SENSITIVITY-BASED OPTIMAL SHAPE DESIGN OF INDUCTION-HEATING DEVICES

P. Di Barba\(^{(1)}\), F. Dughiero\(^{(2)}\), M. Forzan\(^{(2)}\) and E. Sieni\(^{(2)}\)

Paolo Di Barba
University of Pavia, Department of Electrical, Computer and Biomedical Engineering
Via Ferrata, 1
27100 Pavia, Italy
Phone: +39 0382 985265
Fax: +39 0382 422276
e-mail: paolo.dibarba@unipv.it

Fabrizio Dughiero
University of Padova, Department of Industrial Engineering
Via Gradenigo 6/A
35131 Padova, Italy
Phone: +39 049 8277708
Fax: +39 049 8277599
e-mail: fabrizio.dughiero@unipd.it

Michele Forzan
University of Padova, Department of Industrial Engineering
Via Gradenigo 6/A
35131 Padova, Italy
Phone: +39 049 8277591
Fax: +39 049 8277599
e-mail: michele.forzan@unipd.it

Elisabetta Sieni (Corresponding author)
University of Padova, Department of Industrial Engineering
Via Gradenigo 6/A
35131 Padova, Italy
Phone: +39 049 8277514
Fax: +39 049 8277599
e-mail: elisabetta.sieni@unipd.it
**ABSTRACT.** A Design Of Experiment (DOE) strategy applied to multiobjective optimization is proposed in order to evaluate the influence of design variables variations to optimized quantities. A secondary objective function is the sensitivity of a primary objective function to design variable variations evaluated by means of DOE strategy. The optimization problem includes also a third objective function that considers device constraint due to technological limitations on power generator. The proposed case study deals with the design of an electromagnetic device that will be used to carry out laboratory experiments on magneto fluid hyperthermia, that is a clinic treatment for cancer cure. The induction system is designed to apply a controlled time varying magnetic field to biological cells, cultured in Petri dish, mixed with magnetic nanoparticles. The paper presents an original cost-effective method of multi-objective design optimization taking into account design uncertainties.

**Keywords:** finite elements, DOE, multiobjective optimization, design variables uncertain

**1 INTRODUCTION**

Currently, the optimal design of electromagnetic device is largely researched also to include the effect of uncertainties on design variables or parameters [1–7]. In fact, in production process, the device and its components are affected by tolerances that can significantly modify its performance [8–12] and tolerance intervals are given to each geometrical dimension. The design of a device needs to find optimal solutions insensitive to small perturbations of design variables. Various strategies to take into account uncertain variables in the design of a device have been proposed, see e.g. [1–
a comparative review of optimization procedures based on worst case scenario can be found in [2], while an approach based on the approximated Lipschitz constant is proposed in [15]. Another possible strategy is the concept of multidimensional hypercube centred on the current solution [16]. More generally, parametric and non-parametric multiobjective optimization can be used in the design of electromagnetic devices [17–27].

The aim of the proposed multiobjective optimization is to design an inductor to obtain homogenous magnetic field intensity in the bottom of a Petri dish used in some experiments of magneto-fluid hyperthermia [28–31]. In particular, the aim is to obtain a uniform magnetic field in a prescribed region to heat a magnetic nanoparticle fluid by means of a time-varying magnetic field at radio-frequency in the range of 100–400 kHz [32,33]. Accordingly, the main objective function is the field inhomogeneity, to be minimized with respect to the geometric variables of induction-heating device; in this paper, the geometric model depends on five design variables. The second objective function is the sensitivity of the solution against small perturbations in a subset of three design variables, to be minimized too. Perturbations of design variables have been investigated using Design Of Experiment (DOE) strategy [34,35]. Finally, a third objective function is defined in terms of either the voltage at inductor ends or the supplied electrical current. Therefore, an optimal shape-design problem characterized by three-objective space is investigated. A detailed description of the case study is given in Section 3.

In the past, the DOE strategy has been proposed e.g. in [34] to numerically evaluate the sensitivity of a solution with respect to a small perturbation of some parameters not incorporated in the design variable set, like material properties in a problem of optimal shape design. In the paper, in turn, the DOE strategy has been applied to evaluate the
sensitivity of a solution just with respect to variations of design variables in a cost-effective way; moreover, sensitivity is considered as an additional objective function. Methodological aspects are focused on in Section 2.

