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Cosmological constraints from UV/IR mixing

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ABSTRACT: Holography and entropy bounds suggest that the ultraviolet (UV) and infrared (IR) cutoffs of gravitational effective theories are related to one another as a form of UV/IR mixing. Motivated by this, we derive a bound on the allowed scalar field range in theories with cosmic horizons. We show how this bound challenges several inflationary scenarios, such as α -attractors and modular-invariant inflation. Besides, we find a relation between the number of extra spatial dimensions and the tensor-to-scalar ratio.

KEYWORDS: Cosmological models, String and Brane Phenomenology, Effective Field Theories, Models of Quantum Gravity

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1 Introduction

As of today, there is growing evidence that the ultraviolet (UV) and infrared (IR) regimes are not completely decoupled in quantum gravity. In string theory this is certainly the case, as exemplified by modular invariance on the worldsheet; see e.g. refs. [1, 2]. From a bottom-up perspective, one may say that not all sets of IR parameters necessarily flow into a consistent UV completion. This idea lies at the core of the Swampland Program [3, 4], and making it quantitative is a central challenge. In this note, we investigate the consequences of the mixing between UV and IR regimes in models of phenomenological interest for cosmology, such as long-lived inflationary epochs. We will motivate and investigate a specific manifestation of UV/IR mixing in the form of bounds on the UV and IR cutoff scales, Λ_{UV} and Λ_{IR} , and we will argue that these bounds lead to constraints in large classes of models.

Our focus is on d -dimensional effective field theories (EFTs) in the presence of a cosmic horizon, such as models of cosmic acceleration with $d > 2$. The point we make is that, just like the requirement that $\Lambda_{\text{IR}} \leq \Lambda_{\text{UV}}$ induces an upper bound on the allowed inflaton field range [5–7], holographic and entropy bounds [8–11] also imply the existence of a lower bound on the minimum field value. This uniquely fixes a window where the inflationary dynamics should occur. Concretely, we will argue that the inflaton φ must take value within a finite interval, i.e. $\varphi \in [\varphi_1, \varphi_2]$, and that the inflaton excursion during inflation, $\Delta\varphi$, must be such that

$$\kappa_d \Delta\varphi \leq \frac{p}{2\lambda} \ln \frac{(l_{\text{P},d}^d V_1)^{\frac{1}{1-(d-2)q}}}{(l_{\text{P},d}^d V_2)^{d-1}}. \tag{1.1}$$

Here, p and q are parameters governing the amount of UV/IR mixing, λ provides a measure of the decay rate of the UV cutoff with respect to φ , $V_{1,2} = V(\varphi_{1,2})$ the scalar potential evaluated at the endpoints of the allowed field range, and $l_{\text{P},d} = 1/m_{\text{P},d} = \kappa_d^{2/(d-2)}$ is the Planck length. The fact that general quantum gravity principles suggest that the parameters p , q and λ are order-one will be the seed of constraints on cosmological dynamics.

In several models of early cosmic inflation [12–14], including α -attractors [15–21] and modular-invariant proposals [22–24], we will argue that inflation does not stop as the boundary of the allowed inflaton range is reached. A potential for early cosmic inflation needs to be such that the slow-roll parameters ϵ_V and $|\eta_V|$ are small during the inflationary phase. We find that order-one parameters p , q and λ imply that the slow-roll parameters ϵ_V and $|\eta_V|$ are small all over the allowed range $\varphi \in [\varphi_1, \varphi_2]$, including its endpoints, where the EFT breaks down. In other words, the point at which inflation ends lies outside the interval $[\varphi_1, \varphi_2]$. As a consequence, one cannot describe the entire early cosmological dynamics — eventually exiting the inflationary epoch — within the given EFT. From the relation between $\Delta\varphi$ and the number of e-folds N_e , we can also get an upper bound on the latter. We find that this bound is highly sensitive to the UV/IR mixing parameters. For certain values of the parameters, the bound could forbid the minimum number of e-folds required to explain the CMB properties, $N_e \gtrsim 60$ [25].

Our results align with other attempts to constrain EFTs of cosmic acceleration in a model-independent fashion, such as refs. [5–7, 26–34], and we will comment on the relationship with them in detail. More generally, it is believed that transplanckian scalar field excursions are pathological from an EFT point of view [35], at least for pure moduli. Our bound in eq. (1.1) reflects this basic quantum gravity principle in theories with a potential.

It is also tempting to turn the logic around and use experimental inputs on inflationary observables, such as the number of e-folds $N_e \gtrsim 60$ and the tensor-to-scalar ratio $r \lesssim 0.028$ [36], to constrain theoretical parameters. In this way, we derive the bound

$$\sqrt{\frac{n}{n+2}} \leq \frac{1-3q}{1-2q} \frac{60p}{N_e^*} \sqrt{\frac{0.004}{r}} \ln \frac{10^8}{r} \tag{1.2}$$

on the number n of extra spatial dimensions. By virtue of this relation, detection of primordial gravitational waves may constrain the number of extra dimensions.

The rest of the paper is organized as follows. In section 2, we review entropy bounds motivating our proposed UV/IR mixing relationship. In section 3, we elaborate the implications of the latter on single-field inflationary models, discussing concrete examples in section 4 and also in appendix A. A final discussion is in section 5.

2 Entropy bounds and UV/IR mixing

In this section, we review EFT constraints based on holography and entropy bounds and motivate the parameterization of UV/IR mixing that we employ throughout the note. Related ideas are in refs. [29, 37–41].

A lightsheet of a $(d-2)$ -dimensional surface is a $(d-1)$ -dimensional surface generated by light rays that begin on the surface and extend orthogonally away from it with non-positive

expansion. The Covariant Entropy Bound (CEB) is the conjecture that the entropy S on any lightsheet of a surface does not exceed the area A_{d-2} of the surface in Planck units [10, 11], i.e.

$$S \leq \frac{1}{4} m_{\text{P},d}^{d-2} A_{d-2}. \tag{2.1}$$

This bound will serve us as guiding principle to motivate a specific form of UV/IR mixing in accelerated expanding cosmologies. In the rest of the section, we will ignore order-one coefficients, since we are only interested in estimating relative scalings.

Consider an EFT with N light degrees of freedom. If the EFT is spatially localized within a $(d-1)$ -dimensional volume vol_{d-1} , given the UV-cutoff Λ_{UV} , let us assume that the maximal EFT entropy scales as [8, 42]

$$S_{\text{EFT}} = N \Lambda_{\text{UV}}^{d-1} \text{vol}_{d-1}. \tag{2.2}$$

In particular, S_{EFT} is extensive, namely it scales with the volume and with the number of fields. Then, one may get maximal entropy for maximal volume, hence the presence of the cutoff [38, 40]. At the same time, the total vacuum energy scales as

$$E = N \Lambda_{\text{UV}}^d \text{vol}_{d-1}. \tag{2.3}$$

Importantly, we are working in a regime of weak gravitational coupling, which allows us to ignore gravitational interactions among the N fields.

