \bar{y}')(y'_{t+k} - \bar{y}')}{\sum_{t=1}^{n}(y'_t - \bar{y}')^2}$$

is the sample autocorrelation function computed for the break-free series, T is the series length and K is the number of lags included in the summation.
The modified version of the BP statistics proposed by Box and Ljiung (1979) is

$$LB_K = \sum_{k=1}^{K} \frac{T(T+2)}{T-k} (\hat{\rho}(k))^2 . \tag{5}$$

From the theoretical side, Hosking (1996) derive the asymptotic properties of sample autocorrelations in case of long memory processes and proved that they differ in important respects from the corresponding results for the short memory processes whose autocorrelations are absolutely summable.
More in details, Hosking (1996) shows that when $0 < d < 1/4$, $\hat{\rho}(k)$ has the standard normal limiting distribution and asymptotic variance of order n^{-1}; when $d = 1/4$, $\hat{\rho}(k)$ is asymptotically standard normal and asymptotic variance of order $n^{-1}log(n)$; when $1/4 < d < 1/2$, $\hat{\rho}(k)$ has a nonstandard asymptotic behaviour (see Theorem 7 in Hosking's paper).
In our framework, under the null hyphotesis of structural change, the break-free series obtained from the mean plus noise and the Markov switching model is a white noise, then the BP_K and LB_K statistics converge to the usual chi-square distribution with K degrees of freedom. Under the alternative hyphotesis of long memory the statistics diverge to infinity.

4 Monte Carlo study

In this Section we conduct an extensive Monte Carlo experiment to examine the size and power properties in finite samples of the tests presented in the previous sections.

The computational burden of this simulation experiment is considerable, especially for what concernes the application of the Bai and Perron procedure. For this reason, we consider the following three different data generating processes often considered in literature, e.g. Granger and Hyung (2004), Diebold and Inoue (2001):

1. DGP1: mean plus noise model with $p = 0.005, 0.01, 0.05, 0.1$ and $\sigma_\eta^2 = 0.1$;
2. DGP2: Markov switching model, with $(p, q) = (0.95, 0.95;\ 0.95, 0.99;\ 0.99, 0.95;\ 0.99, 0.99)$ and $\sigma_\eta^2 = 0.1$. In this case the initial state s_1 is generated by a Bernoulli random variable with $p = 0.5$;
3. DGP3: $I(d)$ model with $d = 0.1, 0.2, 0.3$ and 0.4 and $\sigma_\epsilon^2 = 1$.

After simulating a series, y_t, from one of the DGPs, we estimate the break dates with the Bai-Perron (1998) procedure and obtain the break-free series, that is $y_t' = y_t - m_t$, where m_t is the sample mean of each regime. Series generated from DGP1 and DGP2 become white noise after filtering out the breaks, whereas series generated from DGP3 are still $I(d)$ series. Hence, when considering DPG1 and DGP2 the esperiment is conducted under H_0 and it is evaluated the size of the tests. On the contrary, when considering DGP3 the experiment is under H_1 and an estimation of the power is provided. For all DGPs we set the sample sizes to $T = 250$, 500 and 1000. The results are based on 2000 independent replications.

The functions we use are written in R language (R Development Core Team, 2006) and are available upon request by the authors.

Table 1 provides the empirical rejection frequencies of the three test statistics given in the previous section. They are obtained under $H_0 : d = 0$ and based on the 5% critical values of the corresponding asymptotic distributions.[2]

We evaluate the power of the BP_K, LB_K and mLW for several values of K and m tests in $[T/16, T/4]$.[3]

The Monte Carlo experiment revealed that the empirical power of the three tests depends on K and m. On the one hand, the BP_K and LB_K tests tend to become too conservative as K increases, so in (4) and (5) we have used $K = T/16$. On the other hand, the mLW test becomes too conservative, decreasing the number of frequencies, so we have chosen $m = T/4$.

Reading Table 1 we can observe that in terms of size (DGP1, DGP2) BP systematically outperforms the other tests. On the other side, mLW tends to display high over -rejection patterns, especially as the value of parameter p in DGP1 increases and for all values of parameters p and q in DGP2. Moreover the over-rejection becomes even more serious with the increasing of the sample size T. Turning to the tests based on the Box-Pierce statistics, we can see that BP_K works better than LB_K especially in DGP2. In particular, BP displays empirical sizes quite close to the nominal level even if the results are sligtly worse for $T = 1000$. This is an important result for practitioners, as it indicates that we have a promisingly reliable inference tool for testing structural breaks vs long memory.