2 SENSITIVITY COMPUTATION METHODS

The sensitivity of design variables was computed using a DOE strategy [34] in order to evaluate the effect of a set of uncertain parameters meant as quantities different from the design variables. In contrast, the same DOE strategy is here applied to a subset of $N_p$ out of $N$ design variables, $N_p < N$ ($N$ total number of design variables, $N_p$ number of uncertain design variables in the subset).

2.1 DOE method

According to the multifactorial DOE strategy [35] in the case of $N_p = 3$, given the current solution of the optimization problem, four extra solutions ($Y_j$, $j=1,4$) are computed by varying the values of the uncertain variables, $p_k$ $k=1,..3$, around their current nominal value, as shown in Table 1. To consider more than three uncertain variables, according to the DOE strategy proposed by Placket-Burmann [35], a table of sign alternance with more experiments has to be considered. Sign alternance follows the Placket-Burmann rule [35]. For instance, when the uncertain variables are between 4 and 7, 8 supplementary experiments, $k=1,...,8$, are required. In Table 1, it is assumed that the $N_p$ uncertain design variables has been attributed an uncertainty range; then the signs ‘+’ and ‘-‘, in Table 1, correspond to select the upper or lower limit in the range of the design variable uncertainty, respectively.
TABLE 1 Table of design: sign alternance of uncertain variables $p_k$, $k=1,3$ for evaluating sensitivity

<table>
<thead>
<tr>
<th></th>
<th>$p_1$</th>
<th>$p_2$</th>
<th>$p_3$</th>
<th>$f_i$ ($i=1$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_1$</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>$f_{i,1}$</td>
</tr>
<tr>
<td>$Y_2$</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>$f_{i,2}$</td>
</tr>
<tr>
<td>$Y_3$</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>$f_{i,3}$</td>
</tr>
<tr>
<td>$Y_4$</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>$f_{i,4}$</td>
</tr>
</tbody>
</table>

So, given a solution, four values of the $f_i$ objective functions, named $f_{i,j}$, $j=1...4$, ($f_{i,1}, f_{i,2}, f_{i,3}, f_{i,4}$), are computed by varying the design variable values as described in the following. Given the step function defined as:

$$U(Y_j, p_k) = \begin{cases} 
1 & \text{if } (Y_j, p_k) = '+' \\
0 & \text{if } (Y_j, p_k) = '-' 
\end{cases}$$  \hspace{1cm} (1)

the sensitivity is evaluated this way [34]: for the $k$-th uncertain design variable, $k=1,3$, the sums of $f_i$ values ($f_{i,j}$) corresponding to a ‘+’ in Table 1, $S_{+,p_k}$, and the ones corresponding to a ‘-’, $S_{-,p_k}$, are computed as follows:

$$S_{+,p_k} = \sum_{j=1}^{4} f_{i,j} U(Y_j, p_k)$$  \hspace{1cm} (2)

$$S_{-,p_k} = \sum_{j=1}^{4} f_{i,j} [1 - U(Y_j, p_k)]$$  \hspace{1cm} (3)

Then, the influence $s_{p_k}$ of a variation of the $k$-th design variable on the value of objective function $f_i$ is evaluated as [34]:

$$s_{p_k} = \frac{S_{+,p_k}}{N_+} - \frac{S_{-,p_k}}{N_-}$$  \hspace{1cm} (4)
where \( N_+ \) and \( N_- \) are the number of sign ‘+’ and ‘-’ in the column corresponding to the considered variable in Table 1. For the \( k \)-th design variable the partial sensitivity, \( s_{pk} \), is estimated just using (4), which is the core equation. After (4) it can be remarked that the multi-factorial DOE (linear number of experiments) is cost-effective with respect to the full-factorial DOE (exponential number of experiments). Finally, the total sensitivity with respect to all the uncertain design variable, \( f_2 \), is given by:

\[
  f_2 = \sqrt{\sum_{k=1}^{3} \frac{x^2_k}{n_k}}
\]

(5)

Under a multi-objective context, \( f_2 \) given by (5) can well be regarded as a secondary objective function in addition to the design criterion \( f_1 \): this is exactly the leading idea of Section 3.