The system we have in mind is the observable universe in a spacetime with a cosmic horizon. This is contained within a sphere of radius R equal to the apparent cosmic horizon, i.e. $\text{vol}_{d-1} = R^{d-1}$ and $R = 1/H$, where H is the Hubble parameter. Examples of such spacetimes are cosmological solutions with accelerated expansion. Because $1/H$ is the largest observable length, we set $\Lambda_{\text{IR}} = H$. To avoid gravitational collapse, we must require the Schwarzschild radius $r_{\text{S}}(E) = (l_{\text{P},d}^{d-2} E)^{1/(d-3)}$ not to be larger than R , giving

$$N \left(\frac{\Lambda_{\text{UV}}}{\Lambda_{\text{IR}}} \right)^d \leq \left(\frac{m_{\text{P},d}}{\Lambda_{\text{IR}}} \right)^{d-2} \tag{2.4}$$

and, correspondingly, $S_{\text{EFT}} \leq N^{1/d} (m_{\text{P},d} R)^{(d-1)(d-2)/d}$. For $N = 1$, we recover the Cohen-Kaplan-Nelson entropy bound [8]. The requirement in eq. (2.4) means that the largest admissible UV cutoff is such that¹

$$(l_{\text{P},d} \Lambda_{\text{UV}})^d = \frac{(l_{\text{P},d} \Lambda_{\text{IR}})^2}{N}. \tag{2.5}$$

Notice that the cutoff depends on N and the size of the box $R = 1/\Lambda_{\text{IR}}$. These quantities are not independent, else one could violate the CEB *e.g.* by increasing N arbitrarily at fixed R .

Having gained intuition on how entropy considerations lead to a link between UV and IR cutoff, we can look again at the CEB and rewrite eq. (2.1) as [41]

$$(l_{\text{P},d} \Lambda_{\text{UV}})^{d-1} \leq l_{\text{P},d} \frac{\Lambda_{\text{IR}}}{N}, \tag{2.6}$$

¹Although ref. [29] made considerations through the concept of a thermal bath, if we identify the temperature as $T = \Lambda_{\text{UV}}$, then the results agree.

where we fixed $S = S_{\text{EFT}}$. It remains to identify Λ_{UV} . This is conceptually harder than with Λ_{IR} , for which a natural option is suggested by simple physical considerations. On the other hand, several effects can concur in the determination of Λ_{UV} and certainly we cannot ignore gravity. If we only look at gravitational interactions, an upper bound on the UV cutoff is given by the species scale $\Lambda_{\text{sp}} = m_{\text{P},d}/N_{\text{sp}}^{1/(d-2)}$ [43, 44]. This is in fact an absolute upper bound on the UV cutoff of any d -dimensional EFT with N_{sp} species; still, Λ_{UV} may be lowered by further (non-gravitational) effects. Hence, at least we certainly have $\Lambda_{\text{UV}} \leq \Lambda_{\text{sp}}$, as well as $N \leq N_{\text{sp}}$ by definition. Note that the requirement $H \leq \Lambda_{\text{UV}}$, in view of eq. (2.5), implies the constraint $\Lambda_{\text{UV}} \leq m_{\text{P},d}/N^{1/(d-2)}$. With these facts in mind, given our ignorance on identifying unequivocally Λ_{UV} , we take a conservative approach and assume the parameterization

$$l_{\text{P},d}\Lambda_{\text{UV}} = (l_{\text{P},d}\Lambda_{\text{sp}})^{\frac{1}{(d-1)p}}, \tag{2.7}$$

where $p \leq 1/(d-1)$ is a positive constant. The highest value $p = 1/(d-1)$ corresponds to taking $\Lambda_{\text{UV}} = \Lambda_{\text{IR}}$, if it is also $N = N_{\text{sp}}$, from eq. (2.5). This is then a limiting situation and gives the strongest constraints on cosmological models.² Identifying a lower bound on p is tricky. At least, we need $p \geq \max[\ln(l_{\text{P},d}\Lambda_{\text{sp}})/\ln(l_{\text{P},d}\Lambda_{\text{IR}})]/(d-1)$ to ensure $\Lambda_{\text{IR}} \leq \Lambda_{\text{UV}}$.

The discussion so far supports the idea that the UV and IR cutoffs of an EFT cannot be chosen independently, which is a form of UV/IR mixing. Although a precise relation between Λ_{UV} and Λ_{IR} may only be derived within a complete theory of quantum gravity — which we lack —, the previous discussion eventually motivates a parameterization of UV/IR mixing such as

$$l_{\text{P},d}\Lambda_{\text{sp}} \leq \frac{(l_{\text{P},d}H)^p}{N_{\text{sp}}^q}, \tag{2.8}$$

where q is another non-negative parameter included for generality.³ It must be bounded as $q \leq (1-p)/(d-2)$ to ensure the Hubble scale to never exceed the species scale. For instance, taking $p = q = 1/(d-1)$ reproduces eq. (2.6), with $\Lambda_{\text{UV}} = \Lambda_{\text{sp}}$. The purpose of this note is to study the consequence of eq. (2.8) on accelerated expanding cosmologies.

We conclude with a comment on an alternative argument to evaluate the entropy stored in EFTs of cosmic inflation. In ref. [45], a central conclusion is that the maximal entropy of a four-dimensional EFT from modes at the horizon scales like $S_{\text{h}} = N\Lambda_{\text{UV}}^2/H^2$, which dominates over the entropy in bulk modes. Identifying $\Lambda_{\text{UV}} = m_{\text{P},4}/\sqrt{N}$, this agrees with the CEB. On the other hand, the entropy in bulk modes is estimated as $S_{\text{b}} = \sqrt{NV}/H^2$, with V the potential. Upon the same identification, we find $S_{\text{b}} = m_{\text{P},4}^2/\Lambda_{\text{UV}}H$, which differs from what we call S_{EFT} . Were it $S_{\text{EFT}} \leq S_{\text{b}}$, we would get further support for the bound in eq. (2.6). However, because $H \leq \Lambda_{\text{UV}} \leq m_{\text{P},4}$, we cannot establish which of the inequalities $S_{\text{b}} = (m_{\text{P},4}H/\Lambda_{\text{UV}}^2)^2 S_{\text{EFT}} \lesseqgtr S_{\text{EFT}}$ holds true. From $S_{\text{b}} \leq S_{\text{h}}$, one may only say that $\Lambda_{\text{IR}} \leq \Lambda_{\text{UV}}$.

²Notice that the fine details such as the order-one constants that we dropped (not being able to know them exactly) might alter this picture slightly, perhaps still allowing for $\Lambda_{\text{IR}} \leq \Lambda_{\text{UV}}$.

³In general, we may write $N = N_{\text{sp}}^\alpha$, for a power $\alpha \in [0, 1]$, since $1 \leq N \leq N_{\text{sp}}$. Hence, eqs. (2.6), (2.7) can be combined into the inequality $l_{\text{P},d}\Lambda_{\text{sp}} \leq (l_{\text{P},d}H/N_{\text{sp}}^\alpha)^p$, and we label $q = \alpha p$. So, for generality we include a second parameter q , but we do not change its relative sign to $p > 0$.