In terms of power (DGP3), all tests perform well, excepted when d takes the smallest valeus and $T = 250$. This is not surprising since when d is small it is considerably difficult

[2] Results for 1% and 10% are not included for brevity's sake, but are available upon request.

[3] Once more, these results are not presented for brevity's sake, but are available upon request.

Table 1: Empirical power of the tests at 5% level

DGP1	T=250			T=500			T=1000		
test	BP_K	LB_K	mLW	BP_K	LB_K	mLW	BP_K	LB_K	mLW
$p = 0.005$	5.0	6.4	6.2	4.8	6.4	6.3	4.7	6.5	7.8
$p = 0.010$	4.1	5.3	6.4	4.9	6.2	7.1	4.8	6.6	8.9
$p = 0.050$	4.6	6.0	9.2	5.3	6.7	9.3	4.8	7.8	11.6
$p = 0.100$	5.5	6.2	11.2	4.8	5.7	14.8	5.9	7.6	22.5
DGP2	**T=250**			**T=500**			**T=1000**		
test	BP_K	LB_K	mLW	BP_K	LB_K	mLW	BP_K	LB_K	mLW
$(p, q) = (0.95, 0.95)$	4.4	6.7	15.9	5.6	9.2	21.8	8.6	15.3	32.1
$(p, q) = (0.95, 0.99)$	4.7	5.6	8.0	5.8	7.3	11.4	5.9	8.9	17.2
$(p, q) = (0.99, 0.95)$	6.2	7.9	19.7	6.9	9.9	26.6	7.1	10.6	37.6
$(p, q) = (0.99, 0.99)$	5.0	6.0	12.8	6.7	8.3	18.7	8.7	12.0	25.8
I(d)	**T=250**			**T=500**			**T=1000**		
test	BP_K	LB_K	mLW	BP_K	LB_K	mLW	BP_K	LB_K	mLW
$d = 0.10$	18.0	19.5	42.0	26.9	31.2	64.8	42.7	47.4	88.4
$d = 0.15$	44.2	47.0	69.2	66.8	69.1	89.3	87.4	89.8	99.0
$d = 0.20$	73.7	75.2	86.7	91.0	92.2	97.5	99.1	99.3	99.9
$d = 0.25$	90.9	91.6	95.5	98.9	99.1	99.8	100	100	100
$d = 0.30$	97.7	98.2	98.4	99.9	99.9	99.9	100	100	100
$d = 0.35$	99.8	99.8	99.7	100	100	100	100	100	100
$d = 0.40$	99.9	99.9	99.7	100	100	100	100	100	100
$d = 0.45$	100	100	100	100	100	100	100	100	100

to distinguish between long and short memory, even for the most common long memory testing procedures.

5 An example: the Italian inflation rates

As an example, we examine the time series relative to the monthly Italian inflation rates. The sample period covers January, 1975 - August, 2006, with 379 observations (the base years is 2000) and the inflation rate exhibits quite different behaviours. Briefly, the second half of the Seventies is characterized by strong fluctuations as a consequence of the two oil shocks and frequent currency rate devaluations. With the beginning of the Eighties the inflation rate tends to vary much less than before, as the Italian Government substained a restrictive monetary policy, that was even more needed in the Nineties, when Italy was slowly getting ready to be part of the European Monetary Union and in this view had to respect Maastrict parameters.
The series of inflation rates is constructed by taking 100 times the first difference of the log-Consumer Price Index series. The series is then seasonally adjusted. Figure 1 reports the seasonally adjusted series together with the empirical autocorrelation functions. We can