2.2 Method comparison and validation

For the sake of a comparison, the sensitivity of \( f_1 \) is computed also by means of a different method, i.e. using Taylor first-order approximation and varying only one design variable at a time inside the interval \( \pm d \), e.g. \( d=1 \text{ mm} \):

\[
  \frac{\partial f_1}{\partial x_i} \approx \frac{f_1(x_i + d) - f_1(x_i - d)}{2d} \left[ \frac{\text{points}}{\text{mm}} \right]
\]

(6)

Given the uncertainty \( \sigma_k \) in [mm] computed for the \( k \)-th design variable, and given the objective function \( f_1 \), computed for the current solution and evaluated in [points], the global effect of a variation of a design variable on the solution \( S_n, \sigma_{i,n}(S_n) \), is calculated using a classical method to evaluate measurement uncertainty [36].
Equation (7) is a more accurate method to evaluate the influence of a variation of a set of \( N_p \) variables on function \( f_i \). In fact, (7) includes the weight of a variation of each variable (with a value <1 or >1) evaluated as a derivative of the examined objective function. In this paper, the derivative is approximated numerically performing two extra simulations for each design variables around the solution evaluated by means of the optimization procedure. It is expected that (5), i.e. an estimation of the solution variability, be comparable to (7).

3 CASE STUDY

In Fig. 1(a) the cross section of the axi-symmetric geometry of the device considered as the case study is shown [37]. The electromagnetic device is composed of an inductor with two copper turns, four ferrite rings and a ferrite disc placed as in Fig. 1 (a). Ferrite blocks allows to shape the magnetic flux lines in order to achieve the prescribed field homogeneity. The size of ferrite blocks has been chosen using some preliminary results presented in [37], and considering ferrite elements commercially available. A Petri dish is placed in a thermally insulated box in order to mitigate the influence of the environment temperature; in Fig. 1 (a) the thermal box is sketched only to show the whole device: in fact, temperature field simulation falls out of the scope of the work. The magnetic problem is solved in time-harmonics conditions using a Finite Element (FE) code [38]. The inductor can be supplied by imposing either a current (e.g. 500 Arms at 350 kHz), or a voltage (600 Vrms at 350 kHz). A typical mesh (Fig. 1 (b)) exhibits 24,000 nodes and 9,900 second-order surface elements.
Table 2 shows the $N=5$ design variables characterizing the case study, with the relevant range; the uncertainty intervals of the $N_p$ design variable subset (height of the ferrite disks in the upper part of the device, i.e. $hf_0$, $hf_1$, $hf_2$ in Fig. 1(a)) are also prescribed. The other two design variables are the $z$-directed size of the inductor turn, $H_s$, and the distance between the two inductor turns, $st$.

3.1 Electromagnetic analysis problem

The electromagnetic axi-symmetric problem is solved using the $A$-$V$ formulation. The problem is solved in terms of the phasor of magnetic vector potential, $A$, coupled with the electric scalar potential, $V$. When the Coulomb gauge is applied on the magnetic vector potential, i.e. $\nabla \cdot A = 0$, the following coupled equations are solved [39]:

\begin{align*}
R &= 85\text{mm} \\
\text{hf}_0 &= [1, 30] \pm 1 \\
\text{hf}_1 &= [1, 30] \pm 1 \\
\text{hf}_2 &= [1, 30] \pm 1 \\
H_s &= [10, 60] \\
st &= [0, 30]
\end{align*}
\begin{equation}
\n\nabla \times \frac{1}{\mu} \nabla \times \hat{A} + j \omega \mu \frac{1}{\rho} \hat{A} = -\frac{1}{\rho} \nabla \hat{V}
\end{equation}

\begin{equation}
\n\nabla \cdot \frac{1}{\rho} (j \omega \mu \hat{A} + \nabla \hat{V}) = 0
\end{equation}

with \( \mu \) being the material permeability, \( \omega \) field pulsation and \( \rho \) material resistivity for copper turns, while \( \hat{A} \) is the phasor of the magnetic vector potential.