3 Bounds on inflationary dynamics

We focus on single-field models of cosmic inflation. In the slow-roll regime, the energy scale is set by the Hubble parameter H , which is related to the scalar potential V as $H^2 = 2l_{\text{P},d}^{d-2}V/[(d-1)(d-2)]$. By the very definition of UV cutoff we must have $\Lambda_{\text{UV}} \geq \Lambda_{\text{IR}}$, implying⁴

$$l_{\text{P},d}\Lambda_{\text{sp}} \geq (l_{\text{P},d}H)^{(d-1)p}. \quad (3.1)$$

In accordance with general quantum gravity principles, we expect the species scale to decay exponentially at the boundary of field space [40, 46, 47], i.e.

$$\Lambda_{\text{sp}} \geq m_{\text{P},d} e^{-\kappa_d \lambda \varphi}, \quad (3.2)$$

for an order-one constant λ . Equivalently, the number of species N_{sp} grows such that

$$N_{\text{sp}} \leq N_0 e^{\kappa_d (d-2) \lambda \varphi}. \quad (3.3)$$

Possible polynomial dependencies in $N_0 = N_0(\varphi)$ are not going to be relevant for our analysis. At the boundary of field space, we expect these inequalities to be close to saturation. From now on, we thus take $\Lambda_{\text{sp}} = m_{\text{P},d} e^{-\kappa_d \lambda \varphi}$ and $N_{\text{sp}} = N_0 e^{\kappa_d (d-2) \lambda \varphi}$. Below, we explore the consequences of combining the bounds in eqs. (2.8), (3.1).

First, a consequence of eqs. (3.1), (3.2) is the upper bound $\varphi \leq \varphi_2$, where

$$\kappa_d \varphi_2 = -\frac{(d-1)p}{2\lambda} \ln(l_{\text{P},d}^d V_2). \quad (3.4)$$

Second, a consequence of eqs. (2.8), (3.2), (3.3) is the lower bound $\varphi \geq \varphi_1$, where

$$\kappa_d \varphi_1 = -\frac{1}{2\lambda} \frac{p}{1-(d-2)q} \ln(l_{\text{P},d}^d V_1), \quad (3.5)$$

where we highlight that $q \leq (1-p)/(d-2) < 1/(d-2)$ because $p > 0$, so $\varphi_2 \geq \varphi_1 \geq 0$. Therefore, an inflationary theory fulfilling both eqs. (2.8), (3.1) must evolve with the field φ ranging at most through the interval

$$\varphi_1 \leq \varphi \leq \varphi_2. \quad (3.6)$$

If a potential does not host an inflationary phase within the interval in eq. (3.6), the model does not comply with the proposed UV/IR-mixing constraints; see figures 1 and 2. In particular, a necessary condition on the inflationary dynamics is the inequality anticipated in eq. (1.1), i.e.

$$\Delta\varphi \leq \varphi_2 - \varphi_1 = \frac{p}{2\kappa_d \lambda} \ln \frac{(l_{\text{P},d}^d V_1)^{\frac{1}{1-(d-2)q}}}{(l_{\text{P},d}^d V_2)^{d-1}}, \quad (3.7)$$

where $\Delta\varphi = \int_{t_i}^{t_e} dt \sqrt{\dot{\varphi}^2(t)} \leq \varphi_2 - \varphi_1$ is the field range spanned through the inflationary phase between the initial time t_i and the final time t_e , assuming a monotonic evolution. In eq. (3.7), we have a simple criterion that can be tested on any inflationary model more directly than the underlying principles in eqs. (2.8), (3.1). If the interval $[\min(\varphi_e, \varphi_i), \max(\varphi_e, \varphi_i)]$ is not

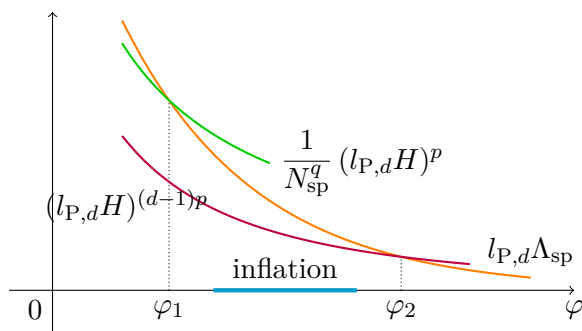


Figure 1. A visualization of the field range singled out by eqs. (2.8), (3.1). Due to UV/IR mixing, cosmic inflation should take place within the interval $\varphi \in [\varphi_1, \varphi_2]$.

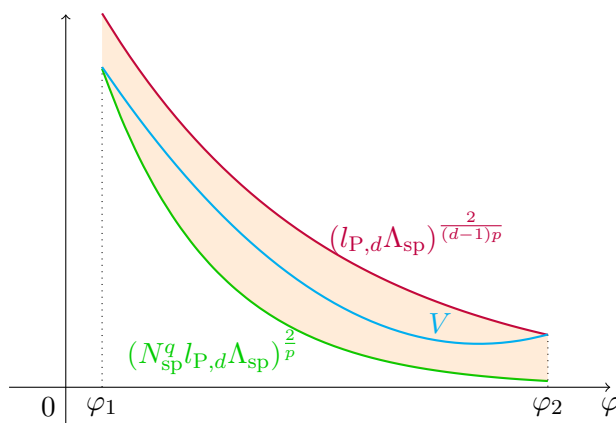


Figure 2. Another visualization of the region (orange area) allowed by the bounds in eqs. (2.8), (3.1). It might even allow for metastable dS minima (cyan curve).

entirely contained within the interval $[\varphi_1, \varphi_2]$, then the inflationary dynamics extend beyond the region in which the EFT is well-defined.

We stress that the content of eq. (3.7) is a global statement on the inflationary potential arising from UV/IR mixing. This means that, although the value of the potential is constrained locally, due to eqs. (2.8), (3.1), we also have a global prediction based on eq. (3.7), which is always very simple to test. Yet, we are unable to draw local conclusions on the derivatives of the potential. A priori, flat regions — including metastable de Sitter (dS) minima and maxima — are allowed, as long as such flat regions are well within the interval in eq. (3.6); see figure 2. A key point in our analysis is that we expect all constants in eqs. (3.4), (3.5) to be order-one: the species-scale falloff rate is conjectured to be bounded as $\lambda \leq 1/\sqrt{d-2}$ [40, 46], and we know that in the limiting case $p = q = 1/(d-1)$. This expectation is also supported by comparison with other theorized general properties of UV-consistent EFTs [29, 30, 32, 47], as we discuss below.

