

# Super-resolution in near-field acoustic time reversal using reverberated elastic waves in skull-shaped antenna

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## 1 Summary

We investigate the potential of using elastic waves for near-field acoustic time reversal, and in doing so evaluate the possibility of reconstructing sound source positions at below-wavelength distances from a skull-shaped acoustic antenna. Our work is based on a conceptual processing model that translates elastic waves conducted and reverberated in an elastic object into source position, through a time reversal analysis. Signals are recorded by passive sensors glued on a replica of a human skull, measuring solely its mechanical vibrations, and not sensitive to airborne sound. The sound source is placed along the azimuthal and sagittal planes for distances to the skull between 5 and 100 cm. We reconstruct the source position for signals with frequencies in the physiological hearing range with a resolution indirectly proportional to the distance between source and skull across all measurements in the far-field. Measurements in the near-field show -3 dB widths smaller than half a wavelength (super-resolution) with highest resolutions of down to  $\lambda/15$  measured in front of the orbital cavities. We infer that these anatomical details give rise to complex features of the skull's Green's function, that in turn enhance resolution in a direction-dependent manner.

## 1 Introduction

It is well known [1] that anatomy contributes to the task of auditory source localization, as its effects on an acoustic signal, described by the head-related transfer function (HRTF) [37, 21], can be seen as a spectral filter and depend on the location of the signal's source. Human auditory source localization mostly relies on differences in the phase and amplitude of signals perceived by the two ears, as well as "spectral cues", or frequency-dependent effects associated with the shape of the pinnae and, possibly, other features of the body ([34]).

Building on the work of Catheline et al. [7], we ex-

plore here the specific role of elastic waves mediated in a skull-shaped object mimicking bone-conducted sound. While this study does not address the issue of whether and how bone conducted sound is employed by the human auditory (ears/brain) system, our goal is to determine whether these reverberated signals contain specific information about the reconstruction of the position of an auditory source, especially in the near-field. This could be relevant to current efforts in the study of bone conduction sound [36, 35, 31, 25, 32]. Using the principle of acoustic time reversal [16, 18], we convert the signal recorded by two receivers into the spatial coordinates of a source in the horizontal and sagittal plane, and evaluate the resolution with which the source position is thus reconstructed.

Catheline et al. [7] showed via a time reversal experiment with a dry skull that in-skull elastic wave propagation provides information about spatial positioning of a sound source. They found that their time reversal algorithm, using elastic waves alone, received at two or only one recording transducer mimicking the ear, successfully reconstructed the source position(s), for single as well as multiple sources. The spatial resolution of this method was found to decrease with increasing distance between the skull and the sound source. This is in good agreement with the far-field diffraction law, which provides a relationship between the spatial resolution and the distance separating the antenna (skull) from the source. Our objective is to expand the early work of Catheline et al. [7] and Ing et al. [22] to (1) analyze the resolution of the same algorithm for a skull-shaped antenna specifically in the near-field, i.e., the sound source is placed closer than one wavelength to the skull, and (2) to evaluate the directionality of the algorithm, i.e. evaluate changes in resolution with respect to angular position of the sound source.

In this study, we conduct a suite of experiments on a simple setup, equivalent to the setup used in Catheline et al. [7], consisting of two recording transducers glued to a replica human skull. Sound is generated

81 by a small speaker deployed at a variety of distances  
 82 and azimuths. Our results show in particular that,  
 83 in the near-field, the resolution with which we re-  
 84 construct the source position changes as a function  
 85 of azimuth with respect to the skull and is clearly  
 86 influenced by complex features of the skull such as  
 87 the orbital cavities. Furthermore we achieve super-  
 88 resolution throughout all angles for sources very close  
 89 to the skull.

90 Similarly minded experiment have been conducted  
 91 in recent years e.g. in the context of optics, where  
 92 imaging with evanescent waves allows to surpass the  
 93 classical diffraction limit; the super-resolution of near-  
 94 field microscopes is piloted by their probe size [28, 24].  
 95 In this context, a source [20, 12, 4] or scatterers [14]  
 96 smaller than a wavelength, placed within the medium  
 97 can be detected in the far-field with super-resolution  
 98 as well. Time reversal experiments can also surpass  
 99 the diffraction limit when resonators are placed near  
 100 a source [23, 29] or when an acoustic sink is used  
 101 [9]. To a lesser degree, near-field details can some-  
 102 times be extracted from the far-field using sophis-  
 103 ticated algorithms such as inverse filter [8] or MU-  
 104 SIC [30]. Experiments with metamaterials, super-  
 105 lenses and hyper-lenses [27] demonstrate moderate  
 106 sub-diffraction imaging down to a quarter of the op-  
 107 tical wavelength. All these techniques use different  
 108 terminology but they all require some near-field mea-  
 109 surements.