### 3.2 Optimization problem

The optimization aim is three-fold: maximizing the magnetic field homogeneity in the bottom of the Petri dish, minimizing the design sensitivity and limiting the voltage supply at inductor ends or, alternatively, the inductor current. Consequently, the following three objective functions have been considered:

\( (f_1) \) the inhomogeneity of the magnetic field, \( H \), on the bottom of the Petri dish, to be minimized as in [37] with a tolerance interval of \( \pm 10 \text{ A/m} \). Once equations (8) and (9) are solved, the \( H \) field intensity can be computed from magnetic vector potential \( A \) in a straight forward way. Therefore, the inhomogeneity of \( H \) in terms of the \( H \)-norm discrepancy, that is dimensionless, is evaluated on the bottom of the Petri dish on a fixed grid of points;

\( (f_2) \) the sensitivity of \( f_1 \) with respect to the set of uncertain design variables shown in Table 2, evaluated according to (5), to be minimized;

\( (f_3) \) the end voltage (or current) when the inductor is supplied by applying a current (or a voltage, respectively). The rationale is that, in general, the end voltage must not exceed the typical value available at the converter output (e.g. 700 Vrms as maximum voltage of capacitance with a current up to 700 Arms). The third objective function, \( f_3 \), can consider the following two cases:
(a) the inductor has been supplied by a current $I$ of 500 Arms at 350 kHz, and the voltage at the inductor ends ($f_3$) is minimized,
or, alternatively:
(b) the inductor has been supplied by a voltage of 600 Vrms at 350 kHz, and the supply current ($f_3$) is minimized.
The aforementioned objective functions, subject to bounds in Table II, have been minimized in the Pareto sense (i.e. search for the front of non-dominated solutions) using a standard evolutionary algorithm (NSGA-II).

An additional remark on voltage and current calculation in the inductor is worthwhile. The FE electromagnetic solution takes into account the actual distribution of current density in the inductor, $I_{\text{turn}}$, that in turn depends on the induced electric field and the voltage applied by an external supply. The last quantity represents the imposed current source:

$$
I_{\text{turn}} = -\frac{1}{\rho} \int_{S_{\text{turn}}} (j\omega \mu \hat{A} + \nabla \hat{V}) dS 
$$

where $S_{\text{turn}}$ is the cross sectional area of the inductor turn, normal to the current flow in a 2D axi-symmetric model. The inductor is composed by two turns series connected, so each turn must carry the same current intensity. When the external supply imposes a current intensity, the applied voltage is calculated for each turn using a circuital approach based on the node-voltage analysis to fulfil the requirements about total imposed current and series connected turns. The same circuital approach is applied to compute the complex voltage values of each turn when the total voltage is the supply value (the solution is trivial when the inductor comprises only 2 turns). As a consequence, the value of voltage (or current) to feed the inductor depends on the actual electromagnetic field distribution that affects the induced term.
The implemented version of NSGA-II algorithm \([15,25,34]\) exploits simulated binary cross-over (SBX algorithm \([40]\)) with a probability of crossover of 0.9 and polynomial mutation, with a mutation probability of 1/N. The distribution indices for crossover and mutation operators, are both equal to 20. The number of individual for each generation is 20 and the number of generations is 50. The optimization process lasted approximately one day using a 64 bit workstation with 24 GB RAM and an Intel Xeon CPU at 3.33 GHz. Results are presented in Section 4.

4 RESULTS

The results of the two optimization case studies defined in Section 3 are summarized. In case (a) the voltage at inductor ends is minimized, whereas in case (b) the current in the inductor is minimized.

4.1 Case (a)

Fig. 2 reports the approximated 3D Pareto front that was obtained by minimizing the three objective functions in the case of the inductor supplied by a constant current. Each point in Fig. 2 corresponds to a different FE analysis. Black crosses represent the non-dominated solutions among all generated individuals.

![Fig. 2: 3D objective space: generated individuals (green points) and approximated Pareto front (black crosses). Solutions in Table 3 have been highlighted.](image)
Fig. 3 reports the corresponding 2D orthogonal projections of the 3D front: sensitivity (f_2) and inductor voltage (f_3) as a function of the magnetic field inhomogeneity (f_1).

![Fig. 3: 2D orthogonal projections of Pareto fronts: (a) sensitivity and (b) voltage vs H inhomogeneity.](image)

In Table 3 a set of four solutions located along the Pareto fronts are reported in terms of design variables and objective functions values. The corresponding geometries are shown in Fig. 4.