The bound described in eq. (3.7) is novel, but related to others in the literature. The arguments for the refined dS conjecture in ref. [29] are based on the CEB. Our results are

⁴Notice that one could impose directly $\Lambda_{\text{sp}} \geq \Lambda_{\text{IR}}$, as in refs. [6, 7], but this would yield weaker bounds than those derived below.

compatible with it, but following a slightly different logic. In this note, we are assuming eq. (2.6) — parameterized in the form of eq. (2.8) — and we are also employing eq. (3.1), i.e. $\Lambda_{\text{IR}} \leq \Lambda_{\text{UV}}$: it is the combination of these two bounds that leads to eq. (3.7). In ref. [29], only eq. (2.6) is made use of explicitly.⁵ Via the identification of the maximal UV-cutoff, the latter reads

$$N^\gamma (m_{\text{P},d} R)^\delta \leq (m_{\text{P},d} R)^\beta, \tag{3.8}$$

with $\gamma = 1/d$ and $\delta = (d-1)(d-2)/d$ and $\beta = d-2$. It is expected for the number of light states to grow like $N = N_0 e^{b\kappa_d \varphi}$, and we stress that $b = (d-2)\lambda$ if we identify $N = N_{\text{sp}}$. Via the relationship $l_{\text{P},d}^d V = 1/(m_{\text{P},d} R)^2$, then, eq. (3.8) implies

$$l_{\text{P},d}^d V \geq N^{-\frac{2\gamma}{\beta-\delta}} = e^{-\frac{2b\gamma}{\beta-\delta} \kappa_d \varphi}. \tag{3.9}$$

The next step in ref. [29] — which we have not taken here — is to saturate the bound in eq. (3.9) in view of the expectation that light degrees of freedom dominate the Hilbert space in the weak-coupling regime. This eventually leads to the refined dS conjecture. While we are not able to infer local predictions on the derivatives of the potential, the refined dS conjecture infers the local bound [28, 29]

$$\gamma_{\text{dS}} \geq c \quad \vee \quad \eta_{\text{dS}} \leq -c', \tag{3.10}$$

for two positive universal order-one constants c and c' , where in the single-field case $\gamma_{\text{dS}} = \sqrt{(\partial V/\partial \varphi)^2}/(\kappa_d V)$ and $\eta_{\text{dS}} = (\partial^2 V/\partial \varphi^2)/(\kappa_d^2 V)$. For a positive non-increasing potential, given any interval $\varphi \in [\varphi_a, \varphi_b]$, the condition $\gamma_{\text{dS}} \geq c$ implies

$$V(\varphi_b) \leq V(\varphi_a) e^{-c\kappa_d(\varphi_b - \varphi_a)}. \tag{3.11}$$

Various arguments support the bound $c \geq 2/\sqrt{d-2}$ in the field-space asymptotics [30, 32, 48]. If we take $\varphi_{a,b} = \varphi_{1,2}$, this corresponds to taking $(d-1)p = \sqrt{d-2}\lambda$ and $q = 1/(d-1)$ in eq. (3.7). Away from the field-space asymptotics, the transplanckian censorship conjecture [30] also leads to the inequality

$$V(\varphi_b) \leq m_{\text{P},d}^d A e^{-\frac{2\kappa_d(\varphi_b - \varphi_a)}{\sqrt{(d-1)(d-2)}}}, \tag{3.12}$$

for an order-one constant A . Demanding $H \leq \Lambda_{\text{sp}}$ leads to the same conclusion, with $A < 1$ [47]; see also refs. [7, 49]. Let k be the constant such that $[l_{\text{P},d}^d V(\varphi_a)]^k = A$. Then, eq. (3.7) would imply $\sqrt{d-1}p = \sqrt{d-2}\lambda$ and $(d-1)q = (d-1-1/k)/(d-2)$.

Let us now discuss some implications of eq. (3.7).

3.1 Limiting case

To begin with, let us discuss the limiting case, where we assume $\Lambda_{\text{UV}} = \Lambda_{\text{sp}}$, i.e. we fix $p = q = 1/(d-1)$.

⁵Of course, eq. (3.1), namely $\Lambda_{\text{IR}} \leq \Lambda_{\text{UV}}$, also underlies ref. [29], but it is not explicitly used to impose an additional constraint.

According to eqs. (2.8), (3.1), the potential is bounded as $l_{\text{P},d} e^{-2\lambda\varphi} \geq l_{\text{P},d}^{d-2} V \geq l_{\text{P},d} e^{-2\lambda\varphi}$. In other words, it is an exponential, with slope $\gamma = 2\lambda$, namely

$$V = \mu^d e^{-2\kappa_d \lambda \varphi}. \tag{3.13}$$

More generally, eq. (3.13) is compatible with general expectations from string compactifications, where the scalar potential may have a non-trivial profile in intermediate regions between the bulk and the field-space asymptotics, but is expected to vanish exponentially in limits such as zero string coupling and infinite volume [6, 29].

3.2 A bound on extra dimensions

The UV/IR mixing parameters p and q and the species-scale decay rate λ are central in our analysis. On the other hand, we also have inflationary observables, such as the tensor-to-scalar ratio, r , the amplitude of scalar perturbations, A_s , the spectral tilt, n_s , and the number of e-folds, N_e , whose values are constrained experimentally [50, 51]. In the spirit of ref. [34], we may use observational data as inputs to constrain p , q and λ , with $d = 4$.

In slow-roll inflation, the power spectrum of curvature fluctuations for the mode wavenumber k reads $\mathcal{P}_{\mathcal{R}}(k) = 2\kappa_4^4 V / (3\pi^2 r) |_{k=1/l_H}$ [25], with $r = 16\epsilon_V$, where l_H is the comoving Hubble radius. Observationally, it is best-fit by the curve $\mathcal{P}_{\mathcal{R}}(k) = A_s (k/k_*)^{n_s-1}$, where $A_s \simeq 2.10 \cdot 10^{-9}$ and $n_s \simeq 0.96$, for the pivot scale $k_* = 0.05 \text{ Mpc}^{-1}$ [50, 51]. By considering V and r to be approximately constant — which means ignoring the k -dependence, as acceptable in a rough approximation being $n_s - 1 \simeq -0.04$ — we may then rewrite eq. (3.7) as

$$\lambda \lesssim \frac{p}{\kappa_4 \Delta\varphi} \frac{1 - 3q}{1 - 2q} \ln \frac{10^8}{r}. \tag{3.14}$$

This bound is different from the one of ref. [34] for two main reasons. First, ref. [34] uses $H_2 \simeq \kappa_4 \sqrt{V_2/3}$, while our bound features the ratio H_1/H_2 . Second, even if we identify $H_1 \simeq 1/\kappa_4$, our bound is more general due to the presence of the UV/IR mixing parameters p and q . Then, one can calculate $\Delta\varphi$ as a function of N_e , r and possibly additional parameters of a given model, and substitute it in eq. (3.14).

