Analysis and control on networks: Trends and perspectives

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In the last twentyfive years, there has been a dramatic increase of the number of works devoted to the analysis of flows dynamics on networks based on hyperbolic partial differential equations (PDEs), which appear versatile tools for the development of control strategies and real-time computational methods more efficient and less expensive than the ones derived by microscopic and discrete models, in particular for large-scale systems.

The vitality of the field is also motivated by a wide range of strategic applications in modern society: from vehicular traffic and urban design to gas and water management, to biological and medical structures, to communication and data systems, to supply-chain planning, or also to social consensus patterns and cultural connections. In order to successfully address the challenging problems arising in these applications, the available analytic techniques of the theory of one-dimensional PDEs must be adapted to tackle the complex features of the specific models taken in consideration and combined with other tools and approaches developed within the control theoretic community. At the same time, a key role in this effort is played by the introduction and analysis of advanced numerical methods that allow to effectively test the theoretic findings in concrete problems.

The workshop *"Analysis and Control on Networks: trends and perspectives"* that took place in Padova on March 9-11, 2016, brought together some of the leading experts in the fields of hyperbolic conservation laws and of control and optimization of PDEs, that have achieved most influential recent advances in these areas. The present and the previous *Networks and Heterogeneous Media* Issue collect some significative recent results related to the research area of continuous flows on network structures, most of which were discussed in the aforementioned workshop, without the pretence of covering all themes of this broad field. In particular, this volume focuses on the following topics:

- Analysis of coupling conditions and of the corresponding solutions at a junction in hyperbolic, kinetic and parabolic models.
- Collective phenomena and self-organization driven by interaction networks arising in quantum synchronization and in opinion dynamics.
- Asymptotic behaviour of elastic-thermoelastic networks and control problems for piecewise deterministic Markow processes describing neuronal dynamics or communication networks.

More precisely, Gugat, Herty and Müller investigate the existence of solutions for a gas pipeline network in the case of transition at a junction from supersonic states in the incoming pipes to subsonic states in the outgoing ones with suitable coupling conditions. They establish an existence result extending the notion of "demand and supply" commonly used in traffic flow models.

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Borsche, Klar and Pham consider macroscopic nonlinear moment approximations of kinetic chemotaxis equations on a network that have the distinctive feature of ensuring the positivity of the cell density. For these macroscopic hydrodynamic models they introduce appropriate coupling conditions at the nodes of the network which are derived from the coupling conditions of the kinetic equations. An asymptotic preserving well-balanced scheme for these macroscopic nonlinear moment models on a graph is also presented and numerical tests are discussed for tripod and more general networks.

The paper by Corli, di Ruvo, Malaguti and Rosini is devoted to the analysis of travelling wave solutions for viscous, possibly degenerate, parabolic approximations of scalar conservation laws on a single node network. They consider coupling conditions which impose as usual the conservation of the total flow, and hence of the total mass, but don't require the continuity of the solution at the node. In this setting, necessary and sufficient algebraic conditions for the existence of travelling waves are provided in terms of the flux functions, the diffusivity coefficients and the end states.

Antonelli, Ha, Lim and Marcati present a kinetic model for a network of quantum oscillators derived from the Schrödinger-Lohe model using the Wigner formalism. They analyze the emerging dynamics and provide sufficient conditions for the complete synchronization of the oscillators.

The design of models for opinion dynamics on a general Riemannian manifold is the object of investigation of Aydoğdu, McQuade and Pouradier Duteil. They explore two different approaches: the first one is based on the local projection of the dynamics onto the tangent space to the manifold while the second one exploits intrinsic geometric properties of the manifold. Their analysis focuses on the long time behaviour, on the various type of equilibria that can be reached by the system as well as on the interaction network structure (i.e. the weights assigned to the interacting agents) that leads to these specific features.

Han and Zuazua analyze the asymptotic behaviour of a star-shaped network composed of elastic and thermoelastic rods with transmission conditions of conservative nature that guarantee the continuity of the temperature and displacement at the node. Necessary and sufficient conditions for the asymptotic stability of the network are given. Furthermore, exponential and polynomial decay rates are derived in the cases where one or more elastic components are present and suitable irrationality conditions on the lengths of the rods are satisfied.

The contribution of Ranault,Thieullen and Trélat falls in a rather different framework from the other papers of the volume, tackling an optimal control problem with finite time horizon for piecewise deterministic Markov processes. Here, the infinite dimensional nature of the deterministic component of the dynamics makes the analysis more delicate than in the usual approaches available in the literature. This study is inspired by neuronal models arising in Optogenetics, a recent and innovative biological technique which involves the use of light stimulation to control the activities of neurons that have been genetically modified to express light-sensitive ion channels. Piecewise deterministic Markov processes are well suited to describe a large variety of quite different phenomena, including the congestion of communication networks.

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