<table>
<thead>
<tr>
<th></th>
<th>h_f0 [mm]</th>
<th>h_f1 [mm]</th>
<th>h_f2 [mm]</th>
<th>Hs [mm]</th>
<th>st [mm]</th>
<th>f_1</th>
<th>f_2</th>
<th>f_3 [V]</th>
</tr>
</thead>
<tbody>
<tr>
<td>S_0</td>
<td>8.02</td>
<td>17.16</td>
<td>7.54</td>
<td>60.00</td>
<td>8.52</td>
<td>188</td>
<td>614.9</td>
<td>656.3</td>
</tr>
<tr>
<td>S_1</td>
<td>1.93</td>
<td>14.76</td>
<td>29.39</td>
<td>42.19</td>
<td>28.84</td>
<td>1080</td>
<td>2.8</td>
<td>735.0</td>
</tr>
<tr>
<td>S_2</td>
<td>13.44</td>
<td>30.00</td>
<td>25.67</td>
<td>59.89</td>
<td>25.36</td>
<td>1048</td>
<td>12.9</td>
<td>616.5</td>
</tr>
<tr>
<td>S_3</td>
<td>20.45</td>
<td>19.82</td>
<td>21.33</td>
<td>49.72</td>
<td>3.79</td>
<td>376</td>
<td>182.6</td>
<td>737.8</td>
</tr>
</tbody>
</table>

![Fig. 4: Geometries of the designed device for solutions on Pareto front for the case (a).](image)

Fig. 5 (a) reports the magnetic field intensity along a line in the bottom of the Petri dish.

The magnetic flux lines and the magnetic field intensity on the Petri dish are shown in...
The typical value of magnetic field, in the examined case, is close to 7 kA/m (peak value).

Fig. 5 (b) Magnetic field in the bottom of the Petri dish as a function of the x coordinate. (b) Magnetic flux line of direct problem and magnetic field color map on the Petri dish for solution $S_0$.

The effects on objective functions due to a positive or negative variation of an uncertain design variable at a time (the ones used in the DOE computation) are reported in Table 4. For each solution, $S_n$, the optimized values are shown (row named ‘Start’). The second row, named (‘round’), shows the effect on objective functions due to rounding the design variable values to the nearest integer and, finally, the effect obtained by applying a perturbation to a single design variables in the set of $N_p$ ones. Considering the objective function $f_i$, the partial derivatives (6) are computed for the design variable $h_{f_0}$, $h_{f_1}$, $h_{f_2}$. In particular ‘$h_{f_k,+}$’ and ‘$h_{f_k,-}$’ corresponds to a positive or negative variation of the design variable $h_{f_k}$, $k=1,3$, respectively.

Table 4 Effect of a variation $d=\pm 1$mm on the solutions listed in Table 3.

<table>
<thead>
<tr>
<th>$S_n$</th>
<th>$h_{f_0}$ [mm]</th>
<th>$h_{f_1}$ [mm]</th>
<th>$h_{f_2}$ [mm]</th>
<th>$H_s$ [mm]</th>
<th>$st$ [mm]</th>
<th>$f_1$</th>
<th>$f_2$</th>
<th>$f_3$ [V]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_0$</td>
<td>Start</td>
<td>8.02</td>
<td>17.16</td>
<td>7.54</td>
<td>60</td>
<td>8.52</td>
<td>188</td>
<td>616.4</td>
</tr>
<tr>
<td>round</td>
<td>8</td>
<td>17</td>
<td>8</td>
<td>60</td>
<td>9</td>
<td>274</td>
<td>572.0</td>
<td>654.8</td>
</tr>
<tr>
<td>$h_{f_{0,+}}$</td>
<td>9</td>
<td>17</td>
<td>8</td>
<td>60</td>
<td>9</td>
<td>529</td>
<td>250.9</td>
<td>654.4</td>
</tr>
<tr>
<td>$h_{f_{0,-}}$</td>
<td>7</td>
<td>17</td>
<td>8</td>
<td>60</td>
<td>9</td>
<td>805</td>
<td>316.3</td>
<td>655.3</td>
</tr>
<tr>
<td>$h_{f_{1,+}}$</td>
<td>8</td>
<td>18</td>
<td>8</td>
<td>60</td>
<td>9</td>
<td>263</td>
<td>584.0</td>
<td>655.1</td>
</tr>
<tr>
<td>$h_{f_{1,-}}$</td>
<td>8</td>
<td>16</td>
<td>8</td>
<td>60</td>
<td>9</td>
<td>284</td>
<td>556.9</td>
<td>654.7</td>
</tr>
</tbody>
</table>
Finally, the sensitivity of each solution in Table 3, computed on design variables using the (4) and data in Table 4, is reported in Table 5. In order to compute (4), four extra FE computations are needed. To compute (7) the extra FE solutions are six. In order to compare the two methods to evaluate the sensitivity of a solution, Table 5 reports also the values of the sensitivity computed by means of DOE strategy. The values of the sensitivity computed using (4) are proportional with the ones computed using the DOE strategy during the optimization process (Fig. 6).