Each specific inflationary model gives specific predictions on $\Delta\varphi$ and, in turn, constraints on p, q and λ . To remain as conservative as possible, we can replace $\Delta\varphi$ by the Lyth bound [52–54]

$$\kappa_4 \Delta\varphi \geq \frac{N_e}{60} \sqrt{\frac{r}{0.002}}, \tag{3.15}$$

leading to

$$\lambda \leq \frac{1 - 3q}{1 - 2q} \frac{60p}{N_e} \sqrt{\frac{0.002}{r}} \ln \frac{10^8}{r}. \tag{3.16}$$

This upper bound can further be combined with the lower bound [40, 46, 47]

$$\lambda \geq \sqrt{\frac{n}{2(n+2)}} \tag{3.17}$$

computed for n extra spacetime dimensions, giving the constraint presented in eq. (1.2), i.e.

$$\sqrt{\frac{n}{n+2}} \leq \frac{1-3q}{1-2q} \frac{60p}{N_e} \sqrt{\frac{0.004}{r}} \ln \frac{10^8}{r}. \quad (3.18)$$

This is a bound on the number n of extra dimensions in terms of inflationary (r, N_e) and UV/IR-mixing (p, q) parameters. In particular, it implies that future detection of primordial gravitational waves might set an upper bound on the number of extra spacetime dimensions. We also see that a smaller r allows for a larger n . For instance, if we saturate the experimental upper limit $r \lesssim 0.056$ and we take the indicative value $N_e \simeq 60$ [50], we find $\sqrt{n/(n+2)} \lesssim 5.7p(1-3q)/(1-2q)$. One can see that if q is close to the maximal value $q = (1-p)/2$, the bound becomes impossible to satisfy for any $n \geq 1$. On the other hand, we may write $p \gtrsim (1/5.7)[(1-2q)/(1-3q)]\sqrt{n/(n+2)}$. For instance, exploiting the bound $1/3 \geq p$, we may infer $q \lesssim 0.29$ or $1/3 < q < 1/2$, computed for the least restrictive case $n = 1$ (larger n gives a lower maximal value).

It is apparent that the most critical dependence is the one on r : finding experimental evidence for a very small r would relax the restrictions on n .

If the assumption on the constant potential is dropped, we can still write

$$\frac{N_e}{60} \sqrt{\frac{r}{0.002}} \leq \frac{p}{2\lambda} \ln \frac{(\kappa_4^4 V_1)^{\frac{1}{1-2q}}}{(\kappa_4^4 V_2)^3}. \quad (3.19)$$

One can then repeat a similar analysis as above.

4 Examples

In this section we overview the implications of eqs. (3.4), (3.5), (3.7) for a number of proposals of slow-roll cosmic inflation [55]. In appendix A, we discuss proposed realizations of string-theoretic dS vacua.

We have bounds constraining the scalar potential, namely in eqs. (2.8), (3.1). As discussed, a (weaker) global statement is the bound in eq. (3.7), relating the largest admissible field excursion to the global variation of the scalar potential. This results in a simple criterion that we can apply to readily raise tensions in given inflationary models.

4.1 Chaotic inflation

Let us consider a prototypical chaotic inflationary model with a power-law potential [56]

$$V(\varphi) = m_{\text{P},d}^d (\beta \kappa_d \varphi)^{2m}, \quad (4.1)$$

where $m, \beta > 0$ are parameters. Such models are in tension with data [57], but it is nevertheless instructive for us to discuss them. Indeed, the problem we will stumble upon will be recurrent in other models, too, that are instead compatible with current observations. This problem is the fact that φ_e lies outside the allowed inflaton field range in eq. (3.6). The advantage of the present setup is that we will be able to work at each step with analytic expressions.

The values of $\varphi_{1,2}$ setting the boundaries of the allowed inflaton range (3.6) are the solutions to the equations

$$e^{-h_{1,2}\lambda\kappa_d\varphi_{1,2}} = (\beta\kappa_d\varphi_{1,2})^m, \quad (4.2)$$

where, for brevity, we defined $h_1 = [1 - (d-2)q]/p$ and $h_2 = 1/[(d-1)p]$. Notice that their ratio is bounded as $h_1/h_2 = [1 - (d-2)q](d-1) \leq d-1$. We also define $\xi_{1,2} = h_{1,2}\lambda$. An analytic solution can be written in terms of the principal branch of the Lambert W -function, i.e.

$$\kappa_d \varphi_{1,2} = \frac{m}{\xi_{1,2}} W_0\left(\frac{\xi_{1,2}}{m\beta}\right). \tag{4.3}$$

Below, we argue that, for generic order-one parameters $\xi_{1,2}$, the point where inflation would stop lies outside the allowed inflaton range. For brevity, we fix $d = 4$.

Inflation can occur as long as the slow-roll parameters ϵ_V and $|\eta_V|$ are small. Since in this class of models $\epsilon_V = 2m^2/(\kappa_4\varphi)^2$ and $\eta_V = (2 - 1/m)\epsilon_V$ we focus on the condition $\epsilon_V \ll 1$ for brevity. In particular, inflation stops at the latest when $\kappa_4\varphi \simeq \sqrt{2}m$. By plugging in the solution for $\varphi_{1,2}$, we find

$$\sqrt{\epsilon_V(\varphi_{1,2})} = \frac{\sqrt{2}\xi_{1,2}}{W_0\left(\frac{\xi_{1,2}}{m\beta}\right)}. \tag{4.4}$$

To have $H \ll m_{\text{P},d}$, one must fix $\beta \ll 1$. Hence, if we assume that $\xi_{1,2} = O(1)$, we may conclude that⁶

$$\sqrt{\epsilon_V(\varphi_{1,2})} \simeq \frac{\sqrt{2}\xi_{1,2}}{\ln\left(\frac{\xi_{1,2}}{m\beta}\right)} \ll 1. \tag{4.5}$$

Hence, as the inflaton rolls towards the origin, it will cross the value $\varphi = \varphi_1$ while ϵ_V is still tiny; in other words, one would have $\varphi_e < \varphi_1$. If instead $\xi_{1,2} \ll m\beta$, eq. (4.4) behaves as $\sqrt{\epsilon_V(\varphi_{1,2})} \simeq \sqrt{2}m\beta \ll 1$. Because the function W_0 is monotonically increasing, we thus learn that the only way to achieve $\sqrt{\epsilon_V(\varphi_1)} > 1$ is if $h_{1,2} \gg O(1)$. Otherwise, we conclude that at $\varphi_{1,2}$, where the EFT violates UV/IR-mixing, inflation is still dynamically allowed since ϵ_V and η_V are small.

Let us give a concrete example. An inflationary scale $H \lesssim 10^{-5}m_{\text{P},4}$ [51] requires $\beta\kappa_4\varphi \lesssim 10^{-5/m}$, so we fix $\beta = 10^{-4/m}$. To be optimistic and get a large range $\varphi_2 - \varphi_1$, we pick the smallest h_2 and then the largest h_1 . This means first picking $p = p_{\text{max}} = 1/3$, i.e. $h_2 = 1$, and then $q = q_{\text{min}} = 0$, i.e. $h_1 = 3$.⁷ The largest ϵ_V -value within the interval in eq. (3.6) is $\epsilon_V(\varphi_1)$. If e.g. $\lambda = 1/\sqrt{2}$, we see that $\epsilon_V(\varphi_1) > 1$ only for $m \geq 3$. In that case, however, $\epsilon_V(\varphi_2)$ is too large for realistic inflationary scenarios; e.g. for $m = 3$, $\epsilon_V(\varphi_1) = 2.20$ and $\epsilon_V(\varphi_2) = 0.56$. Increasing m will increase ϵ_V at both points.

