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The effect of residual stress on fatigue behavior of V-notched components: a review

P. Ferro^a, F. Berto^{b,*}, S.M.J. Razavi^b, M.R. Ayatollahi^c

^a Department of Engineering and Management, University of Padova, Stradella S. Nicola, 3 I-36100 Vicenza, Italy.

^b Department of Mechanical and Industrial Engineering, Norwegian University of Science and Technology (NTNU), Richard Birkelands vei 2b, 7491, Trondheim, Norway.

^c Department of Mechanical Engineering, Iran University of Science and Technology, Narmak, 16846, Tehran, Iran.

Abstract

Fatigue strength of mechanical components in the high cycle regime depends on the intensity of the residual stress field induced by non-homogeneous plastic deformation or the solidification of a local portion of material due to welding operations. In presence of geometric variations modelled as sharp V-notch angle, the residual stress distribution near the notch tip is singular and follows the same solution obtained by Williams in 1952 where the intensity of the asymptotic stress field is quantified by the notch stress intensity factor (NSIF). However, the residual stress varies during fatigue loading until a stationary value is reached. Numerical models were developed for the calculation of the residual NSIFs and their variation under fatigue loading. Taking advantage from these models, new approaches were recently developed which are able to predict the fatigue strength of pre-stressed notched components. A review of such recent advances is described in this work.

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1. Introduction

Starting from William's work (1952), a great number of studies were carried out to evaluate the effect of local stress fields on the static and fatigue strength of mechanical components having a geometry variation modelled as sharp V-notch angle. Examples come from the fatigue strength of welded joints which is quantified in terms of

* Corresponding author. Tel.: +47-735-93831.

E-mail address: filippo.berto@ntnu.no

Notch Stress Intensity Factors (NSIFs) (Atzori and Meneghetti (2001), Atzori et al. (1999), Lazzarin and Livieri (2001), Lazzarin and Tovo (1998)) or local Strain Energy Density (SED) averaged over a control volume of radius R_C (Lazzarin and Zambardi (2001), Livieri and Lazzarin (2005), Berto and Lazzarin (2009)). In all these studies, the residual stress effect on fatigue strength of welded joints is included in reference curves obtained by elaborating a large amount of experimental data. This simplification is due to the difficulties to quantify the intensity and distribution of residual stress near the weld toe both by experiments and numerical models. A further complication is linked to the dependence of residual stress from welding parameters, geometry, clamping conditions, number of cycles and remotely applied stress. Such complications may be sufficient to discourage the use of expensive and time-consuming experimental techniques based on high intensity synchrotron X-ray and neutron radiation sources; but they are not sufficient to discourage the development of numerical models capable to capture the evolution of the as-welded and load-modified residual stress field near the most likely sites of failure initiation. Even if different numerical models were developed in the past with the aim to calculate the residual stress distribution in welded joints, the first work in which the asymptotic nature of the residual stress near the weld toe was revealed is dated 2006 (Ferro et al. (2006)). In that work, the effect of stationary and transient thermal loads on thermal and residual stress fields were described in detail. It was shown that both the thermal and residual stress fields near a V-notch tip are singular; the singularity degree, which depends on the V-notch opening angle, matches the elastic (Williams (1952)) or the elastic-plastic solution (Hutchinson (1968), Rice and Rosengren (1968)), depending on the magnitude of the thermal loads and clamping conditions. More in depth investigations followed that first result. The influence of clamping conditions and phase transformation effects (transformation plasticity (Leblond and Deveaux (1989)), specific volume change) on residual stress distributions were investigated in Ferro's (2012) and Ferro and Petrone's works (2009). In particular, it is worth mentioning that phase transformations affect the sign of residual asymptotic stress field so that, according to the material to be welded, a stress-relief heat treatment may enhance or decrease the fatigue strength of the joint.

In order to evaluate the influence of residual stress on fatigue strength of welded joints, the calculation of NSIFs related to as-welded joints is not sufficient. During cyclic load, a redistribution/relaxation is observed due to the plastic effects. The redistribution occurs during loading in the first cycle and it remains stable during the successive load cycles (Ferro et al. (2016)). This effect has to be considered in the low-cycle regime while it can be neglected in the high-cycle regime where the redistribution of residual stresses induced by plastic effects is negligible (small scale yielding hypothesis) (Ferro et al. (2016)). When the residual and stationary NSIF is calculated by a reliable numerical model, the residual asymptotic stress field can be treated as analogous to a 'mean stress' field as described in Ferro's work (2014). Aim of this work is to review the most recent advances in this field.