Table 5 Approximated partial derivatives computed using (6) and sensitivity, for solutions in Table 3, exploiting (5), (7) and Table 4.
It appears that the better solution in terms of magnetic field uniformity is the more sensitive to design variables variations, whereas the worst case in terms of homogeneity is the less sensitive.

### 4.2 Case (b)

Fig. 7 reports the 3D Pareto front obtained minimizing the three objective functions for the inductor supplied by a constant voltage. Each point in Fig. 7 corresponds to a different FE analysis. Black crosses represent the non-dominated solutions among generated individuals.
Fig. 7 3D objective space: generated individuals (green points) and approximated Pareto front (black crosses). Solutions in Table 6 have been highlighted.

Fig. 8 reports the corresponding 2D orthogonal projections of the 3D front: sensitivity ($f_2$) or current ($f_3$) as a function of the magnetic field inhomogeneity ($f_1$).

![Fig. 8: 2D projections of Pareto front: (a) sensitivity and (b) supply current vs H inhomogeneity.](image)

In Table 6 a set of three solutions on the Pareto front are reported in terms of design variables and objective functions values. The corresponding geometries are in Fig. 9.

**Table 6 Selected solutions on the Pareto front. Design variables, objective functions $f_1$, $f_2$ and $f_3$.**

<table>
<thead>
<tr>
<th></th>
<th>$h_{f0}$ [mm]</th>
<th>$h_{f1}$ [mm]</th>
<th>$h_{f2}$ [mm]</th>
<th>$H_s$ [mm]</th>
<th>$s_t$ [mm]</th>
<th>$f_1$</th>
<th>$f_2$</th>
<th>$f_3$ [A]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_0$</td>
<td>1.0</td>
<td>20.7</td>
<td>26.7</td>
<td>19.8</td>
<td>17.2</td>
<td>1041</td>
<td>13.0</td>
<td>304.2</td>
</tr>
<tr>
<td>$S_1$</td>
<td>5.1</td>
<td>12.1</td>
<td>4.5</td>
<td>30.8</td>
<td>22.0</td>
<td>229</td>
<td>579.3</td>
<td>347.0</td>
</tr>
<tr>
<td>$S_2$</td>
<td>10.3</td>
<td>18.3</td>
<td>15.2</td>
<td>13.8</td>
<td>22.4</td>
<td>357</td>
<td>122.3</td>
<td>290.5</td>
</tr>
</tbody>
</table>
Fig. 9 Geometries of the designed device for solutions on Pareto front for the case (b).

Fig. 10 (a) reports the magnetic field intensity along a line in the bottom of the Petri dish. The magnetic flux lines and the magnetic field intensity on the Petri dish are in Fig. 10 (b). In this case the typical value of magnetic field, in the examined case, is close to 6 kA/m (peak value); correspondingly, the current is close to 300 A.

In Table 7 the effects on objective functions of a positive or negative variation on a single design variable (only the ones used in the DOE computation) are reported. For
each solution, \( S_i \), the same evaluations reported in Table 4 have been repeated. Positive and negative variations on design variables as in case (a) have been applied.

Table 7 Effect of a variation \( d=\pm 1 \text{mm} \) on the solutions listed in Table 6.