4.2 α -attractors

A well-studied class of four-dimensional inflationary models leading to testable predictions collectively go by the name of α -attractors [15–18, 20]. For recent reviews, see refs. [59–61].

⁶At large x , we have $W_0(x) = \ln x - \ln \ln x + O[(\ln \ln x)/\ln x]$ [58].

⁷Of course, maximizing the range $\varphi_2 - \varphi_1$ according to eq. (4.3) is a complicated optimization problem. We stick to the choice in the main text since the example is just for illustrative purposes.

Two basic classes of α -attractors are the so called T-models and E-models, with respective potentials

$$V_T(\varphi) = \left(\frac{\mu}{\kappa_4}\right)^4 \tanh^{2m} \frac{\kappa_4 \varphi}{\sqrt{6\alpha}}, \quad (4.6a)$$

$$V_E(\varphi) = \left(\frac{\mu}{\kappa_4}\right)^4 \left(1 - e^{-\sqrt{\frac{2}{3\alpha}} \kappa_4 \varphi}\right)^{2m}, \quad (4.6b)$$

where $\alpha, \mu, m > 0$ are parameters. For $\alpha \gg 1$, both models reduce to chaotic inflation and our previous discussion applies. Instead, the attractor behavior is manifest when α is order-one, such that the spectral index n_s and tensor-to-scalar ratio r only depend on α and the number of e-folds N_e . In particular, for $\alpha = m = 1$, the E-model reduces to the Starobinsky model [12].⁸

By following the same strategy as for chaotic inflation, we will show that for α -attractor models, too, the inflaton range allowed by eq. (3.6) is such that the EFT loses validity while inflation is still happening. In other words, φ_e is not contained in the allowed interval in eq. (3.6). Again, this will occur generically for order-one parameters, but we cannot exclude the possibility that by appropriately fine-tuning the parameters p, q and λ one can avoid our conclusion.

The starting point is to determine the extrema, $\varphi_{1,2}$ of the interval in eq. (3.6), i.e. the solutions to

$$e^{-2h_{1,2}\lambda\kappa_4\varphi_{1,2}} = V_T(\varphi_{1,2}), \quad (4.7a)$$

$$e^{-2h_{1,2}\lambda\kappa_4\varphi_{1,2}} = V_E(\varphi_{1,2}), \quad (4.7b)$$

again for $h_1 = [1 - (d-2)q]/p$ and $h_2 = 1/[(d-1)p]$. We are unable to find analytic solutions, but the problem can be understood qualitatively and discussed numerically.

To start, we recall that we must fix $\mu \ll 1$ in order to have $H \ll m_{P,4}$. Then, given $\xi_{1,2} = h_{1,2}\lambda$, we identify three possible situations; see also figure 3.

- (a) If $\xi_2 \leq \xi_1 \leq O(1)$, the equations have solutions only away from the origin, in the plateau-like region.
- (b) If $\xi_2 \leq O(1) \leq \xi_1$, φ_1 is in the power-law region and φ_2 still in the plateau.
- (c) If $O(1) \leq \xi_2 \leq \xi_1$, the solutions take place in the power-law region near the origin.

For case (c), we refer directly to section 4.1. Below, we thus discuss only cases (a) and (b). We remark that we are considering $q < p$; if $p = q$, we already know from section 3.1 that the potentials would not be compatible with the bound since they are monotonically-increasing for $\varphi > 0$.

In case (a), both φ_1 and φ_2 are in the plateau region, therefore the solution does not stop inflating when φ crosses the point φ_1 , where the potential is still shallow. Hence, $\varphi_e < \varphi_1$ in

⁸In supergravity embeddings, α is related to the field-space curvature through $\kappa_4^2 R_M = -2/3\alpha$. However, especially when non-linearly realized, supergravity alone does not guarantee UV-consistency. Indeed, embedding α -attractors into a microscopic theory of quantum gravity, such as string theory, remains an open challenge. We refer to refs. [62–64] for recent discussions on the specific case of the Starobinsky model.

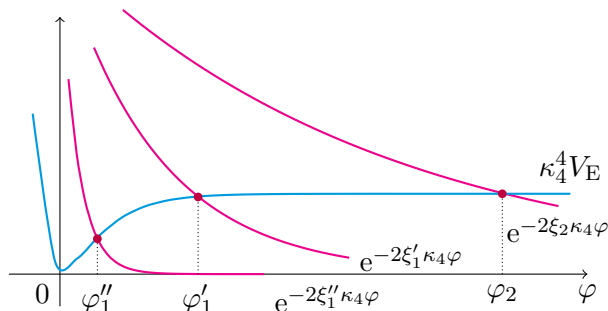


Figure 3. A visualization of the solutions for eqs. (3.4), (3.5) for the E-model potential: for fixed φ_2 , larger values $\xi_1'' > \xi_1'$ provide more room for inflation, being $\varphi_1'' < \varphi_1'$. The plot for the T-model is qualitatively similar for $\varphi > 0$.

this case. Yet, one can ask whether the number of e-folds between φ_2 and φ_1 , N_{21} , is very large. For both classes of α -attractors, at leading order in N_{21} , we can calculate

$$\kappa_4 \varphi_2 \simeq \sqrt{\frac{3\alpha}{2}} \ln N_{21}, \tag{4.8}$$

which depends only on α , but not on m and μ . By plugging in eqs. (4.7a), (4.7b), we find

$$N_{21} \simeq [\kappa_4^4 V_{T,E}(\varphi_2)]^{-\frac{1}{\sqrt{6\alpha}} \frac{1}{\xi_2}}. \tag{4.9}$$

Approximating $V_{T,E} \simeq (\mu/\kappa_4)^4$, we get

$$N_{21} \simeq \mu^{-\sqrt{\frac{8}{3\alpha}} \frac{1}{\xi_2}}, \tag{4.10}$$

which is extremely sensitive to the values p and λ in $\xi_2 = \lambda/3p$. For instance, let $\mu = 10^{-5/2}$, $\alpha = 1$, and $\lambda = 1/\sqrt{2}$: for the limiting case $p = 1/3$ we find $N_{21} \simeq 6 \cdot 10^6$, while for $p = 1/12$ we get $N_{21} \simeq 28$. By keeping the first correction to eq. (4.8), too, we get

$$\ln N_{21} \simeq \ln \left(e^{\sqrt{\frac{2}{3\alpha}} \kappa_4 \varphi_2} - e^{\sqrt{\frac{2}{3\alpha}} \kappa_4 \varphi_1} \right). \tag{4.11}$$

We write $q = (1 - p)/2 - \delta/2$, with $\delta > 0$ parameterizing how far we are from saturating the upper bound on q . Then, we can write $e^{\kappa_4 \varphi_1} = e^{\kappa_4 \varphi_2 / [3(p+\delta)]}$, whence

$$\ln N_{21} \simeq \sqrt{\frac{2}{3\alpha}} \kappa_4 \varphi_2 + \ln \left(1 - e^{\sqrt{\frac{2}{3\alpha}} \frac{1-3(p+\delta)}{3(p+\delta)} \kappa_4 \varphi_2} \right). \tag{4.12}$$

We see that we get a negative contribution from the logarithm in the right-hand side — which is bigger in absolute value for smaller $p + \delta$ —, making N_{21} smaller.