2. Asymptotic residual stress field

Before any model is developed which quantify the influence of residual stresses on fatigue strength of pre-stressed notched components, the distribution of residual stress near a 'geometric singularity' has to be first studied. Consider the problem of the elastic equilibrium in the presence of a V-shaped notch with an opening angle 2β (Fig. 1).

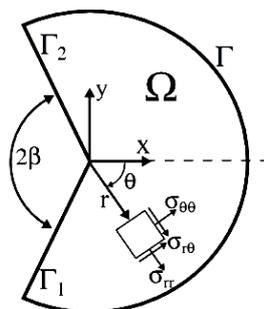


Fig. 1. Domain Ω for the sharp V-notch problem.

If the material is homogeneous and isotropic, under the hypothesis of linear, thermo-elastic theory and plane-strain conditions, the equations representing the stress field near the V-notch, are independent of the thermal terms and match the solution obtained by Williams (1952) (Ferro et al. (2006)). Whatever the load applied (thermal or mechanical), under linear-elastic hypothesis and plane-stress or plane-strain conditions, the induced stress field near the notch tip (by relating only to the first term of the Williams’ solution and mode I of V-notch opening), is described by the following asymptotic equation:

$$\sigma_{ij}(\theta) = \frac{K_I^{th,m}}{r^{1-\lambda_I}} g_{ij}(\theta) \quad (i, j = r, \theta) \tag{1}$$

where $g_{ij}(\theta)$ are the angular functions, λ_I is the first eigenvalue obtained from Eq. (2),

$$\lambda \sin(2\beta) + \sin(2\beta\lambda) = 0 \tag{2}$$

and $K_I^{th,m}$ is the NSIF due to a thermal (*th*) or mechanical (*m*) symmetrical load (opening mode I). According to Gross and Mendelson’s definition (1972):

$$K_I^{th,m} = \sqrt{2\pi} \lim_{r \rightarrow 0} r^{1-\lambda_I} \sigma_{\theta\theta} \quad (r, \theta = 0) \tag{3}$$

The first eigenvalue depends only by the V-notch angle (2β) and varies in the range between 0.5 and 1. The eigenvalue is 0.5 in the crack case ($2\beta = 0$), and increases to 0.674 and 0.757 when the notch opening angles are equal to 135 and 150 degrees, respectively. By simulating the solidification of a fusion zone (FZ) near the tips of a double V-notched plate ($2\beta = 135^\circ$), the asymptotic nature of residual stresses is revealed (Fig. 1).

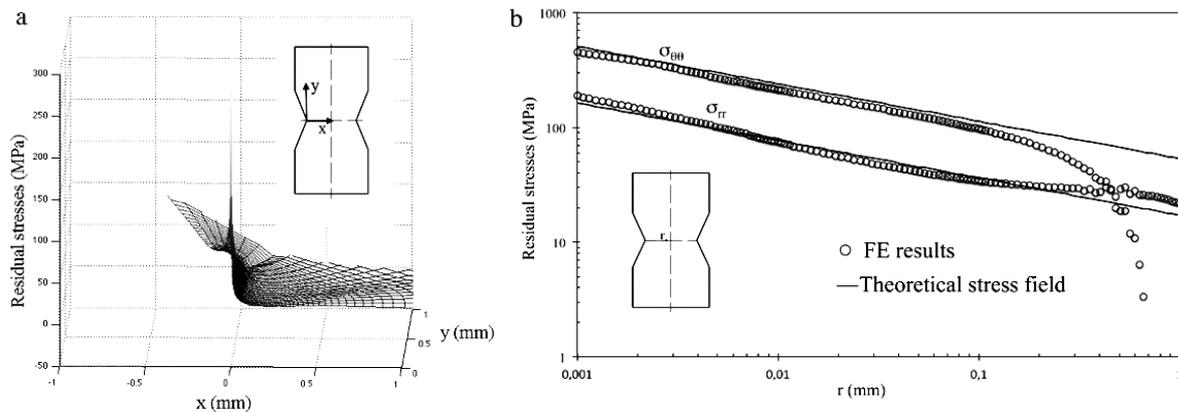


Fig. 2. (a) in-plane distribution of residual stresses (radial component, σ_r) near the notch tip; (b) Tensile residual stresses along the bisector of the V-notch ($2\beta=135^\circ$), ($K_I^{res} = 135 \text{ MPa}\cdot\text{mm}^{0.3264}$) (material: ASTM 11 SA 516, free edges) (Ferro, (2014)).