<table>
<thead>
<tr>
<th>( S_i )</th>
<th>( hf_0 ) [mm]</th>
<th>( hf_1 ) [mm]</th>
<th>( hf_2 ) [mm]</th>
<th>( H_s ) [mm]</th>
<th>( st ) [mm]</th>
<th>( f_1 )</th>
<th>( f_2 )</th>
<th>( f_3 ) [A]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_0 )</td>
<td>Start</td>
<td>1</td>
<td>20.7</td>
<td>26.7</td>
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<td>13.7</td>
</tr>
<tr>
<td>round</td>
<td>1</td>
<td>21</td>
<td>27</td>
<td>20</td>
<td>17</td>
<td>1040</td>
<td>13.5</td>
<td>304.8</td>
</tr>
<tr>
<td>( hf_{0,+} )</td>
<td>2</td>
<td>21</td>
<td>27</td>
<td>20</td>
<td>17</td>
<td>1033</td>
<td>16.5</td>
<td>305.3</td>
</tr>
<tr>
<td>( hf_{0,-} )</td>
<td>0</td>
<td>21</td>
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</table>

Finally, the sensitivity of each solution in Table 6, computed on design variables using (4) and data in Table 7, is reported in Table 8. It can be underline that the values of the sensitivity computed using (4) are proportional to the ones computed using the DOE strategy during the optimization process. The two methods to evaluate the sensitivity are compared in Table 8 and Fig. 11 and also in this case the obtained values are correlated.

Table 8 Approximated partial derivatives computed using (6) and sensitivity, for solutions in Table 3, exploiting (5), (7) and Table 7.

<table>
<thead>
<tr>
<th>( S_i )</th>
<th>( \frac{\partial f_1}{\partial (hf_0)} )</th>
<th>( \frac{\partial f_1}{\partial (hf_1)} )</th>
<th>( \frac{\partial f_1}{\partial (hf_2)} )</th>
<th>( \sigma_{i,n} (S_n) ) (7)</th>
<th>( f_2 ) (5)</th>
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<td>( S_0 )</td>
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<td>-3</td>
<td>0</td>
<td>8.3</td>
<td>13.0</td>
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<td>( S_1 )</td>
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<td>-3</td>
<td>109.5</td>
<td>203.4</td>
<td>579.3</td>
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<td>71.5</td>
<td>115.8</td>
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Also for case (b), the lower sensitivity corresponds to the worst case in terms of magnetic field intensity uniformity, whereas it can be observed that considering solution with a better uniformity, the sensitivity increases. Moreover, considering the third objective function, the better solution shows a lower sensitivity.

The proposed multiobjective optimization gives to the designer the possibility to select the best feasible solutions in terms of field uniformity and in accordance with the power supply characteristics. The designer has to decide to achieve an excellent field uniformity using a weak solution in terms of sensitivity or vice versa accordingly to the his/her experience in practical realizing such a kind of devices. For instance, referring to Table 7, the selection of an optimal solution in terms of robustness could be $S_0$ at the expenses of a higher inhomogeneity; in contrast a solution like $S_1$ would be oriented to a more uniform magnetic field, but with higher sensitivity to fabrication tolerances. The final choice depends on the quality of the available manufacturing technology.

Accordingly to the optimization results, the Laboratory for the Electroheat of Padua University will realize an inductor with high uniformity of magnetic field, with low supplied voltage and accepting a quite high sensitivity. This design has been realized to

Fig. 11 Sensitivity - DOE computed using (5) as a function of sensitivity – analytical computed using (7) for the case (b).
carry out laboratory experiments where a high precision inductor manufacturing is required.

5 CONCLUSION
Optimization algorithms coupled to commercially available numerical tools can be used for the robust design of electromagnetic devices. Sensitivity analysis allows evaluating the influence effect of a variation on design variables on an objective function. This is important because during the production process the device or component is affected by manufacturing tolerances. Sensitivity computation can help the designer to exclude solutions largely affected by tolerance deviations.

Actually, sensitivity is used in manifold ways: e.g. it could be evaluated just at the start of the optimization procedure, by means of a technique of design of computer experiments, in order to identify a reduced set of design variables and so discard the less sensitive ones. Alternatively, it can be evaluated at the end of the optimization procedure, in order to assess the robustness of the optimized solution; moreover, when sensitivity is incorporated in the objective or constraint functions, there is an extra cost at each iteration for simulating the local perturbation. In the paper, sensitivity has been computed in a cost-effective way exploiting a multi-factorial approach to the design of experiments; subsequently, sensitivity has been considered as an auxiliary objective function, in addition to the main design criterion: therefore, a multi-objective design problem is originated that has been solved according to Pareto optimality theory; the proposed method has been validated by means of a real-life case study.

ACKNOWLEDGMENT
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6. Abdallh A, Crevecoeur G, Dupré L. Impact reduction of the uncertain geometrical parameters on magnetic material identification of an EI electromagnetic inductor