In case (b), the value φ_1 is such that ϵ_V may be large. Yet, N_e is dominated by the value of φ_2 , according to the discussion of case (a). This shows that, if p and q are not close to their limiting values, the bound of eq. (3.7) is not extremely restrictive in this case either.

We now illustrate a numerical example. Again, we fix $(h_1, h_2) = (3, 1)$ and $\lambda = 1/\sqrt{2}$. For an inflationary scale $H \simeq 10^{-5} m_{P,4}$, we pick $\mu \simeq 10^{-5/2}$. We choose $\alpha = m = 1$ because it corresponds to the Starobinsky model for the E-potential, but we checked that

for values $(\alpha, m) \in [10^{-1}, 10^2]$ our considerations are qualitatively similar. The solutions to eqs. (4.7a), (4.7b) are $\kappa_4(\varphi_1, \varphi_2) \simeq (5.44, 16.28)$ for the T-model and $\kappa_4(\varphi_1, \varphi_2) \simeq (5.43, 16.28)$ for the E-model. At φ_2 the value of the slow-roll parameters is $(\epsilon_V, |\eta_V|) \simeq (1.51 \cdot 10^{-11}, 4.50 \cdot 10^{-6})$ for the T-model and $(\epsilon_V, |\eta_V|) \simeq (3.79 \cdot 10^{-12}, 2.25 \cdot 10^{-6})$ for the E-model. At φ_1 the value of the slow-roll parameters is $(\epsilon_V, |\eta_V|) \simeq (7.40 \cdot 10^{-4}, 3.00 \cdot 10^{-2})$ for the T-model and $(\epsilon_V, |\eta_V|) \simeq (1.92 \cdot 10^{-4}, 1.58 \cdot 10^{-2})$ for the E-model. Hence, we see that the inflationary EFT stops being valid while inflation is still dynamically allowed. Besides, we find that $N_{21} \simeq 6980$ and $N_{21} \simeq 7037$ for the T- and E-models, respectively. However, even small variations on p and q have a drastic impact on N_{21} . For instance, let us pick the maximal value $p = 1/3$ and q close to the maximal value ($q = 1/3$), e.g. $q = 0.3$. This means $h_1 = 1.2$ and $h_2 = 1$, deviating from the optimistic case above. We find $\kappa_4(\varphi_1, \varphi_2) \simeq (13.57, 16.28)$ for both the T- and the E-model. The values of the slow-roll parameters are still tiny across the entire interval. However, in this case we see that $N_{21} \simeq 9.14$ for both models.

4.3 Modular-invariant cosmologies

Recently, a novel class of inflationary models has been proposed in which the action of the scalar field is required to be modular-invariant [22–24, 65, 66]. In this class, one can construct potentials that at large field value resemble closely the α -attractor shape, but that may differ at small field values. By direct inspection, one can check that several potentials constructed in this way are of the generic type

$$V = \left(\frac{\mu}{\kappa_4}\right)^4 (1 - \rho e^{-a\kappa_d\varphi})^m, \tag{4.13}$$

where $\rho \leq 1$ and a is order-one. In particular, this is the case for the explicit models of refs. [22, 23]. Then, for these potentials, our previous analysis on standard α -attractors can be repeated almost without modifications and our conclusions are unchanged.

5 Discussion

Motivated by general ideas in quantum gravity, such as holography and entropy bounds, we have investigated a specific form of mixing between UV and IR cutoff scales in EFTs describing cosmic acceleration. This resulted in bounds constraining the scalar potential, in eqs. (2.8), (3.1), and in the bound in eq. (3.7), relating the largest admissible field excursion to the global variation of the scalar potential, which represents a simple criterion. For models of early cosmic inflation, we argued that the bound is challenging to obey while describing the whole inflationary epoch within a given EFT. Although with a number of ways out, which we discussed, our results thus hint at a generic tension between four-dimensional inflationary models and general quantum gravity principles. This follows despite more conservative assumptions than earlier studies [29, 30, 67, 68]. The tension emerges when combining large field excursions with comparatively small variations in the potential, as supposedly happens in plateau-like regions. On the other hand, because the dark energy-dominated era has only lasted for less than one e-fold, it is harder to formulate constraints for the present universe that are based on a bound on the total admissible field displacement, like ours. Nevertheless,

constraining models of quintessence with large field excursions may still elucidate the fate of cosmic horizons in string compactifications.

We also note that the assumption of slow-roll can be relaxed. There exist general classes of potentials — expected in string compactifications, like multi-exponential potentials [6, 69] — such that the dynamics is effectively single field-like and that the kinetic and potential energies evolve at a fixed non-negligible ratio [70]. This fact is crucial in extensions of the distance conjecture in the presence of a potential [71, 72] and, likewise, it immediately extends the applicability of eq. (3.7). Since $H^2 = \sigma \kappa_d^2 V$ beyond the slow-roll approximation, for an order-one constant σ , our whole analysis still carries out.

The specific implications of eq. (3.7) depend on the precise values of p , q and λ , which we have refrained from fixing, but we can list some general observations.

- As showed in examples, eq. (3.7) can lead to a situation in which inflation stops neither at φ_1 nor at φ_2 because the potential is still sufficiently flat at those points. This happens generically for order-one parameters p , q and λ . In this case, we are not allowed to describe the whole inflationary dynamics while fulfilling the proposed bounds on UV/IR mixing.
- Another constraint stemming from eq. (3.7) is in the form of an upper bound on the number of e-folds. For certain models, such as e.g. α -attractors, this bound is highly sensitive to the values of p , q and λ .
- An important caveat is that, during reheating, the CEB and entropy bounds in general are not well-motivated due to the lack of a cosmic horizon. However, if the potential is shallow across the whole interval $[\varphi_1, \varphi_2]$, as is actually happening in the examples, the EFT becomes invalid while still in the proper inflationary epoch, before the reheating phase.
- A general limitation of the bound in eq. (3.7) is that it requires knowledge of the potential at $\varphi_{1,2}$, but the inflationary phase may take place well in the interior of the interval in eq. (3.6).
- As a conclusive comment, which applies to all setups considered in this work, the fact that $\kappa_4 \varphi_{1,2}$ are large but not parametrically so suggests that these models do not strictly live at the boundary of field space. In this sense, UV/IR mixing is teaching us that microscopically consistent models of inflation, if they exist, cannot be in asymptotic regions of field space, where one typically gains computational control. Since computational control in the bulk is usually a consequence of supersymmetry, it is not clear to us how it can be gained in these inflationary models; see e.g. refs. [73, 74].