110 Because very few studies in psychoacoustics have  
 111 explored human sound localization performances for  
 112 nearby sources [26], we are unable to determine  
 113 whether the resolution achieved by our algorithm re-  
 114 produces the performance of human listeners using  
 115 bone conducted sound. While we do find that elastic  
 116 waves contain sufficient information to successfully re-  
 117 construct source positions in the near-field, we cannot  
 118 yet establish whether a similar capability is achieved  
 119 by the human auditory system.

## 120 2 Methods

121 The experimental setup is based on the previously  
 122 conducted experiment of Catheline et al. [7]: We  
 123 use a skull-shaped object (for simplicity from now on  
 124 called skull) made of the epoxy resin. The skull is  
 125 mounted on a rotatable rod with a reference (hori-  
 126 zontal) plane chosen approximately as a plane passing  
 127 through the area of the ethmoid bone above the vomer  
 128 and through the zygomatic arch and process of the  
 129 temporal bone. A conventional loudspeaker (RS Pro  
 130 TRG040008) is deployed sequentially at a discrete set  
 131 of positions in the horizontal and vertical plane. The  
 132 loudspeaker shows a flat frequency response between  
 133 200 Hz and 8 kHz. The distance between the source  
 134 (loudspeaker) and the skull (the point on the surface  
 135 of the skull closest to the speaker), denoted  $D$ , varies

136 from 5 to 100 cm, while the source position at each dis-  
 137 tance varies with angle  $\varphi$  between  $-50^\circ$  (i.e. down,left)  
 138 and  $+50^\circ$  (i.e. up, right). The experiment is con-  
 139 ducted in an anechoic chamber. Equipment which  
 140 could possibly reflect sound is covered with multiple  
 layers of sound dampening material. Two passive sen-

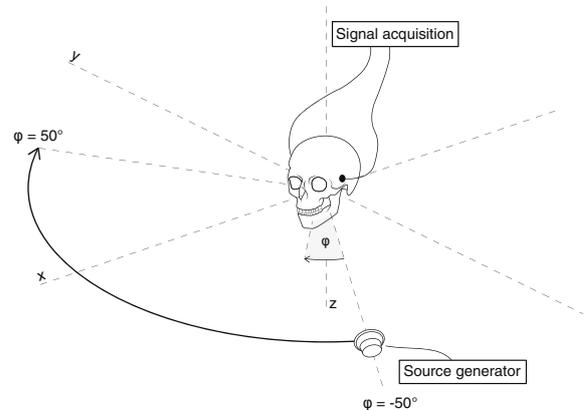


Figure 1: Sketch of the experimental setup in the horizontal plane: A loudspeaker is connected to a source generator (PC) and emits a chirp signal at each angle  $\varphi$  ranging from  $-50^\circ$  to  $50^\circ$  along a half circle at various distances to the skull. The resulting vibration of the skull is recorded through two passive sensors glued to the hypothetical ear locations. They are connected to the signal acquisition system, consisting of a sound card connected to a PC.

141 sors (Murata PKS1-4A), with a working bandwidth  
 142 ranging between 100 Hz and 15 kHz and a diameter  
 143 of 1 cm, are glued close to the hypothetical ear lo-  
 144 cations on both sides of the skull. They are used as  
 145 receivers to record the elastic vibrations and are con-  
 146 nected to a sound card (Soundscape SS8IO-3) which  
 147 has a 140 dB dynamic range and a 44.1 kHz sampling  
 148 frequency.

A sketch of the experimental setup in the horizontal plane is shown in Figure 1.

152 We checked that the sensors solely measure the vi-  
 153 bration of the skull and are unresponsive to airborne  
 154 sound. This ensures that the time reversal algorithm  
 155 will utilize only elastic waves. Additionally, the in-  
 156 fluence of the foam platform used to place the loud-  
 157 speaker at certain distances has been tested to have  
 158 no influence on sound emission of the loudspeaker.