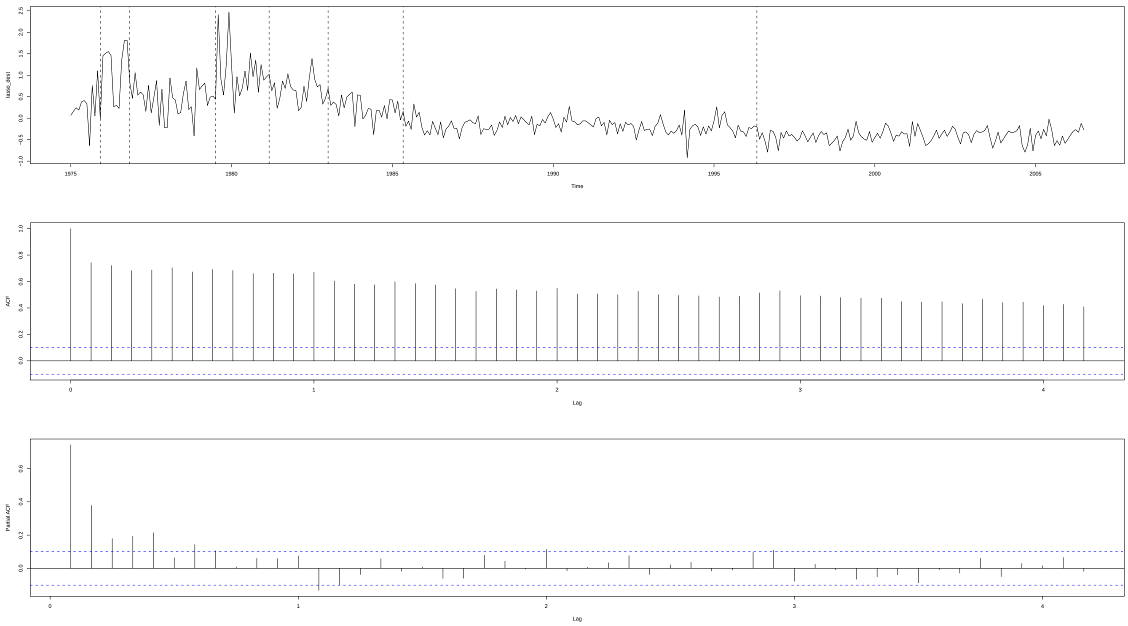

Figure 1: Italian inflation rates

see that the sample autocorrelation function exhibits a very slow decay indicating, perhaps, the presence of long memory. However, bearing in mind what we discussed in the previous sections of this paper, the appearance of strong persistence in this series might also be due to unaccounted structural breaks.

To investigate about the dubious appearance of the sample autocorrelation function, we begin estimating the long memory parameter d in the whole sample. With the Whittle method we obtain $d = 0.418$, with the log-periodogram method the result is $d = 0.933$. Then we employ the Bai and Perron procedure and find the following break points: 1975(12) 1976(11) 1979(7) 1981(3) 1983(1) 1985(5) 1996(5), reported in Figure 1.

By filtering out the estimated break points we obtain the break-free series of the Italian inflation rates that is reported in Figure 2 together with its empirical autocorrelation functions. We can see that the ACF looks now more like the ACF of a white noise process.

Afterwards, we estimate with the Whittle method the long memory parameter in the break-free series and obtain an estimation of $d = 4.583013e - 05$ very close to zero. With the log periodogram method we obtain $d = -0.0666869$. This means that after removing the break points from the series the estimated value of the long memory parameter moves towards zero, whatever estimation method is used. In particular, with Whittle method the estimated value is practically indistinguishable from zero. This suggests that, maybe, the strong persistence in the Italian inflation rates is spurious and can be explained with unaccounted structural breaks.

Finally, we apply to the break-free series the BP_K, LB_K and mLW tests to verify the null hypothesis of structural breaks. The p-values of the tests are 0.0845 for test BP_K, 0.0651 for test LB_K and 0.499 for test mLW. Hence, at least at the level of 5%, all tests accept the null hypothesis of incorrelation in the break free series, i.e. the null hypothesis

