Allegra, Giorda, and Paris Reply: In our Letter [1] we addressed the evolution of photon-number entangled states (PNES) $|\Psi\rangle = \sum_n \Psi_n |n\rangle|n\rangle$, $|n\rangle$ being an n-photon Fock state of a harmonic oscillator in noisy channels described by the master equation $\dot{\varrho} = A\sum_{j=1}^{2} \mathcal{L}[a_j]\varrho + B\sum_{j=1}^{2} \mathcal{L}[a_j^\dagger]\varrho$, $\mathcal{L}[O]\varrho = 2O\varrho O^\dagger - O^\dagger O\varrho - \varrho O^\dagger O$, $A = \frac{1}{2}\Gamma(1+N_T)$, $B = \frac{1}{2}\Gamma N_T$, where Γ is the damping factor of the channel and N_T the average number of thermal excitations of the channel. Upon exploiting several nonequivalent separability criteria we found evidence that entanglement of Gaussian PNES survives longer, and thus we drew a conjecture about the generality of this result. General arguments supporting the conjecture have been then put forward in [2]. On the other hand, the conjecture need not to be true if one enlarges the set of separability criteria, e.g., to include the negativity of the density matrix under partial transposition (NDPT). Indeed, in [3] it has been shown that the conjecture is too strong to be maintainable in the general case.

The preceding Comment [4] presents two examples where the conjecture is violated. The first is the states $|\psi_{01}\rangle = c_0|00\rangle + c_1|11\rangle$ with $0 \le c_1^2 \le 1/2$, whereas the second example concerns photon subtracted squeezed vacuum (PSSV), i.e., the PNES with $\psi_n = \propto (n+1)x^{n+1}$ with initial entanglement $\epsilon_0 \in \{0.1, 1\}$ and initial energy $E_0 \in \{0.013, 0.3\}$. Not surprisingly, these examples confirm that by enlarging the set of separability criteria the conjecture is not maintainable. However, these examples are worth analyzing in order to assess whether their "robustness" compared to Gaussian states has some relevant physical consequences. With this aim we have evaluated the residual NDPT $\mathcal{N}_R[\rho(t_G)]$ displayed by the above states at the Simon (Gaussian) separation time t_G (i.e., at the time in which Gaussian states with the same initial entanglement or energy become separable) for $B/A \equiv N_T/(1+N_T) \lesssim 0.5$ (note that one photon at optical frequency $\nu \sim 10^{15}$ Hz corresponds to a temperature $T \sim 5 \times 10^4$ K).

The results are shown in Fig. 1, where, in order to give an estimate of the level of residual entanglement, we report the ratios $\mathcal{N}_R/\mathcal{N}_G$ and $\mathcal{N}_R/\mathcal{N}_0$, where $\mathcal{N}_G$ is the maximal entanglement at the energy of $\rho(t_G)$, i.e., the entanglement displayed by a pure Gaussian state with same energy, and $\mathcal{N}_0$ is the initial entanglement. For PSSV states (left-hand panels), we show that although their separation time can be larger than t_G, their residual entanglement is very small for all values of B/A and, in particular, for realistic values of N_T, i.e., $B/A \ll 1$. Similar results are obtained for $|\psi_{01}\rangle$ (right-hand panels).

In summary, the conjecture that we put forward in our Letter, based on a correct comparison of several different separability criteria, is not maintainable in a strict sense if one includes the NDPT criterion. Nevertheless, for the examples of non-Gaussian states proposed in the Comment [4], which violate the conjecture, the residual entanglement is extremely low. Therefore, it remains an open question whether these, as well as other kinds of states, could represent a useful resource in quantum communication protocols. The issue of degradation of continuous variable entanglement in noisy channels is indeed an extremely relevant one, and it is worth analyzing in detail with a more substantial analysis.

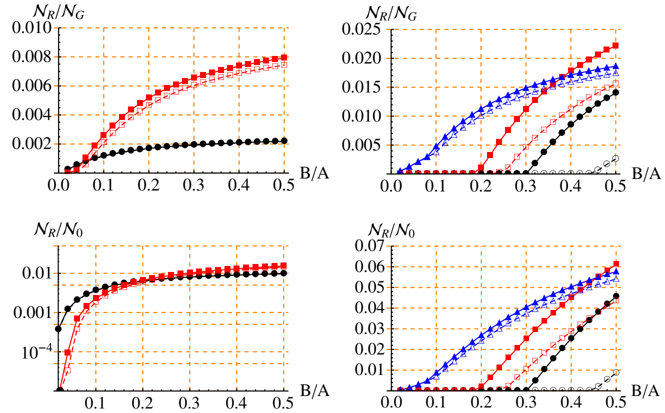

FIG. 1 (color online). Renormalized negativity at separability time t_G for Gaussian states with same initial energy (open symbols) or entanglement (full symbols). (Left) PSSV states: black circles, $\epsilon_0 = 0.1$, $E_0 = 0.013$ (notice that the curves overlap); red squares, $\epsilon_0 = 1.0$, $E_0 = 0.3$. (Right) $|\psi_{01}\rangle$ states: black circles, $|c_1|^2 = 0.5$; red squares, $|c_1|^2 = 0.25$; blue triangles, $|c_1|^2 = 0.05$.

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[1] M. Allegra, P. Giorda, and M. G. A. Paris, Phys. Rev. Lett. **105**, 100503 (2010).
[2] G. Adesso, Phys. Rev. A **83**, 024301 (2011).
[3] K. K. Sabapathy, J. S. Ivan, and R. Simon, Phys. Rev. Lett. **107**, 130501 (2011).
[4] J. Lee, M. S. Kim, and H. Nha, preceding Comment, Phys. Rev. Lett. **107**, 238901 (2011).