The first part of the experiment consists of recording the signals at the sensors for each speaker position. The speaker emits a chirp signal  $c(t)$  with a duration of 1 s and a linear frequency distribution between 0 Hz and 6 kHz. The function in time for such a chirp of duration  $T$ , minimum frequency  $f_0$  and maximum frequency  $f_1$  reads

$$c(t) = \sin \left[ \Phi_0 + 2\pi \left( f_0 t + \frac{k}{2} t^2 \right) \right], \quad (1)$$

with the initial phase  $\Phi_0$  at time  $t = 0$  and the chirpy-

ness  $k = \frac{f_1 - f_0}{T}$  (in our case  $k = 6000 \text{ Hz/s}$ ), also  
 known as the rate of frequency range across the chirp.  
 For each distance  $D$  the source positions in the hori-  
 zontal plane are defined by the azimuth  $\varphi$ .

The recorded signal  $s$  at one of the sensors' location  
 $r$ , writes

$$s(\varphi_0, r, t) = c(t) * G(\varphi_0, r, t), \quad (2)$$

where  $*$  denotes convolution,  $\varphi_0$  is the source position  
 (azimuth) and  $G(\varphi_0, r, t)$  is the acoustic impulse re-  
 sponse of the skull, which is also the Green's function  
 of the signal emitted at  $\varphi_0$  and recorded at  $r$ , assum-  
 ing without loss of generality that emitter and receiver  
 are punctual. A representative waveform of a signal  
 recorded with one of the sensors and its normalized  
 frequency spectrum is shown in Figure 2. Note that

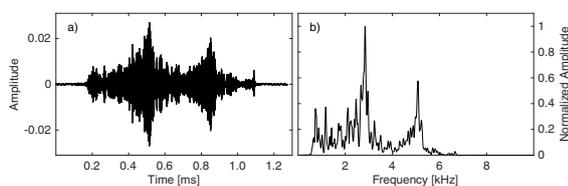


Figure 2: a) Exemplary waveform of a recorded signal  
 at one of the sensors. b) Frequency spectrum of the  
 same signal.

the spectra of all impulse responses (only one shown  
 here) show strong similarity to the results from Cath-  
 eline et al. [7] where a real dry skull was used and  
 its resonance frequencies were confirmed with other  
 studies of dry skulls and cadaver heads [32, 19]. This  
 proves that, in the first approximation and for the  
 purposes of our study, the epoxy skull replica em-  
 ployed here is sufficiently similar to a real skull. It  
 should be noted that, firstly, epoxy can have mechani-  
 cal properties similar to those of bone tissue ([2, 3]);  
 secondly, the most important role in our experiments  
 is presumably played by the outer shape of the skull,  
 driving wave propagation in air around the skull: and  
 the replica is designed to have realistic external shape.

Following Fink [17], the received signal  $s(\varphi_0, r, -t)$   
 is time-reversed, i.e flipped with respect to time. It  
 must then be backward propagated to any possible lo-  
 cation  $\varphi_i$ . This is equivalent to convolving  $s(\varphi_0, r, -t)$   
 with the Green's function  $G(\varphi_i, r, t)$ . Since we do not  
 have access to  $G(\varphi_i, r, t)$ , but we do have a library of  
 recordings of  $s(\varphi_i, r, t)$  for all possible values of  $\varphi_i$ , we  
 implement

$$\begin{aligned} T_i(\varphi_0, r, t) &= s(\varphi_0, r, -t) * s(\varphi_i, r, t) = \\ &= c(-t) * G(\varphi_0, r, -t) * c(t) * G(\varphi_i, r, t). \end{aligned} \quad (3)$$

The term  $G(\varphi_0, r, t) * G(\varphi_i, r, -t)$  is the transfer func-  
 tion of such a time reversal algorithm and, in terms  
 of signal analysis, represents a matched filter [17].  
 This convolution coincides with the cross-correlation

of  $G(\varphi_0, r, t)$  and  $G(\varphi_i, r, t)$  ([13, 11]). For each source  
 position  $\varphi_0$ , the signal processing procedure consists  
 of implementing Equation 3, i.e. analytically cross-  
 correlating the signals, and of finding the maximum  
 value, with respect to time, of the time-reversed wave  
 field  $T_i$  for each  $\varphi_i$ . The resulting function  $F(\varphi_i)$   
 is dubbed "spatial focusing function" (shortly, focus-  
 ing function), as this procedure is equivalent to eval-  
 uating whether (and with what resolution) the time-  
 reversed and backward-propagated wave field is able  
 to reconstruct the original source position  $\varphi_0$ . The  
 focusing function is next normalized with respect to  
 its maximum; It is then reasonable to assume that,  
 the closer  $F(\varphi_i)$  is to 1 (i.e., identical Green's func-  
 tions) for a given value of  $\varphi_i$ , the closer  $\varphi_i$  is to  
 the original source  $\varphi_0$ . This method can be interpreted  
 as a pattern recognition system, that identifies, from an  
 acoustic reference library, the Green's function corre-  
 sponding to the actual position of the source, and so  
 determines the position of the source.