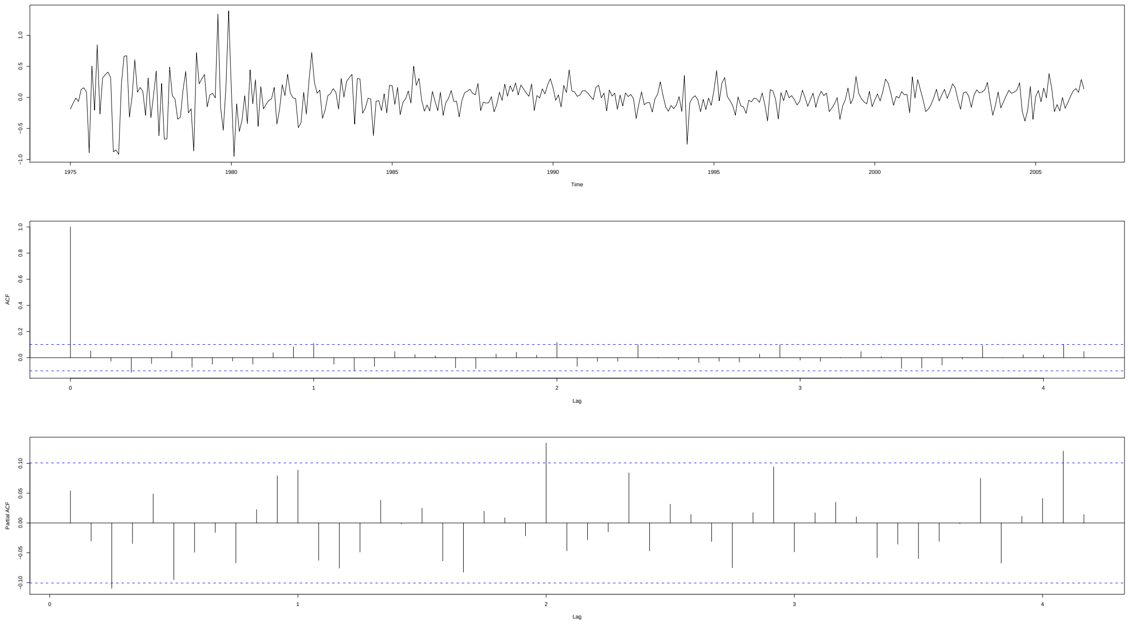

Figure 2: Break-free Italian inflation rates

of structural breaks in the original series. On the whole, we can conclude that the Italian inflation rates series is characterized by structural breaks and not by long memory.

6 Conclusive remarks

The recent literature has given much attention to the issue of discriminating the long memory from occasional breaks phenomena since in practice it is really difficult to distinguish series generated from the two processes.

This paper uses an extensive set of simulation experiments to explore the exact sampling properties of alternative test statistics for the hypothesis of structural breaks vs long memory. In particular, after filtering out the break points from the original series, we propose to use the well-known Box Pierce and Ljung Box statistics. These tests has the advantage of being very easy to implement and also having a known asymptotic distribution.

In a Monte Carlo experiment we compare the performance of the BP_K and LB_K tests with that of Hsu's modified Local Whittle test. The results of our Monte Carlo investigation put in evidence that the simple BP_K test perform well, the LB_K slightly worse, while the behaviour of the mLW test is far poorer, especially in terms of size distortion that is particularly severe with the increase of the series length. In terms of power all test have almost the same the performance.

This finding is encouraging since we have characterized a very simple testing tool that permits to discriminate the long memory from the occasional-break processes. An empirical example with the Italian inflation rates data shows the implementation of the test.

References

Bai, J. and Perron, P., (1998). Estimating and testing linear models with multiple structural changes. *Econometrica*, **66**, 47–78.

Beran, J. (1994). *Statistics for Long-Memory Processes*. Chapman and Hall, London.

Berkes, I. Horvath L., Kokoszka P. and Shao, Q.M. (2006). On discriminating between long-range dependence and changes in mean *The Annals of Statistics*, **34**, 1140–1165.

Box, G.E.P. and Pierce D.A.(1970). Distribution of Residual Autocorrelations in Autoregressive-Integrated Moving Average Time Series Models *Journal of the American Statistical Association*, **65**, 1509–1526.

Chen, C. and Tiao, G.C. (1990). Random level-shift time series models, ARIMA approximations, and level-shift detection *Journal of Business and Economics Statistics*, **8**, 83–97.

Diebold, F.X. and Inoue, A. (2001). Long memory and regime switching *Journal of Econometrics*, **105**, 131–159.

Dolado, J.J, Gonzalo, J. and Mayoral, L. (2004) A Simple Test of Long-Memory vs. Structural Breaks in the Time Domain: What is What? Universidad Carlos III de Madrid (mimeo).

Granger, C.W.J. and Hyung, N. (2004). Occasional structural breaks and long memory with an application to the S$P500 absolute stock returns. *Journal of Empirical Finance*, **11**, 399–421.

Hamilton, J.D. (1989). A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica*, **57**, 357–384.

Hsu, C.-C. (2005) Long memory ore structural change: an empirical examination on inflation rates. *Economics Letters*, **88**, 289–294.

Ljung, G.M. and Box G.E.P., (1979). The likelihood function of stationary autoregressive-moving average models *Biometrika*, **66**, 265–270.

Ohanissian, A., Russell, J.R. and Tsay, R.S. (2004). True or spurious long memory? A new test. University of Chicago (mimeo).

Perron, P. and Qu, Z. (2004). An analytical evaluation of the log-periodogram estimate in the presence of level shift and its implications for stock returns volatility Boston University (mimeo).

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