There are several future directions that one could pursue. The most natural one consists in sharpening our knowledge of the parameters p and q in eq. (2.8), which will provide much more precise predictions on the allowed potential variation across the maximum field range, as well as on the allowed number of e-folds. Another natural one consists in generalizing our analysis to multi-field models. The current conclusions technically hold only for EFTs that are effectively one-dimensional in field space during the inflationary phase. Although this is a feature of a large class of cosmological models, alternatives exist and conclusions might be different.

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A KKLТ and LVS scenarios

In this appendix we apply qualitatively our bounds on two concrete proposals for constructing dS vacua in string theory; for general reviews, see e.g. refs. [75–77].

A.1 KKLТ

The Kachru-Kallosh-Linde-Trivedi (KKLT) scenario [78] is a proposal for constructing dS vacua in string theory. As a four-dimensional supergravity construction, it features a potential generated by 3-form fluxes in a type-IIB Calabi-Yau orientifold, together with non-perturbative corrections and an uplift term associated to a supersymmetry-breaking anti-D3-brane.

A complete understanding of the string embedding of this model is still elusive.⁹ Nevertheless, we will argue that the putative EFT is in tension with the bounds from UV/IR mixing discussed in the main text. For our purposes, we just need the scalar potential [83]

$$\kappa_4^4 V = \frac{a|A|}{2e^{ac}} \left[\frac{a|A|}{3ce^{ac}} \left(1 + \frac{3}{ac} \right) - \frac{|W_0|}{c^2} \right] + \frac{\sigma^2}{c^2}, \quad (\text{A.1})$$

where $\kappa_4\varphi = \sqrt{3/2} \ln c$ is the canonical Einstein-frame radion, with a stabilized axion. All other fields are assumed stabilized at higher energy scales and appear with their expectation values inside all terms: the Gukov-Vafa-Witten superpotential W_0 , the constants a and A , depending on the details of non-perturbative correction, and the doubled tension $2\tau_3 = m_{\text{P},4}^4 \sigma^2 / c^2$ of an uplifting anti-D3-brane at the tip of a warped throat [84]. For appropriate parameters, the potential has a unique metastable vacuum at large volume $c \gg 1$, such that

$$|W_0| = \frac{2|A|}{3} \left(1 + \frac{3}{2ac} \right) \frac{ac}{e^{ac}} + \frac{4\sigma^2}{a^2|A|} \frac{e^{ac}}{\left(1 + \frac{2}{ac} \right) c}. \quad (\text{A.2})$$

If the uplift term is such that

$$\sigma^2 = \frac{a^2|A|^2}{6} \left(1 + \frac{2}{ac} \right) c e^{-2ac}, \quad (\text{A.3})$$

the vacuum is Minkowski. For a tiny (positive) cosmological constant, we can still take the value of c fixed by the previous relation as a good approximation of the volume vacuum expectation value.

To fulfill eq. (3.5), one needs

$$c \geq c_1 = \left[\frac{m_{\text{P},4}^4}{V_1} \right]^s, \quad (\text{A.4})$$

⁹See refs. [79–81] for recent criticisms, and ref. [82] for recent efforts towards complete top-down realizations.

with $s = p/[\sqrt{6}(1 - 2q)\lambda]$, which is expected to be an order-one parameter. If $c_1 \ll c$, the observationally-viable potential is dominated by the σ -independent positive term, and one should solve $(a^2|A|^2/6)^s c_1^{1-s} = e^{2asc_1}$: because of the exponential term, at $c_1 \gg 1$ — which is necessary for perturbative control — there is no solution. If $c_1 \simeq c$, since near the minimum the potential is tiny — i.e. $\langle V \rangle \simeq 10^{-120} m_{\text{P},4}^4$ —, the power should be very small for the volume $c^{3/2} \gtrsim 10^{180s}$ to allow for an observationally-viable KK scale, besides being compatible with the microscopic setup itself. These claims can be supported numerically. Our conclusion that KKLT violates holographic bounds — within the controlled EFT region — is in line with previous results, such as [29, 45, 81].

A.2 LVS

The Large Volume Scenario (LVS) is another proposal for a dS vacuum in string theory which relaxes the degree of fine tuning of KKLT through a specific internal geometry and with the inclusion of α' -corrections [85].

The EFT features two kinetically-mixed moduli τ_b and $\tau_s \ll \tau_b$, such that the Einstein-frame compactification volume scales as $\tau_b^{3/2} - \tau_s^{3/2} \simeq \tau_b^{3/2}$. Although our discussion does not apply directly to a multi-field model, we can explore the bound along the τ_b -direction, if τ_s sits at its minimum. In this case, one can take $\tau_b = e^{\sqrt{2/3} \kappa_4 \varphi}$ and the potential takes the form¹⁰

$$V = \frac{\mu_1^4}{\tau_b^{\frac{3}{2}}} - \frac{\mu_2^4}{\tau_b^3} + \frac{\mu_3^4}{\tau_b^{\frac{9}{2}}} + \frac{\mu_4^4}{\tau_b^{3n}}, \tag{A.5}$$

for four constants $\mu_{1,2,3,4}$, with the last term representing a generic uplift term. It is apparent that the tensions that appear in the KKLT potential are evaded: as we are just balancing different powers, it is conceivable to expect solutions to $\tau_{b,1} = (m_{\text{P},4}^4/V_1)^s$ such that $1 \ll \tau_{b,1} < \langle \tau_b \rangle$, where $\langle \tau_b \rangle$ is the standard LVS solution. In this case, knowledge of all the parameters is crucial.

Data Availability Statement. This article has no associated data or the data will not be deposited.

Code Availability Statement. This article has no associated code or the code will not be deposited.

¹⁰In the absence of a suitable uplift, the LVS scalar potential is

$$\frac{V}{m_{\text{P},4}^4} = 4a|A| \left[\frac{2a|A|}{3} \frac{\tau_s^{\frac{1}{2}}}{\tau_b^{\frac{3}{2}}} e^{-a\tau_s} - |W_0| \frac{\tau_s}{\tau_b^3} \right] e^{-a\tau_s} + \frac{3\xi}{2} \frac{|W_0|^2}{\tau_b^{\frac{9}{2}}},$$

where ξ controls the α' -corrections. In the anti-dS vacuum, with $\langle V \rangle/m_{\text{P},4}^4 = -(3/4a)|W_0|^2/\langle \tau_s \rangle \langle \tau_b \rangle^{9/2}$, the solution reads

$$\begin{aligned} \langle \tau_b \rangle^{\frac{3}{2}} &= \frac{3}{4a} \frac{|W_0|}{|A|} \langle \tau_s \rangle^{\frac{1}{2}} e^{a\langle \tau_s \rangle}, \\ \langle \tau_s \rangle &= \xi^{\frac{2}{3}}. \end{aligned}$$

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