The invariance under time reversal is lost if the  
 propagation medium has frequency-dependent atten-  
 uation. This introduces a first-order time derivative  
 in the governing propagation equation. However, the  
 theorem of spatial reciprocity is still valid, i.e. there  
 is a loss of amplitude in the time-reversed vs. for-  
 ward propagating wave field, but this does not affect  
 source-localization resolution (does not affect the lo-  
 cation of the focus of the time-reversed wave field)

We take both sensors into account by computing  
 the mean of the focusing functions of the two signals.

In order to investigate the role of different frequency  
 contents, the originally measured signals are succes-  
 sively filtered with varying low-pass filters with max-  
 imum frequency  $f_{max}$ .

Following e.g. [22, 5, 33], we estimate the spatial  
 resolution of our time reversal algorithm by analyzing  
 the -3 dB width  $\Delta p$  of  $F(\varphi_i)$  for each given source po-  
 sition (angle  $\varphi$  and distance  $D$  between the source po-  
 sition and the skull) and various smallest wavelengths  
 $\lambda_{min} = c/f_{max}$  (with  $c$  = speed of sound in air).

We compare our resolution estimates against the  
 apparent aperture  $A$  of our skull-shaped antenna, as  
 defined by Catheline et al. [7], through the far-field  
 diffraction law

$$A = \frac{D \cdot \lambda_{min}}{2\Delta p}. \quad (4)$$

While resolution as defined here is known to follow  
 the diffraction-law in the far-field [7], that is not the  
 case in the near-field, where Equation 4 is only used  
 here for the sake of comparison.

## 3 Results

### 3.1 Verification of diffraction law

In this section, we reproduce the results of Catheline et al. [7] and verify that our far-field data are consistent with the diffraction law (Equation 4). The source position is chosen to be at  $\varphi = 0^\circ$ , which is in front of the center of the skull. We calculate the normalized focusing function  $F(\varphi_i)$  along the curvilinear abscissa in the horizontal plane as described previously, for each distance to the skull. This is shown in Figure 3

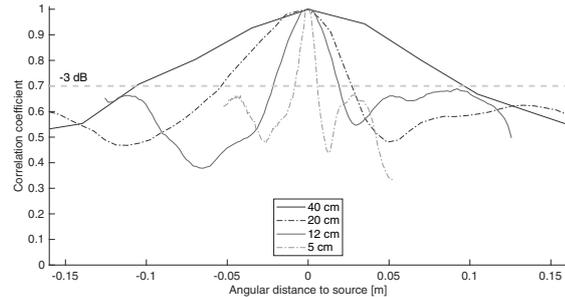


Figure 3: Normalized focusing functions along the curvilinear abscissa for sources in front of the center of the skull ( $\varphi = 0^\circ$ ) and at different distances to the skull. The distance of the measurement points to the skull decreases from 40 cm, down to 20 cm, 12 cm and 5 cm (different curves). There is a clear trend of increasing resolution (decreasing -3 dB width of the curves) with decreasing distance.

as a function of the curvilinear abscissa. The -3 dB (correlation coefficient of 0.7) widths of the curves are in good agreement with the diffraction law, confirming the findings of Catheline et al. [7], where the width of the curve is directly proportional to the distance between skull and sound source. Additionally it can be seen that the maximum peak to ground level (frequently named contrast) of our time reversal scheme lies below -3 dB. This has been confirmed for all measurements and ensures that calculating the resolution is not hindered by a low-contrast focusing function. Figure 4 shows the -3 dB widths of the focusing functions of the signals for sources with different maximum frequencies  $f_{max}$  and at different distances in front of the skull ( $\varphi = 0^\circ$ ). We calculate the values of  $A$  using Equation 4 and the values shown in Figure 4. They are found to be approximately 10 cm for all distances and maximum frequencies proving that the apparent aperture in the far-field is independent of distance or maximum frequency.