2.1. Influence of phase transformation on residual stress field

More in depth investigations were carried out in order to evaluate the influence of phase transformations on residual stresses. During solidification and cooling of a multi-phase material, the variation of the specific volume and the ‘transformation plasticity’ (Leblond and Deveaux (1989)) associated to phase transformation, influence the thermal and residual stresses induced by thermal loads. It was shown that such effects are so high that any numerical model of welding process that doesn’t take into account phase transformation effects fails in calculating the thermal and residual stress field (Ferro et al. (2006)). When the scale of observation is focused on about one tenth of the notch depth, it was observed that phase transformation changes the sign of residual stresses if compared to the

results obtained in a simplified mono-phase material (Ferro (2012)). This means that stress relief heat treatments may decrease the fatigue strength at high-cycle regime when the sign of the asymptotic residual stress is negative. Fig. 2 shows the asymptotic residual stress fields (θ component) along the bisector of the V-shaped weld toe calculated by taking into account phase transformation effects, volume change only and no phase transformations. As a general rule it was found that as-welded mono-phase materials, such as austenitic or ferritic stainless steels, are characterized by a compressive singular residual stress field, while multi-phase material such as carbon steels shows a tensile asymptotic residual stress field (under free-edge clamping condition).

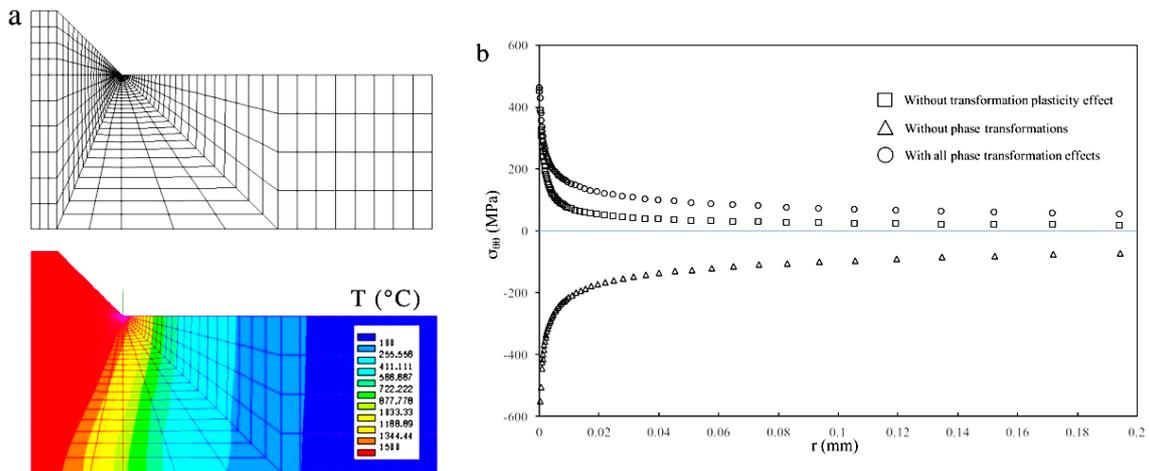


Fig. 3. (a) Mesh of the numerical model and fusion zone dimension and shape (butt-welded joint); (b) Phase transformation effects on residual stresses along the bisector of the V-notch ($2\beta=135^\circ$) (material: ASTM 11 SA 516, free edges).

2.2. Residual stresses redistribution

It is well known that residual stresses redistribute during cyclic load due to plastic effect. However, such redistribution is completed after few cycles and a residual NSIF (R-NSIF) stationary value is reached. It was found that the residual stress redistribution under fatigue loading is negligible in the high-cycle regime since the zone that experiments plastic deformation is restricted to about one tenth of the zone dominated by the elastic asymptotic residual stress distribution (small scale yielding hypothesis) (Ferro (2014)). On the other hand, stress redistribution increases as the remotely applied stress increases.