Measurements in the sagittal plane (not shown here) show smaller slopes of the linear fits evaluated in the same way as in Figure 4 across all results. Compared to the case of the horizontal plane, therefore the apparent aperture size is larger for these measurements (15 cm). This may be related to the different

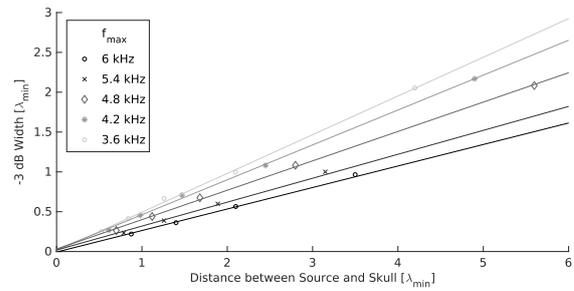


Figure 4: -3 dB width values of the focusing functions for sources at different distances to the skull (x-axis) and maximum frequencies  $f_{max}$  of the signal. The slope of each linear fit, which corresponds to the apparent aperture  $A$  in Equation 4, is approximately 10 cm for all curves.

diameters of the skull, close to 10 and 15 cm, in the horizontal and sagittal planes, respectively.

The measurement points in the near-field (at distances smaller than one wavelength) lie on the same linear fit (i.e. same apparent aperture) as the points for measurements in the far-field although Equation 4 does not hold true in the near-field. In the near-field, i.e. for sources closer than one minimum wavelength away from the skull, source positions can still be resolved with the same angular resolution which results in super-resolution in space, i.e. -3 dB widths below  $0.5 \lambda_{min}$  (see Figure 4). While one could infer that the diffraction limit also holds true in the near-field, our results are purely empirical; any values below the previously formulated diffraction limit are not represented in Equation 4. We speculate that they can be ascribed to the near-field contribution of evanescent waves.

Our far-field data is in agreement with Equation 4 and the previous findings of Catheline et al. [7]. In addition, we are able to achieve the same angular resolution as stated in the far-field diffraction law in the near-field (sound sources at below-wavelength distances) leading to super-resolution.

### 3.2 Directional variation in resolution

We furthermore investigate the directional variation of resolution of the time reversal analysis in the horizontal plane. The angular variations in resolution of our time reversal scheme in the near-field are visualized in Figure 5 showing the values of  $A$  (top) and  $\Delta p$  (bottom) with respect to the source azimuth  $\varphi$  for different source distances (5 cm, 12 cm and 20 cm and 100 cm). All data is filtered to have a maximum frequency of 3 kHz. The reason for an offset of around  $2 - 3^\circ$  to the center ( $\varphi = 0^\circ$ ) is due to a limited accuracy in the manual placement of the center position and the center of the rotation axis.

In the far-field, the apparent aperture does not vary with azimuth (see 100 cm data in Figure 5) and is

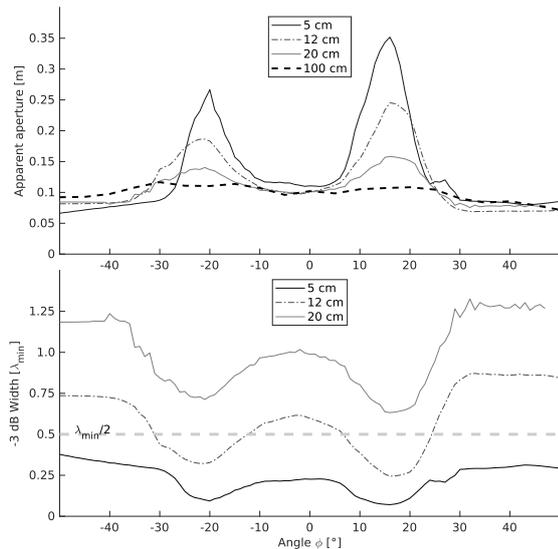


Figure 5: Angular variations of resolution for different source distances. Top: Variation in apparent aperture for different source distances. Maxima are at  $-20^\circ$  and  $15^\circ$  whereas the values decrease for source positions close to the center and further away from the center. Bottom: Variation in  $-3$  dB widths for different source distances. Super-resolution is accomplished throughout all angles at a distance of 5 cm and for certain angles at a distance of 12 cm. Highest resolution (smallest  $-3$  dB width) is accomplished for source positions directly in front of the orbital cavities. This effect is (relatively) enhanced the closer the source to the skull.

314 equal to the value of 10 cm obtained from Figure 4  
 315 throughout all far-field measurements at source az-  
 316 imuth  $\varphi = 0^\circ$ .

317 In the near-field, the largest apparent aperture val-  
 318 ues lie roughly in front of the two orbita, at  $-20^\circ$  and  
 319  $15^\circ$ , and are up to more than three times larger com-  
 320 pared to the aforementioned far-field value, whereas  
 321 source positions in front of the nasal bone or along  
 322 the process of the temporal bone show values closer  
 323 to 10 cm. The closer the source to the skull, the more  
 324 prominent the angular directionality of the apparent  
 325 aperture. Hence, the maximum apparent aperture is  
 326 more than three times larger than the skull diameter  
 327 in the horizontal plane.

328  $-3$  dB widths are smaller than half a wavelength  
 329 (super-resolution) throughout all azimuths at a dis-  
 330 tance between source and skull of 5 cm, down to  
 331  $\lambda_{min}/15$  (i.e. for  $\varphi = -20^\circ$  and  $15^\circ$ ). This shows that  
 332 the skull-shaped antenna enables sub-wavelength fo-  
 333 cusing of near-field sources and, furthermore, anatom-  
 334 ical details of the skull may give rise to differences in  
 335 resolution at certain positions due to the presence of  
 336 evanescent waves. They can be described as a non-  
 337 propagative spatial fluctuation field that decreases ex-  
 338 ponentially over roughly one wavelength [10] and can

339 be created at a boundary between two media through  
 340 certain incident angles of a propagating wave [15].  
 341 Usually, their effect is not measured in the far field  
 342 and the far-field diffraction law (Equation 4) does  
 343 not account for such effects, limiting the resolution  
 344 of time reversal. However, if near-field components of  
 345 the wavefield are measured and incorporated in the  
 346 time-reversal algorithm, subwavelength information,  
 347 that is carried by evanescent waves, is incorporated  
 348 in the time-reversal process, leading to super resolu-  
 349 tion [23].

350 All these results are also approximately achieved  
 351 via a one-sided evaluation of the signals, i.e. when  
 352 only one receiver is used.

353 In summary, our data shows large variations in res-  
 354 olution in the near-field, depending on the position  
 355 of the source relative to the geometric complexities of  
 356 the skull.

## 4 Conclusion

357  
 358 In this study we measured elastic wave signals in a  
 359 replica of a human skull due to an incident airborne  
 360 sound emitted by a source at various distances and  
 361 orientation with respect to the skull. Our goal was to  
 362 investigate the physical limits of a sound-localization  
 363 algorithm that uses full waveform information and the  
 364 information contained in elastic waves propagating in  
 365 the skull bone. While we do not at all claim to directly  
 366 reproduce the sound localization "algorithm" that ex-  
 367 ists in the human ear-brain system, our quantification  
 368 of these limits may be considered as a point of com-  
 369 parison in near-field psychoacoustics experiments.

370 We showed that the resolution of a time reversal  
 371 scheme using a skull-shaped antenna with one or two  
 372 receivers is consistent with the diffraction law in the  
 373 far-field. The apparent apertures in the horizontal  
 374 and sagittal planes are roughly consistent with the  
 375 horizontal and vertical extent of the skull. Inter-  
 376 estingly, the apparent aperture in the near-field is  
 377 markedly increased (more than 3 times its value in  
 378 the far-field) in the horizontal plane and at specific  
 379 angles. In that case we can achieve super-resolution  
 380 that may be associated to the non-negligible contri-  
 381 bution of evanescent waves in the near-field.

382 Our results suggest that anatomical details of the  
 383 skull give rise to complex features of the radiated  
 384 sound field in the near-field, enabling sub-wavelength  
 385 focusing and directional changes in resolution. We  
 386 clearly find the influence of small anatomical geomet-  
 387 ric complexities such as the orbital cavities to posi-  
 388 tively influence resolution using elastic waves. We  
 389 believe that it will be useful, in future studies, to ex-  
 390 plore the performance of our algorithms in other fre-  
 391 quency ranges and for other biological models (e.g.,  
 392 echolocating species such as dolphins or bats).

393 As noted by Parseihian et al. [26], very few stud-  
 394 ies in psychoacoustics have explored human sound lo-

395 calization performances for nearby sources (e.g. [6]).  
 396 It appears to us that further experimental work is  
 397 needed to more robustly evaluate how well humans  
 398 localize nearby sources and if our findings can be re-  
 399 lated to psychoacoustic studies in the near-field.

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