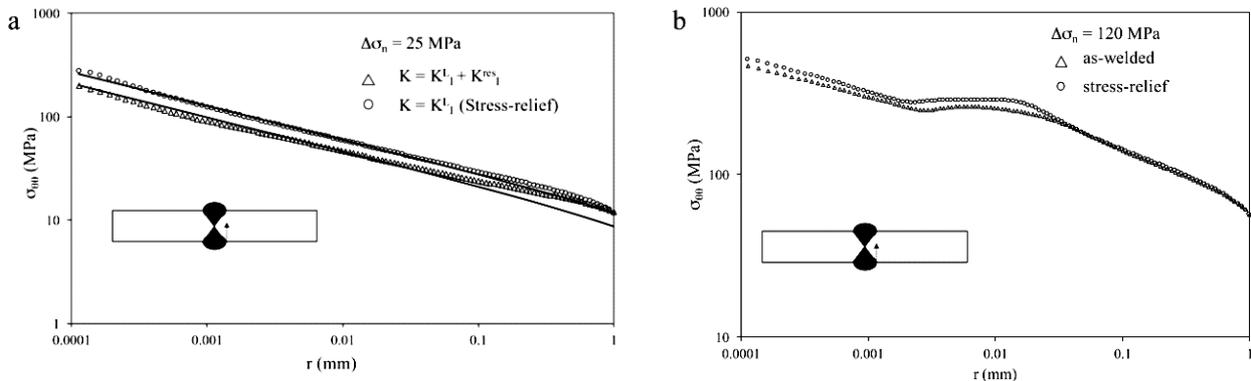


Fig. 4. Maximum transverse stress field of stress-relief and as-welded joint after ten cycles at different values of the remotely applied stress amplitude ($\Delta\sigma_n$) (a) $\Delta\sigma_n = 25$ MPa; (b) $\Delta\sigma_n = 120$ MPa; $2\beta=135^\circ$, $K_I^L =$ NSIF induced by load, $K_I^{res} =$ NSIF induced by the solidification of the FZ: AA 6063, free edges (Ferro et al. 2016).

At high stress amplitudes, plasticity ‘erases’ the pre-existing residual stress field so that there is not difference between the fatigue strength of a stress-relieved joint and an as-welded joint. This means that the superposition

property can be applied only in the high-cycle regime where the experimental results show that fatigue strength is sensitive to pre-existing residual stresses. In this case the R-NSIF can be summed to the stress-induced NSIF as shown in Fig. 4a. It can be noted that in that case, residual stresses are negative (single-phase material, AA 6063) so that they decrease the maximum transverse stress field (Fig. 4a). Fig. 4b shows that at high remotely applied stress amplitude, the plastic effects make the maximum transverse stress field almost insensitive to the pre-existing residual stress field.

3. Quantification of the influence of residual stresses on fatigue strength of welded joints

On the basis of the above mentioned developments, a model which quantifies the influence of residual stresses on fatigue strength of welded joints or pre-stressed notched components was finally developed (Ferro (2014), and experimentally validated (Ferro et al. (2016)). The model uses the concept of strain energy density (SED) averaged over a control volume of radius R_C (Lazzarin and Zambardi (2001), Livieri and Lazzarin (2005), Berto and Lazzarin (2009)), which under plane strain conditions and mode I loading takes the form:

$$\bar{W} = \frac{e_I}{E} \left[\frac{K_I^{th,m}}{R_C^{1-\lambda_I}} \right]^2 \tag{4}$$

In Eq. (4), the parameter e_I depends on V-notch opening angle (2β), Poisson’s ratio (ν) of the material and failure hypothesis. Under Beltrami failure hypothesis (total strain energy density) and plane strain conditions, $\nu = 0.34$ (aluminum alloy AA 6063) and $2\beta = 135^\circ$, e_I is equal to 0.111. The control radius (R_C) is a material characteristic length that for Al-alloy welded joints was found to be equal to 0.12 mm (Livieri and Lazzarin (2005), Berto and Lazzarin (2009)). Now, residual stresses have the effect of modifying the local load ratio (R). As a matter of fact, the following relationships hold true:

$$\left. \begin{aligned} \Delta K &= K_{I_{max}}^m - K_{I_{min}}^m \\ R &= \frac{K_{I_{min}}^m + K_I^{res}}{K_{I_{max}}^m + K_I^{res}} \end{aligned} \right\} K_{I_{min}}^m + K_I^{res} > 0 \tag{5}$$

$$R = \frac{K_{I_{min}}^m}{K_{I_{max}}^m}$$

$$\left. \begin{aligned} \Delta K &= K_{I_{max}}^m + K_I^{res} \\ R &= 0 \end{aligned} \right\} K_{I_{min}}^m + K_I^{res} \leq 0 \tag{6}$$

Where K_I^{res} is the R-NSIF which characterizes the residual stress field. R^m and R correspond to the local load ratio of the nominal and real cycle, respectively. Starting from Eqs (4-6), for $R = 0$, the following equation is obtained (details about the analytical frame employed are published in Ferro (2014)):

$$\Delta\sigma_n = \frac{R_C^{1-\lambda_I} \left[\frac{E}{e_I} \left(\frac{C}{N} \right)^{1/z} \right]^{1/2}}{k_I t^{1-\lambda_I}} - \frac{K_I^{res}}{k_I t^{1-\lambda_I}} \tag{7}$$

where $\Delta\sigma_n$ ($= \sigma_{n,max}$) is the nominal stress amplitude. Similarly, for $R>0$ the following relationship is obtained:

$$\sigma_{\max}^m = \frac{\left((K_I^{res})^2 + \frac{(1+R^m)E}{(1-R^m)} e_I \left(\frac{C}{N} \right)^{1/z} \right)^{1/2}}{k_I t^{1-\lambda_I} (1+R^m)} - \frac{K_I^{res}}{k_I t^{1-\lambda_I} (1+R^m)} \tag{8}$$

where C is a constant and z is the slope of the fatigue data expressed in terms of local strain energy density experimentally calculated ($z = \log(N_{D_1} / N_{D_2}) / \text{Log}(\Delta\bar{W}_{D_2} / \Delta\bar{W}_{D_1})$, subscripts D_1 and D_2 indicate two points of the curve $\Delta\bar{W}(N)$); k_I is a non-dimensional coefficient, analogous to the shape functions of cracked components calculated by using the following equation:

$$K_I^m = k_I \sigma_n t^{1-\lambda_I} \tag{9}$$

where σ_n is the remotely applied stress, and t is a geometrical parameter of the plate, according to Lazzarin and Tovo (1998). Eqs. (7,8) are applied in high-cycle fatigue regime where the redistribution of the pre-existing residual stresses is considered negligible (Ferro (2014)).

By using experimental results of fatigue strength of butt-welded stress-relieved and as-welded joints in AA 6063 (Bertini et al. (1998)), the model was validated in Ferro et al. (2016). Under the condition $K_{I_{min}}^m + K_I^{res} \leq 0$, Fig. 5 shows an estimation of the fatigue resistance of the stress-relieved and as-welded component predicted by means of the proposed model, Eq. (7). Due to the negative value of the R-NSIF, an improvement of fatigue strength of as-welded joints is observed experimentally and predicted by the model compared to the fatigue strength of the stress-relief specimens. It is worth mentioning that in this model the fatigue strength of as-welded joints in the low-cycle regime was set equal to that of stress-relieved specimens according to the redistribution/relaxation induced by high remotely applied stress amplitudes (Fig. 4b).

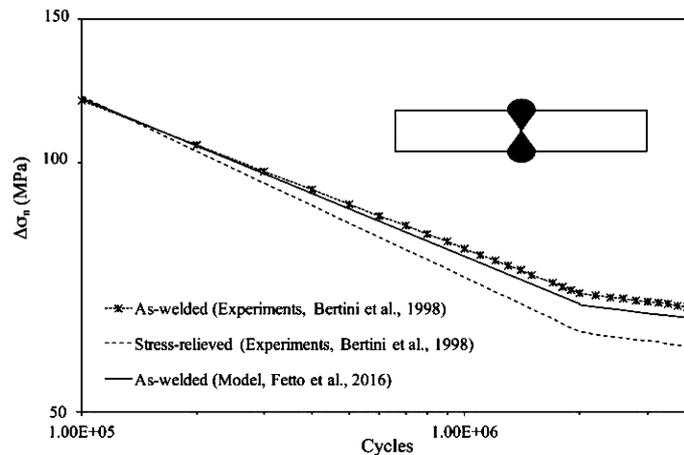


Fig. 5. Residual stress influence on fatigue strength of the AA 6063 butt-welded joint predicted by Eq. (7) (Ferro et al. (2016))

4. Conclusions

Starting from 2006, the asymptotic residual stress fields in notched components have been studied extensively with the aim to develop a model which quantifies the influence of residual stresses on fatigue strength of welded joints. Such asymptotic residual stress fields were found strongly influenced by mechanical constraints, geometry, process parameters and material. In particular, the sign of the residual stress field depends on phase transformations effects such as volume changes and transformation plasticity. For this reason, these effects cannot be neglected in any reliable numerical model of welding process. Furthermore, residual stresses redistribute during the fatigue load application because of the plastic deformation that occurs near the weld toe at high remotely applied stress

amplitudes. On the contrary, when such remotely applied stress amplitudes are low, the stress redistribution is not expected and the superposition principle can be applied. In such conditions, the residual stress field near the notch tip works in the same way as a mean stress and its influence on fatigue strength is quantified by the proposed model.

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